H.O. Pub. No. 9

AMERICAN PRACTICAL NAVIGATOR

AN EPITOME OF NAVIGATION

ORIGINALLY BY

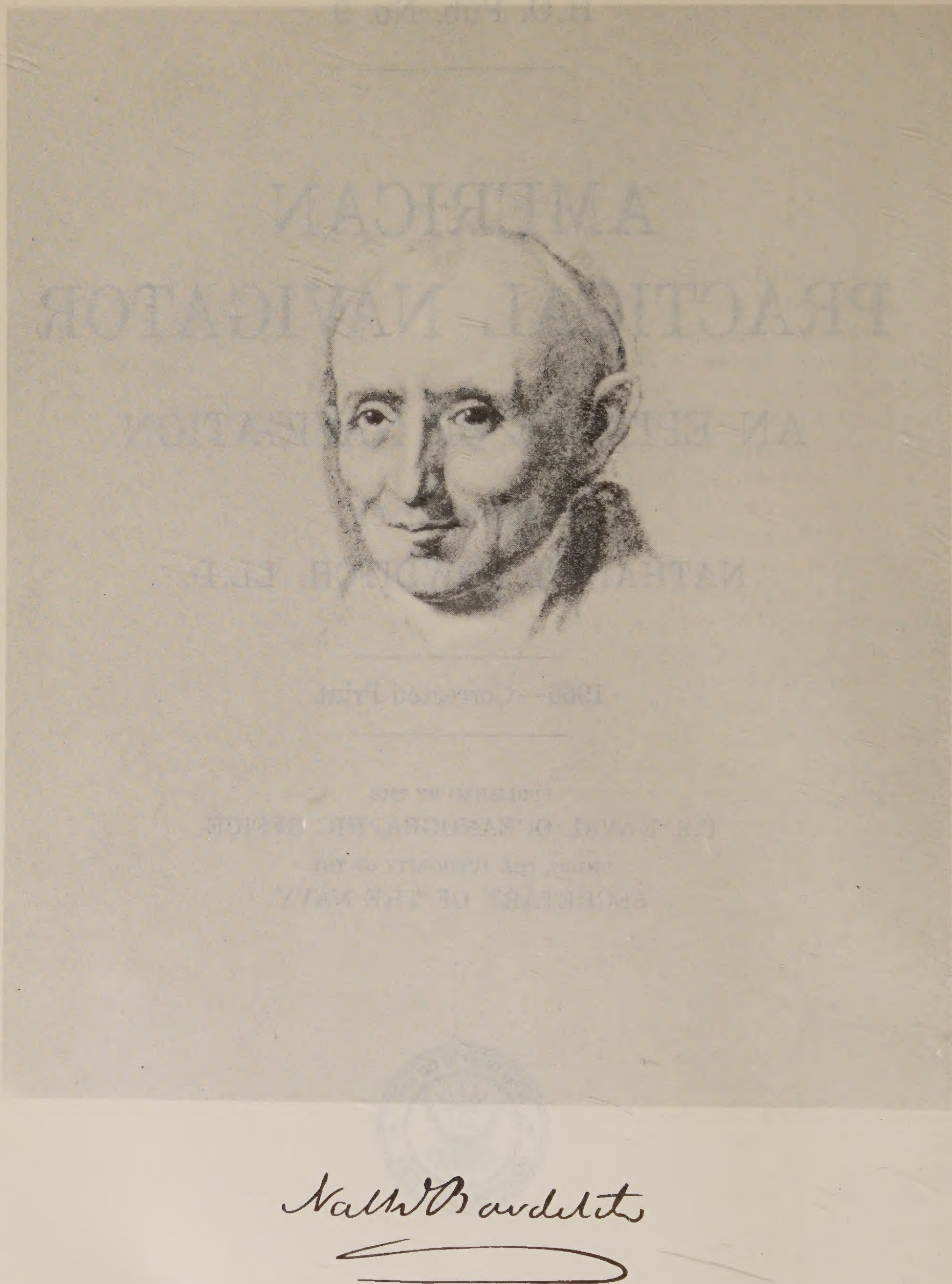
NATHANIEL BOWDITCH, LL.D.

1966—Corrected Print

PUBLISHED BY THE
U.S. NAVAL OCEANOGRAPHIC OFFICE
UNDER THE AUTHORITY OF THE
SECRETARY OF THE NAVY

U.S. GOVERNMENT PRINTING OFFICE
WASHINGTON : 1966

For sale by authorized Sales Agents of the U.S. Naval Oceanographic Office, also the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402. Price $7.



Last painting by Gilbert Stuart (1828). Considered by the family of Bowditch to be the best of various paintings made, although it was unfinished when the artist died.

NATHANIEL BOWDITCH

(1773–1838)

Nathaniel Bowditch was born on March 26, 1773, at Salem, Mass., fourth of the seven children of shipmaster Habakkuk Bowditch and his wife, Mary.

Since the migration of William Bowditch from England to the Colonies in the 17th century, the family had resided at Salem. Most of its sons, like those of other families in this New England seaport, had gone to sea, and many of them became shipmasters. Nathaniel Bowditch himself sailed as master on his last voyage, and two of his brothers met untimely deaths while pursuing careers at sea.

It is reported that Nathaniel Bowditch's father lost two ships at sea, and by late Revolutionary days he returned to the trade of cooper, which he had learned in his youth. This provided insufficient income to properly supply the needs of his growing family, and hunger and cold were often experienced. For many years the nearly destitute family received an annual grant of fifteen to twenty dollars from the Salem Marine Society. By the time Nathaniel had reached the age of ten, the family's poverty necessitated his leaving school and joining his father in the cooper's trade.

Nathaniel was unsuccessful as a cooper, and when he was about 12 years of age, he entered the first of two ship-chandlery firms by which he was employed. It was during the nearly ten years he was so employed that his great mind first attracted public attention. From the time he began school Bowditch had an all-consuming interest in learning, particularly mathematics. By his middle teens he was recognized in Salem as an authority on that subject. Salem being primarily a shipping town, most of the inhabitants sooner or later found their way to the ship chandler, and news of the brilliant young clerk spread until eventually it came to the attention of the learned men of his day. Impressed by his desire to educate himself, they supplied him with books that he might learn of the discoveries of other men. Since many of the best books were written by Europeans, Bowditch first taught himself their languages. French, Spanish, Latin, Greek, and German were among the two dozen or more languages and dialects he studied during his life. At the age of 16 he began the study of Newton's *Principia,* translating parts of it from the Latin. He even found an error in that classic, and though lacking the confidence to announce it at the time, he later published his findings and had them accepted.

During the Revolutionary War a privateer out of Beverly, a neighboring town to Salem, had taken as one of its prizes an English vessel which was carrying the philosophical library of a famed Irish scholar, Dr. Richard Kirwan. The books were brought to the Colonies and there bought by a group of educated Salem men who used them to found the Philosophical Library Company, reputed to have been the best library north of Philadelphia at the time. In 1791, when Bowditch was 18, two Harvard-educated ministers, Rev. John Prince and Rev. William Bentley, persuaded the Company to allow Bowditch the use of its library. Encouraged by these two men and a third—Nathan Read, an apothecary and also a Harvard man—Bowditch studied the works of the great men who had preceded him, especially the mathematicians and the astronomers. By the time he became of age, this knowledge, acquired before and after his long working hours and in his spare time, had made young Bowditch the outstanding mathematician in the Commonwealth, and perhaps in the country.

In the seafaring town of Salem, Bowditch was drawn to navigation early, learning the subject at the age of 13 from an old British sailor. A year later he began studying surveying, and in 1794 he assisted in a survey of the town. At 15 he devised an almanac reputed to have been of great accuracy. His other youthful accomplishments included the construction of a crude barometer and a sundial.

When Bowditch went to sea at the age of 21, it was as captain's writer and nominal second mate, the officer's berth being offered him because of his reputation as a scholar. Under Captain Henry Prince, the ship *Henry* sailed from Salem in the winter of 1795 on what was to be a year-long voyage to the Ile de Bourbon (now called Île de la Réunion) in the Indian Ocean.

Bowditch began his seagoing career when accurate time was not available to the average naval or merchant ship. A reliable marine chronometer had been invented some 60 years before, but the prohibitive cost, plus the long voyages without opportunity to check the error of the timepiece, made the large investment an impractical one. A system of determining longitude by "lunar distance," a method which did not require an accurate timepiece, was known, but this product of the minds of mathematicians and astronomers was so involved as to be beyond the capabilities of the uneducated seamen of that day. Consequently, ships navigated by a combination of dead reckoning and parallel sailing (a system of sailing north or south to the latitude of the destination and then east or west to the destination).

To Bowditch, the mathematical genius, computation of lunar distances was no mystery, of course, but he recognized the need for an easier method of working them in order to navigate ships more safely and efficiently. Through analysis and observation, he derived a new and simplified formula during his first trip, a formula which was to open the book of celestial navigation to all seamen.

John Hamilton Moore's *The Practical Navigator* was the leading navigational text when Bowditch first went to sea, and had been for many years. Early in his first voyage, however, the captain's writer-second mate began turning up errors in Moore's book, and before long he found it necessary to recompute some of the tables he most often used in working his sights. Bowditch recorded the errors he found, and by the end of his second voyage, made in the higher capacity of supercargo, the news of his findings in *The Practical Navigator* had reached Edmund Blunt, a publisher at Newburyport, Mass. At Blunt's request, Bowditch agreed to correct Moore's book. The first edition of *The New Practical Navigator* was published in 1799, with correction of the errors Bowditch had found to that time, and with some additional information. The following year a second edition was published with additional corrections. Bowditch eventually found more than 8,000 errors in the work, however, and it was finally decided to completely rewrite the book and to publish it under his own name. In 1802 the first edition of *The New American Practical Navigator* by Nathaniel Bowditch was published, and his vow to put nothing in the book he could not teach every member of his crew served to keep the work within the understanding of the average seaman. In addition to the improved method of determining longitude, Bowditch's book gave the ship's officer information on winds, currents, and tides; directions for surveying; statistics on marine insurance; a glossary of sea terms; instruction in mathematics; and numerous tables of navigational data. His simplified methods, easily grasped by the intelligent seaman willing to learn, paved the way for "Yankee" supremacy of the seas during the clipper ship era.

Two months before sailing for Cadiz on his third voyage, in 1798, Bowditch married Elizabeth Boardman, daughter of a shipmaster. While he was away, his wife died at

the age of 18. Two years later, on October 28, 1800, he married his cousin, Mary Ingersoll, she, too, the daughter of a shipmaster. They had eight children.

Bowditch made a total of five trips to sea, over a period of about nine years, his last as master and part owner of the three-masted *Putnam*. Homeward bound from a 13-month voyage to Sumatra and the Ile de France (now called Mauritius) the *Putnam* approached Salem harbor on December 25, 1803, during a thick fog without having had a celestial observation since noon on the 24th. Relying upon his dead reckoning, Bowditch conned his wooden-hulled ship to the entrance of the rocky harbor, where he had the good fortune to get a momentary glimpse of Eastern Point, Cape Ann, enough to confirm his position. The *Putnam* proceeded in, past such hazards as "Bowditch's Ledge" (named after a great-grandfather who had wrecked his ship on the rock more than a century before) and anchored safely at 1900 that evening. Word of the daring feat, performed when other masters were hove-to outside the harbor, spread along the coast and added greatly to Bowditch's reputation. He was, indeed, the "practical navigator."

His standing as a mathematician and successful shipmaster earned him a lucrative (for those times) position ashore within a matter of weeks after his last voyage. He was installed as president of a Salem fire and marine insurance company, at the age of 30, and during the 20 years he held that position the company prospered. In 1823 he left Salem to take a similar position with a Boston insurance firm, serving that company with equal success until his death.

From the time he finished the "*Navigator*" until 1814, Bowditch's mathematical and scientific pursuits consisted of studies and papers on the orbits of comets, applications of Napier's rules, magnetic variation, eclipses, calculations on tides, and the charting of Salem harbor. In that year, however, he turned to what he considered the greatest work of his life, the translation into English of *Mécanique Céleste*, by Pierre Laplace. *Mécanique Céleste* was a summary of all the then known facts about the workings of the heavens. Bowditch translated four of the five volumes before his death, and published them at his own expense. He gave many formula derivations which Laplace had not shown, and also included further discoveries following the time of publication. His work made this information available to American astronomers and enabled them to pursue their studies on the basis of that which was already known. Continuing his style of writing for the learner, Bowditch presented his English version of *Mécanique Céleste* in such a manner that the student of mathematics could easily trace the steps involved in reaching the most complicated conclusions.

Shortly after the publication of *The New American Practical Navigator*, Harvard College honored its author with the presentation of the honorary degree of Master of Arts, and in 1816 the college made him an honorary Doctor of Laws. From the time the Harvard graduates of Salem first assisted him in his studies, Bowditch had a great interest in that college, and in 1810 he was elected one of its Overseers, a position he held until 1826, when he was elected to the Corporation. During 1826–27 he was the leader of a small group of men who saved the school from financial disaster by forcing necessary economies on the college's reluctant president. At one time Bowditch was offered a Professorship in Mathematics at Harvard but this, as well as similar offers from West Point and the University of Virginia, he declined. In all his life he was never known to have made a public speech or to have addressed any large group of people.

Many other honors came to Bowditch in recognition of his astronomical, mathematical, and marine accomplishments. He became a member of the American

Academy of Arts and Sciences, the East India Marine Society, the Royal Academy of Edinburgh, the Royal Society of London, the Royal Irish Academy, the American Philosophical Society, the Connecticut Academy of Arts and Sciences, the Boston Marine Society, the Royal Astronomical Society, the Palermo Academy of Science, and the Royal Academy of Berlin.

Nathaniel Bowditch outlived all of his brothers and sisters by nearly 30 years. Death came to him on March 16, 1838, in his sixty-fifth year. The following eulogy by the Salem Marine Society indicates the regard in which this distinguished American was held by his contemporaries:

"In his death a public, a national, a human benefactor has departed. Not this community, nor our country only, but the whole world, has reason to do honor to his memory. When the voice of Eulogy shall be still, when the tear of Sorrow shall cease to flow, no monument will be needed to keep alive his memory among men; but as long as ships shall sail, the needle point to the north, and the stars go through their wonted courses in the heavens, the name of Dr. Bowditch will be revered as of one who helped his fellow-men in a time of need, who was and is a guide to them over the pathless ocean, and of one who forwarded the great interests of mankind."

The New American Practical Navigator was revised by Nathaniel Bowditch several times after 1802 for subsequent editions of the book. After his death, Jonathan Ingersoll Bowditch, a son who made several voyages, took up the work and his name appeared on the title page from the eleventh edition through the thirty-fifth, in 1867. In 1868 the newly organized U. S. Navy Hydrographic Office bought the copyright and has published the book since that time, revisions being made from time to time to keep the work in step with navigational improvements. The name has been altered to the *American Practical Navigator*, Hydrographic Office Publication No. 9, but the book is still commonly known as "Bowditch." A total of more than 700,000 copies has been printed in about 70 editions during the more than a century and a half since the book was first published in 1802. It has lived because it has combined the best thoughts of each generation of navigators, who have looked to it as their final authority.

PREFACE

This epitome of navigation has been maintained since its initial publication in 1802. The account of its origin, immediate success, and perpetuation appears so inseparable from the accomplishments of its original author, Nathaniel Bowditch, that it has been included in the life résumé of this illustrious navigator and author.

In this extensively revised edition, the U. S. Navy Hydrographic Office has included timely information consistent with modern practices and techniques. The text has been completely rewritten. Since a primary objective has been to provide a reference publication, some duplication exists, cross-referencing is extensive, and the index is detailed. All illustrations are new. Color has been added where it serves a useful purpose. Practice problems have been included with some chapters. Selected references have been given where complete coverage would be inappropriate.

The appendix has been enlarged, and the table arrangement improved. Certain tables of previous editions have been omitted, some of those retained have been altered, and new ones have been added.

The intent of the original author to provide a compendium of navigational material understandable to the mariner has been consistently followed. However, navigation is not presented as a mechanical process to be followed blindly. Rather, emphasis has been given to the fact that the aids provided by *science* can be used effectively to improve the *art* of navigation only if a well-informed person of mature judgment and experience is on hand to interpret information as it becomes available. Thus, the facts needed to perform the mechanics of navigation have been supplemented with additional material intended to help the navigator acquire perspective in meeting the various needs that arise.

Many institutions, organizations, groups, and individuals have assisted in the preparation of this publication, but all of the material has been edited by one individual to assure continuity and consistency. Particular acknowledgment is given the following: Mr. Charles L. Petze, Jr. for assistance in preparation of chapter I; the U. S. Navy Bureau of Ships for information relating to chapters VI and VII; the U. S. Naval Research Laboratory for review of part three; the U. S. Naval Observatory for information relating to chapter XIV and for suggestions relating to appendices F, H, I, and X; the Corps of Engineers of the U. S. Army for assistance in preparation of chapter XXVII; the U. S. Coast and Geodetic Survey of the Department of Commerce for preparation of chapter XXXI, and for providing information on geomagnetism and data for appendix M and most of table 5; the U. S. Weather Bureau for assistance in preparation of part seven and tables 16 and 17; the National Bureau of Standards of the Department of Commerce for assistance in preparation of appendix D; the U. S. Naval Institute for permission to use modified versions of work forms published in Dutton's *Navigation and Nautical Astronomy* (copyrighted 1943, 1948, 1951); the U. S. Power Squadrons for suggestions relating to the graph of article 924 for height of tide determination, navigation of small craft (art. 2310), and table 3; and many individuals, especially experienced practicing navigators, who have offered constructive suggestions or directed attention to errors in previous editions.

PREFACE TO THE 1962 REPRINT

This 1962 corrected reprint has presented the opportunity to incorporate new and timely information. Adoption by the United States of new equivalents for length and mass and of new values for absolute zero necessitated a major revision of Appendix D. Minor corrections to tables 6, 11, 17, 20, 21, and the text, resulting from these changes, have been made. Hydrographic Office publication numbers now conform to the numbering system in use since 1960. The sections relating to air navigation publications and space navigation have been rewritten. The revised Loran-A coverage diagram includes all rates operational in 1961. Appendix S, Maritime Positions, now includes revised material for Africa as well as several new ports.

In addition to the correction of errors published in errata sheets for the 1958 edition, other minor unpublished ones have been corrected. Editorial changes have provided clarification of certain parts of the text and some illustrations have been modified to present information more clearly. All appendices now carry their numerical or alphabetical designation at the top of each page. A list of contents now immediately precedes each part of the volume.

For practical navigation, the additions, corrections, and revisions incorporated in this reprint are not considered to be of sufficient scope and magnitude to necessitate replacement of the 1958 printing.

H.O. Pub. No. 9—Tables, including 1 through 34, is now available as a separate volume entitled "Tables from the American Practical Navigator—Bowditch."

PREFACE TO THE 1966 CORRECTED PRINT

This printing includes modifications and minor corrections to the previous 1962 printing. Appendix K has been revised so that it conforms to the nautical chart symbols listed in the September 1963 edition of Chart No. 1. The text, references, appendices, and index have been modified to reflect recent changes in United States Government publications.

The change in the name of the U.S. Navy Hydrographic Office to U.S. Naval Oceanographic Office has not been indicated in this print.

The modifications and corrections incorporated in this 1966 print are not considered sufficient to warrant replacement of either the 1958 edition or the 1962 corrected print for ordinary purposes of navigation.

CONTENTS

PART FIVE

THE PRACTICE OF NAVIGATION

PART SIX

OCEANOGRAPHY

PART SEVEN

WEATHER

PART EIGHT

HYDROGRAPHY

APPENDICES

PART ONE

FUNDAMENTALS

PART ONE

FUNDAMENTALS

CHAPTER I

HISTORY OF NAVIGATION

Introduction

101. Background.—Navigation began with the first man. One of his first conscious acts probably was to home on some object that caught his eye, and thus **land navigation** was undoubtedly the earliest form. His first venture upon the waters may have come shortly after he observed that some objects float, and through curiosity or an attempt at self-preservation he learned that a larger object, perhaps a log, would support him. **Marine navigation** was born when he attempted to guide his craft. **Air navigation** by men, of course, came much later.

The earliest marine navigation was a form of **piloting,** which came into being as man became familiar with landmarks and used them as guides. **Dead reckoning** probably came next as he sought to predict his future positions, or perhaps as he bravely ventured farther from landmarks. **Celestial navigation,** as it is known today, had to await acquisition of information regarding the motions of the heavenly bodies, although these bodies were used to steer by almost from the beginning. **Electronic navigation** is the modern application of a different form of energy to solve an old problem, its principal use being to extend the range of piloting.

102. From art to science.—Navigation is the process of directing the movements of a craft from one point to another. To do this safely is an *art*. In perhaps 6,000 years—some writers make it 8,000—man has transformed this art almost into a *science*, and navigation today is so nearly a science that the inclination is to forget that it was ever anything else. It is commonly thought that to navigate a ship one must have a chart to determine the course and distance, a compass to steer by, and a means of determining the positions of the ship during the passage. *Must* have? The word "must" betrays how dependent the modern navigator has become upon the tools now in his hands. Many of the great voyages of history—voyages that made known much of the world—were made without one or more of these "essentials."

103. Epic voyages.—History records a number of great voyages of varying navigational significance. Little or nothing is known of the navigational accomplishments of the ancient mariners, but the record of the knowledge and equipment used during later voyages serves to illustrate periodic developments in the field.

104. Pre-Christian navigation.—Down through the stream of time a number of voyages have occurred without navigational significance. Noah's experience in the ark is of little interest navigationally, except for his use of a dove to locate land. There is evidence to support the view that at least some American Indians reached these shores by sea, the earliest of several groups probably having come about 2200 BC, the approximate time that a general exodus seems to have occurred from a center in southwestern Asia. This is about the time the Tower of Babel is believed to have been built. It is noteworthy that almost every land reached by the great European explorers was already inhabited.

It is not difficult to understand how a people not accustomed to the sea might make a single great voyage without contributing anything of significance to the advancement of navigation. Not so clear, however, is the fact that the Norsemen and the Poly-

nesians, great seafaring people, left nothing more than conflicting traditions of their methods. The reputed length of the voyages made by these people suggests more advanced navigational methods than their records indicate, although the explanation may be that they left few written accounts of any kind. Or perhaps they developed their powers of perception to such an extent that navigation, to them, was a highly advanced art. In this respect their navigation may not have differed greatly from that of some birds, insects, fishes, and animals.

One of the earliest well-recorded voyages is known today through the book of observations written by Pytheas of Massalia, a Greek astronomer and navigator. Sometime between the years 350 BC and 300 BC he sailed from a Mediterranean port and followed an established trade route to England. From there he ventured north to Scotland and Thule, the legendary land of the midnight sun. He went on to explore Norwegian fiords, and rivers in northwest Germany. He may have made his way into the Baltic.

Pytheas' voyage, and others of his time, were significant in that they were the work of men who had no compasses, no sextants, no chronometers, no electronic devices such as are commonplace today. The explanation of how they did it is not what some historians have said, that before seafaring men had adequate equipment, the compass especially, they hugged the shore and sailed only by daylight in fair weather. Many undoubtedly did use this practice. But the more intrepid did not creep along the coast, venturing nothing more daring than sailing from headland to headland. They were often out of sight of land, and yet knew sufficiently well where they were and how to get home again. They were able to use the sun, the stars, and the winds without the aid of mechanical devices.

Pytheas had none of the equipment considered essential by the modern navigator—none, at least, as it is thought of today. It would be incorrect, however, to say that he had no navigational aids whatever. He was not the first to venture upon the sea, and even in his time man was the inheritor of his predecessors' knowledge.

He must have known what the mariners of his time, Phoenician and Greek, knew about navigation. There was a fair store of knowledge about the movements of the stars, for example, which all seafaring men shared. They had a practical grasp of some part of what is now called celestial navigation, for the moving celestial bodies were their compasses. Pytheas may not have been acquainted with the *Periplus* of Scylax, the earliest known sailing directions, but it is reasonable to suppose that he had similar information.

If there were sailing directions, there may well have been charts of a sort, even though no record of them exists.

Even if Pytheas and his contemporaries had sailing directions and charts, these must have been far from comprehensive, and they undoubtedly did not cover the areas north of Britain. But these early seamen knew direction by day or night if the sky was clear, and they could judge it reasonably well when the sky was overcast, using the wind and the sea. They knew the hot Libyan wind from the desert—today called the **sirocco**—and the northern wind, the **mistral.**

They could estimate distance. Their ships must have carried some means of measuring time—the sand glass was known to the ancients—and they could estimate speed by counting the strokes of the oars, a common practice from galley to modern college racing shell. Mariners who spent their lives traveling the Mediterranean knew what their ships could do, even if today it is not known what they meant by "a day's sail"—whether 35 miles, or 50, or 100.

105. Sixteenth century navigation.—Progress in the art of navigation came slowly during the early centuries of the Christian era, all but stopped during the Dark Ages,

and then spurted forward when Europe entered a golden age of discovery. The circumnavigation of the globe by the expedition organized by Ferdinand Magellan, a disgraced Portuguese nobleman who sailed under the flag of Spain, was a voyage which illustrates the advances made during the 1,800 years following Pytheas.

Magellan was able to find justification for his belief that a navigable pass to the Pacific Ocean existed in high southern latitudes, in Martin Behaim's globe or chart of the world, in the globe constructed by Johann Schoner of Nuremberg in 1515, and in Leonardo da Vinci's map of the world drawn in the same year. He obtained further information for his voyage from Ruy Faleiro, an astronomer and cartographer whose charts, sailing directions, nautical tables, and instructions for use of the astrolabe and cross-staff were considered to be among the best available. Faleiro was also an advocate of the fallacious methods of determining longitude by variation.

When Magellan sailed in 1519, his equipment included sea charts, parchment skins to be made into charts en route, a terrestrial globe, wooden and metal theodolites, wooden and wood-and-bronze quadrants, compasses, magnetic needles, hour glasses and "timepieces," and a log to be towed astern.

So the 16th century navigator had crude charts of the known world, a compass to steer by, instruments with which he could determine his latitude, a log to estimate speed, certain sailing directions, and solar and traverse tables. The huge obstacle yet to be overcome was an accurate method of determining longitude.

106. Eighteenth century navigation.—Little is known today of the "timepieces" carried by Magellan, but surely they were not used to determine longitude. Two hundred years later, however, the chronometer began to emerge. With it, the navigator, for the first time, was able to determine his longitude accurately and fix his position at sea.

The three voyages of discovery made by James Cook of the Royal Navy in the Pacific Ocean between 1768 and 1779 may be said to mark the dawn of modern navigation. Cook's expedition had the full backing of England's scientific organizations, and he was the first captain to undertake extended explorations at sea with navigational equipment, techniques, and knowledge that might be considered modern.

On his first voyage Cook was provided with an astronomical clock, a "journeyman" clock, and a watch lent by the Astronomer Royal. With these he could determine longitude, using the long and tedious lunar distance method. On his second voyage four chronometers were provided. These instruments, added to those already possessed by the mariner, enabled Cook to navigate his vessels with a precision undreamed of by Pytheas and Magellan.

By the time Cook began his explorations, astronomers had made great contributions to navigational advancement, and the acceptance of the heliocentric theory of the universe had led to the publication of the first official nautical almanac. Charts had progressed steadily, and adequate projections were available. With increased understanding of variation, the compass had become reliable. Good schools of navigation existed, and textbooks which reduced the mathematics of navigation to the essentials had been published. Speed through the water could be determined with reasonable accuracy by the logs then in use. Most important, the first chronometers were being produced.

107. Twentieth century navigation.—The maiden voyage of the SS *United States* in July 1952 served to illustrate the progress made in navigation during the 175 years since Cook's voyages. Outstanding because of its record trans-Atlantic passage, the vessel is of interest navigationally in that it carried the most modern equipment available and exemplified the fact that navigation had become nearly a science.

Each of the deck officers owned a sextant with which he could make observations more accurately than did Cook. Reliable chronometers, the product of hundreds of

years of experimental work, were available to determine the time of each observation. The gyro compass indicated true north regardless of variation and deviation.

Modern, convenient almanacs were used to obtain the coordinates of various celestial bodies, to an accuracy greater than needed. Easily used altitude and azimuth tables gave the navigator data for determining his Sumner (celestial) line of position by the method of Marcq St.-Hilaire. Accurate charts were available for the waters plied, sailing directions for coasts and ports visited, light lists giving the characteristics of the various aids to navigation along these coasts, and pilot charts and navigational texts for reference purposes.

Electronics served the navigator in a number of ways. Radio time signals and weather reports enabled him to check his chronometers and avoid foul weather. A radio direction finder was available to obtain bearings, and a radio telephone was used to communicate with persons on land and sea. The electrically operated echo sounder indicated the depth of water under the keel, radar the distances and bearings of objects within range, even in the densest fog. Using loran, the navigator could fix the position of his ship a thousand miles and more from transmitting stations.

Piloting and Dead Reckoning

108. Background.—The history of piloting and dead reckoning extends from man's earliest use of landmarks to the latest model of the gyro compass. In the thousands of years between, navigation by these methods has progressed from short passages along known coast lines to transoceanic voyages during which celestial observations cannot be, or are not, made.

109. Charts.—A form of sailing directions was written several hundred years before Christ. Although charts cannot be traced back that far, they may have existed during the same time. From earliest times men have undoubtedly known that it is more difficult to explain how to get to a place than it is to draw a diagram, and since the first charts known are comparatively accurate and cover large areas, it seems logical that earlier charts served as guides for the cartographers.

Undoubtedly, the first charts were not made on any "projection" (ch. III) but were simple diagrams which took no notice of the shape of the earth. In fact, these "plane" charts were used for many centuries after chart projections were available.

The **gnomonic projection** (art. 317) is believed to have been developed by Thales of Miletus (640–546 BC), who was chief of the Seven Wise Men of ancient Greece; founder of Greek geometry, astronomy, and philosophy; and a navigator and cartographer.

The size of the earth was measured at least as early as the third century BC, by Eratosthenes. He observed that at noon on the day of the summer solstice, a certain well at Syene (Assuan) on the tropic of Cancer was lighted throughout its depth by the light of the sun as it crossed the meridian; but that at Alexandria, about 500 miles to the north, shadows were cast by the sun at high noon. He reasoned that this was due to curvature of the earth, which must be spherical. By means of the shadow of an object of known height at Alexandria, Eratosthenes determined the zenith distance to be about 7°5, or $\frac{1}{48}$ of the earth's circumference. The earth must therefore be $48 \times 500 = 24{,}000$ (statute) miles. The correct value is about 24,900 statute miles.

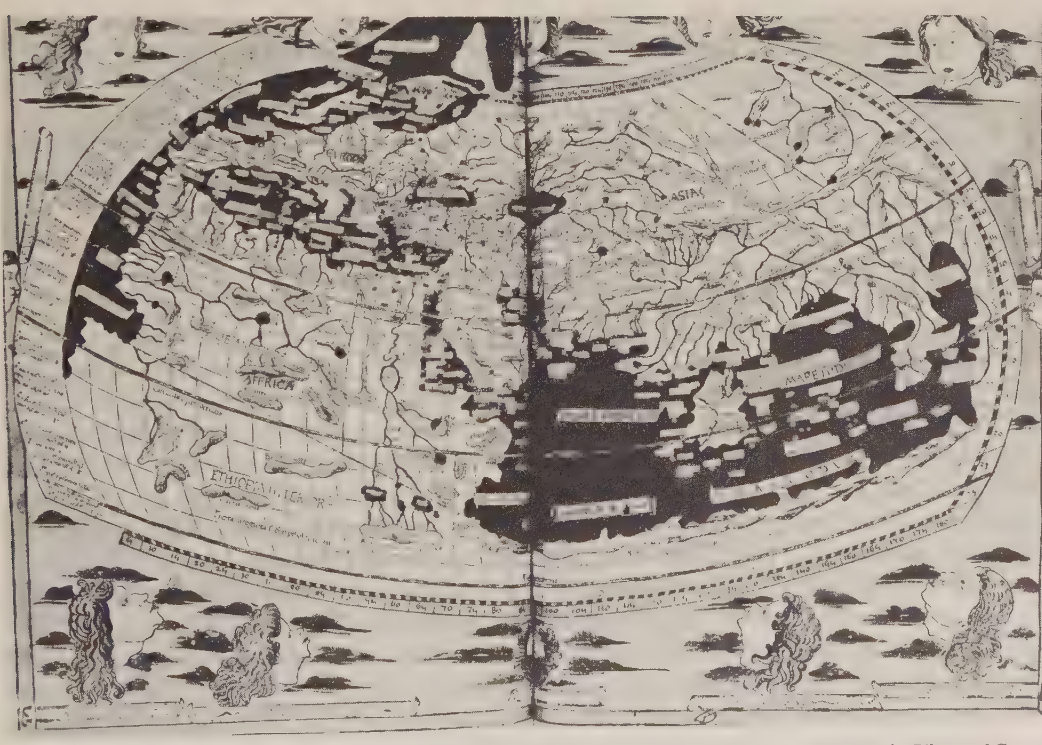
Eratosthenes is believed to have been the first person to measure latitude, using the degree for this purpose. He constructed a 16-point wind rose, prepared a table of winds, and recognized local and prevailing winds. From his own discoveries and from information gleaned from the manuscripts of mariners, explorers, land travelers, historians, and philosophers, he wrote an outstanding description of the known world, which helped elevate geography to the status of a science.

Stereographic (art. 318) and **orthographic** (art. 319) **projections** were originated by Hipparchus in the second century BC.

Ptolemy's World Map. The Egyptian Claudius Ptolemy was a second century AD astronomer, writer, geographer, and mathematician who had no equal in astronomy until the arrival of Copernicus in the 16th century. An outstanding cartographer, for his time, Ptolemy constructed many charts and listed the latitudes and longitudes, as determined by celestial observations, of the places shown. As a geographer, however, he made his most serious mistake. Though Eratosthenes' calculations on the circumference of the earth were available to him, he took the estimate of the Stoic philosopher, Posidonius (circa 130–51 BC), who calculated the earth to be 18,000 miles in circumference. The result was that those who accepted his work—and for many hundreds of years few thought to question it—had to deal with a concept that was far too small. In 1409 the Greek original of Ptolemy's *Cosmographia*, a book in which he declared this doctrine, was discovered and translated into Latin. It served as the basis for future cartographic work, and so it was that Columbus died convinced that he had found a shorter route to the East Indies. Not until 1669, when Jean Picard computed the circumference of the earth to be 24,500 miles, was a more accurate figure generally used.

Ptolemy's map of the world (fig. 109a) was a great achievement, however. It was the original conic projection, and on it he located some 8,000 places by latitude and longitude. It was he who fixed the convention that the top of the map is north.

Asian Charts. Through the Dark Ages some progress was made. Moslem cartographers as well as astronomers took inspiration from Ptolemy. However, they knew that Ptolemy had overestimated the length of the Mediterranean by some 20°. Charts of the Indian Ocean, bearing horizontal lines indicating parallels of latitude,



Courtesy of the Map Division of the Library of Congress.

FIGURE 109a.—The world, as envisioned by Ptolemy about AD 150. This chart was prepared in 1482 by Nicolaus Germanus for a translation of *Cosmographia*.

Courtesy of the Map Division of the Library of Congress.

FIGURE 109b.—A 14th-century Portolan chart.

and vertical lines dividing the seas according to the direction of the wind, were drawn by Persian and Arabian navigators. The prime meridian separated a windward from a leeward region and other meridians were drawn at intervals indicating "three hours sail." This information, though far from exact, was helpful to the sailing ship masters.

Portolan Charts. The mariners of Venezia (Venice), Livorno (Leghorn), and Genova (Genoa) must have had charts when they competed for Mediterranean trade before, during, and after the Crusades. Venice at one time had 300 ships, a navy of 45 galleys, and 11,000 men engaged in her maritime industry. But perhaps the rivalry was too keen for masters carelessly to leave charts lying about. At any rate, the earliest useful charts of the Middle Ages that are known today were drawn by seamen of Catalonia (now part of Spain).

The Portolan charts were constructed from the knowledge acquired by seamen during their voyages about the Mediterranean. The actual courses and dead reckoning distances between land points were used as a skeleton for the charts, and the coasts between were usually filled in from data obtained in land surveys. After the compass came into use, these charts became quite accurate. Some, for example, indicated the distance between Gibraltar and Bayrūt (Beirut) to be 3,000 Portolan miles, or 40°.5 of longitude. The actual difference of longitude is 40°.8.

These charts were distinguished by a group of long rhumb lines intersecting at a common point, surrounded by eight or 16 similar groups of shorter lines. Later *Portolanis* had a *rose dei venti* (rose of the winds), the forerunner of the compass rose, superimposed over the center (fig. 109b). They carried a scale of miles, located nearly all the known hazards to navigation, and had numerous notes of interest to the pilot. They were not marked with parallels of latitude or meridians of longitude, but present-day harbor and coastal charts trace their ancestry directly to them.

Padrón Real. The growing habit of assembling information for charts took concrete form in the *Padrón Real.* This was the pattern, or master, map kept after 1508 by the *Casa de Contratación* at Seville. It was intended to contain everything known about the world, and it was constructed from facts brought back by mariners from voyages to newly discovered lands. From it were drawn the charts upon which the explorers of the Age of Discovery most depended.

World maps of the Middle Ages. In 1515 Leonardo da Vinci drew his famous map of the world. On it, America is represented as extending more to the east and west

Courtesy of the New York Public Library.

FIGURE 109c.—Ortelius' world map, from his atlas *Theatrum Orbis Terra*, published at Antwerp in 1570.

than to the north and south, with only a chain of islands, the largest named Florida, between it and South America. A wide stretch of ocean is shown between South America and *Terra Australis Nondum Cognita*, the mythical south-seas continent whose existence in the position shown was not disproved until 250 years later.

Ortelius' atlas *Theatrum Orbis Terra* was published at Antwerp in 1570. One of the most magnificent ever produced, it illustrates Europe, Africa, and Asia with comparative accuracy. North and South America are poorly depicted, but Magellan's Strait is shown. All land to the south of it, as well as Australia, is considered part of *Terra Australis Nondum Cognita* (fig. 109c).

The Mercator projection (art. 305). For hundreds, perhaps thousands, of years cartographers drew their charts as "plane" projections, making no use of the discoveries of Ptolemy and Hipparchus. As the area of the known world increased, however, the attempt to depict that larger area on the flat surface of the plane chart brought map makers to the realization that allowance would have to be made for the curvature of the earth.

Gerardus Mercator (Latinized form of Gerhard Kremer) was a brilliant Flemish geographer who recognized the need for a better method of chart projection. In 1569 he published a world chart which he had constructed on the principle since known by his name. The theory of his work was correct, but Mercator made errors in his computation, and because he never published a complete description of the mathematics involved, mariners were deprived of the full advantages of the projection for another 30 years.

Then Edward Wright published the results of his own independent study in the matter, explaining the Mercator projection fully and providing the table of meridional parts which enabled all cartographers to make use of the principle.

Wright was a mathematician at Caius College who developed the method and table and gave them to certain navigators for testing. After these proved their usefulness, Wright decided upon publication, and in 1599 *Certaine Errors in Navigation Detected and Corrected* was printed.

The Lambert projections. Johann Heinrich Lambert, 1728–1777, self-educated son of an Alsace tailor, designed a number of map projections. Some of these are still widely used, the most renowned being the **Lambert conformal** (art. 314).

110. Sailing directions.—From earliest times there has been a demand for knowledge of what lay ahead, and this gave rise to the early development of sailing directions (art. 420).

The *Periplus* of Scylax, written sometime between the sixth and fourth centuries BC, is the earliest known book of this type. Surprisingly similar to modern sailing directions, it provided the mariner with information on distances between ports, aids and dangers, port facilities, and other pertinent matters. The following excerpt is typical:

"Libya begins beyond the Canopic mouth of the Nile. . . . The first people in Libya are the Adrymachidae. From Thonis the voyage to Pharos, a desert island (good harbourage but no drinking water), is 150 stadia. In Pharos are many harbors. But ships water at the Marian Mere, for it is drinkable. . . . The mouth of the bay of Plinthine to Leuce Acte (the white beach) is a day and night's sail; but sailing round by the head of the bay of Plinthine is twice as long. . . ."

Parts Around the World, Pytheas' book of observations made during his epic voyage in the fourth century BC, was another early volume of sailing directions. His rough estimates of distances and descriptions of coast lines would be considered crude today, but they served as an invaluable aid to navigators who followed him into these otherwise unknown waters.

Sailing directions during the Renaissance. No particularly noteworthy improvements were made in sailing directions during the Middle Ages, but in 1490 the *Portolano Rizo* was published, the first of a series of improved design. Other early volumes of this kind appeared in France and were called "routiers"—the **rutters** of the English sailor. In 1557 the Italian pilot Battista Testa Rossa published *Brieve Compendio del Arte del Navigar*, which was designed to serve the mariner on soundings and off. It forecast the single, all-inclusive volume that was soon to come, the **Waggoner.**

About 1584 the Dutch pilot Lucas Jans Zoon Waghenaer published a volume of navigational principles, tables, charts, and sailing directions which served as a guide for such books for the next 200 years. In *Spieghel der Zeevaerdt* (The Mariner's Mirror), Waghenaer gave directions and charts for sailing the waters of the Low Countries and later a second volume was published covering waters of the North and Baltic seas.

These "Waggoners" met with great success and in 1588 an English translation of the original book was made by Anthony Ashley. During the next 30 years, 24 editions of the book were published in Dutch, German, Latin, and English. Other authors followed the profitable example set by Waghenaer, and American, British, and French navigators soon had "Waggoners" for most of the waters they sailed.

The success of these books and the resulting competition among authors were responsible for their eventual discontinuance. Each writer attempted to make his work more inclusive than any other (the 1780 *Atlantic Neptune* contained 257 charts of North America alone) and the result was a tremendous book difficult to handle. They were too bulky, the sailing directions were unnecessarily detailed, and the charts too large. In 1795 the British Hydrographic Department was established, and charts and sailing directions were issued separately. The latter, issued for specific waters, were returned to the form of the original *Periplus*.

Modern sailing directions. The publication of modern sailing directions by the U. S. Navy Hydrographic Office is one of the achievements properly attributed to Matthew Fontaine Maury. During the two decades he headed the institution, Maury gathered data that led directly to the publication of eight volumes of sailing directions. Today there are more than 65 volumes providing the mariner with detailed information on almost all foreign coasts, in addition to eight volumes of **coast pilots** of the United States and its possessions, published by the U. S. Coast and Geodetic Survey.

111. The compass.—Early in the history of navigation man noted that the pole star (it may have been α *Draconis* then) remained close to one point in the northern sky. This served as his compass. When it was not visible, he used other stars, the sun and moon, winds, clouds, and waves. The development of the magnetic compass, perhaps a thousand years ago, and the 20th century development of the gyro compass, offer today's navigator a method of steering his course with an accuracy as great as he is capable of using.

The **magnetic compass** (art. 623) is one of the oldest of the navigator's instruments. Its origin is not known. In 203 BC, when Hannibal set sail from Italy, his pilot was said to be one *Pelorus*. Perhaps the compass was in use then; no one can say for certain that it was not. There is little to substantiate the story that the Chinese invented it, and the legend that Marco Polo introduced it into Italy in the 13th century is almost certainly false. It is sometimes stated that the Arabs brought it to Europe, but this, too, is unlikely. Probably it was known first in the west. The Norsemen of the 11th century were familiar with it, and about 1200 a compass used by mariners when the pole star was hidden was described by a French poet, Guyot de Provins.

A needle thrust through a straw and floated in water in a container comprised the earliest compass known. A 1248 writer, Hugo de Bercy, spoke of a new compass construction, the needle "now" being supported on two floats. Petrus Peregrinus de Maricourt, in his *Epistola de Magnete* of 1269, wrote of a pivoted floating compass with a lubber's line, and said that it was equipped with sights for taking bearings.

The reliability of the magnetic compass of today is a comparatively recent achievement. As late as 1820 Peter Barlow reported to the British Admiralty "half of the compasses in the Royal Navy were mere lumber, and ought to be destroyed." Some 75 years ago, Lord Kelvin developed the Admiralty type compass used today.

The **compass card,** according to tradition, originated about the beginning of the 14th century, when Flavio Gioja of Amalfi attached a sliver of lodestone or a magnetized needle to a card. But the rose on the compass card is probably older than the needle. It is the wind rose of the ancients. Primitive man naturally named directions by the winds. The prophet Jeremiah speaks of the winds from the four quarters of heaven (Jer. 49:36) and Homer named four winds—Boreas, Eurus, Notus, and Lephyrus. Aristotle is said to have suggested a circle of 12 winds, and Eratosthenes, who measured the world correctly, reduced the number to eight about 200 BC. The "Tower of Winds" at Athens, built about 100 BC, had eight sides. The Latin rose of 12 points was common on most compasses used in the Middle Ages.

Variation (art. 709) was well understood 200 years ago, and navigators made allowance for it, but earliest recognition of its existence is not known. Columbus and even the 11th century Chinese have been given credit for its discovery, but little proof can be offered for either claim.

The secular change in variation was determined by a series of magnetic observations made at Limehouse, England. In 1580 William Borough fixed the variation in that area at approximately 11°25′ east. Thirty-two years later Edmund Gunter, professor of astronomy at Gresham College, determined it to be 6°13′ east. At first

it was believed that Borough had made an error in his work, but in 1633 a further decrease was found, and the earth's changing magnetic field was established.

A South Atlantic expedition was led by Edmond Halley at the close of the 17th century to gather data and to map, for the first time, lines of variation. In 1724 George Graham published his observations in proof of the diurnal change in variation. Canton determined that the change was considerably less in winter than in summer, and about 1785 the strength of the magnetic force was shown by Paul de Lamanon to vary in different places.

The existence of **deviation** (art. 709) was known to John Smith in 1627 when he wrote of the "bittacle" as being a "square box nailed together with wooden pinnes, because iron nails would attract the Compasse." But no one knew how to correct a compass for deviation until Captain Matthew Flinders, while on a voyage to Australia in HMS *Investigator* in 1801–02, discovered a method of doing so. Flinders did not understand deviation completely, but the vertical bar he erected to correct for it was part of the solution, and the **Flinders bar** (art. 720) used today is a memorial to its discoverer. Between 1839 and 1855 Sir George Airy, then Astronomer Royal, studied the matter further and developed combinations of permanent magnets and soft iron masses for adjusting the compass. The introduction, by Lord Kelvin, of short needles as compass magnets made adjustment more precise.

The gyro compass (art. 631). The age of iron ships demanded a compass which could be relied upon to indicate true north at all times, free from disturbing forces of variation and deviation.

In 1851, at the Pantheon in Paris, Leon Foucault performed his famous pendulum experiment to demonstrate the rotation of the earth. Foucault's realization that the swinging pendulum would maintain the plane of its motion led him, the following year, to develop and name the first gyroscope, using the principle of a common toy called a "rotascope." Handicapped by the lack of a source of power to maintain the spin of his gyroscope, Foucault used a microscope to observe the indication of the earth's rotation during the short period in which his manually operated gyroscope remained in rotation. A gyro compass was not practical until electric power became available, more than 50 years later, to maintain the spin of the gyroscope.

Elmer A. Sperry, an American, and Anschutz-Kampfe, a German, independently invented gyro compasses during the first decade of the 20th century. Tested first in 1911 on a freighter operating off the East Coast of the United States and then on American warships, Sperry's compass was found adequate, and in the years following World War I gyro compasses became standard equipment on all large naval and merchant ships.

Gyro compass auxiliaries commonly used today were added later. These include gyro repeaters, to indicate the vessel's heading at various locations; gyro pilots, to steer vessels automatically; course recorders, to provide a graphic record of courses steered; gyro-magnetic compasses, to repeat headings of magnetic compasses so located as to be least affected by deviation; and others in the fields of fire control, aviation, and guided missiles.

112. The log.—Since virtually the beginning of navigation, the mariner has attempted to determine his speed in traveling from one point to another. The earliest method was probably by estimate.

The oldest speed measuring device known is the **Dutchman's log.** Originally, any object which would float was thrown overboard on the lee side, from a point well forward, and the time required for it to pass between two points on the deck was noted. The time, as determined by sand glass, was compared with the known distance along the deck between the two points to determine the speed.

Near the end of the 16th century a line was attached to the log, and as the line was paid out a sailor recited certain sentences. The length of line which was paid out during the recitation was used to determine the speed. There is record of this method having been used as recently as the early 17th century. In its final form this **chip log, ship log,** or **common log** consisted of the *log chip* (or *log ship*), *log line*, *log reel*, and *log glass*. The chip was a quadrant-shaped piece of wood weighted along its circumference to keep it upright in the water (fig. 112). The log line was made fast to the log chip by means of a bridle, in such manner that a sharp pull on the log line dislodged a wooden peg and permitted the log chip to be towed horizontally through the water, and hauled aboard. Sometimes a *stray line* was attached to the log to veer it clear of the ship's wake. In determining speed, the observer counted the knots in the log line which was paid out during a certain time. The length of line between knots and the number of seconds required for the sand to run out were changed from time to time as the accepted length of the mile was altered.

The chip log has been superseded by patent logs that register on dials. However, the common log has left its mark on modern navigation, as the use of the term **knot** to indicate a speed of one nautical mile per hour dates from this device. There is evidence to support the opinion that the expression "dead reckoning" had its origin in this same device, or perhaps in the earlier Dutchman's log. There is logic in attributing "dead" reckoning to a reckoning relative to an object "dead" in the water.

Courtesy of "Motor Boating."

FIGURE 112.—The common or chip log, showing the log reel, the log line, the log chip, and the log glass.

Mechanical logs first appeared about the middle of the 17th century. By the beginning of the 19th century, the forerunners of modern mechanical logs were used by some navigators, although many years were to pass before they became generally accepted.

In 1773 logs on which the distance run was recorded on dials secured to the taffrail were tested on board a British warship and found reasonably adequate, although the comparative delicateness of the mechanism led to speculation about their long-term worth. Another type in existence at the time consisted of a wheel arrangement made fast on the underside of the keel, which transmitted readings to a dial inside the vessel as the wheel rotated.

An improved log was introduced by Edward Massey in 1802. This log gave considerably greater accuracy by means of a more sensitive rotator attached by a short length of line to a geared recording instrument. The difficulty with this log was that it had to be hauled aboard to take each reading. Various improvements were made, notably by Alexander Bain in 1846 and Thomas Walker in 1861, but it was not until 1878 that a log was developed in which the rotator could be used in conjunction with a dial secured to the after rail of the ship, and although refinements and improvements have been made, the patent log used today is essentially the same as that developed in 1878.

Engine revolution counters (art. 615) had their origin with the observations of the captains of the first paddle steamers, who discovered that by counting the paddle revolutions, they could, with practice, estimate their runs in thick weather as accurately as they could by streaming the log. Later developments led to the modern revolution

counter on screw-type vessels, which can be used with reasonable accuracy if the propeller is submerged and an accurate estimate of slip is made.

Pitot-static and impeller-type logs (arts. 613, 614) are recent mechanical developments in the field of speed measurement. Each utilizes a retractable "rodmeter" which projects through the hull of the ship into the water. In the Pitot-static log, static and dynamic pressures on the rodmeter transmit readings to the master speed indicator. In the impeller-type log an electrical means of transmitting speed indications is used.

113. Units of distance and depth.—The modern navigator is concerned principally with three units of linear measure: the **nautical mile,** the **fathom,** and the **foot** (sometimes also the **meter**). Primitive man, however, used such natural units as the width of a finger, the **span** of his hand, the length of his **foot,** the distance from elbow to the tip of the middle finger (the **cubit** of biblical renown), or the **pace** (sometimes one but usually a double step) to measure short distances.

These ancient measurements varied from place to place, and from person to person. One of the first recorded attempts to establish a tangible standard length was made by the Greeks, who used the length of the Olympic stadium as a unit called a **stadium.** This was set at 600 Greek feet (607.9 modern U. S. feet), or almost exactly one-tenth of a modern nautical mile. The Romans adopted this unit and extended its use to nautical and even astronomical measurements. The Roman stadium was 625 Roman feet, or 606.3 U.S. feet, in length. This approximates the modern British Navy **cable** of 608 feet. The U.S. Navy cable is 720 feet.

The origin of the **Mediterranean mile** of 4,035.43 U.S. feet is attributed to the Greeks. The **Roman mile** of 4,858.60 U.S. feet gradually replaced the shorter Greek unit, and was probably the value in use in Palestine when Christ in his Sermon on the Mount spoke of the "second mile" (Matt. 5:41). It is probable that the **mile** was given its name by the Romans, since the word is derived from the Latin **mille** (thousand). This unit was defined as a thousand paces. However, the Greek unit was similarly defined, as was the Arabian **mille** or **mil** of 6,000 Arabian feet, equal to 1.03 nautical miles.

The **nautical mile** bears little relation to these land measures, which were not associated with the size of the earth. With the emergence of the nautical chart, it became customary to show a scale of miles on the chart, and the accepted value of this unit varied over the centuries with the changing estimates of the size of the earth. These estimates varied widely, ranging from about 44.5 to 87.5 modern nautical miles per degree of latitude, although generally they were too small. Columbus and Magellan used the value 45.3. Actually, the earth is about 32 percent larger. The *Almagest* of Ptolemy considered 62 Roman miles equivalent to one degree, but a 1466 edition of this book contained a chart of southern Asia drawn by Nicolaus Germanus on which 60 miles were shown to a degree. Whether the change was considered a correction or an adaptation to provide a more convenient relationship between the mile and the degree is not clear, but this is the earliest known use of this ratio.

Later, when the size of the earth was determined by measurement, the relationship of 60 Roman miles of 4,858.60 U.S. feet to a degree of latitude was seen to be in error. Both possible solutions to the problem—changing the ratio of miles to a degree, or changing the length of the mile—had their supporters, and neither group was able to convince the other. As a result, the shorter mile remained as the **land** or **statute mile** (now established as 5,280 feet in the United States), and the longer **nautical mile** gradually became established at sea. The earliest known reference to it by this name occurred in 1730.

Finer instruments and new methods make increasingly more accurate determinations of the size of the earth an ever-present possibility. Hence, a unit of length

defined in terms of the size of the earth is undesirable. Recognition of this led, in 1875, to a change in the definition of the **meter** from one ten-millionth of the distance from the pole to the equator of the earth to the distance between two marks (approximately 39.37 U. S. inches) on a standard platinum-iridium bar kept at the Pavillon de Breteuill at Sevres, near Paris, France, by the International Commission of Weights and Measures. In further recognition of this principle, the International Hydrographic Bureau in 1929 recommended adoption of a standard value for the nautical mile, and proposed 1852 international meters. The length of 1852 meters has not changed, but in 1959, U. S. measurements were redefined; the length of one nautical mile was established at 6,076.11549 U. S. feet (approximately). Most major maritime nations now use the international value.

The **fathom** as a unit of length or depth is of obscure origin, but primitive man considered it a measure of the outstretched arms, and the modern seaman still estimates the length of a line in this manner. That the unit was used in early times is indicated by reference to it in the detailed account given of the Apostle Paul's voyage to Rome, as recorded in the 27th chapter of the *Acts of the Apostles*. Posidonius reported a sounding of more than 1,000 fathoms in the second century BC. How old the unit was at that time is unknown.

114. Soundings.—Probably the most dangerous phase of navigation occurs when the vessel is "on soundings." Since man first began navigating the waters, the possibility of grounding his vessel has been a major concern, and frequent soundings have been the most highly valued safeguard against that experience. Undoubtedly used long before the Christian era, the lead line is perhaps the oldest instrument of navigation.

The lead line. The **hand lead** (art. 617), consisting of a lead weight attached to a line usually marked in fathoms, has been known since antiquity and, with the exception of the markings, is probably the same today as it was 2,000 or more years ago. The **deep sea lead,** a heavier weight with a longer line, was a natural outrowgth of the hand lead. A 1585 navigator speaks of soundings of 330 fathoms, and in 1773, in the Norwegian Sea, Captain Phipps had all the sounding lines on board spliced together to obtain a sounding of 683 fathoms. Matthew Fontaine Maury made his deep sea soundings by securing a cannon shot to a ball of strong twine. The heavy weight caused the twine to run out rapidly, and when bottom was reached, the twine was cut and the depth deduced from the amount remaining on the ball.

The sounding machine. The biggest disadvantage of the deep sea lead is that the vessel must be stopped if depths are to be measured accurately. This led to the development of the sounding machine (art. 618).

Early in the 19th century a sounding machine similar to one of the earlier patent logs was invented. A wheel was secured just above the lead and the cast made in such a way that all the line required ran out freely and the lead sank directly to the bottom. The motion through the water during the descent set the wheel revolving, and this in turn caused the depth to be indicated on a dial. Ships sailing at perhaps 12 knots required 20 or 30 men to heave aboard the heavy line with its weight of 50 or more pounds after each cast. A somewhat similar device was the **buoy sounder.** The lead was passed through a buoy in which a spring catch was fitted and both were cast over the side. The lead ran freely until bottom was reached, when the catch locked, preventing further running out of the line. The whole assembly was then brought on board, the depth from the buoy to the lead being read.

The first use of the pressure principle to determine the depth of water occurred early in the 19th century when the "Self-acting Sounder" was introduced. A hollow glass tube open at its lower end contained an index which moved up in the tube as greater water pressure compressed the air inside. The index retained its highest

position when hauled aboard the vessel, and its height was proportional to the depth of the water.

The British scientist, William Thomson (Lord Kelvin) in 1878 perfected the sounding machine after repeated tests at sea. Prior to his invention, fibre line was used exclusively in soundings. His introduction of piano wire solved the problem of rapid descent of the lead and also that of hauling it back aboard quickly. The chemically coated glass tube which he used to determine depth was an improvement of earlier methods, and the worth of the entire machine is evidenced by the fact that it is still used in essentially the same form.

Echo sounding. Based upon the principle that sound travels through sea water at a nearly uniform rate, automatic depth-registering devices (art. 619) have been invented to indicate the depth of water under a vessel, regardless of its speed. In 1911 an account was published of an experiment performed by Alexander Behm of Kiel, who timed the echo of an underwater explosion, testing this theory. High frequency sounds in water were produced by Pierre Langevin, and in 1918 he used the principle for echo depth finding. The first practical echo sounder was developed by the United States Navy in 1922.

The actual time between emission of a sonic or ultrasonic signal and return of its echo from the bottom, the angle at which the signal is beamed downward in order that its echo will be received at another part of the vessel, and the phase difference between signal and echo have all been used in the development of the modern echo sounder.

115. Aids to navigation.—The Cushites and Libyans constructed towers along the Mediterranean coast of Egypt, and priests maintained beacon fires in them. These were the earliest known lighthouses. At Sigeum in the Troad (part of Troy) a lighthouse was built before 660 BC. One of the seven wonders of the ancient world was the lighthouse called the Pharos of Alexandria, which may have been more than 200 feet tall. It was built by Sostratus of Cnidus (Asia Minor) in the third century BC, during the reign of Ptolemy Philadelphus. The word "pharos" has since been a general term for lighthouses. Some time between 1584 and 1611 the light of Cordouan, the earliest wave-swept lighthouse, was erected at the entrance to the Gironde river in western France. An oak log fire illuminated this structure until the 18th century.

Wood or coal fires were used in the many lighthouses built along the European and British coasts in the 17th and 18th centuries. One of these, the oak pile structure erected by Henry Whiteside in 1776 to warn shipmasters of Small's Rocks, subsequently played a major role in navigational history, as it was this light which figured in the discovery of the celestial line of position by Captain Thomas Sumner some 60 years later (art. 131).

In England such structures were privately maintained by interested organizations. One of the most famous of these groups, popularly known as "Trinity House," was organized in the 16th century, perhaps earlier, when a "beaconage and buoyage" fee was levied on English vessels. This prompted the establishment of Trinity House "to make, erect, and set up beacons, marks, and signs for the sea" and to provide vessels with pilots. The organization is now in its fifth century of operation, and its chief duties are to serve as a general lighthouse and pilotage authority, and to supply pilots.

The first lightship was a small vessel with lanterns hung from its yardarms. It was stationed at the Nore, an estuary in the Thames River, England, in 1732.

The pilot's profession is not much younger than that of the mariner. The Bible relates (1 Kings 9:27) that Hiram of Tyre provided pilots for King Solomon. The duties of these pilots are not specified. In the first century AD, fishermen of the

Gulf of Cambay, India, met seagoing vessels and guided them into port. It is probable that pilots were established in Delaware Bay earlier than 1756.

Seafaring people of the United States had erected lighthouses and buoys before the Revolutionary War, and in 1789 Congress passed legislation providing for federal expansion of the work. About 1767 the first buoys were placed in the Delaware River. These were logs or barrels, but about 1820 they were replaced with spar buoys. In that same year, the first lightship was established in Chesapeake Bay.

As the maritime interests of various countries grew, more and better aids to navigation were made available. In 1850 Congress prescribed the present system of coloring and numbering United States buoys (app. J). Conformity as to shape resulted from the recommendations of the International Marine Conference of 1889. The second half of the 19th century saw the development of bell, whistle, and lighted buoys, and in 1910 the first lighted buoy in the United States utilizing high pressure acetylene apparatus was placed in service. Stationed at the entrance to Ambrose Channel in New York, it provided the basis for the high degree of perfection which has been achieved in the lighted buoy since that time. The complete buoyage system maintained by the U. S. Coast Guard today is chiefly a product of the 20th century. In 1900 there were approximately 5,000 buoys of all types in use in the United States, while today there are more than 20,000.

116. The sailings.—The various methods of mathematically determining course, distance, and position arrived at have a history almost as old as mathematics itself. Thales, Hipparchus, Napier, Wright, and others contributed the formulas that led to the tables permitting computation of course and distance by plane, traverse, parallel, middle-latitude, Mercator, and great-circle sailings.

Plane sailing (art. 813). Based upon the assumption that the surface of the earth is plane, or flat; this method was used by navigators for many centuries. The navigator solved problems by laying down his course relative to his meridian, and stepping off the distance run to the new position. This system is used with accuracy today in measuring short runs on a Mercator chart, which compensates for the convergence of the meridians, but on the plane chart, serious errors resulted. Early navigators might have obtained mathematical solutions to this problem, with no greater accuracy, but the graphical method was commonly used.

Traverse sailing (art. 814). Because sailing vessels were subject to the winds, navigators of old were seldom able to sail one course for great distances, and consequently a series of small triangles had to be solved. Equipment was designed to help seamen in maintaining their dead reckoning positions. The modern **rough log** evolved from the *log board*, hinged wooden boards that folded like a book and on which courses and distances were marked in chalk. Each day the position was determined from this data and entered in the ship's journal, today's **smooth log.**

The log board was succeeded by the **travas,** a board with lines radiating from the center in 32 compass directions. Regularly spaced along the lines were small holes into which pegs were fitted to indicate time run on the particular course. In 1627 John Smith described the travas as a "little round board full of holes upon lines like the compasse, upon which by the removing of a little sticke they (seamen) keepe an account, how many glasses (which are but halfe houres) they steare upon every point of the compasse."

These devices were of great value to the navigator in keeping a record of the courses and distances sailed, but still left him the long mathematical solutions necessary to determine the new position. In 1436 what appears to have been the first **traverse table** was prepared by Andrea Biancho. Using this table of solutions of right-angled

plane triangles, the navigator was able to determine his course and distance made good after sailing a number of distances in different directions.

Parallel sailing (art. 815) was an outgrowth of the navigator's inability to determine his longitude. Not a mathematical solution in the sense that the other sailings are, it involved converting the distance sailed along a parallel (departure), as determined by dead reckoning, into longitude.

Middle-latitude sailing (art. 816). The inaccuracies involved in plane sailing led to the improved method of middle-latitude sailing early in the 17th century. A mathematician named Ralph Handsen is believed to have been its inventor.

Middle-latitude sailing is based upon the assumption that the use of a parallel midway between those of departure and arrival will eliminate the errors inherent in plane sailing due to the convergence of the meridians. The assumption is reasonably accurate and although the use of Mercator sailing usually results in greater accuracy, middle-latitude sailing still serves a useful purpose.

Mercator sailing (art. 817). Included in Edward Wright's *Certaine Errors in Navigation Detected and Corrected*, of 1599, was the first published table of meridional parts, which provided the basis for the most accurate of rhumb line sailings—Mercator sailing.

Great-circle sailing (art. 819). For many hundreds of years mathematicians have known that a great circle is the shortest distance between two points on the surface of a sphere, but it was not until the 19th century that navigators began to regularly make use of this information.

The first printed description of great-circle sailing appeared in Pedro Nunes' 1537 *Tratado da Sphera*. The method had previously been proposed by Sebastian Cabot in 1498, and in 1524 Verrazano sailed a great-circle course to America. But the sailing ships could not regularly expect the steady winds necessary to sail such a course, and their lack of knowledge concerning longitude, plus the necessity of stopping at islands along their routes to take supplies, made it impractical for most voyages at that time.

The gradual accumulation of knowledge concerning seasonal and prevailing winds, weather conditions, and ocean currents eventually made it possible for the navigator to plan his voyage with more assurance. Nineteenth century writers of navigational texts recommended the use of great-circle sailing, and toward the close of that century such sailing became increasingly popular, particularly in the Pacific.

The mathematics involved in great-circle sailing may be tedious, but the use of the gnomonic projection in locating points along the great-circle track has simplified the method.

117. Hydrographic offices.—The practice of recording hydrographic data was centuries old before the establishment of the first official hydrographic office, in 1720. In that year the **Depot des Cartes, Plans, Journaux et Memoirs Relatifs a la Navigation** was formed in France with the Chevalier de Luynes in charge. The Hydrographic Department of the British Admiralty, though not established until 1795, has played a major part in European hydrographic work.

The **U. S. Coast and Geodetic Survey** was originally founded when Congress, in 1807, passed a resolution authorizing a survey of the coast, harbors, outlying islands, and fishing banks of the United States. On the recommendation of the American Philosophical Society, President Jefferson appointed Ferdinand Hassler, a Swiss immigrant who had founded the Geodetic Survey of his native land, the first Director of the "Coast Survey."

The approaches to New York were the first sections of the coast charted, and from there the work spread northward and southward along the eastern seaboard. In 1844

the work was expanded and arrangements made to chart simultaneously the Gulf and East Coasts. Investigation of tidal conditions began, and in 1855 the first tables of tide predictions were published. The California gold rush gave impetus to the survey of the West Coast, which began in 1850, the year California became a State. The survey ship *Washington* undertook investigations of the Gulf Stream. Coast pilots, or sailing directions, for the Atlantic coast of the United States were privately published in the first half of the 19th century, but about 1850 the Survey began accumulating data that led to federally produced coast pilots. The 1889 *Pacific Coast Pilot* was an outstanding contribution to the safety of West Coast shipping.

Today the U. S. Coast and Geodetic Survey, as it has been called since 1878, provides the mariner with the charts and coast pilots of all waters of the United States and its possessions, and tide and tidal current tables for much of the world.

U. S. Navy Hydrographic Office. In 1830 the U. S. Navy established a "Depot of Charts and Instruments" in Washington, D. C. Primarily, it was to serve as a storehouse where such charts and sailing directions as were available, together with navigational instruments, could be assembled for issue to Navy ships which required them. Lieutenant L. M. Goldsborough and one assistant, Passed Midshipman R. B. Hitchcock, constituted the entire staff.

The first chart published by the Depot was produced from data obtained in a survey made by Lieutenant Charles Wilkes, who had succeeded Goldsborough in 1834, and who later earned fame as the leader of a United States exploring expedition to Antarctica.

From 1842 until 1861 Lieutenant Matthew Fontaine Maury served as Officer-in-Charge. Under his command the office rose to international prominence. Maury decided upon an ambitious plan to increase the mariner's knowledge of existing winds, weather, and currents. He began by making a detailed record of pertinent matter included in old log books stored at the Depot. He then inaugurated a hydrographic reporting program among shipmasters, and the thousands of answers received, along with the log book data, were first utilized to publish the *Wind and Current Chart of the North Atlantic* of 1847. The United States instigated an international conference in 1853 to interest other nations in a system of exchanging nautical information. The plan, which was Maury's, was enthusiastically adopted by other maritime nations, and is the basis upon which hydrographic offices operate today.

In 1854 the Depot was redesignated the "U. S. Naval Observatory and Hydrographical Office," and in 1866 Congress separated the two, broadly increasing the functions of the latter. The Office was authorized to carry out surveys, collect information, and print every kind of nautical chart and publication, all "for the benefit and use of navigators generally."

One of the first acts of the new Office was to purchase the copyright of *The New American Practical Navigator*. Several volumes of sailing directions had already been published. The first *Notice to Mariners* appeared in 1869. Daily broadcast of navigational warnings was inaugurated in 1907, and in 1912, following the sinking of the SS *Titanic*, Hydrographic Office action led to the establishment of the International Ice Patrol.

The development by the U. S. Navy of an improved depth finder in 1922 made possible the acquisition of additional information concerning bottom topography. During the same year aerial photography was first employed as an aid in chart making. The Hydrographic Office published the first chart for lighter-than-air craft in 1923. Aerial geomagnetic surveys were instituted in 1953 to provide magnetic information for ocean areas. Since World War II various electronic means have been employed to improve and extend surveys.

Meanwhile, numerous books have been published to assist the mariner and aviator in the solution of celestial observations. The initials "H.O." preceding a publication number are familiar to most navigators.

The **International Hydrographic Bureau** is an organization whose purpose is to encourage world-wide uniformity in hydrographic procedures. From the time of the International Marine Conference, held at Washington, D. C. in 1889, a need for such an organization was felt, and in 1919, at the Conference held in London, a French proposal for the establishment of such a body was adopted by delegates from the 24 nations represented. The International Hydrographic Bureau, located at Monaco, has since served as a coordinating agency for hydrographic work throughout the world.

118. Navigation manuals.—Although navigation is as old as man himself, navigation textbooks, as they are thought of today, are a product of the last several centuries. Until the end of the Dark Ages such books, or manuscripts, as were available were written by astronomers for other astronomers. The navigator was forced to make use of these, gleaning what little was directly applicable to his profession. After 1500, however, the need for books on navigation resulted in the publication of a series of manuals of increasing value to the mariner.

Sixteenth century manuals. Frequently a command of Latin was required to study navigation during the 16th century. *Regimento do estrolabio e do quadrante* (fig. 130a), which was published at Lisbon in 1509, or earlier, explained the method of finding latitude by meridian observations of the sun and the pole star, contained a traverse table for finding the longitude by dead reckoning, and listed the longitudes of a number of places. Unfortunately, the author made several errors in transcribing the declination tables published by Abraham Zacuto in 1474, and this resulted in errors being made for many years in determining latitude. Nevertheless, the nameless writer of the *Regimento* performed a great service for all mariners. His "Handbook for the Astrolabe and Quadrant"—to translate the title—had many editions and many emulators.

In 1519 Fernandez de Encisco published his *Suma de Geographia*, the first Spanish manual. The book was largely a translation of the *Regimento*, but new information was included, and revisions were printed in 1530 and 1546.

The Flemish mathematician and astronomer R. Gemma Frisius published a book on navigation in 1530. This manual, entitled *De Principiis Astronomiae*, gave an excellent description of the sphere, although the astronomy was that of Ptolemy, and discussed at length the use of the globe in navigation. Gemma gave courses in terms of the principal winds, proposed that longitude be reckoned from the Fortunate Islands (Canary Islands), and gave rules for finding the dead reckoning position by courses and distances sailed.

Tratado da Sphera, Pedro Nunes' great work, appeared in 1537. In addition to the first printed description of great-circle sailing, Nunes' book included a section on determining the latitude by two altitudes of the sun (taken when the azimuths differed by not less than 40°) and solving the problem on a globe. The method was first proposed by Gemma. *Tratado da Sphera* contained the conclusion of a study of the "plane chart" which Nunes had made. He exposed its errors, but was unable to develop a satisfactory substitute.

During the years that followed, an extensive navigational literature became available. The Spaniards Pedro de Medina and Martin Cortes published successful manuals in 1545 and 1551, respectively. Medina's *Arte de Navegar* passed through 13 editions in several languages and *Breve de la Spera y de la Arte de Navegar*, Cortes' book, was eventually translated into English and became the favorite of the British navigator. Cortes discussed the principle which Mercator used 18 years later in constructing his

THE NEW AMERICAN

PRACTICAL NAVIGATOR;

BEING AN

EPITOME OF NAVIGATION;

CONTAINING ALL THE TABLES NECESSARY TO BE USED WITH THE

NAUTICAL ALMANAC,

IN DETERMINING THE

LATITUDE;

AND THE

LONGITUDE BY LUNAR OBSERVATIONS;

AND

KEEPING A COMPLETE RECKONING AT SEA:

ILLUSTRATED BY

PROPER RULES AND EXAMPLES:

THE WHOLE EXEMPLIFIED IN A

JOURNAL,

KEPT FROM

BOSTON TO MADEIRA,

IN WHICH ALL THE RULES OF NAVIGATION ARE INTRODUCED:

ALSO

The Demonstration of the most useful Rules of TRIGONOMETRY: With many useful Problems in MENSURATION, SURVEYING, and GAUGING: And a Dictionary of SEA-TERMS; with the Manner of performing the most common EVOLUTIONS at Sea.

TO WHICH ARE ADDED,

Some GENERAL INSTRUCTIONS and INFORMATION to MERCHANTS, MASTERS of VESSELS, and others concerned in NAVIGATION, relative to MARITIME LAWS and MERCANTILE CUSTOMS.

FROM THE BEST AUTHORITIES.

ENRICHED WITH A NUMBER OF

NEW TABLES,

WITH ORIGINAL IMPROVEMENTS AND ADDITIONS, AND A LARGE VARIETY OF NEW AND IMPORTANT MATTER:

ALSO,

MANY THOUSAND ERRORS ARE CORRECTED,

WHICH HAVE APPEARED IN THE BEST SYSTEMS OF NAVIGATION YET PUBLISHED.

BY NATHANIEL BOWDITCH,

FELLOW OF THE AMERICAN ACADEMY OF ARTS AND SCIENCES.

ILLUSTRATED WITH COPPERPLATES.

First Edition.

PRINTED AT NEWBURYPORT, (Mass.) 1802,

BY

EDMUND M. BLUNT, (Proprietor)

For CUSHING & APPLETON, Salem.

SOLD BY EVERY BOOKSELLER, SHIP-CHANDLER, AND MATHEMATICAL-INSTRUMENT-MAKER, IN THE UNITED STATES AND WEST-INDIES

FIGURE 118.—Original title page of *The New American Practical Navigator*, written by Nathaniel Bowditch and published in 1802.

famous chart, and he also listed accurately the distance between meridians at all latitudes.

The first western hemisphere navigation manual was published by Diego Garcia de Palacio at Mexico City in 1587. His *Instrucion Nauthica* included a partial glossary of nautical terms and certain data on ship construction.

John Davis' *The Seaman's Secrets* of 1594 was the first of the "practical" books. Davis was a celebrated navigator who asserted that it was the purpose of his book to give "all that is necessary for sailors, not for scholars on shore." Davis' book discussed at length the navigator's instruments, and went into detail on the "sailings." He explained the method of dividing a great circle into a number of rhumb lines, and the work he had done with Edward Wright qualified him to report on the method and advantages of Mercator sailing. He endorsed the system of determining latitude by two observations of the sun and the intermediate bearing.

Although best known for the presentation of the theory of Mercator sailing, Edward Wright's *Certaine Errors in Navigation Detected and Corrected* (1599) was a sound navigation manual in its own right. Particularly, he advocated correcting sights for dip, refraction, and parallax (ch. XVI).

Later manuals. The next 200 years saw a succession of navigation manuals made available to the navigator; so many that only a few can be mentioned. Among those which enjoyed the greatest success were Blundeville's *Exercises*, John Napier's *Mirifici Logarithmorum Canonis Constructio* (which introduced the use of logarithms at sea), the tables and rules of Edmund Gunter, *Arithmetical Navigation* by Thomas Addison, and Richard Norwood's *The Sea-mans Practice* (which gave the length of the nautical mile as 6,120 feet). Robert Dudley filled four volumes in writing the *Arcano del Mare* (1646–47) as did John Robertson with *Elements of Navigation*. Jonas and John Moore, William Jones, and several Samuel Dunns were others who contributed navigation books before Nathaniel Bowditch in America and J. W. Norie in England wrote the manuals which navigators found best suited to their needs.

Bowditch's *The New American Practical Navigator* was first published in 1802 (fig. 118), and Norie's *Epitome of Navigation* appeared the following year. Both were outstanding books which enabled the mariner of little formal education to grasp the essentials of his profession. The Englishman's book passed through 22 editions in that country before losing its popularity to Captain Lecky's famous *"Wrinkles" in Practical Navigation* of 1881. The *American Practical Navigator* is still read widely, more than a century-and-a-half after its original printing.

A number of worthy navigation manuals have appeared in recent years.

Celestial Navigation

119. Astronomy is sometimes called the oldest of sciences. The movements of the sun, moon, stars, and planets were used by the earliest men as guides in hunting, fishing, and farming. The first maps were probably of the heavens.

Babylonian priests studied celestial mechanics at a very early date, possibly as early as 3800 BC, more probably about 1500 years later. These ancient astronomers predicted lunar and solar eclipses, constructed tables of the moon's hour angle, and are believed to have invented the zodiac. The week and month as known today originated with their calendar. They grouped the stars by constellations. It is probable that they were arranged in essentially their present order as early as 2000 BC. The five planets easily identified by the unaided eye were known to the Babylonians, who were apparently the first to divide the sun's apparent motion about the earth into 24 equal parts. They published this and other astronomical data in ephemerides. There is evidence that the prophet Abraham had an excellent knowledge of astronomy.

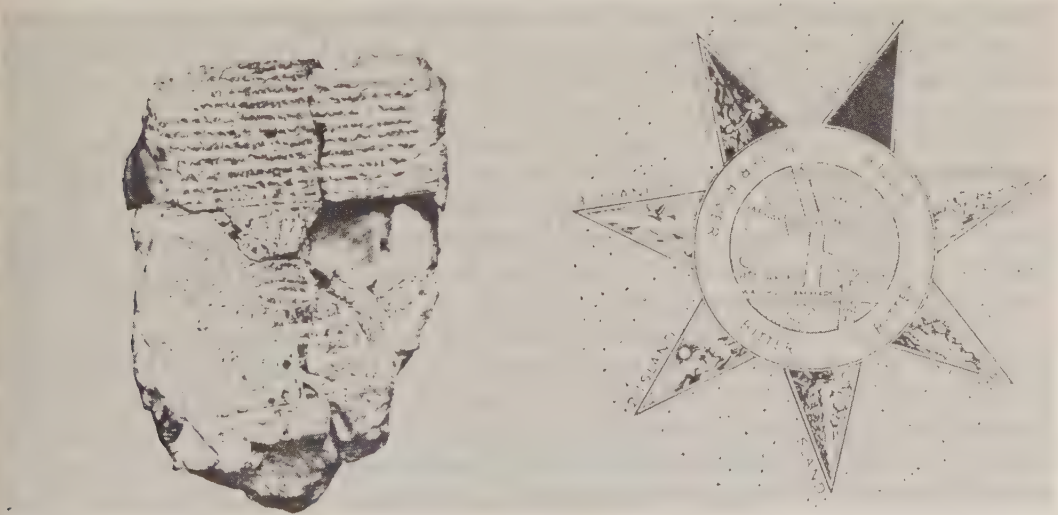
The Chinese, too, made outstanding contributions to the science of the heavens. They may have fixed the solstices and equinoxes before 2000 BC. They had quadrants and armillary spheres, used water clocks, and observed meridian transits. These ancient Chinese determined that the sun made its annual apparent revolution about the earth in 365¼ days, and divided circles into that many parts, rather than 360. About 1100 BC the astronomer Chou Kung determined the sun's maximum declination within about 15′.

Astronomy was used by the Egyptians in fixing the dates of their religious festivals almost as early as the Babylonian studies. By 2000 BC or earlier the new year began with the heliacal rising of Sirius; that is, the first reappearance of this star in the eastern sky during morning twilight after having last been seen just after sunset in the western sky. The heliacal rising of Sirius coincided approximately with the annual Nile flood. The famous Pyramid of Cheops, which was probably built in the 17th century BC, was so constructed that the light of Sirius shone down a southerly shaft when at upper transit, and the light of the pole star shone down a northerly shaft at lower transit, the axes of the two shafts intersecting in the royal burial chamber. When the pyramid was constructed, α *Draconis*, not Polaris, was the pole star.

The Greeks learned of navigational astronomy from the Phoenicians. The earliest Greek astronomer, Thales, was of Phoenician ancestry. He is given credit for dividing the year of the western world into 365 days, and he discovered that the sun does not move uniformly between solstices. Thales is most popularly known, however, for predicting the solar eclipse of 585 BC, which ended a battle between the Medes and the Lydians. He was the first of the great men whose work during the next 700 years was the controlling force in navigation, astronomy, and cartography until the Renaissance.

120. Shape of the earth.—Advanced as the Babylonians were, they apparently considered the earth to be flat. Land surveys of about 2300 BC show a "salt water river" encircling the country (fig. 120).

But seafarers knew that the last to be seen of a ship as it disappeared over the horizon was the masthead. They recognized the longer summer days in England when they sailed to the tin mines of Cornwall, as early as 900 BC. In that "north-



Courtesy of the Map Division of the Library of Congress.

FIGURE 120.—The original and reconstruction of a Babylonian map of about 500 BC. The Babylonians believed the earth to be a flat disk encircled by a salt water river.

land" the Mediterranean sailors noticed that the pole star was higher in the sky and the lower southern constellations were no longer visible. When Thales invented the gnomonic projection, about 600 BC, he must have believed the earth to be a sphere. Two centuries later Aristotle wrote that the earth's shadow on the moon during an eclipse was always circular. Archimedes (287–212 BC) used a glass celestial globe with a smaller terrestrial globe inside it. Although the average man has understood the spherical nature of the earth for only a comparatively short period, learned astronomers have accepted the fact for more than 25 centuries.

121. Celestial mechanics.—Among astronomers the principal question for 2,000 years was not the shape of the earth, but whether it or the sun was the center of the universe. A stationary earth seemed logical to the early Greeks, who calculated that daily rotation would produce a wind of several hundred miles per hour at the equator. Failing to realize that the earth's atmosphere turns with it, they considered the absence of such a wind proof that the earth was stationary.

The belief among the ancients was that all celestial bodies moved in circles about the earth. However, the planets—the "wanderers," as they were called—contradicted this theory by their irregular motion. In the fourth century BC Eudoxus of Cnidus attempted to account for this by suggesting that planets were attached to concentric spheres which rotated about the earth at varying speeds. The plan of **epicycles,** the theory of the universe which was commonly accepted for 2,000 years, was first proposed by Apollonius of Perga in the third century BC. Ptolemy accepted and amplified the plan, explaining it in his famous books, the *Almagest* and *Cosmographia*. According to Ptolemy, the planets moved at uniform speeds in small circles, the centers of which moved at uniform speeds in circles about the earth (fig. 121).

At first the Ptolemaic theory was accepted without question, but as the years passed, forecasts based upon it proved to be inaccurate. By the time the *Alfonsine Tables* were published in the 13th century AD, a growing number of astronomers considered the Ptolemaic doctrine unacceptable. However, Purbach, Regiomontanus, Bernhard Walther of Nuremberg, and even Tycho Brahe in the latter part of the 16th century, were among those who tried to reconcile the earth-centered epicyclic plan to the observed phenomena of the heavens.

As early as the sixth century BC, a brotherhood founded by Pythagoras, a Greek philosopher, proposed that the earth was round and self-supported in space, and that it, the other planets, the sun, and the moon revolved about a central fire which they called *Hestia*, the hearth of the universe. The sun and the moon, they said, shone by reflected light from Hestia.

The central fire was never located, however, and a few hundred years later Aristarchus of Samos advanced a genuine heliocentric theory. He denied the existence of Hestia and placed the sun at the center of the universe, correctly considering it to be a star which shone by itself. The Hebrews apparently understood the correct relationship at least as early as Abraham (about 2000 BC), and the early inhabitants of the Western Hemisphere probably knew of it before the Europeans did.

The Ptolemaic theory was generally accepted until its inability to predict future positions of the planets could no longer be reconciled. Its replacement by the heliocentric theory is credited principally to Nicolaus Copernicus (or Koppernigk). After studying mathematics at the University of Cracow, Copernicus went to Bologna, where he attended the astronomical lectures of Domenicao Maria Novara, an advocate of the Pythagorean theory. Further study in Martianus Copella's *Satyricon*, which includes a discussion of the heliocentric doctrine, convinced him that the sun was truly the center of the universe.

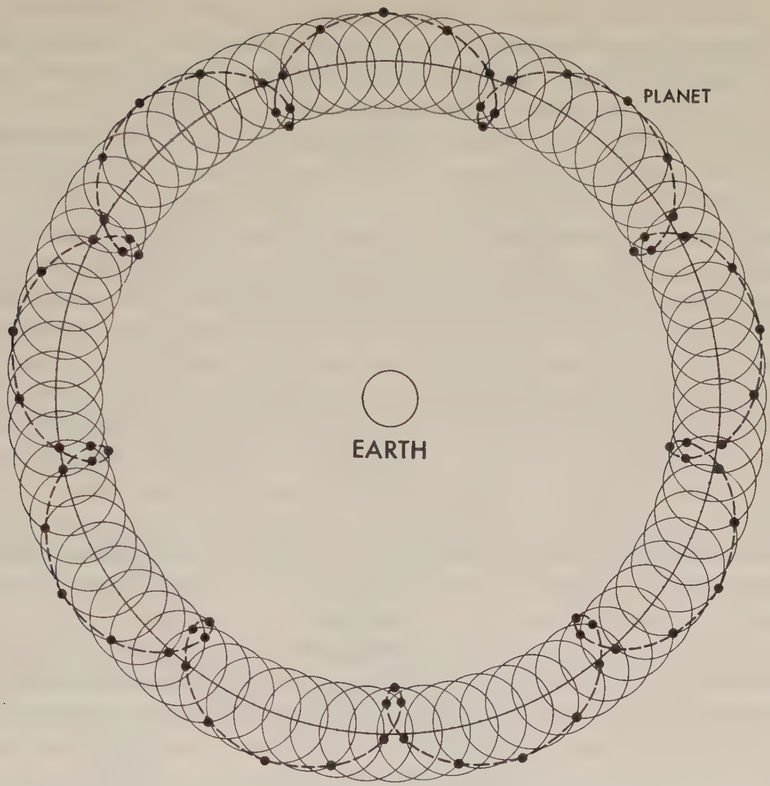


FIGURE 121.—The plan of epicycles, by which the ancients explained the retrograde motion of the planets. The planets were believed to rotate in small circles whose centers moved about the earth in a large circle.

Until the year of his death Copernicus tested his belief by continual observations, and in that year, 1543, he published *De Revolutionibus Orbium Coelestium.* In it he said that the earth rotated on its axis daily and revolved in a circle about the sun once each year. He placed the other planets in circular orbits about the sun also, recognizing that Mercury and Venus were closer than the earth, and the others farther out. He concluded that the stars were motionless in space and that the moon moved circularly about the earth. His conclusions did not become widely known until nearly a century later, when Galileo publicized them. Today, "heliocentric" and "Copernican" are synonymous terms used in describing the character of the solar system.

122. Other early discoveries.—A knowledge of the principal motions of the planets permitted reasonably accurate predictions of future positions. Other, less spectacular data, however, were being established to help round out the knowledge astronomers needed before they could produce the highly accurate almanacs known today.

More than a century before the birth of Christ, Hipparchus discovered the **precession of the equinoxes** (art. 1419) by comparing his own observations of the stars with those recorded by Timocharis and Aristyllus about 300 BC. Hipparchus cataloged more than a thousand stars, and compiled an additional list of time-keeping stars which differed in sidereal hour angle by 15° (one hour), accurate to 15′. A spherical star map, or planisphere, and a celestial globe were among the equipment he designed. However, his instruments did not permit measurements of such precision that stellar parallax could be detected, and, consequently, he advocated the geocentric theory of the universe.

Three centuries later Ptolemy examined and confirmed Hipparchus' discovery of precession. He published a catalog in which he arranged the stars by constellations and gave the magnitude, declination, and right ascension (art. 1426) of each. Following Hipparchus, Ptolemy determined longitudes by eclipses. In the *Almagest* he included the plane and spherical trigonometry tables which Hipparchus had developed, mathematical tables, and an explanation of the circumstances upon which the equation of time (art. 1912) depends.

The next thousand years saw little progress in the science of astronomy. Alexandria continued as a center of learning for several hundred years after Ptolemy, but succeeding astronomers at the observatory confined their work to comments on his great books. The long twilight of the Dark Ages had begun.

Alexandria was captured and destroyed by the Arabs in AD 640, and for the next 500 years Moslems exerted the primary influence in astronomy. Observatories were erected at Baghdād and Damascus during the ninth century. Ibn Yunis' observatory near Cairo gathered the data for the Hakimite tables in the 11th century. Earlier, the Spanish, under Moorish tutelage, set up schools of astronomy at Cordova and Toledo.

123. Modern astronomy may be said to date from Copernicus, although it was not until the invention of the telescope, about 1608, that precise measurement of the positions and motions of celestial bodies was possible.

Galileo Galilei, an Italian, made outstanding contributions to the cause of astronomy, and these served as a basis for the work of later men, particularly Isaac Newton. He discovered Jupiter's satellites, providing additional opportunities for determining longitude on land. He maintained that it is natural for motion to be uniform and in a straight line and that a force is required only when direction or speed is changing. Galileo's support of the heliocentric theory, his use and improvement of the telescope, and particularly the clarity and completeness of his records provided firm footing for succeeding astronomers.

Early in the 17th century, before the invention of the telescope, Tycho Brahe found the planet Mars to be in a position differing by as much as 8′ from that required by the geocentric theory. When the telescope became available, astronomers learned that the apparent diameter of the sun varied during the year, indicating that the earth's distance from the sun varies, and that its orbit is not circular.

Johannes Kepler, a German who had succeeded Brahe and who was attempting to account for his 8′ discrepancy, published in 1609 two of astronomy's most important doctrines, the **law of equal areas,** and the **law of elliptical orbits.** Nine years later he announced his third law, relating the periods of revolution of any two planets to their respective distances from the sun (art. 1407).

Kepler's discoveries provided a mathematical basis by which more accurate tables of astronomical data were computed for the maritime explorers of the age. His realization that the sun is the controlling power of the system and that the orbital planes of the planets pass through its center almost led him to the discovery of the law of gravitation.

Sir Isaac Newton reduced Kepler's conclusions to the **universal law of gravitation** (art. 1407) when he published his three laws of motions in 1687. Because the planets exert forces one upon the other, their orbits do not agree exactly with Kepler's laws. Newton's work compensated for this and, as a result, the astronomer was able to forecast with greater accuracy the positions of the celestial bodies. The navigator benefited through more exact tables of astronomical data.

Between the years 1764 and 1784, the Frenchmen Lagrange and Laplace conclusively proved the solar system's mechanical stability. Early in the 19th century,

Nathaniel Bowditch translated and commented upon Laplace's *Mécanique Céleste*, bringing it up-to-date. Prior to their work this stability had been questioned due to apparent inconsistencies in the motions of some of the planets. After their demonstrations, men were convinced and could turn to other important work necessary to refine and improve the navigator's almanac.

But there were real, as well as apparent, irregularities of motion which could not be explained by the law of gravitation alone. By this law the planets describe ellipses about the sun, and these orbits are repeated indefinitely, except as the other planets influence the orbits of each by their own gravitational pull. Urbain Leverrier, one-time Director of the Paris Observatory, found that the line of apsides of Mercury was advancing 43″ per century faster than it should, according to the law of gravitation and the positions of other known planets. In an attempt to compensate for the resulting errors in the predicted positions of the planet, he suggested that there must be a mass of circulating matter between the sun and Mercury. No such circulating matter has been found, however, and Leverrier's discovery is attributed to a shortcoming of Newton's law, as explained by Albert Einstein.

In Einstein's hands, Leverrier's 43″ became a fact as powerful as Brahe's 8′ had been in the hands of Kepler. Early in the 20th century, Einstein announced the theory of relativity (art. 1407). He stated that for the planets to revolve about the sun is natural, and gravitational force is unnecessary for this, and he asserted that there need be no circulating matter to account for the motion of the perihelion of Mercury as this, too, is in the natural order of things. Calculated from his theory, the correction to the previously computed motion of the perihelion in 100 years is 42″9.

Prior to Einstein's work, other discoveries had helped round out man's knowledge of the universe.

Aberration (art. 1417), discovered by James Bradley about 1726, accounted for the apparent shifting of the stars throughout the year, due to the combined orbital speed of the earth and the speed of light. Twenty years later Bradley described the periodic wobbling of the earth's axis, called **nutation** (art. 1417), and its effect upon precession of the equinoxes.

Meanwhile, in 1718 Edmond Halley, England's second Astronomer Royal, detected a motion of the stars, other than that caused by precession, that led him to conclude that they, too, were moving. By studying the works of the Alexandrian astronomers, he found that some of the most prominent stars had changed their positions by as much as 32′. Jacques Cassini gave Halley's discovery further support when he found, a few years later, that the declination of Arcturus had changed 5′ in the 100 years since Brahe made his observations. This **proper motion** (art. 1414) is motion in addition to that caused by precession, nutation, and aberration.

Sir William Herschel, the great astronomer who discovered the planet Uranus in 1781, proved that the solar system is moving toward the constellation *Hercules*. As early as 1828 Herschel advocated the establishment of a standard time system. Neptune was discovered in 1846 after its position had been predicted by the Frenchman Urbain Leverrier. Based upon the work of Percival Lowell, an American, Pluto was identified in 1930. Uranus, Neptune, and Pluto are of little concern to the navigator.

A more recent discovery may well have greater navigational significance. This is the existence of sources of electromagnetic energy in the sky in the form of **radio stars** (art. 1414). The sun has been found to transmit energy of radio frequency, and instruments have been built which are capable of tracking it across the sky regardless of weather conditions.

124. Sextant.—Prior to the development of the magnetic compass, the navigator used the heavenly bodies chiefly as guides by which to steer. The compass, however,

Courtesy of the John Carter Brown Library, Brown University.

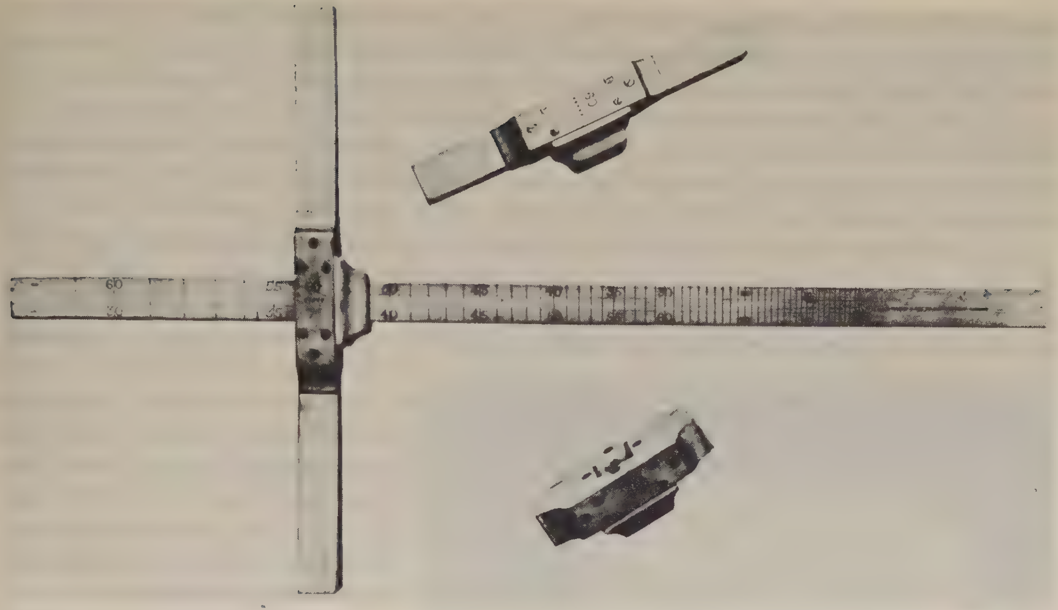
FIGURE 124a.—An ancient astrolabe, one of the earliest kinds of altitude-measuring instruments.

led to more frequent long voyages on the open sea, and the need for a vertical-angle measuring device which could be used for determining altitude, so that latitude could be found.

Probably the first such device used at sea was the **common quadrant,** the simplest form of all such instruments. Made of wood, it was a fourth part of a circle, held vertical by means of a plumb bob. An observation made with this instrument at sea was a two- or three-man job. This device was probably used ashore for centuries before it went to sea, although its earliest use by the mariner is unknown.

Invented perhaps by Apollonius of Perga in the third century BC, the **astrolabe** (fig. 124a)—from the Greek for *star* and *to take*—had been made portable by the Arabs possibly as early as AD 700. It was in the hands of Christian pilots by the end of the 13th century, often as an elaborate and beautiful creation wrought of precious metals. Some astrolabes could be used as **star finders** (art. 2210) by fitting an engraved plate to one side Large astrolabes were among the chief instruments of 15th and 16th century observatories, but the value of this instrument at sea was limited.

The principle of the astrolabe was similar to that of the common quadrant, but the astrolabe consisted of a metal disk, graduated in degrees, to which a movable sight

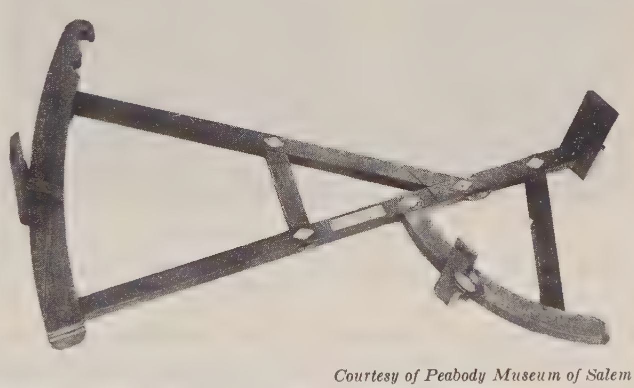


Courtesy of Peabody Museum of Salem.

FIGURE 124b.—The cross-staff, the first instrument to utilize the visible horizon in making celestial observations.

vane was attached. In using the astrolabe, which may be likened to a pelorus held on its side, the navigator adjusted the sight vane until it was in line with the star, and then read the zenith distance from the scale. As with the common quadrant, the vertical was established by plumb bob.

Three men were needed to make an observation with the astrolabe (one held the instrument by a ring at its top, another aligned the sight vane with the body, a third made the reading) and even then the least rolling or pitching of a vessel caused large acceleration errors in observations. Therefore, navigators were forced to abandon the plumb bob and make the horizon their reference.



Courtesy of Peabody Museum of Salem.

FIGURE 124c.—The backstaff, or sea quadrant, a favorite instrument of American colonial navigators.

The **cross-staff** (fig. 124b) was the first instrument which utilized the visible horizon in making celestial observations. The instrument consisted of a long, wooden shaft upon which one of several cross-pieces was mounted perpendicularly. The cross-pieces were of various lengths, the one being used depending upon the angle to be measured. The navigator fitted the appropriate cross-piece on the shaft and, holding one end of the shaft beside his eye, adjusted the cross until its lower end was in line with the horizon and its upper end with the body. The shaft was calibrated to indicate the altitude of the body observed.

In using the cross-staff, the navigator was forced to look at the horizon and the celestial body at the same time. In 1590 John Davis, author of *The Seaman's Secrets*, invented the **backstaff** (fig. 124c) or **sea quadrant.** He was one of the few practical

seamen (Davis Strait is named for him, in honor of his attempt to find the Northwest Passage) to invent a navigational device. The backstaff marked a long advance and was particularly popular among American colonial navigators.

In using this instrument, the navigator turned his back to the sun and aligned its shadow with the horizon. The backstaff had two arcs, and the sum of the values shown on each was the zenith distance of the sun. Later, this instrument was fitted with a mirror to permit observations of bodies other than the sun.

Another instrument developed about the same time was the **nocturnal** (fig. 124d). Its purpose was to provide the mariner with the appropriate correction to be made to the altitude of Polaris to determine latitude. By sighting on Polaris through the hole in the center of the instrument and adjusting the movable arm so that it pointed at Kochab, the navigator could read the correction from the instrument. Most nocturnals had an additional outer disk graduated for the months and days of the year and by adjusting this the navigator could also determine solar time.

British Crown copyright. Science Museum, London, England.

FIGURE 124d.—The nocturnal, an instrument used to determine latitude by an observation of Polaris.

Tycho Brahe designed several instruments with arcs of 60°, having one fixed sight and another movable one. He called the instruments **sextants** and the name is now commonly applied to all altitude-measuring devices used by the navigator (ch. XV). In 1700 Sir Isaac Newton sent to Edmond Halley, the Astronomer Royal, a description of a device having double-reflecting mirrors, the principle of the modern marine sextant. However, this was not made public until after somewhat similar instruments had been made in 1730 by the Englishman John Hadley, and the American Thomas Godfrey.

The original instrument constructed by Hadley was, in fact, an *octant*, but due to the double-reflection principle it measured angles up to one-fourth of a circle, or 90°. Godfrey's instrument is reported to have been a *quadrant*, and so could measure angles through 180°. The two men received equal awards from England's Royal Society, as their work was considered to be a case of simultaneous independent invention, although Hadley probably preceded Godfrey by a few months in the actual construction of his sextant.

In the next few years both instruments were successfully tested at sea, but 20 years or more passed before the navigator gave up his backstaff or sea quadrant for the new device. In 1733 Hadley attached a spirit level to a quadrant, and with it was able to measure altitudes without reference to the horizon. Some years later the first bubble sextant (art. 1513) was developed.

Pierre Vernier, in 1631, had attached to the limb of a quadrant a second, smaller graduated arc, thereby permitting angles to be measured more accurately, and this device was incorporated in all later angle-measuring instruments.

The sextant has remained practically unchanged since its invention more than two centuries ago. The only notable improvements have been the addition of an endless tangent screw and a micrometer drum, both having been added during the 20th century.

125. Determining latitude.—The ability to determine longitude at sea is comparatively modern, but latitude has been available for thousands of years.

Meridian transit of the sun. Long before the Christian era, astronomers had determined the sun's declination for each day of the year, and prepared tables listing the data. This was a comparatively simple matter, for the zenith distance obtained by use of a shadow cast by the sun on the day of the winter solstice could be subtracted from that obtained on the day of the summer solstice to determine the range of the sun's declination, about 47°. Half of this is the sun's maximum declination, which could then be applied to the zenith distance recorded on either day to determine the latitude of the place. Daily observations thereafter enabled the ancient astronomers to construct reasonably accurate declination tables.

Such tables were available long before the average navigator was ready to use them, but certainly by the 15th century experienced seamen were determining their latitude at sea to within one or two degrees. In his 1594 *The Seaman's Secrets*, Davis made use of his experience in high latitudes to explain the method of determining latitude by lower transit observations of the sun.

Ex-meridian observation of the sun. The possibility of overcast skies at the one time each day when the navigator could get a reliable observation for latitude led to the development of the "ex-meridian" sight. Another method, involving two sights taken with a considerable time interval between, had previously been known, but the mathematics were so involved that it is doubtful that many seamen made use of it.

There are two methods by which ex-meridian observations can be solved. The direct process was the more accurate, although it required a trigonometrical solution. By the latter part of the 19th century, tables were introduced which made the method of reduction to the meridian more practical and, when occasion demands such an observation, this is the method generally used today. However, with the development of line of position methods and the modern inspection table, ex-meridian observations have lost much of their popularity.

Latitude by Polaris. First use of the pole star to determine latitude is not known, but many centuries ago seamen who used it as a guide by which to steer were known to comment upon its change of altitude as they sailed north or south.

By Columbus' time some navigators were using Polaris to determine latitude, and with the invention of the nocturnal late in the 16th century, providing corrections to the observed altitude, the method came into more general use. The development of the chronometer in the 18th century permitted exact corrections, and this made determination of latitude by Polaris a common practice. Even today, more than a century after discovery of the celestial line of position, the method is still in use. The modern inspection table has eliminated the need for meridian observations as a special method for determining latitude. Perhaps when the almanacs and sight reduction tables make the same provision for solution of Polaris sights as they do for any other navigational star, this last of the special methods will cease to be used for general navigation. But customs die slowly, and one as well established as that of position finding in terms of separate latitude and longitude observations—instead of lines of position—is not likely to disappear completely for many years to come.

126. The search for a method of "discovering" longitude at sea.—A statement once quite common was, "The navigator always knows his latitude." A more accurate statement would have been, "The navigator never knows his longitude." In 1594 Davis wrote: "Now there be some that are very inquisitive to have a way to get the longitude, but that is too tedious for seamen, since it requireth the deep knowledge of astronomy, wherefore I would not have any man think that the longitude is to be found at sea by any instrument, so let no seamen trouble themselves with any such rule, but let them keep a perfect account and reckoning of the way of their ship." In speaking of conditions of his day, he was correct, for it was not until the 19th century that the average navigator was able to determine his longitude with accuracy.

Parallel sailing. Without knowledge of his longitude, the navigator of old found it necessary on an ocean crossing to sail northward or southward to the latitude of his destination, and then to follow that parallel of latitude until the destination was reached, even though this might take him far out of his way. Because of this practice, parallel sailing was an important part of the navigator's store of knowledge. The method was a crude one, however, and the time of landfall was often in error by a matter of days, and, in extreme cases, even weeks.

Eclipses. Almost as early as the rotation of the earth was established, astronomers recognized that longitude could be determined by comparing local time with that at the reference meridian. The problem was the determination of time at the reference meridian.

One of the first methods proposed was that of observing the disappearance of Jupiter's satellites as they were eclipsed by their planet. This method, originally proposed by Galileo for use on land, required the ability to observe and identify the satellites by using a powerful telescope, knowledge of the times at which the eclipses would take place, and the skill to keep the instrument directed at the bodies while aboard a small vessel on the high seas. Although used in isolated cases for many years, the method was not satisfactory at sea, due largely to the difficulty of observation (some authorities recommended use of a telescope as long as 18 or 19 feet) and the lack of sufficiently accurate predictions.

Variation of the compass was seriously considered as a method of determining longitude for 200 years or more. Faleiro, Magellan's advisor, believed it could be so utilized, and, until the development of the chronometer, work was carried on to perfect the theory. Although there is no simple relationship between variation and longitude, those who advocated the method felt certain that research and investigation would eventually provide the answer. Many others were convinced that such a solution did not exist. In 1676, Henry Bond published *The Longitude Found*, in which he stated that the latitude of a place and its variation could be referred to the prime meridian to determine longitude. Two years later Peter Blackborrow rebutted with *The Longitude Not Found*.

Variation was put to good use in determining the nearness to land by shipmasters familiar with the waters they plied, but as the solution to the longitude problem it was a failure, and with the improvement of lunar distance methods and the invention of the chronometer, interest in the method waned. If it had been possible to provide the mariner with an accurate chart of variation, and to keep it up-to-date, a means of establishing an approximate line of position in areas where the gradient is large would have resulted; in many cases this would have established longitude if latitude were known.

Lunar distances. The first method widely used at sea to determine longitude with some accuracy was that of lunar distances (art. 131), by which the navigator

determined GMT by noting the position of the relatively fast-moving moon among the stars. Both Regiomontanus, in 1472, and John Werner, in 1514, have been credited with being the first to propose the use of the lunar distance method. At least one source states that Amerigo Vespucci, in 1497, determined longitude using the moon's position relative to that of another body. One of the principal reasons for establishing the Royal Observatory at Greenwich was to conduct the observations necessary to provide more accurate predictions of the future positions of the moon. Astronomers, including the Astronomers Royal, favored this method, and half a century after the invention of the chronometer it was still being perfected. In 1802 Nathaniel Bowditch simplified the method and its explanation, thus eliminating much of the mystery surrounding it and making it understandable to the average mariner. By using Bowditch's method, the navigator was able to head more or less directly toward his destination, rather than travel the many additional miles often required in "running down the latitude" and then using parallel sailing. An explanation of the lunar distance method, and tables for its use, were carried in the *American Practical Navigator* until 1914.

The Board of Longitude. The lunar distance method, using the data and equipment available early in the 18th century, was far from satisfactory. Ships, cargoes, and lives were lost because of inaccurately-determined longitudes. During the Age of Discovery, Spain and Holland posted rewards for solution to the problem, but in vain. When 2,000 men were lost as a squadron of British men-of-war ran aground on a foggy night in 1707, officers of the Royal Navy and Merchant Navy petitioned Parliament for action. As a result, the Board of Longitude was established in 1714, empowered to reward the person who could solve the problem of "discovering" longitude at sea. A voyage to the West Indies and back was to be the test of proposed methods which were deemed worthy. The discoverer of a system which could determine the longitude within 1° by the end of the voyage was to receive £10,000; within 40′, £15,000; and within 30′, £20,000. These would be handsome sums today. In the 18th century they were fortunes.

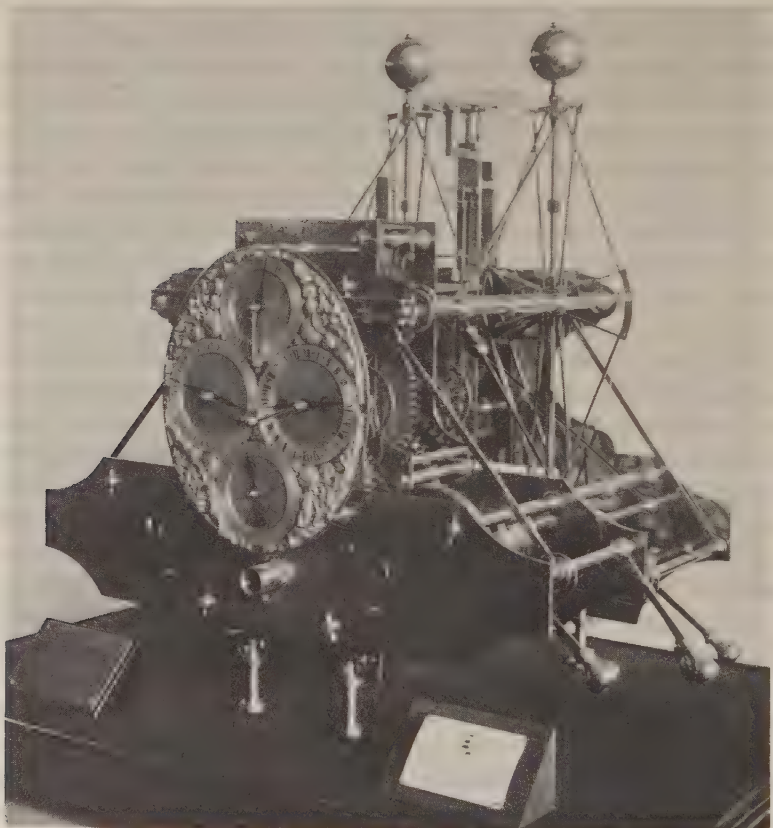
127. Evolution of the chronometer.—Many and varied were the solutions proposed for finding longitude, and as the different methods were found unsatisfactory, it became increasingly apparent that the problem was one of keeping the time of the prime meridian. But the development of a device that would keep accurate time during a long voyage seemed to most men to be beyond the realm of possibility. Astronomers were flatly opposed to the idea and felt that the problem was properly theirs. There is even some evidence to indicate that the astronomers of the Board of Longitude made unfair tests of chronometers submitted to them.

Christian Huygens (1629–95), a Dutch scientist and mathematician, made a number of contributions of great value in the field of astronomy, but his most memorable work, to the navigator, was his attempt at constructing a perfect timepiece. It was probably Galileo who first suggested using a pendulum in keeping time. Huygens realized that an error would result from the use of a simple pendulum, however, and he devised one in which the bob hung from a double cord that passed between two plates in such a way that it traced a cycloidal path.

In 1660 Huygens built his first chronometer. The instrument utilized his cycloidal pendulum, actuated by a spring. To compensate for rolling and pitching, Huygens mounted the clock in gimbals. Two years later the instrument was tested at sea, with promising results. The loss of tension in the spring as it ran down was the major weakness in this clock. Huygens compensated for this by attaching oppositely tapered cones and a chain to the spring. A 1665 sea test of the new timepiece showed greater accuracy, but still not enough for determination of longitude. In 1674 he constructed

a chronometer with a special balance and long balance-spring. Although it was the best marine timepiece then known, Huygens' last clock was also unsuited for use at sea due to the error caused by temperature changes.

John Harrison was a carpenter's son, born in Yorkshire in 1693. He followed his father's trade during his youth, but soon became interested in the repair and construction of clocks. At the age of 20 he completed his first timekeeper, a pendulum-type clock with wooden wheels and pinions. Harrison's gridiron pendulum, one which maintained its length despite temperature changes, was designed about 1720, and contained alternate iron and brass rods to eliminate distortion. Until the time that metal



British Crown copyright. From the original in the National Maritime Museum, London, England. Reproduced by permission of the Admiralty.

FIGURE 127.—Harrison's No. 1 chronometer. The first of four timekeepers constructed by Harrison, this clock weighs 65 pounds.

alloys having small coefficients of temperature expansion were developed, Harrison's invention was the type pendulum used by almost all clockmakers.

By 1728 Harrison felt ready to take his pendulum, an escapement he had invented, and plans for his own marine timepiece before the Board of Longitude. In London, however, George Graham, a famous clockmaker, advised him to first construct the timekeeper. Harrison did, and in 1735 he submitted his No. 1 chronometer (fig. 127). The Board authorized a sea trial aboard HMS *Centurion*. The following year, that vessel sailed for Lisbon with Harrison's clock on board, and upon her return, the error was found to be three minutes of longitude, a performance which astounded members of the Board. But the chronometer was awkward and heavy, being enclosed in glass

and weighing some 65 pounds, and the Board voted to give Harrison only £500, to be used in producing a more practical timepiece.

During the next few years he constructed two other chronometers, which were stronger and less complicated, although there is no record of their being tested by the Board of Longitude. Harrison continued to devote his life to the construction of an accurate clock to be used in determining longitude, and finally, as he approached old age, he developed his No. 4. Again he went before the Board, and again a test was arranged. In November of 1761, HMS *Deptford* sailed for Jamaica with No. 4 aboard, in the custody of Harrison's son, William. On arrival, after a passage lasting two months, the watch was only nine seconds slow (2¼ minutes of longitude). In January of 1762 it was placed aboard HMS *Merlin* for the return voyage to England. When the *Merlin* anchored in English waters in April of that year, the total error shown by the chronometer was 1 minute, 54.5 seconds. This is equal to less than a half degree of longitude, or less than the minimum error prescribed by the Board for the largest prize. Harrison applied for the full £20,000, but the Board, led by the Astronomer Royal, allowed him only a fourth of that, and insisted on another test.

William Harrison sailed again with No. 4 for Barbados in March of 1764, and throughout the almost four-months-long voyage the chronometer showed an error of only 54 seconds, or 13.5 minutes of longitude. The astronomers of the Board reluctantly joined in a unanimous declaration that Harrison's timepiece had exceeded all expectations, but they still would not pay him the full reward. An additional £5,000 were paid on the condition that plans be submitted for the construction of similar chronometers. Even when this was done, the Board delayed payment further by having one of its members construct a timepiece from the plans. Not until 1773, Harrison's 80th year, was the rest of the reward paid, and only then because of intervention by the king himself.

Pierre LeRoy, a great French clockmaker, constructed a chronometer in 1766 which has since been the basis for all such instruments. LeRoy's several inventions made his chronometer a timepiece which has been described as a "masterpiece of simplicity, combined with efficiency." Others to contribute to the art of watchmaking included Ferdinand Berthoud of France and Thomas Mudge of England, each of whom developed new escapements. The balance wheel was improved by John Arnold, who invented the escapement acting in one direction only, substantially that used today. Acting independently, Thomas Earnshaw invented a similar escapement. He built the first reliable chronometer at a relatively low price. The chronometer the Board of Longitude had made from Harrison's plans cost £450; Earnshaw's cost £45.

Timepieces designed to provide the navigator with information other than time were popular a century or more ago. One showed the times of high and low water, the state of the tide at any time, and the phases of the moon; another gave the equation of time and the apparent motions of the stars and planets; a third offered the position of the sun and both mean and sidereal times. But the chronometers produced by LeRoy and Earnshaw were the ones of greatest value to the navigator; they gave him a simple and reliable method of determining his longitude.

Time signals, which permit the mariner at sea to check the error in his chronometer, are essentially a 20th century development. Telegraphic time signals were inaugurated in the United States at the end of the Civil War, and enabled ships to check their chronometers in port by **time ball** signals. Previously, the Navy's "standard" chronometer had been carried from port to port to allow such comparison. In their most advanced form, time balls were dropped by telegraphic action. In 1904 the first official "wireless" transmission of time signals began from a naval station at Navesink,

N. J. These were low-power signals which could be heard for a distance of about 50 miles. Five years later the range had been doubled, and, as other nations began sending time signals, the navigator was soon able to check his chronometer around the world.

The search for longitude was ended.

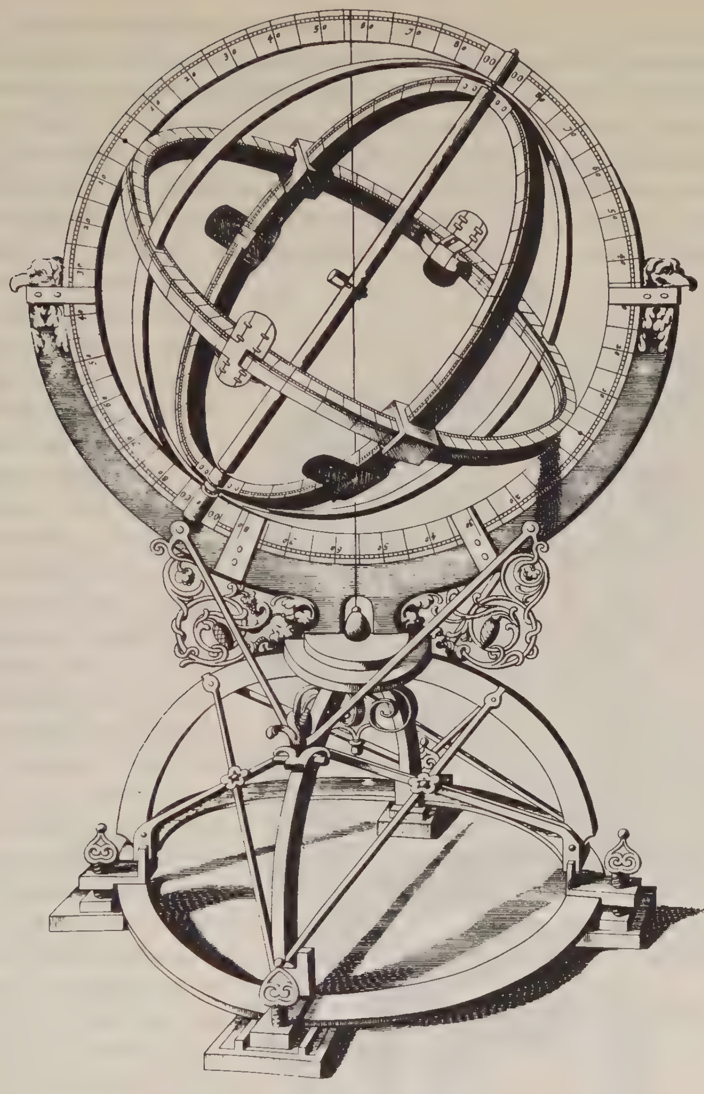
128. Establishment of the prime meridian.—Until the beginning of the 19th century, there was little uniformity among cartographers as to the meridian from which longitude was measured. The navigator was not particularly concerned, as he could not determine his longitude, anyway.

Ptolemy, in the second century AD, had measured longitude eastward from a reference meridian two degrees west of the Canary Islands. In 1493 Pope Alexander VI drew a line in the Atlantic west of the Azores to divide the territories of Spain and Portugal, and for many years this meridian was used by chart makers of the two countries. In 1570 the Dutch cartographer Ortelius used the easternmost of the Cape Verde Islands. John Davis, in his 1594 *The Seaman's Secrets*, said the Isle of Fez in the Canaries was used because there the variation was zero. Mariners paid little attention, however, and often reckoned their longitude from several different capes and ports during a voyage, depending upon their last reliable fix.

The meridian of London was used as early as 1676, and over the years its popularity grew as England's maritime interests increased. The system of measuring longitude both east and west through 180° may have first appeared in the middle of the 18th century. Toward the end of that century, as the Greenwich Observatory increased in prominence, English map makers began using the meridian of that observatory as a reference. The publication by the Observatory of the first British *Nautical Almanac* in 1767 further entrenched Greenwich as the prime meridian. A later and unsuccessful attempt was made in 1810 to establish Washington as the prime meridian for American navigators and cartographers. At an international conference held in Washington in 1884 the meridian of Greenwich was officially established, by the 25 nations in attendance, as the prime meridian. Today all maritime nations have designated the Greenwich meridian the prime meridian, except in a few cases where local references are used for certain harbor charts.

129. Astronomical observatories.—Thousands of years before the birth of Christ, crude observatories existed, and astronomers constructed primitive tables which were the forerunners of modern almanacs. The famous observatory at Alexandria, the first "true" observatory, was constructed in the third century BC, but the Egyptians, as well as the Babylonians and Chinese, had already studied the heavens for many centuries. The **armillary sphere** (fig. 129a) was the principal instrument used by the early astronomers. It consisted of a skeleton sphere with several movable rings which could be adjusted to indicate the orbits of the various celestial bodies. One source attributes the invention of the armillary sphere to Eratosthenes in the third century BC; another says the Chinese knew it 2,000 years earlier, as well as the water clock and a form of astrolabe. The Alexandrian observatory was the seat of astronomical learning in the western world for several centuries, and there Hipparchus discovered the precession of the equinoxes, and Ptolemy did the work which led to his *Almagest*.

Astronomical study did not cease entirely during the Dark Ages. The Arabians erected observatories at Baghdād and Damascus in the ninth century AD, and observatories in Cairo and northwestern Persia followed. The Moors brought the astronomical knowledge of the Arabs into Spain, and the *Toledan Tables* of 1080 resulted from an awakening of scientific interest that brought about the establishment of schools of astronomy at Cordova and Toledo in the tenth century.



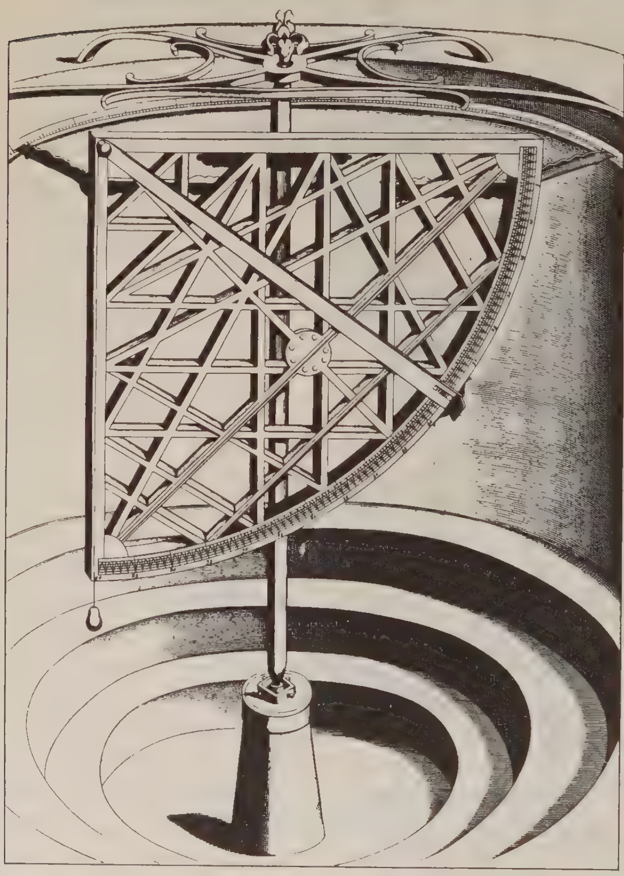
Courtesy of the Map Division of the Library of Congress.

FIGURE 129a.—An armillary sphere, one of the most important instruments of the ancient astronomers.

The great voyages of western discovery began early in the 15th century, and chief among those who recognized the need for greater precision in navigation was Prince Henry "The Navigator" of Portugal. About 1420 he had an observatory constructed at Sagres, on the southern tip of Portugal, so that more accurate information might be available to his captains. Henry's hydrographic expeditions added to the geographical knowledge of the mariner, and he was responsible for the simplification of many navigational instruments.

The Sagres observatory was rudimentary, however, and not until 1472 was the first complete observatory built in Europe. In that year Bernard Walther, a wealthy astronomer, constructed the Nuremberg Observatory, and placed Regiomontanus in charge. Regiomontanus, born Johann Müller, contributed a wealth of astronomical data of the greatest importance to the navigator.

The observatory at Cassel, built in 1561, had a revolving dome and an instrument capable of measuring altitude and azimuth at the same time. Tycho Brahe's Uraniburgum Observatory, located on the Danish island Hveen, was opened in 1576, and the results of his observations contributed greatly to the navigator's knowledge. Prior to the discovery of the telescope, the astronomer could increase the accuracy of his observations only by using larger instruments. Brahe used a quadrant with a radius of 19 feet, with which he could measure altitudes to 0′.6, an unprecedented degree of precision at that time. He also had an instrument with which he could determine altitude and azimuth simultaneously (fig. 129b). After Brahe, Kepler made use of the observatory and his predecessor's records in determining the laws which bear his name.



Courtesy of the Map Division of the Library of Congress.

FIGURE 129b.—A reproduction of Brahe's pelorus. This instrument was used to determine altitude and azimuth simultaneously.

The **telescope,** the modern astronomer's most important tool, was invented by Hans Lippershey about 1608. Galileo heard of Lippershey's invention, and soon improved upon it. In 1610 he discovered the four great moons of Jupiter, which led to the "longitude by eclipse" method successfully used ashore for many years and experimented with at sea. With the 32-power telescope he eventually built, Galileo was able to observe clearly the motions of sun spots, by which he proved that the sun rotates on its axis. In Paris, in 1671, the French National Observatory was established.

Greenwich Royal Observatory. England had no early privately supported observatories such as those on the continent. The need for navigational advancement was ignored by Henry VIII and Elizabeth I, but in 1675 Charles II, at the urging of John Flamsteed, Jonas Moore, Le Sieur de Saint-Pierre, and Christopher Wren, established the Greenwich Royal Observatory. Charles limited construction costs to £500, and appointed Flamsteed the first Astronomer Royal, at an annual salary of £100. The equipment available in the early years of the observatory consisted of two clocks, a "sextant" of seven-foot radius, a quadrant of three-foot radius, two telescopes, and the star catalog published almost a century before by Tycho Brahe. Thirteen years passed before Flamsteed had an instrument with which he could determine his latitude accurately. In 1690 a transit instrument equipped with a telescope and vernier was invented by Romer, and he later added a vertical circle to the device. This enabled the astronomer to determine declination and right ascension at the same

time. One of these instruments was added to the equipment at Greenwich in 1721, replacing the huge quadrant previously used. The development and perfection of the chronometer in the next hundred years added further to the accuracy of observations.

Other national observatories were constructed in the years that followed; at Berlin in 1705, St. Petersburg in 1725, Palermo in 1790, Cape of Good Hope in 1820, Parramatta in New South Wales in 1822, and Sydney in 1855.

U. S. Naval Observatory.—The first observatory in the United States is said to have been built in 1831–1832 at Chapel Hill, N. C. The Depot of Charts and Instruments, established in 1830, was the agency from which the U. S. Navy Hydrographic Office and the Naval Observatory evolved 36 years later. Under Lieutenant Charles Wilkes, the second Officer-in-Charge, the Depot about 1835 installed a small transit instrument for rating chronometers. The Mallory Act of 1842 provided for the establishment of a permanent observatory, and the director was authorized to purchase all such supplies as were necessary to continue astronomical study. The observatory was completed in 1844 and the results of its first observations were published two years later. Congress established the Naval Observatory as a separate agency in 1866. In 1872 a refracting telescope with a 26-inch aperture, then the world's largest, was installed. The observatory, located at Washington, D. C., has occupied its present site since 1893.

The **Mount Wilson Observatory** of the Carnegie Institution of Washington was built in 1904–05. The observatory's 100-inch reflector telescope opened wider the view of the heavens, and enabled astronomers to study the movements of celestial bodies with greater accuracy than ever before. But a still finer tool was needed, and in 1934 the 200-inch reflector for the **Palomar Mountain Observatory** was cast. The six-million-dollar observatory was built by the Rockefeller General Education Board for the California Institute of Technology, which also operates the Mount Wilson Observatory. The 200-inch telescope makes it possible to see individual stars 20,000,000 light-years away and galaxies at least 1,600,000,000 light-years away.

As with earlier instruments, the telescope has about reached the limit of practical size. Present efforts are being directed toward application of the electron microscope to the telescope, to increase the range of present instruments.

130. Almanacs.—From the beginning, astronomers have undoubtedly recorded the results of their observations. Tables computed from such results have been known for centuries. The work of Hipparchus, in the second century BC, and Ptolemy, in his famous *Almagest,* are examples. Then the *Toledan Tables* appeared in AD 1080, and the *Alfonsine Tables* in 1252. Even with these later tables, however, few copies were made, for printing had not yet been invented, and those that were available were kept in the hands of astronomers. Not until the 15th century were the first almanacs printed and made available to the navigator. In Vienna, in 1457, George Purbach issued the first almanac. Fifteen years later the Nuremberg Observatory, under Regiomontanus, issued the first of the ephemerides it published until 1506. These tables gave the great maritime explorers of the age the most accurate information available. In 1474 Abraham Zacuto introduced his *Almanach Perpetuum* (fig. 130a) which contained tables of the sun's declination in the most useful form yet available to the mariner. *Tabulae Prutenicae,* the first tables to be calculated on Copernican principles, were published by Erasmus Reinhold in 1551 and gave the mariner a clearer picture of celestial movements than anything previously available. The work of Brahe and Kepler at the Uraniburgum Observatory provided the basis for the publication of the *Rudolphine Tables* in 1627.

Still, the information contained in these books was intended primarily for the use of the astronomer, and the navigator carried the various tables only that he might

make use of the portions applicable to his work. The first official almanac, *Connaissance des Temps,* was issued by the French National Observatory in 1696. The French Observatory rose to its greatest prominence during the 20 years that Urbain Leverrier held the position of director.

In 1767 the British *Nautical Almanac* was first published. Nevil Maskelyne was then Astronomer Royal, and he provided the navigator with the best information available. The book contained tables of the sun's declination, and corrections to the observed altitude of Polaris. The moon's position relative to other celestial bodies was included at 12-hour intervals, and lunar distance tables gave the angular distance between the moon and certain other bodies at three-hour intervals.

For almost a hundred years the British *Nautical Almanac* was the one used by American navigators, but in 1852 the Depot of Charts and Instruments published the first *American Ephemeris and Nautical Almanac,* for the year 1855.

Early American almanacs were distinguished by their excessive detail in some cases and shortage of data of importance to the navigator in others. Declination was given to the nearest 0".1 and the equation of time to the nearest 0^s.01. Most figures were given only for noon at Greenwich, and a tedious interpolation was involved in converting the information to that at a given time at the longitude of the observer. Lunar distances were given at three-hour intervals. Few star data were listed (fig. 130b).


KL Março tem dias .xxxi. ha lũa .xxx. ho. dia .xij. hõ e no .xij. | Di: Lugar. sol Declinaçã: sol.

		Domes	Pisces	Graã	Minu.
d	Adrian martyr e simplici	1	20	3	59
e	Donado martyr e basilci	2	21	3	35
f	Martinino martyr pêça	3	12	3	11
g	Lucio papa e martyre	4	23	2	48
A	Sam focas martyre.	5	24	2	24
b	Victor e victorino marti	6	25	2	0
c	Thome de aquino confessor	7	26	1	36
d	Appollonio martyre.	8	72	1	12
e	Os quarenta martyres.	9	28	0	48
f	Alexandre e gaio martyr	10	29	0	24
g	Quarenta mil martyres	11	1 ♈	0	0
A	Sam gregorio papa e co	12	2	0	24
b	Sam leandre bispo bispo	13	3	0	48
c	Leo papa e martyr e do	14	4	1	12
d	longinus martyr.	15	5	1	36
e	Gertrudis virgem e mart	16	6	2	0
f	Patricio bispo e confesso	17	6	2	24
g	Anselino bispo e doutor	18	7	2	48
A	Joseph d nossa senhora	19	8	3	11
b	Gutbberto confessor	20	9	3	35
c	Bento abade fundador	21	10	3	59
d	Paulo bispo e confessor	22	11	4	22
e	Pigmete preste e martyr	23	12	4	46
f	Theodoro psbi Dia d Jaã	24	13	5	9
g	Anunciaçã de nossa senhora	25	14	5	33
A	Sam castorio martyre	26	15	5	56
b	Joam yrmitam confessor	27	16	6	19
c	Marcello papa e gronoan	28	17	6	43
d	Quirino martyr eustalio e	29	18	7	6
e	Sam segundo martyre	30	19	7	29
f	Felice pappa e babina vi	31	20	7	51

Courtesy of the John Carter Brown Library, Brown University.

FIGURE 130a.—An excerpt from the Portuguese *Regimento do estrolabio e do quadrante* of about 1509, giving the sun's declination and other data based upon Zacuto's calculations for the month of March. The first day of spring, the 11th by the Julian calendar then in use, is marked by the symbol of Aries, the ram (♈).

Since 1858 the *American Nautical Almanac* has been printed without the ephemeris section, that part of value chiefly to astronomers. Until 1908 the positions of the brighter stars were given only for January 1st, and in relation to the meridian of Washington. Beginning in that year, the apparent places of 55 major stars were given for the first of each month. In 1913, the tables of lunar distances were omitted. In 1919, sunrise and sunset tables were added.

One of the greatest inconveniences involved in using the old almanacs was the astronomical day, which began at noon of the civil day of the same date. This system was abolished in 1925, and the United States adopted the expression "civil time" to designate time by the new system. Greenwich hour angle was first published for the moon in the *Lunar Ephemeris for Aviators* for the last four months of 1929. This publication became a supplement to the *Nautical Almanac* in 1931, and for 1932 the two were merged.

The *Air Almanac,* designed by Lieutenant Commander P. V. H. Weems, was published for 1933, giving Greenwich hour angle for all bodies included. For 1934

this information was given in the *Nautical Almanac*, and the *Air Almanac* was discontinued. The first British air almanac was published for the last quarter of 1937, and modified for 1939 with features followed closely in the first *American Air Almanac*, for 1941. In 1950 a revised *Nautical Almanac* appeared, patterned after the popular *American Air Almanac*. Starting with the 1953 edition, the British and American air almanacs were combined in a single publication. In that year the United States reverted to the expression "mean time" in place of "civil time." In 1958, the British and American nautical almanacs were combined, and in 1960, the name was standardized.

FIXED STARS, 1855.

MEAN PLACES OF 100 PRINCIPAL FIXED STARS, FOR JANUARY 1, 1855.

Star's Name.	Mag.	Right Ascension.	An. Variation.	Declination.	An. Variation.
		h. m. s.	s.	° ′ ″	″
α Andromedæ .	2	0 0 53.97	+ 3.087	+28 17 23.3	+19.93
γ Pegasi (*Algenib*)	3.2	0 5 46.37	3.085	+14 22 38.1	20.05
β Hydri . . .	3	0 18 3.62	3.292	−78 4 23.1	20.23
α Cassiopeæ .	var.	0 32 18.36	3.356	+55 44 29.2	19.83
β Ceti . . .	2	0 36 18.45	3.016	−18 47 0.1	19.86
α Urs. Min. (*Polaris*)	2	1 6 29.82	+18.117	+88 32 11.3	+19.23
θ[1] Ceti . . .	3	1 16 46.57	3.000	− 8 55 58.6	18.74
α Eridani (*Achernar*)	1	1 32 18.42	2.238	−57 58 28.2	18.59
α Arietis . . .	2	1 59 0.44	3.365	+22 46 28.4	17.29
γ Ceti . . .	3.4	2 35 47.42	3.102	2 37 19.4	15.44
α Ceti . . .	2.3	2 54 42.21	+ 3.129	+ 3 31 4.7	+14.40
α Persei . .	2	3 13 59.52	4.243	49 20 26.8	13.25
η Tauri . . .	3	3 38 52.31	3.553	+23 39 11.0	11.54
γ[1] Eridani . .	3	3 51 15.91	2.796	−13 55 26.7	10.59
α Tauri (*Aldebaran*)	1	4 27 36.26	3.436	+16 12 49.4	7.72
α Aurigæ (*Capella*) .	1	5 5 59.03	+ 4.423	+45 50 41.8	+ 4.27
β Orionis (*Rigel*)	1	5 7 34.23	2.884	− 8 22 22.5	4.54
β Tauri . .	2	5 17 7.72	3.791	+28 28 48.3	3.55
δ Orionis . .	2	5 24 36.06	3.066	− 0 24 37.8	3.05
α Leporis . .	3	5 26 20.19	2.648	−17 55 46.0	2.94
ε Orionis . .	2	5 28 51.43	+ 3.044	− 1 17 54.6	+ 2.71
α Columbæ . .	2	5 34 24.05	2.177	−34 9 13.3	2.23
α Orionis . .	var.	5 47 19.35	3.249	+ 7 22 32.6	+ 1.11
μ Geminorum .	3	6 14 11.30	3.636	+22 34 59.9	− 1.37
α Argus (*Canopus*) .	1	6 20 44.13	1.330	−52 37 4.7	− 1.81
51 (Hev.) Cephei .	5	6 31 6.10	+30.650	+87 15 7.9	− 2.80
α Canis Maj. (*Sirius*)	1	6 38 45.60	2.646	−16 31 12.8	4.52
ε Canis Majoris .	2.1	6 52 55.69	2.360	−28 46 40.3	4.58
δ Geminorum .	3.4	7 11 27.65	3.597	+22 14 41.7	6.16
α[2] Geminor. (*Castor*)	2.1	7 25 20.49	3.841	32 12 6.2	7.37
α Can. Min. (*Procyon*)	1	7 31 42.52	+ 3.145	+ 5 35 35.7	− 8.79
β Geminor. (*Pollux*)	1.2	7 36 26.23	3.681	+28 22 19.9	8.26
15 Argus . . .	3	8 1 22.22	2.557	−23 53 20.5	10.06
ε Hydræ . .	3.4	8 39 5.74	3.189	+ 6 56 52.2	12.86
ι Ursæ Majoris .	3	8 49 15.44	4.123	+48 36 26.7	13.78
ι Argus . .	2	9 13 12.52	+ 1.602	−58 40 3.3	−14.91
α Hydræ . . .	2	9 20 27.65	2.951	− 8 1 56.8	15.36
θ Ursæ Majoris .	3	9 23 7.85	4.048	+52 20 6.3	16.13
ε Leonis . . .	3	9 37 36.82	3.424	24 26 22.0	16.34
α Leonis (*Regulus*)	1.2	10 0 38.72	3.205	+12 40 26.4	17.40
η Argus . .	2	10 39 26.75	+ 2.306	−58 55 21.5	−18.74

FIGURE 130b.—Star data from the 1855 *Nautical Almanac*. The annual corrections in declination and right ascension can be used to obtain reasonably correct values today.

131. The navigational triangle.—It is customary for modern navigators to reduce their celestial observations by solving the triangle whose points are the elevated pole, the celestial body, and the zenith of the observer. The sides of this triangle are the polar distance of the body (codeclination), its zenith distance (coaltitude), and the polar distance of the zenith (colatitude of the observer).

Lunar distances. A spherical triangle was first used at sea in solving lunar distance problems. Simultaneous or nearly simultaneous observations were made of the altitudes of the moon and the sun or a star near the ecliptic, and the angular distance between the moon and the other body. The zenith of the observer and the two celestial bodies formed the vertices of the triangle, whose sides were the two coaltitudes and the angular distance between the bodies. By means of a mathematical calculation the navigator "cleared" this distance of the effects of refraction and parallax applicable to each altitude, and other errors. The corrected value was then used as an argument for entering the almanac, which gave the true lunar distance from the sun and several stars at three-hour intervals.

Previously, the navigator had set his watch or checked its error and rate, which could be relied upon for short periods, with the local mean time determined by celestial observations. The local mean time of the watch, properly corrected, applied to the Greenwich mean time obtained from the lunar distance observation, gave the longitude.

The mathematics involved was tedious, and few mariners were capable of solving the triangle until Nathaniel Bowditch published his simplified method in 1802 in *The New American Practical Navigator*. Chronometers were reliable by that time, but their high cost prevented their general use aboard the majority of naval and merchant ships. Using Bowditch's method, however, most navigators, for the first time, could determine their longitude, and so eliminate the need for parallel sailing and the lost time associated with it. The popularity of the lunar distance method is indicated by the fact that tables for its solution were carried in the *American Nautical Almanac* until the second decade of the 20th century.

The determination of latitude was considered a separate problem, usually solved by means of a meridian altitude or an observation of Polaris.

The time sight. The theory of the time sight (art. 2106) had been known to mathematicians since the dawn of spherical trigonometry, but not until the chronometer was developed could it be used by mariners.

The time sight made use of the modern navigational triangle. The codeclination, or polar distance, of the body could be determined from the almanac. The zenith distance (coaltitude) was determined by observation. If the colatitude were known, three sides of the triangle were available. From these the meridian angle was computed. The comparison of this with the Greenwich hour angle from the almanac yielded the longitude.

The time sight was mathematically sound, but the navigator was not always aware that the longitude determined was only as accurate as the latitude, and together they merely formed a point on what is known today as a line of position. If the observed body was on the prime vertical, the line of position ran north and south and a small error in latitude generally had little effect on the longitude. But when the body was close to the meridian, a small error in latitude produced a large error in longitude.

The line of position by celestial observation (art. 1703) was unknown until discovered in 1837 by 30-year-old Captain Thomas H. Sumner, a Harvard graduate and son of a United States Congressman from Massachusetts. The discovery of the "Sumner line," as it is sometimes called, was considered by Maury "the commencement of a new era in practical navigation." In Sumner's own words, the discovery took place in this manner:

"Having sailed from Charleston, S. C., 25th November, 1837, bound to Greenock, a series of heavy gales from the Westward promised a quick passage; after passing the Azores, the wind prevailed from the Southward, with thick weather; after passing Longitude 21° W., no observation was had until near the land; but soundings were had not far, as was supposed, from the edge of the Bank. The weather was now more boisterous, and very thick; and the wind still Southerly; arriving about midnight, 17th December, within 40 miles, by dead reckoning, of Tusker light; the wind hauled S. E., true, making the Irish coast a lee shore; the ship was then kept close to the wind, and several tacks made to preserve her position as nearly as possible until daylight; when nothing being in sight, she was kept on E. N. E. under short sail, with heavy gales; at about 10 A. M. an altitude of the sun was observed, and the Chronometer time noted; but, having run so far without any observation, it was plain the Latitude by dead reckoning was liable to error, and could not be entirely relied on.

"Using, however, this Latitude, in finding the Longitude by Chronometer, it was found to put the ship 15′ of Longitude, E. from her position by dead reckoning; which in Latitude 52° N. is 9 nautical miles; this seemed to agree tolerably well with the dead reckoning; but feeling doubtful of the Latitude, the observation was tried with a Latitude 10′ further N., finding this placed the ship E. N. E. 27 *nautical* miles, of the former position, it was tried again with a Latitude 20′ N. of the dead reckoning; this

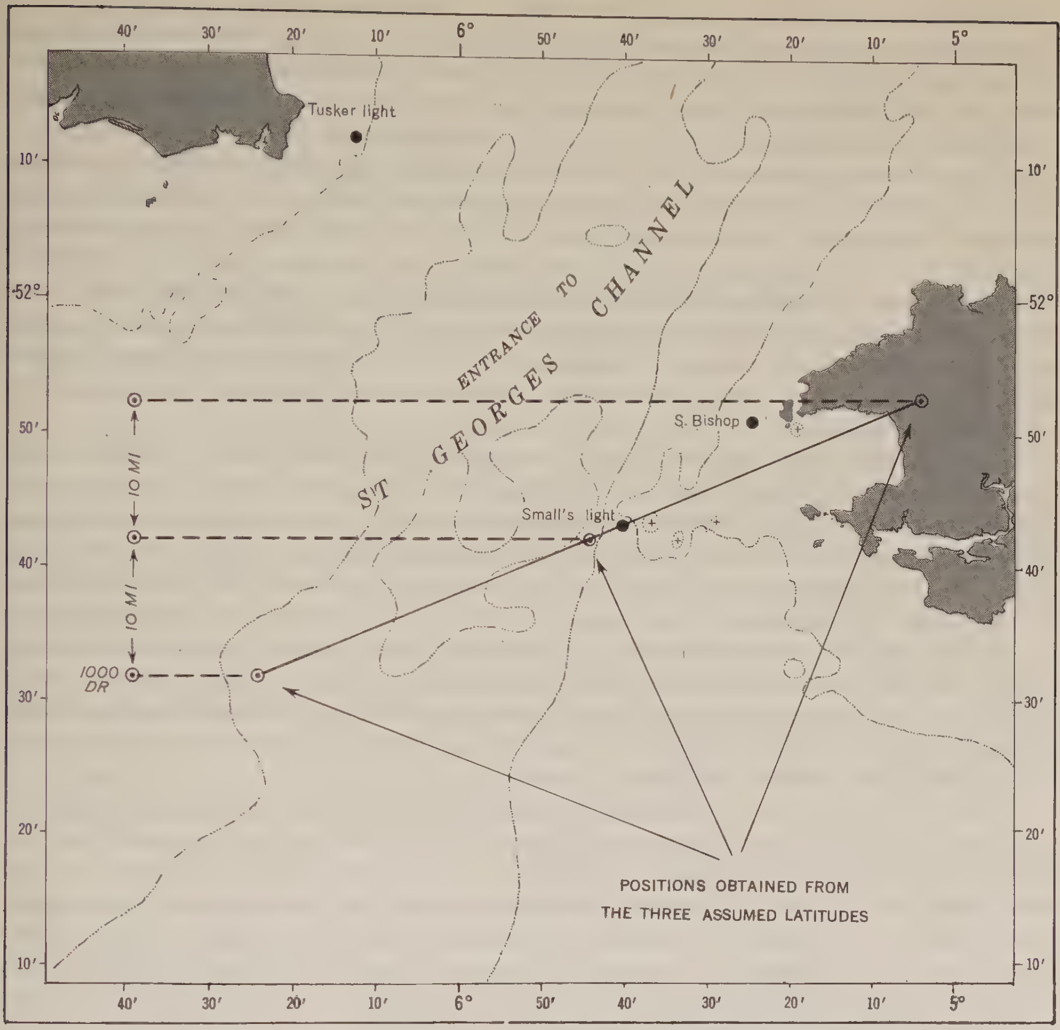


FIGURE 131.—The first celestial line of position, obtained by Captain Thomas Sumner in 1837.

also placed the ship still further E. N. E., and still 27 *nautical miles* further; these three positions were then seen to lie in the direction of *Small's light.* It then at once appeared, that the observed altitude must have happened at *all the three* points, and at *Small's light,* and at the ship, at the *same instant of time*; and it followed, that Small's light must bear E. N. E., if the Chronometer was right. Having been convinced of this truth, the ship was kept on her course, E. N. E., the wind being still S. E., and in less than an hour, Small's light was made bearing E. N. E. ½ E., and close aboard."

In 1843 Sumner published his book, *A New and Accurate Method of Finding a Ship's Position at Sea by Projection on Mercator's Chart,* which met with great acclaim. In it he proposed that a single time sight be solved twice, as he had done (fig. 131), using latitudes somewhat greater and somewhat less than that arrived at by dead reckoning, and joining the two positions obtained to form the line of position. It is significant that Sumner was able to introduce this revolutionary principle without seriously upsetting the method by which mariners had been navigating for years. Perhaps he realized that a better method could be derived, but almost certainly navigators would not have accepted the line of position so readily had he recommended that they abandon altogether the familiar time sight.

The Sumner method required the solution of two time sights to obtain each line of position. Many older navigators preferred not to draw the lines on their charts, but to fix their position mathematically by a method which Sumner had also devised and included in his book. This was a tedious procedure, but a popular one. Lecky recommended the method, and it was still in use early in the 20th century.

The alternative to working two time sights in the Sumner method was to determine the azimuth of the body and to draw a line perpendicular to it through the point obtained by working a single time sight. Several decades after the appearance of Sumner's book, this method was made available to navigators through the publication of accurate azimuth tables, and the system was widely used until comparatively recent times. The 1943 edition of the *American Practical Navigator* included examples of its use. The two-minute azimuth tables still found on many ships were designed principally for this purpose. The mathematical solution for azimuth was not at first a part of the time sight.

132. Modern methods of celestial navigation.—Sumner gave the mariner the line of position; St.-Hilaire the altitude difference or intercept method. Others who followed these men applied their principles to provide the navigator with rapid means for determining his position. The new navigational methods developed by these men, although based upon work done earlier, are largely a product of the 20th century.

Four hundred years ago Pedro Nunes used a globe to obtain a fix by two altitudes of the sun, and the azimuth angles. Fifty years later Robert Hues determined his latitude on a globe by using two observations and the time interval between them. G. W. Littlehales, of the U. S. Navy Hydrographic Office, advocated using a stereographic projection to obtain computed altitude and azimuth in his *Altitude, Azimuth, and Geographical Position*, published in 1906.

Various graphic and mechanical methods have also been proposed. Of these, only one, the *Star Altitude Curves* of Captain P. V. H. Weems, USN (Ret.), has had wide usage, almost entirely among aviators. During World War II, some aircraft were fitted with a device called an "astrograph," which projected star altitude curves from film upon a special plotting sheet. The curves could be moved to allow for the earth's rotation. When they were properly oriented, part of the line of position could be traced on the plotting sheet. More generally, however, the navigational triangle has been solved mathematically or by the use of tables.

Spherical trigonometry is the basis for solving every navigational triangle, and until about 80 years ago the navigator had no choice but to completely solve each triangle himself. The cosine formula is a fundamental spherical trigonometry formula by which the navigational triangle can be conveniently solved. This formula was commonly used in lunar distance solutions when they were first introduced, but, because ambiguous results are obtained when the azimuth is close to 90° or 270°, mathematicians turned to the haversine, which has the advantage of increasing numerically from 0° to 180°. The **cosine-haversine formula** (art. 2109) was used by navigators until recent years.

Toward the end of the 19th century the "short" methods began to appear. About 1875, A. C. Johnson of the British Royal Navy published his book *On Finding the Latitude and Longitude in Cloudy Weather*. No plotting was involved in Johnson's method, but he made use of the principle that a single time sight be worked, rather

than the two that Sumner proposed, and the line of position drawn through the point thus determined.

In 1879 Percy L. H. Davis, of the British Nautical Almanac Office, and Captain J. E. Davis collaborated on a *Sun's True Bearing or Azimuth Table*, which enabled the navigator to lay down a line of position using a computed azimuth. *Chronometer Tables*, published by Percy Davis 20 years later, covered latitudes up to 50° and gave local hour angle values for selected altitudes to one minute of arc. In 1905 his *Requisite Tables* were issued, enabling the mariner to "solve spherical triangles with three variable errors."

These were the first of a large number of "short" solutions which followed the work of Marcq St.-Hilaire. Generally, they consist of adaptations of the formulas of spherical trigonometry, and tables of logarithms in a convenient arrangement. It is customary for such methods to divide the navigational triangle into two right spherical triangles by dropping a perpendicular from one vertex to the side opposite. In some methods, partial solutions are made and the results tabulated. Aquino and Braga of Brazil; Ball, Comrie, Davis, and Smart of England; Bertin, Hugon, and Souillagouet of France; Fuss of Germany; Ogura and Yonemura of Japan; Blackburne of New Zealand; Pinto of Portugal; Garcia of Spain; and Ageton, Driesonstok, Gingrich, Rust, and Weems of the United States are but a few of those providing such solutions. Although "inspection tables" have largely superseded them, many of these "short" methods are still in use, kept alive largely by the compactness of their tables and the universality of their application. They are an intermediate step between the tedious earlier solutions and the fast tabulated ones, and they encouraged the navigator to work to a practical precision. The earlier custom of working to a precision not justified by the accuracy of the information used created a false sense of security in the mind of some navigators, especially those of little experience.

A book of tabulated solutions, from which an answer can be extracted by inspection, is not a new idea. Lord Kelvin, generally considered the father of modern navigational methods, expressed interest in such a method. However, solution of the many thousands of triangles involved would have made the project too costly if done by hand. Electronic computers have provided a practical means of preparing tables. In 1936 the first published volume of H.O. Pub. No. 214 was made available, and later H.O. Pub. No. 249 was provided for air navigators. British Admiralty editions of both these sets of tables have been published. Editions of H.O. Pub. No. 214 have also been published by the Instituto Hidrographico de la Marina, Cadiz, Spain, and by the Istituto Idrografico della Marina, Genova, Italy.

Electronic Navigation

133. Electricity.—Twenty-five hundred years ago Thales of Miletus commented on basic electrical phenomena, but more than two millenniums passed before men first approached an understanding of electricity and the uses to which it could be put.

Until about 1682 the only known method of creating electricity was by rubbing glass with silk or amber with wool. Then Otto von Guericke of Magdeburg invented an "electric machine" and made possible the creation of electricity for experimental work. The Leyden jar, the electrical condenser (or machine) commonly used today, had its origin in 1745 when its principle was accidentally discovered independently by P. van Musschenbroek, of the University of Leyden, and von Kleist.

Stephen Gray, about 1729, demonstrated the difference between conductors and non-conductors, or insulators, and ten years later Hawkesbee and DuFay, working independently, each discovered the positive and negative qualities of electricity.

In the middle of the 18th century Sir William Watson of England, developer of the Leyden jar in essentially its present form, sent electricity more than two miles by wire. Whether Watson was aware of the tremendous possibilities his experiment demonstrated is not known. Twenty-five years later, about 1774, Lesage devised what is believed to have been the first method of electrical communication. He had a separate wire for each letter of the alphabet and momentarily charged the appropriate wire to send each letter.

A German scholar, Francis Aepinus (1728–1802), was the first to recognize the reciprocal relationship of electricity and magnetism. In 1837 Karl Gauss and Wilhelm Weber collaborated in inventing a reflecting galvanometer for use in telegraphic work, which was the forerunner of the galvanometer at one time employed in submarine signaling. Michael Faraday (1791–1867), in a lifetime of experimental work, contributed most of what is known today in the field of electromagnetic induction. In 1864 James Clerk Maxwell of Edinburgh made public his electromagnetic theory of light. Many consider it the greatest single advancement in man's knowledge of electricity.

134. Electronics.—In 1887 Heinrich Hertz provided the proof of Maxwell's theory by producing electromagnetic waves and showing that they could be reflected. A decade later Joseph J. Thomson discovered the electron and so provided the basis for the development of the vacuum tube by Fleming and DeForest. In 1899 R. A. Fessenden pointed out that directional reception of radio signals was possible if a single coil or frame aerial was used as the receiving antenna. In 1895 Guglielmo Marconi transmitted a "wireless" message a distance of about one mile. By 1901 he was able to communicate between stations more than 2,000 miles apart. The following year Arthur Edwin Kennelly and Oliver Heaviside introduced the theory of an ionized layer in the atmosphere and its ability to reflect radio waves. Pulse ranging had its origin in 1925 when Gregory Breit and Merle A. Tuve used this principle to measure the height of the ionosphere.

135. Application of electronics to navigation.—Perhaps the first application of electronics to navigation was the transmission of radio **time signals** (art. 1909) in 1904, thus permitting the mariner to check his chronometer at sea. Telegraphic time signals had been sent since 1865, providing a means of checking the chronometer in various ports.

Next, radio broadcasts providing navigational warnings, begun in 1907 by the U. S. Navy Hydrographic Office, helped increase the safety of navigation at sea.

By the latter part of World War I the directional properties of a loop antenna were successfully utilized in the **radio direction finder** (art. 1202). The first radiobeacon was installed in 1921.

Early 20th century experiments by Behm and Langevin led to the development, by the U. S. Navy, of the first practical **echo sounder** (art. 619) in 1922.

As early as 1904, Christian Hulsmeyer, a German engineer, obtained patents in several countries on a proposed method of utilizing the reflection of radio waves as an obstacle detector and a navigational aid to ships. Apparently, the device was never constructed. In 1922 Marconi said, "It seems to me that it should be possible to design apparatus by means of which a ship could radiate or project a divergent beam of these rays (electromagnetic waves) in any desired direction, which rays if coming across a metallic object, such as another ship, would be reflected back to a receiver screened from the local transmitter on the sending ship, and thereby immediately reveal the presence and bearing of the other ship in fog or thick weather."

In that same year of 1922 two scientists, Dr. A. Hoyt Taylor and Leo C. Young, testing a communication system at the Naval Aircraft Radio Laboratory at

Anacostia, D. C., noted fluctuations in the signals when ships passed between stations on opposite sides of the Potomac River. Although the potential value of the discovery was recognized, work on its exploitation did not begin until March 1934, when Young suggested to Dr. Robert M. Page, an assistant, that this might bear further investigation. By December, Page had constructed a pulse-signal device that determined the positions of aircraft. This was the first **radar** (art. 1208). In the spring of 1935 the British, unaware of American efforts, began work in this field, and developed radar independently. In 1937 the USS *Leary* tested the first seagoing radar. In 1940 United States and British scientists combined their efforts, resulting in more rapid progress. Probably no scientific or industrial development in history expanded so rapidly in all phases—research, development, design, production, trials, and training—and on such a scale. In 1945, at the close of hostilities of World War II, radar was made available for commercial use.

Meanwhile, the pulse technique upon which radar is based was utilized for other navigational aids. Work on **loran** (art. 1302) began at the Radiation Laboratory at the Massachusetts Institute of Technology in 1941. By the end of 1942 the first stations had been established, in the North Atlantic. Installations in the Aleutians and the South Pacific soon followed. With the termination of hostilities, loran, like radar, was made available for public use. A somewhat similar system, **gee** (art. 1308), was developed simultaneously in Great Britain. Another pulse system, **shoran** (art. 1213), was developed by the United States for bombing through undercast. Following World War II this aid was further perfected and used for measurement of distances in surveying. A lower frequency, longer range system called **electronic position indicator** (**EPI**) (art. 1213) was developed by the U. S. Coast and Geodetic Survey for use in locating survey ships a considerable distance offshore. Another American development, **Raydist** (arts. 1214, 1311), is used in accurate measurement of distance for surveying and for ship speed trials. Raydist; **Decca** (art. 1309), a British hyperbolic system of high accuracy used for navigation and surveying; and **lorac** (art. 1310), a somewhat similar American system, use continuous waves, rather than pulses. Not only are such devices improving the accuracy of charted features, but they may well apply directly to geodesy, permitting a more accurate determination of the size and shape of the earth, for they make possible measurement of distances across previously inaccessible terrain.

A rotating electronic beam was utilized during World War II in the German navigation system called **sonne** (art. 1206), later further perfected by the British under the name **consol** (art. 1206).

In air navigation electronics was used to develop an automatic direction finder. Four-course **radio ranges** (art. 1207) and the more recent **vortac** (art. 1207) have been used to mark the federal airways. Electronics has various applications to traffic control in congested areas, and in low-visibility approach systems permitting landings under conditions of reduced horizontal and vertical visibility.

Electronics permits measurement of weather conditions at various heights and distances from observing stations, and the transmission of observations from isolated stations to weather centrals. Radar is permitting study of the structure and movement of thunderstorms.

High-speed electronic computers make practicable the modern inspection table, and rapidly perform lengthy computations which make it possible for loran tables and charts to become available to the navigator almost as soon as new stations are operational.

The application of electronics to navigation is almost limitless. Many systems not mentioned have been suggested, and undoubtedly new ones will be operational in the future.

Conclusion

136. Navigation has come a long way, but there is no evidence that it is nearing the end of its development. Progress will continue as long as man remains unsatisfied with the means at his disposal.

Perhaps the best guides to the future are the desires of the present, for a want usually precedes an acquisition. Pytheas and his contemporaries undoubtedly dreamed of devices to indicate direction and distance. The 16th century navigator had these, and wanted a method of determining longitude at sea. The 18th century navigator could determine longitude, but found the task a tedious one, and perhaps longed to be freed from the *drudgery* of navigation. The modern navigator is still seeking further release from the *work* of navigation, and now wants to be freed from the limitations of weather.

There is little probability of further major development in the simplification of tables for celestial navigation. Further release from the work of navigation is more likely to come through another approach—*automation*.

This process might be said to have started with the application of electronics to computation. The direct use of electronics in navigation is more spectacular, but in this it is vulnerable to jamming by an unfriendly power, intentional or accidental mechanical damage, natural failure, propagation limitations in certain areas and at certain times, and accuracy limitations at long ranges.

In the future, it is likely that electronics will be applied increasingly as an additional source of energy to extend the range of usefulness of other methods, rather than to replace them. To date electronics has been related primarily to piloting, extending its range far to sea, and permitting its use in periods of foul weather. In the future it can be expected to play an increasingly important role in the field of dead reckoning and celestial navigation. **Inertial** and **Doppler** systems (art. 809) are under development for use in guided missiles and aircraft, and a **geomagnetic electrokinetograph (GEK)** (art. 611) has been developed to measure the cross component of a current by means of two electrodes towed astern a vessel, utilizing the earth's magnetic field. **Radio astronomy** (art. 1102) may provide a practical means of determining position astronomically through overcast. Star trackers and electronic recorders and computers may further extend the application of electronics to celestial navigation.

It is not inconceivable that a fix may someday be automatically and continuously available, perhaps on latitude and longitude dials. However, when this is accomplished, by one or a combination of systems, it will be but a short additional step to feed this information electronically to a pen which will automatically trace the path of the vessel across a chart. Another short step would be to feed the information electrically to a device to control the movements of the vessel, so that it would automatically follow a predetermined track.

When this has been accomplished, new problems will undoubtedly arise, for it is not likely that the time will ever come when there will be no problems to be solved.

137. The navigator.—It might seem that when complete automation has been achieved, all of the work of the navigator will have been eliminated. However, advance planning of route and schedule will undoubtedly require human intelligence. So will the interpretation of results en route, and the alteration of schedule when circumstances render this desirable. Unless the automatic system can be made 100 percent reliable—a remote prospect for the foreseeable future—it will need checking from time to time, and provision will have to be made for other, perhaps cruder, methods in the event of failure.

Until such time as mechanization may become complete and perfect, the prudent navigator will not permit himself to become wholly dependent upon "black boxes" which may fail at crucial moments, or ready-made solutions that may not be available when most needed. Today and in the future, as in the past, a knowledge of fundamental principles is essential to adequate navigation. If the navigator contents himself with the ability to read dials or look up answers in a book, he will be of questionable value. His future, if he has one, will be in jeopardy.

Human beings who entrust their lives to the skill and knowledge of a navigator are entitled to expect him to be capable of handling any reasonable emergency. When his customary tools or methods are denied him, they have a right to expect him to have the necessary ability to take them safely to their destination, however elementary the knowledge and means available to him.

The wise navigator uses all reliable aids available to him, and seeks to understand their uses and limitations. He learns to evaluate his various aids when he has means for checking their accuracy and reliability, so that he can adequately interpret their indications when his resources are limited. He stores in his mind the fundamental knowledge that may be needed in an emergency. Machines may reflect much of the *science* of navigation, but only a competent human can practice the *art* of navigation.

References

Collinder, Per. *A History of Marine Navigation.* Tr. Maurice Michael. New York, St. Martin's, 1955.

Hewson, J. B. *A History of the Practice of Navigation.* Glasgow, Brown, 1951.

Petze, C. L., Jr. *The Evolution of Celestial Navigation.* Vol. 26, Ideal Series. New York, Motor Boating, 1948.

Stewart, J. Q., and Pierce, N. L. "The History of Navigation." *Marine and Air Navigation* (Boston, Ginn, 1944). Chap. 29.

Taylor, E. G. R. *The Mathematical Practitioners of Tudor and Stuart England.* London, Cambridge University Press, 1955.

Waters, D. W. *The Art of Navigation in England in Elizabethan and Early Stuart Times.* New Haven, Yale University Press, 1958.

Wroth, L. C. *Some American Contributions to the Art of Navigation, 1519–1802.* Providence, John Carter Brown Library, 1947.

In addition, articles pertaining to the history of navigation are frequently carried in certain periodicals, including:

"The American Neptune." (Salem)
"The Journal of the Institute of Navigation." (London)
"The Nautical Magazine." (Glasgow)
"Navigation, Journal of the Institute of Navigation." (Los Angeles)
"Navigation, Revue Technique de Navigation Maritime et Aérienne." (Paris)
"United States Naval Institute Proceedings." (Annapolis)

CHAPTER II

BASIC DEFINITIONS

201. Navigation is the process of directing the movements of a craft from one point to another. The word *navigate* is from the Latin *navigatus*, the past participle of the verb *navigere*, which is derived from the words *navis*, meaning "ship," and *agere*, meaning "to move" or "to direct." Navigation of water craft is called **marine navigation** to distinguish it from navigation of aircraft, called **air navigation.** Navigation of a vessel on the surface is sometimes called **surface navigation** to distinguish it from **underwater navigation** of a submerged vessel. The expression **submarine navigation** is applicable to a submarine, whether submerged or on the surface. Navigation of vehicles across land or ice is called **land navigation.** The expression **lifeboat navigation** is used to refer to navigation of lifeboats or life rafts, generally involving rather crude methods. The expression **polar navigation** refers to navigation in the regions near the geographical poles of the earth, where special techniques are employed.

The principal divisions of navigation are as follows:

Dead reckoning is the determination of position by advancing a known position for courses and distances. A position so determined is called a **dead reckoning position.** It is generally accepted that the course *steered* and the speed *through the water* should be used, but the expression is also used to refer to the determination of position by use of the course and speed expected to be made good over the ground, thus making an estimated allowance for disturbing elements such as current and wind. A position so determined is better called an **estimated position.** The expression "dead reckoning" probably originated from use of the Dutchman's log, a buoyant object thrown overboard, to determine the speed of the vessel relative to the object, which was assumed to be *dead* in the water. Apparently, the expression **deduced reckoning** was used when allowance was made for current and wind. It was often shortened to *ded reckoning* and the similarity of this expression to *dead reckoning* was undoubtedly the source of the confusion that is still associated with these expressions.

Piloting (or **pilotage**) is navigation involving frequent or continuous determination of position or a line of position relative to geographic points, to a high order of accuracy. It is practiced in the vicinity of land, dangers, aids to navigation, etc., and requires good judgment and almost constant attention and alertness on the part of the navigator.

Electronic navigation involves the use of electronic equipment in any way. It may be called **radio navigation** if any form of radio is used. **Sonic navigation,** involving the use of sound waves, becomes part of electronic navigation when electronic equipment is used in the control, production, transmission, reception, or amplification of sound signals. Electronic navigation overlaps piloting, and as electronic equipment is developed to make celestial observations (art. 1102), it becomes intimately associated with celestial navigation.

Celestial navigation is navigation using information obtained from celestial bodies.

202. The earth is approximately an **oblate spheroid** (a sphere flattened at the poles). Its dimensions and the amount of flattening are not known exactly, but the values determined by the English geodesist A. R. Clarke in 1866, as defined by the

U. S. Coast and Geodetic Survey in 1880, are used for charts of North America. According to these dimensions the longer or equatorial radius, a, is 3,443.96 nautical, or 3,963.23 statute, miles and the shorter or polar radius, b, is 3,432.28 nautical, or 3,949.80 statute, miles. The mean radius $\left(\frac{2a+b}{3}\right)$ is 3,440.07 nautical, or 3,958.76 statute, miles. The "oblateness" or amount of flattening is $\frac{a-b}{a}=\frac{11.68}{3443.96}=\frac{1}{295}$, or $\frac{1}{294.98}$ if a and b are computed to additional decimal places. For many navigational purposes the earth is assumed to be a sphere, without intolerable error.

The **axis of rotation** or **polar axis** of the earth is the line connecting the **north pole** and the **south pole.**

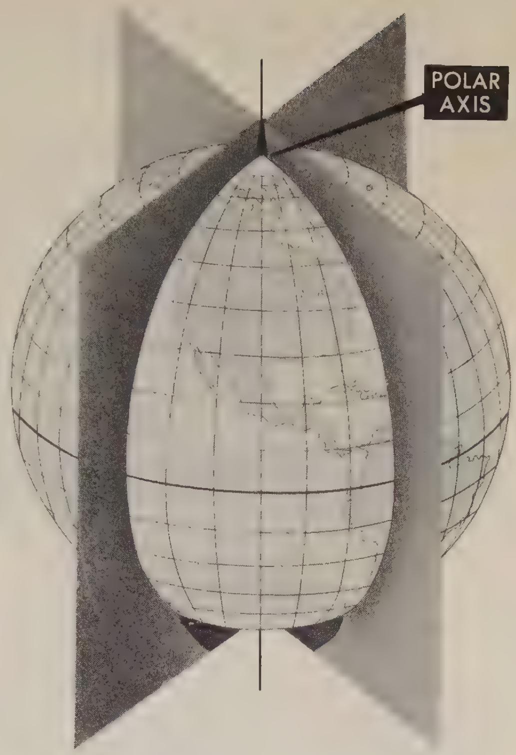


FIGURE 203a.—The planes of the meridians meet at the polar axis.

203. Circles of the earth.—A **great circle** is the line of intersection of a sphere and a plane through the center of the sphere. This is the largest circle that can be drawn on a sphere. The shortest line on the surface of a sphere between two points on that surface is part of a great circle. On the spheroidal earth the shortest line is called a **geodesic.** A great circle is a near enough approximation of a geodesic for most problems of navigation.

A **small circle** is the line of intersection of a sphere and a plane which does not pass through the center of the sphere.

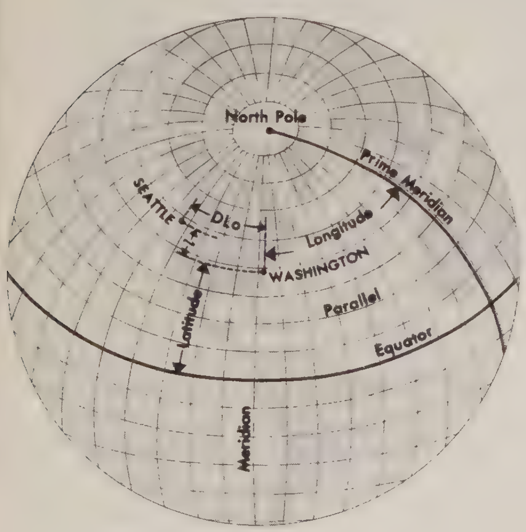


FIGURE 203b.—Circles and coordinates of the earth. All parallels except the equator are small circles; the equator and meridians are great circles.

A **meridian** is a great circle through the geographical poles of the earth. Hence, all meridians meet at the poles, and their planes intersect each other in a line, the **polar axis** (fig. 203a). The term *meridian* is usually applied to the **upper branch** only, that half from pole to pole which passes through a given point. The other half is called the **lower branch.**

The **prime meridian** is that meridian used as the origin for measurement of longitude (fig. 203b). The prime meridian used almost universally is that through the original position of the British Royal Observatory at Greenwich, near London.

The **equator** is the terrestrial great circle whose plane is perpendicular to the polar axis (fig. 203c). It is midway between the poles.

A **parallel** or **parallel of latitude** is a circle on the surface of the earth, parallel to the plane of the equator (fig. 203d). It connects all points of equal latitude. The equator, a great circle, is a limiting case connecting points of 0° latitude. The poles, single points at latitude 90°, are the other limiting case. All other parallels are small circles.

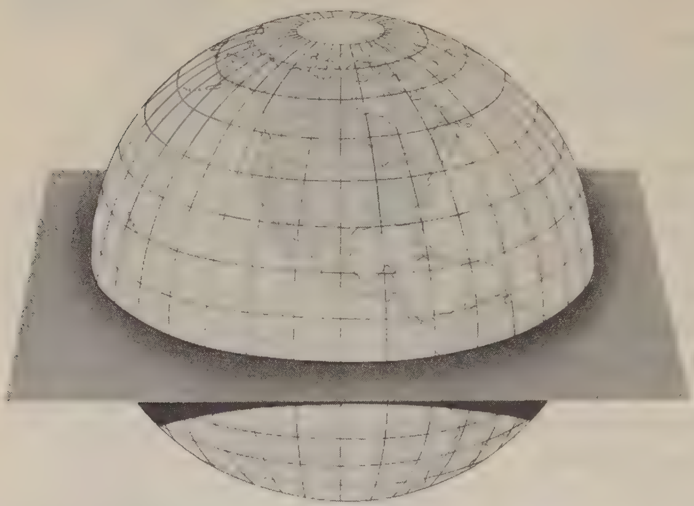


FIGURE 203c.—The equator is a great circle midway between the poles.

204. Position on the earth.—A position on the surface of the earth (except at either of the poles) may be defined by two magnitudes called **coordinates.** Those customarily used are *latitude* and *longitude.* A position may also be expressed in relation to known geographical positions.

Latitude (L, lat.) is angular distance from the equator, measured northward or southward along a meridian from 0° at the equator to 90° at the poles (fig. 203b). It is designated *north* (N) or *south* (S) to indicate the direction of measurement.

The **difference of latitude** (*l*) between two places is the angular length of arc of any meridian between their parallels (fig. 203b). It is the numerical difference of the latitudes if the places are on the same side of the equator, and the sum if they are on opposite sides. It may be designated *north* (N) or *south* (S) when appropriate.

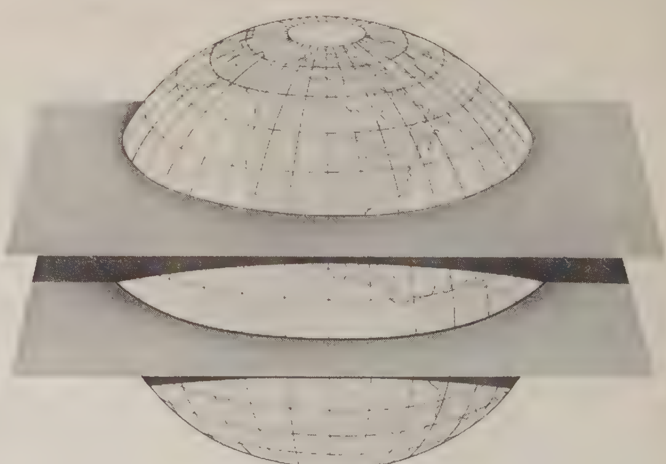


FIGURE 203d.—A parallel of latitude is parallel to the equator.

The **middle** or **mid latitude (Lm)** between two places on the same side of the equator is half the sum of their latitudes. Mid latitude is labeled N or S to indicate whether it is north or south of the equator. The expression is occasionally used with reference to two places on opposite sides of the equator, when it is equal to half the *difference* between the two latitudes, and takes the name of the place farthest from the equator. However, this usage is misleading, as it lacks the significance usually associated with the expression. When the places are on opposite sides of the equator, two mid latitudes are generally used, the average of each latitude and 0°.

Longitude (λ, **long.**) is the arc of a parallel or the angle at the pole between the prime meridian and the meridian of a point on the earth, measured eastward or westward from the prime meridian through 180° (fig. 203b). It is designated *east* (E) or *west* (W) to indicate the direction of measurement.

The **difference of longitude** (**DLo**) between two places is the shorter arc of the parallel or the smaller angle at the pole between the meridians of the two places (fig. 203b). If both places are on the same side (east or west) of Greenwich, DLo is the numerical difference of the longitudes of the two places; if on opposite sides, DLo is the numerical sum unless this exceeds 180°, when it is 360° minus the sum. The distance between two meridians at any parallel of latitude, expressed in distance units, usually nautical miles, is called **departure** (**p**). It represents the distance made good to the east or west as a craft proceeds from one point to another. Its numerical value between any two meridians decreases with increased latitude, while DLo is numerically the same at any latitude. Either DLo or p may be designated *east* (E) or *west* (W) when appropriate.

205. Distance on the earth.—**Distance** (**D, dist.**) is the spatial separation of two points, and is expressed as the length of a line joining them. On the surface of the earth it is usually stated in miles. Navigators customarily use the **nautical mile** (**mi., M**) of 1852 meters exactly. This is the value suggested by the International Hydrographic Bureau in 1929, and since adopted by most maritime nations. It is often called the **international nautical mile** to distinguish it from slightly different values used by some countries. On July 1, 1959, the United States adopted the exact relationship of 1 yard = 0.9144 meter. The length of the international nautical mile is consequently equal to 6,076.11549 U. S. feet (approximately).

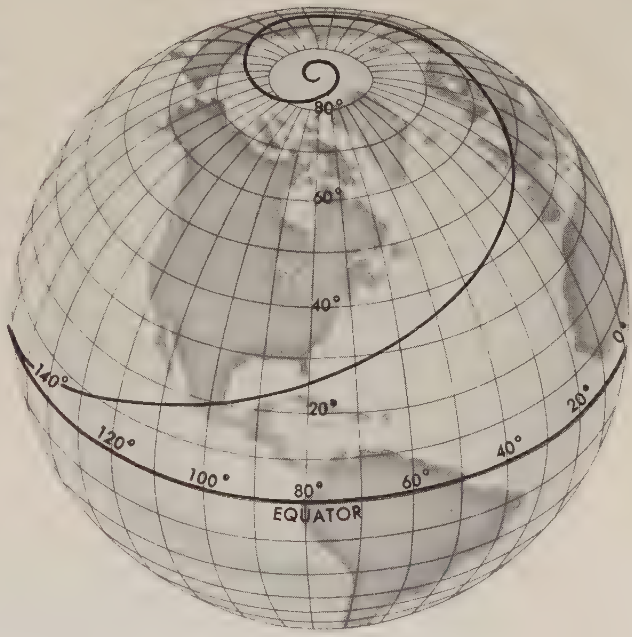


Figure 205.—A rhumb line or loxodrome.

For most navigational purposes the nautical mile is considered the length of one minute of latitude, or of any great circle of the earth, regardless of location. On the Clarke spheroid of 1866, used for mapping North America, the length of one minute of latitude varies from about 6,046 feet at the equator to approximately 6,108 feet at the poles. The length of one minute of a great circle of a sphere having an area equal to that of the earth, as represented by this spheroid, is 6,080.2 United States feet. This was the standard value of the nautical mile in the United States prior to adoption of the international value. A **geographical mile** is the length of one minute of the equator, or about 6,087 feet.

The **land** or **statute mile** (**mi., m**) of 5,280 feet is commonly used for navigation on rivers and lakes, notably the Great Lakes of North America.

The nautical mile is about 38/33 or approximately 1.15 statute miles. A conversion table for nautical and statute miles is given in table 20.

Distance, as customarily used by the navigator, refers to the length of the **rhumb line** connecting two places. This is a line making the same oblique angle with all meridians. Meridians and parallels (including the equator) which also maintain constant true directions, may be considered special cases of the rhumb line. Any other rhumb line spirals toward the pole, forming a **loxodromic curve** or **loxodrome** (fig. 205). Distance along the great circle connecting two points is customarily designated **great-circle distance.**

206. Speed (S) is rate of motion, or distance per unit of time.

A knot (**kn.**), the unit of speed commonly used in navigation, is a rate of one nautical mile per hour. The expression "knots per hour" refers to acceleration, not speed.

Sometimes the expression **speed of advance** (**SOA**) is used to indicate the speed expected to be made good over the ground, and **speed over ground** (**SOG**) the actual speed made good over the ground.

207. Direction on the earth.—Direction is the position of one point relative to another, without reference to the distance between them. In navigation, direction is customarily expressed as the angular difference in degrees from a reference direction, usually north or the ship's head. Compass directions (east, south by west, etc.) or points (of 11¼° or ⅓₂ of a circle) are seldom used by modern navigators for precise directions.

Course (**C, Cn**) is the intended horizontal direction of travel, expressed as angular distance from north, usually from 000° at north, clockwise through 360°. Strictly, the term applies to direction *through the water*, not the direction intended to be made good *over the ground*, but in common American usage it is applied to either. **Course made good** is the single course from the point of departure to point of arrival at any given time. Sometimes the expression **course of advance** (**COA**) is used to indicate the direction expected to be made good over the ground, and **course over ground** (**COG**) the actual direction made good over the ground. **Course line** is a line extending in the direction of a course.

In making computations it is sometimes convenient to express a course as an angle from *either* north or south, through 90° or 180°. In this case it is designated **course angle** (C) and should be properly labeled to indicate the origin (prefix) and direction of measurement (suffix). Thus, C N 35° E = Cn 035° (000°+35°), C N 155° W = Cn 205° (360°−155°), C S 47° E = Cn 133° (180°−47°). But Cn 260° may be either C N 100° W or C S 80° W, depending upon the conditions of the problem.

The symbol C is always used for *course angle*, and is usually used for *course* where there is little or no possibility of confusion.

Track (**TR**) is the path actually followed by a vessel, or the path of proposed travel. It differs from *course* and *course made good* by including the element of distance as well as direction, although the term is occasionally used to refer to direction only. However, the path actually followed is usually a somewhat irregular line. The path of proposed travel consists of one or a series of course lines from the point of departure to the destination, along which it is intended the vessel will proceed. A great circle which a vessel intends to follow approximately is called a **great-circle track.**

Heading (**Hdg., SH**) is the direction in which a vessel is pointed, expressed as angular distance from north, usually from 000° at north, clockwise through 360°. *Heading* should not be confused with *course*. *Heading* is a constantly changing value as a vessel oscillates or yaws back and forth across the course or as the direction of motion is temporarily changed, as in avoiding an obstacle. *Course* is a predetermined value and usually remains constant for a considerable time (fig. 207a).

Bearing (**B, Bn**) is the direction of one terrestrial point from another, expressed as angular distance from a reference direction, usually from 000° at the reference direction, clockwise through 360°. When measured through 90° or 180° from *either*

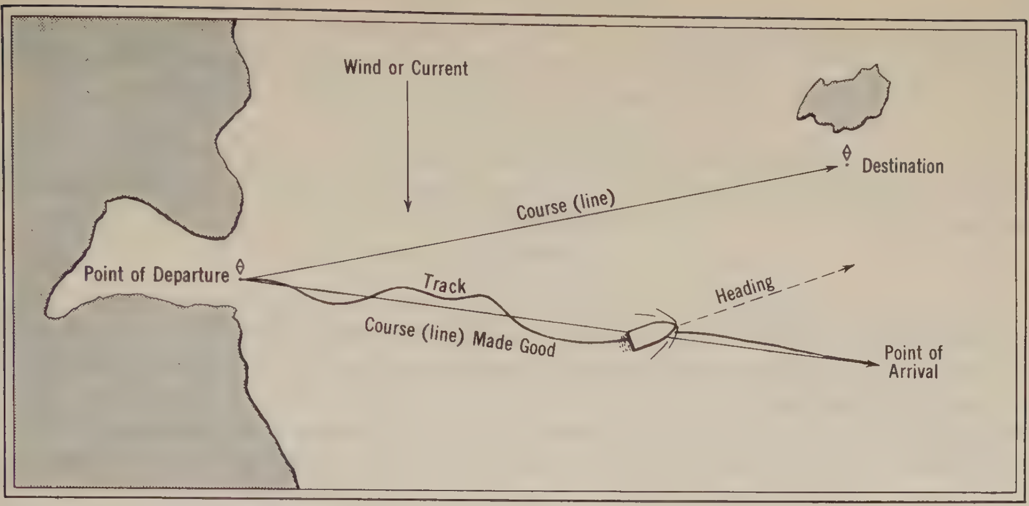


FIGURE 207a.—Course (line), course (line) made good, track, and heading.

north or south, it is called **bearing angle** (**B**), which bears the same relationship to bearing as *course angle* does to *course*. *Bearing* and *azimuth* are sometimes used interchangeably, but the latter is better reserved exclusively for reference to horizontal direction of a point on the celestial sphere from a point on the earth.

A **relative bearing** (**RB**) is one relative to the heading, or to the vessel itself. It is usually measured from 000° at the heading, clockwise through 360°. However, it is sometimes conveniently measured right or left from 0° at the ship's head through 180°. This is particularly true when using table 7. Older methods, such as indicating the number of degrees or points from some part of the vessel (10° forward of the starboard beam, two points on the port quarter, etc.) are seldom used by modern navigators to indicate precise directions, except for bearings dead ahead or astern, or broad on the bow, beam, or quarter.

To convert a relative bearing to a bearing from north (fig. 207b), express the relative bearing in terms of the 0°–360° system and add the heading:

$$Bn = RB + SH$$

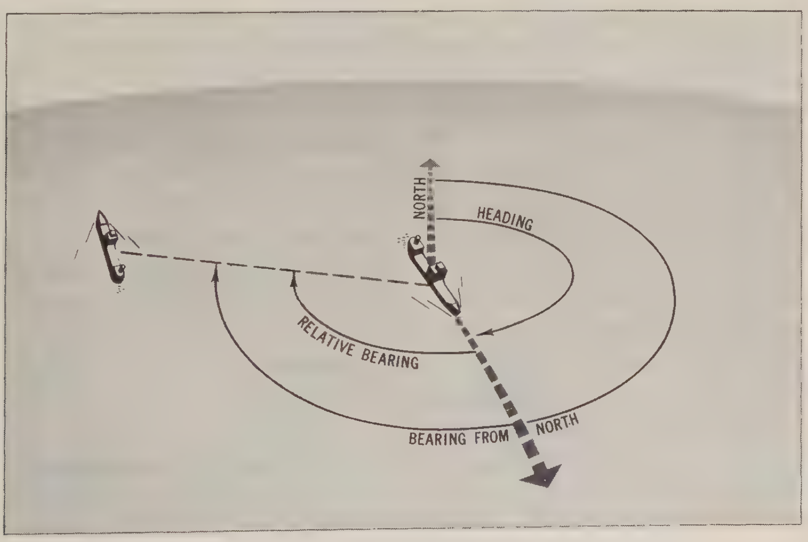


FIGURE 207b.—Relative bearing.

Thus, if another vessel bears 127° relative from a ship whose heading is 150°, the bearing from north is 127°+150°=277°. If the total exceeds 360°, subtract this amount. To convert a bearing from north to a relative bearing, subtract the heading:

$$RB=Bn-SH$$

Thus, a lightship which bears 241° from north bears 241°−137°=104° relative from a ship whose heading is 137°. If SH is larger than Bn, add 360° to Bn before subtracting.

Problems

204. *Given.*—Point *A*: L 37°21′.4 N, λ 143°18′.8 W; Point *B*: L 43°04′.1 N, λ 11°47′.3 E; Point *C*: L 63°24′.4 S, λ 132°06′.9 E; Point *D*: L 2°36′.6 S, λ 168°01′.2 W.

Required.—(1) The difference of latitude between *A* and *B*, between *A* and *C*, and between *C* and *D*.

(2) The difference of longitude between *A* and *B*, *A* and *C*, and *B* and *C*.

Answers.—(1) l_{AB} 5°42′.7 N, l_{AC} 100°45′.8 S, l_{CD} 60°47′.8 N; (2) DLo_{AB} 155°06′.1 E, DLo_{AC} 84°34′.3 W, DLo_{BC} 120°19′.6 E.

205a. The distance between points *E* and *F* is 258.4 nautical miles.

Required.—The distance in statute miles between points *E* and *F* (1) by proportion, using the ratio given in article 205; (2) by conversion factor, using the value given in article 205; (3) by table 20.

Answers.—(1) D 297.6 m, (2) D 297.2 m, (3) D 297.4 m.

205b. The distance between points *G* and *H* is 83.3 statute miles.

Required.—The distance in nautical miles between points *G* and *H* (1) by proportion, using the ratio given in article 205; (2) by conversion factor, using the value given in article 205; (3) by table 20.

Answers.—(1) D 72.3 M, (2) D 72.4 M, (3) D 72.4 M.

206a. A ship is steaming at 18.5 knots.

Required.—The speed in statute miles per hour.

Answer.—S 21.3 mph.

206b. A motorboat is traveling at 30 statute miles per hour.

Required.—The speed in knots.

Answer.—S 26 kn.

207a. *Required.*—Convert the following course angles to courses: (1) N 127° W, (2) S 3° W, (3) N 99° E, (4) S 171° E.

Answers.—(1) Cn 233°, (2) Cn 183°, (3) Cn 099°, (4) Cn 009°.

207b. *Required.*—Convert the following courses to course angles, giving the two possible answers of each: (1) 153°, (2) 257°.

Answers.—(1) C N 153° E or S 27° E, (2) C N 103° W or S 77° W.

207c. A ship is on course 151°. The following relative bearings are observed: (1) 006°, (2) 109°, (3) 255°, (4) broad on the port bow.

Required.—The bearings from north.

Answers.—(1) Bn 157°, (2) Bn 260°, (3) Bn 046°, (4) Bn 106°.

207d. A ship is on course 244°. The following bearings from north are observed: (1) 041°, (2) 188°, (3) 332°.

Required.—The relative bearings.

Answers.—(1) RB 157°, (2) RB 304°, (3) RB 088°.

207e. The captain of a ship on course 055° wishes to change course when a certain lighthouse is broad on the starboard beam.

Required.—The bearing from north when the course is to be changed.

Answer.—Bn 145°.

CHAPTER III

CHART PROJECTIONS

General

301. The navigator's chart.—A **map** is a conventional representation, usually on a plane surface, of all or part of the physical features of the earth's surface or any part of it. A **chart** is such a representation intended primarily for navigation. A **nautical** or **marine chart** is one intended primarily for marine navigation. It generally shows depths of water (by soundings and sometimes also by depth curves), aids to navigation, dangers, and the outline of adjacent land and such land features as are useful to the navigator.

Chart making presents the problem of representing the surface of a spheroid upon a plane surface. The surface of a sphere or spheroid is said to be **undevelopable** because no part of it can be flattened without distortion. A **map projection** or **chart projection** is a method of representing all or part of the surface of a sphere or spheroid upon a plane surface. The process is one of transferring points on the surface of the sphere or spheroid onto a plane, or onto a **developable** surface (one that can be flattened to form a plane) such as a cylinder or cone. If points on the surface of the sphere or spheroid are projected from a single point (including infinity), the projection is said to be **perspective** or **geometric.** Most map projections are not perspective.

302. Selecting a projection.—Each projection has distinctive features which make it preferable for certain uses, no one projection being best for all conditions. These distinctive features are most apparent on charts of large areas. As the area becomes smaller, the differences between various projections become less noticeable until on the largest scale chart, such as of a harbor, all projections become practically identical. Some of the desirable properties are:

1. *True shape* of physical features.
2. *Correct angular relationship.* A projection with this characteristic is said to be **conformal** or **orthomorphic.**
3. *Equal area,* or the representation of areas in their correct relative proportions.
4. *Constant scale* values for measuring distances.
5. *Great circles* represented as straight lines.
6. *Rhumb lines* represented as straight lines.

It is possible to preserve any one and sometimes more than one property in any one projection, but it is impossible to preserve all of them. For instance, a projection cannot be both conformal and equal area, nor can both great circles and rhumb lines be represented as straight lines.

303. Types of projection.—Projections are usually classified primarily as to the type of developable surface to which the spherical or spheroidal surface is transferred. They are sometimes further classified as to whether the projection (but not necessarily the charts made by it) is centered on the equator (**equatorial**), a pole (**polar**), or some point or line between (**oblique**). The name of a projection often indicates its type and sometimes, in addition, its principal feature.

The projection used most frequently by mariners is commonly called **Mercator,** after its inventor (art. 109). Classified according to type this is an **equatorial cy-**

lindrical orthomorphic projection, the cylinder conceived as being tangent along the equator. A similar projection based upon a cylinder tangent along a meridian is called **transverse Mercator** or **transverse cylindrical orthomorphic.** It is sometimes called **inverse Mercator** or **inverse cylindrical orthomorphic.** If the cylinder is tangent along a great circle other than the equator or a meridian, the projection is called **oblique Mercator** or **oblique cylindrical orthomorphic.**

In a **simple conic** projection points on the surface of the earth are conceived as transferred to a tangent cone. In a **Lambert conformal** projection the cone intersects the earth (a **secant** cone) at two small circles. In a **polyconic** projection, a series of tangent cones is used.

An **azimuthal** or **zenithal** projection is one in which points on the earth are transferred directly to a plane. If the origin of the projecting rays is the center of the earth, a **gnomonic** projection results; if it is the point opposite the plane's point of tangency, a **stereographic** projection; and if at infinity (the projecting lines being parallel to each other), an **orthographic** projection (fig. 303). The gnomonic, stereographic, and orthographic are perspective projections. In an **azimuthal equidistant** projection, which is not perspective, the scale of distances is constant along any radial line from the point of tangency.

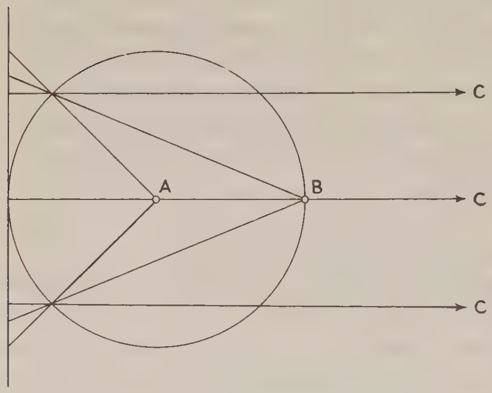


FIGURE 303.—Azimuthal projections: *A*, gnomonic; *B*, stereographic; *C* (at infinity), orthographic.

Cylindrical and plane projections can be considered special cases of conical projections with the heights infinity and zero, respectively.

A **graticule** is the network of latitude and longitude lines laid out in accordance with the principles of any projection.

Cylindrical Projections

304. Features.—If a cylinder is placed around the earth, tangent along the equator, and the planes of the meridians are extended, they intersect the cylinder in a number of vertical lines (fig. 304). These lines, all being vertical, are parallel, or everywhere equidistant from each other, unlike the terrestrial meridians, which become closer together as the latitude increases. On the earth the parallels of latitude are perpendicular to the meridians, forming circles of progressively smaller diameter as the latitude increases. On the cylinder they are shown perpendicular to the projected meridians, but because a cylinder is everywhere of the same diameter, the projected parallels are all the same size.

If the cylinder is cut along a vertical line (a meridian) and spread out flat, the meridians appear as equally spaced, vertical lines, and the parallels as horizontal lines. The spacing of the parallels relative to each other differs in the various types of cylindrical projections.

The cylinder may be tangent along some great circle other than the equator, forming an oblique or transverse cylindrical projection, on which the pattern of latitude and longitude lines appears quite different, since the line of tangency and the equator no longer coincide.

305. Mercator projection.—The only cylindrical projection widely used for navigation is the **Mercator** or **equatorial cylindrical orthomorphic,** named for its inventor Gerhard Kremer (Mercator), a Flemish geographer. It is not perspective and the

parallels cannot be located by geometrical projection, the spacing being derived mathematically. The use of a tangent cylinder to explain the development of the projection has been used, but the relationship of the terrestrial latitude and longitude lines to those on the cylinder is often carried beyond justification, resulting in misleading statements and illustrations.

The distinguishing feature of the Mercator projection (fig. 305) among cylindrical projections is that both the meridians and parallels are expanded at the same ratio with increased latitude. The expansion is equal to the secant of the latitude, with a small correction for the ellipticity of the earth. Since the secant of 90° is infinity, the projection cannot include the poles. Expansion is the same in all directions and angles are correctly shown, the projection being conformal. Rhumb lines appear as straight lines, the directions of which can be measured directly on the chart. Distances can also be measured directly, to practical accuracy, but not by a single distance scale over the entire chart, unless the spread of latitude is small. The latitude scale is customarily used for measuring distances, the expansion of the scale being the same as that of distances at the same latitude. Great circles, except meridians and the equator, appear as curved lines concave to the equator (fig. 310a). Small areas appear in their correct shape but of increased size unless they are near the equator. Plotting of positions by latitude and longitude is done by means of rectangular coordinates, as on any cylindrical projection.

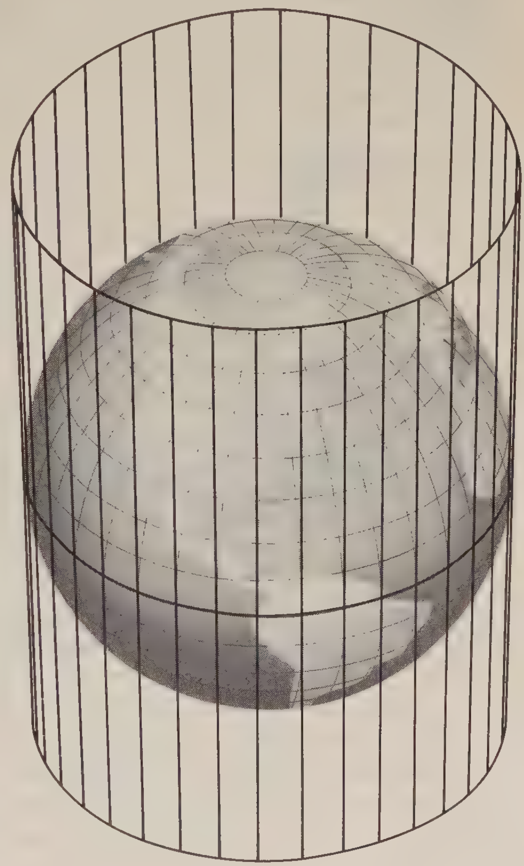


FIGURE 304.—A cylindrical projection.

306. Meridional parts.—At the equator a degree of longitude is approximately equal in length to a degree of latitude. As the distance from the equator increases, degrees of latitude remain approximately the same (not exactly because the earth is not quite a sphere), while degrees of longitude become progressively shorter. Since degrees of longitude appear everywhere the same length in the Mercator projection, it is necessary to increase the length of the meridians if the expansion is to be equal in all directions. Thus, to maintain the correct proportions between degrees of latitude and degrees of longitude, the former are shown progressively longer as the distance from the equator increases (fig. 305).

The length of the meridian, as thus increased between the equator and any given latitude, expressed in minutes of the equator as a unit, constitutes the number of **meridional parts** (**M**) corresponding to that latitude. Meridional parts, given in table 5 for every minute of latitude from the equator to the pole, afford facilities for constructing a Mercator chart and for solving problems in Mercator sailing (art. 817). These values are for the Clarke spheroid of 1866. By means of table 4 they can be converted to the values for certain other spheroids and the sphere.

FIGURE 305.—A Mercator map of the world.

The formula for meridional parts, given in the explanation to table 5, is derived from an integral representing the exact relationship.

307. Mercator chart construction.—To construct a Mercator chart, first select the scale and then proceed as follows:

Draw a series of vertical lines to represent the meridians, spacing them in accordance with the scale selected. If the chart is to include the equator, the distances of the various parallels from the equator are given directly in table 5, although it may be desirable to convert the tabulated values to more convenient units. Thus, if 1°(60′) of longitude is to be shown as one inch, each meridional part will be $\frac{1}{60}$ or 0.01667 inch in length. The distance, in inches, of any parallel from the equator is then determined by dividing its meridional parts by 60 or multiplying them by 0.01667.

If the equator is not to be included, the **meridional difference** (**m**) is used. This is the difference between the meridional parts of the various latitudes and that of the lowest parallel (the one nearest the equator) to be shown. Distances so determined are measured from the lowest parallel.

It is often desired to show a minimum area on a chart of limited size, to the largest possible scale. The scale is then dictated by the limitations.

When the graticule has been completed, the features to be shown are located by means of the latitude and longitude scales.

Example.—A Mercator chart is to be constructed at the maximum scale on a sheet of paper 35 × 46 inches, with a minimum two-inch margin outside the **neat line** limiting

the charted area. The minimum area to be covered is lat. 44°–50° north and long. 56°–68° west.

Solution.—Step one: Determine which dimension to place horizontal. From table 5 the meridional difference is:

$$\begin{array}{lr} M_{50°} & 3456.6 \\ M_{44°} & \underline{2929.6} \\ m & 527.0 \end{array}$$

The chart is to cover at least 12° (68°−56°) of longitude. The longitude is therefore to cover a distance of $12 \times 60 = 720$ meridional parts. Since there are a greater number of meridional parts of longitude to be shown than of latitude, the long dimension is placed horizontal.

Step two: Determine whether the latitude or longitude is the limiting scale factor. The number of inches available for latitude coverage is 31 (35 inches minus a two-inch margin top and bottom). If 527 meridional parts are to be shown in 31 inches, each meridional part will be $\frac{31}{527} = 0.05882$ inch. There are $46-4=42$ inches available for longitude, and therefore the length of each meridional part will be $\frac{42}{720} = 0.05833$ inch. Thus, the longitude is the limiting scale factor, for all of the desired area could not be shown in the available space if the larger scale were to be used. Using the smaller scale, it is found that 30.74 inches (0.05833×527) will be needed to show the desired latitude coverage. The top and bottom margins can be increased slightly, or additional latitude coverage can be shown. If it is desired to include the additional coverage, the amount can be determined by dividing the available space, 31 inches, by the scale, 0.05833. This is 531.5 meridional parts, or 4.5 more than the minimum. By inspection of table 5, it is seen that the latitude can be extended either 3′.3 below 44° or 2′.9 above 50°. Suppose it is decided that the margin will be increased slightly and only the desired minimum coverage shown.

Step three: Determine the spacing of the meridians and parallels. Meridians 1° or 60′ apart will be placed $60 \times 0.05833 = 3.50$ inches apart. Next, determine each degree of latitude separately. First, compute the meridional difference between the lowest parallel and the various parallels to be shown:

$$\begin{array}{lr} M_{45°} & 3013.5 \\ M_{44°} & \underline{2929.6} \\ m & 83.9 \end{array} \quad \begin{array}{lr} M_{46°} & 3098.8 \\ M_{44°} & \underline{2929.6} \\ m & 169.2 \end{array} \quad \begin{array}{lr} M_{47°} & 3185.7 \\ M_{44°} & \underline{2929.6} \\ m & 256.1 \end{array} \quad \begin{array}{lr} M_{48°} & 3274.2 \\ M_{44°} & \underline{2929.6} \\ m & 344.6 \end{array} \quad \begin{array}{lr} M_{49°} & 3364.5 \\ M_{44°} & \underline{2929.6} \\ m & 434.9 \end{array} \quad \begin{array}{lr} M_{50°} & 3456.6 \\ M_{44°} & \underline{2929.6} \\ m & 527.0 \end{array}$$

Next, determine the distance of each parallel from that of L 44° N by multiplying its meridional difference by the scale, 0.05833:

$$\begin{aligned} L\ 44° \text{ to } L\ 45° &= 0.05833 \times 83.9 = 4.89 \text{ in.} \\ L\ 44° \text{ to } L\ 46° &= 0.05833 \times 169.2 = 9.87 \text{ in.} \\ L\ 44° \text{ to } L\ 47° &= 0.05833 \times 256.1 = 14.94 \text{ in.} \\ L\ 44° \text{ to } L\ 48° &= 0.05833 \times 344.6 = 20.10 \text{ in.} \\ L\ 44° \text{ to } L\ 49° &= 0.05833 \times 434.9 = 25.37 \text{ in.} \\ L\ 44° \text{ to } L\ 50° &= 0.05833 \times 527.0 = 30.74 \text{ in.} \end{aligned}$$

Step four: Draw the graticule. Draw a horizontal line 2.13 inches $\left(\frac{35-30.74}{2}\right)$ from the bottom. This is the lower neat line. Label it "44° N." Draw the right-hand neat line two inches from the edge. Label it "56° W." Along the lower parallel measure off distances in units of 3.50 inches from λ 56° W at the right to λ 68° W at the left. Through the points thus located draw vertical lines to represent the meridians.

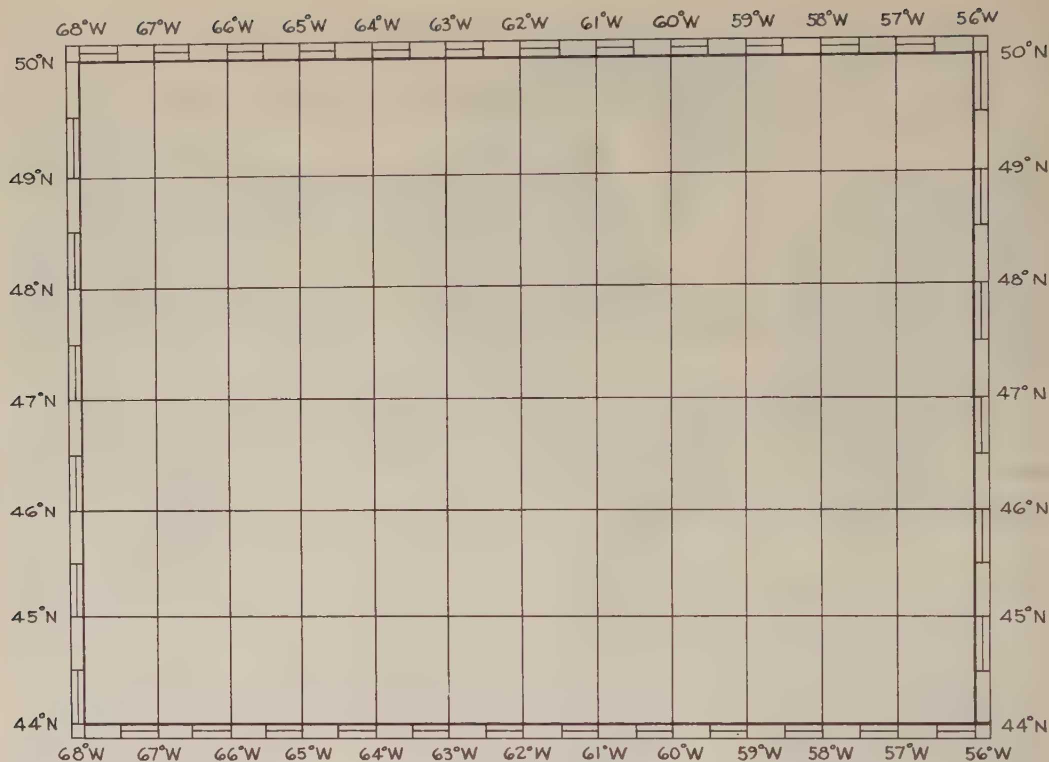


FIGURE 307.—The graticule of a Mercator chart from L 44° N to L 50° N and from λ 56° W to λ 68° W.

Along any meridian measure upward from the horizontal line a series of distances as determined by the calculations above. Through these points draw horizontal lines to represent the parallels. Label the meridians and parallels as shown in figure 307.

Step five: Mark off the latitude and longitude scales around the neat line. The scales can be graduated in units as small as desired. Determine the longitude scale by dividing the degrees into equal parts. Establish the latitude scale by computing each subdivision of a degree in the same manner as described above for whole degrees. In low latitudes degrees of latitude can be divided into equal parts without serious loss of accuracy.

Step six: Fill in the desired detail.

In *south* latitude the distance between consecutive parallels increases toward the *south.* The top parallel is drawn first and distances measured downward from it. Latitude labels increase toward the *south* (down).

In *east* longitude the longitude labels increase toward the *east* (right).

308. Transverse and oblique Mercator projections.—If Mercator principles are used to construct a chart, but with the cylinder tangent along a meridian, a **transverse Mercator** or **transverse cylindrical orthomorphic** projection results. The word "inverse" is sometimes used in place of "transverse" with the same meaning. If the cylinder is tangent at some great circle other than the equator or a meridian (fig. 308a), the projection is called **oblique Mercator** or **oblique cylindrical orthomorphic.** These projections utilize a **fictitious graticule** similar to but offset from the familiar network of meridians and parallels (fig. 308b). The tangent great circle is the **fictitious equator.** Ninety degrees from it are two **fictitious poles.** A group of great circles through these poles and perpendicular to the tangent great circle are the **fictitious**

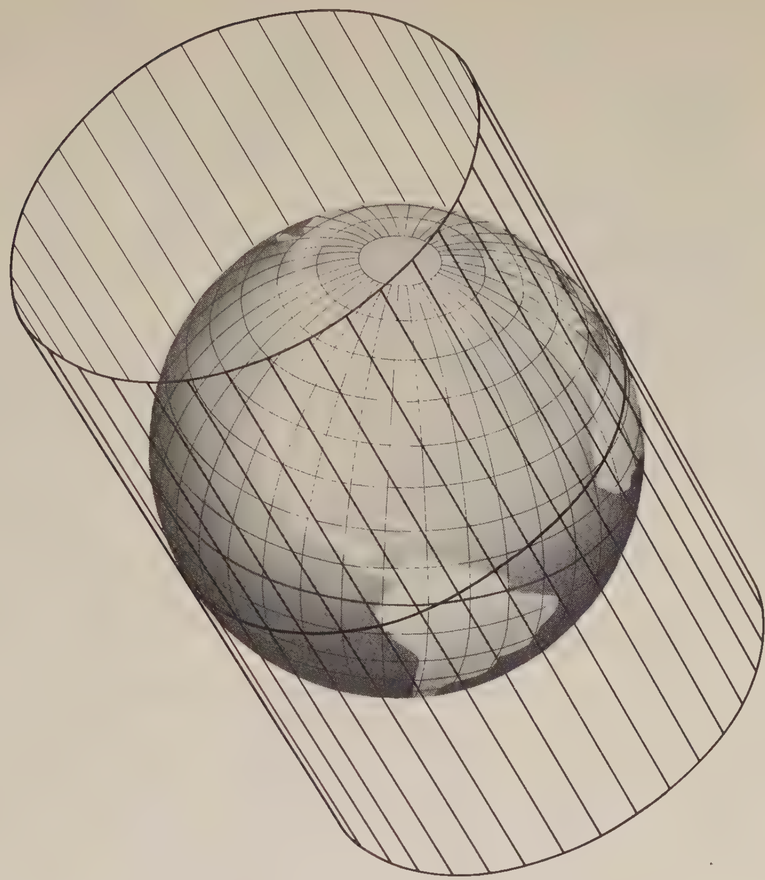


FIGURE 308a.—An oblique Mercator projection.

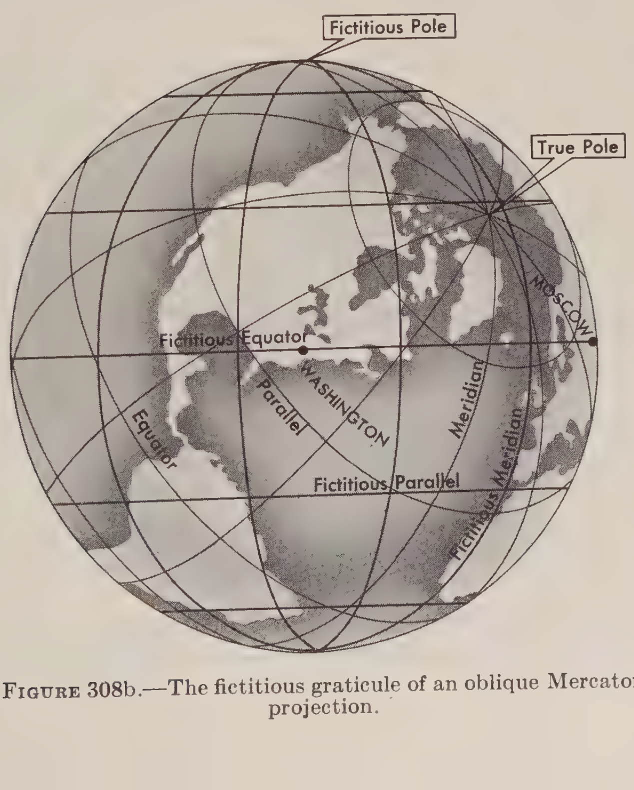


FIGURE 308b.—The fictitious graticule of an oblique Mercator projection.

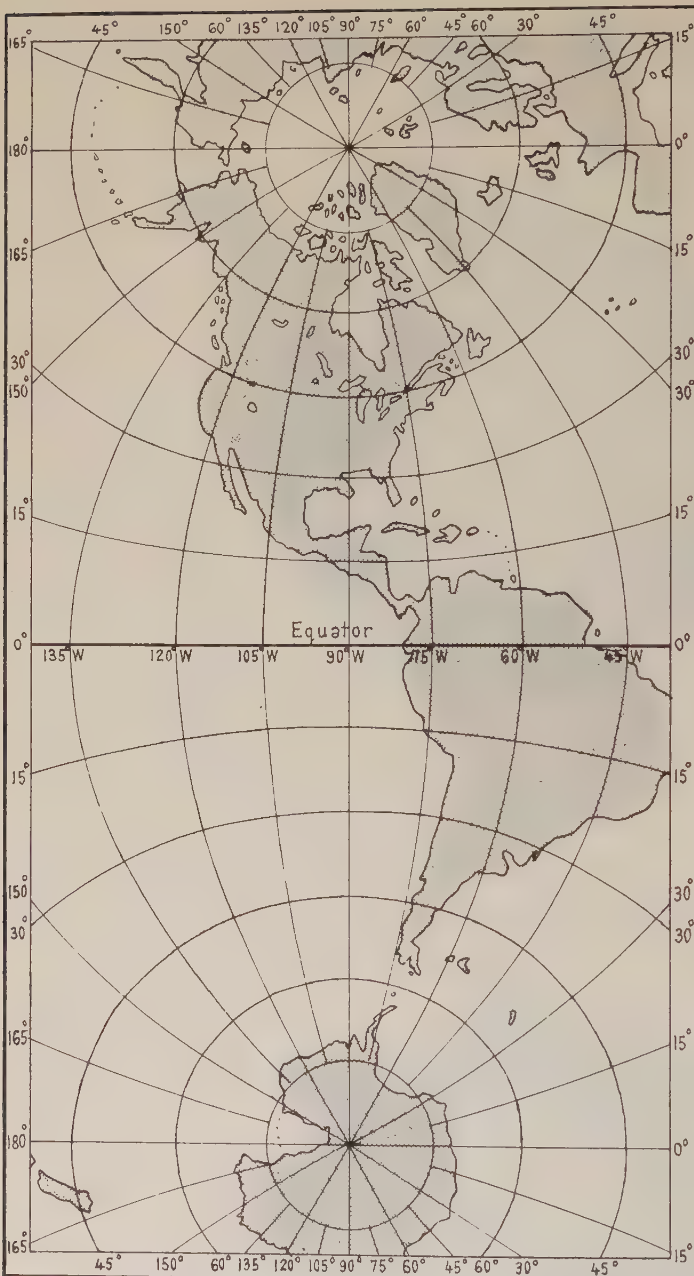


FIGURE 309.—A transverse Mercator map of the western hemisphere.

meridians, while a series of circles parallel to the plane of the tangent great circle form the **fictitious parallels.**

The actual meridians and parallels appear as curved lines (figs. 309, 310b, and 322).

A straight line on the transverse or oblique Mercator projection makes the same angle with all fictitious meridians, but not with the terrestrial meridians. It is therefore a **fictitious rhumb line.** Near the tangent great circle a straight line closely approximates a great circle. It is in this area that the chart is most useful.

The **Universal Transverse Mercator (UTM) grid** is a military grid superimposed upon a transverse Mercator graticule, or the representation of these grid lines upon any graticule.

This grid system and these projections are often used for large-scale (harbor) nautical charts and military charts.

309. Transverse Mercator projection.—A special case of the Mercator projection in which the cylinder is tangent along a meridian is called a **transverse (inverse) Mercator** or **transverse (inverse) cylindrical orthomorphic** projection. Since the area of minimum distortion is near a meridian, this projection is useful for charts covering a large band of latitude and extending a relatively short distance on each side of the tangent meridian (fig. 309) or for charts of the polar regions (fig. 322). It is sometimes used for star charts showing the evening sky at various seasons of the year (figs. 2205–2208).

310. Oblique Mercator projection.—The Mercator projection in which the cylinder is tangent along a great circle other than the equator or a meridian is called an **oblique**

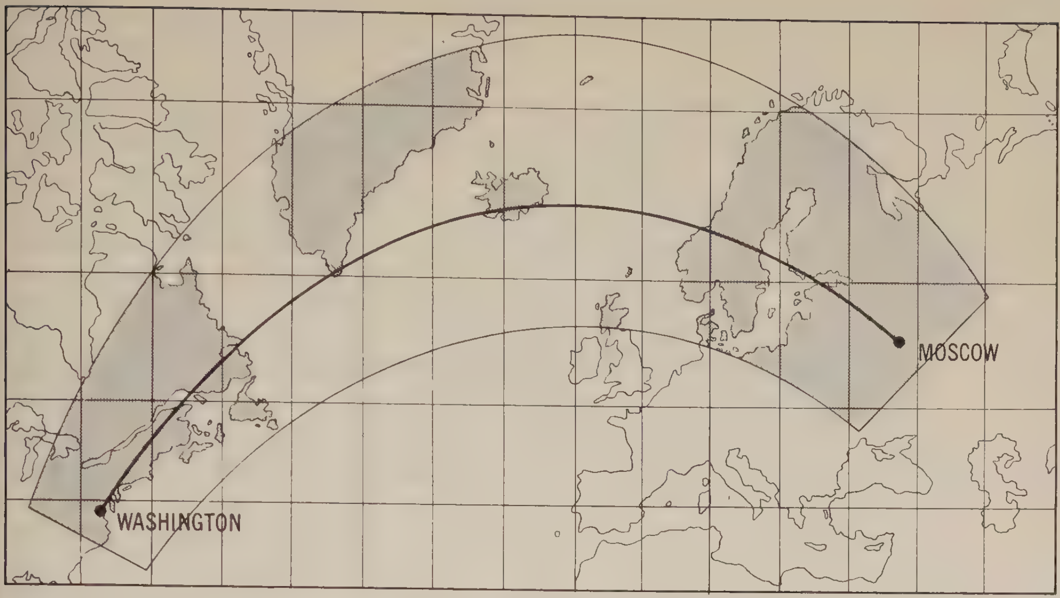


FIGURE 310a.—The great circle between Washington and Moscow as it appears on a Mercator map. See figures 308b and 310b.

Mercator or **oblique cylindrical orthomorphic** projection. This projection is used principally where it is desired to depict an area in the near vicinity of an oblique great circle, as, for instance, along the great-circle route between two important, widely separated centers. Figure 310a is a Mercator map showing Washington and Moscow and the great circle joining them. Figure 310b is an oblique Mercator map with the great circle between these two centers as the tangent great circle or fictitious equator (as in fig. 308b). The limits of the chart of figure 310b are indicated in figure 310a. Note the large variation in scale as the latitude changes.

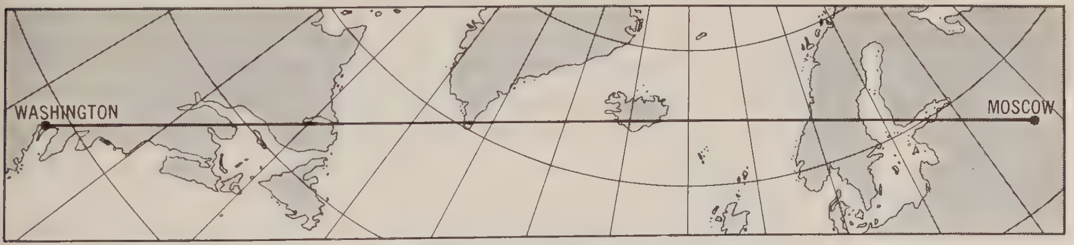


FIGURE 310b.—An oblique Mercator map based upon a cylinder tangent along the great circle through Washington and Moscow. The map includes an area 500 miles on each side of the great circle. The limits of this map are indicated on the Mercator map of figure 310a.

311. Rectangular projection.—A cylindrical projection similar to the Mercator but with uniform spacing of the parallels is called a **rectangular** projection (fig. 311). It is convenient for graphically depicting information where distortion is not important. The principal navigational use of this projection is for the star chart of the *Air Almanac* (art. 2204), where positions of stars are plotted by rectangular coordinates representing declination (ordinate) and sidereal hour angle (abscissa). Since the meridians are parallel, the parallels of latitude (including the equator and the poles) are all represented by lines of equal length.

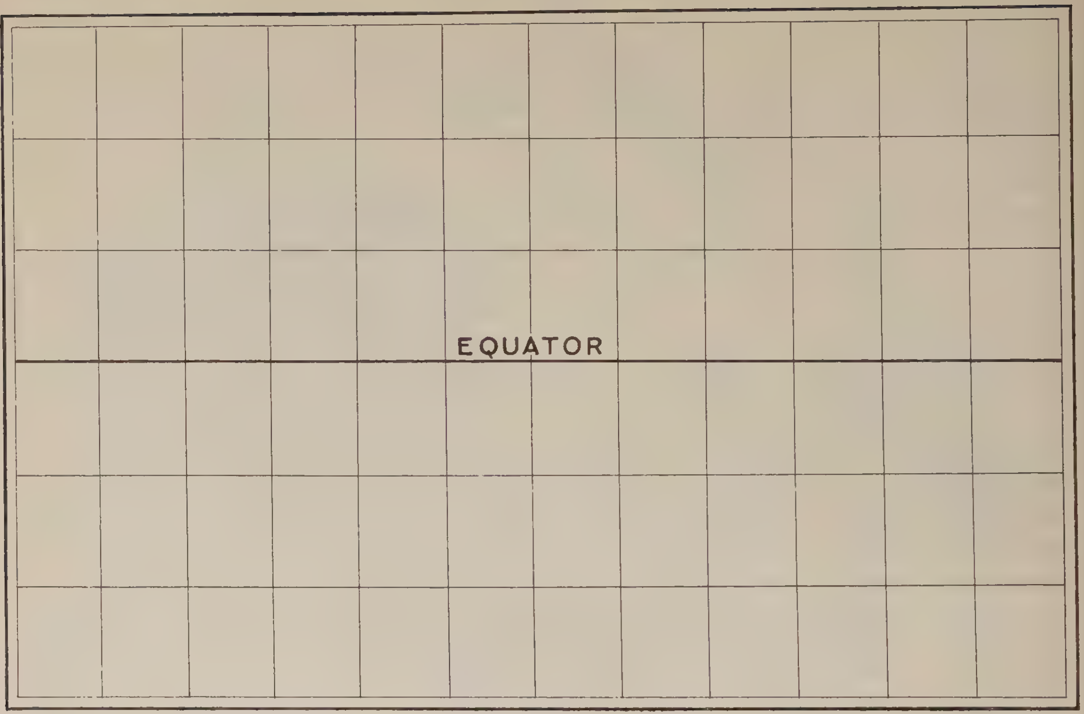


FIGURE 311.—A rectangular graticule. Compare with figure 305.

Conic Projections

312. Features.—A conic projection is produced by transferring points from the surface of the earth to a cone or series of cones which are then cut along an element and spread out flat to form the chart. If the axis of the cone coincides with the axis of the earth, the usual situation, the parallels appear as arcs of circles and the meridians as either straight or curved lines converging toward the nearer pole. Excessive distortion is usually avoided by limiting the area covered to that part of the cone near the surface of the earth. A parallel along which there is no distortion is called a **standard parallel.** Neither the **transverse conic** projection, in which the axis of the cone is in the equatorial plane, nor the **oblique conic** projection, in which the axis of the cone is oblique to the plane of the equator, are ordinarily used for navigation, their chief use being for illustrative maps.

The appearance and features of conic projections are varied by using cones tangent at various parallels, using a secant (intersecting) cone, or by using a series of cones.

313. Simple conic projection.—A conic projection using a single tangent cone is called a **simple conic** projection (fig. 313a). The height of the cone increases as the latitude of the tangent parallel decreases. At the equator the height reaches infinity and the cone becomes a cylinder. At the pole its height is zero and it becomes a plane. As in the Mercator projection, the simple conic projection is not perspective, as only the meridians are projected geometrically, each becoming an element of the cone. When this is spread out flat to form a map, the meridians appear as straight lines converging at the apex of the cone. The standard parallel, or that at which the cone is tangent to the earth, appears as the arc of a circle with its center at the apex of the cone, or the common point of intersection of all the meridians. The other parallels are concentric circles, the distance along any meridian between consecutive parallels being in correct relation to the distance on the earth, and hence derived

mathematically. The pole is represented by a circle (fig. 313b). The scale is correct along any meridian and along the standard parallel. All other parallels are too great in length, the error increasing with increased distance from the standard parallel. Since the scale is not the same in all directions about every point, the projection is not conformal, its principal disadvantage for navigation. Neither is it an equal-area projection.

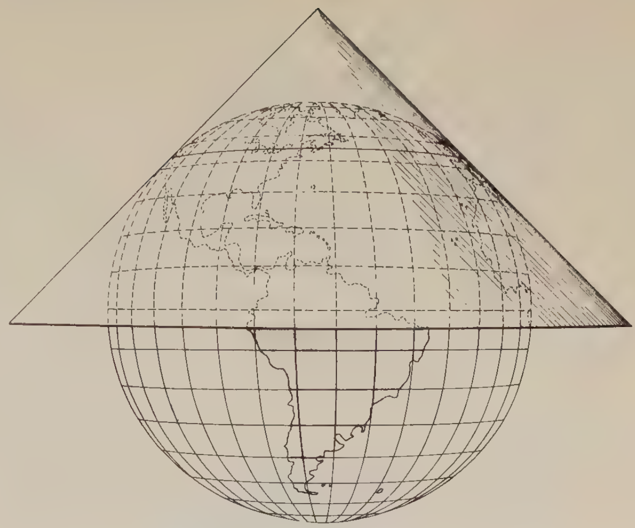


Figure 313a.—A simple conic projection.

Since the scale is correct along the standard parallel and varies uniformly on each side, with comparatively little distortion near the standard parallel, this projection is useful for mapping an area covering a large spread of longitude and a comparatively narrow band of latitude. It was developed by Claudius Ptolemy in the second century after Christ to map just such an area, the Mediterranean.

314. Lambert conformal projection.—The useful latitude range of the simple conic projection can be increased by using a secant cone intersecting the earth at two

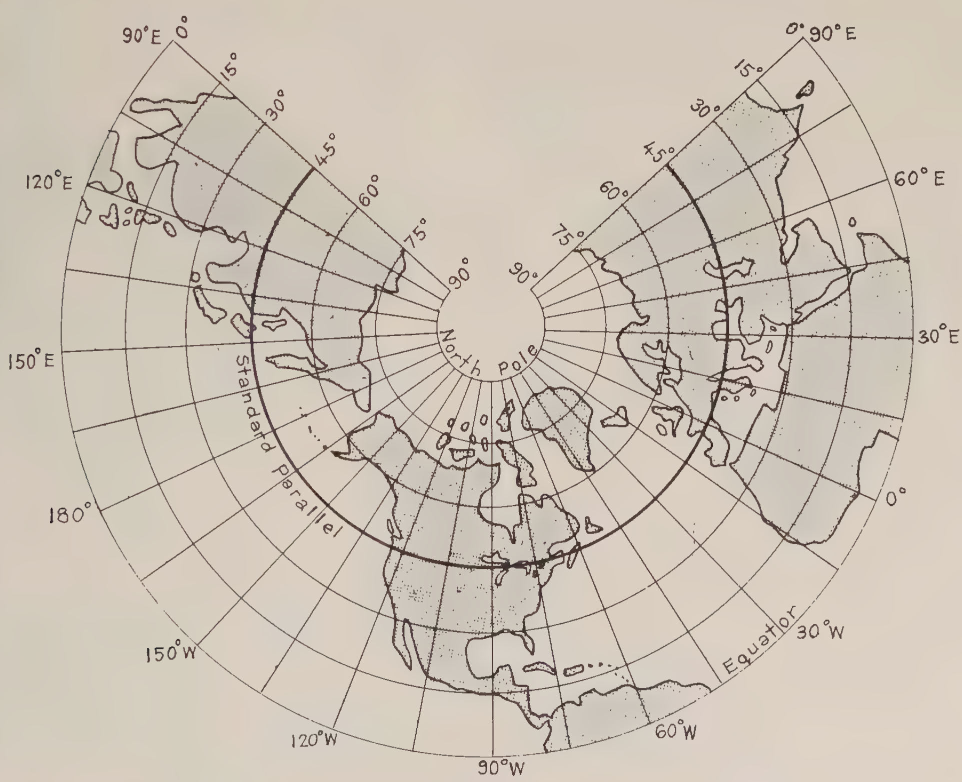


Figure 313b.—A simple conic map of the northern hemisphere.

FIGURE 314.—A secant cone for a conic projection with two standard parallels.

standard parallels (fig. 314). The area between the two standard parallels is compressed, and that beyond is expanded. Such a projection is called a **secant conic** or **conic projection with two standard parallels.**

If, in such a projection, the spacing of the parallels is altered so that the distortion is the same along them as along the meridians, the projection becomes conformal. This is known as the **Lambert conformal** projection, after its eighteenth century Alsatian inventor, Johann Heinrich Lambert. It is the most widely used conic projection for navigation, though its use is more common among aviators than mariners. Its appearance is very much the same as that of the simple conic projection. If the chart is not carried far beyond the standard parallels, and if these are not a great distance apart, the distortion over the entire chart is small. A straight line on this projection so nearly approximates a great circle that the two can be considered identical for many purposes of navigation. Radio bearings, from signals which are considered to travel great circles, can be plotted on this projection without the correction needed when they are plotted on a Mercator chart. This feature, gained without sacrificing conformality, has made this projection popular for aeronautical charts, since aircraft make wide use of radio aids to navigation. It has made little progress in replacing the Mercator projection for marine navigation, except in high latitudes. In a slightly modified form this projection has been used for polar charts (art. 321).

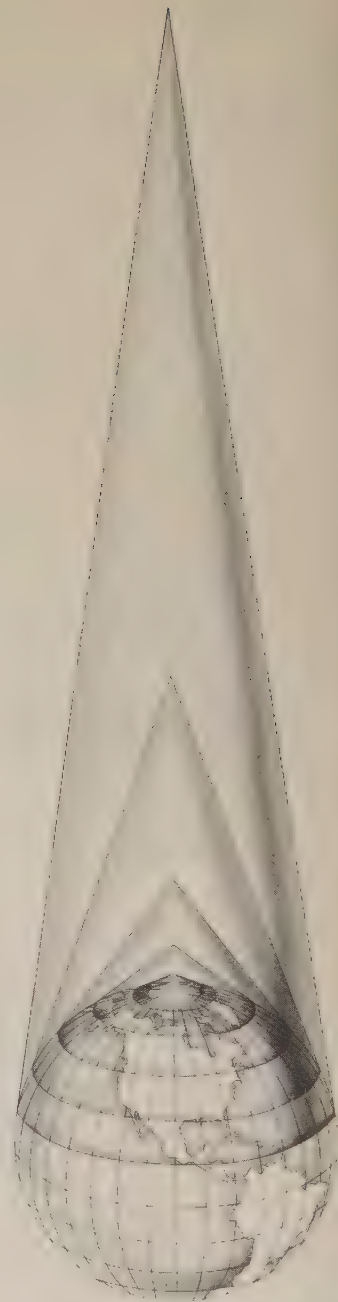


FIGURE 315a.—A polyconic projection.

315. Polyconic projection.—The latitude limitations of the secant conic projection can be essentially eliminated by the use of a series of cones, resulting in a **polyconic** projection. In this projection each parallel is the base of a tangent cone (fig. 315a). At the edges of the chart the area between parallels is expanded to eliminate gaps. The scale is correct along any parallel and along the central meridian of the projection. Along other meridians the scale increases with increased difference of longitude from the central meridian. Parallels appear as nonconcentric circles and meridians as curved lines converging toward the pole and concave to the central meridian.

The polyconic projection is widely used in atlases, particularly for areas of large range in latitude and reasonably large range in longitude, as for a continent such as North America (fig. 315b). However, since it is not conformal, this projection is not customarily used in navigation, except for **boat sheets** used in hydrographic surveying (art. 4118).

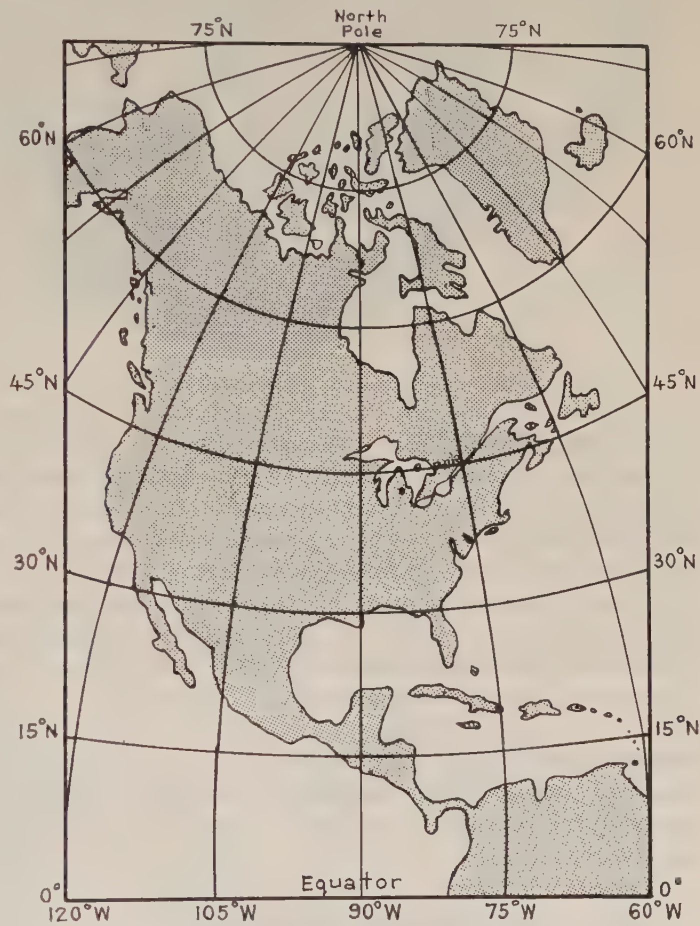


FIGURE 315b.—A polyconic map of North America.

Azimuthal Projections

316. Features.—If points on the earth are projected directly to a plane surface, a map is formed at once, without cutting and flattening, or "developing." This can be considered a special case of a conic projection in which the cone has zero height.

The simplest case of the azimuthal projection is one in which the plane is tangent at one of the poles. The meridians are straight lines intersecting at the pole, and the parallels are concentric circles with their common center at the pole. Their spacing depends upon the method of transferring points from the earth to the plane.

If the plane is tangent at some point other than a pole, straight lines through the point of tangency are great circles, and concentric circles with their common center at the point of tangency connect points of equal distance from that point. Distortion, which is zero at the point of tangency, increases along any great circle through this point. Along any circle whose center is the point of tangency, the distortion is constant. The bearing of any point from the point of tangency is correctly represented. It is for this reason that these projections are called **azimuthal.** They are also called **zenithal.** Several of the common azimuthal projections are perspective.

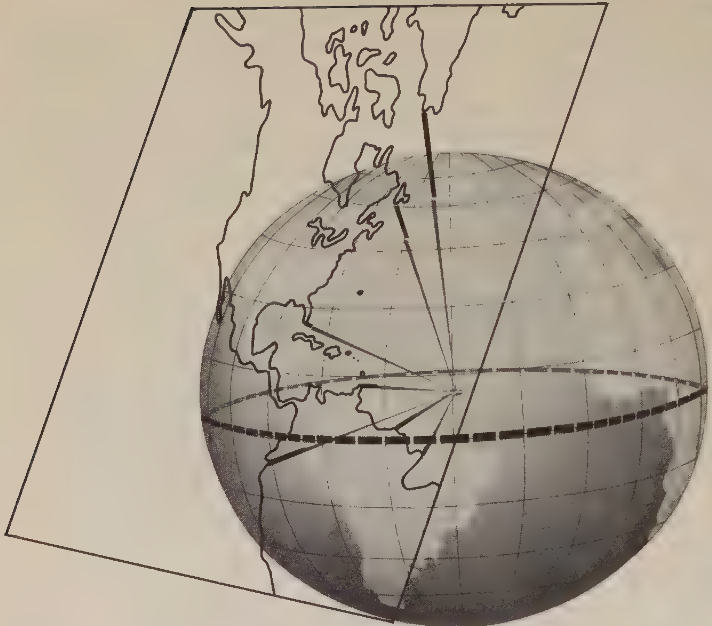


FIGURE 317a.—An oblique gnomonic projection.

317. Gnomonic projection.—If a plane is tangent to the earth, and points are projected geometrically from the center of the earth, the result is a **gnomonic** projection (fig. 317a). This is probably the oldest of the projections, believed to have been developed by Thales about 600 BC. Since the projection is perspective, it can be demonstrated by placing a light at the center of a transparent terrestrial globe and holding a flat surface tangent to the sphere.

For the oblique case the meridians appear as straight lines converging toward the nearer pole. The parallels, except the equator, appear as curves (fig. 317b). As in all azimuthal projections, bearings from the point of tangency are correctly represented. The distance scale, however, changes rapidly. The projection is neither conformal nor equal area. Distortion is so great that shapes, as well as distances and areas, are very poorly represented, except near the point of tangency.

The usefulness of the projection rests upon the one feature that *any* great circle appears on the map as a straight line. This is apparent when it is realized that a great circle is the line of intersection of a sphere and a plane through the center of the sphere, this center being the origin of the projecting rays for the map. This plane intersects any other nonparallel plane, including the tangent plane, in a straight line. It is this one useful feature that gives charts made on this projection the common name **great-circle charts.**

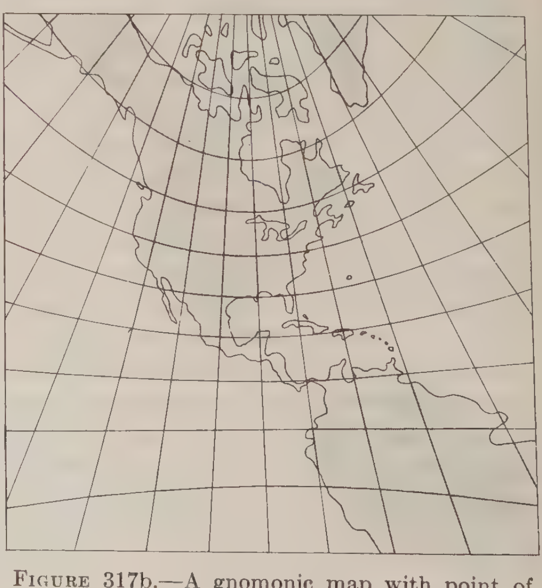


FIGURE 317b.—A gnomonic map with point of tangency at latitude 30° N, longitude 90° W.

Gnomonic charts published by the U. S. Navy Hydrographic Office bear instructions for determining direction and distance on the charts. The principal navigational use of such charts is for plotting the great-circle track between points, for planning purposes. Points along the track are then transferred, by latitude and longitude, to the navigational chart, usually one on the Mercator projection. The great circle is then followed approximately by following the rhumb line from one point to the next (art. 820).

318. Stereographic projection.—If points on the surface of the earth are pro-

jected geometrically onto a tangent plane, from a point on the surface of the earth opposite the point of tangency, a **stereographic** projection results (fig. 318a). It is also called an **azimuthal orthomorphic** projection.

The scale of the stereographic projection increases with distance from the point of tangency, but more slowly than in the gnomonic projection. An entire hemisphere can be shown on the stereographic projection without excessive distortion (fig. 318b). As in other azimuthal projections, great circles through the point of tangency appear as straight lines. All other circles, including meridians and parallels, appear as circles or arcs of circles.

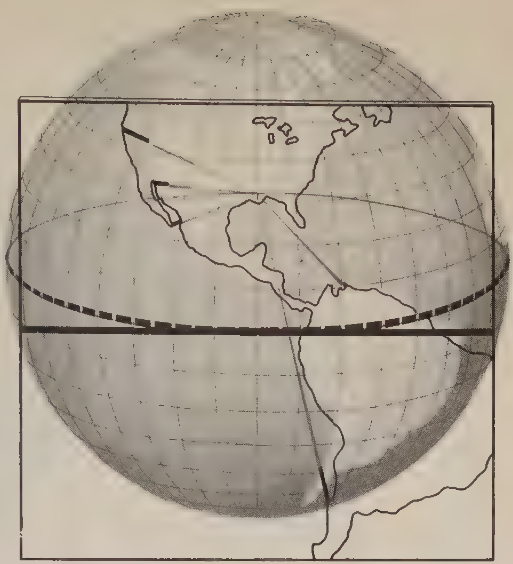


FIGURE 318a.—An equatorial stereographic projection.

The principal navigational use of the stereographic projection is for charts of the polar regions and devices for mechanical or graphical solution of the navigational triangle (art. 2122).

319. Orthographic projection.—If terrestrial points are projected geometrically from infinity (projecting lines parallel) to a tangent plane, an **orthographic** projection results (fig. 319a). This projection is neither conformal nor equal area and has no advantages as a map projection. Its principal navigational use is in the field of navigational astronomy, where it is useful for illustrating or graphically solving the navigational triangle and for illustrating celestial coordinates. If the plane is tangent at a point on the equator, the usual case, the parallels (including the equator) appear as straight lines and the meridians as ellipses, except that the meridian through the point of tangency appears as a straight line and the one 90° away as a circle (fig. 319b).

FIGURE 318b.—A stereographic map of the western hemisphere.

320. Azimuthal equidistant projection.—An azimuthal projection in which the distance scale along any great circle through the point of tangency is constant is called an **azimuthal equidistant** projection. If a pole is the point of tangency, the meridians appear as straight radial lines and the parallels as concentric circles, equally spaced. If the plane is tangent at some point other than a pole, the concentric circles represent distance from the point of tangency. In this case meridians and parallels appear as curves. The projection can be used to portray the entire earth, the

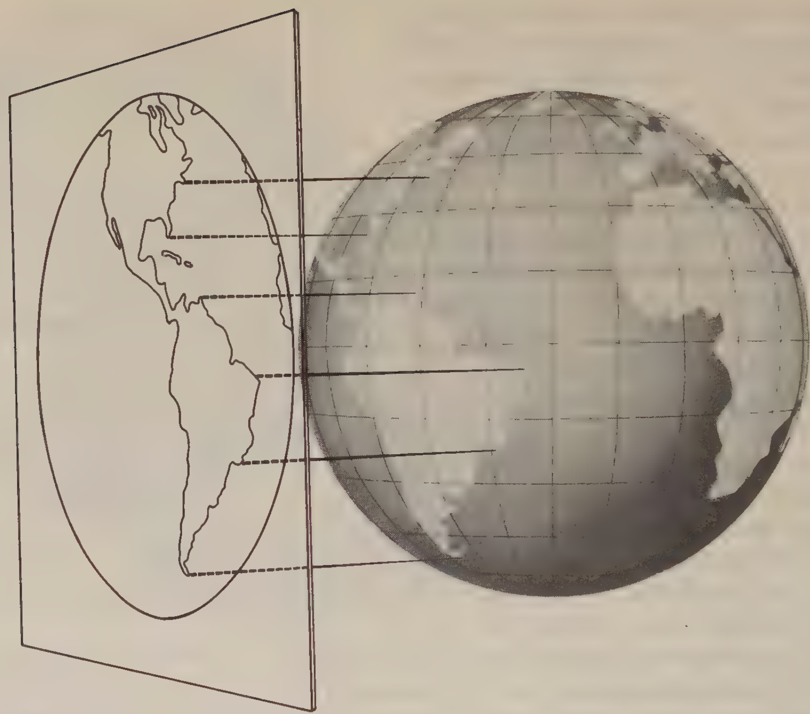


FIGURE 319a.—An equatorial orthographic projection.

point 180° from the point of tangency appearing as the largest of the concentric circles. The projection is neither conformal nor equal area, nor is it perspective. Near the point of tangency the distortion is small, but it increases with distance until shapes near the opposite side of the earth are unrecognizable (fig. 320).

FIGURE 319b.—An orthographic map of the western hemisphere.

The projection is useful because it combines the three features of being azimuthal, having a constant distance scale from the point of tangency, and permitting the entire earth to be shown on one map. Thus, if an important harbor or airport is selected as the point of tangency, the great-circle course, distance, and track from that point to any other point on the earth are quickly and accurately determined. For communication work at a fixed point, the point of tangency, the path of an incoming signal is at once apparent if the direction of arrival has been determined. The direction to train a directional antenna for desired results can

FIGURE 320.—An azimuthal equidistant map of the world with the point of tangency at latitude 40° N, longitude 100° W.

be determined easily. The projection is also used for polar charts and for the familiar star finder and identifier, H. O. 2102–D (art. 2210).

Polar Charts

321. Polar projections.—Special consideration is given to the selection of projections for polar charts, principally because the familiar projections become special cases with unique features.

In the case of cylindrical projections in which the axis of the cylinder is parallel to the polar axis of the earth, distortion becomes excessive and the scale changes rapidly. Such projections cannot be carried to the poles. However, both the transverse and oblique Mercator projections are used.

Conic projections with their axes parallel to the earth's polar axis are limited in their usefulness for polar charts because parallels of latitude extending through a full 360° of longitude appear as arcs of circles rather than full circles. This is because a

cone, when cut along an element and flattened, does not extend through a full 360° without stretching or resuming its former conical shape. The usefulness of such projections is also limited by the fact that the pole appears as an arc of a circle instead of a point. However, by using a parallel very near the pole as the higher standard parallel, a conic projection with two standard parallels can be made which requires little stretching to complete the circles of the parallels and eliminate that of the pole. Such a projection, called the **modified Lambert conformal** or **Ney's** projection, is useful for polar charts. It is particularly acceptable to those accustomed to using the ordinary Lambert conformal charts in lower latitudes.

Azimuthal projections are in their simplest form when tangent at a pole, since the meridians are straight lines intersecting at the pole, and parallels are concentric circles with their common center at the pole. Within a few degrees of latitude of the pole they all look essentially alike, but as the distance becomes greater, the spacing of the parallels becomes distinctive in each projection. In the polar azimuthal equidistant it is uniform; in the polar stereographic it increases with distance from the pole until the equator is shown at a distance from the pole equal to twice the length of the radius of the earth, or about 27% too much; in the polar gnomonic the increase is considerably greater, becoming infinity at the equator; in the polar orthographic it decreases with distance from the pole (fig. 321). All of these but the last are used for polar charts.

322. Selection of a polar projection.—The principal considerations in the choice of a suitable projection for polar navigation are:

1. *Conformality.* It is desirable that angles be correctly represented so that plotting can be done directly on the chart, without annoying corrections.

2. *Great-circle representation.* Since great circles are more useful than rhumb lines in high latitudes, it is desirable that great circles be represented by straight lines.

3. *Scale variation.* Constant scale over an entire chart is desirable.

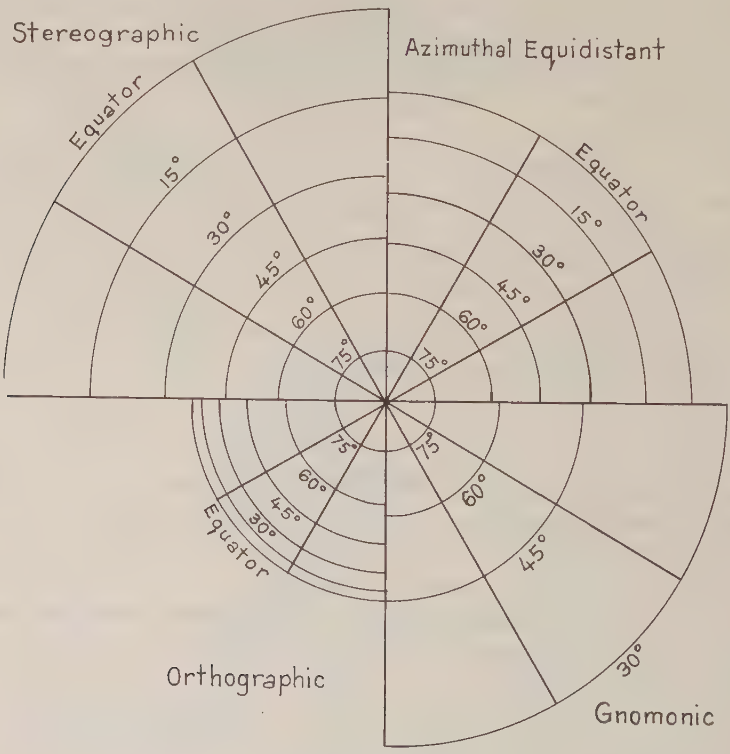


FIGURE 321.—Expansion of polar azimuthal projections.

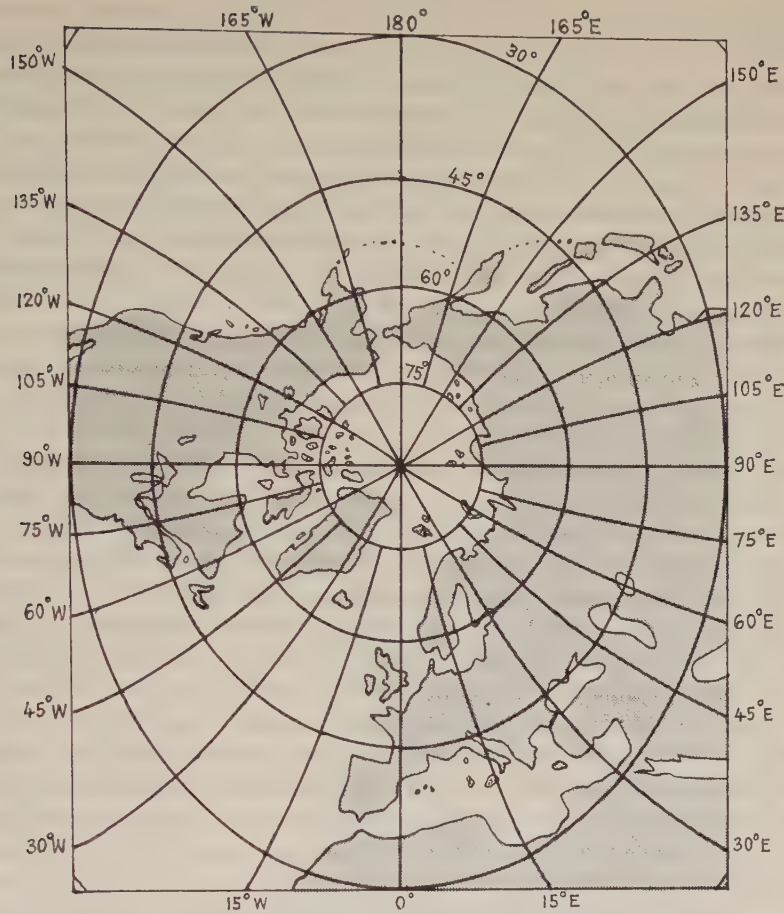


FIGURE 322.—A polar transverse Mercator map with the cylinder tangent at the 90° E-90° W meridian.

4. *Meridian representation.* Straight meridians are desirable for convenience and accuracy of plotting, and for grid navigation (art. 2510).

5. *Limits of utility.* Wide limits are desirable to reduce to a minimum the number of projections needed. The ideal would be a single projection for world coverage.

The projections commonly used for polar charts are the transverse Mercator, modified Lambert conformal, gnomonic, stereographic, and azimuthal equidistant. Near the pole there is little to choose between them. Within the limits of practical navigation all are essentially conformal and on all a great circle is nearly a straight line.

As the distance from the pole increases, however, the distinctive features of each projection become a consideration. The transverse Mercator is conformal and its type of distortion is familiar to one accustomed to using a Mercator chart. Distances can be measured in the same manner as on any Mercator chart. The tangent meridian and all straight lines perpendicular to it are great circles. All other great circles, including the meridians, are curves. The departure of a great circle from a straight line becomes a maximum at the outer edges parallel to the tangent meridian, where the straight lines are nearer the pole than the arcs of great circles between the same points. A slight inconvenience in measurement of angles may result from the curvature of the meridians (fig. 322). The projection is excellent for a narrow band along the tangent meridian and for use with automatic navigation equipment generating transverse latitude and transverse longitude.

The modified Lambert conformal projection is virtually conformal over its entire extent, and the amount of its scale distortion is comparatively little if it is carried only to about 25° or 30° from the pole. Beyond this, the distortion increases rapidly. A great circle is very nearly a straight line anywhere on the chart. Distances and directions can be measured directly on the chart in the same manner as on a Lambert conformal chart. However, for highly accurate work this projection is not suitable, for it is not strictly conformal, and great circles are not exactly straight lines.

The polar gnomonic projection is the one polar projection on which great circles are exactly straight lines. The excessive distortion and lack of conformality of this projection make it unsuitable for ordinary navigation.

The polar stereographic projection is conformal over its entire extent, and a great circle differs but little from a straight line. The scale distortion is not excessive for a considerable distance from the pole, but is greater than that of the modified Lambert conformal projection.

The polar azimuthal equidistant projection is useful for showing a large area such as a hemisphere, because there is no expansion along the meridians. However, the projection is not conformal, and distances cannot be measured accurately in any but a north-south direction. Great circles other than the meridians differ somewhat from straight lines. The equator is a circle centered at the pole.

The three projections most commonly used for charts for ordinary navigation near the poles are the transverse Mercator, modified Lambert conformal, and the polar stereographic. The transverse Mercator permits use of automatic dead reckoning equipment designed for use on a Mercator projection, transverse coordinates being substituted for geographical coordinates. However, for accuracy, it requires input of a constant transverse rhumb direction, which no present instrument provides. When a directional gyro is used as a directional reference, as in many aircraft, the track of the craft is approximately a great circle. A desirable chart is one on which a great circle is represented as a straight line with a constant scale and with angles correctly represented. These requirements are not met entirely by any single projection, but they are approximated by both the modified Lambert conformal and the polar stereographic. The scale is more nearly constant on the former, but the projection is not strictly conformal. The polar stereographic is conformal, and its maximum scale variation can be reduced by using a plane which intersects the earth at some parallel intermediate between the pole and the lowest parallel, so that that portion within this standard parallel is compressed, and that portion outside is expanded.

The selection of a suitable projection for use in polar regions, as in other areas, depends upon the requirements, which establish relative importance of the various features. For a relatively small area, any of several projections is suitable. For a large area, however, the choice is more critical. If grid directions (art. 2510) are to be used, it is important that all units in related operations use charts on the same projection, with the same standard parallels, so that a single grid direction exists between any two points. Nuclear powered submarine operations under the polar icecap have increased the need for grid directions in marine navigation. Increasing installations of gyro compasses with directional gyro modes in surface ships should increase the need for grid directions further.

Plotting Sheets

323. Definition and use.—A **plotting sheet** is a blank or incomplete chart. It has the latitude and longitude lines, and it may have one or more *compass roses* (art. 516) for measuring direction, but little or no additional information. The meridians are

usually unlabeled by the publisher so the plotting sheet can be used for any longitude. If it is suitable for use in any latitude, the parallels, also, may be unlabeled.

Plotting sheets are less expensive to produce than charts and are equally suitable or superior for some purposes. They are used primarily for plotting lines of position from celestial observations and for dead reckoning, particularly when land, aids to navigation, and depth of water are not important.

Any projection can be used for constructing a plotting sheet, but that used for the navigator's charts is customarily employed also for his plotting sheets.

324. Small area plotting sheets.—A Mercator plotting sheet can be constructed by the method explained in article 307. For a relatively small area a good approximation can be more quickly constructed by the navigator by either of two alternative methods based upon a graphical solution of the secant of the latitude, which approximates the expansion.

First method (fig. 324a). *Step one.* Draw a series of equally spaced, vertical lines at any spacing desired. These are the meridians; label them at any desired interval, as 1′, 2′, 5′, 10′, 30′, 1°, etc.

Step two. Through the center of the sheet draw a horizontal line to represent the parallel of the mid latitude of the area to be covered, and label it.

Step three. Through any convenient point, such as the intersection of the central meridian and the parallel of the mid latitude, draw a line making an angle with the *horizontal* equal to the mid latitude. In figure 324a this angle is 35°.

Step four. Draw in and label additional parallels. The length of the oblique line between consecutive meridians is the perpendicular distance between consecutive parallels, as shown by the dashed arc. The number of minutes of arc between consecutive parallels thus drawn is the same as that between the meridians shown.

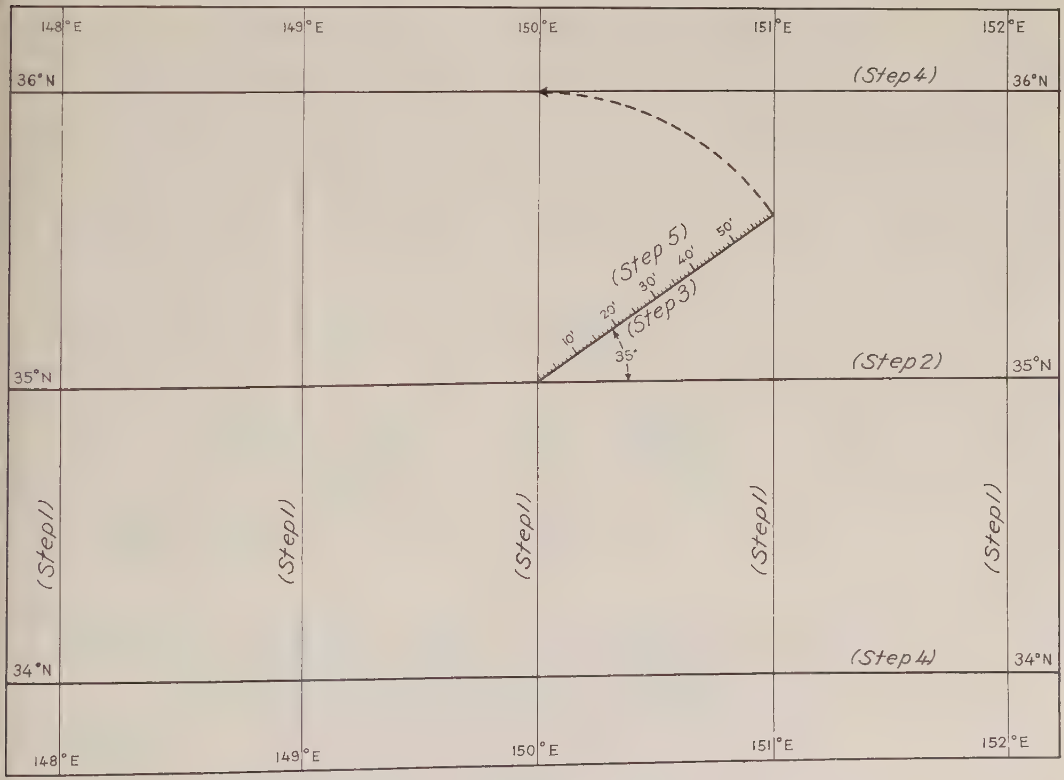


FIGURE 324a.—Small area plotting sheet with selected longitude scale.

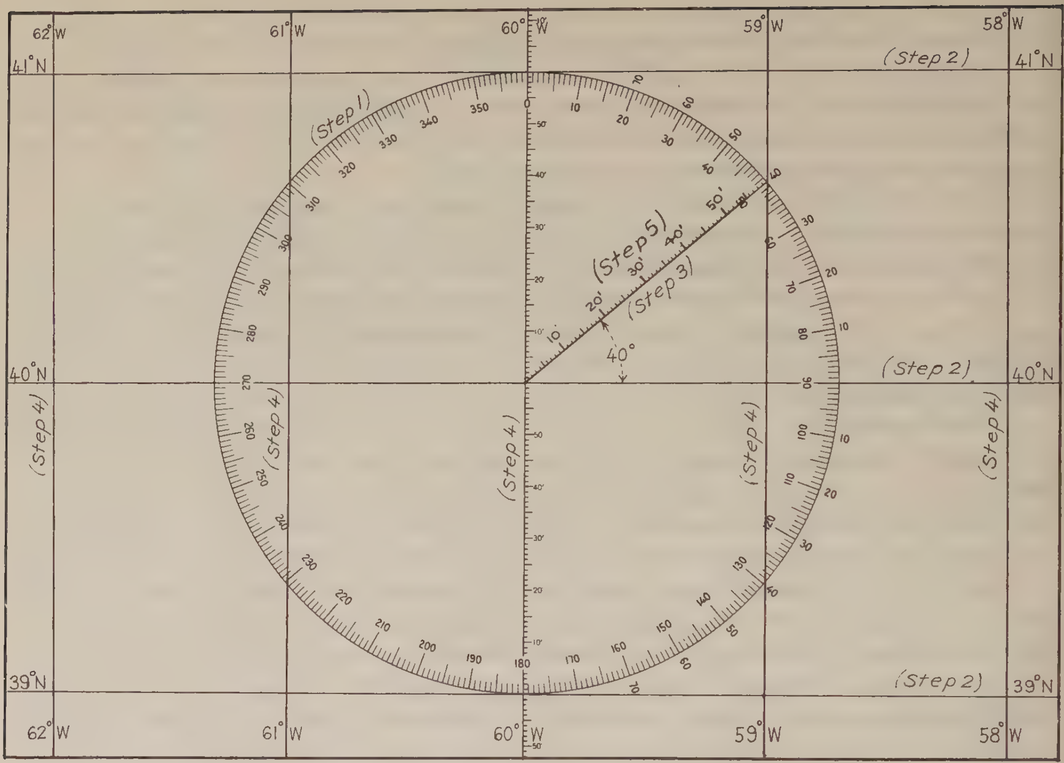


FIGURE 324b.—Small area plotting sheet with selected latitude scale.

Step five. Graduate the oblique line into convenient units. If 1′ is selected, this scale serves as both a latitude and mile scale. It can also be used as a longitude scale by measuring horizontally from a meridian instead of obliquely along the line.

Second method (fig. 324b). *Step one.* At the center of the sheet draw a circle with a radius equal to 1° (or any other convenient unit) of latitude at the desired scale. If a sheet with a compass rose is available, as in figure 324b, the compass rose can be used as the circle and will prove useful for measuring directions. It need not limit the scale of the chart, as an additional concentric circle can be drawn and desired graduations extended to it.

Step two. Draw horizontal lines through the center of the circle and tangent at the top and bottom. These are parallels of latitude; label them accordingly, at the selected interval (as every 1°, 30′, etc.).

Step three. Through the center of the circle draw a line making an angle with the *horizontal* equal to the mid latitude. In figure 324b this angle is 40°.

Step four. Draw in and label the meridians. The first is a vertical line through the center of the circle. The second is a vertical line through the intersection of the oblique line and the circle. Additional meridians are drawn the same distance apart as the first two.

Step five. Graduate the oblique line into convenient units. If 1′ is selected, this scale serves as a latitude and mile scale. It can also be used as a longitude scale by measuring horizontally from a meridian instead of obliquely along the line.

The same end result is produced by either method. The first method, starting with the selection of the longitude scale, is particularly useful when the longitude limits of the plotting sheet determine the scale. When the latitude coverage is more important, the second method may be preferable. If a standard size is desired, part

of the sheet can be printed in advance, forming what is called a **universal plotting sheet.** This is done by the U. S. Navy Hydrographic Office (art. 431). In either method a central compass rose might be printed. In the first method the meridians may be shown at the desired interval and the mid parallel may be printed and graduated in units of longitude. In using the sheet it is necessary only to label the meridians and draw the oblique line and from it determine the interval and draw in and label additional parallels. If the central meridian is graduated, the oblique line need not be. In the second method the parallels may be shown at the desired interval, and the central meridian may be printed and graduated in units of latitude. In using the sheet it is necessary only to label the parallels, draw the oblique line and from it determine the interval and draw in and label additional meridians. If the central meridian is graduated, as shown in figure 324b, the oblique line need not be.

Both methods use a constant relationship of latitude to longitude over the entire sheet and both fail to allow for the ellipticity of the earth. For practical navigation these are not important considerations for a small area. If a larger area is to be shown or if more precise results are desired, the method of article 307 should be used.

Grids

325. Purpose and definition of grid.—No system has been devised for showing the surface of the earth *on a flat surface,* without distortion. Moreover, the appearance of any portion of the surface varies with the projection and, in many cases, with the location of the portion with respect to the point or line of tangency. For some purposes (particularly military) it is desirable to be able to identify a location or area by rectangular coordinates, using numbers or letters, or a combination of numbers and letters, without the necessity of indicating the units used or assigning a name (north, south, east, or west), thus reducing the possibility of a mistake. This is accomplished by means of a **grid.** In its usual form this consists of two series of lines which are mutually perpendicular *on the chart,* with suitable designators. The grid used in **grid navigation** (art. 2510) is a similar network, or a single series of parallel lines, used to provide a uniform directional reference, particularly in polar regions. In any system the difference in direction between true north at any point and grid north at that same point is called **grid declination.**

326. Types of grids.—A grid may use the rectangular graticule of the Mercator projection, or a set of arbitrary lines on a particular projection. The most widely used system of the first is called the **World Geographic Referencing System (Georef).** It is merely a method of designating latitude and longitude by a system of letters and numbers instead of by angular measure, and therefore is not strictly a grid, except on a Mercator projection. It is particularly useful for operations extending over a wide area. Examples of the second type of grid are the **Universal Transverse Mercator (UTM) grid,** the **Universal Polar Stereographic (UPS) grid,** and the **Temporary Geographic Grid (TGG).** Since these systems are used primarily by military forces, they are sometimes called **military grids.**

References

Chamberlin, Wellman. *The Round Earth on Flat Paper.* Washington, National Geographic Society, 1947.

Deetz, C. H., and Adams, O. S. *Elements of Map Projection.* 5th ed. U.S. Coast and Geodetic Survey Special Publication No. 68. Washington, U.S. Govt. Print. Off., 1945.

Greenhood, David. *Down to Earth: Mapping for Everybody.* New York, Holiday House, 1944.

Hinks, A. R. *Map Projections.* 2nd ed. London, Cambridge University Press, 1921.

Jameson, A. H., and Ormsby, M. T. M. *Mathematical Geography.* Vol. I, *Elementary Surveying and Map Projection.* London, Pitman, [1942?].

Jervis, W. W. *The World in Maps.* New York, Oxford, 1937.

Mainwaring, James. *An Introduction to the Study of Map Projection.* London, Macmillan, 1942.

Raisz, Erwin. *General Cartography.* New York, McGraw-Hill, 1938.

Steers, J. A. *An Introduction to the Study of Map Projections.* Rev. ed. London, U. of London Press, 1929.

CHAPTER IV

CHARTS AND PUBLICATIONS

Sources

401. Introduction.—Charts and publications are important navigational aids. It is desirable that the navigator have a knowledge of what is available in this field, how he can obtain the various items, how he can be sure they are accurate and up-to-date, and what information he can expect to get from each. Instructions for the use of a number of these items are given elsewhere in this book.

402. Sources of charts and publications.—There is no central government office from which the navigator can satisfy all of his chart and publication requirements. The principal sources are the U. S. Navy Hydrographic Office, U. S. Coast and Geodetic Survey, U. S. Coast Guard, and U. S. Naval Observatory. Other sources include the U. S. Geological Survey, Mississippi River Commission, U. S. Engineer Office of the Department of the Army, U. S. Weather Bureau, USAF Aeronautical Chart and Information Center, Federal Aviation Agency, and various commercial sources.

403. The U. S. Navy Hydrographic Office maintains liaison with foreign hydrographic departments; makes hydrographic, topographic, oceanographic, and geomagnetic surveys in international waters and along foreign coasts; conducts research in oceanography and in navigational methods (both marine and air); systematically collects data in these fields from public and private institutions and persons in all parts of the world; prepares, prints, and distributes nautical and aeronautical charts; and prepares and issues publications and timely advice, including radio broadcasts, for the safe navigation of surface and subsurface vessels and of aircraft.

The products of the U. S. Navy Hydrographic Office include nautical and aeronautical charts of the high seas and foreign waters, sailing directions for foreign shores, light lists, various navigational manuals and tables, weather summaries, various oceanographic charts and publications, pilot charts, loran and radar charts, plotting sheets, a number of special charts, and several periodical publications to notify navigators of changes to their charts and publications.

404. The U. S. Coast and Geodetic Survey, of the Department of Commerce, conducts research in hydrography, cartography, tides, currents, geodesy, geomagnetism, and seismology. It publishes coast and harbor charts of the United States and its possessions and aeronautical charts of the United States, tide and tidal current tables for both United States and foreign waters, coast pilots (sailing directions) for coasts of the United States and its possessions (including the intracoastal waterway) and a number of special publications covering results of its research.

405. The U. S. Coast Guard has charge of the inspection of merchant marine vessels, licensing of merchant marine officers, and the installation and maintenance of aids to marine navigation (lighthouses, beacons, buoys, etc.). It publishes light lists for the waters of the United States and its possessions, and international and inland rules of the road and pilot rules.

406. The U. S. Naval Observatory conducts research in various branches of astronomy, including measurement and dissemination of time. It furnishes time sig-

nals, publishes nautical and air almanacs and an ephemeris, as well as tables of sunrise, sunset, and twilight.

407. Miscellaneous sources.—The U. S. Geological Survey, Department of the Interior, publishes topographic maps of the United States. The Mississippi River Commission publishes charts of the Mississippi River from Cairo, Illinois, to the delta. District offices of the U. S. Corps of Engineers, Department of the Army, publish charts of the Ohio River and other United States rivers, Illinois Waterway system, the Great Lakes (but not Canadian harbor charts nor charts of Georgian Bay), Lake Champlain, Oneida Lake, New York canals, and the St. Lawrence River above St. Regis and Cornwall. The U. S. Weather Bureau, Department of Commerce, publishes a chart and booklet showing principal types of clouds, instructions for marine meteorological observers, a glossary of weather terms, and other meteorological publications. The USAF Aeronautical Chart and Information Center produces aeronautical charts and publications. The Federal Aviation Agency produces a number of publications of interest chiefly to aviators. Various other publications and their sources are listed in appendix N.

408. Obtaining charts and publications.—In most instances charts and publications are distributed directly by the publishing agency. A notable exception is the U. S. Navy Hydrographic Office, which, except for official distribution, distributes its charts and publications through authorized sales agents throughout the world. These agents are listed in (H.O.) Pub. No. 1–N, Part I. Publications of the U.S. Coast Guard, U.S. Naval Observatory, and the U.S. Weather Bureau are sold by the Superintendent of Documents, U.S. Government Printing Office. Some of the publications of other agencies are sold both by this office and the publisher (or its agents).

The U. S. Navy Hydrographic Office, U. S. Coast and Geodetic Survey, and the Superintendent of Documents have sales agents in various United States and foreign ports. In addition, the U. S. Navy Hydrographic Office maintains a number of branch offices at major ports to collect, compile, and distribute timely information to afford the maximum safety and facility of operation to vessels of the Navy and the merchant marine. These branch offices issue pilot charts, the *Daily Memorandum*, and *Notice to Mariners*. The U. S. Coast and Geodetic Survey maintains district offices at which their charts and publications can be purchased.

Appendix N lists sources of charts and publications of interest to the navigator.

409. Numbering of charts.—Each chart is given a number by its publishing agency. Vessels of the U. S. Navy use charts of various publishers, with some duplication of numbers. To avoid confusion, charts issued to these vessels are given a **consecutive number** and arranged in convenient groups in **chart portfolios.** This system is not available to commercial or private users.

410. Terminology.—The following terminology applies generally to charts and publications of government agencies:

A **new chart** or **publication** is the first edition.

A **new edition** is a revision that supersedes previous issues, containing changes of such importance that earlier issues are obsolete.

A **corrected (new) print** is a revision that does not supersede previous issues, containing minor changes and corrections, principally those published in the *Notice to Mariners* since the preceding edition.

A **reprint** is a reproduction without change.

A **supplement** contains corrections and additions to an existing publication.

A **change** consists of corrections and additions to a loose-leaf publication.

A **summary** is a collection in one publication of related items of a specified class.

Oceanographic and Meteorological Charts and Publications

411. Tide tables are published annually by the U. S. Coast and Geodetic Survey. In them are tabulated the predicted times and heights of high and low waters for every day in the year for a number of **reference stations,** and differences for obtaining similar predictions for numerous other places. They also give other useful information such as a method for obtaining the height of the tide at any time, local mean time of sunrise and sunset for various latitudes, reduction of local mean time to standard time, zone time of moonrise and moonset for certain ports, and other astronomical data. The use of these tables is explained in chapter IX.

Tide tables are available in separate volumes for (1) east coast of North and South America, including Greenland, (2) west coast of North and South America, including the Hawaiian Islands, (3) Europe and west coast of Africa, including the Mediterranean Sea, and (4) central and western Pacific Ocean and Indian Ocean.

412. Tidal current tables, published annually by the U. S. Coast and Geodetic Survey, tabulate daily predictions of the times of slack water and the times and speeds of maximum flood and ebb currents for a number of waterways, together with differences for obtaining predictions at numerous other places. They also include other useful information on tidal currents, such as a method for obtaining the speed of current at any time and one for determining the duration of slack water, coastal tidal currents, the combination of currents, and current diagrams. Information on the Gulf Stream is included in the tidal current tables for the Atlantic coast of North America. The use of these tables is explained in chapter IX.

Tidal current tables are available in separate volumes for (1) Atlantic coast of North America, and (2) Pacific coast of North America and Asia. For places not covered by these tables the navigator must rely upon notes, tables, and arrows on charts, special current charts, sailing directions, and any other available sources.

413. Tidal current charts for various United States harbors are published by the U.S. Coast and Geodetic Survey. Each of the nine sets consists of about 12 charts which depict the direction and speed of the tidal current for each hour of the tidal cycle, thus presenting a comprehensive view of the tidal current movement in the respective waterways as a whole, and supplying a means for readily determining for any time the direction and speed of the current at various localities throughout the areas covered. The charts are intended for use in connection with the tidal current tables for the same areas, except for New York Harbor, where the tide tables are to be used.

414. Pilot charts are published by the U.S. Navy Hydrographic Office for each month for (1) the North Atlantic Ocean, and (2) the North Pacific Ocean. Pilot charts are published in atlas form for (1) the Northern North Atlantic Ocean, (2) the South Atlantic Ocean and Central American Waters, and (3) the South Pacific and Indian Oceans.

These charts present in graphical form the available facts or conclusions obtained from many years of research in navigation, oceanography, and meteorology, to assist the mariner in selecting the safest and quickest routes and avoiding dangers. Their principal features are monthly averages for: prevailing winds and currents; percentage of gales, calms, and fog; lines of equal air and water temperature, and atmospheric pressure; and limits of the drift of both field ice and icebergs. Also presented are lines of equal magnetic variation, location of ocean station vessels, and recommended routes or steamer tracks. Timely articles are printed on the backs of many pilot charts.

Pilot charts of the North Atlantic and North Pacific are furnished without charge to cooperating observers.

415. Miscellaneous oceanographic publications.—The U. S. Navy Hydrographic Office promotes basic oceanographic research and collects and codifies data which it makes available in the form of charts, manuals, and special reports. Charts in this category include those showing bottom sediment, surface temperature, currents, sea and swell, bathymetric charts (showing bottom gradient tints), as well as the pilot charts (art. 414). Representative books are *Breakers and Surf, Principles in Forecasting* (H.O. Pub. No. 234); *Oceanographic Atlas of the Polar Seas* (Pub. No. 705), Part I, Antarctica and Part II, Arctic; *Wind, Sea, and Swell: Theory of Relations for Forecasting* (H.O. Pub. No. 601). This Office also publishes several instruction manuals of use to the navigator. These include *Manual of Ice Seamanship* (Pub. No. 551), *Sonic Soundings* (H.O. Pub. No. 606–b), *Bathythermograph Observations* (H.O. Pub. No. 606–c), *Ice Observations* (H.O. Pub. No. 606–d), and *Sea and Swell Observations* (H.O. Pub. No. 606–e). The U.S. Navy Hydrographic Office *Catalog of Nautical Charts and Publications* (Pub. No. 1–N series) lists the various oceanographic charts and publications available.

The U.S. Coast and Geodetic Survey conducts research in tides and currents and makes available several publications relating to them.

Electronic Navigation Charts and Publications

416. Loran.—Tables for plotting loran lines of position are published by the U.S. Navy Hydrographic Office as H.O. Pub. No. 221, in a number of volumes. Loran lines of position are printed on certain nautical and aeronautical charts by the U.S. Navy Hydrographic Office and the Coast and Geodetic Survey. Information on loran charts and publications is contained in H.O. Pub. 1–N, *Catalog of Nautical Charts and Publications*, the *Catalog of U.S. Navy Aeronautical Charts and Related Publications*, the *Coast and Geodetic Survey catalog of Aeronautical Charts and Related Publications*, and the *DOD Catalog of Aeronautical Charts and Flight Information Publications* which is available only to military users. H.O. Pub. No. 1–V, *Catalog of Aeronautical Charts and Publications*, has been canceled.

417. Radar.—The U.S. Coast and Geodetic Survey has published several experimental nautical charts showing a great number of land contours and gradient tints, for use with radar.

418. Miscellaneous.—The U.S. Navy Hydrographic Office publications entitled *Radio Navigational Aids* (H.O. Pubs. Nos. 117–A, Atlantic and Mediterranean Area and 117–B, Pacific and Indian Oceans Area) contain detailed information on radiobeacons and other aids to navigation. The light lists includes some details of radiobeacons. Volume II of *International Code of Signals* (H.O. Pub. No. 104) deals with radio communication. Various other publications relating to radio navigation are listed in appendix N.

419. Information by radio.—H.O. Pubs. Nos. 117–A and 117–B, *Radio Navigational Aids*, contain complete lists of the radio stations that perform services of value to the mariner, and give general and detailed information concerning these services, and present the regulations of various nations on this subject.

In addition to its information on radiobeacons and radio direction finder stations, H.O. Pub. No. 117 gives full information on time signals, navigational warnings, distress signals, medical advice, quarantine report stations, long-range navigational aids, wartime emergency procedures for U. S. merchant vessels, and plain language weather reports and storm and hurricane warnings. For information concerning radio traffic stations, the mariner should consult the lists published by the Bureau of the International Telecommunication Union, Berne, Switzerland.

H.O. Pubs. Nos. 118–A and 118–B, *Radio Weather Aids*, contain general information, marine broadcasts, synoptic broadcasts, facsimile broadcasts, weather codes and code forms, and miscellaneous conversion tables.

H.O. Pub. No. 119, *Weather Station Index*, contains a complete list of international index numbers with locations of stations, key groups, and call signs, and includes a supplemental listing of U.S. meteorological reporting stations.

Corrections to these volumes are published in *Notice to Mariners*. The publications themselves are corrected by "Changes" which are issued quarterly for H.O. Pubs. Nos. 117–A, 117–B, 118–A, and 118–B.

Navigational Publications

420. Sailing Directions or **pilots** are books containing descriptions of coast lines, harbors, dangers, aids to navigation, winds, currents, and tides; instructions for navigating narrow waterways and for approaching and entering harbors; information on port facilities, signal systems, and pilotage services; and other data that cannot be conveniently shown on charts. Those covering the coasts of the United States and its possessions, including the Intracoastal Waterway, are called **coast pilots,** and are published by the U. S. Coast and Geodetic Survey. Those covering foreign coasts, called **sailing directions,** are published in looseleaf form by the U. S. Navy Hydrographic Office.

Supplements to coast pilots are published annually, and change pages to sailing directions are published periodically. The more important changes are published in *Notice to Mariners*.

421. Light lists for the United States and its possessions, including the Intracoastal Waterway, the Great Lakes (both United States and Canadian shores), and the Mississippi River and its navigable tributaries, are published annually by the U. S. Coast Guard. Similar publications covering foreign coasts are published in looseleaf form by the U.S. Navy Hydrographic Office in seven volumes (H.O. Pubs. Nos. 111–A, 111–B and 112 through 116). "Changes" are published at appropriate intervals. Light lists give detailed information regarding navigational lights, light structures, radiobeacons, and fog signals. Corrections to both sets of light lists are published weekly in the *Notice to Mariners*. Coast Guard light lists also give unlighted buoys.

422. Navigational tables.—Many types of navigational tables are published. While many of these appear as parts of other books, such as those at the back of this volume, a number of separate books of tables are available. Nearly all of these are published by the U. S. Navy Hydrographic Office. The ones of principal interest to the navigator are:

H.O. Pub. No. 260, *Azimuth Tables*, lists the azimuth angle of the sun at intervals of 10^m between sunrise and sunset for each degree of latitude between the equator and 70° (north or south). It is also applicable to other bodies having declinations of 0° to 23°. Azimuth angles are tabulated to a precision of 1′. These are popularly known as the "Red Azimuth Tables" to distinguish them from H.O. Pub. No. 261. The use of these tables is explained in article 2126.

H.O. Pub. No. 151, *Table of Distances between Ports*, tabulates about 40,000 distances between various ports throughout the world.

Distances between United States Ports, published by the U. S. Coast and Geodetic Survey, tabulates approximately 10,000 distances along the shortest routes marked by aids to navigation between various United States ports. It also includes conversion tables similar to table 20 and parts of table 21 of this volume.

H.O. Pub. No. 261, *Azimuths of Celestial Bodies*, is similar to H.O. Pub. No. 260, but for declinations of 24° to 70°. These are popularly known as the "Blue Azimuth Tables." Their use is explained in article 2126.

H.O. Pub. No. 208, *Navigation Tables for Mariners and Aviators*, is a set of trigonometric tables arranged in convenient form for solving the navigational triangle by the formulas of Dreisonstok (art. 2110).

H.O. Pub. No. 211, *Dead Reckoning Altitude and Azimuth Table*, is a trigonometric table arranged in convenient form for solving the navigational triangle by the formulas of Ageton (art. 2111).

H.O. Pub. No. 214, *Tables of Computed Altitude and Azimuth*, tabulates the solution of the navigational triangle for each 1° of latitude from the equator to 89° (north or south), each 1° of meridian angle where the altitude is 5° or more, and each 0°5 of declination from 0° to 29° with selected values above 29°. Altitude is given to the nearest 0'1 and azimuth angle to the nearest 0°1. There are nine volumes, each covering 10° of latitude, with separate tabulations for same and contrary names of declination and latitude. These are the basic tables for marine celestial navigation, and are explained fully in chapter XX.

H.O. Pub. No. 218, *Astronomical Navigation Tables*, is somewhat similar to H.O. Pub. No. 214, but designed primarily for aviators. Altitudes are tabulated to the nearest 1' and azimuth angles to the nearest 1°. Altitudes are corrected for refraction at a height of eye of 5,000 feet. In addition to the tabulation for same and contrary names of declination and latitude there is a section giving altitude and azimuth of 22 selected stars, the name of the star being given as one of the entering arguments to eliminate interpolation for declination. There are 14 volumes, each covering 5° of latitude, the total coverage extending from the equator to latitude 69° (north or south). These tables have been largely superseded by H.O. Pub. No. 249.

H.O. Pub. No. 221, *Loran Tables*, is discussed in article 416.

H.O. Pub. No. 249, *Sight Reduction Tables*, is intended primarily for air navigation. Volume I tabulates the altitude to the nearest 1', and *azimuth* (not azimuth angle) to the nearest 1°, for seven selected stars. Entries are given for each 1° (2° beyond latitude 69°) of local hour angle of the vernal equinox for each 1° of latitude from 89° N to 89° S, in a single volume.

Volumes II and III tabulate the altitude to the nearest 1' and *azimuth angle* to the nearest 1°, for each 1° of meridian angle (2° beyond 69°) and 1° of declination, from 0° to 29° (with separate tabulations for same and contrary name), for each 1° of latitude from 89° N to 89° S. Volume II covers latitudes 0° to 39°, and volume III covers latitudes 40° to 89°. Altitudes extend to negative values to provide for observation of bodies near the horizon from aircraft in flight.

423. Almanacs.—The positions of celestial bodies on the celestial sphere; times of sunrise, sunset, moonrise, moonset, and beginning and ending of twilight; sextant altitude corrections; and other astronomical information of particular interest to navigators are published by the U. S. Naval Observatory in books called "almanacs." *The Nautical Almanac*, published annually, tabulates the basic information to the nearest 0'1 at hourly intervals. *The Air Almanac*, published three times a year, tabulates essentially the same information to the nearest 1' at time intervals of ten minutes. Since 1960, *The American Ephemeris and Nautical Almanac* and *The Astronomical Ephemeris* have also been unified. Published annually, they tabulate a great amount

of astronomical information of interest primarily to astronomers. The information is generally tabulated to a precision much greater than needed by either marine or air navigators. All of these publications are published jointly by the United States and Great Britain.

424. Manuals.—Reference or instruction books are published by many sources, both governmental and commercial. Some of these are general, such as the present volume, and others are limited to particular aspects of the subject, such as H.O. Pub. No. 257, *Radar Plotting Manual*, H.O. Pub. No. 217, *Maneuvering Board Manual*, and H.O. Pub. No. 226, *Handbook of Magnetic Compass Adjustment and Compensation*. There are a great number and variety of such books. In general, they can be obtained from stores handling nautical publications, government agencies having cognizance over the subjects of the manuals, or instrument makers (in the case of manuals describing specific instruments). The U.S. Government Printing Office publishes lists of publications on a number of subjects and most other government agencies and commercial publishing companies have similar lists for distribution.

Periodical Publications and Broadcasts

425. *Notice to Mariners,* published weekly by the U.S. Navy Hydrographic Office, lists changes in aids to navigation throughout the world, newly reported dangers such as wrecks, important new soundings, and official regulations affecting navigation. It is the official publication for the correction of charts, sailing directions, light lists, etc. It also carries announcements of new charts, new editions of charts, and new publications. Two editions are published, one for the Atlantic and Mediterranean, and one for the Pacific and Indian Oceans. *Notice to Mariners* is distributed without charge to qualified users. It can be consulted at offices of sales agents for products of the U. S. Navy Hydrographic Office, U. S. Coast and Geodetic Survey, and U. S. Coast Guard; Branch Hydrographic Offices; District Offices of the Coast Guard; United States consulates abroad; and Centralization Offices in various ports of the world.

426. *Daily Memorandum,* published each working day by the U.S. Navy Hydrographic Office, gives a synopsis of late information relating to aids to navigation and dangers to vessels, including reports of ice, derelicts, etc. The urgent items are also broadcast by radio (art. 427). It also contains advance information of the more important material that will appear in the *Notice to Mariners*. This publication is distributed locally by the Branch Hydrographic Offices. An East Coast edition is published at Washington, a West Coast edition by the Branch Hydrographic Office in San Francisco, a Pacific edition by the Branch Hydrographic Office in Honolulu, a Far East edition by the Branch Hydrographic Office in Yokosuka, and a Canal Zone edition by the Branch Hydrographic Office in Cristobal, C. Z.

427. Radio broadcasts.—Nearly all maritime nations broadcast radio navigational warnings. In general, such broadcasts contain information of importance to the safety of vessels at sea, such as the position of ice and derelicts, inadequacy and changes in aids to navigation, mine fields, etc. Most of the information is furnished by cooperating observers at sea.

As a general rule, each nation broadcasts only those navigational warnings affecting its own coasts. In the United States the broadcasts are made by Navy and Coast Guard radio stations. The information is compiled by the U. S. Navy Hydrographic Office and the U. S. Coast Guard. Frequently, broadcasts include warnings from both agencies. The major items affecting the Atlantic and Gulf coasts and, occasionally, important Pacific and foreign notices are broadcast daily by station NSS, Washington. The major Pacific items are broadcast daily by station NPG, San Francisco. Usually the information contained in these general broadcasts is adequate for offshore

navigation, but before nearing the coast, vessels should obtain the information available in local broadcasts from the area to be entered. In general, the information contained in a local broadcast affects only the area in which the broadcasting station is located and, occasionally, adjacent areas.

Urgent messages, such as those concerning tsunamis (art. 3310), hurricanes, etc., are broadcast immediately upon receipt and at frequent intervals thereafter as long as they are applicable. In some countries provision is made whereby navigational warnings can be obtained upon request. In the majority of cases this information is a repetition of scheduled broadcasts.

Urgent messages pertaining to the Atlantic area are called **hydrolants** and those pertaining to the Pacific **hydropacs.** These terms refer to messages broadcast by the United States. Urgent messages pertaining to the Eastern Atlantic and Mediterranean and broadcast by the British Admiralty are called **naveams.**

In addition to navigational warnings, radio services include time signals, weather and ice reports and predictions, distress information, and medical advice. Full information on navigational radio broadcasts, including the times, stations, frequencies, and instructions for utilizing this service, is given in H.O. Pubs. Nos. 117–A and 117–B, *Radio Navigational Aids,* and H.O. Pubs. Nos. 118–A and 118–B, *Radio Weather Aids.*

Miscellaneous Charts and Publications

428. Isomagnetic charts.—The U. S. Navy Hydrographic Office publishes a series of charts of magnetic information. There are five groups, each consisting of one chart for each polar region and one for the remainder of the world, plus grid variation charts for each polar region. There is one group each showing lines of equal dip (inclination), horizontal intensity, vertical intensity, total intensity, and variation of the compass (magnetic declination). In addition to these charts, the Hydrographic Office publishes a 12-sheet series of charts showing world coverage of magnetic variation of the compass. The U. S. Coast and Geodetic Survey publishes charts showing lines of equal variation for the United States and Alaska, and other isomagnetic charts.

Variation is also shown on the navigator's regular nautical charts.

429. Great-circle charts.—The U. S. Navy Hydrographic Office publishes a number of charts on the gnomonic projection with the points of tangency selected so as to make the charts suitable for planning ocean voyages. These charts are customarily used in connection with regular nautical charts, the desired great circle being plotted as a straight line on the gnomonic chart and various points along the line being transferred by means of its geographical coordinates (latitude and longitude) to the nautical chart to mark the ends of a series of rhumb lines. Points along any desired great circle can also be established by computation (art. 822).

430. Aeronautical charts, although designed primarily for air navigation, are sometimes useful to the marine navigator as well. They often show more details of adjacent land than do nautical charts, and they show aeronautical beacons (lighted and radio) which can be of value to the marine navigator who understands their use. Such charts are published principally by the Aeronautical Chart and Information Center of the U. S. Air Force, the U. S. Coast and Geodetic Survey, and the U. S. Navy Hydrographic Office. These agencies publish catalogs listing their aeronautical charts and publications.

431. Plotting charts and plotting sheets are published by the U. S. Navy Hydrographic Office.

Plotting charts show the land area in outline. Soundings, aids to navigation, and other information customarily shown on nautical charts are not given. The charts are used principally for planning.

Plotting sheets are blank charts showing only the graticule of latitude and longitude lines, at a specified range of latitude, and compass roses. The meridians are not labeled, permitting the plotting sheet to be used at any longitude. A **universal plotting sheet** (art. 324) shows only the parallels of latitude, a central compass rose, and a single mid meridian. The user draws in the meridians at the correct interval, depending upon the latitude. In addition to the universal plotting sheet, there are four series of ordinary plotting sheets, each series of a different size. Details are given in Pub. No. 1–N, Introduction Part I (*Catalog of Nautical Charts and Publications*). Plotting sheets are used primarily for plotting celestial fixes and dead reckoning at sea. The navigator customarily uses a chart when near land.

432. Miscellaneous charts.—Both the U. S. Navy Hydrographic Office and the U. S. Coast and Geodetic Survey publish a number of special charts listed in their catalogs. Some of the H.O. charts are listed in Pub. No. 1–N. Among these are track charts, an air route chart of the world, an airline distance map of the United States, a time zone chart of the world, outline charts and maps, azimuthal equidistant charts centered on certain strategic cities, star charts, special charts for the fishing industry, and others.

433. Miscellaneous publications.—There are numerous other publications of greater or less interest to navigators. Among these are:

Chart No. 1, *Nautical Chart Symbols and Abbreviations.* A pamphlet showing the standard symbols in color and the various abbreviations which have been approved for use on nautical charts published by the United States of America. Much of this information is reproduced in appendix K.

H.O. Pub. No. 103, *International Code of Signals,* Vol. I (visual).

H.O. Pub. No. 110, *DAPAC* (Danger Areas in the Pacific).

Nemedri. A publication giving routing instructions for areas in northern European, Mediterranean, and Black Sea waters declared dangerous because of mines. This publication is distributed by the British Admiralty for the International Routing and Reporting Authority. A United States reprint is distributed without cost by the U. S. Navy Hydrographic Office.

H.O. Pub. No. 150, *World Port Index.*

H.O. Pub. No. 216, *Air Navigation.*

H.O. Pub. No. 220, *Navigation Dictionary.*

Weather maps and reports, available from the U. S. Weather Bureau.

H.O. Pub. No. 606–a, *Navigational Observations.* An instruction manual for the various observations on which the U. S. Navy Hydrographic Office desires reports.

H.O. Pub. No. 609, *A Functional Glossary of Ice Terminology.*

H.O. 2102–D, *Star Finder and Identifier.*

H.O. Misc. 10578, *Eskimo Place Names and Aids to Conversation,* compiled by Commander Donald B. MacMillan, USNR.

Rules of the Road—International—Inland. A pamphlet giving the international and inland rules of the road in parallel columns, followed by pilot rules for certain inland waters, published by the U.S. Coast Guard. Additional pamphlets or individual sheets published by the same agency give the specific rules applying to U.S. waterways. An example is the booklet entitled *Rules of the Road—Western Rivers.*

Aids to Marine Navigation of the United States, published by the U.S. Coast Guard.

CHAPTER V

THE NAUTICAL CHART

General Information

501. Introduction.—A nautical chart is a conventional graphic representation, on a plane surface, of a navigable portion of the surface of the earth. It shows the depth of water by numerous soundings, and sometimes by depth curves, the shore line of adjacent land, topographic features that may serve as landmarks, aids to navigation, dangers, and other information of interest to navigators. It is designed as a work sheet on which courses may be plotted, and positions ascertained. It assists the navigator in avoiding dangers and arriving safely at his destination. The nautical chart is one of the most essential and reliable aids available to the navigator.

502. Projections.—Nearly all nautical charts used for ordinary purposes of navigation are constructed on the Mercator projection (art. 305). Large-scale harbor charts on standard scales (1:12,500, 1:25,000, 1:50,000) are often constructed on the transverse Mercator projection. Charts for special purposes, such as great-circle sailing or polar navigation, often are on some other projection. Many aeronautical charts are constructed on the Lambert conformal projection (art. 314). The principal projections, with their navigational uses, are discussed in chapter III.

503. Scale.—The *scale* of a chart is the ratio of a given distance on the chart to the actual distance which it represents on the earth. It may be expressed in various ways. The most common are:

Natural scale, expressed as a simple ratio or fraction. For example, 1:80,000 or $\frac{1}{80,000}$ means that one unit (such as an inch) on the chart represents 80,000 of the same unit on the surface of the earth.

Numerical scale, or a statement of that distance on the earth shown in one unit (usually an inch) on the chart, or vice versa. For example, "30 miles to the inch" means that one inch on the chart represents 30 miles of the earth's surface. Similarly, "2 inches to a mile" indicates that 2 inches on the chart represent 1 mile on the earth.

Graphic scale. A line or bar may be drawn at a convenient place on the chart and subdivided into nautical miles, yards, etc. All charts vary somewhat in scale from point to point, and in some projections the scale is not the same in all directions about a single point. A single subdivided line or bar for use over an entire chart is shown only when the chart is of such scale and projection that the scale varies a negligible amount over the chart, usually one of about 1:50,000 or larger. Since one minute of latitude is very nearly equal to one nautical mile, the latitude scale serves as an approximate graphical scale. On most nautical charts the east and west borders are subdivided to indicate the latitude scale.

On a Mercator chart the scale varies with the latitude. This is noticeable on a chart covering a relatively large distance in a north-south direction. On such a chart the scale at the latitude in question should be used for measuring distances.

Of the various methods of indicating scale, the graphical method is normally available in some form on the chart. In addition, the natural scale is customarily stated on charts on which the scale does not change appreciably over the chart.

The natural and numerical scales of a chart are readily interchangeable. For instance, in a nautical mile there are about 6,076.11549 feet or $6{,}076.11549\times 12=72{,}913.39$ inches. If the natural scale of a chart is 1:80,000, one inch of the chart represents 80,000 inches of the earth, or a little more than a mile. To find the exact amount, divide the scale by the number of inches in a mile, or $\frac{80{,}000}{72{,}913.39}=1.097$. Thus, a natural scale of 1:80,000 is the same as a numerical scale of 1.097 (or approximately 1.1) miles to an inch. Stated another way, there are $\frac{72{,}913.39}{80{,}000}=0.911$ (approximately 0.9) inch to a mile. Similarly, if the numerical scale is 60 nautical miles to an inch, the natural scale is $1{:}(60\times 72{,}913.39)=1{:}4{,}374{,}803$.

It should be clearly understood that scale, as discussed above, refers to *distances*, not *areas*. If the area scale is desired, it is found by squaring the natural scale. Thus, if the natural scale of a chart is 1:50,000, the corresponding area scale is $1{:}(50{,}000\times 50{,}000)=1{:}2{,}500{,}000{,}000$ or one square inch on the chart represents 2,500,000,000 square inches on the earth, or a square 50,000 inches on a side.

A chart covering a relatively large area is called a *small-scale* chart and one covering a relatively small area is called a *large-scale* chart. Since the terms are relative, there is no sharp division between the two. Thus, a chart of scale 1:100,000 is large scale when compared with a chart of 1:1,000,000 but small scale when compared with one of 1:25,000.

504. Chart classification by scale.—Charts are constructed on many different scales, ranging from about 1:2,500 to 1:14,000,000 (and even smaller for some world charts). Small-scale charts covering large areas are used for planning and for offshore navigation. Charts of larger scale, covering smaller areas, should be used as the vessel approaches pilot waters. Several methods of classifying charts according to scale are in use in various nations. The following classifications of nautical charts are those used by the U. S. Navy Hydrographic Office and the U. S. Coast and Geodetic Survey:

Sailing charts are the smallest scale charts used for planning, fixing position at sea, and for plotting the dead reckoning while proceeding on a long voyage. The scale is generally 1:600,000 or smaller. The shore line and topography are generalized and only offshore soundings, the principal navigational lights, outer buoys, and landmarks visible at considerable distances are shown.

General charts are intended for coastwise navigation outside of outlying reefs and shoals. The scales range from about 1:100,000 to 1:600,000.

Coast charts are intended for inshore coastwise navigation where the course may lie inside outlying reefs and shoals, for entering or leaving bays and harbors of considerable width, and for navigating large inland waterways. The scales range from about 1:50,000 to 1:100,000.

Harbor charts are intended for navigation and anchorage in harbors and small waterways. The scale is generally larger than 1:50,000.

In addition, there are special series of charts, such as the 1:40,000 U. S. Coast and Geodetic Survey charts of the Intracoastal Waterway (inside route) and various series of river and canal charts.

505. Accuracy.—The accuracy of a chart depends upon:

1. *Thoroughness and up-to-dateness of the survey and other navigational information.* Some estimate of the accuracy of the survey can be formed by an examination of the source notes given in the title of the chart. If the chart is based upon very old surveys, it should be used with caution. Many of the earlier surveys were made under

conditions that were not conducive to great accuracy. It is safest to question every chart based upon surveys of doubtful accuracy.

The number of soundings and their spacing is some indication of the completeness of the survey. Only a small fraction of the soundings taken in a thorough survey are shown on the chart, but sparse or unevenly distributed soundings indicate that the survey was probably not made in detail. Large or irregular blank areas, or absence of depth curves, generally indicate lack of soundings in the area. If the water surrounding such a blank area is deep, there is generally considerable depth in the blank; conversely, shallow water surrounding such an area indicates the strong possibility of shoal water. If neighboring areas abound in rocks or are particularly uneven, the blank area should be regarded with additional suspicion. However, it should be kept in mind that relatively few soundings are shown when there is a large number of depth curves or where the bottom is flat or gently and evenly sloping. Additional soundings are shown when they are helpful in indicating the uneven character of a rough bottom (figs. 505a and 505b).

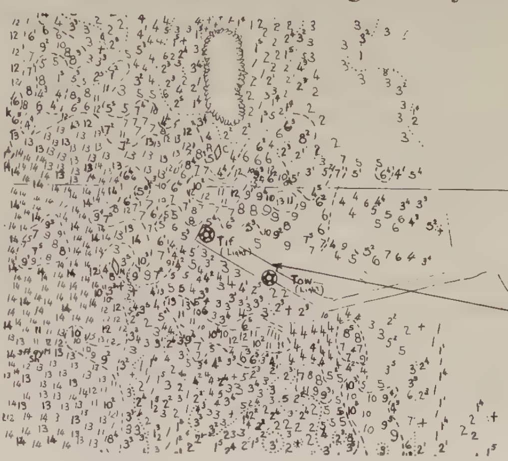


Figure 505a.—Part of a boat sheet, showing the soundings obtained in a survey.

Even a detailed survey may fail to locate every rock or pinnacle, and in waters where their existence is suspected, the best methods for determining their presence are wire drag surveys or use of electronic underwater obstacle detection gear. Areas that have been dragged may be indicated on the chart and a note added to show the effective depth at which the drag was operated.

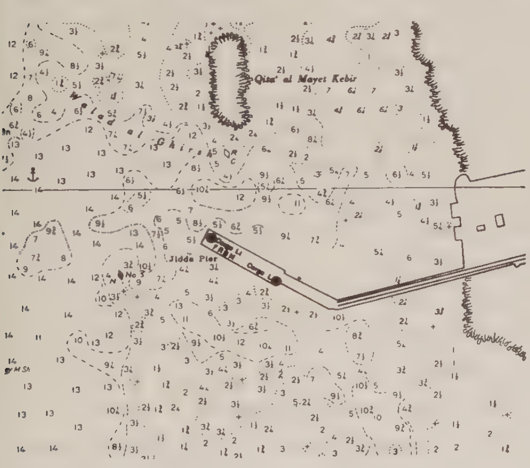


Figure 505b.—Part of a nautical chart made from the boat sheet of figure 505a. Compare the number of soundings in the two figures.

Changes in the contour of the bottom are relatively rapid in areas where there are strong currents or heavy surf, particularly when the bottom is composed principally of soft mud or sand. The entrances to bar harbors are especially to be regarded with suspicion. Similarly, there is sometimes a strong tendency for dredged channels to shoal, especially if they are surrounded by sand or mud, and cross currents exist. Notes are sometimes shown on the chart when the bottom contours are known to change rapidly. However, the absence of such a note should not be regarded as evidence that rapid change does not occur.

Changes in aids to navigation, structures, etc., are more easily determined, and charts are generally corrected in this regard to the date of distribution. However, there is always the possibility of a change having occurred since the chart was mailed or received. The date to which the chart was corrected is stamped on it

before distribution. All issues of *Notice to Mariners* printed after that date should be checked to ensure accuracy in this respect.

2. *Suitability of the scale for the design and intended navigational use.* The same detail cannot be shown on a small-scale chart as on one of a larger scale. For this reason it is good practice to use the largest scale chart available when in the vicinity of shoals or other dangers.

3. *Presentation and adequacy of data.* The amount and kind of detail to be shown, and the method of presentation, are continually under study by charting agencies. Development of a new navigational aid may render many previous charts inadequate. An example is radar. Many of the charts produced before radar became available lack the detail needed for confident identification of targets.

Part of the responsibility for the continuing accuracy of charts lies with the user. If charts are to remain reliable, they must be corrected as indicated by the *Notice to Mariners*. In addition, the user's reports of errors and changes and his suggestions often are useful to the publishing agencies in correcting and improving their charts. Navigators and maritime activities have contributed much to the reliability and usefulness of the modern nautical chart. If a chart becomes wet, the expansion and subsequent shrinkage when the chart dries are likely to cause distortion.

506. Dates on charts.—The system of dates now used on charts published by the U. S. Coast and Geodetic Survey and the U. S. Navy Hydrographic Office is as follows:

First edition. The original date of issue of a new chart is shown at the lower left-hand corner and at the top center margin, thus:

1st Ed., Sept. 1901

New edition. A new edition is made when, at the time of printing, the corrections are too numerous or too extensive to be reported in *Notice to Mariners*, making previous printings obsolete. The date of the first edition is retained at the top margin. At the lower left-hand corner it is replaced by the number and date of the new edition, thus:

5th Ed., July 11, 1955

Corrected (New) print. A corrected print contains corrections which have been published in the *Notice to Mariners*, and other information which is not of sufficient importance to justify a new edition. The date of a corrected print is the date on which the last check is made to see that all important corrections have been applied. Normally, this date is the Monday following the date of the last *Notice to Mariners* used. It is added at the lower left-hand corner of the chart, thus:

5th Ed., July 11, 1955; Revised 2/4/57

For any subsequent corrected prints the date is replaced by the later one, thus:

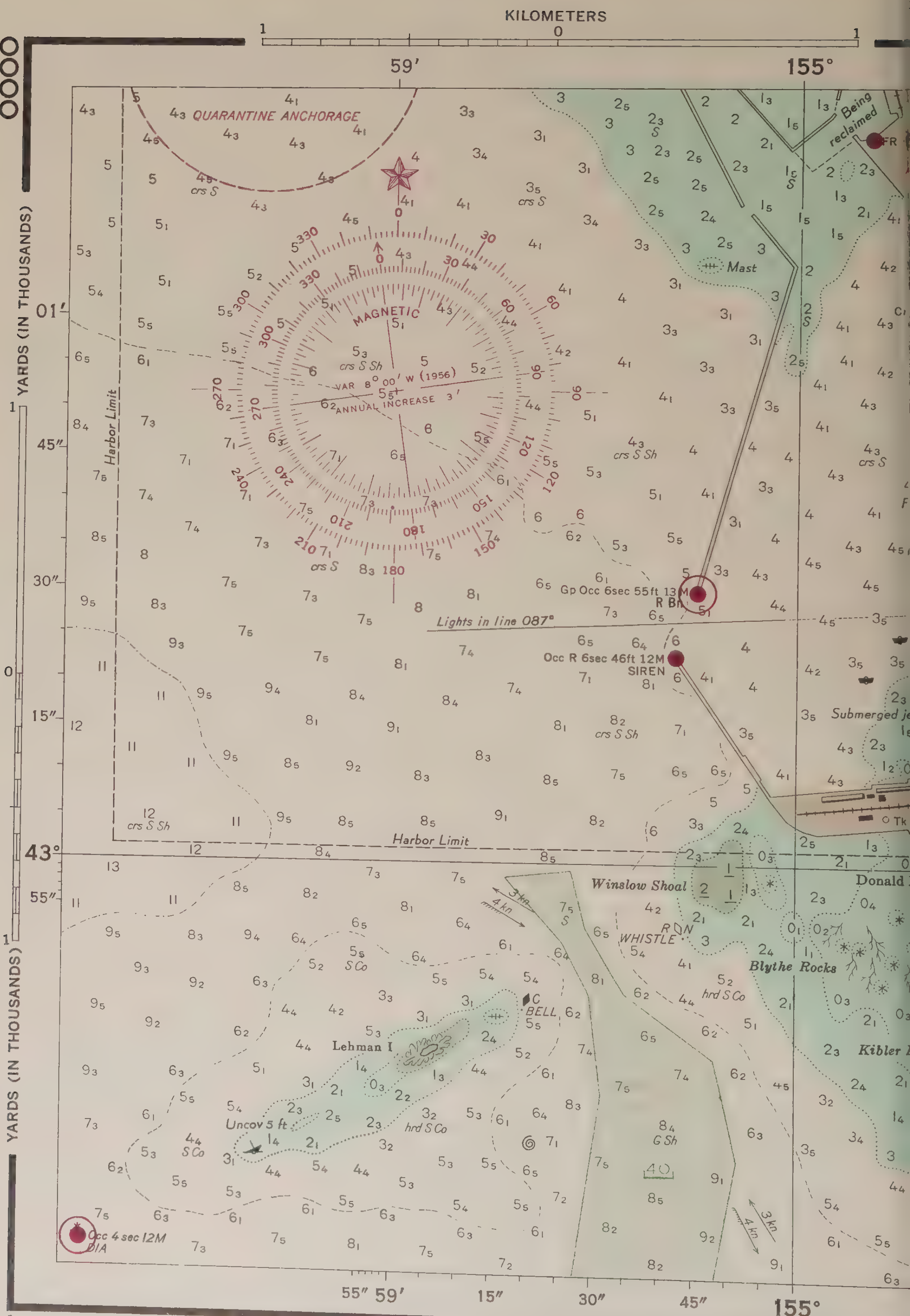
5th Ed., July 11, 1955; Revised 2/17/58

Hydrographic Office chart terminology is discussed in article 4406.

Hand-correction date. Stocks of charts kept on hand by the publishing agencies or their distribution centers are hand corrected for changes shown in *Notice to Mariners* published prior to the date of distribution. The date of the latest issue for which hand corrections have been made is stamped in the margin. This is the most important date shown on the chart. Important changes after the date of the latest hand-corrected change are published in the weekly *Notice to Mariners* and should be applied immediately by the user.

Chart Reading

507. Chart symbols.—Much of the information contained on charts is shown by conventional symbols which make no attempt at accuracy in scale or detail, but are



1st Ed., June 18, 1956

0000 PRICE 00 CENTS

STANDARD NAUTICAL CHART
(Area Fictitious)

Published a
by the U. S. NAVY
under the authority of t

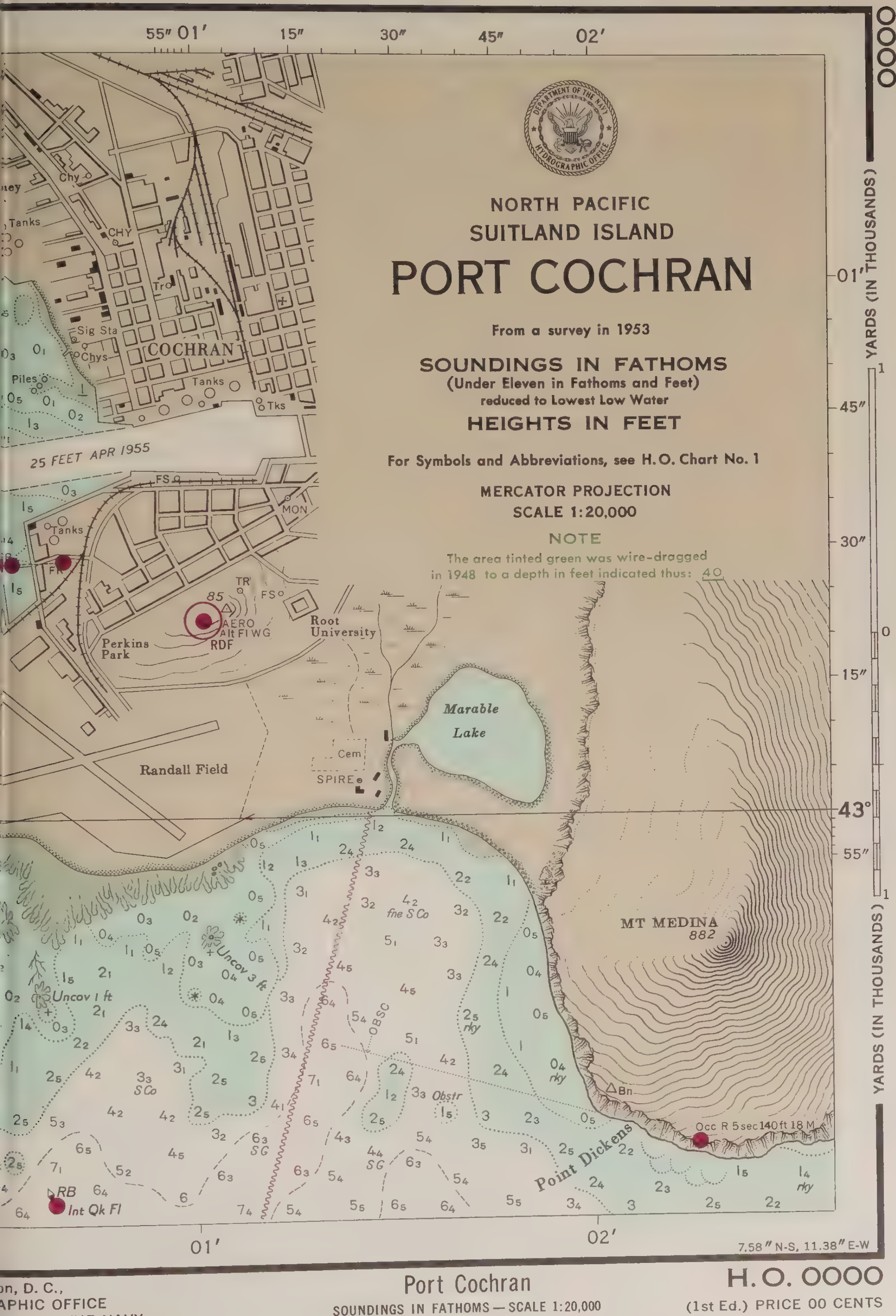


FIGURE 507.— A fictitious sample nautical chart.

shown at the correct location and make possible the showing of a large amount of information without congestion or confusion. The standard symbols and abbreviations which have been approved for use on regular nautical charts published by the United States of America are shown in appendix K. A knowledge of the meanings of these symbols is essential to a full understanding of charts. A fictitious sample chart showing a number of these symbols is given in figure 507.

Most of the symbols and abbreviations shown in appendix K are in agreement with those recommended by the International Hydrographic Bureau. Where there is disagreement, the number of the symbol or abbreviation is in leaning figures. Vertical figures enclosed in parentheses indicate that the symbol or abbreviation is an addition to those recommended by the International Hydrographic Bureau.

The symbols and abbreviations on any given chart may differ somewhat from those shown in appendix K because of a change in the standards since printing of the chart or because the chart was published by an agency having a different set of standards.

508. Lettering.—Certain standards regarding lettering have been adopted, except on charts made from reproducibles furnished by foreign nations.

Vertical type is used for features which are dry at high water and not affected by movement of the water, except for heights above water.

Leaning type is used for water, underwater, and floating features, except soundings.

The type of lettering used may be the only means of determining whether a feature may be visible at high tide. For instance, a rock might bear the title "________ Rock" whether or not it extends above the surface. If the name is given in vertical letters, the rock constitutes a small islet; if in leaning type, the rock constitutes a reef.

509. The shore line shown on nautical charts is the boundary between water and land at high tide (usually mean high water). A broken line indicates that the charted position is approximate only. The nature of the shore may be indicated, as shown by the symbols in part A of appendix K.

Where the low-water line differs considerably from the high-water line, the low-water line may be indicated by dots in the case of mud, sand, gravel, or stones, with the kind of material indicated, and by a characteristic symbol in the case of rock or coral. The area alternately covered and uncovered may be shown by a tint which is a combination of the land tint and a blue water tint.

In marsh or mangrove areas, the outer edge of vegetation is used as the shore line. The inner edge is marked by a broken line when no other symbol (such as a cliff, levee, etc.) furnishes such a limit. The area between inner and outer limits may be given the combined land-water tint or the land tint.

510. Water areas.—Soundings or depths of water are shown in several ways. Individual soundings are shown by numbers. These do not follow the general rule for lettering. They may be either vertical or leaning, or both may be used on the same chart to distinguish between the data based upon different surveys, different datums, or furnished by different authorities.

On all charts produced from surveys by United States vessels, soundings are shown in English units (feet or fathoms). The unit used is shown in the chart legend. Foreign charts may show depths in meters, and occasionally in other units. The units used on charts of various nations are shown in appendix L.

"No bottom" soundings are indicated by a number with a line over the top and a dot over the line, thus: $\dot{\overline{45}}$. This indicates that the spot was sounded to the depth indicated without reaching the bottom. Areas which have been wire dragged are shown by a limiting line, and the clear effective depth is indicated, with a characteristic symbol under the numbers, thus: $\llcorner\underline{24}\lrcorner$.

The soundings are supplemented by a series of bottom contours or *depth curves* connecting points of equal depth. These lines present a graphic indication of the configuration of the bottom. The types of lines used for various depths are shown in part R of appendix K. On some of the recent charts of the U. S. Coast and Geodetic Survey an increased number of depth curves have been shown in solid blue or black lines, the depth represented by each being shown by numbers placed in breaks in the lines, as with land contours. This type chart, presenting a more detailed indication of the bottom configuration with fewer numerical soundings, is particularly useful to the vessel equipped with an echo sounder permitting continuous determination of a profile of the bottom. Such a chart, to be reliable, can be made only for areas which have been surveyed in great detail.

Areas which uncover at low tide are tinted as indicated in article 509. Those areas out to a given depth, usually one, two, or three fathoms, often are given a blue tint, and occasionally a lighter blue is carried to some greater depth, usually five fathoms. On older charts the one-, two-, and three-fathom curves have stippled edges. Charts designed to give maximum emphasis to the configuration of the bottom show depths over the entire chart by a series of blue gradient tints similar to the tints sometimes shown on land areas to indicate graduations in height. These are called *bathymetric* charts.

The side limits of dredged channels are indicated by broken lines. The controlling depth and the date it was determined, if known, are shown by a statement in or along the channel. The *controlling* depth is not necessarily an indication of the *least* depth in the channel on the date of determination. For channels less than 100 feet in width, at least 80 percent at the center is clear to the charted (controlling) depth. For channels more than 100 feet in width, at least the 50 percent at the center is clear to the charted depth. The possibility of shoaling since the controlling depth was determined should be considered.

The chart scale is generally too small to permit all soundings to be shown. In the selection of soundings to be shown, *least* depths are generally chosen first and a sounding pattern worked out to provide safety, a practical presentation of the bottom configuration, and a neat appearance. Depths greater than those indicated may be found close to charted depths, but steep changes in depth are given every consideration in sounding selection. Also, the state of the tide affects the depth at any given moment. An isolated shoal sounding should be approached with caution, or avoided, unless it is known that the area has been wire dragged, for there is always the possibility that a depth less than the least shown may have escaped detection. Also, the shoal area near a coast little frequented by vessels is sometimes not surveyed with the same thoroughness as other areas. Such areas and those where rocks, coral, etc., are known to exist should be entered with caution, or avoided.

The substance forming the bottom is shown by abbreviations, as listed in part S of appendix K. The meaning of some of the less-well-known terms is given below:

Ooze is a soft, slimy, organic sediment composed principally of shells or other hard parts of minute organisms.

Marl is a crumbling, earthy deposit, particularly one of clay mixed with sand, lime, decomposed shells, etc. A layer of marl may become quite compact.

Shingle consists of small, rounded, waterworn stones. It is similar to gravel but with the average size of stone generally larger.

Schist is crystalline rock of a finely laminated nature.

Madrepore is a stony coral which often forms an important building material for reefs.

Lava is rock in the fluid state, or such material after it has solidified. It is formed at very high temperature and issues from the earth through volcanoes.

Pumice is cooled volcanic glass with a great number of minute cavities caused by the expulsion of water vapor at high temperature, resulting in a very light material.

Tufa is a porous rocky deposit sometimes formed in streams and in the ocean near the mouths of rivers.

Scoria (plural *scoriae*) is rough, cinderlike lava.

Seatangle is any of several species of seaweed, especially those of large size.

Spicules are the small skeletons of various marine animals such as sponges.

Foraminifera (plural) are small marine animals with hard shells of from one to many chambers.

Globigerina is a very small marine animal of the foraminifera order, with a chambered shell, or the shell of such an animal. In large areas of the ocean the calcareous shells of these animals are very numerous, being the principal constituent of a soft mud or **globigerina ooze,** forming part of the ocean bed.

Diatom is a microscopic animal with external skeletons of silica, often found in both fresh and salt water. Part of the ocean bed is composed of a sedimentary ooze consisting principally of large collections of the skeletal remains of diatoms.

Radiolaria (plural) are minute sea animals with a siliceous outer shell. The skeletons of these animals are very numerous, especially in the tropics.

Pteropod is a small marine animal with or without a shell and having two thin, winglike feet. These animals are often so numerous they cover the surface of the sea for miles. In some areas their shells cover the bottom.

Polyzoa (plural) are very small marine animals which reproduce by budding, many generations often being permanently connected by branchlike structures.

Cirripeda (plural) are barnacles and certain other parasitic marine animals.

Fucus is a coarse seaweed growing attached to rocks.

Matte is a dense, twisted growth of a sea plant such as grass.

"Calcareous" is an adjective meaning "containing or composed of calcium or one of its compounds."

511. Chart datum.—*Depths.* All depths indicated on charts are reckoned from some selected level of the water, called the *chart datum.* The various chart datums are explained in chapter XXXI. On charts made from surveys conducted by the United States the chart datum is selected with regard to the tides of the region, so that depths might be shown in their least favorable aspect. On charts based upon those of other nations the datum is that of the original authority. When it is known, the datum used is stated on the chart. In some cases where the chart is based upon old surveys, particularly in areas where the range of tide is not great, the actual chart datum may not be known.

For U. S. Coast and Geodetic Survey charts of the Atlantic and Gulf coasts of the United States and Puerto Rico the chart datum is *mean low water.* For charts of the Pacific coast of the United States, including Alaska, it is *mean lower low water.* Most U. S. Navy Hydrographic Office charts are based upon *mean low water, mean lower low water,* or *mean low water springs.* The chart datum for British Admiralty charts based upon British surveys is *mean low water springs* in areas where the daily inequality is small, and *Indian spring low water* where the daily inequality is large. The chart datum for charts published by other countries varies greatly, but is usually lower than mean low water. On charts of the Baltic Sea, Black Sea, the Great Lakes, and other areas where tidal effects are small or without significance, the datum adopted is an arbitrary height approximating the mean water level. Chart datums used in various areas are shown in appendix M.

The chart datum of the largest-scale charts of an area is generally the same as the reference level from which height of tide is tabulated in the tide tables.

The height of a chart datum is usually only an approximation of the actual mean value specified, for determination of the actual mean height usually requires a longer series of tidal observations than is available to the cartographer, and the height changes somewhat over a period of time.

Since the chart datum is generally a computed mean or average height at some state of the tide, the depth of water at any particular moment may be less than shown on the chart. For example, if the chart datum is *mean lower low water*, the depth of water at *lower low water* will be less than the charted depth about as often as it is greater. A lower depth is indicated in the tide tables by a minus sign (—).

Heights. The shore line shown on charts is the high-water line, generally the level of mean high water. The heights of lights, rocks, islets, etc., are generally reckoned from this level. However, heights of islands, especially those at some distance from the coast, are often taken from sources other than hydrographic surveys, and may be reckoned from some other level, often mean sea level. The plane of reference for topographic detail is frequently not stated on the chart.

Since heights are usually reckoned from high water and depths from some form of low water, the reference levels are seldom the same. This is generally of little practical significance, but it might be of interest under some conditions, particularly where the range of tide is large.

512. Dangers are shown by appropriate symbols, as indicated in part O of appendix K.

A rock which is uncovered at mean high water is shown as an islet enclosed by a dotted line to make it more prominent. If an isolated, offlying rock is known to uncover at the chart datum but to be covered at high water, the appropriate symbol is shown and the height above the chart datum, if known, is usually given, either by statement such as "*Uncov 2 ft*" or by the figure indicating the number of feet above the chart datum underlined and usually enclosed in parentheses, thus: ($\underline{2}$). This is illustrated in figure 512a. A rock which does not uncover is shown by the appropriate symbol. If it is considered a danger to surface vessels, the symbol is enclosed by a dotted curve.

A distinctive symbol is used to show a detached coral reef which uncovers at the chart datum. For a coral or rocky reef which is submerged at chart datum, the sunken rock symbol or an appropriate statement is used, enclosed by a dotted or dashed line if the limits have been determined.

Several different symbols are used for wrecks, depending upon the nature of the wreck or scale of the chart. The usual symbol for a visible wreck is shown in figure 512b. A sunken wreck with less than ten fathoms of water over it is considered dangerous and its symbol is surrounded by a dotted curve. The safe clearance depth found over a wreck is indicated by a standard sounding number placed at the wreck. If this depth is determined by a wire drag, the sounding is underscored by the wire drag symbol (art. 510).

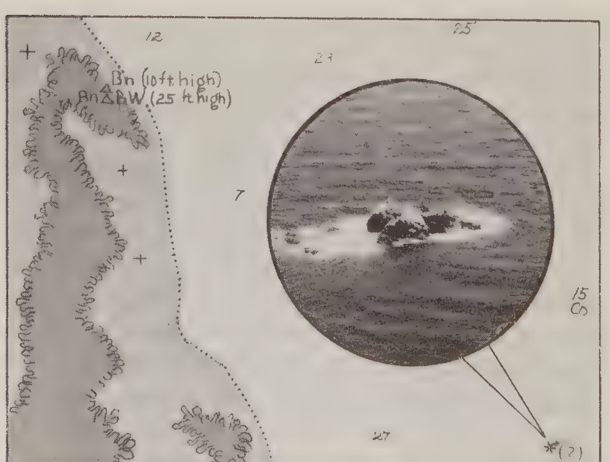


FIGURE 512a.—A rock awash.

Tide rips, eddies, and kelp are shown by symbol or lettering.

Piles, dolphins (clusters of piles), snags, stumps, etc., are shown by small circles and a label

identifying the type of obstruction. If such dangers are submerged, the letters "Subm" precede the label.

513. Aids to navigation are shown by symbol, as given in appendix K, usually supplemented by abbreviations and sometimes by additional descriptive text. In order to render the symbols conspicuous it is necessary to show them in greatly exaggerated size relative to the scale of the chart. It is therefore important that the navigator know which part of the symbol represents the actual position of the aid. For floating aids (lightships and buoys), the position part of the symbol marks the location of the anchor or sinker, the aid swinging in an orbit around this position.

FIGURE 512b.—A visible wreck.

The principal charted aids to navigation are lighthouses, beacons, lightships, radiobeacons, and buoys. The number of aids shown and the amount of information concerning them varies with the scale of the chart. Wherever distance of visibility is given, it is computed for a height of eye of the observer of 15 feet. Unless otherwise indicated, lights which do not alternate in color are white, and alternating lights are red and white. Light lists give complete navigational information concerning them.

Lighthouses and *lighted beacons* are shown as black dots surrounded by magenta disks. The disks for primary lighthouses are a little larger than those for beacons. In either case, the center of the black dot within the magenta disk is the position of the light. On older charts a six-pointed star symbol was used for primary lighthouses and a five-pointed star symbol for beacons. The center of the star symbol marks the position of the light.

On large-scale charts the characteristics of lights are shown in the following order:

Characteristic	*Example*	*Meaning*
1. Character	Gp. Fl.	group flashing
2. Color	R	red
3. Period	(2) 10 sec.	two flashes every 10 seconds
4. Height	160 ft.	160 feet
5. Visibility	19M	visible 19 nautical miles (15 ft. height of eye)
6. Number	"6"	light number 6

The legend for this light would appear on the chart:

Gp. Fl. R (2) 10 sec. 160 ft. 19 M "6"

On older charts this form is varied slightly. As the chart scale becomes smaller the six items listed above are omitted in the following order: first, height; second, period (seconds); third, number (of flashes, etc.) in group; fourth, light number; fifth, visibility. Names of unnumbered lights are shown when space permits.

Daybeacons (unlighted beacons) are shown by small triangles, the center of the triangle marking the position of the aid. Except on Intracoastal Waterway charts the abbreviation Bn is shown beside the symbol, with the appropriate abbreviation for color. For black beacons the triangle is solid black and there is no color abbreviation. All

beacon abbreviations are in vertical lettering, as appropriate for fixed aids (fig. 513a).

Lightships are shown by ship symbol, the center of the small circle at the base of the symbol indicating the position of the lightship's anchor. On recent charts the circle is overprinted by a small magenta disk as shown in figure 513b. As a floating aid, the light characteristics and the name of the lightship are given in leaning letters.

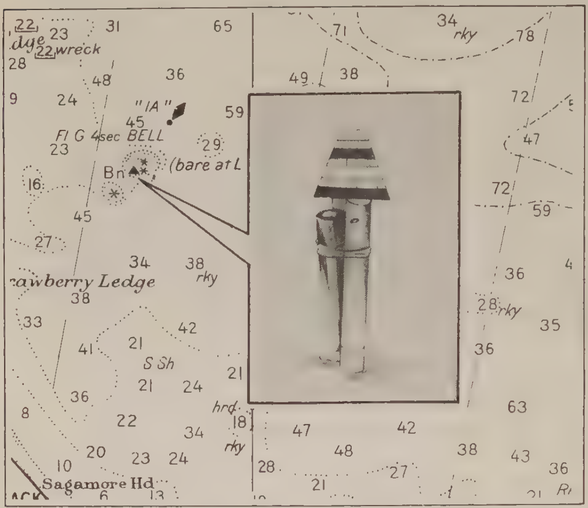


FIGURE 513a.—A daybeacon.

Radiobeacons are indicated on the chart by a small magenta circle, as shown in figure 513b, accompanied by the appropriate abbreviation to indicate whether an ordinary radiobeacon (R Bn) or a radar beacon (Racon). The same symbol is used for a radio direction finder station with the abbreviation "RDF" and a coast radar station with the abbreviation Ra. Other radio stations are indicated by a small black circle with a dot in the center, or a smaller circle without a dot, and the appropriate abbreviation. In every case the center of the circle marks the position of the aid.

Buoys, except mooring buoys, are shown by a diamond-shaped symbol and a small dot outside the diamond and near one of its points (at one of its acute angles). The dot indicates the position of the buoy's sinker. A mooring buoy is shown by a distinctive symbol as indicated at number 22 of part L, appendix K. The small circle interrupting the symbol's base line indicates the position of the sinker.

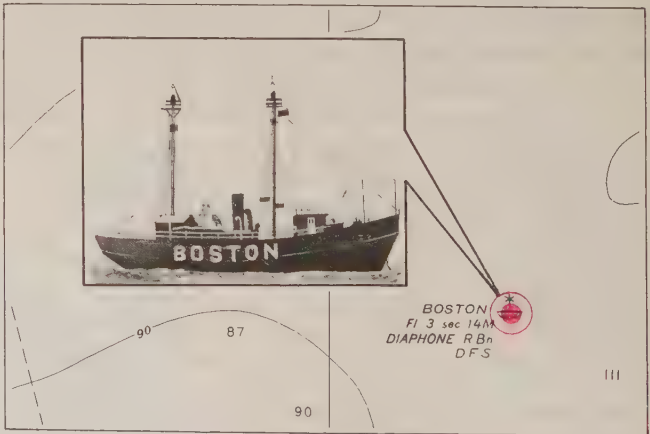


FIGURE 513b.—A lightship with a radiobeacon.

A black buoy is shown by a solid black diamond symbol, without abbreviation. For all other buoys, color is indicated by an abbreviation, or in full by a note on the chart. In addition, the diamond shape of symbols of red buoys are colored magenta. A buoy symbol with a line connecting the side points (shorter axis), half of the symbol being magenta or open and the other half black, indicates horizontal bands. A line connecting the upper and lower points (longer axis) represents vertical stripes. Two lines connecting the opposite *sides* of the symbol indicate a checkered buoy.

There is no significance to the angle at which the diamond-shape appears on the chart. The symbol is placed so as to avoid interference with other features of the chart.

Lighted bouys are indicated by a small magenta disk centered on the dot or small

circle indicating the position of the buoy's sinker, as shown in figure 513c. On older charts a series of radiating lines representing light rays was drawn about the dot or circle, and no magenta disk was given.

Abbreviations for light characteristics, type and color of buoy, number of the buoy, and any other pertinent information given near the symbol are in leaning letters. The letter C, N, or S, indicates a can, nun, or spar, respectively (art. 917). The words "bell," "gong," and "whistle," are shown as BELL, GONG, and WHIS, respectively. The number or letter designation of the buoy is given in quotation marks on small-scale charts. On large-scale charts they are given without quotation marks or punctuation, thus: No 1, No 2, etc.

Station buoys are not shown on small-scale charts, but are given on some large-scale charts.

Aeronautical lights included in the light lists are shown by the lighthouse symbol, accompanied by the abbreviation "AERO". The completeness to which the characteristics are shown depends principally upon the effective range of other navigational lights in the vicinity, and the usefulness of the light for marine navigation.

Ranges are indicated by a dashed or solid line. If the direction is given, it is expressed in degrees clockwise from true north.

Fog signal apparatus is indicated by the appropriate word in capital letters (HORN, BELL, GONG, etc.) or an abbreviation indicating the type of sound. The letters "D.F.S." indicate a **distance finding station** having synchronized sound and radio signals. The location of a fog signal which does not accompany a visual aid, either lighted or unlighted, is shown by a small circle and the appropriate word in vertical block letters.

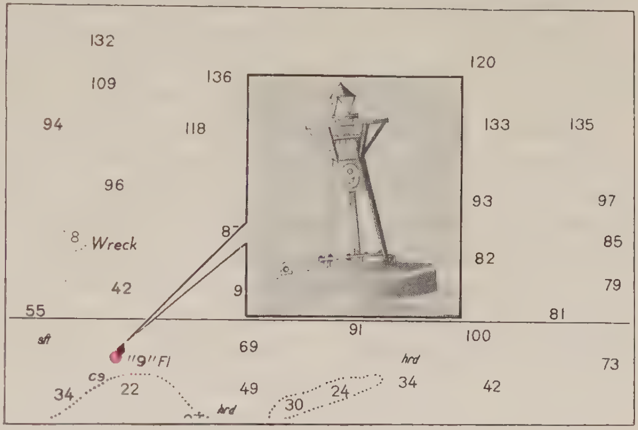


FIGURE 513c.—A lighted buoy.

Private aids are not indicated on the chart except in special cases. When they are shown, they are marked "Privately maintained" or "Priv. maintd." Any privately maintained unlighted aid is indicated by a small circle accompanied by the word "Marker," or a larger circle with a dot in the center and the word "MARKER," the symbols for any landmark or conspicuous object not having a distinctive symbol. The center of the circle indicates the position of the aid. A privately maintained lighted aid has the light symbol and is accompanied by the characteristics and the usual indication of its private nature.

Floats are indicated by the open buoy symbol accompanied by the word "FLOAT." Either the lighted or unlighted symbol is used, as appropriate, to indicate whether or not the float displays a light.

A *light sector* is the sector or area bounded by two radii and the arc of a circle in which a light is visible or in which it has a distinctive color different from that of adjoining sectors. The limiting radii are indicated on the chart by dotted lines.

Colors of the sectors are indicated by words spelled out if space permits, or by abbreviation (W, R, etc.) if it does not.

Limits of light sectors and arcs of visibility *as observed from a vessel* are given in the light lists, in clockwise order.

514. Land areas.—The amount of detail shown on the land areas of nautical charts depends upon the scale, the intended purpose of the chart, and available information. Since the advent of radar, topographical details have increased and have been extended farther inland, where this information has been available.

Relief is shown by contours, form lines, hachures, or tint shading. Tint shading is used principally to stress those terrain features affecting surface radar returns. It may be shown with or without contours and spot elevations.

Contours are lines connecting points of equal elevation. The heights represented by the contours are indicated in leaning figures at suitable places along the lines. Heights are usually expressed in feet (or in meters with means for conversion to feet on certain special charts). The interval between contours is uniform over any one chart, except that certain intermediate contours are sometimes shown by dashed line. When contours are broken, their locations are approximate.

Form lines are approximations of contours used for the purpose of indicating relative elevations. They are used in areas where accurate information is not available in sufficient detail to permit exact location of contours. Elevations of individual form lines are not indicated on the chart.

Hachures are short lines, or groups of lines, indicating the direction and extent of steep slopes. The lines generally follow the direction of the slope, the length of the lines indicating the height of the slope. Distinctive symbols somewhat resembling hachures are used for cliffs or other steep slopes on or near the coast line, where contours or form lines, being virtually over each other, would be difficult to interpret or would fail to give a true indication of the nature of the terrain.

Spot elevations are generally given only for summits or for tops of conspicuous landmarks. The heights of spot elevations and contours are given with reference to mean high water when this information is available.

When there is insufficient space to show the heights of islets or rocks, they are indicated by leaning figures enclosed in parentheses in the water area nearby.

Cities and roads. Except on the smaller scale charts, cities are usually represented by their street systems or a conventional system of intersecting lines. The symbol for large cities approximates their extent and shape. Street names are generally not charted except those along the waterfront on the largest scale charts. Only the more important streets are shown on smaller scale charts. In general, only the important through highways and roads leading from them to the waterfront are shown. Occasionally, highway numbers are given. When shown, trails are indicated by a light broken line. Buildings along the waterfront or individual ones back from the waterfront but of special interest to the mariner are shown on large-scale charts. Special symbols are used for certain kinds of buildings, as indicated in part I of appendix K. Both single and double track railroads are indicated by a single line with cross marks. In general, city electric railways are not charted. A fence or sewer extending into the water is shown by a broken line, usually labeled. Airports are shown on small-scale charts by symbol and on large-scale charts by shape and extent of runways. Breakwaters and jetties are shown by single or double lines depending upon the scale of the chart. A submerged portion and the limits of the submerged base are shown by broken lines.

515. Landmarks are shown by symbols, as given in appendix K. Some of the accompanying labels encountered on a chart are interpreted as follows:

Building or **house.** One of these terms, as appropriate, is used when the entire structure is the landmark, rather than an individual feature of it.

A **spire** is a slender pointed structure extending above a building. It is seldom less than two-thirds of the entire height of the structure, and its lines are rarely broken by stages or other features. The term is not applied to a short pyramid-shaped structure rising from a tower or belfry.

A **cupola** (kū′pō·lȧ) is a small dome-shaped tower or turret rising from a building (fig. 515).

A **dome** is a large, rounded, hemispherical structure rising above a building, or a roof of the same shape. A prominent example is that of the Capitol of the United States, in Washington.

A **chimney** is a relatively small, upright structure projecting above a building for the conveyance of smoke.

A **stack** is a tall smokestack or chimney. The term is used when the stack is more prominent as a landmark than accompanying buildings.

FIGURE 515.—A cupola.

A **flagpole** is a single staff from which flags are displayed. The term is used when the pole is not attached to a building.

The term **flagstaff** is used for a flagpole rising from a building.

A **flag tower** is a scaffold-like tower from which flags are displayed.

A **radio tower** is a tall pole or structure for elevating radio antennas.

A **radio mast** is a relatively short pole or slender structure for elevating radio antennas, usually found in groups.

A **tower** is any structure with its base on the ground and high in proportion to its base, or that part of a structure higher than the rest, but having essentially vertical sides for the greater part of its height.

A **lookout station** or **watch tower** is a tower surmounted by a small house from which a watch is kept regularly.

A **water tower** is a structure enclosing a tank or standpipe so that the presence of the tank or standpipe may not be apparent.

A **standpipe** is a tall cylindrical structure, in a waterworks system, the height of which is several times the diameter.

The term **tank** is used for a water tank elevated high above the ground by a tall skeleton framework.

The expression **gas tank** or **oil tank** is used for the distinctive structures described by these words.

516. Miscellaneous.—*Measured mile.* A measured nautical mile indicated on a chart is accurate to within six feet of the correct length. Most measurements in the United States were made before 1959, when the United States adopted the international nautical mile. The new value is within six feet of the previous standard length of 6,080.20 feet, adjustments not having been made. If the measured distance differs from the standard value by more than six feet, the actual measured distance is stated and the words "measured mile" are omitted.

Periods after abbreviations in water areas are omitted, as these might be mistaken for rocks. However, a lower case *i* or *j* is dotted.

Courses shown on charts are given in true directions, to the nearest minute of arc.

Bearings shown are in true directions *toward* (not from) the objects.

Commercial radio broadcasting stations are shown on charts when they are of value to the mariner either for obtaining radio bearings or as landmarks.

Rules of the road. Lines of demarcation between the areas in which international and inland rules apply are shown only when they cannot be adequately described in notes on the chart.

Compass roses are placed at convenient locations on Mercator charts to facilitate the plotting of bearings and courses. The outer circle is graduated in degrees with zero at true north. The inner circle is graduated in points and degrees with the arrow indicating magnetic north.

Magnetic information. On many charts magnetic variation is given to the nearest 15′ by notes in the centers of compass roses. When this is done, the annual change is given to the nearest 1′ to permit correction of the given value at a later date. However, since the annual change is a variable quantity, and since the values given are rounded off, as well as for other reasons, it is wise to use a chart of recent date. On other charts the variation is given by a series of **isogonic lines** connecting points of equal variation, usually a separate line being given for each degree of variation. The line of zero variation is called the **agonic line.** Many plans and insets show neither compass roses nor isogonic lines, but indicate magnetic information by note. A local magnetic disturbance of sufficient force to cause noticeable deflection of the magnetic compass, called **local attraction,** is indicated by a note on the chart.

Currents are sometimes shown on charts by means of arrows giving the directions, and figures giving the speeds. The information thus given refers to the usual or average conditions, sometimes based upon very few observations. It is not safe to assume that conditions at any given time will not differ considerably from those shown.

Longitudes are reckoned eastward and westward from the meridian of Greenwich, England, unless otherwise stated. Nearly all modern charts use Greenwich.

Notes on charts should be read with care, as they may give important information not graphically presented. Several types of notes are used. First, those in the margin give such information as the chart number and (sometimes) price, publication and edition notes, identification of adjoining charts, etc. Second, notes in connection with the chart title include such information as scale, sources of charted data, tidal information, the unit in which soundings are given, cautions, etc. A third class of notes is that given in proximity to the detail to which it refers. Examples of this type of note are those referring to local magnetic disturbance, controlling depths of channels, measured miles, dangers, dumping grounds, anchorages, etc.

Title. The chart title may be at any convenient location, usually in some area not important to navigation. It is composed of several distinctive parts as shown in figure 516.

Use of Charts

517. Advance preparation.—Before a chart is to be used, it should be studied carefully. All notes should be read and understood. There should be no question of the meanings of symbols or the unit in which depths are given, for there may not be time to determine such things when the ship is underway, particularly if an emergency should arise. Since the graduations of the latitude and longitude scales differ considerably on various charts, those of the chart to be used should be noted carefully. Dangers and abnormal conditions of any kind should be noted.

GENERAL GEOGRAPHIC AREA → ASIA
PERSIAN GULF—SAUDI ARABIA

APPROACHES TO

SPECIFIC LOCALITY → RA'S AT TANNURAH

SOURCE AND DATE(S) OF SURVEY → From U. S. Navy surveys between 1940 and 1952 with additions from British surveys in 1949 and 1950

CHART DATUM → SOUNDINGS IN FATHOMS
(Under Eleven in Fathoms and Feet)
reduced to the approximate level of Indian Spring Low Water

HEIGHTS IN METERS ABOVE MEAN SEA LEVEL
Value of heights in feet shown thus: *(102 ft)*

LEGEND → For Symbols and Abbreviations, see Chart No. 1

PROJECTION → MERCATOR PROJECTION

HORIZONTAL DATUM → NAHRWAN DATUM

SCALE → SCALE 1:150,000 AT LAT. 27°

FIGURE 516.—A chart title.

Particular attention should be given to soundings. It is good practice to select a realistic danger sounding (art. 911) and mark this prominently with a colored pencil.

It may be desirable to place additional information on the chart. Arcs of circles might be drawn around navigational lights to indicate the limit of visibility at the height of eye of an observer on the bridge. Notes regarding the appearance of light structures, tidal information, prominent ranges, or other information from the light lists, tide tables, tidal current tables, and sailing directions might prove helpful.

The particular preparation to be made depends upon the requirements and the personal preferences and experience of the individual navigator. The specific information selected is not important. But it *is* important that the navigator familiarize himself with his chart so that in an emergency the information needed will be available and there will be no question of its meaning.

518. Maintaining charts.—When a chart is received, the date to which it has been hand corrected will be found stamped in the margin. Responsibility for maintaining it after this date lies with the user. *An uncorrected chart is a menace.* The various issues of *Notice to Mariners* subsequent to the stamped hand correction date contain all the information needed for maintaining charts. The more urgent items are also given in advance in the *Daily Memorandum* or by radio broadcast. A convenient way of keeping a record of the *Notice to Mariners* corrections made to each chart on hand is by means of 5×8-inch chart correction record cards (PRNC–NHO–5610/2, formerly form N.H.O. 1278), which can be purchased for a nominal sum.

When a new edition of a chart is published, it should be obtained and the old one retired from use. The very fact that a new edition has been prepared generally indicates that there have been changes that cannot adequately be shown by hand correction.

519. Use and stowage of charts.—Charts are among the most important aids of the navigator, and should be treated as such. When in use they should be spread out flat on a suitable chart table or desk, and properly secured to prevent loss or

damage. Every effort should be made to keep charts dry, for a wet chart stretches and may not return to the original dimensions after drying. The distortion thus introduced may cause inaccurate results when measurements are made on the chart. If a chart does become wet, the distortion may be minimized by ironing the chart with a warm iron until it is dry.

Permanent corrections to charts should be made in ink so that they will not be inadvertently erased. All other lines should be drawn lightly in pencil so that they can be easily erased without removing permanent information or otherwise damaging the chart. To avoid possible confusion, lines should be drawn no longer than necessary, and adequately labeled. When a voyage is completed, the charts should be carefully and thoroughly erased unless there has been an unusual incident such as a grounding or collision, when they should be preserved without change, as they will undoubtedly be requested by the investigating authority. After a chart has been erased, it should be inspected carefully for possible damage and for incompletely erased or overlooked marks that might prove confusing when the chart is next used.

When not in use charts should be stowed flat in their proper drawers or portfolios, with a minimum of folding. The stowed charts should be properly indexed so that any desired one can be found when needed. In removing or replacing a chart, care should be exercised to avoid damage to it or other charts.

A chart that is given proper care in use and stowage can have a long and useful life.

520. Chart lighting.—In the use of charts it is important that adequate lighting be provided. However, the light on the bridge of a ship underway at night should be such as to cause the least interference with the darkness-adaptation of the eyes of bridge personnel who watch for navigational lights, running lights, dangers, etc. Experiments by the Department of the Navy have indicated that red light is least disturbing to eyes which have been adapted to maximum vision during darkness. In some instances red lights, filters, or goggles have been provided on the bridges or in chartrooms of vessels. However, the use of such light seriously affects the appearance of a chart. Red, orange, and buff disappear. Other colors may appear changed. This has led to the substitution of magenta or purple for red and orange, and gray for buff on some charts. However, before a chart is used in any light except white, a preliminary test should be made and the effect noted carefully. If a glass or plastic top is provided for the chart table or desk, a dim white light *below* the chart may provide sufficient illumination to permit chart reading, without objectionable disturbance of night vision.

PART TWO

PILOTING AND DEAD RECKONING

PART TWO

PILOTING AND DEAD RECKONING

CHAPTER VI

INSTRUMENTS FOR PILOTING AND DEAD RECKONING

Introduction

601. Kinds of instruments.—The word "instrument" has several meanings, at least two of which apply to navigation: (1) an implement or tool, and (2) a device by which the present value of a quantity is measured. Thus, a straightedge and a mechanical log are both instruments, the first serving as a tool, and the second as a measuring device. This chapter is concerned with the navigational instruments used for plotting, and those for measuring distance or speed, depth, and direction. Instruments for measuring time are discussed in chapter XV. These quantities are the basic data in dead reckoning (ch. VIII) and piloting (ch. IX). Other instruments are discussed in chapters XI, XV, XXXVII, and XL.

In addition to the instruments discussed, several others are important to the navigator. Binoculars are helpful in observing landmarks. A flashlight has many uses, the principal one being to illuminate the scales of instruments when they are to be read at night. Erasers should be soft, and pencils should not be so hard that they damage the surface of the chart. The navigator's chart is discussed in chapter V.

Plotting Instruments

602. Dividers and compasses.—**Dividers,** or "pair of dividers," is an instrument originally used for dividing a line into equal segments. The instrument consists essentially of two hinged legs with pointed ends which can be separated to any distance from zero to the maximum imposed by physical limitations. The setting is retained either by friction at the hinge, as in the usual navigational dividers, or by means of a screw acting against a spring.

If one of the legs carries a pencil or ruling pen, the instrument is called **compasses.** The two legs may be attached to a bar of metal or wood instead of being hinged, thus permitting greater separation of the points. Such an instrument is called **beam compasses** or **beam dividers** (fig. 4011b).

The principal use of dividers in navigation is to measure or transfer distances on a chart, as described in article 804. Compasses are used for drawing distance circles (art. 905), circles of visibility (art. 916), or any plotting requiring an arc of a circle.

The friction at the hinge of most dividers and compasses can be varied, and should be adjusted so that the instrument can be manipulated easily with one hand, but will retain the separation of the points in normal handling. A drop of oil on the hinge may be required occasionally. The points should be sharp, and should have equal length, permitting them to be brought close together for the measurement of very short distances.

For navigation, it is desirable to have dividers and compasses with comparatively long legs, to provide adequate range for most requirements. It is desirable to learn to manipulate dividers or compasses with one hand.

603. Parallel rulers are an instrument for transferring a line parallel to itself. In its most common form it consists of two parallel bars or rulers, connected in such a manner that when one is held in place on a flat surface, the other can be moved, remain-

ing parallel to its original direction. Firm pressure is required on one ruler while the other is being moved, to prevent slippage. The principal use of parallel rulers in navigation is to transfer the direction of a charted line to a compass rose, and vice versa.

The edges of the rulers should be truly straight; and in the case of double-edged rulers, should be parallel to each other in order that either edge can be used. Parallelism can be tested by comparison of all edges with the same straight line, as a meridian or parallel of a Mercator chart. The linkage can be tested for looseness and lack of parallelism by "walking" the rulers between parallel lines on opposite sides of the chart and back again.

Some metal parallel rulers have a protractor engraved on the upper surface to permit orientation of the ruler at any convenient meridian.

In one type of instrument, parallelism during transfer is obtained by supporting a single ruler on two knurled rollers. Both rollers have the same diameter, and the motion of one is transmitted to the other by an axle having a cover which provides a convenient handle. This type of ruler is convenient and accurate, and is less likely to slip than the linked double-ruler type. However, care is necessary to prevent its rolling off the chart table when the vessel is rolling or pitching.

Directions can also be transferred by means of two triangles such as are used in drafting, or by one triangle and a straightedge. One edge of a triangle is aligned in the desired direction and the triangle is then moved along a straightedge held firmly against one of its other edges until the first edge is at the desired place on the chart. Some triangles have protractors (art. 604) engraved on them to assist in transferring lines. Such a triangle becomes a form of plotter (art. 605).

604. Protractor.—A **protractor** is a device for measuring angles on a chart or other surface. It consists essentially of a graduated arc, usually of 180°, on suitable material such as metal or plastic.

A **three-arm protractor** consists essentially of a circular protractor with three radial arms attached. This instrument, discussed in greater detail in article 4011, is used primarily in hydrographic surveying.

605. Plotters.—The increased popularity of graphical methods in practical navigation during recent decades has resulted in the development of a wide variety of devices to facilitate plotting. In its most common form, such a device consists essentially of a protractor combined with a straightedge. There are two general types, one having no movable parts, and

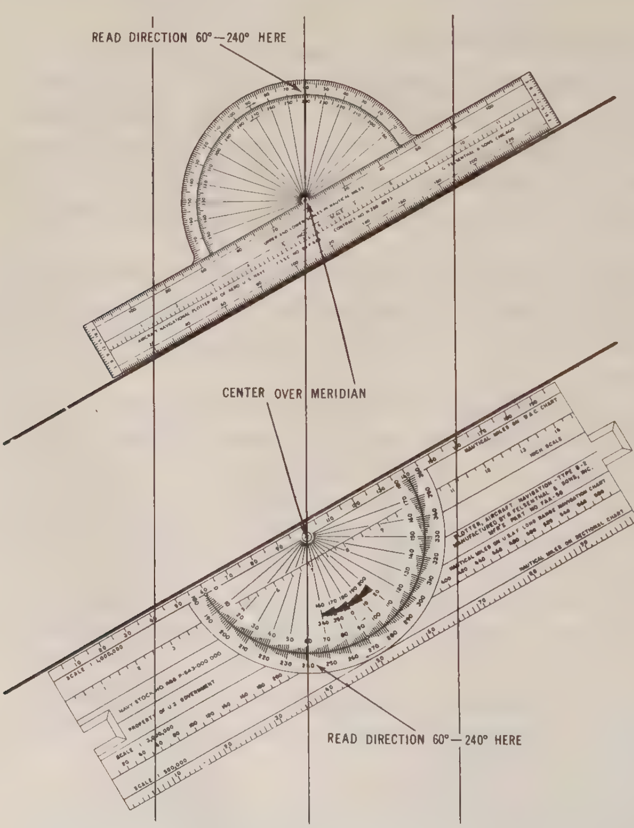


FIGURE 605.—Two plotters having no movable parts.

the other having a pivot at the center of the arc of the protractor, to permit rotation of the straightedge around the protractor. Examples of the fixed type are illustrated in figure 605. Those shown were designed for air navigation, but are applicable to many processes of marine navigation. The direction of the straightedge is controlled by placing the center of the protractor arc and the desired scale graduation on the same reference line. If the reference line is a meridian, the directions shown by the straightedge are true geographic directions. If, as in some processes of celestial navigation, it is desired to plot a line perpendicular to another line, the direction may be measured from a parallel of latitude or its equivalent, instead of adding or subtract-

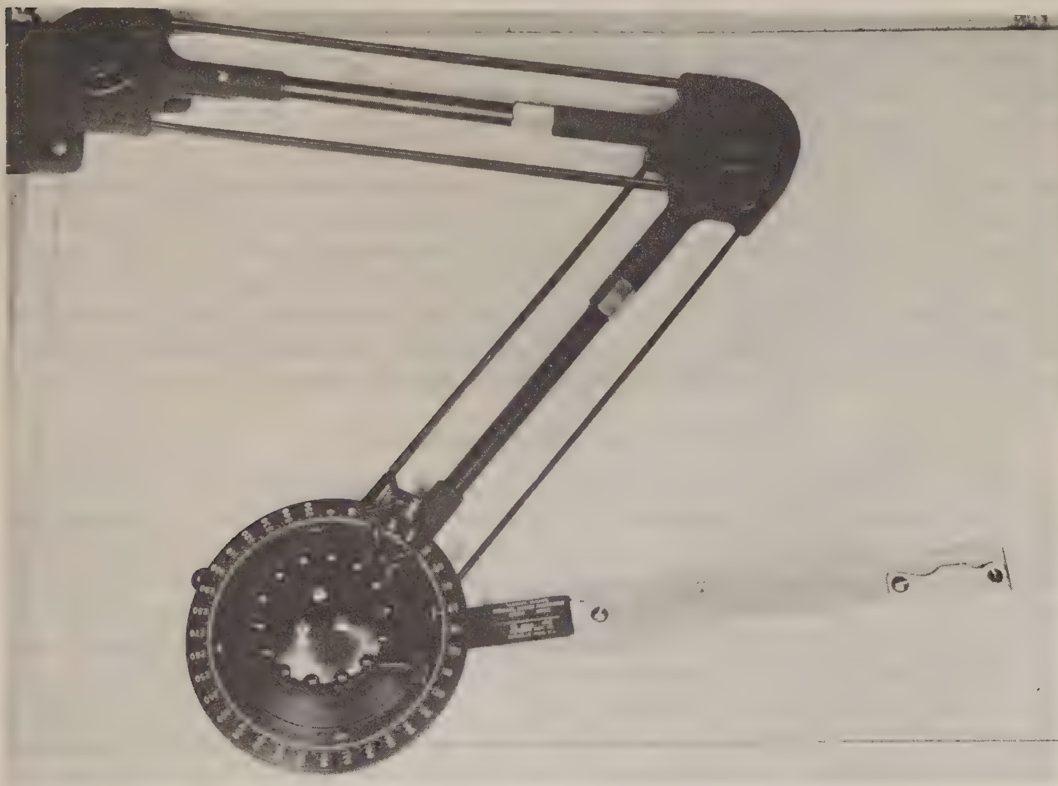


FIGURE 606.—Drafting machine.

ing 90° from the value and measuring from a meridian. Some fixed-type plotters have auxiliary scales labeled to indicate true direction if a parallel is used as the reference.

Most plotters also provide linear distance scales, as shown in figure 605. In the movable-arm type of plotter, a protractor is aligned with a meridian, and the movable arm is rotated until it is in the desired direction.

606. Drafting machine.—If a chart table of sufficient size is available, a **drafting machine** (fig. 606) is probably the most desirable plotting instrument. The straightedge of this instrument can be clamped so as to retain its direction during movement over the entire plotting area. Straightedges of various lengths and linear scales are interchangeable. Some models make provision for mounting two straightedges perpendicular to each other. However, for most purposes of navigation, the perpendicular is more conveniently obtained by the use of a triangle with a single straightedge. The movable protractor also retains its orientation, and can be adjusted to conform to the

compass rose of a chart secured in any position on the chart table. Directions of the straightedge can then be read or set on the protractor without reference to charted compass roses. Use of the clamped protractor requires that charted meridians be straight and parallel, as on a Mercator chart (art. 305). Its use is restricted with projections such as the Lambert conformal (art. 314), on which meridians converge.

When a drafting machine is used, the chart or plotting sheet is first secured to the chart table. The straightedge is aligned with a meridian (or parallel) and clamped in position. The protractor is then adjusted so that 000° and 180° (090° and 270° if a parallel is used) are at the ruler indices, and clamped. With this setting, any subsequent position of the ruler is indicated as a true direction. If the protractor is offset by the amount of the compass error (ch. VII), true directions can be plotted by setting the straightedge at the compass direction on the protractor, without need for applying compass error arithmetically. However, it is generally preferable to keep it set to true directions, and apply compass error mentally.

If accurate results are to be obtained, the anchor base must be rigidly fastened to the chart table. This should be checked from time to time, as the base may be loosened by vibration or normal use. The pivots in the anchor base should be firm without binding. The endless belts of the parallel motion mechanism should be taut if rigidity of the ruler is to be preserved. Provision is usually made for adjusting each of the various rulers to uniformity of alignment so that any other ruler can be substituted without changing the setting. As with parallel rulers, the device can be checked for parallelism by means of meridians or parallels on opposite sides of a Mercator chart.

Distance and Speed Measurement

607. Units of measurement.—Mariners generally measure horizontal distances in nautical miles (art. 205), but occasionally in yards or feet. Either feet or fathoms are used for measuring depth of water, and feet for measuring height above water. The British yard is now equal to that of the United States (art. 205). However, the difference was never significant in the ordinary practice of navigation. Nations which have adopted the metric system use meters in place of yards, feet, and fathoms, and for some purposes they use kilometers in place of nautical miles. Conversion factors for these and other units are given in appendix D. Nautical miles of 6,076.11549 feet (approximately) and land or statute miles of 5,280 feet can be interconverted by means of table 20. Meters, feet, and fathoms can be interconverted by means of table 21.

Speed is customarily expressed in knots (art. 206), or for some purposes, in kilometers per hour, or yards or feet per minute. For short distances, a nautical mile can be considered equal to 2,000 yards or 6,000 feet. This is a useful relationship because $\frac{6{,}000 \text{ feet}}{60 \text{ minutes}}=100$ feet per minute. Thus, speed in knots is equal approximately to hundreds of feet per minute or, hundreds of yards per 3-minute interval.

608. Distance, speed, and time are related by the formula

$$\text{distance}=\text{speed}\times\text{time}.$$

Therefore, if any two of the three quantities are known, the third can be found. The units, of course, must be consistent. Thus, if speed is measured in knots, and time in hours, the answer is in nautical miles. Similarly, if distance is measured in yards, and time in minutes, the answer is in yards per minute.

Table 19 is a speed, time, and distance table which supplies one of the three values if the other two are known. It is intended primarily for use in finding the distance steamed in a given time at a known speed. Table 18 is for use in determining speed by measuring the time needed to steam exactly one mile.

The solution of problems involving distance, speed, and time can easily be accomplished by means of a slide rule (art. O15). If the index of scale C is set opposite speed in knots on scale D, the distance in nautical miles appears on scale D opposite time in hours on scale C. If 60 of scale C is set opposite speed in knots on scale D, the distance covered in any number of *minutes* is shown on scale D opposite the minutes on scale C. Several circular slide rules particularly adapted for solution of distance, speed, and

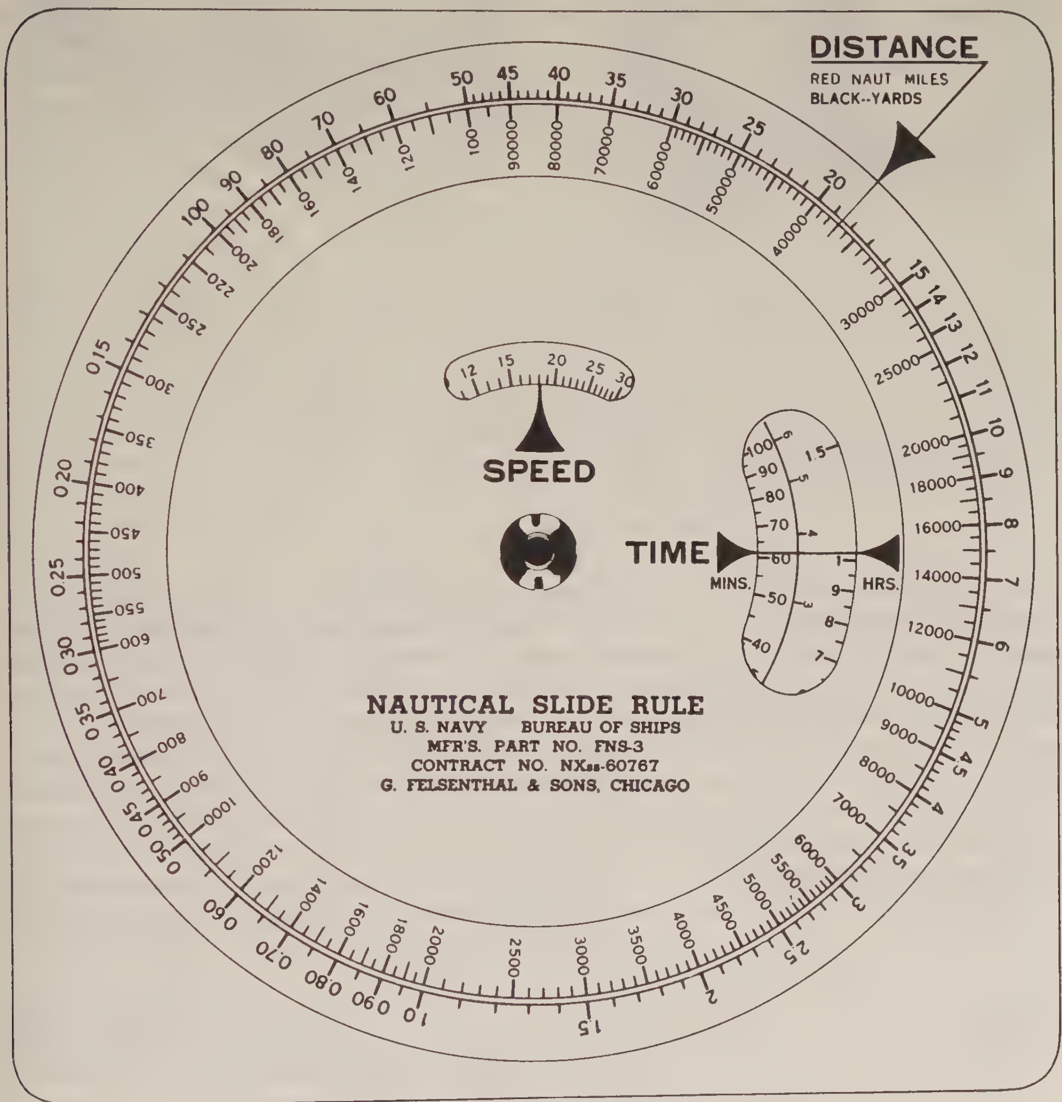


FIGURE 608.—The nautical slide rule.

time problems have been devised. One of these, called the "Nautical Slide Rule" is shown in figure 608.

609. Measurement of distance to an object can be made in a variety of ways, as by radar (art. 1208), sonar (art. 1103), RAR beacon (art. 1205), distance finding station (art. 1205), sextant angle (art. 905), range finder, or by several indirect methods. Another method used principally for measuring distance between ships in formation, but useful in measuring other distances, is by means of a small, hand-held instrument called a **stadimeter.**

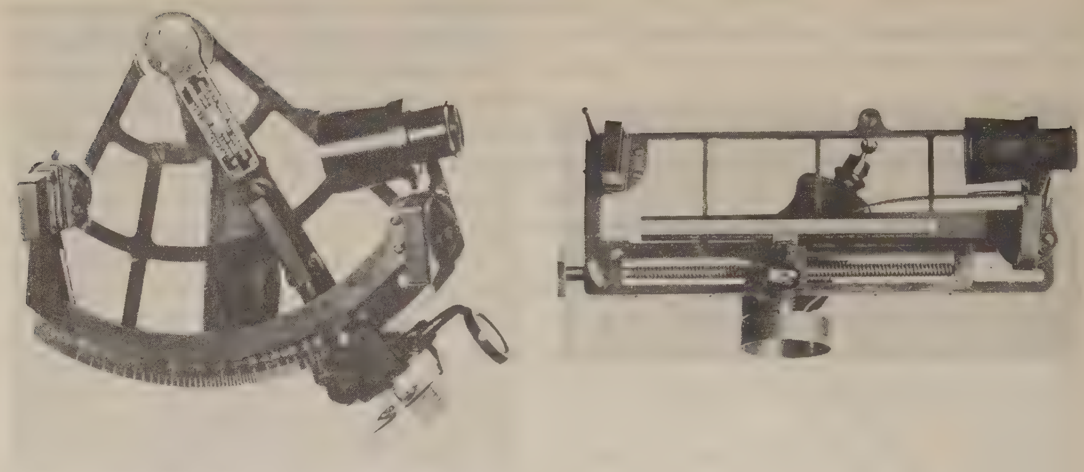


FIGURE 609a.—Stadimeters. Brandon (sextant) type at left; Fisk type at right.

Two types of stadimeters are illustrated in figure 609a. Both the Brandon or sextant type and the Fisk type operate on the principle used in table 9:

In a plane right triangle, *ABC*, having opposite sides *a*, *b*, and *c*,

$$\tan A = \frac{a}{b}, \text{ and } b = a \cot A.$$

This is applied to the stadimeter as shown in figure 609b. The height of the object is set on the height scale of the instrument, and the measured subtended angle is expressed in yards on the distance (range) scale. To measure the angle, one directs the line of sight through the instrument to the water line of the object observed, and adjusts the range index until the reflection of the top of the object is seen in coincidence with the water line. If the readings are not within the scale of the instrument, some fraction or multiple of the height can be used and a corresponding adjustment made to the answer. Thus, if *half* the height is set on the instrument, the distance indicated is *half* the correct value.

Since the observer's eye is not at the water level, a right angle is not necessarily formed between the line of sight and the top of the observed object. However, the

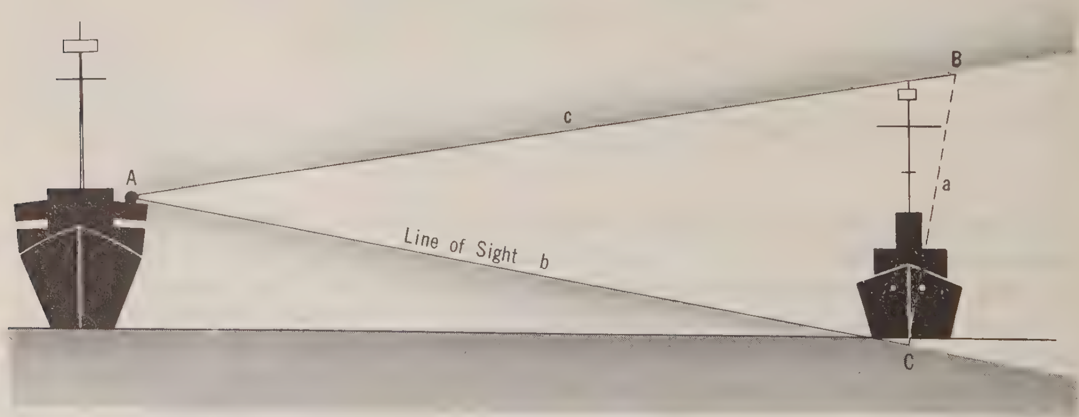


FIGURE 609b.—Geometry of a stadimeter measurement. The distance $b = a \cot A$.

resulting error is so small that it can be neglected under ordinary circumstances.

The aspect of a ship observed should be considered in stadimeter ranges. Thus, little error is introduced if the observer is broad on the beam of the other vessel, as in figure 609b, but less accuracy is obtained if the other vessel presents an end-on view, unless the water line directly below the masthead is correctly estimated.

A stadimeter can be used to indicate that a *change* in distance has occurred, even when the height of the object is not known. Similar indication of a change in distance can be obtained by a sextant (art. 905), or the actual distance can be determined by the measured angle and table 9 if the height is known.

610. Measurement of distance traveled may be made directly, or the distance can be determined indirectly by means of the speed and time, using the relationship given in article 608.

One of the simplest mechanical distance-measuring devices is the **taffrail log,** consisting of (1) a *rotator* which turns like a screw propeller when it is towed through the water; (2) a braided *log line,* up to 100 fathoms in length, which tows the rotator and transmits its rotation to an indicator on the vessel; and (3) a dial and pointer mechanism which registers the distance traveled through the water. In some installations, the readings of the register are transferred electrically to a dial on or near the bridge.

The taffrail log is usually streamed from the ship's quarter, although it may be carried at the end of a short boom extending outboard from the vessel. The log line should be sufficiently long, and attached in such position, that the rotator is clear of the disturbed water of the wake of the vessel; otherwise an error is introduced. Errors may also be introduced by a head or following sea; by mechanical wear or damage, such as a bent fin; or by fouling of the rotator, as by seaweed or refuse.

An accurately calibrated taffrail log in good working order provides information of sufficient reliability for most purposes of navigation. Its readings should be checked at various speeds by towing it over a known distance in an area free from currents. Usually, the average of several runs, preferably in opposite directions, is more accurate than a single one. If an error is found, it is expressed as a percentage and applied to later readings. The calibration should be checked from time to time.

Although a taffrail log is included in the equipment carried by many oceangoing vessels, the convenience and reliability of other methods of determining distance or speed have reduced the dependence formerly placed upon this instrument.

611. Measurement of speed.—Speed can be determined indirectly by means of distance and time, or it can be measured directly. All instruments now in common use for measuring speed determine rate of motion *through the water.* This is done (1) electromagnetically, (2) by measuring the water pressure due solely to the forward motion of the vessel, (3) by means of a small screw propeller having a speed of rotation proportional to speed of the vessel, and (4) by determining the relationship between ship speed and speed of rotation of its screw or screws. Instruments for measuring speed, like those for measuring distance, are called **logs.**

Before the development of modern logs, speed was determined in a number of ways. Perhaps the most common primitive device is the **chip log** (art. 112), although a **ground log** (a weight, with line attached, which was thrown overboard and rested on the bottom in shallow water) and a **Dutchman's log** (art. 112) have also been used. These devices are rarely used by modern navigators.

Speed over the bottom can be determined (1) by direct measurement; (2) by measuring on the chart or plotting sheet the distance made good between fixes, and dividing this by the time; or (3) by finding the vector sum of velocity through the water and velocity of the current. A suitable instrument for measuring speed over the bottom is not generally available, although some developmental work along this

line has been done. Measurement of the distance between two fixes requires a method of obtaining accurate fixes. The third method requires knowledge of velocity through the water, and current. An estimate of the current can be used to determine the approximate course and speed over the bottom. However, estimates are not always accurate, and an instrument to measure current would provide better results. Such an instrument, called the **geomagnetic electrokinetograph (GEK)**, has been developed. By means of two electrodes towed astern, beyond the magnetic influence of the vessel, the component of current perpendicular to the course is measured. By measurement of two such components, preferably on perpendicular courses, one can determine the total current. This device has given satisfactory results in experimental work, having been used primarily by oceanographers in their study of ocean currents, but has not been adapted for use in ordinary navigation.

612. The electromagnetic type underwater log consists essentially of a rodmeter, an oscillator-amplifier, and an indicator-transmitter. The rodmeter, which protrudes below the hull of the vessel, contains an electromagnetic sensing element which produces a voltage directly proportional to speed through the water. This voltage is amplified in the oscillator-amplifier, and is converted to pointer and synchro indications of speed in the indicator-transmitter. The speed signals are also converted to distance, by means of a roller-and-disk mechanism in the indicator-transmitter. This system has no orifice or moving parts external to the vessel, and has high precision and accuracy from zero speed to full scale.

613. Speed measurement by dynamic water pressure.—When an object is moving through a fluid such as water or air, its forward side is exposed to a *dynamic* pressure which is proportional to the speed at which the object is moving, in addition to the *static* pressure due to depth and density of the fluid above the object. When the fluid is water, and ship speeds are involved, dynamic pressure is equal to

$$P=\frac{S^2}{1.832g}$$

where P is dynamic pressure, S is speed through the water, and g is the acceleration due to gravity (32.2 feet per second per second, approximately). If this formula is solved for S, it becomes

$$S=1.353\sqrt{gP}.$$

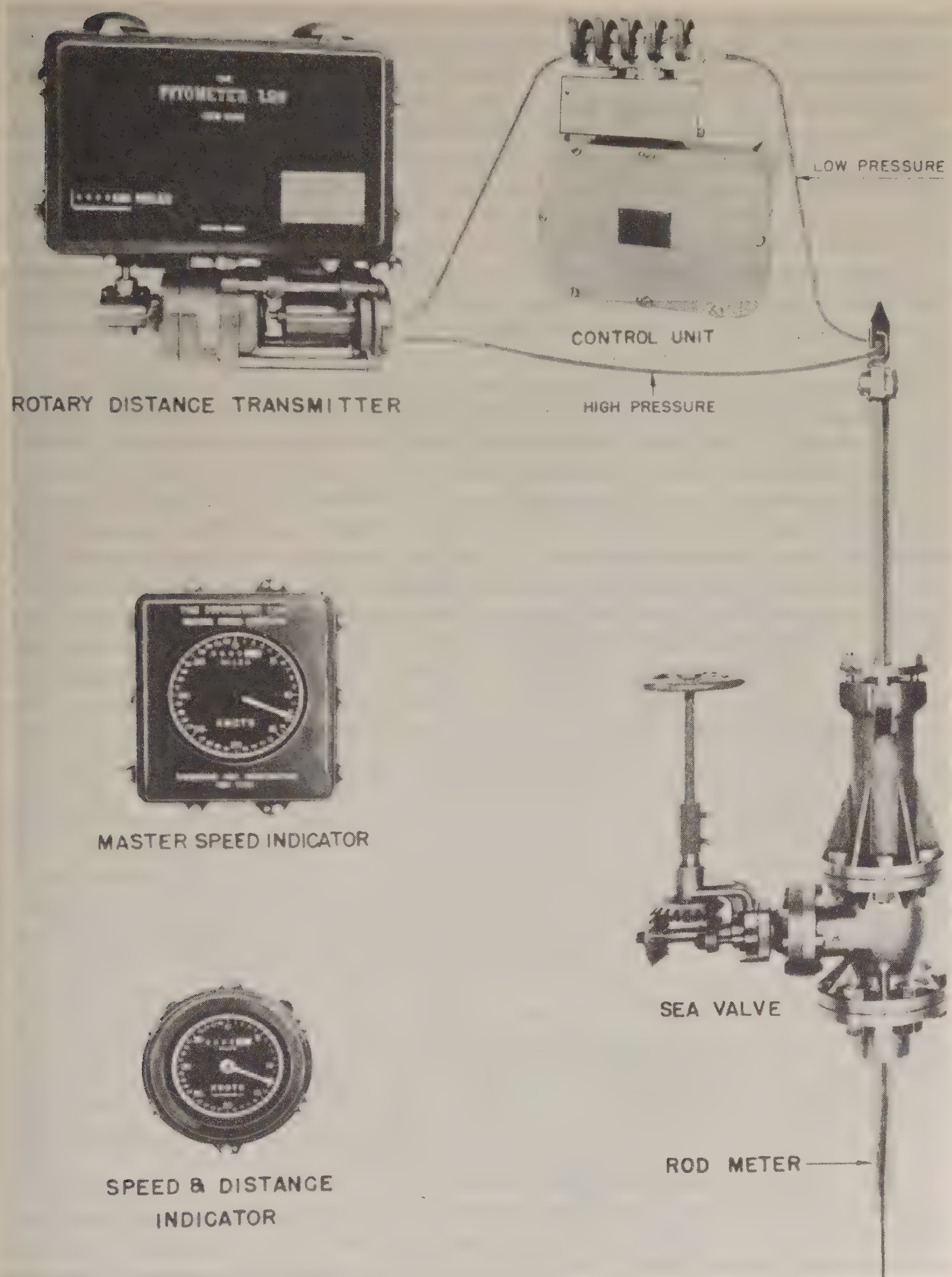
If 32.2 is substituted for g:

$$S=7.68\sqrt{P}.$$

Therefore, if dynamic pressure can be measured, this principle can be used for determining speed.

One of the most widely used means of measuring dynamic pressure is by a **Pitot tube.** This device consists of a tube having an opening on its forward side or end. If the tube is stationary in the water, this opening is subject to static pressure only. But when the tube is in motion, the pressure at the opening is the sum of static and dynamic pressures. This is called **Pitot pressure** or **total pressure.** The Pitot tube is surrounded by an outer tube which has openings along its athwartship sides. Whether the tube is stationary or in motion, these openings are subject to static pressure only.

In the **Pitot-static log** (fig. 613) the Pitot tube is in the form of a vertical "rodmeter" which extends through and is supported by a sea valve in the vessel's bottom. The tube extends 24 to 30 inches below the bottom of the vessel, into water relatively undisturbed by motion of the hull. The two pressures, Pitot and static, are led to separate bellows attached to opposite ends of a centrally pivoted lever. This lever is



Courtesy of Pitometer Log Corporation.

FIGURE 613.—A Pitot-static log.

electrically connected to a mechanism which controls the speed of a pump. When the vessel is dead in the water, the pressures are equal, and the pump is stopped. When the ship is moving, the pump speed is regulated so that the pressures in the two bellows are equalized. Thus, the pump speed is proportional to the ship speed.

Through suitable gearing, the pump provides a mechanical output at the rate of 60 revolutions per nautical mile. This rotation is transmitted electrically to the master indicator, where the distance traveled appears on the distance counter in units of 0.01 nautical mile. In addition, the master indicator has a timing device which transforms distance and time into speed, the latter appearing on the speed dial

Both speed and distance indications may be transmitted to various **repeaters** throughout the ship. Logs of this type have been replaced in ships of the U.S. Navy by those of the electromagnetic type to provide both greater accuracy at each calibration speed and over a given range of speeds.

In an early model of this type log, the two pressures from the Pitot tube are led to opposite sides of a manometer. The difference in pressure is indicated by a pressure gage graduated to read directly in knots. Distance is determined by a mechanical integrator and cam attached to the speed dial.

Various less accurate instruments have been devised for determining speed by measuring water pressure due to forward motion of the vessel. These are relatively simple, inexpensive instruments intended primarily for use by small craft. One instrument has a finger which the water pressure forces aft against a calibrated spring. A flexible hydraulic cable transmits the motion to a speed indicator. Another instrument uses a small scoop attached to the hull of the vessel. The pressure of the water scooped up is transmitted by tubing to the speed indicator, which is essentially a pressure gage graduated in knots. A third type measures the drag of a small towed object. The accuracy of such devices depends to a large extent upon the refinements of design, manufacture, installation, maintenance, and calibration.

614. Impeller-type log.—An impeller-type log has a small propeller-driven alternating-current generator located near the outer end of a rodmeter which extends through a sea valve on the hull plating, and projects approximately two feet into the water. The propeller rotates as it moves through the water. The number of its revolutions is proportional to the distance traveled through the water, and its speed of rotation is proportional to the ship's speed. The output of the generator is amplified, and passed to the master indicator-transmitter, where the number of cycles, reduced by gearing, is recorded on mileage counter dials in units of 0.01 nautical mile. The frequency of the alternating current, being proportional to ship speed, is transmitted to a tachometer mechanism geared to the pointer of the speed indicator. Calibration is accomplished by adjusting the position of driving rollers along the radius of a driven disk.

The speed and distance indications of the master indicator can be transmitted to remote indicators. Speed indications of this equipment are accurate to approximately 0.15 knot at speeds between 0.25 knot and 25 knots.

615. Speed by engine revolution counter.—The number of turns of a propeller shaft is proportional to the distance traveled. If the element of time is added, speed can be determined. If the screw were advancing through a solid substance, the distance it would advance in one revolution would be the **pitch** of the screw. Thus, if a propeller having a pitch of ten feet turns at 200 revolutions per minute, it advances 2,000 feet in one minute, equivalent to a speed of 19.75 knots. It does not do so in water because of **slip**, the difference between the distance it would advance in a solid substance and actual distance traveled, expressed as a percentage of the former. For example, if slip is 18 percent, both the ship's speed and distance covered are reduced by this percentage. Thus, instead of 19.75 knots, the speed is only $19.75 \times 0.82 = 16.2$ knots.

Slip depends upon the type and speed of rotation of the propeller, the type of ship, the condition of loading and ship's bottom, the state of the sea and the ship's course relative to it, and the apparent wind. Despite the many variables, slip can be determined with sufficient accuracy for practical navigation. This is usually accomplished by steaming a known distance and noting the time of passage. The speed corresponding to the number of revolutions being used can then be determined by means of the formula of article 608, in the form

$$\text{speed}=\frac{\text{distance}}{\text{time}}$$

or by reference to table 18 (if the distance is exactly one mile). Thus, speed can be determined directly, without computing slip, and a table or curve of ship speed for various engine revolution speeds can be made. In determining speed in this manner, the average speed of two runs (one in each direction) should be used. The vessel should be on course and speed long enough to stabilize slip before starting each run. Any suitable distance can be used, but a distance of one nautical mile has been measured at various convenient locations. Each such **measured mile** is suitably marked on the beach, and shown on the chart, with the course to steer.

This method of determining speed is widely used in the merchant marine. By means of an **engine revolution counter** the number of revolutions during any suitable time interval can be measured. If a **tachometer** is available, the *rate* of shaft revolution is determined, usually in revolutions per minute. For best results, allowance should be made for condition of the bottom, draft and trim of the vessel, and the state of the sea.

Depth Measurement

616. Importance.—Accurate knowledge of the depth of water under a vessel is of such navigational importance that there is a legal requirement that American merchant vessels of 500 gross tons or more engaged in ocean and coastwise service "shall be fitted with an efficient mechanical deep-sea sounding apparatus in addition to the deep-sea hand leads."

617. The lead (lĕd) is a device consisting of a suitably marked line having a weight attached to one of its ends. It is used for measuring depth of water. Although the lead is probably the oldest of all navigational aids, it is still a highly useful device, particularly in periods of reduced visibility. Although its greatest service is generally in the shoal water near the shore, it sometimes can provide valuable information when the vessel is out of sight of land.

Two types of lead are in common use, the **hand lead,** weighing from 7 to 14 pounds and having a line marked to about 25 fathoms; and the **deep-sea (dipsey) lead,** weighing from 30 to 100 pounds and having a line marked to 100 fathoms or more in length. The markings commonly used on lead lines are as follows:

Distance from lead in fathoms	*Marking*	*Distance from lead in fathoms*	*Marking*
2	two strips of leather	20	short line with two knots
3	three strips of leather	25	short line with one knot
5	white rag (usually cotton)	30	short line with three knots
7	red rag (usually wool)	35	short line with one knot
10	leather with hole	40	short line with four knots
13	same as three fathoms	45	short line with one knot
15	same as five fathoms	50	short line with five knots
17	same as seven fathoms		etc.

Fathoms marked on the lead line are called **marks.** The intermediate whole fathoms are called **deeps.** In reporting depths it is customary to use these terms, as "by the mark five," "deep six," etc. The only fractions of a fathom usually reported are halves and quarters, the customary expressions being "and a half, eight," "less a quarter, four," etc. A practice sometimes followed is to place distinctive markings on the hand lead line at each foot near the critical depths of the vessel with which it is to be used. The markings should be placed on the lead line when it is wet, and the accuracy of the marking should be checked from time to time to detect any changes in the length of the line. The distance from the hand of the leadsman to the surface of the water under various conditions of loading should be determined so that correct allowance can be made when the marking nearest the surface cannot be observed.

The lead itself has a recess in its bottom. If this recess is filled with tallow or other suitable substance, a sample of the bottom can sometimes be obtained. This information can prove helpful in establishing the position of the vessel. If tallow is not available, some other substance can be used. Soap is suitable if it is replaced from time to time. When the recess is filled for obtaining a sample, the lead is said to be **armed** with the substance used.

618. The sounding machine, developed by William Thomson (Lord Kelvin) as a substitute for the deep-sea lead, provides means for obtaining approximate soundings to a depth of about 100 fathoms without slowing or stopping the vessel. This is accomplished by attaching a depth-registering device to the lead so that a vertical cast is unnecessary.

Several types of depth-registering device have been developed, all depending upon the increase of pressure with depth (art. 3008). The most common form is a slender glass tube coated on the inside with a chemical which changes color when contacted by sea water, or having a ground surface on the inside so that it appears white when dry and clear when wet. The chemical tube requires recoating after each sounding, but the ground glass tube can be used over and over again, if it is cleaned thoroughly and allowed to dry after each using. The top of the tube is closed. As the lead sinks, water is forced into the tube, compressing the entrapped air. The height to which the water rises is an indication of depth. The upper end of a ground glass tube is sealed with a cap which can be removed to facilitate cleaning and drying.

Errors are introduced if the inside diameter of the glass tube is not uniform throughout its length, if the chemical has deteriorated, or if salt is not washed out of the ground glass tube after use. Errors from these sources may be as much as 20 percent. If the indicated depth is too great, a dangerous situation exists. A slight error may be introduced by atmospheric pressure, but since the scale is calibrated for a lower-than-normal pressure, the usual error is on the side of safety. For usual pressures, readings are about three percent too little. If the temperature of the air is different from that of the sea, the entrapped air will expand or contract as it is immersed in the water, causing an error of about one percent for each 3° change of temperature. If the air is cooled, the indicated depth is too great. Error will also be introduced if the cap permits leakage of air.

A mechanical depth recorder is sometimes used. This consists essentially of a pointer which is attached to a piston forced against the tension of a spring as the water pressure increases. The pointer remains at the greatest depth reached, requiring resetting before the next sounding is made.

In critical areas, it is wise to check the readings of a sounding machine, or to use another method of sounding. If an echo sounder (art. 619) is not available, a check can be made by stopping the vessel, running out a measured length of sounding wire

(at least 50 fathoms if depth permits), and comparing this measured length with the indication of the depth-registering device.

619. Echo sounder.—Most soundings are now made by means of an **echo sounder.** This instrument produces an underwater sound-wave signal, and measures the elapsed time until return of an echo from the bottom. It is a form of **sonar** (art. 1108), although this term is usually applied only to similar equipment which directs the sound-wave signal horizontally to measure range. An echo sounder operating within the range of audible sound (about 20 to 20,000 cycles per second) may be called a **sonic depth finder.** One using sound waves of a higher frequency may be called an **ultrasonic depth finder.** The trend has been toward higher frequency, to reduce interference from ship noise.

FIGURE 619.—The indicator of an echo sounder.

There are many forms of echo sounder. In a typical installation (fig. 619) a light tube is mounted vertically behind an opaque shield which rotates at a predetermined speed. A narrow slot in the shield permits the light to be seen at one place only. This slot is under or adjacent to a circular scale graduated in depth. The sound-wave signal is transmitted when the slot is at the top or zero of the scale. At this instant, the light flashes. When an echo is received, the light again flashes. The graduation adjacent to the second flash indicates the depth. Several different scales may be available for use in various depths. The scale is controlled by adjusting the speed of rotation of the opaque shield. If the depth is greater than the maximum graduation of the scale in use, an erroneous reading may be obtained unless the operator is alert. Thus, if the maximum reading is 400 fathoms, and the depth of the water is 600 fathoms, the shield will make a complete rotation and half of another before the echo returns. The scale would indicate a reading of 200 fathoms. If allowance is made for the number of complete rotations, accurate soundings can be obtained at a relatively large scale. However, there is less possibility of error if the correct scale is used. Doubt as to the correct scale can be removed by switching to a smaller scale (greater depth), if one is available. In some models the light itself rotates. Some of the newer echo sounders are equipped with a recording device that produces a written trace of the bottom, called a **bottom profile** (fig. 4206a). This is accomplished by means of an arm which moves across a graduated tape, making one transit for each sounding. When the echo is received, a short line is produced on a moving tape graduated in time and depth units.

Echo sounders of American manufacture are calibrated for a speed of sound of 4,800 feet per second. The actual speed varies primarily with the temperature, pressure, and salinity, as discussed in article 3503, but in the ocean is nearly always faster than the speed of calibration. The error thus introduced is on the side of safety unless the water is fresh or very cold. Soundings shown on charts of the U. S. Navy Hydrographic Office are those obtained by an echo sounder without correction, and can therefore be compared directly with the readings obtained aboard ship since the variation in speed from mean conditions is not great. Only in precise scientific work should it be necessary to correct the readings for actual sound speed under prevailing conditions. Accurate adjustment can be made only if information is available on conditions at various depths.

Errors are sometimes introduced by false bottoms. If soft mud covers the ocean floor, some of the sound-wave energy may penetrate to a harder layer beneath, resulting in indication of two bottoms. It is not unusual in deep water to receive a strong return at a depth of about 200 fathoms during the day, and somewhat nearer the surface at night. This is called the **phantom bottom** or **deep scattering layer.** It is believed to be due to large numbers of tiny marine animals. Schools of fish return an echo sufficiently strong to make the echo sounder a valuable aid to commercial fishermen.

In modern equipment the sound waves, whether sonic or ultrasonic, are produced electrically by means of a **transducer,** a device for converting electrical energy to sound waves, or vice versa. The transducer utilizes either the piezo-electric properties of quartz or the magnetostriction properties of nickel and its alloys.

Early models produced sound signals by striking the ship's hull with a mechanical hammer in the forward part of the vessel. The echo was received by a microphone in the after part of the vessel, depth being determined by the angle at which the signal returned.

Direction Measurement

620. Reference directions.—A horizontal direction is generally expressed as an angle between a line extending in some **reference direction** and a line extending in the given direction. The angle is numerically equal to the difference between the two directions, called the **angular distance** from the reference direction. Unless the reference direction is stated or otherwise understood, the intended direction is in doubt. Thus, to a navigator, direction 135° is southeast. To an astronomer or surveyor, it may be northwest.

A number of reference directions are used in navigation. If a direction is stated in three figures, without designation of reference direction, it is generally understood that the direction is related to true (geographical) north. When grid navigation (art. 2510) is being used, particularly in high latitudes, grid north is generally used as the reference direction. The reference direction for magnetic directions is magnetic north, and that for compass directions is compass north. For relative bearings it is the heading of the ship. For amplitudes, the reference direction is east or west, usually 090° or 270° true, but magnetic, compass, or even grid east or west may be used. In maneuvering situations, the heading of another vessel might be used as the reference direction.

The primary function of an instrument used for measuring direction is to determine the reference direction. This having been done, other directions can be indicated by a compass rose oriented in the reference direction. North is established by some form of compass. A compass rose is attached to the north-seeking element so that other directions can be determined directly. However, if one always keeps in mind that the primary function of the instrument is to indicate a reference direction, he should be able to avoid some of the mistakes commonly made in the application of compass errors.

621. Desirable characteristics of a navigational compass.—To adequately serve its purpose, a navigational compass needs to have certain characteristics to permit it to meet requirements of accuracy, reliability, and convenience.

The most important characteristic is accuracy. No other quality, however important or to whatever extent it may be possessed, compensates for the lack of accuracy. This does not mean that the compass need be without error, but that such errors as it may possess can be readily determined. Provision should be made for removing deviation or reducing it to a minimum (ch. VII). If accurate horizontal directions are to be determined, the compass needs to be provided with some type of compass rose maintained in a horizontal position (art. 2903). Adequate sighting equipment is needed if bearings are to be observed, and an index is needed to mark the forward direction parallel to the keel if heading is to be measured. Accurate readings cannot be expected from a compass that **hunts** (oscillates) excessively. A characteristic closely related to accuracy is precision (art. O3). The amount of precision required varies somewhat with the use and depends as much upon the steadiness of the compass and its design as upon its inherent qualities.

A compass is reliable when its operation is not often interrupted; when its indications are relatively free from unknown or unsuspected disturbances; when it is little affected by extremes of temperature, moisture, vibration, or the shock of gunfire; and when it is not so sensitive that large errors are introduced by ordinary changes in conditions or equipment near the compass.

The value of a compass is dependent somewhat upon the convenience with which it can be used. Accuracy, too, may be involved. Thus, a compass should not be installed in such a position that one must be in an unnatural or uncomfortable position to use it. A compass intended for use in obtaining bearings is of reduced value if it is installed at a location that does not permit an unobstructed view in most directions. The compass graduations and index should be clean, adequately lighted if the instrument is to be used at night, and clearly marked.

622. Kinds of compasses.—The compasses commonly used by the mariner are (1) *magnetic* and (2) *gyroscopic.* The magnetic compass tends to align itself with the magnetic lines of force of the earth, while the gyro compass seeks the true (geographic) meridian. The word "compass" is also applied to instruments which do not continuously indicate some form of north. Thus, an aircraft directional gyro (art. 2803) tends to remain approximately aligned with any great circle to which it is set. An astro compass, sun compass, or sky compass (art. 2515) is used to determine the heading or other reference directly at any given moment, by means of celestial bodies.

A compass may be designated to indicate its principal use, as a **standard, steering,** or **boat compass.** The compass designated as standard is usually a magnetic compass installed in an exposed position having an unobstructed view in most directions, permitting accurate determination of error. Preferably, it is located at a magnetically favorable position near the bridge. Before the development of a reliable gyro compass, the standard compass was used for navigation of the vessel and for determining the error of the steering compass.

Although the modern, reliable gyro compass has largely superseded the magnetic compass for most purposes, directional information is so important to a vessel that the availability of a second method is considered justified. It is wise to understand both types, keep a record of errors and the performance of all compasses, and to compare the indications of magnetic and gyro compasses at frequent intervals, as every half hour when underway.

623. Magnetic compasses.—If a small magnet is pivoted at its center of gravity in such manner that it is free to turn and dip, it will tend to align itself with the magnetic

field of the earth (art. 706). It thus provides a directional reference and becomes a simple compass. However, such a compass would not be adequate for use aboard ship. For this purpose a compass should have a stronger directive element than that provided by a single, pivoted magnet, should have provision for measuring various directions, should have some means of damping the oscillations of the directive element, should be approximately horizontal, and should have some means of neutralizing local magnetic influences.

In a mariner's compass, several magnets are mounted parallel to each other. To them is attached a **compass card** having a compass rose to indicate various directions (art. 624). Both magnets and compass card are enclosed in a bowl having a glass top through which the card can be seen. The bowl is weighted at the bottom and is suspended in gimbals in such manner that it remains nearly horizontal as the vessel rolls and pitches. In nearly all modern compasses the bowl is filled with a liquid that supplies a buoyant force almost equal to the force of gravity acting upon the directive element and card. This reduces the friction on the pivot (a metal point in a jeweled bearing), and provides a means of damping the oscillations of the compass card. The card is mounted in such manner as to remain in an essentially horizontal position. A mark called a **lubber's line** is placed on the inner surface of the bowl, adjacent to the compass card, to indicate the forward direction parallel to the keel when the bowl is correctly installed. The gimbals used for mounting the compass bowl are attached to a stand called a **binnacle,** which in most installations is permanently and rigidly attached to the deck of the vessel, usually on its longitudinal center line. Most binnacles provide means for neutralization of local magnetic influences due to magnetism within the vessel. A cover or "hood" is provided to protect the compass from the elements, dust, etc.

Directional information is of such importance that selection and installation of a suitable compass should be made carefully, seeking such guidance as may be needed. In the U. S. Navy this is covered by Bureau of Ships' directives. For merchant vessels and yachts, one would do well to consult a dependable compass adjustor before selecting and installing a compass or making any alteration in the vicinity of the compass. Common errors are the use of a compass designed for a different type craft (as an aircraft compass in a boat), permitting chrome plating of a binnacle by someone who does not know how to do this without creating a magnetic field, authorizing electric welding of steel near the compass, improper installation of magnetic equipment or electric appliances near the compass, allowing short circuits to occur in the vicinity of the compass, etc.

After the compass has been selected and installed, proper adjustment and compensation (ch. VII) are important, and future care of the instrument should not be neglected. It should be checked and overhauled at regular intervals, and any indication of malfunctioning or deterioration, however slight, should not be overlooked. Discoloration of the liquid or the presence of a bubble, for instance, indicates a condition that should be investigated and corrected at once. If it becomes necessary to add liquid, one should be certain that he has the correct substance, and should attempt to determine the source of the leak. Except as a temporary expedient, this is best done by a professional. Some compasses should be protected from prolonged exposure to sunlight, to prevent discoloration of the card and liquid.

If a vessel is to be inactive for a long period of time—at least several months—it is good practice to remove the magnetic compass from its binnacle and store it in a place relatively free from magnetic influences, and of approximately even temperature. Unless instructions indicate otherwise, the compass should be stored upside down, to remove the weight from the pivot, and prevent the card from swinging.

624. The compass card is composed of light, nonmagnetic material. In nearly all modern compasses the card is graduated in 360°, increasing clockwise from north through east, south, and west. An older system still used somewhat is to graduate the card through 90° in each quadrant, increasing from both north and south. Some compass cards are graduated in "points," usually in addition to the degree graduations. There are 32 **points of the compass,** 11¼° apart. The four **cardinal points** are north, east, south, and west. Midway between these are four **intercardinal points** at northeast, southeast, southwest, and northwest. These eight points are the only ones appearing on the cards of compasses used by the U. S. Navy. The eight points between cardinal and intercardinal points are named for the two directions between which they lie, the cardinal name being given first, as north northeast, east northeast, east southeast, etc. The remaining 16 points are named for the nearest cardinal or intercardinal point "by" the next cardinal point in the direction of measurement, as north by east, northeast by north, etc. Smaller graduations are provided by dividing each point into four "quarter points," thus producing 128 graduations altogether. There are several systems of naming the quarter points. That used in the U. S. Navy when quarter points were used is given in table 2.

The naming of the various graduations of the compass card in order is called **boxing the compass,** an important attainment by the student mariner of earlier generations. The point system of indicating relative bearings (art. 904) survived long after degrees became almost universally used for compass and true directions. Except for the cardinal and intercardinal points, and occasionally the two-point graduations, all of which are used to indicate directions generally (as "northwest winds," meaning winds from a general northwesterly direction), the point system has become largely historical.

625. The U. S. Navy 7½-inch compass has a liquid-filled bowl in which a 7½-inch aluminum card is pivoted. There is provision for either one or two pairs of magnets, symmetrically placed. The card and magnet assembly is provided with a central float or air chamber to reduce the weight on the pivot to between 60 and 90 grains (0.14 and 0.21 oz.) at 60° F when the correct compass fluid is used. Older compasses use a fluid consisting of 45 percent ethyl alcohol and 55 percent distilled water. Newer compasses use a highly refined petroleum distillate similar to varsol. Use of this oil increases the stability and efficiency of the compass. A hollow cone extends into the underside of the float. The bottom of this cone is open. The pointed top has a jewel bearing of synthetic sapphire. The card-float-magnet assembly rests on an osmium-iridium tipped pivot at the jewel center. This pivot extends upward from the bottom of the bowl. This compass is illustrated in figure 625.

The compass bowl is made of cast bronze, and has a tightly gasketed glass top cover to prevent leakage of the liquid. A bellows-type expansion chamber is provided to allow for changes in volume of the liquid as the temperature changes. The top rim or bezel of the bowl is accurately machined so that an azimuth or bearing circle can be placed over it. The compass is equipped with a gimbal ring for keeping the compass level when mounted in a binnacle. In addition to providing support for the compass, the binnacle has provision for housing the correctors used to neutralize local magnetic effects within the vessel.

626. The U. S. Navy six-inch compass is a newly developed instrument which differs in a number of respects from older magnetic compasses. It is lighter in weight, requires less space, and is expected to prove more reliable with less maintenance than the 7½-inch compass. The six-inch diameter card is of magnesium foil, strengthened by concentric and radial ribs. This card and the small, powerful Alnico V magnets are sufficiently light in weight that a float is unnecessary. An osmium-tipped pivot

attached to the underside of the card, at its center, rests on a concave synthetic sapphire jewel in the top of a spindle attached to the bottom of the compass bowl. Expansion of the liquid, an oil similar to varsol, is taken care of by a bubble-trap type expansion chamber. This chamber is in the form of a cylinder surrounding the card area and connected to it by a single, small opening at the bottom. The top half, approximately, of the expansion chamber is filled with air which is compressed as the liquid expands. As the liquid contracts, the trapped air pushes more of it into the card area. This arrangement eliminates the need for the troublesome bellows of older compasses. The light for illuminating the compass card and lubber's line is housed at the bottom of the compass. Its intensity can be adjusted by a rheostat at the base of the binnacle.

Both the binnacle and pedestal upon which it stands are of cast aluminum. The binnacle has provision for neutralizing the effects of the magnetism within the vessel,

Figure 625.—U. S. Navy 7½-inch compass.

and the pedestal houses electrical coils and resistor panels for reducing or eliminating the magnetic effects introduced by degaussing (ch. VII). The soft iron correctors (ch. VII), both quadrantal and Flinders bar, are thin-walled tubes supported in aluminum spacers with heavier aluminum housings bracketed to the outer wall of the binnacle. The quadrantal correctors can be slewed to reduce E error (ch. VII). Provision is made for mounting the Flinders bar on either the forward or after side of the compass.

627. Other magnetic compasses.—In addition to the 7½-inch and 6-inch compasses, the U. S. Navy has a five-inch alcohol-and-water filled compass, and two three-inch varsol-filled compasses. One of the three-inch compasses is top-reading like the larger compasses, and the other has the graduations on the side of the beveled outer edge of the card, so that the reading can be made through a window on the after side of the compass bowl, in a manner similar to the reading of an aircraft compass mounted on an instrument panel.

A wide variety of magnetic compasses are used in merchant ships and yachts. The basic principles of operation of all magnetic compasses are the same, the various

types differing only in details of construction. A feature which is widely used in commercial compasses is a hemispherical top (fig. 627) which provides magnification of the graduations.

An older type which is now rarely encountered is the **dry compass,** so called because it does not have a liquid-filled bowl. A typical dry compass has a card of strong paper, with the central part cut away and the outer edge stiffened by a thin aluminum ring. The weight of the paper card is sustained by 32 radial silk threads. Eight small, magnetized steel needles are suspended by silk threads from the aluminum ring.

Courtesy of Wilfrid O. White and Sons, Inc.

FIGURE 627.—A compass with a hemispherical top.

628. Magnetic compass limitations.—Because of its essential simplicity, a magnetic compass does not easily become totally inoperative. Being independent of any power supply or other service, a magnetic compass may survive major damage to its ship without losing its utility. Small boat compasses often remain serviceable under the most rigorous conditions.

Despite its great reliability, however, a magnetic compass is subject to some limitations. Since it responds to *any* magnetic field, it is affected by any change in the local magnetic situation. Hence, the undetected presence or change of position of magnetic material near the compass may introduce an unknown error. Thus, an error might be introduced by a steel wrench or paint can left near the compass, or by a change in position of a steel boom or gun in the vicinity of the compass. Even such small amounts of magnetic material as might be included in a pocketknife or steel keys are sufficient to affect the compass if brought as close as they are when on the person of an individual standing by a compass. Nylon clothing may also introduce error in a magnetic compass. As distance from the compass increases, the strength of the magnetic field needed to introduce an error increases. A cargo of large amounts of iron or steel may be sufficient to affect the compass. The compass may also be affected by changes of the magnetic characteristics of the vessel itself. Such changes may occur during a protracted docking period, during a long sea voyage on substantially the same course, when repairs or changes of equipment are made, if the ship sustains heavy shock as by gunfire or riding out a heavy sea, if the vessel is struck by lightning, or if a short circuit occurs near the compass.

The directive force acting upon a magnetic compass is the horizontal component of the earth's magnetic field. This component is strongest at or near the magnetic equator, decreasing to zero at the magnetic poles (ch. VII). Near the magnetic poles, therefore, the magnetic compass is useless (art. 2513), and in a wider area its indications are of questionable reliability. The magnetic field of the earth has a number of local anomalies due to the presence of magnetic material within the earth. During magnetic

storms (art. 2526) it may be altered considerably. Changes in the magnetic field surrounding a vessel, due either to changes of the field itself or to change of position of the vessel within the field, affect the magnetism of the vessel and the correctors used to neutralize this effect, with a possible disturbance of the balance set up between them.

For these and other reasons, frequent determination of compass error is necessary for safe navigation. Methods of determining and correcting compass error are discussed in chapter VII.

629. Magnetic compass accessories.—Compass heading is indicated by the lubber's line. Compass bearings may be measured by sighting across the compass, bringing the object and the vertical axis of the compass in line. Accuracy in making this alignment is increased by the use of a device to direct the line of sight across the center of the compass. Perhaps the simplest device of this kind is a **bearing bar,** consisting of two vertical **sighting vanes** mounted at opposite ends of a horizontal bar having a small pivot which fits into a hole drilled part way through the glass cover of the compass, at its center. The "near" vane (nearer the eye of the observer) has a very thin, open, vertical slot through which the line of sight is directed; the "far" vane has a thin, vertical wire or thread mounted on a suitable frame. The bar is rotated until the object is in line with the two vanes. The bearing is the reading of the compass in line with the vanes, on the far side from the observer. If a reflecting surface is pivoted to the far vane to permit observation of the azimuth (art. 1428) of a celestial body, the device is called an **azimuth instrument.** Bearing bars and azimuth instruments are usually used only with smaller compasses, and never with an after-reading compass (art. 627).

Larger compasses or repeaters (art. 641) are usually provided with a **bearing circle** or **azimuth circle** (fig. 629). These devices take a variety of forms, but consist essentially of two parts: (1) a pair of sighting vanes attached to a ring which fits snugly over the compass, and (2) a mirror to reflect the compass graduation into the line of sight. The use of these devices is similar to that of the bearing bar and azimuth instrument. The azimuth circle has a pivoted reflecting surface attached to the far vane, to permit observation of celestial bodies. In most cases it also has a reflecting mirror and prism mounted on opposite sides of the ring, midway between the vanes. The prism is covered with opaque material except for a thin, vertical slot at its center. The surface of the mirror is curved so that reflection of sunlight falling upon it is in the form of a slender vertical line (at the distance of the prism) of about the same width as the slot. When the azimuth circle is adjusted so that this line of light falls upon the slot, a thin, bright line appears on the compass card graduations at the bearing of the sun. Most bearing and azimuth circles are provided with reverse compass rose graduations to permit reading of relative bearings or azimuths (by the vanes) at a mark on top of the compass bowl, in line with the lubber's line; bubbles for indicating the level position during observation; means for adjusting the snugness of the fit over the compass bowl; and handles for turning the device.

If a bearing or azimuth circle does not fit snugly over the compass bowl, an error might be introduced. Inaccuracy may also result from tilting of the reflecting surface of an azimuth circle with respect to the vertical plane through the line of sight. This can be checked by comparing an azimuth of the sun observed by means of the prism with one observed with the sighting vanes (with suitable protection being provided for the eyes). If the prism attachment is not available, a check can be made by comparing observed (compass) azimuths at different altitudes with computed (true) values at the time of observation. If both observed and computed azimuths are correct, the difference between them will be constant (if the compass error remains constant throughout the observation).

FIGURE 629.—An azimuth circle.

None of the bearing or azimuth instruments described above can be used with a compass not designed for it, as one having a hemispherical top, or an after-reading compass.

Some modern magnetic compasses are provided with electrical pick-offs of sufficient sensitivity that the instrument can be used to control such devices as remote indicators, automatic steering equipment, course recorders, and dead reckoning equipment without disturbing the reliability of the compass. However, these devices are more commonly controlled by a gyro compass and hence are considered later in the chapter, after a discussion of this type compass.

630. The gyroscope.—Leon Foucault, a French physicist, first demonstrated the rotation of the earth by means of a pendulum. However, the pendulum was not entirely acceptable as proof of rotation because it required the earth's gravity for operation. In 1852, he gave the name **gyroscope** to a toy top which had been known for a quarter of a century as a "rotascope." By means of the gyroscope, Foucault illustrated the earth's rotation without the use of gravity.

A conventional gyroscope consists of a comparatively massive, wheel-like rotor balanced in gimbals which permit rotation in any direction about three mutually perpendicular axes through the center of gravity. The three axes are called the **spin axis,** the **torque axis,** and the **precession axis,** as shown in figure 630.

Since the rapidly spinning rotor is balanced at its center of gravity, it is in a state

of neutral rotational equilibrium. If the gimbal bearings were completely frictionless, the spin axis would retain its direction in space despite any motion applied to the system as a whole, as by the rotation of the earth. This property is called **gyroscopic inertia.** Thus, if the spin axis were directed toward a star, the axis would continue to point toward the star during its apparent motion across the sky. To an observer on the earth, the spin axis would appear to change direction as the earth rotated eastward. If the spin axis were placed parallel to the earth's axis, the earth's rotation would have no effect and the device would become a kind of compass, since the spin axis would be in the plane of a meridian. However, such a device would require frictionless bearings and perfect balance. Even if these obstacles could be overcome, the device would not be suitable as a compass because it would not be *north-seeking.*

The method by which a gyroscope is made to *seek* north involves the surprising behavior exhibited by any rotating mass, when a force is applied which tends to change the *direction* in space of the spin axis. The motion resulting from such a force is not in line with the force, as might be expected, but *perpendicular* to it. This property is called **gyroscopic precession.**

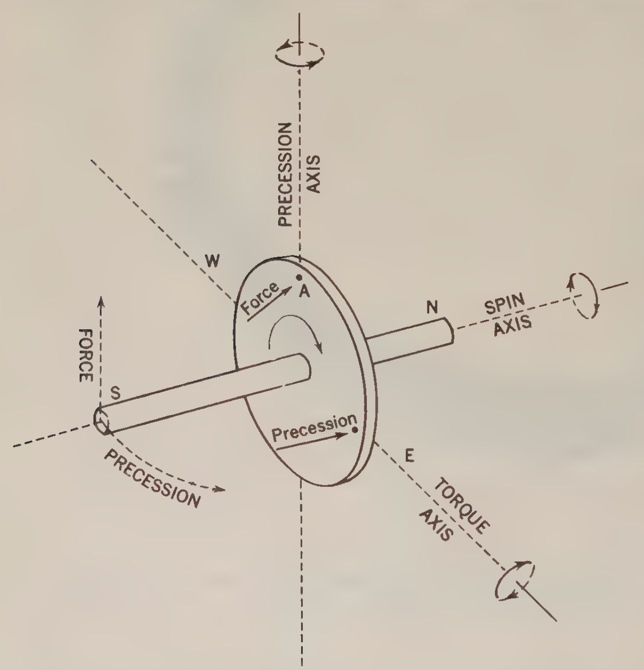


FIGURE 630.—Axes of a gyroscope, and the direction of precession.

Refer to figure 630 and suppose that the torque and spin axes are horizontal, that the spin axis is directed north and south, and that the rotation about the spin axis is clockwise, looking north. If a force is applied to the rotor at A tending to raise the south end of the spin axis, the south end, if free to move, will turn or "precess" to the east, as shown. The direction of precession is such that it appears as though a force applied to the rotor at A is, instead, applied at a point *90° away in the direction of spin* from point A. Precession tends to move the plane *and direction* of rotation of the gyroscope into alignment with the force applied to the rotor. If precession is prevented, as by restraining motion of the spin axis, this axis will rotate in the direction of the applied force, as if the rotor were not spinning. Thus, in figure 630, a force applied to the rotor at A causes the south end of the spin axis to rise. The reason for this is that if precession is blocked, the force thus introduced causes precession in the direction of the original force. This effect is used to stabilize some types of gyro compasses and avoid cumulative errors due to rolling while the vessel is on intercardinal headings.

A recently developed gyroscope called a **Gyrotron vibratory gyro** uses a vibrating mass instead of a rotating one. It is based upon the same principle used in the halteres of certain two-winged insects, such as the common housefly, to give them the sensing needed to achieve stability in flight. Instead of a single vibrating reed, the vibratory gyro uses a two-pronged device similar to a tuning fork. The vibratory gyro has no bearings, and so is free from the errors introduced by bearing friction. It is a rugged

device having long life and requiring little attention. Its most promising application is for measurement of rate of turn, which it performs more accurately than a rotating gyro, and over a wide range from a rate of as little as one or two degrees per hour to 100 revolutions or more per minute. The Gyrotron vibratory gyro has a number of possible applications to navigation.

631. The gyro compass.—A gyro compass is essentially one or more north-seeking gyroscopes with a suitable compass rose, housing, etc.

One method of utilizing precession to cause a gyroscope to seek north is illustrated in figure 631a. Two reservoirs connected by a tube are attached to the bottom of the case enclosing the gyro rotor, with one reservoir north of the rotor and the other south of it. The reservoirs are filled with mercury to such a level that the weight below the spin axis is equal to the weight above it, so that the gyroscope is nonpendulus. The system of reservoirs and connecting tubes is called a **mercury ballistic.** In practice, there are usually four symmetrically placed reservoirs.

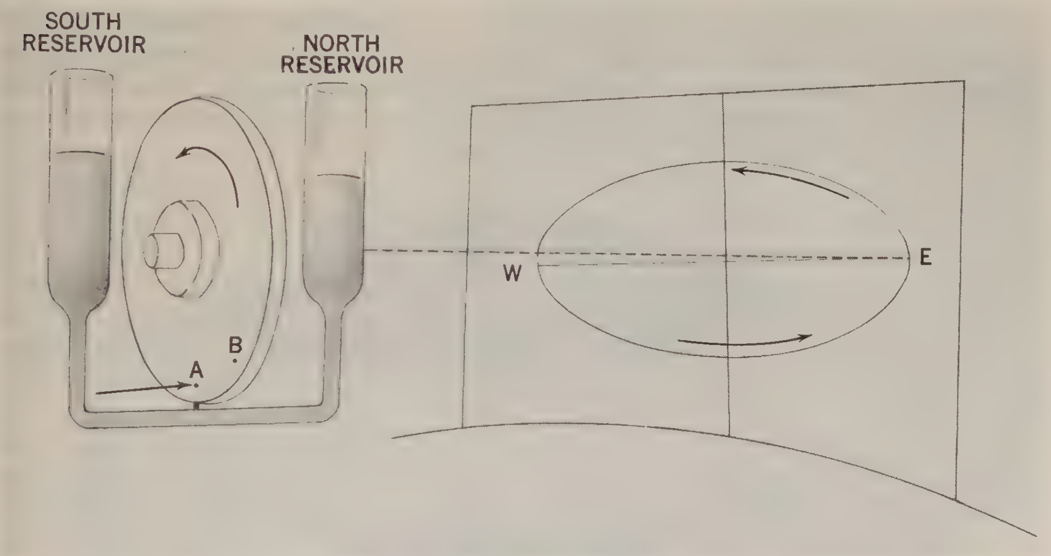


FIGURE 631a.—The mercury ballistic (left) and the elliptical path (right) of the axis of spin without damping.

Suppose that the spin axis is horizontal but is directed to the eastward of north. As the earth rotates eastward on its axis, the spin axis tends to maintain its direction in space; that is, it appears to follow a point, such as a star rising in the northeastern sky. With respect to the earth, the north reservoir rises and some of the mercury flows under the force of gravity into the south reservoir. The south side becomes heavier than the north side, and a force is applied to the bottom of the rotor case at point A. If the gyro rotor is spinning in the direction shown, the north end of the spin axis precesses slowly to the westward, following an elliptical path. When it reaches the meridian, upward tilt reaches a maximum. Precession continues, so that the axis is carried past the meridian and commences to sink as the earth continues to rotate. When the sinking has continued to the point where the axis is horizontal again, the excess mercury has returned to the north reservoir and precession stops. As sinking continues, due to continued rotation of the earth, an excess of mercury accumulates in the north reservoir, thus reversing the direction of precession and causing the spin axis to return slowly to its original position with respect to the earth, following the path shown at the right of figure 631a. One circuit of the ellipse requires about 84 minutes.

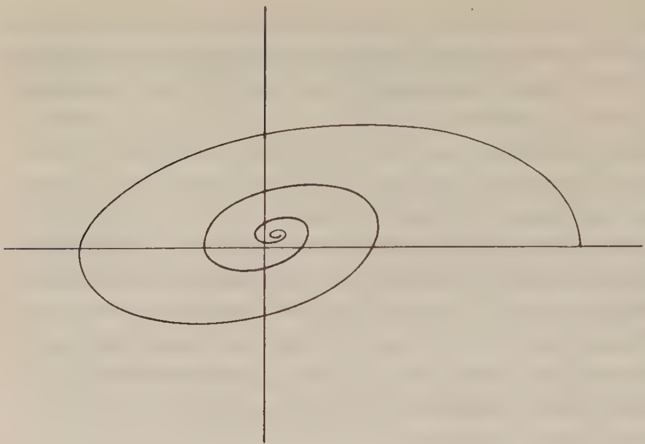
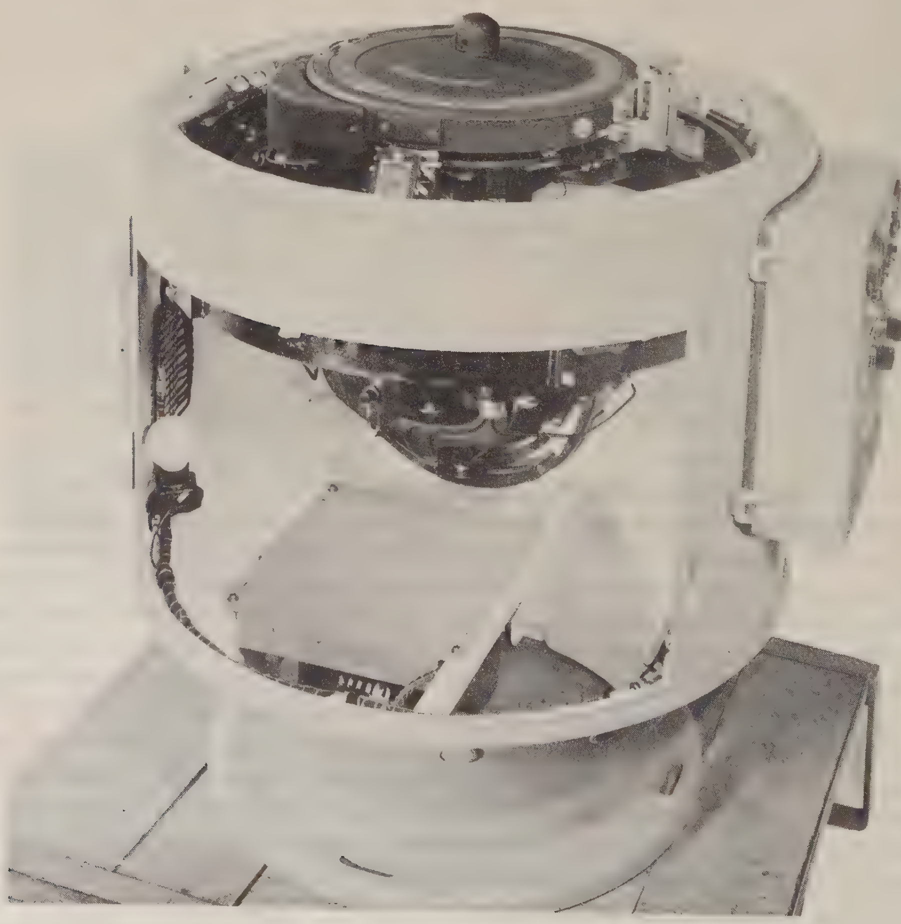


FIGURE 631b.—Spiral path of the axis of spin with damping.

The elliptical path is symmetrical with respect to the meridian, and, neglecting friction, would be retraced indefinitely, unless some method of damping the oscillation were found. One method is by offsetting the point of application of the force from the mercury ballistic. Thus, if the force is applied not in the vertical plane, but at a point to the eastward of it, as at *B* in figure 631a, the resulting precession causes the spin axis to trace a spiral path as shown in figure 631b, and eventually to settle near the meridian. The gyroscope is now north-seeking and can be used as a compass. Some compasses are provided with means for automatically moving the point of application to the center line during a large change of course or speed, to avoid introduction of a temporary error.



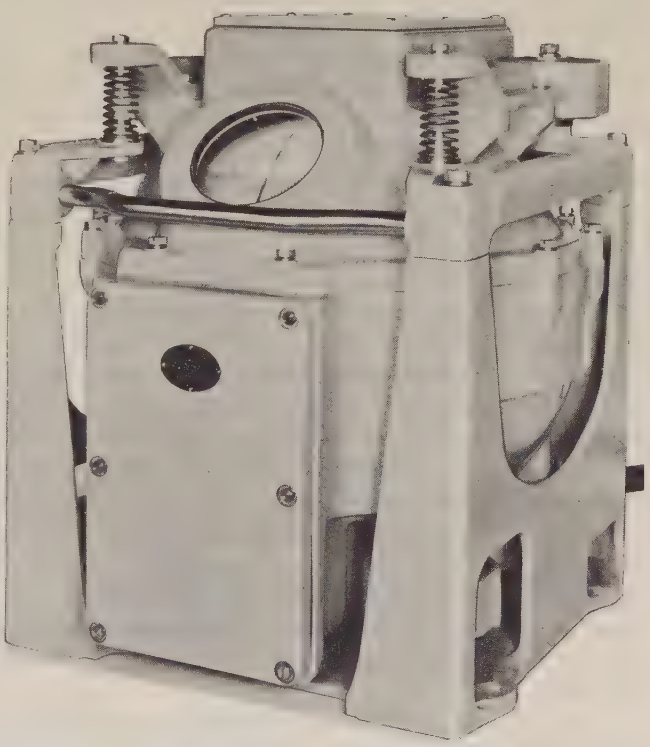
Courtesy of the Sperry Gyroscope Co.

FIGURE 631c.—The Mark 14 Mod 2 gyro compass.

Another method of damping the oscillations caused by the rotation of the earth is to reduce the precessing force of a pendulous gyro as the spin axis approaches the meridian. One way of accomplishing this is to cause oil to flow from one damping tank to another in such a manner as to counteract some of the tendency of an offset pendulous weight to cause precession. Oscillations are completely damped out in approximately one and one-half swings.

Details of construction differ considerably in the various compasses. Each instrument is provided with a manual giving such information and operating instructions. Figure 631c illustrates the Mark 14, Mod 2 gyro compass, a type that is widely used in the U. S. Navy and the merchant marine. The Mark 23 Mod 0 gyro compass, illustrated in figure 631d, is a much smaller compass recently developed in accordance with U. S. Navy specifications to provide an instrument that can be used in vessels of many types.

632. Desirable characteristics of the gyro compass.—Since a gyro compass is not affected by a magnetic field, it is not subject to magnetic compass errors (ch. VII), nor is it useless near the earth's magnetic poles. If an error is present, it is the same on all headings, and no table of corrections is needed. The directive force is sufficiently strong to permit directional pick-off for use in remote-indicating repeaters, automatic steering, dead reckoning and fire-control equipment, course recorders, etc.



Courtesy of the Sperry Gyroscope Co.

FIGURE 631d.—The Mark 23 Mod 0 gyro compass.

633. Undesirable characteristics of the gyro compass.—A gyro compass is dependent upon a source of suitable electric power.

If operation of the compass is interrupted long enough to permit uncertainty in its indications, a considerable period (as much as four hours for some gyro compasses) may be needed for it to settle on the meridian after it reaches operating speed. This period can be reduced by orienting the compass in the proper direction before it is started. If this is not practicable, the settling period can be hastened by leveling the compass when it reaches the meridian (one-fourth of a cycle or 21 minutes after starting at maximum deflection) or by leveling *and* precessing the gyro to the approximate meridian after its direction and rate of precession are observed for several minutes. Either process may need to be repeated several times and followed by a settling period.

The gyro compass is subject to certain errors requiring applications of corrections, either manually or automatically (art. 634).

The compass is an intricate mechanism of many parts. Thus, it requires some maintenance. In heavy seas a gyro compass may become unreliable unless certain

features are included in the design—features which are generally omitted from the smaller, simpler compasses.

The directive force of a gyro compass decreases with latitude, being maximum at the equator and zero at the geographical poles. The compass remains usable at all latitudes thus far attained by surface vessels, except those which have become beset and drifted with the ice across the Arctic Ocean. The use of the gyro compass in high latitudes is discussed in articles 640 and 2514.

A gyro compass has generally been considered unsuitable for use in aircraft because of its weight and the question of whether it will operate at the high speeds (approaching or exceeding that of rotation of the earth) and accelerations to which it would be subjected in aircraft. A gyro compasss weighing only nine pounds has been developed for use in small craft. A light compass designed for use in aircraft is being developed and evaluated.

634. Gyro compass errors.—Gyro compasses are subject to several systematic errors (art. 2903). Some of these can be eliminated or offset in the design of the compass, while others require manual adjustment for their correction.

The total combined error (the resultant error) at any time is called **gyro error (GE)**, which is expressed in degrees east or west to indicate the direction in which the axis of the compass is offset from true north. If the gyro error is east, the readings are too low; and if it is west, they are too high. Thus, if GE is 1° W, 1° is subtracted from all readings of the compass, either headings or bearings, to determine the equivalent true directions. One degree is added to all true directions to determine the equivalent gyro directions. The gyro error of modern compasses is generally so small that it can be ignored for practical navigation. However, significant errors can be introduced in several ways, and it is good practice to compare the gyro heading with the magnetic heading at frequent intervals (as every half hour and after each change of course) and to check the accuracy of the gyro compass by celestial observation or landmarks from time to time (as every morning and afternoon when means are available).

The errors generally associated with the gyro compass are speed error, damping error, ballistic deflection error, quadrantal error, and gimballing error. In addition, gyro compasses are subject to the errors common to directional instruments, such as those introduced by inaccurate graduation of the compass rose or incorrectly located lubber's line. Error may also be introduced, of course, by malfunctioning of the compass.

635. Speed error is introduced by motion of the vessel along its track. Refer to figure 635a. If a vessel is at anchor at any point *A*, it is being carried eastward by rotation of the earth at the rate of 902.46 minutes of longitude per hour (with respect to the stars). In terms of knots, this is equal to 902.46 times the cosine of the latitude, approximately. Because of the ellipticity of the earth, the actual value is a little more than this in low latitudes, and a little less in high latitudes. The actual value at any latitude can be found by multiplying the length of a degree of longitude at that latitude (from table 6) by $\frac{902.46}{60}=15.041$.

This eastward motion due to rotation of the earth is shown in figure 635a by the vector *AB*. The north-south axis of the gyro compass settles in a direction 90° from the direction of motion. Therefore, if the vessel is stationary with respect to the earth, 0° on the compass card coincides with a true meridian, and no error is introduced. This is also true if the vessel is moving due east or due west. In this case the speed of the ship over the surface of the earth is added to or subtracted from the motion due to rotation of the earth, but the direction of motion is unchanged (unless the speed of the vessel is

greater than the rotational speed of the earth, and in the opposite direction). The only effect, therefore, is to strengthen or weaken the directive force, usually by a small amount.

If the vessel is on course north or south, as shown by the vector AC in figure 635a, the motion in space is tilted toward the north or south of due east. In this case, it is the vector sum (art. O18) of the motion due to rotation of the earth and the velocity of the vessel over the surface of the earth, or AD in figure 635a. Since AD is not due east, the perpendicular to it does not lie in the true meridian, but at some angle δ to it, along AM_v. Since the axis of the gyro lies along AM_v, the "virtual meridian," the angle is the error introduced by the motion of the vessel along its track. Since AD is perpendicular to AM_v, and AB is perpendicular to AC, angle BAD is equal to angle δ. Therefore, the angle δ can be found by the formula

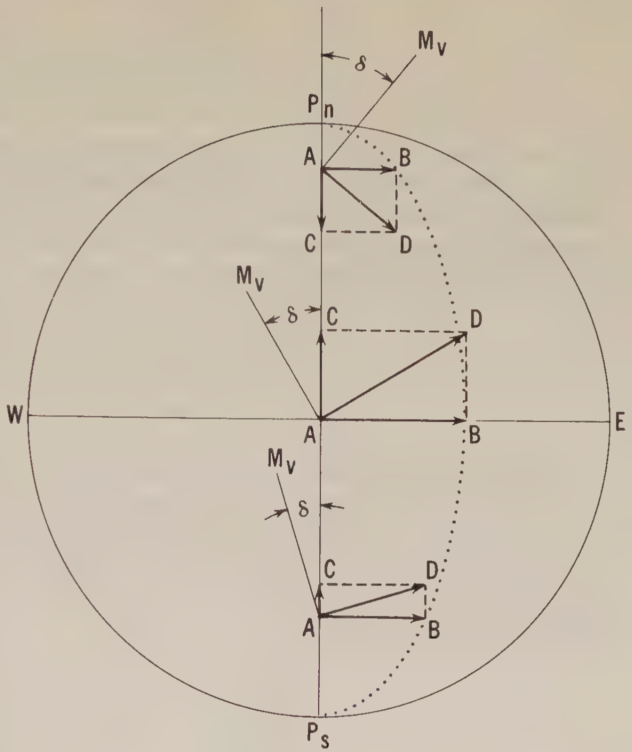


FIGURE 635a.—Speed error.

$$\tan \delta = \frac{AC}{AB}.$$

Since AC is the speed of the vessel and AB is 902.46 cos L, approximately, the formula can be written

$$\tan \delta = \frac{S}{902.46 \cos L}$$

where S is the speed and L the latitude of the vessel.

If the course of the vessel is not a cardinal direction, the resultant is still the vector sum of the two speed vectors, and can be found graphically or by computation. One method is to resolve the vessel's speed vector into two components, as shown in figure 635b, obtaining the N–S component along the true meridian, and the E–W component in the direction of rotation of the earth. The N–S component is equal to S cos C, and the E–W component to S sin C, where C is the true course angle. The total N–S motion is then S cos C. The total easterly motion is that due to rotation of the earth plus or minus the E–W component of the ship's speed across the surface of the earth, or 902.46 cos L±S sin C, approximately. The term S sin C is positive (+) for easterly courses and negative (−) for westerly courses. The formula for finding δ now becomes

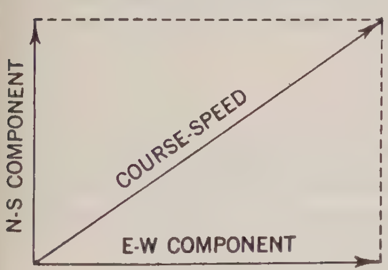


FIGURE 635b.—Components of vessel's motion.

$$\tan \delta=\frac{S \cos C}{902.46 \cos L \pm S \sin C} \text{ (approximately).}$$

At ship speeds in latitudes less than 70°, the term S sin C is much smaller than 902.46 cos L and has so little effect upon the answer that it can be ignored. The angle δ is small enough that its tangent can be considered the angle itself (expressed in radians). That is, a tangent to a circle can be considered of the same length as an arc of the circle over a short distance from the point of tangency. Therefore, the formula for δ can be written

$$\delta=\frac{57.3 \text{ S} \cos C}{902.46 \cos L}$$

or

$$\delta=0.0635 \text{ S} \cos C \sec L.$$

As shown in this formula, the speed error δ is affected by the three variables, speed, course, and latitude. If the course has a northerly component, the error is westerly; and if it has a southerly component, the error is easterly.

Example.—A ship at latitude 30° N is steaming on true course 045°, at a speed of 20 knots.

Required.—Speed error.

Solution.—

0.0635	log	8. 80277
S 20 kn.	log	1. 30103
C N45° E	*l* cos	9. 84949
L 30° N	*l* sec	10. 06247
δ 1°.04 W	log	10. 01576

Answer.— δ 1°.04 W.

In most gyro compasses this error is corrected mechanically. Speed and latitude are set in by hand, and the cosine of the course is introduced automatically by means of a "cosine cam" running in an eccentric groove on the underside of the azimuth gear. In some compasses these corrections combine to offset the lubber's line by the correct amount. Small changes in speed or latitude have relatively little effect upon the result. Therefore, in normal operations, infrequent changes are sufficient for satisfactory results. If no provision is made for mechanically applying this correction, a table or curves can be used to indicate the correction to be applied mathematically to readings of the compass. These are made up from the formula given above, and are entered with the speed, course, and latitude (art. 640).

636. Damping error applies only to those gyro compasses in which damping is accomplished by offsetting the point of application of the force from a mercury ballistic (art. 631). For this reason it is sometimes called **ballistic damping error.** It can be found from the equation

$$\alpha=r \tan L$$

in which α is the damping error, r is the angle between the vertical through the spin axis of the gyro rotor and a line through this axis and the point of application of the force from the mercury ballistic (1°.7 for Sperry compasses), and L is the latitude. The error is easterly in north latitude and westerly in south latitude.

Example.—A gyro compass having a value of r of 1°.7 is at latitude 50° N.

Required.—The damping error.

Solution.—

$$\alpha = r \tan L$$
$$= 1^\circ 7 \times 1.1918$$
$$= 2^\circ 03 \text{ E}$$

Answer.—$\alpha = 2^\circ 03$ E.

As in the case of speed error, provision is made in most compasses (to which it applies) for correcting this error. An auxiliary latitude-correction scale is provided for this purpose. In some compasses this offsets the lubber's line. In others, it alters the position of a small weight attached to the casing near one end of the axle. The first method is preferable because it is unaffected by changes of gyro speed of rotation.

If this error is not corrected mechanically, it can be combined algebraically with speed error and a single set of tables or graphs made up. This is a method sometimes used in polar regions, beyond the scale of the latitude corrections (arts. 640, 2514).

637. Ballistic deflection error.—When the north-south component of the speed changes, an accelerating force acts upon the compass, causing a surge of mercury from one part of the system to another, or a deflection (along the meridian) of the mass of a pendulous compass. In either case, this is called **ballistic deflection.** It results in a precessing force which introduces a temporary **ballistic deflection error** in the readings of the compass unless it is corrected.

A change of course or speed also results in a change in the speed error, and unless the correcting mechanism responds promptly to this change, a temporary error from this source is also introduced. The sign of this error is opposite that of the ballistic deflection, and so the two tend to cancel each other. If they are of equal magnitude and equal duration, the cancellation is complete and the compass responds immediately and automatically to changes of speed error. This can be accomplished by designing the compass so that

$$\frac{B}{H} = 0.0211 \sec L$$

in which B is the pendulous moment of a pendulous compass and the couple per unit angle applied by a mercury ballistic, H is the angular momentum of the gyro rotor, and L is the latitude.

It is customary to design a gyro compass so that the ratio $\frac{B}{H}$ is correct for some particular latitude (as 41° or 45°) and accept the small residual error that is temporarily present at other latitudes. This is satisfactory for vessels which remain within relatively narrow limits of latitude, or which are seldom subjected to large accelerating forces. However, where these conditions are not met, provision is made for varying the ratio with latitude. In a compass having a mercury ballistic, this is customarily accomplished by moving the mercury reservoirs radially toward or away from the center of the compass, thus altering the value of B. In a pendulous gyro, the value of H is changed by altering the rotational speed of the gyro.

When the ratio $\frac{B}{H}$ is as given in the equation above, the period of oscillation about the vertical axis is given by the equation

$$T = \frac{\pi}{30}\sqrt{\frac{R}{g}}$$

in which T is the period in minutes, R is the radius of the earth in feet (approximately 20,900,000) and g is the acceleration due to gravity (approximately 32.2 feet per second per second). Substituting in the formula,

$$T=0.1047\sqrt{\frac{20,900,000}{32.2}}$$

$$=84 \text{ minutes (approximately).}$$

This is sometimes stated as the period of a pendulum having a radius equal to the radius of the earth, since the equation for a short pendulum is the same as that given above with l (length) being substituted for R. More accurately, it is the period of a pendulum of infinite length with its bottom at the surface of the earth, or the largest period that a simple pendulum can have when acting under the gravitational force of the earth. When a device is adjusted so as to have this period it is said to be "Schuler tuned," after Ivan Schuler, a German scientist who discovered the relationship. It is because of this tuning of the gyro compass that one oscillation occurs in about 84 minutes, and that the maximum effect of certain disturbing forces occurs about 21 minutes (one-fourth cycle) after application of the force.

638. Quadrantal error.—If a body mounted in gimbals is not suitably balanced, a disturbing force causes it to swing from side to side. A swinging body tends to rotate so that its long axis of weight is in the plane of the swing. The rolling of a vessel introduces the force needed to start a gyro compass swinging. The effect reaches a maximum on intercardinal headings, midway between the two horizontal axes of the compass, and changes direction of error in consecutive quadrants. This is called **quadrantal error,** or sometimes **intercardinal rolling error.** It is corrected by the addition of weights to balance the compass so that the weight is the same in all directions from the center. Without a long axis of weight, there is no tendency to rotate during a swing.

A second cause of quadrantal error is more difficult to eliminate. As a vessel rolls, the apparent vertical is displaced first to one side and then to the other, due to the accelerations involved. The vertical axis of the gyro compass tends to align itself with the apparent vertical. If the vessel is on a northerly or southerly course, the pivot of the compass is displaced from the vertical, resulting in a precession first to one side, then to the other. The effect is negligible and would be exactly balanced if successive rolls on opposite sides were equal. On an easterly or westerly heading, the pivot remains under the gyro axle, but the dynamic effect of the roll, acting upon the damping mechanism, introduces a precessing force which causes an error. However, the period is short and the error is in opposite directions on opposite rolls, so the effect is negligible. On noncardinal headings, both effects are present, and the relationship is such that the error is in the same direction regardless of the direction of roll. Thus, a persistent error is introduced, which changes direction in successive quadrants. This error is generally eliminated by the use of a second gyroscope. In some compasses, this is in the form of a small gyroscope called a **floating ballistic** which stabilizes the point of application of the mercury ballistic with respect to the true vertical as the vessel rolls. In others, two gyroscopes are used for the directive element and these are so installed that they tend to precess in opposite directions. Thus, they neutralize each other. Another way of eliminating this error is to design the mercury ballistic system so that the surge of liquid due to north-south component of the roll is diminished in amount and delayed so that it is about a quarter of a cycle out of phase with the roll.

639. Gimballing error is that due to tilt of the compass rose. Directions are measured in the horizontal plane. If the compass card is tilted, the projection of its

outer rim onto the horizontal is an ellipse, and the graduations are not equally spaced with respect to a circle. This error, which applies to all instruments making use of a compass rose that can be tilted, is discussed in article 2903. For normal angles of tilt, this error is small and can be neglected. For accurate results, readings should be made when the card is horizontal. This error applies to the reading of the compass *or its repeaters* (art. 641), rather than to the compass itself. If the compass and its repeaters are installed so that the outer gimbals are in the longitudinal axis of the vessel, this error is minimized.

640. Use of the gyro compass in polar regions is discussed in article 2514. If means are not available for determining an equivalent setting or correction, a correction graph can be constructed. Ballistic deflection error, quadrantal error, and gimballing error are temporary or corrected in the design of the compass, and so can be ignored. Speed error and damping error (if it applies to the particular compass involved) can be combined into a single table or curve of corrections, using the formulas of articles 635 and 636. In high latitudes the east-west component of the vessel's speed is significant, and the error may be too large to consider its tangent equal to the angle itself expressed in radians. Therefore, the applicable formulas are:

$$\tan \delta = \frac{S \cos C}{902.46 \cos L \pm S \sin C} \tag{1}$$

$$\alpha = r \tan L. \tag{2}$$

The only approximation remaining is the use of 902.46, which varies slightly with latitude. The error thus introduced is not significant. The U. S. Navy Bureau of Ships' curves for latitude 80° are shown in figure 640. From the intersection of the appropriate speed curve and the radial line representing the *true* course (interpolating if necessary) a horizontal line is drawn to the vertical line through the origin, where the correction is indicated. To construct the curve for speed 35 knots, proceed as follows:

(1) Compute the speed error, δ, for true courses at intervals of perhaps 30°. As an example, the error for course 210° (C S30°W) is:

$$\tan \delta = \frac{35 \times 0.86603}{902.46 \times 0.17365 - 35 \times 0.50000}$$

$$= 0.21773.$$

$$\delta = 12^\circ\!.3 \text{ E}.$$

The error is easterly because the course has a southerly component (art. 635).

(2) Compute the damping error. The curves of figure 640 are for a value of r of 1°.7:

$$\alpha = 1^\circ\!.7 \times 5.6713 = 9^\circ\!.6 \text{ E}.$$

In northern latitudes damping error is easterly.

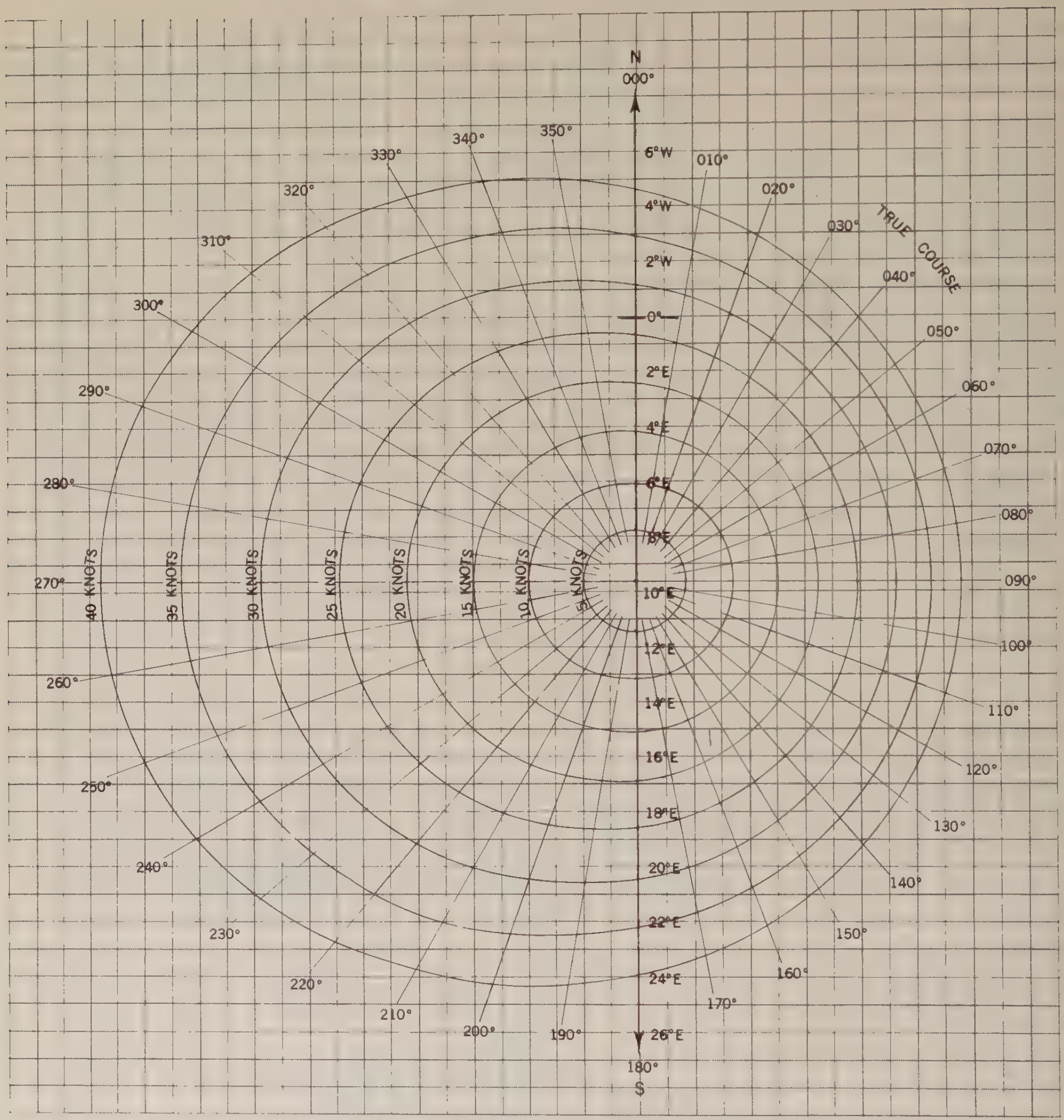


FIGURE 640.—Gyro compass error curves for latitude 80°.

(3) Combine δ and α algebraically to obtain gyro error (GE):

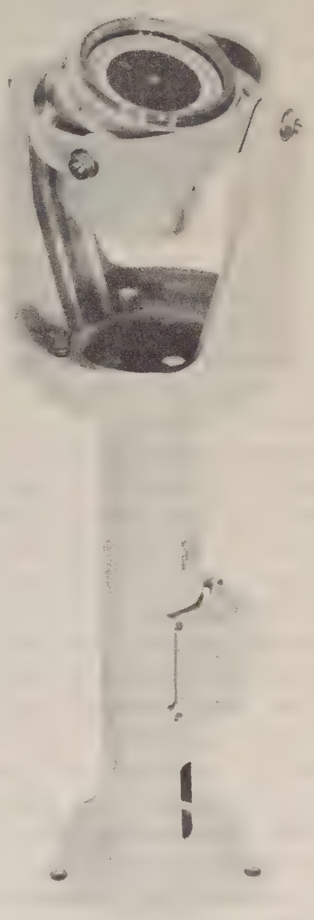
TC °	δ °	α °	*GE* °
000	12. 6 W	9. 6 E	3. 0 W
030	9. 9 W	9. 6 E	0. 3 W
060	5. 3 W	9. 6 E	4. 3 E
090	0. 0	9. 6 E	9. 6 E
120	5. 3 E	9. 6 E	14. 9 E
150	9. 9 E	9. 6 E	19. 5 E
180	12. 6 E	9. 6 E	22. 2 E
210	12. 3 E	9. 6 E	21. 9 E
240	7. 9 E	9. 6 E	17. 5 E
270	0. 0	9. 6 E	9. 6 E
300	7. 9 W	9. 6 E	1. 7 E
330	12. 3 W	9. 6 E	2. 7 W

(4) To draw the curve, select a convenient origin and label this with the value of α. Draw a vertical line through the origin and mark off a convenient scale such that all values of δ can be shown both above and below the origin. The zero on this scale is at point α units above the origin (below in the southern hemisphere). Label the scale according to GE. Through the origin draw various radial lines at any convenient interval to represent true courses. For each computed course draw a horizontal construction line from the GE on the central scale to the appropriate radial line. The intersection of each pair of lines is one point on the curve. Connect all such points with a smooth curve, and erase the construction lines. If a straightedge or graph paper is used, the construction lines need not be drawn.

It is good practice to draw the curve for the highest speed first, to be sure that succeeding curves will fit on the paper. From such curves the gyro courses corresponding to various true courses can be determined and the radial lines labeled with these values for converting gyro directions to true directions.

The curves described in this article are for use *when all correctors are set on zero*, or if no provision is made for mechanically correcting for speed and damping errors. If the compass does not have a mercury ballistic, the damping error is omitted from the calculations and curves.

641. Gyro compass repeaters.—A gyro compass is customarily located at a favorable position below decks, and its indications transmitted electrically to various positions throughout the vessel. Each repeater consists of a compass rose on a suitable card so mounted that the direction of the ship's head is indicated at a lubber's line. Although the repeater may be mounted in any position, including vertically on a bulkhead, it is generally placed in gimbals in a bowl, similar to the mounting of a compass, which it resembles (fig. 641). This is true particularly of repeaters used for obtaining bearings. A gyro repeater used primarily to indicate the gyro heading is sometimes called a **ship's course indicator.**



Courtesy of Ahrendt Instrument Co.

FIGURE 641.—A gyro repeater used as a ship's course indicator (Mark 2 Mod 5).

Gyro compass indications are also used in automatic steering devices, direction-stabilized radar scopes, wind indicators, fire control equipment, etc.

A compass used to control other equipment, particularly repeaters, is sometimes called a **master compass.** In the case of a gyro compass, it is usually called a **master gyro compass.** It is good practice to check all repeaters periodically with the master compass to insure continued synchronization.

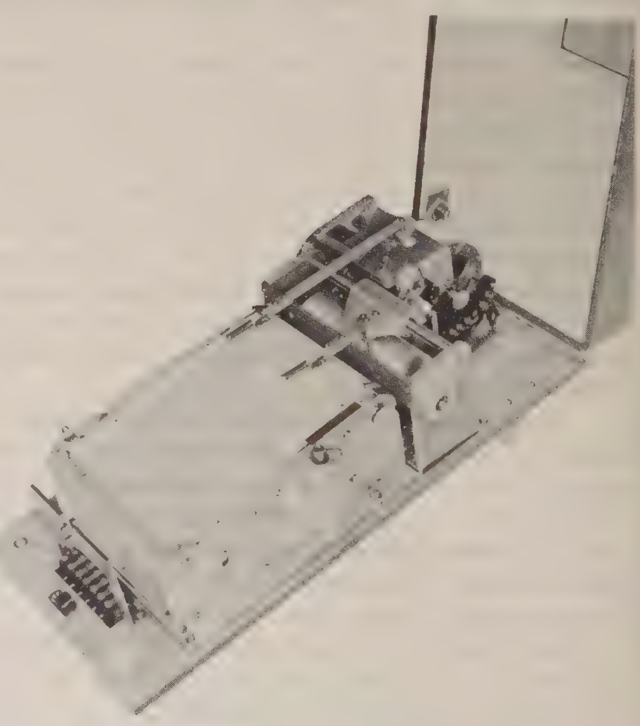
642. Alidade.—A gyro repeater with a telescopic sight mounted over it is called an **alidade.** If the telescopic sight is mounted so that it remains pointed in the same gyro direction regardless of motions of the vessel, the instrument is called a **self-synchronous alidade.** This instrument will retain its setting until oriented to a new gyro direction.

Courtesy of Wilfrid O. White and Sons, Inc.

FIGURE 643.—A pelorus.

643. Pelorus.—Although it is desirable to have a compass, a compass repeater, or an alidade for obtaining bearings, satisfactory results can be obtained by means of an inexpensive device known as a **pelorus** (fig. 643). In appearance and use this device resembles a compass or compass repeater, with sighting vanes or a sighting telescope attached, but it has no directive properties. That is, it remains at any *relative* direction to which it is set. It is generally used by setting 000° at the lubber's line. Relative bearings are then observed. They can be converted to bearings true, magnetic, grid, etc., by *adding* the appropriate heading. The direct use of relative bearings is sometimes of value. A pelorus is useful, for instance, in determining the moment at which an aid to navigation is broad on the beam. It is also useful in measuring pairs of relative bearings for use with table 7 or for determining distance off and distance abeam without a table (art. 910).

If the true heading is set at the lubber's line, true bearings are observed directly. Similarly, compass bearings can be observed if the compass heading is set at the lubber's line, etc. However, the vessel must be on the heading to which the pelorus is set if accurate results are to be obtained, or else a correction must be applied to the observed results. Perhaps the easiest way of avoiding error is to have the steersman indicate when the vessel is on course. This is usually done by calling out "mark, mark, mark" as long as the vessel is within a specified fraction of a degree of the desired heading. The observer, who is watching a distant object across the pelorus, selects an instant when the vessel is steady and is on course. An alternative method is to have the observer call out "mark" when the relative bearing is steady, and the steersman note the heading. If the compass is swinging at the moment of observation, the observation should be rejected. The number of degrees between the desired and actual headings is *added* if the vessel is to the *right* of the course, and *subtracted* if



Courtesy of Sperry Gyroscope Co.

FIGURE 644.—A course recorder.

to the *left*. Thus, if the course is 060° and the heading is 062° at the moment of observation, a correction of 2° is added to the bearing.

Each observer should determine for himself the technique that produces the most reliable results.

644. Course recorder.—A continuous graphical record of the headings of a vessel can be obtained by means of a **course recorder** (fig. 644). In its usual form, paper with both heading and time graduations is slowly wound from one drum to another, its speed being controlled by a spring-powered clockwork mechanism. A pen is in contact with the paper, tracing a line to indicate the heading at each moment. The pen is attached to an arm controlled by indications from a compass, usually the master gyro compass.

645. Dead reckoning equipment.—The primary navigational functions of **dead reckoning equipment** (DRE) are to (1) provide continuous indications of the vessel's present latitude and longitude, and (2) provide a graphical record of the vessel's dead reckoning track. In addition, most types of dead reckoning equipment provide means for tracking one or more other craft, to obtain a graphical record of the other craft's course and speed. This equipment is generally installed only on warships.

Dead reckoning equipment consists in general of four components: (1) an analyzer; (2) latitude and longitude indicator dials; (3) a desk-size unit called a **dead reckoning tracer** (**DRT**), usually installed in the chart house; and (4) a glass plotting surface over the dead reckoning tracer.

The analyzer receives directional signals from the vessel's gyro compass, and distance signals from the underwater log. The course and distance data are transformed automatically to electrical signals proportional to the north-south and east-west components of the vessel's movement. These distance signals are transmitted to the latitude and longitude indicators, changing their readings by the correct amount to indicate the new latitude and the new longitude in degrees and minutes. Since the number of miles in the north-south component of distance traveled is nearly equal to the change in latitude expressed in minutes, the latitude indicator is fed directly. Departure (art. 204) is automatically transformed to difference of longitude before being registered on the longitude indicator dials. If the indicator dials are correctly set to latitude and longitude, they continuously show subsequent dead reckoning positions of the vessel.

The north-south and east-west component signals from the analyzer are also transmitted to the DRT (fig. 645), where they control the motion of a pencil which moves across a chart or plotting sheet attached to the DRT base. The pencil draws a line which conforms to the maneuvers of the vessel. The mechanism can be set to plot the track at any scale from ¼ mile per inch (1/10 mile on some) to 16 miles per inch. A clock-controlled contact lifts the pencil from the paper for 15 seconds of each minute and for a longer period each 10 minutes, thus providing automatic time measurement. The pencil carriage can be moved manually to any part of the chart for initial setting and the direction of travel can be adjusted so that the chart can be placed with any cardinal direction "up."

The cover of the DRT is a sheet of glass to which a plotting sheet or blank paper can be fastened. An electric lamp on the top of the pencil carriage throws a spot of light through the paper directly over the carriage, thus providing a moving reference scaled to the course and speed of the vessel. If the position of the spot of light is marked periodically on the paper, a second record of the vessel's track is obtained. However, the principal use of this sheet is for plotting successive positions of another craft, using the spot of light as the origin. A polar grid centered on the light may be projected onto the paper to facilitate measurement. The course of the other vessel

Courtesy of Ahrendt Instrument Co.

FIGURE 645.—A dead reckoning tracer.

can be measured directly from the plot, and its speed can be determined by means of the time needed to travel any distance measured on the plot. This process is called **tracking.** If the ranges and bearings are plotted from a fixed point, *relative* movement is determined, a practice commonly followed in connection with radar (art. 1212).

While dead reckoning equipment is a great convenience, particularly when changes of course or speed are numerous, its indications should be checked by graphical plot on the chart or plotting sheet. Reliable dead reckoning is too important to be left entirely to mechanical equipment without an independent check.

Problems

634. Gyro error is 1° E.

Required.—(1) True heading when the gyro heading is 155°.

(2) The course to steer by gyro compass if the desired true course is 211°.

(3) The true bearing of a lighthouse if the bearing by gyro compass is 043°.

Answers.—(1) TH 156°, (2) C_{pgc} 210°, (3) TB 044°.

635. A ship at latitude 53° N is steaming on true course 205°, at a speed of 18 knots.

Required.—Speed error.

Answer.— δ 1°72 E.

636. A gyro compass having a value of r of 1°.0 is at latitude 20° S.

Required.—The damping error.

Answer.— α 0°.36 W.

643a. A pelorus is set with 000° at the lubber's line, and a bearing of 216° is observed when the heading is 155° true.

Required.—The true bearing.

Answer.—TB 011°.

643b. A pelorus is set with 070° at the lubber's line. A bearing of 030° is observed when the compass heading is 068°.

Required.—The compass bearing.

Answer.—CB 028°.

CHAPTER VII

COMPASS ERROR

Magnetism

701. Theory of magnetism.—The fact that iron can be magnetized (given the ability to attract other iron) has been known for thousands of years, but the explanation of this phenomenon has awaited the recently acquired knowledge of atomic structure. According to present theory, the magnetic field around a current-carrying wire and the magnetism of a permanent **magnet** are the same phenomenon—fields created by moving electrical charges. This occurs whether the charge is moving along a wire, flowing with the magma of the earth's core, encircling the earth at high altitude as a stream of charged particles, or rotating around the nucleus of an atom.

It has recently been shown that microscopically small regions, called **domains,** exist in iron and other ferromagnetic substances. In each domain the fields created by electrons spinning around their atomic nuclei are parallel to each other, causing the domain to be magnetized to saturation. In a piece of unmagnetized iron, the directions of the various domains are arranged in a random manner with respect to each other. If the substance is placed in a weak magnetic field, the domains rotate somewhat toward the direction of that field. Those domains which are more nearly parallel to the field increase in size at the expense of the more non parallel ones. If the field is made sufficiently strong, entire domains rotate suddenly by angles of as much as 90° or 180° so as to become parallel to that "crystal axis" which is most nearly parallel to the direction of the field. If the strength of the field is increased to a certain value depending upon individual conditions, all of the domains rotate into parallelism with the field, and the iron itself is said to be magnetically **saturated.** If the field is removed, the domains have a tendency to rotate more or less rapidly to a more natural direction parallel to some crystal axis, and more slowly to random directions under the influence of thermal agitation.

Magnetism which is present only when the material is under the influence of an external field is called **induced magnetism.** That which remains after the magnetizing force is removed is called **residual magnetism.** That which is retained for long periods without appreciable reduction, unless the material is subjected to a demagnetizing force, is called **permanent magnetism.**

Certain substances respond readily to a magnetic field. These **magnetic materials** are principally those composed largely of iron, although nickel and cobalt also exhibit magnetic properties. The best magnets are made of an alloy composed mostly of iron, nickel, and cobalt. Aluminum and some copper may be added. Platinum and silver, properly alloyed with other material, make excellent magnets, but for ordinary purposes the increased expense is not justified by the improvement in performance. Permanent magnets occur in nature in the form of **lodestone,** a form of magnetite (an oxide of iron) possessing magnetic properties. A piece of this material constitutes a **natural magnet.**

702. Hard and soft iron.—In some alloys of iron, the crystals can be so arranged and internally stressed that the domains remain parallel to each other indefinitely, and the metal thus becomes a **permanent magnet.** Such alloys are used for the magnets of a compass. In other kinds of iron, the domains reorient themselves rapidly to conform to the direction of a changing external field, and soon take random directions if the field is removed. A ferromagnetic substance which retains much of its magnetism in the

absence of an external field, is said to have high **remanence** or **retentivity.** The strength of a reverse field (one of opposite polarity) required to reduce the magnetism of a magnet to zero is called the **coercivity** or **coercive force** of the magnet. Hence, a compass magnet should have high remanence in order to be strong, and high coercivity so that stray fields will not materially affect it. For convenience, iron is called "hard" if it has high remanence, and "soft" if it has low remanence. **Permeability** (μ) is the ratio of the strength of the magnetic field inside the metal (B) to the strength of the external field (H), or $\mu=\frac{B}{H}$.

703. Lines of force.—The direction of a magnetic field is usually represented by lines, called **lines of force.** Relative intensity in different parts of a magnetic field is indicated by the spacing of the lines of force, a strong field having the lines close together. If a piece of unmagnetized iron is placed in a magnetic field, the lines of force tend to crowd into the iron, following its long axis, and the field is stronger in the vicinity of the iron, somewhat as shown in figure 703a. If the iron becomes permanently magnetized

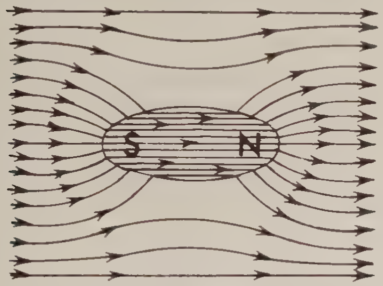


FIGURE 703a.—Lines of force crowd into ferromagnetic material placed in a magnetic field.

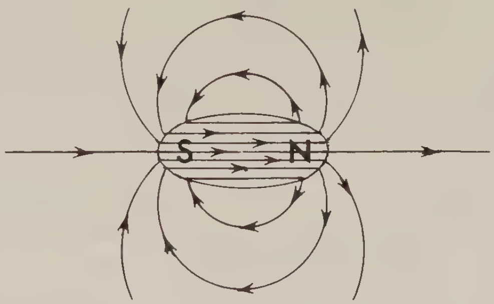


FIGURE 703b.—Field of a permanent magnet.

and is removed from this field, the lines of force around the iron follow paths about as shown in figure 703b.

704. Magnetic poles.—The region in which the lines of force enter the iron is called the **south pole,** and the region in which they leave the iron is called the **north pole.** Thus, the lines of force are directed from south to north within the magnet, and and from north to south in the external field. Every magnet has a north pole and a south pole. If a magnet is cut into two pieces, each becomes a magnet with a north pole and south pole. A single pole cannot exist independently. If two magnets are brought close together, *unlike poles attract each other and like poles repel.* Thus, a north pole attracts a south pole but repels another north pole.

The earth itself has a magnetic field (art. 706), with its magnetic poles being some distance from the geographical poles. If a permanent bar magnet is supported so that it can turn freely, both horizontally and vertically, it aligns itself with the magnetic field of the earth, which at most places is in a general north-south direction and inclined to the horizontal. Since the north pole of the magnet points in a northerly direction, the earth's magnetic pole in the northern hemisphere has *south* magnetism. Nevertheless, it is called the **north magnetic pole** because of its geographical location. For a similar reason, the pole in the southern hemisphere, although it has north magnetism, is called the **south magnetic pole.** To avoid confusion, north magnetism is usually called "red," and south magnetism, "blue." The red (north) pole of a magnet is usually painted red, and in some cases the south (blue) pole is painted blue. The north magnetic pole of the earth is a blue pole, and the south magnetic pole is a red pole.

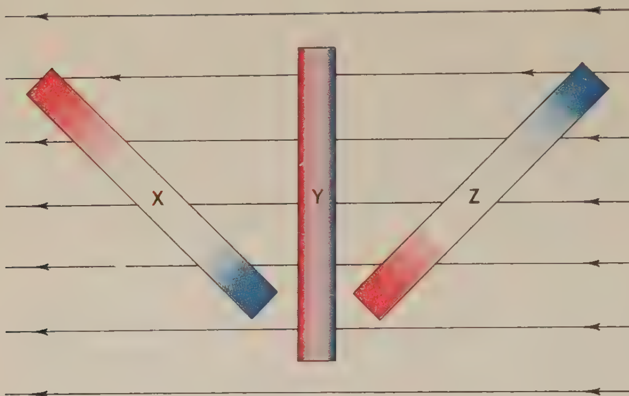


FIGURE 705.—The polarity of a soft iron bar in a magnetic field.

705. The magnetism of soft iron, in which remanence is low, depends upon the position of the iron with respect to an external field. It is strongest if the long axis is parallel to the lines of force, and decreases to a minimum if the material is rotated so that the long axis is perpendicular to the lines of force. Figure 705 shows three positions of a bar magnet with respect to a magnetic field. At position *X* the pole at the upper end of the bar is red and relatively strong. As the bar is rotated toward position *Y*, the upper end remains red, but its strength decreases. At position *Y*, no pole is apparent at either end, but a red pole extends along the entire left side of the bar, and a blue pole along the right side. Poles are strongest when concentrated into a small area. Hence, when spread over an entire side, as at position *Y*, they are relatively weak. At position *Z*, the upper end is blue.

The change in polarity as a bar of soft iron is rotated in a magnetic field can easily be demonstrated. If a bar of soft iron is placed vertical in northern magnetic latitudes (as in any part of the United States), the north (red) end of a compass magnet brought near it will be attracted by the upper end of the bar, and repelled by the lower end. If the bar is inverted, so that its ends are interchanged, the upper end (which as the lower end previously repelled the compass needle) will attract the north end of the needle, and the lower end will repel it. Thus, the polarity of the rod is reversed, *either* end having blue magnetism if it is at the top. This changing polarity of soft iron in the earth's field is a major factor affecting the magnetic compasses of a steel vessel.

706. Terrestrial magnetism.—The earth itself can be considered to be a gigantic magnet. Although man has known for many centuries that the earth has a magnetic field, the origin of the magnetism is not completely understood. Nevertheless, the horizontal component of this field is a valuable reference in navigation, for it provides the directive force for the magnetic compass, which indicates the ship's heading *in relation to the horizontal component of this field.*

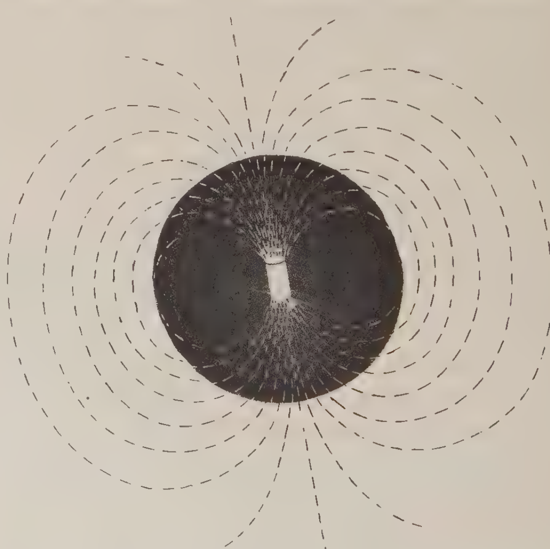


FIGURE 706.—The magnetic field of the earth.

The world-wide pattern of the earth's magnetism is roughly like that which would result from a short, powerful, bar magnet near the earth's center, as shown in figure 706. The geographical poles are at the top and bottom, and the magnetic poles are offset somewhat from them. This representation, however, is greatly simplified. The actual field is more

complex, and requires measurement of its strength and direction at many places (art. 707) before it can be defined accurately enough to be of practical use to the navigator. Not only are the magnetic poles offset from the geographical poles, but the magnetic poles themselves are not 180° apart and, in general, a magnetic compass aligned with the lines of force does not point toward either magnetic pole. In 1960, the north magnetic pole was located at latitude 74°9 N, longitude 101°0 W, approximately, to the northward of Prince of Wales Island; and the south magnetic pole was at latitude 67°1 S, longitude 142°7 E, approximately, in the northeastern part of Wilkes Land. However, the magnetic poles are not stationary. The entire magnetic field of the earth, including the magnetic poles, undergoes a small daily or **diurnal change,** and a very slow, progressive **secular change.** In addition, temporary sporadic changes occur from time to time during magnetic storms (art. 2526). During a severe storm, variation may change as much as 5°, or more. However, such disturbances are never so rapid as to cause noticeable deflection of the compass card, and in most navigable waters the change is so little that it is not significant in practical navigation. Even when there is no temporary disturbance, the earth's field is considerably more intricate than indicated by an isomagnetic chart (art. 708). Natural magnetic irregularities occurring over relatively small areas are called **magnetic anomalies** by the magneticians, but the navigator generally refers to these phenomena as **local disturbances.** Notes warning of such disturbances are shown on charts. In addition, artificial disturbances may be quite severe when a vessel is in close proximity to other vessels, piers, machinery, electric currents, etc.

The elements of the earth's field are as follows:

Total intensity (F) is the strength of the field at any point, measured in a direction parallel to the field. Intensity is usually measured in **oersteds,** one oersted being equal to a force of one dyne acting on a unit pole. The range of intensity of the earth's field is about 0.25 to 0.70 oersted. For convenience in geomagnetic surveying, a smaller unit is used, called the **gamma.** One oersted equals 100,000 gammas, so that the range of intensity of the earth's field is about 25,000 to 70,000 gammas.

Horizontal intensity (H) is the horizontal component of the total intensity. At the **magnetic equator,** which corresponds roughly with the geographic equator, the field is parallel to the surface of the earth, and the horizontal intensity is the same as total intensity. At the magnetic poles of the earth, the field is vertical and there is no horizontal component. The direction of the horizontal component at any place defines the **magnetic meridian** at that place. This component provides the desired directive force of a magnetic compass.

North component (X) is the horizontal intensity's component along a geographic (true) meridian.

East component (Y) is the horizontal intensity's component perpendicular to the north component.

Vertical intensity (Z) is the vertical component of the total intensity. It is zero at the magnetic equator. At the magnetic poles it is the same as the total intensity. While the vertical intensity has no direct effect upon the direction indicated by a magnetic compass, it does induce magnetic fields in vertical soft iron, and these may affect the compass.

Variation (V, Var.), called **declination** (D) by magneticians, is the angle between the geographic and magnetic meridians at any place. The expression **magnetic variation** is used when it is necessary to distinguish this from other forms of variation. This element is measured in angular units and named east or west to indicate the side of true north on which the (magnetic) northerly part of the magnetic meridian lies. For computational purposes, easterly variation is sometimes designated positive (+),

and westerly variation negative (−). **Grid variation** (GV) or **grivation** is the angle between the grid and magnetic meridians at any place, measured and named in a manner similar to variation.

Magnetic dip (I), called **inclination** (I) by magneticians, is the vertical angle, expressed in angular units, between the horizontal at any point and a line of force through that point. The **magnetic latitude** of a place is the angle having a tangent equal to half that of the magnetic dip of the place.

At a distance of several hundred miles above the earth's surface, the magnetic field surrounding the earth is believed to be uniform, as it appears in figure 706, and centered around two **geomagnetic poles.** These do not coincide with either the magnetic poles (art. 704) or the geographical poles. However, they are 180° apart, the north geomagnetic pole being at latitude 78°.5 N, longitude 69° W (near Etah, Greenland) and the south geomagnetic pole being at latitude 78°.5 S, longitude 111° E. The great circles through these poles are called **geomagnetic meridians.** That geomagnetic meridian passing through the south geographical pole is the origin for measurement of **geomagnetic longitude,** which is measured eastward through 360°. The complement of the arc of a geomagnetic meridian from the nearer geomagnetic pole to a place is called the **geomagnetic latitude.** When the sun is over the upper branch of the geomagnetic meridian of a place, it is **geomagnetic noon** there, and when it is over the lower branch of the geomagnetic meridian, it is **geomagnetic midnight.** The angle between the lower branch of the geomagnetic meridian of a place and the geomagnetic meridian over which the sun is located is called **geomagnetic time.** The diurnal change is related to geomagnetic time. The auroral zones (art. 2526) are centered on the geomagnetic poles.

707. Measurement of the earth's magnetic field is made continuously at about 70 permanent **magnetic observatories** throughout the world. In addition, large numbers of temporary stations are occupied for short periods to add to man's knowledge of the earth's field. In the past, measurements at sea have been made by means of nonmagnetic ships constructed especially for this purpose. However, this is a slow and expensive method, and quite inadequate to survey properly the 71 percent of the earth's surface covered with water. Since World War II, a satisfactory **airborne magnetometer** has been developed by the U. S. Navy. By means of this instrument, continuous readings can be recorded automatically during long overwater flights.

708. Isomagnetic charts showing lines of equality of some magnetic element are published by the U. S. Navy Hydrographic Office in collaboration with the U. S. Coast and Geodetic Survey. Formerly, three charts of each element were published, in addition to a north polar grid variation chart, making a total of 22 in the series. Beginning with the series for epoch 1955, charts for the north and east components (X and Y) are not published, as this information is of limited use and can be determined easily from the horizontal component and variation. The three charts of each element consist of one on the Mercator projection (art. 305) covering most of the world, and one on a polar projection (azimuthal equidistant (art. 320) or stereographic) for each of the two polar areas. All charts now included in the series are published at intervals of 10 years, showing the values for the beginning of each year ending in five. Charts showing variation are also published for the years ending in zero (1950, 1960, etc.).

The isomagnetic chart of most concern to a navigator is H.O. Chart No. 1706, *The Variation of the Compass*, a simplified version of which is shown in figure 708a. The lines connecting points of equal magnetic variation are called **isogonic lines.** *These are not magnetic meridians (lines of force).* The line connecting points of zero variation is called the **agonic line.** Variation is also shown on nautical charts. Those of relatively small scale generally show isogonic lines. Those of scale larger than 1:100,000 generally give the information in the form of statements inside compass roses placed at various

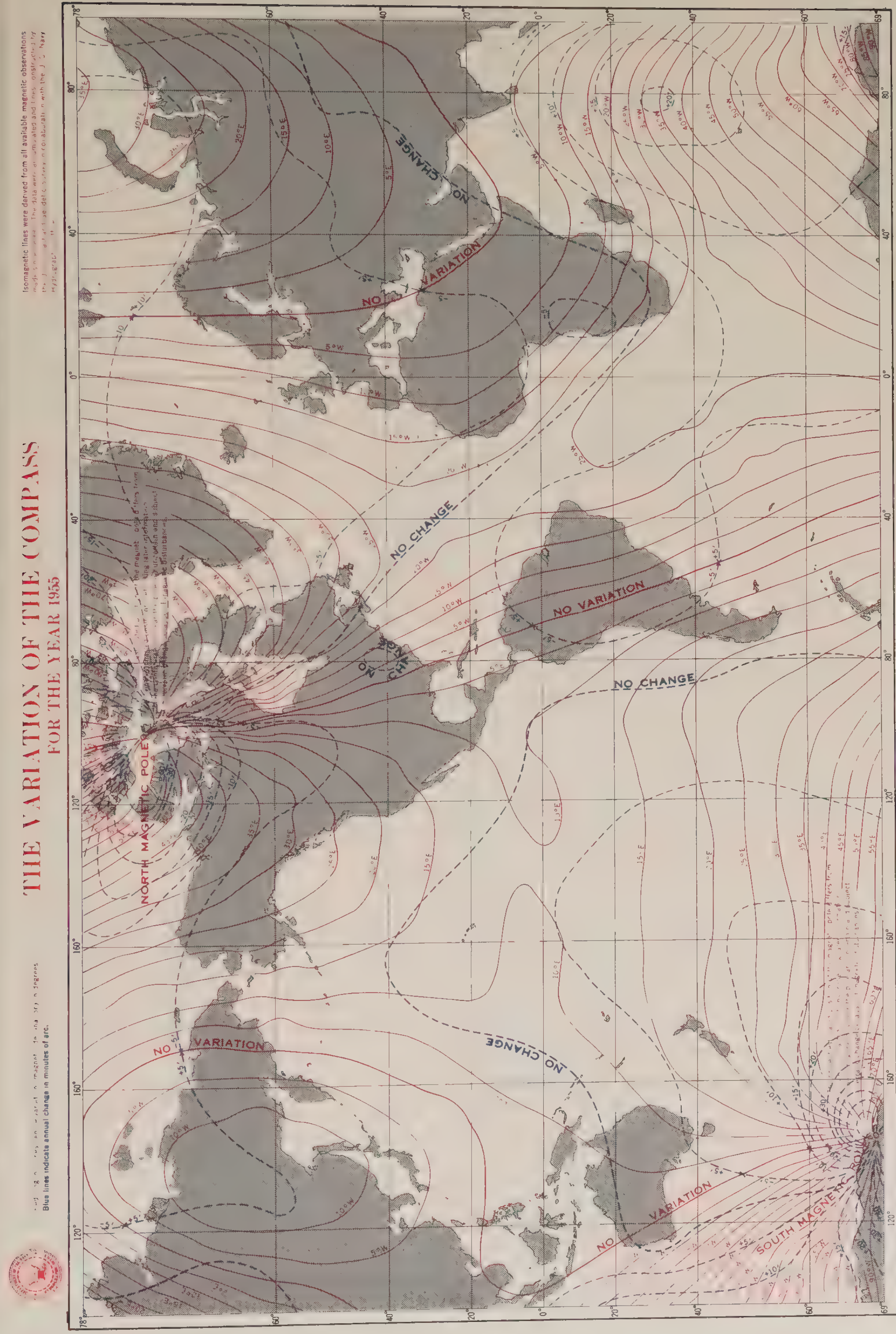


FIGURE 708a.—Variation. A simplification of H.O. Chart No. 1706.

places on the chart, and sometimes, also, by a **magnetic compass rose** within the true compass rose and offset from it by the amount of the variation. By means of this arrangement, true directions can be plotted without arithmetically applying variation to magnetic directions, or magnetic directions can be read directly from the chart. The magnetic compass rose is generally graduated in both degrees and points. Variation is given to the nearest 15′, and the annual change to the nearest 1′. However, since the rate of change is not constant, a very old chart should not be used, even though it has been corrected for all changes shown in *Notices to Mariners*.

Another isomagnetic chart of value to the mariners is H.O. Chart No. 1700, *Magnetic Dip*, figure 708b. Lines connecting points of equal magnetic dip are called **isoclinal lines.** The line connecting points of zero dip is called the **magnetic equator.**

Other isomagnetic charts are H.O. Chart No. 1701, showing horizontal intensity; H.O. Chart No. 1702, showing vertical intensity; and H.O. Chart No. 1703, showing total intensity. Lines connecting points of equal intensity on any of these charts are called **isodynamic lines.**

In all series of isomagnetic charts, the same number is used for polar charts, but with N or S following the number (and N–G or S–G for the grid variation charts) to indicate the north or south polar region, respectively. All of the isomagnetic charts also show **isopors,** in a distinctive color, connecting points of equal **annual change** of the element at the epoch of the chart.

The charts are as accurate as can be made with available information, except that the lines are smoothed somewhat, rather than depicting every small irregularity. The larger irregularities are reflected in the information shown on nautical charts, but local disturbance is indicated by warning notes at appropriate places. In areas where measurements of the magnetic field have not been made for a long period, the previous information is altered in accordance with the best information available on secular change, with some adjustment to provide continuous smooth curves. When information is thus carried forward for many years, errors may be introduced, particularly in areas where the rate of change is large and variable. Magneticians have not detected a recognizable world-wide pattern in secular change, such as would occur if it were due only to shifting of the positions of the magnetic poles. Rather, these shifts are part of the general complex, little-understood secular change.

The Compass Error

709. Magnetic compass error.—Directions relative to the northerly direction along a geographic meridian are **true.** In this case, true north is the **reference direction.** If a compass card is horizontal and oriented so that a straight line from its center to 000° points to true north, any direction measured by the card is a true direction and has no error (assuming there is no calibration or observational error). If the card remains horizontal but is rotated so that it points in any other direction, the amount of the rotation is the **compass error.** Stated differently, compass error is the angular difference between true north and **compass north** (the direction north as indicated by a magnetic compass). It is named east or west to indicate the side of true north on which compass north lies.

If a magnetic compass is influenced by no other magnetic field than that of the earth, and there is no instrumental error, its magnets are aligned with the magnetic meridian at the compass, and 000° of the compass card coincides with **magnetic north.** All directions indicated by the card are **magnetic.** As stated in article 706, the angle between geographic and magnetic meridians is called **variation (V** or **Var.).** Therefore, if a compass is aligned with the magnetic meridian, compass error and variation are the same.

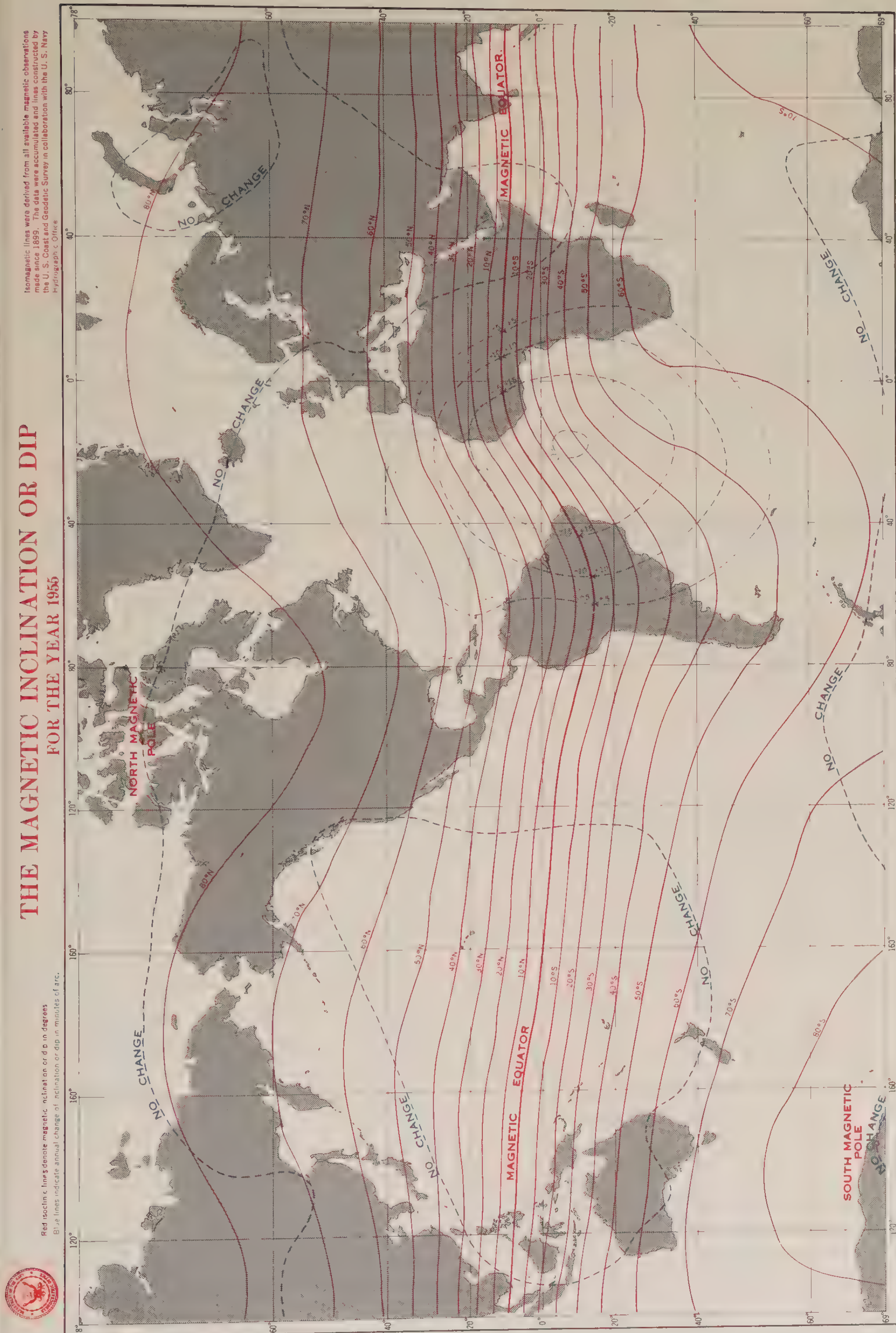


FIGURE 708b.—Magnetic dip. A simplification of H. O. Chart No. 1700.

When a compass is mounted in a vessel, it is generally subjected to various magnetic influences other than that of the earth. These arise largely from induced magnetism in metal decks, bulkheads, masts, stacks, boat davits, guns, etc., and from electromagnetic fields associated with direct current in electrical circuits. Some metal in the vicinity of the compass may have acquired permanent magnetism. The actual magnetic field at the compass is the vector sum, or resultant (art. O18), of all individual fields at that point. Since the direction of this resultant field is generally not the same as that of the earth's field alone, the compass magnets do not lie in the magnetic meridian, but in a direction that makes an angle with it. This angle is called **deviation (D** or **Dev.)**. Thus, deviation is the angular difference between magnetic north and compass north. It is expressed in angular units and named east or west to indicate the side of magnetic north on which compass north lies. Thus, deviation is the error of the compass in pointing to magnetic north, and all directions measured with compass north as the reference direction are **compass directions.** Since variation and deviation may each be either east or west, the effect of deviation may be to either increase or decrease the error due to variation alone. The algebraic sum (art. O6) of variation and deviation is the total compass error.

For computational purposes (art. 727), deviation and compass error, like variation, may be designated positive (+) if east and negative (−) if west.

Variation changes with location, as indicated in figure 708a. Deviation depends upon the magnetic latitude and also upon the individual vessel, its trim and loading, whether it is pitching or rolling, the heading (orientation of the vessel with respect to the earth's magnetic field), and the location of the compass within the vessel. Therefore, deviation is not published on charts.

710. Deviation table.—In practice aboard ship, the deviation is reduced to a minimum, as explained later in this chapter. The remaining value, called **residual deviation,** is determined on various headings and recorded in some form of **deviation table.** Figure 710 shows both sides of the form used by the United States Navy. This table is entered with the magnetic heading, and the deviation on that heading is determined from the tabulation, separate columns being given for degaussing (DG) off and on (art. 740). If the deviation is not more than about 2° on any heading, satisfactory results may be obtained by entering the values at intervals of 45° only.

If the deviation is small, no appreciable error is introduced by entering the table with either magnetic or compass heading. If the deviation on some headings is large, the desirable action is to reduce it, but if this is not practicable, a separate deviation table for compass heading entry may be useful. This may be made by applying the tabulated deviation to each entry value of magnetic heading, to find the corresponding compass heading, and then interpolating between these to find the value of deviation at each 15° compass heading. Another method is to plot the values on cross-section paper and select the desired values graphically.

A nomogram especially designed for interconversion of magnetic and compass headings is called a **Napier diagram,** having been devised by James Robert Napier (1821–79). It consists of a dotted, vertical center line graduated from 000° to 360° (usually in two parallel parts of 180° each), with two series of cross lines making angles of 60° with the dotted vertical line and with each other. If magnetic headings are used, deviation is measured along a solid cross line; and if compass headings are used, deviation is measured along a dotted cross line. A deviation curve is drawn through the various points. To convert a magnetic heading to a compass heading, one finds the magnetic heading on the vertical center line, moves parallel to a solid cross line until the curve is reached, and returns to the center line by moving parallel to a dotted line.

The compass heading is the value at the point of return. The reverse process is used for converting a compass heading to a magnetic heading. This nomogram is of particular value where the deviation is large and changing rapidly. It is now possible, however, to reduce deviation to such small values that the Napier diagram has lost much of its appeal and is seldom used.

MAGNETIC COMPASS TABLE
NAVSHIPS 1104 (REV. 10-57) REPORT-SHIPS-3530-2

U.S.S. Blank NO. – – – (PF, CL, DD, etc.)

[X] PILOT HOUSE [ ] SECONDARY CONNING STATION [ ] OTHER

BINNACLE TYPE: [X] NAVY ST'D [ ] OTHER

COMPASS 7-1/2" MAKE C. G. Conn SERIAL NO. 8560

TYPE CC COILS "K" DATE 10/11/65

READ INSTRUCTIONS ON BACK BEFORE STARTING ADJUSTMENT

SHIPS HEAD MAGNETIC	DEVIATIONS		SHIPS HEAD MAGNETIC	DEVIATIONS	
	DG OFF	DG ON		DG OFF	DG ON
0	0.5E	0.5E	180	0.5W	0.0
15	1.0E	1.0E	195	1.0W	0.5W
30	1.5E	1.5E	210	1.0W	1.0W
45	2.0E	1.5E	225	1.5W	1.5W
60	2.0E	2.0E	240	2.0W	2.0W
75	2.5E	2.5E	255	2.0W	2.5W
90	2.5E	3.0E	270	1.5W	2.0W
105	2.0E	2.5E	285	1.0W	1.5W
120	1.5E	2.0E	300	1.0W	1.0W
135	1.5E	1.5E	315	0.5W	0.5W
150	1.0E	1.0E	330	0.5W	0.5W
165	0.0	0.5E	345	0.0	0.0

DEVIATIONS DETERMINED BY: [ ] SUN'S AZIMUTH [X] GYRO [ ] SHORE BEARINGS

B 6 MAGNETS RED [ ] FORE [X] AFT AT 12" FROM COMPASS CARD

C 4 MAGNETS RED [ ] PORT [X] STBD AT 6" FROM COMPASS CARD

D 2-7" [X] SPHERES [ ] CYLS AT 12" [X] ATHWARTSHIP [ ] SLEWED – ° [ ] CLOCKWISE [ ] CTR. CLKWISE

HEELING MAGNET: [ ] RED UP [X] BLUE UP 6" FROM COMPASS CARD FLINDERS BAR: [X] FORE [ ] AFT 12"

[X] LAT 18°00'N [X] LONG 120°-00'E
[ ] H 0.385 [ ] Z 0.140

SIGNED (Adjuster or Navigator) APPROVED (Commanding)

VERTICAL INDUCTION DATA
(Fill out completely before adjusting)

RECORD DEVIATION ON AT LEAST TWO ADJACENT CARDINAL HEADINGS

BEFORE STARTING ADJUSTMENT: N 8 W; E 0; S 4 E; W 9 E.

RECORD BELOW INFORMATION FROM LAST NAVSHIPS 1104 DEVIATION TABLE:

DATE 4/22/66 [ ] LAT 32-53N [ ] LONG 117-18W
[ ] H .260 [ ] Z .420

12" FLINDERS BAR [X] FORWARD [ ] AFT DEVIATIONS N 2.5W; E 7E; S 6.5E; W 5W.

RECORD HERE DATA ON RECENT OVERHAULS, GUNFIRE, STRUCTURAL CHANGES, FLASHING, DEPERMING, WITH DATES AND EFFECT ON MAGNETIC COMPASSES

APPROXIMATELY 30 DAYS ALONGSIDE DOCK FOR OVERHAUL

PERFORMANCE DATA

COMPASS AT SEA:	[ ] UNSTEADY	[X] STEADY
COMPASS ACTION:	[ ] SLOW	[X] SATISFACTORY
NORMAL DEVIATIONS:	[X] CHANGE	[ ] REMAIN RELIABLE
DEGAUSSED DEVIATIONS:	[X] VARY	[ ] DO NOT VARY

REMARKS

INSTRUCTIONS

1. This form shall be filled out by the Navigator for each magnetic compass as set forth in Chapter 24, Part 2, and Chapter 81, Section III, of Bureau of Ships Manual.
2. When a swing for deviations is made, the deviations should be recorded both with degaussing coils off and with degaussing coils energized at the proper currents for heading and magnetic zone.
3. Each time this form is filled out after a swing for deviations, a copy shall be submitted to the Bureau of Ships. A letter of transmittal is not required.
4. When choice of box is given, check applicable box.
5. Before adjusting, fill out section on "Vertical Induction Data" above.

NAVSHIPS-1104 (REV. 10-57) BACK D-20867

FIGURE 710.—Deviation table.

Another solution is to make a deviation table with one column for magnetic heading, a second column for deviation, and a third for compass heading. Still another solution, most popular among yachtsmen, is to center a compass rose inside a larger one so that an open space is between them and a radial line would connect points of the same graduation on both roses. Each magnetic heading for which deviation has been determined is located on the outer rose, and a straight line is drawn from this point to the corresponding compass heading on the inner rose.

A variation of this method is to draw two parallel lines a short distance apart, and graduate each from 0 to 360 so that a perpendicular between the two lines connects points of the same graduation. Straight lines are drawn from magnetic directions on one line to the corresponding compass directions on the other. If the lines are horizontal and the upper one represents magnetic directions, the slope of the line indicates the direction of the deviation. That is, for westerly deviation the upper part of the connecting line is left (west) of the bottom part, and for easterly deviation it is right.

An important point to remember regarding deviation is that it varies with the *heading*. Therefore, a deviation table is *never* entered with a bearing (art. 904). If the deviation table converts directly from one type heading to another, deviation is found by taking the difference between the two values. On the compass rose or straight-line type, the deviation can be written alongside the connecting line, and the intermediate values determined by estimate. If one has trouble determining whether to add or subtract deviation when bearings are involved, he has only to note which heading, magnetic or compass, is larger. The same relationship holds between the two values of bearing.

The deviation table should be protected from damage due to handling or weather, and placed in a position where it will always be available when needed. A method commonly used is to mount it on a board, cover it with shellac or varnish, and attach it to the binnacle. Another method is to post it under glass near the compass. It is good practice for the navigator to keep a second copy available at a convenient place for his use.

711. Applying variation and deviation.—As indicated in article 709, a single direction may have any of several numerical values depending upon the reference direction used. One should keep clearly in mind the relationship between the various expressions of a direction. Thus, true and magnetic directions differ by the variation, magnetic and compass directions differ by the deviation, and true and compass directions differ by the compass error. Other relationships are also useful. Thus, grid (art. 2510) and magnetic directions differ by the grid variation or grivation, and true and relative directions differ by the true heading. The use of variation and deviation is considered here. Other relationships are discussed elsewhere in this volume.

If variation or deviation is easterly, the compass card is rotated in a clockwise direction. This brings smaller numbers opposite the lubber's line. Conversely, if either error is westerly, the rotation is counterclockwise and larger numbers are brought opposite the lubber's line. Thus, if the heading is 090° true (fig. 711, A) and variation is 6° E, the magnetic heading is $090°-6°=084°$ (fig. 711, B). If the deviation on this heading is 2° W, the compass heading is $084°+2°=086°$ (fig. 711, C). Also, compass error is $6°\text{ E}-2°\text{ W}=4°\text{ E}$, and compass heading is $090°-4°=086°$. If compass error is easterly, the compass reads too low (in comparison with true directions), and if it is westerly, the reading is too high. Many rules-of-thumb have been devised as an aid to the memory, and any which assist in applying compass errors in the right direction are of value. However, one may forget the rule or its method of application, or may wish to have an independent check. If he understands the explanation given above, he can determine the correct sign without further information. The same rules apply to the use of gyro error. Since variation and deviation are compass errors, the process of removing either from an indication of a direction (converting compass to magnetic or magnetic to true) is often called **correcting.** Conversion in the opposite direction (inserting errors) is then called **uncorrecting.**

Example.—A vessel is on course 215° true in an area where the variation is 7° W. The deviation is as shown in figure 710. Degaussing is off. The gyro error (GE) is 1° E. A lighthouse bears 306°5 by magnetic compass.

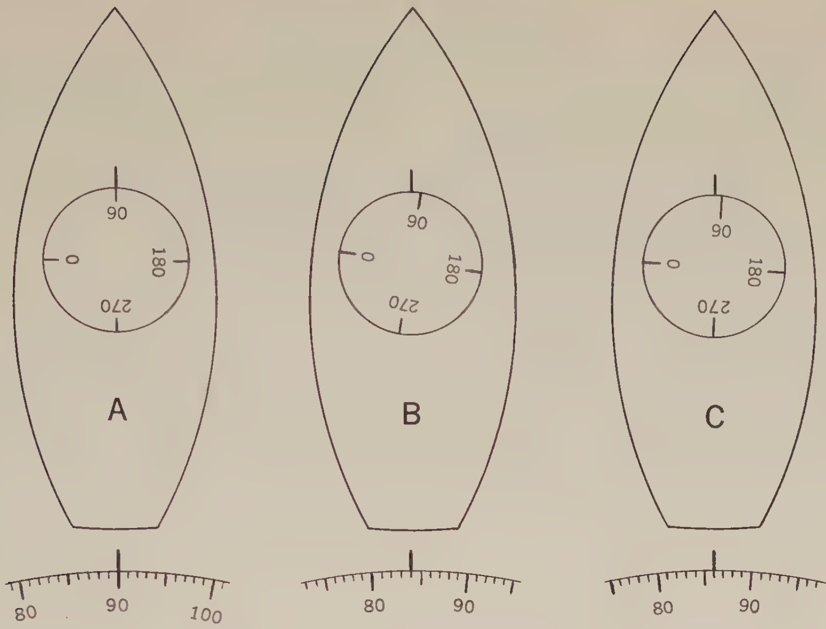


FIGURE 711.—Effect of variation and deviation on the compass card.

Required.—(1) Magnetic heading (MH).
(2) Deviation.
(3) Compass heading (CH).
(4) Compass error.
(5) Gyro heading.
(6) Magnetic bearing of the lighthouse.
(7) True bearing of the lighthouse.
(8) Relative bearing (art. 904) of the lighthouse.
Solution.—

TH 215°
V 7° W
(1) MH 222°
(2) D 1°5 W
(3) CH 223°5

The deviation is taken from the deviation table (fig. 710), to the nearest half degree.
(4) Compass error is 7° W+1°5 W=8°5 W.

TH 215°
GE 1° E
(5) Hpgc 214°

CB 306°5
D 1°5 W
(6) MB 305°
V 7° W
(7) TB 298°

(8) RB=TB−TH=298°−215°=083°.

Answers.—(1) MH 222°, (2) D 1°5 W, (3) CH 223°5, (4) CE 8°5 W, (5) Hpgc 214°, (6) MB 305°, (7) TB 298°, (8) RB 083°.

Deviation and Its Reduction

712. Magnetism of a steel vessel.—The materials of which a vessel is constructed are not, in general, selected for their magnetic properties. As a result, many degrees of permeability, remanence, and coercivity (art. 702) exist within its structure. Detailed analysis of the complex field existing at a magnetic compass is a specialized study not ordinarily required of the navigator. However, a general knowledge of the basic principles involved is of value to the navigator in helping him understand better the behavior of his magnetic compasses.

For most purposes, a vessel can be considered to be composed of two types of material: "hard iron" and "soft iron."

"Hard iron" is all material having some degree of permanent magnetism. This magnetism is acquired largely during construction of the vessel, when the rearrangement of the domains (art. 701) is facilitated by the bending, riveting, welding, and other violent mechanical processes. Since a vessel remains on a constant magnetic heading

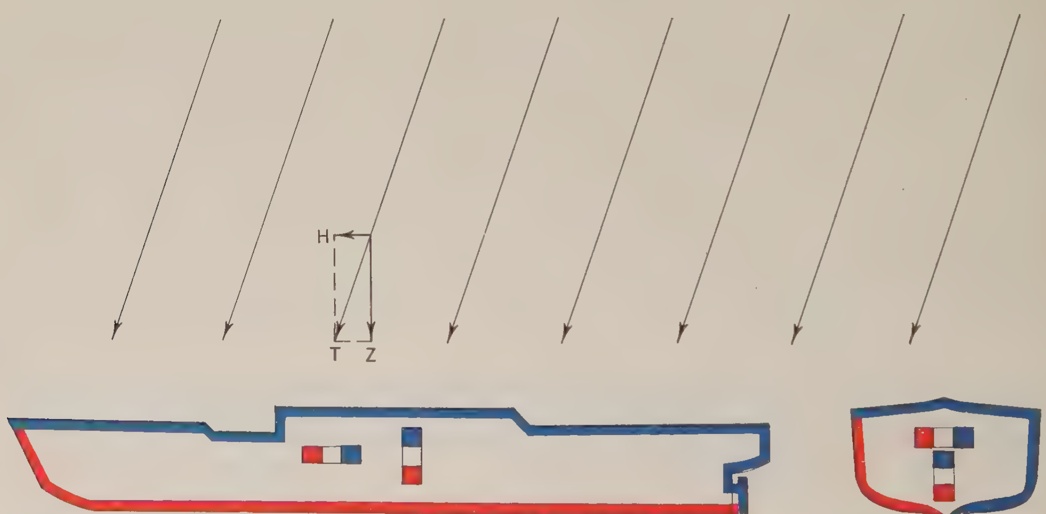


FIGURE 712.—Permanent magnetism of a vessel built on heading magnetic north (left) and magnetic east (right) at a place where the magnetic dip is 70°N.

while it is on the building ways, a field of permanent magnetism becomes established, the positions of the poles being dependent largely upon the orientation of the hull with respect to the magnetic field of the earth. If a vessel is constructed on a heading of magnetic north, at a place where the magnetic dip is 70° N (the approximate value at the midpoint of the east coast of the United States), its field of permanent magnetism is about as shown at the left of figure 712. The upper and stern portions are magnetically blue, while the lower and forward portions are magnetically red. If the vessel is built on a heading of magnetic east, the starboard and upper portions are blue, and the port and lower portions are red, as shown by the stern view at the right of figure 712. If the heading is magnetic northeast, the upper, starboard, and stern portions are blue, and the lower, port, and forward portions red. The red and blue portions for any given vessel can be visualized by drawing a sketch similar to that of figure 712, with the correct orientation

The "permanent" magnetism thus acquired during construction is less permanent than that of a permanent magnet such as one of those used in a compass, and is modified somewhat after launching, particularly if the vessel remains on another heading for a

considerable time during fitting out. The change is especially rapid during the first few days after launching, when the domains of the softer iron become reoriented. At this stage, deviation due to permanent magnetism may change several degrees. Further changes in the permanent magnetism may occur during long periods of being tied up or moored on a constant heading, or during a run of several days on nearly the same heading. This change is gradual and affects the strength, but usually not the polarity, of the magnetic field. The permanent field may be changed quickly, in polarity as well as in strength, if the vessel grounds, collides with another vessel, is struck by lightning, undergoes magnetic treatment (art. 744), fires its guns, or is struck by shells or bombs, etc.

The effect that the permanent magnetism of hard iron has upon a compass depends upon the position and strength of the poles relative to the compass. When the poles are in line with the north-south axis of the compass card, the only effect is to strengthen or weaken the directive force of the compass. When the compass heading is approximately 90° away, so that the poles are east and west of the compass, the deviating effect is maximum. The direction of the deviation is the same as that of the blue pole with respect to the compass.

"Soft iron" is all that material in which induced magnetism (art. 701) is present. With respect to its effect upon the magnetic compass, it is classed as either vertical or horizontal. Unlike hard iron, its magnetic field changes quickly as its orientation with respect to the earth's field changes. It also changes as the strength of the earth's field changes. For some purposes induced magnetism can be treated as if it were concentrated in two bars of soft iron, one vertical and the other horizontal. The polarity depends upon the position of the vessel relative to the earth's magnetic field, and the strength depends upon the strength of the vertical and horizontal components of the earth's field. This is illustrated in figure 712. In north magnetic latitude the bottom of the vertical rod has red magnetism and the top has blue magnetism. In south magnetic latitude these are reversed. In both north and south magnetic latitudes the magnetic north end of the horizontal bar has red magnetism, and the magnetic south end has blue magnetism. Thus, whatever the position of the rod, that part in the direction of magnetic north has red magnetism, and that part in the direction of magnetic south has blue magnetism. That is, each end has magnetism opposite to that of the magnetic pole indicated by the direction in which it is pointed.

The effect upon a magnetic compass of the induced magnetism in soft iron depends upon the strength and direction of the field relative to the compass. The cumulative effect of the induced magnetism in vertical soft iron is generally on the center line of the vessel (if of conventional construction), and for a compass located forward, as on the bridge, is aft of the compass. In magnetic north latitude the effect is generally that of a blue pole at the level of the compass card. In magnetic south latitude the pole is red. On a heading of compass north or south the pole is in line with the magnets of a center line compass and serves only to strengthen or weaken the directive force. On a heading of compass east or west the pole is perpendicular to the north-south axis of the compass card, and the deviating force is greatest.

For a compass located on the center line of a vessel of conventional construction, the horizontal soft iron close enough to have appreciable effect upon the compass is arranged in a more-or-less symmetrical manner with respect to the compass. Thus, on any cardinal compass heading, the fore-and-aft and athwartship horizontal soft iron is either in line with the compass magnets or equally and similarly arranged on both sides. No error is introduced by such symmetrical horizontal soft iron because the iron north and south of the compass magnets serves only to strengthen or weaken the directive force, and that east and west of the compass sets up an equal and opposite

field on each side. On intercardinal headings, the poles of the induced magnetism are offset and a maximum deviating force occurs. That part of horizontal soft iron which is not symmetrically arranged with respect to the compass—the asymmetrical soft iron—produces deviation which is maximum on the cardinal headings and zero on the intercardinal headings (by compass). This type of deviation is particularly great in a compass not mounted on the center line of the vessel. It may also produce deviation which is constant on all headings.

In wooden-hulled vessels such as certain yachts and small fishing vessels, one or more of these types of magnetism may be weak or entirely missing, but this does not justify the omission of any part of the correction procedure.

As far as its effect upon the compass is concerned, the magnetic field at a center line compass located forward on a vessel of conventional construction, and on an even keel, is essentially the same as that which would result from four sources: (1) the earth's magnetism; (2) a single blue pole the location and strength of which depends upon the magnetic history of the vessel; (3) a single pole which is blue in north magnetic latitude and red in south magnetic latitude, is on the center line aft of the compass, and increases in strength with higher magnetic latitude; and (4) a single blue pole on the starboard side for easterly headings and on the port side for westerly headings, being of zero strength on a heading of north or south and decreasing in strength with increased magnetic latitudes. The single pole concept assumes that the effect of one pole predominates. The locations of the poles depend partly upon the position of the compass to which they apply. The actual field surrounding any magnetic compass may be considerably more complex than indicated.

713. Compass adjustment.—There are at least two possible solutions to the problem of compass error. The error can be permitted to remain, and the various directions interconverted by means of variation and deviation, or compass error, as explained in article 711; or the error can be removed. In practice, a combination of both of these methods is used.

Variation depends upon location of the vessel, and the navigator has no control over it. Provision could be made for offsetting the lubber's line, but this would not be effective in correcting magnetic compass bearings, and this practice is not generally followed. Variation does not affect the operation of the compass itself, and so is not objectionable from this standpoint.

Deviation is undesirable because it is more troublesome to apply, and the magnetic field which causes it partly neutralizes the directive force acting upon the compass, causing it to be unsteady and sluggish. As the vessel rolls and pitches, or as it changes magnetic latitude, the magnetic field changes, producing a corresponding change in the deviation of an unadjusted compass.

Deviation is eliminated, as nearly as practicable, by introducing at the compass a magnetic field that is equal in magnitude and opposite in polarity to that of the vessel. This process is called **compass adjustment,** or sometimes **compass compensation,** although the latter designation is now more generally applied to the process of neutralizing the effect due to degaussing of the vessel (art. 745).

In general, the introduced field is of the same kind of magnetism as well as of the same intensity as those of the field causing deviation. That is, permanent magnets are used to neutralize permanent magnetism, and soft iron to neutralize induced magnetism, so that the adjustment remains effective with changes of heading and magnetic latitude. A relatively small mass of iron near the compass introduces a field equal to that of a much larger mass at a distance.

When a compass is properly adjusted, its remaining or **residual deviation** is small and practically constant at various magnetic latitudes, the directive force is as strong

as is obtainable on all headings, and the compass returns quickly from deflections and is comparatively steady as the vessel rolls and pitches.

714. Effect of latitude.—As indicated in article 706, the magnetic field of the earth is horizontal at the magnetic equator, and vertical at the magnetic poles, the change occurring gradually as a vessel proceeds away from the magnetic equator. At any place the relative strength of the horizontal and vertical components depends upon the magnetic dip. The directive force of a magnetic compass, provided by the horizontal component of the earth's magnetic field, is maximum on or near the magnetic equator and gradually decreases to zero at the magnetic poles. Within a certain area surrounding each magnetic pole the directive force is so weak that the compass is unreliable (art. 2513).

Deviation changes with a change of the relative strength of either the deviating force or the directive force. Thus, with *either* an increase in deviating force or a decrease in directive force, the deviation increases. However, if both the deviating and directive forces change by the same proportion, and with the same sign, there is no change in deviation. Also, if a deviating force is neutralized by an equal and opposite force *of the same kind*, there is no change of deviation with a change of magnetic latitude.

Permanent magnetism is the same at any latitude. If the permanent magnetism of the vessel is neutralized by properly placed permanent magnets of the correct strength, a change of magnetic latitude can be made without introduction of deviation. But if residual deviation due to permanent magnetism is present, it increases with a change to higher latitude. The deviating force remains unchanged while the directive force decreases, resulting in an increase in the relative strength of the deviating force.

As magnetic latitude increases, the vertical component of the earth's magnetic field becomes stronger, increasing the amount of induced magnetism in vertical soft iron. At the same time the directive force of the compass decreases. Both effects result in increased deviation unless the deviating force is neutralized by induced magnetism in vertical soft iron.

As magnetic latitude increases, the induced magnetism in the horizontal soft iron decreases in the same proportion as the decrease in the directive force of the compass, since both are produced by the horizontal component of the earth's magnetic field. Therefore, any deviation due to this cause is the same at any latitude.

715. Parameters.—Compass adjustment might be accomplished by locating the pole of each magnetic field, and establishing another pole of opposite polarity and equal intensity at the same place, or of less intensity and nearer to the compass; or a pole of opposite polarity and suitable intensity might be established at the correct distance on the opposite side of the compass. Thus, a blue pole east of a compass attracts the red northern ends of the compass magnets and repels the blue southern ends. Both effects cause rotation of the compass magnets and the attached compass card in a clockwise direction, producing easterly deviation. Either a red pole east of a compass, or a blue pole west of it, causes westerly deviation. If there are two fields of opposite polarity, one will tend to neutralize the other. If the intensities of the two fields are equal at the compass, one will cancel the other, and no deviation occurs.

Because of the complexities of the magnetic field of a vessel, and the fact that each individual field making up the total is present continuously, the process of isolating individual poles would be a difficult and time-consuming one. Fortunately, this is unnecessary. The vessel's field is resolved into certain specified components. Each of these components, regardless of its origin or the number of individual fields contributing to it, can be neutralized separately. Each component is called a **parameter,** and the various parameters are designated by letter, as follows:

Permanent magnetism. **Parameter *P*** is the fore-and-aft component. It is positive (+) if it is the equivalent of a blue pole forward of the compass, and negative (−) if red.

Parameter *Q* is the athwartship component. It is positive if it is the equivalent of a blue pole to starboard.

Parameter *R* is the vertical component. It is positive if it is the equivalent of a blue pole below the compass.

Induced magnetism has nine parameters, each the equivalent of that produced by a slender rod of soft iron. Each *end* of a rod is positive if it is forward, to starboard, or below the compass. Each rod is positive if both ends are positive or if both ends are negative, and negative if the two ends are of opposite sign. The rods are as follows:

a, b, c—one end level with the compass and in its fore-and-aft axis, either forward or aft. It is an *a* rod if it extends fore-and-aft, a *b* rod if athwartships, and a *c* rod if vertical.

d, e, f—one end level with the compass and in its athwartships axis, either to starboard or to port. It is a *d* rod if it extends fore-and-aft, an *e* rod if athwartships, and an *f* rod if vertical.

g, h, k—one end in the vertical axis of the compass, either above it or below it. It is a *g* rod if it extends fore-and-aft, an *h* rod if athwartships, and a *k* rod if vertical.

716. Coefficients.—Deviation which is easterly throughout approximately 180° of heading and westerly throughout the remainder is called **semicircular deviation,** indicating that its sign remains unchanged throughout a semicircle. Deviation caused by permanent magnetism and that caused by induced magnetism in vertical soft iron are semicircular. Deviation which changes sign in each quadrant, being easterly in two opposite quadrants and westerly in the other two, is called **quadrantal deviation.** It is caused by induced magnetism in horizontal soft iron. The types of deviation resulting from the various parameters are called coefficients. There are six, as follows:

Coefficient *A* is constant on all headings. If its cause is magnetic, as from an asymmetrical combination of parameters, it is a "true" constant. If its cause is mechanical, as from an incorrectly placed lubber's line, or mathematical, as from an error in computation of magnetic azimuth, it is an "apparent" constant.

Coefficient *B* is semicircular deviation which is proportional to the sine of the compass heading. It is maximum on compass headings east or west, and zero on compass headings north or south. Coefficient *B* is caused by permanent magnetism, and also by induced magnetism in asymmetrical vertical soft iron.

Coefficient *C* is semicircular deviation which is proportional to the cosine of the compass heading. It is maximum on compass headings north or south, and zero on compass headings east or west. Coefficient *C* is caused by permanent magnetism or by induced magnetism in asymmetrical vertical soft iron athwartship of the compass.

Coefficient *D* is quadrantal deviation which is proportional to the sine of twice the compass heading. It is maximum on intercardinal compass headings, and zero on cardinal compass headings. Coefficient *D* is caused by induced magnetism in horizontal soft iron which is symmetrical with respect to the compass.

Coefficient *E* is quadrantal deviation which is proportional to the cosine of twice the compass heading. It is maximum on cardinal compass headings, and zero on intercardinal compass headings. Coefficient *E* is caused by induced magnetism in horizontal soft iron which is asymmetrical with respect to the compass.

Coefficient *J* is the change of deviation for a heel of 1° while the vessel is on compass heading 000°.

The determination and use of the approximate coefficients in the analysis of compass deviation are discussed in article 727. The force components producing these coefficients are called **exact coefficients.** They are designated by the corresponding upper case German letters. The exact coefficients are now little used in practical

navigation. They are fully discussed in various books on compass adjustment.

717. Effect of compass location.—The location of a magnetic compass greatly influences the amount and type of deviation, as well as the adjustment. Thus, if a compass is on the center line, forward, the effective pole of vertical soft iron is aft of it; but if the compass is in the after part of the vessel, the effective pole is forward. If the compass is not on the center line, as the steering compass of an aircraft carrier, the magnetic field of the vessel is not symmetrical with respect to the compass. If a compass is located in a steel pilot house, the surrounding metal acts as a shield and reduces the strength of the magnetic field of the earth. This is of particular significance in high magnetic latitudes, where the directive force is weak.

Many factors influence the selection of a position for the compass. The most important consideration is the use to be made of it. A steering compass is of little use unless it is located so that it can be seen by the steersman. A compass to be used for emergency steering should be at the emergency steering station. A compass to be used for observing bearings or azimuths, or a standard compass to be used for checking other compasses, should be located so as to have a clear view in most directions.

However, some choice is possible. A compass should not be placed off the center line if it can be placed on the center line and still serve its purpose. It should not be placed near iron or steel equipment that will frequently be moved, if this can be avoided. Thus, a location near a gun, boat davit, or boat crane is not desirable. The immediate vicinity should be kept free from sources of deviation—particularly those of a changing nature—if this can be done. That is, no source of magnetism, other than the structure of the vessel, should be permitted within a radius of several feet of the magnetic compass. Some sources which might be overlooked are electric wires carrying direct current; magnetic instruments, searchlights, windshield wipers, electronic equipment, or motors; steel control rods, gears, or supports associated with the steering apparatus; fire extinguishers, gas detectors, etc.; and metal coat hangers, flashlights, keys, pocketknives, metal cap devices, or nylon clothing. The effect of some items such as an ammeter or electric windshield wiper varies considerably at different times. If direct current is used to light the compass, the wires should be twisted.

A magnetic compass cannot be expected to give reliable service unless it is properly installed and protected from disturbing magnetic influences.

718. The binnacle.—The compass is housed in a **binnacle.** This may vary from a simple wooden box to an elaborate device of bronze or other nonmagnetic material. Most binnacles provide means for housing or supporting the various objects used for compass adjustment, as well as the equipment for compensating for deviation caused by degaussing. The standard binnacle for the U. S. Navy 7½-inch compass is shown in figure 718. The trays for holding the fore-and-aft and

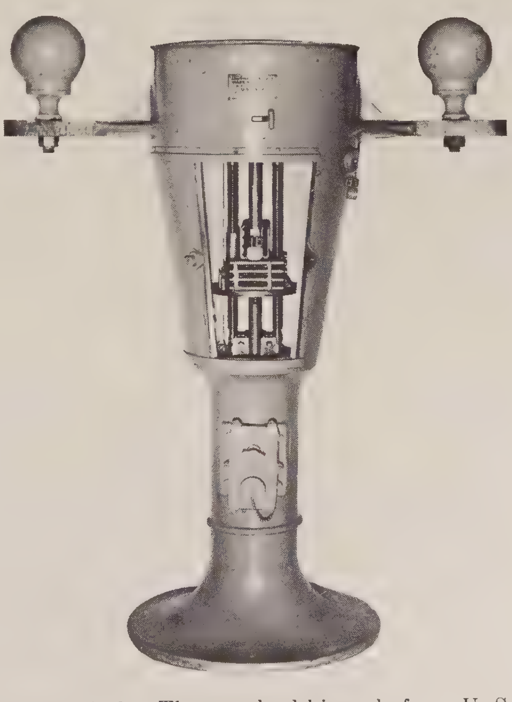


FIGURE 718.—The standard binnacle for a U. S. Navy 7½-inch compass.

athwartship magnets (art. 719), and the tube for the heeling magnet (art. 724), can be seen through the open door.

719. Adjustment for deviation due to permanent magnetism.—Permanent magnetism can be considered concentrated in a single pole, the position of which depends upon the magnetic heading upon which the vessel was constructed, and the subsequent magnetic history of the vessel. Figure 719a indicates the condi-

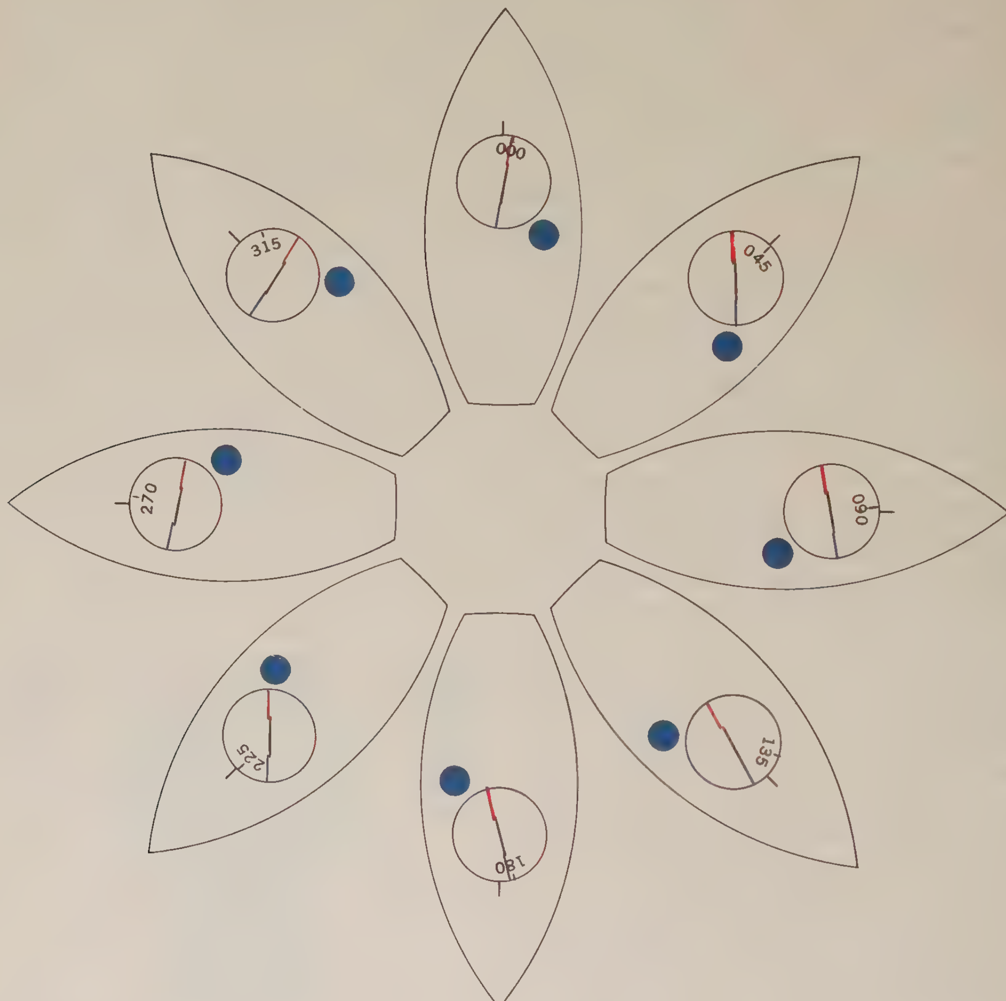


FIGURE 719a.—Deviation due to permanent magnetism if the resultant field is that of a blue pole on the starboard quarter of the vessel.

tion if the permanent magnetism can be considered concentrated in a single blue pole which is directly south of the compass when the vessel is headed magnetic northeast. The only effect on this heading is to weaken the directive force. No deviation is produced because the pole is in line with the compass magnets. On heading magnetic southwest, the pole is also in line with the compass magnets and there is no deviation, but the directive force is strengthened. On any other heading, the pole is not in line with the compass magnets, and deviation occurs, being in the same direction as that of the blue pole from the compass, since the blue pole attracts the red

northerly ends of the compass magnets and repels the blue southerly ends. The maximum effect occurs when the *compass* heading is approximately 90° from that of zero deviation. In figure 719a the headings shown on the compass card are the magnetic headings of the vessel. Their offset from the lubber's line shows the direction and relative magnitude of deviation.

If there were no other magnetism in the vessel, the poles might easily be located and neutralized by placing a magnet in such a position that a field of permanent magnetism but opposite polarity would occur at the compass. Although this method of adjustment has been used, it has not proven entirely satisfactory.

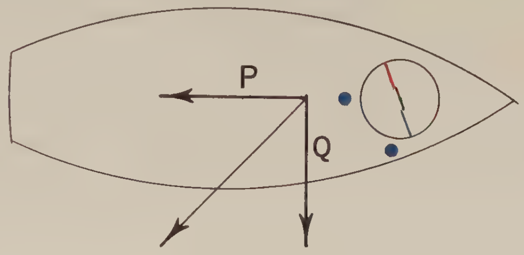


Figure 719b.—The horizontal component of the permanent field of figure 719a resolved into its components, parameters *P* and *Q*.

The usual method is to adjust for the fore-and-aft (parameter *P*) and athwartship (parameter *Q*) components separately. These are shown in figure 719b. The vertical parameter *R* does not produce deviation while the vessel is on an even keel. Its effect when the vessel heels is discussed in article 724. Thus, the effect of a single blue pole at the position shown in figure 719a is the same as that which would be produced by two weaker poles as shown in figure 719b. On heading east or west by the compass, parameter *Q* does not produce deviation directly. However, on easterly headings it does weaken the directive force due to the earth's magnetic field and therefore the deviating force of parameter *P* (causing deviation coefficient *B*) is relatively stronger and has a greater deviating effect. On a westerly heading the directive force would be strengthened, with a corresponding decrease in the *B* coefficient of deviation. By weakening the directive force on easterly headings, parameter *Q* also makes the compass sluggish on these headings. In high latitudes, where the horizontal component of the earth's magnetic field is weak, the compass may lose its directivity at a greater distance from the magnetic pole. Nearer the pole, it might point in the opposite direction.

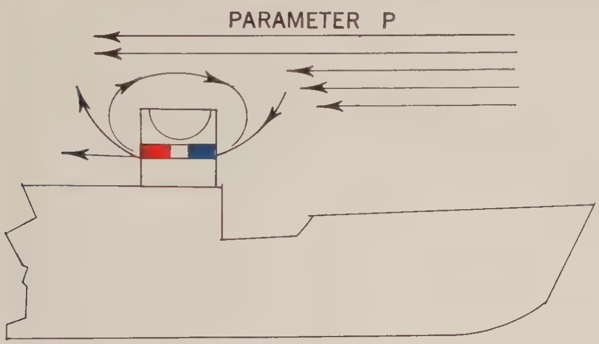


Figure 719c.—The field of a permanent magnet below the compass and opposing parameter *P* of figure 719b.

Many binnacles provide a group of several small tubes or "trays" extending in a fore-and-aft direction below the compass. One or more permanent magnets can be inserted in these trays, and the whole assembly moved up or down to vary the effect upon the compass. Figure 719c shows the situation if a single magnet is placed with its red end aft. The field at the compass is in the opposite direction of that of parameter *P*, and if it is of equal strength, the effect of this parameter is eliminated.

If now the vessel is headed north or south by the compass, the only pole remaining is that due to parameter *Q* (causing deviation coefficient *C*), as shown in figure 719d. A set of trays in an athwartship direction below the compass permits insertion of one or more permanent magnets to neutralize the remaining permanent magnetism. The

effect of inserting a single magnet with red end to starboard is shown in figure 719e. With both components removed, the field at the compass is completely neutralized.

Both the fore-and-aft (B) and athwartship (C) trays are in pairs with an equal number of trays on each side of the vertical axis of the compass. In each set of trays it is generally desirable to use an *even* number of magnets equally distributed on each side, to produce a symmetrical field at the compass. However, under some conditions, maximum reduction of deviation occurs with an *odd* number of magnets, particularly when two magnets at maximum distance from the compass overcorrect. If there is a choice, a greater number of magnets at a distance is preferable to a lesser number close to the compass.

With each parameter, the trays to use are those which are approximately perpendicular to the compass magnets. The magnets are placed so that the red ends will be on that side of the compass corresponding to the deviation. Thus, if deviation is easterly, the magnets should be placed so that the red ends will be east of the compass

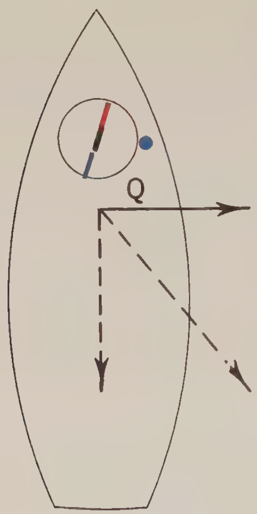


FIGURE 719d.—The permanent field of figure 719a after neutralization of parameter P.

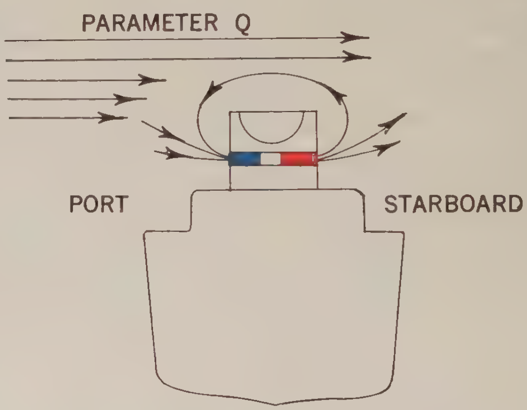


FIGURE 719e.—The field of a permanent magnet below the compass and opposing parameter Q of figure 719b.

(forward if the heading is east, and to starboard if the heading is north). However, if the wrong end is inserted in the trays, the fact will be immediately apparent because the compass card will rotate in the wrong direction. If the binnacle is not constructed to receive appropriate corrector magnets, these might be secured to some supporting surface near the compass.

During adjustment, the unused magnets should be kept far enough from the compass so that they will not affect it.

720. Adjustment for deviation due to induced magnetism in vertical soft iron.—Figure 720 shows the effect upon the compass of a single blue pole on the center line of the vessel, aft of the compass. This is a typical situation for induced magnetism in vertical soft iron, for a centerline compass located in the forward part of a vessel in magnetic north latitude. On heading north by compass there is no deviating force, but the directive force is weakened. In high northern latitudes, where this pole becomes strong and the directive force becomes weak, magnetism of this type, if not neutralized, can cause the compass to be unreliable in a much larger area than if the force is neutralized. On a heading of south by compass there is no deviation, but the

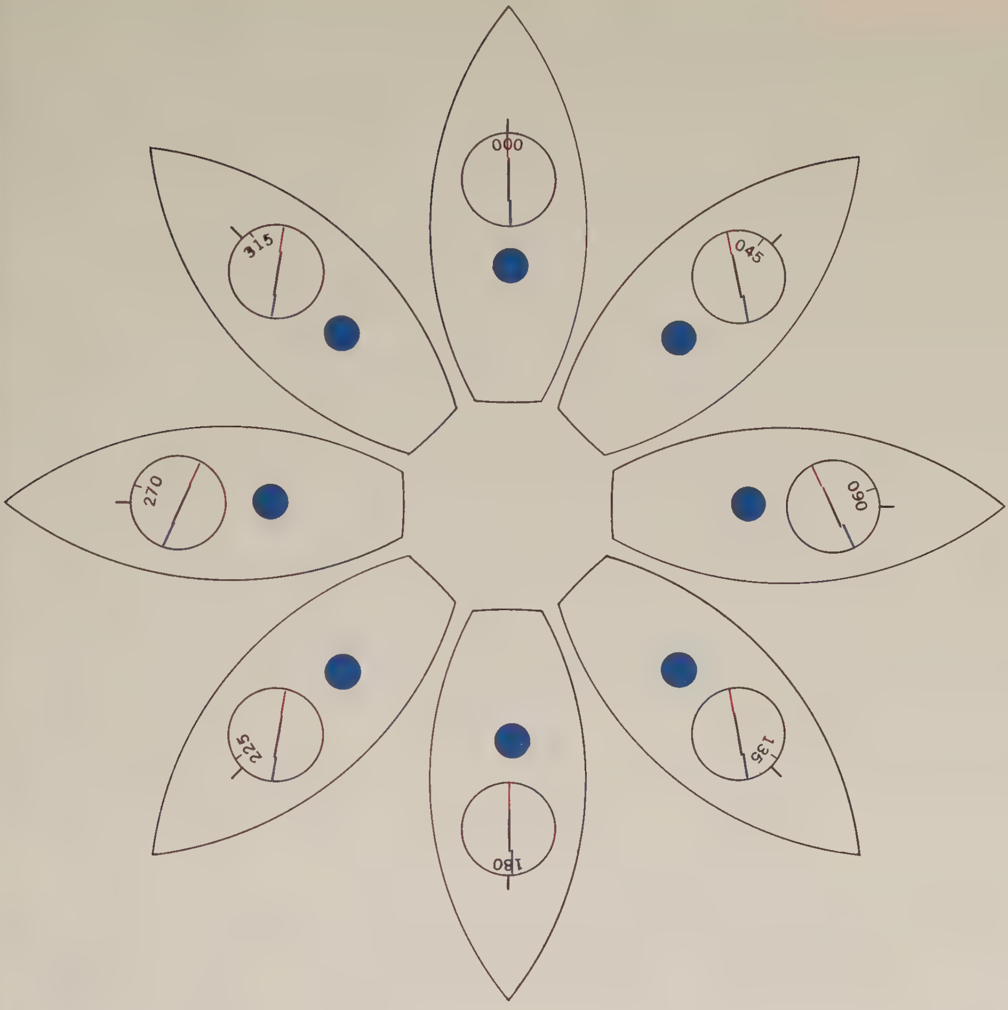


FIGURE 720.—Deviation due to induced magnetism in vertical soft iron if the resultant field is that of a blue pole on the center line aft of the compass.

directive force is strengthened. On headings with an easterly component the deviation is westerly, and on headings with a westerly component the deviation is easterly. In each case the maximum occurs when the vessel is on compass heading approximately east or west. Thus, the deviation due to induced magnetism in vertical soft iron is semicircular, coefficient *B*. In figure 720 the headings shown on the compass card are the magnetic headings of the vessel. Their offset from the lubber's line shows the direction and relative magnitude of deviation.

The deviating force due to induced magnetism in vertical soft iron is neutralized by placing a bar of soft iron in a vertical position on the opposite side of the compass from the effective pole due to the field of the vessel. This piece of metal is called a **Flinders bar,** after Captain Matthew Flinders, RN (1774–1814), an English navigator and explorer who is generally given credit for discovering both the effect and method of adjustment (art. 111). Today, most binnacles for large ships provide a tube for insertion of a Flinders bar. The bar consists of various lengths of soft iron placed end to end; with the remainder of the tube being filled with spacers of nonmagnetic mate-

rial, usually wood, brass, or aluminum. The standard Flinders bar is two inches in diameter and is divided into six sections, one each of 12, 6, 3, and 1½ inches, and two of ¾ inch. This permits use of any multiple of ¾ inch to 24 inches. All the iron pieces should be above the spacers in the tube, without a gap between pieces, the largest piece being on top. The upper end is then about two inches above the level of the compass card. For short lengths, one or more spacers should be omitted so that about $\frac{1}{12}$ of the length of the bar is above the level of the compass card.

The various pieces should be inserted in the tube carefully. If they are dropped, they may acquire some permanent magnetism. This reduces their effectiveness for the purpose intended. Each piece should be tested from time to time to determine whether or not it has acquired permanent magnetism. This can be done by holding it vertical with one end east or west of the compass and very near the compass magnets, noting the reading of the compass, and then inverting the piece so that the ends are interchanged. If the reading differs, permanent magnetism has been acquired by the iron rod. The temporary change of reading while the rod is being inverted should be ignored. In making the test, one should be careful to place the rod in the same position relative to the compass before and after inversion. On an easterly or westerly heading the Flinders bar holder can be used. A small amount of permanent magnetism can be removed by holding the rod approximately parallel to the lines of force of the earth's field, with the blue pole of the rod toward the north, and tapping one end of the rod gently with a hammer. Several alternate tests and treatments may be needed to make the rod magnetically neutral. If this process is not effective in removing the permanent magnetism, the rod should be heated to a dull red and allowed to cool slowly.

An older type Flinders bar, rarely encountered with modern compasses, consists of a number of slender rods of equal length, the *number* of rods being varied rather than the length of a single rod. Another old system consists of using a single rod of fixed length, and varying its distance from the compass.

721. Determination of Flinders bar length.—As indicated in articles 719 and 720, coefficient B magnetism may be introduced both by permanent magnetism of the vessel and by induced magnetism in asymmetrical vertical soft iron. A problem thus arises as to what part of the deviation on headings magnetic east and west is due to each cause. If the vessel remains on an even keel at about the same magnetic latitude, adjustment can be made without this knowledge. However, satisfactory performance under all conditions requires separate adjustment for each cause.

There are several possible solutions to this problem. The two sources can be separated by use of the fact that a change of magnetic latitude affects them differently. On the magnetic equator there is no vertical component of the earth's magnetic field, and consequently no induced magnetism in vertical soft iron. Therefore, if the compass is adjusted on the magnetic equator, all coefficient B deviation is due to permanent magnetism, and is removed by the fore-and-aft magnets. After a considerable change of magnetic latitude, the deviation on a heading of magnetic east or west is again measured. By means of the curves of figure 721, A, the required amount of standard two-inch Flinders bar is determined. Accurate results will be obtained only if the vessel is magnetically the same at both latitudes. That is, a structural change, an alteration in the number or position of magnets or other devices used in the adjustment, magnetic treatment, etc., invalidates the measurement. After the required amount of Flinders bar has been inserted, some deviation may be present due to mutual induction among the various devices used for adjustment. This should be removed by means of the permanent magnets. Once the correct amount of Flinders bar has been

installed, no change should be needed unless there is a substantial change in the amount or location of vertical soft iron, or unless the compass is relocated.

This method is not always practical. If the correct length and location of Flinders bar for another vessel of similar construction and compass location have been determined previously, the same length can be used for the compass being adjusted. If a large change in magnetic latitude can be made without appreciable change of deviation on headings east and west, the amount of Flinders bar is correct. If the deviation changes,

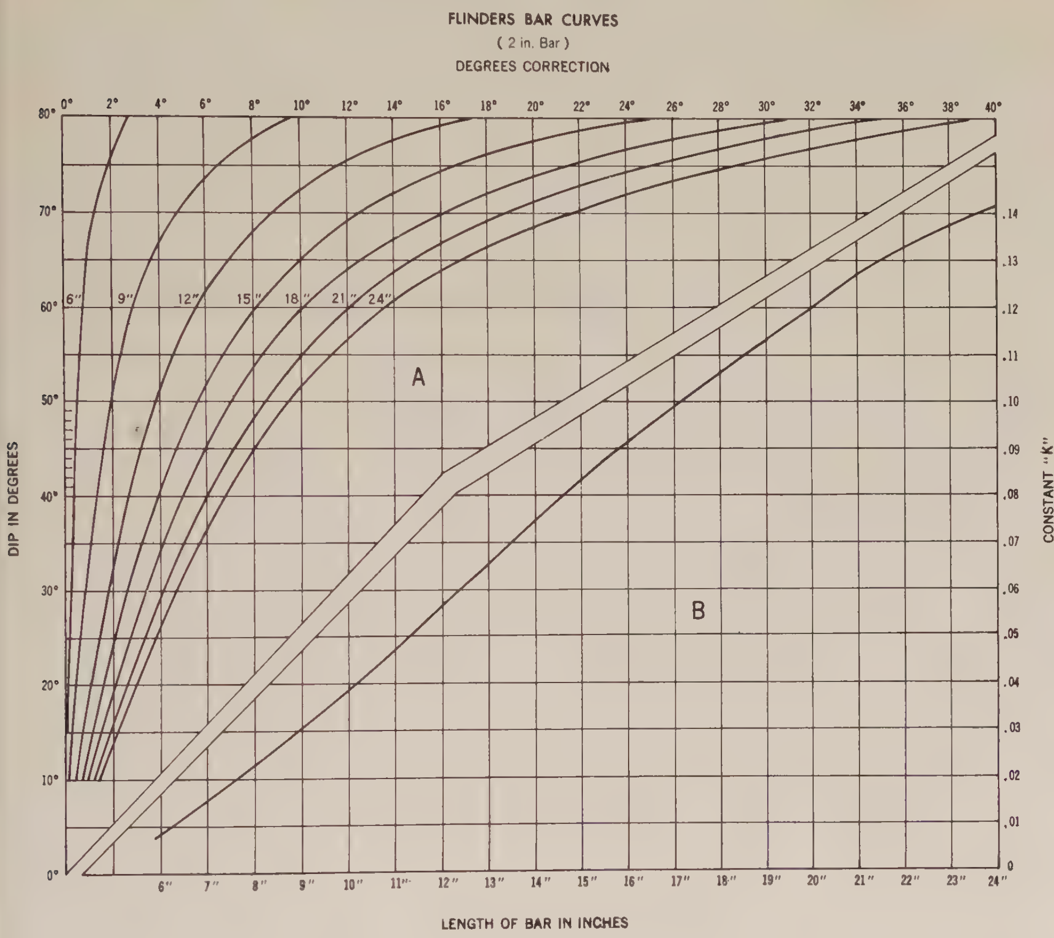


FIGURE 721.—Flinders bar curves: A, if deviation due to induced magnetism in vertical soft iron is known; B, if coefficient K is known.

readjustment is needed. By studying the structure of the vessel, an experienced compass adjuster may be able to make a reasonably accurate estimate of the length to use.

In the absence of enough reliable information to permit a reasonably accurate determination of the correct length, the Flinders bar may be omitted entirely, and the deviation on east and west headings removed by means of the fore-and-aft permanent magnets. This is common practice for yachts, fishing vessels, and even for some coastal vessels which do not change magnetic latitude more than a few degrees.

The correct length of Flinders bar can be determined by figure 721, B, if reliable data are available on the deviation occurring on magnetic east or west headings at two

widely separated magnetic latitudes. The constant K is determined by computation, using the formula

$$K=\lambda \frac{H_2 \tan d_2-H_1 \tan d_1}{Z_2-Z_1}$$

in which

K=a constant proportional to the required length of Flinders bar.

λ=shielding factor, or the proportion of the earth's field effective at the compass. Generally, it varies from about 0.7 to 1.0, averaging about 0.9 for compasses in exposed positions, and 0.8 for those surrounded by metal deck houses.

H_1=horizontal intensity of earth's magnetic field at place of first deviation reading.

H_2=horizontal intensity of earth's magnetic field at place of second deviation reading.

d_1=total deviation on heading magnetic east or west at place of first deviation reading.

d_2=total deviation on heading magnetic east or west at place of second deviation reading.

Z_1=vertical intensity of earth's magnetic field at place of first deviation reading.

Z_2=vertical intensity of earth's magnetic field at place of second deviation reading.

The values of horizontal and vertical intensity (H and Z) can be obtained from H.O. charts No. 1701 and 1702, respectively.

The constant K represents a mass of vertical soft iron (the *c* rod) causing deviation. From the intersection of the curve of figure 721, B, and a horizontal line through the value of constant K, draw a vertical line to the bottom scale, which shows the required length of Flinders bar.

If some length of Flinders bar was in place when the two deviation readings were made, enter the graph of figure 721, B, with this length and determine the corresponding value of K. Call this K_2 and that obtained by computation K_1. Algebraically add K_1 and K_2 to determine the value of K to use for finding the total length of Flinders bar required. If the Flinders bar is forward of the compass, K_2 is negative (−), and if aft of the compass, K_2 is positive (+). In the computation of K_2, both Z_1 and Z_2 are positive in north magnetic latitude and negative in south magnetic latitude. Also, d_1 and d_2 are positive if deviation is east on magnetic heading east in north latitude or magnetic heading west in south latitude. If *either* the heading or direction of the deviation is reversed, the sign of d_1 or d_2 is negative. If both are reversed, the sign is positive. If the value of K is negative, the Flinders bar should be installed forward of the compass, and if positive, it should be installed aft.

Example.—The deviation of a magnetic compass of a ship on heading magnetic east is 1°E at New York (H 0.170, Z 0.539) and 9°E at Panama (H 0.311, Z 0.260). The shielding factor is 0.8.

Required.—The correct length of Flinders bar if (1) no Flinders bar is in place during observations, (2) six inches of Flinders bar is in place forward of the compass during observations.

Solution.—

$$(1)\ K_1=0.8\left(\frac{0.311\times0.15838-0.170\times0.01746}{0.260-0.539}\right)$$

$$=(-)\ 0.133$$

$$K_2=0$$

$$K=K_1+K_2=(-)\ 0.133$$

From figure 721, B, the correct amount of Flinders bar is 22 inches. Since the amount used must be a multiple of ¾ inch, the amount to use is 21¾ inches. Since K is negative, the bar should be installed forward of the compass.

(2) From figure 721, B, the value of K_2 corresponding to six inches of Flinders bar is 0.009. The value is negative because the bar is forward of the compass. Therefore, $K_1+K_2=(-)\,0.133+(-)\,0.009=(-)\,0.142$. From figure 721, B, the *total* amount of Flinders bar required is 24 inches, which should be installed forward of the compass.

Answers.—(1) 21¾ inches of Flinders bar installed forward of the compass, (2) 24 inches of Flinders bar installed forward of the compass.

When the length of Flinders bar is determined in this way, accurate results can be expected only if the vessel is magnetically unchanged between deviation readings.

Lord Kelvin suggested the following rule for improving the adjustment for coefficient B if no better method is available:

Remove the deviation observed on magnetic east or west headings by means of fore-and-aft B *magnets when the vessel has arrived at places of weaker vertical magnetic field, and by means of Flinders bar when it has arrived at places of stronger vertical magnetic field, whether in the northern or southern hemisphere.*

After a number of applications of this rule following alternate passage from weaker to stronger fields and then stronger to weaker fields, the amount of Flinders bar should be very nearly correct.

722. Adjustment for deviation due to induced magnetism in symmetrical horizontal soft iron.—That part of horizontal soft iron which is symmetrically arranged with respect to the compass can be considered equivalent to two rods extending through the compass, one in a fore-and-aft direction ($-a$ rod) and the other in an athwartship direction ($-e$ rod). The deviation caused by both of these rods is quadrantal, but of opposite sign. If both rods were equally effective in causing deviation, they would cancel each other and no deviation would result on any heading. In most vessels, however, the athwartships iron dominates, and deviation due to all horizontal soft iron can generally be considered to be that which would result from a single $(-)\,e$ rod. In figure 722a the deviation resulting from such a rod is shown for various magnetic headings in any latitude. There is no deviation on any cardinal heading, but the directive force is weakened on heading magnetic east or west. The maximum deviation occurs on intercardinal headings by compass, being easterly in the northeast and southwest quadrants, and westerly in the other two quadrants. This is coefficient D deviation. In figure 722a the headings shown on the compass card are the magnetic headings of the vessel. Their offset from the lubber's line shows the direction and relative magnitude of deviation.

The field causing this deviation is neutralized by installing two masses of soft iron abeam of the compass, on opposite sides and equidistant from its center. Such iron is usually in the form of hollow spheres or cylinders, called **quadrantal correctors.** These can be moved in or out in an athwartship direction along brackets on the sides of the binnacle.

Quadrantal correctors act as $(+)\,e$ parameters which neutralize the $(-)\,e$ parameter of the athwartships iron. As shown in figure 722b, the portion of the corrector adjacent to the compass is always of opposite polarity to the deflecting force. The amount of the correction can be adjusted by moving the correctors toward or away from the compass card. If the inboard limit of travel is reached without fully removing the deviation, larger correctors are needed. If overcorrection occurs at the outboard limit, smaller correctors are needed. A single corrector can be used, but this produces an unbalanced field which is less desirable than a balanced one. In general, large correctors at a greater distance are preferable to small correctors close up because there is less mutual induction

between the correctors if they are widely separated. In the rare case when quadrantal deviation is *westerly* on heading northeast (coefficient D is negative, the fore-and-aft horizontal soft iron predominating), the quadrantal correctors should be mounted fore-and-aft on the binnacle.

Figure 722c shows the approximate amount of deviation correction to be expected from correctors of various sizes, shapes, and distance from the center of a standard

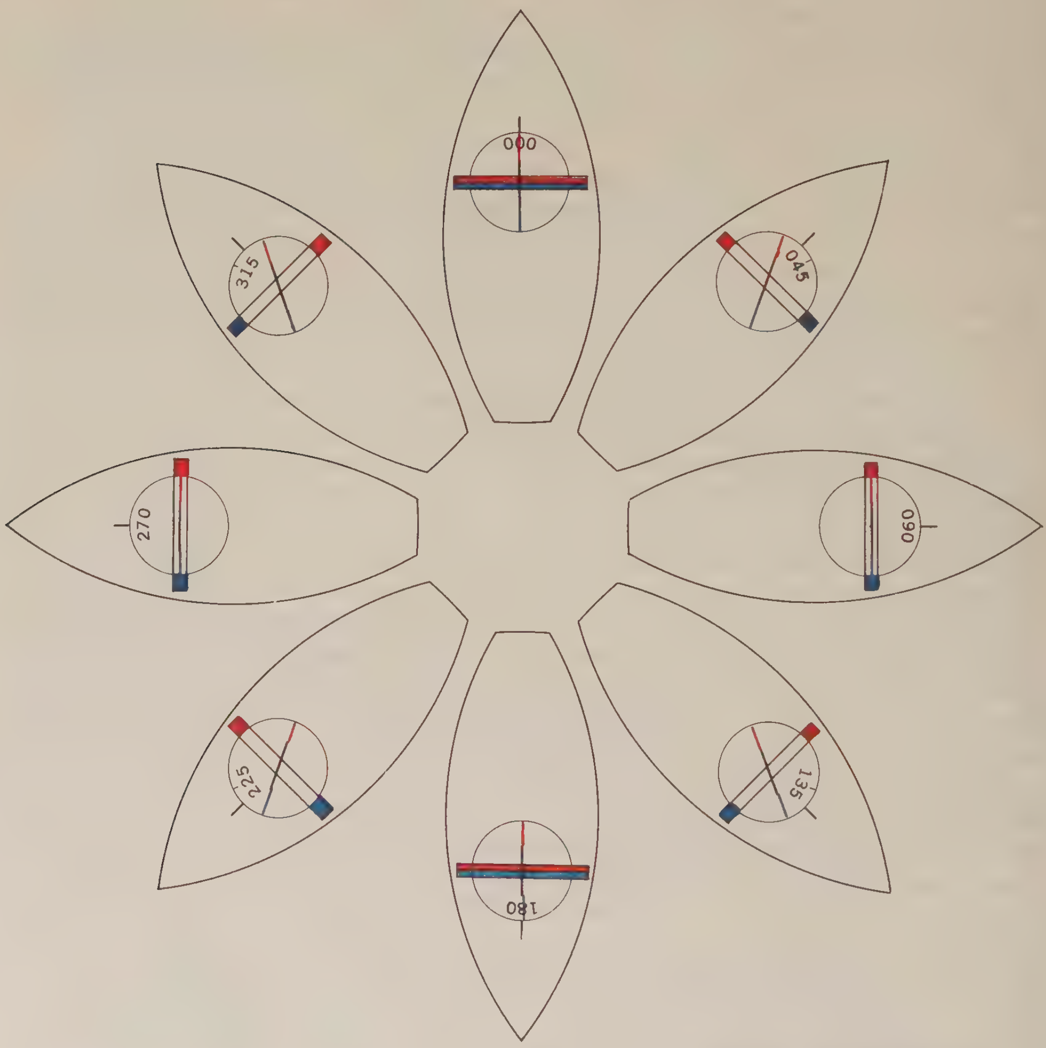


FIGURE 722a.—Deviation caused by induced magnetism in symmetrical horizontal soft iron.

Navy 7½-inch compass. The data apply to either the athwartships or fore-and-aft position.

Like the Flinders bar (art. 720), the quadrantal correctors should be handled carefully, and checked from time to time to see if they have acquired permanent magnetism. The test can be made by rotating each corrector through 180° without altering its distance from the center. If the compass heading changes, the correctors have acquired permanent magnetism which can be removed by tapping with a hammer when

the blue pole is toward the north, or by removing the spheres, heating them to a dull red, and permitting them to cool slowly.

723. Adjustment for deviation due to induced magnetism in asymmetrical horizontal soft iron.—If the horizontal soft iron is not arranged symmetrically with respect to the compass, resulting in an effective pole which is on neither the fore-and-aft nor athwartships axis through the compass, quadrantal deviation with its maximum values

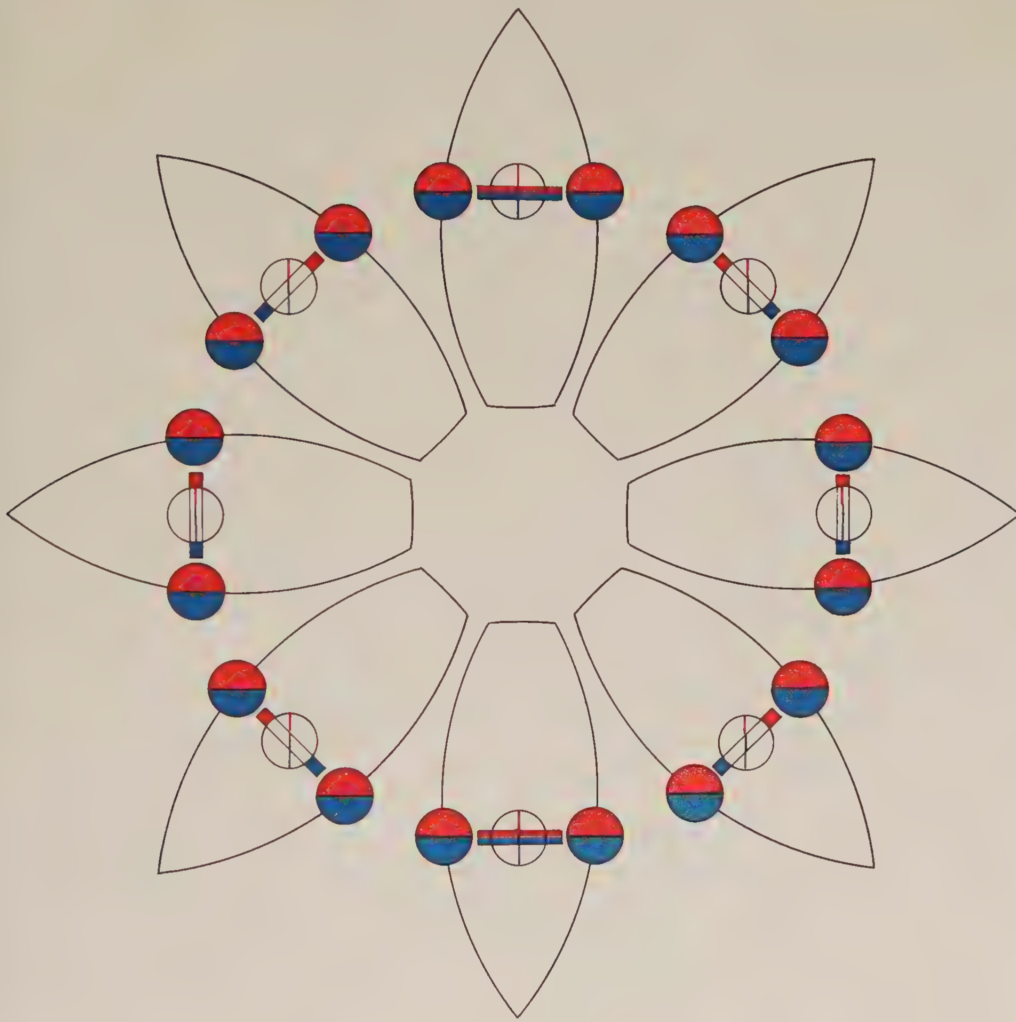


FIGURE 722b.—Adjustment for symmetrical horizontal soft iron.

on cardinal headings (coefficient E) results. Constant deviation (coefficient A) may also be caused by this arrangement. Either coefficient E or A is due to a combination of parameters.

For a centerline compass on a ship of conventional construction, any deviation due to induced magnetism in asymmetrical horizontal soft iron is small, and many installations make no provision for neutralizing the effect. However, some binnacles are provided with a pair of ***E*-links,** which are bars that can be attached to the side brackets to permit the quadrantal correctors to be slewed somewhat with respect to the com-

pass. When this has been done, the horizontal axis through the correctors and the compass makes an angle with the athwartship axis of the compass.

After a compass has been adjusted, any remaining constant deviation due to magnetic coefficient *A* is likely to be very small. If such deviation exists, its cause is likely to be chiefly mechanical. If a compass is used primarily for determining the heading (as a steering compass), all constant deviation can be removed by realignment of the binnacle so as to rotate the lubber's line by the required amount. However, if a compass is to be used for observing bearings or azimuths, only the mechanical *A*-error should be removed in this manner. This is because such readings are taken on the face of the card itself, and are therefore not affected by misalignment of the lubber's line. The two components of constant deviation can be separated in the following manner: Measure the deviation on various headings by means of bearings or azimuths (art. 1428). This includes only magnetic coefficient *A*. Then measure the deviation on various headings by means of the lubber's line, comparing the heading by compass with the magnetic heading determined by pelorus or gyro compass. This includes the combined effect of magnetic and mechanical coefficient *A* deviation. The difference between the two values is the mechanical coefficient *A*. For a properly adjusted compass the magnetic coefficient *A* deviation is so small that provision is not made for its removal.

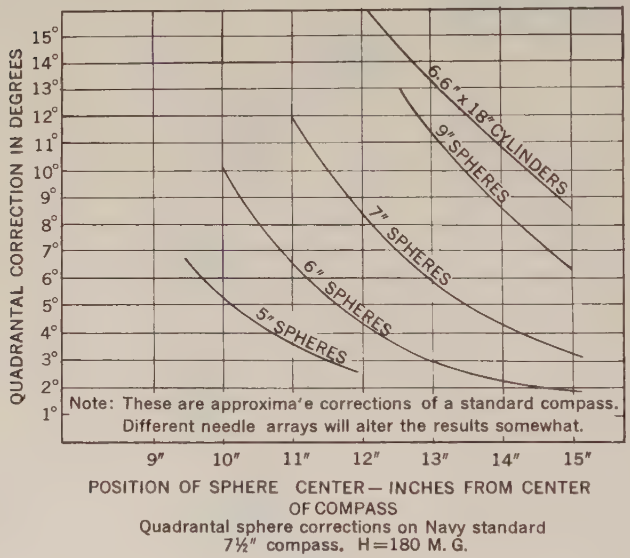


FIGURE 722c.—Effect of various quadrantal correctors.

724. Heeling error.—All of the effects discussed previously refer to a vessel on an even keel. When the vessel heels, conditions are altered. Deviation which now appears, or the *change* of deviation from that when the vessel was on an even keel, is called **heeling error.** For a constant angle of heel and a steady heading, this error remains essentially unchanged. However, it tends to increase as the heel becomes greater, and to reverse sign as the heel changes from one side to another. Therefore, if a vessel is rolling or pitching, the compass tends to oscillate. This increases the difficulty of reading the compass.

The cause of heeling error is the displacement of the permanent and induced magnetic fields with respect to the compass. Figure 724 shows a vessel heeled to starboard on heading magnetic north or south, in north magnetic latitude. The vessel was constructed in north magnetic latitude. On an even keel the vertical parameter *R* of permanent magnetism for a centrally located compass is directly below the compass, with the blue pole nearer the compass. When the vessel is heeled as shown at *A*, the blue pole is to port of the compass, causing deviation toward that side. A vertical rod of soft iron below the compass (parameter *k*) exerts a similar influence, as shown at *B*. An athwartship horizontal rod through the compass has no deviating effect while the vessel is on an even keel, but when it heels as shown in figure 724, the vertical component of the earth's field causes the port end to acquire a blue pole and the starboard end a red pole (parameter *e*), as shown at *C*. Each of the three causes shown in figure 724 results in a blue pole being established on the port or high side of the vessel. This causes

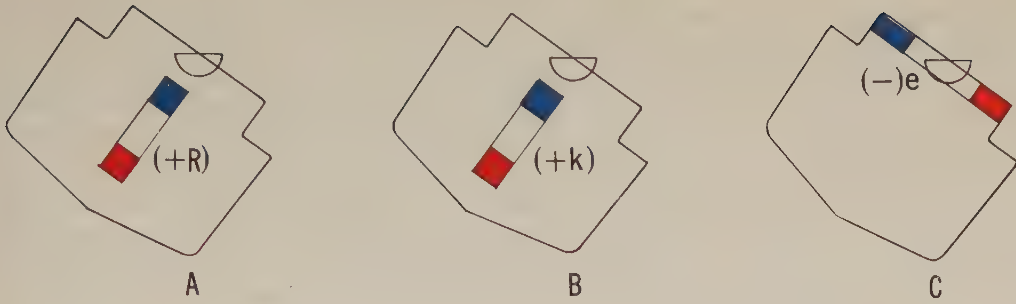


FIGURE 724.—Effect of heel.

the red north ends of the compass magnets to be attracted to this side. If the heading is magnetic north, the deviation is westerly, and if magnetic south, it is easterly. This effect is offset somewhat by the changed magnetic field surrounding the quadrantal correctors. On heading magnetic east or west, these components have no deviating effect, but the directive force of the compass is strengthened or weakened. When the vessel pitches, the effects described for north-south and east-west headings are reversed. On a heading other than a cardinal direction (magnetic) the effect is some combination of the two. The magnetic situation varies not only with the heading, but also with the magnetic latitude and the magnetic history of the vessel.

Although heeling error is due in part to permanent magnetism and in part to induced magnetism, the induced magnetism generally exerts the greater influence. The most effective method of neutralizing this effect would be to attack each parameter separately. This would require the placement of soft iron *above* the compass. Since this would not be a convenient arrangement, the condition is improved by placing a vertical permanent magnet, called a **heeling magnet,** centrally *below* the compass, and adjusting its height until the error is minimized. In north magnetic latitude, the red end is placed uppermost in most installations. As the vessel proceeds to lower magnetic latitudes, parameter R becomes less effective in producing deviation because of the stronger directive force due to the horizontal component of the earth's magnetic field. Parameters k and e become weaker because of decreased intensity of the vertical component of the earth's field, and the strengthening of the horizontal component also reduces their effect. Therefore, the heeling magnet requires readjustment as the magnetic latitude changes. As the vessel approaches the magnetic equator, the heeling magnet should be lowered. After the vessel crosses the magnetic equator, it may be necessary to invert the heeling magnets, so that the opposite end is uppermost. A change in the setting of the heeling magnet may introduce deviation on headings of compass east or west because of altered induction between the heeling magnet and the Flinders bar. This should be removed by means of the fore-and-aft (B) magnets in the trays below the compass.

If adjustment for heeling error is made when the vessel is tied up or at anchor, it is best done by listing the vessel on a northerly or southerly heading, and adjusting the heeling magnet until the reading of the compass is restored to what it was before the vessel heeled. If the adjustment is made at sea, the vessel should be placed on a heading of compass north or south. If there is little rolling, the vessel can be listed and the compass reading restored, as at dockside. If the vessel rolls moderately on this heading, the heeling magnet should be placed at that height at which oscillation of the compass card is minimum. If the setting for minimum oscillation is different on north and south headings, the mean position should be used. Any yawing of the vessel should be considered when reading the compass under rolling conditions.

The approximate position of the heeling magnet can be determined by means of

an instrument known as a **heeling adjuster** or a **vertical force instrument,** a form of **dip needle.** This consists of a small magnet balanced about a horizontal axis by means of a small adjustable weight. A scale indicates the distance of the weight from the axis. The instrument is taken ashore and balanced at a place where the earth's field is undisturbed, the magnet being in a magnetic north-south direction, approximately. The instrument is then taken aboard ship, the compass removed from its binnacle, and the heeling adjuster installed in its place. The weight is set to a distance equal to the distance determined ashore, multiplied by λ, the shielding factor (art. 721). The heeling magnet is then moved up or down until the magnet of the instrument is level. This should be approximately the correct setting. This method is used principally when the listing of a vessel is difficult or impractical.

725. Soft iron correctors and nearby magnets.—The soft iron correctors used in compass adjustment are near enough to the compass magnets and the magnets used in compass adjustment to be influenced by them.

The Flinders bar acquires a certain amount of induced magnetism from the fields of the heeling magnet and the fore-and-aft (*B*) corrector magnets. The approximate amount of deviation caused by induced magnetism from the heeling magnet of a 7½-inch compass when H=0.165 is shown in figure 725. Because of such induced magnetism, the "drop-in" method of determining the amount of Flinders bar is not accurate. By this method, Flinders bar lengths are added until the compass reading changes by the required amount. Better adjustment is achieved by using the required amount of Flinders bar and removing any remaining deviation on east-west headings by means of the fore-and-aft magnets. The principal reason that it is preferable to use a larger number of magnets at a distance from the compass than a smaller number near it, is that the former arrangement produces less induced magnetism in the Flinders bar and quadrantal correctors. If the Flinders bar length is changed, the deviation on headings of magnetic east and west should be checked, and any needed adjustment made by means of fore-and-aft magnets. When all correctors have been put in place, their positions relative to each other are constant. Therefore, the Flinders bar acts as a permanent magnet, and the resulting deviation is semicircular (coefficient *B*). The Flinders bar may also introduce a small amount of quadrantal deviation (coefficient *D*), its action being somewhat like that of a quadrantal corrector placed in the fore-and-aft axis of the compass.

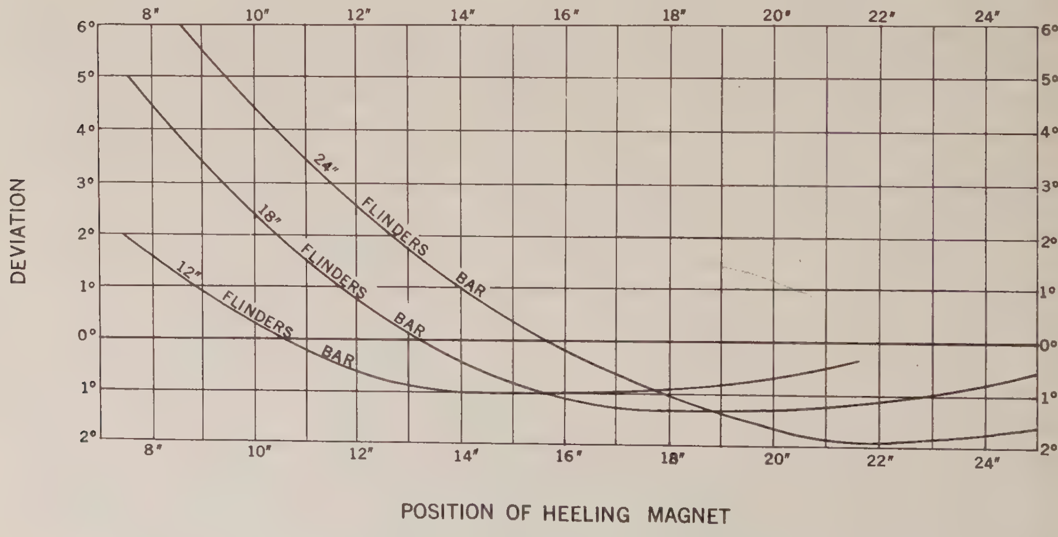


FIGURE 725.—Deviation due to inductive effect of heeling magnet on Flinders bar.

The quadrantal correctors acquire induced magnetism from the fields of the fore-and-aft (B) magnets, the athwartship (C) magnets, and the compass magnets. The magnetism acquired from the B and C magnets is semicircular (coefficient B from the B magnets, and coefficient C from the C magnets), and that acquired from the field of the compass magnets is quadrantal (coefficient D). The semicircular deviation is minimized by keeping the B and C magnets as far away from the quadrantal correctors as practicable, and any deviation that does exist is removed by means of these magnets. The quadrantal deviation is removed by means of the quadrantal correctors themselves. The compass magnets of most modern compasses have little effect upon the quadrantal correctors.

Because of the interaction between the various correctors, it is good practice to insert the required amount of Flinders bar, and to install the quadrantal correctors and heeling magnet at their approximate positions before adjusting the compass. If a radical change is subsequently made in any of these adjustments, the settings of the B and C magnets should be checked and altered if necessary.

Analysis of Deviation

726. Nature and purpose of analysis.—An analysis consists of determining the approximate value of each of the six coefficients, and studying the results. The purpose of the analysis is to give the compass adjuster an understanding of the magnetic properties of the vessel. This provides the basis for the approximate placement of the various correctors, and suggests possibilities for further refinement in the adjustment. Without an analysis, compass adjustment is a more-or-less mechanical process. Fewer mistakes are likely to be made by the person who understands the nature of the magnetic field he seeks to neutralize.

727. The analysis.—The first step in an analysis is to record the deviation on each cardinal and intercardinal heading *by the compass to be analyzed.* For the purpose of analysis, easterly deviation is considered positive (+), and westerly deviation negative (−). Approximate values of the various coefficients are:

Coefficient A—mean of deviation on all headings.

Coefficient B—mean of deviation on headings 090° and 270°, with sign at 270° reversed.

Coefficient C—mean of deviation on headings 000° and 180°, with sign at 180° reversed.

Coefficient D—mean of deviation on intercardinal headings, with signs at headings 135° and 315° reversed.

Coefficient E—mean of deviation on cardinal headings, with signs at 090° and 270° reversed.

Coefficient J—change of deviation for a heel of 1° while the vessel heads 000° by compass. It is considered *positive* if the north end of the compass card is drawn toward the *low* side, and *negative* if toward the *high* side.

Example.—A magnetic compass which has not been adjusted has deviation on cardinal and intercardinal compass headings as follows:

Compass heading	*Deviation*	*Compass heading*	*Deviation*
000°	1°5 W	180°	8°0 E
045°	34°0 E	225°	1°5 W
090°	31°0 E	270°	29°0 W
135°	13°5 E	315°	36°0 W

On heading compass north the deviation is 13°5 W when the vessel heels 10° to starboard.

Required.—The approximate value of each coefficient.

Solution.—

$$A=\frac{-1^\circ.5+34^\circ.0+31^\circ.0+13^\circ.5+8^\circ.0-1^\circ.5-29^\circ.0-36^\circ.0}{8}=(+)\ 2^\circ.3$$

$$B=\frac{31^\circ.0+29^\circ.0}{2}=(+)\ 30^\circ.0$$

$$C=\frac{-1^\circ.5-8^\circ.0}{2}=(-)\ 4^\circ.8$$

$$D=\frac{34^\circ.0-13^\circ.5-1^\circ.5+36^\circ.0}{4}=(+)\ 13^\circ.8$$

$$E=\frac{-1^\circ.5-31^\circ.0+8^\circ.0+29^\circ.0}{4}=(+)\ 1^\circ.1$$

$$J=\frac{-13^\circ.5+1^\circ.5}{10}=(-)\ 1^\circ.2$$

Answers.—*A* (+) 2°.3, *B* (+) 30°.0, *C* (−) 4°.8, *D* (+) 13°.8, *E* (+) 1°.1, *J* (−) 1°.2.

On any compass heading (CH) the deviation (d) from each coefficient acting alone is:

Coefficient A: $d_A=A$
Coefficient B: $d_B=B \sin CH$
Coefficient C: $d_C=C \cos CH$
Coefficient D: $d_D=D \sin 2CH$
Coefficient E: $d_E=E \cos 2CH$
Coefficient J: $d_J=J \cos CH$.

For a vessel on an even keel, the total deviation on any compass heading is the algebraic sum of the deviation due to each of the first five coefficients:

$$d=d_A+d_B+d_C+d_D+d_E=A+B \sin CH+C \cos CH+D \sin 2CH+E \cos 2CH.$$

For the compass of the example given above, the deviation due to each component, and the total, on various headings is:

CH	A	B	C	D	E	d
°	°	°	°	°	°	°
000	+2.3	0.0	−4.8	0.0	+1.1	−1.4
015	+2.3	+7.8	−4.6	+6.9	+1.0	+13.4
030	+2.3	+15.0	−4.2	+12.0	+0.6	+25.7
045	+2.3	+21.2	−3.4	+13.8	0.0	+33.9
060	+2.3	+26.0	−2.4	+12.0	−0.6	+37.3
075	+2.3	+29.0	−1.2	+6.9	−1.0	+36.0
090	+2.3	+30.0	0.0	0.0	−1.1	+31.2
105	+2.3	+29.0	+1.2	−6.9	−1.0	+24.6
120	+2.3	+26.0	+2.4	−12.0	−0.6	+18.1
135	+2.3	+21.2	+3.4	−13.8	0.0	+13.1
150	+2.3	+15.0	+4.2	−12.0	+0.6	+10.1
165	+2.3	+7.8	+4.6	−6.9	+1.0	+8.8
180	+2.3	0.0	+4.8	0.0	+1.1	+8.2
195	+2.3	−7.8	+4.6	+6.9	+1.0	+7.0
210	+2.3	−15.0	+4.2	+12.0	+0.6	+4.1
225	+2.3	−21.2	+3.4	+13.8	0.0	−1.7
240	+2.3	−26.0	+2.4	+12.0	−0.6	−9.9
255	+2.3	−29.0	+1.2	+6.9	−1.0	−19.6
270	+2.3	−30.0	0.0	0.0	−1.1	−28.8
285	+2.3	−29.0	−1.2	−6.9	−1.0	−35.8
300	+2.3	−26.0	−2.4	−12.0	−0.6	−38.7
315	+2.3	−21.2	−3.4	−13.8	0.0	−36.1
330	+2.3	−15.0	−4.2	−12.0	+0.6	−28.3
345	+2.3	−7.8	−4.6	−6.9	+1.0	−16.0

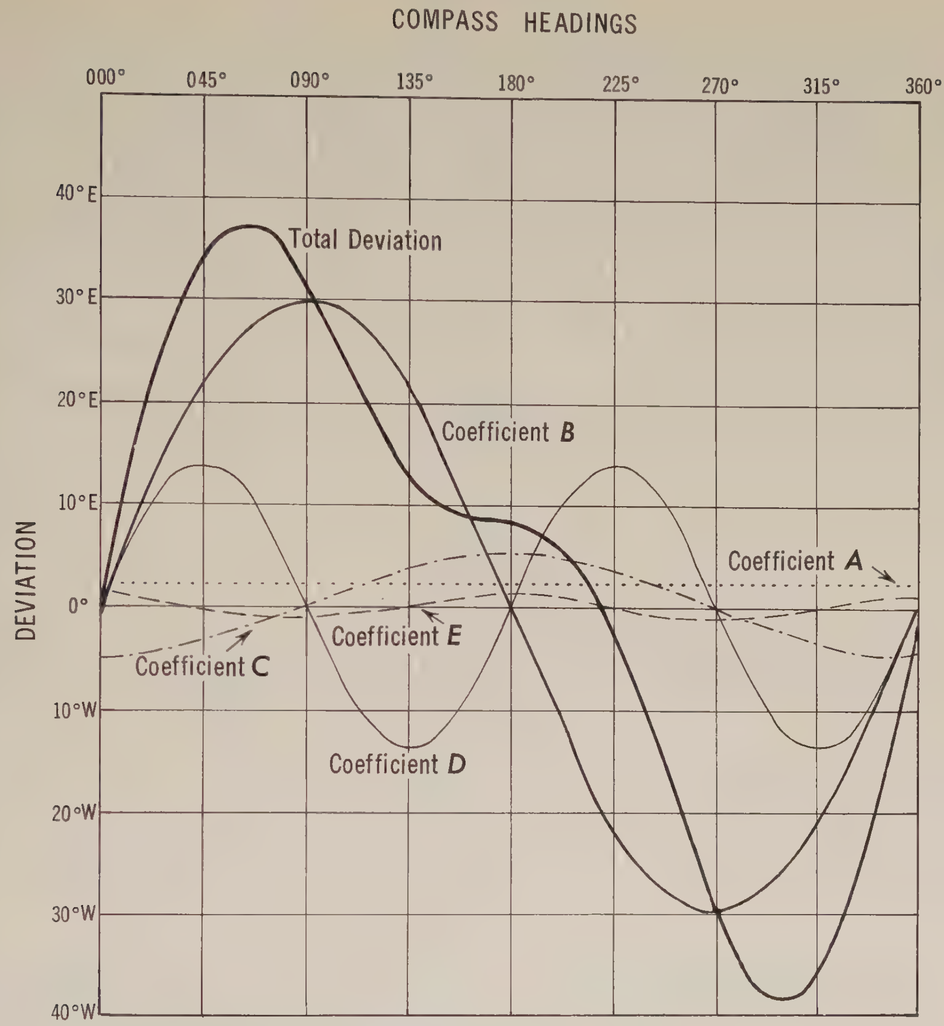


FIGURE 727.—Coefficients and total deviation of an unadjusted magnetic compass.

The various components and the total deviation are shown in graphical form in figure 727. Since the various coefficients are only approximated by the method given above, the curve of total deviation found in this way should not be expected to coincide exactly with a curve drawn from values found by measurement on the various headings.

The *shapes* of the curves of figure 727 are typical of those of an unadjusted compass of a large steel ship. However, an analysis of the results indicates the following:

Coefficient *A* is normally negligible. The presence of more than 2° of constant error indicates an abnormal condition which should be discovered and corrected. If the vessel has been in service for some time without major structural change, and no misalignment of the lubber's line of the compass or the pelorus or gyro compass used for measuring deviation has been noted previously, it is probable that a mistake has been made in determining the azimuth or bearing used for establishing deviation.

Coefficient *E* is normally negligible for a compass located on the center line of the vessel. This vessel has an excessive amount, which should be corrected by slewing the quadrantal correctors, using an *E*-link.

Since deviation is east on heading 090° and west on 000°, it is probable that the blue pole of the vessel's permanent field is on the port bow.

The compass being unadjusted, no Flinders bar is in place, and the large *B* deviation on heading 090° is a combination of deviation from induced magnetism in vertical soft iron and that due to the permanent magnetism of the vessel. Since the deviation on heading 270° is nearly the same as that on 090°, but of opposite sign, adjustment on one of these headings should result in nearly correct adjustment on the other. Since some *B* and *C* deviation occurs on intercardinal headings, while no *D* deviation occurs on cardinal headings, adjustment for *B* and *C* should be made before that for final *D* adjustment.

A second analysis made after adjustment may reveal possibilities for further refinement in the adjustment.

If heeling error is measured on any heading other than compass north or south, the value of coefficient *J* can be found by means of the formula:

$$\mathrm{d}=J\cos \mathrm{CH}$$

converted to

$$J=\frac{\mathrm{d}}{\cos \mathrm{CH}}$$

or

$$J=\mathrm{d}\sec \mathrm{CH}.$$

If HE is the total observed change of deviation (heeling error), and i is the angle of heel in degrees (for relatively small angles), the formula becomes

$$J=\frac{\mathrm{HE}\sec \mathrm{CH}}{i}.$$

If heeling error is sought, the formula becomes

$$\mathrm{HE}=Ji\cos \mathrm{CH}.$$

Adjustment Procedure

728. Preliminary steps.—Efficient and accurate adjustment is preceded by certain preliminary steps best made while a vessel is tied up or at anchor.

The magnetic environment of the compass should be carefully inspected. Stray magnetic influences such as those caused by tools, direct current electric appliances, personal equipment (such as keys, pocketknives, or steel belt buckles), nylon clothing, etc., should be eliminated. Permanently installed equipment of magnetic material (such as cargo booms, boat davits, cranes, or guns) should be placed in the positions they normally occupy at sea. The degaussing coils should be secured by the reversing process (art. 743) if this has not already been done.

The compass itself should be checked carefully for bubbles, and to be sure it is centered on the vertical axis of the binnacle. If it is, and the vessel is on an even keel, there is no change of reading as the heeling magnet is raised and lowered in its tube. An adjustment should be made to the gimbal rings if the compass is off center. There should be no play in the position of the compass once it is centered.

The lubber's line, too, should be checked to be sure it is in line with the longitudinal axis of the vessel. This can be done by sighting on the jackstaff if the compass is on the center line. If it is not, a batten might be erected at a distance from the center line equal to the distance from the center of the compass to the center line. Another way is to determine the distance from the compass to the center line and from this point to the jackstaff. The first distance divided by the second is the natural tangent of the

angle at the compass between the line of sight to the jackstaff and the line of sight through the lubber's line. If the compass is in an exposed position where bearings can be taken, and the true heading is known, the observed relative bearing of a distant object can be compared with that obtained by careful measurement on the chart. If the vessel is at anchor or underway, the method explained in article 723 can be used.

If a pelorus or gyro compass or repeater is to be used in determining deviation of the compass, its lubber's line should be checked in the same manner, or by comparing a relative bearing of a distant object taken by two instruments, the lubber's line of one having previously been checked. If a gyro compass is to be used, it is checked to see that it is synchronized with a repeater. With accurate synchronization, any error in one will also be present in the other. The speed and latitude adjustments of the gyro compass should be checked carefully.

All devices to be used in the adjustment should be checked to see that they are on hand and in good condition. The trays for *B* and *C* permanent magnets, the quadrantal correctors, and heeling magnet should be checked for freedom of motion. The Flinders bar and quadrantal correctors should be checked for permanent magnetism. The correct amount of Flinders bar should be placed in its tube. The quadrantal correctors should be placed in their approximate positions, being centered if no better information is available. The heeling magnet is generally placed with the red end uppermost in north magnetic latitude, and the blue end uppermost in south magnetic latitude. If no better information is available, the heeling magnet should be placed near the bottom of the tube.

Plans for the actual adjustment should be made carefully. A suitable time and location should be selected. If landmarks are to be used, suitable ones should be selected to provide the information desired. Areas of heavy traffic should be avoided. If azimuths of the sun are to be used, a time should be selected when the sun will not be too high in the sky for suitable observation. A curve of magnetic azimuths (art. 731) should be made, and just before adjustment begins a comparing watch should be checked and set, if possible, to correct time. Local variation should be checked carefully, and corrected for date, if necessary. Any necessary recording and work forms should be made up. Each person to participate in the adjustment should be instructed regarding the general plan and his specific duties.

729. Underway procedure.—When everything is in order and the vessel has arrived at its adjusting area, final adjustment can begin. Trim should be normal, and the vessel free from list, so that no heeling error is present.

All adjustment headings should be *magnetic*. Compass headings can be used, but this results in a slight turn being required every time an adjustment is altered. Also, the coefficients are not completely separated unless the vessel is on magnetic headings.

Turns to each new heading should be made slowly, swinging slightly beyond the desired heading before steadying on it. If steering is by gyro, the gyro error should be checked on each heading if time and facilities permit. The vessel should remain on each heading for at least two minutes before the deviation is determined or an adjustment made, to permit the compass card to come to rest and the magnetic condition of the vessel to become settled. If observations are made before the vessel's magnetism becomes settled, the reading will be incorrect by an amount called the **Gaussin error.**

Adjustments should be carried out in the correct order, as follows:

1. Steady on magnetic heading 090° (or 270°) and adjust the fore-and-aft permanent magnets until the compass heading coincides with the magnetic heading, thus

removing all coefficient B on this heading. Use magnets in pairs, from the bottom up, with the trays at the lowest point of travel. When overcorrection occurs, remove the two highest magnets and raise the trays until all deviation has been removed. If two magnets overcorrect, use a single magnet. It is not necessary to determine in advance which direction the red ends should occupy, for a mistake will be immediately apparent by an *increase* in the deviation.

2. Steady on magnetic heading 180° (or 000°) and adjust the athwartship permanent magnets until the compass heading coincides with the magnetic heading, thus removing all coefficient C on this heading. Use the same technique as in step 1.

3. Steady on magnetic heading 270° (090° if 270° was used in step 1) and remove *half* the deviation with the fore-and-aft magnets.

4. Steady on magnetic heading 000° (180° if 000° was used in step 2) and remove *half* the deviation with the athwartship magnets.

5. Steady on any intercardinal magnetic heading and adjust the position of the quadrantal correctors until the compass heading coincides with the magnetic heading, thus removing all coefficient D on this heading. Leave the quadrantal correctors at equal distances from the compass.

6. Steady on either intercardinal magnetic heading 90° from that used in step 5 and remove *half* the deviation by adjusting the positions of the quadrantal correctors, leaving them at equal distances from the compass.

7. Secure all correctors in their final positions and record their number, size, positions, and orientation, as appropriate, on the bottom of the deviation table form (if a standard form such as that shown in fig. 710 is used).

8. **Swing ship** for residual deviation. That is, determine the remaining deviation on a number of headings at approximately equal intervals. Every 15° is preferable, but if the maximum deviation is small, every 45° (cardinal and intercardinal headings) may suffice.

9. If the vessel has degaussing, energize the degaussing coils and repeat the swing.

10. Make a deviation table (art. 710) for each condition (degaussing off and on), giving values for headings at 15° intervals if the maximum deviation is large (more than about 2°), or at 45° intervals if the maximum deviation is small. Record values to the nearest half degree.

If preferred, the adjustment may be started on a north or south heading, thus reversing steps 1 and 2 and also 3 and 4.

With patience and skill, the readings can be made at exact headings. However, if some of the headings are off slightly during the swing, this need not invalidate the results. The exact headings should be recorded, and the deviation determined for these values. The results can then be plotted on cross-section paper with the deviation being one coordinate and the heading the other. The deviation at each heading to be recorded can then be read from the curve. This is good practice even when readings are made at exact headings, for if any large errors have been made, the fact will be immediately apparent. Also, such a curve may be of assistance in making an analysis. If a reason cannot be found for any marked irregularity in the curve, readings might be made again at the headings involved.

The deviation of all compasses aboard the vessel can be determined from a single swing if the heading by each compass is recorded at the moment the magnetic direction is noted. If deviation of one compass is determined by means of a magnetic bearing or azimuth (arts. 733–735), the readings of this compass can then be used to establish the magnetic headings for determining the deviation of each other compass (art. 732).

Compass adjustment is best made when the sea is relatively smooth, so that steady headings can be steered, and heeling error is absent. The setting of the heeling magnet can be checked later, preferably at the next time that the vessel is on a north or south heading and rolling moderately.

An analysis of deviation can be made either before or after adjustment. If this reveals an excessive amount of *A* (constant) deviation, the source of the error should be found and corrected (art. 723), if mechanical or mathematical. If an appreciable amount of *E* deviation is present, *E*-links should be used and the spheres slewed. This is particularly to be anticipated for compasses which are not on the center line.

The procedure outlined above is for initial adjustment aboard a new or radically modified vessel. Deviation on the heading being used for navigation should be checked from time to time and any important differences from the values shown on the deviation table should be investigated. At sea, it is good practice to compare the magnetic and gyro compasses at intervals not exceeding half an hour. The error of one or both of these compasses should be checked twice a day when means are available. In pilot waters deviation checks should be made as convenient opportunities present themselves.

Whenever there is reason to question the accuracy of the deviation table, the ship should be swung at the first opportunity and a new table made up if there are significant changes in the old one. Suitable occasions for swinging ship would be after a deviation check indicates a significant error or after any event that might result in changes in the magnetic field of the vessel (art. 712). Intervals of swing should not exceed three months even when there is no reason to question the accuracy of the deviation table.

If a swing indicates the presence of large maximum deviation, the compass should be readjusted. Unless there is reason to change it, the Flinders bar length should remain the same. Other adjustments are altered as needed, none of the correctors being removed at the beginning of adjustment. Whenever the vessel crosses the magnetic equator, the opportunity should be used to check the deviation on magnetic headings east and west. Any adjustment needed should be made by means of the fore-and-aft (*B*) magnets. Upon crossing the magnetic equator, the heeling magnet should be inverted.

The Flinders bar and quadrantal correctors should be checked for permanent magnetism at intervals of about a year, or oftener if such magnetism is suspected.

Finding the Deviation

730. Placing a vessel on a desired magnetic heading.—As indicated in article 729, compass adjustment is best made with the vessel on *magnetic* headings. The compass being adjusted cannot be used for placing the vessel on a desired magnetic heading because its deviation is unknown, and is subject to change during the process of adjustment. A number of methods are available, including use of (1) another magnetic compass of known deviation, (2) a gyro compass, (3) bearing of a distant object, and (4) azimuth (art. 1428) of a celestial body.

Magnetic compass. The deviation at the desired magnetic heading is determined from the deviation table for that compass, and applied to the magnetic heading to determine the equivalent compass heading.

Example 1.—It is desired to place a vessel on magnetic heading east, using the standard compass. The deviation table for this compass is shown in figure 710. Degaussing is off.

Required.—Heading per standard compass (psc).

Solution.—From figure 710 the deviation on heading 090° magnetic with degaussing

off is found to be 2°.5E. Therefore, the equivalent compass heading is 090°−2°.5=087°.5.

Answer.—Hpsc 087°.5.

Gyro compass. The variation is applied to the desired magnetic heading, to determine the equivalent true heading. Any gyro error is then applied to determine the equivalent gyro heading. This is the method commonly used by vessels equipped with a reliable gyro compass.

Example 2.—It is desired to place a vessel on magnetic heading north, using the gyro compass. The variation in this area is 6° W, and the gyro error is 1° E.

Required.—Heading per gyro compass (pgc).

Solution.—The equivalent true heading is 000°−6°=354°. The gyro heading is 354°−1°=353°.

Answer.—Hpgc 353°.

Bearing of distant object. If a vessel remains within a small area during compass adjustment, the bearing of a distant object is essentially constant. The required distance of the object in miles is found by multiplying the cotangent of the maximum tolerable error by the *radius* in miles of the maneuvering circle. Thus, if the maximum error that can be tolerated is 0°.5 (cotangent 114.6), and the vessel can be maneuvered within 200 yards (0.1 mile) of a fixed position such as a buoy, the object selected should be at least 114.6×0.1=11.5 miles away. The 200-yard limit is within radial lines centered at the distant object and tangent to a circle having a radius of 200 yards and its center at the center of the maneuvering area. Thus, a vessel has considerable maneuvering space along the line of sight, but very limited room across this line. However, it is not necessary that the vessel *stay* within the required area, but only that it be there when readings are made. Thus, if the center of the area is marked by a buoy, the vessel might steady on each heading while still some distance away, and note the required readings as the buoy is passed. In this way, a small radius may be practical even for a large vessel.

The object selected should be conspicuous and should have a clearly defined feature of small visible width upon which to observe bearings. The object having been selected, its true bearing from the center of the maneuvering area should be measured on the chart. To this, the variation *at the center of the maneuvering area* should be applied to determine the equivalent magnetic bearing. The desired magnetic heading should be set at the lubber's line of the pelorus, and the far vane set at the magnetic bearing of the distant object. The vessel should then be maneuvered until the object is in line with the vanes.

Example 3.—It is desired to place a vessel on magnetic heading northeast in an area where the variation is 4° E. The true bearing of a distant object is 219°.

Required.—The setting of the pelorus.

Solution.—Set 045° at the lubber's line, and set the far vane at 219°−4°=215°.

If preferred, 000° can be set at the lubber's line, and the far vane at the relative bearing, 170° (magnetic bearing minus desired magnetic heading). If a gyro repeater or a magnetic compass is used instead of a pelorus, the true (or magnetic) bearing should be converted to the equivalent gyro (or compass) bearing.

If the distant object selected is not charted, or the position of the vessel is not known accurately, the approximate magnetic bearing of the object can be determined by measuring its *compass* bearing on each cardinal and intercardinal compass heading, and finding the mean of these readings. The value so determined will be incorrect by the amount of any constant deviation (coefficient *A*).

Example 4.—The compass bearings of a distant object are as shown below.

Required.—The magnetic bearing of the object, assuming no constant deviation (coefficient A).

Solution.—

CH °	*CB* °
000	324. 8
045	320. 7
090	312. 6
135	306. 8
180	304. 9
225	310. 8
270	316. 2
315	320. 0
sum	2516. 8
mean	314. 6

Answer.—MB 314°6.

Azimuth of celestial body. The true azimuth of the celestial body selected should be computed (arts. 2125–2127) for the time of observation. The magnetic variation should then be applied to determine the equivalent magnetic azimuth. The desired magnetic heading should then be set at the lubber's line of the pelorus, and the far vane set at the magnetic azimuth of the celestial body. The vessel should then be maneuvered until the body is in line with the vanes.

Example 5.—It is desired to place a vessel on magnetic heading west in an area where the variation is 17° W, and at a time when the computed true azimuth of the sun is 098°.

Required.—The setting of the pelorus.

Solution.—Set 270° at the lubber's line, and set the far vane at 098°+17°=115°.

If preferred, 000° can be set at the lubber's line, and the far vane at the relative azimuth (magnetic azimuth minus desired magnetic heading). If a gyro repeater or a magnetic compass is used instead of a pelorus, the true (or magnetic) azimuth should be converted to the equivalent gyro (or compass) azimuth.

731. Curve of magnetic azimuths.—During the course of compass adjustment and swinging ship, a magnetic direction is needed many times, either to place the vessel on desired magnetic headings or to determine the deviation of the compass being adjusted. If a celestial body is used to provide the magnetic reference, the azimuth is continually changing as the earth rotates on its axis. Frequent and numerous computations can be avoided by preparing, in advance, a table or **curve of magnetic azimuths.** True azimuths at frequent intervals are computed by any of the methods of computation discussed in chapters XX and XXI. The variation at the center of the maneuvering area is then applied to determine the equivalent magnetic azimuths. These are plotted on cross-section paper, with time as the other argument, using any convenient scale. A curve is then faired through the points.

Points at intervals of half an hour (with a minimum of three) are usually sufficient unless the body is near the celestial meridian and relatively high in the sky, when additional points are needed. If the body *crosses* the celestial meridian, the direction of curvature of the line reverses.

Unless extreme accuracy is required, the Greenwich hour angle and declination can be determined for the approximate mid time, the same value of declination used for all computations, and the Greenwich hour angle considered to increase 15° per hour.

An illustration of a curve of magnetic azimuths of the sun is shown in figure 731. This curve is for the period 0700–0900 zone time on May 31, 1958, at latitude 23°09!5 N,

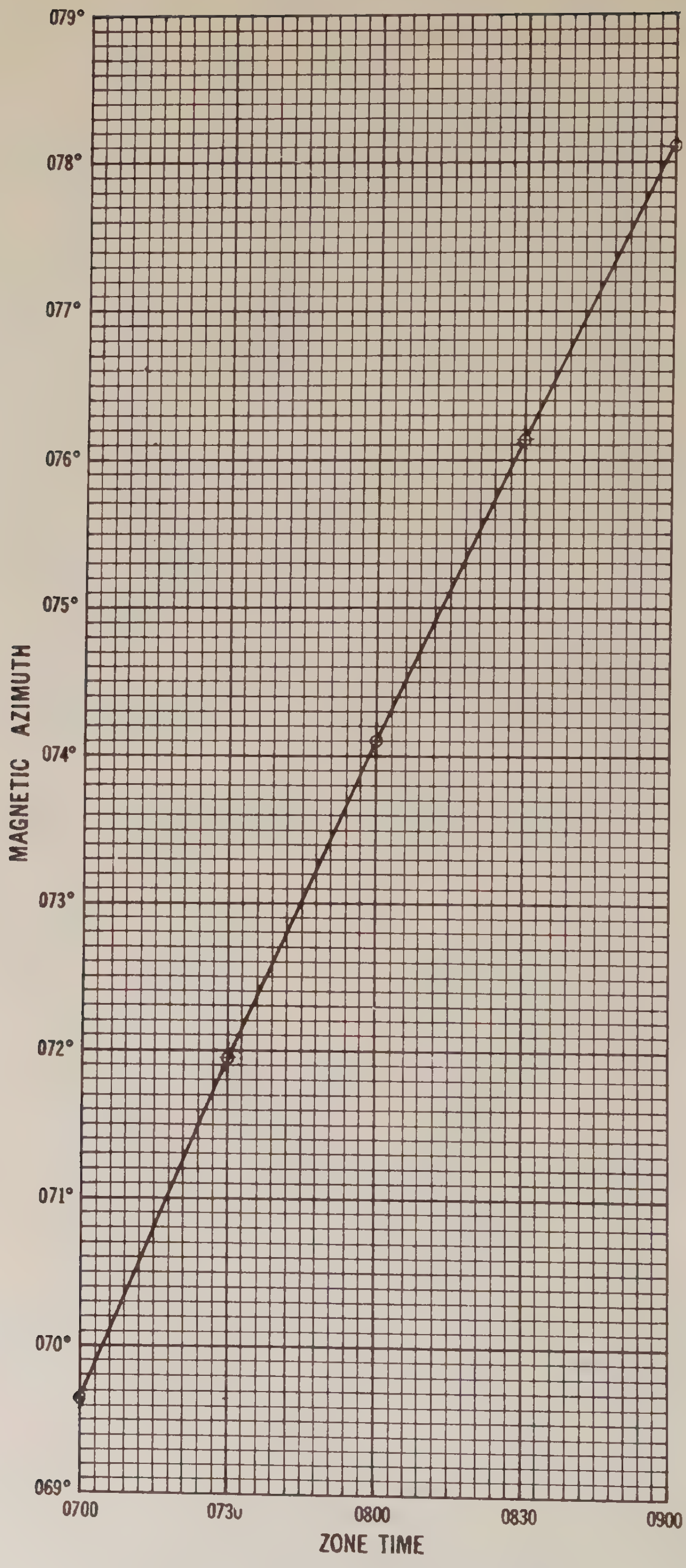


FIGURE 731.—Curve of magnetic azimuths.

longitude 82°24′.1 W, about a mile north of Battery No. 5, Havana, Cuba. The variation in this area is 2°47′ E. At the midtime, the meridian angle of the sun is 66°46′.9 E, and the declination is 21°53′.3 N. Azimuths were computed by H.O. Pub. No. 260 (art. 2126) at half-hour intervals, as follows:

Zone time	*Meridian angle* ° ′ (h m)	*Declination* °	*Latitude* °	*Magnetic azimuth* ° ′
0700	81 46.9 E (5 27.1 E)	21.9 N	23.2 N	069 39
0730	74 16.9 E (4 57.1 E)	21.9 N	23.2 N	071 57
0800	66 46.9 E (4 27.1 E)	21.9 N	23.2 N	074 06
0830	59 16.9 E (3 57.1 E)	21.9 N	23.2 N	076 08
0900	51 46.9 E (3 27.1 E)	21.9 N	23.2 N	078 07

This curve was constructed on the assumption that the vessel would remain in approximately the same location during the period of adjustment and swing. If the position changes materially, this should be considered in the computation.

732. Deviation by magnetic headings.—If the vessel is placed on a magnetic heading by any of the methods of article 730, compass deviation on that heading is the difference between the magnetic heading and the compass heading. If the compass heading is less than the magnetic heading, deviation is easterly, if the compass heading is greater than the magnetic heading, deviation is westerly.

Example.—A vessel is being maneuvered to determine the deviation of the magnetic steering compass on cardinal and intercardinal headings. The gyro compass, which has an error of 0°.5 W, is used for placing the vessel on each of the magnetic headings. Variation in the area is 27°.5 E.

Required.—Deviation on each magnetic heading, using the compass headings given below:

Solution.—

MH °	*V* °	*TH* °	*GE* °	*Hpgc* °	*CH* °	*Dev.* °
000	27.5 E	027.5	0.5 W	028	000.3	0.3 W
045	27.5 E	072.5	0.5 W	073	046.1	1.1 W
090	27.5 E	117.5	0.5 W	118	093.6	3.6 W
135	27.5 E	162.5	0.5 W	163	136.7	1.7 W
180	27.5 E	207.5	0.5 W	208	179.6	0.4 E
225	27.5 E	252.5	0.5 W	253	223.8	1.2 E
270	27.5 E	297.5	0.5 W	298	266.5	3.5 E
315	27.5 E	342.5	0.5 W	343	313.2	1.8 E

733. Deviation by magnetic bearing or azimuth.—Deviation can be found by comparing a magnetic bearing or azimuth with one measured by compass. The magnetic direction can be obtained as explained in articles 730–731. If the compass direction is less than the magnetic direction, deviation is easterly; if the compass direction is greater than the magnetic direction, deviation is westerly. This method is used for determining deviation on a given *compass* heading. The equivalent magnetic heading can be determined by applying the deviation thus determined. If this method is used for swinging ship, the values can be plotted as explained in article 729. For a well-adjusted compass, the deviation may be so small that the compass headings can be considered magnetic headings, without introducing significant errors.

Example.—The standard compass of a vessel has been adjusted, and the vessel is to be swung for residual deviation during the period and for the place for which the curve of magnetic azimuths of figure 731 has been constructed.

Required.—Find the deviation on each heading given below, at the times indicated.
Solution.—

CH °	*Time* h m s	*CZn* °	*MZn* °	*Deviation* °
000	7 35 20	073.2	072.4	0.8 W
045	7 41 12	074.0	072.8	1.2 W
090	7 50 15	074.2	073.4	0.8 W
135	7 57 36	074.0	073.9	0.1 W
180	8 04 44	073.7	074.4	0.7 E
225	8 10 10	073.5	074.8	1.3 E
270	8 16 33	074.3	075.2	0.9 E
315	8 24 51	075.8	075.7	0.1 W

The magnetic azimuth (MZn) is determined from figure 731, and the deviation from compass azimuth (CZn) and magnetic azimuth.

734. Deviation by a range is a special case of deviation by magnetic bearing. Two objects appearing in line, one behind the other, constitute a **range.** Range markers are established in many places to mark important channels, the extremities of measured miles, etc. In addition, numerous good ranges occur naturally, as when a lighthouse is in line with a tank, or a tower with a chimney. The true direction of such a range can be determined by measurement on the chart, and variation applied to determine the equivalent magnetic direction. In the case of a natural range, the objects should preferably be at least an inch apart as they appear on the chart, to minimize any plotting errors.

A range is superior to the bearing of a single object because it provides a critical indication of when the vessel is in the correct position to take a reading. The vessel crosses the range on various compass headings. At each crossing, the compass bearing of the range is observed, and also the compass heading. It is well to use two ranges nearly 90° apart, if available, because of the difficulty of crossing at small angles.

Example.—A vessel maneuvering to adjust its compass in the Lower Bay of New York Harbor finds the true direction of the range between West Bank Light and Coney Island Light to be 032°. The variation in this area is 11°2 W. The vessel steams across the range on various compass headings, noting the compass direction of the range at the times of crossing, as shown below.

Required.—The deviation on each compass heading indicated.

Solution.—The magnetic bearing of the range is 032°+11°2=043°2.

CH °	*MB Range* °	*CB Range* °	*Deviation* °
000	043.2	032.9	10.3 E
045	043.2	023.7	19.5 E
090	043.2	031.9	11.3 E
135	043.2	044.2	1.0 W
180	043.2	048.5	5.3 W
225	043.2	051.0	7.8 W
270	043.2	055.6	12.4 W
315	043.2	049.8	6.6 W

The analysis of these results (art. 727) indicates a constant error of 1°0 E. The mean compass bearing is 042°2, differing from the correct magnetic bearing by the amount of the constant error.

Ranges are widely used to check the deviation on the heading in use as a vessel

proceeds through pilot waters. In this manner several checks can be made without advance preparation as a vessel enters or leaves port.

735. Deviation by reciprocal bearings.—Another method of using magnetic bearings is by means of a compass on the beach. This method is particularly useful when no suitable distant object or range is available, or where it may not be practical to remain close to a given bearing line.

A reliable compass is taken ashore to a location which is free from magnetic disturbance. If the location is not marked by a conspicuous object, such as a beacon, flagpole, prominent tree, etc., a temporary marker should be erected. A staff with a flag or bunting should be adequate. The marker should be of sufficient size and nature to be conspicuous at the vessel. At suitable visual or radio signals from the vessel, bearings are observed simultaneously aboard the vessel and ashore. The bearings of the vessel observed by the shore compass are magnetic. The reciprocals of these can be considered magnetic bearings of the shore station from the vessel. The bearings measured aboard the vessel are compass bearings. The difference is deviation. To avoid confusion in the sequence of bearings, the time of each bearing is recorded. Timepieces should be synchronized before the start of observations.

Example.—Simultaneous bearings are observed by a shore compass and the standard compass aboard a vessel, as shown below.

Required.—The deviation of the standard compass on each heading.

Solution.—

CH	*Time*	*MB of vessel*	*MB of shore position*	*CB of shore position*	*Deviation*
°		°	°	°	°
000	1112	307	127	137	10 W
045	1120	309	129	131	2 W
090	1126	312	132	130	2 E
135	1018	296	116	113	3 E
180	1029	295	115	109	6 E
225	1039	288	108	096	12 E
270	1052	288	108	113	5 W
315	1104	289	109	115	6 W
		mean	118	118	

The analysis of these results indicates no constant deviation. This is further indicated by the fact that the means of the bearings aboard and ashore are equal.

Adjustment by Deflector

736. Principles involved.—As indicated in article 713, the magnetic field of a vessel causes deviation of a magnetic compass, and also alters its directive force, strengthening it on some headings and weakening it on others. The purpose of compass adjustment is to neutralize the effect of the vessel's magnetic field on the compass. If this is done completely, all deviation is removed, and the directive force is the same on all headings. The usual procedure, described earlier in this chapter, is to adjust by reducing or eliminating the deviation. By the deflector method, the various correctors are adjusted until the directive force is the same on all cardinal headings. Deviation is then a minimum.

The *relative* directive force on various headings is determined by means of an instrument called a **deflector.** Actual measurement is of the setting of the instrument when the compass card has been rotated or "deflected" through 90° under certain standard conditions. The units are arbitrary "deflector units" which are used only for comparison with readings on other headings.

The deflector method provides a quick adjustment with only four headings being needed, without need for bearings, azimuths, or comparison with other compasses. It is easy to use. However, it is not as thorough as the method described in article 729, and should not be used when the usual method is available. The deflector method makes no provision for determination of coefficient A (art. 716), the amount of Flinders bar needed, the setting of the heeling magnet, or the residual deviation. Coefficient E can be determined, but is usually ignored. The method has never been popular in the United States. It offers little or no advantage for a vessel equipped with a reliable gyro compass.

737. Adjustment by deflector.—The preliminary steps of adjustment are the same as indicated in article 728, omitting those relating to peloruses and other compasses. Preparations having been completed, the adjustment should be carried out as follows:

1. Steady on heading 000° (or 180°) *by the compass being adjusted.* Note the heading by another compass and keep the vessel on this heading, steering by means of the second compass. Put the deflector in place over the first compass, and deflect the compass card 90°. Record the reading on the deflector scale, and remove the deflector.

2. Steady on heading 090° (or 270°) by the compass being adjusted, and follow the procedure of step 1.

3. Steady on heading 180° (000° if 180° was used in step 1) by the compass being adjusted, and determine the deflector reading by the procedure of step 1. Leave the deflector in place and set it to the mean of the readings on headings 000° and 180°. Adjust the fore-and-aft permanent magnets until the deflection is 90°. This corrects for coefficient B, and the deflector readings on compass headings 000° and 180° should now be the same. Remove the deflector.

4. Steady on heading 270° (090° if 270° was used in step 2) by the compass being adjusted, and determine the deflector reading by the procedure of step 1. Leave the deflector in place and set it to the mean of the readings on headings 090° and 270°. Adjust the athwartship permanent magnets until the deflection is 90°. This corrects for coefficient C, and the deflector readings on compass headings 090° and 180° should now be the same.

5. Without changing the heading, set the deflector to the mean of the N–S and E–W means. Adjust the quadrantal correctors until the deflection is 90°. This corrects for coefficient D, and the deflector readings on all cardinal headings should be the same. Remove the deflector.

Adjustment is now complete. It can be checked by repeating the five steps, a procedure which is particularly recommended if the difference between deflector readings on opposite headings is more than ten units. If means are available, and time permits, the vessel should be swung for residual deviation. If preferred, a heading of east or west can be used, reversing steps 1 and 2 and also steps 3 and 4.

This method is particularly useful when a quick adjustment is needed following some change that affects the magnetic environment of the compass.

738. The Kelvin deflector was developed in Great Britain by Sir William Thomson (Lord Kelvin). It consists essentially of two permanent magnets hinged like a pair of dividers, with opposite poles at the hinge. The magnets are mounted vertically over the center of the compass, with the hinged end on top. The separation of the lower ends can be varied by means of a screw. The amount of separation, indicated by a scale and vernier drums, is the reading used in the adjustment.

The deflecting force increases as the separation becomes greater. When the deflector is in place over the compass, the blue pole is in line with the north (red) end

of the compass magnets, as indicated by a pointer. As the deflecting magnets are rotated around the vertical axis of the instrument, the compass card rotates in the same direction, but at a slower rate. The separation is adjusted until the rotation of the instrument is 170° when the deflection of the compass card is 90°. These are the standard conditions under which readings are made.

The Kelvin type deflector, which provides adjustment to an accuracy of 2° to 3°, is used on many British merchant vessels. Deflectors are seldom used on British Navy vessels.

739. The De Colong deflector was developed in Russia, and is standard equipment on naval vessels of the USSR. It provides an accuracy of 0°.5 to 1°.0. Essentially, this instrument consists of two horizontal magnets which are perpendicular to each other. The small magnet is held in a fixed position close to the compass card. The large magnet is mounted in a small tray which can be moved up and down along a vertical spindle mounted over the center of the compass. The red end of this magnet is placed toward the north. When it is positioned so that the directive force is exactly neutralized, the small magnet causes the compass card to be deflected 90°. The height of the large magnet is the deflector reading, the scale being on the vertical spindle, and the index on the movable tray.

Provision is made for mounting the large magnet vertically, to measure the vertical force of the magnetic field at the compass. A separate scale is provided for this purpose. Additional magnets are generally provided for use near the magnetic equator, where the vertical intensity is very small.

In practice, a separate deflector is provided for each compass, and they are not interchangeable. By the addition of an auxiliary scale, the instrument could be made usable for any compass.

Degaussing Compensation

740. Degaussing.—As indicated in article 712, a steel vessel has a certain amount of permanent magnetism in its "hard" iron, and induced magnetism in its "soft" iron. Whenever two or more magnetic fields occupy the same space, the total field is the vector sum (art. O18) of the individual fields. Thus, within the effective region of the field of a vessel, the total field is the combined total of the earth's field and that due to the vessel. Consequently, the field due to earth's magnetism alone is altered or distorted due to the field of the vessel. This is indicated by a tendency of the lines of force to crowd into the metal of the vessel (art. 703), as shown in figure 741a.

Certain mines and other explosive devices are designed to be triggered by the magnetic influence of a vessel passing near them. It is therefore desirable to reduce to a practical minimum the magnetic field of a vessel. One method of doing this is to neutralize each component by means of an electromagnetic field produced by direct current of electricity in electric cables installed so as to form coils around the vessel. A unit sometimes used for measuring the strength of a magnetic field is the **gauss**. The reduction of the strength of a magnetic field decreases the number of gauss in that field. Hence, the process is one of **degaussing** the vessel.

When a vessel's degaussing coils are energized, the magnetic field of the vessel is completely altered. This introduces large deviation in the magnetic compasses. This is removed, as nearly as practicable, by introducing at each compass an equal and opposite force of the same type—one caused by direct current in a coil—for each component of the field due to the degaussing currents. This is called **compass compensation.** When there is a possibility of confusion with compass adjustment to neutralize the effects of the natural magnetism of the vessel, the expression **degaussing compensation** is used. Since the neutralization may not be perfect, a small amount

of deviation due to degaussing may remain on certain headings. This is the reason for swinging ship twice—once with degaussing off and once with it on—and having two separate columns in the deviation table (fig. 710).

741. A vessel's magnetic signature.—A simplified diagram of the distortion of the earth's magnetic field in the vicinity of a steel vessel is shown in figure 741a. The strength of the field is indicated by the spacing of the lines, being stronger as the lines are closer together. If a vessel passes over a device for detecting and recording the strength of the magnetic field, a certain pattern is traced, as shown in figure 741b. Since the magnetic field of each vessel is different, each has a distinctive trace, known as its **magnetic signature.** The simplified signature shown in figure 741b is one that might result from an uncomplicated field such as that shown in figure 741a.

Several degaussing stations have been established to determine magnetic signatures and recommend the currents needed in the various degaussing coils. Since a vessel's induced magnetism varies with heading and magnetic latitude, the current settings of the coils which neutralize induced magnetism need to be changed to suit the conditions. A "degaussing folder" is provided each vessel to indicate the changes, and to give other pertinent information.

A vessel's permanent magnetism changes somewhat with time and the magnetic history of the vessel. Therefore, the information given in the degaussing folder should be checked from time to time by a return to the magnetic station.

742. Degaussing coils.—For degaussing purposes, the total field of the vessel is divided into three components: (1) vertical, (2) horizontal fore-and-aft, and (3) horizontal athwartships. The positive directions are considered downward, forward, and to port, respectively. These are the normal directions for a vessel headed north or east in north latitude. Each component is opposed by a separate degaussing field just strong enough to neutralize it. Ideally, when this has been done, the earth's field passes through the vessel smoothly and without distortion. The opposing degaussing fields are produced by direct current flowing in coils of wire. Each of the degaussing coils is placed so that the field it produces is directed to oppose one component of the ship's field.

The number of coils installed depends upon the magnetic characteristics of the vessel, and the degree of safety desired. The ship's permanent and induced magnetism may be neutralized separately so that control of induced magnetism can be varied as heading and latitude change, without disturbing the fields opposing the vessel's permanent field. The principal coils employed are the following:

Main (M) coil. The *M*-coil is placed horizontal, and completely encircles the vessel, usually at or near the water line. Its function is to oppose the vertical component of the vessel's permanent and induced fields combined. Generally the induced field predominates. Current in the *M*-coil is varied or reversed according to the change of the *induced* component of the vertical field with latitude.

Forecastle (F) and quarterdeck (Q) coils. The *F*- and *Q*-coils are placed horizontal just below the forward and after thirds (or quarters), respectively, of the weather deck. The designation "*Q*" for quarterdeck is reminiscent of the days before World War II when the "quarterdeck" of naval vessels was aft along the ship's quarter. These coils, in which current can be individually adjusted, remove much of the fore-and-aft component of the ship's permanent and induced fields. More commonly, the combined *F*- and *Q*-coils consist of two parts; one part the *FP*- and *QP*-coils, to take care of the permanent fore-and-aft field, and the other part, the *FI*- and *QI*-coils, to neutralize the induced fore-and-aft field. Generally, the forward and after coils of each type are connected in series, forming a split-coil installation and designated *FP-PQ* coils and *FI-QI* coils. Current in the *FP-QP* coils is generally constant, but in the *FI-QI* coils

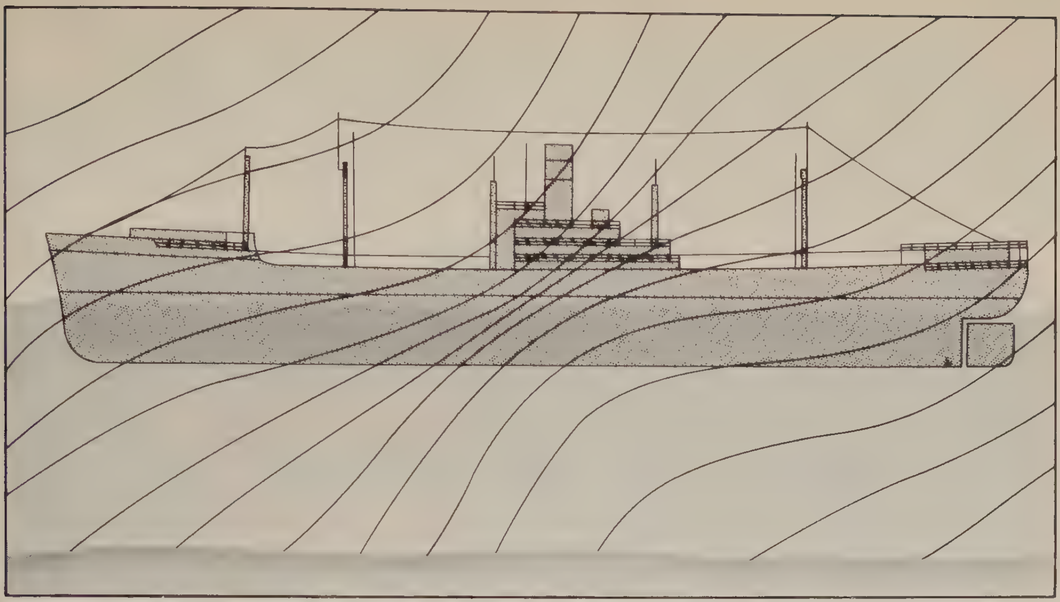


Figure 741a.—Simplified diagram of distortion of earth's magnetic field in the vicinity of a steel vessel.

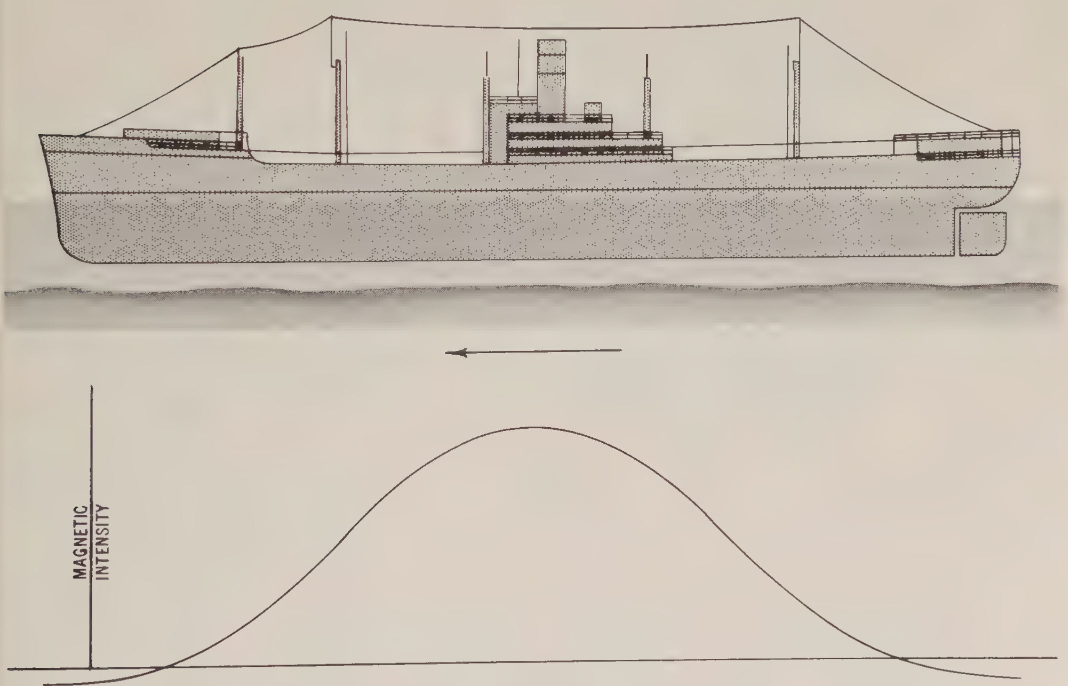


Figure 741b.—Simplified signature of vessel of figure 741a.

is varied according to the heading and magnetic latitude of the vessel. In split-coil installations, the coil designations are often contracted to P-coil and I-coil.

Longitudinal (L) *coil.* Better control of the fore-and-aft components, but at greater installation expense, is provided by placing a series of vertical, athwartships coils along the length of the ship. It is the *field*, not the coils, which is longitudinal. Current in an L-coil is varied as with the FI-QI coils. It is maximum on north and south headings, and zero on east and west headings.

Athwartship (A) *coil.* The A-coil is in a vertical fore-and-aft plane, thus producing a horizontal athwartship field which neutralizes the athwartship component of the vessel's field. In most vessels, this component of the permanent field is small and can be ignored. Since the A-coil neutralizes the induced field, primarily, the current is changed with magnetic latitude and with heading, being maximum on east or west headings, and zero on north or south headings.

The strength and direction of the current in each coil is indicated and adjusted at a control panel which is normally accessible to the navigator. Current may be controlled directly by rheostats at the control panel or remotely by push buttons which operate rheostats in the engine room.

Since degaussing fields oppose the vessel's fields, the positive directions of the degaussing fields are upward, aft, and to starboard. For positive fields in M, F, FI, FP, Q, QI, and QP coils, current flows forward on the starboard side of the vessel; and the north end of a small compass placed *above* any of these coils is deflected outboard. For a positive field in the L-coil, current flows upward on the starboard side, and the north end of a compass is deflected aft when placed *below* an upper, athwartship portion of the coil. For a positive field in the A-coil, current in the upper, fore-and-aft portion flows aft, and the north end of a compass is deflected to starboard when placed *below* this portion of the coil. The FI-QI coils are generally connected so that the field in the FI-coil is negative when that in the QI-coil is positive.

Appropriate values of the current in each coil are determined at a degaussing station, the various currents being adjusted until the vessel's signature is made as flat as possible. Recommended current values and directions for all headings and magnetic latitudes are set forth in the vessel's degaussing folder. This document is normally retained by the navigator, whose responsibility it is to see that the recommended settings are maintained whenever the degaussing system is energized.

743. Securing the degaussing system.—Unless the degaussing system is properly secured, residual magnetism may remain in the metal of the vessel. During degaussing compensation and at other times, as recommended in the degaussing folder, the "reversal" method is used. The steps in the reversal process are as follows:

1. Start with maximum degaussing current used since the system was last energized.
2. Decrease current to zero and increase it in the opposite direction to the same value as in step 1.
3. Decrease the current to zero and increase it to three-fourths maximum value in the original direction.
4. Decrease the current to zero and increase it to one-half maximum value in the opposite direction.
5. Decrease the current to zero and increase it to one-fourth maximum value in the original direction.
6. Decrease the current to zero and increase it to one-eighth maximum value in the opposite direction.
7. Decrease the current to zero and open switch.

744. Magnetic treatment of vessels.—In some instances, the degaussing can be made more effective by changing the magnetic characteristics of the vessel by a process known as **deperming.** Heavy cables are wound around the vessel in an athwartship direction, forming vertical loops around the longitudinal axis of the vessel. The loops are run beneath the keel, up the sides, and over the top of the weather deck at closely-spaced equal intervals along the entire length of the vessel. Predetermined values of direct current are then passed through the coils. When the desired magnetic characteristics have been acquired, the cables are removed.

A vessel which does not have degaussing coils, or which has a degaussing system which is inoperative, can be given some temporary protection by a process known as **flashing.** A horizontal coil is placed around the outside of the vessel and energized with large predetermined values of direct current. When the vessel has acquired a vertical field of permanent magnetism of the correct magnitude and polarity to reduce to a minimum the resultant field below the vessel for the particular magnetic latitude involved, the cable is removed. This type protection is not as satisfactory as that provided by degaussing coils because it is not adjustable for various headings and magnetic latitudes, and also because the vessel's magnetism slowly readjusts itself following treatment.

During magnetic treatment it is a wise precaution to remove all magnetic compasses and Flinders bars from the vessel. Permanent adjusting magnets and quadrantal correctors are not materially affected, and need not be removed. If for any reason it is impractical to remove a compass, the cables used for magnetic treatment should be kept as far as practical from it.

745. Degaussing compensation.—The magnetic fields created by the degaussing coils would render the vessel's magnetic compasses useless unless compensated. This is accomplished by subjecting the compass to compensating fields along three mutually perpendicular axes. These fields are provided by small compensating coils adjacent to the compass. In nearly all installations, one of these coils, the heeling coil, is horizontal and on the same plane as the compass card. Current in the heeling coil is adjusted until the vertical component of the total degaussing field is neutralized. The other compensating coils provide horizontal fields perpendicular to each other. Current is varied in these coils until their resultant field is equal and opposite to the horizontal component of the degaussing field. In early installations, these horizontal fields were directed fore-and-aft and athwartships by placing the coils around the Flinders bar and the quadrantal spheres. Compactness and other advantages are gained by placing the coils on perpendicular axes extending 045°–225° and 315°–135° relative to the heading. A frequently used compensating installation, called the type "K," is shown in figure 745. It consists of a heeling coil extending completely around the top of the binnacle, four "intercardinal" coils, and three control boxes. The intercardinal coils are named for their positions relative to the compass when the vessel is on a heading of north, and also for the compass headings on which the current in the coils is adjusted to the correct amount for compensation. The NE–SW coils operate together as one set, and the NW–SE coils operate as another. One control box is provided for each set, and one for the heeling coil.

The compass compensating coils are connected to the power supply of the degaussing coils, and the currents passing through the compensating coils are adjusted by series resistances so that the compensating field is equal to the degaussing field. Thus, a change in the degaussing currents is accompanied by a proportional change in the compensating currents. Each coil has a separate winding for each degaussing circuit it compensates.

Degaussing compensation is carried out while the vessel is moored at the shipyard where the degaussing coils are installed. This is usually done by personnel of the yard, using the following procedure:

1. The compass is removed from its binnacle and a dip needle is installed in its place. The *M*-coil and heeling coil are then energized, and the current in the heeling coil is adjusted until the dip needle indicates the correct value for the magnetic latitude of the vessel. The system is then secured by the reversing process.

2. The compass is restored to its usual position in the binnacle. By means of auxiliary magnets, the compass card is deflected until the compass magnets are parallel to one of the compensating coils or set of coils used to produce a horizontal field. The compass magnets are then perpendicular to the field produced by that coil. One of the degaussing circuits producing a horizontal field, and its compensating winding, are then energized, and the current in the compensating winding is adjusted until the compass reading returns to the value it had before the degaussing circuit was energized. The system is then secured by the reversing process. The process is repeated with each additional circuit used to create a horizontal field. The auxiliary magnets are then removed.

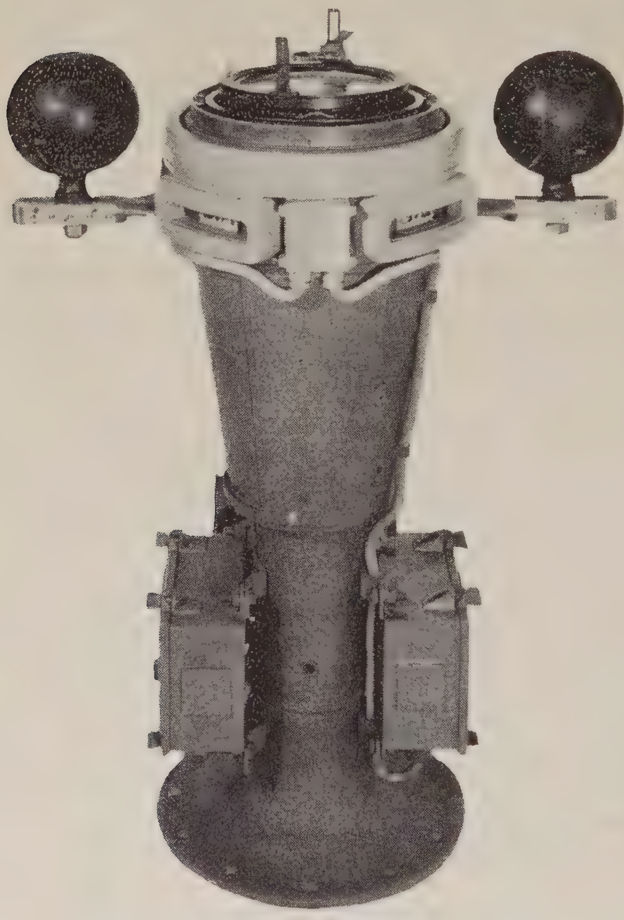


FIGURE 745.—Type "K" degaussing compensation installation.

3. The auxiliary magnets are placed so that the compass magnets are parallel to the other compensating coils or set of coils used to produce a horizontal field. The procedure of step 2 is then repeated for each circuit producing a horizontal field.

When the vessel gets under way, it proceeds to a suitable maneuvering area. The vessel is then headed so that the compass magnets are parallel first to one compensating coil or set of coils and then the other, and any needed adjustment is made in the compensating circuits to reduce the error to a minimum. The vessel is then swung for residual deviation, first with degaussing off and then with degaussing on, and the correct current settings for each heading at the magnetic latitude of the vessel. From the values thus obtained, the "DG OFF" and "DG ON" columns of the deviation table (fig. 710) are filled in. If the results indicate satisfactory compensation, a record is made of the degaussing coil settings and the resistances, voltages, and currents in the compensating coil circuits. The control boxes are then secured.

Under normal operating conditions, the settings need not be changed unless changes are made in the degaussing system, or unless an alteration is made in the amount of Flinders bar or the setting of the quadrantal correctors. However, it is possible for a ground to occur in the coils or control box if the circuits are not adequately protected from sea water or other moisture. If this occurs, it should be reflected by a change in deviation with degaussing on, or by a decreased installation resistance. Under these conditions, compensation should be carried out again. If the compass is to be needed with degaussing on before the ship can be returned to a shipyard where the compensation can be made by experienced personnel, the compensation should be made at sea on the actual headings needed, rather than by deflection of the compass needles by magnets. More complete information related to this process is given in H.O. Pub. No. 226 and the degaussing folder.

If a vessel has been given magnetic treatment, its magnetic properties have been changed. This necessitates readjustment of each magnetic compass. This is best delayed for several days to permit stabilization of the magnetic characteristics of the vessel. If this cannot be delayed, the vessel should be swung again for residual deviation after a few days. Degaussing compensation should not be made until after compass adjustment has been completed.

Problems

711a. Fill in the blanks in the following:

	TC °	*V* °	*MC* °	*D* °	*CC* °	*CE* °
(1)	105	15 E	—	5 W	—	—
(2)	—	—	—	4 E	215	14 E
(3)	—	12 W	—	—	067	7 W
(4)	156	—	166	—	160	—
(5)	222	—	216	3 W	—	—
(6)	009	—	357	—	—	10 E
(7)	—	2 W	—	6 E	015	—
(8)	—	—	210	—	214	1 W

Answers.—(1) MC 090°, CC 095°, CE 10°E; (2) TC 229°, V 10°E, MC 219°; (3) TC 060°, MC 072°, D 5°E; (4) V 10°W, D 6°E, CE 4°W; (5) V 6°E, CC 219°, CE 3°E; (6) V 12°E, D 2°W, CC 359°; (7) TC 019°, MC 021°, CE 4°E; (8) TC 213°, V 3°E, D 4°W.

711b. A vessel is on course 150° by compass in an area where the variation is 19°E. The deviation is as shown in figure 710. Degaussing is on.

Required.—(1) Deviation.

(2) Compass error.

(3) Magnetic heading.

(4) True heading.

Answers.—(1) D 1°E, (2) CE 20°E, (3) MH 151°, (4) TH 170°.

711c. A vessel is on course 055° by gyro and 041° by magnetic compass. The gyro error is 1°W. The variation is 15°E.

Required.—The deviation on this heading.

Answer.—D 2°W.

711d. A vessel is on course 177° by gyro. The gyro error is 0.°5 E. A beacon bears 088° by magnetic compass in an area where variation is 11°W. The deviation is as shown in figure 710, degaussing off.

Required.—The true bearing of the beacon.

Answer.—TB 076°.

721a. A magnetic compass is adjusted on the magnetic equator, without any Flinders bar being used. The residual deviation on heading 090° magnetic is 1° E. Some days later, at latitude 37° N, dip 70°, the deviation on heading 090° is 12° W.

Required.—The length and location of Flinders bar required to restore a residual deviation of 1° E (using fig. 721, A) if the magnetic properties of the vessel are unchanged.

Answer.—Fifteen inches of Flinders bar forward of the compass.

721b. The deviation of a magnetic compass of a vessel on heading 270° magnetic is 2° E at Sydney, Australia (south magnetic latitude) and 12° W at Seattle, Wash. (north magnetic latitude). At Sydney, H=0.258 and Z=0.51. At Seattle, H=0.188 and Z=0.53. The shielding factor is 0.9.

Required.—The length of Flinders bar to use if (1) no Flinders bar is in place during observations, (2) 12 inches of Flinders bar is in place forward of the compass during observations.

Answers.—(1) 8¼ inches (8.5 inches by computation) of Flinders bar aft of the compass, (2) nine inches (8.8 inches by computation) of Flinders bar forward of the compass.

727. A magnetic compass which has not been adjusted has deviation on cardinal and intercardinal compass headings as follows:

Compass heading °	*Deviation* °	*Compass heading* °	*Deviation* °
000	2.0 E	180	6.0 E
045	20.5 E	225	5.5 W
090	18.5 E	270	22.0 W
135	8.0 E	315	23.5 W

On heading compass north the deviation is 6°0 W when the vessel heels 7° to starboard.

Required.—(1) The approximate value of each coefficient.

(2) The total deviation to be expected on compass heading 300°, with the vessel on an even keel.

(3) Heeling error on compass heading 060°, with a heel of 10°.

Answers.—(1) *A* (+)0°5, *B* (+)20°2, *C* (−)2°0, *D* (+)7°6, *E* (+)2°9, *J* (−)1°1; (2) d 26°0 W; (3) HE 5°5.

730a. It is desired to place a vessel on magnetic heading west, using the magnetic steering compass. The deviation table for this compass is shown in figure 710. Degaussing is on.

Required.—Heading per steering compass (p stg c).

Answer.—$H_{p\ stg\ c}$ 272°.

730b. It is desired to place a vessel on magnetic heading south, using the gyro compass. The variation in this area is 12° E, and the gyro error is 0°5 E.

Required.—Heading per gyro compass.

Answer.—H_{pgc} 191°5.

730c. It is desired to place a vessel on magnetic heading southeast in an area where the variation is 6° W. The true bearing for a distant object is 047°.

Required.—(1) The magnetic bearing of the object.

(2) The relative bearing of the object when the vessel is on the desired magnetic heading.

Answers.—(1) MB 053°, (2) RB 278°.

730d. The compass bearings of a distant object are as follows:

CH °	*CB* °	*CH* °	*CB* °
000	358	180	002
045	357	225	006
090	351	270	012
135	353	315	009

Required.—The magnetic bearing of the object, assuming no constant deviation (coefficient *A*).

Answer.—MB 001°.

730e. It is desired to place a vessel on magnetic heading east in an area where the variation is 13° E, and at a time when the computed true azimuth of the sun is 218°.

Required.—(1) The magnetic azimuth of the sun.

(2) The relative azimuth when the vessel is on the desired magnetic heading.

(3) The azimuth by a magnetic compass having deviation as shown in figure 710 (DG on).

(4) The azimuth by a gyro compass having a gyro error of 1° W.

Answers.—(1) MZn 205°, (2) RZn 115°, (3) CZn 202°, (4) Zn_{pgc} 219°.

732. A vessel is being maneuvered to determine the residual deviation of a magnetic compass. The gyro compass, which has an error of 1° E, is used for placing the vessel on the magnetic headings indicated below. Variation in the area is 7.°8 W. The following readings are obtained:

MH °	*CH* °	*MH* °	*CH* °
000	000.0	180	180.1
045	044.1	225	225.8
090	088.5	270	271.4
135	134.2	315	315.9

Required.—Gyro heading and deviation on each magnetic heading.

Answers.—

MH °	*Hpgc* °	*Dev.* °	*MH* °	*Hpgc* °	*Dev.* °
000	351.2	0.0	180	171.2	0.1 W
045	036.2	0.9 E	225	216.2	0.8 W
090	081.2	1.5 E	270	261.2	1.4 W
135	126.2	0.8 E	315	306.2	0.9 W

733. A vessel is being swung for residual deviation during the period and at the place for which the curve of magnetic azimuths of figure 731 has been constructed. The following readings are obtained:

CH °	*Time* h m s	*CZn* °	*CH* °	*Time* h m s	*CZn* °
000	7 56 13	73.7	180	8 16 36	75.2
045	8 01 22	72.9	225	8 22 19	76.8
090	8 04 55	71.9	270	8 27 12	78.7
135	8 11 01	74.0	315	8 33 27	77.2

Required.—Deviation on each compass heading.

Answers.—

CH °	*Deviation* °	*CH* °	*Deviation* °
000	0.1 E	180	0.0
045	1.3 E	225	1.2 W
090	2.6 E	270	2.8 W
135	0.9 E	315	0.8 W

734. A vessel being swung for residual deviation crosses a range on various compass headings as indicated below, the compass bearing of the range being observed at each crossing. The true direction of the range is 255°. The variation in the vicinity is 24°5 E.

CH °	*CB* °	*CH* °	*CB* °
000	230.3	180	230.6
045	228.7	225	232.4
090	227.4	270	233.8
135	228.0	315	232.3

Required.—Deviation on each compass heading.

Answers.—

CH °	*Deviation* °	*CH* °	*Deviation* °
000	0.2 E	180	0.1 W
045	1.8 E	225	1.9 W
090	3.1 E	270	3.3 W
135	2.5 E	315	1.8 W

735. Bearings of a vessel are taken by means of a compass ashore, and simultaneous bearings of the shore position are taken from the vessel, as follows:

CH °	*CB of shore position* °	*MB of vessel* °	*CH* °	*CB of shore position* °	*MB of vessel* °
000	020	198	180	003	184
045	013	189	225	009	194
090	004	174	270	013	204
135	001	172	315	017	205

Required.—(1) Deviation on each heading.
(2) The value of coefficient *A*.

Answers.—

(1)

CH °	*Deviation* °	*CH* °	*Deviation* °
000	2 W	180	1 E
045	4 W	225	5 E
090	10 W	270	11 E
135	9 W	315	8 E

(2) Coefficient *A* is zero.

CHAPTER VIII

DEAD RECKONING

801. Introduction.—**Dead reckoning** (**DR**) is the determination of position by advancing a known position for courses and distances. It is reckoning relative to something stationary or "dead" in the water, and hence applies to courses and speeds *through the water*. Because of leeway due to wind, inaccurate allowance for compass error, imperfect steering, or error in measuring speed, the actual motion through the water is seldom determined with complete accuracy. In addition, if the water itself is in motion, the course and speed over the bottom differ from those through the water. It is good practice to use the true course *steered* and the best determination of *measured* speed, which is normally speed *through the water*, for dead reckoning. Hence, geographically, a **dead reckoning position** is an approximate one which is corrected from time to time as the opportunity presents itself. Although of less than the desired accuracy, dead reckoning is the only method by which a position can be determined at *any* time and therefore might be considered *basic* navigation, with all other methods only appendages to provide means for correcting the dead reckoning. The prudent navigator keeps his direction- and speed- or distance-measuring instruments in top condition and accurately calibrated, for his dead reckoning is no more accurate than his measurement of these elements.

If a navigator can accurately assess the disturbing elements introducing geographical errors into his dead reckoning, he can determine a better position than that established by dead reckoning alone. This is properly called an **estimated position** (**EP**). It may be established either by applying an estimated correction to a dead reckoning position, or by estimating the course and speed being made good over the bottom. The expression "dead reckoning" is sometimes applied loosely to such reckoning, but it is better practice to keep this "estimated reckoning" distinct from dead reckoning, if for no other reason than to provide a basis for evaluating the accuracy of one's estimates. When good information regarding current, wind, etc., is available, it should be used, but the practice of applying corrections based upon information of uncertain accuracy is, at best, questionable, and may introduce an error. Estimates should be based upon judgment and experience. Positional information which is incomplete or of uncertain accuracy may be available to assist in making the estimate. However, before adequate experience is gained, one should be cautious in applying corrections, for the estimates of the inexperienced are often quite inaccurate.

Dead reckoning not only provides means for continuously establishing an approximate position, but also is of assistance in determining times of sunrise and sunset, the celestial bodies available for observation, the predicted availability of electronic aids to navigation, the suitability and interpretation of soundings for checking position, the predicted times of making landfalls or sighting lights, estimates of arrival times, and in evaluating the reliability and accuracy of position-determining information. Because of the importance of accurate dead reckoning, a careful log is kept of all courses and speeds, times of all changes, and compass errors. These may be recorded directly in the log or first in a **navigator's notebook** for later recording in the log, but whatever the form, a careful record is important.

Modern navigators almost invariably keep their dead reckoning by plotting directly on the chart or plotting sheet, drawing lines to represent the direction and distance of

travel and indicating dead reckoning and estimated positions from time to time. This method is simple and direct. Large errors are often apparent as inconsistencies in an otherwise regular plot. Before the advent of power vessels, when frequent course and speed changes were common, and when charts were sometimes of questionable accuracy, it was common practice to keep the dead reckoning mathematically by one, or a combination, of the "sailings" (arts. 811–825). Except for great-circle sailing, and occasionally composite and Mercator sailings, these are of little more than historical interest to modern navigators, other than those of small boats.

In determining distance run in a given time, one may find table 19 useful. Similar information is given in a somewhat different form in an auxiliary table in H.O. Pub. No. 214.

802. Plotting position on the chart.—A position is usually expressed in units of latitude and longitude, generally to the nearest 0.′1, but it may be expressed as bearing and distance from a known position, such as a landmark or aid to navigation.

To plot a position on a Mercator chart, or to determine the coordinates of a point on such a chart, proceed as follows:

To plot a position when its latitude and longitude are known: Mark the given latitude on a convenient latitude scale along a meridian, being careful to note the unit of the smallest division on the scale. Place a straightedge at this point and parallel to a parallel of latitude (perpendicular to a meridian). Holding the straightedge in place, set one point of a pair of dividers at the given longitude on the longitude scale at the top or bottom of the chart (or along any parallel) and the other at a convenient printed meridian. Without changing the spread of the dividers, place one point on the same printed meridian at the edge of the straightedge, and the second point at the edge of the straightedge in the direction of the given longitude. This second point is at the given position. *Lightly* prick the chart. Remove first the straightedge and then the dividers, watching the point to be sure of identifying it. Make a dot at the point, enclose it with a small circle or square as appropriate (art. 805), and label it. If the dividers are set to the correct spread for longitude *before* the latitude is marked, one point of the dividers can be used to locate the latitude and place the straightedge, if one is careful not to disturb the setting of the dividers.

To determine the coordinates of a point on the chart: Place a straightedge at the given point and parallel to a parallel of latitude. Read the latitude where the straightedge crosses a latitude scale. Keeping the straightedge in place, set one leg of a pair of dividers at the given point and the other at the intersection of the straightedge and a convenient printed meridian. Without changing the spread of the dividers, place one end on a longitude scale, at the same printed meridian, and the other point on the scale, in the direction of the given point. Read the longitude at this second point.

Several variations of these procedures may suggest themselves. That method which seems most natural and is least likely to result in error should be used.

803. Measuring direction on the chart.—Since the Mercator chart, commonly used by the marine navigator, is *conformal* (art. 302), directions and angles are correctly represented. It is customary to orient the chart with 000° (north) at the top; other directions are in their correct relations to north and each other.

As an aid in measuring direction, **compass roses** are placed at convenient places on the chart or plotting sheet. A desired direction can be measured by placing a straightedge along the line from the center of a compass rose to the circular graduation representing the desired direction. The straightedge is then in the desired direction, which may be transferred to any other part of the chart by parallel motion, as by parallel rulers or two triangles (art. 603). The direction between two points is determined by transferring that direction to a compass rose. If a drafting machine (art. 606) or some

form of plotter (art. 605) or protractor (art. 604) is used, measurement can be made directly at the desired point, without using the compass rose.

Measurement of direction, whether or not by compass rose, can be made at any convenient place on a Mercator chart, since meridians are parallel to each other and a line making a desired angle with any one makes the same angle with all others. Such a line is a **rhumb line,** the kind commonly used for course lines, except in polar regions. For direction on a chart having nonparallel meridians, measurement can be made at the meridian involved if the chart is conformal, or by special technique if it is not conformal. Explanation of the former is given in article 2511. The only nonconformal chart commonly used by navigators is the gnomonic, and instructions for measuring direction on this chart are usually given on the chart itself.

Compass roses for both true and magnetic directions may be given. A drafting machine can be oriented to any reference direction—true, magnetic, compass, or grid. When a plotter or protractor is used for measuring an angle with respect to a meridian, the resulting direction is true unless other than true meridians are used. For most purposes of navigation it is good practice to plot true directions only, and to label them in true coordinates.

804. Measuring distance on the chart.—The length of a line on a chart is usually measured in nautical miles, to the nearest 0.1 mile. For this purpose it is customary to use the latitude scale, considering one minute of latitude equal to one nautical mile. The error introduced by this assumption is not great over distances normally measured. It is maximum near the equator or geographical poles. Near the equator a ship traveling 180 miles by measurement on the chart would cover only 179 miles over the earth. Near the pole a run of 220 miles by chart measurement would equal 221 miles over the earth.

Since the latitude scale on a Mercator chart expands with increased latitude, measurement should be made at approximately the mid latitude. For a chart covering a relatively small area, such as a harbor chart, this precaution is not important because of the slight difference in scale over the chart. On such charts a separate mile scale may be given, and it may safely be used over the entire chart. However, habit is strong, and mistakes can probably be avoided by *always* using the mid latitude.

For long distances the line should be broken into a number of parts or *legs*, each one being measured at its mid latitude. The length of a line that should be measured in a single step varies with latitude, decreasing in higher latitudes. No realistic numerical value can be given, since there are too many considerations. With experience a navigator determines this for himself. On the larger scale charts this is not a problem because the usual dividers used for this purpose will not span an excessively long distance.

In measuring distance, the navigator spans with his dividers the length of the line to be measured and then, without altering the setting, transfers this length to the latitude scale, carefully noting the graduations so as to avoid an error in reading. This precaution is needed because of the difference from chart to chart. In measuring a desired length along a line, the navigator spans this length on the latitude scale opposite the line and then transfers his dividers to the line, without changing the setting. For a long line the navigator sets his dividers to some convenient distance and steps off the line, counting the number of steps, multiplying this by the length of the step, and adding any remainder. If the line extends over a sufficient spread of latitude to make scale difference a factor, he resets his dividers to the scale for the approximate mid latitude of each leg. The distance so measured is the length of the rhumb line.

For measuring distance on a nearly-constant-scale chart, such as the Lambert conformal, the mid-latitude precaution is usually unnecessary. Such charts generally

have a mile scale independent of the latitude scale. On a gnomonic chart a special procedure is needed, and this is usually explained on the chart.

805. Plotting and labeling the course line and positions.—**Course** is the intended horizontal direction of travel. A **course line** is a line extending in the direction of the course. From a known position of the ship the course line is drawn in the direction indicated by the course. It is good practice to label all lines and points of significance as they are drawn, for an unlabeled line or point can easily be misinterpreted later. Any simple, clear, logical, unambiguous system of labels is suitable. The following is widely used and might well be considered standard.

Label a course line with direction and speed. *Above* the course line place a capital C followed by three figures to indicate the course steered. It is customary to label and steer courses to the nearest whole degree, although they are generally computed to the nearest 0°1. The course label should indicate *true* direction, starting with 000° at true north and increasing clockwise through 360°. *Below* the course line, and under the direction label, place a capital S followed by figures representing the speed in knots. Since the course is *always* given in degrees true and the speed in knots, it is not necessary to indicate the units or the reference direction (fig. 805).

A point to be labeled is enclosed by a small circle in the case of a **fix** (an accurate position determined without reference to any former position) or dead reckoning position, and by a small square in the case of an estimated position. It is labeled with the time, usually to the nearest minute, and the nature of the position (FIX, EP, DR). Time is usually expressed in four figures without punctuation, on a 24-hour basis (art. 1903). Zone time (art. 1907) is usually used, but Greenwich mean time (art. 1907) may be employed. A course line is a succession of an infinite number of dead reckoning positions. Only selected points are labeled.

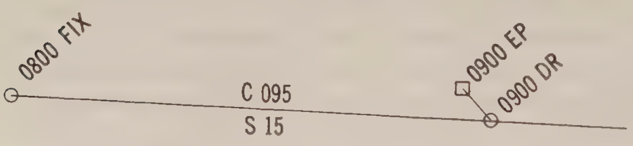


FIGURE 805.—A course line with labels.

The labels of a *line* are placed *along* the line, and those of a *point* are at an *angle to the line.*

806. Dead reckoning by plot.—As a vessel clears a harbor and proceeds out to sea, the navigator obtains one last good fix while identifiable landmarks are still available. This is called **taking departure,** and the position determined is called the **departure.** Piloting (ch. IX) comes to an end and the course is set for the open sea. The course line is drawn and labeled, and some future position is indicated as a DR position. The number of points selected for labeling depends primarily upon the judgment and individual preference of the navigator. It is good practice to label each point where a change of course or speed occurs. If such changes are frequent, no additional points need be labeled. With infrequent changes, it is good practice to label points at some regular interval, as every two hours. From departure, the dead reckoning plot continues unbroken until a new well-established position is obtained, when both DR and fix are shown. The fix serves as the start of a new dead reckoning plot. Although estimated positions are shown, it is generally not good practice to begin a new DR at these points.

A typical dead reckoning plot is shown in figure 806, indicating procedures both when there are numerous changes of course and speed and when there is a long continuous course. It is assumed that no fix is obtained after the initial one at 0800 on September 8. Note that course lines are not extended beyond their limits of usefulness. One should keep a neat plot and leave no doubt as to the meaning of each line and

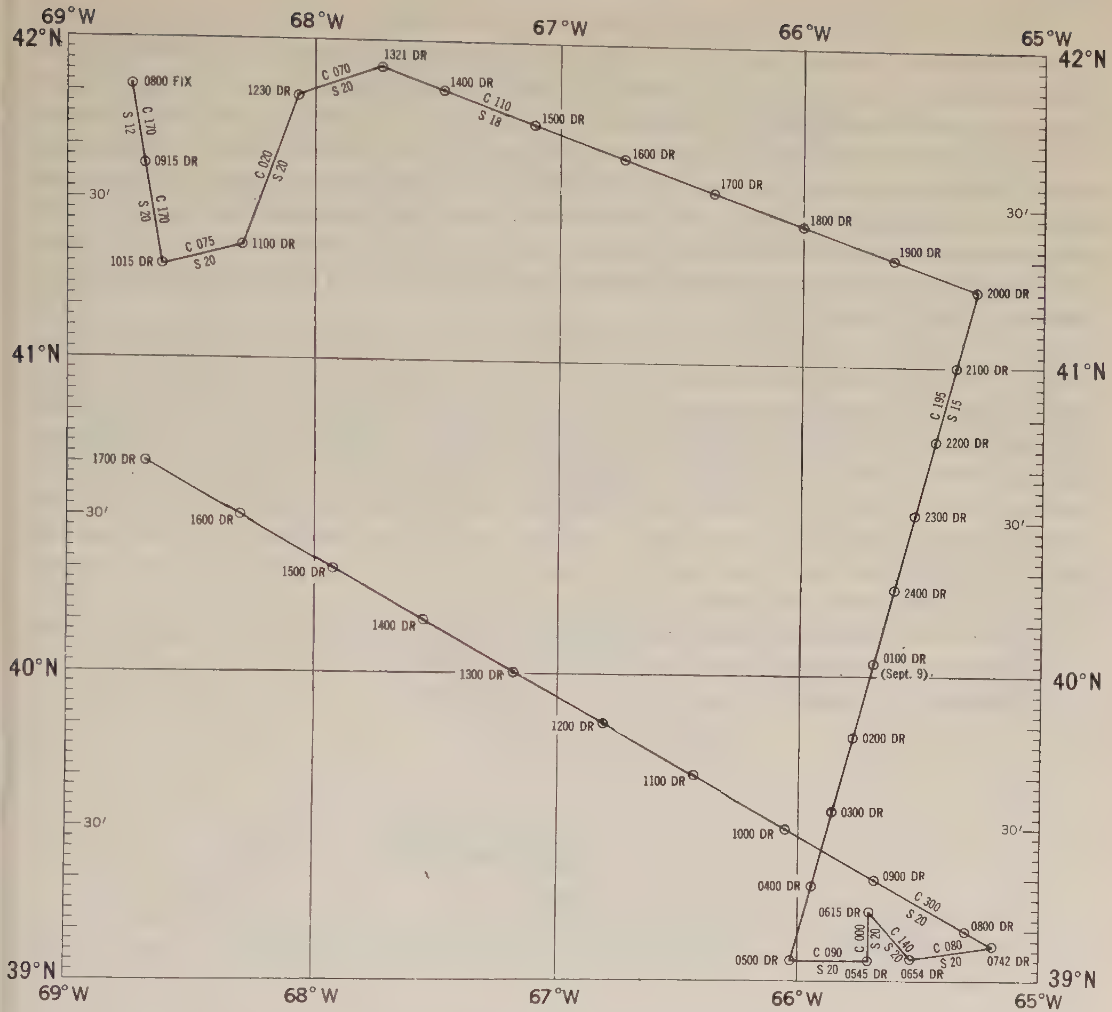


FIGURE 806.—A typical dead reckoning plot.

marked point. *A neat, accurate plot is the mark of a good navigator.* The plot should be kept extended to some future time. A good navigator is always ahead of his ship. In shoal water or when near the shore, aids to navigation, dangers, etc., it is customary to keep the dead reckoning plot on a chart. A chart overprinted with loran or other electronic position lines may be used at a considerable distance from shore. But on the open sea, with only dead reckoning and celestial navigation available, it is good practice to use a plotting sheet (art. 323).

807. Current.—Water in essentially horizontal motion over the surface of the earth is called **current.** The direction in which the water is moving is called the **set,** and the speed is called the **drift.** In navigation it is customary to use the term "current" to include all factors introducing geographical error in the dead reckoning, whether their immediate effects are on the vessel or the water. When a fix is obtained, one assumes that the current has set *from* the DR position at the same time *to* the fix, and that the drift is equal to the distance in miles between these positions, divided by the number of hours since the last fix. This is true regardless of the number of changes of course or speed since the last fix.

If set and drift since the last fix are known, or can be estimated, a better position can be obtained by applying a correction to that obtained by dead reckoning. This is conveniently done by drawing a straight line in the direction of the set for a distance equal to the drift multiplied by the number of hours since the last fix, as shown in figure 805. The direction of a straight line from the last fix to the EP is the estimated **course made good,** and the length of this line divided by the time is the estimated **speed made good.** These estimated values are sometimes called the **course of advance** (**COA**) and **speed of advance** (**SOA**), respectively. The course and speed actually made good over the ground are then called the **course over the ground** (COG) and **speed over the ground** (SOG), respectively.

If a current is setting in the same direction as the course, or its reciprocal, the course over the ground is the same as that through the water. The effect on the speed can be found by simple arithmetic. If the course and set are in the same direction, the speeds are added; if in opposite directions, the smaller is subtracted from the larger. This situation is not unusual when a ship encounters a tidal current while entering or leaving port. If a ship is *crossing* a current, solution can be made graphically by vector diagram (arts. O17, O18) since velocity over the ground is the vector sum of velocity *through* the water and velocity *of* the water. Although *distances* can be used, it is generally easier to use *speeds.*

Example 1.—A ship on course 080°, speed ten knots, is steaming through a current having an estimated set of 140° and drift of two knots.

Required.—Estimated course and speed made good.

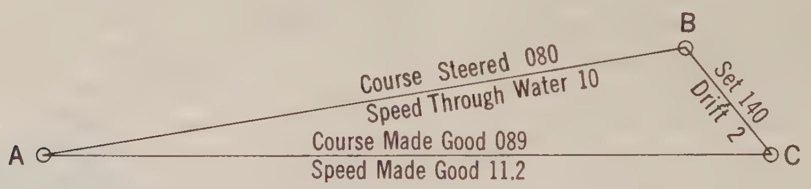


FIGURE 807a.—Finding course and speed made good through a current.

Solution (fig. 807a).—(1) From *A*, any convenient point, draw *AB*, the course and speed of the ship, in direction 080°, for a distance of ten miles.

(2) From *B* draw *BC*, the set and drift of the current, in direction 140°, for a distance of two miles.

(3) The direction and length of *AC* are the estimated course and speed made good. Determine these by measurement.

Answers.—Estimated course made good 089°, estimated speed made good 11.2 kn.

If it is required to find the course to steer at a given speed to make good a desired course, plot the current vector from the origin, *A*, instead of from *B*.

Example 2.—The captain desires to make good a course of 095° through a current having a set of 170° and a drift of 2.5 knots, using a speed of 12 knots.

Required.—The course to steer and the speed made good.

Solution (fig. 807b).—(1) From *A*, any convenient point, draw line *AB* extending in the direction of the course to be made good, 095°.

(2) From *A* draw *AC*, the set and drift of the current.

(3) Using *C* as a center, swing an arc of radius *CD*, the speed through the water (12 knots), intersecting line *AB* at *D*.

(4) Measure the direction of line *CD*, 083°5. This is the course to steer.

(5) Measure the length *AD*, 12.4 knots. This is the speed made good.

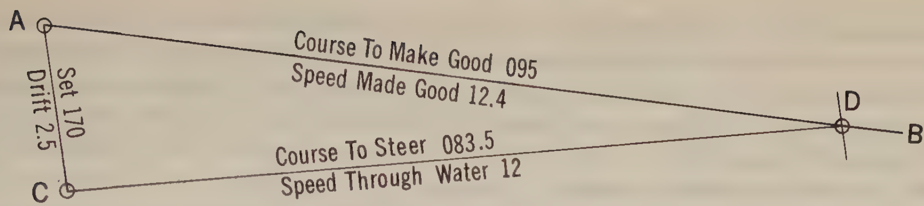


FIGURE 807b.—Finding the course to steer at a given speed to make good a given course through a current.

Answers.—Course to steer 083°5, speed made good 12.4 kn.

If it is required to find the course to steer and the speed to use to make good a desired course and speed, proceed as follows:

Example 3.—The captain desires to make good a course of 265° and a speed of 15 knots through a current having a set of 185° and a drift of three knots.

Required.—The course to steer and the speed to use.

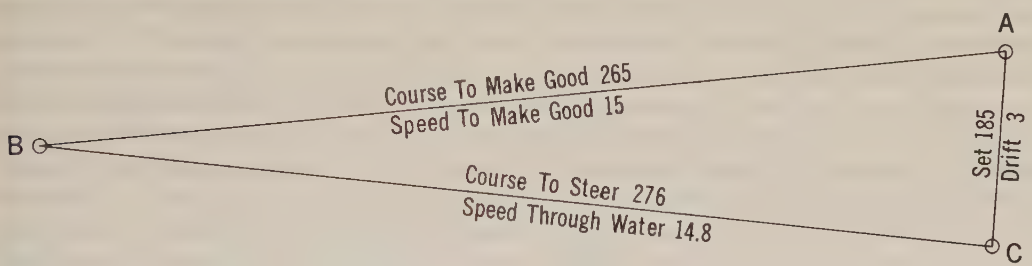


FIGURE 807c.—Finding the course to steer and the speed to use to make good a given course and speed through a current.

Solution (fig. 807c).—(1) From *A*, any convenient point, draw *AB* in the direction of the course to be made good, 265°, and for a length equal to the speed to be made good, 15 knots.

(2) From *A* draw *AC*, the set and drift of the current.

(3) Draw a straight line from *C* to *B*. The direction of this line, 276°, is the required course to steer; and the length, 14.8 knots, is the required speed.

Answers.—Course to steer 276°, speed to use 14.8 kn.

Such vector solutions can be made to any convenient scale and at any convenient place, such as the center of a compass rose, an unused corner of the plotting sheet, a separate sheet, or directly on the plot.

808. Leeway is the leeward motion of a vessel due to wind. It may be expressed as distance, speed, or angular difference between course steered and course through the water. However expressed, its amount varies with the speed and relative direction of the wind, type of vessel, amount of freeboard, trim, speed of the vessel, state of the sea, and depth of water. If information on the amount of leeway to be expected under various conditions is not available for the type vessel involved, it should be determined by observation. When sufficient data have been collected, suitable tables or graphs can be made for quick and convenient estimate. The accuracy of the information should be checked whenever convenient, and corrections made when sufficient evidence indicates the need.

Leeway is most conveniently applied by adding its effect to that of current and other elements introducing geographical error in the dead reckoning. It is customary to consider the combined effect of all such elements as current, and to make allowance

for this as explained in article 807. In sailing ship days it was common practice to consider leeway in terms of its effect upon the course only, and to apply it as a correction in the same manner that variation and deviation are applied. While this method has merit even with power vessels, it is generally considered inferior to that of considering leeway as part of current.

809. Automatic dead reckoning.—Several types of devices are in use for performing automatically all or part of the dead reckoning. Perhaps the simplest is the automatic **course recorder,** which provides a graphical record of the various courses steered. In its usual form this device is controlled by the gyro compass, and so indicates gyro courses.

Dead reckoning equipment receives inputs from the compass, usually the gyro compass, and a mechanical log or engine revolution counter. It determines *change* in latitude and longitude, the latter by first determining departure and then mechanically multiplying this by the secant of the latitude. The device is provided with counters on which latitude and longitude can be set. As the vessel proceeds, the changes are then mechanically added to or subtracted from these readings to provide a continuous, instantaneous indication of the dead reckoning position. The navigator or an assistant reads these dials at intervals, usually each hour, and records the values in a notebook. Most models of dead reckoning equipment are provided, also, with a tracer for keeping a graphical record of dead reckoning in the form of a plot by moving a pencil or pen across a chart or plotting sheet. This part of the device is called a **dead reckoning tracer.** Whatever the form, dead reckoning equipment is a great convenience, particularly when a ship is maneuvering. However, such mechanical equipment is subject to possible failure. The prudent navigator keeps a hand plot and uses the dead reckoning equipment as a check. In navigation it is never wise to rely upon a single method if a second method is available as a check.

If it were possible to measure, with complete accuracy, the direction and distance traveled *with respect to the earth,* an accurate geographical position could be known at all times. Several methods of doing so have been suggested, and while developments along these lines relate principally to aircraft and guided missiles, it is possible that from these or other developments may come some method suitable for shipboard use. The two methods most prominently suggested are (1) Doppler and (2) inertial. By the **Doppler** method one or more beams of radiant energy are directed downward at an angle. The return echo from the bottom is of a slightly different frequency due to the motion of the craft. The amount of the change, or Doppler, is proportional to the speed. By proper selection of beams, it is possible to measure speed in a lateral direction as well as in a forward direction. Distance can be determined by mechanical or electronic integration of these measurements, and this can be converted into position. By the **inertial** method, accelerometers measure the acceleration in various directions, and by double integration this is converted to distance, from which position can be determined. Either of these methods can provide considerable accuracy over a period of several hours, but since the error increases with time, they are not yet suitable for general shipboard use over long distances.

810. Dead reckoning by computation.—Dead reckoning involves the determination of position by means of course and distance from a known position. A closely related problem is that of finding the course and distance from one point to another. Although both of these problems are customarily solved by plotting directly on the chart, it occasionally becomes desirable to solve by computation, usually by logarithms (arts. O10–O15) or traverse table (art. 812). The various methods of solution are

collectively called the **sailings.** Computation should be carried to the precision shown in the examples, even though this in some instances exceeds the usable precision, and sometimes the accuracy.

811. The sailings.—In the solution of problems involved in the sailings, the following quantities are used:

1. *Latitude* (L). The latitude of the point of departure is designated L_1; that of the point of arrival or the destination, L_2; mid latitude, Lm; latitude of the vertex of a great circle, L_v; and latitude of any point on a great circle, L_x.
2. *Difference of latitude* (l).
3. *Meridional parts* (M). The meridional parts of the point of departure are designated M_1, and of the point of arrival or the destination, M_2.
4. *Meridional difference* (m).
5. *Longitude* (λ). The longitude of the point of departure is designated λ_1; that of the point of arrival or the destination, λ_2; of the vertex of a great circle, λ_v; and of any point on a great circle, λ_x.
6. *Difference of longitude* (DLo).
7. *Departure* (p).
8. *Course* or *course angle* (Cn or C).
9. *Distance* (D).

The various kinds of sailings are:

1. **Plane sailing.** The earth, or that part traversed, is regarded as a plane surface. A single course and distance, difference of latitude, and departure are the only items involved. Hence, the method provides solution for latitude of the point of arrival, but not for longitude of this point, one of the spherical sailings being needed for this problem. Because of the basic assumption that the earth is flat, this method should not be used for distances of more than a few hundred miles.
2. **Traverse sailing** combines the plane sailing solutions when there are two or more courses.
3. **Parallel sailing** is the interconversion of departure and difference of longitude when a vessel is proceeding due east or due west. This was a common occurrence when the sailings were first employed several hundred years ago, but only an incidental situation now.
4. **Middle-** (or **mid-**) **latitude sailing** involves the use of the mid latitude for converting departure to difference of longitude when the course is not due east or due west.
5. **Mercator sailing** provides a mathematical solution of the plot as made on a Mercator chart. It is similar to plane sailing, but uses meridional difference and difference of longitude in place of difference of latitude and departure, respectively.
6. **Great-circle sailing** involves the solution of courses, distances, and points along a great circle between two points, the earth being regarded as a sphere.
7. **Composite sailing** is a modification of great-circle sailing to limit the maximum latitude.

In addition, **meridian sailing** might be added to this list to cover the special case of a vessel following a course of due north or due south (true). However, no solution is needed for this case because there is no departure or difference of longitude, and the distance is considered equal to the difference of latitude in minutes. The true course is 000° or 180°.

Except for great-circle sailing and the great-circle part of composite sailing, the various problems normally arising under the sailings can be solved (1) by plane trigonometry, either using natural functions (tab. 31) or logarithms (tabs. 32 and 33);

(2) by traverse table (tab. 3), or (3) graphically. For the graphical solution, cross-section paper is helpful. The triangle of each method is drawn, and the parts are measured. Solution by computation is most accurate.

In the mathematical solution of navigational problems, the use of standard work forms is desirable to provide orderly computations and to minimize errors. This subject is further discussed in appendix Q, which gives recommended forms for many of the common problems of navigation.

Great-circle sailing and occasionally composite sailing and Mercator sailing are the only ones commonly used, except by small-boat navigators.

812. Traverse tables, such as table 3, providing a solution for any plane right triangle, can be used in the solution of the usual problems encountered in any of the sailings except great-circle and composite. A separate table is given for each degree of course if the lower line of column headings is used, and for each degree of latitude if the upper line of column headings is used. For intermediate values interpolation should be made between tables. The main part of each table involves solution for the various sides of a plane triangle. The auxiliary table to the right of each main table provides a tabulated solution for the course. The manner of using the table in specific problems is illustrated in the examples given in the explanations of the various sailings.

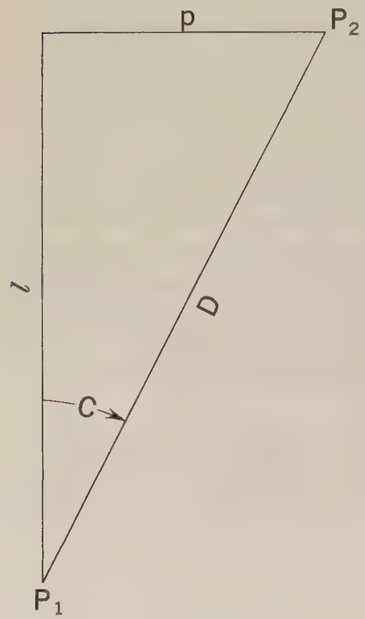


FIGURE 813.—The plane sailing triangle.

813. Plane sailing.—In plane sailing the figure formed by the meridian through the point of departure, the parallel through the point of arrival, and the course line is considered a plane right triangle. This is illustrated in figure 813, in which P_1 and P_2 are the points of departure and arrival, respectively. The course angle and the three sides are as labeled. From this triangle:

$$\cos C=\frac{l}{D} \qquad \sin C=\frac{p}{D} \qquad \tan C=\frac{p}{l}$$

From the first two of these formulas the following relationships can be derived:

$$l=D\cos C \qquad D=l\sec C \qquad p=D\sin C$$

The usual problems solved by plane sailing are: (1) given the course and distance, find the difference of latitude and the departure; and (2) the reverse of this. It is good practice to label l, N or S, and p, E or W, to aid in identification of the quadrant of the course. Logarithmic and traverse table solutions are illustrated in the following examples:

Example 1.—A vessel steams 188.4 miles on course 005°.

Required.—(1) Difference of latitude, (2) departure.

Solution.—By computation:

	D 188.4 mi.	log 2.27508	log 2.27508
	C 005°	l cos 9.99834	l sin 8.94030
(1)	l 187′.7 N	log 2.27342	
(2)	p 16.4 mi. E		log 1.21538

By traverse table:

D	l	p
100.0	99.6	8.7
80.0	79.7	7.0
8.0	8.0	0.7
0.4	0.4	0.0
188.4	(1) 187.7 N	(2) 16.4 E

Example 2.—A ship has steamed 136.6 miles north and 203.1 miles west.

Required.—(1) Course, (2) distance.

Solution.—*By computation:*

p 203.1 mi. W	log	2.30771		
l 136′.6 N	log (−)	2.13545	log	2.13545
C N 56°04′.6 W	l tan	0.17226	l sec	0.25330
(2) D 244.8 mi.			log	2.38875
(1) Cn 303°.9				

By traverse table:

			l	D (303°)	D (304°)
p 203.1 mi. W	log	2.30771	100.0	183.6	178.8
l 136′.6 N	log (−)	2.13545	30.0	55.1	53.6
p÷l 1.487	log	0.17226	6.0	11.0	10.7
C N 56°.1 W			0.6	1.1	1.1
(1) Cn 303°.9			136.6	250.8	244.2
(2) D 244.9 mi.					

In the solution, the navigational form (art. O11) is used, with the basic quantity being on the left, and related information on the same line. Thus, 2.27508 is the logarithm ("log") of 188.4, and 9.99834 is the logarithmic cosine ("l cos") of 5°.

The labels (N, S, E, W) of l, p, and C are determined by noting the direction of motion or the relative positions of the two places.

In the solution of example 2 by traverse table, it is first necessary to solve for p÷l. If this is done by logarithms, as shown above, the solution is similar to that by computation, with one additional step. Solution for p÷l can be made by any method, or course can be found as shown in the first solution, and this value used for entering the traverse table to determine the distance.

The distance in the traverse table solution is found by interpolation between the values for 303° and 304°.

When the course is near 090° or 270°, the solution of C to the nearest 0°.1 only, as by traverse table, may introduce a large error in distance.

814. Traverse sailing.—A **traverse** is a series of courses, or a track consisting of a number of course lines, as might result from a sailing vessel beating into the wind. **Traverse sailing** is the finding of a single equivalent course and distance. If the effect of an estimated current is to be considered, the set is treated as an additional course, the drift times the number of hours involved being used as the distance. If direction and distance from some point, such as a lighthouse, other than the point of departure is desired, the bearing of the point of departure from the selected position is used as the first course and the distance between these points as the first distance.

Solution is usually made by means of the traverse tables, the distance to the north or south and that to the east or west on each course being tabulated, the algebraic sum of difference of latitude and departure being found, and the result being converted to course and distance.

Example.—A ship steams as follows: course 158°, distance 15.5 miles; course 135°, distance 33.7 miles; course 259°, distance 16.1 miles; course 293°, distance 39.0 miles; course 169°, distance 40.4 miles.

Required.—Equivalent single (1) course, (2) distance.

Solution.—Solve for each leg as in example 1, article 813. Tabulate the answers as follows:

Course	Dist.	N	S	E	W
°	*mi.*	*mi.*	*mi.*	*mi.*	*mi.*
158	15.5		14.4	5.9	
135	33.7		23.8	23.8	
259	16.1		3.0		15.8
293	39.0	15.2			35.9
169	40.4		39.7	7.7	
		15.2	80.9	37.4	51.7
(1)	(2)		15.2		37.4
192.3	67.2		65.7		14.3

Convert *l* 65′.7 S, p 14.3 mi.W to equivalent single course and distance as shown in example 2, article 813.

815. Parallel sailing consists of the interconversion of departure and difference of longitude. It is the simplest form of spherical sailing (other than meridional sailing). The formulas for these transformations are:

$$\text{DLo} = \text{p sec L} \qquad\qquad \text{p} = \text{DLo cos L}$$

When solution is made by table 3, enter the table for the latitude and use the *upper* line of column headings.

Example 1.—The DR latitude of a ship on course 090° is 49°40′.2 N.

Required.—The change in longitude if the ship steams for 136.4 miles.

Solution.—By computation:

p	136.4 mi. E	log	2. 13481
L	49°40′.2 N	*l* sec	0. 18897
DLo	210′.8 E	log	2. 32378
DLo	3°30′.8 E		

By traverse table:

p	DLo (49°)	DLo (50°)
100. 0	152. 4	155. 6
30. 0	45. 7	46. 7
6. 0	9. 1	9. 3
0. 4	0. 6	0. 6
136. 4	207. 8	212. 2

DLo for L 49°40′.2 N: 210′.7=3°30′.7 E.

Example 2.—The DR latitude of a ship on course 270° is 37°50′.1 S. The ship steams on this course until the longitude changes 4°33′.5W.

Required.—The distance steamed.

Solution.—By computation:

DLo	273′.5W	log	2. 43696
L	37°50′.1S	*l* cos	9. 89751
p	216.0 mi.W	log	2. 33447

By traverse table:

DLo	p (37°)	p (38°)
200. 0	159. 7	157. 6
70. 0	55. 9	55. 2
3. 0	2. 4	2. 4
0. 5	0. 4	0. 4
273. 5	218. 4	215. 6

p for L 37°50′.1 S: 216.1 mi. W.

The labels (E or W) of p and DLo agree with the direction of motion.

816. Middle-latitude sailing, popularly called **mid-latitude sailing,** combines plane sailing and parallel sailing. Plane sailing is used to find difference of latitude and departure when course and distance are known, or vice versa. Parallel sailing is used to interconvert departure and difference of longitude, the middle or mean latitude (Lm) being used. If a course line crosses the equator, that part on each side (the north latitude and south latitude portions, respectively) should be solved separately.

This sailing, like most elements of navigation, contains certain simplifying approximations which produce answers somewhat less accurate than those yielded by more rigorous solutions. For ordinary purposes, however, the results are more accurate than the navigation of the vessel using them. From time to time suggestions have been made that a correction be applied to eliminate the error introduced by assuming that the meridians of the point of departure and of the destination converge uniformly (as the two sides of a plane angle), rather than as the sine of the latitude (approximately). The proposed correction usually takes the form of some quantity to be added to or subtracted from the middle latitude to obtain a "corrected middle latitude" for use in the solution. Tables giving such a correction have been published for both spherical and spheroidal earths. However, the actual correction is not a simple function of the middle latitude and the difference of longitude, as assumed, because the basic formulas of the sailing are themselves based upon a sphere, rather than a spheroid. Hence, the use of such a correction is misleading, and may introduce more error than it eliminates. The use of any correction is not considered justified; if highly accurate results are required, a different method should be used.

Example 1.—A vessel steams 1,253.4 miles on course 070° from lat. 15°17′.4 N, long. 151°37′.8 E.

Required.—(1) Latitude and (2) longitude of the point of arrival.

Solution.—By computation:

D	1253.4 mi.	log 3.09809	log 3.09809
C	070°	l cos 9.53405	l sin 9.97299
l	428.7 N	log 2.63214	
p	1177.8 mi. E		log 3.07108
Lm	18°51′.8 N		l sec 0.02397
DLo	1244′.7 E		log 3.09505

L_1	15°17′.4 N	L_1	15°17′.4 N
l	7°08′.7 N	½ l	3°34′.4 N
(1) L_2	22°26′.1 N	Lm	18°51′.8 N

λ_1	151°37′.8 E
DLo	20°44′.7 E
(2) λ_2	172°22′.5 E

By traverse table:

D	l	p
1000.0	342.0	940.0
200.0	68.4	187.9
50.0	17.1	47.0
3.0	1.0	2.8
0.4	0.1	0.4
1253.4	428.6	1178.1

				p	DLo(18°)	DLo(19°)
L_1	15°17!4 N	L_1	15°17!4 N			
l	7°08!6 N	$\frac{1}{2}l$	3°34!3 N			
(1) L_2	22°26!0 N	Lm	18°51!7 N	1000.0	1051.0	1058.0
				100.0	105.1	105.8
λ_1	151°37!8 E			70.0	73.6	74.0
DLo	20°45!3 E	DLo	1245!3	8.0	8.4	8.5
(2) λ_2	172°23!1 E			0.1	0.1	0.1
				1178.1	1238.2	1246.4

Example 2.—A vessel at lat. 8°48!9 S, long. 89°53!3 W is to proceed to lat. 17°06!9 S, long. 104°51!6 W.

Required.—(1) Course, (2) distance.

Solution.—*By computation:*

L_1	8°48!9 S	λ_1	89°53!3 W
L_2	17°06!9 S	λ_2	104°51!6 W
l	8°18!0 S	DLo	14°58!3 W
$\frac{1}{2}l$	4°09!0 S	DLo	898!3 W
Lm	12°57!9 S		

DLo	898!3 W	log	2.95342		
Lm	12°57!9 S	l cos	9.98878		
p	875.4 mi. W	log	2.94220		
l	498!0 S	log (−)	2.69723	log	2.69723
C	S 60°21!9 W	l tan	0.24497	l sec	0.30586
(2) D	1007.1 mi.			log	3.00309
(1) Cn	240°4				

By traverse table:

				DLo	p(12°)	p(13°)
L_1	8°48!9 S	λ_1	89°53!3 W			
L_2	17°06!9 S	λ_2	104°51!6 W			
l	8°18!0 S	DLo	14°58!3 W	800.0	782.5	779.5
$\frac{1}{2}l$	4°09!0 S	DLo	898!3	90.0	88.0	87.7
Lm	12°57!9 S			8.0	7.8	7.8
				0.3	0.3	0.3
				898.3	878.6	875.3

				l	D(240°)	D(241°)
p	875.4 mi. W	log	2.94221	400.0	800.0	825.1
l	498!0 S	log (−)	2.69723	90.0	180.0	185.6
p÷l	1.758	log	0.24498	8.0	16.0	16.5
C	S 60°4 W			0.0	0.0	0.0
(1) Cn	240°4			498.0	996.0	1027.2
(2) D	1008.5 mi.					

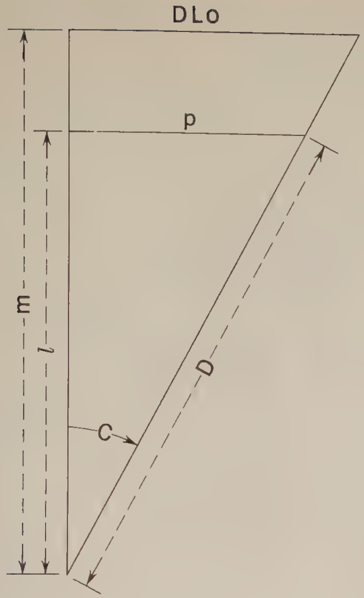


FIGURE 817.—Mercator sailing relationships.

The labels (N, S, E, W) of l, p, DLo, and C are determined by noting the direction of motion or the relative positions of the two places.

When the course is near 090° or 270°, the solution of C to the nearest 0°.1 only, as by traverse table, may introduce a large error in distance.

817. Mercator sailing problems are solved graphically when measurement is made on a Mercator chart. Graphical solution can also be made as shown in figure 817. The lower part is identical with the plane sailing triangle, figure 813. For mathematical solution the formulas of Mercator sailing are:

$$\tan C = \frac{DLo}{m} \qquad D = l \sec C$$

$$l = D \cos C \qquad DLo = m \tan C$$

Another formula sometimes of use is:

$$p = \frac{l \times DLo}{m}$$

Solution can be made by computation or by traverse table.

Example 1.—A ship at lat. 32°14′.7 N, long. 66°28′.9 W is to head for Chesapeake Lightship, lat. 36°58′.7 N, long. 75°42′.2 W.

Required.—(1) Course, (2) distance.

Solution.—By computation:

L_1	32°14′.7 N	M_1	2033.3	λ_1	66°28′.9 W
L_2	36°58′.7 N	M_2	2377.0	λ_2	75°42′.2 W
l	4°44′.0 N	m	343.7	DLo	9°13′.3 W
l	284′.0 N			DLo	553′.3 W

DLo	553′.3 W	log	2.74296		
m	343.7	log (−)	2.53618		
C	N 58°09′.1 W	l tan	0.20678	l sec	0.27764
l	284′.0 N			log	2.45332
(2) D	538.2 mi.			log	2.73096
(1) Cn	301°.8				

By traverse table:

L_1	32°14′.7 N	M_1	2033.3	λ_1	66°28′.9 W
L_2	36°58′.7 N	M_2	2377.0	λ_2	75°42′.2 W
l	4°44′.0 N	m	343.7	DLo	9°13′.3 W
l	284′.0 N			DLo	553′.3 W

				l	D (301°)	D (302°)
DLo	553′.3 W	log	2.74296	200.0	388.3	377.4
m	343.7	log (−)	2.53618	80.0	155.3	151.0
DLo÷m	1.610	log	0.20678	4.0	7.8	7.5
C	N 58°.2 W			0.0	0.0	0.0
(1) Cn	301°.8			284.0	551.4	535.9
(2) D	539.0 mi.					

Example 2.—A ship at lat. 75°31′.7 N, long. 79°08′.7 W, in Baffin Bay, steams 263.5 miles on course 155°.

Required.—(1) Latitude and (2) longitude of point of arrival.

Solution.—By computation:

D 263.5 mi.	log 2.42078	
C 155°	l cos 9.95728	l tan 9.66867
l 238′.8 S	log 2.37806	
m 846.3		log 2.92752
DLo 394′.6 E		log 2.59619

L_1 75°31′.7 N	M_1 7072.4	λ_1 79°08′.7 W
l 3°58′.8 S		DLo 6°34′.6 E
(1) L_2 71°32′.9 N	M_2 6226.1	(2) λ_2 72°34′.1 W
	m 846.3	

By traverse table:

D	l	m	DLo
200.0	181.3	800.0	373.0
60.0	54.4	40.0	18.6
3.0	2.7	6.0	2.8
0.5	0.5	0.6	0.3
263.5	238.9	846.6	394.7

L_1 75°31′.7 N	M_1 7072.4	λ_1 79°08′.7 W
l 3°58′.9 S		DLo 6°34′.7 E
(1) L_2 71°32′.8 N	M_2 6225.8	(2) λ_2 72°34′.0 W
	m 846.6	

The labels (N, S, E, W) of l, DLo, and C are determined by noting the direction of motion or the relative positions of the two places.

If the course is near 090° or 270°, a small error in C introduces a large error in DLo. The solution for C to the nearest 0°.1 only, as by traverse table, may introduce a large error in distance if the course is near 090° or 270°.

818. Rhumb lines and great circles.—The principal advantage of a **rhumb line** is that it maintains constant true direction. A ship following the rhumb line between two places does not change true course. A rhumb line makes the same angle with all meridians it crosses and appears as a straight line on a Mercator chart. It is adequate for most purposes of navigation, bearing lines (except long ones, as those obtained by radio) and course lines both being plotted on a Mercator chart as rhumb lines, except in high latitudes. The equator and the meridians are great circles, but may be considered special cases of the rhumb line. For any other case, the difference between the rhumb line and the great circle connecting two points increases (1) as the latitude increases, (2) as the difference of latitude between the two points decreases, and (3) as the difference of longitude increases. It becomes very great for two places widely separated on the same parallel of latitude far from the equator.

A **great circle** is the intersection of the surface of a sphere and a plane through the center of the sphere. It is the largest circle that can be drawn on the surface of the sphere, and is the shortest distance, along the surface, between any two points on the sphere. Any two points are connected by only one great circle unless the points are antipodal (180° apart on the earth), and then an infinite number of great circles passes through them. Thus, two points on the same meridian are not joined by any great circle other than the meridian, unless the two points are antipodal. If they are the

poles, *all* meridians pass through them. Every great circle bisects every other great circle. Thus, except for the equator, every great circle lies half in the northern hemisphere and half in the southern hemisphere. Any two points 180° apart on a great circle have the same latitude numerically, but contrary names, and are 180° apart in longitude. The point of greatest latitude is called the **vertex.** For each great circle there is one of these in each hemisphere, 180° apart. At these points the great circle is tangent to a parallel of latitude, and hence its direction is due east-west. On each side of these vertices the direction changes progressively until the intersection with the equator is reached, 90° away, where the great circle crosses the equator at an angle equal to the latitude of the vertex. As the great circle crosses the equator, its change in direction reverses, again approaching east-west, which it reaches at the next vertex.

On a Mercator chart a great circle appears as a sine curve extending equal distances each side of the equator. The rhumb line connecting any two points of the great circle on the same side of the equator is a chord of the curve, being a straight line nearer the equator than the great circle. Along any intersecting meridian the great circle crosses at a higher latitude than the rhumb line. If the two points are on opposite sides of the equator, the direction of curvature of the great circle relative to the rhumb line changes at the equator. The rhumb line and great circle may intersect each other, and if the points are equal distances on each side of the equator, the intersection takes place at the equator.

819. Great-circle sailing is used when it is desired to take advantage of the shorter distance along the great circle between two points, rather than to follow the longer rhumb line. The arc of the great circle between the points is called the **great-circle track.** If it could be followed exactly, the destination would be dead ahead throughout the voyage (assuming course and heading were the same). The rhumb line *appears* the more direct route on a Mercator chart because of chart distortion. The great circle crosses meridians at higher latitudes, where the distance between them is less.

The decision as to whether or not to use great-circle sailing depends upon the conditions. The saving in distance should be worth the additional effort, and of course the great circle should not cross land, or carry the vessel into dangerous waters or excessively high latitudes. A slight departure from the great circle or a modification called composite sailing (art. 825) may effect a considerable saving over the rhumb line track without leading the vessel into danger. If a fix indicates the vessel is a considerable distance to one side of the great circle, the more desirable practice often is to determine a new great-circle track, rather than to return to the original one.

Since a great circle is continuously changing direction as one proceeds along it, no attempt is customarily made to follow it exactly, except in polar regions (ch. XXV). Rather, a number of points are selected along the great circle, and rhumb lines are followed from point to point, taking advantage of the fact that for short distances a great circle and a rhumb line almost coincide.

The number of points to use is a matter of personal preference, a large number of points providing closer approximation to the great circle but requiring more frequent change of course. As a general rule, each 5° of longitude is a convenient length. Legs of equal length are not provided in this way, but this is not objectionable under normal conditions.

If a magnetic compass is used, the variation for the middle of the leg is usually used for the entire leg. In some areas the change in variation and the change in course due to convergence of the meridians are in opposite directions and of about the same magnitude. In these areas the same magnetic course can be used for relatively long distances. The change of deviation with change of heading may also be a consideration.

The problems of great-circle sailing can be solved by (1) chart (art. 820), (2) conversion angle (art. 821), (3) computation (art. 822), (4) table (art. 823), (5) graphically, or (6) mechanically. Of these, (5) and (6) are but graphical or mechanical solutions of (3). They usually provide solution only for initial course and the distance, and are not in common use.

820. Great-circle sailing by chart.—Problems of great-circle sailing, like those of rhumb line sailing, are most easily solved by plotting directly on a chart. For this purpose the U. S. Navy Hydrographic Office publishes a number of charts on the gnomonic projection (art. 317), covering the principal navigable waters of the world. On this projection any straight line is a great circle, but since the chart is not conformal (art. 302), directions and distances cannot be measured directly, as on a Mercator chart. An indirect method is explained on each chart.

The usual method of using a gnomonic chart is to plot the great circle and, if it provides a satisfactory track, to determine a number of points along the track, using the latitude and longitude scales in the immediate vicinity of each point. These points are then transferred to a Mercator chart or plotting sheet and used as a succession of destinations to be reached by rhumb lines. The course and distance for each leg is determined by measurement on the Mercator chart or plotting sheet. This method is illustrated in figure 820, which shows a great circle plotted as a straight line on a gnomonic chart and a series of points transferred to a Mercator chart. The arrows represent corresponding points on the two charts. The points can be plotted directly on plotting sheets without the use of a small-scale chart, but the use of the chart provides a visual check to avoid large errors, and a visual indication of the suitability of the track.

Since gnomonic charts are normally used only because of their great-circle properties, they are often popularly called **great-circle charts.**

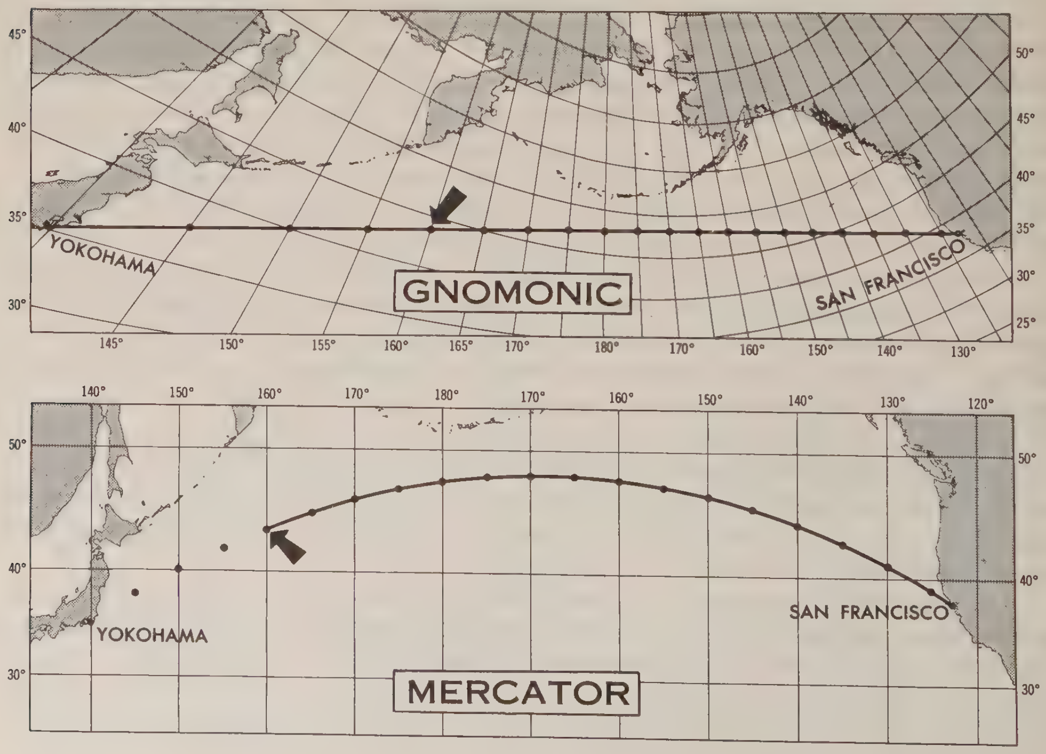


FIGURE 820.—Transferring great-circle points from a gnomonic chart to a Mercator chart.

A projection on which a straight line is *approximately* a great circle can be used in place of a gnomonic chart with negligible error. If such a projection is conformal, as in the case of the Lambert conformal (art. 314), measurement of course and distance of each leg can be made directly on the chart, as explained in article 2511.

Some great circles are shown on pilot charts and certain other charts, together with the great-circle distances. Where tracks are recommended on charts or in sailing directions, it is good practice to follow such recommendations.

821. Great-circle sailing by conversion angle.—The direction of the great circle at the point of departure is called the **initial great-circle course,** and its direction at the destination is called the **final great-circle course.** The *difference* between the initial great-circle course and the single rhumb line course is called **conversion angle.** This is usually about half the difference between initial and final great-circle courses.

Conversion angles for difference of longitude to 120°, sufficient for virtually all situations in which great-circle sailing is likely to be used by ships, are given in table 1. To use the table, measure the rhumb line course on a Mercator chart (or compute it by Mercator or mid-latitude sailing) and apply the conversion angle to find the initial great-circle course. The sign of the correction can be determined by means of the tabulation at the bottom of the table. With a little practice, one can determine the sign mentally by remembering that the great-circle course always lies nearer the pole (in the hemisphere of the point of departure) than the rhumb line course, except for those values given in italics.

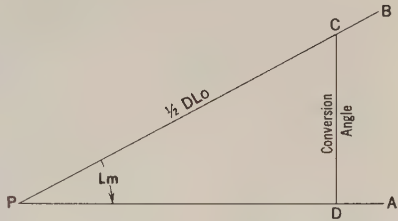


FIGURE 821.—Graphical solution for conversion angle.

The use of the conversion angle as taken directly from the table results in a course line tangent to the great circle (as plotted on a Mercator chart) and hence one that carries the vessel to higher latitudes than the great circle. To convert this to the corresponding chord, as in great-circle sailing by chart, divide the conversion angle by the number of legs, and *subtract* this value from the tabulated conversion angle before applying the correction to the rhumb line course. At the end of each leg make a new solution, using the position of the vessel as the point of departure.

This method does not indicate the suitability of the route unless the entire solution is made in advance and the results plotted on a chart.

Approximate values of conversion angle can be found by the formula:

$$\tan \text{conversion angle} = \frac{\sin \text{Lm} \tan \frac{1}{2} \text{DLo}}{\cos \frac{1}{2} l}$$

if both points are on the same side of the equator. For small differences of latitude, $\cos \frac{1}{2} l$ can be considered 1 without introducing a significant error. The tangent of a small angle equals, approximately, the angle itself (in radians). Therefore, for small values of DLo and l (up to 15° to 20°) the formula can be simplified:

$$\text{conversion angle} = \tfrac{1}{2} \text{DLo} \sin \text{Lm}.$$

This formula can be solved graphically (fig. 821). Draw any line *PA*, and from *P* draw *PB* making an angle with *PA* equal to Lm. Along *PB* measure ½ DLo, letting any convenient linear unit equal 1°. From *C*, the point thus found, draw *CD* perpendicular to *PA*. The length of *CD* in the units used for ½ DLo is the conversion angle in degrees. Conversion angle can also be determined by table 3, using mid latitude as course, ½ DLo as *D*, and conversion angle as p. The value found by formula, however solved, may not be accurate for large differences of latitude.

822. Great-circle sailing by computation.—In figure 822, 1 is the point of departure, 2 the destination, P the pole nearer 1, $1XV2$ the great circle through 1 and 2, V the vertex, and X any point on the great circle. The arcs $P1$, PX, PV, and $P2$ are the colatitudes of points 1, X, V, and 2, respectively. If 1 and 2 are on opposite sides of the equator, $P2$ is $90° + L_2$. The length of arc 1–2 is the great-circle distance between 1 and 2. Arcs 1–2, $P1$, and $P2$ form a spherical triangle. The angle at 1 is the initial great-circle course from 1 to 2, that at 2 the supplement of the final great-circle course (or the initial course from 2 to 1), and that at P the DLo between 1 and 2.

Great-circle sailing by computation usually involves solution for the initial great-circle course; the distance; latitude and longitude, and sometimes the distance, of the vertex; and the latitude and longitude of various points (X) on the great circle. The computation for initial course and the distance involves solution of an oblique spherical triangle, and any method of solving such a triangle can be used. If 2 is the **geographical position** (**GP**) of a celestial body (the point at which the body is in the zenith), this triangle is solved in celestial navigation, except that 90°−D (the altitude) is desired instead of D. The solution for the vertex and any point X usually involves the solution of right spherical triangles.

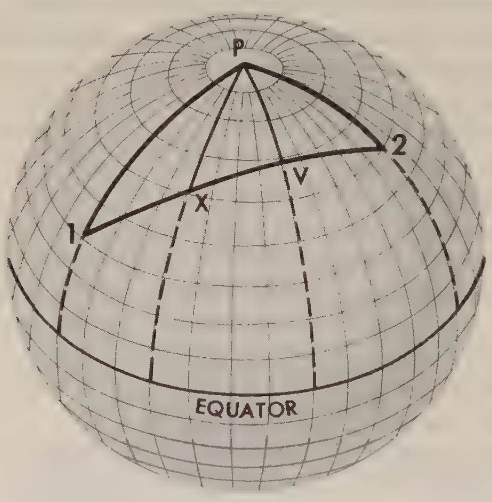


FIGURE 822.—The navigational triangle of great-circle sailing.

Although various formulas can be used, haversine formulas are considered most suitable for determining initial course and the distance, as these avoid the ambiguity that may arise through the use of trigonometric functions which do not indicate the quadrant in which the answer lies. In the formulas given below, the subscripts refer to the points indicated in figure 822. All terms without subscripts are from 1 to 2, D_v and DLo_v are from 1 to V, and D_{vx} and DLo_{vx} are from V to X. Other quantities can be computed by interchanging 1 and 2 in figure 822 and using the same formulas. The following formulas are suitable for great-circle sailing by computation:

$$\text{hav D} = \text{hav DLo} \cos L_1 \cos L_2 + \text{hav } l$$

which may be written $\text{hav D} = \text{hav } \theta + \text{hav } l$ (where $\text{hav } \theta = \text{hav DLo} \cos L_1 \cos L_2$)

$$\text{hav C} = \sec L_1 \csc D\,[\text{hav co}L_2 - \text{hav }(D \sim \text{co}L_1)]$$

$$\cos L_v = \cos L_1 \sin C$$

$$\sin DLo_v = \cos C \csc L_v$$

$$\sin D_v = \cos L_1 \sin DLo_v$$

$$\tan L_x = \cos DLo_{vx} \tan L_v$$

Example.—A ship is proceeding from Manila to Los Angeles. The captain wishes to use great-circle sailing from lat. 12°45′.2 N, long. 124°20′.1 E, off the entrance to San Bernardino Strait, to lat. 33°48′.8 N, long. 120°07′.1 W, five miles south of Santa Rosa Island.

Required.—(1) The initial great-circle course.

(2) The great-circle distance.

(3) The latitude and longitude of the vertex.

(4) The distance from the point of departure to the vertex.

(5) The latitude and longitude of points at DLo intervals of 12° each side of the vertex.

Solution.—

λ_1	124°20′.1 E					D	103°05′.9
λ_2	120°07′.1 W					coL_1	77°14′.8
DLo	115°32′.8 E	l hav	9.85468			D $\sim$ coL_1	25°51′.1
L_1	12°45′.2 N	l cos	9.98915			l sec	0.01085
L_2	33°48′.8 N	l cos	9.91953				
θ		l hav	9.76336	n hav	0.57991		
l	21°03′.6 N			n hav	0.03340		
D	103°05′.9			n hav	0.61331	l csc	0.01145
coL_2	56°11′.2	n hav	0.22175				
D $\sim$ coL_1	25°51′.1	n hav(−)	0.05004				
		n hav	0.17171			l hav	9.23480
(1) Cn	050°.3			C	N 50°19′.3 E	l hav	9.25710
(2) D	6185.9 mi.						
L_1	12°45′.2 N	l cos	9.98915			l cos	9.98915
C	N 50°19′.3 E	l sin	9.88629	l cos	9.80514		
(3) L_v	41°21′.2 N	l cos	9.87544	l csc	0.17999		
(3) λ_v	160°34′.4 W	DLo_v	75°05′.5 E	l sin	9.98513	l sin	9.98513
D_v	70°28′.5					l sin	9.97428
(4) D_v	4228.5 mi.						

DLo_{vx}	12°00′.0	24°00′.0	36°00′.0	48°00′.0	60°00′.0	72°00′.0
l cos DLo_{vx}	9.99040	9.96073	9.90796	9.82551	9.69897	9.48998
l tan L_v	9.94457	9.94457	9.94457	9.94457	9.94457	9.94457
l tan L_x	9.93497	9.90530	9.85253	9.77008	9.64354	9.43455
(5) L_x	40°43′.6 N	38°48′.1 N	35°27′.2 N	30°29′.8 N	23°45′.2 N	15°12′.9 N
(5) λ_x	172°34′.4 W	175°25′.6 E	163°25′.6 E	151°25′.6 E	139°25′.6 E	127°25′.6 E
(5) λ_x	148°34′.4 W	136°34′.4 W	124°34′.4 W	———	———	———

CoL_1 is always 90° − L_1. CoL_2 is 90° − L_2 if L_1 and L_2 are of same name, and 90° + L_2 if of contrary name.

D $\sim$ coL_1 is always the numerical difference between D and coL_1.

C is labeled N or S to agree with L_1 and E or W to agree with DLo. This is not the same as in rhumb line sailings. In great-circle sailing L_2 may be south of L_1, yet the initial course may have a northerly component.

L_v is always numerically equal to or greater than L_1 or L_2.

If C is less than 90°, the nearer vertex is toward L_2; but if C is greater than 90°, the nearer vertex is in the opposite direction.

DLo_v and D_v of the nearer vertex are never greater than 90°. However, when L_1 and L_2 are of contrary name, the other vertex, 180° away, may be the preferable one to use in the solution for various points along the great circle if it is nearer the mid point of the great circle.

The vertex nearer L_1 has the same name (N or S) as L_1.

L_x has the same name (N or S) as L_v if DLo_{vx} is less than 90°, but the opposite name if DLo_{vx} is greater than 90°.

The great circle is a symmetrical curve about the vertex. Hence, any given DLo can be applied to λ_v in both directions (E and W) to find two points having the same

latitude. However, if whole degrees of λ_x are desired, different E and W intervals are needed unless λ_v is a whole degree or an exact half degree.

Only those points on the portion of the great circle between the point of departure and destination are recorded.

The following formulas are sometimes useful in great-circle sailing:

$$\sin C = \sin DLo \cos L_2 \csc D$$

This offers a simpler solution than the haversine formula, but unless L_2 is of the same name and equal to or greater than L_1, it leaves doubt as to whether C is less or greater than 90°.

$$\cos C = \sin L_v \sin DLo_v$$

This offers an even simpler solution, but has the same limitations as those given above. Further, it requires a knowledge of the position of the vertex. It is particularly useful in determining the direction of the great circle at any given point along the circle.

$$\sin L_x = \sin L_v \cos D_{vx}$$

$$\sin DLo_{vx} = \sec L_x \sin D_{vx}$$

These formulas are useful for finding points at approximately equal *distances*, along the great circle, from the vertex, should this be considered more desirable than finding points of equal DLo. The method of selecting the longitude (or DLo_{vx}) and determining the latitude at which the great circle crosses the selected meridian provides shorter legs in higher latitudes and longer legs in lower latitudes, where the difference between the great circle and rhumb line is smaller. In using these formulas, D_{vx} is expressed in degrees. If it is greater than 90° (5,400 miles), L_x is of contrary name to L_v, and DLo_{vx} is greater than 90°.

$$\cos DLo_{vx} = \tan L_x \cot L_v$$

This formula is useful in determining the longitude (or DLo_{vx}) at which the great circle crosses selected parallels of latitude. If L_x is of contrary name to L_v, DLo_{vx} is greater than 90°. This formula is also used in composite sailing (art. 825).

823. Great-circle sailing by table.—Although tables designed to facilitate the computations of great-circle sailing have been published, no such table is in common use today. However, any method of solving the astronomical triangle of celestial navigation can be used for solving great-circle sailing problems. When such an adaptation is made, the point of departure replaces the assumed position of the observer, the destination replaces the geographical position of the body, difference of longitude replaces meridian angle, initial course angle replaces azimuth angle, and great-circle distance replaces zenith distance (90°−altitude). Therefore, any table of azimuths (if the entering values are meridian angle, declination, and latitude) can be used for determining initial great-circle course. H.O. Pubs. Nos. 208, 211, 214, 249, 260, and 261 are examples of tables that can be used for this purpose. Tables which provide solution for altitude, such as H.O. Pubs. Nos. 208, 211, 214, and 249, can be used for determining great-circle distance. The required distance is 90°−altitude (90°+negative altitudes).

In inspection tables such as H.O. Pubs. Nos. 214, 249, 260, and 261, the given combination of L_1, L_2, and DLo may not be tabulated. In this case reverse the name of L_2 and use 180°—DLo for entering the table. The required course angle is then 180° minus the tabulated azimuth, and distance is 90° plus the altitude. If neither com-

bination can be found, solution cannot be made by that method. By interchanging L_1 and L_2, one can find the supplement of the final course angle.

Solution by table often provides a rapid approximate check, but accurate results usually require triple interpolation (art. P4). Inspection tables do not provide solution for points along the great circle, and therefore are of limited usefulness.

An example of the use of H.O. Pub. No. 214 for great-circle sailing is given as example 8, near the front of each volume.

824. Altering a great-circle track to avoid obstructions.—Great-circle sailing cannot be used unless the great-circle track is free from obstructions. It does not start until one clears the harbor and takes his departure (art. 806), and often ends near the entrance to the destination. However, islands, points of land, or other obstructions may prevent the use of great-circle sailing over the entire distance. One of the principal advantages of solution by great-circle chart is that the presence of any obstructions is immediately apparent.

Often a relatively short run by rhumb line is sufficient to reach a point from which the great-circle track can be followed. Where a choice is possible, the rhumb line selected should conform as nearly as practicable to the direct great circle.

If the great circle crosses a small island, one or more legs may be altered slightly, or perhaps the drift of the vessel will be sufficient to make any planned alteration unnecessary. The possible use of the island in obtaining an en route fix should not be overlooked. If a larger obstruction is encountered, as in the case of the Aleutian Islands on a great circle from Seattle to Yokohama, some judgment may be needed in selecting the track. It may be satisfactory to follow a great circle to the vicinity of the obstruction, one or more rhumb lines along the edge of the obstruction, and another great circle to the destination. Another possible solution is the use of composite sailing (art. 825), and still another the use of two great circles, one from the point of departure to a point near the maximum latitude of unobstructed water, and the second from this point to the destination.

It is sometimes desirable to alter a great-circle track to avoid unfavorable winds or currents. The shortest route is not always the quickest.

Whatever the problem, a great-circle chart can be helpful in its solution.

825. Composite sailing.—When the great circle would carry a vessel to a higher latitude than desired, a modification of great-circle sailing, called **composite sailing,** may be used to good advantage. The composite track consists of a great circle from the point of departure and tangent to the limiting parallel, a course line along the parallel, and a great circle tangent to the limiting parallel and through the destination.

Solution of composite sailing problems is most easily made by means of a great-circle chart. Lines from the point of departure and the destination are drawn tangent to the limiting parallel. The coordinates of various selected points along the composite track are then measured and transferred to a Mercator chart, as in great-circle sailing (art. 820).

Composite sailing problems can also be solved by computation. For this purpose the last formula of article 822 is used:

$$\cos DLo_{vx} = \tan L_x \cot L_v.$$

In the computation, the point of departure and the destination are used successively as point X.

Example.—A ship leaves Baltimore, bound for Bordeaux (Royan), France. The captain desires to use composite sailing from lat. 36°57′.7 N, long. 75°42′.2 W, one mile

south of Chesapeake Lightship, to lat. 45°39′.1 N, long. 1°29′.8 W, near the entrance to Grande Passe de l'Ouest, limiting the maximum latitude to 47° N.

Required.—(1) The longitude at which the limiting parallel is reached. (2) The longitude at which the limiting parallel should be left.

Solution.—

L_1	36°57′.7 N	l tan 9.87651	
L_2	45°39′.1 N		l tan 0.00988
L_v	47°00′.0 N	l cot 9.96966	l cot 9.96966
DLo_{v1}	45°26′.1 E	l cos 9.84617	
DLo_{v2}	17°27′.0 W		l cos 9.97954
(1) λ_{v1}	30°16′.1 W		
(2) λ_{v2}	18°56′.8 W		

Composite sailing applies only when the vertex lies between the point of departure and the destination.

The remainder of the problem is one of solving the two great circles by great-circle sailing and the east-west portion by parallel sailing. Since both great circles have vertices at the same parallel, computation for C, D, and DLo_{vx} can be made by considering them parts of the *same* great circle with L_1, L_2, and L_v as given and $DLo = DLo_{v1} + DLo_{v2}$. The total distance is the sum of the great-circle and parallel distances. In finding λ_x be careful to apply DLo_{vx} to the correct vertex and in the correct direction.

Problems

806a. Draw a small area plotting sheet by either method explained in article 324, covering the area between latitude 32°-34° N and longitude 118°-122° W. Plot the following points:

A	L 33°49′.1 N	λ 120°52′.0 W	*C*	L 33°38′.0 N	λ 118°38′.6 W
B	L 32°17′.4 N	λ 121°28′.0 W	*D*	L 32°30′.6 N	λ 118°36′.2 W

Required.—(1) The bearings of *B*, *C*, and *D* from *A*.

(2) The course and distance of *A*, *B*, and *C* from *D*.

Answers.—(1) B_{AB} 198°.5, B_{AC} 095°.5, B_{AD} 124°; (2) C_{DA} 304°, D_{DA} 138.8 mi., C_{DB} 264°.5, D_{DB} 145.7 mi., C_{DC} 358°.5, D_{DC} 67.2 mi.

806b. Use the plot of problem 806a. A ship starts from *A* at 1200, and steams as follows:

Time	*Course*	*Speed*
1200	120°	15 kn.
1330	240°	15 kn.
1500	240°	17 kn.
1800	125°	20 kn.
2000	090°	20 kn.
2300	015°	10 kn.
0500		

Plot and label the dead reckoning course line and DR positions.

Required.—(1) The dead reckoning position of the ship at 0500.

(2) The bearing and distance of *D* from the 2300 DR position.

(3) The course and distance from the 0500 DR position to *C*.

(4) Estimated time of arrival (ETA), to the nearest minute, at *C* if the ship proceeds directly from the 0500 DR position at 20 knots.

Answers.—(1) 0500 DR: L 33°35′.1 N, λ 119°35′.8 W; (2) B 096°, D 66.0 mi.; (3) C 086°, D 48.1 mi.; (4) ETA 0724.

807a. A ship on course 120°, speed 12 knots, is steaming through a current having a set of 350° and a drift of 1.5 knots.

Required.—Course and speed made good.

Answers.—Course made good 114°, speed made good 11.1 kn.

807b. The captain desires to make good a course of 180° through a current having a set of 090° and a drift of two knots, using a speed of 11 knots.

Required.—The course to steer and the speed made good.

Answers.—Course to steer 190°.5, speed made good 10.8 kn.

807c. The captain desires to make good a course of 325° and a speed of 20 knots through a current having a set of 270° and a drift of one knot.

Required.—The course to steer and the speed to use.

Answers.—Course to steer 327°, speed to use 19.4 kn.

813a. A vessel steams 117.3 miles on course 214°.

Required.—(1) Difference of latitude, (2) departure, by plane sailing.

Answers.—(1) l 97′.2 S, (2) p 65.6 mi. W.

813b. A steamer is bound for a port 173.3 miles south and 98.6 miles east of the vessel's position.

Required.—(1) Course, (2) distance, by plane sailing.

Answers.—(1) C 150°.4; (2) D 199.4 mi. by computation, 199.3 mi. by traverse table.

814a. A ship steams as follows: course 359°, distance 28.8 miles; course 006°, distance 16.4 miles; course 266°, distance 4.9 miles; course 144°, distance 3.1 miles; course 333°, distance 35.8 miles; course 280°, distance 19.3 miles.

Required.—(1) Course, (2) distance, by traverse sailing.

Answers.—(1) C 334°.4, (2) D 86.1 mi.

814b. A lightship bears 020°, distant 3.4 miles from a ship standing out to sea at a speed of 12 knots. The ship steams on the following courses for the times indicated: course 090° for 18^m, course 114° for 1^h12^m, course 070° for 3^h45^m, course 095° for 54^m, course 050° for 36^m. The navigator estimates that during this entire time the ship has been in a current having a set of 315° and a drift of 0.5 knot.

Required.—The bearing and distance of the estimated position from the lightship, by traverse sailing.

Answers.—B 080°.3, D 73.0 mi. by computation, 73.1 mi. by traverse table.

815a. The 1530 DR position of a ship is lat. 44°36′.3 N, long. 31°18′.3 W. The ship is on course 270°, speed 17 knots.

Required.—The 2000 DR position, by parallel sailing.

Answer.—2000 DR: L 44°36′.3 N, λ 33°05′.7 W.

815b. The captain of the ship of problem 815a desires to change course when the ship arrives at long. 38°00′.0 W.

Required.—Estimated time of arrival (ETA) at the turning point, by parallel sailing.

Answer.—ETA 0819 the following day.

816a. A vessel steams 263.3 miles on course 340° from lat. 16°32′.2 S, long. 1°04′.4 E.

Required.—(1) Latitude and (2) longitude of the point of arrival, by middle-latitude sailing.

Answer.—By computation or traverse table: (1) L 12°24′.8 S, (2) λ 0°28′.6 W.

816b. A vessel leaves lat. 45°00′.0 N, long. 150°00′.0 W and arrives at lat. 38°18′.7 N, long. 137°14′.6 W.

Required.—(1) Course made good, (2) distance made good, by middle-latitude sailing.

Answers.—(1) C 125°.1; (2) D 698.6 mi. by computation, 697.9 mi. by traverse table.

816c. A vessel is at lat. 1°08′.3 S, long. 175°24′.5 E. It steams at 13.5 knots on course 075° for 22^h15^m. Twenty-four hours after that it is at lat. 0°06′.6 S, long. 174°20′.0 W.

Required.—(1) Latitude and (2) longitude of the point of arrival after 22^h15^m; (3) course and (4) distance made good during the last 24 hours of steaming; and (5) course and (6) distance made good over entire period, by middle-latitude sailing.

Answers.—(1) L 0°09′.5 N by computation, 0°09′.4 N by traverse table; (2) λ 179°45′.3 W; (3) C 092°.8; (4) D 325.7 mi. by computation, 338.3 mi. by traverse table; (5) C 084°.3; (6) D 618.5 mi. by computation, 625.6 mi. by traverse table.

817a. A ship at lat. 33°53′.3 S, long. 18°23′.1 E, leaving Cape Town, heads for Ambrose Lightship, lat. 40°27′.1 N, long. 73°49′.4 W.

Required.—(1) Course and (2) distance, by Mercator sailing.

Answers.—(1) C 310°.9; (2) D 6,811.5 mi. by computation, 6,812.8 mi. by traverse table. Compare these answers with those of problem 822, the great-circle sailing solution between the same points.

817b. A ship at lat. 15°03′.7 N, long. 151°26′.8 E steams 57.4 miles on course 035°.

Required.—(1) Latitude and (2) longitude of the point of arrival, by Mercator sailing.

Answers.—(1) L 15°50′.7 N, λ 152°00′.7 E.

821. Point *A* is at lat. 35°24′.2 N, long. 125°02′.6 W. Point *B* is at lat. 41°09′.2 N, long. 147°22′.6 E.

Required.—The conversion angle from *A* to *B* (1) by table 1, (2) by complete formula, (3) by construction; (4) rhumb line course by Mercator sailing; (5) initial great-circle course (using table 1, conversion angle); (6) chord course for first leg, if there are to be 17 legs (using table 1, conversion angle).

Answers.—Conversion angle (1) 29°.8, (2) 30°.7, (3) 27°.1, (4) Cn 274°.8, (5) Cn 304°.6, (6) Cn 302°.8.

822. A ship leaves Cape Town bound for New York City. The captain decides to use great-circle sailing from lat. 33°53′.3 S, long. 18°23′.1 E (near Green Point Light) to Ambrose Lightship, lat. 40°27′.1 N, long. 73°49′.4 W.

Required.—(1) The initial great-circle course.

(2) The great-circle distance.

(3) The latitude and longitude of both vertices.

(4) The distance from the point of departure to each vertex.

(5) The latitude and longitude of points on the great circle at longitude 15° E and at each 5° of longitude thereafter to longitude 70° W.

Answers.—(1) C 304°.5. (2) D 6,762.7 mi. (3) L_v 46°49′.4 S, λ_v 69°18′.8 E; L_v 46°49′.4 N, λ_v 110°41′.2 W. (4) D 2,407.5 mi. to eastern hemisphere vertex, 8,392.5 mi. to western hemisphere vertex. (5) L_1 33°53′.3 S, λ_1 18°23′.1 E (point of departure); L_x 31°52′.2 S, λ_x 15°00′.0 E; L_x 28°32′.5 S, λ_x 10°00′.0 E; L_x 24°47′.7 S, λ_x 5°00′.0 E; L_x 20°37′.8 S, λ_x 0°00′.0; L_x 16°04′.5 S, λ_x 5°00′.0 W; L_x 11°10′.8 S, λ_x 10°00′.0 W; L_x 6°01′.7 S, λ_x 15°00′.0 W; L_x 0°43′.9 S, λ_x 20°00′.0 W; L_x 4°35′.0 N, λ_x 25°00′.0 W; L_x 9°47′.2 N, λ_x 30°00′.0 W; L_x 14°45′.7 N, λ_x 35°00′.0 W; L_x 19°25′.0 N, λ_x 40°00′.0 W; L_x 23°41′.5 N, λ_x 45°00′.0 W; L_x 27°33′.3 N, λ_x 50°00′.0 W; L_x 30°59′.8 N, λ_x 55°00′.0 W; L_x 34°01′.7 N, λ_x 60°00′.0 W; L_x 36°40′.1 N, λ_x 65°00′.0 W; L_x 38°56′.6 N, λ_x 70°00′.0 W; L_2 40°27′.1 N, λ_2 73°49′.4 W (destination).

825. A ship is to steam from Valparaiso, Chile, to Wellington, New Zealand. The captain wishes to use composite sailing from lat. 32°58′.0 S, long. 71°41′.2 W, off Punta Angeles Light, to lat. 42°00′.0 S, long. 175°00′.0 E, near Cape Palliser, limiting the maximum latitude to 50° S.

Required.—(1) The longitude at which the limiting parallel is reached.

(2) The longitude at which the limiting parallel should be left.

(3) The initial great-circle course.

(4) The total distance.

(5) The latitude and longitude of points along the great circles at intervals of 10° of DLo from the vertices.

Answers.—(1) λ_{v1} 128°42′.9 W. (2) λ_{v2} 144°04′.3 W. (3) C 230°.0. (4) D 5,024.6 mi. (5) L_1 32°58′.0 S, λ_1 71°41′.2 W (point of departure); L_x 37°27′.2 S, λ_x 78°42′.9 W; L_x 42°23′.6 S, λ_x 88°42′.9 W; L_x 45°54′.3 S, λ_x 98°42′.9 W; L_x 48°14′.2 S, λ_x 108°42′.9 W; L_x 49°34′.1 S, λ_x 118°42′.9 W; L_{v1} 50°00′.0 S, λ_{v1} 128°42′.9 W (first vertex); L_{v2} 50°00′.0 S, λ_{v2} 144°04′.3 W (second vertex); L_x 49°34′.1 S, λ_x 154°04′.3 W; L_x 48°14′.2 S, λ_x 164°04′.3 W; L_x 45°54′.3 S, λ_x 174°04′.3 W; L_x 42°23′.6 S, λ_x 175°55′.7 E; L_2 42°00′.0 S, λ_2 175°00′.0 E (destination).

CHAPTER IX

PILOTING

General

901. Introduction.—On the high seas, where there is no immediate danger of grounding, navigation is a comparatively leisurely process. Courses and speeds are maintained over relatively long periods, and fixes are obtained at convenient intervals. Under favorable conditions a vessel might continue for several days with no positions other than those obtained by dead reckoning, or by estimate, and with no anxiety on the part of the captain or navigator. Errors in position can usually be detected and corrected before danger threatens.

In the vicinity of shoal water the situation is different. Frequent or continuous positional information is usually essential to the safety of the vessel. An error which at sea may be considered small, may in pilot waters be intolerably large. Frequent changes of course and speed are common. The proximity of other vessels increases the possibility of collision. Navigation under these conditions is called **piloting** or **pilotage.**

No other form of navigation requires the continuous alertness needed in piloting. At no other time is navigational experience and judgment so valuable. The ability to work rapidly and to correctly interpret all available information, always keeping "ahead of the vessel," may mean the difference between safety and disaster.

In piloting, positions are commonly obtained by reference to nearby landmarks, or the bottom. Advancements in electronics have provided additional aids which are of particular value in piloting, and have extended the range of piloting techniques far to sea.

902. Preparation for piloting.—Because the time element is often of vital importance in piloting, adequate preparation is important. Long-range preparation includes the acquisition of a thorough knowledge of the methods and techniques of piloting, and the organization and training of those who will assist in any way. This includes the steersman, who will be granted less tolerance in straying from the prescribed course than when farther offshore.

The more immediate preparation includes a study of the charts and publications of the area to familiarize oneself with the channels, shoals, tides, currents, aids to navigation, etc. One seldom has time to seek such information once he is proceeding in pilot waters. The more detailed preparation required for leaving or entering port is given in chapter XXIII.

Position

903. Lines of position.—As in electronic and celestial navigation, piloting makes extensive use of **lines of position.** Such a line is one on some point of which the vessel may be presumed to be located, as a result of observation or measurement. It may be highly reliable, or of questionable accuracy. Lines of position are of great value, but one should always keep in mind that *they can be in error* because of imperfections in instruments used for obtaining them and human limitations in those who use the instruments and utilize the results. The extent to which one can have confidence in various lines of position is a matter of judgment acquired from experience.

A line of position might be a straight line (actually a part of a great circle), an arc of a circle, or part of some other curve such as a hyperbola (art. O34). An appropriate label should be placed on the plot of a line of position *at the time it is drawn*, to avoid possible error or confusion. A label should include all information essential for identification, but no extraneous information. The labels shown in this volume are recommended.

904. Bearings.—A **bearing** is the horizontal direction of one terrestrial point from another. It is usually expressed as the angular difference between a reference direction and the given direction. In navigation, north is generally used as the reference direction, and angles are measured clockwise through 360°. It is customary to express all bearings in three digits, using preliminary zeros where needed. Thus, north is 000° or 360°, a direction 7° to the right of north is 007°, east is 090°, southwest is 225°, etc.

For plotting, *true* north is used as the reference direction. A bearing measured from this reference is called a **true bearing (TB)**. A **magnetic, compass,** or **grid bearing (MB. CB,** or **GB)** results from using magnetic, compass, or grid north, respectively, as the reference direction. This is similar to the designation of courses. In the case of bearings, however, one additional reference direction is often convenient. This is the heading of the ship. A bearing expressed as angular distance from the heading is called a **relative bearing (RB)**. It is usually measured clockwise through 360°, but for some purposes it is more conveniently measured either clockwise or counterclockwise through 180°, and designated *right* or *left*, respectively. A relative bearing may be expressed in still another way, as indicated in figure 904. Except for dead ahead and points at 45° intervals from it, this method is used principally for indicating directions obtained visually, without precise measurement. An even more general indication of relative bearing may be given by such directions as "ahead," "on the starboard bow," "on the port quarter," "astern." The term *abeam* may be used as the equivalent of either the general "on the beam" or, sometimes, the more precise "broad on the beam." Degrees are sometimes used instead of points to express relative bearings by the system illustrated in figure 904. However, if degrees are used, a better practice is to use the 360° or 180° system. Thus, a relative bearing of "20° forward of the port beam" is better expressed as "290°," or "70° left."

True, magnetic, and compass bearings are interconverted by the use of variation and deviation, or compass error, in the same manner as courses. Interconversion of relative and other bearings is accomplished by means of the heading. If true heading is added to a relative bearing, true bearing results (RB+TH=TB). If magnetic, compass, or grid heading (MH, CH, GH) is added to relative bearing, the corresponding magnetic, compass, or grid bearing is obtained.

A **bearing line** extending in the direction of an observed bearing of a charted object is one of the most widely used lines of position. If one knows that an identified landmark has a certain bearing from his vessel, the vessel can only be on the line at which such a bearing might be observed, for at any other point the bearing would be different. This line extends outward from the landmark, along the *reciprocal* of the observed bearing. Thus, if a lighthouse is *east* of a ship, that ship is *west* of the lighthouse. If a beacon bears 156°, the observer must be on a line extending 156°+180°=336° from the beacon. Since observed bearing lines are great circles, this relationship is not strictly accurate, but the error is significant only where the great circle departs materially from the rhumb line, as in high latitudes (ch. XXV).

Bearings are obtained by compass, gyro repeater, pelorus, alidade, radar, etc. One type of bearing can be obtained by eye without measurement. When two objects appear directly in line, one behind the other, they are said to be "in range," and together they constitute a **range.** For accurately charted objects, a range provides the most

accurate line of position obtainable, and one of the easiest to observe. Tanks, steeples, towers, cupolas, etc., sometimes form natural ranges. A navigator should be familiar with prominent ranges in his operating area, particularly those which can be used to mark turning points, indicate limits of shoals, or define an approach heading or let-go point of the anchorage of a naval vessel. So useful is the range in marking a course that artificial ranges, usually in the form of two lighted beacons, have been installed in line with channels in many ports. A vessel proceeding along the channel has only

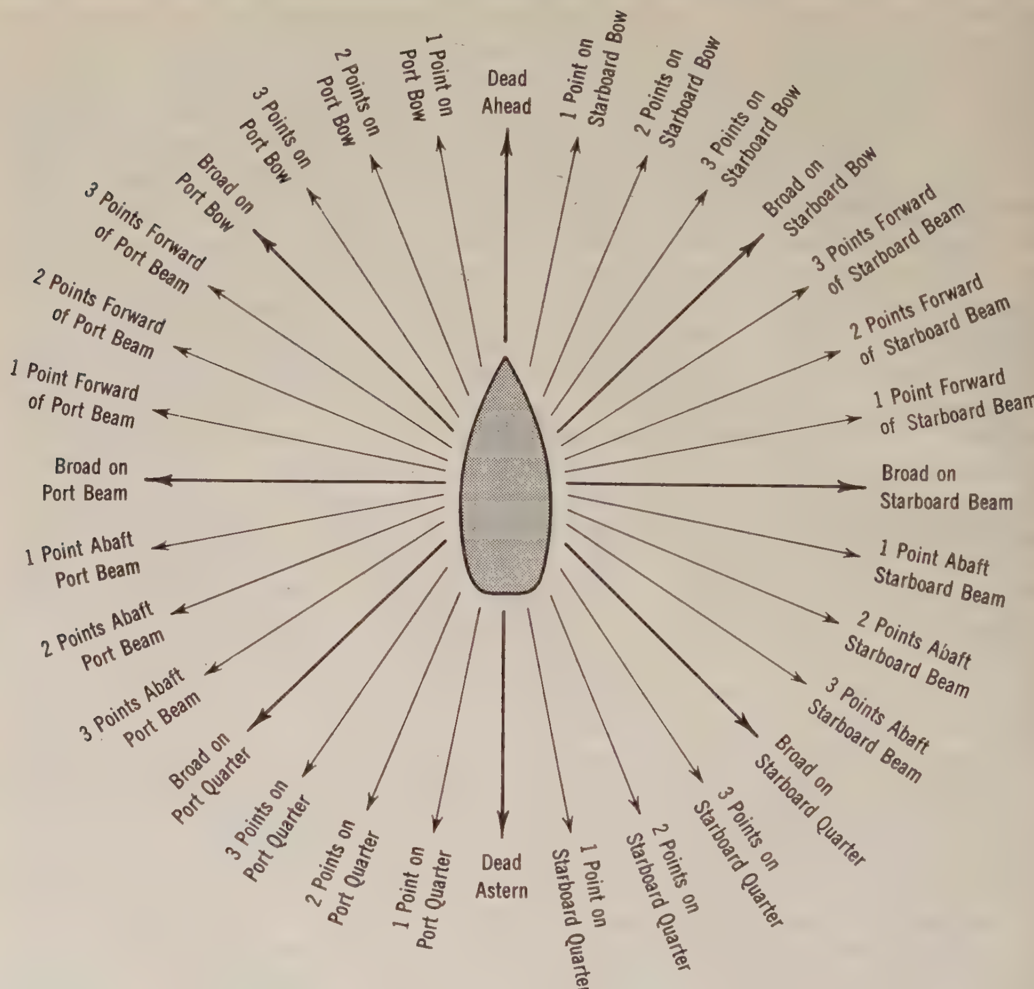


FIGURE 904.—One method of expressing relative bearings.

to keep the beacons in range to remain in the center of the channel. If the *farther* beacon (customarily the higher one) appears to "open out" (move) to the right of the forward (lower) beacon, one knows that he is to the right of his desired course line. Similarly, if it opens out to the left, the vessel is off course to the left.

It is good practice to plot only a short part of a line of position in the vicinity of the vessel, to avoid unnecessary confusion and to reduce the chart wear by erasure. Particularly, one should avoid the drawing of lines through the chart symbol indicating the landmark used. In the case of a range, a straightedge is placed along the two objects, and the desired portion of the line is plotted. One need not know the numerical

value of the bearing represented by the line. However, if there is any doubt as to the identification of the objects observed, the measurement of the bearing should prove useful.

A bearing line is labeled with the time above the line, and the bearing below the line. A range is labeled with the time only.

905. Distance.—If a vessel is known to be a certain distance from an identified point on the chart, it must be somewhere on a circle with that point as the center and the distance as the radius. An arc of the circle can be drawn and labeled with the time above the line and the distance below the line.

Distances are obtained by radar, range finder, stadimeter, synchronized sound and radio signals, synchronized air and water sounds, vertical sextant angles (table 9), etc. If vertical sextant angles are used, measurement should be made from the top of the object to the visible sea horizon, if it is available. If measurement is made to a water line not vertically below the top of the object, a problem may be encountered because distance from table 9 is to the point vertically below the top of the object, while the distance used for entering table 22 to determine dip short of the horizon is to the water line. Generally, any differences in these two distances can be determined from the chart. This problem may, in some cases, be avoided by decreasing the height of eye sufficiently to bring the horizon between the observer and the object.

906. The fix.—A line of position, however obtained, represents a series of possible positions, but not a single position. However, if *two simultaneous, nonparallel* lines of position are available, the only position that satisfies the requirements of being on both lines at the same time is the intersection of the two lines. This point is one form of **fix.** Examples of several types of fix are given in the illustrations. In figure 906a a fix is obtained from two bearing lines. The fix of figure 906b is obtained by two distance circles. Figure 906c illustrates a fix from a range and a distance. In figure 906d a bearing and distance of a single object are used.

Some consideration should be given to the selection of objects to provide a fix. It is essential, for instance, that the objects be identified. The angle between lines of position is important. The ideal is 90°. If the angle is small, a slight error in measur-

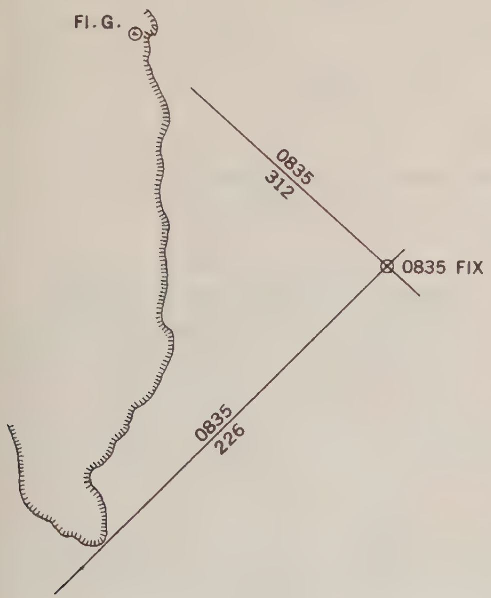


Figure 906a.—A fix by two bearing lines.

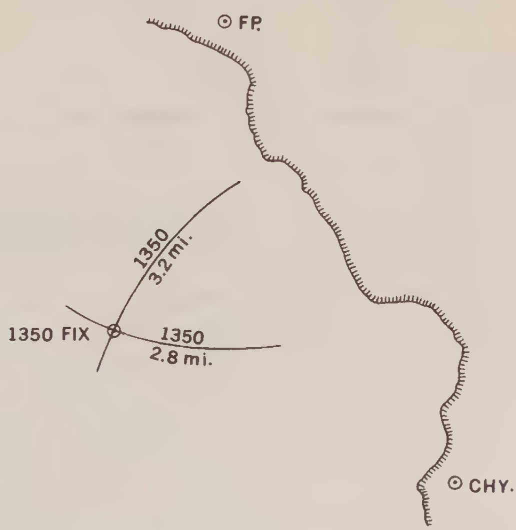


Figure 906b.—A fix by two distances.

ing or plotting either line results in a relatively large error in the indicated position. In the case of a bearing line, nearby objects are preferable to those at a considerable distance, because the linear (distance) error resulting from an angular error increases with distance. Thus, an error of 1° represents an error of about 100 feet if the object is one mile distant, 1,000 feet if the object is ten miles away, and one mile if the object is 60 miles from the observer.

Another consideration is the type of object. Lighthouses, spires, flagpoles, etc., are good objects because the point of observation is well defined. A large building, most nearby mountains, a point of land, etc., may leave some reasonable doubt as to the exact point used for observation. If a tangent is used (fig. 906a), there is a possibility that a low spit may extend seaward from the part observed. A number of towers, chimneys, etc., close together require careful identification. A buoy or a lightship may drag anchor and be out of position. Most buoys are secured by a single anchor and so have a certain radius of swing as the tide, current, and wind change.

Although two accurate nonparallel lines of position completely define a position, if they are taken at the same time, an element of doubt always exists as to the accuracy of the lines. Additional lines of position can serve as a check on those already obtained, and, usually, to reduce any existing error. If three lines of position cross at a common point, or form a small triangle, it is usually a reasonable assumption that the position is reliable, and defined by the center of the figure. However, this is not *necessarily* so, and one should be aware of the possibility of an erroneously indicated position. Some-

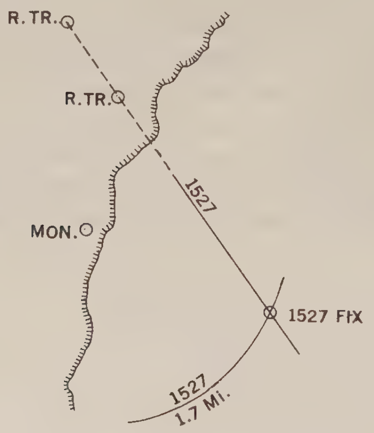


FIGURE 906c.—A fix by a range and distance.

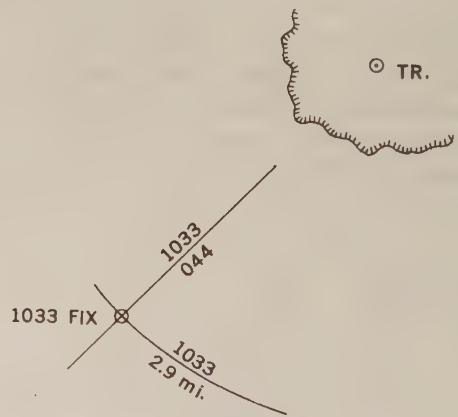


FIGURE 906d.—A fix by distance and bearing of single object.

times an error can be identified. For instance, if several fixes obtained by bearings on three objects produce triangles of about the same size, one might reasonably suspect a constant error in the observation of the bearings, particularly if the same instrument is used for all observations, or in the plotting of the lines. If the application of a constant error to all bearings results in a point, or near-point fix, the navigator is usually justified in applying such a correction. This situation is illustrated in figure 906e, where the solid lines indicate the original plot, and the broken lines indicate each line of position moved 3° in a clockwise direction. If different instruments are used for observation, one of them might be consistently in error. This might be detected by altering all bearings observed by that instrument by a fixed amount and producing good fixes. However, one seldom has time for much experimentation of this kind while piloting. If an error is suspected, such experiments might better be conducted while

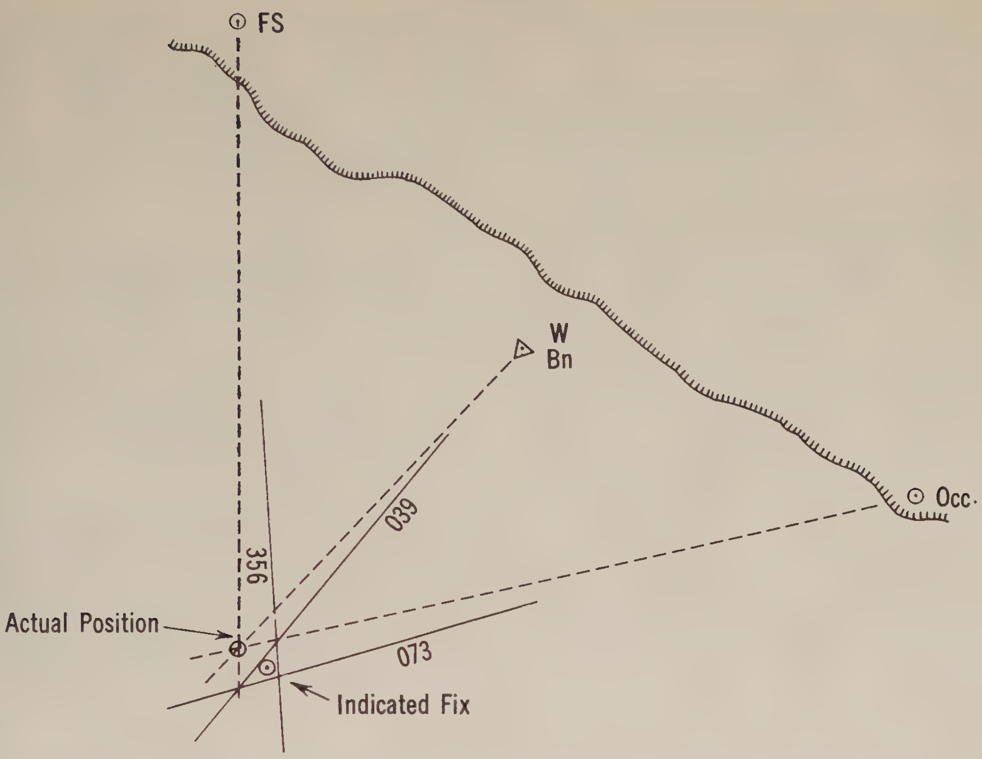


FIGURE 906e.—Adjusting a fix for inaccurate bearings.

at anchor or moored alongside a pier. Underway, other instruments, such as radar, or another method, such as that explained in article 907, might better be used.

Lines of position obtained by observation of bearings or distances can be used with lines of position obtained in any other manner, as by radio direction finder, loran, celestial navigation, etc. If an object such as a buoy is passed close aboard, a fix is obtained without plotting. Similarly, there should be no doubt as to the position of a vessel which is observed to be midway between two channel buoys a short distance apart.

907. Horizontal angles.—A fix may be obtained by means of the *difference* in bearing of several objects. If a constant error is present in the instrument used for measuring directions, it will not be reflected in the difference between bearings. Therefore, the differences may be more accurate than the bearings. Horizontal sextant angles, however, are usually the most accurate source of such information.

Customarily, two angles are obtained on three objects. These angles are then plotted from a single point on a sheet of transparent material, or set on a mechanical device called a **three-arm protractor** (fig. 4011c) in United States usage and a **station pointer** in British terminology. The three lines or arms are then fitted to the chart by trial and error until all three pass through the objects used for observation. The observer is then at the common intersection of the three lines. This method provides accurate results unless the three objects lie on or near a circle which passes through the observer. The best way to avoid this is to select objects nearly in a straight line, a group with the center one nearer than the other two, or objects so widely separated that the angle between the two end ones approaches or exceeds 180°.

Because of its high accuracy, this method is frequently used in hydrographic surveying (ch. XLI). It has fallen into virtual disuse in the ordinary course of navi-

gation because of the convenience and reliability of other methods. However, it may be used when a position of greater than normal accuracy is required, as for finding the position of letting go an anchor. Simultaneous angles are required for accurate results. These are customarily obtained by two observers, each with a sextant. If a single observer is available, he first observes the angle changing more slowly, then the other, and then makes a second observation of the first angle. The average of the two readings of the first angle is used with the second angle.

908. Nonsimultaneous observations.—For fully accurate results, observations made to fix the position of a moving vessel should be made simultaneously, or nearly so. On a slow-moving vessel, relatively little error is introduced by making several observations in quick succession. A wise precaution is to observe the objects more nearly ahead or astern first, since these are least affected by the motion of the observer. For more accurate results, all readings but the last can be repeated in reverse order, and the mean of each pair used. Thus, if three bearings (on objects *A*, *B*, and *C*) are to be used, five readings can be taken, as follows: *A*, *B*, *C*, *B*, *A*. The two *B* readings are averaged, and also the two *A* readings. The time of the *C* reading is considered the time of all readings. Approximately equal intervals should elapse between readings. An indication of the error introduced by *not* observing this precaution is the fact that at ten knots a vessel moves 1,000 feet in one minute. At 20 knots, this distance is doubled. If the angle between lines of position is small, and the earlier bearings are of objects near the beam, the position indicated by the fix might be in error by more than the motion of the vessel, particularly if the objects on which later bearings are taken are abaft those of earlier bearings.

Sometimes it is not possible or desirable to make simultaneous or nearly simultaneous observations. Such a situation may arise, for instance, when a single object is available for observation, or when all available objects are on nearly the same or reciprocal bearings, and there is no means of determining distance. Under such conditions, a period of several minutes or more may be permitted to elapse between observations to provide lines of position crossing at suitable angles. When this occurs, the lines can be adjusted to a common time to obtain a **running fix.** Refer to figure 908a. A ship is proceeding along a coast on course 020°, speed 15 knots. At 1505 lighthouse *L* bears 310°. If the line of position is accurate, the ship is somewhere on it at the time of observation. Ten minutes later the ship will have traveled 2.5 miles in direction 020°. If the ship was at *A* at 1505, it will be at *A′* at 1515. However, if the position at 1505 was *B*, the

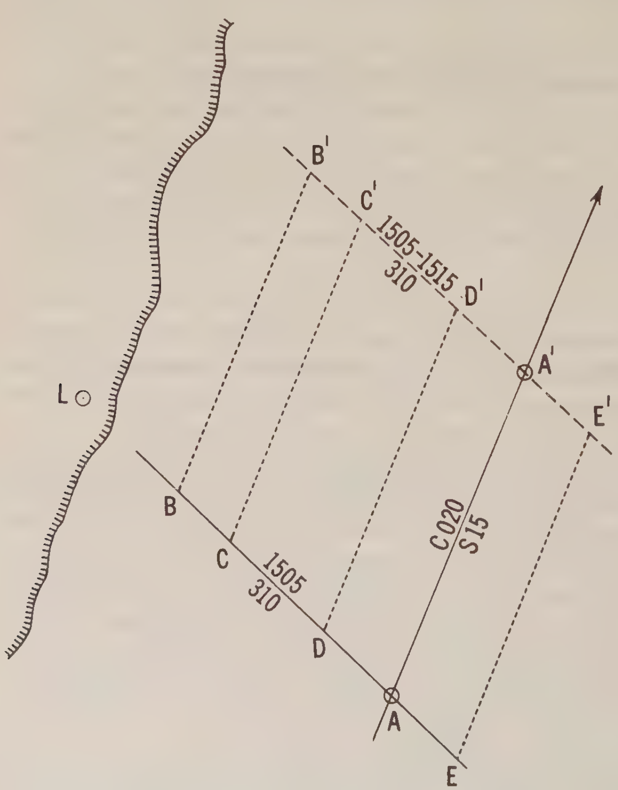


FIGURE 908a.—Advancing a line of position.

position at 1515 will be B'. A similar relationship exists between C and C', D and D', E and E', etc. Thus, if *any* point on the original line of position is moved a distance equal to the distance run, and in the direction of the motion, a line through this point, parallel to the original line of position, represents all possible positions of the ship at the later time. This process is called **advancing** a line of position. The moving of a line *back* to an *earlier* time is called **retiring** a line of position.

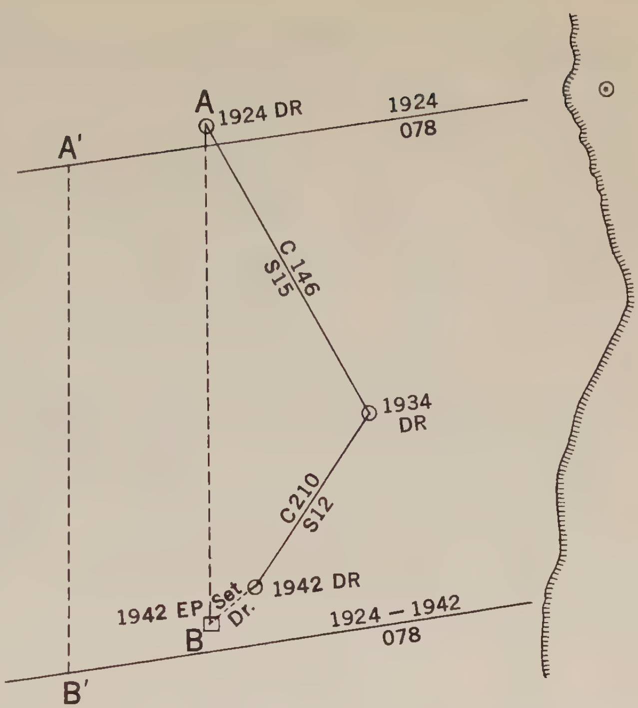


FIGURE 908b.—Advancing a line of position with a change in course and speed, and allowing for current.

The accuracy of an adjusted line of position depends not only upon the accuracy of the original line, but also upon the reliability of the information used in moving the line. A small error in the course made good has little effect upon the accuracy of a bearing line of an object near the beam, but maximum effect upon the bearing line of an object nearly ahead or astern. Conversely, the effect of an error in speed is maximum upon the bearing line of an object abeam. The opposite is true of circles of position. The best estimate of course and speed made good should be used in advancing or retiring a line of position.

If there are any changes of course or speed, these should be considered, for the motion of the line of position should reflect as accurately as possible the motion of the observer between the time of observation and the time to which the line is adjusted. Perhaps the easiest way to do this is to measure the direction and distance between dead reckoning or estimated positions at the two times, and use these to adjust some point on the line of position. This method is shown in figure 908b. In this illustration allowance is made for the estimated combined effect of wind and current, this effect being plotted as an additional course and distance. If courses and speeds made good over the ground are used, the separate plotting of the wind and current effect is not used. In the illustration, point A is the DR position at the time of observation, and point B is the estimated position (the DR position adjusted for wind and current) at the time to which the line of position is adjusted. Line $A'B'$ is of the same length and in the same direction as line AB.

Other techniques may be used. The position of the object observed may be advanced or retired, and the line of position drawn in relation to the adjusted position. This is the most satisfactory method for a circle of position, as shown in figure 908c. When the position of the landmark is adjusted, the advanced line of position can be laid down without plotting the original line, which need be shown only if it serves a useful purpose. This not only eliminates part of the work, but reduces the number of lines on the chart, and thereby decreases the possibility of error. Another method is

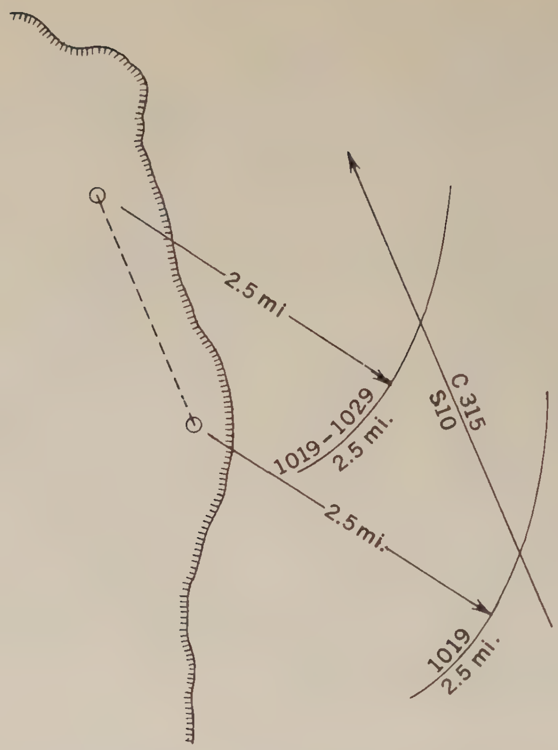


FIGURE 908c.—Advancing a circle of position.

to draw any line, such as a perpendicular, from the dead reckoning position at the time of observation to the line of position. A line of the same length and in the same direction, drawn from the DR position or EP at the time to which the line is adjusted, locates a point on the adjusted line, as shown in figure 908d. If a single course and speed is involved, common practice is to measure from the intersection of the line of position and the course line. If the dividers are set to the distance run between bearings and placed on the chart so that one point is on the first bearing line and the other point is on the second bearing line, and the line connecting the points is parallel to the course line, the points will indicate the positions of the vessel at the times of the bearings.

An adjusted line of position is labeled the same as an unadjusted one, except that both the time of observation and the time to which the line is adjusted are shown, as in the illustrations of this article and article 909. Because of additional sources of error in adjusted lines of position, they are not used when satisfactory simultaneous lines can be obtained.

909. The running fix.—As stated in article 908, a fix obtained by means of lines of position taken at different times and adjusted to a common time is called a **running fix.** In piloting, common practice is to *advance* earlier lines to the time of the last observation. Figure 909a illustrates a running fix obtained from two bearings of the same object. In figure 909b the ship changes course and speed between observations of two objects. A running fix by two circles of position is shown in figure 909c.

When simultaneous observations are not available, a running fix may provide the most reliable position obtainable. The time between observations should be no longer than necessary, for the uncertainty of course and distance made good increases with time.

The errors applicable to a running fix are those resulting from errors of the individual lines of position. However, a given error may have quite a different effect upon

the fix than upon the line of position. Consider, for example, the situation of an unknown head current. In figure 909d a ship is proceeding along a coast, on course 250°, speed 12 knots. At 0920 lighthouse *A* bears 190°, and at 0930 it bears 143°. If the earlier bearing line is advanced a distance of two miles (ten minutes at 12 knots) in the direction of the course, the running fix is as shown by the solid lines. However, if there is a head current of two knots, the ship is making good a speed of only ten knots, and in ten minutes will travel a distance of only 1⅔ miles. If the first bearing line is advanced this distance, as shown by the broken line, the actual position of the ship is at *B*. *This is nearer the beach than the running fix*, and therefore a dangerous situation. A following current gives an indication of position too far from the object. Therefore, if a current *parallel* to the course (either head or following) is suspected, a *minimum* estimate of speed made good will result in a possible margin of safety. If the second bearing is of a different object, a *maximum* estimate of speed should be made if the second object is on the same side and farther forward, or on the opposite side and farther aft, than the first object was when observed. All of these situations assume that danger is on the same side as the object observed first. If there is either a head or following current, a series of running fixes based upon a number of bearings of the same object will plot in a straight line parallel to the course line, as shown in figure 909e. The plotted line will be too close to the object observed if there is a following current, and too far out if there is a head current. The existence of the current will not be apparent unless the actual speed over the ground is known. The position of the plotted line relative to the dead reckoning course line is not a reliable guide.

A current oblique to the course will result in an incorrect position, but the direction of the error is indeterminate. In general, the effect of a current with a strong head or following component is similar to that of a head or following current, respectively. The existence of an oblique current, but not its amount, can be detected by observing and

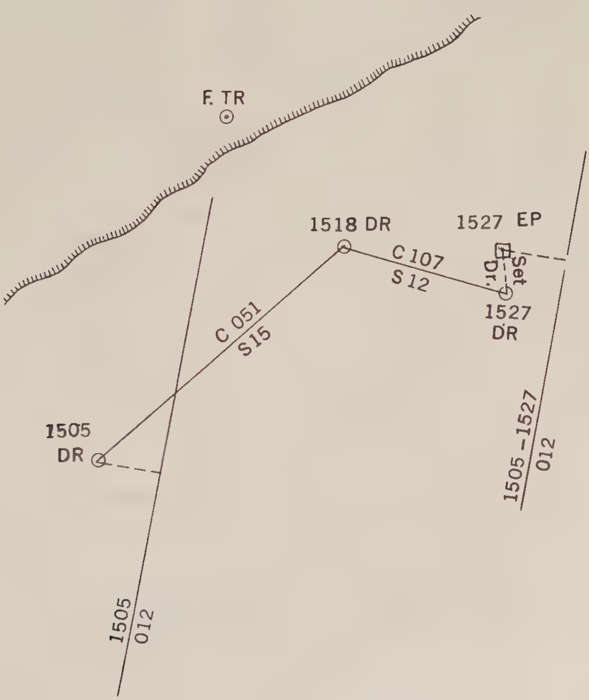


FIGURE 908d.—Advancing a line of position by its relation to the dead reckoning.

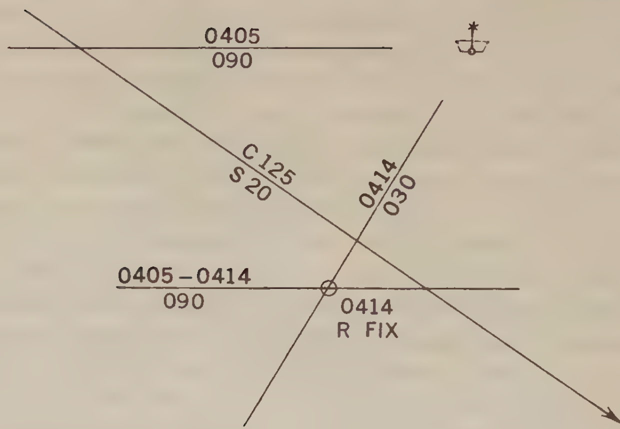


FIGURE 909a.—A running fix by two bearings on the same object.

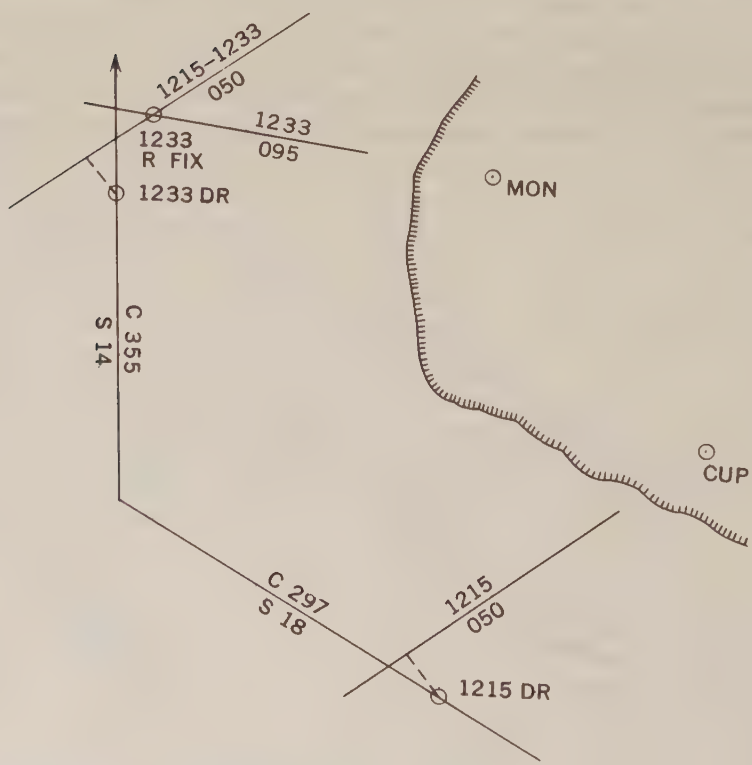


FIGURE 909b.—A running fix with a change of course and speed between observations on separate landmarks.

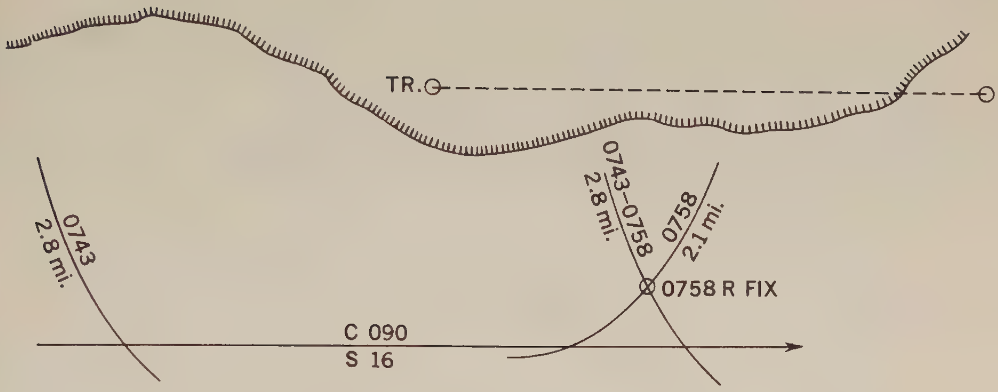


FIGURE 909c.—A running fix by two circles of position.

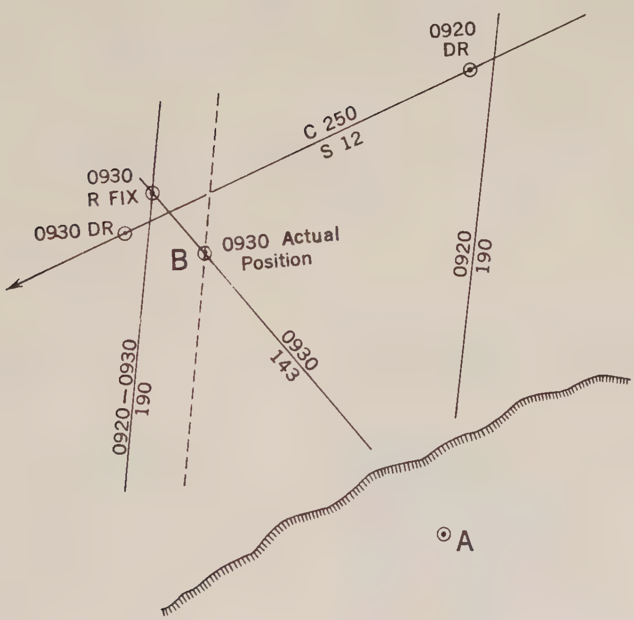


FIGURE 909d.—Effect of a head current on a running fix.

plotting several bearings of the same object. The running fix obtained by advancing one bearing line to the time of the next one will not agree with the running fix obtained by advancing an earlier line. Thus, if bearings *A*, *B*, and *C* are observed at five-minute intervals, the running fix obtained by advancing *B* to the time of *C* will not be the same as that obtained by advancing *A* to the time of *C*, as shown in figure 909f.

Whatever the current, the *direction* of the course made good (assuming constant current) can be determined. On the chart, plot the various bearing lines (fig. 909g, left). Draw a straight line on a piece of transparent material, and along it mark off the distances run (using any assumed speed) between bearings (fig. 909g, right). If transparent material is not available, mark off the distances along the edge of a piece

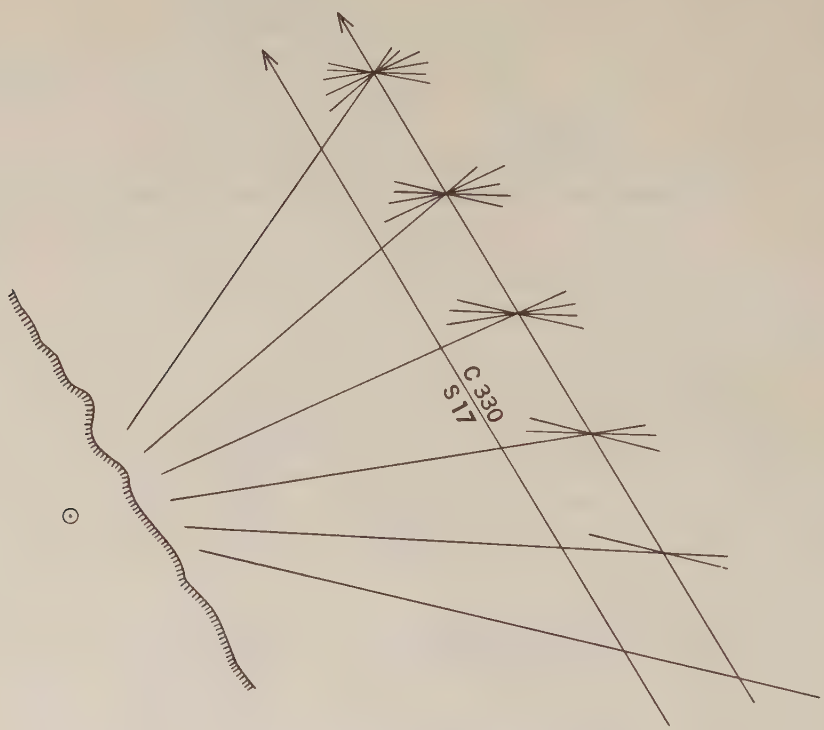


FIGURE 909e.—A number of running fixes with a following current.

of paper. By trial and error, fit the distances to the bearing lines on the chart, so that each mark falls on its bearing line (fig. 909h). The direction of the line is the course being made good. Its distance from the track is in error by an amount proportional to the ratio of the speed being made good to the speed assumed for the solution. If a good fix (not a running fix) is obtained at some time before the first bearing for the running fix, and the current has not changed, the track can be determined by drawing a line from the fix, in the direction of the course made good. The intersection of the track with any of the bearing lines is an actual position.

The current can be determined whenever a dead reckoning position and fix are available for the same time. The direction *from* the dead reckoning position *to* the fix is the set of the current. The distance between these two positions, divided by the time (expressed in hours and tenths) since the last fix, is the drift of the current in knots. For accurate results, the dead reckoning position must be run up from the previous fix without any allowance for current. Any error in either the dead reckoning

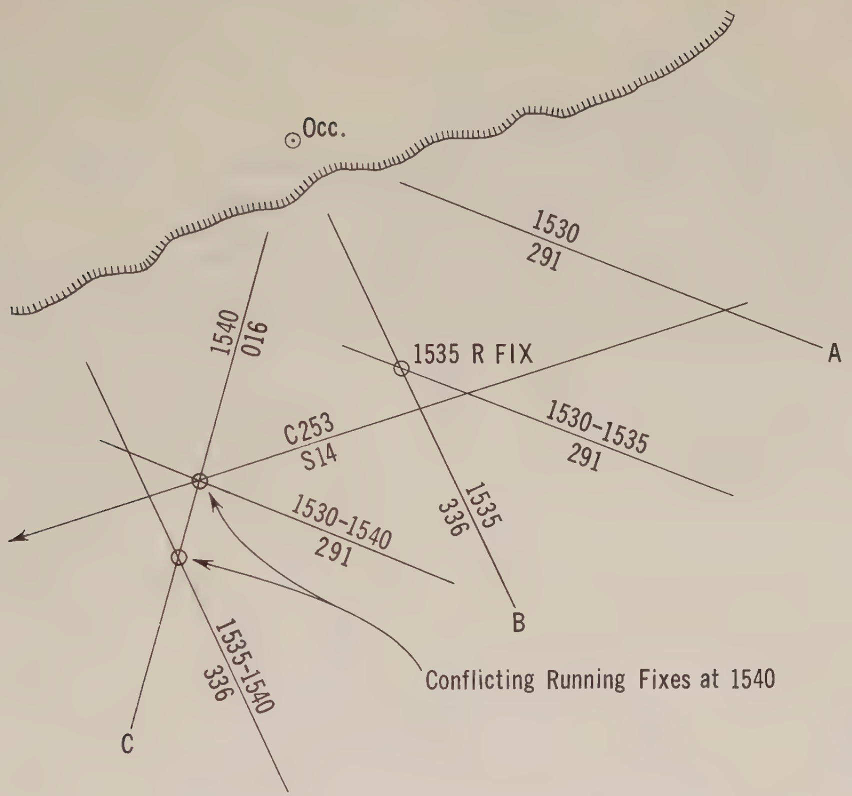


FIGURE 909f.—Detecting the existence of an oblique current, by a series of running fixes.

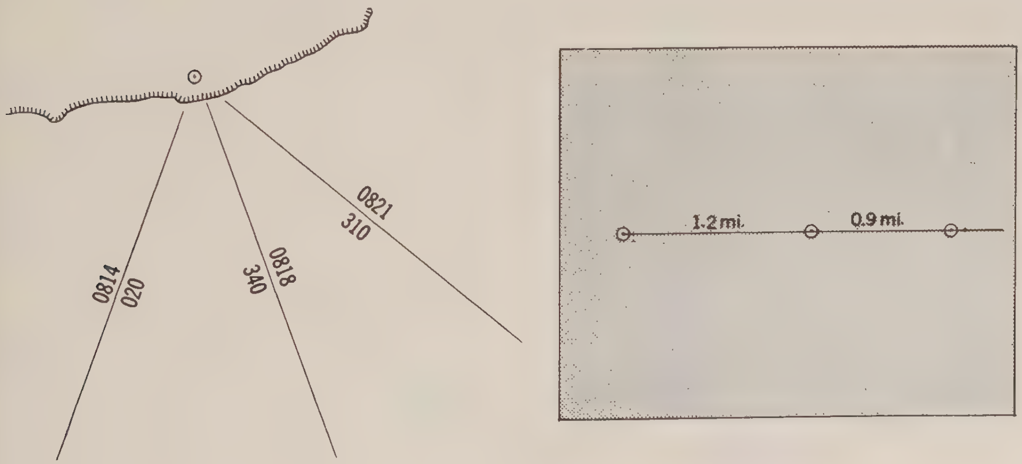


FIGURE 909g.—Preparing to determine the course made good.

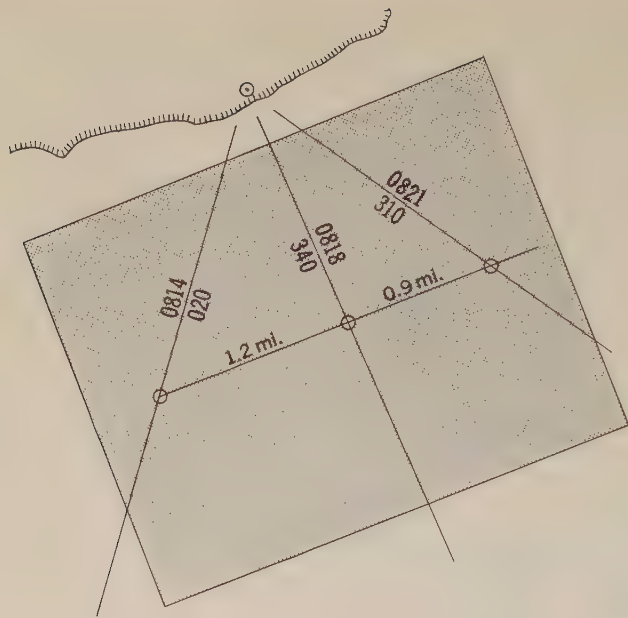


FIGURE 909h.—Determining the course made good.

position (such as poor steering, unknown compass error, inaccurate log, wind, etc.) or the fix will be reflected in the determination of current. When the dead reckoning position and fix are close together, a relatively small error in either may introduce a large error in the apparent set of the current.

910. Solution without a plot.—A running fix can be obtained by utilizing the mathematical relationships involved. Refer to figure 910. A ship steams past landmark *D*. At any point *A* a bearing of *D* is observed and expressed as degrees right or left of the course (a relative bearing if the ship is on course). At some later time, at *B*, a second bearing of *D* is observed and expressed as before. At *C* the landmark is broad on the beam. The angles at *A*, *B*, and *C* are known, and also the distance run between points. The various triangles could be solved by trigonometry (app. O) to find the distance from *D* at any bearing. Distance and bearing provide a fix.

Table 7 provides a quick and easy solution. The table is entered with the difference between the course and first bearing (angle *BAD* in fig. 910) along the top of the table, and the difference between the course and second bearing (angle *CBD*) at the left of the table. For each pair of angles listed, two numbers are given. To find the distance from the landmark at the time of the second bearing (*BD*), multiply the distance run between bearings by the first number from table 7. To find the distance when the object is abeam (*CD*), multiply the distance run between *A* and *B* by the second number from the table. If the run between bearings is exactly one mile, the tabulated values are the distances sought.

Example.—A ship is steaming on course 050°, speed 15 knots. At 1130 a lighthouse bears 024°, and at 1140 it bears 359°.

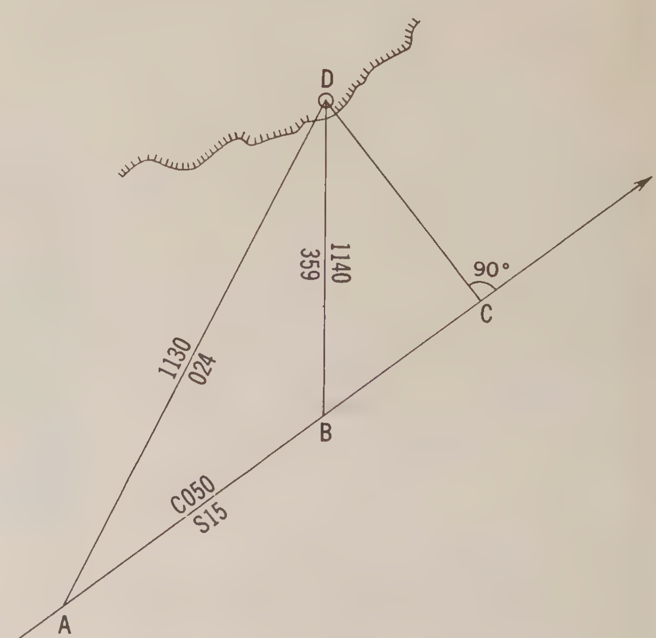


FIGURE 910.—Triangles involved in a running fix.

Required.—(1) Distance from the light at 1140.

(2) Distance from the light when it is broad on the port beam.

Solution (fig. 910).—(1) The difference between the course and the first bearing (050°–024°) is 26°, and the difference between the course and the second bearing (050°+360°−359°) is 51°.

(2) From table 7 the two numbers (factors) are 1.04 and 0.81, found by interpolation.

(3) The distance run between bearings is 2.5 miles (10 minutes at 15 knots).

(4) The distance from the lighthouse at the time of the second bearing is 2.5×1.04=2.6 miles.

(5) The distance from the lighthouse when it is broad on the beam is 2.5×0.81=2.0 miles.

Answers.—(1) D 2.6 mi., (2) D 2.0 mi.

Certain combinations of angles provide quick mental solution without the need for table 7. If the second difference (angle *CBD*) is double the first difference (angle *BAD*), triangle *BAD* is isosceles (art. O28), with equal angles at *A* and *D*. Therefore side *AB* (the run) is equal to side *BD* (the distance off at the time of the second bearing). This is called **doubling the angle on the bow.** If the first angle is 45° and the second 90°, the distance run equals the distance when broad on the beam. These are called **bow and beam bearings.** If the first angle is 63°.5 and the second 90°, the distance off when abeam is about twice the distance run. If the angles are 71°.5 and 90°, the distance off when abeam is about three times the distance run. If the first angle is 22°.5 and the second 45°, the distance at which the object will be passed abeam is about 7/10 of the distance run between bearings. If the angles are 22°.5 and 26°.5, the distance abeam will be about 7/3 of the distance run. If the angles are 30° and 60°, the distance of the object when abeam will be about 7/8 of the distance run between bearings. If the two angles are such that their natural cotangents differ by unity, the distance abeam will be equal approximately to the distance run between bearings. Some combinations having approximately this relationship are 22°–34°, 25°–41°, 27°–46°, 32°–59°, and 40°–79°.

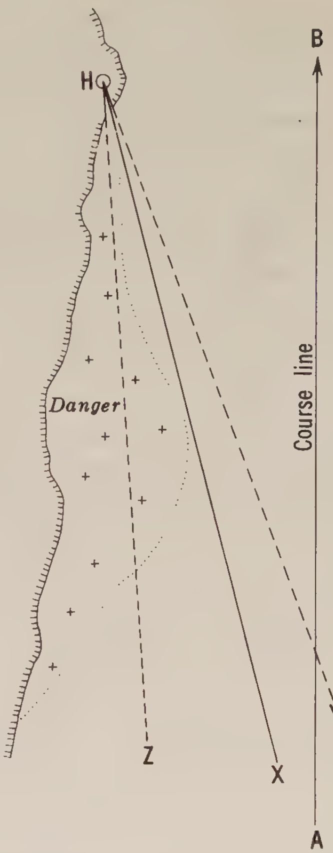


FIGURE 911a.—A danger bearing.

If either the course or speed is in error, the result will be inaccurate.

911. Safe piloting without a fix.—A fix or running fix is not always necessary to insure safety of the vessel. If a ship is proceeding up a dredged channel, for instance, the only knowledge needed to prevent grounding is that the ship is within the limits of the dredged area. This information might be provided by a range in line with the channel. A fix is not needed except to mark the point at which the range can no longer be followed with safety. Such a point is usually marked by a buoy.

Under favorable conditions a **danger bearing** might be used to insure safe passage past a shoal or other danger. Refer to figure 911a. A vessel is proceeding along a coast, on course line *AB*. A shoal is to be avoided. A line *HX* is drawn from lighthouse *H*, tangent to the outer edge of the danger. As long as the bearing of light *H* is

less than *XH*, the danger bearing, the vessel is in safe water. An example is *YH*, no part of the bearing line passing through the danger area. Any bearing *greater* than *XH*, such as *ZH*, indicates a *possible* dangerous situation. If the object is passed on the port side, the safe bearing is *less* than the danger bearing, as shown in figure 911a. If the object is passed on the starboard side, the danger bearing represents the minimum bearing, safe ones being *greater*. To be effective, a danger bearing should not differ greatly from the course, and the object of which bearings are to be taken should be easily identifiable and visible over the entire area of usefulness of the danger bearing. A margin of safety might be provided by drawing line *HX* through a point a short

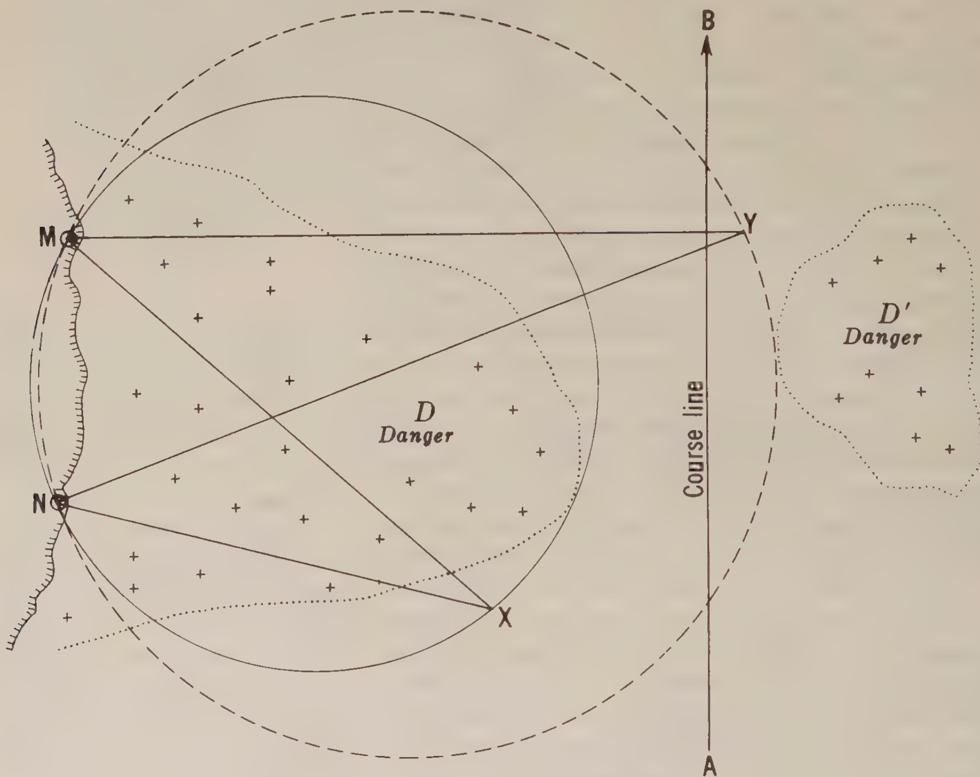


FIGURE 911b.—Horizontal danger angles.

distance off the danger. If a natural or artificial range is available as a danger bearing, it should be used.

A vessel proceeding along a coast may be in safe water as long as it remains a minimum distance off the beach. This information may be provided by any means available. One method useful in avoiding particular dangers is the use of a **danger angle.** Refer to figure 911b. A ship is proceeding along a coast on course line *AB*, and the captain wishes to remain outside a danger *D*. Prominent landmarks are located at *M* and *N*. A circle is drawn through *M* and *N* and tangent to the outer edge of the danger. If *X* is a point on this circle, angle *MXN* is the same as at any other point on the circle (except that part between *M* and *N*). Anywhere within the circle the angle is *larger* and anywhere outside the circle it is *smaller*. Therefore, any angle smaller than *MXN* indicates a safe position and any angle larger than *MXN* indicates possible danger. Angle *MXN* is therefore a maximum **horizontal danger**

angle. A minimum horizontal danger angle is used when a vessel is to pass *inside* an offlying danger, as at D' in figure 911b. In this case the circle is drawn through M and N and tangent to the *inner* edge of the danger area. The angle is kept larger than MYN. If a vessel is to pass *between* two danger areas, as in figure 911b, the horizontal angle should be kept smaller than MXN but larger than MYN. The minimum danger angle is effective only while the vessel is inside the larger circle through M and N. Bearings on either landmark might be used to indicate the entering and leaving of the larger circle. A margin of safety can be provided by drawing the circles through points a short distance off the dangers. Any method of measuring the angles, or difference of bearing of M and N, can be used. Perhaps the most accurate is by horizontal sextant angle. If a single landmark of known height is available, similar procedure can be used with a **vertical danger angle** between top and bottom of the object. In this case the charted position of the object is used as the center of the circles.

A vessel may sometimes be kept in safe water by means of a **danger sounding.** The value selected depends upon the draft of the vessel and the slope of the bottom. It should be sufficiently deep to provide adequate maneuvering room for the vessel to reach deeper water before grounding, once the minimum depth is obtained. In an area where the shoaling is gradual, a smaller margin of depth can be considered safe than in an area of rapid shoaling. Where the shoaling is very abrupt, as off Point Conception, California, no danger sounding is practical. It is good practice to prominently mark the danger sounding line on the chart. A colored pencil is useful for this purpose.

If it is desired to round a point marked by a prominent landmark, without approaching closer than a given minimum distance, this can be done by steaming until the minimum distance is reached and then immediately changing course so as to bring the landmark broad on the beam. Frequent small changes of course are then used to keep the landmark near, *but not forward of*, the beam. This method is not reliable if the vessel is being moved laterally by wind or current.

An approximation of the distance off can be found by noting the rate at which the bearing changes. If the landmark is kept abeam, the change is indicated by a change of heading. During a change of 57°5, the distance off is about the same as the distance run. For a change of 28°5, the distance is about twice the run; for 19° it is about three times the run; for 14°5 it is about four times the run; and for 11°5 it is about five times the run. Another variation is to measure the number of seconds required for a change of 16°. The distance off is equal to this interval multiplied by the speed in knots and divided by 1,000. That is, $D=\frac{St}{1{,}000}$, where D is the distance in nautical miles, S is the speed in knots, and t is the time interval in seconds. This method can also be used for straight courses (with bearings 8° forward and abaft the beam), but with somewhat reduced accuracy.

912. Soundings.—The most important use of soundings is to determine whether the depth is sufficient to provide a reasonable margin of safety for the vessel. For this reason, soundings should be taken continuously in pilot waters. A study of the chart and the establishment of a danger sounding (art. 911) should indicate the degree of safety of the vessel at any time.

Under favorable conditions, soundings can be a valuable aid in establishing the position of the vessel. Their value in this regard depends upon the configuration of the bottom, the amount and accuracy of information given on the chart, the type and accuracy of the sounding equipment available aboard ship, and the knowledge and skill of the navigator. In an area having a flat bottom devoid of distinctive features, or in an

area where detailed information is not given on the chart, little positional information can be gained from soundings. However, in an area where depth curves run roughly parallel to the shore, a sounding might indicate distance from the beach. In any area where a given depth curve is sharply defined and relatively straight, it serves as a line of position which can be used with other lines, such as those obtained by bearings of landmarks, to obtain a fix. The 100-fathom curve at the outer edge of the continental shelf might be crossed with a line of position from celestial observation or loran. The crossing of a sharply defined trench, ridge, shoal, or flat-topped seamount (a **guyot**) might provide valuable positional information.

In any such use, identification of the feature observed is important. In an area of rugged underwater terrain, identification might be difficult unless an almost continuous determination of position is maintained, for it is not unusual for a number of features within a normal radius of uncertainty to be similar. If the echo sounder produces a continuous recording of the depth, called a **bottom profile,** this can be matched to the chart in the vicinity of the course line. If no profile is available, a rough approximation of one can be constructed as follows: Record a series of soundings at short intervals, the length being dictated by the scale of the chart and the existing situation. For most purposes the interval might be each minute, or perhaps each half-mile or mile. Draw a straight line on transparent material and, at the scale of the chart, place marks along the line at the distance intervals at which soundings were made. For this purpose the line might be superimposed over the latitude scale or a distance scale of the chart. At each mark record the corresponding sounding. Then place the transparency over the chart and, by trial and error, match the recorded soundings to those indicated on the chart. Keep the line on the transparency parallel or nearly parallel to the course line plotted on the chart. A current may cause some difference between the plotted course line and the course made good. Also, speed over the bottom might be somewhat different from that used for the plot. This should be reflected in the match. This method should be used with caution, because it may be possible to fit the **line of soundings** to several places on the chart.

Exact agreement with the charted bottom should not be expected at all times. Inaccuracies in the soundings, tide, or incomplete data on the chart may affect the match, but general agreement should be sought. Any marked discrepancy should be investigated, particularly if it indicates less depth than anticipated. If such a discrepancy cannot be reconciled, the wisest decision might well be to haul off into deeper water or anchor and wait for more favorable conditions or additional information.

913. Most probable position (**MPP**).—Since information sufficient to establish an *exact* position is seldom available, the navigator is frequently faced with the problem of establishing the most probable position of the vessel. If three reliable bearing lines cross at a point, there is usually little doubt as to the position, and little or no judgment is needed. But when conflicting information or information of questionable reliability is received, a decision is required to establish the MPP. At such a time the experience of the navigator can be of great value. Judgment can be improved if the navigator will continually try to account for all apparent discrepancies, even under favorable conditions. If a navigator habitually analyzes the situation whenever positional information is received, he will develop judgment as to the reliability of various types of information, and will learn something of the conditions under which certain types should be treated with caution.

When complete positional information is lacking, or when the available information is considered of questionable reliability, the most probable position might well be considered an **estimated position** (**EP**). Such a position might be determined from a single line of position, from a line of soundings, from lines of position which are some-

what inconsistent, from a dead reckoning position with a correction for current or wind, etc.

Whether the most probable position is a fix, running fix, estimated position, or dead reckoning position, it should be kept continually in mind, together with some estimate of its reliability. The practice of continuing a dead reckoning plot from one good fix to another is advisable, whether or not information is available to indicate a most probable position differing from the dead reckoning position, for the DR plot provides an indication of current and leeway. A series of estimated positions may not be consistent because of the continual revision of the estimate as additional information is received. However, it is good practice to plot all MPP's, and sometimes to maintain a separate EP plot based upon the best estimate of course and speed being made good over the ground, for this should furnish valuable information to indicate whether the present course is a safe one.

Aids to Navigation

914. Kinds of aids.—When piloting, a navigator is concerned primarily with the position of his vessel relative to nearby land, shoals, and other dangers. It is natural, therefore, that he make extensive use of **landmarks,** which are conspicuous objects, structures, or lights serving as indicators for guidance or warning of a craft. Such an object visible from a distance to seaward is called a **seamark.** Either type of mark may be called a **daymark** if useful only during daylight, or a **nightmark** if useful primarily during darkness. A natural or artificial mark used to assist a vessel in avoiding a particular hazard may be called a **clearing mark.** If an uncharted landmark is discovered, its position might be established from available information or by triangulation (art. 4110) from known positions, and plotted on the chart for future use. A permanent feature might well be reported to the appropriate government charting agency.

A mark established by man, to serve as a landmark, is called an **aid to navigation.** This should not be confused with the expression **navigational aid,** which includes, in addition to aids to navigation, such items as instruments, charts, tables, etc. The principal aids to navigation are:

Lighthouse, a structure exhibiting a major light designed to serve as an aid to navigation. Lighthouses vary in appearance because of location, relative importance, the type of soil upon which they are constructed, prevalence of violent storms, backgrounds against which they are seen, distances the lights are to be seen, etc. Some are located on land, and some in the water. Figure 914 illustrates several typical light structures. The type of structure and its coloring assist in daylight identification. There are about 400 lighthouses in United States waters, being located along all coasts, the Great Lakes, and many of the inland waterways.

Beacon. In a general sense, a beacon is anything serving as a signal or indication, either for guidance or warning. However, as a distinctive type of aid to navigation, a beacon is either a *fixed* aid (not a floating one) or an unlighted aid (sometimes called a **daybeacon**). As thus defined, a lighthouse is a beacon. However, the term "beacon" is generally applied particularly to secondary fixed structures, whether lighted or not. There are about 15,000 beacons of this type in United States waters.

Lightship, a distinctively marked vessel anchored or moored at a charted point, to serve as an aid to navigation. By night it displays a characteristic masthead light and a less brilliant light on the forestay. The forestay indicates the direction in which the vessel is headed, and hence the direction of the current (or wind), since lightships head into the wind or current. By day a lightship displays the International Code signal of the station when requested, or if an approaching vessel does not seem to recognize it. The name of the station is painted in large letters on the side of the

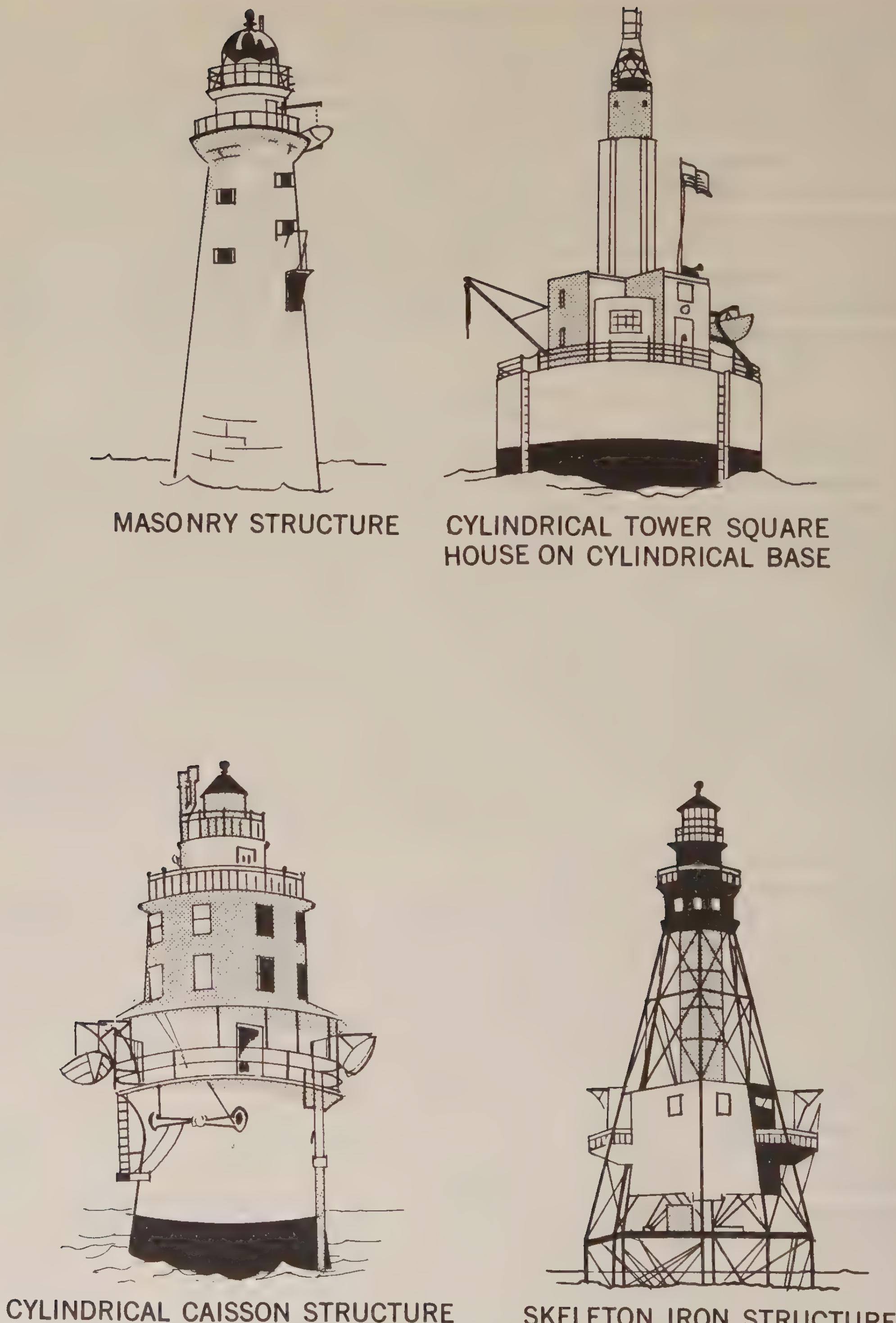


FIGURE 914.—Typical light structures.

vessel. All lightships except Lake Huron Lightship have red hulls; white lettering and superstructure; and buff masts, lantern galleries, ventilators, and stacks. Relief lightships have the same coloring, but carry the name "Relief." Lake Huron Lightship has a black hull. A lightship may be equipped with certain auxiliary devices such as a fog signal, submarine sound signal, and radiobeacon. When under way or off station a lightship displays the lights and sounds the signals prescribed by the rules of the road, and flies the International Code signal flags "PC," signifying that it is not then serving as an aid to navigation. It does not then show or sound any of the signals of a lightship. A lightship is the floating equivalent of a lighthouse. Most U. S. lightships eventually will be replaced by structures similar to Texas towers.

Buoy, a floating object, other than a lightship, moored to the bottom as an aid to navigation. There are many different types of buoys to serve different purposes. Buoys are the most numerous aid to navigation, about 20,000 unlighted and 3,000 lighted buoys being maintained by the United States Coast Guard alone.

Any lighted aid to navigation may be called a **light.**

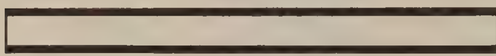
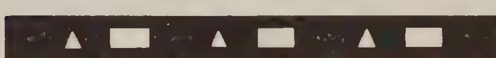
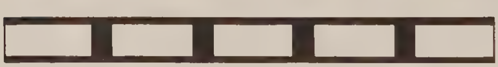
Along the coasts of the United States, and in the Great Lakes (United States side) most aids to navigation are installed and maintained by the United States Coast Guard. Along certain rivers they are under the control of other government agencies, notably the Corps of Engineers of the United States Army. A number of privately maintained aids are in use. An example of such aids are lights at the ends of privately owned piers.

915. Lights.—A light extends the use of various aids to navigation to periods of darkness. If such a light is to serve its purpose, the user must be able to distinguish it from the general background of shore lights, and to determine *which* navigational light it is. That is, one must be able to *identify* the light. For this purpose each light is given a distinctive sequence of light and dark periods, and in some cases a distinctive color, or color sequence. These features are called the **characteristics** of the light (fig. 915). No two lights are given the same characteristics if they are so located that one might be mistaken for the other.

The U.S. Coast Guard publishes a *Light List* in five volumes: two volumes for the Atlantic coast and Gulf of Mexico, and one volume each for the Pacific coast and islands, Great Lakes, and Mississippi River system. The U.S. Navy Hydrographic Office publishes seven *Lists of Lights* giving characteristics of lights of foreign waters and the principal lights along the coasts of the United States. Characteristics are also indicated on charts. The letter *W* indicates a white light, *R* a red light, and *G* a green light. Other colors are not used by the United States Coast Guard. If no color indication is given, a light is white. Colors are produced by glass shades or screens. The **period** of a light is the time required for one complete sequence of characteristics.

Some lights provide bearing indication by a system of **light sectors,** different colors being exhibited in the various sectors. As an observer crosses the boundary between sectors, he can note the change of color. The boundaries are indicated in the light lists and by broken lines on charts. The bearings given in these indications are those *of* the light as observed at a distance, not the direction outward *from* the light. In general, red sectors are used to indicate obstruction areas. In using light sectors to determine bearing, one should remember that the line of demarcation is not always sharply defined, and that when haze or smoke is present, a white light might have a reddish hue.

916. Visibility of lights.—Usually a navigator wants to know not only the identity of a light, but also the area in which he might reasonably expect to observe it. His track is planned to take him within range of lights which can prove useful during

Illustration	Symbols and meaning		Phase description
	Lights which do not change color	Lights which show color variations	
	F. = Fixed	Alt. = Alternating.	A continuous steady light.
	F. Fl. = Fixed and flashing.	Alt. F. Fl. = Alternating fixed and flashing.	A fixed light varied at regular intervals by a flash of greater brilliance.
	F. Gp. Fl. = Fixed and group flashing.	Alt. F. Gp. Fl. = Alternating fixed and group flashing	A fixed light varied at regular intervals by groups of 2 or more flashes of greater brilliance.
	Fl. = Flashing	Alt. Fl. Alternating flashing.	Shows a single flash at regular intervals, the duration of light always being less than the duration of darkness. Shows not more than 30 flashes per minute.
	Gp. Fl. = Group flashing.	Alt. Gp. Fl. = Alternating group flashing.	Shows at regular intervals groups of 2 or more flashes.
	Qk. Fl. = Quick flashing.	----------------	Shows not less than 60 flashes per minute.
	I. Qk. Fl. = Interrupted quick flashing.	----------------	Shows quick flashes for about 4 seconds, followed by a dark period of about 4 seconds.
	S-L. Fl. = Short-long flashing.	----------------	Shows a short flash of about 0.4 second, followed by a long flash of 4 times that duration.
	Occ. = Occulting	Alt. Occ. = Alternating occulting.	A light totally eclipsed at regular intervals, the duration of light always equal to or greater than the duration of darkness.
	Gp. Occ. = Group occulting.	----------------	A light with a group of 2 or more eclipses at regular intervals.

Light colors used and abbreviations: W = white, R = red, G = green.

FIGURE 915.—Light characteristics.

periods of darkness. If lights are not sighted within a reasonable time after prediction, a dangerous situation may exist, requiring resolution or action to insure safety of the vessel.

The area in which a light can be observed is normally a circle with the light as the center, and the range of visibility as the radius. However, on some bearings the range may be reduced by obstructions. In this case the obstructed arc might differ with height of eye and distance. Also, lights of different colors may be seen at different distances. This fact should be considered not only in predicting the distance at which a light can be seen, but also in identifying it. The condition of the atmosphere has a considerable effect upon the distance at which lights can be seen. Sometimes lights are obscured by fog, haze, dust, smoke, or precipitation which may be present at the light, or between it and the observer, but not at the observer, and possibly unknown

to him. There is always the possibility of a light being extinguished. In the case of unwatched lights, this condition might not be detected and corrected at once. During periods of armed conflict, certain lights might be deliberately extinguished if they are considered of greater value to the enemy than to one's own vessels.

On a dark, clear night the range of visibility is limited primarily in two ways: (1) luminosity and (2) curvature of the earth. A weak light cannot normally be expected to be seen beyond a certain range, regardless of the height of eye. This distance is called **luminous range.** Light travels in almost straight lines, so that an observer below the visible horizon of the light should not expect to see the light, although the loom extending upward from the light can sometimes be seen at greater distances. Table 8 gives the distance to the horizon at various heights. The tabulated distances assume normal refraction. Abnormal conditions might extend this range somewhat (or in some cases reduce it). Hence, the **geographic range,** as the luminous range, is not subject to exact prediction at any given time.

The geographic range depends upon the height of both the light and the observer, as shown in figure 916. In this illustration a light 150 feet above the water is shown. At this height, the distance to the horizon, by table 8, is 14.0 miles. Within this range the light, if powerful enough and atmospheric conditions permit, is visible regardless of the height of eye of the observer (if there is no obstruction). Beyond this range, the visibility depends upon the height of eye. Thus, by table 8 an observer with height of eye of five feet can see the light on his horizon if he is 2.6 miles beyond the horizon of the light, or a total of 16.6 miles. For a height of 30 feet the distance is $14.0+6.3 = 20.3$ miles. If the height of eye is 70 feet, the geographic range is $14.0+9.6 = 23.6$ miles.

The range of important lights is given in the light lists, and also on the charts. *The tabulated or charted range is for mean high water and a height of eye of 15 feet,* and because of various uncertainties is given only to the nearest whole mile. Where the luminous range is less than the charted range, the shorter distance is given. This fact is not indicated. Therefore, in predicting the range at which a light can be seen, one should first determine the geographic range and compare this with the charted range.

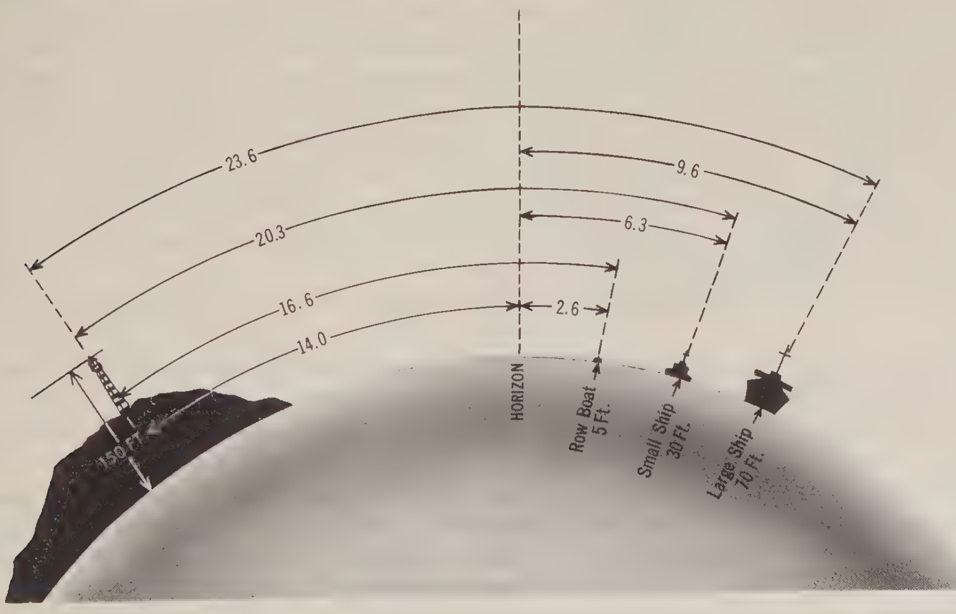


FIGURE 916.—Geographic range of visibility of a light.

This is done by *adding* 4.4 miles (the distance to the horizon at a height of 15 feet) to the value from table 8 for the height of the light. If this value approximates the charted range, one is generally safe in assuming that the charted range is the geographic range. The predicted range is then found by adding the distance to the horizon for both the light and the observer, as indicated above, or, approximately, by taking the *difference* between 4.4 and the distance for the height of eye of the observer (a constant for any given height) and *adding* this value to the charted range (subtracting if the height of eye is less than 15 feet). In making a prediction, one should keep in mind the possibility of the luminous range being *between* the charted range and the predicted range. The *power* of the light should be of some assistance in identifying this condition.

If one is approaching a light, and wishes to predict the *time* at which it should be sighted, he first predicts the range. It is then good practice to draw a circle indicating the limit of visibility. The point at which the course line crosses the circle of visibility is the predicted position of the vessel at the time of sighting the light. The predicted time of arrival at this point is the predicted time of sighting the light. The direction of the light from this point is the predicted bearing at which the light should be sighted. Conversion of the true bearing to a relative bearing is usually helpful in sighting the light. The accuracy of the predictions depends upon the accuracy of the predicted range, and the accuracy of the predicted time and place of crossing the circle of visibility. If the course line crosses the circle of visibility at a small angle, a small lateral error in track may result in a large error of prediction, both of bearing and time. This is particularly apparent if the vessel is *farther* from the light than predicted, in which case the light might be passed without being sighted. Thus, if a light is not sighted at the predicted time, the error *may* be on the side of safety. However, such an interpretation should not be given unless confirmed by other information, for there is always the possibility of reduced range of visibility, or of the light being extinguished.

When a light is first sighted, one might determine whether it is on the horizon by immediately reducing the height of eye by several feet, as by squatting or changing position to a lower height. If the light disappears, and reappears when the original height is resumed, it is on the horizon. This process is called **bobbing a light.** If a vessel has considerable vertical motion due to the condition of the sea, a light sighted on the horizon may alternately appear and disappear. This may lead the unwary to assign faulty characteristics and hence to err in its identification. The true characteristics should be observed after the distance has decreased, or by increasing the height of eye of the observer.

917. Buoys.—The primary functions of buoys are to delineate channels, indicate shoals, mark obstructions, and warn the mariner of danger.

The principal types of buoys used by the United States are illustrated in figure 917, and described as follows:

Can, a buoy built up of steel plates, in the shape of an ordinary cylindrical "tin" can.

Nun, a buoy built up of steel plates, the above-water portion being in the shape of a truncated cone.

Spar, a large log, trimmed, shaped, and appropriately painted. Some spar buoys of the same shape are constructed of steel.

Bell, a steel float surmounted by a short skeleton tower in which the bell is located. Older bell buoys are sounded by the motion of the buoys in the sea. In newer types the bells are struck by compressed gas or electrically operated hammers. A **gong buoy** is similar to a bell buoy, but instead of a bell it has a set of gongs, each of which has a distinctive tone.

Whistle, a steel float surmounted by a small tower in which a whistle is located. Older whistle buoys are sounded by the motion of the sea. In some newer buoys a trumpet is sounded electrically.

Lighted, a steel float surmounted by a skeleton tower with the light at the top. Energy for the light is provided by electric batteries or a tank of acetylene gas, located in the metal float.

Combination, a buoy having more than one means of conveying intelligence, as a **lighted bell buoy** or **lighted whistle buoy.**

Some unlighted buoys are fitted with reflectors to assist in their location and identification by searchlight at night. The colors of the reflectors have the same significance as those of lights. Radar reflectors are fitted to some buoys to increase the strength of the returned signal. A buoy may be equipped with a marker radiobeacon.

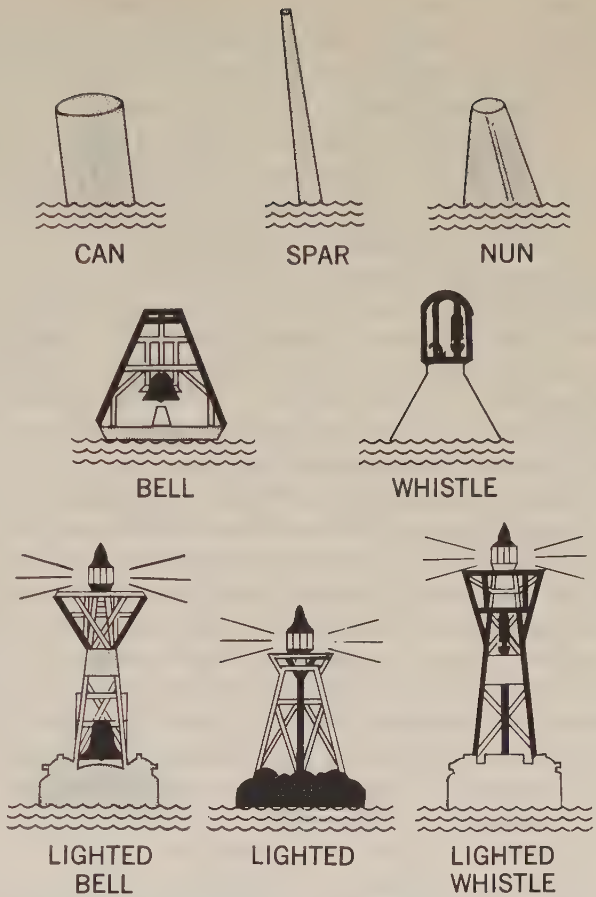


FIGURE 917.—Principal buoy types used by United States.

Most maritime countries use either the **lateral system** of buoyage or the **cardinal system,** or both. In the lateral system, used on all navigable waters of the United States, the coloring, shape, numbering, and lighting of buoys indicate the direction to a danger relative to the course which should be followed. In the cardinal system the coloring, shape, and lighting of buoys indicate the direction to a danger relative to the buoy itself. The color, shape, lights, and numbers of buoys in the lateral system as used by the United States are determined relative to a direction *from* seaward. Along the *coasts* of the United States, the *clockwise* direction around the country is arbitrarily considered to be the direction "from seaward." Some countries using the lateral system have methods of coloring their buoys and lights opposite to that of the United States. Appendix J treats this subject in greater detail.

In United States waters the following distinctive system of identification is used:

Red nun buoys mark the *right* side of channels for an inbound vessel, and obstructions which should be kept to starboard. They have *even* numbers which increase from seaward.

Black can buoys mark the *left* side of channels for an inbound vessel, and obstructions which should be kept to port. They have *odd* numbers which increase from seaward.

Red and black horizontally banded buoys mark junctions or bifurcations of channels, or an obstruction that can be passed on either side. The color (red or black) of the

top band and the shape (nun or can) indicate the side on which the buoy should be passed by a vessel proceeding along the principal channel.

Black and white vertically striped buoys mark the center of a channel and should be passed close aboard. These "mid-channel buoys" may be either nuns or cans.

In fairways and channels solid red or solid black buoys have numbers. Others may have letter designations. Along channels certain numbers may be omitted to maintain the approximate sequence on both sides of the channels.

Lights. Red lights are used only on red buoys and buoys with a red band at the top, green lights are used only on black buoys and buoys with a black band at the top. White lights are used without any color significance. Lights on red and black buoys, if not fixed, are always regularly flashing or regularly occulting. Quick flashing lights are used when a light of distinct cautionary significance is desired, as at a sharp turn or constriction in the channel. Interrupted quick flashing lights are used on red and black horizontally banded buoys. White short-long flashing lights are used on black and white vertically striped buoys.

Special purpose buoys. White buoys mark anchorages. Yellow buoys mark quarantine anchorages. White buoys with green tops are used in dredging and survey operations. Black and white horizontally banded buoys mark fish net areas. Yellow and black vertically striped buoys mark seadromes. White and orange banded, either horizontally or vertically, are used for special purposes.

Wreck buoys are generally placed on the seaward or channel side, as near the wreck as conditions permit. The possibility of the wreck having shifted position due to sea action since the buoy was placed should not be overlooked.

Station buoys, are placed close to some lightships and important buoys to mark the position if the regular aid is not at the assigned position. Such buoys are colored and numbered the same as the regular aid, lightship station buoys having the letters "LS" above the initials of the station.

Buoys are secured by anchor, and swing in a circle of small radius as the current changes. In this respect, they are inferior to fixed aids for position fixing. They may be shifted, carried away, capsized, or sunk. Lighted buoys may be extinguished and sound buoys may not function.

Dates shown in light lists for seasonal buoys are approximations which vary with local conditions.

918. Fog signals.—Any sound-producing device may serve as a fog signal to warn the mariner of danger or to assist him in establishing the position of his vessel. If a fog signal is to be fully effective for the second of these functions, the mariner must be able to recognize it as a fog signal and to know from what point it is sounded. Bells, whistles, etc., which are sounded by action of the sea are erratic in operation, and positive identification is not always possible. At most lighthouses and lightships fog signals are operated by mechanical means, providing a definite sequence of sounds and silent periods resembling the characteristics of lights. The sequence, stated in the light list, is an aid to identification. The distinctive sound of each type of signal apparatus is helpful in this respect. About 600 fog signals are maintained by the United States Coast Guard. These are of the following types:

Bell, sounded by means of a hammer actuated by hand, descending weight, compressed gas, or electricity.

Diaphone, a device producing sound by means of a slotted reciprocating piston actuated by compressed air. "Two-tone" diaphones produce a blast with a high pitch followed by a lower pitch.

Diaphragm horn, a device producing sound by means of a disk diaphragm vibrated by compressed air, steam, or electricity. Duplex or triplex units produce simultaneous sounds of different pitch, resulting in a chime signal.

Reed horn, a device producing sound by means of a steel reed vibrated by compressed air.

Siren, a device producing sound by means of either a disk or a cup-shaped rotor actuated by compressed air, steam, or electricity.

Whistle, producing sound by compressed air or steam emitted through a circular slot into a cylindrical bell chamber.

Radiobeacons, radar, and other electronic aids to navigation can be of considerable assistance to a vessel equipped to use them.

Tides and Currents

919. Tidal effects.—The daily rise and fall of the **tide,** with its attendant flood and ebb of **tidal current,** is familiar to every mariner. He is aware, also, that at **high water** and **low water** the depth of water is momentarily constant, a condition called **stand.** Similarly, there is a moment of **slack water** as a tidal current reverses direction. As a general rule, the *change* in height or the current *speed* is at first very slow, increasing to a maximum about midway between the two extremes, and then decreasing again. If plotted against time, the height of tide or speed of a tidal current takes the general form of a sine curve. Sample curves, and more complete information about causes, types, and features of tides and tidal currents, are given in chapter XXXI. Ocean (nontidal) currents are discussed in chapter XXXII. The present chapter is concerned primarily with the application of tides and currents to piloting, and predicting the tidal conditions that might be encountered at any given time.

Although tides and tidal currents are caused by the same phenomena, the time relationship between them varies considerably from place to place. For instance, if an estuary has a wide entrance and does not extend far inland, the time of maximum speed of current occurs at about the mid time between high water and low water. However, if an extensive tidal basin is connected to the sea by a small opening, the maximum current may occur at about the time of high water or low water outside the basin, when the difference in height is maximum.

The *height of tide* should not be confused with *depth of water.* For reckoning tides a reference level is selected. Soundings shown on the largest scale charts are the vertical distances from this level to the bottom. At any time the actual depth is this charted depth *plus* the height of tide. In most places the reference level is some form of low water. But all low waters at a place are not the same height, and the selected reference level is seldom the *lowest* tide that occurs at the place. When lower tides occur, these are indicated by a negative sign. Thus, at a spot where the charted depth is 15 feet, the actual depth is 15 feet plus height of tide. When the tide is three feet, the depth is $15+3=18$ feet. When it is $(-)$ 1 foot, the depth is $15-1=14$ feet. It is well to remember that *the actual depth can be less than the charted depth.* In an area where there is a considerable **range of tide** (the difference between high water and low water), the height of tide might be an important consideration in using soundings to assist in determining position, or whether the vessel is in safe water.

One should remember that heights given in the tide tables are *predictions*, and that when conditions vary considerably from those used in making the predictions, the heights shown may be considerably in error. Heights lower than predicted are particularly to be anticipated when the atmospheric pressure is higher than normal, or when there is a persistent strong offshore wind. Along coasts where there is a large

inequality between the two high or two low tides during a tidal day the height predictions are less reliable than elsewhere.

The current encountered in pilot waters is due primarily to tidal action, but other causes are sometimes present. The tidal current tables give the best prediction of total current, regardless of cause. The predictions for a river may be considerably in error following heavy rains or a drought. The effect of current is to alter the course and speed made good over the bottom (art. 807). Due to the configuration of land (or shoal areas) and water, the set and drift may vary considerably over different parts of a harbor. Since this is generally an area in which small errors in position of a vessel are of considerable importance to its safety, a knowledge of predicted currents can be critical, particularly if the visibility is reduced by fog, snow, etc. If the vessel is proceeding at reduced speed, the effect of current with respect to distance traveled is greater than normal. Strong currents are particularly to be anticipated in narrow passages connecting larger bodies of water. Currents of more than five knots are encountered from time to time in the Golden Gate at San Francisco. Currents of more than 13 knots sometimes occur at Seymour Narrows, British Columbia.

In straight portions of rivers and channels the strongest currents usually occur in the middle, but in curved portions the swiftest currents (and deepest water) usually occur near the outer edge of the curve. Countercurrents and eddies may occur on either side of the main current of a river or narrow passage, especially near obstructions and in bights.

In general, the range of tide and the speed of tidal current are at a minimum upon the open ocean or along straight coasts. The greatest tidal effects are usually encountered in rivers, bays, harbors, inlets, bights, etc. A vessel proceeding along a coast can be expected to encounter stronger sets toward or away from the shore while passing an indentation than when the coast is straight.

920. Predictions of tides and currents to be expected at various places are published annually by the U. S. Coast and Geodetic Survey. These are supplemented by nine sets of **Tidal Current Charts,** each set consisting of 12 charts, one for each hour of the tidal cycle. On these charts the set of the current at various places in the area is shown by arrows, and the drift by numbers. Since these are *average* conditions, they indicate in a general way the tidal conditions on any day and during any year. They are designed to be used with the tidal current tables (except those for New York Harbor, which are used with the tide tables). These charts are available for Boston Harbor, Narragansett Bay to Nantucket Sound, Long Island Sound and Block Island Sound, New York Harbor, Delaware Bay and River, Tampa Bay, San Francisco Bay, Puget Sound (northern part), and Puget Sound (southern part). Current arrows are sometimes shown on nautical charts. These represent average conditions and should not be considered reliable predictions of the conditions to be encountered at any given time. When a strong current sets over an irregular bottom, or meets an opposing current, ripples may occur on the surface. These are called **tide rips.** Areas where they occur frequently are shown on charts.

Usually, the mariner obtains tidal information from tide and tidal current tables. However, if these are not available, or if they do not include information at a desired place, the mariner may be able to obtain locally the **mean high water lunitidal interval** or the **high water full and change.** The approximate *time* of high water can be found by adding either interval to the time of transit (either upper or lower) of the moon (art. 2104). Low water occurs approximately ¼ tidal day (about 6^h12^m) before and after the time of high water. The actual interval varies somewhat from day to day, but approximate results can be obtained in this manner. Similar information for tidal currents (**lunicurrent interval**) is seldom available.

921. Tide tables for various parts of the world are published in four volumes by the U. S. Coast and Geodetic Survey. Each volume is arranged as follows:

Table 1 contains a complete list of the predicted times and heights of the tide for each day of the year at a number of places designated as **reference stations.**

Table 2 gives differences and ratios which can be used to modify the tidal information for the reference stations to make it applicable to a relatively large number of **subordinate stations.**

Table 3 provides information for use in finding the approximate height of the tide at any time between high water and low water.

Table 4 is a sunrise-sunset table at five-day intervals for various latitudes from 76°N to 60°S (40°S in one volume).

Table 5 provides an adjustment to convert the local mean time of table 4 to zone or standard time.

Table 6 (two volumes only) gives the zone time of moonrise and moonset for each day of the year at certain selected places.

Table 7 gives certain astronomical data. In the two volumes not having moonrise-moonset tables, this is table 6.

Extracts from tables 1, 2, and 3 for the East Coast of North and South America are given in appendix T. Before the edition having predictions for 1958, the arrangement of tables 1 and 2 were somewhat different than shown.

922. Tide predictions for reference stations.—The first page of appendix T is the table 1 daily predictions for New York (The Battery) for the first quarter of 1958. As indicated at the bottom of the page, times are for Eastern Standard Time (+5 zone, time meridian 75° W). Daylight saving time is not used. Times are given on the 24-hour basis. The tidal reference level for this station is mean low water.

For each day, the date and day of week are given, and the time and height of each high and low water are given in chronological order. Although high and low waters are not labeled as such, they can be distinguished by the relative heights given immediately to the right of the times. Since *two* high tides and *two* low tides occur each tidal day, the type of tide at this place is *semidiurnal* (art. 3105). The *tidal* day being longer than the *civil* day (because of the revolution of the moon eastward around the earth), any given tide occurs *later* from day to day. Thus, on Saturday, March 22, 1958, the first tide that occurs is the lower low water (−0.3 foot at 0333). The following high water (lower high water) is 4.1 feet above the reference level (a 4.4 foot rise from the preceding low water), and occurs at 0929. This is followed by the higher low water (−0.2 feet) at 1540, and then the higher high water of 4.4 feet at 2143. The cycle is repeated on the following day with variations in height, and later times.

Because of later times of corresponding tides from day to day, certain days have only one high water or only one low water. Thus, on January 9 high tides occur at 1051 and 2329. The next following high tides are at 1146 on January 10 and 0024 on January 11. Thus, only one high tide occurs on January 10, the previous one being shortly before midnight on the ninth, and the next one occurring early in the morning of the eleventh, as shown.

923. Tide predictions for subordinate stations.—The second page of appendix T is a page of table 2 of the tide tables. For each subordinate station listed, the following information is given:

Number. The stations are listed in geographical order and given consecutive numbers. At the end of each volume an alphabetical listing is given, and for each entry the consecutive number is shown, to assist in finding the entry in table 2.

Place. The list of places includes both subordinate and reference stations, the latter being given in bold type.

Position. The approximate latitude and longitude are given to assist in locating the station. The latitude is north or south, and the longitude east or west, depending upon the letters (N, S, E, W) next *above* the entry. These may not be the same as those at the *top* of the column.

Differences. The differences are to be applied to the predictions for the reference station shown in bold capitals next *above* the entry on the page. Time and height differences are given separately for high and low waters. Where differences are omitted, they are either unreliable or unknown.

The time difference is the number of hours and minutes to be applied to the time at the reference station to find the time of the corresponding tide at the subordinate station. This interval is added if preceded by a plus sign (+), and subtracted if preceded by a minus sign (−). Special conditions occurring at a few stations are indicated by footnotes on the applicable pages. In some instances, the corresponding tide falls on a different date at reference and subordinate stations.

Height differences are shown in a variety of ways. For most entries separate height differences in feet are given for high water and low water. These are applied to the height given for the reference station. In many cases a *ratio* is given for either high water or low water, or both. The height at the reference station is multiplied by this ratio to find the height at the subordinate station. For a few stations, *both* a ratio and difference are given. In this case the height at the reference station is first multiplied by the ratio, and the difference is then applied. An example is given in each volume of tide tables. Special conditions are indicated in the table or by footnote. Thus, a footnote on the second page of appendix T indicates that "Values for the Hudson River above George Washington Bridge are based upon averages for the six months May to October, when the fresh-water discharge is a minimum."

Ranges. Various ranges are given, as indicated in the tables. In each case this is the difference in height between high water and low water for the tides indicated.

Example.—List chronologically the times and heights of all tides at Yonkers, (No. 1581) on January 10, 1958.

Solution.—

Date	January 10, 1958
Subordinate station	Yonkers
Reference station	New York
High water time difference	(+) 1^h09^m
Low water time difference	(+) 1^h10^m
High water height difference	(−) 0.7 ft.
Low water height difference	0.0 ft.

	New York		*Yonkers*	
HW	2329 (9th)	4.4 ft.	0038	3.7 ft.
LW	0525	(−) 0.6 ft.	0635	(−) 0.6 ft.
HW	1146	4.6 ft.	1255	3.9 ft.
LW	1758	(−) 0.8 ft.	1908	(−) 0.8 ft.

924. Finding height of tide at any time.—Table 3 of the tide tables provides means for determining the approximate height of tide at any time. It is based upon the assumption that a plot of height versus time is a sine curve (art. O40). Instructions for use of the table are given in a footnote below the table, which is reproduced in appendix T.

Example 1.—Find the height of tide at Yonkers (No. 1581) at 1000 on January 10, 1958.

Solution.—The given time is between the low water at 0635 and the high water at 1255 (example of art. 923). Therefore, the tide is rising. The duration of rise is $1255-0635=6^h20^m$. The range of tide is $3.9-(-0.6)=4.5$ feet. The given time is 2^h55^m *before* high water, the nearest tide. Enter the upper part of the table with duration of rise 6^h20^m, and follow the line horizontally to 2^h57^m (the nearest tabulated value to 2^h55^m). Follow this column vertically downward to the entry 2.0 feet in the line for a range of tide of 4.5 feet. This is the correction to be applied to the nearest tide. Since the nearest tide is high water, subtract 2.0 from 3.9 feet. The answer, 1.9 feet, is the height of tide at the given time.

Answer.—Ht. of tide at 1000, 1.9 ft.

Interpolation in this table is not considered justified.

It may be desired to know at what time a given depth of water will occur. In this case, the problem is solved in reverse.

Example 2.—The captain of a vessel drawing 22 feet wishes to pass over a temporary obstruction near Days Point, Weehawken (No. 1571), having a charted depth of 21 feet, passage to be made during the morning of January 10, 1958.

Required.—The earliest time after 0800 that this passage can be made, allowing a safety margin of two feet.

Solution.—The least acceptable depth of water is 24 feet, which is three feet more than the charted depth. Therefore, the height of tide must be three feet or more. At the New York reference station a low tide of (—) 0.6 foot occurs at 0525, followed by a high tide of 4.6 feet at 1146. At Days Point the corresponding low tide is (—) 0.6 foot at 0548, and the high tide is 4.4 feet at 1210. The duration of rise is 6^h22^m, and the range of tide is 5.0 feet. The least acceptable tide is 3.0 feet, or 1.4 feet less than high tide. Enter the *lower* part of table 3 with range 5.0 feet and follow the horizontal line until 1.5 feet is reached (the nearest tabulated value to 1.4 feet). Follow this column vertically *upward* until the value of 2^h19^m is reached on the line for a duration of 6^h20^m (the nearest tabulated value to 6^h22^m). The minimum depth will occur about 2^h19^m *before* high water, or at about 0951.

Answer.—A depth of 24 feet occurs at 0951.

If the range of tide is more than 20 feet, *half* the range (*one third* if the range is greater than 40 feet) is used to enter table 3, and the correction to height is *doubled* (*trebled* if one third is used).

A diagram for a graphical solution is given in figure 924. Eye interpolation can be used if desired. The steps in this solution are as follows:

1. Enter the upper graph with the duration of rise or fall. This is represented by a horizontal line.
2. Find the intersection of this line and the curve representing the interval from the nearest *low* water (point *A*).
3. From *A*, follow a vertical line to the sine curve of the lower diagram (point *B*).
4. From *B*, follow horizontally to the vertical line representing the range of tide (point *C*).
5. Using *C*, read the correction from the series of curves.
6. Add (algebraically) the correction of step 5 to the *low* water height, to find the height at the given time.

The problem illustrated in figure 924 is similar to that of example 1 given above. The duration of rise is 6^h25^m, and the interval from *low* water is 5^h23^m. The range of tide is 6.1 feet. The correction (by interpolation) is 5.7 feet. If the height of the preceding low tide is (—)0.2 foot, the height of tide at the given time is $(-)0.2+5.7=5.5$ feet. To solve example 2 by the graph, enter the lower graph and find the intersection of the vertical line representing 5.0 feet and the curve representing 3.6 feet (the mini-

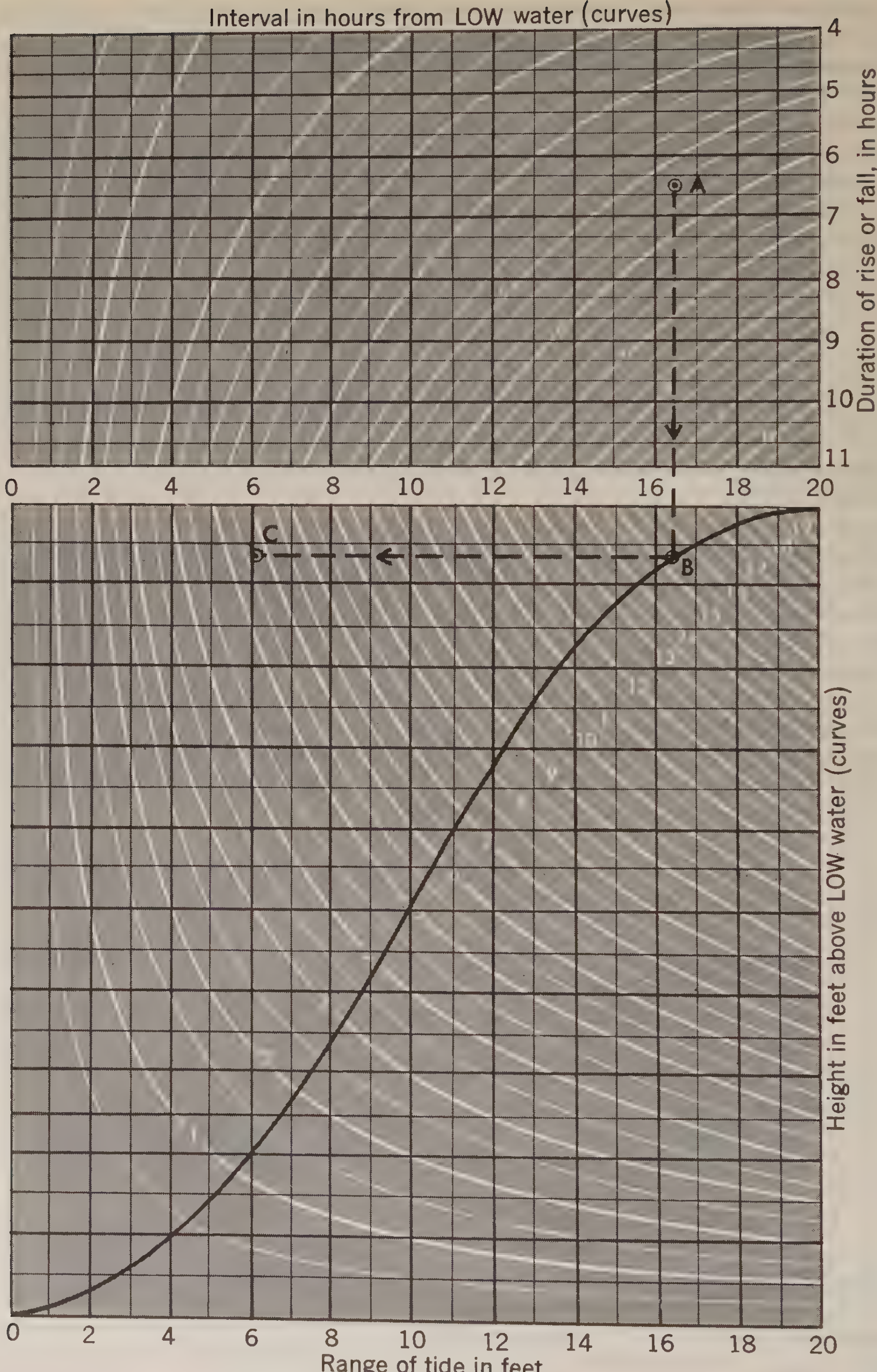


FIGURE 924.—Graphical solution for height of tide at any time.

mum acceptable height above low water). From this point follow horizontally to the sine curve, and then vertically to the horizontal line in the upper figure representing the duration of rise of 6^h22^m. From the curve, determine the interval 4^h10^m. The earliest time is about 4^h10^m *after* low water, or at about 0958.

925. Tidal current tables are somewhat similar to tide tables, but the coverage is less extensive, being given in two volumes. Each volume is arranged as follows:

Table 1 contains a complete list of predicted times of maximum currents and slack, with the velocity (speed) of the maximum currents, for a number of reference stations.

Table 2 gives differences, ratios, and other information related to a relatively large number of subordinate stations.

Table 3 provides information for use in finding the speed of the current at any time between tabulated entries in tables 1 and 2.

Table 4 gives the number of minutes the current does not exceed stated amounts, for various maximum speeds.

Table 5 (Atlantic Coast of North America only) gives information on rotary tidal currents.

Each volume contains additional useful information related to currents. Extracts from the tables for the Atlantic Coast of North America are given in appendix U. Before the edition having predictions for 1958, the arrangement of tables 1 and 2 were somewhat different than shown.

926. Tidal current predictions for reference stations.—The extracts of appendix U are for The Narrows, New York Harbor. Times are given on the 24-hour basis, for meridian 75° W.

For each day, the date and day of week are given, with complete current information. Since the cycle is repeated twice each tidal day, currents at this place are semidiurnal. On most days there are four slack waters and four maximum currents, two of them floods (f) and two of them ebbs (e). However, since the tidal day is longer than the civil day, the corresponding condition occurs later from day to day, and on certain days there are only three slack waters or three maximum currents. At some places, the current on some days runs maximum flood twice, but ebb only once, a minimum flood occurring in place of the second ebb. The tables show this information.

As indicated by appendix U, the sequence of currents at The Narrows on Wednesday, February 26, 1958, is as follows:

0000 Flood current, 8^m after maximum velocity (speed).
0300 Slack, ebb begins.
0625 Maximum ebb of 1.6 knots, setting 160°.
1019 Slack, flood begins.
1217 Maximum flood of 1.1 knots, setting 340°.
1512 Slack, ebb begins.
1840 Maximum ebb of 1.5 knots, setting 160°.
2216 Slack, flood begins.
2400 Flood current, 42^m before maximum velocity (speed).

Only one maximum flood occurs on this day, the previous one having occurred 8 minutes before the day began, and the following one predicted for 42 minutes after the day ends.

927. Tidal current predictions for subordinate stations.—For each subordinate station listed in table 2 of the tidal current tables, the following information is given:

Number. The stations are listed in geographical order and given consecutive numbers, as in the tide tables (art. 923). At the end of each volume an alphabetical listing is given, and for each entry the consecutive number is shown, to assist in finding the entry in table 2.

Place. The list of places includes both subordinate and reference stations, the latter being given in **bold type**.

Position. The approximate latitude and longitude are given to assist in locating the station. The latitude is north or south and the longitude east or west as indicated by the letters (N, S, E, W) next *above* the entry. The current given is for the center of the channel unless another location is indicated by the station name.

Time difference. Two time differences are tabulated. One is the number of hours and minutes to be applied to the tabulated times of slack water at the reference station to find the times of slack waters at the subordinate station. The other time difference is applied to the times of maximum current at the reference station to find the times of the corresponding maximum current at the subordinate station. The intervals, which are added or subtracted in accordance with their signs, include any difference in time between the two stations, so that the answer is correct for the standard time of the subordinate station. Limited application and special conditions are indicated by footnotes.

Velocity (speed) ratios. Speed of the current at the subordinate station is found by multiplying the speed at the reference station by the tabulated ratio. Separate ratios may be given for flood and ebb currents. Special conditions are indicated by footnotes.

As indicated in appendix U, the currents at Chelsea Docks (No. 1005) can be found by *adding* 1^h30^m for slack water and 1^h40^m for maximum current to the times for The Narrows, and multiplying flood currents by 0.9 and ebb currents by 1.2. Applying these to the values for Wednesday, February 26, 1958, the sequence is as follows:

0000 Flood current, 1^h32^m before maximum velocity (speed).
0132 Maximum flood of 1.4 knots, setting 010°.
0430 Slack, ebb begins.
0805 Maximum ebb of 1.9 knots.
1149 Slack, flood begins.
1357 Maximum flood of 1.0 knots, setting 010°.
1642 Slack, ebb begins.
2020 Maximum ebb of 1.8 knots.
2346 Slack, flood begins.
2400 Flood current, 14^m after slack.

928. Finding speed of tidal current at any time.—Table 3 of the tidal current tables provides means for determining the approximate velocity (speed) at any time. Instructions for its use are given below the table, which is reproduced in appendix U.

Example 1.—Find the speed of the current at Chelsea Docks at 1500 on February 26, 1958.

Solution.—The given time is between the maximum flood of 1.0 knots at 1357 and the slack at 1642 (art. 927). The interval between slack and maximum current (1642 —1357) is 2^h45^m. The interval between slack and the desired time (1642—1500) is 1^h42^m. Enter the table (A) with 2^h40^m at the top, and 1^h40^m at the left side (the nearest tabulated values to 2^h45^m and 1^h42^m, respectively), and find the factor 0.8 in the body of the table. The approximate speed at 1500 is $0.8 \times 1.0 = 0.8$ knot, and it is flooding.

Answer.—Speed 0.8 kn.

It may be desired to determine the period during which the current is less (or greater) than a given amount. Table 4 of the tidal current tables can be used to determine the period during which the speed does not exceed 0.5 knot. For greater speeds, and for more accurate results under some conditions, table 3 of the tidal current tables can be used, solving by reversing the process used in example 1.

Example 2.—During what period on the evening of February 26, 1958, does the ebb current equal or exceed 1.0 knot at Chelsea Docks?

Solution.—The maximum ebb of 1.8 knots occurs at 2020. This is preceded by a slack at 1642, and followed by the next slack at 2346. The interval between the earlier slack and the maximum ebb is 3^h38^m, and the interval between the ebb and following slack is 3^h26^m. The desired factor is $\frac{1.0}{1.8}=0.6$. Enter table A with 3^h40^m (the nearest tabulated value to 3^h38^m) at the top, and follow down the column to 0.6 (midway between 0.5 and 0.7). At the left margin the interval between slack and the desired time is found to be 1^h30^m (midway between 1^h20^m and 1^h40^m). Therefore, the current becomes 1.0 knot at $1642+1^h30^m=1812$. Next, enter table A with 3^h20^m (the nearest tabulated value to 3^h26^m) at the top, and follow down the column to 0.6. Follow this line to the left margin, where the interval between slack and desired time is found to be 1^h20^m. Therefore, the current is 1.0 knot or greater until $2346-1^h20^m=2226$. If the two intervals between maximum current and slack were nearest the same 20^m interval, table A would have to be entered only once.

Answer.—The speed equals or exceeds 1.0 knot between 1812 and 2226.

929. Current diagrams.—A current diagram is a graph showing the speed of the current along a channel at different stages of the tidal current cycle. The current tables include such diagrams for Boston Harbor; Vineyard and Nantucket Sounds (one diagram); East River, New York; New York Harbor; Delaware Bay and River (one diagram); Chesapeake Bay; South San Francisco Bay; and North San Francisco Bay. The diagram for New York Harbor is reproduced in appendix U.

On this diagram each vertical line represents a given instant identified in terms of the number of hours before or after slack *at The Narrows.* Each horizontal line represents a distance from Ambrose Channel Entrance, measured along the usually-traveled route. The names along the left margin are placed at the correct distances from Ambrose Channel Entrance. The current is for the center of the channel opposite these points. The intersection of any vertical line with any horizontal line represents a given moment in the current cycle at a given place in the channel. If this intersection is in a shaded area, the current is flooding; if in an unshaded area, it is ebbing. The speed in knots can be found by interpolation (if necessary) between the numbers given in the body of the diagram. The given values are *averages.* To find the value at any given time, multiply the speed found from the diagram by the ratio of *maximum speed of the current involved* to the *maximum shown on the diagram,* both values being taken for The Narrows. If the diurnal inequality is large, the accuracy can be improved by altering the width of the shaded area to fit conditions. The diagram covers 1½ current cycles, so that the right-hand third is a duplication of the left-hand third.

If the current for a single station is desired, table 1 or 2 should be used. The current diagrams are intended for use in either of two ways: First, to determine a favorable time for passage through the channel. Second, to find the average current to be expected during any passage through the channel. For both of these uses a number of "speed lines" are provided. When the appropriate line is transferred to the correct part of the diagram, the current to be encountered during passage is indicated along the line.

Example.—During the morning of January 10, 1958, a ship is to leave Pier 83 at W. 42nd St., and proceed down the bay at ten knots.

Required.—(1) Time to get underway to take maximum advantage of a favorable current, allowing 15 minutes to reach mid channel.

(2) Average speed over the bottom during passage down the bay.

Solution.—(1) Transfer the line (slope) for ten knots southbound to the diagram, locating it so that it is centered on the unshaded ebb current section between W. 42nd

St. and Ambrose Channel Entrance. This line crosses a horizontal line through W. 42nd St. about one-third of the distance between the vertical lines representing three and two hours, respectively, after ebb begins at The Narrows. The setting is not critical. Any time within about half an hour of the correct time will result in about the same current. Between the points involved, the entire speed line is in the ebb current area.

(2) Table 1 indicates that on the morning of January 10 ebb begins at The Narrows at 0050. Two hours forty minutes after ebb begins, the time is 0330. Therefore, the ship should reach mid channel at 0330. It should get underway 15 minutes earlier, at 0315.

(3) To find the average current, determine the current at intervals (as every two miles), add, and divide by the number of entries.

Distance	*Current*
18	1.2
16	1.4
14	1.9
12	1.5
10	2.0
8	1.9
6	1.3
4	1.2
2	1.4
0	1.2
sum	14.8

The sum of 14.8 is for ten entries. The average is therefore $14.8 \div 10 = 1.5$ knots.

(4) This value of current is correct only if the ebb current is an average one. From table 1 the maximum ebb involved is 2.2 knots. From the diagram the maximum value at The Narrows is 2.0 knots. Therefore, the average current found in step (3) should be increased by the ratio $2.2 \div 2.0 = 1.1$. The average for the run is therefore $1.5 \times 1.1 = 1.6$ knots. Speed over the bottom is $10 + 1.6 = 11.6$ knots.

Answers.—(1) T 0315, (2) S 11.6 kn.

In the example, an ebb current is carried throughout the run. If the transferred speed line had been partly in a flood current area, all ebb currents (those increasing the ship's speed) should be given a positive sign (+), and all flood currents a negative sign (−). A separate ratio should be determined for each current (flood or ebb), and applied to the entries for that current. In Chesapeake Bay it is not unusual for an outbound vessel to encounter three or even four separate currents during passage down the bay. Under the latter condition, it is good practice to multiply *each* current taken from the diagram by the ratio for the current involved.

If the time of starting the passage is fixed, and the current during passage is desired, the starting time is identified in terms of the reference tidal cycle. The speed line is then drawn through the intersection of this vertical time line and the horizontal line through the place. The average current is then determined in the same manner as when the speed line is located as described above.

Miscellaneous

930. Allowing for turning characteristics of vessel.—When precise piloting is necessary (as in an area where maneuvering space is limited, when a specified anchorage is approached, or when steaming in formation with other ships), the turning char-

acteristics of the vessel should be considered. That is, a ship does not complete a turn instantaneously, but follows a curve the characteristics of which depend upon the vessel's length, beam, underwater contour, draft, etc. From the moment the rudder is put over until the new course is reached, the vessel moves a certain distance in the direction of the original course. This distance is called **advance.** The distance the vessel moves perpendicular to the original course line during the turn is called **transfer.** These terms are illustrated in figure 930a. The amount of advance and transfer for a given vessel depends primarily upon the amount of rudder used and the angle through which the ship is to be turned. The speed of the vessel has little effect. Allowance for advance and transfer is illustrated in the following example.

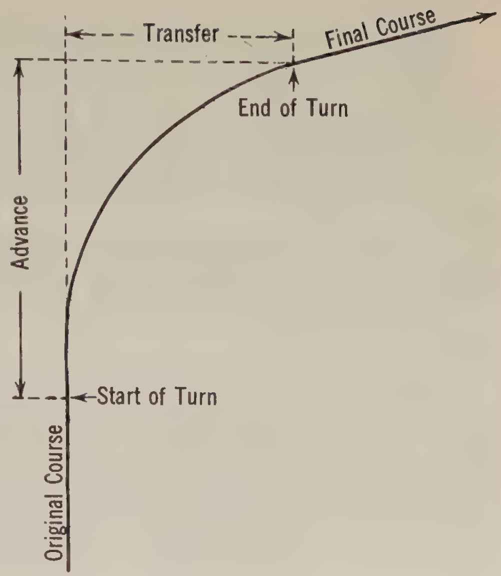


FIGURE 930a.—Advance and transfer.

Example (fig. 930b).—A ship proceeding on course 100° is to turn 60° to the left to come on a range which will guide it up a channel. For a 60° turn and the amount of rudder used, the advance is 920 yards and the transfer is 350 yards.

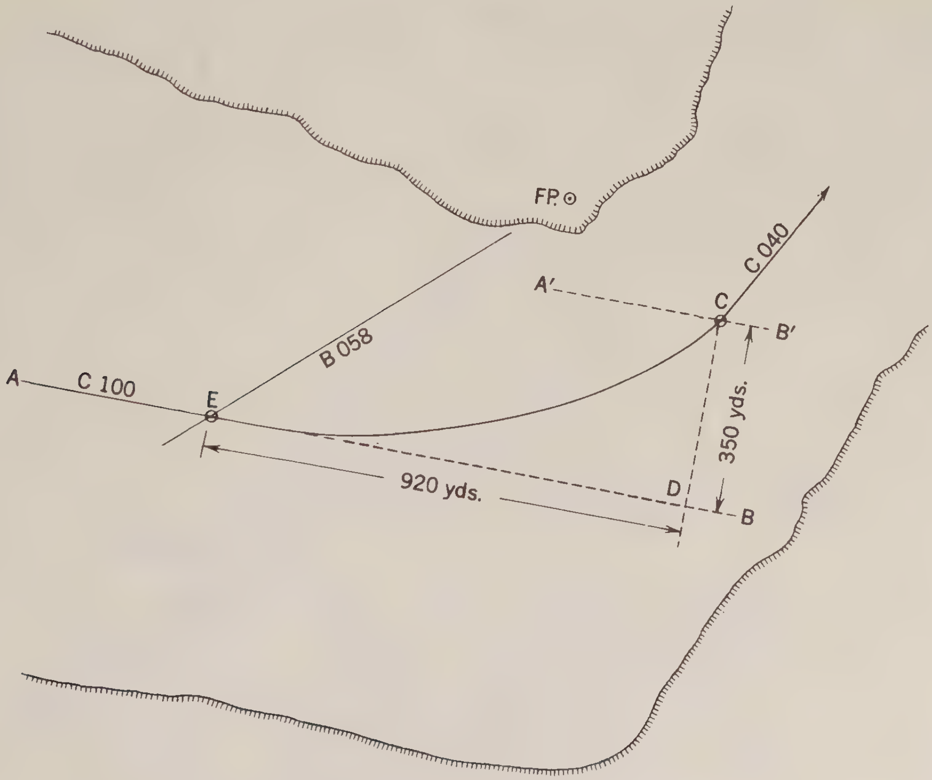


FIGURE 930b.—Allowing for advance and transfer.

Required.—The bearing of flagpole "FP." when the rudder is put over.

Solution.—(1) Extend the original course line, *AB*.

(2) At a perpendicular distance of 350 yards, the transfer, draw a line *A′B′* parallel to the original course line *AB*. The point of intersection, *C*, of *A′B′* with the new course line (located by the range) is the place at which the turn is to be completed.

(3) From *C* draw a perpendicular, *CD*, to the original course line, intersecting it at *D*.

(4) From *D* measure the advance, 920 yards, *back* along the original course line. This locates *E*, the point at which the turn should be started.

(5) The direction of "FP." from *E*, 058°, is the bearing when the turn should be started.

Answer.—B 058°.

931. Anchoring.—If a vessel is to anchor at a predetermined point, as in an assigned berth, an established procedure should be followed to insure accuracy of placing the anchor. In the case of naval vessels, it is desired that the error not exceed ten yards. This requires a high order of navigational accuracy. Several procedures have been devised. The following is representative (fig. 931).

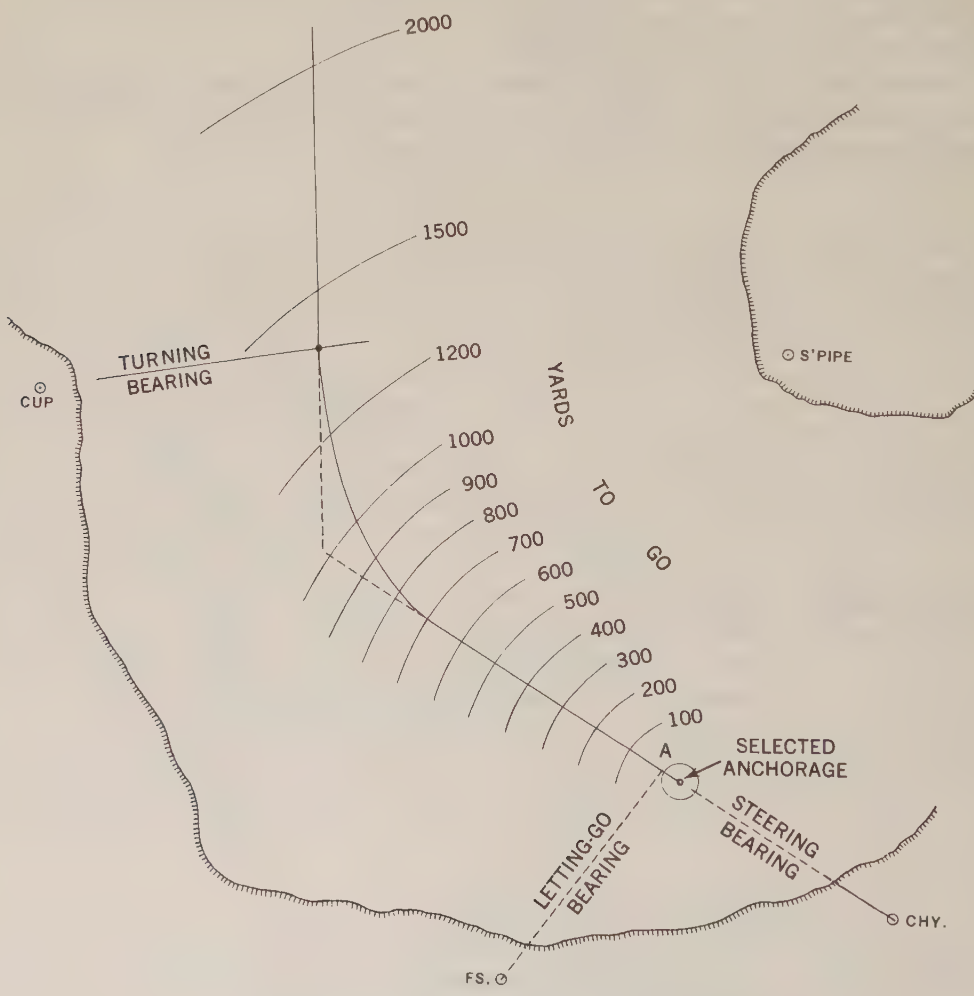


Figure 931.—Anchoring.

The position selected for anchoring is located on the chart. The direction of approach is then determined, considering limitations of land, shoals, other vessels, etc. Where conditions permit, the approach should be made heading into the current or, if the wind has a greater effect upon the vessel, into the wind. It is desirable to approach from such direction that a prominent object, or preferably a range, is available dead ahead to serve as a steering guide. It is also desirable to have a range or prominent object near the beam at the point of letting go the anchor. If practicable a straight approach of at least 500 yards should be provided to permit the vessel to steady on the required course. The track is then drawn in, allowing for advance and transfer during any turns.

Next, a circle is drawn with the selected position of the anchor as the center, and with a radius equal to the distance between the hawsepipe and pelorus, alidade, etc., used for measuring bearings. The intersection of this circle and the approach track, point *A*, is the position of the vessel (bearing-measuring instrument) at the moment of letting go. A number of arcs of circles are then drawn and labeled as shown in figure 931. The desired position of the anchor is the common center of these arcs. The selected radii may be chosen at will. Those shown in figure 931 have been found to be generally suitable. In each case the distance indicated is from the small circle. Turning bearings may also be indicated.

During the approach to the anchorage, fixes are plotted at frequent intervals, the measurement and plotting of bearings going on continuously, usually to the nearest half or quarter degree. The navigator advises the captain of any tendency of the vessel to drift from the desired track, so that adjustments can be made. The navigator also keeps the captain informed of the distance to go, to permit adjustment of the speed so that the vessel will be nearly dead in the water when the anchor is let go.

At the moment of letting go, the position of the vessel should be determined as accurately as possible, preferably by two simultaneous horizontal sextant angles, or by simultaneous or nearly simultaneous bearings of a number of prominent landmarks.

A number of variations may suggest themselves. One occasionally mentioned is as follows: An inverted compass rose (0° at south) is placed around each landmark used. A thumb tack with an attached thread is inserted at the symbol of two landmarks. One observer continually notes the bearing of each object. Alternately they call out the bearings. The navigator takes one thread in each hand and maintains a slight strain. As each bearing is called out, he adjusts the appropriate thread by means of the reverse compass rose. The point of intersection of the two threads is the position of the vessel. By this means the ship can be "walked in" to the anchorage. This method is particularly to be recommended when one landmark is on each bow.

The exact procedure to use depends upon local conditions, number and training of available personnel, equipment, and personal preference of individuals concerned. Whatever the procedure, it should be carefully planned, and any needed advance preparations should be made early enough to avoid haste and the attendant danger of making a mistake. Teamwork is important. Each person involved should understand precisely what is expected of him.

932. Piloting and electronics.—Many of the familiar electronic aids to navigation are used primarily in piloting. The radio direction finder provides bearings through fog and at greater distance from the aids. Distance finding stations provide distances which might not otherwise be available. The sonic depth finder provides frequent or continuous soundings. Radar provides bearings, distances, and information on the location and identity of various targets. Some of the longer range systems such as loran, consol, and Decca extend piloting techniques far to sea, where nearness of shoals and similar dangers is not a problem. Electronic aids, whether applicable primarily to piloting or to other forms of navigation, are discussed in part three (chs. X–XIII).

933. Practical piloting.—In pilot waters, navigation is primarily an art. It is essential that the principles explained in this chapter be mastered and applied intelligently. From every experience the wise navigator acquires additional knowledge and improves his judgment. The mechanical following of a set procedure should not be expected to produce satisfactory results always.

While piloting, the successful navigator is somewhat of an opportunist, fitting his technique to the situation at hand. If a vessel is steaming in a large area having relatively weak currents and moderate traffic, like Chesapeake Bay, fixes may be obtained at relatively long intervals, with a dead reckoning plot between. In a narrow channel with swift currents and heavy traffic, like the East River between Manhattan and Long Island, New York City, an almost continuous fix is needed. In such an area the navigator may draw the desired track on the chart and obtain fixes every few minutes, or even seconds, directing the vessel back on the track as it begins to drift to one side.

If the navigator is to traverse unfamiliar waters, he studies the chart, sailing directions or coast pilot, tide and tidal current tables, and light lists to familiarize himself with local conditions. The experienced navigator learns to interpret the signs around him. The ripple of water around buoys and other obstructions, the direction and angle of tilt of buoys, the direction at which vessels ride at anchor, provide meaningful information regarding currents. The wise navigator learns to interpret such signs when the position of his vessel is not in doubt. When visibility is poor, or available information is inconsistent, the ability developed at favorable times can be of great value.

With experience, a navigator learns when a danger angle or danger bearing is useful, and what ranges are reliable and how they should be used. However familiar one is with an area, he should not permit himself to become careless in the matter of timing lights for identification, plotting his progress on a chart, or keeping a good recent position. Fog sometimes creeps in unnoticed, obscuring landmarks before one realizes its presence. A series of frequent fixes obtained while various aids are visible provides valuable information on position and current.

Practical piloting requires a thorough familiarity with principles involved and local conditions, constant alertness, and judgment. A study of avoidable groundings reveals that in most cases the problem is not lack of knowledge, but failure to use or interpret available information. Among the more common errors are:

1. Failure to obtain or evaluate soundings.
2. Failure to identify aids to navigation.
3. Failure to use all available navigational aids.
4. Failure to correct charts.
5. Failure to adjust a magnetic compass or maintain an accurate table of corrections.
6. Failure to apply deviation, or error in its application.
7. Failure to apply variation, or to allow for change in variation.
8. Failure to check gyro and magnetic compass readings at frequent and regular intervals.
9. Failure to keep a dead reckoning plot.
10. Failure to plot information received.
11. Failure to properly evaluate information received.
12. Poor judgment.
13. Failure to do own navigating (following another vessel).
14. Failure to obtain and use information available on charts and in various publications.

15. Poor ship organization.
16. Failure to "keep ahead of the vessel."

Further discussion on practical piloting is given in chapter XXIII.

Problems

904. The navigator of a vessel on gyro course 214° obtains a relative bearing on a lighthouse of 86° left. The gyro error is 1° W, deviation of the standard compass is 3° W, and variation is 13° E.

Required.—The (1) gyro, (2) true, (3) standard-compass, and (4) magnetic bearings of the lighthouse.

Answers.—(1) Bpgc 128°, (2) TB 127°, (3) CB 117°, (4) MB 114°.

Problems 906a–909g, 911a–911c, and 916b are based upon the following fictitious aids to navigation, which can be plotted on a large-scale plotting sheet (art. 324) or a clear part of a coast chart for the appropriate latitude. The latitude range of these problems is from 40°20′ N to 40°40′ N. Approach charts to New York include this range. The longitude range is from 164°20′ W to 164°40′ W. In these problems the gyro compass is the reference for all courses and bearings, and it is considered to be without error.

	Latitude	*Longitude*
Jones Point Light	40°20′.6 N	164°20′.5 W
Parker Point Light	40°23′.7 N	164°21′.2 W
Point Carlson Light	40°22′.0 N	164°28′.3 W
North Baker Range Light	40°33′.9 N	164°38′.2 W
South Baker Range Light	40°31′.5 N	164°34′.7 W
Hanford Mid-channel Buoy	40°22′.9 N	164°34′.1 W
Water tower	40°36′.2 N	164°27′.9 W
West Bank Lightship	40°39′.5 N	164°20′.3 W
Cupola	40°25′.4 N	164°21′.3 W

906a. A navigator of a vessel on course 075° observes Jones Point Light to bear 56° right at the same time an assistant observes Parker Point Light to bear 46° left.

Required.—The fix at the time of the bearings.

Answer.—Fix: L 40°21′.9 N, λ 164°22′.5 W.

906b. A vessel under way in fog obtains simultaneous radar ranges (distances) on Point Carlson Light bearing southerly (3.0 miles) and Parker Point Light (5.5 miles).

Required.—The fix at the time of the distance measurements.

Answer.—Fix: L 40°25′.0 N, λ 164°28′.2 W.

906c. A vessel under way with the Baker Range dead ahead observes Point Carlson Light broad on the port beam.

Required.—The fix at the time Point Carlson Light is abeam.

Answer.—Fix: L 40°24′.9 N, λ 164°25′.0 W.

906d. The navigator of a vessel on course 110° measures the vertical sextant angle between the top of Point Carlson Light and the horizon at the same time the light bears dead ahead. The top of the light is 230 feet above the water and the sextant angle is 0°18′.3. The height of eye of the observer is 30 feet. There is no IC.

Required.—The fix at the time the angle is measured.

Answer.—Fix: L 40°24′.4 N, λ 164°37′.0 W.

906e. A vessel is under way with the Baker Range dead ahead. The South Baker Range Light is 5.0 miles off by radar.

Required.—The fix at the time of measurement.

Answer.—Fix: L 40°28′.2 N, λ 164°29′.8 W.

906f. A southbound vessel passes Hanford Mid-channel Buoy close aboard at 0327.

Required.—The fix at this time.

Answer.—Fix: L 40°22′.9 N, λ 164°34′.1 W.

907a. Using horizontal sextant angles, a navigator measures the angle between South Baker Range Light and Point Carlson Light to be 54°14′. At the same time an assistant measures the angle between Parker Point Light and Point Carlson Light to be 25°04′.

Required.—The fix at the time of observation.

Answer.—Indeterminable because the three lights and the vessel are all located on the circumference of the same circle.

907b. Using horizontal sextant angles, a navigator measures the angle between South Baker Range Light and Point Carlson Light to be 85°45′. At the same time an assistant measures the angle between Parker Point Light and Point Carlson Light to be 35°10′.

Required.—The fix at the time of observation.

Answer.—Fix: L 40°31′.6 N, λ 164°27′.6 W.

908. About 0229 the navigator of a vessel observes the following bearings:

Jones Point Light	150°
Point Carlson Light	263°
Parker Point Light	020°
Point Carlson Light	266°
Jones Point Light	154°

The time intervals between bearings are approximately equal. The bearing on Parker Point Light is obtained at 0229.

Required.—(1) The bearings to plot for an 0229 fix.

(2) The 0229 fix.

Answers.—(1) Jones Point Light, B 152°; Point Carlson Light, B 264½°; Parker Point Light, B 020°. (2) 0229 fix: L 40°22′.5 N, λ 164°21′.8 W.

909a. A vessel is underway on course 071°, speed 15.0 knots. At 1150 the water tower bears 051°. At 1200 the tower bears 009°.

Required.—The 1200 running fix.

Answer.—1200 R fix: L 40°34′.9 N, λ 164°28′.2 W.

909b. At 1205 the vessel of problem 909a changes course to 047°, and at 1210 the water tower bears 270°.

Required.—The 1210 running fix, by advancing the 1200 bearing line and crossing it with the 1210 bearing line.

Answer.—1210 R fix: L 40°36′.2 N, λ 164°25′.4 W.

909c. Under way in fog, a vessel on course 188°, speed 5.0 knots, passes west of the West Bank Lightship and at 0613 it is 1.2 miles off by distance finding signals. At 0622 the distance is 1.8 miles.

Required.—The 0622 running fix.

Answer.—0622 R fix: L 40°38′.1 N, λ 164°21′.8 W.

909d. The navigator of a vessel on course 052°, speed 13.5 knots, observes Point Carlson Light bearing 079° at 2117.

Required.—The distance off when abeam if Point Carlson Light is abeam at (1) 2126, (2) 2129, (3) 2132.

Answers.—(1) D 1.1 mi., (2) D 1.4 mi., (3) D 1.7 mi.

909e. The navigator of a vessel on course 000° observes South Baker Range Light bearing 324° at 0551 and 270° at 0600.

Required.—The distance off when abeam if the vessel is making good (1) 15.0 kn., (2) 16.0 kn., (3) 17.0 kn.

Answers.—(1) D 1.6 mi., (2) D 1.8 mi., (3) D 1.9 mi.

909f. The navigator of a vessel obtains bearings on West Bank Lightship, as follows: 033° at 1423, 021° at 1435, 010° at 1443.

Required.—The course being made good over the ground.

Answer.—COG 073°.

909g. A vessel on course 214°, speed 14.0 knots, fixes its position at latitude 40°33!0 N, longitude 164°21!5 W at 1200. At 1254 a second fix places the vessel at latitude 40°22!0 N, longitude 164°32!5 W.

Required.—The set and drift of the current during the run.

Answers.—Set 249°, drift 1.6 kn.

910a. The navigator of a vessel steaming at 17 knots observes the following pairs of relative bearings on different landmarks and seamarks as indicated:

	Time	*Object*	*Relative bearing*		*Time*	*Object*	*Relative bearing*
(1)	1237	*A*	318°	(3)	1325	*C*	335°
	1258	*A*	292°		1350	*C*	303°
(2)	1306	*B*	040°	(4)	1401	*D*	281°
	1321	*B*	059°		1452	*D*	257°

Required.—In each case, the distance off at the time of the second bearing, and the distance when abeam, using table 7.

Answers.—

	Object	*Dist. at 2nd bearing*	*Dist. abeam*
(1)	*A*	9. 2	8. 5
(2)	*B*	8. 3	7. 1
(3)	*C*	5. 7	4. 8
(4)	*D*	34. 8	34. 0

910b. A vessel is steaming on course 193° at 20 knots. The following true bearings are observed on the objects indicated:

	Time	*Object*	*True Bearing*		*Time*	*Object*	*True Bearing*
(1)	0800	*A*	229°	(4)	0912	*D*	215°5
	0836	*A*	265°		0927	*D*	238°
(2)	0840	*B*	238°	(5)	0929	*E*	223°
	0855	*B*	283°		0954	*E*	253°
(3)	0855	*C*	256°5	(6)	0959	*F*	233°
	0906	*C*	283°		1031	*F*	272°

Required.—Without plotting, and without the use of table 7, determine the distances off *A*, *B*, *D*, and *E* at the time of the second bearing, and the distances off *B*, *C*, *D*, *E*, and *F* when abeam.

Answers.—

	Object	*Dist. at 2nd bearing*	*Dist. abeam*
(1)	*A*	12. 0	—
(2)	*B*	5. 0	5. 0
(3)	*C*	—	7. 3
(4)	*D*	5. 0	3. 5
(5)	*E*	8. 3	7. 3
(6)	*F*	—	10. 7

911a. Two shoals to the south of South Baker Range Light are marked by buoys. The positions of the buoys are reported to be unreliable because of the recent passage of a storm. At the narrowest point in the channel the position of the danger sounding on each side is lat. 40°25′.5 N, long. 164°37′.2 W; and lat. 40°25′.7 N, long. 164°35′.6 W.

Required.—The maximum and minimum danger bearings on South Baker Range Light to clear the shoals.

Answers.—B 018° and B 007°.

911b. A vessel is to pass along the coast north of Parker Point and inside a submerged rock a short distance offshore. In this area the five-fathom curve (the danger sounding) is farthest offshore at lat. 40°24′.5 N, long. 164°22′.3 W. The closest safe approach to the rock is at lat. 40°24′.4 N, long. 164°22′.9 W.

Required.—The maximum and minimum horizontal sextant angles between Parker Point Light and the prominent cupola to the north of it which will permit safe passage between the five-fathom curve and the rock.

Answers.—Danger angles: 92° and 68°.

911c. A vessel steaming at 13.0 knots has West Bank Lightship abeam at 0311, and immediately begins a course change to keep the lightship broad on the beam. At 0316 the change in heading is noted.

Required.—(1) Distance off if the heading change is 19°.

(2) Distance off if the heading change is 16°.

Answers.—(1) D 3.2 mi., (2) D 3.9 mi.

916a. A lookout at a height of eye of 55 feet observes a flashing light on the horizon. The light is timed and identified as a navigational light 117 feet above sea level.

Required.—The distance of the vessel from the light when it is first observed.

Answer.—D 20.9 mi.

916b. The light of Hanford Mid-channel Buoy is located 11 feet above sea level and has a charted range of 6 miles. At 0207 a vessel on course 221°, speed 15 knots, passes West Bank Lightship abeam, 0.9 mile to starboard. The height of eye on the bridge is 48 feet.

Required.—(1) The distance at which the navigator, on the bridge, can expect to see the buoy light in clear weather with good visibility.

(2) The time and bearing at which the light should be sighted.

Answers.—(1) D 6.0 mi., (2) T 0302, B 202°.

916c. A navigational light 120 feet above sea level has a charted range of 17 miles.

Required.—The distance at which an observer at a height of eye of 60 feet can expect to see the light.

Answer.—D 17 to 21.4 mi., depending upon the luminous range of the light.

920. The mean high water lunitidal interval at a certain port is 2^h17^m.

Required.—The approximate times of each high and low water on a day when the moon transits the local meridian at 1146.

Answers.—HW at 0139 and 1403, LW at 0751 and 2015.

922. List chronologically the times and heights of all tides at New York (The Battery) on February 11, 1958.

Answer.—

Time	*Tide*	*Height*
0157	HW	4.5 ft.
0831	LW	(−) 0.1 ft.
1423	HW	3.8 ft.
2049	LW	(−) 0.1 ft.

923. List chronologically the times and heights of all tides at Castle Point, Hoboken, N. J. (No. 1569) on March 18, 1958.

Answer.—

Time	*Tide*	*Height*
0113	LW	(−) 0.3 ft.
0723	HW	4.4 ft.
1336	LW	(−) 0.4 ft.
1945	HW	4.3 ft.

924a. Find the height of tide at Union Stock Yards, New York (No. 1573) at 0600 on February 6, 1958.

Answer.—Ht. of tide at 0600, 0.3 ft.

924b. The captain of a vessel drawing 24 feet wishes to pass over a temporary obstruction near Bayonne, N. J. (No. 1555) having a charted depth of 22 feet, passage to be made during the evening of March 5, 1958.

Required.—The earliest and latest times that the passage can be made, allowing a safety margin of two feet.

Answers.—Earliest time 1806, latest time 2148.

926. Determine the sequence of currents at The Narrows on January 8, 1958.

Answer.—

0000 Ebb current, 56^m after slack.
0222 Maximum ebb of 2.3 knots.
0543 Slack, flood begins.
0821 Maximum flood of 2.2 knots.
1135 Slack, ebb begins.
1454 Maximum ebb of 2.5 knots.
1828 Slack, flood begins.
2051 Maximum flood of 1.9 knots.
2357 Slack, ebb begins.
2400 Ebb current, 3^m after slack.

927. Determine the sequence of currents at Highlands Bridge, Shrewsbury River (No. 1083) on January 5, 1958.

Answer.—

0000 Ebb current, 22^m before maximum velocity (speed).
0022 Maximum ebb of 2.7 knots.
0351 Slack, flood begins.
0628 Maximum flood of 3.3 knots, setting 170°.
0940 Slack, ebb begins.
1302 Maximum ebb of 3.0 knots.
1644 Slack, flood begins.
1855 Maximum flood of 2.6 knots, setting 170°.
2150 Slack, ebb begins.
2400 Ebb current, 1^h11^m before maximum velocity (speed).

928a. Find the speed of the current at Bear Mountain Bridge (No. 1029) at 0900 on February 19, 1958.

Answer.—Speed 0.5 kn.

928b. At about what time during the morning of February 3, 1958, does the flood current northwest of The Battery (No. 1001) reach a speed of 1.0 knot?

Answer.—T 0525.

929. A vessel arrives at Ambrose Channel Entrance two hours after flood begins at The Narrows on the morning of February 16, 1958.

Required.—(1) The speed through the water required to take fullest advantage of the flood tide in steaming to Chelsea Docks.

(2) The average current to be expected.

(3) Estimated time of arrival off Chelsea Docks.

Answers.—(1) S 9 kn., (2) S 1.4 kn., (3) ETA 0608.

930. A vessel on course 337° begins a 90° turn at half right rudder when a buoy is close aboard. When the heading is 067°, the buoy bears 192°, distant 1225 yards by radar.

Required.—Advance and transfer.

Answers.—Advance 1,000 yards, transfer 712 yards.

931. In the solution of this problem, a plotting sheet covering the area between latitudes 40°29′35″ N and 40°30′25″ N, and longitudes 164°19′35″ W and 164°20′50″ W will be needed. The plotting sheet or chart used for other problems of this chapter will be suitable if the scale is adjusted so that one graduation equals 50 feet (two graduations equal one second of arc).

A vessel in convoy receives orders to anchor in lat. 40°29′50″ N, long. 164°20′25″ W. After studying the chart, the navigator and captain select two landmarks to use as reference points in making the approach. Landmark *A* is located at lat. 40°29′40″ N, long. 164°20′50″ W; landmark *B* at lat. 40°30′20″ N, long. 164°20′30″ W. It is decided that the approach will be made with landmark *A* dead ahead. The gyro repeater to be used in taking bearings is located 200 feet abaft the hawsepipes.

Required.—(1) The course during approach. The bearings of landmark *B* when there are (2) 1,000 yards, (3) 600 yards, (4) 400 yards, (5) 200 yards, (6) 100 yards, (7) 0 yards (the let-go point) to go, assuming the vessel keeps landmark *A* dead ahead.

Answers.—(1) C 242°.5, (2) B 295°.5, (3) B 314°.0, (4) B 325°.5, (5) B 337°.5, (6) B 343°.5, (7) B 349°.5.

PART THREE

ELECTRONIC NAVIGATION

PART THREE

ELECTRONIC NAVIGATION

CHAPTER X

RADIO WAVES

1001. Source of radio waves.—All matter is made up of tiny particles called **atoms.** Each atom has a central **nucleus** composed principally of subatomic particles called **protons** and **neutrons.** One or more **electrons** revolve around the nucleus in orbits resembling those of planets around the sun (art. 1407). The number and arrangement of the particles constituting an atom of each element of matter determine the properties of that element. Electrons, each having a mass of only about 1/1,840 that of a proton or neutron, are kept in their orbits principally by means of an attractive electrical force, each electron carrying one negative "charge" and each proton one positive "charge." Like charges repel and unlike charges attract. This electrical attraction is additional to the gravitational attraction existing between all particles in the universe. The neutron is electrically neutral.

Under suitable conditions, some electrons become detached from their atoms. An excess or deficiency of electrons in a nonconductor is called **static electricity.** A substance which provides a path for electron movement with relatively little resistance is called a **conductor.** A *flow* of electrons along such a conductor constitutes an **electric current,** although the current direction is conventionally considered to be opposite to the direction of flow of the electrons. A **direct current** flows continuously in the same direction. If the strength of the current varies rhythmically but does not change direction, the current is said to be **pulsating.** If the direction of flow periodically reverses, an **alternating current** results.

In addition to its electrical and gravitational forces, a moving electron is accompanied by a **magnetic force.** As long as the flow is steady, the magnetic force is constant. If a conductor is in the region of influence or **field** of magnetism, there is no noticeable effect unless the strength of the field is changing, or relative motion exists between the conductor and the field, when an **induced current** flows in the conductor. The extent to which a substance has electrons free to move under suitable influence determines its value as a conductor. One which offers great resistance to such flow is called an **insulator.**

In a suitable electrical system, an electric charge creates a magnetic field which builds up to a maximum. If the electric current is then discontinued, the magnetic field collapses. This change in the strength of the magnetic field induces an electric current in the conductor, but in the opposite direction to the original current. This current creates a new magnetic field, and the cycle repeats. Thus, an alternating current is produced, the strength increasing to a maximum in one direction, decreasing to zero, increasing to a maximum in the opposite direction, and again decreasing to zero to start a new cycle. This cycle is repeated many times each second, the number depending upon the characteristics of the system. Such a system is called an **oscillating circuit.**

A relatively small amount of energy is dissipated as heat in overcoming the resistance of the circuit. The remainder continues to oscillate between electric and magnetic fields. The build-up and collapse of each field occurs at about the speed of light, which is approximately 186,000 statute miles (300,000,000 meters) per second. If a relatively long period of time is available for the cycle to occur, the energy is fully transferred before the next step occurs. However, if the cycle is speeded until the time needed

for each field to build up or collapse is more than about one-half cycle, some of the energy becomes detached and is radiated into space, through which it travels at about the speed of light. This **electromagnetic radiation,** having both electrical and magnetic properties, is known as **radio waves,** if of a frequency suitable for radio communication.

1002. Radio wave terminology.—The build-up and collapse of the electric and magnetic fields are proportional to the *sine* of the portion of the cycle completed, as shown in figure 1002. This representation has led to the use of the term "wave" when referring to electromagnetic propagation. The highest point on the curve (in the direction considered positive) is the **crest,** and the lowest point the **trough.** Either point may be called the **peak,** considered positive or negative if a distinction is desired. The displacement of a peak from zero is called the **amplitude.** The forward side of any wave is called the **wave front.** For a nondirectional antenna each wave proceeds outward as an expanding sphere (or hemisphere).

One **cycle** is a complete sequence of values, as from crest to crest. The distance traveled by the energy during one cycle is the **wave length,** usually expressed in metric units (meters, centimeters, etc.). The number of cycles repeated during unit time (usually one second) is the **frequency.** This is given in **cycles per second (cps), kilocycles per second (kc), megacycles per second (mc),** or occasionally **kilomegacycles per second (kmc).** A kilocycle is 1,000 cycles, a megacycle is 1,000,000 cycles, and a kilomegacycle is 1,000,000,000 cycles. Wave length and frequency are inversely proportional. The approximate value of either may be found by dividing 300,000,000 by the other quantity, if wave length is expressed in meters and frequency in cycles per second. Thus, if the frequency is 1,500 kilocycles per second (1,500,000 cps), the wave length is $\frac{300,000,000}{1,500,000}=200$ meters. If the wave length is ten centimeters (0.1 meter), the frequency is about $\frac{300,000,000}{0.1}=3,000,000,000$ cycles per second or three kilomegacycles (usually expressed as 3,000 megacycles). A more precise value for the speed of propagation in air is 299,708,000 meters per second. This is equivalent to 186,230 statute miles, or 161,829 nautical miles, per second. The exact value varies slightly with density of the medium through which the wave travels, and frequency. The speed in a vacuum is a little more than that in air.

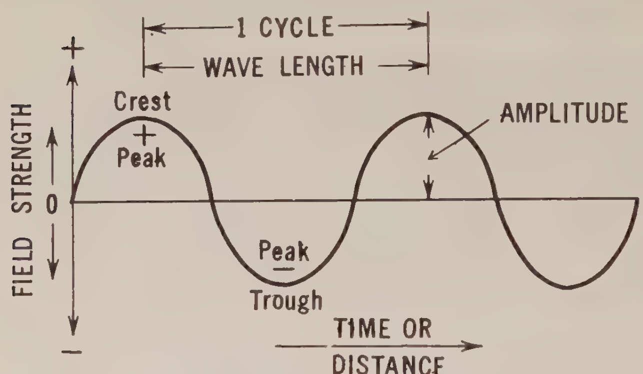


FIGURE 1002.—Radio wave terminology.

The **phase** of a wave is the amount by which the cycle has progressed from a specified origin. For most purposes it is stated in circular measure, a complete cycle being considered 360°. Generally, the origin is not important, principal interest being the phase relative to that of some other wave. Thus, two waves having crests one-fourth cycle apart are said to be 90° "out of phase." If the crest of one wave occurs at the trough of another, the two are 180° out of phase.

1003. Electromagnetic spectrum.—The entire range of electromagnetic radiation frequencies is called the **electromagnetic spectrum.** The range of frequencies suitable for radio transmission, called the **radio spectrum,** extends from ten kilocycles per second

to 300,000 megacycles per second, approximately. For convenience, it is divided into a number of **bands,** as shown in table 1003. Below the radio spectrum, but overlapping it, is the **audio frequency** band, extending from 20 to 20,000 cycles per second, approximately. This is the range of frequencies that can be heard by the human ear. Above the radio spectrum are heat and infrared, the visible spectrum (light in its various colors), ultraviolet, X-rays, gamma rays, and cosmic rays. These are included in table 1003. Waves shorter than one meter are sometimes called **microwaves.**

Band	Abbreviation	Range of frequency	Range of wave length
Audio frequency	AF	20 to 20,000 cps	15,000,000 to 15,000 m
Radio frequency	RF	10 kc to 300,000 mc	30,000 m to 0.1 cm
Very low frequency	VLF	10 to 30 kc	30,000 to 10,000 m
Low frequency	LF	30 to 300 kc	10,000 to 1,000 m
Medium frequency	MF	300 to 3,000 kc	1,000 to 100 m
High frequency	HF	3 to 30 mc	100 to 10 m
Very high frequency	VHF	30 to 300 mc	10 to 1 m
Ultra high frequency	UHF	300 to 3,000 mc	100 to 10 cm
Super high frequency	SHF	3,000 to 30,000 mc	10 to 1 cm
Extremely high frequency	EHF	30,000 to 300,000 mc	1 to 0.1 cm
Heat and infrared *		10^6 to 3.9×10^8 mc	0.03 to 7.6×10^{-5} cm
Visible spectrum *		3.9×10^8 to 7.9×10^8 mc	7.6×10^{-5} to 3.8×10^{-5} cm
Ultraviolet *		7.9×10^8 to 2.3×10^{10} mc	3.8×10^{-5} to 1.3×10^{-6} cm
X-rays *		2.0×10^9 to 3.0×10^{13} mc	1.5×10^{-5} to 1.0×10^{-9} cm
Gamma rays *		2.3×10^{12} to 3.0×10^{14} mc	1.3×10^{-8} to 1.0×10^{-10} cm
Cosmic rays *		$>4.8\times10^{15}$ mc	$<6.2\times10^{-12}$ cm

* Values approximate.

Table 1003.—Electromagnetic spectrum.

1004. Polarization.—As indicated in article 1001, radio waves have both electrical and magnetic properties. The two fields are conceived as having direction associated with the orientation of the vibrations. The direction of the electric component of the field is called the **polarization** of the electromagnetic field. Thus, if the electric component is vertical, the wave is said to be "vertically polarized," and if horizontal, "horizontally polarized." A wave traveling through space may be polarized in any direction. One traveling along the surface of the earth is always vertically polarized because the earth, a conductor, short-circuits any horizontal component. The magnetic field and the electrical field are always mutually perpendicular.

1005. Reflection.—When radio waves strike a surface, they are reflected in the same manner as light waves, if conditions are favorable. Radio waves of all frequencies are reflected by the surface of the earth. The strength of the **reflected wave** depends upon **grazing angle** (the angle between the incident ray and the horizontal), type of polarization, frequency, reflecting properties of the surface, and divergence of the reflected ray. Lower frequency results in greater penetration. At very low frequencies usable radio signals can be received some distance below the surface of the sea.

A change of phase takes place when a wave is reflected from the surface of the earth. The amount of the change varies with the conductivity of the earth and the polarization of the wave, reaching a maximum of 180° for a horizontally polarized wave reflected from sea water (considered to have infinite conductivity). When **direct waves** (those traveling from transmitter to receiver in a relatively straight line, without reflection) and reflected waves arrive at a receiver, the total signal is the vector sum of the two. If the signals are in phase, they reinforce each other, producing a stronger signal. If there is a phase difference, the signals tend to cancel each other, the cancellation being complete if the phase difference is 180° and the two signals have the same amplitude. This interaction of waves is called **wave interference.** A

phase difference may occur because of the change of phase of a reflected wave, or because of the longer path followed by it. The second effect decreases with greater distance between transmitter and receiver, for under these conditions the difference in path lengths is smaller. At lower frequencies there is no practical solution to interference caused in this way. For VHF and higher frequencies the condition can be improved by elevating the antenna, if the wave is vertically polarized. Also, interference at higher frequencies can be more nearly eliminated because of the greater ease of beaming the signal to avoid reflection.

Reflections may also occur from mountains, trees, and other obstacles. Such reflection is negligible for lower frequencies, but becomes more prevalent as frequency increases. In radio communication it can be reduced by using directional antennas, but this solution is not always available for navigational systems.

Various reflecting surfaces occur in the atmosphere. At high frequencies, reflections take place from rain. At still higher frequencies, reflections are possible from clouds, particularly rain clouds. Reflections may even occur at a sharply defined boundary surface between air masses, as when warm, moist air flows over cold, dry air. When such a surface is roughly parallel to the surface of the earth, radio waves may travel for greater distances than normal. A somewhat similar condition is described in article 1006. The principal source of reflection in the atmosphere is the ionosphere (arts. 1007, 1008).

1006. Refraction of radio waves is similar to that of light waves (art. 1613). Thus, as a signal passes from air of one density to that of a different density, the direction of travel is altered. The principal cause of refraction in the atmosphere is the difference in temperature and pressure occurring at various heights and in different air masses.

Refraction occurs at all frequencies, but at those below 30 mc the effect is small as compared with ionospheric effects (art. 1008), diffraction (art. 1009), and absorption (art. 1010). At higher frequencies, refraction in the lower layer of the atmosphere extends the **radio horizon** to a distance about 15 percent greater than the visible horizon. The effect is the same as if the radius of the earth were about one-third greater than it is, and there were no refraction.

Sometimes the lower portion of the atmosphere becomes stratified with horizontal layers of air having certain characteristics, resulting in nonstandard temperature and moisture changes with height. If there is a marked temperature inversion (art. 3815) or a sharp decrease in water vapor content with increased height, a horizontal radio **duct** may be formed. High frequency radio waves traveling horizontally within the duct are refracted to such an extent that they remain within the duct, following the curvature of the earth for phenomenal distances. This is called **super-refraction.** Maximum results are obtained when both transmitting and receiving antennas are within the duct. There is a lower limit to the frequency affected by ducts. It varies from about 200 mc to more than 1,000 mc.

At night, surface ducts may occur over land due to cooling of the surface. At sea, surface ducts about 50 feet thick may occur at any time in the trade wind belt. Surface ducts 100 feet or more in thickness may extend from land out to sea when warm air from the land flows over the cooler ocean surface. Elevated ducts from a few feet to more than 1,000 feet in thickness may occur at elevations of 1,000 to 5,000 feet, due to the settling of a large air mass. This is a frequent occurrence in Southern California and certain areas of the Pacific Ocean.

Refraction effects associated with the ionosphere are discussed in article 1008.

A bending in the horizontal plane occurs when a ground wave (art. 1008) crosses a coast at an oblique angle. This is due to a marked difference in the conducting and

reflecting properties of the land and water over which the wave travels. The effect is known as **coastal refraction** or **land effect.**

1007. The ionosphere.—Since an atom normally has an equal number of negatively charged electrons and positively charged protons, it is electrically neutral. An **ion** is an atom or group of atoms which has become electrically charged, either positively or negatively, by the loss or gain of one or more electrons.

Loss of electrons may occur in a variety of ways. In the atmosphere, ions are usually formed by collision of atoms with rapidly moving particles, or by the action of cosmic rays or ultraviolet light. In the lower portion of the atmosphere, recombination soon occurs, leaving a small percentage of ions. In thin atmosphere far above the surface of the earth, however, atoms are widely separated and a large number of ions may be present. The region of numerous positive and negative ions and unattached electrons is called the **ionosphere.** The extent of ionization depends upon the kinds of atoms present in the atmosphere, the density of the atmosphere, and the position relative to the sun (time of day and season). After sunset, ions and electrons recombine faster than they are separated, decreasing the ionization of the atmosphere.

An electron can be separated from its atom only by the application of greater energy than that holding the electron. Since the energy of the electron depends primarily upon the kind of an atom of which it is a part, and its position relative to the nucleus of that atom, different kinds of radiation may cause ionization of different substances.

In the outermost regions of the atmosphere the density is so low that oxygen exists largely as separate atoms, rather than combining as molecules as it does nearer the surface of the earth. At great heights the energy level is low and ionization from solar radiation is intense. This is known as the **F layer.** Above this level the ionization decreases because of the lack of atoms to be ionized. Below this level it decreases because the ionizing agent of appropriate energy has already been absorbed. During daylight, two levels of maximum F ionization can be detected, the F_2 layer at about 125 statute miles above the surface of the earth, and the F_1 layer at about 90 statute miles. At night, these combine to form a single F layer.

At a height of about 60 statute miles the solar radiation not absorbed by the F layer encounters, for the first time, large numbers of oxygen *molecules.* A new maximum ionization occurs, known as the **E layer.** The height of this layer is quite constant, in contrast with the fluctuating F layer. At night the E layer becomes weaker, sometimes completely disappearing.

Below the E layer a weak **D layer** forms at a height of about 45 statute miles, where the incoming radiation encounters ozone (O_3) for the first time. The D layer is the principal source of absorption of HF waves, and of reflection of LF and VLF waves during daylight.

1008. The ionosphere and radio waves.—When a radio wave encounters a particle having an electric charge, it causes that particle to vibrate. The vibrating particle absorbs electromagnetic energy from the radio wave and reradiates it. The net effect is a change of polarization and an alteration of the path of the wave. That portion of the wave in a more highly ionized region travels faster, causing the wave front to tilt and the wave to be directed toward a region of less intense ionization.

Refer to figure 1008a, in which a single layer of the ionosphere is considered. Ray 1 enters the ionosphere at such an angle that its path is altered, but it passes on through and proceeds outward into space. As the angle with the horizontal decreases, a critical value is reached where the ray (2) is bent or reflected back toward the earth. As the angle is still further decreased, as at 3, the return to earth occurs at a greater distance from the transmitter.

A wave reaching a receiver by way of the ionosphere is called a **sky wave.** This expression is also appropriately applied to a wave reflected from an air mass boundary. In common usage, however, it is generally associated with the ionosphere. The wave which travels along the surface of the earth is called a **ground wave.** At angles greater than the critical angle, no sky-wave signal is received. Therefore, there is a minimum distance from the transmitter at which sky waves can be received. This is called the **skip distance,** shown in figure 1008a. If the ground wave extends out for less distance than the skip distance, a **skip zone** occurs, in which no signal is received.

The critical radiation angle depends upon the intensity of ionization, and the frequency of the radio wave. As the frequency increases, the angle becomes smaller. At frequencies greater than about 30 mc virtually all of the energy penetrates through or is absorbed by the ionosphere. Therefore, at any given receiver there is a maximum usable frequency if sky waves are to be utilized. The strongest signals are received at or slightly below this frequency. There is also a lower practical frequency beyond which signals are too weak to be of value. Within this band the optimum frequency

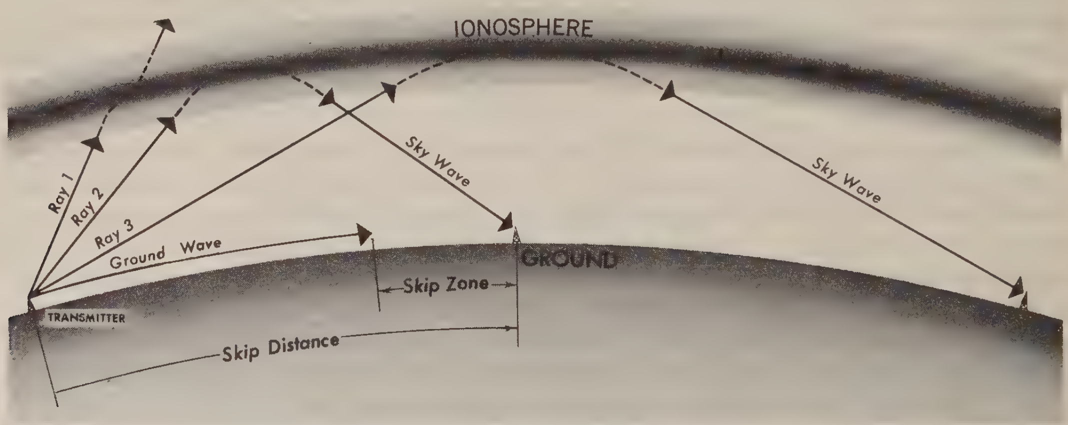


FIGURE 1008a.—The effect of the ionosphere on radio waves.

can be selected to give best results. It cannot be too near the maximum usable frequency because this frequency fluctuates with changes of intensity within the ionosphere. During magnetic storms the ionospheric density decreases. The maximum usable frequency decreases, and the lower usable frequency increases. The band of usable frequencies is thus narrowed. Under extreme conditions it may be completely eliminated, isolating the receiver and causing a radio **blackout.**

Sky-wave signals reaching a given receiver may arrive by any of several paths, as shown in figure 1008b. A signal which undergoes a single reflection is called a "one-hop" signal, one which undergoes two reflections with a ground reflection between is called a "two-hop" signal, etc. A "multihop" signal undergoes several reflections. The layer at which the reflection occurs is usually indicated, also, as "one hop E," "two hop F," etc.

Because of the different paths and phase changes occurring at each reflection, the various signals arriving at a receiver have different phase relationships. Since the density of the ionosphere is continually fluctuating, the strength and phase relationships of the various signals may undergo an almost continuous change. Thus, the various signals may reinforce each other at one moment and cancel each other at the next, resulting in fluctuations of the strength of the total signal received. This is

called **fading.** This phenomenon may also be caused by interaction of components within a single reflected wave, or changes in its strength due to changes in the reflecting surface. Ionospheric changes are associated with fluctuations in the radiation received from the sun, since this is the principal cause of ionization. Signals from the F layer are particularly erratic because of the rapidly fluctuating conditions within the layer itself.

The maximum distance at which a one-hop-E signal can be received is about 1,400 miles. At this distance the signal leaves the transmitter in approximately a horizontal direction. A one-hop-F signal can be received out to about 2,500 miles. At low frequencies ground waves extend out for great distances.

A sky wave may undergo a change of polarization during reflection from the ionosphere, accompanied by an alteration in the direction of travel of the wave. This is

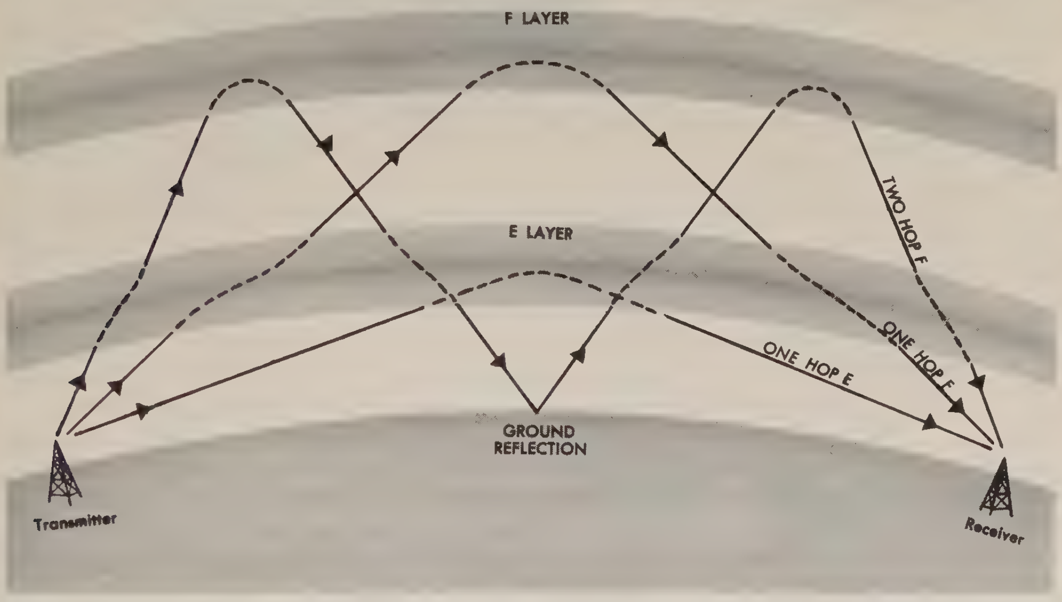


FIGURE 1008b.—Various paths by which a sky wave signal might be received.

called **polarization error.** Near sunrise and sunset, when rapid changes are occurring in the ionosphere, reception may become erratic and polarization error a maximum. This is called **night effect.**

1009. Diffraction.—When a radio wave encounters an obstacle, its energy is reflected or absorbed, causing a **shadow** beyond the obstacle. However, some energy does enter the shadow area because of **diffraction.** This is explained by **Huygens' principle,** which states that *every point on the surface of a wave front is a source of radiation, transmitting energy in all directions ahead of the wave.* No noticeable effect of this principle is observed until the wave front encounters an obstacle, which intercepts a portion of the wave. From the edge of the obstacle, energy is radiated into the shadow area, and also outside of the area. The latter interacts with energy from other parts of the wave front, producing alternate bands in which the secondary radiation reinforces or tends to cancel the energy of the primary radiation. Thus, the practical effect of an obstacle is a greatly reduced signal strength in the shadow area,

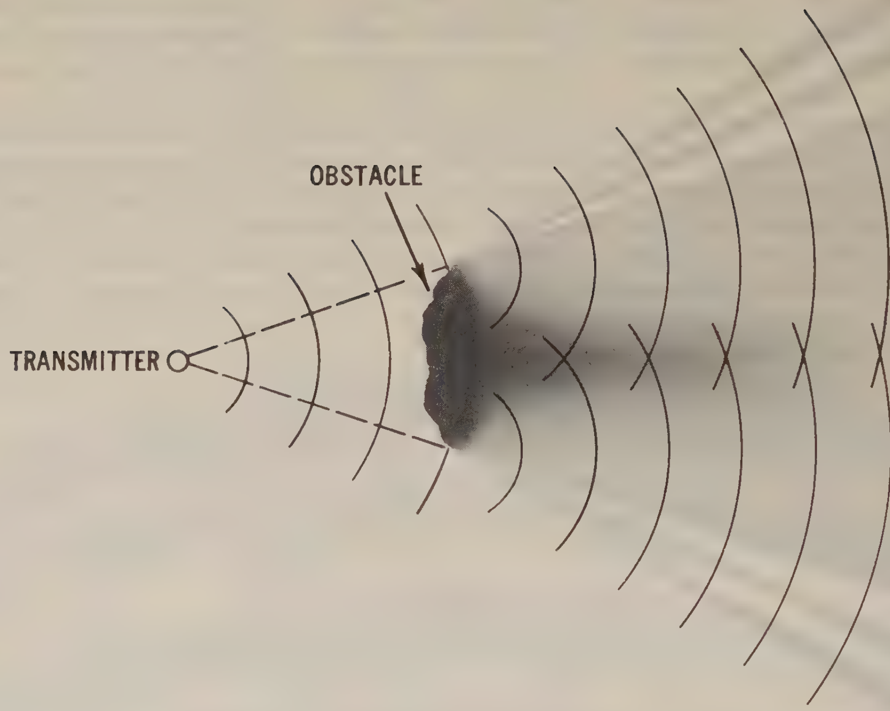


FIGURE 1009.—Diffraction.

and a disturbed pattern for a short distance outside the shadow area. This is illustrated in figure 1009.

The amount of diffraction is inversely proportional to the frequency, being greatest at very low frequencies.

1010. Absorption and scattering.—The amplitude of a radio wave expanding outward through space varies inversely with distance. That is, it gets weaker with increased distance. The decrease of strength with distance is called **attenuation.** Under certain conditions the attenuation is greater than in free space.

A wave traveling along the surface of the earth loses a certain amount of energy to the earth. The wave is diffracted downward and absorbed by the earth. As a result of this absorption, the remainder of the wave front tilts downward, resulting in further absorption by the earth. Attenuation is greater over a surface that is a poor conductor. Relatively little absorption occurs over sea water, which is an excellent conductor at low frequencies, and low frequency ground waves travel great distances over water.

A sky wave suffers an attenuation loss in its encounter with the ionosphere. The amount depends upon the height and composition of the ionosphere, as well as the frequency of the radio wave. Maximum ionospheric absorption occurs at about 1,400 kc.

In general, atmospheric absorption increases with frequency, being a problem only at SHF and EHF. At these frequencies, attenuation is further increased by **scattering** due to reflection by oxygen, water vapor, water droplets, and rain in the atmosphere.

1011. Noise.—Unwanted signals in a receiver are called **interference.** The intentional production of such interference to obstruct communication is called **jamming.** Unintentional interference is called **noise.**

Noise may originate within the receiver. **Hum** is usually the result of induction from neighboring circuits carrying alternating current. **Microphonic noise** is the result of vibration of elements in an electron tube. Irregular crackling or sizzling sounds may be caused by poor contacts or faulty components within the receiver. Electron movement in normal components causes some noise. This source sets the ultimate limit of sensitivity (art. 1018) that can be achieved in a receiver. It is the same at any frequency.

Noise originating outside the receiver may be either man-made or natural. Man-made noises originate in electrical appliances, motor and generator brushes, ignition systems, and other sources of sparks which transmit electromagnetic signals that are picked up by the receiving antenna.

Natural noise is caused principally by discharge of static electricity in the atmosphere. This is called **atmospheric noise, atmospherics,** or **static.** An extreme example is a thunderstorm. An exposed surface may acquire a considerable charge of static electricity. This may be caused by friction of water or solid particles blown against or along such a surface. It may also be caused by splitting of a water droplet which strikes the surface, one part of the droplet acquiring a positive charge and the other a negative charge. These charges may be transferred to the surface. The charge tends to gather at points and ridges of the conducting surface, and when it accumulates to a sufficient extent to overcome the insulating properties of the atmosphere, it discharges into the atmosphere. Under suitable conditions this becomes visible and is known as **St. Elmo's fire,** which is sometimes seen at mastheads, the ends of yardarms, etc.

Atmospheric noise occurs to some extent at all frequencies, but decreases with higher frequencies. Above about 30 mc it is not generally a problem.

Since most of the noise occurs at low frequencies, it travels great distances and the accumulation may reach troublesome proportions at these frequencies, particularly during the summer in mountainous regions.

1012. Antenna characteristics.—Antenna design and orientation have a marked effect upon radio wave propagation. For a single-wire antenna, strongest signals are transmitted along the perpendicular to the wire, and virtually no signal in the direction of the wire. For a vertical antenna, the signal strength is the same in all horizontal directions. Unless the polarization undergoes a change during transit, the strongest signal *received* from a vertical transmitting antenna occurs when the receiving antenna is also vertical.

For lower frequencies the radiation of a radio signal takes place by interaction between the antenna and the ground. For a vertical antenna, efficiency increases with greater length of the antenna. For a horizontal antenna, efficiency increases with greater distance between antenna and ground. Near-maximum efficiency is attained when this distance is one-half wave length. This is the reason for elevating low frequency antennas to great heights. However, at the lowest frequencies, the required height becomes prohibitively great. At 10 kc it would be about eight nautical miles for a half-wave-length antenna. Therefore, lower frequency antennas are inherently inefficient. This is partly offset by the greater range of a low frequency signal of the same transmitted power as one of higher frequency.

At higher frequencies, the ground is not used, both conducting portions being included in a **dipole** antenna. Not only can such an antenna be made efficient, but it can also be made sharply directive, thus greatly increasing the strength of the signal transmitted in a desired direction.

The power received is inversely proportional to the square of the distance from the transmitter, assuming there is no attenuation due to absorption or scattering.

1013. Range.—The range at which a usable signal is received depends upon the power transmitted, the sensitivity of the receiver, frequency, route of travel, noise level, and perhaps other factors. For the same *transmitted* power, both the ground-wave and sky-wave ranges are greatest at the lowest frequencies, but this is somewhat offset by the lesser efficiency of antennas for these frequencies. At higher frequencies, only direct waves are useful, and the effective range is greatly reduced. Attenuation, skip distance, ground reflection, wave interference, condition of the ionosphere, atmospheric noise level, and antenna design all affect the distance at which useful signals can be received.

1014. Frequency and radio wave propagation.—Frequency is an important consideration in radio wave propagation, as indicated previously. The following summary indicates the principal effects associated with the various frequency bands, starting with the lowest and progressing to the highest usable radio frequency.

Very low frequency (VLF, 10 to 30 kc). For a given transmitted power, sky-wave signals travel tremendous distances, the ionosphere being most effective in reflecting waves of the lowest frequency. Diffraction is also maximum. However, because of the long wave length, large antennas are needed, and even these are inefficient, permitting radiation of relatively small amounts of power. Relatively little energy is reflected by the ground or other obstacles. Magnetic storms have little effect upon transmission because of the efficiency of the ionosphere in reflecting VLF waves. During such storms, VLF signals may constitute the only source of radio communication over great distances. However, interference from atmospheric noise may be troublesome. Signals may be received below the surface of the sea.

Low frequency (LF, 30 to 300 kc). As frequency is increased to the LF band, the ionosphere becomes less efficient as a reflector, diffraction decreases, ground losses increase, and range for a given power output falls off rapidly. However, this is partly offset by more efficient transmitting antennas, which can be made of a size practical for use aboard ship. The LF band is useful for radio direction finding (art. 1202) and ground-wave transmission over medium distances.

Medium frequency (MF, 300 to 3,000 kc). Ground waves provide dependable service, but the range for a given power is reduced greatly, varying from about 400 miles at the lower portion of the band to about 15 miles at the upper end for a transmitted signal of one kilowatt. These values are influenced, however, by the power of the transmitter, the directivity and efficiency of the antenna, and the nature of the terrain over which signals travel. Elevating the antenna to obtain direct waves may improve the transmission. At the lower frequencies of the band, sky waves are available both day and night. As the frequency is increased, ionospheric absorption increases to a maximum at about 1,400 kc. At higher frequencies the absorption decreases, permitting increased use of sky waves. Since the ionosphere changes with the hour, season, and sunspot cycle, the reliability of sky-wave signals is variable. By careful selection of frequency, one can obtain ranges of as much as 8,000 miles with one kilowatt of transmitted power, using multihop signals. However, the frequency selection is critical. If it is too high, the signals penetrate the ionosphere and are lost in space. If it is too low, signals are too weak. In general, sky-wave reception is equally good by day or night, but lower frequencies are needed at night. The standard broadcast band for commercial stations (535 to 1,605 kc) and the authorized frequencies for loran (art. 1302) are in the MF band.

High frequency (HF, 3 to 30 mc). As with higher medium frequencies, the ground-wave range of HF signals is limited to a few miles, but the elevation of the antenna may increase the direct-wave distance of transmission. Also, the height of the antenna

does have an important effect upon sky-wave transmission because the antenna has an "image" within the conducting earth. The distance between antenna and image is related to the height of the antenna, and this distance is as critical as the distance between elements of an antenna system. Maximum usable frequencies (art. 1008) fall generally within the HF band. By day this may be 10 to 30 mc, but during the night it may drop to eight to ten mc. The HF band is widely used for ship-to-ship and ship-to-shore communication.

Very high frequency (VHF, 30 to 300 mc). Communication is limited primarily to the direct wave, or the direct wave plus a ground-reflected wave. Elevating the antenna to increase the distance at which direct waves can be used results in increased distance of reception, even though some wave interference between direct and ground-reflected waves is present. Diffraction is much less than with lower frequencies, but is most evident when signals cross sharp mountain peaks or ridges. Under suitable conditions, reflections from the ionosphere are sufficiently strong to be useful, but generally they are unavailable. There is relatively little interference from atmospheric noise in this band. Reasonably efficient directional antennas are possible with VHF. The VHF band is much used for communication with aircraft and for radio aids to air navigation. The first television and FM channels were within this band.

Ultra high frequency (UHF, 300 to 3,000 mc). Sky waves are not used in the UHF band because the ionosphere is not sufficiently dense to reflect the waves, which pass through it into space. Ground waves and ground-reflected waves are used, although there is some wave interference. Diffraction is negligible, but the radio horizon extends about 15 percent beyond the visible horizon, due principally to refraction. Reception of UHF signals is virtually free from fading and interference by atmospheric noise. Sharply directive antennas can be produced for transmission in this band, which is coming into wider use for television and other line-of-sight transmission.

Super high frequency (SHF, 3,000 to 30,000 mc). There are no sky waves in the SHF band, transmission being entirely by direct and ground-reflected waves. Diffraction and interference by atmospheric noise are virtually nonexistent. Highly efficient, sharply directive antennas can be produced. Thus, transmission in this band is similar to that of UHF, but with the effects of shorter waves being greater. Reflection by clouds, water droplets, dust particles, etc., increases, causing greater scattering, increased wave interference, and fading. At the upper end of the band, absorption in the atmosphere increases as the frequency approaches that of molecular motion. Use of this band is largely experimental.

Extremely high frequency (EHF, 30,000 to 300,000 mc). The effects of shorter waves are more pronounced in the EHF band, transmission being free from wave interference, diffraction, fading, and interference by atmospheric noise. Only direct and ground-reflected waves are available. Scattering and absorption in the atmosphere are pronounced and may produce an upper limit to the frequency useful in radio communication. The EHF band is a region of experimentation.

1015. Regulation of frequency use.—While the characteristics of various frequencies are important to the selection of the most suitable one for any given purpose, these are not the only considerations. Confusion and extensive interference would result if every user had complete freedom of selection. Some form of regulation is needed. The allocation of various frequency bands to particular uses is a matter of international agreement. Within the United States the Federal Communications Commission has responsibility for authorizing use of particular frequencies. In some cases a given frequency is allocated to several widely separated transmitters, but only under conditions which minimize interference, as during daylight hours. Interference between stations is further reduced by the use of **channels,** each of a narrow band of

frequencies. That is, assigned frequencies are separated by an arbitrary band of frequencies that are not authorized for use. In the case of radio aids to navigation, ship communications, etc., bands of several channels are allocated, permitting selection of band and channel by the user.

1016. Kinds of radio transmission.—A series of waves transmitted at constant frequency and amplitude is called a **continuous wave (CW).** This cannot be heard except at the very lowest radio frequencies, when it may produce, in a receiver, an audible hum of high pitch.

Although a continuous wave may be used directly, as in radio direction finding (art. 1202) or Decca (art. 1309), it is more commonly modified in some manner. This

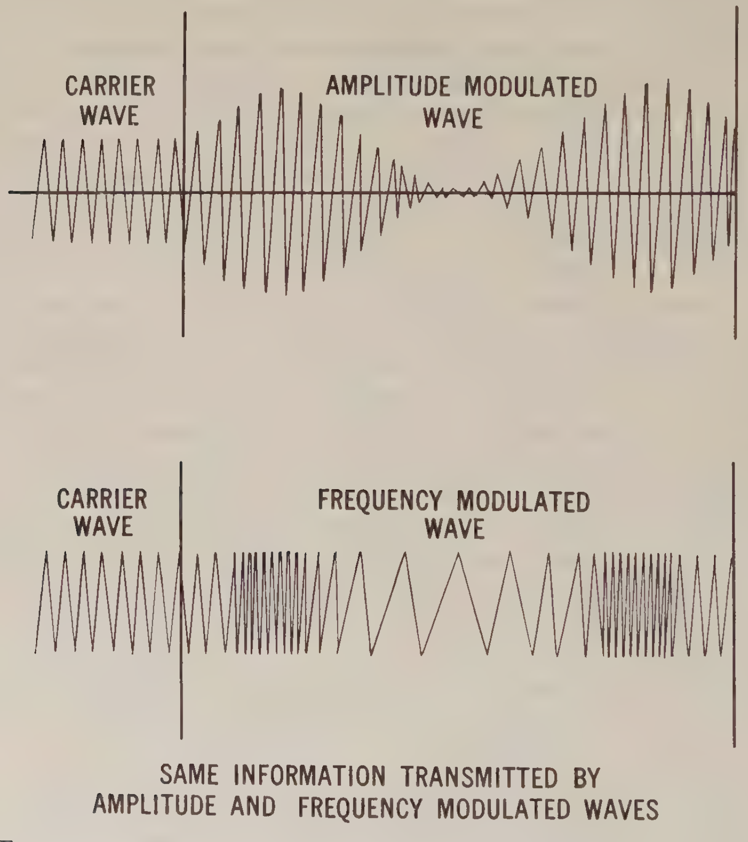


FIGURE 1016a.—Amplitude modulation (upper figure) and frequency modulation (lower figure) by the same modulating wave.

is called **modulation.** When this occurs, the continuous wave serves as a **carrier wave** for information. Any of several types of modulation may be used.

In **amplitude modulation (AM)** the amplitude of the carrier wave is altered in accordance with the amplitude of a **modulating wave,** usually of audio frequency, as shown in figure 1016a. In the receiver the signal is **demodulated** by removing the modulating wave and converting it back to its original form. This form of modulation is widely used in voice radio, as in the standard broadcast band of commercial broadcasting.

If the *frequency* instead of the *amplitude* is altered in accordance with the amplitude of the impressed signal, as shown in figure 1016a, **frequency modulation (FM)** occurs. This is used for FM broadcasts and the sound portion of television broadcasts.

Pulse modulation (PM) is somewhat different, there being no impressed modulating wave. In this form of transmission, very short bursts of carrier wave are transmitted, separated by relatively long periods of "silence," during which there is no transmission. This type of transmission, illustrated in figure 1016b, is used in some common radio navigational aids, including radar (art. 1208) and loran (art. 1302).

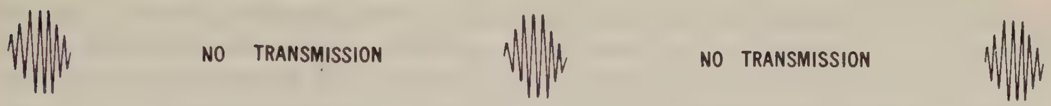


FIGURE 1016b.—Pulse modulation.

1017. Transmitters.—A radio **transmitter** consists essentially of (1) a **power supply** to furnish direct current, (2) an **oscillator** to convert direct current into radio-frequency oscillations (the carrier wave), (3) a device to control the generated signal, (4) an **amplifier** to increase the output of the oscillator. For some transmitters a **microphone** is needed with a **modulator** and final amplifier to modulate the carrier wave. In addition, an **antenna** and **ground** (for lower frequencies) are needed to produce electromagnetic radiation. These components are illustrated diagrammatically in figure 1017.

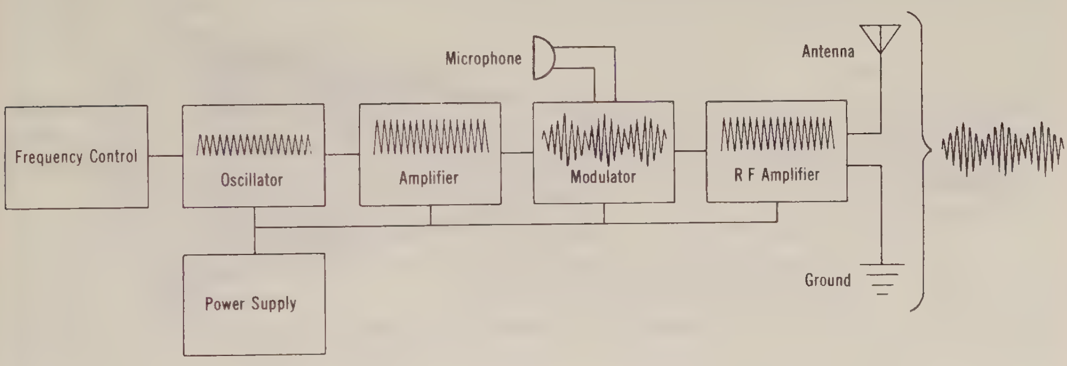


FIGURE 1017.—Components of a radio transmitter.

1018. Receivers.—When a radio wave passes a conductor, a current is induced in that conductor. A radio **receiver** is a device which accepts the power thus generated in an antenna, and transforms it into usable form. It should be able to select signals of a single frequency (actually a narrow band of frequencies) from among the many which may reach the receiving antenna. If necessary, the receiver should be able to demodulate the signal, and always it should provide adequate amplification. The output of a receiver may be presented audibly by earphones or loud speaker; or visually on a dial, cathode ray tube (art. 1019), counter, or other display. Thus, the useful reception of radio signals requires three components: (1) an antenna, (2) a receiver, and (3) a display unit.

Radio receivers differ mainly in (1) **frequency range,** the range of frequencies to which they can be tuned; (2) **selectivity,** the ability to confine reception to signals of the desired frequency and avoid others of nearly the same frequency; (3) **sensitivity,** the ability to amplify a weak signal to usable strength against a background of noise; (4) **stability,** the ability to resist drift from conditions or values to which set; and (5) **fidelity,** the completeness with which the essential characteristics of the original signal are reproduced. Receivers may have additional features such as an automatic frequency control, automatic noise limiter, etc.

Some of these characteristics are interrelated. For instance, if a receiver lacks selectivity, signals of a frequency differing slightly from those to which the receiver is tuned may be received. This condition is called **spillover,** and the resulting interference is called **cross talk.** If the selectivity is increased sufficiently to prevent spillover, it may not permit receipt of a great enough band of frequencies to obtain the full range of those of the desired signal. Thus, the fidelity may be reduced.

1019. The cathode ray tube is a useful device for presenting certain types of information. This tube, with its associated controls, is often called an **oscilloscope,** or **scope** for short. In television receivers it is usually called the **picture tube.**

The essential components of a cathode ray tube are shown in figure 1019. At the left is a **cathode** which serves as a source of electrons. In this usage it is called an **electron gun.** The electrons are collected and focused into a beam by a **focusing anode,** and then speeded up by an **accelerating anode.** If there were no other controls, the beam of electrons would travel the remainder of the length of the tube and strike the enlarged, curved surface of the tube **face** at its center, approximately. The inside of the face is coated with a material known as a **phosphor** (such as zinc sul-

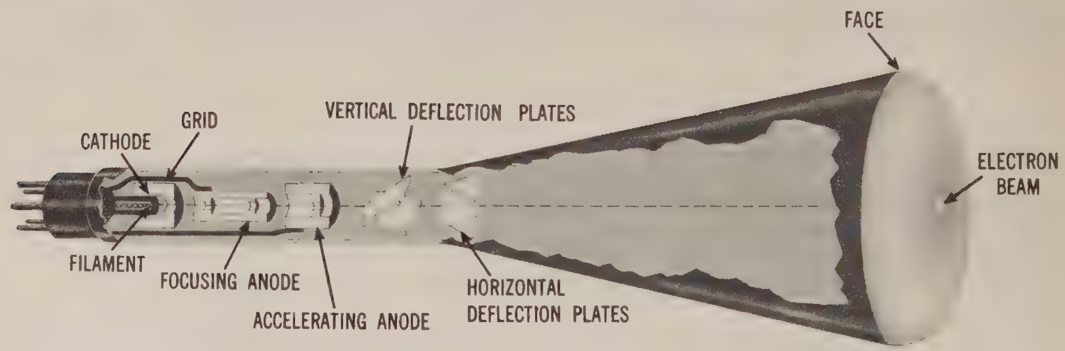


Figure 1019.—A cathode ray tube.

phide or calcium sulphide) which becomes luminous (**phosphorescent**) where a beam of electrons impinges upon it. If the beam is sharply focused, a dot of light appears at the point of impact.

By means of the **vertical deflection plates,** the beam is bent upward or downward. This is accomplished by impressing electric charges on these plates. The beam, being negatively charged, is repelled by the negative plate and attracted by the positive plate. If an alternating current is used, the strength and polarity of the electric charge on each plate changes continually, causing the beam to be deflected alternately up and down. This results in vertical motion of the spot of light on the face of the tube. If the motion is sufficiently rapid, a vertical *line* appears on the face of the tube. This is true not only because of the persistency of vision within the eye, but also because the tube face does not immediately fade when the stream of electrons is moved to another point. This visible line is called a **trace,** and the motion of the dot in producing it, a **sweep.** A horizontal trace can be made by means of the **horizontal deflection plates** which operate in a manner similar to that of the vertical deflection plates.

If both sets of plates are energized at the same time, the spot of light can be moved to various places on the face of the tube. If two alternating currents are properly synchronized, the spot can be made to trace repeatedly some pattern, such as a sine

wave. It is generally desirable to have one trace repeated in accordance with a prearranged plan, having the deflection such that motion in one direction across the face of the tube is relatively slow, and that in the opposite direction is very fast, so that the return of the spot to a starting point is almost instantaneous. Such a return is called **flyback,** and the faint trace that may be visible is called a **retrace.** The position of the spot along the trace can be used as a measurement of elapsed time since the spot was at some reference point. This is usually accomplished by having a received signal impress a momentary charge on the *other* set of deflecting plates, causing a deflection of the trace as the spot is momentarily moved to one side of the line; or by causing the received signal to intensify the spot, causing it to glow brighter.

By suitable controls, the trace can be divided into two or more parts, made to rotate, or take any of a great variety of motions and shapes.

In a **dark trace tube** the spot appears dark on a lighter background.

The cathode ray tube has many applications in electronic navigational equipment.

References

International Hydrographic Bureau. *Radio Aids to Maritime Navigation and Hydrography.* 2d ed. International Hydrographic Bureau Special Publication 39. Monaco, 1965.

Reintjes, J. F., and Coate, G. T. *Principles of Radar.* 3d ed. M. I. T. Radar School. New York, McGraw-Hill, 1952.

Schelkunoff, S. A., and Friis, H. T. *Antenna Theory and Practice.* New York, Wiley, 1952.

Sheingold, A. *Fundamentals of Radio Communication.* New York, Van Nostrand, 1951.

Terman, F. E. *Radio Engineers' Handbook.* New York, McGraw-Hill, 1943.

U.S. Department of Commerce. *Ionospheric Radio Propagation.* National Bureau of Standards Monograph 80. Washington, U.S. Govt. Print. Off., 1965.

U.S. Department of the Navy. *Elements of Electricity and Radio.* NAVSHIPS 900,012. Washington, 1944.

CHAPTER XI

ELECTRONICS AND NAVIGATION

1101. Electronics is the science and technology relating to the emission, flow, and effects of electrons in a vacuum or through a semiconductor such as a gas, and to systems using devices in which this action takes place. The widest use of electronics is in radio in its various forms. However, by the definition given above, electronics may be used in a secondary sense in a great many devices which are otherwise unrelated to radio.

1102. Use of electronics in navigation.—The expression "electronic navigation" may imply a distinct type of navigation comparable to celestial navigation, piloting, and dead reckoning. However, the use of electronics by the navigator is nearly always in one of these fields, although it is true that piloting techniques have been extended far from shore.

In celestial navigation, electronics is used for transmission of radio time signals to ships at sea, permitting the frequent checking of chronometers. A more direct application is the **radio sextant.** If the body is above the horizon, this instrument can measure altitudes of the sun and moon through an overcast or in clear weather, day or night. With further development, it may be possible to use this instrument for measurement of altitudes of other celestial bodies.

In piloting, electronics has its widest application. In addition to the various radio aids commonly associated with navigation, electronics is used in the echo sounder (art. 619), sonar (art. 1108), and sofar (art. 1313).

In dead reckoning, electronics is used in some devices for automatically determining dead reckoning position. These may be essentially *recording* or *indicating* devices, or instruments for *measuring* speed and direction, as well as indicating the results of the measurements (art. 809).

In addition to these applications of electronics to navigation, radio communication is helpful to the mariner. Weather maps and other information may be sent by facsimile (art. 3828). Various navigational warnings are broadcast, as well as weather and ice reports and predictions, distress information, and even medical advice. Information concerning the various services available is given in H.O. Pubs. Nos. 117–A and 117–B, *Radio Navigational Aids*, and in 118–A and 118–B, *Radio Weather Aids*.

The use of electronics for direct determination of positional information is related primarily to measurement of direction and measurement of distance or difference in distance.

1103. Direction measurement at the receiving site is accomplished by means of a directional antenna. Nearly all antennas have some directional properties, but in the usual antenna used for radio communication, these properties are not sufficiently critical for navigational use.

A widely used directional antenna is in the form of a **loop.** Suppose a transmitted radio signal encounters such a loop oriented in the direction of travel of the radio signal, as shown in figure 1103a. If the diameter of the loop is half the wave length, the crest of one wave arrives at one side of the loop at the same time that the trough arrives at the opposite side, as shown. Thus, the currents induced in the two sides reinforce each other, causing maximum output from the antenna. A short time later, as the wave continues to move past the antenna, the crest reaches the other side of the

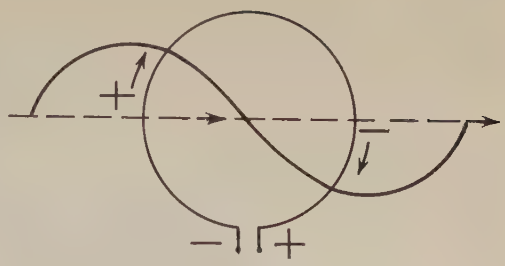


FIGURE 1103a.—Principle of the loop antenna.

loop, and a new trough reaches the approach side. A maximum current now flows in the opposite direction. Therefore, with the antenna in this orientation, an alternating current flows in the loop. If the loop diameter is less than half a wave length, the current is less than maximum.

If the antenna is rotated 90°, the alternate crests and troughs arrive at both sides at the same time, tending to cause currents to flow in opposite directions around the loop. Under these conditions the two parts cancel each other, resulting in zero antenna output. This condition is called a **null.**

As the antenna is rotated, its output varies with the angle relative to the direction of motion of the radio signal. This condition is illustrated in figure 1103b. The length of a line from the center to the outer edge of the shaded area represents the strength of the antenna output at that bearing, relative to the direction of motion of the radio wave. Thus, when it is in line, *with either side of the loop toward the approaching signal,* the output is maximum, and at 90° it is minimum. Since the change with bearing is most rapid near the region of minimum signal, this is the portion used for determination of direction.

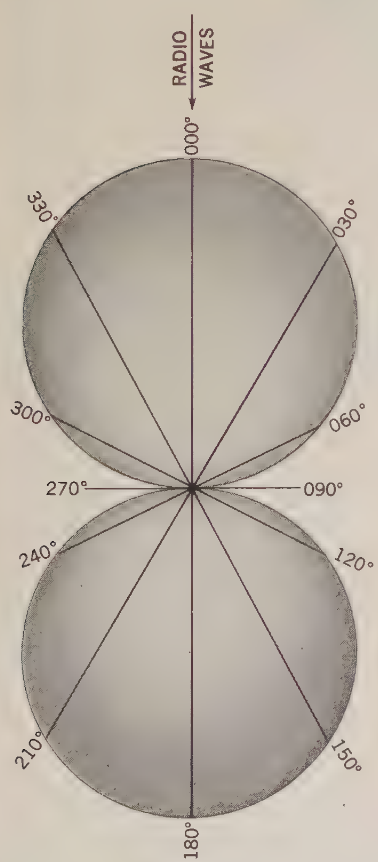


FIGURE 1103b.—Polar diagram of output of loop antenna.

Because of the characteristics of the simple loop antenna, a 180° **ambiguity** exists. That is, a signal approaching from either of two directions 180° apart would cause the same antenna output. This ambiguity can be resolved by using a vertical **sense antenna** in connection with a loop. The output from this wire, if the direction of motion of the signal is horizontal, is the same in all directions. Therefore, the polar diagram of its output is a circle, with the same polarity in all directions. If this output is exactly equal to the maximum of the loop, it will cancel the output from one side and double that from the other, since the polarity in the two sides is opposite. The resulting diagram of antenna output is shown in figure 1103c. With this arrangement, a single minimum exists, permitting the determination of which of the two reciprocal bearings is correct, thereby removing the ambiguity. The loop antenna is then used for making the reading. This is the type of equipment commonly used with a radio direction finder (art. 1202).

Two variations of the loop antenna are also used in radio direction finders. In one of these, the **crossed loop** type, two loops are rigidly mounted in such manner that one is rotated 90° with respect to the other. The *relative* output of the two antennas is related to the orientation of each with

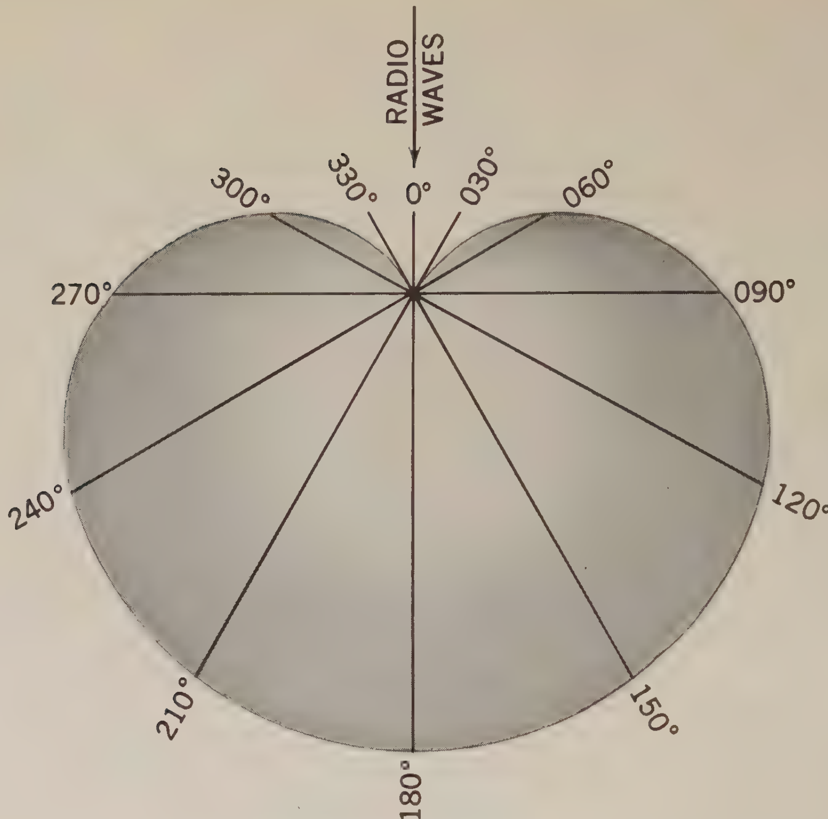


FIGURE 1103c.—Polar diagram of output of loop antenna with vertical sense antenna.

respect to the direction of travel of the radio wave, and is measured by a device called a **goniometer.** This is the type antenna used in an automatic direction finder. In the other variation, the **rotating loop** type, a single loop is kept in rapid rotation by means of a motor. The antenna output is shown on a cathode ray tube, and the resulting display shows the direction of the signal.

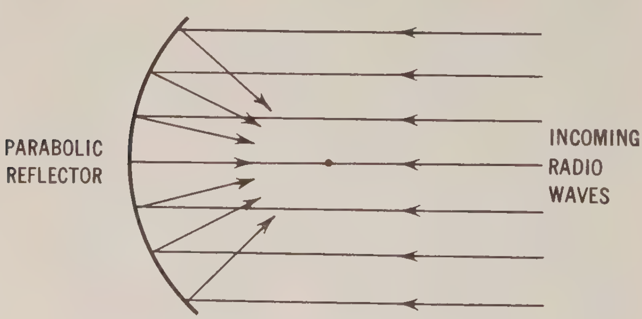


FIGURE 1103d.—Principle of the parabolic reflector.

With higher frequencies, for which a dipole antenna is used, a different method of achieving directional properties is employed. The antenna is placed at the focus of a reflecting parabola (art. O34). Incoming parallel beams are all reflected to the antenna, which receives a concentration of energy, as shown in figure 1103d. When the parabola is turned away from the approaching signal, little or no signal is received. The effectiveness of such an arrangement increases with higher frequency, for which an efficient antenna decreases in size, approaching a single point. This type antenna is used for radar (art. 1208), and the ramark beacon (art. 1210) depends upon it.

1104. Directional transmission.—The simple loop antenna, with or without a vertical sensing antenna, can be used for transmitting signals. The polar diagram of the strength of the transmitted signal is similar to that of the output of a receiving antenna, as shown in figure 1103b or figure 1103c.

Where it is desired to maintain the same direction or directions of transmission, permanent large installations can be made and properly designed for maximum efficiency at the frequency used. This is called an **Adcock antenna,** which is similar in principle to the loop except that it is not connected across the top.

For higher frequencies, the parabolic reflector is used to produce a beam of radio energy. The effect is similar to that of a searchlight.

Various combinations of antennas and phase relationships are used to produce patterns of signals serving as a navigational system. Some of these are discussed in articles 1105 and 1106.

1105. Radio tracks.—A track defined by radio may be called a "radio track." One of the simplest methods is to use two Adcock antennas placed 90° with respect to each other. As shown in figure 1105, one antenna can be used to produce a "figure 8" pattern with its axis in a north-south direction, and a second one to produce a similar pattern in an east-west direction. If each antenna transmits a characteristic signal, the lines along which these two signals are received with equal intensity represent radio tracks. This system, used in the **radio ranges** (art. 1207) which for many years constituted the primary guidance along the federal airways of the United States, has a 90° ambiguity. The directions of the tracks can be altered by changing the orientation of the antennas, or by changing the phases of the signals from the two antennas.

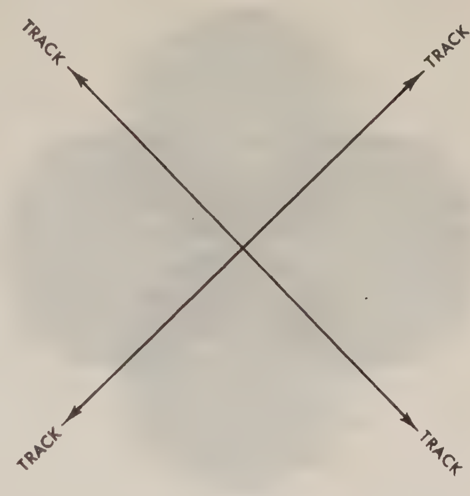


FIGURE 1105.—Radiation pattern of two Adcock antennas rotated 90° with respect to each other.

A variation of this system is the use of three or more antennas equally spaced along a straight line, the distance between consecutive antennas being three wave lengths. By a combination of amplitudes and phase shifts, a series of equisignal tracks are produced. This system, known as **elektra,** was used by the Germans during the early part of World War II. It was the predecessor of the German **sonne** and British **consol** systems (art. 1206).

At higher frequencies, radio tracks can be provided by parabolic reflectors. The disadvantage of such a system is that virtually no signal is received unless one is almost directly in line with the beam.

Although ships have occasionally used radio tracks, particularly the four-course radio ranges, such systems have been designed primarily for use by aircraft. Simple track guidance, as here described, has been largely replaced by rotating beacons providing multiple track guidance.

1106. Rotating beacons may be used to provide an indication of direction without actual direction measurement at the receiver. In the earliest installations, a directional antenna was mounted on a vertical axis and rotated slowly at uniform speed. When a distinctive phase of the pattern, such as a null, passed through a reference direction (usually true or magnetic north), a nondirectional signal was transmitted. When this

signal was received, a stopwatch was started. The elapsed time from this moment until the distinctive phase was received was an indication of the direction of the receiver from the transmitter.

In later installations the antenna remains stationary and the radiation pattern is caused to rotate. In the **vortac** ranges (art. 1207) used for air navigation with respect to the federal airways, two signal patterns are transmitted by VHF antennas similar in principle to the Adcock antenna. The pattern of one remains fixed, and that of the other rotates. The result is a change of *phase* with direction. Along the reference direction (magnetic north) from the transmitter the signals are in phase. The phase difference along any other radial is constant. The receiver measures the phase difference and indicates the direction on a dial. The receiver is also provided with a knob for selecting a desired phase difference (direction) and a pointer to indicate whether the craft should go right or left to reach the desired radial.

With three antennas in line, as in the elektra system, rotation is accomplished by slowly shifting the phase of the current in the two outer antennas. This, in combination with periodic reversal of the direction of the current, produces alternate sectors of dot and dash signals. During the cycle of operation, the patterns rotate so that a portion of each pattern sweeps past the receiver. The relative number and order of dots and dashes is an indication of direction when referred to a table or special chart for interpretation. However, identical readings can be obtained in a number of sectors. A radio direction finder bearing, dead reckoning position, or other positional information can be used to resolve the ambiguity. As developed by the Germans, this system was known as **sonne**. The British further developed the system under the name **consol** (art. 1206). The American development is known as **consolan.**

1107. Speed measurement can be accomplished electronically by utilizing the Doppler principle. A beam of electromagnetic energy can be transmitted from a moving craft. If this energy strikes an obstacle and some of the energy returns as an echo, it will have a slightly different apparent frequency because of the motion of the transmitter. The difference is proportional to the speed *in the direction of the beam.* If the beam is directed ahead or astern, the speed of the craft is indicated. If two beams are used with a fixed angle between them, and the two rotated about a vertical axis until both readings are the same, direction of motion can also be measured. In this case the measured speed is a fixed proportion of the actual speed. Thus, **Doppler navigation** (art. 809) is a dead reckoning system, since it provides measurement of both speed and direction of motion. This method is particularly applicable to aircraft.

Another method of measuring speed and direction of motion is by **inertial navigation** (art. 809). By this principle, fore-and-aft and athwartship accelerations are measured and automatically integrated once to provide a measurement of speed in each direction, and a second time to provide an indication of distance.

Since both Doppler and inertial systems provide dead reckoning information, their errors are cumulative, tending to increase with time.

1108. Distance measurement.—Since the speed of travel of radio waves is nearly constant, the time of travel between two points is directly proportional to the distance between the points. Therefore, it provides a possible method of determining distance if a means is available for measuring very small intervals of time. Considering the speed of radio waves as 186,230 statute miles per second, or 983,294,400 feet per second, a wave travels approximately 983 feet in one-*millionth* of a second. This small unit is called a **microsecond** (μs). About 6.18 μs are needed for a wave to travel one nautical mile.

If signals are transmitted from a known point at established times, as every second

of GMT, and the time of reception at a second point is measured, the difference between the two times is an indication of distance. Such a system requires clocks that can be kept synchronized to a very small unit of time, perhaps one microsecond.

Another method is to measure a time *interval* by means of a cathode ray tube (art. 1019). The reference or starting time needed for measurement of the interval is commonly provided by originating the signal at or near the receiving antenna. The signal travels to the "target" and back, the time required for the round trip being measured. This is the principle of radar (art. 1208). In **primary radar,** a reflected signal or **echo** is returned. In **secondary radar,** the transmitted signal serves as an interrogator to **trigger** a **transponder,** which immediately (or after a known delay) transmits a return signal. This is the principle of shoran (art. 1213), hiran (art. 1213), electronic position indicator (art. 1213), distance measuring equipment used with vortac (art. 1207), and racons (art. 1210).

In order to utilize this principle, it is necessary to be able to transmit very short bursts or "pulses" of energy. Otherwise, the return signal would be lost in the stronger outgoing signal. This is accomplished by means of pulse modulation (art. 1016), which permits transmission of signals during a period as short as a fraction of a microsecond, if needed.

Distance through the water is measured in a similar manner, using sound waves. The short bursts of energy, usually in the ultrasonic range above audible frequencies, are produced electronically. Because of the much slower speed of sound waves, as compared with radio signals, the lengths of the individual pulses are correspondingly greater, and simpler means are generally used for measuring the time interval. This principle is used in **sonar** (from **so**und **n**avigation **a**nd **r**anging) to measure horizontal distances, and in **echo sounders** (art. 619) to measure vertical distances. The term "sonar" is sometimes used in a general sense to include echo sounders.

Another method of measuring distance electronically is by comparison of the phase difference between signals derived from two continuous wave transmissions of different frequency. A transmitter and a receiver are located at each of the points between which distance is to be measured. At each station the interaction between the transmitted and received signals produces signals of two additional frequencies, called **beat frequencies,** equal to the sum and difference, respectively, of the two signals. If one of these additional signals is transmitted from one station to the other and compared with the corresponding signal there, the phase difference is an indication of distance. If the distance between the stations is changing, a Doppler effect occurs, permitting measurement of speed. This is the operating principle of pure-range **Raydist** (art. 1214).

Distance can also be measured by a combination of radio and sound signals. Simultaneous signals are transmitted by radio and by sound, either through the air or through the water. The difference in speed is so great that the travel time of the radio signal can be considered zero. Thus, the time interval between reception of the radio and sound signals is an indication of distance. This method is used only over relatively short distances, where the distance in nautical miles can be considered equal to the elapsed time in seconds divided by 1¼ if the sound travels through water, and by 5½ if it travels through air. This was the first electronic method of determining distance and is still utilized in a number of **distance finding stations** (art. 1205). The method is sometimes used by surveyors, who have a special beacon for this purpose. The finding of distance by this beacon is called **radio acoustic ranging (RAR),** further discussed in article 1205.

1109. Distance-difference measurement.—If synchronized signals from two stations are transmitted, the *difference* in distance from the stations can be measured, either by means of the elapsed time interval between the arrival of the two signals, or

by measurement of the phase difference between the signals. If beat frequencies are used, synchronization may not be needed.

Refer to figure 1109. Let *M* and *S* be two stations. Synchronization is achieved by letting the signals of *M*, the **master,** control those of *S*, the **slave.** Circles *M*1, *M*2, *M*3, etc., are units of distance from *M*; and circles *S*1, *S*2, *S*3, etc., are units of distance from *S*. If both signals are transmitted at the same instant, they will arrive together at any point along a line equidistant from the two stations. This **center line** is the perpendicular bisector of the **base line** joining the two stations. On a sphere, both center line and base line are great circles.

The center line is the zero time difference line. If the *M* signal arrives at some point before the *S* signal, the time difference can be found by subtracting the *M* signal travel time (or distance) from the *S* signal travel time (or distance). If a line is drawn through all intersections at which units of distance from *S* are greater by *one* than those from *M* (*S*8, *M*7; *S*7, *M*6; *S*6, *M*5; etc.), a curve is formed, as shown at "+1" in figure 1109. A similar curve labeled "−1" is formed if all points at which units of distance from *S* are *less* by *one* than those from *M* (*M*8, *S*7; *M*7, *S*6; etc.) are connected. The minus sign indicates that the *M* signal arrives (−)1 time unit *before* the *S* signal, or $S-M=(-)1$. On a plane surface, such curves are **hyperbolas** (art. O34) because they connect points of equal *difference* of distance between two fixed points. On a sphere, such curves are called **spherical hyperbolas.** On the spheroidal earth they are not plane hyperbolas, and differ somewhat from spherical hyperbolas.

Other, more sharply curving hyperbolas are formed by connecting lines of greater time (distance) difference, as at (+)2, (−)2, (+)3, (−)3, etc. The maximum difference occurs along the **base line extensions** beyond the transmitters. This difference depends upon the distance between stations. A pattern of all positive readings can be obtained by delaying the start of the *S* signal until the *M* signal is received at *S*, or longer. Suppose the *S* signal is transmitted ten units *after* the *M* signal. The *M* signal for a base line six units long will already have traveled four units beyond *S* when the *S* signal leaves the transmitter. Therefore, the reading along the base line extension from *S* is (+)4, or ten *more* than shown in figure 1109. By the time the *S* signal arrives at the master transmitter, the *M* signal will be at ten (the delay) plus six (the number of units between *M* and *S*) units, or 16 units away. Therefore, the reading along the base line extension beyond *M* is $10+6=(+)16$. Similarly it can be shown that all other readings are also increased by (+)10.

Each hyperbola becomes more nearly a straight line (great circle) as distance from the base line increases. At a distance from the center of the base line of five times the length of the base line, the departure of the hyperbola from a great circle becomes very small. For a "long" base line of several hundred miles, as in loran (art. 1302), the lines are considered curves over their entire length. This is also true of a "medium" base line as used in gee (art. 1308), Decca (art. 1309), Lorac (art. 1310), and hyperbolic Raydist (art. 1311), which are not used over such an extensive area. If the base line is very short, as in sonne and consol (art. 1312), the system is considered directional rather than hyperbolic, beyond a distance of a few miles from the station.

Each hyperbola is a line of position. Accuracy of such a system is greatest along the base line, where the hyperbolas are most closely spaced. As the distance between consecutive lines increases, the accuracy decreases, being so low along the base line extensions that use of this part of the pattern is normally avoided.

A hyperbolic system has the disadvantage of requiring two stations for a single family of lines of position. This can be partly overcome by using a series or **chain** of stations, so that each station except the end ones operates with the station on either side to form an intersecting **lattice** of position lines. This method is used with loran

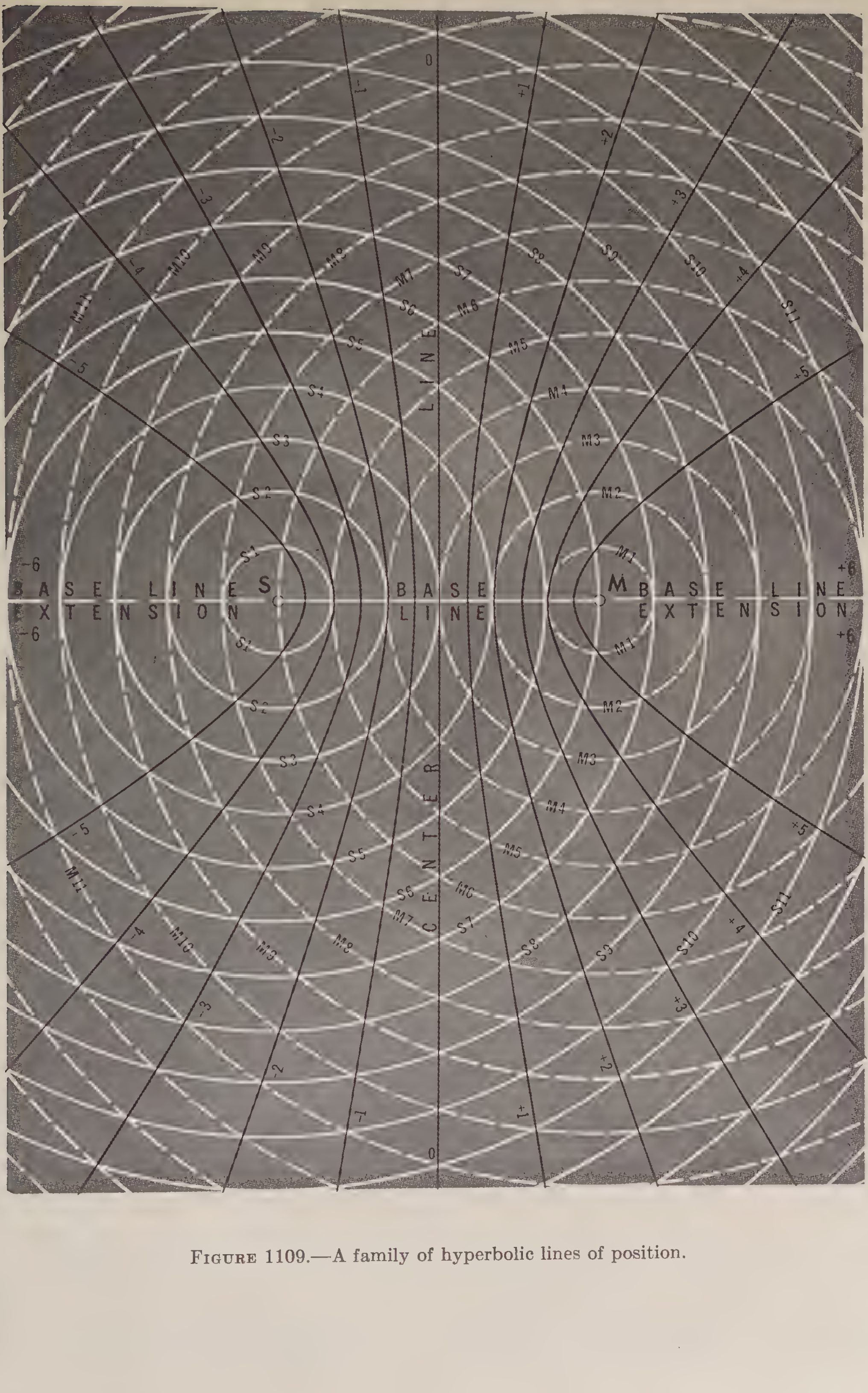


FIGURE 1109.—A family of hyperbolic lines of position.

(art. 1302). With Decca (art. 1309), a central master operates with three slaves surrounding it. Another disadvantage of a hyperbolic system is the need for computation of points along the hyperbolas. These points are computed in advance and tabulated, or plotted and connected by curves on special charts. This task is not normally performed by the user, but it does add to the cost of the system.

An advantage of a hyperbolic system is that it may not require transmission from the craft, an important consideration in time of war.

Hyperbolic lines of position may also be established by means of sound signals. Such a system, called **sofar,** is described in article 1313.

References

Hall, J. S. *Radar Aids to Navigation.* M. I. T. Radiation Laboratory Series. New York, McGraw-Hill, 1947.

Reintjes, J. F., and Coate, G. T. *Principles of Radar.* 3rd ed. M. I. T. Radar School. New York, McGraw-Hill, 1952.

Schelkunoff, S. A., and Friis, H. T. *Antenna Theory and Practice.* New York, Wiley, 1952.

Terman, F. E. *Radio Engineers' Handbook.* New York, McGraw-Hill, 1943.

U. S. Department of the Navy. *Radar System Fundamentals.* NAVSHIPS 900,016 and 900,017. Washington, 1944.

CHAPTER XII

DIRECTION AND DISTANCE BY ELECTRONICS

1201. Introduction.—Many systems have been proposed to utilize the various techniques described in chapter XI. Only the more prominent ones put to practical use are discussed in this chapter and the following one.

1202. Radio direction finder (**RDF**).—The type radio direction finder commonly used aboard ship consists essentially of a loop antenna (art. 1103) that can be turned in any direction around a vertical axis, and some kind of indicator. In its usual form, the indicator consists of a compass rose with a pointer pivoted at its center. The pointer is so oriented to the antenna that it points toward the direction from which the signal is coming when the null is received. If the compass rose is oriented with 0° in line with the ship's head, the usual orientation in a permanent installation, measured directions are *relative* bearings. In many permanent installations, there is a course input from the gyro compass so that approximately true bearings are measured.

Radio bearings may be taken on any received radio signal within frequency range of the receiver. At many locations **radiobeacons** are provided for this purpose. Their locations and identifying signals are shown on the chart by appropriate symbol (app. K) and the abbreviation "R. Bn.", and are tabulated in H.O. Pub. No. 117, *Radio Navigational Aids*. When bearings are taken on other stations, one should be careful to determine the location of the *transmitting* antenna from which the signal is coming. This may not always be the same as a receiving antenna associated with the same station, and the signal may possibly be rebroadcast from another station.

Along some foreign coasts **direction finder stations** are provided to obtain bearings, upon request, and transmit the information to the vessel requesting it. These stations are indicated on the chart by the letters "R.D.F.", and listed in H.O. Pub. No. 117.

1203. Errors of radio bearings.—Bearings obtained by radio direction finder are subject to certain errors, as follows:

Quadrantal error. When radio waves arrive at a receiver, they are influenced somewhat by the environment, resulting in an erroneous indication of direction. Aboard ship this is a function of the *relative* bearing, normally being maximum for bearings broad on the bow and broad on the quarter. Its value for various bearings can be determined, and a **calibration table** made. The usual method of calibration is to obtain a series of simultaneous radio and visual bearings on a transmitter. This may be done while a ship swings at anchor, or more quickly by steaming in a circle within sight of a transmitter. Another method, when two ships are available, is for the second ship to transmit while circling the first. Naval vessels sometimes use this method while both ships are underway, proceeding between ports. A vector diagram solution, usually on a maneuvering board (art. 1212), can be used to determine the courses and speeds of the maneuvering vessel. Metal booms, cranes, etc., should be in their normal positions during calibration. If their positions are changed when the radio direction finder is used, an error may be introduced.

Coastal refraction. As indicated in article 1006, a radio wave crossing a coast line at an oblique angle undergoes a change of direction due to difference in conducting and reflecting properties of land and water. This is sometimes called **land effect.** It is avoided by not using, or regarding as of doubtful accuracy, bearings of waves which cross a shore line at an oblique angle. If the transmitter is near the coast, negligible

error is introduced because of the short distance the waves travel before undergoing refraction.

Polarization error. As indicated in article 1008, the direction of travel of radio waves may undergo an alteration during the confused period near sunrise or sunset, when great changes are taking place in the ionosphere. This error is sometimes called **night effect.** The error can be minimized by averaging several readings, but any radio bearings taken during this period should be considered of doubtful accuracy.

Reciprocal bearings. Unless a radio direction finder has a vertical sensing wire (art. 1103), there is a possible 180° ambiguity in the reading. If such an error is discovered, one should take the reciprocal of the *uncorrected* reading, and apply the correction for the new direction. If there is doubt as to which of the two possible directions is the correct one, one should wait long enough for the bearing to change appreciably and take another reading. The transmitter should draw *aft* between readings. If the reciprocal is used, the station will appear to have drawn *forward.* A reciprocal bearing furnished by a direction finder station should not be used because the quadrantal error is not known, either on the given bearing or its reciprocal.

In general, good radio bearings should not be in error by more than 2°. However, conditions vary considerably, and skill is an important factor. By practicing frequently when results can be checked by visual observation or by other means, one can develop skill and learn to what extent radio bearings can be relied upon under various conditions. Bearings taken ashore should be of slightly greater accuracy than those taken aboard ship. Shore stations indicate bearings of doubtful accuracy. These stations should not be asked to estimate the size of the probable error.

1204. Using radio bearings.—A bearing obtained by radio, like one determined in any other manner, provides means for establishing a line of position. By heading in the direction from which the signal is coming, one can proceed toward, or **home** on, the transmitter. In thick weather one should avoid heading directly toward the source of radiation unless he has reliable information to indicate that he is some distance away. In 1934 the Nantucket Lightship was rammed and sunk by a ship homing on its radiobeacon.

Radio waves, like light, travel along great circles. Except in high latitudes, visual bearings can usually be plotted as straight lines on a Mercator chart, without significant error. Radio bearings, however, are often observed at such positions with respect to the transmitter that the use of a rhumb line is not satisfactory. Under these conditions it is customary to apply the **conversion angle** (art. 821) as a correction to the observed angle, to find the equivalent rhumb line. Such a correction is not needed when a bearing is plotted on a gnomonic chart or one on which a straight line is a good approximation of a great circle. In other situations, a correction may be necessary.

If the transmitter and receiver are on the same meridian, or are both on the equator no correction is needed because rhumb lines and great circles coincide under these conditions. The size of the correction increases with degree of departure from these conditions, and with greater distance between transmitter and receiver.

Conversion angles are given in table 1. This table is used to convert great circle to rhumb line directions or vice versa as in great circle sailing, radio, and consol bearings (art. 1206). If the difference of longitude is not more than 4°5, and the mid latitude between transmitter and receiver is not more than 85°, the first part of the table should be used. The simplifying assumptions used in the computation of this part of the table do not introduce a significant error within the limits of the table.

The sign of the correction can be determined by referring to the rules given at the bottom of each page of table 1. These follow from the fact that the great circle is

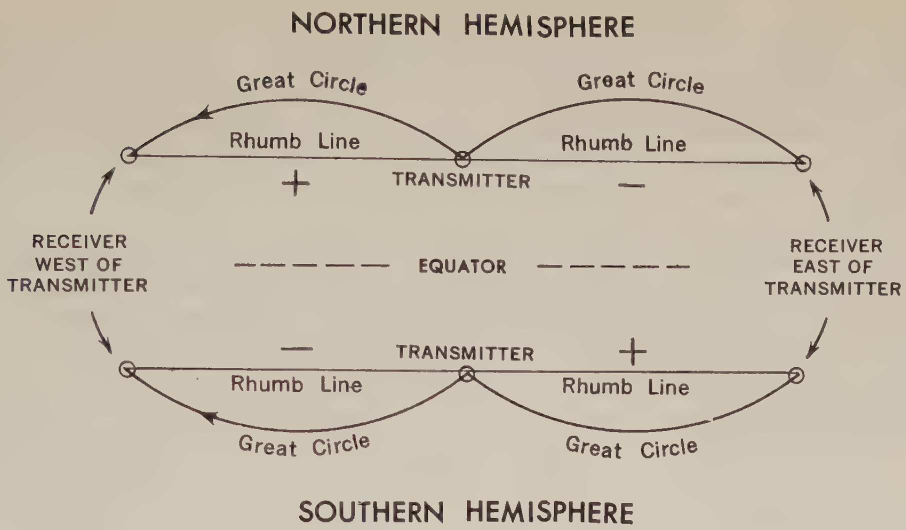


FIGURE 1204.—Sign of conversion angle correction to radio bearings.

nearer the pole than the rhumb line. It can be visualized by means of a simple sketch, as shown in figure 1204.

Example.—The DR position of a ship is lat. 42°15′.2 N, long. 9°48′.6 W. A radio bearing is taken on Cabo Montedor Light Station, at lat. 41°45′00″ N, long. 8°52′20″ W. The reading, corrected for calibration error, is 125°.5.

Required.—The equivalent rhumb line bearing.

Solution.—

	Latitude	*Longitude*
Receiver	42°15′.2 N	9°48′.6 W
Transmitter	41°45′.0 N	8°52′.3 W
Difference	30′.2	56′.3=0°.9
Mid latitude	42°.0	
Correction	(+) 0°.3 (from table 1)	
Great-circle bearing	125°.5	
Rhumb line bearing	125°.8	

Answer.—B 125°.8.

Radio bearings are plotted and labeled as any other bearing line (art. 904). If it is desired to indicate the nature of the bearing, the word "radio" might be added to the label, preferably below the line. Since radio bearings are generally somewhat less accurate than visual bearings, and often are observed at greater distances, positions obtained by them are generally considered of insufficient accuracy to be termed fixes, and so are usually considered estimated positions (art. 913). However, judgment should govern the reliance to be placed upon such positional information. A series of such positions may provide the basis for elimination of random errors, giving a reliable fix unless systematic errors (art. 2903) are present.

Some navigators estimate or assume a probable error (usually of ±2° unless conditions suggest another value) and plot lines on each side of the bearing line to indicate the probable area within which the vessel is presumed to be located.

Radio bearings furnished by a direction finder station have been corrected for known errors at the receiver, but not for conversion angle. The latter should be applied by the user.

1205. Distance finding stations.—At some locations a radio signal is synchronized with a sound signal which may be transmitted through either air or water. The

travel time of the radio signal is negligible compared to that of the sound signal. Consequently, the difference in time between reception of the two signals is proportional to the distance from the station. The distance in nautical miles is equal to the number of seconds of time interval divided by 5½ if the sound travels through air, or by 1¼ if through water (or multiplied by 0.18 or 0.8, respectively). The distance so found is from the origin of the *sound* signal, which might differ somewhat from that of the radio signal. The distance may be in error by as much as ten percent.

A light, portable, floating beacon, designed for use primarily in surveying, transmits radio signals through the air when triggered by a suitable sound signal. Determination of distance by the use of such a beacon is called **radio acoustic ranging (RAR).**

1206. Consol is a long-range, short-base-line, hyperbolic system operating in the 250–350 kc frequency range. The three antennas constituting a station are spaced at intervals of about three wave lengths. Beyond a distance of about 25 miles from the center station the lines of position can be considered great circles with negligible error. In use, the system is considered a directional one, the hyperbolic portion of the lines not being used.

The radiation pattern of each station consists of alternate sectors of dot and dash signals, the sectors averaging 15° in width. During the "keying" cycle of 30 or 60 seconds, this pattern rotates, the **equisignal** between dots and dashes moving through one sector. During this period, 60 signals (either dots or dashes) are transmitted. At any point along the dividing bearing between sectors at the beginning of the cycle, 60 dots or 60 dashes should be received. Along any other bearing line the count of 60 is distributed between the two types of signal. The relative number of each, and their order, is related to the bearing of the receiver.

The total count is generally less than 60 because the dot and dash sectors overlap, one type signal gradually fading as the other becomes stronger. The equisignal boundary is the line along which both signals are of equal intensity and neither can be distinguished. Several signals may be lost during passage of this sector. The number of signals lost (60 minus the actual count) should be distributed equally between the dots and dashes. If the difference is an odd number, the smaller correction should be applied to the count received first, because the ear can generally follow a fading signal to a lower degree of contrast than it can detect the first signal of a new series. This is particularly true when dots are received first. Thus, if the count is 25 dots and 30 dashes, the total is 55, and 5 signals have been lost in the equisignal sector. The corrected count is 25+2=27 dots, and 30+3=33 dashes.

The great-circle bearing corresponding to the count is determined by referring to H.O. Pubs. Nos. 117–A and 117–B, a separate table being given for the dot and dash sectors of each station. If plotting is to be done on a Mercator chart, the conversion angle correction should be applied using the table given in H.O. Pubs. Nos. 117–A and 117–B. Table 1 may also be used; however, the entering arguments of transmitter and receiver must be reversed. Special charts showing the lines or graduations have been prepared by some foreign countries.

The time of observation of a consol line of position is the moment at which the equisignal is heard.

Under favorable conditions, the coverage area for a consol station extends outward for about 1,000 to 1,200 miles by day, and 1,200 to 1,500 miles by night, over water. Over land, or at any time that the noise level is high, these ranges may be reduced materially. The accuracy varies considerably over the pattern. Directionally, it is greatest along the great circle through the center antenna and perpendicular to the line of antennas. At an angle of 60° to this perpendicular, the accuracy drops to a minimum usable value. Therefore, there is a usable sector of about 120° on each side of the line of antennas, with an unusable sector of about 60° at each end of this line.

In terms of distance, the greatest angular error occurs within the range in which sky waves and ground waves mingle, between about 250 and 400 miles from the station.

As a very general rule, for 95 percent of the time when ground waves are received, the error over water is not more than about one-third degree along the perpendicular, increasing to about twice this value at an angle of 60° to the perpendicular. In terms of miles, this is about one mile error for each 180 miles from the station along the perpendicular, and each 90 miles along the bearing line 60° from the perpendicular. For sky waves these values are about doubled, and when sky waves and ground waves are near the same amplitude, the error may be considerably larger. These values refer to single observations. The error is generally reduced by taking the average of several readings. On many occasions a good dead reckoning position is more reliable than a position obtained by consol. However, the method is valuable when the position is considerably in doubt, and is a useful check to prevent gross errors by other methods. If the position is so seriously in doubt that the sector is uncertain, a bearing by radio direction finder should resolve the ambiguity.

A reading cannot be made oftener than once every one or two minutes, depending upon the cycle of operation. Each 30- or 60-second "keying cycle" is followed by a period of equal length, during which a continuous tone and identification are transmitted. No special equipment is required beyond an ordinary medium frequency communication receiver, and very little skill or training is needed.

Consol is a British development of a German system known as **sonne,** which in turn evolved from the nonrotating **elektra.** There are several consol installations along the coasts of western and northern Europe. Two stations of an American version, called **consolan,** are installed on the east and west coasts of the United States. The Japanese also have a version.

1207. Radio ranges.—The airways of the United States and some other countries are marked by a series of distinctive radiobeacons called **ranges.** Under suitable conditions, these are useful in marine navigation.

Two different types of ranges are in use. The older low frequency **four-course range** consists of two Adcock antennas (art. 1104) so oriented that their signal areas occupy sectors, usually about 90° each. The edges of the sectors overlap to form narrow equisignal sectors or "beams" directed along the airways. One antenna transmits the Morse code letter *A* (• ▬), and the other the letter *N* (▬ •). These signals are so synchronized that when they are received with equal intensity, they interlock to form a single monotone "on-course" signal. As the equisignal sector is left, one signal predominates. As the angle from the center of the equisignal sector increases, the predominating signal becomes more prominent and the monotone fades. The area near each side of the equisignal sector is called a "twilight sector."

Some equisignal sectors extend out to sea. Their locations and the identification of the *A* and *N* sectors are shown on appropriate aeronautical charts. A marine navigator equipped with such information may find the ranges useful for determining bearings, or even for homing.

A newer type range eliminates the four-course limitations of the older ones by transmitting a rotating pattern, using very high frequency signals. Two such systems, **tacan** for military aircraft and **omnirange** (**VOR**) for others, together with electronic equipment for determining distance by an interrogator-transponder (art. 1108), are located at each installation, called a **vortac** station. By means of the special receiving and indicating equipment needed, one can determine either (1) bearing and distance at any time by automatic dial and meter indications, or (2) direction to turn to arrive at a selected "radial" (bearing). Because of its limited application to marine navigation, the special equipment is not normally carried aboard ship.

1208. Radar determines distance by measuring the time required for a radio signal to travel from a transmitter to a "target" and return, either as a reflected "echo" (primary radar) or as a retransmitted signal from a transponder (art. 1108) triggered by the original signal (secondary radar). The name is derived from **ra**dio **d**etection **a**nd **r**anging. Since radar uses a directional antenna, the direction of the target is also determined, but with somewhat less accuracy than the distance.

In a radar set, signals are generated in a transmitter by a timing circuit so that energy leaves the antenna in very short bursts or "pulses." During transmission of a pulse, the antenna is connected to the transmitter but not the receiver. As soon as the pulse leaves, an electronic switch disconnects the antenna from the transmitter and connects it to the receiver. Another pulse is not transmitted until after the preceding one has had time to travel to the most distant target within range, and return. Since the interval between pulses is long compared with the length of a pulse, strong signals can be provided with low *average* power.

From the receiver, the return signal goes to the indicator. This consists of a cathode ray tube (art. 1019) and appropriate circuits. Many types of display have been devised, a number of them to meet specialized requirements. For navigational use, the earliest type of display was the **A-scope.** The principle of this scope is illustrated in figure 1208a. At A a pulse leaves the antenna of a ship, and a vertical deflection appears at the start of the horizontal trace on the scope face. At B the pulse has traveled some distance outward from the antenna. A short horizontal line appears after the vertical deflection on the scope face. The length of this line is directly proportional to the distance traveled by the pulse. At C the pulse encounters a target with a reflecting surface. At D the original pulse has moved on beyond the target, but part of its energy has been reflected back toward the transmitter. At E the echo has arrived back at the transmitting craft, causing a vertical deflection of the horizontal trace. The height of this deflection is directly proportional to the strength of the returning signal. At F the echo has proceeded on past the transmitting ship, and the trace is completed.

This sequence is repeated a great many times, perhaps 1,000 per second, the rate being called the **pulse repetition rate** (**PRR**) or **pulse recurrence rate.** The start of each trace is synchronized with transmission of the signal so that each trace is a repetition of the previous one, if slight changes in relative positions of transmitting ship, target, and antenna orientation are neglected. Therefore, the trace and all deflections appear as a continuous line. The distance between leading edges of the vertical deflections, or "pips," is directly proportional to range. A change of range alters the position of the second pip. The orientation of the antenna is an indication of direction. A pip appears only when the antenna is pointed toward the target.

The type of presentation now most commonly used for navigational radar is called the **plan position indicator** (**PPI**). On this presentation the sweep starts at the center of the tube face and moves outward along a radial line which rotates in synchronization with the antenna. Instead of being deflected, the trace glows with greater intensity (brightness) at the appropriate places. Because of the persistence of the tube face coating, the glow continues after the trace rotates on past the target, resulting in a maplike presentation on the scope. This presentation is shown in figure 1208b.

On a PPI, the range of a target is proportional to the distance of its echo signal from the center of the scope. This may be measured by a series of visible concentric circles at established distances from the center, or by means of an adjustable ring synchronized with a counter. Bearing is indicated by the direction of an echo signal from the center of the scope. To facilitate measurement of direction, a movable, radial, guide line or **cursor** is provided, and a compass rose is placed around

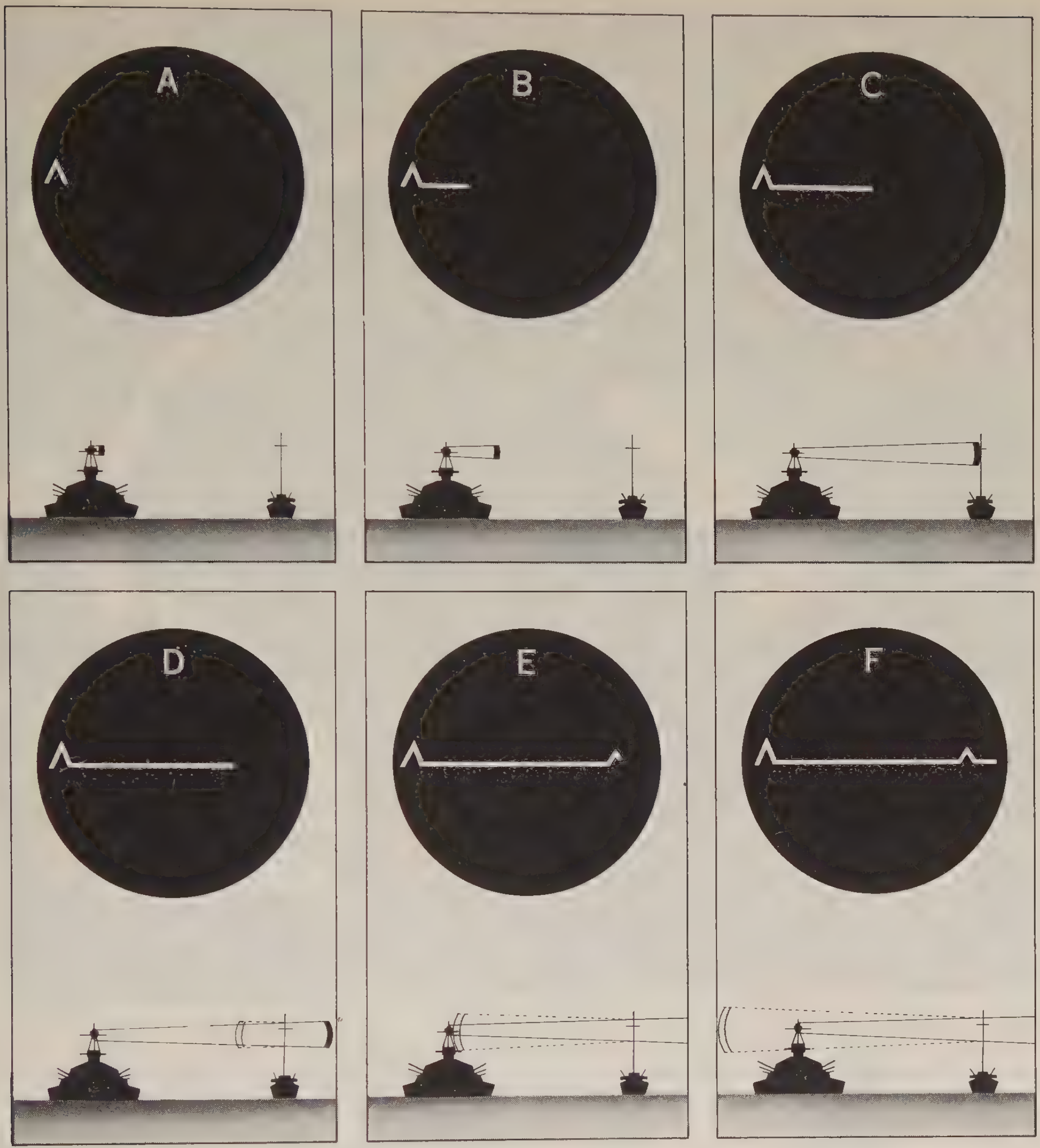


FIGURE 1208a.—A-scope.

the outside of the scope. In the "heading-upward" presentation, relative bearings are indicated, the top of the scope representing the direction of the ship's head. In the "north-upward" presentation, gyro north is always at the top, regardless of the heading. True bearings are indicated if there is no gyro error. On this type presentation a radial line is customarily provided at the heading of the vessel.

Provision may be made for offsetting the center of the PPI presentation from the center of the tube face, to permit large-scale observation of distant targets in one direction. With "true motion" radar, the center of the tube face continues to represent the same geographical position until reset. The actual motion of all moving objects, including one's own vessel, appears on the scope, instead of the relative movement usually shown.

Other modifications have been devised. In some installations a **repeater** duplicates the presentation, making the information available at a distance from the radar.

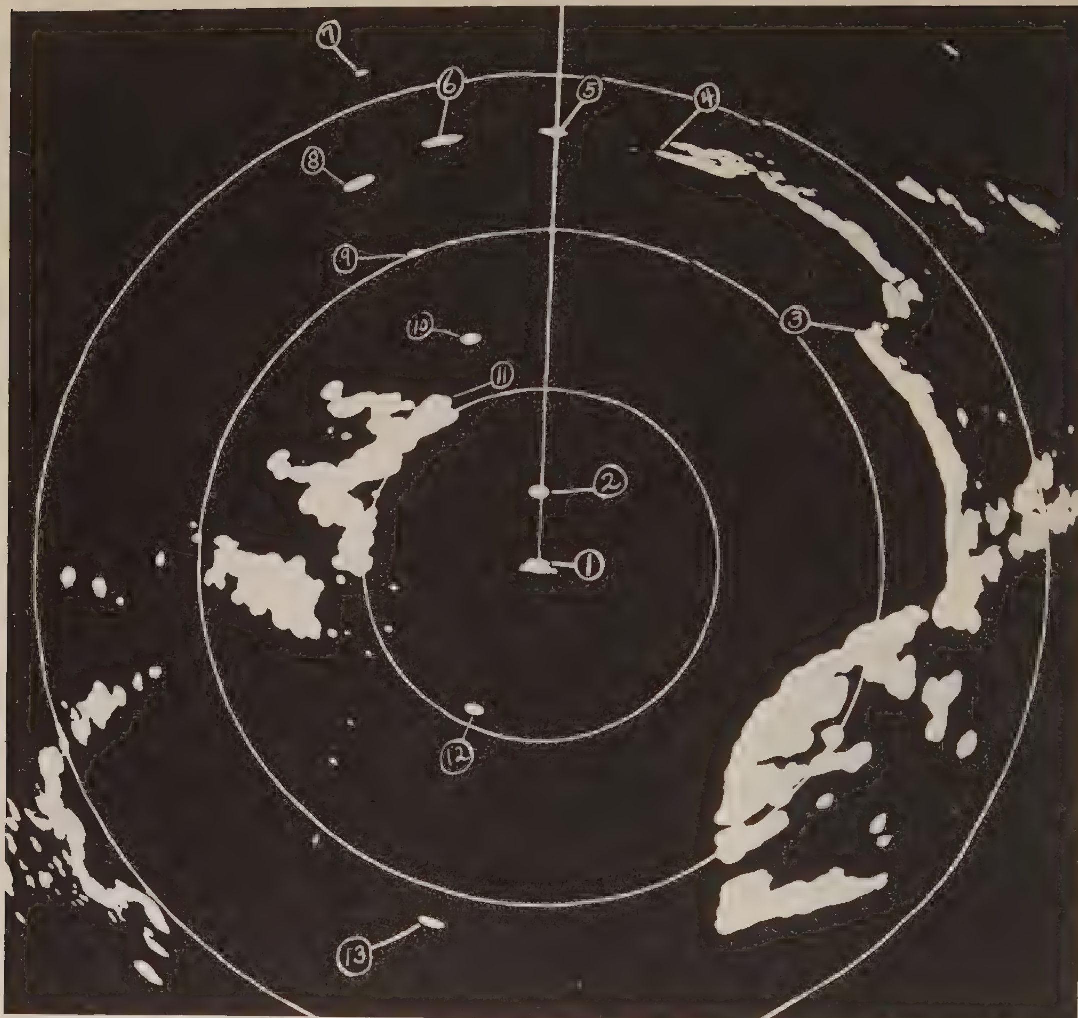


FIGURE 1208b.—Plan Position Indicator. 1. Ship's position. 2. Ship dead ahead. 3. Isone Misaki. 4. Futtsu Saki. 5. Fort No. 1. 6. Fort No. 2. 7. Small craft. 8. Ship. 9. Fort No. 3. 10. Small craft. 11. Kannon Saki. 12. Ship. 13. Ship. See figure 2308 showing chart of this area.

Since the receiver is disconnected during transmission of a signal, no echo can be received during this period. As a result, there is a minimum range at which objects can be detected. The shortest pulses are about 0.1 microsecond in duration, or approximately 98 feet long. Since the time measurement is of the *round trip* as the signal travels to the target and the echo returns, the range is *half* the distance corresponding to the measured time interval. Therefore, a minimum range of about 49 feet is theoretically possible with a pulse of 0.1 microsecond. However, the *practical* minimum range is somewhat greater because of **sea return** of echoes from the water near the ship, where the signals strike the surface of the sea almost vertically. A practical minimum of 50 yards is considered excellent.

The maximum range is limited by the power, nature of the target, and by the curvature of the earth, since radar operates in the higher frequencies that are essentially line-of-sight. The **radar horizon,** at which rays from the transmitting antenna are tangent to the surface of the earth, is at a distance about 15 percent greater than that

of the visible horizon (tab. 8). Under conditions of abnormal refraction, both visible and radar horizons may be extended to greater distances.

1209. Scope interpretation.—With practice, one can acquire considerable skill in interpreting the signals appearing on the radar scope face. Some of the factors to be kept in mind in interpretation are the following:

Resolution in range. In part *A* of figure 1209 a transmitted pulse has arrived at the second of two targets of insufficient size or density to absorb or reflect all of the energy of the pulse. While the pulse has traveled from the first to the second target, the echo from the first has traveled an equal distance in the opposite direction. At *B* the transmitted pulse has continued on beyond the second target, and the two echoes are returning toward the transmitter. The distance between leading edges of the two echoes is *twice* the distance between targets. The correct distance will be shown on the scope, which is calibrated to show *half* the distance traveled out and back. At *C* the targets

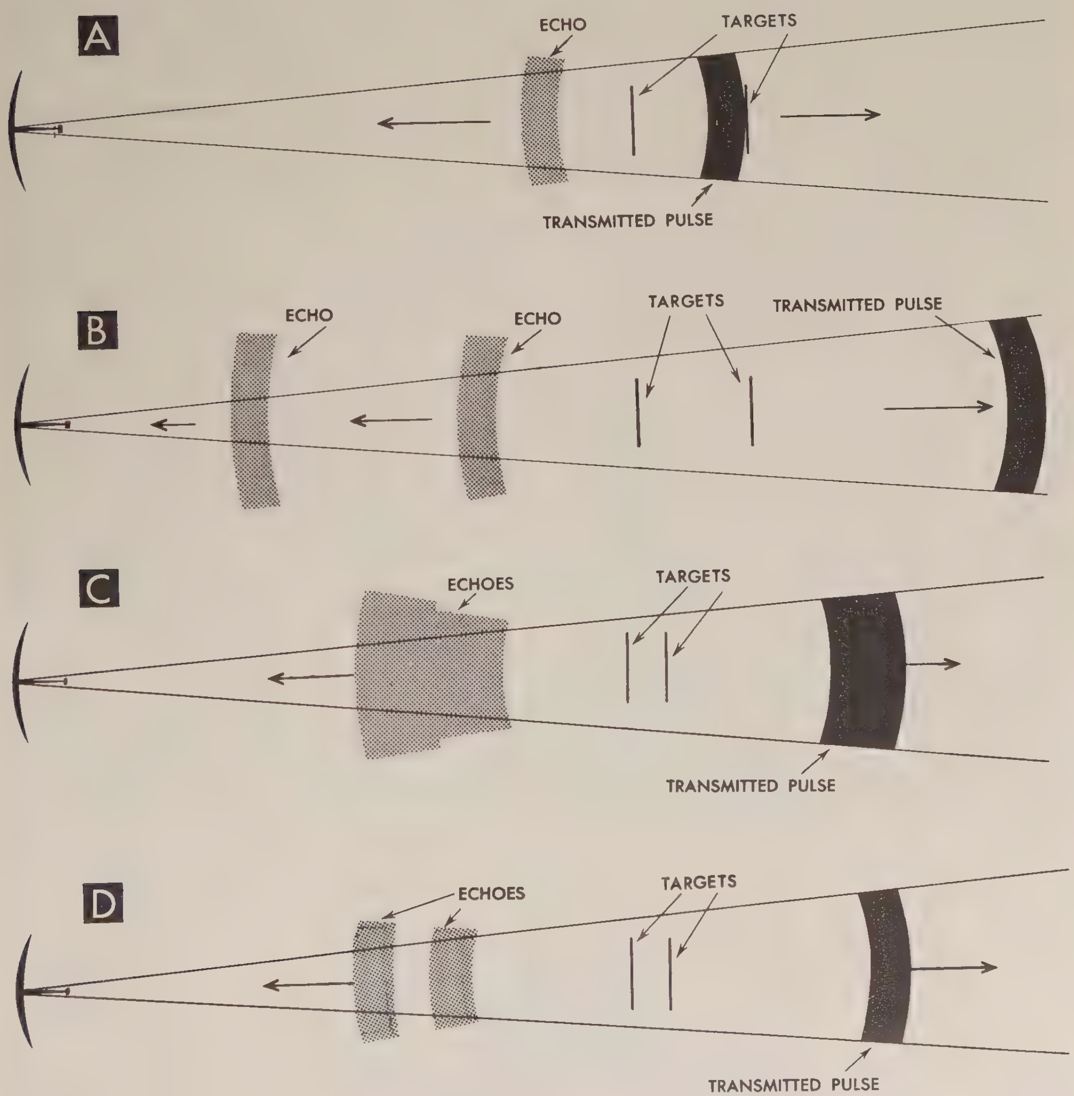


FIGURE 1209.—Resolution in range.

are closer together and the pulse length has been increased. The two echoes merge, and on the scope will appear as a single, large target. At D the pulse length has been decreased, and the two echoes appear separated. The ability of a radar to separate targets close together on the same bearing is called **resolution in range.** It is related primarily to pulse length, the minimum distance between targets that can be distinguished as separate ones being *half* the pulse length. This (half the pulse length) is the apparent depth or thickness of a target presenting a flat perpendicular surface to the radar beam. Thus, several ships close together may appear as an island. Echoes from a number of small boats, piles, breakers, or even large ships close to the shore may blend with echoes from the shore, resulting in an incorrect indication of the position and shape of the shore line.

Resolution in bearing is similar to that in range. A pulse proceeds outward along a narrow sector. As the beam rotates, energy is returned during the entire time that a target is "illuminated," the same as with a searchlight. A vertical target such as a mast is "seen" over the arc in which there is sufficient illumination to render it visible. On a radar PPI a target appears widened by an amount equal to the **beam width,** half the beam width being added to each side. Thus, the echoes from two or more targets close together at the same range may merge to form a single, wider echo. The ability to separate such targets is called **resolution in bearing.** In angular units it is dependent primarily upon beam width, a narrower beam having a higher resolution. In terms of distance between targets, range is also important, resolution increasing as range decreases.

Height of antenna and target. If the radar horizon (art. 1208) is between the transmitting vessel and the target, the lower part of the target will not be visible. A large vessel may appear as a small craft, a shore line may appear at some distance inland. Areas within radar shadows (art. 1009) may not be visible at all.

Reflecting quality of target. Echoes from several targets of the same size may be quite different in appearance. A metal surface is a better reflector of radio waves than a wooden surface. A surface perpendicular to the beam returns a stronger echo than a nonperpendicular one. For this reason, a gently sloping beach may not be visible. A vessel encountered broadside returns a stronger echo than one heading toward or away from the radar vessel. In some instances, the strength of an echo can be increased by means of a **corner reflector.** This is a device with several reflecting surfaces so arranged that a radar signal from any direction is returned toward its source. Corner reflectors are fitted to a number of buoys (labeled "Ra Ref" on the chart), and are carried in some lifeboats. The strength of a returning echo can be *reduced* by coating a surface with radar absorbent material.

Frequency. As the frequency is increased, reflections occur from smaller targets. Thus, a ten-centimeter radar generally penetrates fog, rain, snow, etc., while a three-centimeter radar receives returns from such obstacles, and can be used to track storms. Radar frequencies are sometimes indicated by "band," as follows:

Band	*Frequency (mc)*	*Approx. wave length (cm)*
P	225–390	100
L	390–1,550	30
S	1,550–5,200	10
X	5,200–11,000	3
K	11,000–36,000	1
Q	36,000–46,000	0.75
V	46,000–56,000	0.6

A C-band extending from 3,500 mc to 5,850 mc is sometimes mentioned.

Scope interpretation is complicated somewhat by the presence of unwanted signals from atmospheric noise, sea return, precipitation, etc. Collectively, this is called **clutter.** Generally, it is strongest near the vessel and gradually decreases with increased range, because of reduced sea return. Strong echoes can sometimes be detected by reducing the volume or "gain" of the receiver (not the image intensity of the indicator), so that weaker signals will not appear. Even when the amplitude of the clutter is about the same as that of desired signals, the latter can sometimes be detected by watching the scope during several rotations of the antenna. At each rotation the signals from targets remain at about the same place, and of about the same magnitude, while those from waves, noise, etc., fluctuate, appearing different on each revolution. Floating ice or a small boat may not be detected at any range if the waves are high. A rough surface returns a stronger echo than a smooth surface.

Sometimes a signal appears on a radar screen when there is no visible object at the point indicated, and no apparent source of the signal. This is called a **ghost.** It may be due to faulty operation of the radar set, or to an actual echo returned from a discontinuity in the atmosphere. Sometimes such discontinuities reflect light, also, producing images or apparent images similar to mirages and of seeming apparent reality. A similar condition occasionally occurs in the sea. This phenomenon is undoubtedly the basis of many reports of strange objects sighted visually or by radar. Sometimes such apparent objects exhibit incredible speed or maneuverability.

1210. Radar navigation.—Radar provides a means of establishing position, or keeping a vessel in safe water during periods of reduced visibility, or at considerable distance from shore, when other methods may not be available. Since both range and bearing can be obtained, a single identifiable object is needed. However, if a visual bearing is available, it should be more reliable than one obtained by radar. Since radar range is usually more accurate than radar bearing, a fix by two or more ranges is generally preferable to one obtained by two bearings or by range and bearing. However, accurate range requires reliable identification of the part of the target returning the echo. This is not always apparent when natural objects are used.

Radar beacons have been installed at some places. One type, called **ramark** (from **ra**dar **mark**), transmits continuously in all directions. On the scope of a radar receiving the signal a radial line appears at the bearing of the beacon. The beacon does not have to be within the range to which the scope is adjusted. A limited number of this type beacon has been installed for experimental use by ships.

Another type beacon, called **racon** from the words **ra**dar bea**con,** consists essentially of a transponder (art. 1108) which returns a coded signal when triggered by a signal from a radar transmitter. The code, consisting of a series of dots and dashes, provides identification of the beacon. The range and bearing are indicated by the position of the first character of the code on the PPI. This type beacon is used principally by aviators. Information on these installations is given in various aeronautical publications (art. 2802). Because the return signal is of a different frequency than the outgoing signal, radar equipment must provide for the change in frequency if racon signals are to be used. Echoes returning at the frequency of the outgoing signals do not appear on the scope.

In addition to the usual methods of piloting, radar is adapted to several methods of somewhat limited application. If a single prominent target is available in an operating area, a series of concentric circles and radial lines—a polar plot similar to that of a maneuvering board (art. 1212)—can be drawn on the chart and suitably labeled. If bearing and distance are measured frequently, an almost continuous fix can be obtained by spotting in the positions by eye. If a polar plot is made on a piece of transparent material to the same scale as the chart, the ranges and bearings of a number

of points can be plotted in quick succession, and the transparent material fitted to the chart by trial and error. The center of the plot is then the position of the radar.

Several models of **chart comparison unit** (**CCU**) have been devised. By means of this device, an image of the chart is superimposed over the PPI, or an image of the PPI is superimposed over the chart. Either method permits direct comparison of radar image and chart, if the two are of the same scale. Although distortion of the PPI presentation is not the same as that of the chart, an experienced person can usually effect a reliable match, providing reasonably accurate determination of position. A chart comparison unit designed to produce a virtual image of the chart on the face of the scope is sometimes called a **virtual PPI reflectoscope** (**VPR**).

Early models of the chart comparison unit were used with white-on-black charts designed especially for the purpose. Later models can be used with ordinary nautical charts. Various other special chart presentations have been devised for radar, but the present trend is toward modification of nautical charts to make relief and radar-conspicuous objects more prominent. This is accomplished primarily by shading and the use of additional contours.

Useful information can sometimes be obtained from radar scope photographs made at known positions on previous runs or by other vessels with comparable installations. In certain confined waters, notably along certain stretches of the Ohio River, a series of such photographs made with a typical radar installation have been combined to form a mosaic which presents a continuous maplike presentation. In some cases this mosaic has been printed in fluorescent ink on the regular chart. When the chart is illuminated by fluorescent light, the mosaic glows in a manner that resembles a PPI.

1211. Harbor radar.—At a number of ports, shore-based radar has been installed to assist in the movement of traffic during periods of low visibility. Each installation is tailored to fit its surroundings and requirements. A typical installation consists of a large antenna installed at a prominent point in the harbor, and one or more scopes manned by competent personnel with knowledge of local conditions. The installations are not intended to *control* shipping in the vicinity, but are considered advisory only. Upon their own request, vessels about to enter or leave port, or shift berth, are advised of traffic conditions, and other matters of concern. During passage between harbor entrance and the berthing area or anchorage, they may be warned of possible danger. Customarily, communication with the vessel is through the pilot, who comes aboard equipped with a portable radio. Resolution of present radars is not sufficiently great to permit docking a vessel by radar alone.

A secondary use of harbor radar is to detect drift of aids to navigation from their assigned stations. It is also used to assist a pilot vessel locate an entering ship, or to direct a vessel to a craft in distress or to any other desired point.

One of the principal problems associated with harbor radar is the identification of the echo from a vessel with which radio communication has been established. At least two systems for accomplishing this are under development.

1212. Radar as an anticollision device.—Radar has not materially reduced the number of collisions, as might have been anticipated. This may be due to any of a number of reasons, or probably to a combination of several. Among these are the following: uncertainty as to whether the other vessel has radar, failure to use radar information, lack cf confidence in radar, lack of appreciation of the limitations of radar, failure to act promptly, failure to establish prompt communication with the other vessel, uncertainty as to obligation under rules of the road, misinterpretation of radar information, difficulty of adequately visualizing a situation presented on a radar scope, and lack of knowledge of use to be made of radar information. Most of these can be summed up as lack of adequate training. There is record of radar actually

having been removed from vessels because it was considered a collision hazard. A better remedy would undoubtedly have been to instruct ships' personnel in proper use of this valuable aid.

Neither the international nor inland rules of the road provide special procedure for a radar-equipped vessel, which is therefore expected to obey the same rules applicable to other vessels. This is particularly important in view of the fact that radar is not infallible in detecting the presence of small vessels. In some cases the mere presence of radar is somehow believed to offer a protection or provide an immunity which does not in fact exist. If a vessel has radar, the equipment should be kept in good working condition and used whenever visibility is reduced. Even in clear weather it can be a valuable aid in evaluating a situation, although it is not a substitute for visual observation.

The principal value of radar as an anticollision device is its ability to give *early* information on the locations and movements of other vessels. Two fundamental problems are involved. The first is the determination of relative motion of two or more vessels if they maintain courses and speeds. The second is the determination of the action to take to produce a desired result. Both problems can be solved by a simple plot. Either a navigational or relative movement plot will suffice.

In the navigational plot, positions of one's own vessel are plotted at intervals of a few minutes. From each position the bearing and distance of the other vessel are plotted. From these positions the course and speed of the other vessel can be determined. The dead reckoning of both ships can be run ahead to determine where they will be at any future time. By trial and error, the point of nearest approach and the distance and bearing at this point can be determined. Similarly, the effect of changing course or speed can also be determined.

A somewhat simpler and more direct solution can be made by means of a relative movement plot. This is most easily performed on a polar plotting diagram such as a **radar plotting sheet** (H.O. 4665 series) or **maneuvering board,** H.O. 2665–20 (large size) or H.O. 2665–10 (small size, usually used). If such a plotting sheet is not available, one can easily be constructed, or any compass rose can be used. On these forms, position of one's own ship *remains* at the center, as on the usual PPI. Positions of the other ship are plotted from the position of one's own vessel.

Example 1.—A ship underway obtains the following radar bearings and ranges of another vessel at the times indicated:

Time	*Bearing*	*Range*
1510	030°	8,500 yds.
1512	029°	7,600 yds.
1514	026°.5	6,700 yds.
1516	024°	5,800 yds.
1518	023°.5	5,000 yds.

Required.—(1) The nearest approach of the two vessels.

(2) The bearing of the other vessel at the point of nearest approach.

(3) Time of arrival at the point of nearest approach.

Solution (fig. 1212a).—Let the distance between consecutive circles represent 1,000 yards.

(1) Plot each of the given positions from the center, and label each with the time. If the course and speed of each vessel are constant, the points should plot in approximately a straight line. This is the **relative movement line.** At any moment the other vessel is at *some* point on this line. The direction of this point is the bearing of the other vessel at the moment the point is occupied, and the distance between this point and the

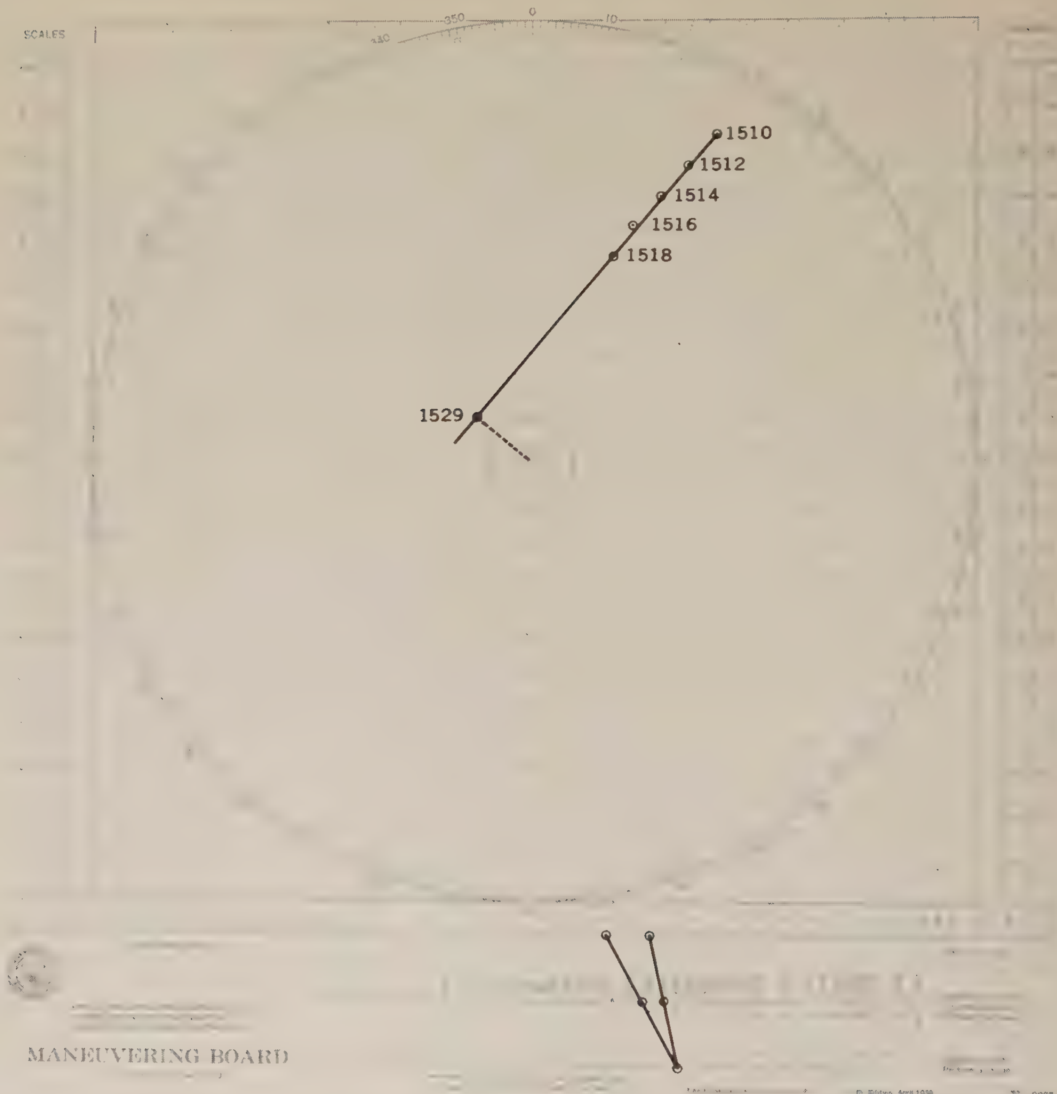


FIGURE 1212a.—Finding time, bearing, and range of nearest approach by relative plot.

center is the range. The direction of the relative movement line is the **direction of relative movement** (**DRM**) of the other vessel with respect to one's own vessel. The DRM of one's own ship with respect to the other vessel is the reciprocal of this line (as are the bearings). The length of the relative movement line in one hour is the relative speed or **speed of relative movement** (**SRM**). The DRM and SRM, representing *relative* motion, should not be confused with the actual course and speed of the other vessel. The nearest approach of the other vessel is on the relative movement line, at its nearest point to the center. The required distance is therefore the perpendicular distance from the center (own ship) to this line (extended). The graduations indicate this to be 1,500 yards.

(2) The direction of the perpendicular, 310°, is the bearing of the other vessel at the point of nearest approach.

(3) The time at which the other vessel is at the foot of the perpendicular is the time of arrival at the point of nearest approach. In this problem it can be determined by using a pair of dividers and stepping off a succession of two-minute runs along

the relative movement line. Another way is by means of the nomogram at the bottom of the diagram. By dividers, the distance between the 1510 and 1518 positions is measured, and also the distance between the 1518 position and the point of nearest approach. These are marked on the *center* line of the nomogram, being careful to distinguish between the "yards" and "miles" scales. A mark is placed at eight on the top line of the nomogram to represent the interval between 1510 and 1518. A straight line connecting the eight-minute mark on the top line with the corresponding distance mark on the middle line, if extended, intersects the bottom line at a point indicating the *relative* speed. A second line from this point, through the second distance mark on the middle line, if extended, intersects the top line at 11^m, the time needed for the other vessel to cover the distance along the relative movement line from the 1518 position to the point of nearest approach. Therefore, time of arrival at this point is $1518+11^m=1529$.

Answers.—(1) D 1,500 yds., (2) B 310°, (3) T 1529.

With the information given in example 1, and the course and speed of one's own vessel, a person can determine course and speed of the other vessel, a process called **tracking.** A speed vector diagram (art. O18) is used.

Example 2.—Find the course and speed of the other vessel of example 1, if own ship is on course 110°, speed 12 knots.

Solution (fig. 1212b).—Let the distance between consecutive circles represent two knots.

Draw the speed vector of own ship (12 knots in direction 110°), starting at the center. Label the outer end of this line *r*. Read the relative speed (SRM), 13.5 knots, from the bottom line of the nomogram of figure 1212a. Relative speed might also be determined by arithmetic. In eight minutes (1510–1518) the other vessel moves 1.8 miles (3,600 yards) relative to own ship. In 60 minutes it will travel $1.8\times\frac{60}{8}=13.5$ miles. From *r*, draw a line parallel to, and in the same direction as, the relative movement line, and measure off a speed of 13.5 knots. Label the end of the vector *m*. The line *rm* is the **relative speed vector,** its length representing the speed of relative movement, and its direction representing the direction of relative movement. A line from the center to *m* is the speed vector of the other vessel, its length representing the actual speed, and its direction the course.

Answers.—C 170°, S 14.7 kn.

It is good practice to continue plotting relative positions of the other vessel until it has passed. Any change in the direction of the relative movement line indicates a change of course or speed. After enough positions have been plotted to establish the new direction of the line, a new solution can be made. If the bearing becomes constant and the distance is decreasing, the two vessels are on collision courses, and unless remedial action is taken, a collision will take place.

It is good practice to start such a plot at the earliest practicable time, remembering that if ships are approaching head on, the *relative* speed is equal to the sum of their individual speeds. If the situation is seen to be a dangerous one, action can be taken in time to prevent a close situation. Many accidents are caused by waiting until the vessels are so close that a change by the other vessel, which may not have radar, brings the vessels together before there is time to detect the change and take action. Since the rules of the road regarding passing or crossing are not applicable until the vessels are *in sight* of each other, any action based upon radar information before the other vessel is sighted is in harmony with the law. Unless the intentions of the other vessel are known, it is good practice to prevent his close approach, if possible, by taking bold action early.

Example 3.—The "own ship" of examples 1 and 2 is capable of a maximum speed

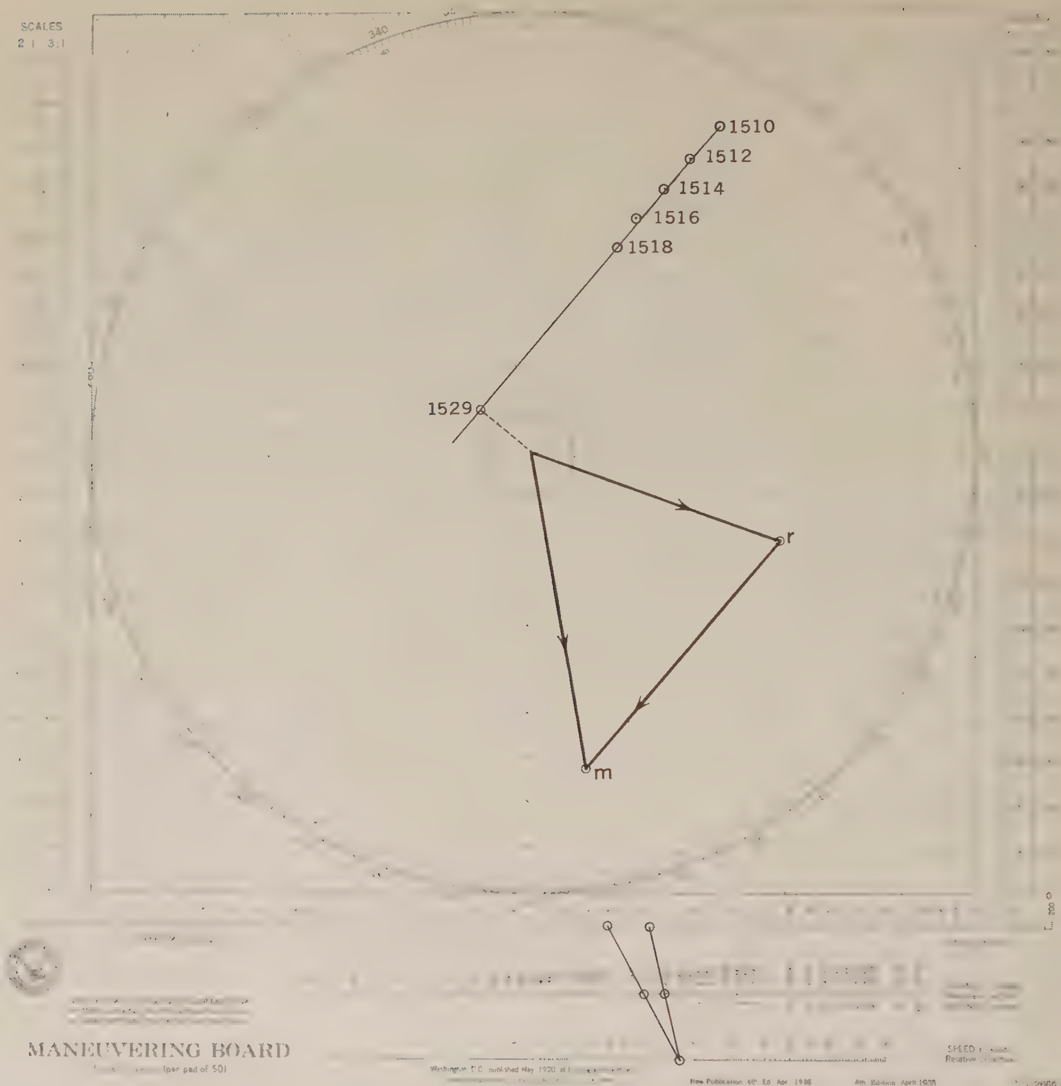


FIGURE 1212b.—Finding course and speed of other vessel, by relative plot.

of 16 knots. It is decided that at 1521 full speed will be used, and the course will be changed to prevent the ships from approaching closer than 3,000 yards.

Required.—The new course.

Solution (fig. 1212c).—Find the relative position of the other ship at 1521. This can be estimated from the previous plot, or determined accurately by connecting the relative speed (13.5 knots) on the bottom line of the nomogram with three minutes on the top line, and noting the point at which this line crosses the center line (1,350 yards), or mathematically, taking ⅜ the relative distance covered in eight minutes. The relative distance (1,350 yards) is measured off along the relative movement line from the 1518 position. From this point draw a new relative movement line tangent to the 3,000-yard circle. From m on the speed vector diagram, draw a line parallel but in the direction *opposite* to the new relative movement line. Label the intersection of this line and the 16-knot speed circle r'. A line from the center to this point is the speed vector of own ship. Its direction is the required course.

Answer.—C 134°.

The *minimum* speed at which the desired result can be obtained may be found by drawing a perpendicular from the center to the relative speed vector $r'm$. The direction of this perpendicular is the course at minimum speed. For this problem the values (not shown in the illustration) are 14.4 knots on course 160°. Since the relative speed vector would then be very short (the ships would be on nearly parallel courses at nearly the same speed), the distance between ships would change slowly. If the relative speed is known, the time to cover any relative distance can be found by nomogram or by arithmetic.

A number of variations of this problem may suggest themselves. With practice, one can acquire the ability to make approximate solutions mentally. Such mental solutions should be checked by plot. This is particularly important if the bearing is changing slowly. It is good practice to have the plot kept by one person who can observe the changing relationship as the vessels proceed. However, one should keep in mind the fact that although a plot adds to the value of radar, it is not a magic solution to all radar problems. It may not reflect small changes in course, and its indications are not instantaneous.

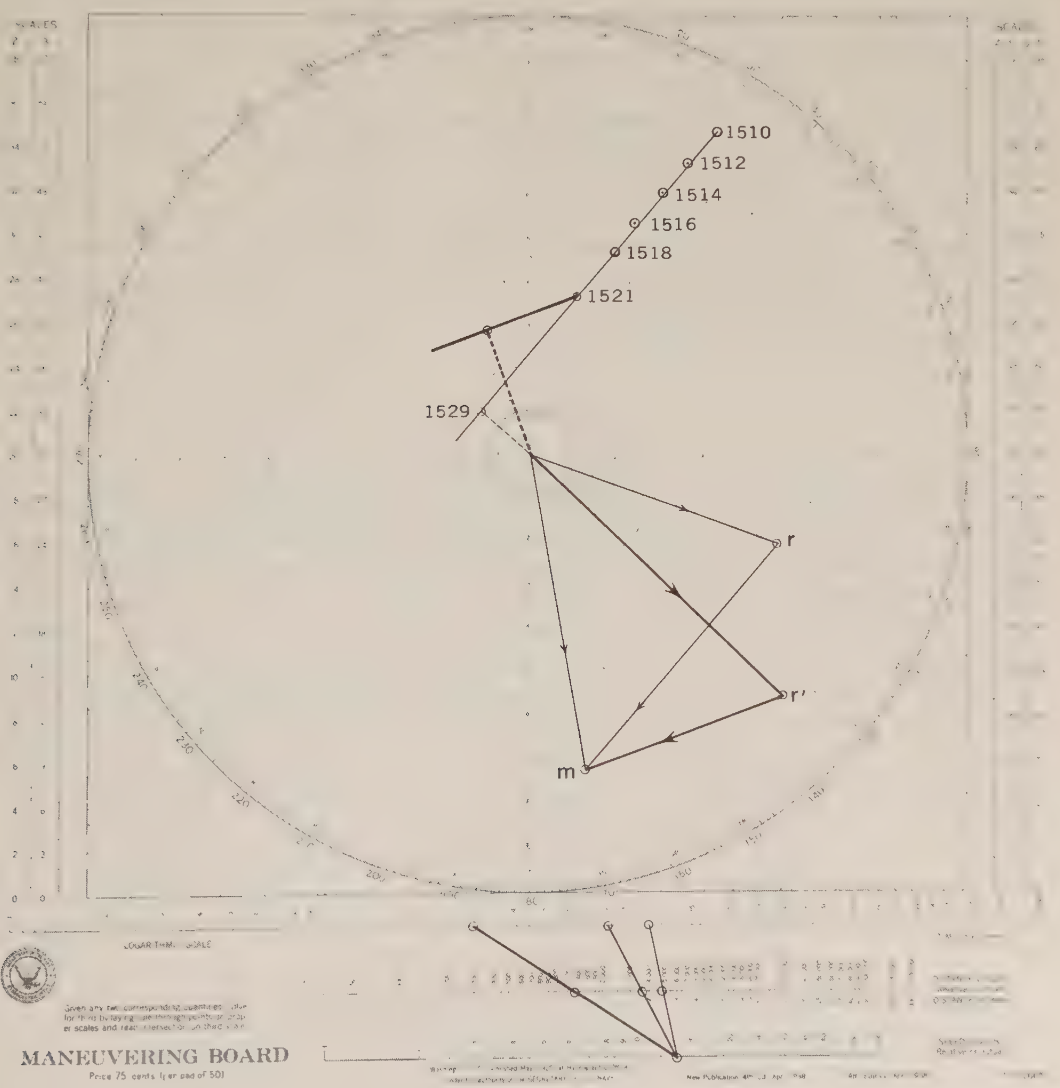


FIGURE 1212c.—Finding course at given speed to produce desired result, by relative plot.

1213. Shoran is a form of secondary radar (art. 1108) using two transponder beacons located ashore and a single indicator aboard ship to measure the distance from each beacon. By this means two distances are continually available, permitting rapid determination of position. Special charts are not needed, but where they have been provided, they show a number of concentric circles centered upon each beacon. Approximate positions can be plotted by inspection.

Shoran was developed during World War II to permit bombing through undercast. It provided such high accuracy that after the war it was further developed for possible use in surveying. Its use permitted measurement of distances over water or inaccessible terrain, thus providing means for more accurate positioning of offshore islands and other features inaccessible by previous methods.

The name **shoran** was derived from **sho**rt **ra**nge **n**avigation. A higher precision version used to meet the most exacting survey requirements is called **hiran,** from **hi**gh precision sho**ran.** Because of the high frequency used (230–310 mc), shoran is limited in range by the curvature of the earth. A lower frequency (1,900 kc) version permitting use by ships at distances of several hundred miles from shore was developed by the U. S. Coast and Geodetic Survey and called **electronic position indicator** (**EPI**). A British system similar to shoran, but with transmitters at the fixed ground stations and a transponder beacon at the mobile station, is known as **oboe.**

Since these systems provide simultaneous measurement of two distances, the *difference* in the two measurements might be used to provide a hyperbolic system (ch. XIII). However, for the use generally made of such equipment, the need for establishing hyperbolas would be a disadvantage.

1214. Pure-range Raydist measures distance electrically by phase comparison of beat frequency signals (art. 1108) resulting from transmission of signals at the two points between which the distance is to be measured. This method has had limited use, primarily in survey operations.

Hyperbolic Raydist is discussed in article 1311.

Problems

1204. The DR position of a ship is lat. 44°08ʹ.2 S, long. 62°56ʹ.9 W. A radio bearing is taken on Isla Leones Light Station, at lat. 45°03′03″ S, long. 65°36′33″ W. The uncorrected reading is 039°.5 relative, the ship being on true heading 205° at the moment the bearing is observed. The calibration table indicates a correction of (—) 2° should be applied.

Required.—The equivalent true rhumb line bearing.

Answer.—B 243°.5.

1212a. A ship on course 230°, speed 15 knots, obtains the following radar bearings and ranges of another vessel at the times indicated:

Time	*Bearing*	*Range*
0820	215°	24.0 mi.
0824	215°.5	23.4 mi.
0828	216°.5	22.8 mi.
0832	217°	22.3 mi.
0836	218°	21.7 mi.
0840	219°	21.2 mi.

Required.—(1) The nearest approach of the two vessels.

(2) The bearing of the other vessel at the point of nearest approach.

(3) Direction of relative movement (DRM).

(4) Speed of relative movement (SRM).

(5) Time of arrival at the point of nearest approach.

(6) Course and speed of the other vessel.

Answers.—(1) D 10.6 mi.; (2) B 278°; (3) DRM 009°; (4) SRM 9.7 kn.; (5) T 1033; (6) C 270°, S 10 kn.

1212b. At 0848 the other vessel of problem 1212a changes course to 034° and increases speed to 20 knots.

Required.—(1) The nearest approach of the two vessels if both maintain course and speed.

(2) New relative speed.

(3) Time of arrival at point of nearest approach.

Answers.—(1) D 0 (collision), (2) SRM 34.6 kn., (3) T 0923.

1212c. At 0858 the "own ship" of problem 1212b changes course to the *right,* coming to the course that will result in a nearest approach of five miles without changing speed.

Required.—(1) The new course.

(2) New relative speed.

(3) Time and distance at which "own ship" will be dead ahead of the other vessel.

(4) Time of arrival at the point of nearest approach if both vessels maintain course and speed.

(5) Bearing of the other vessel at nearest approach.

Answers.—(1) C 278°; (2) SRM 29.7 kn.; (3) T 0905, D 10.9 mi.; (4) T 0925; (5) B 152°.

References

Hall, J. S. *Radar Aids to Navigation.* M. I. T. Radiation Laboratory Series. New York, McGraw-Hill, 1947.

Penrose, H. E., and Boulding, R. S. H. *Principles and Practice of Radar.* New York, Van Nostrand, 1950.

Reintjes, J. F., and Coate, G. T. *Principles of Radar.* 3rd ed. M. I. T. Radar School. New York, McGraw-Hill, 1952.

Roberts, Arthur. *Radar Beacons.* M. I. T. Radiation Laboratory Series. New York, McGraw-Hill, 1947.

U.S. Department of the Navy. *Radar System Fundamentals.* NAVSHIPS 900,016 and 900,017. Washington, 1944.

U.S. Department of the Navy. *Radarman 3 & 2.* NAVPERS 10144. Washington, 1964.

U.S. Navy Hydrographic Office. *Maneuvering Board Manual.* H.O. Pub. No. 217. Washington, U.S. Govt. Print. Off., 1963.

U.S. Navy Hydrographic Office. *Radar Plotting Manual.* H.O. Pub. No. 257. Washington, U.S. Govt. Print. Off., 1964.

U. S. Office of Scientific Research and Development. *Electronic Navigation Systems.* National Defense Research Committee, Division 13. OSRD Report No. 6279. Cambridge, Harvard University, 1945.

Wylie, F. J. *The Use of Radar at Sea.* Institute of Navigation (British). London, Hollis and Carter, 1952.

LORAN-A COVERAGE DIAGRAM

* STANDBY STATUS

FIGURE 1302a.—LORAN-A coverage.

CHAPTER XIII

HYPERBOLIC SYSTEMS

1301. Introduction.—The principles of hyperbolic systems are discussed in article 1109. The present chapter describes the distinctive features of some of the more widely used systems.

1302. Loran is a hyperbolic system of navigation by which difference in distance from two fixed points on shore is determined by measurement of the time interval between reception of pulse-modulated (art. 1016), synchronized signals from transmitters at the two points. The name **loran** is derived from **lo**ng **ra**nge **n**avigation. Since it operates in the 1,750 to 1,950 kc frequency range, both ground waves and sky waves can be used to provide coverage over an extensive area with relatively few stations. Since ships do not transmit, they can use loran without breaking radio silence.

Usually, stations of a pair are located from 200 to 400 miles apart, although they may be as close as 100 miles or as far as 700 miles. At one time several station pairs separated by distances of 1,000 to 1,400 miles were operated. In this **SS loran,** sky waves only were used. Generally, a number of stations are located so as to form a **chain,** with all but the end stations in the group being "double pulsing." In most parts of the coverage area (fig. 1302a), signals can be received from at least two pairs of stations, thus making it possible to obtain a fix by loran alone.

FIGURE 1302b.—A typical train of loran signals from a single transmitted pulse.

The range at which signals are received varies considerably with kind of signal (ground wave or sky wave), route of the signal (over land or water), time of day, atmospheric noise level, geographic region, ionospheric conditions, and possible directional properties of the receiving antenna.

As a general rule, ground-wave coverage during the day extends to about 700 miles in the Atlantic and 800 miles in the Pacific. At night the range is about two-thirds this amount. During daylight hours, relatively few sky-wave signals are received, but at night, signals arrive by so many different paths that a **train** of signals may be received from a single transmitted pulse. Figure 1302b shows a typical scope appearance of such a train near the limit of ground-wave coverage. All of the signals are from a single transmitted pulse. One-hop-E signals are received to a maximum distance of about 1,400 miles. Curvature of the earth prevents their reception at greater distances regardless of power of the transmitter. Beyond this, strong signals may be received by multihop-E waves or by one or more reflections from the F layer. Because of relatively large uncertainties in the lengths of the paths of such signals, and the increased uncertainty of identification, loran tables and charts do not provide facilities for their use. The extending of lines to provide coverage for such signals is not recommended. Reception of reliable signals on some occasions is no assurance that those received at other times can be trusted. Typical variation in appearance of ground-wave and sky-wave signals with time of day is shown in figure 1302c.

The range at which a ground-wave signal can be received is much less if the path is across land than if it is across water. For this reason loran stations are located so that signal paths are as much as possible across water in the direction of greatest importance, and it is desirable that the base line also be across water. The retarding effect varies greatly with the type of land, and is somewhat less when the land is not adjacent to the transmitter. The paths of sky waves are so high that signal strengths are not noticeably affected by land unless it is within about 20 or 30 miles of the transmitter or receiver.

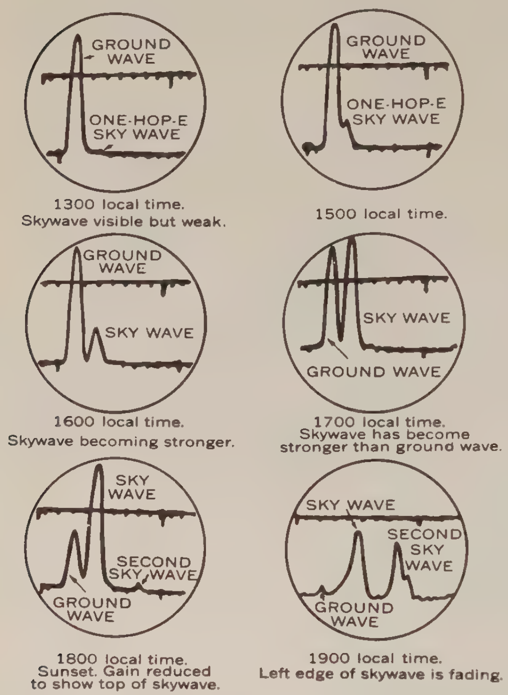


FIGURE 1302c.—Typical variation in appearance of signals with local time.

When the atmospheric noise level is high, signals which may otherwise be usable are lost in the clutter.

The areas near the base line extensions are excluded from the diagram of figure 1302a because of the relatively large error of position for a small error in the time difference reading.

Transmitting antennas are vertical, to avoid directional properties in the horizontal plane. Vertical receiving antennas are desirable for the same reason.

Pulse signals from each pair of stations are transmitted continually. Identification is by means of frequency and **pulse repetition rate** (**PRR**), sometimes called **pulse recurrence rate.** Frequency is identified by channel number, as follows:

Channel No.	*Frequency (kc)*	*Channel No.*	*Frequency (kc)*
1	1950	3	1900
2	1850		

The same frequency can be used for signals from a number of different station pairs, by varying the rate at which the signals are transmitted. Three **basic pulse repetition rates** are available, as follows:

Special (S) ---------------------------- 20 pulses per second,
Low (L) ---------------------------- 25 pulses per second,
High (H) ---------------------------- 33⅓ pulses per second.

The interval between the start of consecutive pulses is 50,000 μs for the special rate, 40,000 μs for the low rate, and 30,000 μs for the high rate. The special rate is retained for future use.

A further breakdown of repetition rate can be accomplished by varying the basic rate slightly. In practice, the difference between consecutive **specific pulse repetition rates** is 100 μs. The specific rates in use are identified by number, starting with 0 for the basic rate and increasing to 7 (eight rates), each higher number *increasing* slightly the rate at which signals are transmitted, and *decreasing* by 100 μs the interval between signals.

Thus, a total of 24 rates is available (if the special basic rate is used) for each of the four frequencies. The same rate may be used in areas so widely separated that

interference is not likely to occur. Each rate is identified by three characters. The first is a number identifying the frequency channel, the second a letter identifying the basic pulse repetition rate, and the third a number identifying the specific pulse repetition rate. Thus, the designation 1L7 indicates frequency channel 1, low basic pulse repetition rate, and specific pulse repetition rate 7. Stated differently, pulses are transmitted at intervals of 39,300 μs, on a frequency of 1950 kc. The term **rate,** implying the number of pulses per unit time, is now used for the full three-character designation, and even for the station pair, their signals, and the resulting hyperbolic lines of position and the tables and curves by which they are represented.

The system described in this article is sometimes called **standard loran** to distinguish it from a 100 kc or 180 kc experimental system called **low frequency loran,** which might provide ground-wave coverage over very great ranges but at reduced accuracy, and **loran-C,** an operational system of great range and high accuracy.

1303. The loran receiver-indicator.—The receiver used for loran signals is similar to that used in ordinary radio communication, except that it has no speaker. Signals are sent to an indicator consisting of a cathode ray tube (art. 1019) and the necessary timing circuits and controls. The major portion of the space needed for the equipment is occupied by the indicator.

On the face of the scope a visible line is produced by the spot of light formed at the point of impact of the moving beam of electrons. This line is divided into two parts, one above the other. The upper part is called the **A trace,** and the lower part the **B trace.** When the controls are set for a given rate, the length of the combined trace, in microseconds, is adjusted to the interval between beginning of pulses. Thus, if a reading is desired on rate 2H5, separate switches are set on 2, H, and 5 to control the frequency, basic pulse repetition rate, and specific pulse repetition rate, respectively. The combined length of the two traces is then 29,500 μs.

When the controls are thus set for a given rate, the signals of that rate appear as vertical deflections which remain stationary because a signal is received at the same part of each trace. Signals of the same basic pulse repetition rate, but of a different specific pulse repetition rate, appear to drift along the trace. Those of a *lower* rate drift to the right and those of a *higher* rate drift to the left. The greater the difference between the given rate and that of the signal, the faster the rate of drift.

The drift is due to the difference between the length of the combined trace and the time interval between the start of consecutive signals. Suppose the indicator is set for rate 2H3. The length of the combined A and B traces is 29,700 μs. A rate 2H2 signal is received at intervals of 29,800 μs. The spot of light forming the traces completes a cycle in 29,700 μs and moves an additional 100 μs before the next 2H2 signal is received. Each succeeding 2H2 signal appears 100 μs to the *right* (motion is left-to-right) of the previous one, and after 297 signals have been received (9 seconds), will have moved the entire length of both traces and returned to its original position. Signals of rate 2H5 will move to the *left* at twice the speed, completing the circuit in 4½ seconds. On some scopes a faint line called a **retrace** (fig. 1304a) can be seen connecting the ends of the two traces. This indicates the path of the spot of light in moving from the end of one trace to the beginning of the next, during a period of about 70 μs. These two periods of 70 μs are part of the total length of the combined trace.

Signals of the same frequency but another basic pulse repetition rate can be seen, but they appear as flickering signals called **ghosts,** which may drift faster than other signals. Each succeeding signal appears at a point 10,000 μs from the preceding one. Thus, every third or fourth signal may appear at about the same place, but the rate at any given place is so slow (approximately six or eight per second) that the deflection does not appear continuous. Since the spot of light is not deflected in most of its pas-

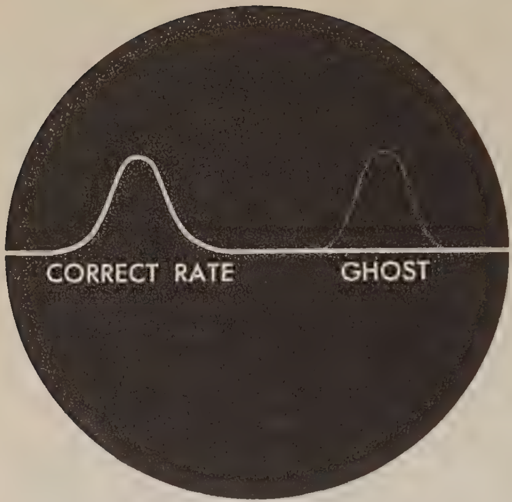


FIGURE 1303.—A signal of the correct basic pulse repetition rate, and a ghost.

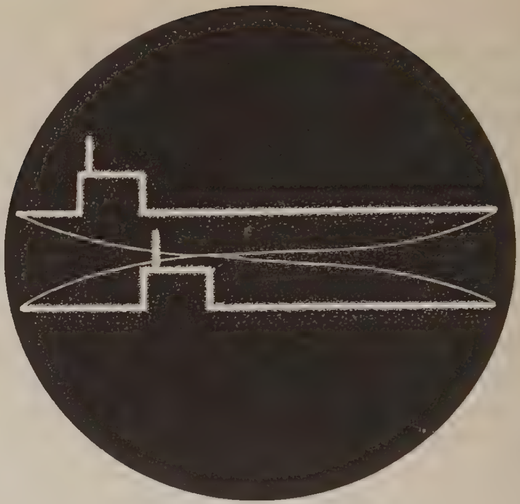


FIGURE 1304a.—The loran scope.

sages, the line appears continuous with the deflection superimposed on it. The appearance of a signal of the correct rate and a ghost is shown in figure 1303.

Strong signals from a frequency channel different from that to which the receiver is tuned may be received. This is called **spillover.** It can be detected by tuning to a different frequency. The frequency at which the signal appears strongest is the correct one.

1304. A loran reading.—Details of loran receiver-indicators differ, but the principles of all are the same. Near the start of each trace of a typical indicator, a portion of the visible line is raised to form **a pedestal,** as shown in figure 1304a. The pedestal of the A trace is fixed, but that of the B trace can be moved to nearly any location along the line.

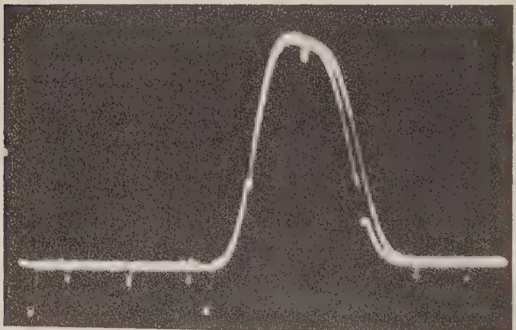


FIGURE 1304b.—Two loran signals properly matched.

When the entire cycle is shown, as in figure 1304a, a signal of 40 μs duration appears as a vertical line, as indicated. It can be moved right or left by means of a switch which temporarily shortens or lengthens the trace by a small amount, causing the signal to drift. After the correct signals have been identified, they are moved, if necessary, until the signal on the A trace is to the *left* of that on the B trace, and **mounted** near the left edge of the pedestal. The pedestal of the B trace is moved until the other signal is mounted near its left edge. By a series of successive magnifications, the left portions of the two pedestals are placed under each other and made to occupy the entire length of the original trace. The two traces are then brought to the same horizontal line, and one signal superimposed over the other, a process called **matching.** Figure 1304b is a photograph of a loran scope with signals properly matched, at greatest magnification. When the signals are matched, they occupy the same position with respect to the two pedestals. The reading is the distance (time separation) between the two pedestals, indicated by *downward* deflections of the traces, or by dial. At greatest amplification, the signals appear as in figure 1303 or figure 1304b.

A loran reading is influenced by three delays introduced in the transmission of the slave signal, as follows:

Half pulse repetition rate delay. A delay equal to half the interval between start of consecutive pulses is introduced so that one signal can be placed on each trace at approximately the same relative position. If this were the only delay, and a receiver were at some point on the center line, one signal would be directly under the other. Without the delay they would appear at the same place on the same trace. This delay is introduced for convenience in making a reading, and is not included in the reading.

Base line delay. If the half pulse repetition rate delay were the only one, readings would increase from zero along the center line to a maximum along each base line extension. Since both master and slave signals look alike, there would be no way of identifying them if the position of the receiver was sufficiently in doubt that it might be on either side of the center line. The base line delay, equal to the length of time needed for a signal to travel the length of the base line (6.18 μs times the length of the base line in nautical miles), causes the readings to increase from zero along the base line extension beyond the slave to a value of twice the base line delay along the base line extension beyond the master station. Because of this delay, the master signal can never appear to the *right* of the slave signal if one signal is placed on each trace.

Coding delay. With a reading near zero one might find difficulty at small scale in determining which signal was left and which was right. An additional delay of 500, 950, or 1,000 μs is provided to increase all readings by this amount. This increases the distance between the master signal and the slave signal when one is on each trace. This delay can be changed easily at the slave transmitter according to a prearranged schedule, to provide some measure of security in time of war.

The reading at any point is equal to 6.18 times the difference in distance (in nautical miles) of the receiver from the two stations (considered negative if nearer the slave), plus the base line delay, plus the coding delay. However, it is not necessary for the navigator to compute readings, because this is done electronically for the whole coverage area of each rate, and the information given in tables and special charts (art. 1307).

1305. Identification and use of various waves.—Travel times of ground waves and various sky waves differ, resulting in reception of a wave train (fig. 1302b) from a single transmitted signal. Since different readings are obtained with different combinations of signals, identification is important.

If a single wave is received, it is almost surely a ground wave. If a ground wave is received as part of a train of waves, it is the first or *left-hand* wave of the group. The position of the receiver relative to the transmitter is some guide. Within a few hundred miles of the station, the first signal is nearly always a ground wave, unless there is intervening land. Near the extreme limit of the coverage area, ground waves are not received. Between these limits is a **critical range** in which the first signal may be either a ground wave or sky wave. This critical range varies with time of day, location, and conditions, as discussed in article 1302. In general, it can be considered to be between about 600 and 900 miles by day, and between about 500 and 700 miles by night.

The *appearance* of the waves can be helpful in their identification. A ground wave is characteristically steady in shape and amplitude. Sky waves may at times appear as steady as ground waves, but such steadiness seldom lasts for more than a few minutes. Because of constant changes in the intensity (reflecting power) and height of the ionosphere (arts. 1007, 1008), and changing phase relationships, sky waves are subject to two characteristic fluctuations.

Changes in intensity, and changing phase relationships, cause changes in the strength of the reflected signal arriving at the receiver. This is called **fading.** It may

be a relatively small change in the amplitude of the signal, or it may be so severe that the signal disappears altogether for a short time. The complete cycle of fading from full strength to minimum and back to full strength may be completed in a period of less than a minute, or it may extend over several minutes.

Changes in height of the ionosphere cause the signal to move right or left along the trace. This motion is not apparent by itself, and equal changes in those parts of the ionosphere reflecting signals from the two transmitters has little effect on the reading. However, a change in intensity may result in shifting the reflecting surface to a higher level. When there are two or more such surfaces a short distance apart, **splitting** of the signal occurs, resulting in more than one crest of the same signal, close together. As the various reflecting surfaces change in intensity and height, the different crests move up and down relative to each other, and change their spacing.

It is good practice to watch the signals for several minutes before making a reading, to be sure of their identification and also to be sure that the *leading* edge of each is visible, for it is this edge, however weak, that should be matched. In a loran area the best practice is to make readings at regular intervals, at least once each hour. The changing appearance with time of day (fig. 1302c) should be helpful in identifying signals. Also, an inconsistency of one loran fix relative to such a series is an indication of possible error of identification.

In general, sky waves are steadier at greater distances from the transmitter, because reflection takes place over a larger area and local variations have less effect, and also because changes in height have less effect upon the length of the path. Therefore, the changes are less extreme. One-hop-E waves are usually steadier than multihop-E waves, or those reflected from the F layer (fig. 1302b). Changes in these signals are so great that intolerably large errors in readings may be introduced. For this reason and the uncertainty in identification of these waves, it is generally considered advisable to limit readings to ground waves and one-hop-E waves.

If a vessel is rolling heavily, all signals of a train may fade somewhat in synchronism with the roll. A weak ground-wave signal may flicker due to random noise signals which appear as continually-fluctuating **grass** on the trace. This momentary change is not easily confused with the slower fading.

For most rates, ground waves should always be matched if available. If ground waves are available from one station, but not from the other, the one-hop-E sky waves of both stations should be matched. In general, multihop-E waves and F waves should not be used. In some instances, where the base line is long, a correction table is provided for matching a ground wave from one station with a sky wave from the other. These corrections are given in the *Loran Tables*, H.O. Pub. No. 221, and in the *Catalog of Aeronautical Charts and Publications*, H.O. Pub. No. 1-V.

1306. Accuracy.—The accuracy of a loran fix depends upon the accuracy of the individual lines of position, and the angle at which the lines intersect (art. 906). The accuracy of individual lines of position depends upon the following factors:

Synchronization of signals. Transmission of loran signals is continuously monitored. Normally, the timing is correct to a fraction of one microsecond, but if the signals get out of synchronization by as much as two microseconds (five microseconds for rate 1L4), either the master or slave signals, or both, are made to **blink** to warn the user of the situation, so that readings on this rate can be avoided until the synchronization is restored, usually in a matter of minutes. Blinking is the shifting of signals right and left about 1,000 microseconds, at intervals of two seconds.

Position relative to transmitting stations. Accuracy is related to the spacing between consecutive lines of position separated by a constant difference of reading, as every microsecond. Lines are most closely spaced, giving highest accuracy, along the base

line between the stations, where an error of one microsecond in the reading produces an error of 0.081 mile, or 492 feet. From this the lines of position fan out, as shown in figure 1109. Near the base line extensions, an error of one microsecond in the reading produces an error of several miles in position. Any ground-wave reading within 25 μs of those of the base line extensions, or any sky-wave reading within 200 μs of those along these lines, should be considered of doubtful value.

Uncertainty in travel time of signal. The time needed for a signal to travel from the transmitter to the receiver depends upon the speed and distance. The speed is so nearly constant that the slight variations involved do not introduce a significant error. The distance between two points, however, depends upon the path followed by the wave. Ground waves follow the curvature of the earth with little variation, so that any error introduced by variations in the path is negligible. This is not true, however, of sky waves. Continual changes in the height and intensity of the ionosphere, as well as tilting of it from the horizontal, produce changes in the length of the path of the radio signal. The increased length of the sky-wave path over the ground-wave path *decreases* with greater distance from the transmitter. Along the center line, where the distance from the two transmitters is the same, the time *difference* is the same for sky waves as for ground waves. At other places, signals from one station are delayed more than those from the other. A **sky-wave correction** is provided in the loran tables and on the loran charts to convert a sky-wave reading to the equivalent ground-wave reading. At distances of 800 miles or more, carefully made sky-wave readings have an average error of about two microseconds. The error increases as the stations are approached, reaching an average value of about seven microseconds at a distance of 250 miles from one of the transmitters. This increased error is partly offset by closer spacing of the lines of position. However, since individual errors can be more than twice the average, the use of sky waves is not generally recommended within 250 miles of either station, and corrections for these areas are not usually tabulated.

Skill in making a reading. The principal source of error in making a reading is in identifying the signals. Patience and judgment are needed to avoid an error due to use of the wrong wave or failure to detect the true leading edge. With a reasonable signal-to-noise ratio, a careful operator should be able to match signals and read the indicator with an error not to exceed one microsecond. With patience, even very weak signals can be matched with an error of not more than a few microseconds.

Alignment of the indicator. Instructions for checking the "alignment" (adjustment) of the indicator are included in the instruction manual provided with each loran receiver-indicator. If the alignment is incorrect, errors may be introduced in the readings.

Incorrect location of transmitters. Computations are made for carefully determined positions of transmitters. However, where isolated stations require independent position determinations, the *relative* positions of the two stations may not be correct, however carefully determined, because of deflection of the vertical (art. 1610). When errors are established through usage, correction chartlets are provided in the loran tables and on loran charts. If the position of one station is found to be in error, the corrections are applicable in radial sectors around that station. If the positions of both stations are incorrect, the pattern is more involved.

Errors in loran tables and charts. Errors due to imperfections in tables and charts are negligible.

Plotting errors. Plotting of loran lines of position requires the same care as plotting of other navigational information if accurate results are to be obtained. For maximum accuracy, a large scale should be used.

1307. Loran lines of position.—Computation of the coordinates of points along various loran lines of position is performed electronically, allowance being made for

the spheroidal shape of the earth. The results are published in H.O. Pub. No. 221, *Loran Tables*. Several rates may be given in each volume, although a change is being made to publication of each rate in a separate pamphlet. From these computations, loran charts are prepared showing the lines of position at suitable intervals. Aeronautical loran charts are available for the entire coverage area, but relatively few nautical loran charts have been published. In areas where nautical loran charts are not available, either the tables or aeronautical charts can be used and the information transferred to the nautical chart.

The loran tables for each rate consist of a small-scale chartlet showing the pattern of the loran lines of position, and any corrections due to incorrect locations of the stations, a sky-wave correction table for one-hop-E waves, and the principal table giving coordinates of points on the lines of position. This table is entered with the loran reading in microseconds, and the latitude *or* longitude. For a line running in a generally north-south direction, the table is entered with the latitude, and the corresponding longitude is taken from the table. For an east-west line, the table is entered with longitude, and latitude is taken from the table. Two such points are thus determined and plotted, usually one on each side of the dead reckoning position. The straight line connecting them is an approximation of a small part of the line of position. Latitude and longitude are given at intervals of whole degrees, half degrees, or quarter degrees, depending upon the degree of curvature of the line. A separate column is given for each tabulated reading, at suitable intervals. An auxiliary tabulation labeled Δ (delta) gives the change in longitude or latitude (to 0ʹ.01) for a one-microsecond change in the reading. The main table should be entered with the *nearest* reading. If interpolation is toward a *smaller* reading, the printed sign of Δ should be reversed. Sample pages of a loran table are given in appendix BB.

Tabulated readings are for ground waves. Sky-wave readings are corrected to the equivalent ground-wave readings before entering the tables. A ground-wave reading is designated T_G, and a sky-wave reading T_S. If a ground wave is matched with a sky wave, the reading is labeled T_{GS} if the ground wave is from the master station, and T_{SG} if from the slave station. A line of position may appropriately be labeled with the time above the line and the identification below the line. It is good practice to give full identification, as 2H3 T_G 2154 or 1L0 T_S 1893 (T_G 1891).

Example 1.—The 1900 DR position of a ship is lat. 42°48ʹ.3 N, long. 62°28ʹ.3 W. About this time loran readings are obtained, as follows:

1859	1H4	T_S 6258
1900	1H2	T_G 2229

Required.—The 1900 fix, using appendix BB.

Solution.—Enter the sky-wave correction table of 1H4 with the dead reckoning position, and find the correction, (+)37, by double interpolation (art. P3). The equivalent ground-wave reading is 6258+37=6295. Enter the 6300 column of the 1H4 table, with the following results:

Long.	*Tab. lat.*	Δ	*Corr.*	*Lat.*
62° W	43°02ʹ.0 N	(+)56	(−)2ʹ.8	42°59ʹ.2 N
63° W	42°43ʹ.1 N	(+)51	(−)2ʹ.6	42°40ʹ.5 N

Next, enter the 2220 column of the 1H2 table, with the following results:

Lat.	*Tab. long.*	Δ	*Corr.*	*Long.*
42°30ʹ N	62°13ʹ.9 W	(−)18	(−)1ʹ.6	62°12ʹ.3 W
43°00ʹ N	62°33ʹ.9 W	(−)16	(−)1ʹ.4	62°32ʹ.5 W

Plot the two points of each line of position, and draw and label the lines. The common intersection of the two lines is the required fix, as shown in figure 1307a.

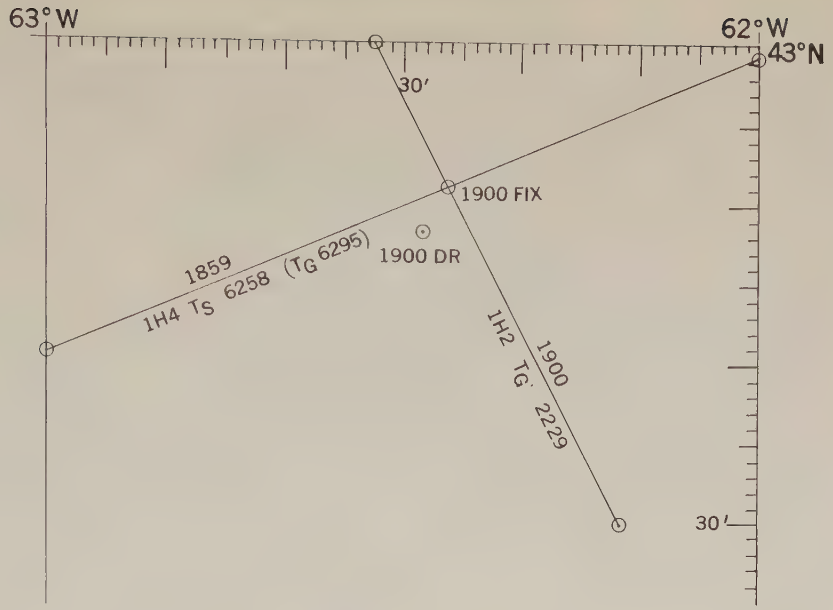


FIGURE 1307a.—A loran fix by table and plotting sheet.

Answer.—1900 fix: L 42°51.′0 N, λ 62°26.′2 W.

It is good practice to watch the scope for a few minutes before making a reading to be sure of correct identification of signals. If this is done for all rates before a reading is made, and sky-wave readings are made first, the intervals between readings can be kept to a minimum, and a skillful operator can often obtain two or three readings, over such a short period of time that the run between them can be ignored. However, where necessary, loran lines of position should be advanced or retired in the same manner as other lines of position (art. 908). If all readings are made within an interval of a few minutes, as customary, the position is considered a fix, rather than a running fix, following the practice of celestial navigation (art. 1707) rather than that of piloting (art. 909).

Figure 1307b is a reproduction, at half scale, of a small part of Hydrographic Office loran chart VRL–201. This small scale was chosen for illustration because it shows the pattern of loran lines in an area that is not congested by a large number of rates. A larger scale is recommended for marine navigation.

The plotted lines are for ground-wave readings. The small numbers near the intersections of printed meridians and parallels are one-hop-E sky-wave corrections at the intersections. On older charts the rate to which each applies is indicated both by color and by superscript. Italic type is used for high basic pulse repetition rate sky-wave corrections, and roman type for the low rate. On newer charts the rate is indicated in full, and both the rate indication and correction are printed in black (as 2L7 + 09), as shown in figure 1307b.

Eye interpolation can be used to locate lines between those printed. Graphs to facilitate such interpolation have been devised. They are available on a card published by the U. S. Navy Hydrographic Office as H.O. Misc. 11,691, and on some loran charts. When the correct position has been located, a short line is drawn parallel to the printed lines. The common intersection of the various lines of position, advanced or retired as necessary, is the fix.

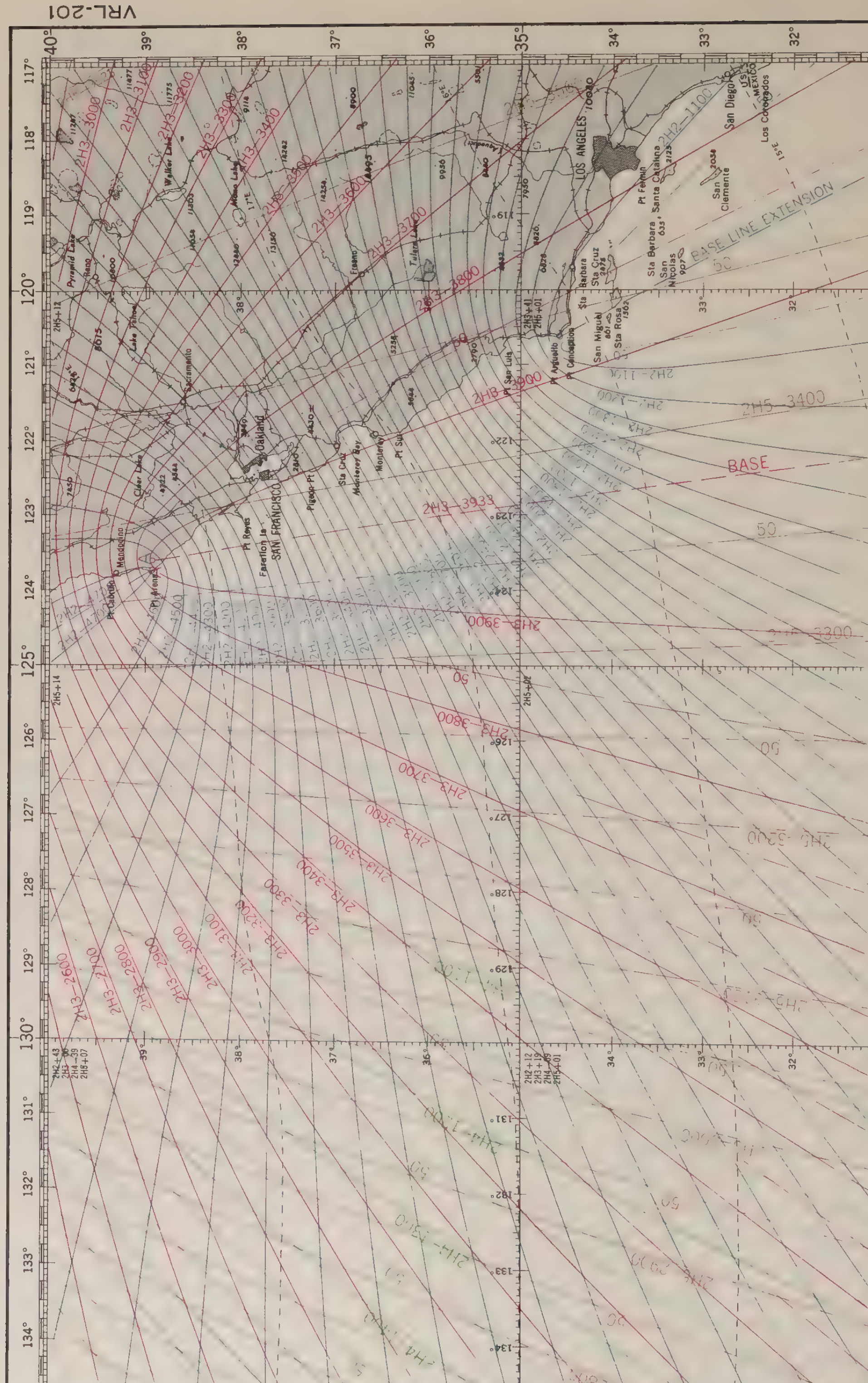


FIGURE 1307b.—Part of Hydrographic Office loran chart VRL-201, reduced 50%.

Example 2.—The 0600 DR position of a ship is lat. 39°06′ N, long. 126°41′ W. About this time loran readings are made in quick succession, as follows:

0559 2H5 T_S 3205
0600 2H2 T_G 4436
0601 2H3 T_G 3225.

Required.—The 0600 fix, using figure 1307b.

Solution.—By interpolation, find the sky-wave correction for the 2H5 reading. This is (+)10 μs, making the equivalent ground-wave reading 3205+10=3215 μs.

Locate the three readings by eye interpolation and draw short lines of position. At their common intersection, read the latitude and longitude.

Answer.—0600 fix: L 38°57′ N, λ 126°20′ W.

Do not expect high accuracy at such small scale.

Where a number of rates are available, only the three or four most useful ones may be shown on the chart. Thus, all or part of useful rates in an area may be omitted. Full information on the reliable coverage areas of all rates is included in the tables.

1308. Gee is a British hyperbolic navigation system in many respects resembling loran (arts. 1302–1307). In both systems the difference of the distances from two transmitters is determined by measurement of the time interval between reception of synchronized pulse-modulated signals (art. 1016). Gee operates in the 20–85 mc frequency range, and is therefore limited essentially to line-of-sight distances. However, refraction and ducting (art. 1006) sometimes extend the range somewhat, and sky waves are occasionally available. Because of this line-of-sight feature, the system is used largely by aircraft. At a height of 30,000 feet the operational range is considered to be about 400 miles.

Transmitting stations are arranged in groups of four, each group being considered a chain. One of the four is a master station controlling synchronization of the group. All stations of a chain operate on the same frequency. Pulses are two to ten microseconds in length. The master transmits at a pulse repetition rate of 500 per second. Two of the slaves transmit at a rate of 250 pulses per second, being synchronized with alternate pulses from the master. The third slave transmits at the rate of $\frac{500}{3}$ per second, being synchronized with each third pulse from the master. Signals from the third slave, and each alternate one from the master, consist of two pulses with a very short interval between them, the two being considered a double pulse constituting a single unit in the pulse repetition rate. Assuming a difference of 700 μs between reception of the master and synchronized slave signals, the sequence of transmitted signals would be as shown in figure 1308a. The spacing of signals at the receiver would depend upon its position relative to the transmitters.

On the scope of the indicator the trace is divided into two parts, as in loran, each part being 2,000 μs in length. When the single-pulse master signal is placed at the left part of the upper trace, the other signals might appear as shown in figure 1308b. The first part of the double-pulse master signal would be directly below the single-pulse master signal. The second part of this double-pulse signal is called a **ghost,** and is used only to identify that master signal used with the second slave. It therefore serves as identification of the first two slaves, which are *downward* deflections on small steps serving the same function as the pedestals of loran (art. 1304).

Since the first two slaves transmit at *half* the rate of the master, one appears on each trace. Each can be matched with the master signal with which it is synchronized, permitting two readings to be made with a single setting. Two magnifications are

provided, one of the **strobe** (the pedestal-like step) and the other of the central portion of the strobe. At greatest magnification, correctly matched signals appear as shown in figure 1308c. The third slave appears alternately on the two traces, but at such a rate that it seems continuous at both places. It is used only when a check is needed on the position determined by the first two slaves, or when these do not provide a reliable fix.

In the gee system, base lines are about 70 to 80 miles long, the appropriate length for the coverage area. Gee is considered a medium-base-line system. Readings

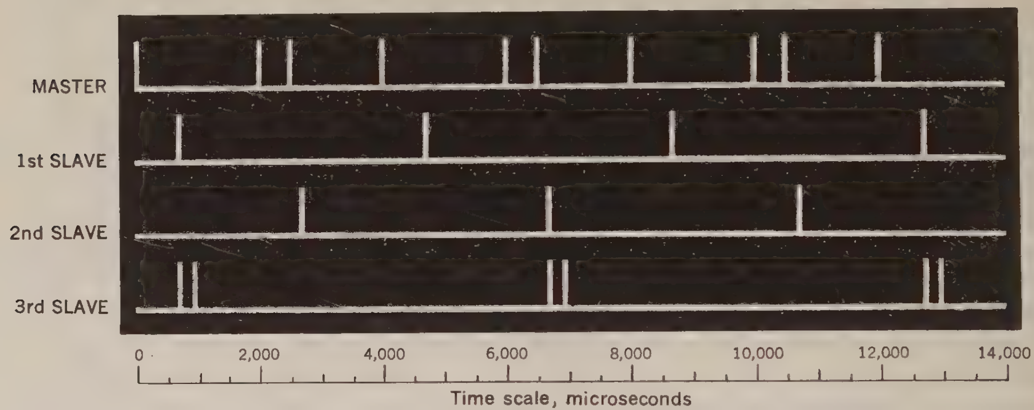


FIGURE 1308a.—Typical sequence of gee signals.

FIGURE 1308b.—Typical appearance of signals on a gee scope.

FIGURE 1308c.—Gee signals correctly matched.

are based upon direct waves. Under good conditions readings can be made with an error not exceeding 2/3 microsecond. As in other hyperbolic systems, maximum accuracy of position occurs along the base line, where a 2/3 microsecond time difference represents a distance of 0.054 mile, or 328 feet.

The Germans at one time used gee under the title **hyperbol.**

1309. Decca is a British hyperbolic navigation system using phase comparison for determining difference of distances from the transmitters. Each chain consists of one master and three slaves. In the ideal installation the slaves are equally spaced around the circumference of a circle 70 to 80 miles in *radius*, with the master at the center.

Each station transmits a continuous wave at a different frequency, the four fre-

quencies for a chain being in the ratio 5, 6, 8, and 9, and the entire group being in the 70–130 kc band. In a typical installation the master uses 85 kc and the slaves 70.833, 113.333, and 127.500 kc. For purposes of identification, these slaves are designated purple, red, and green, respectively.

The receiving unit consists of four receivers, one for each frequency, and circuits for comparing the phase of each slave signal with that of the master signal. If the signals are in phase at the time of transmission, they will also be in phase along the center line of each pair of transmitters. If a receiving unit were at the intersection of the center line and the base line, zero phase difference would be measured. If the unit then moved along the base line, the phase of signals from the station approached would decrease, and that of signals from the other station would increase. At a distance equal to *half* the wave length of the **comparison frequency** (the least common multiple of the two transmission frequencies), the signals would again be in phase, one signal being half a cycle less, and the other half a cycle more than at the center line. A line through all points having this phase relationship would be a hyperbola (assuming a plane surface). A series of such lines could be drawn, each representing a specific phase relationship.

Along the base line, the distance the receiving unit would travel from one in-phase condition to the next would be about 1157, 1446, and 1928 feet, respectively, for the three slaves operating at the frequencies stated above. The distance between in-phase hyperbolas becomes greater as the curves fan out from the base line. The area between any two consecutive in-phase hyperbolas is called a **lane.** Within each lane all phase-difference readings are available. The measurement is shown automatically on a dial called a **Decometer,** one being provided for each slave. If there were no way of determining in which lane the receiving unit was located, the position would need to be known to a high degree of accuracy to resolve the ambiguity. Lane identification is provided by periodic transmission of signals producing a coarser pattern. At short intervals each Decometer identifies the lane.

Each Decometer indicates hundredths of a lane width, and a series of Decca charts having hyperbolas printed in colors agreeing with the identification colors of the slaves permit determination of position by direct plot, as on loran charts (art. 1307). Two slaves provide a fix, the third serving as a check and permitting fixing in areas unfavorable to one of the slaves. To obtain a position, one has merely to read the Decometers and locate the common intersection of the two or three lines indicated. There is no manipulation of dials or matching of signals.

Since the reading is to a precision of 0.01 lane width, the theoretical accuracy is about 12, 14, and 19 feet, respectively, along the base lines between the master and each of the three slaves. The practical accuracy is considerably less, but still very good, the average error along the center line being about as follows, according to The Decca Navigator Company, Ltd.:

Nautical miles from master station	*Line of position errors in yards*	
	Day	*Night*
100	30	100
150	60	250
200	100	500
250	150	800

American tests indicate a somewhat greater error, with further increase with distance from the center line. The greater error by night is due to mingling of sky waves and ground waves. This factor reaches a maximum at a distance of about 350 miles. Signals of reasonable strength have been received at distances as great as 1,000 miles,

but the reliable day-and-night range is considered to be 240 miles. Even if good signals are received at greater distances, good fixes are not available because of the small angle of intersection of the lines of position, unless readings are taken from different chains.

Decca coverage extends over much of western Europe and parts of eastern Canada, the Persian Gulf, and the Bay of Bengal.

1310. Lorac is a hyperbolic system using phase comparison of beat frequencies (art. 1108) to measure difference of distances from transmitters. Each chain consists of a central station and two side stations. Some installations also have an additional station called the reference station. These stations provide two families of curves by which position can be determined.

The continuous-wave signals from the central station and one of the side stations are received at the second side station and also at the vessel. At each receiver, the two signals are combined to obtain a beat frequency signal in the audio frequency range (art. 1003). At the second side station, the beat frequency signal is used to modulate (art. 1016) the carrier wave of the transmitter at that station. This transmitter then sends a phase wave signal to the vessel. This received phase wave signal is compared with another phase wave signal produced at the vessel. The signal produced at the vessel is obtained from signals received from the central station and the first side station. The phase difference of these two phase wave signals varies with the position of the vessel, and depends on the difference in distances from the central station and the first side station. Thus, phase differences determine a family of hyperbolas. A second family of hyperbolas is produced similarly but from signals originated from the central station and the second side station.

As in Decca (art. 1309), the readings appear automatically and continuously on dials. Charts showing the lines of position are needed. Because of the frequencies used, in the 1,700 to 2,500 kc region, the lanes are very narrow, providing accuracies of the order of three feet along the base lines. However, the system does not provide a method of lane identification. It is intended primarily for use in surveying, where the survey vessel starts from a known position. *Changes* in lane are indicated automatically.

The name **Lorac** is derived from **lo**ng **r**ange **ac**curacy. It is intended for use for distances up to 100 to 150 miles by day and 75 to 100 miles by night. Accordingly, the base lines are about 35 miles long. The intended distances are considered long range for the surveying accuracy claimed.

1311. Hyperbolic Raydist is basically similar to Lorac (art. 1310), but differs in several respects. Raydist places one of the transmitters at the mobile station (the vessel), and uses frequency modulation (art. 1016) for relaying the audible signal. Indication can be provided at either the mobile station or one of the fixed stations, but a limited number of mobile stations can be accommodated simultaneously. The frequency range is 1,600 to 2,500 kc, although Raydist can also be used in the 100–150 kc and 30–40 mc regions. An accuracy of 25 feet has been attained at 50 miles. Under favorable conditions readings can be obtained at distances as great as 250 miles. As with Lorac, hyperbolic Raydist does not provide lane identification.

Pure-range Raydist is discussed in article 1214.

1312. Consol (art. 1206) is a short-base-line hyperbolic system providing a rotating pattern of dot-dash signals. Because of the short base line and the long ranges at which the signals are available, the system is used as a directional one.

1313. Sofar is a hyperbolic system using sound transmissions in the ocean. The speed of sound in sea water generally decreases with depth until a minimum is reached, below which the speed increases (art. 3503). The existence of such a minimum-speed

level permits transmission of sound over great distances, a range of more than 3,000 miles having been achieved. If a sound, as that of an explosion, is created in or near the minimum-speed level, and microphones are located at the correct depth, a single signal may be received at several widely spaced listening stations. The differences in time of reception at these stations define hyperbolas. The origin of the sound can be located by reference to a chart on which the hyperbolas have been printed. The name **sofar** is derived from **so**und **f**ixing **a**nd **r**anging.

Sofar was developed by the U. S. Navy, for possible use in search and rescue operations. One set of four sofar listening stations has been installed in the California-Hawaii area for experimental purposes. A small depth charge, dropped overboard by the craft, explodes at the proper depth. The time of reception of the signal at the four stations is automatically timed to an accuracy of about 0.1 second. Comparison of the times at the various stations provides readings which can be translated into position by reference to a sofar chart. Location of the stations is such that position can be determined within an elongated area about one and one-half miles wide and four miles long. About 20 minutes are needed for the sound to travel 1,000 miles. At this distance, a signal is heard over a period of about 12 seconds, gradually building up intensity to a maximum, with a sharp cut-off announcing the arrival of the direct signal, the instant of time measurement.

An intervening obstruction such as an island or seamount produces "shadows" which interfere with reception of sofar signals. One reason for selection of the California-Hawaii area as the site for the first installation is its freedom from obstructions.

Rafos ("sofar" spelled backwards) is the reverse of sofar, sound signals being produced at the shore stations and the differences in reception times being determined at the vessel, using a microphone lowered to the correct depth.

1314. Omega is an experimental, very low frequency, hyperbolic navigation system. The predicted fix accuracy of this system is 0.5 mile or better at a range of 5,000 miles. The basic system will have lanes approximately 15 miles wide over most of the coverage area. Lane identification can be provided by the use of a second frequency.

Problems

1307a. The 0630 DR position of a ship is lat. 42°52′.2 N, long. 62°28′.5 W. The ship is on course 330°, speed 20 knots. About this time loran readings are obtained, as follows:

0621	1H4	T_S	6254
0630	1H2	T_G	2193

Required.—The 0630 fix, using appendix BB. (Plot can be made directly on fig. 1307a.)

Answer.—0630 fix: L 42°50′.3 N, λ 62°32′.0 W.

1307b. The 2000 DR position of a ship is lat. 35°26′ N, long. 125°29′ W. About this time loran readings are made in quick succession, as follows:

1958	2H5	T_S	3115
2000	2H2	T_G	3356
2002	2H3	T_G	3523

Required.—The 2000 fix, using figure 1307b.

Answer.—2000 fix: L 35°29′ N, λ 128°25′ W.

References

Burmister, C. A. "Shoran in Hydrographic Surveying." *International Hydrographic Review.* Vol. XXIV. Monaco, International Hydrographic Bureau, 1947.

Decca Navigator Company, Ltd. *Decca Navigator System.* London, 1948.

Hall, J. S. *Radar Aids to Navigation.* M. I. T. Radiation Laboratory Series. New York, McGraw-Hill, 1947.

Pierce, J. A., McKenzie, A. A., and Woodward, R. H. *Loran.* M. I. T. Radiation Laboratory Series. New York, McGraw-Hill, 1948.

Seismograph Service Corporation. *Lorac.* Tulsa, 1948.

Sonnenberg, G. J. *Radar and Electronic Navigation.* London, Newnes, 1951

U. S. Office of Scientific Research and Development. *Electronic Navigation Systems.* National Defense Research Committee, Division 13. OSRD Report No. 6279. Cambridge, Harvard University, 1945.

PART FOUR

CELESTIAL NAVIGATION

PART FOUR

CELESTIAL NAVIGATION

CHAPTER XIV

NAVIGATIONAL ASTRONOMY

Preliminary Considerations

1401. Introduction.—**Astronomy** is that science which deals with the size, constitution, motions, relative positions, etc., of celestial bodies. **Navigational astronomy** is that part of astronomy of direct use to a navigator, comprising principally celestial coordinates, time, and the apparent motions of celestial bodies with respect to the earth. Sometimes it is called **nautical astronomy.**

1402. Apparent and absolute motions.—All celestial bodies of which man has knowledge are in motion. Since the earth itself is one of these moving bodies, the motion of other bodies, as seen by an observer on the earth, is **apparent motion.** If the earth were stationary in space, any change in the position of another body, relative to the earth, would be due only to the motion of that body. This would be **absolute motion,** or motion relative to a fixed point. But since it has been impossible to identify a fixed point in space, all motion of which man is aware is apparent, made up of a combination of the movement of the other body and the motions of the observer. A person without suitable instruments is not aware of motion in the line of sight, and therefore only motions across the line of sight are observed.

Since all motion is relative, one should be cognizant of the position of the observer when motions are discussed. When one speaks of planets following their orbits around the sun, he is placing the observer at some distant point in space, usually one of the poles of the ecliptic (art. 1419). When he speaks of a body rising or setting, the observer is on the earth. If he refers to a particular rising or setting, he must locate the observer at a particular point on the earth, since the setting sun for one observer may be the rising sun for another. At the same time it may be crossing the meridian of a third observer.

1403. The celestial sphere.—As one looks at the sky on a dark night, he is not aware of the differences in the distances to the various celestial bodies. They might easily be imagined as being equally distant from the earth, all located on the inner surface of a vast hollow sphere of infinite radius, with the earth at its center. This is the **celestial sphere** (fig. 1403). For most purposes of navigation it can be considered an actuality. Since the navigator is concerned primarily with apparent motion for an observer on the earth, this geocentric universe of Ptolemy (art. 121) is a useful concept. While the motions of various bodies relative to each other are important to the astronomer who predicts future positions of celestial bodies, and perhaps to the navigational scientist who designs navigation tables, the navigator speaks of bodies rising, crossing the celestial meridian, and setting, as though these were absolute motions.

1404. Units of astronomical distance.—The distances between celestial bodies, even those within a single family such as the solar system, are so great that terrestrial units are unsatisfactory to express them. The units commonly used for astronomical distances are:

Astronomical unit (AU), the mean distance between the earth and the sun, approximately 92,900,000 statute miles. The astronomical unit is often used as a unit of measurement of distance within the solar system.

FIGURE 1403.—The celestial sphere.

Light-year, the distance light travels in one year. Since the speed of light is about 186,000 statute miles per *second* and there are about 31,600,000 seconds per year, the length of one light-year is about 5,880,000,000,000 (5.88×10^{12}) statute miles, or 63,300 astronomical units. The light-year is commonly used for expressing distances to the stars and galaxies. Alpha Centauri and its neighbor Proxima, generally considered the nearest stars, are 4.3 light-years away. Relatively few stars are less than 100 light-years away, and the most distant galaxies thus far observed are 1.6 billion light-years away. However, most navigational stars are relatively close. Light travels from the sun to the earth in about 8⅓ *minutes*, and from the moon to the earth in about 1¼ *seconds*.

Parsec, the distance at which the **heliocentric parallax** (difference in apparent position as viewed from the earth and the sun) is 1″. At this distance a star would appear to change its position 2″ among the distant stars, if observed from points 180° apart on the earth's orbit. The name is derived from the first letters of the words **par**allax and **sec**ond. One parsec is equal to about 3.26 light-years Hence, even the

nearest star is more than one parsec away. This unit is used to express distances to stars and galaxies.

The difficulty of illustrating astronomical distances and sizes is indicated by the fact that if the earth were represented by a circle one inch in diameter, the moon would be a circle one-fourth inch in diameter at a distance of five feet, the sun would be a circle nine feet in diameter at a distance of nearly a fifth of a mile, and Pluto would be a circle half an inch in diameter at a distance of about seven miles. The nearest star would be one-fifth the actual distance to the moon.

1405. Magnitude.—The relative brightness of celestial bodies is indicated by a scale of stellar magnitudes. In the *Almagest* (art. 121) Ptolemy divided the stars into six groups according to brightness, the 20 brightest being classified as of the first magnitude, and the dimmest being of the sixth magnitude. In modern times, when it became desirable to define more precisely the limits of magnitude, a first magnitude star was considered 100 times brighter than one of the sixth magnitude, the approximate value of Ptolemy's ratio. Since the fifth root (art. O9) of 100 is 2.512, this number is considered the **magnitude ratio.** A first magnitude star is 2.512 times as bright as a second magnitude star, which is 2.512 times as bright as a third magnitude star, etc. A second magnitude star is $2.512 \times 2.512 = 6.310$ times as bright as a fourth magnitude star. A first magnitude star is $2.512^{20} = 100^4 = 100{,}000{,}000$ times as bright as a star of the twenty-first magnitude, the dimmest that can be seen through the 200-inch telescope.

Brightness is normally tabulated to the nearest 0.1 magnitude, about the smallest change that can be detected by the unaided eye of a trained observer. In the *American Ephemeris and Nautical Almanac* it is given to the nearest 0.01 magnitude, for precise astronomical purposes. All stars of magnitude 1.50 or brighter are popularly called "first magnitude" stars. Those between 1.51 and 2.50 are called "second magnitude" stars, those between 2.51 and 3.50 are called "third magnitude" stars, etc. Sirius, the brightest star, has a magnitude of (—) 1.6. The only other star with a negative magnitude is Canopus, (—) 0.9. At greatest brilliance Venus has a magnitude of about (—) 4.4. Mars, Jupiter, and Saturn are sometimes of negative magnitude. The full moon has a magnitude of about (—) 12.6, but varies somewhat. The magnitude of the sun is about (—) 26.7.

The Universe

1406. The solar system.—The **sun,** the most conspicuous celestial object in the sky, is the central body of the solar system. Associated with it are at least nine principal **planets,** of which the earth is one; a number of **satellites** accompanying some of the planets; thousands of **minor planets** or **asteroids;** multitudes of **comets;** and vast numbers of **meteors.**

1407. Motions of bodies of the solar system.—Astronomers distinguish between the two principal motions of celestial bodies, as follows: **rotation** is a spinning motion about an axis within the body, while **revolution** is the motion of a body in its elliptical **orbit** around another body, called its **primary.** For the satellites, the primary is a planet. For the planets and other bodies of the solar system, the primary is the sun. The entire solar system is held together by the gravitational force of the sun. The whole system revolves around the center of its galaxy (art. 1415) as a unit, and the galaxy is probably in motion relative to its neighboring galaxies. The motion of bodies of the solar system relative to surrounding stars is called **space motion.**

Rotation and revolution may be further classified as **synodic** or **sidereal.** During one synodic *rotation* the body makes one complete turn relative to the sun. On the earth it is called an **apparent solar day.** During one sidereal rotation the body makes

one complete turn relative to the stars. Because of motion of the body in its orbit, a sidereal rotation is either longer or shorter, by a small amount, than a synodic rotation. If both rotation and revolution are in the same direction (in the solar system they are both *east* for most bodies, that is, counterclockwise as seen from above the north pole) the sidereal rotation is shorter. During a synodic *revolution* a celestial body makes one trip around the sun, *as viewed from the earth.* Hence, the earth cannot have a synodic revolution. During a sidereal revolution, a celestial body makes one trip around its orbit with respect to the stars; to an observer on the celestial body, the sun would appear to make one trip around the celestial sphere, with respect to the stars. On the earth this is one year.

All of the planets are believed to be in rotation, although this point is in doubt in the case of Venus and, to a lesser extent, Mercury. The periods of rotation of these bodies have not been established because of the absence of visible surface markings of sufficient constancy to permit measurement. The period of Mercury has been established tentatively as 88 days. The rotation of all planets is from west to east, with the possible exception of Uranus (ū′rȧ·nŭs) (art. 1411).

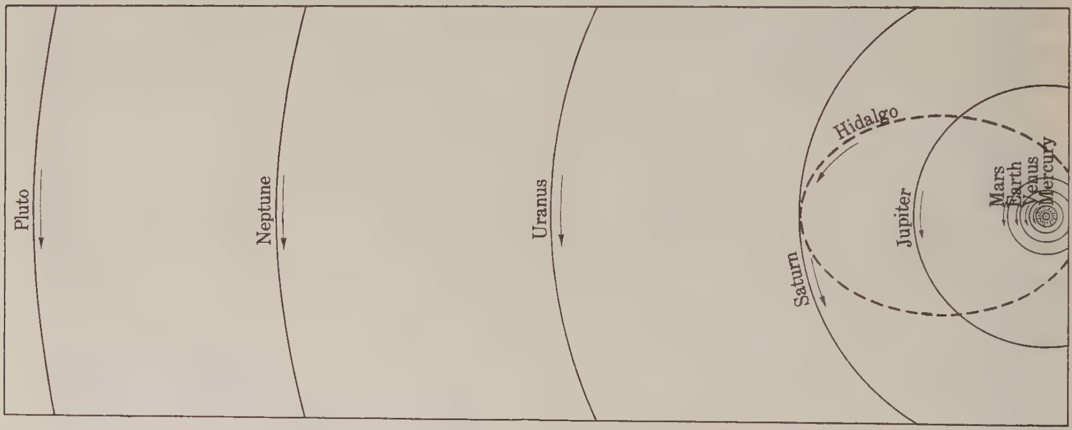


FIGURE 1407a.—Relative size of planetary orbits.

All of the planets revolve around the sun in nearly circular orbits. The flattening or **eccentricity** of the earth's orbit is only 0.017 (zero would be a circle). Some of the minor planets have orbits more eccentric than that of any principal planet (note the orbit of Hidalgo in fig. 1407a). The orbits of comets are highly eccentric. The orbits of all known planets except Pluto are in nearly the same plane, that of the ecliptic (art. 1419). The orbit of Pluto is inclined more than 17° to the ecliptic

The laws governing the motions of planets in their orbits were discovered by Johannes Kepler, and are now known as **Kepler's laws:**

1. *The orbits of the planets are ellipses, with the sun at a common focus.*

2. *The straight line joining the sun and a planet (the* **radius vector***) sweeps over equal areas in equal intervals of time.*

3. *The squares of the sidereal periods of any two planets are proportional to the cubes of their mean distances from the sun.*

In 1687 Isaac Newton stated three "laws of motion," which he believed were applicable to the planets. **Newton's laws of motion** are:

1. *Every body continues in a state of rest or of uniform motion in a straight line unless acted upon by an external force.*

2. *When a body is acted upon by an external force, its acceleration is directly proportional to that force, and inversely proportional to the mass of the body, and acceleration takes place in the direction in which the force acts.*

3. *To every action there is an equal and opposite reaction.*

From Kepler's laws and his own, Newton fashioned a single **universal law of gravitation**, which he believed applied to all bodies, although it was based upon observation within the solar system only:

Every particle of matter attracts every other particle with a force that varies directly as the product of their masses and inversely as the square of the distance between them.

According to these laws the planets remain in their orbits because of a balance of forces between the gravitational attraction of the sun and the tendency of the planet to continue in motion along a straight line. As a planet approaches closer to the sun, its gravitational attraction increases, but by Kepler's second law the speed increases, resulting in stronger centrifugal force. These laws (Newton's laws of motion) have been modified very slightly by Albert Einstein's **theory of relativity.**

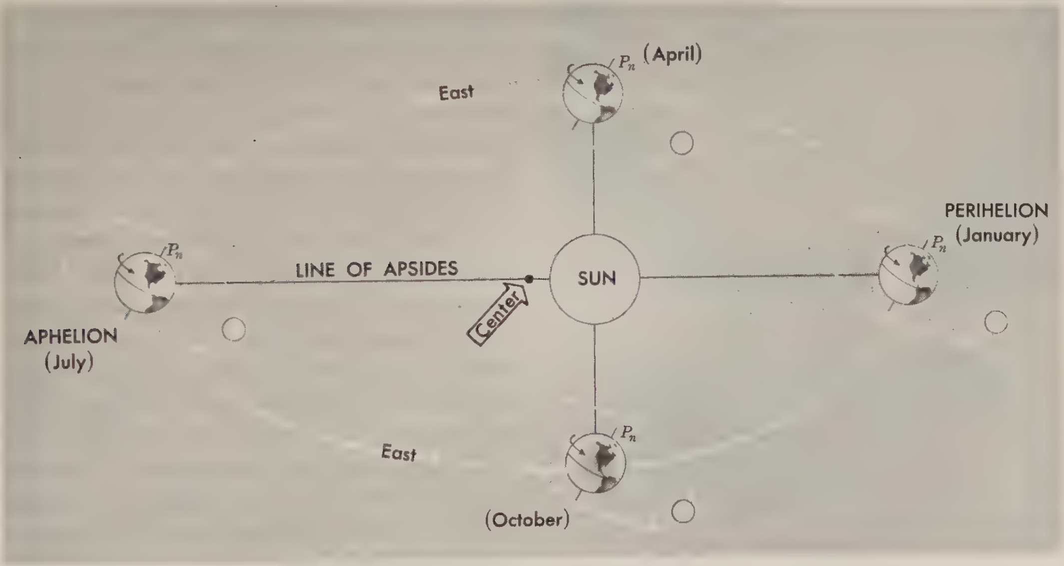


FIGURE 1407b.—Orbits of the earth and moon.

Both the sun and each body revolve about their common center of mass. Because of the preponderance of the mass of the sun over that of the individual planets, the common center of the sun and each planet except Jupiter lies within the sun. The common center of the combined mass of the solar system moves in and out of the sun.

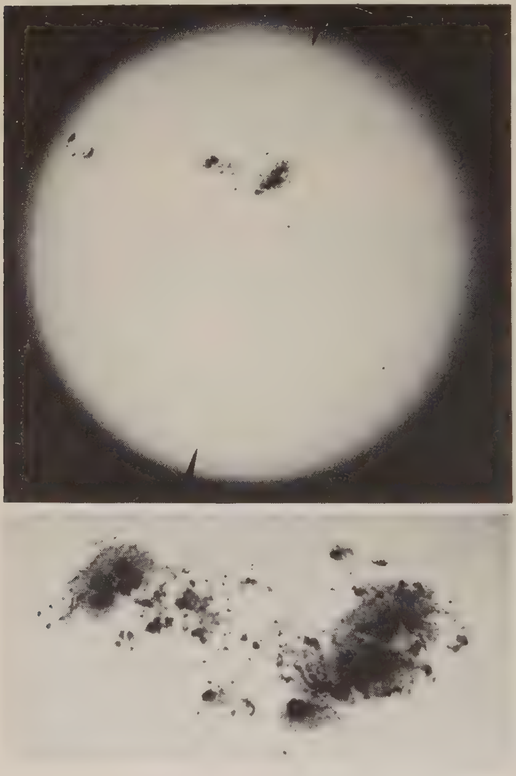
The various laws governing the orbits of planets apply equally well to the orbit of any body with respect to its primary.

In each planet's orbit that point nearest the sun is called the **perihelion.** That point farthest from the sun is called the **aphelion** (ă·fē′lē·ŏn). The line joining perihelion and aphelion is called the **line of apsides** (ăp′sĭ·dēz). In the orbit of the moon, that point nearest the earth is called the **perigee,** and that point farthest from the earth is called the **apogee.** Figure 1407b shows the orbit of the earth (with exaggerated eccentricity), and the orbit of the moon around the earth.

1408. The sun is the dominant member of the solar system because its mass is nearly a thousand times that of all other bodies of the solar system combined. It supplies heat and light to the entire system.

The diameter of the sun is about 866,000 miles. At the distance of the earth, varying between 91,300,000 and 94,500,000 miles, the visible diameter is about 32′. At the closest approach early in January the sun appears largest, being 32′.6 in diameter. Six months later the apparent diameter is 31′.5, the minimum.

Of the various physical features of the sun, one of particular interest is the appearance from time to time of **sun spots** on the surface (fig. 1408). These spots are apparently areas of cooler gas which have risen to the surface and appear dark in contrast to the hotter gases around them. In size they vary from perhaps 50,000 miles in diameter to the smallest spots that can be detected (a few hundred miles in diameter), and perhaps smaller. They generally appear in groups. At the start of each cycle of about 11 years the spots appear at a maximum distance of about 40° on each side of the solar equator. Succeeding spots of the cycle appear progressively closer to the solar equator, until a minimum solar latitude of 5° may be reached. The maximum number of sun spots occurs about midway in the cycle, when the spots are about 16° from the solar equator. The present cycle began in 1954, and should reach maximum activity late in 1959, with a new cycle beginning about 1965. Large sun spots can be seen without a telescope if the eyes are protected, as by the shade glasses of a sextant. Sun spots have magnetic properties. For one cycle all spots north of the solar equator are of positive polarity, and all those to the south are of negative polarity. During the next cycle, which may begin before the last spots of the old cycle have disappeared, the polarity is reversed. Sun spots are related to magnetic storms which adversely affect radio, including radio aids to navigation, on the earth. At such times the **auroras** (art. 2526) are particularly brilliant and widespread.



Courtesy of Mt. Wilson and Palomar Observatories.

FIGURE 1408.—Whole solar disk and an enlargement of the great spot group of April 7, 1947.

The sun rotates on its axis, the period of rotation varying from about 25 days at the solar equator to 34 days at the poles, but this fact has little or no navigational significance beyond its effect upon the changing positions of sun spots relative to the earth. The sun is moving approximately toward Vega at about 12 miles per second, or about two-thirds as fast as the earth moves in its orbit around the sun. The path of the sun toward Vega is called the **sun's way.** This is in addition to the motion of the sun around the center of its galaxy (art. 1415).

1409. Planets.—The principal bodies having nearly circular orbits around the sun are called **planets,** from a Greek word meaning "wandering." They were so named because they were observed to change position or "wander" among the "fixed stars"

which remained in about the same positions relative to each other. Because the sun and moon had a similar wandering motion, the ancients considered them planets, also.

Nine principal planets are known. In order of increasing distance from the sun, these are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto. Of these, only four are commonly used for celestial navigation. These are Venus, Mars, Jupiter, and Saturn, sometimes called the **navigational planets.** The two planets with orbits smaller than that of the earth are called **inferior planets,** and those with orbits larger than that of the earth are called **superior planets.** The four planets nearest the sun are sometimes called the **inner planets,** and the others the **outer planets.** Jupiter, Saturn, Uranus, and Neptune are so much larger than the others that they are sometimes classed as **major planets.** Neptune and Pluto are not visible to the unaided eye, and Uranus is barely so, being of the sixth magnitude.

The orbits of the many thousand tiny **minor planets** lie chiefly between the orbits of Mars and Jupiter.

Six of the planets are known to have satellites, a total of 31 having been discovered. Mercury, Venus, and Pluto have no known satellites.

Various items of general interest regarding the planets are given in appendix F.

1410. The earth as a planet.—In common with other planets, the earth rotates on its axis and revolves in its orbit around the sun. These actual motions (discussed in articles 1416 and 1417) are the principal source of the apparent motions of other celestial bodies. Also, the rotation of the earth results in a deflection of water and air currents to the right in the northern hemisphere and to the left in the southern hemisphere. Because of the earth's rotation, the high tides on the open sea lag behind the meridian transit of the moon.

For most navigational purposes, the earth can be considered a sphere, but, like the other planets, the earth is approximately an **oblate spheroid,** or **ellipsoid of revolution,** being flattened at the poles and bulged at the equator. Therefore, the polar diameter is less than the equatorial diameter, and the meridians are slightly elliptical, rather than circular. The dimensions of the earth are recomputed from time to time, as additional and more precise measurements become available. Since the earth is not *exactly* an ellipsoid, results differ slightly when equally precise and extensive measurements are made on different parts of the surface. Hence, different "spheroids" are used for mapping various parts of the earth. That used for charts of North America was computed by the English geodesist A. R. Clarke in 1866. However, since Clarke did not clearly define his units, the U. S. Coast and Geodetic Survey in 1880 considered it desirable to adopt standard values which probably added about 170 feet to the diameter computed by Clarke. In 1880, also, Clarke himself made a new estimate of the size and shape of the earth but this has not been adopted by the United States. Although the Clarke spheroid of 1866 is still used for charting North America, the International Spheroid, based upon work done by Hayford in 1909–10, is considered a slightly better approximation of the size and shape of the earth. According to these calculations, the dimensions of the earth are:

Equatorial diameter $(2a)$	$=7{,}926.694$ statute miles	$=6{,}888.110$ nautical miles
Polar diameter $(2b)$	$=7{,}900.004$ statute miles	$=6{,}864.918$ nautical miles
Mean diameter	$=7{,}917.797$ statute miles	$=6{,}880.379$ nautical miles
$2(a-b)$	$=\quad 26.690$ statute miles	$=\quad 23.192$ nautical miles

$$\text{Oblateness}=\frac{a-b}{a}=\frac{2(a-b)}{2a}=\frac{26.690}{7{,}926.694} \text{ or } \frac{23.192}{6{,}888.110}=\frac{1}{297}.$$

The mean diameter is the average of the polar diameter and two equatorial diameters perpendicular to each other (the three dimensions of the solid), or $\frac{2(2a+b)}{3}$.

Because of unequal distribution of mass near the surface of the earth, the direction of gravity is tilted slightly at various places. The amount of tilt is called **deflection of the vertical** (art. 1610). If the surface of the spheroid is altered so as to be everywhere perpendicular to the direction of gravity, the earth is considered a **geoid.**

The average density (ratio of mass of the earth to mass of an equal volume of water) is 5.517. This is greater than that of any other planet, as far as is known (the density of Pluto has not been determined). The total mass is about 6,600,000,000,000,000,000,000 (6.6×10^{21}) short tons. Virtually all statements regarding the interior of the earth are matters of conjecture, but it is believed that the density increases from about three at the surface to about ten at the center, where the temperature is believed to be close to 5,000° F. The earth is generally considered to be composed of a solid shell several hundred miles thick surrounding a molten interior, but there is some evidence to support the belief that it is solid throughout.

Since gravity acts approximately toward the center of the planet, the direction "up" varies with the observer, being nearly perpendicular to the spheroid at all places. In general, gravity increases with latitude, because both the distance from the center of the earth and the centrifugal force decrease.

One of the conditions considered essential to life on any celestial body is the existence of an **atmosphere.** Whether or not a body has an atmosphere depends at least partly upon its **velocity of escape,** or the speed the molecules of the gas making up the atmosphere must attain to overcome the force of gravity. The velocity of escape of the earth is about 6.94 statute miles per second, at the surface, and decreases slowly with distance from the earth. Since the molecules of the earth's atmosphere rarely exceed this value for a sufficiently long time to escape, the earth has lost relatively little of its atmosphere. The velocity of escape is approximated by a space ship leaving the earth.

The total mass of air surrounding the earth is 5,800,000,000,000,000 (5.8×10^{15}) short tons. This is less than a millionth part of the mass of the entire earth. The average pressure exerted by this envelope of air is 14.696 pounds per square inch. The pressure decreases rapidly with altitude. About half of the atmosphere is within 18,000 feet (about 3.5 miles) of the surface. Breathing begins to be labored at 10,000 feet. Twilight extends to about 50 miles. Meteors generally appear at about 50 miles. Auroral phenomena may be as low as 40 miles, and may extend as high as above 500 miles (fig. 1410).

The lower portion of the atmosphere, the troposphere, is composed of the following elements, in addition to dust and water vapor:

Element	*Percent*
Nitrogen	78.08
Oxygen	20.95
Argon	0.93
Carbon	0.03
Neon	0.0018
Helium	0.000524
Krypton	0.0001
Hydrogen	0.00005
Xenon	0.000008
Ozone	0.000007 (increasing with altitude)
Radon	0.000000000000000006 (decreasing with altitude)

To the precision given, the first four elements total 99.9 percent.

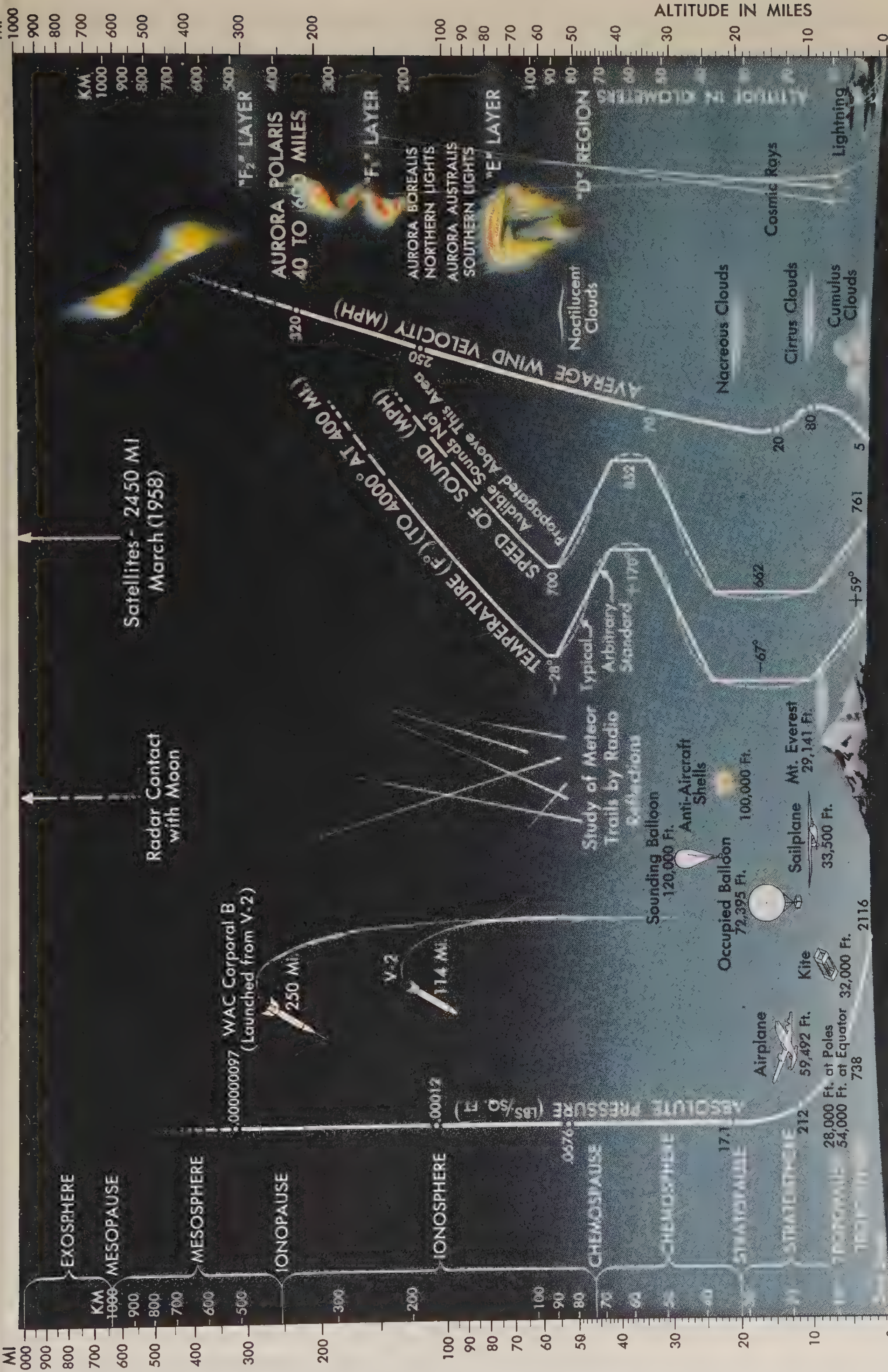


FIGURE 1410.—The earth's atmosphere. (A number of the records indicated have been exceeded since the illustration was prepared.)

Redrawn from art furnished by the Douglas Aircraft Co.

The atmosphere is considered to be composed of several distinctive layers, as follows (fig. 1410):

Layer	*Height* *Kilometers*	*Statute miles*	*Upper limit*
Troposphere	0–11	0–6.8	Tropopause
Stratosphere	11–32	6.8–19.9	Stratopause
Chemosphere	32–80	19.9–45.7	Chemopause
Ionosphere	80–400	45.7–248.5	Ionopause
Mesosphere	400–1000	248.5–621.4	Mesopause
Exosphere	Above 1000	Above 621.4	

In addition to providing life-sustaining oxygen, the atmosphere makes the earth a more habitable place by holding the moisture that produces rain, preventing an excessive change of temperature of several hundred degrees between day and night, shielding the surface from an overdose of cosmic rays, providing a medium to permit sound to occur, and providing the sky and cloud coloring that adds beauty to man's surroundings. If there were no atmosphere, stars would shine with a steady light day and night, the sky would be black, complete darkness would prevail in shadows, and there would be no twilight. It is the atmosphere that produces the refraction which causes celestial bodies to appear elevated in the sky (art. 1613).

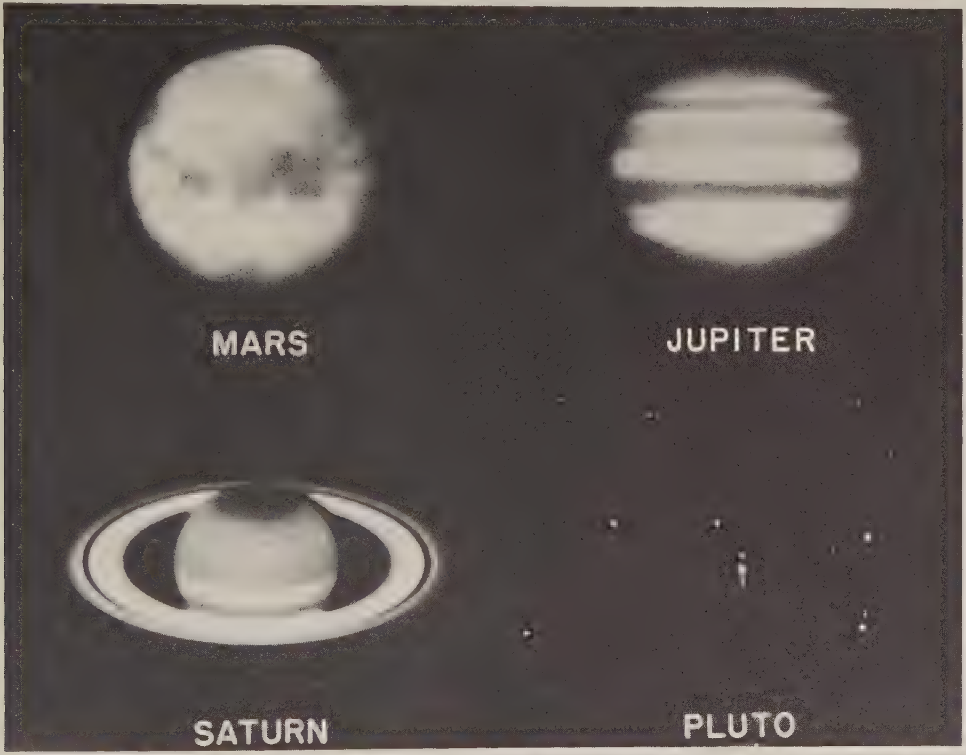
1411. Other planets and the minor planets.—**Mercury** in some ways resembles the moon more than it does other planets. Its diameter is only about 50 percent larger than that of the moon and about the same as those of Jupiter's two largest satellites. Like the moon it has little or no atmosphere, and is believed to keep the same side turned toward its primary. Mercury's mass is only four percent that of the earth, and its orbit is so small that the planet is never seen more than about 28° from the sun. It is for this reason that Mercury is not commonly used for navigation. Near greatest elongation (art. 1422) it appears near the western horizon after sunset or the eastern horizon before sunrise. At these times it resembles a first magnitude star, and is sometimes reported as a new or strange object in the sky. As seen from the earth, Mercury goes through all the phases of the moon, and occasionally **transits** (crosses) the face of the sun, appearing as a tiny, dark, inconspicuous dot on the surface. Mercury has no known satellite.

Venus, like Mercury, has no known satellite, goes through the various phases of the moon, and may transit the sun. In size of orbit, sidereal period of revolution, diameter, volume, mass, density, and surface gravity it resembles the earth more than any other planet. Its orbit is more nearly circular than that of any other planet (eccentricity 0.007). At maximum brilliance, about five weeks before and after inferior conjunction (art. 1422), it has a magnitude of about (−)4.4 and is brighter than any other object in the sky except the sun and moon. At these times it can be seen during the day, and is sometimes observed for a celestial line of position. The surface of the planet has not been observed because it is covered by a layer of dense clouds or gases. Its period of rotation is believed to be of the order of four or five weeks.

Mars (fig. 1411) has a diameter only a little more than half that of the earth, and a mass of 11 percent as much, although its density is nearly 72 percent that of the earth. It has a thin atmosphere, but few clouds. Its day is only slightly longer than that on the earth, but its year is nearly twice as long. Being a superior planet (art. 1409), it is seen only in the full or gibbous phase (art. 1423). When nearest the earth, its apparent diameter is about eight times that at conjunction (art. 1422). Mars has two satellites. **Phobos** is about ten miles in diameter and has an orbit only about 50 percent greater than the diameter of Mars. To an observer on Mars it would appear about a third as large as the moon does to an observer on the earth, and would appear

to rise in the west and set in the east, going through three-fourths of its cycle of phases while above the horizon. It would do this twice each day, since its sidereal period of revolution is only about half the period of rotation of Mars. No other natural satellite is known to *revolve* faster than its primary *rotates*. **Deimos** is only about five miles in diameter, and at greatest brilliance would appear as a very bright star. About two days would elapse between rising and setting, during which it would go through the various phases twice.

Jupiter (fig. 1411), largest of the known planets, has more than twice the mass of all other known planets combined. Its density is low and its rotation fast (9^h50^m), resulting in a pronounced equatorial bulge. It is believed to have a dense, solid core, surrounded by lighter material, and a deep atmosphere of ammonia, methane, helium,



Courtesy of Mt. Wilson and Palomar Observatories.

FIGURE 1411.—Mars, Jupiter, Saturn, and Pluto. First three photographed with 100-inch telescope, Pluto with 200-inch telescope.

and hydrogen. Two of Jupiter's twelve known satellites are about the same size as Mercury, and may have atmospheres. The four outermost satellites revolve from east to west, opposite to the general direction of revolution within the solar system.

Saturn (fig. 1411) is the only planet having a density less than that of water, yet it has a mass of nearly one-third that of Jupiter, and nearly three times that of all other known planets combined. Its composition is believed to be similar to that of Jupiter. It is more oblate than any other known planet. Perhaps the most interesting feature of this planet is its rings, composed of a great number of small solid particles spread out in three thin, flat rings more than 170,000 miles in diameter. The particles nearest the planet revolve more rapidly than those farther out, the innermost ones completing a revolution in less time than the planet completes a rotation. During half the 29.5-year sidereal period of revolution of the planet one side of the rings is

visible to observers on the earth, and during the second half of the period the opposite side is visible. Saturn has nine known satellites, the outermost one of which revolves from east to west.

Uranus is barely visible to the unaided eye, being of the sixth magnitude. It is a comparatively large planet, and probably is similar in composition to Jupiter and Saturn. The inclination of the equator of Uranus to the plane of the ecliptic is 98°, or 82° if the revolution is considered from east to west. Its five known satellites, all small, revolve in the equatorial plane, in the same direction as that of rotation of the planet.

Neptune is slightly smaller than Uranus, but has greater mass, and a longer period of rotation. Relatively little is known of this remote planet of the eighth magnitude. However, it is known to have two satellites, the larger (probably bigger than the moon) revolving from east to west.

Pluto (fig. 1411) was identified in 1930. It is of the 15th magnitude, and cannot be seen in small telescopes. In all but the 200-inch telescope it appears as a point of light. Its diameter is less than half that of the earth. Its orbit is the most eccentric and has the greatest inclination to the ecliptic of any of the known planets. At perihelion it is closer to the sun than Neptune, and there is some evidence to support the view that it was at one time a satellite of the larger planet.

Minor planets. About 1,500 of these tiny planets have been discovered, but it is estimated that there may be as many as 40,000 bright enough to be seen by the largest telescopes, when they are nearest the earth. The largest, **Ceres,** has a diameter of about 480 miles. All but a few are less than 100 miles in diameter. Since there is no known lower limit, there may be no distinction between minor planets and meteors. The combined mass of all minor planets probably does not exceed 0.1 percent that of the earth. The orbits, of various degrees of eccentricity and inclination to the ecliptic, lie mostly between those of Mars and Jupiter. However, at perihelion some of the minor planets are inside the earth's orbit. The orbit of **Hidalgo** is shown in figure 1407a.

1412. The moon is the only satellite of direct navigational interest, although the satellites of Jupiter were at one time used to determine Greenwich mean time, so that longitude could be found (art. 126). The rotation and revolution of the moon are both west to east, and both are of the same duration, $27^d07^h43^m11\overset{s}{.}5$ with respect to the stars (the **sidereal month**) and $29^d12^h44^m02\overset{s}{.}8$ with respect to the sun (the **synodical month**). Because there is no difference in the periods of rotation and revolution, the same side of the moon is always turned toward the earth. However, about 59 percent of the moon's surface has been seen, due to **libration. Libration in latitude** occurs because the axis of rotation is tilted about $6\overset{\circ}{.}5$ with respect to the axis of revolution. **Libration in longitude** occurs because the speed of revolution varies in accordance with Kepler's second law (art. 1407), while the rotational speed is essentially constant. **Diurnal libration** occurs because of the changing position of the observer relative to the moon, due to rotation of the earth. **Physical libration** is a small pendulum-like rotational oscillation of the moon with respect to its radius vector.

At **perigee** the moon is about 221,000 statute miles from the earth's center, and at **apogee** it is about 253,000 miles distant. The average distance is about 238,862 miles. Because of the relative nearness of the moon, its **geocentric parallax** (difference in position relative to the background of stars, as observed from the surface and center of the earth) is comparatively large. It is a maximum when the moon is on the horizon, when it is called **horizontal parallax.** The **equatorial horizontal parallax** for an observer at the equator, where the maximum radius of the earth is involved, is tabulated in the *Nautical Almanac* and the *American Ephemeris and Nautical Almanac*, and used in sextant altitude corrections given in the nautical and air almanacs. The

parallax varies from a maximum at the horizon to zero at the zenith. The parallax at any altitude is sometimes called **parallax in altitude.** The apparent diameter of the moon is approximately the same as that of the sun, but varies through wider limits. Because the moon is so near, the radius of the earth is an appreciable percentage of the distance between earth and moon, and the apparent diameter of the moon increases a measurable amount as its altitude increases (decreasing the distance from the observer). This apparent increase is called **augmentation** (fig. 1412). A similar effect for the sun is very small.

As with the planets and sun, the moon and earth both revolve around their common center of mass, which is about 2,900 miles from the center of the earth. It is this center of mass that describes the orbit of the earth (and moon) around the sun.

Because of its relative nearness and size, the moon is the principal source of the gravitational attraction that causes tides, although the sun has an appreciable effect, also. The action of these bodies in causing tides is described in article 3103. Because of the frictional action of tides, the rotation of the earth is slowing, the length of the day increasing about 0ˢ001 per century.

On the moon, the day is equal in length to the synodical month (about 29½ days). The earth would remain almost stationary in the sky for an observer on the 41 percent of the moon's surface always visible from the earth, would rise and set at about the same point on the horizon for one on the 18 percent which is sometimes visible, and would never appear for one on the 41 percent not seen from the earth.

Because of the relatively low velocity of escape (art. 1410) on the moon, 1.49 miles per second, this satellite has lost virtually all of its atmosphere, if it ever had one. Even water vapor is nonexistent, indicating an absence of water on the moon. Since there is practically no atmosphere, there is no sound, no twilight, and the temperature change between day and night is sudden and large, changing from perhaps (+) 200° F or more by day to about (−) 250° F by night. The sky is black and stars would be visible in broad daylight. Without water or air the moon has no clouds, no rain, no wind, no life. The numerous conspicuous "craters" are probably the results of meteor falls. There is no atmosphere to slow the meteors, and no erosion to erase the marks they leave. The "seas" are relatively flat plains. Some of the mountains are high, the maximum height on the visible side being nearly 30,000 feet. Gravity on the surface of the moon is about one-sixth that on the surface of the earth. The mass of the moon is about 1/82 that of the earth. The diameter of the moon is 2,160 miles. The size of the moon relative to the earth is greater than that of any other satellite relative to its planet.

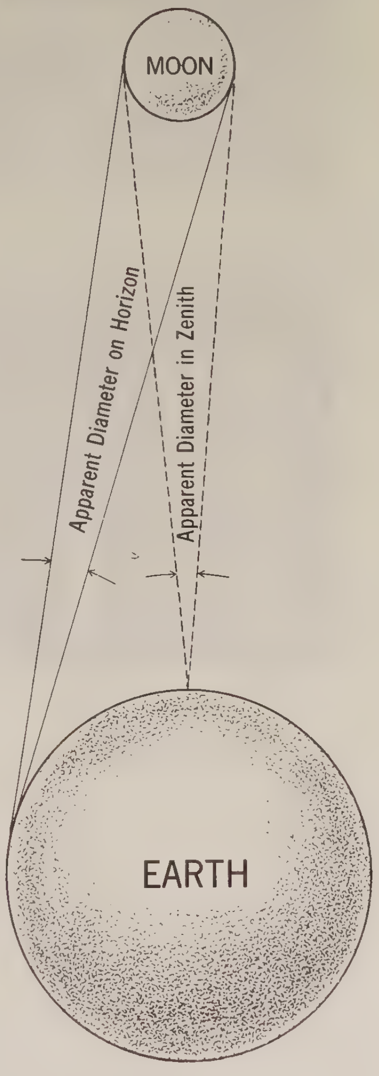
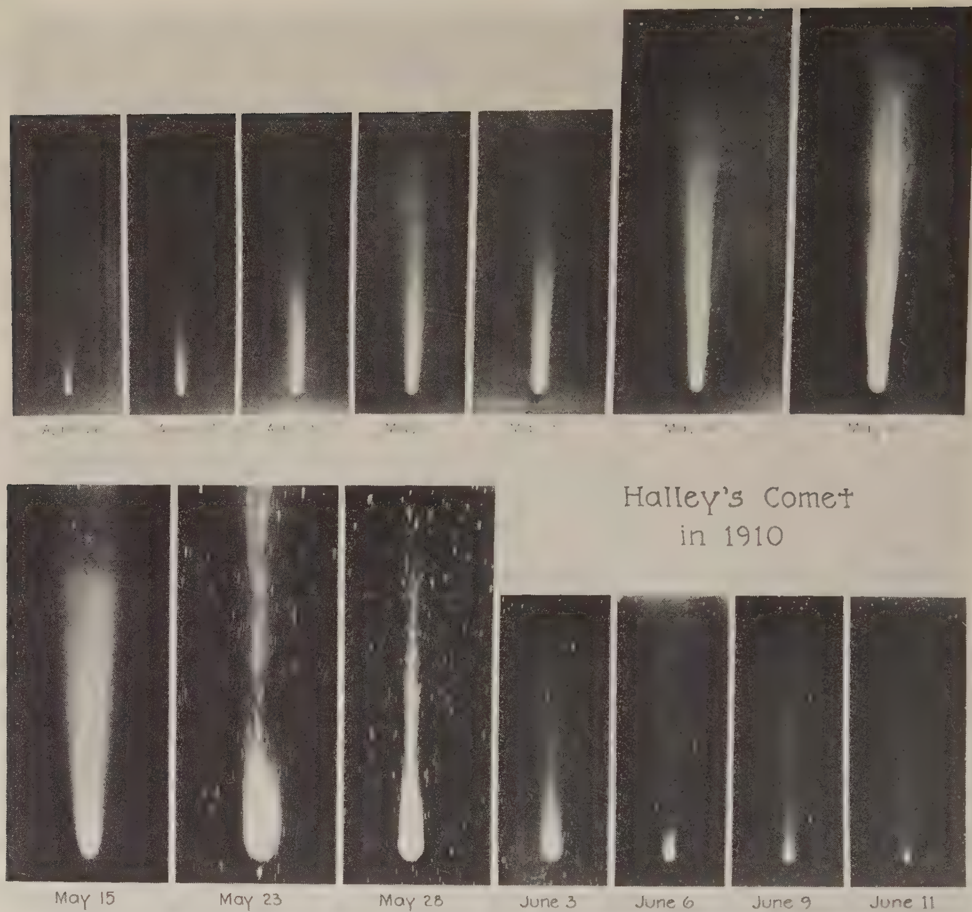


FIGURE 1412.—Augmentation.

1413. Comets and meteors.—Comets are swarms of relatively small, widely separated, solid bodies held together by mutual attraction. Around this **nucleus,** a



Courtesy of Mt. Wilson and Palomar Observatories.

FIGURE 1413.—Halley's Comet; fourteen views, made between April 26 and June 11, 1910.

more spectacular, gaseous head or **coma** and **tail** may form as the comet approaches the sun. The tail is directed *away* from the sun, so that it follows the head while the comet is approaching the sun, and precedes the head while the comet is receding. The total mass of a comet is very small, and the tail is so thin that stars can easily be seen through it. In 1910 the earth passed through the tail of Halley's comet (fig. 1413) without noticeable effect.

Comets are erratic and inconsistent. Some travel east to west and some west to east, in highly eccentric orbits inclined at any angle to the ecliptic. The shortest period of revolution is about 3.3 years. Some periods are so long that astronomers speculate as to whether some comets may not come in from outside the solar system for a single trip around the sun, and then leave the solar system, never to return. In such a case the orbit would be approximately a **parabola** (art. O34).

Without their tails, which exist only when near the sun, comets are not spectacular. Because of the small size of their nuclei, which shine by reflected light from the sun, comets are visible for only a small part of their period of revolution, and this is the part of most rapid motion, in accordance with Kepler's second law (art. 1407). An average of about five comets is observed each year, and about two-thirds of these are identified as previously observed comets. Very few comets are ever visible without a telescope. The spectacular Halley's comet reached aphelion in 1948 and started back toward the sun. It is expected to reach perihelion about February, 1986.

Because of the great distances of the aphelion of some comets, a few astronomers have speculated that additional planets may exist beyond Pluto. This curiosity is heightened by the attempt of some astronomers to identify a **family of comets** with orbits of nearly equal size, similar to those associated with Jupiter, Saturn, Uranus, and Neptune. Massive planets may influence the orbits of comets, particularly at great distances from the sun.

Meteors, popularly called **shooting stars,** are tiny, solid bodies too small to be seen until heated to incandescence by air friction while passing through the earth's atmosphere. A particularly bright meteor is called a **fireball.** One that explodes is called a **bolide.** A meteor that is not consumed during its fall through the atmosphere, but lands as a solid particle, is called a **meteorite.** These are composed principally of iron, with some nickel, and smaller quantities of other material.

Vast numbers of meteors exist. It has been estimated that an average of about 1,000,000 bright enough to be seen enter the earth's atmosphere each hour, and many times this number undoubtedly enter, but are too small to attract attention. A faint glow sometimes observed extending upward approximately along the ecliptic before sunrise and after sunset has been attributed to the reflection of sunlight from quantities of such material. This glow is called **zodiacal light.** A faint glow at that point of the ecliptic 180° from the sun is called the **gegenschein** or **counterglow.** Comets may be an assemblage of a large number of meteors traveling together, and minor planets (art. 1411) may be larger meteors. **Meteor showers** occur at certain times of the year when the earth is believed to be passing through **meteor swarms,** the scattered remains of comets that have broken up. At these times the number of meteors observed is many times the usual number.

Since such large amounts of this material are in existence, much of it in an orbit near the ecliptic, and since the orbits of most minor planets lie between those of Mars and Jupiter, where astronomers compute the orbit of another planet should be located, it is possible that another planet may have existed there at one time and been disrupted, perhaps by an atomic explosion of hydrogen or other material. The estimated total mass of all meteors, comets, and minor planets would make a small planet, but if the material which has fallen on other planets and satellites, and perhaps some or all of the satellites themselves, are added, a sizeable planet might be accounted for.

1414. Stars are distant suns, in many ways resembling the body which provides the earth with most of its light and heat. Even the nearest star is too distant to be seen as more than a point of light in the largest telescope. If planets, satellites, comets, etc., accompany those distant suns, as they do the one nearby, they have not been detected. However, comparatively dark companions of planetary size are known to accompany some stars. Nonluminous stars may exist, since most of the **radio stars** (points from which radio energy emanates) are not marked by a body visible to observers on the earth. The distance of the stars is so great that none is known to have a **heliocentric parallax** (difference in apparent position as observed from the earth and the sun) of as much as 1″.

Stars differ in size from gaseous giants having diameters greater than that of the *orbit* of the earth, to dense dwarfs which may be no larger than the major planets. Although the size and density cover wide ranges, the mass does not differ greatly. Relatively few stars have more than five times or less than one-fifth the mass of the sun, which is also about average in size, density, and temperature. The color varies with the temperature. A very hot star, having a surface temperature of perhaps 20,000° K (Celsius absolute) or more, is bluish-white; while a cooler star, having a temperature of perhaps 2,000° K, is faintly reddish. In *Orion*, blue Rigel and red Betelgeuse, located on opposite sides of the belt, constitute a noticeable contrast.

Under ideal viewing conditions, the dimmest star that can be seen with the unaided eye is of the sixth magnitude. In the entire sky there are about 6,000 stars of this magnitude or brighter. Half of these are below the horizon at any time. Because of the greater absorption of light near the horizon, where the path of a ray travels for a greater distance through the atmosphere, not more than perhaps 2,500 stars are visible to the unaided eye at any time. The 200-inch telescope on Palomar Mountain permits stars as dim as the twenty-first magnitude to be seen. It has been estimated that there are about 1,000,000,000 of this magnitude or brighter. A long-term photographic exposure with the 200-inch telescope permits observation of about twice this number. There is no indication that this is more than a tiny fraction of the total number. However, the average navigator seldom uses more than perhaps 20 or 30 of the brighter stars. Stars which exhibit a noticeable change of magnitude are called **variable stars.** A star which suddenly becomes several magnitudes brighter and then gradually fades is called a **nova.** A particularly bright one is called a **supernova.**

Two stars which appear to be very close together are called a **double star.** If more than two stars are included in the group, it is called a **multiple star;** and if a large number appear in approximately spherical shape, it is called a **globular cluster.** A group of stars moving through space together, but not exhibiting the intimate relationship of a globular cluster, is called an **open cluster.** The Pleiades and some stars of the big dipper (with certain other stars) are examples of open clusters. A group of stars which *appear* close together, regardless of actual distances, is popularly called a **constellation,** particularly if the group forms a striking configuration. Among astronomers a constellation is now considered a region of the sky having precise boundaries so arranged that all of the sky is covered, without overlap. The ancient Greeks recognized 48 constellations covering only certain groups of stars. Modern astronomers recognize 88 constellations. The constellation names and meanings are given in appendix I.

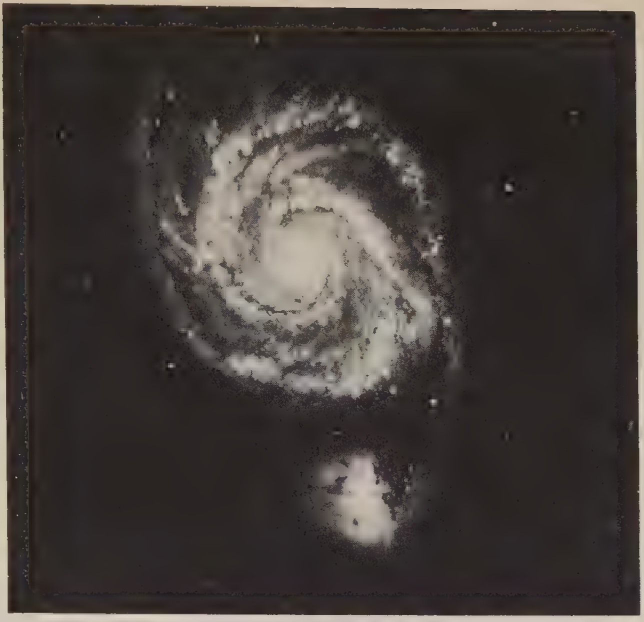
A cloudy patch of matter in the heavens is called a **nebula** (plural *nebulae*). If it is within the galaxy of which the sun is a part, it is called a **galactic nebula;** if outside, it is called an **extragalactic nebula.**

Stars rotate on their axes, and revolve around the center of their galaxy, in addition to influencing and being influenced by surrounding stars. Motion of a star through space, like that of any celestial body, is called **space motion.** That component in the line of sight is called **radial motion;** while that component across the line of sight, causing a star to change its apparent position relative to the background of more distant stars, is called **proper motion.**

1415. Galaxies.—A great number of the nebulae have been identified as extragalactic, and as telescopes became more powerful, it was discovered that these small cloudy patches are groups of stars, in many ways resembling the group of stars of which the sun is a part. Each such vast assemblage of stars constitutes an island universe as widely separated from others, comparatively, as individual stars in one group. Such a group is called a **galaxy.** It was not until well within the twentieth century that the sun was recognized as a part of such a galaxy, the **Milky Way.** In a galaxy the stars tend to congregate in groups called **star clouds** arranged in long spiral arms. The spiral nature is believed due to revolution of the stars about the center of the galaxy, the inner stars revolving more rapidly than the outer ones (fig. 1415). At the position of the sun, about two-thirds of the way out from the center, and nearly midway between "top" and "bottom," the period of revolution is about 200,000,000 years at the present speed of about 175 miles per second. This is nearly ten times the speed of the earth in its orbit. An average estimate of the size of a galaxy is that it is about 100,000 light

years in diameter, 15,000 light years thick at the center, and 5,000 light years thick near the outer edge, and that it contains perhaps 100,000,000,000 stars. This is about 100 times the number of stars that can be seen through the 200-inch telescope. Within the radius of 1,600,000,000 light years that man is able to penetrate there are perhaps 100,000,000 galaxies, although only a small fraction of this number has been actually observed.

The galaxies which have been discovered are observed to congregate in groups, somewhat similar to stars in a galaxy. Whether the part seen is but a small portion of a larger unit too vast to be seen with present instruments has not been established. Development work is being done to attempt to adapt the electron microscope for use with the telescope. By this means man hopes to see much more of what surrounds him in space, and perhaps to answer some of the questions which confront him.



Courtesy of Mt. Wilson and Palomar Observatories.

FIGURE 1415.—Spiral nebula Messier 51, in *Canes Venetici.* Satellite nebula is NGC 5195.

Apparent Motion

1416. Apparent motion due to rotation of the earth is much greater than any other observed motion of celestial bodies. It is this motion that causes celestial bodies to appear to rise somewhere along the eastern half of the horizon, climb to maximum altitude as they cross the meridian, and set along the western horizon, at about the same point relative to due west as the rising point was to due east. This apparent motion along the daily path, or **diurnal circle,** of the body is approximately parallel to the plane of the equator. It would be exactly so if rotation of the earth were the only motion, and the axis of rotation of the earth were stationary in space (arts. 1417 and 1419).

The apparent effect due to rotation of the earth varies with the latitude of the observer. At the equator, where the equatorial plane is vertical (since the axis of rotation of the earth is parallel to the plane of the horizon), bodies appear to rise and set vertically. Every celestial body is above the horizon approximately half the time. The celestial sphere as seen by an observer at the equator is called the **right sphere,** shown in figure 1416a. Several unique relationships of the right sphere are discussed in article 1432.

For an observer at one of the poles, bodies having constant declination neither rise nor set (neglecting precession of the equinoxes and changes in refraction), but circle the sky, always at the same altitude, making one complete trip around the horizon each day. At the north pole the motion is left to right, and at the south pole it is right to left. Approximately half the stars are always above the horizon and the other half never are. This is modified somewhat by actual conditions, a description of which is given in chapter XXV. The **parallel sphere** at the poles is illustrated in figure 1416b.

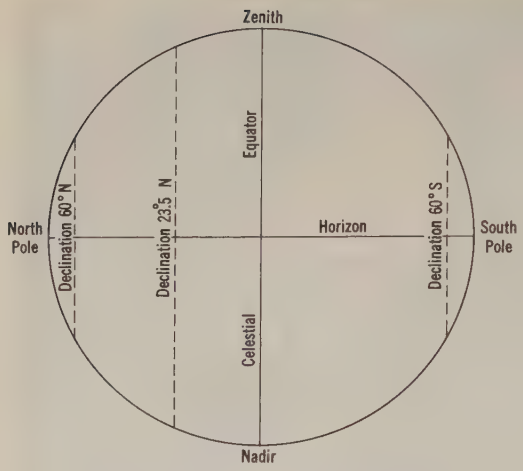


FIGURE 1416a.—The right sphere.

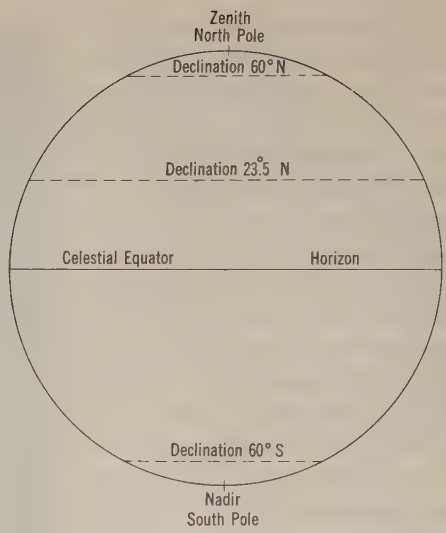


FIGURE 1416b.—The parallel sphere.

Between these two extremes, the apparent motion is a combination of the two. On this **oblique sphere,** illustrated in figure 1416c, **circumpolar** celestial bodies remain above the horizon during the entire 24 hours, circling the elevated celestial pole (art. 1426) each day. The stars of the big dipper and *Cassiopeia* are circumpolar for many observers in the United States. An approximately equal part of the celestial sphere remains below the horizon during the entire day. The southern cross is not visible to most observers in the United States. Other bodies rise obliquely along the eastern horizon, climb to maximum altitude at the celestial meridian, and set along the western horizon. The length of time above the horizon, and the altitude at meridian transit, vary with both the latitude of the observer and the declination of the body. Several useful relationships of the oblique sphere are indicated in article 1432. The relative portions of the celestial sphere that remain either above or below the horizon varies with the latitude, from none at the equator to 100 percent at the poles. At the polar circles (art. 1419) of the earth and beyond, even the sun becomes circumpolar. This is the land of the **midnight sun,** where the sun does not set during part of the summer, and does not rise during part of the winter.

This increased obliquity at higher latitudes explains why days and nights are always about the same length in the tropics, and the change of length of the day becomes greater as the latitude increases. It also explains why twilight lasts longer in higher latitudes. **Twilight** is that period of incomplete darkness following sunset and preceding sunrise. **Evening twilight** starts at sunset, and **morning twilight** ends at sunrise. The darker limit of twilight occurs when the center of the sun is a stated number of degrees below the celestial horizon. Three kinds of twilight are defined, depending upon the darker limit. These are:

Twilight	*Lighter limit*	*Darker limit*	*At darker limit*
civil	0°	−6°	Horizon clear and bright stars visible
nautical	0°	−12°	Horizon vague
astronomical	0°	−18°	Full night

The conditions at the darker limit are relative and vary considerably under different atmospheric conditions.

In figure 1416d the twilight band is shown, with the darker limits of the various kinds indicated. The nearly vertical celestial equator line is for an observer at latitude

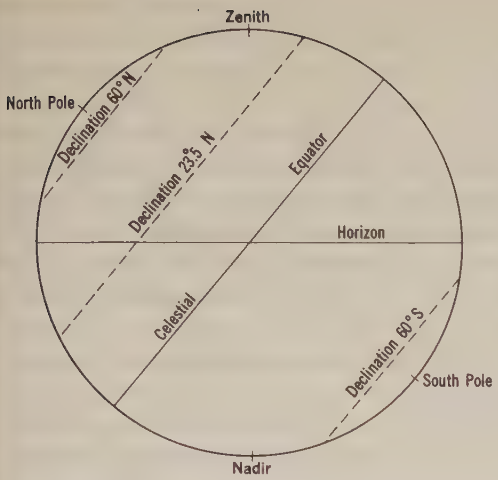


FIGURE 1416c.—The oblique sphere at lat. 40° N.

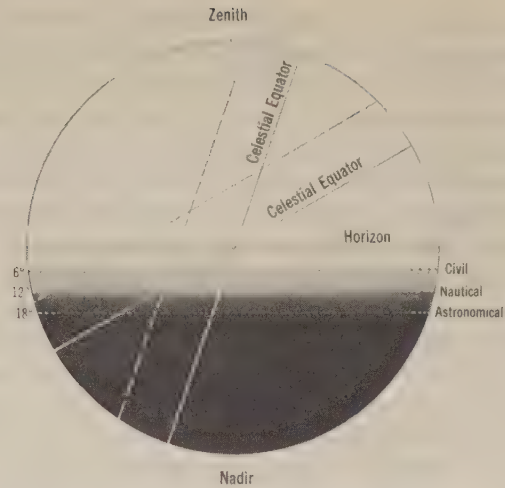


FIGURE 1416d.—The various twilights at lat. 20° N and lat. 60° N.

20° N. The nearly horizontal celestial equator line is for an observer at latitude 60° N. The broken line in each case is the diurnal circle of the sun when its declination is 15° N. The relative duration of any kind of twilight at the two latitudes is indicated by that portion of the diurnal circle between the horizon and the darker limit, although it is not directly proportional to the relative length of line shown, since the projection is orthographic (art. 319). The duration of twilight at the higher latitude is longer, proportionally, than shown. Note that complete darkness does not occur at latitude 60° N when the declination of the sun is 15° N.

1417. Apparent motion due to revolution of the earth.—If it were possible to stop the *rotation* of the earth so that the celestial sphere would appear stationary, the effects of the *revolution* of the earth would become more noticeable. In one year the sun would appear to make one complete trip around the earth, from west to east. Hence, it would seem to move eastward a little less than 1° per day. This motion can be observed by watching the changing position of the sun among the stars. But since both sun and stars generally are not visible at the same time, a better way is to observe the constellations at the same time each night. On any night a star rises nearly four minutes earlier than on the previous night. Thus, the celestial sphere appears to shift westward nearly 1° each night, so that different constellations are associated with different seasons of the year.

Apparent motions of planets and the moon are due to a combination of their motions and those of the earth. If the rotation of the earth were stopped, the combined apparent motion due to the revolutions of the earth and other bodies would be similar to that occurring if both rotation and revolution of the earth were stopped, as discussed in article 1418, but with different timing. Stars would appear nearly stationary in the sky, but would undergo a small annual cycle of change due to **aberration.** The motion of the earth in its orbit is sufficiently fast to cause the light from stars to appear to shift slightly in the direction of the earth's motion. This is similar to the illusion one has when walking in rain that is falling vertically, but appearing to come from ahead due to his own motion. The apparent direction of the light ray from the star is the vector difference (art. O18) of the motion of light and the motion of the earth, similar to that of apparent wind on a moving vessel (art. 3709). This effect is most apparent for a body perpendicular to the line of travel of the earth

in its orbit, for which it reaches a maximum value of 20″.5. The effect of aberration can be noted by comparing the coordinates (declination and sidereal hour angle) of various stars throughout the year. A change is observed in some bodies as the year progresses, but at the end of the year the values have returned almost to what they were at the beginning. That they do not return exactly is due to proper motion (art. 1418), precession of the equinoxes (art. 1419), and **nutation,** which is an irregularity in the motion of the earth due to the disturbing effect of other celestial bodies, principally the moon. **Eulerian motion** is a slight wobbling of the earth about its axis of rotation, often called **polar motion,** and sometimes **wandering of the poles.** This motion, which does not exceed 40 feet from the mean position, produces slight **variation of latitude and longitude** of places on the earth.

By the **calendar,** one year is of 365 days duration for **common years** and 366 days for **leap years.** A leap year is any year divisible by four, unless it is a century year, which must be divisible by 400 to be a leap year. Thus, 1900 was not a leap year, but 2000 will be. This calendar, now in general use, is called the **Gregorian calendar.** Astronomically, the year is not divisible into a whole number of days, and the present system will introduce an error of three days in about 10,000 years. The length of the year with respect to the vernal equinox (art. 1419) is about 365 days, 5 hours, 48 minutes, 46 seconds. This is the **tropical, astronomical, equinoctial, natural,** or **solar year.** Since the vernal equinox is in motion on the celestial sphere (art. 1419), this does not quite agree with the **sidereal year** of about 365 days, 6 hours, 9 minutes, 10 seconds, with respect to the stars. The period of revolution from perihelion to perihelion, about 365 days, 6 hours, 13 minutes, 53 seconds, is called the **anomalistic year.** These values vary slightly from year to year, and progressively over the years, as shown in appendix D.

1418. Apparent motion due to movement of other celestial bodies.—Even if it were possible to stop both the rotation and revolution of the earth, celestial bodies would not appear stationary on the celestial sphere. The moon would make one revolution about the earth each sidereal month (art. 1412), rising in the west and setting in the east. The inferior planets would appear to move eastward and westward relative to the sun, as explained in article 1422, staying within the zodiac. Superior planets would appear to make one revolution around the earth, from west to east, each sidereal period (app. F).

Since the sun (and the earth with it) and all other stars, as far as is known, are in motion relative to each other, slow apparent motions would result in slight changes of the positions of the stars relative to each other. This **space motion** (art. 1414) is, in fact, observed by telescope. That component of such motion across the line of sight, called **proper motion,** produces a change in the apparent position of the star. The maximum which has been observed is that of "Barnard's Star," which is moving at the rate of 10″.3 per year. This is a tenth-magnitude star, and hence not visible to the unaided eye. Of the 57 stars listed on the daily pages of the almanacs, Rigil Kentaurus has the greatest proper motion, about 3″.7. Arcturus, with 2″.3, has the greatest proper motion of the navigational stars in the northern hemisphere. In a few thousand years proper motion will be sufficient to materially alter some familiar configurations of stars, notably the big dipper.

1419. The ecliptic is the path the sun appears to take among the stars due to the annual revolution of the earth in its orbit. It is considered a great circle of the celestial sphere, inclined at an angle of about 23°27′ to the celestial equator, but undergoing a continuous slight change. This angle is called the **obliquity of the ecliptic.** This inclination is due to the fact that the axis of rotation of the earth is not perpendicular to its orbit. It is this inclination which causes the sun to appear to move north and

south during the year, giving the earth its **seasons,** and changing lengths of periods of daylight. This seasonal variation is one of the factors making the earth a desirable place on which to live.

Refer to figure 1407b. The earth is at perihelion early in January and at aphelion six months later. On or about June 21, about ten or eleven days before reaching aphelion, the northern part of the earth's axis is tilted toward the sun. The north polar regions are having continuous sunlight; the northern hemisphere is having its **summer** with long, warm days and short nights; the southern hemisphere is having winter with short days and long, cold nights; and the south polar region is in continuous darkness. This is the **summer solstice.** Three months later, about September 23, the earth has moved a quarter of the way around the sun, but its axis of rotation still points in about *the same direction in space.* The sun shines equally on both hemispheres, and days and nights are the same length over the entire world. The sun is setting at the north pole, and rising at the south pole. The northern hemisphere is having its **autumn,** and the southern hemisphere its spring. This is the **autumnal equinox.** In another three months, on or about December 22, the southern hemisphere is tilted toward the sun and conditions are the reverse of those six months earlier, the northern hemisphere having its **winter,** and the southern hemisphere its summer. This is the **winter solstice.** Three months later, when both hemispheres again receive equal amounts of sunshine, the northern hemisphere is having **spring** and the southern hemisphere autumn, the reverse of conditions six months before. This is the **vernal equinox.**

The word "equinox," meaning "equal nights," is applied because it occurs at the time when days and nights are of approximately equal length all over the earth. The word "solstice," meaning "sun stands still," is applied because the sun stops its apparent

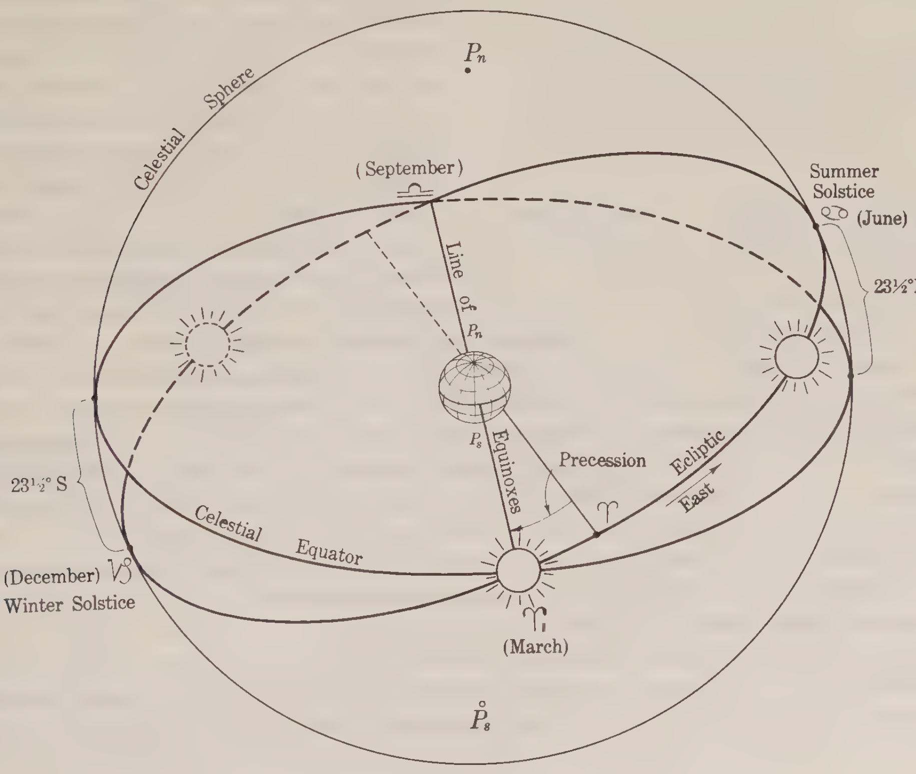
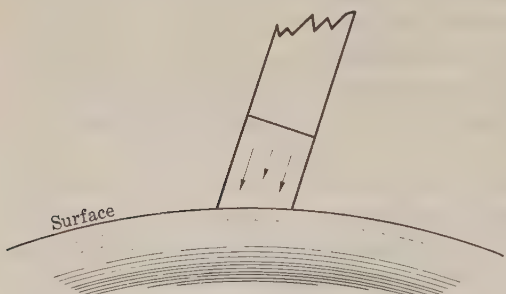
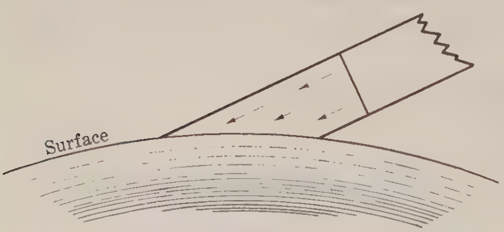


FIGURE 1419a.—Apparent motion of the sun in the ecliptic.

northward or southward motion and momentarily "stands still" before it starts in the opposite direction. This action, somewhat analogous to the "stand" of the tide (art. 3104), refers to the motion in a north-south direction only, and not to the daily apparent revolution around the earth. Note that it does not occur when the earth is at perihelion and aphelion (fig. 1407b). Refer to figure 1419a. At the time of the vernal equinox, the sun is directly over the equator, crossing from the southern hemisphere to the northern hemisphere. It rises due east and sets due west, remaining above the horizon about 12 hours. It is not exactly 12 hours because of refraction, semidiameter, and the height of the eye of the observer. These cause it to be above the horizon a little longer than below the horizon. Following the vernal equinox, the northerly declination increases, and the sun climbs higher in the sky each day (at the latitudes of the United States), until the summer solstice, when a declination of about 23°27′ north of the celestial equator is reached. The sun then gradually retreats southward until it is again over the equator at the autumnal equinox, at about 23°27′ south of the celestial equator at the winter solstice, and back over the celestial equator again at the next vernal equinox.



June 22



December 22

FIGURE 1419b.—Sunlight in summer and winter. Compare the surface covered by the same amount of sunlight on the two dates.

The sun is nearest the earth during the northern hemisphere winter. Hence, it is not the distance that is responsible for the difference in temperature during the different seasons. The reason is to be found in the altitude of the sun in the sky and the length of time it remains above the horizon. During the summer the rays are more nearly vertical, and hence more concentrated, as shown in figure 1419b. Since the sun is above the horizon more than half the time, heat is being added by absorption during a longer period than it is being lost by radiation. This explains the lag of the seasons. Following the longest day, the earth continues to receive more heat than it dissipates, but at a decreasing proportion. Gradually the proportion decreases until a balance is reached, after which the earth cools, losing more heat than it gains. This is analogous to the day, when the highest temperatures normally occur several hours after the sun reaches maximum altitude at meridian transit, and for the same reason. A similar lag occurs at other seasons of the year. *Astronomically*, the seasons *begin* at the equinoxes and solstices. *Meteorologically*, they differ from place to place.

By Kepler's second law, the earth travels faster when nearest the sun, as shown in figure 1419c. Hence, the northern hemisphere (astronomical) winter is shorter than its summer, the difference being about seven days.

Everywhere between the parallels of about 23°27′ N and about 23°27′ S the sun is directly overhead at some time during the year. Except at the extremes, this occurs twice, once as the sun appears to move northward, and the second time as it moves southward. This is the **torrid zone.** The northern limit is the **tropic of Cancer,** and

the southern limit the **tropic of Capricorn.** These names come from the constellations which the sun entered at the solstices when the names were first applied, more than 2,000 years ago. Today, the sun is in the next constellation toward the west, because of precession of the equinoxes, described below. The parallels about 23°27′ from the poles, marking the approximate limits of the circumpolar sun, are called **polar circles,** the one in the northern hemisphere being the **arctic circle** and the one in the southern hemisphere the **antarctic circle.** The areas inside the polar circles are the north and south **frigid zones.** The regions between the frigid zones and the torrid zones are the north and south **temperate zones.**

The expression "vernal equinox," and associated expressions, are applied both to the *times* and *points* of occurrence of the various phenomena. Navigationally, the vernal equinox is sometimes called the **first point of Aries,** because, when the name was given, the sun entered the constellation *Aries*, the ram (♈), at this time. This point is of interest to navigators because it is the origin of measurement of sidereal hour angle (art. 1426). The expressions **March equinox, June solstice, September equinox,** and **December solstice** are occasionally applied as appropriate, because the more common names are associated with the seasons in the northern hemisphere, and are six months out of step for the southern hemisphere.

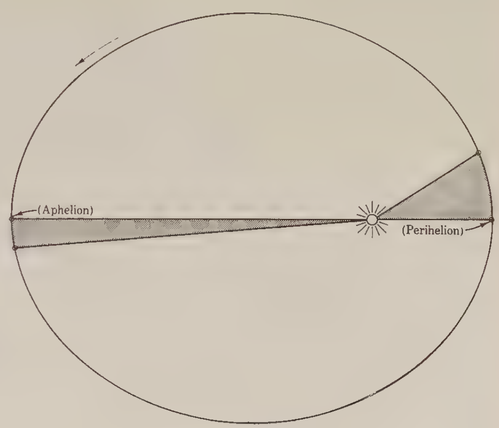


FIGURE 1419c.—Kepler's second law. Since the shaded areas are equal, speed at perihelion is greater than at aphelion.

The axis of the earth is undergoing a precessional motion similar to that of a top spinning with its axis tilted. In about 25,800 years the axis completes a cycle and returns to the position from which it started. Since the celestial equator is 90° from the celestial poles, it too is moving. The result is a slow westward movement of the equinoxes and solstices, which has already carried them about 30°, or one constellation, along the ecliptic from the positions they occupied when named more than 2,000 years ago. Since sidereal hour angle (art. 1426) is measured from the vernal equinox, and declination (art. 1426) from the celestial equator, the coordinates of celestial bodies would be changing even if the bodies themselves were stationary. This westward motion of the equinoxes along the ecliptic is called **precession of the equinoxes** (fig. 1419a). The total amount, called **general precession,** is about 50″27 per year (in 1958). It may be considered divided into two components, **precession in right ascension** (about 46″10 per year) measured along the celestial equator, and **precession in declination** (about 20″05 per year) measured perpendicular to the celestial equator. The annual change in the coordinates of any given star, due to precession alone, depends upon its position on the celestial sphere, since these coordinates are measured relative to the polar axis while the precessional motion is relative to the ecliptic axis (art. 1429).

Due to precession of the equinoxes, the celestial poles are describing circles in the sky. The north celestial pole is moving closer to Polaris, which it will pass at a distance of approximately 28′ about the year 2102. Following this, the polar distance will increase, and eventually other stars, in their turn, will beome the pole star. Similarly, the south celestial pole will some day be marked by stars of the false southern cross.

1420. The zodiac is a circular band of the sky extending 8° on each side of the ecliptic. The navigational planets and the moon are within these limits. The zodiac is divided into 12 sections of 30° each, each section being given the name and symbol ("sign") of the constellation within it. These are shown in figure 1420. The complete list of signs and names is given in appendix A.

The sun remains in each part for approximately one month. When the names were assigned, more than 2,000 years ago, the sun entered *Aries* (♈) at the vernal equinox, *Cancer* (♋) at the summer solstice, *Libra* (♎) at the autumnal equinox, and *Capricornus* (♑) at the winter solstice. Even though this is no longer true because of precession of the equinoxes, *The American Ephemeris and Nautical Almanac* still lists the sun as entering these constellations at the times of the equinoxes and solstices, for this has

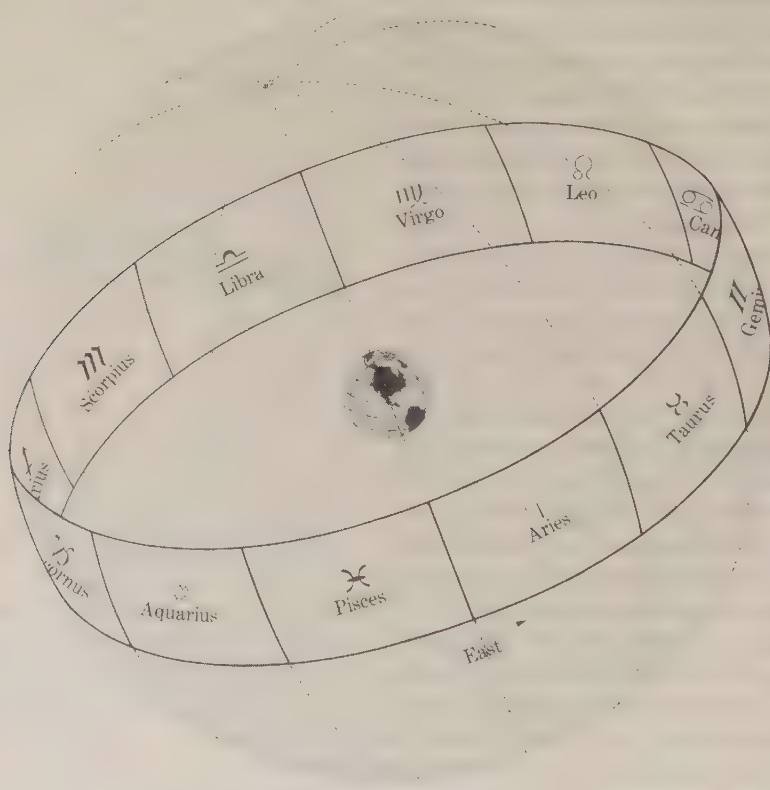


FIGURE 1420.—The zodiac.

come to be their principal astronomical significance. The pseudo science of **astrology** assigns additional significance, not recognized by scientists, to the positions of the sun and planets among the signs of the zodiac.

1421. Time.—Traditionally, astronomy has furnished the basis for measurement of time, a subject of primary importance to the navigator. The **year** is associated with the revolution of the earth in its orbit. The **day** is one rotation of the earth about its axis.

The *duration* of one rotation of the earth depends upon the external reference point used. One rotation relative to the sun is called a **solar day.** However, rotation relative to the **apparent sun** (the actual sun that *appears* in the sky) does not provide time of uniform rate, because of variations in the rate of revolution and rotation of the earth. The error due to lack of uniform rate of revolution is removed by using a fictitious **mean sun.** Thus, **mean solar time** is nearly equal to the *average* **apparent solar time.** Because

the accumulated difference between these times, called **equation of time,** is continually changing, the period of daylight is shifting slightly, in addition to its increase or decrease in length due to changing declination. Apparent and mean suns seldom cross the celestial meridian at the same time. The earliest sunset (in latitudes of the United States) occurs about two weeks *before* the winter solstice, and the latest sunrise about two weeks *after* winter solstice. A similar apparent discrepancy occurs at the summer solstice.

With an increase in precision of the instruments used for measuring the rotation of the earth, it became apparent that the speed of rotation is not constant, increasing slightly during the northern hemisphere spring, and decreasing during the opposite season. Other changes, more erratic, are also noted. These are in addition to the slowing due to tidal action (art. 1412), and are not fully explained. These changes have led the International Bureau of Weights and Measures to adopt the *year* as the basic unit for time, rather than the day, so that daily irregularities can be reduced or eliminated. Time based upon uniform division of the year is called **ephemeris time.** The atomic clock developed by the U. S. Bureau of Standards provides time which in some respects is superior to that based upon the daily rotation of the earth, but is inferior to that based upon the annual revolution of the earth around the sun. This device is based upon the motion of the atoms of ammonia molecules.

If the vernal equinox is used as the reference, a **sidereal day** is obtained, and from it, **sidereal time.** This indicates the approximate positions of the stars, and for this reason is the basis of star charts (art. 2204) and star finders (art. 2210). Because of the revolution of the earth around the sun, a sidereal day is about 3^m56^s *shorter* than a solar day, and there is one more sidereal than solar days in a year. One mean solar day equals 1.00273791 mean sidereal days. Because of precession of the equinoxes, one rotation of the earth with respect to the stars is not quite the same as one rotation with respect to the vernal equinox. One mean solar day averages 1.0027378118868 rotations of the earth with respect to the stars.

In tide analysis, the moon is sometimes used as the reference, producing a **lunar day** averaging 24^h50^m (mean solar units) in length, and **lunar time.**

Since each kind of day is divided arbitarily into 24 hours, each hour having 60 minutes of 60 seconds, the length of each of these units differs somewhat in the various kinds of time.

Time is also classified according to the terrestrial meridian used as a reference. **Local time** results if one's own meridian is used, **zone time** if a nearby reference meridian is used over a spread of longitudes, and **Greenwich** or **universal time** if the Greenwich meridian is used.

The subject of time is discussed in more detail in chapter XIX.

1422. Planetary configurations.—Since the orbit of an inferior planet lies within that of the earth, the planet and sun are nearly in line twice each synodic period of revolution of the inferior planet. When the sun is between the earth and the other planet, that planet is at **superior conjunction.** When the planet is between the earth and sun, it is at **inferior conjunction.** If the orbit of the planet had no inclination to the ecliptic, the planet would cross or **transit** the face of the sun at inferior conjunction and be eclipsed or **occulted** by the sun at superior conjunction. Occasionally this does occur.

Refer to figure 1422, showing orbits of the earth, Venus (an inferior planet), and Mars (a superior planet). As shown, the relative sizes of the orbits are correct, and the relative sizes of the planets are correct, but the planets are too large for their orbits and the sun, and the sun is too large for the orbits of the planets. The earth is considered stationary in its orbit. The positions of Venus are shown at superior and inferior

conjunctions. In moving eastward from one to the other, Venus appears to move to the left of the sun. As observed from the earth, the angle between lines to the sun and a planet, particularly an inferior planet, is called the planet's **elongation,** which may be designated east or west to indicate the apparent position of the planet relative to the sun. As Venus continues along its orbit, its elongation increases slowly until the planet arrives at the point where a straight line from the earth is tangent to its orbit, when the elongation becomes maximum. Here it is called **greatest elongation east.** As Venus continues along its orbit, its elongation decreases rapidly, becoming zero at inferior conjunction. Through the second half of its synodic period its elongation increases rapidly to **greatest elongation west,** and then decreases slowly to zero at the next

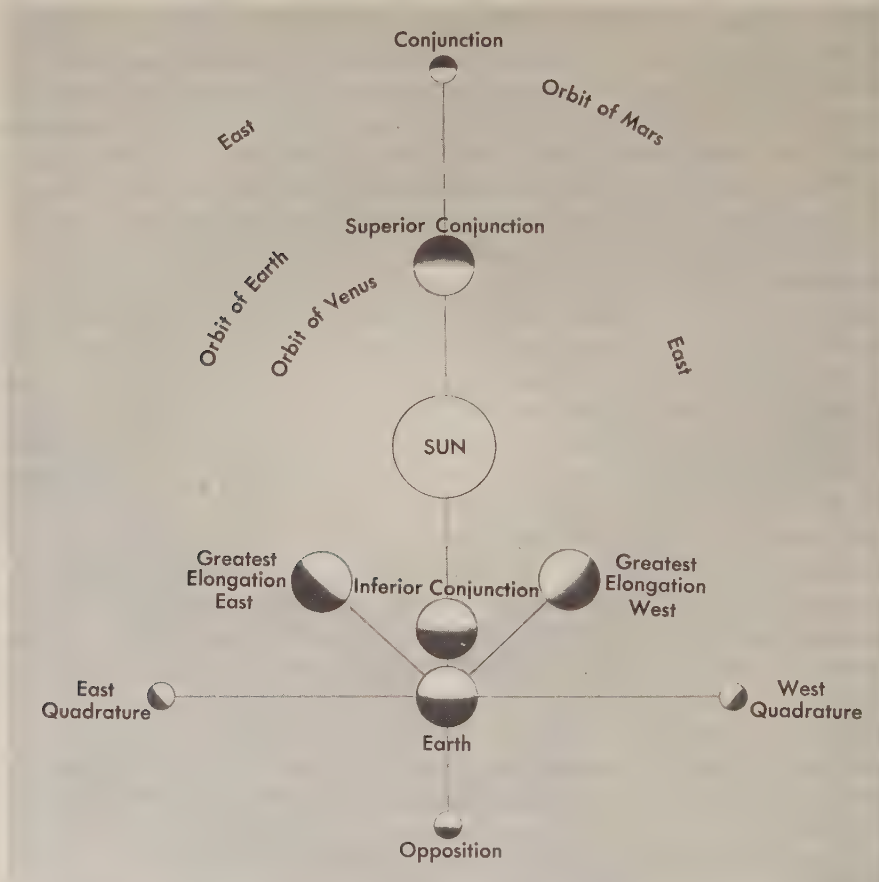


FIGURE 1422.—Planetary configurations.

superior conjunction. The greatest elongation of Venus is about 46°, but varies because its orbit and that of the earth are elliptical, and the phenomenon occurs at different points on the orbits.

The orbit of the planet Mercury lies inside that of Venus, and hence the greatest elongation is not as great, being about 28°. It is because the apparent position of Mercury is never far from the sun that this planet is not considered navigationally important. Since its synodic period of revolution is only 115.9 days, it is at conjunction a little oftener than once every two months. By comparison, Venus is at conjunction a little oftener than once every ten months, having a synodic period of revolution of 583.9 days.

As shown in figure 1422, an inferior planet goes through all phases of the moon (art. 1423), being "full" at superior conjunction, "new" at inferior conjunction, and at "quarter" when it reaches greatest elongation. A telescope is needed to see the phases.

For a superior planet the situation is different. Refer again to figure 1422. When the sun is between the earth and the planet, that planet (Mars in the illustration) is at **conjunction** (☌). The adjective "superior" is not needed because a superior planet, when on the opposite side is *away* from the sun, or at **opposition** (☍) and can never be at inferior conjunction. When its elongation is 90°, a superior planet is at east or west **quadrature** (□), depending upon its apparent position relative to the sun. Since a superior planet has a longer period of revolution than the earth, it *appears* to move westward around the sun, being at conjunction, east quadrature, opposition, west quadrature, and back to conjunction. It is at "full" phase at conjunction and opposition, and gibbous between.

Unless a planet is in the ecliptic, it is not directly in line with the earth and sun at conjunction and opposition. These points are defined as those at which either the sidereal hour angles (art. 1426) or the celestial longitudes (art. 1429) are the same (in the case of conjunction) or 180° apart (at opposition).

The apparent positions of the planets in relation to other members of the solar system, particularly the relationships shown in figure 1422, are called **planetary configurations.** The motions of planets with respect to the sun would be true, generally, with respect to the stars, also, if the earth were stationary in its orbit, as shown. However, because of the earth's motion around the sun, the sun appears to move eastward among the stars. This is usually the direction of apparent motion of the planets, too, and is called **direct motion.** When a planet is near opposition or inferior conjunction, its apparent *westerly* motion relative to the sun is greater than the apparent *easterly* motion of the sun relative to the stars, and the planet appears to move in a *westerly* direction relative to the stars. This is called **retrograde motion.**

The brightest planet in the western sky following sunset is popularly called the **evening star,** and the brightest planet in the eastern sky preceding sunrise is popularly called the **morning star.**

1423. Phases of the moon.—Relative to the sun, the moon makes one complete trip around the celestial sphere each synodical month (about 29½ days). As it does so, it goes through a cycle of aspects or **phases** to an observer on the earth, because the moon, like the planets, shines chiefly by reflected light from the sun. The orbit of the moon is inclined about 5° to the ecliptic, and undergoes a precessional motion called **regression of the nodes.** It is similar to precession of the equinoxes of the earth (art. 1419), and is chiefly responsible for nutation (art. 1417). However, the cycle is completed in a little more than 18 years, as compared with about 25,800 years for the earth.

Because of the small inclination of its orbit, the moon is never far from the ecliptic. At conjunction, when the moon passes nearly between the earth and sun, its illuminated portion is *away* from the earth (*toward* the sun), as shown in figure 1423. (In this illustration, the outer figures show various positions of the moon relative to the earth and sunlight. The inner circle of moons shows the appearance from the earth.) It is then a **new moon,** and may be barely visible because of **earthshine,** which is sunlight reflected from the illuminated side of the earth. To an observer on the moon, the "full earth" would be visible at this time, three and one-half times as great in diameter and nearly 40 times as bright as the full moon appears to an observer on the earth. Since it is at conjunction, the new moon rises, transits the celestial meridian, and sets at approximately the same time as the sun.

A day later the moon has moved about 12°2 eastward of the sun and a thin **crescent** appears on the side toward the sun, with the horns or **cusps** pointing away from the sun.

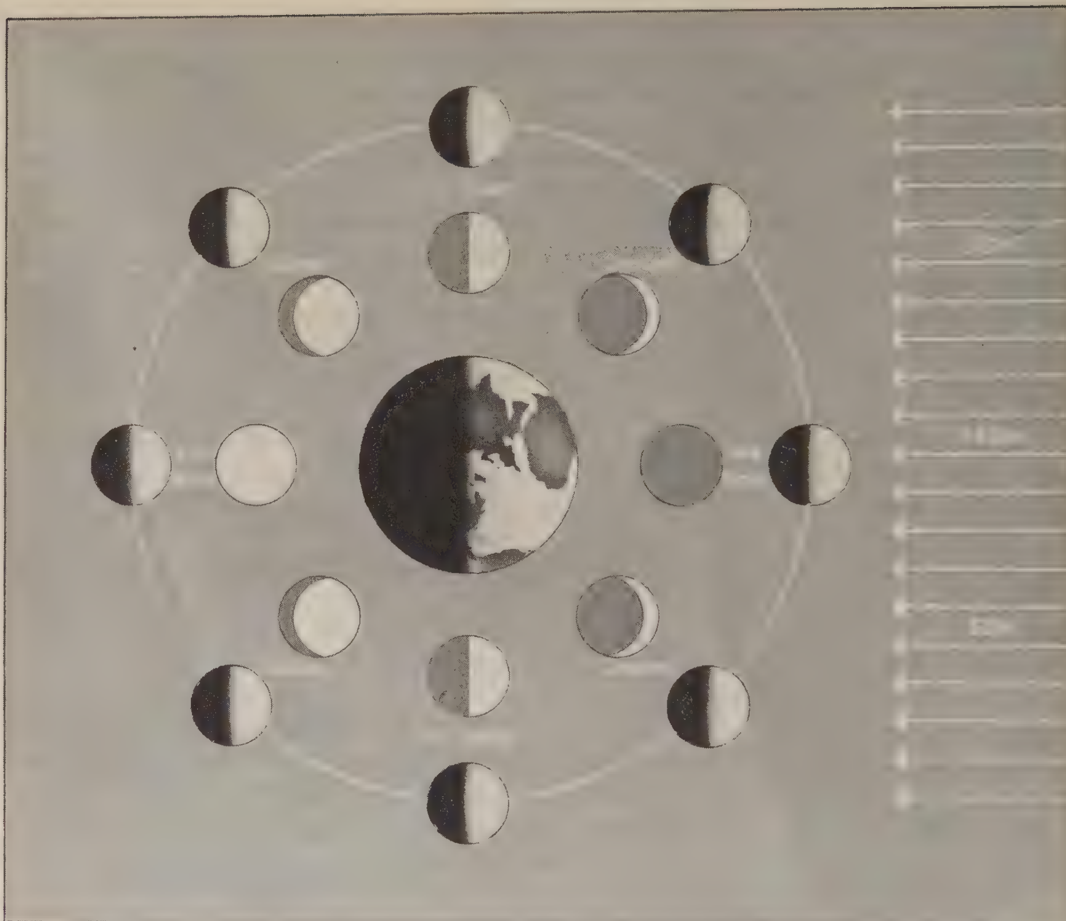


FIGURE 1423.—Phases of the moon. The inner figures of the moon represent its appearance from the earth.

The moon is low in the western sky after sunset. Because of glow from this illuminated portion, and the fact that the side of the earth toward the moon is not quite "full," that part of the moon illuminated by earthshine is not quite as bright. Each day the moon moves approximately 12°2 east, relative to the sun. As it does so, the crescent grows fatter, and the earthshine less conspicuous.

When the moon reaches quadrature, about a week after new moon, it is at **first quarter.** That half of the moon toward the sun is illuminated. The moon is now about 90° or six hours *behind* the sun. It rises about noon, is on the celestial meridian about 6 PM, and sets about midnight.

As the moon continues eastward on successive days, the line separating the illuminated and dark portions, called the **terminator,** moves on across the moon. The moon is now in the **gibbous** phase, which continues until the moon is at opposition, or **full moon.** It now rises about the time of sunset, reaches the celestial meridian about midnight, and sets about the time of sunrise.

On succeeding days the moon again becomes gibbous, and at quadrature it is at **last quarter,** rising about midnight, crossing the celestial meridian about 6 AM, and setting about noon. During the remainder of its cycle the moon again goes through the crescent phase and returns to new moon to start another cycle.

During the first half of the cycle, the moon is **waxing,** and during the second half it is **waning.** The elapsed time since new moon, usually expressed as days and tenths

of a day is called **age of the moon.** Since the moon appears to move *eastward* relative to the sun, crossing the meridian *later* each day, one day each synodical month is without a moonrise, and another is without a moonset.

The times of moonrise and moonset indicated above are approximate only. When the difference between the declination of the sun and moon is considerable, the times given may be in error by as much as several hours, particularly in high latitudes. The times of crossing the celestial meridian vary through smaller limits.

At full moon, the sun and moon are on opposite sides of the ecliptic. Therefore, in the winter the full moon rises early, crosses the celestial meridian high in the sky, and sets late; as the sun does in the summer. In the summer the full moon rises in the southeastern part of the sky (northern hemisphere), remains relatively low in the sky, and sets along the southwestern horizon after a short time above the horizon.

At the time of the autumnal equinox, that part of the ecliptic opposite the sun is most nearly parallel to the horizon. Since the eastward motion of the moon is approximately along the ecliptic, the delay in the time of rising of the full moon from night to night is less than at other times of the year. The full moon nearest the autumnal equinox is called the **harvest moon.** The full moon occurring about a month later is called the **hunter's moon.**

1424. Eclipses.—Because of the inclination of the moon's orbit with respect to the ecliptic, the sun, earth, and moon are usually not so nearly in line at conjunction and opposition of the moon that either the earth or moon passes through the shadow of the other. However, when this does occur, an **eclipse** takes place. Since the sun and moon are of nearly the same apparent size to an observer on the earth, an eclipse is a much more spectacular occurrence than the transit of an inferior planet across the face of the sun, or the occultation of a star or planet by the sun or moon (art. 1422).

When conditions are suitable, the moon passes between the sun and earth, as shown in figure 1424a. If the moon's apparent diameter is larger than that of the sun, the moon being near perigee, its shadow reaches the earth as a nearly round dot only a few miles in diameter. The dot moves rapidly across the earth, from west to east, as the moon continues in its orbit. Within the dot, the sun is completely hidden from view, and a **total eclipse** of the sun occurs. For a considerable distance around the shadow, part of the surface of the sun is obscured, and a **partial eclipse** occurs. In the line of travel of the shadow a partial eclipse occurs as the round disk of the moon appears to move slowly across the surface of the sun, hiding an ever-increasing part of it, until the total eclipse occurs. Because of the uneven edge of the mountainous moon, the light is not cut off evenly, but several last illuminated portions appear through the valleys or passes between the mountain peaks. These are called **Baily's Beads.**

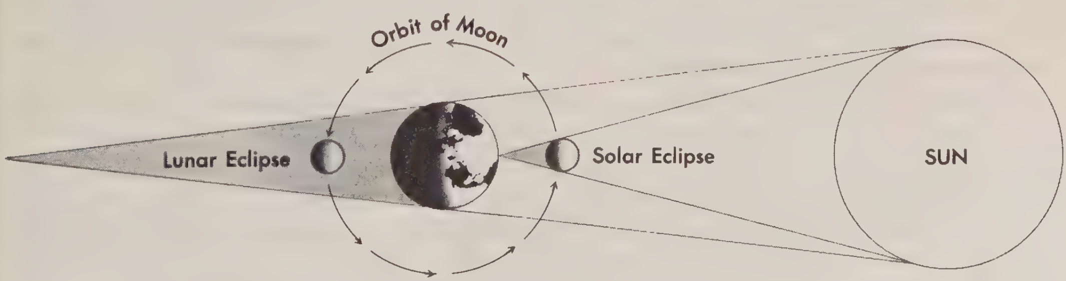
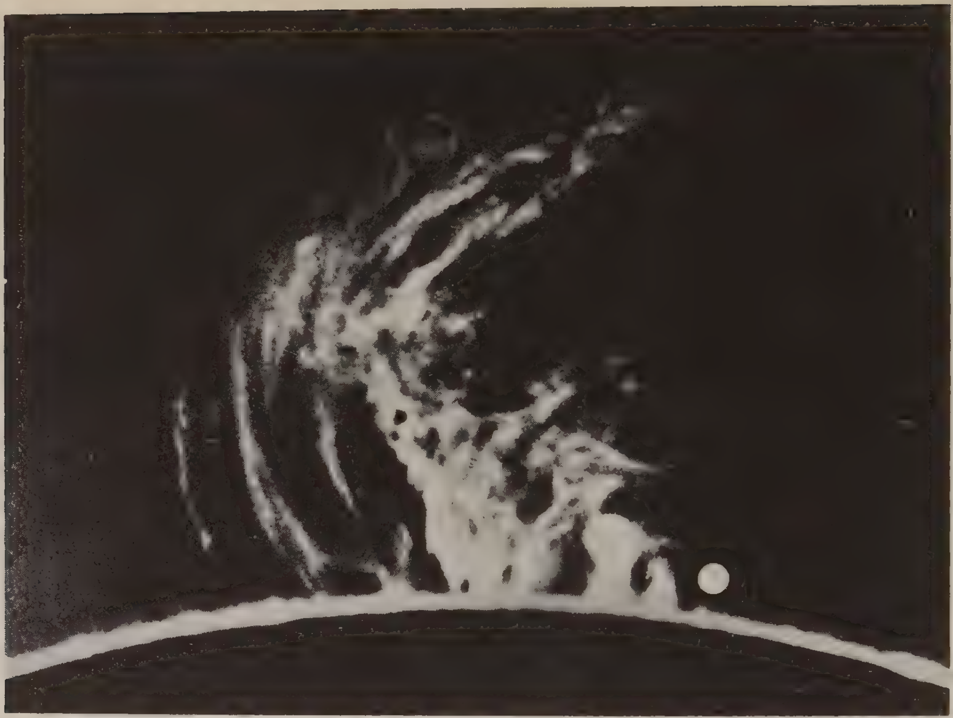


FIGURE 1424a.—Eclipses of the sun and moon.



Courtesy of Mt. Wilson and Palomar Observatories.

FIGURE 1424b.—Solar prominence, 140,000 miles high, photographed in light of calcium. July 9, 1917. Small white disk shows relative size of earth.

A total eclipse is a spectacular phenomenon. As the last light from the sun is cut off, the solar **corona,** or envelope of thin, illuminated gas around the sun, becomes visible. Wisps of more dense gas may appear as **solar prominences** (fig. 1424b). The only light reaching the observer is that diffused by the atmosphere surrounding the shadow. As the moon appears to continue on across the face of the sun, the sun finally emerges from the other side, first as Baily's Beads, and then as an ever widening crescent until no part of its surface is obscured by the moon.

The duration of a total eclipse depends upon how nearly the moon crosses the center of the sun, the location of the shadow on the earth, the relative orbital speeds of the moon and earth, and (principally) the relative apparent diameters of the sun and moon. The maximum length that can occur is a little more than seven minutes.

If the apparent diameter of the moon is less than that of the sun, its shadow does not quite reach the earth. Over a small area of the earth directly in line with the moon and sun, the moon appears as a black disk almost covering the surface of the sun, but with a thin ring of the sun around its edge. This is an **annular eclipse,** and occurs a little oftener than a total eclipse.

If the shadow of the moon passes close to the earth, but not directly in line with it, a partial eclipse may occur without a total or annular eclipse.

An eclipse of the moon occurs when the moon passes through the shadow of the earth, as shown in figure 1424a. Since the diameter of the earth is about three and one-half times that of the moon, the earth's shadow at the distance of the moon is much larger than that of the moon. A total eclipse of the moon can last nearly one and three-quarters hours, and some part of the moon may be in the earth's shadow for almost four hours. During a total **solar eclipse** no part of the sun is visible because a

body (the moon) intervenes in the line of sight. During a **lunar eclipse** some light does reach the moon because of diffraction by the atmosphere of the earth, and hence the eclipsed full moon is visible as a faint reddish disk. A lunar eclipse is visible over the entire hemisphere of the earth facing the moon. Anyone who can see the moon can see the eclipse.

During any one year there may be as many as five eclipses of the sun, and always there are at least two. There may be as many as three eclipses of the moon, or none. The total number of eclipses during a single year does not exceed seven, and can be as few as two. There are more solar than lunar eclipses, but the latter are more numerous at any one place because of the restricted areas over which solar eclipses are visible.

The two points of intersection of the moon's orbit and the ecliptic are called **nodes,** and the line connecting them, the **line of nodes.** Eclipses occur when the sun, earth, and moon are nearly on this line, twice each **eclipse year** of 346.6 days. This is less than a calendar year because of regression of the nodes (art. 1423). In a little more than 18 years the line of nodes returns to approximately the same position with respect to the sun, earth, and moon. During an almost equal period, called the **saros,** a cycle of eclipses occurs. During the following saros the cycle is repeated with only minor differences.

Eclipses have considerable value in establishing additional facts about the sun and moon, and in determining distances between two widely separated points on the earth, at which accurate timing of the eclipse is made.

Coordinates

1425. Latitude and longitude are coordinates used for locating positions on the earth. Several types, differing slightly from each other, are defined. Three of these are discussed here.

Astronomical latitude is the angle (*ABQ*, fig. 1425) between a line in the direction of gravity (*AB*) and the plane of the equator (*QQ′*). **Astronomical longitude** is the angle between the plane of the celestial meridian and the plane of the prime meridian. These coordinates are customarily found by means of celestial observations. If the earth were perfectly homogeneous and level, these positions would be consistent and satisfactory. However, because of **deflection of the vertical** (art. 1610) due to uneven distribution of the mass of the earth, lines of equal astronomical latitude and longitude are not circles, although the irregularities are small. In the United States the prime-vertical component (affecting longitude) may be a little more than 18″, and the meridional component (affecting latitude) as much as 25″.

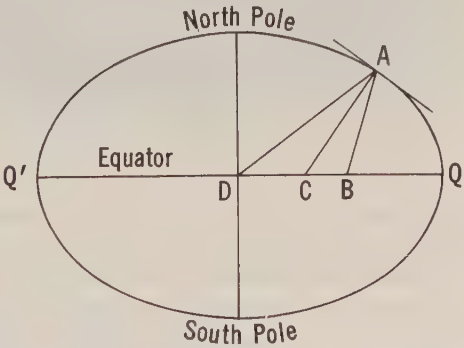


FIGURE 1425.—Three kinds of latitude at point *A*.

Geodetic latitude is the angle (*ACQ*, fig. 1425) between a normal to the spheroid (*AC*) and the plane of the equator (*QQ′*). **Geodetic longitude** is the angle between the plane defined by the normal to the spheroid and the axis of the earth, and the plane of the prime meridian. These values are obtained when astronomical latitude and longitude are corrected for deflection of the vertical. These coordinates are the ones used for charting, and are frequently referred to as **geographic latitude** and **geographic longi-**

tude, although these expressions are sometimes used to refer to astronomical latitude and longitude.

Geocentric latitude is the angle (*ADQ*, fig. 1425) between a line to the center of the earth (*AD*) and the plane of the equator (*QQ'*). This differs from geodetic latitude because the earth is a spheroid, rather than a sphere, and the meridians are ellipses. Since the parallels of latitude are considered to be circles, geodetic longitude is geocentric, and a separate expression is not used. The difference between geocentric and geodetic latitudes is a maximum of about 11.'6 at latitude 45°.

Because of the oblate shape of the earth, the length of a degree of geodetic latitude is not everywhere the same, increasing from about 59.7 nautical miles at the equator to about 60.3 nautical miles at the poles, as shown by table 6. The value of 60 nautical miles customarily used everywhere by the navigator is correct at about latitude 45°.

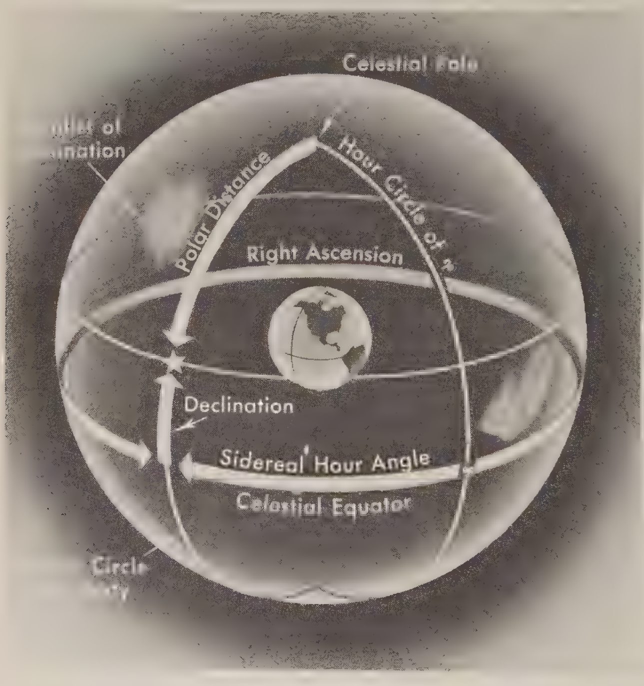


FIGURE 1426a.—Declination and sidereal hour angle.

1426. Celestial equator system.—Positions on the celestial sphere are located by any of several sets of coordinates analogous to latitude and longitude on the earth. The most directly related system is based upon the **celestial equator** (sometimes called **equinoctial**). This is the great circle formed on the celestial sphere by extension of the plane of the equator of the earth (fig. 1426a). **Declination** (**d**) is measured northward or southward from the celestial equator, similar to latitude on the earth. Like latitude, it is labeled N or S (sometimes (+) instead of N and (−) instead of S) to indicate the direction of measurement, and has a maximum value of 90° at the north and south **celestial poles,** which are directly over corresponding poles of the earth. The celestial pole above the horizon is called the **elevated pole,** and that below the horizon the **depressed pole.** The angular distance from a celestial pole, usually the elevated pole, is called **polar distance** (**p**). Polar distance is 90° *minus* declination if the body is between the celestial equator and the pole; otherwise it is 90° *plus* declination. A circle of the celestial sphere, parallel to the plane of the celestial equator, is called a **parallel of declination.** This would be the diurnal circle (art. 1416) of a body having constant declination.

A great circle through the celestial poles and the zenith (art. 1428) of an observer is his **celestial meridian** (fig. 1426b). That half which includes his zenith, and ends at the celestial poles, is called the **upper branch.** The other half is the **lower branch.** A reference to a celestial meridian is generally understood to mean the upper branch unless the lower branch is specified. The celestial meridian remains stationary over a meridian on the earth, and does not participate in the daily apparent rotation of the celestial sphere. A similar great circle of the celestial sphere, but related to a point on that sphere, or to a celestial body, is called an **hour circle** or **circle of declination**

(fig. 1426a). The hour circle through the point vertically above an observer coincides with his celestial meridian.

The hour circle through the vernal equinox (art. 1419) is used as a reference somewhat analogous to the prime meridian on the earth. It is the origin from which **sidereal hour angle** (**SHA**) is measured, westward through 360°. Thus, sidereal hour angle is similar to longitude on the earth, except that longitude is measured either eastward or westward through 180°. If the vernal equinox and all celestial bodies were fixed points, both declination and sidereal hour angle of celestial bodies would remain the same, but these coordinates change as a body alters its position on the celestial sphere, and also as precession of the equinoxes (art. 1419) takes place, resulting in movement of the vernal equinox and celestial equator. Sidereal hour angle is used primarily by navigators. Astronomers usually measure *eastward* in time units, through 24 hours. This quantity is called **right ascension** (**RA**). Thus, converted to the same units, $SHA+RA=360°=24^h$.

Measurement is often made from a celestial meridian rather than from the hour circle of the first point of Aries. This is another form of **hour angle** (**HA**) which, like sidereal hour angle, is measured westward through 360° (fig. 1426b). It is usually designated **Greenwich hour angle** (**GHA**) or **local hour angle** (**LHA**) depending upon whether the Greenwich or local celestial meridian is used as the reference. If measurement is made from the local celestial meridian, *either* eastward or westward through 180°, similar to measurement of longitude on the earth, the quantity is called **meridian angle** (**t**). This is the angle between the plane of the celestial meridian of the observer and the plane of the hour circle of the body. Because of the apparent daily rotation of the celestial sphere, hour angle continually increases, but meridian angle increases from 0° at the celestial meridian to 180° W, which is also 180° E, and then decreases to 0° again. The rate of change for the mean sun (art. 1421) is 15° per hour. The rate of all other bodies except the moon is within 3′ of this value. The average rate of the moon is about 14°5.

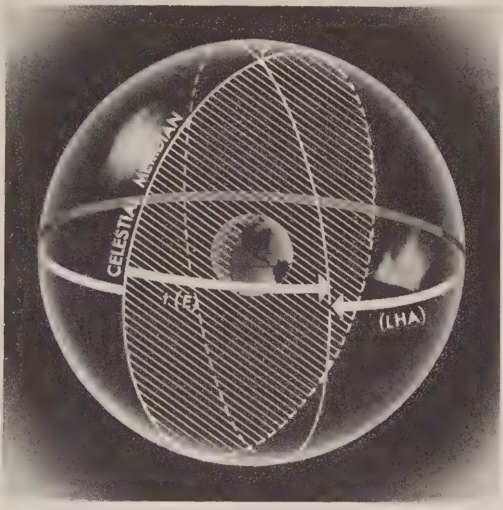


FIGURE 1426b.—Local hour angle and meridian angle.

As the celestial sphere rotates, each body crosses each branch of the celestial meridian approximately once a day. This crossing is called **meridian transit** (sometimes called **culmination**). It may be called **upper transit** to indicate crossing of the upper branch of the celestial meridian, and **lower transit** to indicate crossing of the lower branch.

1427. Time diagram.—To an observer outside the celestial sphere (if this were possible), at such a distance that his view would be orthographic, the outer limit of the sphere would appear as a great circle. If he were over one of the celestial poles, the circle would be the celestial equator. Parallels of declination would appear as circles concentric with, but usually smaller than, the celestial equator. Hour circles would appear as radials. If the observer were over the *south* celestial pole and rotating with the earth, the celestial sphere would appear to rotate in a counterclockwise direction. The difference in hour angle of any two bodies would be indicated by the angle between

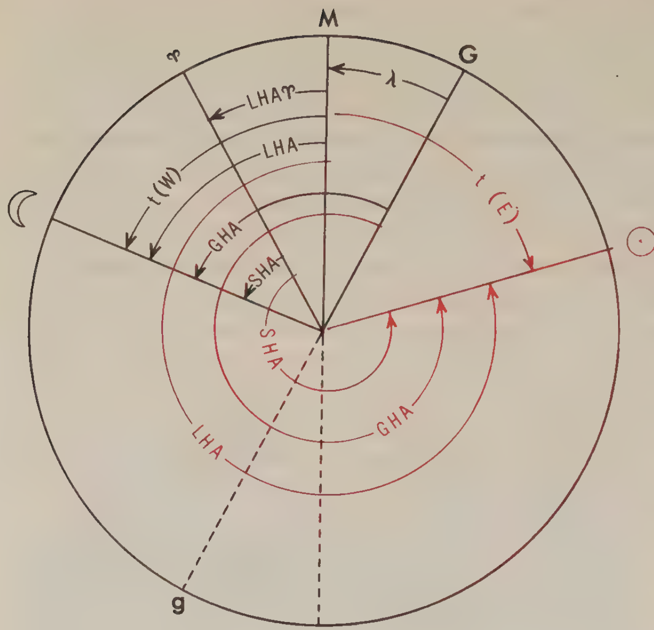


FIGURE 1427a.—Time diagram.

their hour circles. The hour angles themselves would be indicated by the angles, at the pole, between a celestial meridian and the hour circles. On a reduced scale, and without benefit of actual circles showing in the sky, such a view is available to a northern-hemisphere observer who looks at the north celestial pole (approximately at Polaris) and observes stars of high northerly declination, such as those of the big dipper and *Cassiopeia*, circle the sky.

A diagram based upon this concept, called a **time diagram** or **diagram on the plane of the celestial equator,** can be useful in visualizing the relationships of the various "longitude" terms of article 1426. Refer to figure 1427a. The circle represents the celestial equator. The parallels of declination are not of concern, and are omitted. By convention, the diagram is oriented so that the upper branch of the local celestial meridian is at the top, labeled M. The lower branch is shown as a broken line, labeled m. The observer in this case is at longitude 27° W. Therefore, the Greenwich meridian is 27° *east*, as shown. It is labeled G and g to indicate the upper and lower branches, respectively. The vernal equinox is 30° west of the meridian, as shown, and labeled ♈. The moon is 70° west of the celestial meridian, and the sun is 75° east of the celestial meridian. Various quantities are shown and labeled. To one who knows these relationships, a time diagram is often useful in visualizing conditions of a given situation, particularly when some quantities are to be found from others.

Example.—An observer is at longitude 104° W. GHA ♈ is 195°. The SHA of a star is 206°.

Required.—(1) LHA, (2) t, (3) LHA ♈.

Solution.—Draw the diagram, as shown in figure 1427b. From the diagram determine the required relationships: (1) LHA=GHA ♈+SHA−λ. (2) t=360°−LHA. (3) LHA ♈=GHA ♈−λ.

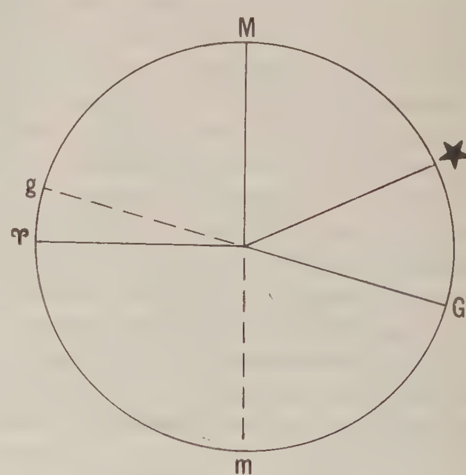


FIGURE 1427b.—Solution by time diagram.

Answers.—(1) LHA 297°, (2) t 63° E, (3) LHA ♈ 91°.

1428. Horizon system.—The celestial equator system of coordinates is not convenient for locating celestial bodies relative to an observer. For this purpose the horizon system is preferable. In this system, the **horizon** is the primary great circle analogous to the equator (fig. 1428a). The **zenith** (Z) is that point directly overhead

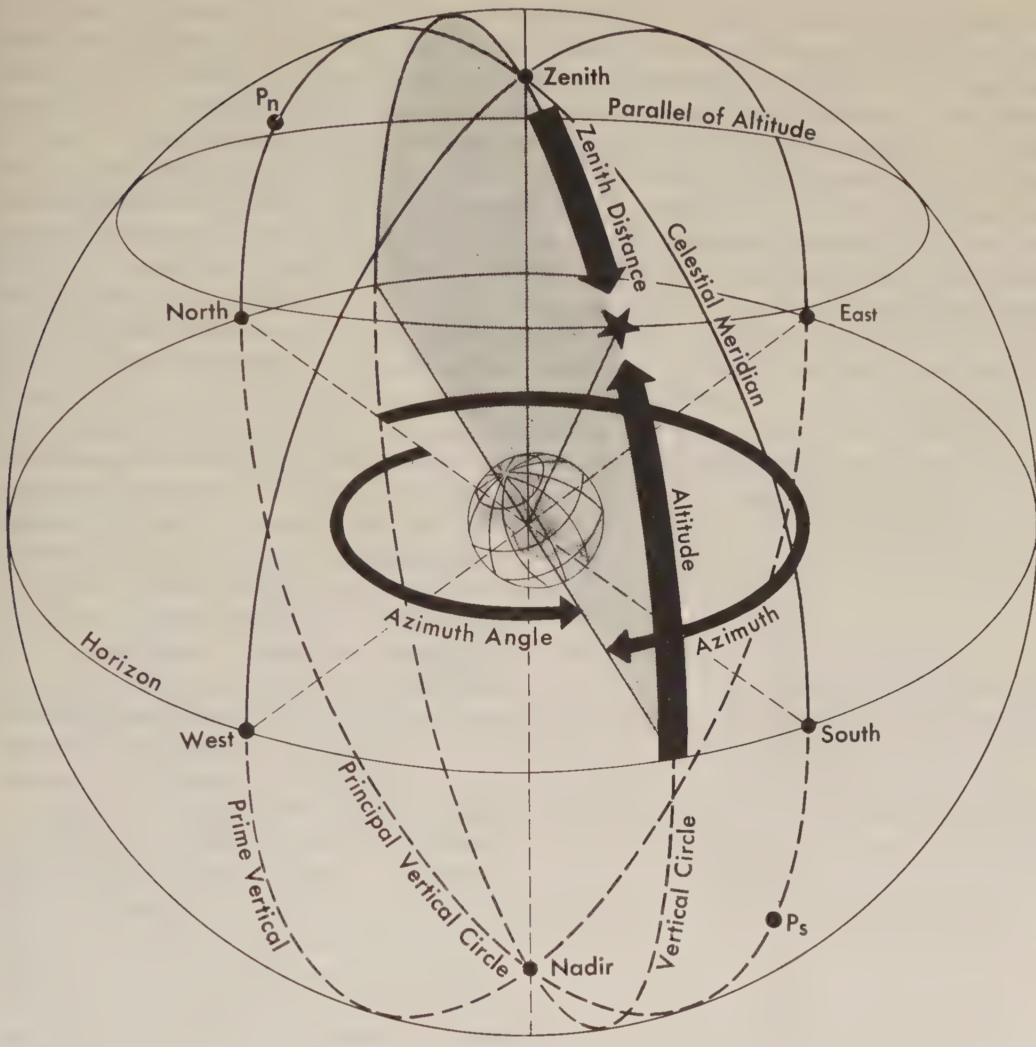


FIGURE 1428a.—The horizon system of coordinates.

(opposite to the direction of gravity). The **nadir** (**Na**) is 180° from the zenith. The zenith and nadir are the poles of the system. A circle parallel to the plane of the horizon is a **parallel of altitude** (sometimes called an **altitude circle** or **almucantar**). Angular distance above the horizon is **altitude** (**h**); angular distance from the zenith is **zenith distance** (**z**). Great circles through the zenith and nadir, and therefore perpendicular to the horizon, are **vertical circles.** The vertical circle through the east and west points of the horizon is the **prime vertical circle,** or, as usually stated, the **prime vertical** (**PV**). The vertical circle through the north and south points of the horizon is the **principal vertical circle.** It is also the celestial meridian (art. 1426).

The angular difference between north and any other horizontal direction (the bearing) is called **azimuth** (**Zn**) when referred to a celestial body. Azimuth, like bearing, is measured clockwise around the horizon from 000° at north through 360°. It is usually expressed in three figures. Sometimes it is convenient to express direction in terms of **azimuth angle** (**Z**) which is usually measured from the direction of the elevated pole (north or south to agree with the latitude), through 180°, but occasionally from the *nearer* north or south point, through 90°. Azimuth angle is labeled to avoid

ambiguity. It is given a prefix, N or S, to indicate the origin of measurement, and a suffix, E or W, to indicate the direction of measurement. By means of these labels azimuth angle can be converted to azimuth. Thus, N 37° W means that the given direction is found by starting at north and measuring 37° in a westerly direction, or 360°−37°=323°. Similarly S 164° W is 164° west of south, or 180°+164°=344°. In converting from azimuth to azimuth angle, one must know the name (N or S) of the latitude if the 180° system is used. Thus, Zn 068° is equal to N 68° E in north latitude, and S 112° E in south latitude. If the 90° system is used, it could be N 68° E only, since this system is without ambiguity except at east and west. In either the 90° or 180° system the suffix agrees with meridian angle, for if a body is east by meridian angle, it is also east by azimuth angle. If LHA is less than 180°, the body is west of the meridian, and hence has an azimuth of more than 180°. Thus, *both* LHA and Zn cannot be either less or greater than 180°.

At rising or setting, a body is not on the prime vertical unless its declination is zero. The arc of the horizon between the prime vertical and the body is its **amplitude** (**A**). This is given the prefix E (east) if the body is rising and W (west) if setting. It is given the suffix N if the body rises or sets north of the prime vertical (which it does if it has northerly declination) and S if it rises or sets south of the prime vertical (having southerly declination). Interconversion of amplitude and azimuth is similar to that of azimuth angle and azimuth. Thus, if A=E 15° S, the body is 15° south of east, or 90°+15°=105°. For any given body, the numerical value of amplitude would be the same at rising and setting if the declination did not change. Amplitudes to a declination range of 24° are given in table 27. A correction to convert the observed value when the body is on the apparent horizon to the corresponding value it would have if the body were on the celestial horizon (tab. 27) is given in table 28.

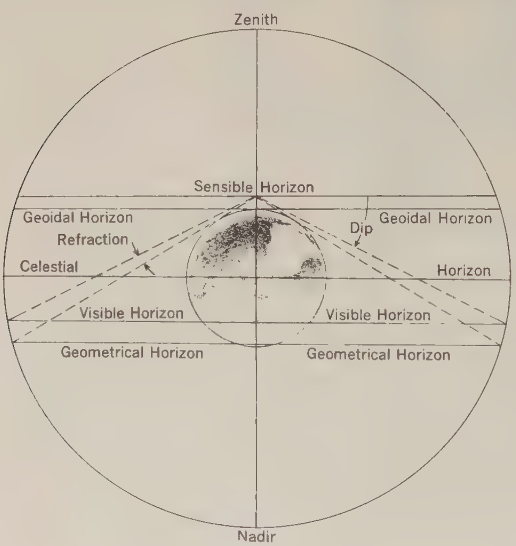


FIGURE 1428b.—The horizons.

The horizon system of coordinates is based upon the **celestial horizon** (sometimes called **rational horizon**). This is a great circle (of the celestial sphere) midway between the zenith and nadir. Its plane passes through the center of the earth, and is perpendicular to the zenith-nadir line (fig. 1428b). At the infinite distance of the celestial sphere this is considered identical with the **sensible horizon,** one having a plane parallel to that of the celestial horizon, but through the eye of the observer. At heights of eye used in marine navigation, the sensible horizon may be considered identical with **geoidal horizon,** the plane of which is parallel to that of the sensible horizon, but through the point on the **geoid** (the sea-level surface of the earth) vertically below the observer. Only the sun and moon are near enough to the earth that the difference of altitude measured from the celestial and sensible horizons has practical significance to the navigator. None of these horizons is marked by a line visible to an observer. In practice, the marine navigator usually measures altitude from the **visible horizon** and converts his readings to the corresponding value from the sensible horizon by means of dip (art. 1606). The visible horizon is the line where earth and sky appear to meet. Over land it is a somewhat irregular line, but at sea this line appears as a circle. It is

approximately a small circle of the celestial sphere differing from the sensible horizon because of height of the observer's eye above the surface, and atmospheric refraction. The **geometrical horizon** is below the visible horizon by the amount of terrestrial refraction. A straight line from the eye of the observer tangent to the earth leads to the geometrical horizon. Occasionally the expression *geometrical horizon* is used as the equivalent of *celestial horizon.*

Altitude measured from the celestial horizon is the complement of zenith distance. Celestial bodies *below* the celestial horizon have **negative altitude,** or a zenith distance of more than 90°. In this case, $z=90°-(-h)=90°+h$, when h is measured downward or negatively from the celestial horizon. Because of dip a body may have slight negative altitude and still be above the visible horizon.

1429. Ecliptic system.—The ecliptic system is based upon the **ecliptic** as the primary great circle, analogous to the equator. The points 90° from the ecliptic are the north and south **ecliptic poles.** The series of great circles through these poles, analogous to meridians, are **circles of latitude.** The circles parallel to the plane of the ecliptic, analogous to parallels on the earth, are **parallels of latitude** or **circles of longitude.** Angular distance north or south of the ecliptic, analogous to latitude, is **celestial latitude. Celestial longitude** is measured *eastward* along the ecliptic through 360°, starting at the vernal equinox. This system of coordinates is of interest chiefly to astronomers.

1430. Galactic system.—Another system of interest primarily to astronomers is based upon a great circle called the **galactic equator,** considered to be in the plane of the galaxy. The north and south **galactic poles** are 90° from the galactic equator. **Galactic latitude** is measured north and south from the galactic equator. **Galactic longitude** is measured eastward from a point on the galactic equator at about SHA 84°24′, declination 28°55′ S in 1950.

1431. Summary of coordinate systems.—The four systems of celestial coordinates are analogous to each other and to the terrestrial system, although each has distinctions such as differences in directions, units, and limits of measurement. The following table indicates the analogous term or terms under each system. For differences, see the description of each system, given earlier in the chapter, or appendix E.

Earth	Celestial Equator	Horizon	Ecliptic	Galactic
equator	celestial equator	horizon	ecliptic	galactic equator
poles	celestial poles	zenith, nadir	ecliptic poles	galactic poles
meridians	hour circles, celestial meridians	vertical circles	circles of latitude	
prime meridian	hour circle ♈, Greenwich celestial meridian, local celestial meridian	principal vertical circle, prime vertical circle	circle of latitude through ♈	great circle through galactic poles and intersection of galactic equator at about SHA 84°24′, declination 28°55′ S (1950)
parallels	parallels of declination	parallels of altitude	parallels of latitude	
latitude	declination	altitude	celestial latitude	galactic latitude
colatitude	polar distance	zenith distance	celestial colatitude	galactic colatitude
longitude	SHA, RA, GHA, LHA, t	azimuth, azimuth angle, amplitude	celestial longitude	galactic longitude

1432. Diagram on the plane of the celestial meridian.—From a point outside the celestial sphere (if this were possible) and over the celestial equator, at such a distance that the view would be orthographic, the great circle appearing as the outer limit would be a celestial meridian. Other celestial meridians would appear as ellipses. The celestial equator would appear as a diameter 90° from the poles, and parallels of declina-

tion as straight lines parallel to the equator. The view would be similar to the orthographic view of the earth, as shown in figure 319b.

A number of useful relationships can be demonstrated by drawing a **diagram on the plane of the celestial meridian** showing this orthographic view. Arcs of circles can be substituted for the ellipses without destroying the basic relationships. Refer to figure 1432a. In the lower diagram the circle represents the celestial meridian, QQ' the celestial equator, Pn and Ps the north and south celestial poles, respectively. If a star has a declination of 30° N, an angle of 30° can be measured from the celestial equator, as

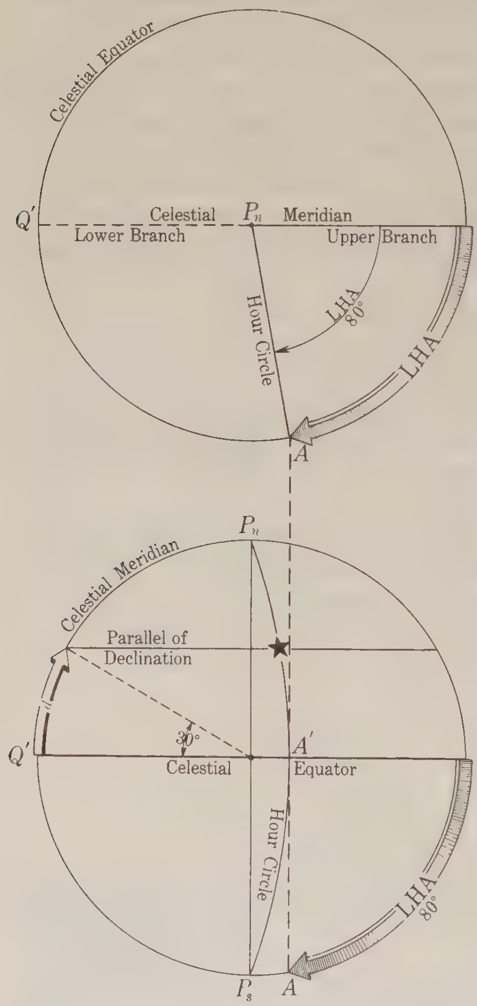


FIGURE 1432a.—Measurement of celestial equator system of coordinates.

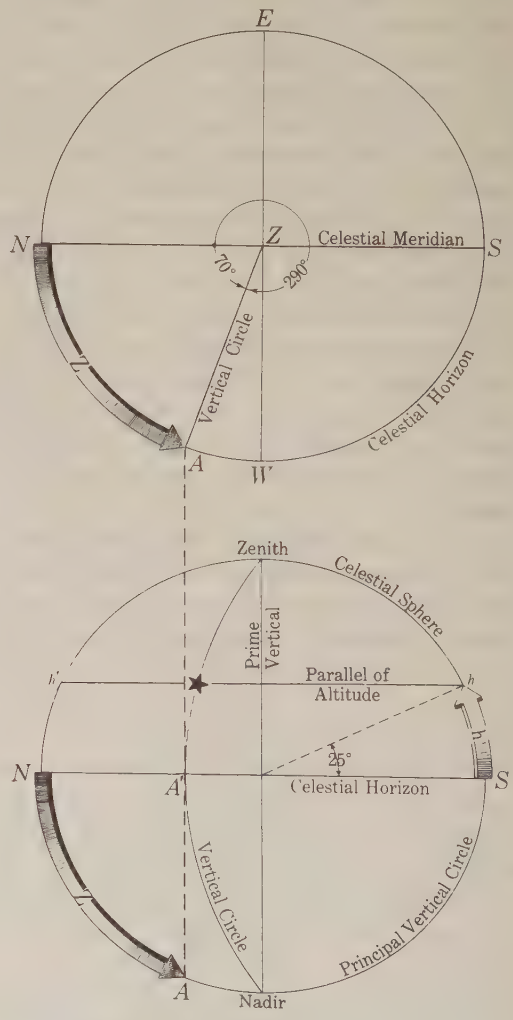


FIGURE 1432b.—Measurement of horizon system of coordinates.

shown. It could be measured either to the right or left, and would have been toward the south pole if the declination had been south. The parallel of declination is a line through this point and parallel to the celestial equator. The star is somewhere on this line (actually a circle viewed on edge).

To locate the hour circle, draw the upper diagram so that Pn is directly above Pn of the lower figure (in line with the polar axis Pn Ps), and the circle is of the same diameter as that of the lower figure. This is the plan view, looking down on the celestial sphere from the top. The circle is the celestial equator. Since the view is from above

the *north* celestial pole, west is clockwise. The diameter QQ' is the celestial meridian shown as a circle in the lower diagram. If the *right* half is considered the upper branch, local hour angle is measured clockwise from this line to the hour circle, as shown. In this case the LHA is 80°. The intersection of the hour circle and celestial equator, point A, can be projected down to the lower diagram (point A') by a straight line parallel to the polar axis. The elliptical hour circle can be represented approximately by an arc of a circle through A', Pn, Ps. The center of this circle is somewhere along the celestial equator line QQ', extended if necessary. It is usually found by trial and error. The intersection of the hour circle and parallel of declination locates the star.

Since the upper diagram serves only to locate point A' in the lower diagram, the two can be combined. That is, the LHA arc can be drawn in the lower diagram, as shown, and point A projected *upward* to A'. In practice, the upper diagram is not drawn, being shown here for illustrative purposes only.

In this example the star is on that half of the sphere toward the observer, or the *western* part. If LHA had been greater than 180°, the body would have been on the *eastern* or "back" side.

From the east or west point over the celestial horizon, the orthographic view of the horizon system of coordinates would be similar to that of the celestial equator system from a point over the celestial equator (fig. 1432a), since the celestial meridian is also the principal vertical circle. The horizon would appear as a diameter, parallels of altitude as straight lines parallel to the horizon, the zenith and nadir as poles 90° from the horizon, and vertical circles as ellipses through the zenith and nadir, except for the principal vertical circle, which would appear as a circle, and the prime vertical, which would appear as a diameter perpendicular to the horizon.

A celestial body can be located by altitude and azimuth in a manner similar to that used with the celestial equator system. If the altitude is 25°, this angle is measured from the horizon toward the zenith and the parallel of altitude is drawn as a straight line parallel to the horizon, as shown at hh' in the lower diagram of figure 1432b. The plan view from above the zenith is shown in the upper diagram. If north is taken at the left, as shown, azimuths are measured clockwise from this point. In the figure the azimuth is 290° and the azimuth angle is N 70° W. The vertical circle is located by measuring either arc. Point A thus located can be projected vertically downward to A' on the horizon of the lower diagram, and the vertical circle represented approximately by the arc of a circle through A' and the zenith and nadir. The center of this circle is on NS, extended if necessary. The body is at the intersection of the parallel of altitude and the vertical circle. Since the upper diagram serves only to locate A' on the lower diagram, the two can be combined, point A located on the lower diagram and projected upward to A', as shown. Since the body of the example has an azimuth greater than 180°, it is on the western or "front" side of the diagram.

Since the celestial meridian appears the same in both the celestial equator and horizon systems, the two diagrams can be combined and, if properly oriented, a body can be located by one set of coordinates, and the coordinates of the other system can be determined by measurement.

Refer to figure 1432c, in which the black lines represent the celestial equator system, and the red lines the horizon system. By convention, the zenith is shown at the top and the north point of the horizon at the left. The west point on the horizon is at the center, and the east point directly behind it. In the figure the latitude is 37° N. Therefore, the zenith is 37° north of the celestial equator. Since the zenith is established at the top of the diagram, the equator can be found by measuring an arc of 37° toward the south, along the celestial meridian. If the declination is 30° N and the LHA is 80°, the body can be located as shown by the black lines, and described above.

The altitude and azimuth can be determined by the reverse process to that described above. Draw a line *hh′* through the body and parallel to the horizon, *NS*. The altitude, 25°, is found by measurement, as shown. Draw the arc of a circle through the body and the zenith and nadir. From *A′*, the intersection of this arc with the horizon, draw a vertical line intersecting the circle at *A*. The azimuth, N 70° W, is found by measurement, as shown. The prefix N is applied to agree with the latitude. The body is left (*north*) of *ZNa*, the prime vertical circle. The suffix *W* applies because the LHA, 80°, shows that the body is west of the meridian.

If altitude and azimuth are given, the body is located by means of the red lines. The parallel of declination is then drawn parallel to *QQ′*, the celestial equator, and the declination determined by measurement. Point *L′* is located by drawing the arc of a circle through *Pn*, the star, and *Ps*. From *L′* a line is drawn perpendicular to *QQ′*, locating *L*. The meridian angle is then found by measurement. The declination is known to be north because the body is between the celestial equator and the north celestial pole. The meridian angle is west to agree with the azimuth, and hence LHA is numerically the same.

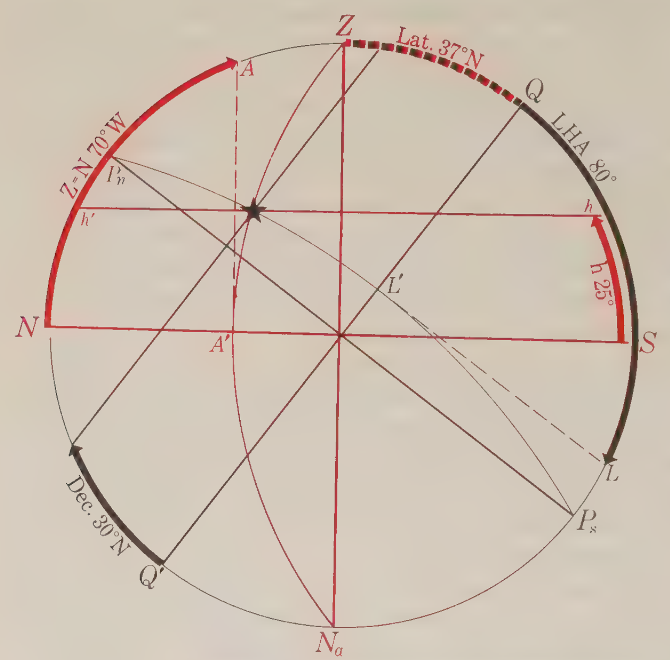


FIGURE 1432c.—Diagram on the plane of the celestial meridan.

Since *QQ′* and *PnPs* are perpendicular, and *ZNa* and *NS* are also perpendicular, arc *NPn* is equal to arc *ZQ*. That is, *the altitude of the elevated pole is equal to the declination of the zenith, which is equal to the latitude.* This relationship is the basis of the method of determining latitude by an observation of Polaris (art. 2105).

The diagram on the plane of the celestial meridian is useful in approximating a number of relationships. Consider figure 1432d. The latitude of the observer (*NPn* or *ZQ*) is 45° N. The declination of the sun (*Q4*) is 20° N. Neglecting the change in declination for one day, note the following: At sunrise, position 1, the sun is on the horizon (*NS*), at the "back" of the diagram. Its altitude, h, is 0°. Its azimuth angle, Z, is the arc *NA*, N 63° E. This is prefixed N to agree with the latitude and suffixed E to agree with the meridian angle of the sun at sunrise. Hence, Zn=0°+63°=063°. The amplitude, A, is the arc *ZA*, E 27° N. The meridian angle, t, is the arc *QL*, 110° E. The suffix E is applied because the sun is east of the meridian at rising. The LHA is 360°−110°=250°.

As the sun moves upward along its parallel of declination, its altitude increases. It reaches position 2 at about 0600, when t=90° E. At position 3 it is on the prime vertical, *ZNa*. Its azimuth angle, Z, is N 90° E, and Zn=090°. The altitude is *Nh′* or *Sh*, 27°.

Moving on up its parallel of declination, it arrives at position 4 on the celestial meridian about noon—when t and LHA are both 0°, by definition. On the celestial meridian a body's azimuth is 000° or 180°. In this case it is 180° because the body is south of the zenith. The maximum altitude occurs at meridian transit, in this case

the arc $S4$, 65°. The zenith distance, z, is the arc $Z4$, 25°. A body is not in the zenith at meridian transit unless its declination is numerically, and by name, the same as the latitude.

Continuing on, the sun moves downward along the "front" or western side of the diagram. At position 3 it is again on the prime vertical. The altitude is the same as when previously on the prime vertical, and the azimuth angle is numerically the same, but now measured toward the west. The azimuth is 270°. The sun reaches position 2 six hours after meridian transit, and sets at position 1, when the azimuth angle is numerically the same as at sunrise, but westerly, and Zn=360°−63°=297°. The amplitude is W 27° N.

After sunset the sun continues on downward along its parallel of declination until it reaches position 5, on the lower branch of the celestial meridian, about midnight. Its negative altitude, arc $N5$, is now greatest, 25°, and its azimuth is 000°. At this point it starts back up along the "back" of the diagram, arriving at position 1 at the next sunrise, to start another cycle.

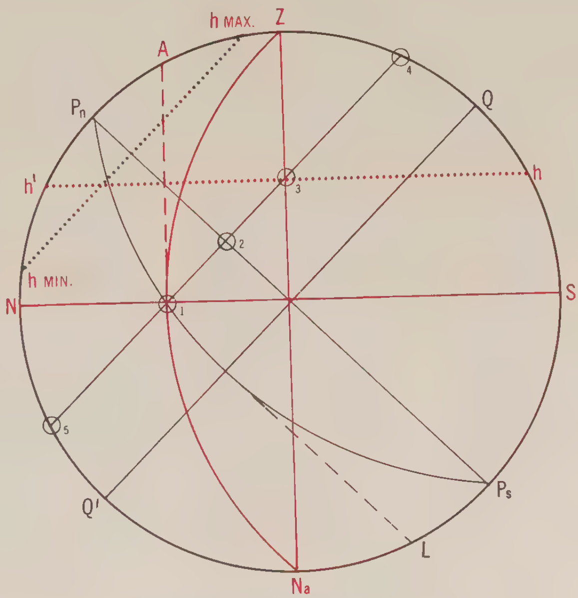


FIGURE 1432d.—A diagram on the plane of the celestial meridian for lat. 45° N.

Half the cycle is from the crossing of the 90° hour circle (the $PnPs$ line, position 2) to the upper branch of the celestial meridian (position 4) and back to the $PnPs$ line (position 2). When the declination and latitude have the **same name** (both north or both south), more than half the parallel of declination (position 1 to 4 to 1) is above the horizon, and the body is above the horizon more than half the time, crossing the 90° hour circle above the horizon. It rises and sets on the same side of the prime vertical as the elevated pole. If the declination is of the same name but numerically smaller than the latitude, the body crosses the prime vertical above the horizon. If the declination and latitude have the same name and are numerically equal, the body is in the zenith at upper transit. If the declination is of the same name but numerically *greater* than the latitude, the body crosses the upper branch of the celestial meridian between the zenith and elevated pole, and does not cross the prime vertical. If the declination is of the same name as the latitude and complementary to it (d+L=90°), the body is on the horizon at lower transit, and does not set. If the declination is of the same name as the latitude and numerically *greater* than the colatitude, the body is above the horizon during its entire daily cycle, and has maximum and minimum altitudes, as shown by the black dotted line in figure 1432d.

If the declination is 0° at any latitude, the body is above the horizon half the time, following the celestial equator QQ', and rising and setting on the prime vertical. If the declination is of **contrary name** (one north and the other south), the body is above the horizon less than half the time, and crosses the 90° hour circle below the horizon. It rises and sets on the opposite side of the prime vertical from the elevated pole. If the declination is of contrary name and numerically smaller than the latitude, the body

crosses the prime vertical below the horizon. This is the situation with the sun in winter, when days are short. If the declination is of contrary name and numerically equal to the latitude, the body is in the nadir at lower transit. If the declination is of contrary name and complementary to the latitude, the body is on the horizon at upper transit. If the declination is of contrary name and numerically greater than the colatitude, the body does not rise.

All of these relationships, and those that follow, can be derived by means of a diagram on the plane of the celestial meridian. They are modified slightly by atmospheric refraction, height of eye, semidiameter, parallax, changes in declination, and apparent speed of the body along its diurnal circle.

It is customary to keep the same orientation in south latitude, as shown in figure 1432e. In this illustration the latitude is 45° S, and the declination of the body is 15° N. Since *Ps* is the elevated pole, it is shown above the southern horizon, with both *SPs* and *ZQ* equal to the latitude, 45°. The body rises at position 1, on the opposite side of the prime vertical from the elevated pole; moves upward along its parallel of declination to position 2, on the upper branch of the celestial meridian, bearing north; and then downward along the "front" of the diagram to position 1, where it sets; remaining above the horizon for less than half the time because declination and latitude are of contrary name. The azimuth at rising is arc *NA*, the amplitude *ZA*, and the azimuth angle *SA*. The altitude circle at meridian transit is shown at *hh′*.

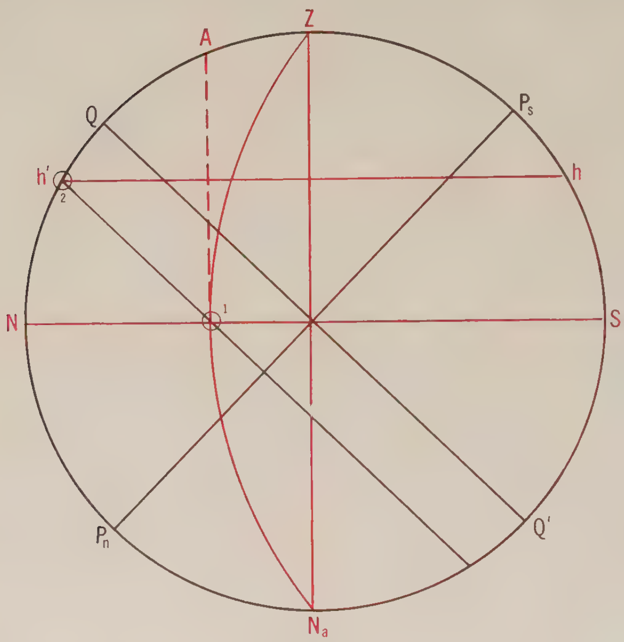


FIGURE 1432e.—A diagram on the plane of the celestial meridian for lat. 45° S.

A diagram on the plane of the celestial meridian can be used to demonstrate the effect of a change in latitude. As the latitude increases, the celestial equator becomes more nearly parallel to the horizon. The colatitude becomes smaller, increasing the number of circumpolar bodies and those which neither rise nor set, and also increasing the difference in the length of the days between summer and winter. At the poles (fig. 1416b), celestial bodies circle the sky, parallel to the horizon. At the equator (fig. 1416a) the 90° hour circle coincides with the horizon. Bodies rise and set vertically; and are above the horizon half the time. At rising and setting the amplitude is equal to the declination. At meridian transit the altitude is equal to the codeclination. As the latitude changes name, the same-contrary name relationship with declination reverses. This accounts for the fact that one hemisphere has winter while the other is having summer.

The error arising from showing the hour circles and vertical circles as arcs of circles instead of ellipses increases with increased declination or altitude. More accurate results can be obtained by measurement of azimuth on the parallel of altitude instead of the horizon, and of hour angle on the parallel of declination instead

of the celestial equator. Refer to figure 1432f. The vertical circle shown is for a body having an azimuth angle of S 60° W. The arc of a circle is shown in black, and the ellipse in red. The black arc is obtained by measurement around the horizon, locating A' by means of A, as previously described. The intersection of this arc with the altitude circle at 60° places the body at M. If a semicircle is drawn with the altitude circle as a diameter, and the azimuth angle measured around this, to B, a perpendicular to the hour circle locates the body at M', on the ellipse. By this method the altitude circle, rather than the horizon, is, in effect, rotated through 90° for the measurement. This refinement is seldom used because actual values are usually found mathematically, the diagram on the plane of the meridian being used primarily to indicate relationships.

With experience, one may mentally visualize the diagram on the plane of the celestial meridian without making an actual drawing. Devices with two sets of spherical coordinates, on either the orthographic (art. 319) or stereographic (art. 318) projection, pivoted at the center, have been produced commercially to provide a mechanical diagram on the plane of the celestial meridian. However, since the diagram's principal use is to illustrate certain relationships, such a device is not a necessary part of the navigator's equipment.

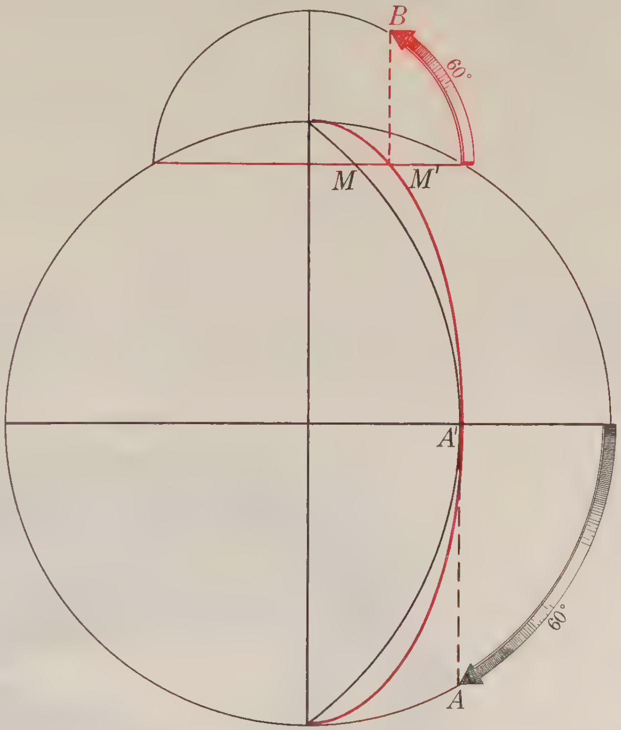


FIGURE 1432f.—Locating a point on an ellipse of a diagram on the plane of the celestial meridian.

1433. The navigational triangle.—A triangle formed by arcs of great circles of a sphere is called a **spherical triangle.** A spherical triangle on the celestial sphere is called a **celestial triangle.** The spherical triangle of particular significance to navigators is called the **navigational triangle.** It is formed by arcs of a celestial meridian, an hour circle, and a vertical circle. Its vertices are the elevated pole, the zenith, and a point on the celestial sphere (usually a celestial body). The terrestrial counterpart is also called a navigational triangle, being formed by arcs of two meridians and the great circle connecting two places on the earth, one on each meridian. The vertices are the two places and a pole. In great-circle sailing these places are the point of departure and the destination. In celestial navigation they are the **assumed position** (**AP**) of the observer and the **geographical position** (**GP**) of the body (the place having the body in its zenith). The GP of the sun is sometimes called the **subsolar point,** that of the moon the **sublunar point,** that of a satellite (either natural or artificial) the **subsatellite point,** and that of a star its **substellar** or **subastral point.** When used to solve a celestial observation, either the celestial or terrestrial triangle may be called the **astronomical triangle.**

The navigational triangle is shown in figure 1433a on a diagram on the plane of the celestial meridian, labeled as in article 1432, but with the hour circle and vertical

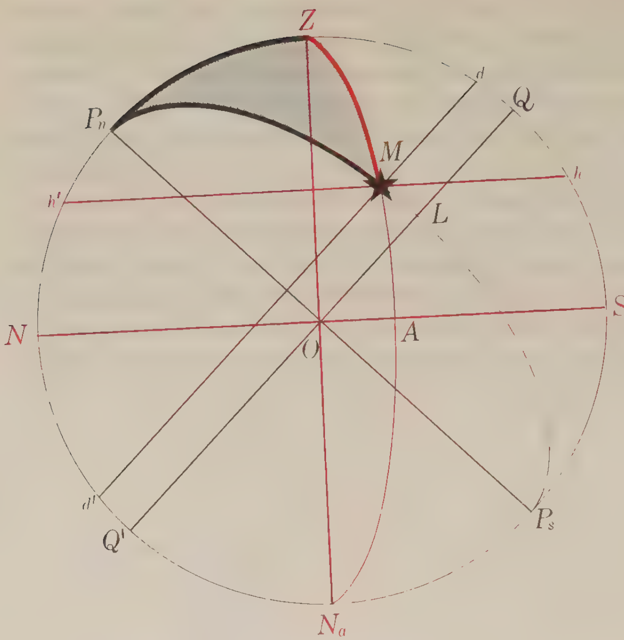


FIGURE 1433a.—The navigational triangle.

circle properly shown as ellipses. The earth is at the center, *O*. The star is at *M*, *dd'* is its parallel of declination, and *hh'* its altitude circle.

In the figure, arc *QZ* of the celestial meridian is the latitude of the observer, and *PnZ*, one side of the triangle, is the **colatitude.** Arc *AM* of the vertical circle is the altitude of the body, and side *ZM* of the triangle is the zenith distance, or **coaltitude.** Arc *LM* of the hour circle is the declination of the body, and side *PnM* of the triangle is the polar distance, or **codeclination.**

The angle at the elevated pole, *ZPnM*, having the hour circle and the celestial meridian as sides, is the meridian angle, t. The angle at the zenith, *PnZM*, having the vertical circle and that arc of the celestial meridian which includes the elevated pole as sides, is the azimuth angle. The angle at the celestial body, *ZMPn*, having the hour circle and the vertical circle as sides, is

FIGURE 1433b.—The navigational triangle in perspective.

the **parallactic angle** (X) (sometimes called the **position angle**), which is not generally used by the navigator.

A number of problems involving the navigational triangle are encountered by the navigator, either directly or indirectly. Of these, the most common are:

1. Given latitude, declination, and meridian angle, to find altitude and azimuth angle. This is used in the reduction of a celestial observation, to establish a line of position (ch. XX).
2. Given latitude, altitude, and azimuth angle, to find declination and meridian angle. This is used to identify an unknown celestial body (ch. XXII).
3. Given meridian angle, declination, and altitude, to find azimuth angle. This may be used to find azimuth when the altitude is known (ch. XX).
4. Given the latitude of two places on the earth and the difference of longitude between them, to find the initial great-circle course and the great-circle distance (ch. VIII). This involves the same parts of the triangle as in 1, above, but in the terrestrial triangle, and hence defined differently.

Both celestial and terrestrial navigational triangles are shown in perspective in figure 1433b.

Problems

1427. *Given.*—An observer is at longitude 77°E. The sun is 60° east of the meridian. GHA ♈ is 37°.

Required.—(1) LHA of the sun.

(2) GHA of the sun.

(3) SHA of the sun.

(4) Approximate time at the local meridian.

Answers.—(1) LHA 300°, (2) GHA 223°, (3) SHA 186°, (4) T 0800.

1428a. *Required.*—Convert Z to Zn in the following:

(1) N 174° E	(4) S 39° E
(2) S 1° E	(5) N 106° W
(3) S 90° W	(6) N 90° W

Answers.—(1) Zn 174°, (2) Zn 179°, (3) Zn 270°, (4) Zn 141°, (5) Zn 254°, (6) Zn 270°.

1428b. *Required.*—Convert Zn to Z in the following, using the 180° system:

Zn	*Lat.*	*Zn*	*Lat.*
(1) 214°	N	(4) 333°	S
(2) 163°	S	(5) 206°	N
(3) 007°	N	(6) 206°	S

Answers.—(1) Z N 146° W, (2) Z S 17° E, (3) Z N 7° E, (4) Z S 153° W, (5) Z N 154° W, (6) Z S 26° W.

1428c. *Required.*—Convert Zn to Z in the following, using the 90° system:

(1) 051°	(3) 251°
(2) 151°	(4) 351°

Answers.—(1) Z N 51° E, (2) Z S 29° E, (3) Z S 71° W, (4) Z N 9° W.

1428d. *Given.*—The following amplitudes:

A	*Lat.*	*A*	*Lat.*
(a) W 24° N	N	(c) E 55° S	N
(b) E 18° N	S	(d) W 4° S	S

Required.—(1) Zn, (2) Z (180° system), (3) Z (90° system).

Answers.—(1) (a) Zn 294°, (b) Zn 072°, (c) Zn 145°, (d) Zn 266°; (2) (a) Z N 66° W, (b) Z S 108° E, (c) Z N 145° E, (d) Z S 86° W; (3) (a) Z N 66° W, (b) Z N 72° E, (c) Z S 35° E, (d) Z S 86° W.

1428e. *Given.*—The following azimuth angles at rising and setting:

(1) N 80° E	(3) S 110° E
(2) N 95° W	(4) S 90° W

Required.—Amplitude.

Answers.—(1) A E 10° N, (2) A W 5° S, (3) A E 20° N, (4) A 0°.

Solve the following problems by diagrams on the plane of the celestial meridian:

1432a. *Given.*—L 32° N, t 71° W, d 27° N.

Required.—Altitude and azimuth.

Answers.—h 28°, Zn 288°.

1432b. *Given.*—L 17° S, t 64° E, d 28° S.

Required.—Altitude and azimuth.

Answers.—h 28°, Zn 115°.

1432c. *Given.*—L 59° N, h 27°, Zn 052°.

Required.—Declination and meridian angle.

Answers.—d 41° N, t 111° E.

1432d. *Given.*—L 31° N, declination of sun 18° S.

Required.—(1) Azimuth at sunrise, (2) maximum altitude, (3) altitude when the azimuth is 234°, (4) azimuth angle when the altitude in the afternoon is 10°, (5) amplitude at sunset.

Answers.—(1) Zn 111°, (2) h 41°, (3) h 18°, (4) Z N 118° W, (5) A W 21° S.

1432e. *Given.*—The declination of the star Dubhe is approximately 62° N. When observed at lower transit, its altitude is 43°.

Required.—(1) Latitude of the observer, (2) azimuth at upper transit.

Answers.—(1) L 71° N, (2) Zn 180°.

1432f. *Required.*—For an observer at latitude 39° N, find for the sun at summer and winter solstices, respectively: (1) LHA at sunrise, (2) LHA when on the prime vertical during the morning, (3) maximum altitude, (4) LHA at sunset, (5) length of daylight if the sun moves 15° per hour.

Answers.—

	Summer	*Winter*
(1) LHA	248°	292°
(2) LHA	304°	236° (below horizon)
(3) h	74°	28°
(4) LHA	112°	68°
(5) T	14^h56^m	9^h04^m

1432g. *Given.*—L 83° N, sun's declination 4° S.

Required.—(1) LHA at sunrise, (2) maximum altitude, (3) LHA at sunset, (4) length of daylight (sun moving 15° per hour).

Answers.—(1) LHA 305°, (2) max h 3°, (3) LHA 55°, (4) T 7^h20^m.

References

ELEMENTARY

Alter, D., and Cleminshaw, C. H. *Pictorial Astronomy.* New York, Crowell, 1952.

Baker, R. H. *An Introduction to Astronomy.* 6th ed. Princeton, Van Nostrand, 1961.

Hood, P. *Observing the Heavens.* New York, Oxford, 1951.

Mayall, R. N. and Mayall, M. W. *A Beginner's Guide to the Skies.* New York, Putnam, 1960.

Struve, O., Lynds, B., and Pillans, H. *Elementary Astronomy.* New York, Oxford, 1959.

Wright, H. *Palomar.* New York, Macmillan, 1952.

ADVANCED

Baker, R. H. *Astronomy.* 7th ed. Princeton, Van Nostrand, 1959.

Brouwer, D., and Clemence, G. M. *Methods of Celestial Mechanics.* New York, Academic, 1961.

Jones, Sir Harold Spencer. *General Astronomy.* 3d ed. New York, Longmans, 1951.

Krogdahl, W. S. *The Astronomical Universe.* New York, Macmillan, 1952.

Rudaux, L., and De Vaucouleurs, G. *Larousse Encyclopedia of Astronomy.* New York, Prometheus, 1959.

Russell, H. N., Dugan, R. S., and Stewart, J. Q. *Astronomy.* Vol. I, *The Solar System.* Rev. ed. 1945. Vol. II, *Astrophysics and Stellar Astronomy.* Boston, Ginn, 1938.

CHAPTER XV

INSTRUMENTS FOR CELESTIAL NAVIGATION

1501. The marine sextant is a hand-held instrument for measuring the angle between the lines of sight to two points by bringing into coincidence at the eye of the observer the direct ray from one point, and a double-reflected ray from the other, the measured angle being twice the angle between the reflecting surfaces. Its principal use is to measure the altitudes of celestial bodies above the visible sea horizon. Sometimes it is turned on its side and used for measuring the *difference* in bearing of two terrestrial objects. Because of its great value for determining position at sea, the sextant has been a symbol of navigation for more than 200 years. The quality of his instrument, the care he gives it, and the skill with which he makes observations are to the navigator matters of professional pride.

The name "sextant" is from the Latin *sextans*, "the sixth part." The arc of early marine sextants is approximately the sixth part of a circle, but because of the optical principle involved (art. 1502), the instrument measures angles of 120°. Most modern instruments measure something more than this.

1502. Principle of operation.—When a ray of light is reflected from a plane surface, the **angle of reflection** is equal to the **angle of incidence** (fig. 1502a). When the reflecting

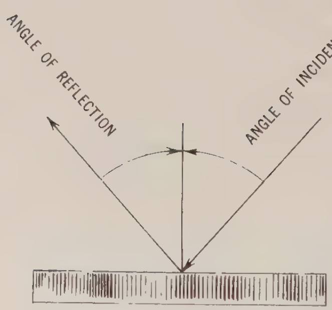


FIGURE 1502a.—Angle of reflection equals angle of incidence.

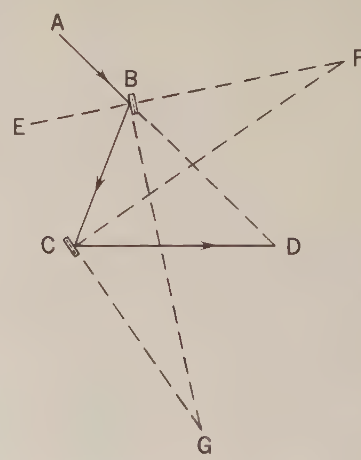


FIGURE 1502b.—Optical principle of the marine sextant.

surface is rotated toward or away from the incident ray, each angle is changed by the amount of rotation, so that the total angle between the incident and reflected rays is altered by twice the change in the reflecting surface. With the sextant, the ray of light is reflected by two mirrors; one movable and the other fixed. The angle between the first and last directions is twice the angle between the mirrors.

In figure 1502b, AB is a ray of light from a celestial body. The index mirror of the sextant is at B, the horizon glass at C, and the eye of the observer at D. Construction lines EF and CF are perpendicular to the index mirror and horizon glass, respectively, and lines BG and CG are parallel to these mirrors. Therefore, angles

BFC and *BGC* are equal because their sides are mutually perpendicular (art. O27). Angle *BGC* is the inclination of the two reflecting surfaces. The ray of light *AB* is reflected at mirror *B*, proceeds to mirror *C*, where it is again reflected, and then continues on to the eye of the observer at *D*. Since the angle of reflection is equal to the angle of incidence,

$$ABE=EBC, \text{ and } ABC=2EBC$$
$$BCF=FCD, \text{ and } BCD=2BCF.$$

Since an exterior angle of a triangle equals the sum of the two nonadjacent interior angles (art. O28),

$$ABC=BDC+BCD, \text{ and } EBC=BFC+BCF.$$

Transposing,

$$BDC=ABC-BCD, \text{ and } BFC=EBC-BCF.$$

Substituting $2EBC$ for *ABC*, and $2BCF$ for *BCD* in the first of these equations,

$$BDC=2EBC-2BCF, \text{ or } BDC=2(EBC-BCF).$$

Since

$$BFC=EBC-BCF, \text{ and } BFC=BGC,$$

therefore

$$BDC=2BFC=2BGC.$$

That is, *BDC*, the angle between the first and last directions of the ray of light, is equal to $2BGC$, twice the angle of inclination of the reflecting surfaces. Angle *BDC* is the altitude of the celestial body.

1503. Micrometer drum sextant.—A modern marine sextant, called a **micrometer drum sextant,** is shown in figure 1503a. In most marine sextants, the **frame.** *A*, is

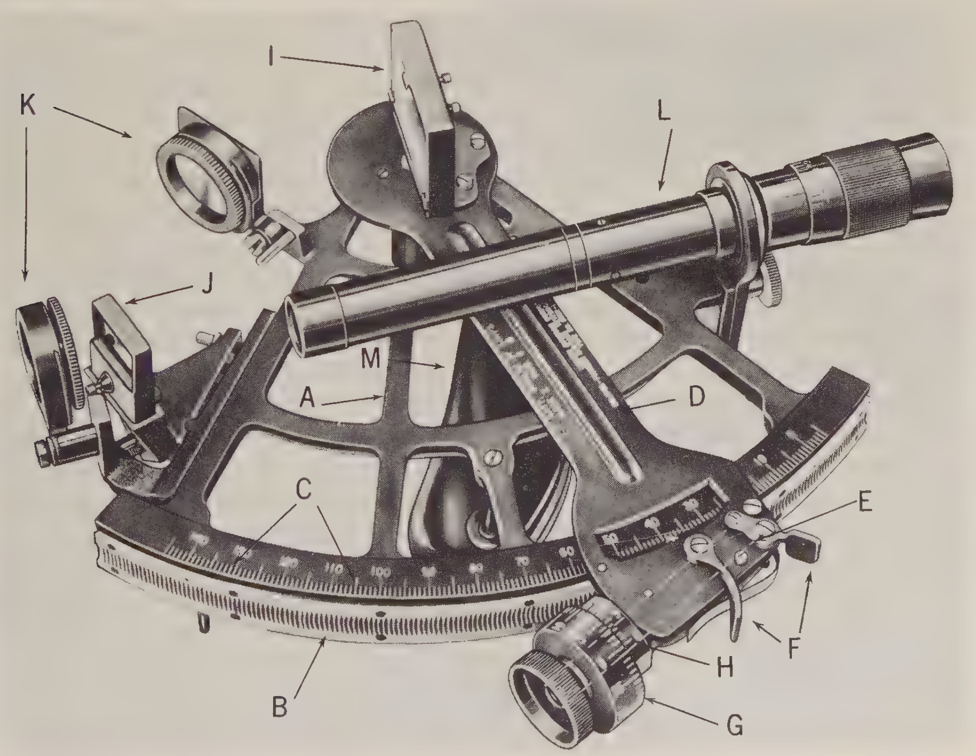


FIGURE 1503a.—U. S. Navy standard micrometer drum sextant.

made of brass or aluminum. There are several variations of the design of the frame, nearly all conforming generally to that shown. The **limb,** *B*, is cut on its outer edge with teeth, each representing one degree of altitude. The altitude graduations, *C*, along the limb, are called the **arc.** Some sextants have an arc marked in a strip of brass, silver, or platinum inlaid in the limb.

The **index arm,** *D*, is a movable bar of the same material as the frame. It is pivoted about the center of curvature of the limb. The **tangent screw,** *E*, is mounted perpendicularly on the end of the index arm, where it engages the teeth of the limb. Because the index arm can be moved through the length of the arc by rotating the tangent screw, this is sometimes called an "endless tangent screw," in contrast with the limited-range device on older instruments. The **release,** *F*, is a spring-actuated clamp which keeps the tangent screw engaged with the teeth of the limb. By applying pressure on the legs of the release, one can disengage the tangent screw. The index arm can then be moved rapidly along the limb. Mounted on the end of the tangent screw is a **micrometer drum,** *G*, which is graduated in minutes of altitude. One complete turn of the drum moves the index arm one degree of altitude along the arc. Adjacent to the micrometer drum and fixed on the index arm is a **vernier,** *H*, which aids in reading fractions of a minute. The vernier shown is graduated into ten parts, permitting readings to six seconds. Other sextants (generally of European manufacture) have verniers graduated into only six parts, permitting readings to ten seconds. The most expensive sextant in common use has no vernier, and readings more precise than one minute can only be estimated.

The **index mirror,** *I*, is a piece of silvered plate glass mounted on the index arm, perpendicular to the plane of the instrument, with the center of the reflecting surface directly over the pivot of the index arm. The **horizon glass,** *J*, is a piece of plate glass silvered on its half nearer the frame. It is mounted on the frame, perpendicular to the plane of the sextant. The index mirror and horizon glass are mounted so that their surfaces are parallel when the micrometer drum is set at 0°, if the instrument is in perfect adjustment. **Shade glasses,** *K*, of varying or variable darkness, are mounted on the frame of the sextant in front of the index mirror and horizon glass. They can be moved into the line of sight at will, to reduce the intensity of light reaching the eye of the observer. Older sextants have two sets of shade glasses, as shown in figure 1504. Many modern sextants are fitted with a single Polaroid **filter** of variable darkness in place of each set of shade glasses, as shown in figure 1503a.

The **telescope,** *L*, screws into an adjustable collar in line with the horizon glass, and should then be parallel to the plane of the instrument. Most modern sextants are provided with only one telescope, but some are equipped with two or more. When only one telescope is provided, it is of the "erect image type," either such as shown or one with a wider "object glass" (far end of telescope), which generally is shorter in length and gives a greater field of view. The second telescope, if provided, is of the "inverting type." The inverting telescope, having one lens less than the erect type, absorbs less light, but at the expense of producing an inverted image. A small colored glass cap is usually provided, to be placed over the "eyepiece" (near end of telescope) to reduce the glare. With this in place, shade glasses are generally not needed. A "peep sight" may be provided. It is a clear tube which serves to direct the line of sight of the observer when no telescope is used.

The telescope shown in figure 1503a is fitted with a "spiral focusing mechanism." Other sextants substitute a "draw" for this mechanism. The draw is fitted inside the telescope tube without threads and is slid in or out as necessary to focus the instrument.

The spiral focusing mechanism is easily adjusted each time the sextant is used, but on the draw type, the navigator should mark the draw to indicate the correct extension for his eyes.

The **handle,** *M*, of most sextants is made of wood or plastic. Sextants are designed to be held in the right hand. Some are equipped with a small light on the index arm to assist in reading altitudes. The batteries for this light are fitted inside a recess in the sextant handle.

Figure 1503b shows a sextant with a silver arc inserted in the limb, a micrometer drum graduated oppositely to the one in figure 1503a, a vernier graduated into six parts, a shorter telescope with a wider object glass than that in figure 1503a, a telescope draw substituted for a spiral focusing mechanism, and a light fitted on the index arm.

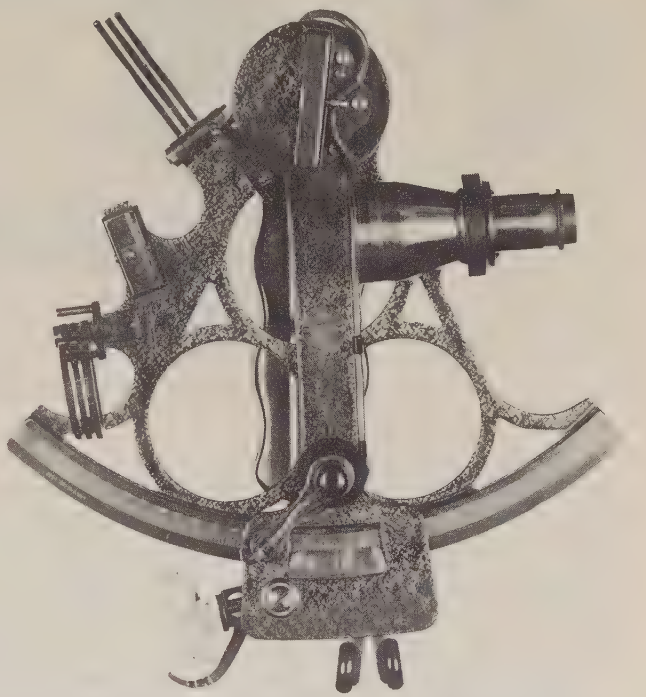


FIGURE 1503b.—A micrometer drum sextant used in the merchant marine.

1504. Vernier sextant.—Nearly all marine sextants of recent manufacture are of the type described in article 1503. At least two older-type sextants are still in use. These differ from the micrometer drum sextant principally in the manner in which the final reading is made. They are called **vernier sextants.**

The **clamp screw vernier sextant** is the older of the two. In place of the modern "release," a **clamp screw** is fitted on the underside of the index arm. To move the index arm, one loosens the clamp screw, releasing the arm. When the arm is placed at the approximate altitude of the body being observed, the clamp screw is tightened. Fixed to the clamp screw and engaged with the index arm is a long tangent screw. When this screw is turned, the index arm moves slowly, permitting accurate setting. Movement of the index arm (by the tangent screw) is limited to the length of the screw (several degrees of arc). Before an altitude is measured, this screw should be set to the approximate mid-point of its range. The final reading is made on a vernier set in the index arm below the arc. A small microscope or magnifying glass fitted to the index arm is used in making the final reading. Figure 1504 shows a clamp screw vernier sextant.

The **endless tangent screw vernier sextant** is identical with the micrometer drum sextant, except that it has no drum, and the fine reading is made by a vernier along the arc, as with the clamp screw vernier sextant. The release is the same as on the micrometer drum sextant and teeth are cut into the underside of the limb which engage with the endless tangent screw. The vernier itself is explained in article 1506.

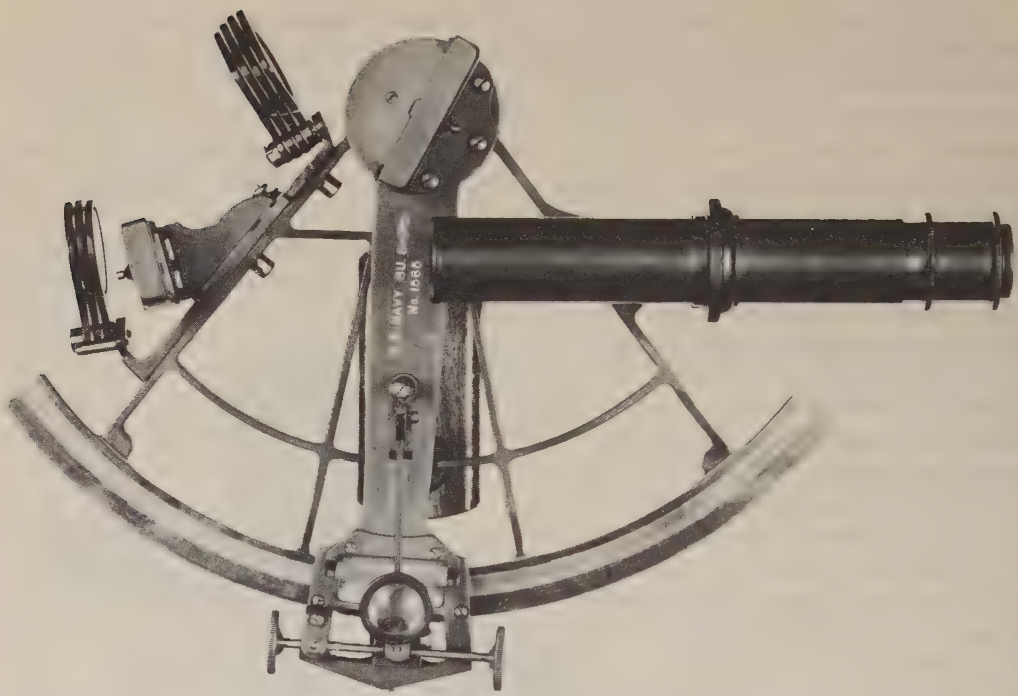


FIGURE 1504.—A clamp screw vernier sextant.

1505. Use of the sextant.—When the *sun* is observed, the sextant is held vertically in the right hand, and the line of sight is directed at the point on the horizon directly below the body. Suitable shade glasses are moved into the line of sight, and the index arm is moved outward from near the 0° point until the reflected image of the sun appears in the horizon glass, near the direct view of the horizon. The sextant is then tilted slightly to the right and left to check its perpendicularity. As the sextant is tilted, the image of the sun appears to move in an arc, and the observer may have to change slightly the direction in which he is facing, to prevent the image from moving out of the horizon glass. When the sun appears at the *bottom* of its apparent arc resulting from this **swinging the arc,** or **rocking the sextant,** the sextant is vertical, and in the correct position for making the observation. If the sextant is tilted, too *great* an angle will be measured. When the sextant is vertical, and the observer is facing directly toward the sun, its reflected image appears at the center of the horizon glass, half on the silvered part, and half on the clear part. The index arm is then moved slowly until the sun appears to be resting exactly on the horizon, which is tangent to the **lower limb.** Occasionally, the sun image is brought *below* the horizon, and the **upper limb** observed. It is good practice to make several observations, moving the limb away from the horizon, alternately above and below it, between readings. Practice is needed to determine the appearance at tangency, which occurs at only one point, to avoid the common error of beginners of bringing the image down too far (too little for an upper-limb observation). Some navigators get more accurate observations by letting the body contact the horizon by its own apparent motion, bringing it slightly below the horizon if rising, and above if setting. At the instant the horizon is tangent to the disk, the time is noted. The **sextant altitude** is the uncorrected reading of the sextant. Figure 1505a illustrates the major steps in making an observation of the sun.

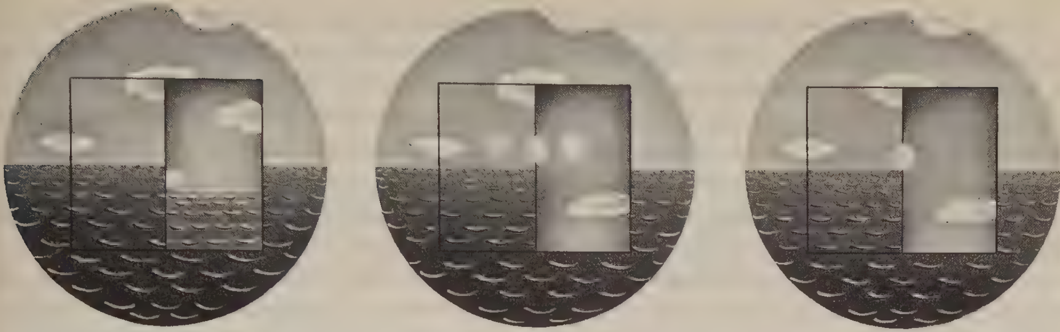


FIGURE 1505a.—*Left*, view through telescope with index arm set near zero. *Center*, "swinging the arc" after the sun has been brought close to the horizon. *Right*, sun at the instant of tangency.

At the left, the index arm has been moved a short distance from 0°. In the center, it has been clamped with the sun in the approximate position for a reading, and the sextant is being rocked. At the right, the sun is in the correct position for a reading.

When the *moon* is observed, the procedure is the same as for the sun, except that shade glasses are usually not required. The upper limb of the moon is observed more often than that of the sun, because of the phases of the moon. When the terminator (art. 1423) is nearly vertical, care should be exercised in selecting the limb that is illuminated, if an inaccurate reading is to be avoided. Sights of the moon are best made during daylight hours, or during that part of twilight in which the moon is least luminous. During the night, false horizons nearly always appear below the moon, due to illumination of the water by moonlight.

When a *star* or *planet* is observed, three methods of making the initial approximation of the altitude are in common use. In the most common, the index arm and micrometer drum are set on zero and the line of sight is directed at the body to be observed. Then, while keeping the reflected image of the body in the mirrored half of the horizon glass, the index arm is slowly swung *out* and the frame of the sextant is rotated *down*. The reflected image of the body is kept in the mirror until the horizon appears in the clear part of the horizon glass.

When there is little contrast between brightness of the sky and the body, this procedure is difficult, for if the body is "lost" while it is being brought down, it may not be recovered without starting again at the beginning of the procedure. An alternative method sometimes used consists of holding the sextant upside down in the left hand, directing the line of sight at the body, and slowly moving the index arm out until the horizon appears in the horizon glass. This is illustrated in figure 1505b. After contact is made, the sextant is inverted and the sight taken in the usual manner.

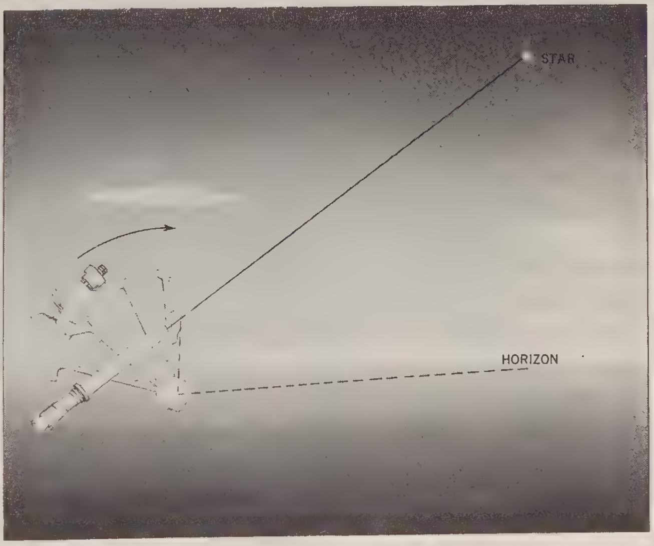


FIGURE 1505b.—Method of bringing horizon "up" to body.

A third method consists of determining in advance the approximate altitude and azimuth of the body by a **star finder** such as H.O. 2102–D (art. 2210). The sextant is set at the indicated altitude, and the observer faces in the direction indicated by the azimuth. After a short search, during which the index arm is moved backward and forward a few degrees, and the azimuth in which the observer faces is changed a little to each side, the image of the body should appear in the horizon glass. The best method to use for any observation is that which produces the desired result with the least effort. It is largely a matter of personal preference.

Measurement of the altitude of a star or planet differs from that of the sun or moon in that the *center* of a star or planet, rather than a limb, is brought into coincidence with the horizon. Figure 1505c shows the reflected image of a star as it should appear at the time of observation. Because of this difference, and the limited time usually available for observation during twilight, the method of letting a star or planet intersect the horizon by its own motion is little used. As with the sun and moon, however, the navigator should not forget to swing the arc to establish perpendicularity of the sextant.

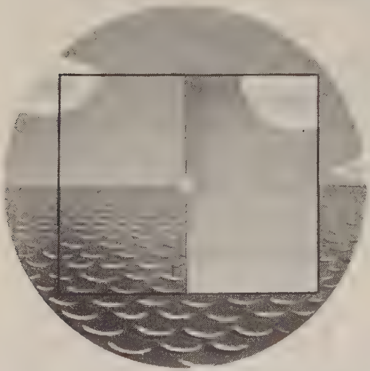


FIGURE 1505c.—Correct position of planet or star at moment of observation.

Occasionally, fog, haze, or other ships may obscure the horizon directly below a body which the navigator wishes to observe. If the arc of the sextant is sufficiently long, a **back sight** might be obtained, using the opposite point of the horizon as the reference. The observer faces *away* from the body and observes the *supplement* of the altitude. If the sun or moon is observed in this manner, what appears in the horizon glass to be the lower limb is in fact the upper limb. In the case of the sun, it is usually preferable to observe what appears to be the upper limb. The arc that appears when rocking the sextant for a back sight is inverted; that is, the *highest* point indicates the position of perpendicularity.

If more than one telescope is furnished with the sextant, the erecting telescope is used to observe the sun. Generally, the inverting telescope will produce the best results when observing the stars, although some navigators prefer not to use any telescope, thus obtaining a wider field of view. The collar into which the sextant telescope fits may be adjusted in or out in relation to the frame. When moved in, more of the mirrored half of the horizon glass is visible to the navigator, and a star or planet is more easily observed when the sky is relatively bright. Near the darker limit of twilight, the telescope can be moved out, giving a broader view of the clear half of the glass, and making the less distinct horizon more easily discernible. If both eyes are kept open until the last moments of an observation, eye strain will be lessened. But in making the final measurement, the nonsighting eye should be closed to permit full ocular concentration. Practice will permit observations to be made quickly, reducing inaccuracy due to eye fatigue. If several observations are made in succession, with a short rest between them, the best results should be obtained. With experience, the observer should be able to "call his shots," identifying the better ones.

When an altitude is being measured, it is desirable to have an assistant note the time, so that simultaneous values of time and altitude will be available. He should be given a warning "stand-by" when the measurement is nearly completed, and a "mark" at the moment a reading is made. He should be instructed to read the three hands in order of their rapidity of motion; the second hand first, then the minute hand, and

finally the hour hand. If it is sufficiently dark that a light is needed to make the reading, the assistant should read both the time, and then the altitude, *behind* the observer and facing away from him, to avoid impairment of the observer's eye adaption to sky and horizon lighting conditions.

If an assistant is not available to time the observations, the observer holds the watch in the palm of his left hand, leaving his fingers free to manipulate the tangent screw of the sextant. After making the observation, he quickly shifts his view to the watch, and notes the positions of the second, minute, and hour hands, respectively. The delay between completing the altitude observation and noting the time should not be more than one or two seconds. The average time should be determined by having someone measure it for several observations, or by counting the half seconds (learning to count with the half-second beats of a chronometer). This interval can then be subtracted from the observed time of each sight.

1506. Reading the sextant.—The reading of a micrometer drum sextant is made in three steps. The degrees are read by noting the position of the arrow on the index arm in relation to the arc. The minutes are read by noting the position of the zero on the vernier with relation to the graduations on the micrometer drum. The fraction of a minute is read by noting which mark on the vernier most nearly coincides with one of the graduations on the micrometer drum. This is similar to reading the time by means of the hour, minute, and second hands of a watch. In both, the relationship of one part of the reading to the others should be kept in mind. Thus, if the hour hand of a watch were *about* on "4," one would know that the time was about four o'clock. But if the minute hand were on "58," one would know that the time was 0358 (or 1558), not 0458 (or 1658). Similarly, if the arc indicated a reading of

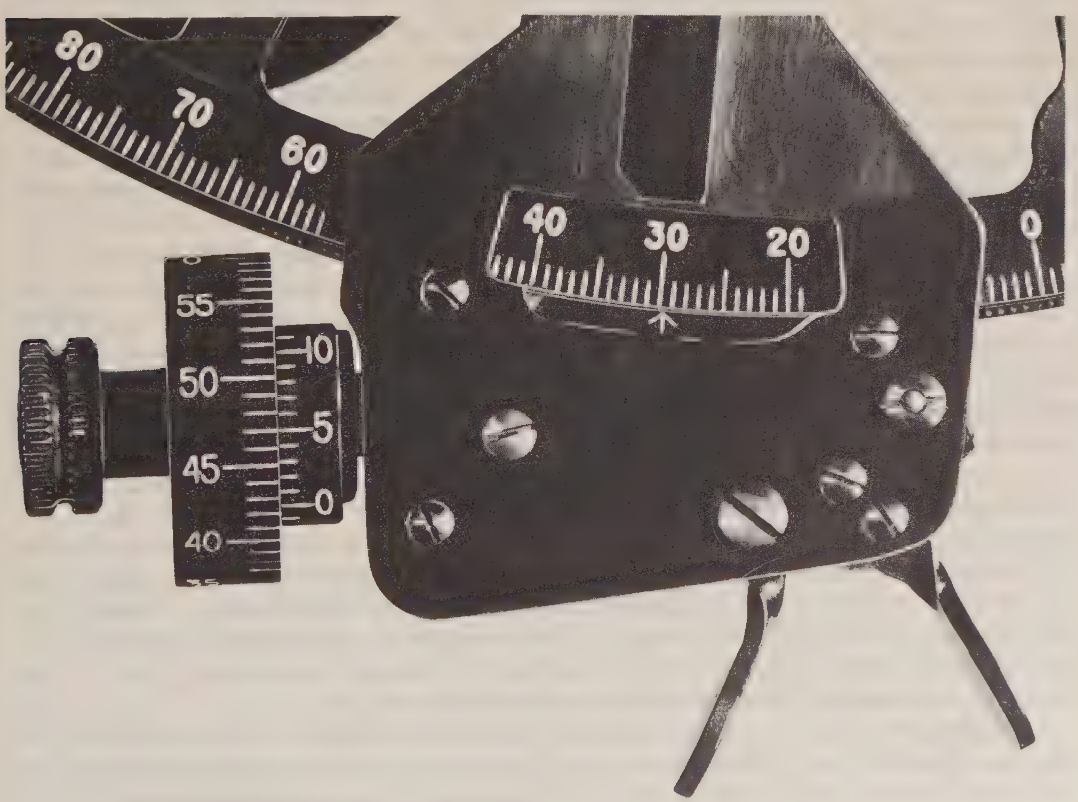


FIGURE 1506a.—Micrometer drum sextant set at 29°42′.5.

FIGURE 1506b.—Vernier sextant set at 29°42′30″.

about 40°, and 58′ on the micrometer drum were opposite zero on the vernier, one would know that the reading was 39°58′, not 40°58′. Similarly, any doubt as to the correct minute can be removed by noting the fraction of a minute from the position of the vernier. In figure 1506a the reading is 29°42′.5. The arrow on the index mark is between 29° and 30°, the zero on the vernier is between 42′ and 43′, and the "0′.5" graduation on the vernier coincides with one of the graduations on the micrometer drum.

The principle of reading a vernier type sextant is the same, but the reading is made in two steps. Figure 1506b shows a typical altitude setting on this type sextant. Each degree on the arc of this sextant is graduated into three parts, permitting an initial reading by the reference mark on the index arm to the nearest full 20 minutes of arc. In this illustration the reference mark lies between 29°40′ and 30°00′, indicating a reading between these values. The reading for the fraction of 20′ is made by means of the vernier, which is engraved on the index arm and has the small reference mark as its zero graduation. On this vernier, 40 graduations coincide with 39 graduations on the arc. Each graduation on the vernier is equivalent to $\frac{1}{40}$ of one graduation (20′) on the arc, or 0′.5 (30″). In the illustration, the vernier graduation representing $2\frac{1}{2}$ minutes (2′30″) most nearly coincides with one of the graduations on the arc. Therefore, the reading is 29°42′30″, or 29°42′.5, as before. When a vernier of this type is used, any doubt as to which mark on the vernier coincides with a graduation on the arc can usually be resolved by noting the position of the vernier mark on each side of the one that seems to be in coincidence.

Negative readings (as in determining index correction, art. 1603), are made in the same manner as positive readings, the various parts being added algebraically (art. O6). Thus, if the three parts of a micrometer drum reading are (−)1°, 56′, and 0′.3, the total reading is (−)1°+56′+0′.3=(−)3′.7.

1507. Developing observational skill.—A well-constructed marine sextant is capable of measuring angles with an instrument error not exceeding 0.'1. Lines of position from altitudes of this accuracy would not be in error by more than about 200 yards. However, there are various sources of error, other than instrumental, in altitudes measured by sextant. One of the principal sources is the observer himself. There is probably no single part of his work that the navigator regards with the same degree of professional pride as his ability to make good celestial observations. Probably none of his other tasks requires the same degree of skill.

The first fix a student navigator obtains by his observation of celestial bodies is likely to be disappointing. Most navigators require a great amount of practice to develop the skill needed to make good observations. But practice alone is not sufficient, for if a mistake is repeated many times, it will be difficult to eradicate. Early in his career a navigator would do well to establish good observational technique—and continue to develop it during the remainder of his days as navigator. Many good pointers can be obtained from experienced navigators, but it should be remembered that each develops his own technique, and a practice that proves highly successful for one observer may not help another. Also, an experienced navigator is not necessarily a good observer, although he may consider himself such. Navigators have a natural tendency to judge the accuracy of their observations by the size of the figure formed when the lines of position are plotted. Although this is some indication, it is an imperfect one, because it does not indicate the errors of individual observations, and may not reflect constant errors. Also, it is a compound of a number of errors, some of which are not subject to control by the navigator.

When a student first begins to use the sextant, he can eliminate gross errors of principle in its use, and gain some ability in making observations, by accepting the coaching of an experienced navigator. By watching the novice make observations, the experienced navigator can observe a tendency to hold the instrument incorrectly, swing the arc improperly, or make other mistakes. When a celestial body is near the celestial meridian, the experienced navigator might make an observation and quickly transfer the sextant to the inexperienced one, who can see how the sight should appear. The two might make simultaneous observations and compare results. At first it is well to select bodies of low altitude, if they are available.

This procedure is helpful in detecting gross mistakes, but since the observations of the experienced navigator are not without error, this method is not suitable for final polishing of technique. For this purpose, observations should be compared with a more exact standard. Lines of position from celestial observations can be compared with good positions obtained by electronics or by piloting, if near a shore. Although this is good practice and provides a means of checking one's skill from time to time, it does not provide the large number of comparisons in a short time needed if technique is to be perfected.

This can sometimes be accomplished when a vessel is at anchor, or at a pier, if a stretch of open horizon is available. In advance, the altitude of a celestial body which will be over the open horizon at a time favorable for observation is computed at intervals of perhaps eight minutes (change in hour angle of 2°). If the body will be near the meridian, a smaller interval should be used. The altitude is determined for the position of the vessel, and all sextant altitude corrections (ch. XVI) are applied with reversed sign. These altitudes are then plotted versus time on cross-section paper, to a large scale, and a curve drawn through the points. At the selected time, a large number of observations are made at short intervals, allowing only enough time between observations for resting the eyes and arms. These observations are then plotted on the cross-section paper and compared with the curve.

An analysis of the results should be instructive. Erratic results indicate poor observational conditions or the need for practice and more care in making observations. If the measured altitudes are consistently too great, the sextant may not be rocked properly, the condition of tangency of the lower limb of the sun or moon may not be judged accurately, a false horizon in the water may have been used, subnormal refraction (dip) might be present, the eye might be higher above water than estimated, time might be in error, the index correction may have been determined incorrectly, the sextant might be out of adjustment, an error may have been made in the computation, the horizontal (vertical) may be tilted slightly by nearby mountains, etc. If the measured altitudes are consistently too low, the condition of tangency of the upper limb of the sun or moon may not be judged accurately, a low cloud may have been used as the horizon, abnormal refraction (dip) might be present, height of eye might be lower than estimated, time might be in error, the index error may have been determined incorrectly, the sextant might be out of adjustment, an error may have been made in the computation, the waves or swell at the horizon might be higher than at the ship, the horizontal (vertical) may be tilted slightly, a planet or bright star may have been placed "tangent" to the horizon rather than centered on the horizon, etc.

A single test of this type, while instructive, may not be conclusive. Several tests should be made with different celestial bodies, at various altitudes, under various conditions of weather and sea, and at different places. Generally, it is possible and desirable to correct any errors being made in the technique of observation, but occasionally a **personal error** (sometimes called **personal equation**) will persist. This might be different for the sun and moon than for planets and stars, and might vary with degree of fatigue of the observer, and other factors. For this reason, a personal error should be applied with caution. However, if a relatively constant personal error persists, and experience indicates that observations are improved by applying a correction to remove its effect, better results might be obtained by this procedure than by attempting to eliminate it from one's observations.

When lines of position of great reliability are desired, even an experienced navigator can usually improve his results by averaging to reduce random error (art. 2904). A number of observations, preferably not less than ten, are made in quick succession. These can then be plotted versus time, on cross-section paper, and a curve faired through the points. Unless the body is near the celestial meridian, this curve should be very nearly a straight line. *Any* point on the curve can be used as the observation, using the time and altitude indicated by the point. It is best to use a point near the middle of the line, to avoid possible errors in its slope.

The slope can be determined by means of H.O. Pub. No. 214, using Δt, which is the change of altitude relative to change in meridian angle (time). Meridian angle changes at the rate of $1'$ in 4^s. Therefore, the change in altitude, in minutes of arc per second of time, is equal to Δt (expressed as minutes of arc) divided by 4^s, or $\frac{\Delta t'}{4^s}$. Thus, if Δt is 0.66, the altitude changes $\frac{0'.66}{4^s}=0'.165$ per second, or $15'\times 0'.66=9'.9$ per minute of time, increasing if the body is rising, and decreasing if it is setting. This rate may be altered by motion of the ship, the amount being the distance traveled in one minute, multiplied by the natural cosine of the relative azimuth of the body. Thus, if the speed is 15 knots, the ship moves 0.25 mile per minute. If the body is 30° on the bow, the altitude changes $0.25\times 0.86603=0'.2$ per minute due to motion of the ship, in addition to its own apparent motion due to rotation of the earth. If the body is forward of the beam, the effect of the ship's motion is to *increase* the altitude; if abaft the beam, to decrease it. The total effect is the algebraic sum of the separate effects due to

rotation of the earth and motion of the vessel, since rate *at the vessel* is desired. Rapid change of Δt indicates a curved rate line. If a large number of observations is made, the slope of the line should be apparent from the plotted points.

A somewhat simpler variation is generally available if observations are made at equal intervals, unless the body is near the meridian. It is based upon the assumption that the change in altitude should be equal for equal intervals of time. A number of observations might be made by having an assistant give a warning "stand-by" and then a "mark" at equal intervals of time, as every ten or 20 seconds. Perhaps a better procedure is to make the observations at equal altitude increments. After the first observation, the altitude is changed by a set amount according to its rate of change, as 5′. The setting is *increased* if the body is rising, and *decreased* if it is setting. The body is then permitted to cross the horizon by its own motion, and at the instant of doing so, the time is noted. If time intervals are constant, the *mid time* and the *average altitude* are used as the observation. If altitude increments are constant, the *average time* and *mid altitude* are used. An uneven number of observations simplifies the finding of the mid value, but with ten observations the finding of the average value is easier.

If only a small number of observations is available, as three, it is usually preferable to solve all observations and plot the resulting lines of position, adjusting them to a common time. The *average* position of the line might be used, but it is generally better practice to use the middle line (or a line midway between the two middle ones if there are an even number).

In this discussion of averaging, it has been assumed that all observations are considered of nearly equal value. Any observation considered unreliable, either in the judgment of the observer or as a result of a plot, should be rejected in finding an average.

1508. Care of the sextant.—The modern marine sextant is a well-built, precision instrument capable of rendering many years of reliable service, with minimum attention. However, its usefulness can easily be impaired by careless handling or neglect. If it is ever dropped, it may never again provide reliable information. If this occurs, the instrument should be taken to an expert for careful testing and inspection.

When not in use, a sextant should invariably be kept in its case and properly stowed. The sextant case should be a well-constructed hardwood box fitted on its exterior with a lock, a handle, and two hooks, preferably the type having safety catches. The interior of the case should be fitted with blocks in which the handle or legs, or both, are placed when the sextant is stowed. Some sextant cases are fitted with catches which clamp over the handle when the sextant is stowed, and some are fitted with felt-lined blocks on the inside of the cover, to clamp down on the extreme ends of the arc when the case is closed. The case should be so constructed that it can be closed with the shade glasses and index arm in nearly any normal position, and preferably with the telescope in place. The last is particularly valuable to the navigator on an overcast day when only one opportunity to observe the sun may present itself, and the sight may have to be taken quickly.

The case itself should be securely stowed in a convenient place away from excessive heat, dampness, and vibration. A shelf with built-up sides into which the case fits snugly is a good stowage place. The practice of leaving the sextant in its case on a chart room settee is a bad one, and the instrument should *never* be left unattended on the chart table.

To remove the sextant from its case, grasp the frame firmly with the left hand, making sure that no pressure is applied to the index arm, and lift the instrument from the box. Then take the sextant in the right hand, by its handle, leaving the left hand

free to make any adjustments necessary before taking a sight. The instrument should never be held by its limb, index arm, or telescope.

Next to careless handling, the greatest enemy of the sextant is moisture. The mirrors, especially, and the arc should be wiped dry after each use. A new sheet of plain lens paper is best to use for this purpose, and linen second best. Over a period of time, however, linen collects dust, which may contain abrasives that will scratch the surface of the mirrors. For this reason, linen, if it is used, should be kept in a small bag to protect it from dust in the air. Chamois leather and silk are particularly likely to collect abrasive dusts from the air and they should not be used to clean the mirrors or telescope lenses. Should the mirrors become particularly dirty, they can be cleaned with a small amount of alcohol, applied with a clean piece of lens paper. The arc can be cleaned, when necessary, with ammonia, but never with a polishing compound. In cleaning or drying the mirrors and arc, care should be taken that excessive pressure is not applied to any part of the instrument.

A small bag of silica gel kept in the sextant case will help in keeping the air in the case free from moisture, and will help to preserve the mirrors. Occasionally, the silica gel should be heated in an oven to remove the adsorbed moisture.

The tangent screw and the teeth on the side of the limb should be kept clean and lightly oiled, using the oil provided with the sextant. It is good practice to set occasionally the index arm of an endless tangent screw at one extremity of the limb and then to rotate the tangent screw over the length of the arc. This will clean the teeth and spread the oil through them. At any time that the sextant is to be stowed for a long period, the arc should be protected with a thin coat of petroleum jelly.

If the mirrors need resilvering, they are best taken to an instrument shop where a professional job can be done. However, on rare occasions it may be necessary to resilver the mirrors of a sextant at sea. In anticipation of this possibility, the navigator should obtain the necessary materials in advance, as makeshift substitutes cannot be relied upon to do the job adequately. The required materials are xylene (available in most pharmacies), dilute nitric acid (optional), alcohol, cotton, tin foil about 0.005 inch thick, a small amount of mercury, a clean blotter, and some tissue paper. Do not substitute aluminum foil commonly used in packaging candy and cigarettes.

First, remove the protective coating with alcohol (or better, acetone) from the back of the mirror to be resilvered, and clean the glass with xylene or acid. If the old silvering is difficult to remove, soak it in water. Place the blotter on a flat surface and turn up and seal the edges to form a tray. This will serve to contain the mercury if the vessel should roll during the operation. Using cotton, clean and smooth out both sides of a piece of tin foil slightly larger than the glass to be silvered, first with alcohol and then with xylene (do not use acid). Make certain that no lint adheres to the foil, and place it on the blotter. Clean the mercury by squeezing it through cheese cloth, and apply a drop to the foil. Carefully spread it over the surface with a finger, making sure that none of the mercury gets under the foil. Add a few more drops of mercury until the entire surface of the foil is covered and tacky. The mercury combines with some of the tin to form an amalgam. Place the chemically cleaned glass on a piece of clean tissue paper with the side to be silvered face down. Then place the glass and the paper on the amalgam. Apply slight pressure to the glass and withdraw the tissue paper. Following this, grasp the edge of the tin foil and lift it and the mirror from the blotter. Invert the glass and the tin foil and place in an inclined position, silvered side up. Any mercury remaining on the blotter is no longer pure and should be disposed of. Five or six hours later any loose foil may be scraped from the sides of the mirror, and the following day a coat of commercial varnish or lacquer should be applied to the silvered

surface. Should the mirrored half of the horizon glass require silvering, the clear half may be protected by a strip of cellulose or adhesive tape.

1509. Sextant adjustments.—There are at least seven sources of error in the marine sextant, three nonadjustable by the navigator, and four adjustable.

The **nonadjustable errors** are: "prismatic error," "graduation error," and "centering error."

The **prismatic error** is present if the two faces of the shade glasses and mirrors are not parallel. Error due to lack of parallelism in the shade glasses may be called **shade error.** Shade error in the shade glasses near the index mirror can be determined by comparison of an angle measured when a shade glass is in the line of sight with the same angle measured when the glass is not in the line of sight. In this manner, the error for each shade glass can be determined and recorded. If shade glasses are used in combination, their combined error should be determined separately. If additional shading is needed for the observations, use the colored telescope eyepiece cover. This does not introduce an error because direct and reflected rays are traveling together when they reach it, and are therefore affected equally by any lack of parallelism of its two sides.

Lack of parallelism of the two faces of the index mirror can be detected by carefully measuring a series of angles; then removing the index mirror, inverting it, and replacing it; and then measuring the same angles again. Half the difference is the prismatic error. After the index mirror has been inverted, it should be checked carefully for perpendicularity to the frame of the sextant, as explained below.

Lack of parallelism of the two faces of the horizon glass will appear as part of the index error, and so need not have separate attention. The same is true of prismatic error in the shade glasses located near the horizon glass, but unless index error is determined with the shade glasses in place, the measured index error will not be the correct value for the combined error.

Graduation errors occur in the arc, micrometer drum, and vernier of a sextant which is improperly cut or incorrectly calibrated. Normally, the navigator cannot determine whether the arc of a sextant is improperly cut, but the principle of the vernier makes it possible to determine the existence of graduation errors in the micrometer drum or vernier and is a useful guide in detecting a poorly made instrument. The first and last markings on any vernier should align perfectly with one less graduation on the adjacent micrometer drum. In figure 1503a, the vernier is graduated in ten units. When the zero point is aligned with any graduation on the micrometer drum, the "ten" graduation should be in perfect alignment with a micrometer graduation nine units greater than the one in line with zero on the vernier. In figure 1503b, the vernier is graduated in six units and should align perfectly with any two graduations five units apart on the micrometer.

Centering error results if the index arm is not pivoted at the exact center of curvature of the arc. It can be determined by measuring known angles, after the adjustable errors have been removed. Horizontal angles can be used by determining the accurate value by careful measurement with a theodolite (art. 4004). Several readings by both theodolite and sextant should minimize errors. An alternative method is to measure angles between the lines of sight to stars, comparing the measured angles with computed values. To minimize refraction errors, one should select stars at about the same altitude, and avoid stars near the horizon.

The same shade glasses, if any, used for determining or eliminating index error should be used for measuring centering error. The errors determined in this manner include any error due to faulty graduation, and prismatic error of the index mirror,

unless corrections are applied for these errors. However, since all vary with the angle measured, they need not be separated. Usually, it is preferable to make a single correction table for all three errors, called **instrument error.** Customarily, such a table is determined by the manufacturer and attached to the inside cover of the sextant case. The sign of the error is reversed, so that the values given are for **instrument correction (I).**

The **adjustable errors** in the sextant are those related to *perpendicularity* of (1) the frame and the index mirror, and (2) the frame and the horizon glass, and *parallelism* of (3) the index mirror and horizon glass to each other at zero setting, and of (4) the telescope to the frame. Each of these errors, if it exists, can be removed from the sextant by careful adjustment. In making these adjustments, *never tighten one adjusting screw without first loosening the other screw which bears on the same surface.* The adjustments should be made in the order indicated.

The first adjustment is for **perpendicularity of the index mirror** to the frame of the sextant. To test for perpendicularity, place the index arm at about 35° on the arc,

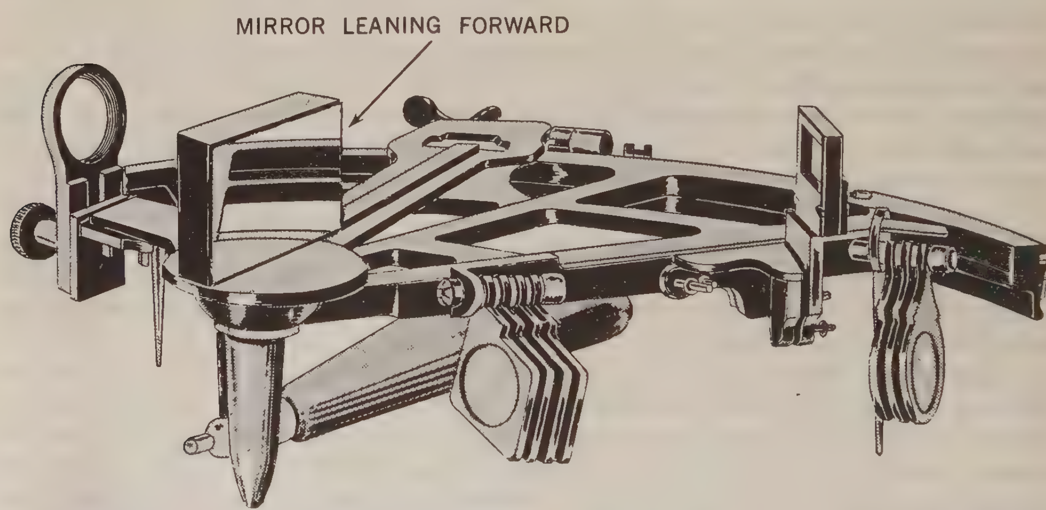


FIGURE 1509a.—Testing the perpendicularity of the index mirror. Here the mirror is not perpendicular.

and hold the sextant on its side, with the index mirror "up" and toward the eye. Observe the direct and reflected views of the sextant arc, as illustrated in figure 1509a. If the two views do not appear to be joined in a straight line, the index mirror is not perpendicular. If the reflected image is above the direct view, the mirror is inclined forward. If the reflected image is below the direct view, the mirror is inclined backward. An alternative and sometimes more satisfactory method of determining perpendicularity involves the use of two small vanes, or similar objects, of exactly the same height. Figure 1509b illustrates this method. Again the index arm is set at about 35°. The vanes are placed upright on the extremities of the limb, in such a way that the observer can, by placing his eye near the index mirror, see the direct view of one vane and the reflected image of the other. The tops of the objects are then observed for alignment. The use of vanes permits observation in the plane of adjustment, rather than at an angle. Adjustment is made by means of two screws at the back of the index mirror.

The second adjustment is for **perpendicularity of the horizon glass** to the frame of the sextant. An error resulting from the horizon glass not being perpendicular is

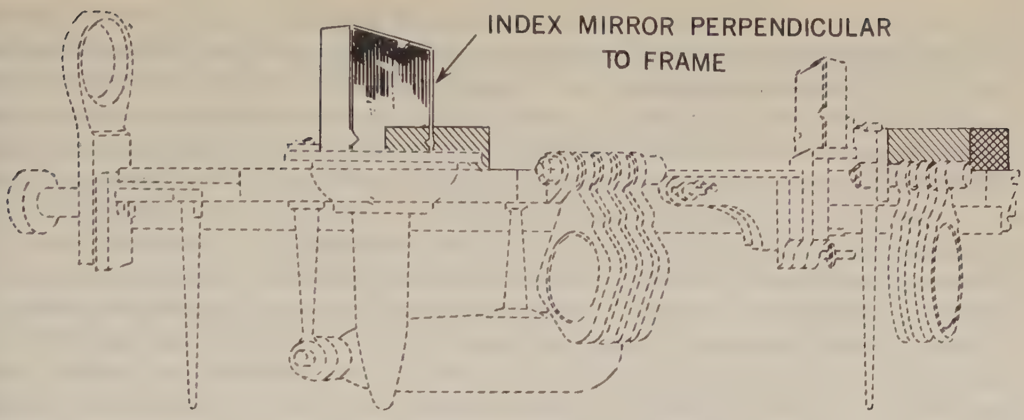


FIGURE 1509b.—Alternative method of testing the perpendicularity of the index mirror. Here the mirror is perpendicular.

called **side error.** To test for perpendicularity, set the index arm at zero and direct the line of sight at a star. Then rotate the tangent screw back and forth so that the reflected image passes alternately above and below the direct view. If, in changing from one position to the other, the reflected image passes directly over the star as seen without reflection, no side error exists, but if it passes to one side, the horizon glass is not perpendicular to the frame of the sextant. Figure 1509c illustrates observations without side error (left) and with side error (right). Whether the sextant reads zero when the true and reflected images are in coincidence is immaterial in this test. An alternative method is to observe a vertical line, such as one edge of the mast of another vessel (or the sextant can be held on its side and the horizon used). If the direct and reflected portions do not form a continuous line, the horizon glass is not perpendicular to the frame of the sextant. A third method is to hold the sextant vertical, as in observing the altitude of a celestial body, and bring the reflected image of the horizon into coincidence with the direct view, so that it appears as a continuous line across the horizon glass. Then tilt the sextant right or left. If the horizon still appears continuous, the horizon glass is perpendicular to the frame, but if the reflected portion appears above or below that part seen direct, the glass is not perpendicular. Adjustment is made by means of two screws near the base of the horizon glass.

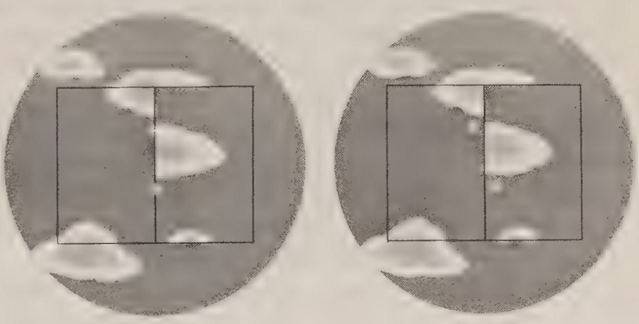


FIGURE 1509c.—Testing the perpendicularity of the horizon glass. *Left*, side error does not exist. *Right*, side error does exist.

The third adjustment is to make the **index mirror and horizon glass parallel** when the index arm is set exactly at zero. The error which results when the two are not parallel is the principal cause of **index error,** the total error remaining after the four adjustments have been made. Index error should be determined each time the sextant is used and need not be removed if its value is known accurately. To make the test for parallelism of the mirrors, set the instrument at zero, and direct the line of sight at the horizon or a star. Side error having been eliminated, the direct view and reflected image of the horizon appear as a continuous line, or the star as a single point,

if the two mirrors are parallel. If the mirrors are not parallel, the horizon appears broken at the edge of the mirrored part of the horizon glass, one part being higher than the other. The reflected image of a star appears above or below the star seen without reflection. If the star appears as a single point, move the tangent screw a small amount to be sure both direct view and reflected image are in the range of vision. The sun can be used by noting the reading when the reflected image is tangent to the sun as seen direct, first above it and then below. These should be numerically equal but of opposite sign (one positive and the other negative). To avoid variations in refraction, do not use low altitudes; or turn the sextant on its side and use the two sides of the sun. Adjustment is made by two screws near the base of the horizon glass. If the error is not to be removed, turn the tangent screw until direct view and reflected image of the horizon or a star are in coincidence. The reading of the sextant is the index error. It is positive if the reading is "on the arc" (positive angle), and negative if "off the arc" (negative angle). In the case of the sun it is *half* the numerical difference (algebraic sum) of the readings, positive or negative to agree with the larger reading. **Index correction (IC)** is numerically the same as index error, but of opposite sign. Since both the second and third adjustments involve the position of the horizon glass, it is good practice to recheck for side error after index error has been eliminated. Index error should always be checked after adjustment for side error.

The fourth adjustment is to make the **telescope parallel** to the frame of the sextant. If the line of sight through the telescope is not parallel to the plane of the instrument, an **error of collimation** will result, and altitudes will be measured as greater than their actual values. To check for parallelism of the telescope, insert it in its collar, and observe two stars 90° or more apart, bringing the reflected image of one into coincidence with the direct view of the other, near either the right or left edge of the field of view (the upper or lower edge if the sextant is horizontal). Then tilt the sextant so that the stars appear near the opposite edge. If they remain in coincidence, the telescope is parallel to the frame, but if they separate, it is not. An alternative method is to place the telescope in its collar and then lay the sextant on a flat table. Sight along the frame of the sextant and have an assistant place a mark on the opposite bulkhead, in line with the frame. Place another mark above the first at a distance equal to the distance from the center of the telescope to the frame. This second line should be in the center of the field of view of the telescope if the telescope is parallel to the frame. Adjustment for nonparallelism is made to the collar, by means of the two screws provided for this purpose.

Determination of any of the errors should be based upon a series of observations, rather than a single one. This is particularly true in the case of index error, which should be determined by approaching coincidence from opposite directions (up and down) on alternate readings. If adjustments are made carefully, and the sextant is given proper handling, it should remain in adjustment over a long period of time. Unless the navigator has reason to question the accuracy of the adjustments, they need not be checked at intervals of less than several months, except in the case of index error, which has the greatest effect on accuracy of readings, and should be checked each time the sextant is used. If the horizon is used for determining index error, this check should be made *before* evening twilight observations, and *after* morning twilight observations, while the horizon is sharp and distinct. If a star is used, the index error should be determined *after* evening observations and *before* morning sights are taken. During the day, it should be checked both before and after observations.

Frequent manipulation of the adjusting screws should be avoided, as it may cause excessive wear. Except in the case of index error, slight lack of adjustment has little

effect on the results, and should be ignored. If adjustments are needed at frequent intervals, the sextant is not receiving proper care, or has worn parts which should be replaced at a navigation instrument shop. If index error is not constant, it should not be removed, but index correction should be determined before or after every observation and applied to the readings, until the sextant can be repaired. A small variable error might well be accepted, but should be watched to see that it does not become unduly large.

1510. Selection of a sextant.—For satisfactory results a sextant should be selected carefully. For accurate work the radius of the arc should be about 7½ inches or more. The instrument should be light, but strongly built. The various moving parts should fit snugly, but move freely without binding or gritting. If the index arm is either too loose or too tight at either end of the arc, the pivot may not be perpendicular to the frame of the sextant. The telescope should be easy to insert or remove from its holder, and to focus.

The use to be made of a sextant should be considered in its selection. For ordinary use in measuring altitudes of celestial bodies, an arc of 90° or slightly more is sufficient. A longer arc is desirable if back sights are to be made, or if horizontal angles are to be measured. If use of the sextant is to be limited to horizontal angles, less accuracy is required. The arc can be of smaller radius, and small nonadjustable errors are unimportant.

If practicable, a sextant should be examined by an expert, and tested for non-adjustable errors before acceptance.

1511. Octants, quintants, and quadrants.—Originally, the term "sextant" was applied to the navigator's double-reflecting, altitude-measuring instrument only if its arc was 60° in length—a sixth of a circle—permitting measurement of angles from 0° to 120°. In modern usage the term is applied to all navigational altitude-measuring instruments, regardless of angular range or principles of operation, although some are octants (angular range 90°), some quintants (144°), some quadrants (180°), and many have an intermediate range.

1512. The artificial horizon.—Measurement of altitude requires a horizontal reference. In the case of the marine sextant this is commonly provided by the visible sea horizon. If this is not clearly visible, reliable altitudes cannot be measured unless a different horizontal reference is available. Such a reference is commonly called an **artificial horizon.** If it is attached to, or part of, the sextant, altitudes can be measured at sea, on land, or in the air, whenever celestial bodies are available for observations. On land, where the visible horizon is not a reliable indication of the horizontal, an external artificial horizon can be devised.

Any horizontal reflecting surface will serve the purpose. A pan of mercury, heavy oil, molasses, or other viscous liquid sheltered from the wind is perhaps simplest. A piece of plate glass fitting snugly across the top of the container is usually the best shelter. If there is any reasonable doubt as to the parallelism of the two sides of the glass, two readings should be made with the glass turned 180° in azimuth between readings, and the average value taken. The pan and liquid should be clean, as foreign material on the surface of the liquid is likely to distort the image and introduce an error in the reading.

To use an external artificial horizon, the observer stands or sits in such a position that the celestial body to be observed is reflected in the liquid, and is also visible by direct view. By means of the sextant, the double-reflected image is brought into coincidence with the image appearing in the liquid. In the case of the sun or moon the *bottom* of the double-reflected image is brought into coincidence with the *top* of the

image in the liquid, if a lower-limb observation is desired. For an upper-limb observation, the opposite sides are brought into coincidence. If one image is made to cover the other, the observation is of the *center* of the body.

When the observation has been made, apply the index correction and any other instrumental correction, as well as any correction for personal error. Then take *half* the remaining angle and apply all other corrections except dip (height of eye) correction, since this is not applicable. If the *center* of the sun or moon is observed, omit, also, the correction for semidiameter. Chapter XVI explains the various corrections and their applications.

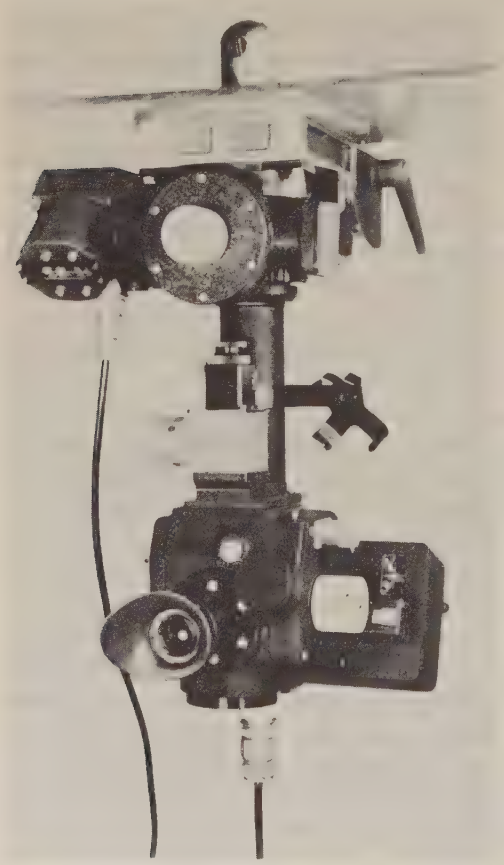


FIGURE 1513a.—Modern periscopic sextant.

A commercial artificial horizon consisting of a metal tray, mercury, cover of two sloping glass sides held in a metal frame, metal bottle to hold the mercury when not in use, and a funnel for pouring, was at one time a familiar part of a navigator's equipment, but the modern navigator might experience difficulty in locating such a device.

1513. Artificial-horizon sextants.—Shortly after the marine sextant was invented (art. 124), an attempt was made to extend its use to periods of darkness. This was done by providing a spirit level attachment. The observer brought the double-reflected image of the celestial body being observed into coincidence with the bubble of the spirit level. Such devices have been made available from time to time, and are still being manufactured. However, they have never come into general use, and are of questionable value.

Charles A. Lindbergh's historic solo flight across the North Atlantic in 1927 demonstrated the practicability of long over-water flights. The development of a suitable instrument for observing altitudes of celestial bodies during darkness and when the horizon was obscured by clouds or haze became a virtual requirement. Various forms of artificial horizon have been used, including a bubble, gyroscope, and pendulum. Of these, the bubble has been most widely used. Figure 1513a illustrates a modern periscopic sextant permitting observation with only a small tube protruding through the top of the airplane. Figure 1513b shows the optical principle of a different type aircraft sextant.

With an artificial horizon of the bubble or pendulum type, considerable skill is needed to make an observation. The image of the horizontal reference (a circle or horizontal line) and the celestial body both appear in the field of view, and both may seem unsteady. An observation is made by matching the two near the center of the field of view. The appearance at coincidence depends upon the instrument. Some bubbles appear dark and are placed on a level with the body. Others have a clear center and are placed over the body. One pendulum type has a horizontal line that is customarily

placed directly across the body, although a limb observation can be made if desired. Bubbles can be regulated in size, and the instructions provided with the instrument should be followed. In general, the bubble diameter should be about one-sixth to one-fourth the size of the field of view. This is about three to four times the size of the sun or full moon as seen through the eyepiece. A very small bubble should be avoided because it tends to lag sextant movements so much that it is unreliable as a horizontal reference.

A considerable amount of practice is needed to develop skill in making reliable observations with an artificial-horizon sextant, even on land or other steady platform. At sea or in the air the motions of the craft greatly increase the difficulty of observation. In addition to compounding the difficulty of making coincidence, the craft motion introduces a sometimes large and rapidly varying **acceleration error.** That is, motions of the craft produce an acceleration on the pendulum or the liquid of the bubble chamber, causing false indication of the horizontal. In smooth air the accelerations tend to follow a cycle of about one to two minutes in length. They are largely eliminated by use of an averaging device. In making an observation, the observer attempts to maintain coincidence continuously over a period, usually two minutes. The *average* altitude, generally indicated on a dial or drum, is used with the *mid* time of observation. Thus, perhaps 60 individual observations, or a continuously integrated altitude, are available to smooth out errors of individual observations.

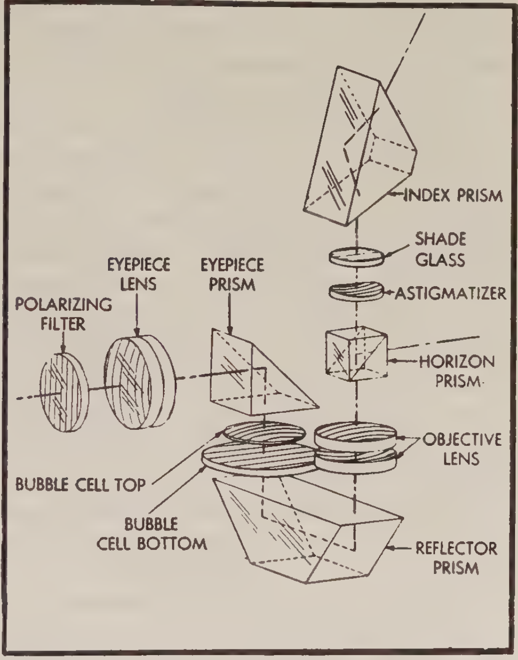


FIGURE 1513b.—Optical principle of a typical bubble sextant.

On land or other steady platform a skillful observer using a two-minute averaging bubble or pendulum sextant can measure altitudes to an accuracy of perhaps 2′ (two miles). This, of course, refers to the accuracy of measurement only, and does not include additional errors such as abnormal refraction, deflection of the vertical, computational and plotting errors, etc. In steady flight through smooth air the error of a two-minute observation is increased to perhaps five to ten miles. At sea, conditions are different. In a glassy sea with virtually no roll or pitch, results should approach those on land. However, with even a slight, gentle roll the accelerations to which a vessel is subjected are quite complex, as indicated by the difficulty one not accustomed to the sea has in getting his "sea legs" during the early part of a voyage. If the vessel is yawing, a large Coriolis error (art. 1611) may be introduced. Under these conditions observational errors of 10–15 miles are not unreasonable. With a moderate sea, errors of 30 miles or more are common. In a heavy sea, any useful observations are virtually impossible to obtain. Single altitude observations in a moderate sea can be in error by a matter of *degrees.*

Because of the difficulty of observing, and the large acceleration errors encountered aboard a vessel, bubble and pendulum type sextants have very limited use at sea. A submarine on war patrol, surfacing only during darkness, may have use for such an instrument. A large number of observations on a reasonably calm night can produce

results of some value. However, even under these conditions some navigators report better results with a marine sextant and dark-adapted eyes. In pack ice a ship generally provides a reasonably steady platform. When the horizon is obscured by ice or haze, polar navigators can sometimes obtain better results with an artificial-horizon sextant than with a marine sextant. Some artificial-horizon sextants have provision for making observations with the natural horizon as a reference, but since this is a secondary usage, results are not generally as satisfactory as by marine sextant. Because of their more complicated optical systems, and the need for providing a horizontal reference, artificial-horizon sextants are generally much more costly to manufacture than marine sextants. Designed for use in the air, they serve a useful purpose there, but for ordinary use aboard ship they have little to recommend them.

Altitudes observed by artificial-horizon sextant are subject to the same errors as those observed by marine sextant, except that dip (height of eye) correction does not apply. Also, when the center of the sun or moon is observed, no correction for semidiameter should be made. Chapter XVI explains the various sextant altitude corrections and their applications.

Adjustment of an artificial-horizon sextant should not be attempted by other than an instrument man qualified to handle the particular type instrument involved. An exception is the adjustment of the size of the bubble. Also, with some instruments an easily movable index permits elimination or reduction of index error. This error can best be determined in an instrument shop equipped with a collimator. If one is not available, the error can be determined by comparing the average of a number of observations made at a known point on land with the computed values. A precomputed curve of altitude versus time is useful for this purpose. Altitude corrections equal to the errors but with reversed sign should be applied to computed altitudes. With normal usage, the index error should not change. In most artificial-horizon sextants there is no index error.

The care and operation of various types of instruments vary considerably. The instruction booklet provided with each instrument should serve as the guide. Information on certain artificial-horizon sextants, and a general guide to artificial-horizon sextant observation, is given in H.O. Pub. No. 216, *Air Navigation*, and other texts.

1514. The marine chronometer is a timepiece having a nearly constant rate. It is used aboard ship to provide accurate time, primarily for timing celestial observations for lines of position, and secondarily for setting the ship's other timepieces. It differs from a watch principally in that it contains a variable lever device to maintain even pressure on the mainspring, and a special balance designed to compensate for temperature variations. A ship in which celestial navigation is used carries one or more chonometers.

A chronometer is set approximately to Greenwich mean time (GMT) and is not reset until the instrument is overhauled and cleaned, usually at three-year intervals. Resetting might disturb the rate. Instead, the difference between GMT and **chronometer time** (C) is carefully determined, and applied as a correction to all chronometer readings. This difference, called **chronometer error** (CE), is "fast" (F) if chronometer time is later than GMT, and "slow" (S) if earlier. The amount by which chronometer error changes in one day is called **chronometer rate,** or sometimes **daily rate,** considered "gaining" or "losing" as the chronometer is running faster or slower than the correct rate. An erratic rate indicates a defective instrument, or need for overhaul. The methods of determining and applying chronometer error and chronometer rate are explained in chapter XIX.

A chronometer is mounted in gimbals in a box, which should be carefully stowed to protect the instrument from damage due to heavy rolling and pitching, vibration, temperature variations, and electrical and magnetic influences. Usually this is done by fitting the box snugly into a heavily padded case suitably located in the chart room of merchant ships, and below decks, near the center of motion, in U. S. Navy ships.

The principal maintenance requirement aboard ship is regular winding at about the same time each day. Aboard United States naval vessels this is customarily done at about 1130 each morning, and reported to the commanding officer at 1200. Aboard merchant ships it is usually wound at about 0800. Although a chronometer is designed to run for more than two days, daily winding helps insure a uniform rate, and constitutes a daily routine that decreases the possibility of letting the instrument run down. On the face of each chronometer is a small dial that indicates the number of hours before the chronometer will be run down. To wind the chronometer, gently turn the instrument on its side, and slide back the guard covering the keyhole. Insert the key and carefully wind in a counterclockwise direction. Seven half-turns should suffice. If a chronometer should run down, wait until GMT is nearly the same as the time indicated before winding. If the chronometer does not start after winding, move the case back and forth gently. Check the error and rate carefully.

At maximum intervals of about three years, a chronometer should be sent to a good chronometer repair shop for cleaning and overhaul. When transported by hand, a chronometer should be clamped in its gimbals and stowed in the large case provided. When shipped, it should be allowed to run down, and the balance secured by a cork before the chronometer is stored in the large case.

Detailed instructions for the care and handling of chronometers are available to U. S. Navy ships in the *Bureau of Ships Manual* and current directives.

1515. Watches.—In the interest of accuracy, a chronometer is not disturbed more than necessary. Celestial observations are timed and ship's clocks set by means of a **comparing watch.** This is a high-grade pocket watch which is set by comparison with a chronometer, and then carried to the place where accurate time is needed. For celestial navigation, a comparing watch should have a large sweep-second hand which can be set. A comparing watch used for timing celestial observations should preferably be set to Greenwich mean time, to avoid the necessity of applying a correction for each observation.

If the second hand cannot be set, the watch should be set to the nearest whole minute, being sure that the second hand is in synchronism with the minute hand, and the **watch error** (**WE**) determined. If a watch is to be used for other purposes than timing of celestial observations, it might preferably be set to zone time. A comparing watch should be set, or watch error determined, immediately before or after celestial observations are made, to avoid the necessity for determining and applying a correction for **watch rate,** and to eliminate a possible error due to an inaccurate or variable rate. If a watch set to GMT is used for timing celestial observations, care should be taken to avoid a possible error of 12 hours or 24 hours. The mental application of zone description to ship's time indicates the approximate GMT and the Greenwich date. The subject of time is discussed more fully in chapter XIX. A stop watch may also be used for celestial observations.

Watches rated to sidereal time (art. 1913) can be purchased, but these have limited use aboard ship.

1516. Other instruments.—The sextant, chronometer, and comparing watch (or stop watch) are the principal instruments of celestial navigation. The azimuth circle

for observing azimuths of celestial bodies is discussed in article 629. Plotting equipment is the same as that for dead reckoning (arts. 602–606). A flashlight might be needed for reading the sextant and the comparing watch. A pocket notebook is desirable for recording predicted positions of celestial bodies if a star finder is used, and for recording the observations. A workbook is desirable for solving celestial observations, so that a permanent record is available. Work forms are desirable, but should form part of the work book, and not be kept separately. These might be provided by rubber stamp, or by printing. In the latter case a looseleaf work book may be desirable to permit arrangement of the various papers in chronological order.

CHAPTER XVI

SEXTANT ALTITUDE CORRECTIONS

1601. Need for correction.—Altitudes of celestial bodies, obtained aboard ship for the purpose of establishing lines of position, are normally measured by a hand-held **sextant,** described in chapter XV. The uncorrected reading of a sextant after such an operation is called **sextant altitude (hs).** If the sextant is in proper adjustment, certain sources of error are eliminated, as explained in article 1509. There remains, however, a number of sources of error over which the observer has little or no control. For each of these he applies a correction. When all of these **sextant altitude corrections** have been applied, the value obtained is the altitude of the center of the celestial body above the celestial horizon, for an observer at the center of the earth. This value, called **observed altitude (Ho),** is compared with the **computed altitude (Hc)** to find the **altitude difference** (*a*) used in establishing a line of position, as explained in chapter XVII.

Articles 1602–1620 describe the various corrections. For highly accurate results, all of these are needed to the greatest accuracy obtainable. The needs of ordinary practical navigation, however, make no such exacting requirements, and in the course of his usual day's work at sea, the navigator has relatively few corrections to apply, from conveniently-arranged tables readily accessible to him. The detailed information in articles 1602–1620 is given to (1) provide the basis for a better understanding of the problem, (2) furnish the information needed for evaluation of results, and (3) provide a source of reference material beyond that given in the usual navigation text.

1602. Instrument correction (I) is the combined correction for nonadjustable errors (prismatic error, graduation error, and centering error) of the sextant, as explained in article 1509. Usually, this correction is determined by the manufacturer, and recorded on a card attached to the inside of the top of the sextant box. It varies with the angle, may be either positive or negative, and is applied to all angles measured by that instrument. For a well-made instrument, the maximum value is so small that this correction can be ignored for all except the most accurate work. Normally, instrument error of artificial-horizon sextants is so small, considering the precision to which angles can be measured by such instruments, that no correction is provided.

1603. Index correction (IC), due to lack of parallelism of the horizon glass and index mirror at zero reading, is discussed in article 1509. Until the adjustment is disturbed, the index correction remains constant for all angles, and is applicable to all angles measured by the instrument. It may be either positive or negative. Normally, artificial-horizon sextants do not have index corrections. Some navigators prefer to adjust their marine sextants so as to eliminate index correction. This is good practice *if* one remembers to check the value each time the sextant is used. Other navigators prefer to retain an index correction to serve as a reminder to check the values. Since dip (art. 1606) at any given height of eye is also a constant for all altitudes, the need for applying both IC and dip can be eliminated by adjusting the sextant so that IC is numerically equal to dip, but of opposite sign. Such a practice should not be used unless the observer has some positive system of reminding himself that the value should be checked each time the instrument is used, and changed if the height of eye changes. It is of little value if observations are not generally made from the same height of eye. If personal correction (art. 1604) is constant, it can also be combined with the index

correction. However, it is generally preferable to keep each of these corrections separate, to avoid possible error.

1604. Personal correction (**PC**) is numerically the same as personal error (art. 1507), but of opposite sign, either positive or negative. If experience indicates the need for such a correction, it should be made to altitudes of the bodies to which it applies. However, the observer should be sensitive to changes in its value. Unless the observer has sufficient evidence to be sure of the existence and relative constancy of a personal error, no correction should be applied. The possibility of combining this correction with dip is explained in article 1603.

1605. Tilt correction (**N**).—The altitude of a celestial body is the *vertical* angle above the horizon. The angular distance from the body to any point on the horizon other than that vertically below the body is greater than the altitude. Therefore, if the frame of a marine sextant is not held vertical during observation, the angle measured is too great, and a negative **tilt correction** is needed. Tables of this correction have been prepared, but they are generally not applicable because **tilt error** can be eliminated by rocking the sextant (art. 1505). If this is not done accurately, an error may remain, but the observer is not aware of it, and therefore does not know the size of the angle of tilt. A "ball recording" artificial-horizon sextant used to some extent during World War II measured the tilt angle, and a tilt correction table was provided with the sextant. Bubble sextants are kept vertical by centering the bubble. With an artificial horizon, there is no tilt error because the celestial body is aligned with its own image.

Tilt correction increases with greater angle of tilt. For the same angle it also increases with greater altitude of the body. There is difference of opinion as to whether the value continues to increase after an altitude of 45°, or whether it then begins to decrease. This question resolves itself into one of whether the axis of tilt is horizontal or in the line of sight to the body. Evidence seems to favor the line of sight axis, with the error being maximum at altitude 45°. The correction, if there is one, applies equally to all celestial bodies. In rough weather, when observation may be difficult, this error can be minimized by observing bodies that are not high in the sky.

1606. Dip (**D**) of the horizon is the angle by which the visible horizon (art. 1428) differs from the horizontal at the eye of the observer (the sensible horizon, art. 1428). Thus, it applies only when the visible horizon is used as a reference, and not when an artificial horizon, either internal or external to the sextant, is used. It applies to all celestial bodies. If the eye of the observer were at the surface of the earth, visible and sensible horizons would coincide, and there would be no dip. This is never the situation aboard ship, however, and at any height *above* the surface, the visible horizon is normally *below* the sensible horizon, as shown in figure 1428b. Normally, then, an altitude measured from the visible horizon is too *great*, and the correction is *negative*. It increases with greater height of the observer's eye. Because of this, it is sometimes called **height of eye correction.**

If there were no atmospheric refraction, dip would be the angle between the horizontal at the eye of the observer, and a straight line from this point tangent to the surface of the earth. In figure 1606a, the eye of the observer is at A, at some point above the surface of the earth. The line AB is the horizontal through A, and AC is the tangent through this point. Angle BAC is the dip at A, neglecting refraction. Since OA is perpendicular to AB, and OD is perpendicular to AC (art. O30), angle AOD is equal to angle BAC (art. O27). If r is the radius of the earth, and h is the height of the observer's eye above the surface, the cosine of angle AOD is $\frac{r}{r+h}$.

The line of sight to the horizon, however, passes through the lowest layers of the earth's atmosphere, where the density of the atmosphere normally decreases as

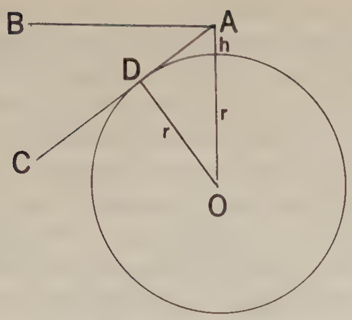


FIGURE 1606a.—Dip without refraction.

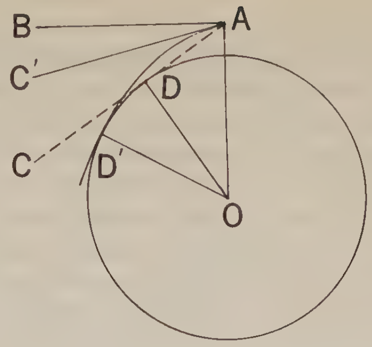


FIGURE 1606b.—Dip with refraction.

height above the surface increases. Consequently, the ray of light from the horizon to the observer's eye is bent by refraction. The result of this **terrestrial refraction** is to increase the distance to the horizon, which is at D' (fig. 1606b) instead of at D. This actual distance, under normal conditions, is given in table 8. Although the horizon is farther away than it would be if there were no refraction, it appears *higher*, for the eye of the observer does not detect the curvature of the line of sight. Therefore, the horizon *appears* to be at C' instead of at C. The dip shown in the tables is BAC'. The effect of refraction, $C'AC$, is shown exaggerated for purposes of illustration.

The amount by which refraction alters dip varies with changing atmospheric conditions. Even the *average* value has not been established with certainty, and several methods of computing dip have been proposed. The values given in the critical table on the inside front cover of the *Nautical Almanac* were computed by the equation

$$D=0.97\sqrt{h},$$

where D is the dip, in minutes of arc; and h is the height of eye of the observer, in feet. Part of this table is repeated on the page facing the inside back cover. The *Air Almanac* table was computed independently by a different method, to a precision of whole minutes. The minor discrepancies thus introduced are not important in practical navigation.

The values given in the table are satisfactory for practical navigation under most conditions. An investigation by the Carnegie Institution of Washington showed that of 5,000 measurements of dip at sea, no value differed from the tabulated value by more than 2′.5, except for one difference of 10′.6. Extreme values of more than 30′ have been reported, and even values of several *degrees* have been encountered in polar regions. Greatest variations from tabulated values can be expected in calm weather, with large differences between sea and air temperatures, particularly if mirage effects are present. Irregularities in the shape of the rising or setting sun may indicate abnormal conditions. Large variations may also be present shortly after passage of a squall line, when errors of as much as 15′ have been reported. When a temperature inversion is known to exist, the tabulated dip may be too small, numerically. The effect of sea-air temperature difference is discussed in greater detail in article 1607.

In the determination of height of eye, position on the ship should be considered, and also the condition of loading and trim. If an observation is made from a position differing from the usual place, the altered height of eye should not be overlooked. Momentary changes due to rolling and pitching can be neutralized, to a large extent, by making observations from a point on the center line of the vessel, at the axis of pitch. The possibility of combining dip and index corrections is discussed in article 1603.

Instruments and marine sextant attachments for measurement of dip have been devised, but are not generally available to the navigator. However, the sextant can

be used without special attachment if it has an arc of sufficient length. The method is to measure the "altitude" of the opposite horizon; that is, the angle (through the zenith) between the lines of sight to the horizon on reciprocal bearings. This is equal to 180° plus the sum of the dip in the two directions. If these can be considered equal, and under stable conditions the difference is probably not great, dip is equal to *half* the difference between 180° and the measured angle (corrected as necessary). However, direct measurement of an angle greater than 180° cannot be made with an ordinary sextant because the reflecting surfaces of the sextant mirrors would have an angle of more than 90° with respect to each other, and a ray of light could not be reflected from the index mirror to the surface of the horizon glass. Satisfactory results can sometimes be obtained by observing the altitude of a body first by facing toward it, in the usual manner, and then by facing 180° from it (a back sight, art. 1633). In the case of the sun or moon, the same limb should be observed in both directions. (It will appear as the opposite limb.) Instrument correction, index correction, and personal correction, as applicable, are applied, and the two altitudes added. The difference between this value and 180° is the sum of the dip in the two directions, if allowance is made for the change of altitude between observations. Unless the body is near the celestial meridian, this is best done by taking a direct sight, a back-sight, and another direct sight, with equal time intervals between observations. The average of the two direct sights is used.

Since variations from normal dip may be one of the principal sources of error in celestial observations, a method of determing dip at sea can be of considerable value. If such a method is not available, the observer should be alert to conditions affecting terrestrial refraction. *Any* observation taken within half an hour after passage of a squall line should be regarded as unreliable. If dip cannot be measured, the effects of abnormal conditions can be minimized by observing three bodies differing in azimuth by about 120° (or four bodies by 90°, five bodies by 72°, etc.). If the error is constant in all directions, its effect is to increase (or possibly to decrease) the size of the closed figure formed by the lines of position without altering the position of its center. Hence, the size of the figure is not necessarily an indication of the accuracy of the fix.

Recent evidence accumulated by the Office of Naval Research indicates that dip may fluctuate somewhat erratically over a range of at least several tenths of a minute, in addition to the slower changes associated with abnormal conditions. This may be caused by irregularities in the atmosphere, producing variations in the refraction. This effect cannot be removed either by measurement of dip or by observations of bodies equally spaced in azimuth, because the dip is likely to change between observations. Over a period of several minutes the variation from the mean value can probably be considered a random error (art. 2904) and therefore might be reduced by making a large number of observations of a single body, and plotting the results on cross-section paper in the manner explained in article 1507.

If land, another ship, or other obstruction is between the observer and his horizon, an altitude can be measured by using the water line of the obstruction as a horizontal reference, if its distance from the observer is known. In this case the dip is greater than that given in the almanacs. Table 22 gives the values to be used.

Further discussion of refraction is given in article 1613. When abnormal astronomical refraction occurs, abnormalities in terrestrial refraction can be expected.

1607. Sea-air temperature difference correction (S).—Under normal atmospheric conditions, the temperature and pressure both decrease at standard rates with increase in height above the surface. Accordingly, the *density* of the atmosphere also decreases at a standard rate, which is uniform over the height encountered aboard ship. The effect of refraction upon dip, as given in the tables, is based upon this standard rate.

Usually, the difference between standard and actual conditions is not great enough to introduce important errors in the assumption that standard conditions exist.

However, when there is a difference between sea and air temperatures at the surface, the air in contact with the sea is warmed or cooled by the sea water, upsetting the normal rate of decrease near the surface. The effect is greater as the temperature difference increases. It may extend only a few inches above the surface, or for hundreds of feet. Under extreme conditions, if the air is very much colder than the water, the surface may steam. The **frost smoke** rising from the water may obscure the horizon, and under the most severe conditions it may rise to such heights as to interfere with visibility. Celestial bodies can be seen, but altitudes cannot be measured with a marine sextant because of lack of a horizon.

Under less extreme conditions, the dip is altered, but observations may seem normal. If the water is warmer than the air, the horizon is depressed and dip is increased. Under these conditions the measured altitudes are too great. Therefore, as a correction to the *altitude*, the sea-air temperature difference correction is negative when the water is warmer than the air. When the air is warmer, the reverse is true, and the altitude correction is positive.

Various attempts have been made to establish a simple relationship between the sea-air temperature difference and the correction, but the results reported by different investigators differ considerably. This is due, in part, to difference of opinion as to the height and depth at which measurements should be made, difficulties in obtaining accurate readings near the surface, variations of temperature differences at the ship and along the line of sight to the horizon, and influence of the vessel on temperatures in its immediate vicinity. Wind, too, has a considerable effect. On a calm day, the lower portion of the atmosphere tends to form in layers, without mixing. If there is a strong, gusty wind, turbulence in the air minimizes the effect due to temperature difference. Actually, sea temperature serves only to indicate temperature at the surface, but temperature gradient in the water may be large, as in the air. Therefore, the ideal would seem to be the measurement of *air* temperature *at the surface*, and at some greater height, since it is the abnormal **lapse rate** (decrease of air temperature with height) that produces the change in normal terrestrial refraction.

Suggested factors based upon difference between temperature of the sea and air vary from about 0.′11 per degree Fahrenheit (0.′20 per degree Celsius) to 0.′21 per degree Fahrenheit (0.′37 per degree Celsius). The average of these is about 0.′16 per degree Fahrenheit (0.′28 per degree Celsius). Thus, the correction is about one-sixth of a minute per degree Fahrenheit, or one minute for each six degrees. The methods of measuring sea and air temperature are discussed in article 3712.

This correction applies to all bodies when the sea horizon is used. However, it should be used with caution, and only under conditions which indicate that better results will be obtained if it is used. Under normal conditions, it is not used. If abnormal conditions are suspected, observations are avoided or considered of questionable reliability; or the precautions indicated in article 1606 are used. If allowance for abnormal conditions is made by using an altered value of dip, as one obtained by measurement, the sea-air temperature difference correction is not used. That is, if allowance is made for abnormal dip, *either* the tabulated value of dip is altered *or* the sea-air temperature correction is applied, *but not both.*

1608. Wave height correction (W).—Corrections for dip are based upon the assumption of a calm sea. Waves disturb this condition, causing the surface to be alternately raised and lowered. At the horizon, the troughs of waves are usually not visible. Through binoculars, irregularities in the line forming the horizon can sometimes be seen, but observations are made from the *tops* or nearly the tops of the waves.

If it is assumed that the vessel is not raised and lowered by the waves, the line of sight to the horizon is *raised* by waves. Refer to figure 1608. The wavy line represents the surface of the sea, with the size of the waves exaggerated. The equivalent still water level is shown by the broken line. The line AB is the curved tangent to the still water level, representing the line of sight if there were no waves. The line AC represents the actual line of sight to the top of a wave. If the slight curvature of AB and AC is neglected, the tangent of the wave height correction (CAB) is $\frac{CB}{AB}$.

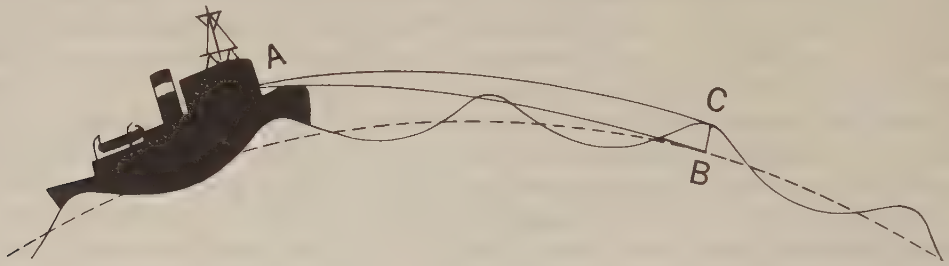


FIGURE 1608.—Effect of wave height on line of sight to horizon.

In this case, CB is approximately one-half the wave height, and AB is the distance to the horizon, both in feet. That is, AB is the value from table 8 multiplied by 6076.11549 . . . if nautical miles are used, or 5,280 if statute miles are used. The increased height of the sea decreases this distance slightly, but the decrease is too small to be a consideration except at low heights of eye or with very high waves. For waves two feet high, the correction is 0′.2 for a height of eye of seven feet. For waves six feet high, the correction is 0′.3 for a height of eye of 30 feet. For waves 20 feet high, the correction is 1′.3 for height of eye of 15 feet, 0′.9 for 30 feet, and 0′.7 for 50 feet. For waves 40 feet high, the correction is 2′.0 at 30 feet, and 1′.1 at 80 feet.

This correction is always positive, and applies to all celestial bodies, but only if the sea horizon is used as a reference. Normally, it is not applied because of the difficulty of determining (1) wave height at the horizon, and (2) height of eye above the equivalent calm level of the sea. Better practice is usually to estimate height of eye above the wave tops, allowing for motions of the vessel, and make no correction.

1609. Sea tilt correction (H).—The height of the sea at any place is affected by the density of the sea water, its temperature, and atmospheric pressure. Because of differences in these values, the height varies from place to place. This results in tilting of the surface of the sea, which is "downhill" from the ridge of high water to that of low water. The maximum tilt due to these causes is probably a little more than 1″.5. The wave caused by the tides also tilts the sea surface. However, on the open sea, tides are seldom more than about two and one-half feet high, and the distance from crest to trough is about 5,400 miles, or one-quarter of the great-circle distance around the earth. Under these conditions, the maximum tilting of the sea surface due to tides is about 0″.025. This may be increased somewhat by storm waves (art. 3311) or tsunamis (art. 3310). In confined waters, particularly in a funnel-shaped area where tides enter from the wide end and progress up a narrowing estuary, the error may be very much greater, possibly reaching a value of half a *minute*. The expression **tide correction** may be used instead of "sea tilt correction."

The correction is positive in the direction of high water and negative in the direction of low water. Between these directions it is equal approximately to the value in these directions multiplied by the cosine of the angle between the wave axis (the line perpendicular to the wave front) and the azimuth of the body. It applies equally to all bodies when the visible horizon is used as the reference. In practice it is not applied.

1610. Deflection of the vertical (V).—Usually, the direction of gravity is assumed to be normal (vertical) to the spheroidal surface of the earth. This assumption is not quite correct. Irregularities in density and height of the material making up the surface crust of the earth result in slight alterations of the direction of gravity. This **deflection of the vertical** is most apparent near high mountains bordering a deep sea (fig. 1610) where an extreme value of more than 1'.1 might be encountered. An experienced geodesist can predict the value with an average error of perhaps 50 percent

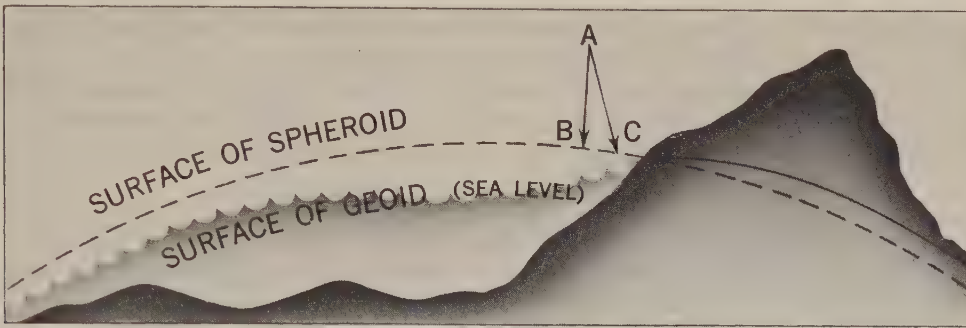


FIGURE 1610.—Deflection of the vertical. *AB* is normal to spheroid. *AC* is normal to geoid. Angle *BAC* is deflection of the vertical.

of the true value. On land, deflection of the vertical can be measured by carefully determining a highly accurate position by celestial observation, called an **astronomical position** (composed of **astronomical latitude** and **astronomical longitude**), and comparing this position with one determined by measurement (either by triangulation or trilateration) from a "known" position. Deflection of the vertical is always a relative value depending upon the position considered "known." A position expected to be relatively free from deflection is usually used as the starting point for a system of measurements, and is known as a **datum.** The **North American Datum of 1927,** used for surveying most of North America, is a station known as Meades Ranch, Kansas.

It has not been possible to measure deflection of the vertical by means of a single observation at sea, due largely to lack of a stable platform and the inability to extend triangulation or trilateration to ships at sea, with the required accuracy. However, this correction is of interest only when establishing a position relative to a fixed point on land, as when a shore-based electronic aid is used. For normal purposes of celestial navigation, it is not significant, for it is usually quite small. Moreover, it changes gradually from the position of the vessel to the destination, so that as land is approached, deflection of the vertical tends to approach the value on shore.

The shape of the earth, if surface irregularities (mountains, etc.) are neglected, is considered a **spheroid** if deflection of the vertical is neglected, and a **geoid** if that deflection is considered. The surface of the geoid is everywhere perpendicular to the direction of gravity. In general, the geoidal surface is *higher* than the spheroidal surface ashore, and *lower* at sea, as shown in figure 1610.

Normally, values of deflection of the vertical are not available to the navigator, and are not needed by him. In precise work, however, such values for a particular area might be furnished. The correction is negative in the direction toward which the zenith is deflected, and positive in the opposite direction. In any other direction it is equal approximately to the maximum deflection times the cosine of the angle between the given direction and the direction of maximum deflection, taking the sign of the nearest maximum deflection. It is applicable to all celestial bodies, whether the natural sea horizon or an artificial horizon is used.

1611. Coriolis correction (Z).—When a body is in motion over the surface of the earth, its motion *in space* is a combination of its motion relative to the earth, and the motion of the earth. Because of the rotation of the earth, principally, the path is a curved one. As a result, there is an apparent force causing deflection to the right in the northern hemisphere and to the left in the southern hemisphere. Because of this **Coriolis force,** ocean currents set in motion by wind flow in a direction to the right (northern hemisphere) of the direction in which the wind blows. Wind, too, is deflected. Instead of blowing directly from an area of high pressure to one of low pressure, and soon neutralizing the pressure difference, it moves toward one side. The result is the characteristic circulation around highs and lows.

The liquid of the bubble chamber of a bubble sextant, and a pendulum, are similarly affected, causing them to give a false indication of the vertical (or horizontal). The same is true of an artificial horizon. The equation for the deflection is

$$Z=2'.62\ S \sin L+0'.146\ S^2 \sin C \tan L-5'.25\ SC',$$

where Z is the Coriolis correction, S the speed over the surface of the earth in units of hundreds of knots, L the latitude, C the true course angle, and C′ the rate of change of true course angle in degrees per minute. The first term, $2'.62\ S \sin L$, corrects for motion along a great circle; the second term, $0'.146\ S^2 \sin C \tan L$, is an additional correction for the difference between motion along a rhumb line, and equivalent motion along a great circle; and the third term, $5'.25\ SC'$, is an additional correction for departures from the course, being negative (as shown) if the departure is right in the northern hemisphere or left in the southern hemisphere. Coriolis corrections (first term only) are given on the inside back cover of the *Air Almanac*.

Coriolis correction may be either positive or negative, and varies with speed and latitude. It applies to all bodies equally, and therefore can be applied to the altitude, the assumed position, or even as an adjustment to the plotted line of position or fix. If the AP or fix is adjusted, it is moved perpendicular to the course line, to the *right* in the northern hemisphere and to the *left* in the southern hemisphere, unless the third term is of such magnitude and sign as to make the entire correction negative, when it is applied in the opposite direction. If the correction is applied to the altitude, the value obtained by formula is multiplied by the sine of the *relative* azimuth. In the northern hemisphere, the resulting altitude correction is positive if the celestial body is on the starboard side, and negative if on the port side. In the southern hemisphere these signs are reversed. These signs assume that the value obtained by the formula is positive. If it is negative, all signs are reversed.

At ship speeds, the Coriolis correction is not large, unless the vessel is yawing considerably. For a ship steaming at 20 knots on a steady course of 090° at latitude 40°, the maximum Coriolis correction is 0′.3 for a celestial body which is abeam. Acceleration error due to rolling and pitching of the vessel is usually much greater than this, and is the principal reason why bubble or pendulum sextants are not often used aboard ship, as indicated in article 1513.

There is no Coriolis correction when the visible horizon is the horizontal reference.

1612. Acceleration correction (C).—If an artificial horizon-sextant with a bubble or pendulum is used, the liquid of the bubble chamber or the pendulum is affected by all accelerations of the instrument. The same is true of the free surface of the liquid of an artificial horizon. With high accelerations such as those due to rolling and pitching of a vessel, or changes of course or speed, the error can be very large. It is for this reason that such instruments are not customarily used aboard ship. Under normal conditions at sea the navigator does not have the information needed to compute the correction. The error is minimized by making observations at the center of roll

and pitch of the ship, or averaging the values taken at both ends of a roll or pitch. Observations should not be made during a turn or when the speed is being changed. Even with these precautions the error is usually unacceptably large except with an almost flat, calm sea. The effect on the level of the sea surface due to accelerations of the earth in its rotation or revolution is considered negligible.

The correction may be either positive or negative, and applies equally to all bodies observed with a bubble or pendulum sextant.

1613. Refraction (**R**).—Light, or other radiant energy, is assumed to travel in a straight line at uniform speed, if the medium in which it is traveling has uniform

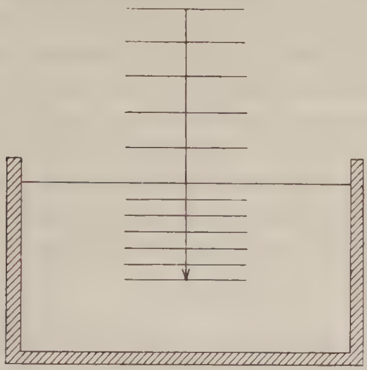


Figure 1613a.—No refraction occurs when light enters denser medium normal to the surface.

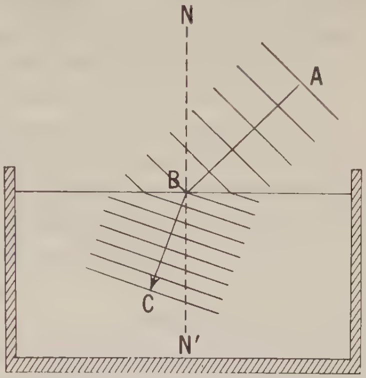


Figure 1613b.—A ray entering a denser medium at an oblique angle is bent toward the normal.

properties. But if light enters a medium of different properties, particularly if the density is different, the speed of light changes somewhat. Light from a single point source travels outward in all directions, in an expanding sphere. At great distances, a small part of the surface of this sphere can be considered flat, and light continuing to emanate from the source can be considered similar to a series of waves, in some respects resembling the ocean waves encountered at sea. If these light "waves" enter a more dense medium, as when they pass from air into water, the speed decreases. If the light is traveling in a direction perpendicular to the surface separating the two media (in this case vertically downward), all parts of each wave front enter the new medium at the same time, and so all parts change speed together, as shown in figure 1613a. But if the light enters the more dense medium at an oblique angle, as shown in figure 1613b, the change in speed occurs progressively along the wave front as the different parts enter the more dense medium. This results in a change in the direction of travel, as shown. This change in direction of motion is called **refraction.** If light enters a more dense medium, it is refracted *toward* the normal (NN'), as in figure 1613b. If it enters a less dense medium, it is refracted *away* from the normal, as light traveling in the opposite direction to that shown in figure 1613b.

The *amount* of the change in direction is directly proportional to the angle between the direction of travel and the normal (angle ABN in figure 1613b). The ratio of this angle to the similar angle after refraction takes place (angle CBN' in figure 1613b) is constant, so that as one increases, the other increases at the same *rate*. Hence, the *difference* between them (the change in direction) also increases at the same rate. Therefore, if the *incident* ray (AB) is nearly parallel to the surface at which refraction takes place, relatively large amounts of refraction occur.

The amount of refraction is also directly proportional to the relative speed of travel in the two media. Various substances are compared by means of a number called the

index of refraction (μ), which depends primarily upon the density of the substance. In figure 1613b, angle *ABN* is called the **angle of incidence** (ϕ) and angle *CBN′* the **angle of refraction** (θ). These are related by Snell's law, which states that *the sines of the angle of incidence and angle of refraction are inversely proportional to the indices of refraction of the substances in which they occur.* Thus, if μ_1 is the index of refraction of the substance in which ϕ occurs, and μ_2 is the index of refraction of the substance in which θ occurs

$$\frac{\sin \phi}{\sin \theta}=\frac{\mu_2}{\mu_1}.$$

If the index of refraction changes suddenly, as along the surface separating water and air (as shown in fig. 1613b), the change in direction is equally sudden. However, if a ray of light travels through a medium of gradually changing index of refraction, its path is curved, undergoing increased refraction as the index of refraction continues to change. This is the situation in the earth's atmosphere, which generally decreases in density with increased height. The gradual change of direction occurring there is called **atmospheric refraction.** The bending of a ray of light traveling from a point on or near the surface of the earth, to the eye of the observer, is called **terrestrial refraction.** This affects dip of the horizon, as discussed in article 1606. A ray of light entering the atmosphere from outside, as from a star, undergoes a similar bending called **astronomical refraction.**

The effect of astronomical refraction is to make a celestial body appear *higher* in the sky than it otherwise would, as shown in figure 1613c. If a body is in the zenith, its light is not refracted, except for a very slight amount when the various layers of the atmosphere are not exactly horizontal. As the zenith distance increases, the refraction becomes greater. At an altitude of 20° it is about 2′.6; at 10°, 5′.3; at 5°, 9′.9; and at the horizon, 34′.5. A table of refraction is given on the inside front cover and facing page of the *Nautical Almanac*, in the columns headed "Stars and Planets." As height above the surface of the earth increases, light from an outside source travels through less of the atmosphere, and refraction decreases. At shipboard heights the difference is negligible, but at aircraft heights the change is a consideration. Therefore, the table given on the inside back cover of the *Air Almanac* is a double-entry table.

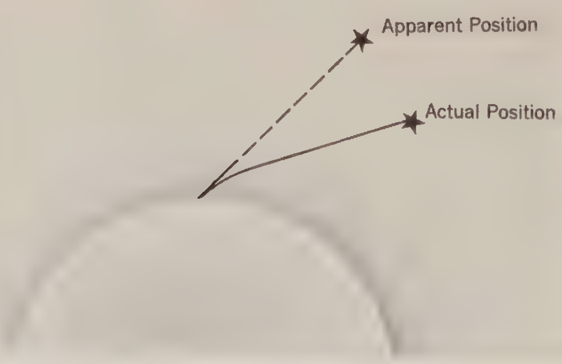


Figure 1613c.—Astronomical refraction.

The values given in the tables are for average conditions. This is called **mean refraction.** A considerable amount of research has been conducted to determine the mean values, the conditions under which values differ from the mean, and the amount of such differences. A number of different mean refraction tables have been produced. Values in the various tables differ slightly because of different assumptions, different methods of observation, and different observed results under apparently similar conditions. This last source of difference is due primarily to the fact that conditions could be determined at the position of the observer, but not at various points along the line traveled by the ray of light in passing through the atmosphere. Nevertheless, the various tables agree very well down to a minimum altitude of 2°. Below this, the refraction is erratic, and differences between values

in the various tables are not as important as differences between mean and instantaneous values. The values given in the almanac tables are in excellent agreement with those actually measured.

Because of their variability, refraction and dip (also affected by refraction) are the principal uncertainties in the accuracy of celestial observations of a careful observer. As a result of this uncertainty, navigators formerly avoided all observations below some arbitrary altitude, usually 15°. While this is still good practice if higher bodies are available, the growing knowledge of atmospheric refraction has increased the confidence with which navigators can use low-altitude sights. There is little reason for lack of confidence in sights as low as 5°. Below this, other available corrections should be applied (art. 1632). If altitudes below 2° are used, larger probable errors should be anticipated, even with the use of additional corrections. Generally, the error in tabulated refraction should not exceed two or three minutes, even at the horizon. However, a knowledge of conditions affecting refraction is helpful in determining the confidence to be placed in such observations. Since refraction elevates both the celestial body and the visible horizon, the error due to abnormal refraction is minimized if the visible horizon is used as a reference.

The atmosphere contains many irregularities which are erratic in their influence upon refraction. Normally, the navigator has not the information needed to correct for such conditions, but only to recognize their existence. As indicated in article 1606, observations made within half an hour after passage of a squall might be considerably in error. The passage of any front might have a similar effect. A temperature inversion (art. 3815) may upset normal refraction. Abnormal values may be expected when there is a large difference between the temperature of the sea and air. With an absence of wind, the air tends to form in layers. When this condition becomes extreme, mirage effects occur. Sometimes the rising or setting sun or moon appears distorted. Multiple horizons may appear, and other ships or islands may seem to float a short distance above the water. Under any such conditions large errors in refraction might be encountered.

Conditions causing abnormal refraction can be expected to occur with considerable frequency in the vicinity of the Grand Banks, along the west coast of Africa from Mogador to Cap Blanc and from the Congo to the Cape of Good Hope, in the Red Sea and the Persian Gulf, and over ice-free water in polar regions. Abnormal refraction may be encountered when offshore winds blow from high, snow-covered mountains to nearby tropical seas, as along the west coast of South America; where cold water from large rivers such as the Mississippi flows into warm sea water; when a strong current flows past a bay or coast, causing colder water to be drawn to the surface, as in the Bay of Rio de Janeiro and Santos, and along the Atlantic coast of Africa between Cape Palmas and Cape Three Points during the time of the southwest monsoon; and along the east coast of Africa in the vicinity of Capo Guardafui during the summer. In the temperate zones abnormal refraction is most common during the spring and summer.

Of the more systematic errors which affect refraction, two can be evaluated, and corrections applied. These are for air temperature (art. 1614) and atmospheric pressure (art. 1615). However, these corrections are based upon assumed standard *gradients* (changes) with height. Temperature gradients are known to vary with type of weather, time of day, season, etc., as well as in a more irregular manner. The various layers of the atmosphere are assumed to be horizontal to the surface, but this is not always the situation. When they tilt, refraction changes. No correction for this cause is available.

Humidity has a relatively slight effect on refraction. In completely dry air, astronomical refraction at the horizon (sometimes called **horizontal refraction**) is perhaps 0.′1 *greater* than the tabulated value. In very moist air, sometimes encountered in the tropics, the maximum refraction might possibly *decrease* by as much as 0.′2.

Wind speed is believed to have some effect upon refraction and dip. Apparently, refraction increases as wind speed becomes greater, the amount of change increasing in direct proportion to the *square* of the wind speed. At 20 knots the change is believed to be about 0.′1 at the horizon, and 0.″15 at altitude 2°. At 30 knots these values are approximately doubled.

Latitude has a slight effect upon refraction because of the decrease in the radius of the earth and the increase of gravity as latitude increases. Both radius and gravity affect density of the atmosphere, and hence refraction. Because of this, mean horizontal refraction is decreased about 0.′2 to 0.′3 at the equator, and increased about an equal amount at the poles. For normal altitudes the change is negligible.

Azimuth may have an effect at some locations. The reason for this is not entirely clear, but is believed to be due to a somewhat permanent tilt of certain atmospheric layers. A series of observations at a location in Germany indicated a difference of as much as 0.′5 between northerly and southerly observations of 0° altitude. At altitude 2° the difference was only 3″. The navigator normally does not have information required to apply such a correction, nor is it of navigational significance at normal altitudes. However, differences should be anticipated when the appearance of the horizon varies with azimuth, or large sea temperature differences exist within a few miles, as near the edge of the Gulf Stream. The same might be true near land, particularly in the tropics.

Dispersion of light of various colors results in light from blue stars being refracted more than light from red stars. At the horizon the maximum correction would be about 2″ for blue stars, and 8″ for red stars. At 5° the amount would be less than one-third these values.

Errors in the tables due to incorrect assumptions are probably too small to be of practical interest to the navigator. If increased knowledge indicates errors exist in the tables, corrected values will undoubtedly be provided.

Since refraction causes celestial bodies to appear elevated in the sky, they are above the horizon longer than they otherwise would be. The mean diameter of the sun and moon are each about 32′, and horizontal refraction is 34.′5. Therefore, the entire sun or moon is actually below the visible horizon when the lower limb appears tangent to the horizon. The effect of dip is to further increase the time above the horizon. Near the horizon the sun and moon appear flattened because of the rapid change of refraction with altitude, the lower limb being raised by refraction to a greater extent than the upper limb.

As a correction to sextant altitudes, refraction is negative because it causes the measured altitude to be too *great*. It *decreases* with increased altitude, and applies to all celestial bodies, regardless of sextant or horizon used.

1614. Air temperature correction (**T**).—The *Nautical Almanac* refraction table is based upon an air temperature of 50°F (10°C) at the surface of the earth. At other temperatures the refraction differs somewhat, becoming greater at lower temperatures, and less at higher temperatures. Table 23 provides the correction to be applied to the altitude to correct for this condition. If preferred, this correction can be applied with *reversed sign* to the refraction from the almanac, and a single refraction applied to the altitude. A combined correction for nonstandard air temperature and nonstandard atmospheric pressure (art. 1615) is given on page A4 of the *Nautical Almanac*. The correction for air temperature varies with the temperature of the air and the altitude of the celestial body, and applies to all celestial bodies, regardless of the method of observation. However, except for extreme temperatures or low altitudes, this correction is not usually applied unless results of unusual accuracy are desired.

1615. Atmospheric pressure correction (**B**).—The *Nautical Almanac* refraction table is based upon an atmospheric pressure of 29.83 inches of mercury (1010 millibars)

at sea level. At other pressures the refraction differs, becoming greater as pressure increases, and smaller as it decreases. Table 24 provides the correction to be applied to the altitude for this condition. A combined correction for nonstandard air temperature (art. 1614) and nonstandard atmospheric pressure is given on page A4 of the *Nautical Almanac*. If the correction is to be applied to the refraction, reverse the sign. This correction varies with atmospheric pressure and altitude of the celestial body, and is applicable to all celestial bodies, regardless of the method of observation. However, except for extreme pressures or low altitudes, this correction is not usually applied unless results of unusual accuracy are desired.

1616. Irradiation correction (**J**).—When a bright surface is adjacent to a darker one, an optical illusion takes place and the bright area appears to be larger than is actually the case. This is called **irradiation.** Thus, when the sky is considerably brighter than the water, the horizon appears slightly depressed. The apparent diameter of the sun is increased slightly by irradiation, and the bright stars appear to have a measurable diameter. Opinions differ on the need for a sextant altitude correction for irradiation. It is probable that during twilight there is insufficient contrast between sky and water to warrant the use of a correction. During the day a slight apparent depression of the horizon may occur. Altitudes of the lower limb of the sun should not be altered because the irradiation effect of sun and horizon are in the same direction and cancel out, approximately. The effect on the upper limb, however, is opposite to that on the horizon. The table of upper limb corrections of the sun given on the inside front cover and facing page of the *Nautical Almanac* includes an irradiation correction of (−) 1′.2, half for the apparent lowering of the horizon, and half for the apparent raising of the upper limb. No irradiation correction is given for bodies other than the sun. Thus, in terms of available corrections, irradiation is negative, is essentially constant, and applies only to the upper limb of the sun when the visible horizon is used as the reference. If an artificial horizon is used, it applies to either limb.

1617. Semidiameter (**SD**) of a celestial body is half the angle, at the observer's eye, subtended by the visible disk of the body. The position of the lower or upper limb of the sun or moon with respect to the visible horizon can be judged with greater precision than that of the center of the body. For this reason it is customary, when using a marine sextant and the visible horizon, to observe one of the limbs of these two bodies, and apply a correction for semidiameter. Normally, the lower limb is used if it is visible. In the case of a gibbous or crescent moon, however, only the upper limb may be available.

The semidiameter of the sun varies from a little less than 15′.8 early in July, when the earth is at its greatest distance from the sun, to nearly 16′.3 early in January, when the earth is nearest the sun. In the *Nautical Almanac* the semidiameter of the sun at GMT 12^h on the middle day of each page opening of the daily page section is given to the nearest 0′.1 at the bottom of the sun's GHA column. The altitude correction tables of the sun, given on the inside front cover and facing page, are divided into two parts, to be used during different periods of the year. The mean semidiameter of each period is included in the tables of both upper and lower limb corrections. The semidiameter each day is listed to the nearest 0″.01 in the *American Ephemeris and Nautical Almanac*. In the *Air Almanac* the semidiameter to the nearest whole minute (always 16′) is given near the lower right-hand corner of each odd-numbered daily page.

The moon undergoes a similar change in semidiameter as its distance from the earth varies. However, because of the greater eccentricity of the moon's orbit than that of earth, the variation in semidiameter is also greater, varying between about 14′.7 and 16′.8. The variation is more rapid, partly because of the greater spread of values, but principally because the moon completes its revolution in approximately

one month, while the earth makes one revolution per year. In the *Nautical Almanac*, semidiameter of the moon at 12^h each day is given to the nearest $0'.1$ at the bottom of the moon data columns. The correction for semidiameter of the moon is included in the corrections given on the inside back cover and facing page. In the *Air Almanac*, semidiameter is given to the nearest whole minute, being shown on the daily pages, immediately below the value for the sun. The semidiameter at intervals of half a day is given to the nearest $0''.01$ in the *American Ephemeris and Nautical Almanac.*

The navigational planets have small semidiameters. For Venus it varies between about $5''$ and $32''$; for Mars, $2''.7$ to $12''.6$; for Jupiter, $16''$ to $25''$; and for Saturn, $7''$ to $10''$. The value for any date is given in the *American Ephemeris and Nautical Almanac*, but not in the *Nautical Almanac* or *Air Almanac* because the apparent *centers* of these bodies are customarily observed.

Stars have no measurable semidiameter.

The computed altitude of a body refers to the *center* of that body, since the coordinates listed in the almanacs are for the center. If the *lower* limb is observed, the sextant altitude is *less* than the altitude of the center of the body, and hence the correction is *positive.* If the *upper* limb is observed, the correction is *negative.* The correction does not apply when the center of the body is observed, which is usually the case when an artificial-horizon sextant is used. With a marine sextant and either the natural or an artificial horizon, semidiameter is customarily applied to observations of the sun and moon, but not other celestial bodies.

1618. Phase correction (F).—Because of phase (art. 1423), the actual centers of planets and the moon may differ somewhat from the apparent centers. Average corrections for this difference are included in the additional corrections for Venus and Mars given on the inside front cover of the *Nautical Almanac.* They should be applied only when these bodies are observed during twilight. At other times the magnitude and even the sign of the correction might differ from those tabulated, because of a different relationship between the body and the horizon. The phase correction for navigational planets other than Venus and Mars is too small to be significant.

A phase correction may apply to observations of the moon if the apparent center of the body is observed, as with an artificial-horizon sextant. However, no provision is made for a correction in this case; the need for it can be avoided by observing one of the limbs of the body.

Phase correction does not apply to observations of the sun or stars.

1619. Augmentation (A).—As indicated in article 1617, semidiameter changes with distance of the celestial body from the observer, becoming greater as the distance decreases. The semidiameter given in the ephemeris and used in the almanacs is for a fictitious observer at the center of the earth. If the celestial body is on the actual observer's horizon, its distance is approximately the same as from the center of the earth; but if the body is in the zenith, its distance is less by about the radius of the earth (fig. 1412). Therefore, the semidiameter *increases* as the altitude becomes greater. This increase is called **augmentation.** For the moon, the augmentation from horizon to zenith is about $0'.3$ at the mean distance of the moon. At perigee it is about $2''$ greater, and at apogee about $2''$ less. Augmentation of the sun from horizon to zenith is about 1/24 of one second of arc. For planets it is correspondingly small, varying with the positions of the planets and the earth in their orbits. At any altitude the augmentation is equal to the sine of the altitude times the value at the zenith.

Augmentation increases the size of the semidiameter correction, whether positive or negative. It is included in the moon correction tables on the inside back cover and facing page of the *Nautical Almanac.* It is not included in the correction tables of other bodies or in the *Air Almanac* tables.

1620. Parallax (P) is the difference in apparent position of a point as viewed from two different places. If a finger is held upright at arm's length and the right and left eyes closed alternately, the finger appears to move right and left a short distance. Similarly, if one of the nearer stars were observed from the earth and from the sun, it would appear to change slightly with respect to the background of more distant stars. This is called **heliocentric parallax** or **stellar parallax.** The nearest star has a parallax of less than 1″. Even if the value were greater, no correction to sextant altitudes would be needed, for the difference would be reflected in the tabulated position of the body.

However, positions of celestial bodies are given relative to the *center* of the earth, while observations are made from its surface. The difference in apparent position from these two points is called **geocentric parallax.** If a body is in the zenith, at Z in figure 1620, there is virtually no parallax, for the line from the body to the center of the earth passes approximately through the observer at *A*. Suppose, however, the moon is at *M*. From *A* it appears to be along the line *AM*, while at the center of the earth it would appear to be along *OM*. The altitude at *A* would be the angle *SAM*, and that at *O* the angle *COM*. Angle *COM* is equal to angle *SBM* (art. O27), which is exterior to the triangle *ABM*, and hence equal to the sum of angles *SAM* and *AMO* (art. O28).

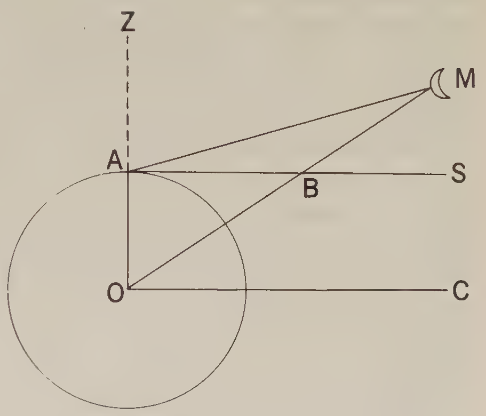


FIGURE 1620.—Geocentric parallax.

Since

$$\angle COM = \angle SBM = \angle SAM + \angle AMO,$$

then

$$\angle AMO = \angle COM - \angle SAM.$$

That is, the angle at the body between lines to the observer and the center of the earth is equal to the difference in altitude at the two places. Angle *AMO* is the geocentric parallax. Since it varies with altitude, it is sometimes called **parallax in altitude (P in A).** The maximum value for a visible body occurs when that body is on the horizon, at *S*. At this position the value is called **horizontal parallax (HP).**

The sine of horizontal parallax is equal to $\frac{r}{D}$, where r is the radius of the earth, and D the distance of the body from the center of the earth. Thus, the sine of the horizontal parallax is directly proportional to the radius of the earth, and inversely proportional to the distance of the body. Since the earth is an oblate spheroid, and not a sphere, the parallax varies slightly over different parts of the earth. The value at the equator, called **equatorial horizontal parallax,** is greatest, and the value at the poles, called **polar horizontal parallax,** is least. The difference is not enough to be of practical navigational significance. The parallax in altitude is equal almost exactly to the horizontal parallax times the cosine of the altitude (h). That is,

$$\text{P in A} = \text{HP} \cos h.$$

The moon, being nearest the earth, has the greatest parallax of any celestial body used for navigation. The equatorial horizontal parallax at mean distance is 57′02″.70. As the distance of the moon varies, so does the parallax, becoming greater as the moon

approaches closer to the earth, and less as it recedes, horizontal parallax varying several minutes each side of the value at mean distance. For the sun, mean equatorial horizontal parallax, called **solar parallax,** is 8″.80. Differences in position on the earth, and distance from the sun, have small effect, the maximum variation due to the latter being about 0″.15. Horizontal parallax of the planets varies considerably because of the large differences in their distances from the earth. For Venus the value varies between 5″ and 32″; for Mars, 3″ and 24″; for Jupiter, 1″ and 2″; and for Saturn, 0″.8 and 1″.0. The geocentric parallax of stars is too small to be measured, even by the most precise telescopes, since the value for the nearest star is only 0″.00003.

Daily values of horizontal parallax for the sun, moon, and planets are given in the *American Ephemeris and Nautical Almanac,* to a precision of 0″.01. In the *Nautical Almanac,* mean values for the sun are included in the two sun correction tables given on the inside front cover and facing page. Horizontal parallax of the moon is tabulated at intervals of one hour on the daily pages. This value is used to enter the lower part of the moon correction tables on the inside back cover and facing page. The additional corrections for Venus and Mars given on the inside front cover are partly for parallax. No correction is given for parallax of Jupiter and Saturn. The *Air Almanac,* giving values only to the nearest whole minute of arc, includes parallax corrections only for the moon. These values are given in the "Moon's P in A" column on the right-hand daily page.

Because of geocentric parallax, a body appears too *low* in the sky. Therefore, the correction is always positive. It applies regardless of the method of observation.

1621. Summary of corrections.—The essential information regarding the application of the various corrections may be tabulated as shown below. In the "Bodies" column, the symbols are: ⊙, sun; ☾, moon; P, planets; ☆, stars. In the "Sextants" column, M refers to a marine sextant with visible horizon, A refers to a marine sextant with artificial horizon, and B refers to an artificial-horizon sextant. The tabulation assumes that completely accurate results are desired and that corrections are to be made in the usual manner, where they are available. Some of the entries need qualification, which may be found in the preceding articles.

Correction	*Symbol*	*Sign*	*Increases with*	*Bodies*	*Sextants*	*Source*
Instrument	I	±	changing altitude	⊙, ☾, P, ☆	M, A, B	sextant box
Index	IC	±	constant	⊙, ☾, P, ☆	M, A, B	measurement
Personal	PC	±	constant	⊙, ☾, P, ☆	M, A, B	measurement
Tilt	N	—	greater tilt angle	⊙, ☾, P, ☆	M	computation
Dip	D	—	higher height of eye	⊙, ☾, P, ☆	M	almanacs
Sea-air temp. diff.	S	±	greater temp. diff.	⊙, ☾, P, ☆	M	computation
Wave height	W	+	higher waves	⊙, ☾, P, ☆	M	computation
Sea tilt	H	±	greater tilt of surface	⊙, ☾, P, ☆	M	computation
Deflection of vert.	V	±	position, azimuth	⊙, ☾, P, ☆	M, A, B	geodesist
Coriolis	Z	±	higher lat., greater speed	⊙, ☾, P, ☆	A, B	*Air Almanac*
Acceleration	C	±	greater acceleration	⊙, ☾, P, ☆	A, B	computation
Refraction	R	—	lower altitude	⊙, ☾, P, ☆	M, A, B	almanacs
Air temp.	T	±	greater diff. from 50° F	⊙, ☾, P, ☆	M, A, B	almanacs, table 23
Atmospheric pressure	B	±	greater diff. from 29.83 inches of mercury	⊙, ☾, P, ☆	M, A, B	*Nautical Almanac,* table 24
Irradiation	J	—	constant	⊙	M, A	*Nautical Almanac*
Semidiameter	SD	±	lesser dist. from earth	⊙, ☾	M, A	almanacs
Phase	F	±	phase	P	M, A, B	*Nautical Almanac*
Augmentation	A	±	higher altitude	☾	M, A	*Nautical Almanac*
Parallax	P	+	lower altitude	⊙, ☾, P	M, A, B	almanacs

These corrections can be considered to fall into five groups:

1. *Corrections for inaccuracies in reading.* Instrument correction, index correction*, personal correction, and tilt correction.

2. *Corrections for inaccuracies in reference level.* Dip*, sea-air temperature difference, wave height, sea tilt, deflection of the vertical, Coriolis, acceleration.

3. *Corrections for bending of ray of light from body.* Refraction*, air temperature, atmospheric pressure.

4. *Adjustment to equivalent reading at center of body.* Irradiation, semidiameter*, phase, augmentation.

5. *Adjustment to equivalent reading at center of earth.* Parallax*.

In the ordinary practice of seamen, extreme accuracy is not required, and only the principal correction of each group is applied (except that irradiation is applied for the upper limb of the sun, and augmentation for the moon). These principal corrections are indicated by asterisks. For low altitudes, additional corrections are applied, as indicated in article 1632.

1622. Order of applying corrections.—For purposes of ordinary navigation, sextant altitudes can be applied in any order desired, using sextant altitude for the entering argument whenever altitude is required. This practice is not strictly accurate, but for altitudes usually observed, the error thus introduced is too small to be of practical significance. When extreme accuracy is desired, however, or at low altitudes, where small changes in altitude result in significant changes in correction, the order of applying corrections is important. Corrections from the first two groups of article 1621 are applied to sextant altitude (hs) to obtain **rectified altitude (hr),** called "apparent altitude" by astronomers, which is then used as an entering argument for obtaining corrections of the third group. For strictest accuracy, all corrections of the first three groups and, in addition, irradiation and semidiameter, should be applied before augmentation, and all other corrections before parallax.

1623. Marine sextant corrections.—As shown in the tabulation of article 1621, all corrections except Coriolis and acceleration apply to marine sextant observations when the visible horizon is used. Of the five corrections ordinarily used, index correction can be eliminated by sextant adjustment (art. 1509), or it can be combined with dip in such manner as to eliminate both (art. 1603). Of the remaining three corrections, only refraction and parallax apply to planets; and only refraction applies to stars.

1624. Artificial-horizon corrections.—When an artificial horizon is used, index correction (and any others of the first group of article 1621) is first applied. The result is then divided by two. Other corrections are then applied to the result, as applicable, in the same manner as for observations using the visible horizon. The sun and full moon are normally observed by bringing the lower limb of one image tangent to the upper limb of the other image. The lower limb is observed if the image seen in the horizon mirror is *above* the image seen in the artificial horizon, unless an inverting telescope is used, when the opposite relationship holds. With a gibbous or crescent moon, judgment may be needed to establish the positions of the limbs. In some cases better results may be obtained by superimposing one image over the other, as with a planet or star. When this is done, the *center* of the body has been observed, and no correction is applied for semidiameter (or irradiation, phase, or augmentation). If the *lower* limb of the sun is observed, an irradiation correction of (+) 1′.2 may be applicable, if experience so indicates. There is no correction for dip (or sea-air temperature or wave height) when an artificial horizon is used.

1625. Artificial-horizon sextant corrections are the same as those for observations made by the use of the visible horizon, with two notable exceptions. First, there is

no correction for dip (or sea-air temperature difference or wave height), none for semidiameter (or irradiation, phase, or augmentation), and usually none for index correction (or instrument correction). Second, because of the lower accuracy normally obtainable by artificial-horizon sextant, corrections are normally made only to the nearest whole minute of arc. As a result of these differences, refraction is the only correction normally applied, except in the case of the moon, where parallax is also applied. Many air navigators, principal users of artificial-horizon sextants, avoid altitudes below 20° and accept the error introduced by refraction, so that no correction is needed. As flight altitudes increase, the error thus introduced becomes correspondingly smaller. However, at modern aircraft speeds, Coriolis correction is of increased importance.

1626. Corrections by *American Nautical Almanac*.—In the *Nautical Almanac*, certain corrections or parts of corrections are combined. Index correction, of course, is not included because this depends upon adjustment of the sextant. The various correction tables are as follows:

"*Sun*," on the inside front cover and facing page, gives mean refraction, mean semidiameter for each of two periods during the year, and mean solar parallax. Irradiation is also included in the upper limb corrections, given in separate columns from lower limb corrections. The table on the inside front cover, and repeated on the loose bookmark, is of the critical type, with altitude as the entering value. Thus, a tabulated correction applies to any value of altitude between that given half a line above it and that half a line below it. If an exact tabulated altitude is used to enter the table, the correction half a line *above* it should be used. In ordinary navigation, index correction, dip, and the correction from this table are needed for correcting marine sextant observations of the sun. For low altitudes or extremes of temperature or atmospheric pressure, a correction from the table on almanac page A4 (or tables 23 and 24 of this volume) should be applied.

"*Stars and planets*," on the inside front cover and repeated on the loose bookmark, gives mean refraction only, for the main tabulation. This is a critical type table, with altitude as the entering argument. The correction is always negative. In ordinary navigation, index correction, dip, and the correction from this table are the only ones needed for stars and the planets Jupiter and Saturn. For Venus and Mars, an additional correction for parallax and phase is given to the right of the main tabulation. The entering altitudes are limited to those occurring during twilight. If observations are made at other times, this additional correction should not be applied even though the altitude may fall within the tabulated range.

"*Dip*," on the inside front cover and repeated on the loose bookmark, is for dip of the horizon. An abbreviated dip table is also given on the page facing the inside back cover. The tables are of the critical type, and the entering argument is the height of the observer's eye, in feet, above the surface of the sea. The correction, always negative, applies to all observations made with the visible sea horizon as a reference.

"*Additional Correction Tables*" for *nonstandard conditions*, given on almanac page A4, provides an additional correction for nonstandard temperature and atmospheric pressure. The sign of each correction is indicated. Equivalent information is given, with increased range of entering values, in tables 23 and 24 of this volume.

"*Altitude Correction Tables—Moon*," on the inside back cover and facing page, gives mean refraction, semidiameter, augmentation, and parallax. The entering argument is altitude for the upper portion of the table, and altitude and horizontal parallax for the lower portion. The combined correction is always positive, but 30′ is to be subtracted from the altitude of the upper limb. In ordinary navigation, index correction, dip, and the correction from this table are needed in correcting marine sextant observations of the moon.

The various separate corrections available from the *Nautical Almanac* can be found as follows:

Dip. Dip table on inside front cover and repeated on loose bookmark, and on the page facing the inside back cover.

Refraction. Mean refraction from "Stars and Planets" table on inside front cover and repeated on loose bookmark, and on the facing page.

Irradiation. A value of (−) 1′.2 is included in the tables of corrections for the upper limb of the sun, given on the inside front cover and repeated on the loose bookmark, and on the facing page, to allow (−) 0′.6 for the upper limb and (−) 0′.6 for the horizon.

Semidiameter. For the sun, the semidiameter for the middle day of each page opening of this daily page section is given at the bottom of the sun GHA column. For the moon, semidiameter for each day is given at the bottom of the moon data columns. The values given are for GMT 1200 on the dates indicated.

Parallax. For the sun, parallax in altitude can be considered 0′.1 for altitudes 0° to 70°07′, and 0′.0 for higher altitudes, with negligible error. This is based upon the mean value of 8″.80. For the moon, horizontal parallax each hour is tabulated on the daily pages. Parallax in altitude is this value multiplied by the cosine of the altitude.

If artificial-horizon sextant altitudes of the sun or moon are corrected by *Nautical Almanac*, the upper and lower limb corrections can be found and the average computed.

1627. Corrections by *Air Almanac*.—In the *Air Almanac*, various corrections are given separately in critical type tables, to the nearest whole minute (nearest two or five minutes of refraction for low altitudes), as follows:

Dip. Outside back cover.

Refraction. Inside back cover. Aboard ship use the values for zero height. A special table for H.O. Pub. No. 218 is given on the page facing the inside back cover. A dome refraction table for use in aircraft is given on the same page.

Coriolis. Inside back cover, below the refraction table.

Air temperature. Inside back cover. This is shown, not as a separate correction, but as an adjustment to mean refraction. Instructions for use of the table are given on the inside back cover of the *Air Almanac*.

Semidiameter. For the sun and moon, on the right-hand daily page, below the moon's P in A. Values given are for GMT 1200. The value given for the sun is always 16′.

Parallax. For the moon, in the P in A table on the right-hand daily page. Horizontal parallax is the value for 0° altitude. Values given are for GMT 1200.

1628. Correcting altitudes of the sun.—In the normal practice of navigation, sun observations obtained by marine sextant with the visible horizon as reference are corrected as shown in the following examples:

Example 1.—On June 2, 1958, the lower limb of the sun is observed with a marine sextant having an IC of (−) 2′.0, from a height of eye of 38 feet. The hs is 51°28′.4.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac*.

Solution.—

(1)

	+ $\underline{\odot}$	−
IC		2′.0
D		6′.0
$\underline{\odot}$	15′.2	
sum	15′.2	8′.0
corr.		(+) 7′.2
hs		51°28′.4
Ho		51°35′.6

(2)

	+ $\underline{\odot}$	−
IC		2′
D		6′
R		1′
SD	16′	
sum	16′	9′
corr.		(+) 7′
hs		51°28′
Ho		51°35′

Example 2.—On June 2, 1958, the upper limb of the sun is observed with a marine sextant having an IC of (+) 1′.0, from a height of eye of 45 feet. The hs is 32°47′.9.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac.*

Solution.—

(1)

	+	☉̄	−
IC	1′.0		
D			6′.5
☉̄			18′.5
sum	1′.0		25′.0
corr.			(−) 24′.0
hs			32°47′.9
Ho			32°23′.9

(2)

	+	☉̄	−
IC	1′		
D			7′
R			2′
SD			16′
sum	1′		25′
corr.			(−) 24′
hs			32°48′
Ho			32°24′

A convenient work form is helpful in the solution. Once the form is prepared, the corrections can be entered in any order desired. The symbols ☉̲ and ☉̄ are used for the corrections from the sun table on the inside front cover of the *Nautical Almanac.* If additional corrections are used, they are included in the same manner as those shown. Observations by artificial horizon and by artificial-horizon sextant, and low-altitude observations and back sights, are discussed elsewhere in this chapter.

1629. Correcting altitudes of the moon.—Moon observations by marine sextant with the visible horizon as reference are normally corrected as shown in the following examples:

Example 1.—At about GMT 1100 on June 2, 1958, the lower limb of the moon is observed with a marine sextant having an IC of (+) 3′.2, from a height of eye of 32 feet. The hs is 18°04′.6.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac.*

Solution.—

(1)

	+	☾̲	−
IC	3′.2		
D			5′.5
☾	62′.5		
L	6′.7		
sum	72′.4		5′.5
corr.			(+)1°06′.9
hs			18°04′.6
Ho			19°11′.5

(2)

	+	☾̲	−
IC	3′		
D			6′
R			3′
SD	16′		
P	56′		
sum	75′		9′
corr.			(+)1°06′
hs			18°05′
Ho			19°11′

Example 2.—At about GMT 0900 on June 2, 1958, the upper limb of the moon is observed with a marine sextant having an IC of (−)1′.6, from a height of eye of 70 feet. The hs is 66°47′.3.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac.*

Solution.—

(1)

	+	☾̄	−
IC			1′.6
D			8′.1
☾	33′.0		
U	3′.8		
add'l			30′.0
sum	36′.8		39′.7
corr.			(−) 2′.9
hs			66°47′.3
Ho			66°44′.4

(2)

	+	☾̄	−
IC			2′
D			8′
R			—
SD			16′
P	23′		
sum	23′		26′
corr.			(−) 3′
hs			66°47′
Ho			66°44′

The typical work forms shown are useful in problems of this type. The symbol ☾ is used for the correction from the upper part of the moon correction table on the inside back cover, and facing page, of the *Nautical Almanac*. The symbols L and U are used for the corrections from the lower part of this table. Observations by artificial horizon, and by artificial-horizon sextant, and low-altitude observations and back sights, are discussed elsewhere in this chapter, as are additional corrections for use when unusual accuracy is desired.

1630. Correcting altitudes of planets.—When Venus and Mars are observed by marine sextant using the visible horizon as reference, sextant altitudes are normally corrected as shown in the following example:

Example.—On May 19, 1958, Venus is observed with a marine sextant having no IC, from a height of eye of 28 feet. The hs is 44°21′.3.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac*.

Solution.—

(1)

	+	V –
IC	—	—
D		5′.1
☆-P		1′.0
add'l	0′.2	
sum	0′.2	6′.1
corr.		(–) 5′.9
hs		44°21′.3
Ho		44°15′.4

(2)

	+	V –
IC	—	—
D		5′
R		1′
sum	—	6′
corr.		(–) 6′
hs		44°21′
Ho		44°15′

For Jupiter and Saturn, no additional correction is given. Correction of observations of these bodies is the same as corrections of star observations (art. 1631). Work forms are useful. The symbol ☆-P is used for the correction taken from the "Star-Planet" table on the inside front cover of the *Nautical Almanac*. If additional corrections are to be used, for results of unusual accuracy or low altitudes, they are included in the form in the same manner as those shown. Observations by artificial horizon and by artificial-horizon sextant, and low-altitude observations and back sights are discussed elsewhere in this chapter.

1631. Correcting altitudes of stars.—Star observations by marine sextant, using the visible horizon as reference, are normally corrected as shown in the following example:

Example.—Miaplacidus is observed with a marine sextant having an IC of (+) 1′.0, from a height of eye of 50 feet. The hs is 27°54′.0.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac*.

Solution.—

(1)

	+	☆ –
IC	1′.0	
D		6′.9
☆-P		1′.8
sum	1′.0	8′.7
corr.		(–) 7′.7
hs		27°54′.0
Ho		27°46′.3

(2)

	+	☆ –
IC	1′	
D		7′
R		2′
sum	1′	9′
corr.		(–) 8′
hs		27°54′
Ho		27°46′

Work forms for such problems are helpful. Additional corrections, used when unusual accuracy is desired, are included in the same manner as those shown. Observations by artificial horizon and by artificial-horizon sextant, and low-altitude observations and back sights, are discussed elsewhere in this chapter.

1632. Low altitudes are normally avoided because of large and variable refraction. But sometimes these are the only observations available. This is particularly true in polar regions, where the sun may be the only celestial body available, and may not reach an altitude of more than a few degrees over a considerable period. In lower latitudes the sun may appear briefly just before sunset or just after sunrise. Low-altitude observations can supply useful information if additional corrections are applied. Reliable lines of position can generally be obtained from low-altitude observations, but when conditions are abnormal, the errors introduced are generally larger than for higher altitudes, and the precautions of article 1613 should be particularly observed.

In correcting low-altitude observations, which for normal conditions can be defined as those less than 5°, first apply corrections from the first two groups of article 1621 to obtain rectified altitude (hr), called "apparent altitude" in the almanac. Normally, this includes only index correction and dip. Then apply the remaining corrections, using rectified altitude when an altitude is needed for entering correction tables. The corrections normally applied are mean refraction, air temperature, atmospheric pressure, semidiameter (as applicable), and parallax (for the sun and moon).

In practice, sextant altitudes are corrected in the usual manner, except that additional corrections are applied, and the process is divided into two parts. The use of rectified altitude for finding parallax introduces an error but this is too small (less than 0'.1) for practical consideration. If the *Nautical Almanac* is used, corrections for altitudes between the horizon and 10° are given in a noncritical type table on almanac page A3. The correction for a negative altitude can be obtained by extrapolation without introducing a significant error for values obtained at ship heights of eye. A combined temperature-atmospheric pressure correction can be obtained from the table on almanac page A4. This table is intended for use without interpolation between columns. Separate corrections can be obtained from tables 23 and 24 of the present volume, which provide interpolated values for greater accuracy. They also provide greater range of temperature and atmospheric pressure.

To correct a low altitude of the sun, then, apply index correction and dip to sextant altitude to find rectified altitude. Using this altitude as an entering value, find the following corrections and apply them to rectified altitude:

sun correction (☉̲ or ☉̄), from page A3 of the *Nautical Almanac;*

combined temperature-atmospheric pressure correction (TB), from page A4 of the *Nautical Almanac* (separate corrections for temperature (T) and atmospheric pressure (B) from tables 23 and 24, respectively, can be used *in place of* the combined correction).

If the *Air Almanac* is used, the mean refraction and air temperature corrections can be combined by using the factor on the inside back cover. A semidiameter correction of 16' is added if the lower limb is observed, and subtracted if the upper limb is observed. Since corrections are to whole minutes only, parallax is not used for the sun. In summary, apply index correction and dip to sextant altitude to find rectified altitude. Using this altitude as an entering value, where needed, apply the following corrections to rectified altitude:

refraction (adjusted for air temperature) (R), from inside back cover of *Air Almanac;*

atmospheric pressure (B), from table 24;

semidiameter (SD), 16' (add if lower limb, and subtract if upper limb).

Example 1.—On June 2, 1958, the lower limb of the sun is observed with a marine sextant having an IC of (+) 1'.8 from a height of eye of 45 feet. The hs is 1°24'.4, air temperature 88°F, and atmospheric pressure 29.78 inches.

Required.—Ho using (1) *Nautical Almanac*, (2) tables 23 and 24, and (3) *Air Almanac.*

Solution:

(1)

	+	$\underline{\odot}$	−
IC	1′.8		
D			6′.5
sum	1′.8		6′.5
corr.			(−) 4′.7
hs			1°24′.4
hr			1°19′.7
$\underline{\odot}$			6′.0
TB	2′.5		
sum	2′.5		6′.0
corr.			(−) 3′.5
hr			1°19′.7
Ho			1°16′.2

(2)

	+	$\underline{\odot}$	−
IC	1′.8		
D			6′.5
sum	1′.8		6′.5
corr.			(−) 4′.7
hs			1°24′.4
hr			1°19′.7
$\underline{\odot}$			6′.0
T	1′.5		
B	—		
sum	1′.5		6′.0
corr.			(−) 4′.5
hr			1°19′.7
Ho			1°15′.2

(3)

	+	$\underline{\odot}$	−
IC	2′		
D			7′
sum	2′		7′
corr.			(−) 5′
hs			1°24′
hr			1°19′
R			18′
B	—		
SD	16′		
sum	16′		18′
corr.			(−) 2′
hr			1°19′
Ho			1°17′

The larger intervals given in the *Air Almanac* refraction table may introduce additional error. In this example, the temperature is changed to Celsius (centigrade), giving a value of 31°. The factor at a height of 0 feet corresponding to this temperature is 0.9. With this and the rectified altitude, the combined refraction and air temperature correction is found to be as shown. Approximately the same result would have been obtained by correcting for mean refraction (without the factor) and temperature (from table 23) separately.

If the moment at which either limb is tangent to the horizon is noted, an observation of 0° altitude has been made without a sextant.

Example 2.—On June 2, 1958, the sun is observed at sunset as the upper limb drops below the horizon, from a height of eye of 38 feet. The air temperature is (−) 10°F, and atmospheric pressure 30.06 inches. Double extrapolation would be needed to solve this problem by the *Nautical Almanac*. A better solution is provided by means of tables 23 and 24.

Required.—Ho using (1) tables 23 and 24, and (2) *Air Almanac.*

Solution.—

(1)

	+	$\overline{\odot}$	−
IC	—		—
D			6′.0
sum	—		6′.0
corr.			(−) 6′.0
hs			0°00′.0
hr			(−)0°06′.0
$\overline{\odot}$			52′.6
T			4′.8
B			0′.3
sum	—		57′.7
corr.			(−) 57′.7
hr			(−) 0°06′.0
Ho			(−) 1°03′.7

(2)

	+	$\overline{\odot}$	−
IC	—		—
D			6′
sum	—		6′
corr.			(−) 6′
hs			0°00′
hr			(−) 0°06′
R			42′
B			—
SD			16′
sum	—		58′
corr.			(−) 58′
hr			(−) 0°06′
Ho			(−) 1°04′

Corrections are applied algebraically. Therefore, for negative altitudes a negative correction is *numerically* added, and a positive correction is *numerically* subtracted.

To correct low altitudes of the moon, apply index correction and dip to sextant altitude to find rectified altitude. Using this altitude as an entering value, find the following corrections and apply them to rectified altitudes:

moon correction (☾), from inside back cover, and facing page, of *Nautical Almanac;*
lower or upper limb correction (L or U), from inside back cover, and facing page, of *Nautical Almanac;*
additional correction (add'l, (−) 30′, for upper limb observation only);
combined temperature-atmospheric pressure correction (TB), from page A4 of the *Nautical Almanac* (separate corrections for temperature (T) and atmospheric pressure (B) from tables 23 and 24, respectively, can be used in place of the combined correction).

If the *Air Almanac* is used, correct the rectified altitude by applying the following corrections:
refraction (adjusted for air temperature) (R), from inside back cover of *Air Almanac;*
atmospheric pressure (B), from table 24;
semidiameter, from right-hand daily page;
parallax, from right-hand daily page.

Example 3.—At GMT $17^h14^m27^s$ on June 2, 1958, the upper limb of the moon is observed with a marine sextant having no IC, from a height of eye of 33 feet. The hs is 2°35′.4, air temperature 63° F, and atmospheric pressure 29.81 inches.

Required.—Ho using (1) *Nautical Almanac*, (2) tables 23 and 24, and (3) *Air Almanac.*

Solution.—

(1)	+ ☾	−
IC	—	—
D		5′.6
sum	—	5′.6
corr.		(−) 5′.6
hs		2°35′.4
hr		2°29′.8
☾	52′.1	
U	4′.5	
add'l		30′.0
TB	0′.4	
sum	57′.0	30′.0
corr.		(+) 27′.0
hr		2°29′.8
Ho		2°56′.8

(2)	+ ☾	−
IC	—	—
D		5′.6
sum	—	5′.6
corr.		(−) 5′.6
hs		2°35′.4
hr		2°29′.8
☾	52′.1	
U	4′.5	
add'l		30′.0
T	0′.4	
B	—	
sum	57′.0	30′.0
corr.		(+) 27′.0
hr		2°29′.8
Ho		2°56′.8

(3)	+ ☾	−
IC	—	—
D		6′
sum	—	6′
corr.		(−) 6′
hs		2°35′
hr		2°29′
R		16′
B	—	
SD		16′
P	59′	
sum	59′	32′
corr.		(+) 27′
hr		2°29′
Ho		2°56′

A lower limb solution would be the same, except that an L correction would have been used from the *Nautical Almanac* and there would be no "add'l" correction, and in the *Air Almanac* solution the sign of the semidiameter correction would be reversed. The moon correction table on the inside back cover, and facing page, of the *Nautical Almanac* extends to a minimum altitude of 0°. The corrections for negative altitudes can be found by extrapolation.

To correct low altitudes of the planets Venus and Mars, apply index correction and dip to sextant altitude to find rectified altitude. Using this altitude as an entering value, find the following corrections and apply them to rectified altitude:

star-planet correction (☆-P), from page A3 of the *Nautical Almanac;*
additional correction (add'l), from page A2 of the *Nautical Almanac;*
combined temperature-atmospheric pressure correction (TB), from page A4 of the *Nautical Almanac* (separate corrections for temperature (T) and atmospheric pressure (B) from tables 23 and 24, respectively, can be used *in place of* the combined correction).

If the *Air Almanac* is used, correct the rectified altitude by applying the following corrections:

refraction (adjusted for air temperature) (R), from inside back cover of *Air Almanac;*
atmospheric pressure (B), from table 24.

Example 4.—On September 28, 1958, Mars is observed with a marine sextant having an IC of (+) 3′.5, from a height of eye of 17 feet. The hs is 4°02′.6, air temperature 2°F, and atmospheric pressure 29.67 inches.

Required.—Ho using (1) *Nautical Almanac*, (2) tables 23 and 24, and (3) *Air Almanac.*

Solution.—

(1)	+	M −
IC	3′.5	
D		4′.0
sum	3′.5	4′.0
corr.		(−) 0′.5
hs		4°02′.6
hr		4°02′.1
☆-P		11′.7
add'l	0′.3	
TB		1′.5
sum	0′.3	13′.2
corr.		(−) 12′.9
hr		4°02′.1
Ho		3°49′.2

(2)	+	M −
IC	3′.5	
D		4′.0
sum	3′.5	4′.0
corr.		(−) 0′.5
hs		4°02′.6
hr		4°02′.1
☆-P		11′.7
add'l	0′.3	
T		1′.2
B	0′.1	
sum	0′.4	12′.9
corr.		(−) 12′.5
hr		4°02′.1
Ho		3°49′.6

(3)	+	M −
IC	4′	
D		4′
sum	4′	4′
corr.		—
hs		4°03′
hr		4°03′
R		14′
B	—	
sum	—	14′
corr.		(−) 14′
hr		4°03′
Ho		3°49′

The solution for Jupiter and Saturn, and for stars, is identical with that of example 4, except that the additional correction (phase and parallax) is omitted.

1633. Back sights.—An altitude measured by facing *away* from the celestial body being observed is called a **back sight.** It may be used when an obstruction, such as another vessel, obscures the horizon under the body; when that horizon is indistinct; or when observations are made in both directions, either to determine dip or to avoid error due to suspected abnormal dip. Such an observation is possible only when the arc of the sextant is sufficiently long to permit measurement of the angle, which is the supplement of the altitude. For such an observation of the sun or moon, the lower limb is observed when the image is brought below the horizon, appearing as a normal upper limb observation, and vice versa. To correct such an altitude, subtract it from 180° and reverse the sign of corrections of the first two groups of article 1621 (normally only index correction and dip).

Example.—On June 2, 1958, a back sight is taken of the lower limb of the sun, with a marine sextant having an IC of (−) 2′.0, from a height of eye of 24 feet. The measured sextant altitude is 118°41′.4.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac.*

Solution.—

(1)

	+	☉ −
IC	2′.0	
D	4′.8	
☉	15′.4	
sum	22′.2	—
corr.	(+)22′.2	
180°−hs	61°18′.6	
Ho	61°40′.8	

(2)

	+	☉ −
IC	2′	
D	5′	
R		1′
SD	16′	
sum	23′	1′
corr.	(+)22′	
180°−hs	61°19′	
Ho	61°41′	

1634. Correcting horizontal angles.—When a marine sextant is used to measure the horizontal angle between two objects, the result is not usually desired to a precision that makes correction necessary, unless the sextant has an unusually large index error. However, if precise results are desired, corrections of the first group only of article 1621 are applied. If a personal error exists, it is not likely to be the same as for altitudes. For measuring angles between two objects differing widely in altitude, as between two stars, it is not likely that results will be required to such precision that additional correction for the third, fourth, and fifth groups of article 1621 will be needed. If they are, the method of application can be determined from the principles of spherical trigonometry (app. O). In this case, the altitudes of both bodies will also be needed. Corrections for the second group of article 1621 are not applicable.

Problems

1624. At about GMT 0800 on June 2, 1958, the following bodies are observed with marine sextants having an IC of (+)2′.2, using an artificial horizon: sun (lower limb) hs 134°33′.9, moon (upper limb) hs 77°23′.4, Venus hs 98°04′.6, Schedar hs 43°24′.4.

Required.—Ho of each observation using (1) *Nautical Almanac*, and (2) *Air Almanac.*

Answers.—(1) Sun Ho 67°33′.6, moon Ho 39°11′.7, Venus Ho 49°02′.6, Schedar Ho 21°40′.9; (2) sun Ho 67°34′, moon Ho 39°12′, Venus Ho 49°03′, Schedar Ho 21°41′.

1625. At about GMT 0300 on June 2, 1958, the following bodies are observed with bubble sextants having no IC: sun hs 23°51′, moon hs 52°20′, Jupiter hs 63°18′, Eltanin hs 24°45′.

Required.—Ho of each observation using (1) *Nautical Almanac*, and (2) *Air Almanac.*

Answers.—(1) and (2) Sun Ho 23°49′, moon Ho 52°56′, Jupiter Ho 63°18′, Eltanin Ho 24°43′.

1628a. On June 2, 1958, the lower limb of the sun is observed with a marine sextant having an IC of (+)1′.8, from a height of eye of 34 feet. The hs is 41°34′.8.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac.*

Answers.—(1) Ho 41°45′.8; (2) Ho 41°46′.

1628b. On June 2, 1958, the upper limb of the sun is observed with a marine sextant having no IC, from a height of eye of 30 feet. The hs is 15°21′.7.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac.*

Answers.—(1) Ho 14°56′.0; (2) Ho 14°57′.

1628c. On June 2, 1958, the lower limb of the sun is observed with a marine sextant having an IC of (−)1′.3, from a height of eye of 43 feet. Another ship is between the observer and the horizon, at a distance of 1.4 miles from the observer. The water line of this ship is used as the horizontal reference. The hs is 25°18′.2.

Required.—Ho using table 22 and (1) *Nautical Almanac*, and (2) *Air Almanac*.

Answers.—(1) Ho 25°13.′0; (2) Ho 25°13′.

1629a. At about GMT 2100 on June 2, 1958, the lower limb of the moon is observed with a marine sextant having an IC of (−) 2.′5, from a height of eye of 55 feet. The hs is 47°35.′5.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac*.

Answers.—(1) Ho 48°20.′5; (2) Ho 48°22′.

1629b. At about GMT 2300 on June 2, 1958, the upper limb of the moon is observed with a marine sextant having an IC of (+)4.′0, from a height of eye of 12 feet. The hs is 22°58.′3.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac*.

Answers.—(1) Ho 23°34.′7; (2) Ho 23°35′.

1630a. On June 18, 1958, Mars is observed with a marine sextant having an IC of (+) 2.′2, from a height of eye of 60 feet. The hs is 34°11.′7.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac*.

Answers.—(1) Ho 34°05.′1; (2) Ho 34°05′.

1630b. Jupiter is observed with a marine sextant having an IC of (−)1.′0, from a height of eye of 27 feet. The hs is 11°23.′9.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac*.

Answers.—(1) Ho 11°13.′2; (2) Ho 11°13′.

1631. Alpheratz is observed with a marine sextant having no IC, from a height of eye of 42 feet. The hs is 38°20.′3.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac*.

Answers.—(1) Ho 38°12.′8; (2) Ho 38°13′.

1632a. On June 2, 1958, the lower limb of the sun is observed with a marine sextant having an IC of (−) 2.′3, from a height of eye of 24 feet. The hs is 2°04.′6, air temperature 65° F, and atmospheric pressure 30.81 inches.

Required.—Ho using (1) *Nautical Almanac*, (2) tables 23 and 24, and (3) *Air Almanac*.

Answers.—(1) Ho 1°55.′1; (2) Ho 1°55.′1; (3) Ho 1°56′.

1632b. On July 2, 1958, the sun is observed as the upper limb drops below the horizon at sunset, from a height of eye of 19 feet. The air temperature is 16° F, and atmospheric pressure 29.90 inches.

Required.—Ho using (1) *Nautical Almanac*, (2) tables 23 and 24, and (3) *Air Almanac*.

Answers.—(1) Ho (−)1°01.′4; (2) Ho (−)0°59.′2; (3) Ho (−)0°58′.

1632c. At GMT $6^h03^m29^s$ on June 2, 1958, the upper limb of the moon is observed with a marine sextant having an IC of (+)2.′6, from a height of eye of 35 feet. The hs is 1°12.′6, air temperature (−) 23° F, and atmospheric pressure 29.04 inches.

Required.—Ho using (1) tables 23 and 24, and (2) *Air Almanac*.

Answers.—(1) Ho 1°26.′1; (2) Ho 1°24′.

1632d. At GMT $12^h44^m01^s$ on June 2, 1958, the lower limb of the moon is observed with a marine sextant having an IC of (+)3.′2, from a height of eye of 22 feet. The hs is 0°24.′4, air temperature 40° F, and atmospheric pressure 29.94 inches.

Required.—Ho using (1) *Nautical Almanac*, (2) tables 23 and 24, and (3) *Air Almanac*.

Answers.—(1) Ho 1°07.′4; (2) Ho 1°07.′6; (3) Ho 1°04′.

1632e. On January 19, 1958, Venus is observed with a marine sextant having an IC of (−)0.′5, from a height of eye of 31 feet. The hs is 3°29.′8, air temperature 55° F, and atmospheric pressure 30.15 inches.

Required.—Ho using (1) *Nautical Almanac*, (2) tables 23 and 24, and (3) *Air Almanac*.

Answers.—(1) Ho 3°11′.3; (2) Ho 3°11′.1; (3) Ho 3°11′.

1632f. Saturn is observed with a marine sextant having an IC of (−) 2′.3, from a height of eye of 37 feet. The hs is 4°39′.2, air temperature 76° F, and atmospheric pressure 28.89 inches.

Required.—Ho using (1) *Nautical Almanac*, (2) tables 23 and 24, and (3) *Air Almanac*.

Answers.—(1) Ho 4°21′.4; (2) Ho 4°21′.1; (3) Ho 4°21′.

1632g. Gienah is observed with a marine sextant having no IC, from a height of eye of 44 feet. The hs is 2°46′.1, air temperature 35° F, and atmospheric pressure 29.92 inches.

Required.—Ho using (1) *Nautical Almanac*, (2) tables 23 and 24, and (3) *Air Almanac*.

Answers.—(1) Ho 2°23′.4; (2) Ho 2°23′.6; (3) Ho 2°21′.

1633. On June 2, 1958, a back sight is taken of the lower limb of the sun, with a marine sextant having an IC of (+) 1′.7, from a height of eye of 49 feet. The measured sextant altitude is 141°04′.9.

Required.—Ho using (1) *Nautical Almanac*, and (2) *Air Almanac*.

Answers.—(1) Ho 39°15′.0; (2) Ho 39°15′.

1634. The horizontal angle between two objects is measured with a marine sextant having an IC of (+) 4′.0. The measured angle is 85°14′.6.

Required.—Corrected angle.

Answer.—Corrected angle 85°18′.6.

CHAPTER XVII

LINES OF POSITION FROM CELESTIAL OBSERVATIONS

1701. Circles of equal altitude.—For every point on the earth there is a zenith (art. 1428) vertically overhead on the celestial sphere (art. 1403). Likewise, every point on the celestial sphere is vertically over *some* terrestrial point, called its **geographical position (GP)**. However, since the earth rotates on its axis, causing *apparent* rotation of the celestial sphere, the GP of any point on the celestial sphere is continually moving to the westward, at the rate of about 15° per hour. If a celestial body is changing its apparent position on the celestial sphere, this motion is added to that caused by rotation, so that the rates of motion of the GP's of various bodies differ slightly. Further, this motion may not be exactly westward, having a small northerly or southerly component as the body changes declination, due either to its own proper motion or precession of the equinoxes (art. 1419), or a combination of the two.

At any moment the declination of a celestial body is equal to the latitude of its GP. The Greenwich hour angle (GHA) of the body, if not greater than 180°, is equal to the longitude (west) of the GP. If the GHA is greater than 180°, its explement (art. O27) is equal to the longitude (east). Thus, if it is established that a body of known coordinates is in the zenith of an observer, the position of the observer is known. However, for the celestial bodies used in navigation, this condition rarely occurs for any individual observer, and is difficult to determine when it does occur.

More commonly, the altitude (art. 1428) is measured, and from this the zenith distance (art. 1428) can be determined. This value defines a circle on the earth, as shown in figure 1701a. Thus, if the observer is one mile from the GP, in *any* direction, he is 1′ from it, and his zenith is 1′ from the celestial body. Anywhere on a circle of one mile (1′) radius, with the GP as the center, the zenith distance is 1′. Similarly, if the zenith distance is 10°, the observer may be anywhere on a circle (assuming a spherical earth) of radius $10 \times 60 = 600$ miles, with the GP as the center. If the zenith distance is 30°, the radius is 1,800 miles; if 60°, the radius is 3,600 miles; and if 90° (body on the celestial horizon), the radius is 5,400 miles. This is a great circle dividing the earth into two hemispheres. Anywhere within that hemisphere having the GP as its center the celestial body is above the celestial horizon. Anywhere within the opposite hemisphere the body is below the celestial horizon.

These **circles of equal altitude** are **circles of position,** or circular **lines of position.** Two such circles for different celestial bodies, or for the same body at different times, may intersect at two points, as shown in figure 1701b. If these circles have radii equal to the zenith distances at the observer, the position of the observer is established at one of the two intersections. Normally, these intersections are separated by such great distances that no question arises as to which represents the position of the observer. However, uncertainty can be removed if additional altitude circles can be established by observation of other celestial bodies. It would be a rare coincidence for a third such circle to pass through both intersections of the first two. The third observation also serves as a check on the accuracy of the first two. The ambiguity might also be resolved by noting the azimuth of either or both of the bodies, for the azimuth should be in the same direction as the radius of the circle of position, measured at the intersection.

1702. Utilizing circles of equal altitude.—For most altitudes conveniently observed, the plotting of circles of equal altitude involves certain difficulties. Because of the long radii of such circles, a chart of very small scale would be needed, and virtually any chart distortion would introduce some error, unless an azimuthal projection (ch. III) centered upon the GP were used, an impractical procedure with a moving GP for each body. The appearance of two circles of equal altitude plotted on a Mercator chart is shown in figure 1702.

It has been suggested that the second difficulty, that of distortion, might be overcome by plotting directly on a sphere, using equipment designed for this purpose. While theoretically sound, this procedure does not overcome the first difficulty, that of scale, and has not proved practical. A variation of this has been the use of movable arcs, by which a small-scale model of one or more navigational triangles (art. 1433) is mechanically produced. The coordinates are carefully measured by means of sliding indices controlled by verniers or micrometers. Another variation has been a graphical solution based upon the drawing of a diagram according to any of various principles. Although a number of mechanical and graphical solutions have been devised, and some have proved practical (ch. XXI), none has been generally accepted as superior to the commonly used tabular methods of solution.

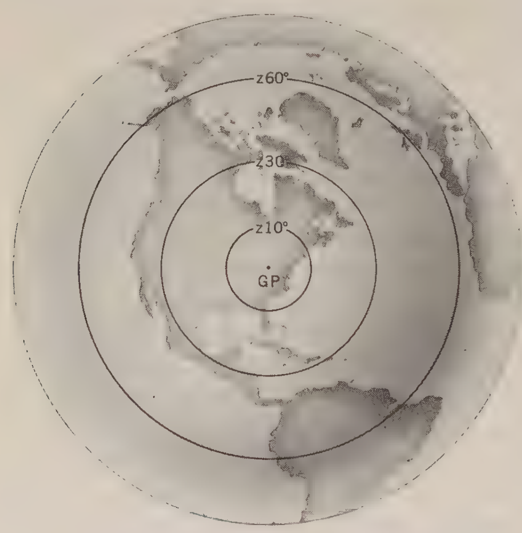


Figure 1701a.—Circles of equal altitude.

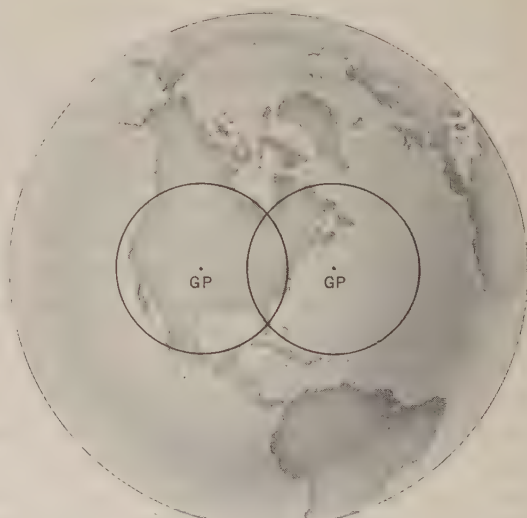


Figure 1701b.—Intersections of two circles of equal altitude.

However, as the altitude of a body increases, reducing the zenith distance, both distortion and scale difficulties decrease. Also, on a Mercator chart, they decrease as the GP approaches the equator. The observation of a celestial body near the zenith is difficult, but in the case of the sun no alternative may be available near noon in the tropics. Such a situation does provide an easy solution and may permit obtaining of a fix from two observations of the same body, with only a few minutes between observations. This solution is discussed further in article 2011.

1703. The line of position.—For zenith distances too great to plot conveniently, a line of position can be laid down in another manner.

The altitude of a celestial body may be measured. After appropriate corrections are applied, this is called **observed altitude (Ho)**. For the instant of observation, the altitude and azimuth at some convenient **assumed position (AP)** near the actual position of the observer are determined by calculation or equivalent process. The difference between this **computed altitude (Hc)** and Ho is the **altitude difference** (*a*), some-

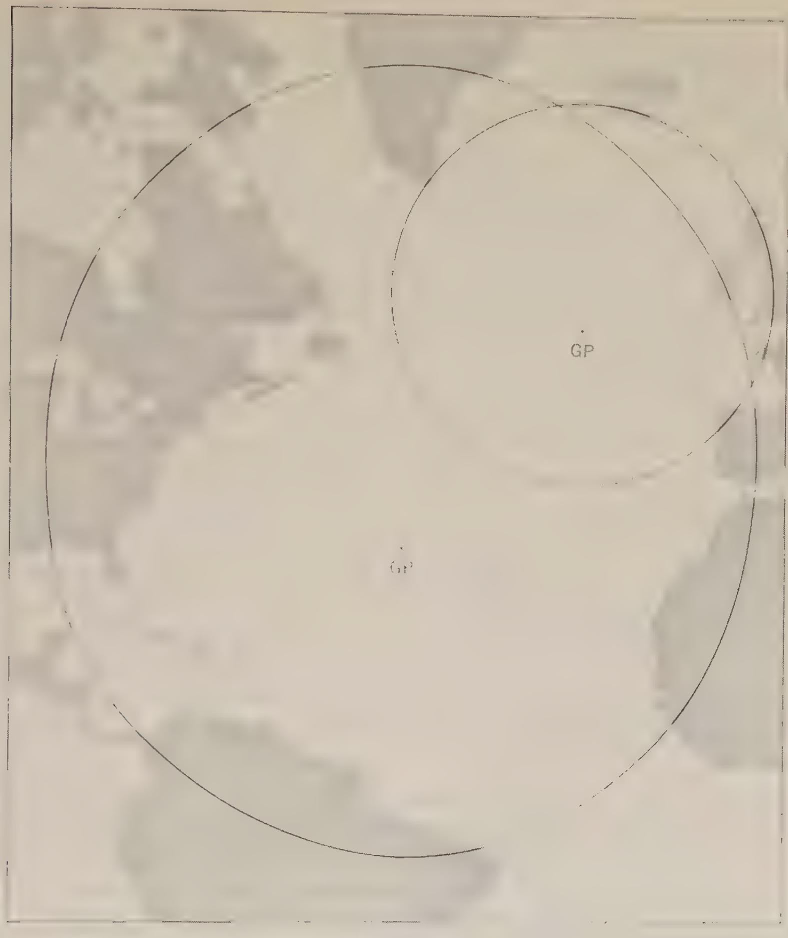


FIGURE 1702.—Circles of equal altitude on a Mercator chart.

times called **altitude intercept.** Since *a* is the difference in altitude at the assumed and actual positions, it is also the difference in zenith distance, and therefore the difference in radii of the circles of equal altitude at the two places. The position having the greater altitude is on the circle of smaller radius, and hence is closer to the GP of the body. In figure 1703a the AP is shown on the inner circle. Hence, Hc is greater than Ho.

The line of position can be plotted by using part of the information within the broken circle of figure 1703a, as shown in figure 1703b. First, the AP is plotted. The circle of equal altitude through this position is not needed, and is not plotted. From the AP the azimuth line is measured toward or away from the GP as appropriate, and the altitude difference is measured along this line. At the point thus located, a line is drawn perpendicular to the azimuth line. For several miles on each side of the azimuth line, this perpendicular can be considered part of the circle of position through the observer, as shown in figure 1703a. This perpendicular is the line of position. It is labeled with the time of observation above the line, and the name of the celestial body below the line, as shown in figure 1703b.

For neatness of plot the azimuth line should not be extended beyond the line of position or the AP, unless it is extended a short distance in the direction of the body, and the symbol of the body observed is shown to indicate whether a "toward" or

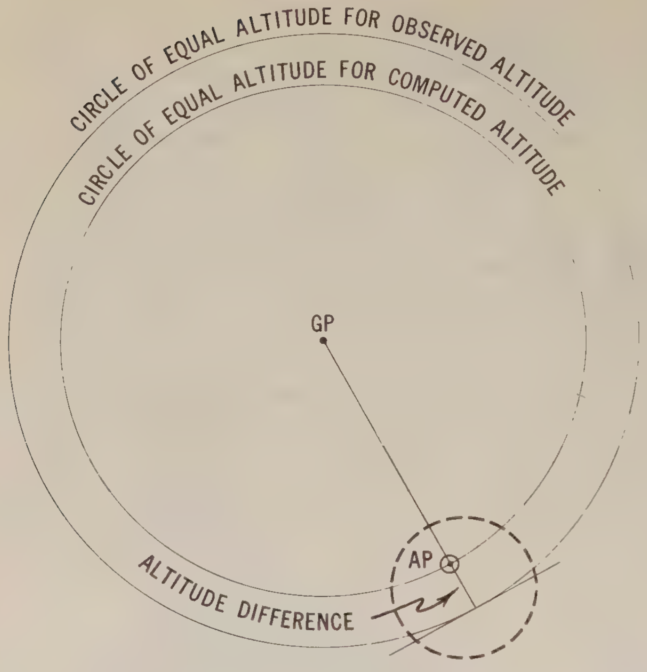


FIGURE 1703a.—The basis for the line of position from a celestial observation.

"away" observation. This method is used in the examples of H.O. Pub. No. 214. Some navigators omit the azimuth line, showing only the AP and line of position, and using a straightedge as a guide for the dividers in measuring the altitude difference. This is good practice, for it reduces the number of lines on the plotting sheet, and therefore minimizes the possibility of making an error. However, until one gains confidence in plotting lines of position, it is desirable to show the azimuth line.

For plotting a line of position from a celestial observation, then, only the assumed position, altitude difference (with an indication of which altitude is greater), and azimuth are needed.

The assumed position is chosen somewhat arbitrarily. It may be the dead reckoning position, an estimated position, or any arbitrarily chosen position nearby. Most commonly, however, the **assumed latitude** (*a*L) is taken as the *nearest* whole degree of latitude to the DR or EP; and the **assumed longitude** ($a\lambda$) is selected so that the local hour angle is a whole degree. The location of the line of position is independent of the location of the AP (within reasonable limits), assuming only that the altitude difference is measured from the AP used for determining Hc. That is, each AP has its own altitude difference, depending upon its distance from the line of position.

The altitude difference, the numerical difference between Hc and Ho, is customarily expressed in nautical miles (minutes of arc), and labeled **T** or **A** to indicate whether the line of position is **toward** or **away** from the GP, as measured from the AP:

Hc	37°51.′6	Hc	61°57.′3
Ho	37°43.′9	Ho	62°12.′7
a	7.7A	*a*	15.4T

The azimuth is customarily determined by computation or table at the time of determining Hc.

This method of plotting a line of position from a celestial observation was suggested by Marcq St.-Hilaire (art. 2108), and generally bears his name. It is used almost universally by modern navigators. The method is based upon knowledge of one point on the line, and the direction of the line. Another method of utilizing the same principle is to assume the latitude and compute the longitude at which the line of position crosses that parallel (the time sight method, art. 2106), or vice versa. When this method is used, the azimuth is customarily found separately, from a table or graph.

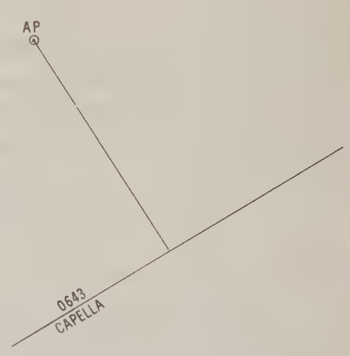


FIGURE 1703b.—A line of position from observation of the star Capella at 0643.

A third method is to compute two points on the line of position and draw a straight line through them. This line is a chord, rather than a tangent, of the circle of position, but in most cases the difference is negligible. This third method was that originally proposed by Captain Thomas H. Sumner (art. 131), and for this reason the resulting line of position is sometimes called a **Sumner line,** although the expression may be applied to any line of position resulting from celestial observation.

When celestial navigation is used, plotting is generally done on plotting sheets (art. 323) published by the U. S. Navy Hydrographic Office. These are less expensive than charts, and the absence of detail eliminates a possible source of confusion and error.

1704. Using lines of position from celestial observations.—Like any other line of position, one resulting from a celestial observation does not pinpoint the position of the craft, but may provide all the information needed to insure safety of the vessel. The selection of a celestial body and the time of observation to provide the desired information is based upon the fact that the line of position is perpendicular to the azimuth line. If the celestial body is on or near the celestial meridian, the line of position is a **latitude line,** indicating the latitude at the time of observation, sometimes called the **observed latitude.** Similarly, a body on or near the prime vertical provides a **longitude line,** indicating the **observed longitude.** One ahead or astern provides a **speed line,** since the line of position is perpendicular to the course, and hence is an indication of the speed made good since the last speed line or fix. Similarly, a body on the beam provides a **course line** which indicates to what extent the course is being made good. If the azimuth line is perpendicular to a coast line, shoal, or other hazard, the line of position indicates the distance of the vessel from the danger. Passage parallel to such a danger, or between two of them, might be made safely by means of a series of observations of a body on the beam during passage, without fixing the position of the vessel. This problem might be simplified by precomputing the sextant altitude at intervals during passage, and plotting this versus time on cross-section paper, so that sextant altitudes can be compared immediately with the values taken from the curve to determine any deviation from the desired track. In a perpendicular approach to a coast, the point at which landfall will be made can be predicted with considerable accuracy if a body having an azimuth parallel to the beach is observed.

During twilight, with clear skies, the selection of a celestial body to provide desired information is simply a matter of choosing the body with azimuth nearest that desired, remembering that bodies having azimuths differing by 180° should provide the same line of position. Observation of bodies in opposite directions provides a check, and a better one than two observations of the same body, or observations of two bodies having nearly the same azimuth, for any constant error in the observations, such as might be caused by abnormal dip, can be eliminated by observing bodies on opposite azimuths and using a line midway between the two plotted lines of position.

When a limited number of bodies is available for a considerable period, as during daylight, the best time to make an observation to obtain a line of position in a desired direction can be determined by means of an azimuth table or diagram, or an inspection table such as H.O. Pub. No. 214. The azimuth is located, and the corresponding meridian angle is recorded. The meridian angle can then be converted to GHA, and the time at which this GHA occurs can be determined from the almanac (art. 2104).

Lines of position can be used for determining an estimated position (art. 1705), or they can be advanced or retired (art. 1706) to obtain a fix (art. 1707) or running fix (art. 1708). If a single body is available for observation, increased accuracy can usually be obtained by making three or more observations, adjusting all lines to a common time (art. 1706), and using either the middle line, or the average position of all lines.

1705. Estimated position.—As indicated in chapter VIII, a **dead reckoning (DR) position** is determined by advancing a known position for courses and distances. In the absence of additional information, the DR position is the best estimate of the position of the vessel. However, the expression **estimated position (EP)** is generally applied to one determined by using additional but inconclusive information. If the effects of wind and current can be estimated, and these effects have not been considered in establishing the DR position, they can be applied separately to establish an EP. As each additional item of information is received, an improved estimate might be made.

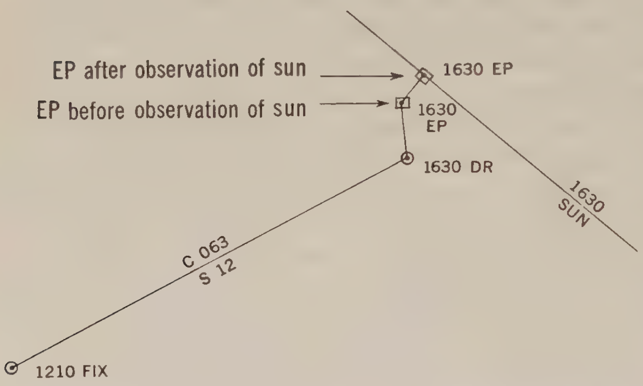


FIGURE 1705a.—Estimated positions before and after observation of the sun for a line of position, allowing for current.

A single line of position can be useful in establishing an estimated position. If an accurate line is obtained, the actual position is somewhere on this line. In the absence of better information, a perpendicular from the previous DR position or EP to the line of position establishes the new EP, as shown in figure 1705a. The foot of the perpendicular from the AP has no significance in this regard, since it is used only to locate the line of position.

The establishment of a good EP is dependent upon accurate interpretation of all information available. Generally, such ability can be acquired only by experience. If, in the judgment of the navigator or captain, the course has been made good, but the speed has been uncertain, the best estimate of the position might be at the intersection of the course line and the line of position, as shown in figure 1705b. If the speed since the last fix is considered accurate, but the course is considered uncertain, the EP might be at the intersection of the line of position and an arc centered on the previous fix and of radius equal to distance traveled, as shown in figure 1705c.

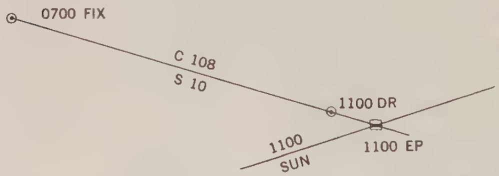


FIGURE 1705b.—An estimated position when the course and a line of position are considered accurate.

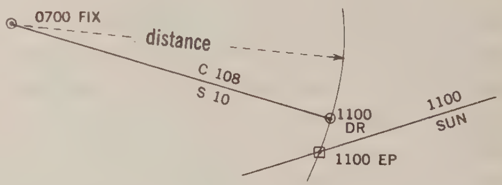


FIGURE 1705c.—An estimated position when the speed and a line of position are considered accurate.

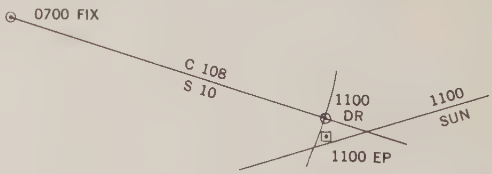


FIGURE 1705d.—An estimated position when a line of position is considered of first accuracy, speed of second accuracy, and course of third accuracy.

More often, neither course nor speed is known to be entirely accurate, but if one is considered more accurate than the other, the EP may be located accordingly. Even the line of position might properly be considered of questionable accuracy, and some estimate of its reliability established. Figure 1705d shows an EP that might be established by considering the line of position of greatest but incomplete accuracy, the speed of secondary accuracy, and the course as least accurate.

The expression **most probable position** (**MPP**) is sometimes used as the equivalent of *estimated position*. However, the former is of somewhat broader application, since it may apply equally well to establishment of the fix when more than two lines of position are available.

Further discussion of navigational accuracy is included in chapter XXIX.

1706. Advancing and retiring lines of position.—For a stationary observer, lines of position resulting from observations made at different times are equally applicable without adjustment. However, for a moving observer, as one aboard a vessel underway at sea, any line of position (except a course line) applies only to the position at the time of observation. If lines resulting from observations made at different times are to be utilized for determining position, they should be adjusted for the motion of the observer between observations.

A line of position resulting from observation of a celestial body can be advanced or retired in the same manner as other lines of position (ch. IX), by selecting any point associated with the line of position and running it forward or backward by dead reckoning, or by estimate. For most accurate results, the best estimate of course and speed made good (over the bottom) between the time of observation and the time to which the line is to be adjusted should be used. Any error in determining these values is reflected in the adjusted line of position. However, error in speed does not affect the accuracy of an adjusted course line, nor does error in course introduce an appreciable error in the accuracy of an adjusted speed line. The time label of an adjusted line of position includes both the time of observation and the time to which the line is adjusted.

As in the case of a line of position resulting from observation of the bearing of an identifiable, charted object (art. 904), the number of lines on the chart can be kept to a minimum, reducing the possibility of confusion, by adjusting the point from which the line is drawn. In the case of celestial navigation, this is the assumed position. This method applies equally well to all observations, and avoids some possible difficulty which might arise in advancing a line of position nearly parallel to the course line. When the AP is advanced or retired, the initial line of position need not be drawn unless it serves some useful purpose.

1707. The fix.—The common intersection of two or more lines of position constitutes a fix, regardless of the source of the position lines, provided only that the lines are based upon simultaneous observations. Celestial observations are seldom simultaneous because all sights of a group are customarily taken by a single observer, usually the navigator. If observations are made a few minutes apart (a **round of sights**), as during a twilight period, all lines are adjusted to a common time, and the position is considered a fix, rather than a running fix. Many navigators advance earlier lines to the time of the last observation, and consider the fix applicable at this time, as shown in figure 1707a. An alternative procedure, which is gaining in acceptance, is to *advance* earlier sights and *retire* later ones to an intermediate time, either the time of the mid observation or a convenient time during the period of observation, such as a whole, half, or quarter hour. This results in a more accurate and convenient time of the fix. In figure 1707b the lines of figure 1707a are adjusted to a common time at a whole hour. With any procedure, the time of the fix is the common time to which the lines of position are adjusted.

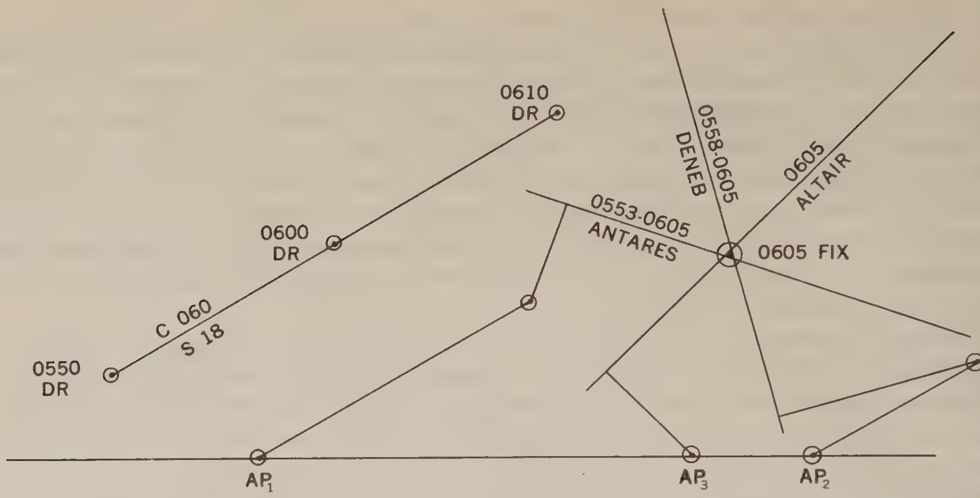


FIGURE 1707a.—A fix obtained by advancing earlier lines of position to the time of the last observation.

In figures 1707a and 1707b the assumed positions are typical of those which might be used with a modern method of sight reduction such as H.O. Pub. No. 214 (ch. XX). *Any* position in the vicinity might be used. If the dead reckoning (or estimated) position at the time of each observation is used as the assumed position for that sight, all sights are plotted from the DR position (or EP) at the time for which the fix is desired. If the same AP is used for all sights, the advanced or retired AP's are along a straight line extending in the direction of the course line, the AP corresponding to the earliest observation being farthest advanced along this line, and others progressing along it in a direction *opposite* to that of the course. If there is any change of course or speed between observations, this should be considered in advancing or retiring a line of position, as it would in running forward the dead reckoning. Under normal conditions, lines of position adjusted for a short interval to obtain a fix are moved by dead reckoning, without separate allowance for current.

Two lines of position provide a fix, but when additional celestial bodies are available, it is good practice to observe them. Additional lines serve as a check on the accuracy of the first two, and should decrease the error of the fix. However, the increased accuracy

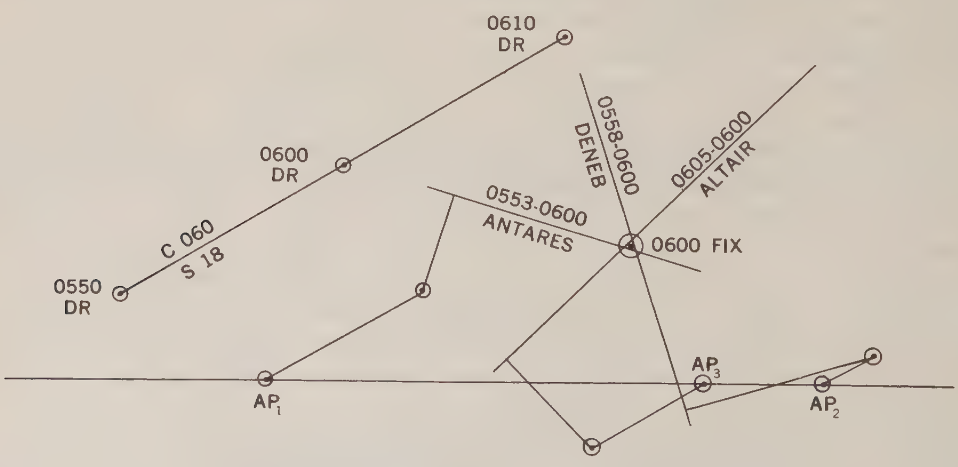


FIGURE 1707b.—A fix obtained by adjusting the lines of position of figure 1707a to a convenient time during the period of observation.

of a fix resulting from a number of lines of position, over that resulting from only two, is not great under normal conditions, and the principal reason for the additional observations is the increased confidence the navigator has in the reliability of his fix.

In selecting bodies for observation, one should generally consider azimuth primarily, and such factors as brightness, altitude, etc., secondarily. Individual circumstances, however, may dictate departures from this procedure. During twilight, when skies are clear and the entire horizon is good, one generally has ample choice of bodies to observe. It is good practice to make several more observations than the minimum considered acceptable, so that additional lines of position will be available, if needed, to resolve possible ambiguities or confirm doubtful results.

Sights need not be solved in the order taken. During evening twilight the brightest bodies should be observed first, as soon as they can be "brought down" successfully to the horizon. During morning twilight the reverse is true, the dimmer stars being observed while they are still visible. However, with advance planning, one can include in the list of bodies to be observed those which should provide the best fix.

If all observations were precisely correct, in every detail, the resulting lines of position would meet at a point. However, this is rarely the case. Three observations generally result in lines of position forming a triangle. If this triangle is not more than two or three miles on a side under good conditions, and five to ten miles under unfavorable conditions, there is normally no reason to suppose that a mistake has been made. Even a point fix, however, is not *necessarily* accurate. An uncorrected error in time, for instance, would move the entire fix eastward if early and westward if late, at the rate of 1′ of longitude for each 4^s of error in time.

With two or four observations, the ideal is to have them crossing at angles of 90°. With three observations, the ideal is angles of 60°. With three observations it is good practice to observe bodies differing in azimuth by 120°, as nearly as possible. This provides lines of position crossing at angles of 60°, and, in addition, any constant error in altitude is eliminated, serving only to increase or decrease the size of the triangle, but not affecting the position of its center. If the azimuths differ by 60°, a large constant error in altitude would result in a fix *outside* the triangle, as shown in figure 1707c.

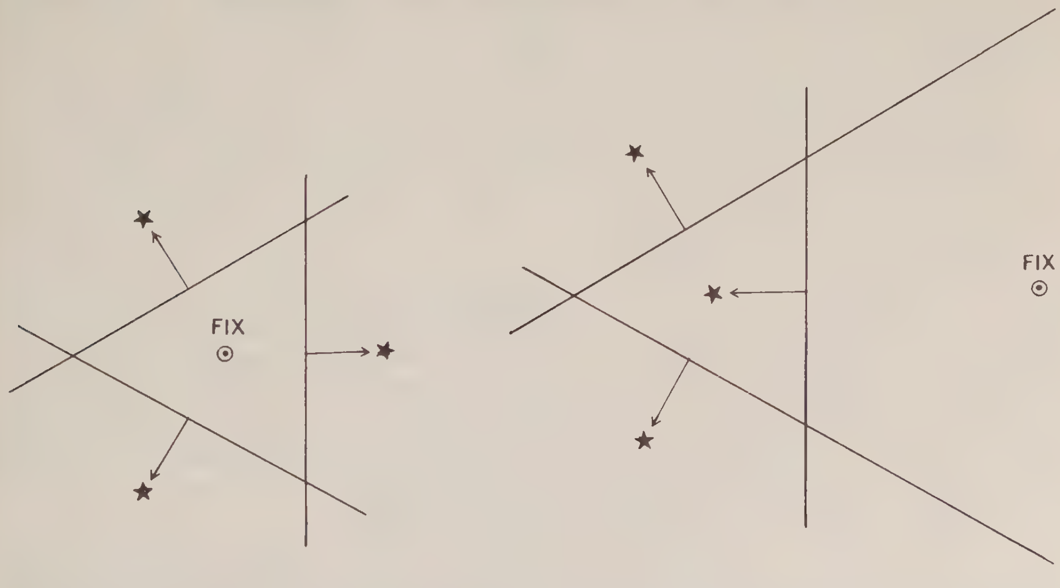


Figure 1707c.—A fix from three lines of position, assuming a constant error in altitude. If all lines are moved away (in this case) from the bodies observed, they would meet in a point which might be either inside (left) or outside (right) the triangle.

With lines of position crossing at 60°, the assumed constant error for a fix outside the triangle is three times that for a fix inside the triangle. With four bodies, azimuths differing by 90° produce a box fix, with constant error eliminated by using the mid point as the fix. With more than four observations, the selection of the fix becomes more complex, and general rules are probably undesirable. The evaluation of each observation and the exercise of judgment become of greater importance. Whatever the number of observations, common practice, backed by logic, is to take the center of the figure formed unless there is reason for deviating from this procedure. By "center" is meant the point representing the least total error of all lines considered reliable. With three lines of position, the center is considered that point, within the triangle, which is equidistant from the three sides. It may be found by bisecting the angles, but more commonly it is located by eye. If a fix outside the triangle is to be used, and eye interpolation is not considered sufficiently reliable, the point can be found by bisecting two external angles and the internal angle at the third intersection. If a constant error is assumed, the most probable position of the fix can always be found, whether within or outside the triangle, by bisecting the angle formed by azimuth lines originating at each intersection.

The matter of navigational errors as applied to this problem is further discussed in chapter XXIX.

1708. A running fix (**R fix**), in celestial navigation, is a position obtained by observations separated by a considerable time interval, usually several hours. The usual occasion for a running fix is the availability of a single celestial body for observation, generally the sun. The delay between observations is usually to permit the azimuth to change sufficiently to provide a good angle of cut between lines of position. Thus, the sun may be observed about 0900, and again about noon.

Generally, a longer wait results in a more nearly perpendicular intersection of the two lines of position, but it may also increase the error of the advanced line. The earlier line is advanced for the course and distance made good. The ability with which these can be predicted determines the accuracy of the running fix, assuming accurate observation, sight reduction, and plotting. For this reason it is impractical to set a specific time limit upon the advancement of a line of position. This should be determined by the conditions of each situation, in the best judgment of the navigator. Experience is valuable in acquiring such judgment.

When an observation of a single body is made, with the intent of later advancing it to obtain a running fix with a second observation, the line of position should be plotted for the time of observation, regardless of the method used for advancing it, for the single line usually provides some useful information, as indicated in article 1704.

Allowance for current, when advancing a line of position, can be made by solving a vector diagram, as indicated in article 807, to determine the course and speed made good. An alternative method is to advance the AP or line without allowance for current, and then to advance it a second time in the direction of set of the current, for a distance equal to the drift multiplied by the number of hours between the time of observation and the time to which the line is advanced. This method is illustrated in figure 1708a. The distance *AB* is equal to the distance between the 0800 and 1152 DR positions. The direction *BC* is the estimated set of the current, and the length *BC* is the distance through which the current is assumed to act.

A third method provides accurate results even when a reliable estimate of the current is not available, provided (1) a good fix was obtained several hours before the time of observation, and (2) the average current between the time of the previous fix and the time of observation can be assumed to continue until the time to which the

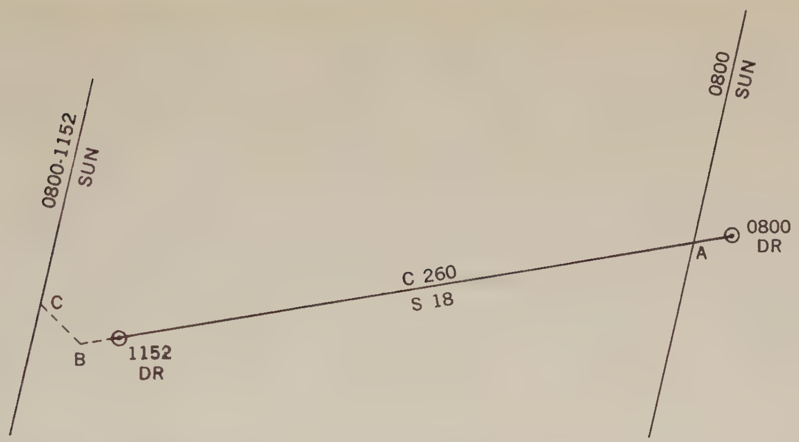


FIGURE 1708a.—Advancing a line of position with allowance for current, without determining course and speed made good.

line is to be advanced. This method is illustrated in figure 1708b. The 0510 fix is shown at the left, and the DR positions at 0830 and 1215, the ship being on course 074°, speed 12 knots. The sun is observed at 0830 and again at 1215, and it is desired to advance the earlier line to obtain a running fix at 1215. The lines of position at 0830 and 1215 are plotted. To advance the 0830 line of position, the distance *AB* is assumed to increase uniformly with time interval from 0510. The interval to 0830 is 3^h20^m, and that to 1215 is 7^h05^m. Therefore, $A'B'=AB\times\frac{7^h05^m}{3^h20^m}=AB\times 2.1$. The advanced line of position is drawn through B', parallel to the original line through B. The running fix is at the intersection of the 1215 line and the advanced 0830 line.

The set of the average current *between 0510 and 0830* is the direction from A' to the 1215 running fix, and the drift is equal to this distance divided by 7^h05^m. The direction of a straight line (not shown) from the 0510 fix to the 1215 running fix is

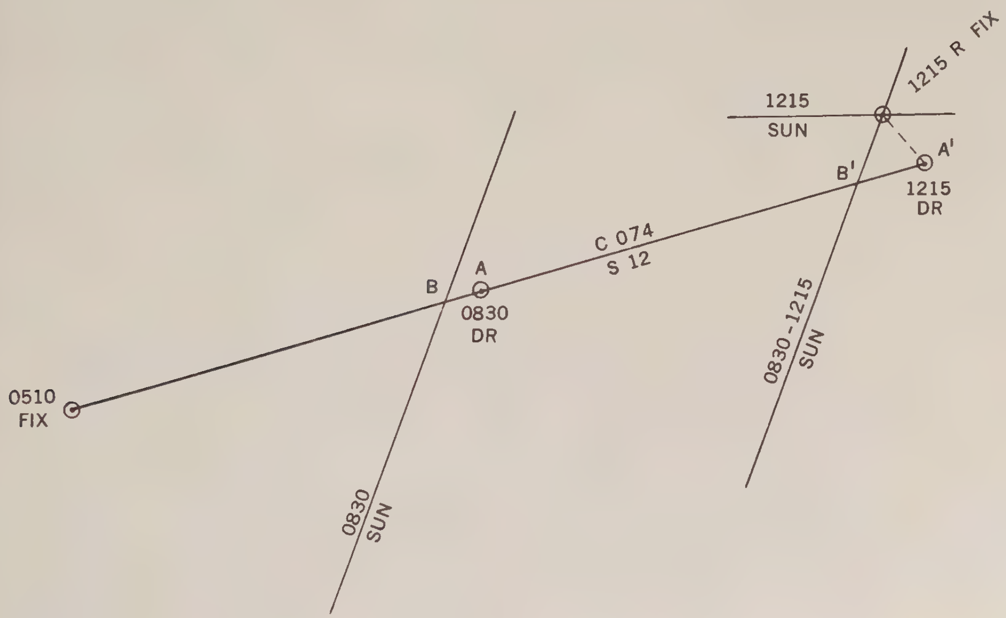


FIGURE 1708b.—Advancing a line of position without previous knowledge of the current.

the course made good between 0510 and 0830, and the length of this line divided by the time (7^h05^m) is the speed made good to 0830.

The points B and B' need not be at the intersection of the lines of position and the course line. *Any* point on the line of position can be used, and the line $A'B'$ drawn parallel to AB. Changes of course and speed do not affect the accuracy of the solution as long as $A'B'$ is parallel to AB.

Several other variations are possible. A convenient one is to measure the distance from the earlier fix to point B, and divide this by the time to determine an "assumed" speed (based upon the assumption that point B represents the position of the vessel at the time of observation), and then to use this speed to advance the line of position. This variation should not be used without adjustment if a change of course or speed is involved between the earlier fix and the time to which the line is to be advanced.

This method should be used with caution. Any error in either the earlier fix or the first line of position is *increased* in proportion to the elapsed time. Thus, in figure 1708b, if AB is in error by one mile, $A'B'$ is in error by 2.1 miles. It should not be used when there is reason to suspect a change in current between fixes.

1709. Celestial navigation and dead reckoning.—As indicated in chapter VIII, dead reckoning consists of advancing a known position for courses and speeds. Some difference in technique arises from a difference of opinion among navigators on the definition of (1) "known position" and (2) courses and speeds.

Regarding the first, no position determined by celestial navigation as commonly practiced at sea is known with perfect accuracy. An average error of two miles is realistic. Because of the varying conditions encountered, it is difficult to establish limits of a "known" position. In general, however, a reasonably reliable fix or running fix is considered sufficiently accurate to justify a new start in the dead reckoning. An estimated position or a fix or running fix of doubtful accuracy is considered an indication, but an inconclusive one, of the error in the dead reckoning. Therefore, it is considered good practice to avoid starting a new dead reckoning track from such a position unless there is a compelling reason for doing so. After long experience and the development of sound judgment, a navigator might acquire great skill in establishing a most probable position of sufficient reliability to justify more frequent breaks in the continuity of the dead reckoning, but even under these conditions any reasonable element of doubt should be given great respect.

What has been said regarding "known position" applies, also, in large measure to course and speed. The course steered and the speed at which a ship is being driven forward by its engines can be determined with relatively little error. Allowance for wind and current is a matter largely of judgment based upon experience. If the dead reckoning is to be meaningful, considerable caution should be exercised in allowing for wind and current when determining the course and speed to use for plotting. In the absence of information of a high degree of reliability, it is considered prudent to determine dead reckoning without allowing for estimated effects of wind and current.

In the absence of better information, then, it is considered good practice to start a new dead reckoning track only from a reliable fix or running fix, and to use courses and speeds without allowance for wind and current. This does not mean, however, that the navigator should not continually be aware of the possibility of error in his position as determined by dead reckoning, nor should he fail to make an estimate of the size and direction of the error. In this ability, and that of accurately interpreting all navigational information received, lies the test of a good navigator. This is largely the *art* of navigation, as distinguished from the somewhat mechanical process of making

observations and computing and plotting the results, and also from the *science* of devising the aids that are used in modern navigation.

When it is desired to determine "average current," this expression being used to mean the resultant of all dead reckoning errors, the dead reckoning should be run forward from a fix (not a running fix) to the time of the next fix (or running fix if the method of art. 1708 is used). A dead reckoning position determined in any other way is not usable, unless it is adjusted to provide a "no-current" position. A straight line connecting such a dead reckoning position and the fix at the same time indicates the current. The direction of the line *from* the DR position *to* the fix is the **set** of the current, and the length of this line divided by the number of hours since the last fix is the **drift,** as in piloting.

Problems

A plotting sheet such as H.O. 3000–9Z (or 3000–5), covering latitudes 27° to 30° north and south is needed for most of the problems of this chapter. If this is not available, one can be constructed by means of table 5, as explained in article 307; or small area plotting sheets can be constructed as explained in article 324.

1703a. In each of the following, determine the altitude difference, *a*, and label it T or A, as appropriate:

	Hc	*Ho*
(1)	18°21′.4	18°25′.9
(2)	53°02′.7	52°35′.5
(3)	(−)0°05′.2	(−)0°12′.7
(4)	(−)0°11′.1	0°01′.1

Answers.—(1) *a* 4.5 T, (2) *a* 27.2 A, (3) *a* 7.5 A, (4) *a* 12.2 T.

1703b. The 0930 DR position of a ship is lat. 29°20′.4 N, long. 130°25′.2 W. At this time the navigator observes the sun, and computes Hc and Zn for the 0930 DR position, as follows: Hc 45°42′.9, Ho 45°50′.2, Zn 157°.3. As a check, he also solves the same sight for an assumed position of lat. 29°00′.0 N, long. 130°30′.0 W, with the following results: Hc 46°00′.0, Zn 157°.0.

Required.—Plot the two lines of position, and account for the result.

Answer.—The two lines of position plot as approximately the same line, which is not dependent upon the assumed position, but only upon the observed altitude and the time of observation.

1705a. The 0500 fix of a ship is lat. 27°10′.0 N, long. 142°55′.5 W. The ship is on course 068°, speed 9 knots. At 0800 the navigator observes the sun, with the following results:

a 6.6 T	*a*L 27°00′.0 N
Zn 105°.0	*a*λ 142°39′.1 W

The current since the morning fix is estimated to set 130°, at a drift of 1.4 knots.

Required.—(1) The 0800 DR position.

(2) The 0800 EP if there were no observation, and no current was anticipated.

(3) The 0800 EP using the current, if there were no observation.

(4) The 0800 EP using the line of position, but not the current.

(5) The 0800 EP using all available information.

Answers.—(1) 0800 DR: L 27°20′.0 N, λ 142°27′.2 W; (2) 0800 EP without current and line of position: L 27°20′.0 N, λ 142°27′.2 W; (3) 0800 EP with current but no line of position: L 27°17′.4 N, λ 142°23′.5 W; (4) 0800 EP with line of position but no current: L 27°19′.5 N, λ 142°25′.3 W; (5) 0800 EP with current and line of position: L 27°18′.7 N, λ 142°25′.8 W.

1705b. The 0530 fix of a ship is lat. 28°55′.8 N, long. 161°51′.7 E. The ship is on course 060°, speed 10 knots. At 0830 the navigator observes the sun, with the following results:

a	6.7 A	*a*L	29°00′.0 N
Zn	110°.0	*a*λ	162°28′.9 E

Required.—(1) The 0830 EP if the course is believed to have been made good, and the line of position is considered accurate.

(2) The 0830 EP if the speed is believed to be correct, and the line of position is considered accurate.

(3) The 0830 EP if the course and speed are considered of equal reliability, and the line of position is considered accurate.

(4) The 0830 EP if the course is of questionable accuracy, but considered more reliable than the speed, and the line of position is considered accurate.

(5) The 0830 EP if the speed is of questionable accuracy, but considered more reliable than the course, and the line of position is considered accurate.

(6) The 0830 EP if the course is believed to have been made good, and the error contributed by the uncertainty of the line of position is believed to be twice that contributed by the uncertainty of the speed.

Answers.—(1) 0830 EP: L 29°13′.5 N, λ 162°26′.3 E; (2) 0830 EP: L 29°06′.5 N, λ 162°23′.6 E; (3) 0830 EP: L 29°09′.8 N, λ 162°25′.0 E; (4) 0830 EP: any place between (1) and (3); (5) 0830 EP: any place between (2) and (3); (6) 0830 EP: L 29°11′.4 N, λ 162°22′.8 E.

1707a. At 1740 the navigator and two assistants observe simultaneously three stars, with the following results:

	Fomalhaut	*Deneb*	*Aldebaran*
Hc	28°10′.3	34°59′.6	39°52′.8
Ho	28°05′.3	35°05′.6	39°46′.8
Zn	210°.0	308°.7	089°.3
*a*L	28°00′.0 N	28°00′.0 N	28°00′.0 N
*a*λ	42°31′.7 W	42°29′.0 W	42°23′.2 W

Required.—The 1740 fix.

Answer.—1740 fix: L 28°06′.6 N, λ 42°30′.5 W.

1707b. The 1800 DR position of a ship is lat. 27°02′.2 N, long. 170°17′.0 W. The ship is on course 045°, speed 14 knots. During evening twilight the navigator observes three stars, with the following results:

	Dubhe	*Altair*	*Spica*
Time	1815	1821	1830
Hc	34°45′.2	22°11′.8	47°24′.8
Ho	34°51′.3	22°15′.7	47°20′.4
Zn	331°.5	090°.3	219°.9
*a*L	27°00′.0 N	27°00′.0 N	27°00′.0 N
*a*λ	170°10′.2 W	170°05′.0 W	169°54′.8 W

Required.—The 1830 fix.

Answer.—1830 fix: L 27°11′.5 N, λ 170°00′.5 W.

1707c. The 1930 DR position of a ship is lat. 29°10′.5 S, long. 122°35′.4 W. The ship is on course 320°, speed 16 knots. During evening twilight the navigator observes a planet and two stars, with the following results:

	Saturn	*Regulus*	*Rigil Kent.*
Time	1931	1942	1951
Hc	46°58′.5	53°04′.0	24°19′.5
Ho	46°55′.5	53°09′.3	24°30′.0
Zn	023°.5	170°.2	297°.6
*a*L	29°00′.0 S	29°00′.0 S	29°00′.0 S
*a*λ	122°55′.0 W	122°45′.1 W	122°35′.2 W

Required.—The 1942 fix.

Answer.—1942 fix: L 29°05′.3 S, λ 122°47′.4 W.

1707d. The 0500 DR position of a ship is lat. 29°53′.9 N, long. 69°32′.1 W. The ship is on course 130°, speed 13 knots. During morning twilight the navigator observes a planet and two stars, with the following results:

	Mars	*Kochab*	*Spica*
Time	0451	0502	0511
Hc	17°14′.1	38°26′.2	33°35′.2
Ho	17°24′.5	38°19′.2	33°47′.8
Zn	130°.1	353°.2	237°.9
*a*L	30°00′.0 N	30°00′.0 N	30°00′.0 N
*a*λ	69°41′.7 W	69°30′.0 W	69°18′.3 W

Required.—The 0500 fix.

Answer.—0500 fix: L 29°54′.0 N, λ 69°30′.5 W.

1707e. The 0930 DR position of a ship is lat. 28°40′.4 N, long. 125°30′.4 E. The ship is on course 220°, speed 25 knots. The navigator observes the sun and moon, and solves each sight from the DR position at the time of sight, with the following results:

	Sun	*Moon*
Time	0936	0943
Hc	54°24′.3	37°07′.9
Ho	54°26′.3	37°14′.7
Zn	200°.2	142°.6

Required.—The 0943 fix.

Answer.—0943 fix: L 28°32′.1 N, λ 125°32′.3 E.

1707f. A ship is on course 314°, speed 24 knots. During evening twilight the navigator observes two stars and the moon, and solves all three sights using assumed latitude 28°00′.0 S, assumed longitude 41°19′.5 W as the AP, with the following results:

	Peacock	*Moon*	*Alpheratz*
Time	1855	1900	1905
Hc	57°12′.6	66°58′.2	23°00′.5
Ho	57°17′.9	67°01′.2	22°53′.7
Zn	194°.7	300°.5	038°.2

Required.—The 1900 fix.

Answer.—1900 fix: L 28°03′.5 S, λ 41°26′.5 W.

1707g. The 0400 DR position of a ship is lat. 27°01′.8 N, long. 51°36′.0 E. The ship is on course 037°, speed 20 knots. At 0545 the course is changed to 309°. During morning twilight the navigator observes two stars, with the following results:

	Vega	*Alpheratz*
Time	0537	0602
a	4.5 T	7.8 T
Zn	300°.5	075°.7
*a*L	27°00′.0 N	27°00′.0 N
*a*λ	51°45′.2 E	51°50′.1 E

Required.—The 0602 fix.

Answer.—0602 fix: L 27°28′.1 N, λ 51°51′.1 E.

1707h. The 0600 DR position of a ship is lat. 27°50′.3 N, long. 20°58′.2 W. The ship is on course 000°, speed 20 knots. During morning twilight the navigator observes four stars, with the following results:

	Dubhe	*Kaus Aust.*	*Spica*	*Vega*
Time	0551	0554	0558	0604
Hc	29°01′.1	21°57′.8	37°59′.4	54°33′.1
Ho	28°53′.4	22°11′.7	38°03′.5	54°28′.5
Zn	330°.0	149°.7	233°.3	057°.3
*a*L	28°00′.0 N	28°00′.0 N	28°00′.0 N	28°00′.0 N
*a*λ	20°54′.6 W	21°08′.4 W	20°56′.7 W	20°51′.3 W

Required.—The 0600 fix.

Answer.—0600 fix: L 27°53′.5 N, λ 20°55′.0 W.

1707i. The 1815 DR position of a ship is lat. 29°41′.5 S, long. 163°52′.3 W. The ship is on course 295°, speed 18 knots. During evening twilight the navigator observes three stars, with the following results:

	Regulus	*Pollux*	*Aldebaran*
Time	1810	1815	1821
Hc	45°18′.6	35°50′.7	22°50′.8
Ho	45°26′.2	36°03′.4	22°57′.7
Zn	040°.2	350°.7	300°.5
*a*L	30°00′.0 S	30°00′.0 S	30°00′.0 S
*a*λ	163°45′.0 W	163°49′.8 W	163°54′.0 W

Required.—(1) The 1815 fix, assuming random errors.

(2) The 1815 fix, assuming a constant error.

Answers.—(1) 1815 fix: L 29°47′.2 S, λ 163°51′.2 W; (2) 1815 fix: L 29°51′.4 S, λ 163°50′.6 W.

1708a. The 0830 DR position of a ship is lat. 29°25′.4 S, long. 9°34′.7 E. The ship is on course 326°, speed 22 knots. The sun is observed during the morning, and again at 1200, with the following results:

	Sun	*Sun*
Time	0830	1200
a	15.2 A	28.4 A
Zn	062°.3	169°.5
*a*L	29°00′.0 S	29°00′.0 S
*a*λ	9°37′.0 E	8°52′.1 E

Required.—The 1200 running fix.

Answer.—1200 R fix: L 28°31′.6 S, λ 8°50′.0 E.

1708b. The 0900 DR position of a ship is lat. 28°05′.6 N, long. 93°44′.0 W. The ship is on course 220°, speed 20 knots, and is believed to be in a current with set of 110° and a drift of 1.5 knots. The sun is observed during the morning, and again at 1200, with the following results:

	Sun	*Sun*
Time	0900	1200
a	11.2 T	17.0 A
Zn	103°.2	172°.0
*a*L	28°00′.0 N	27°00′.0 N
*a*λ	93°54′.0 W	94°38′.9 W

Required.—The 1200 running fix.

Answer.—1200 R fix: L 27°19′.8 N, λ 94°17′.5 W.

1708c. The 0715 fix of a ship is lat. 28°28′.9 S, long. 81°14′.8 W. The ship is on course 120°, speed 15 knots. During the morning the sun is observed twice, with the following results:

	Sun	*Sun*
Time	0945	1200
a	9.4 A	0
Zn	095°.0	005°.0
*a*L	29°00′.0 S	29°00′.0 S
*a*λ	80°26′.1 W	80°11′.2 W

Required.—(1) The 1200 running fix, allowing for current.

(2) Set and drift of the current.

(3) Course made good between 0715 and 0945.

Answers.—(1) 1200 R fix: L 29°01′.0 S, λ 80°00′.2 W; (2) set 049°, drift 1.1 kn.; (3) course made good 116°.0.

1708d. The 0500 fix of a ship is lat. 28°36′.5 N, long. 143°22′.0 E. The courses and speeds during the morning are as follows:

Time	*Course*	*Speed*
0500	047°	24 kn.
0600	102°	20 kn.
0715	038°	16 kn.
0845	075°	19 kn.
1000	030°	23 kn.
1045	085°	25 kn.

During the morning the sun is observed twice, with the following results:

	Sun	*Sun*
Time	0915	1200
a	8.8 A	20.0 A
Zn	125°.0	191°.7
*a*L	29°00′.0 N	29°00′.0 N
*a*λ	144°44′.8 E	145°29′.8 E

Required.—(1) The 1200 running fix, allowing for current.

(2) Set and drift of the current.

(3) Course and speed made good between fixes, assuming no change in current.

Answers.—(1) 1200 R fix: L 29°20′.0 N, λ 145°33′.0 E; (2) set 200°, drift 1.7 kn.; (3) course made good 070°, speed made good 17.7 kn.

1709a. The 0400 DR position of a ship is lat. 27°41′.8 S, long. 64°54′.0 E. This position has been run forward from a fix at 1715 the previous evening. The ship is on course 215°, speed 19 knots, but at 0600 the course is changed to 125°. At 0715 a fix locates the ship at lat. 28°23′.0 S, long. 65°04′.3 E.

Required.—Set and drift of the current between fixes.

Answers.—Set 073°, drift 1.0 kn.

1709b. The 0500 fix of a ship is lat. 27°09′.0 N, long. 158°09′.5 W. The ship is on course 310°, speed 14 knots. At 1155 a running fix locates the ship at lat. 28°01′.2 N, long. 159°33′.2 W. A new dead reckoning plot is started from this position. At 1900 a star fix is obtained, locating the ship at lat. 28°57′.8 N, long. 160°54′.9 W.

Required.—Set and drift of the average current between morning and evening fixes.

Answers.—Set 167°, drift 1.2 kn.

CHAPTER XVIII

THE ALMANAC

1801. Introduction.—A requirement of celestial navigation is the availability of accurate predictions of the positions of the celestial bodies used. These predictions, with respect to the celestial equator system of coordinates (art. 1426), are contained in three publications of the United States Naval Observatory. Recent minor modifications to these publications have not been incorporated in this printing. The solution for a celestial line of position consists principally of the conversion of tabulated coordinates to those on the horizon system of coordinates (art. 1428).

The ***American Ephemeris and Nautical Almanac*** gives, to a high precision, detailed information on a large number of celestial bodies. This annual publication is arranged to suit the convenience of the astronomer, for whom it is primarily intended. The ephemeris is not needed for ordinary purposes of navigation, although it contains some information of general interest, such as various astronomical constants, details of eclipses, information on planetary configurations (art. 1422), and miscellaneous phenomena. Each volume of the ephemeris contains instructions for its use.

The ***American Nautical Almanac,*** an annual publication, contains the astronomical information needed by the marine navigator. It is conveniently arranged to suit his needs, and the information is tabulated to a practical degree of precision (art. O3), in general to the nearest 0'.1 of arc and 1^s of time, at hourly intervals. Beginning with the edition for 1958, this volume is a joint publication of the U. S. Naval Observatory and the British Admiralty, and incorporates a number of changes from previous editions. Extracts from the *Nautical Almanac* for that year are given in appendix V. These extracts, illustrating the various features of that publication, can be used in the solution of the various illustrative and sample problems of the present volume.

The ***Air Almanac,*** published three times per year, is intended primarily for air navigators. In general, the information is similar to that of the *Nautical Almanac*, but is given to a precision of 1′ of arc and 1^s of time, at intervals of 10^m (recent editions give values for the sun and Aries to a precision of 0'.1). This publication is suitable for ordinary navigation at sea, but may lack the precision that is sometimes needed. The *Air Almanac* is a joint publication of the U.S. Naval Observatory and the British Air Council. Extracts from the *Air Almanac* are given in appendix W.

A highly abbreviated, long-term almanac is given in appendix X. Because of the large intervals between entries, and the fact that no provision is made for nutation, information taken from this almanac may be of reduced accuracy. Although this accuracy is sufficient for most purposes of navigation, the almanac is not as convenient to use as either of those published by the U. S. Naval Observatory, and is not recommended when one of them is available. Instructions for its use are included in appendix X.

1802. *American Nautical Almanac.*—The major portion of the *Nautical Almanac* is devoted to hourly tabulation of Greenwich hour angle and declination, to the nearest 0'.1 of arc. On each set of facing pages, information is given for three consecutive days. On the left-hand page, successive columns give GHA of Aries and both GHA and declination of Venus, Mars, Jupiter, and Saturn, followed by the SHA and declination of 57 stars. The GHA and declination of the sun and moon, and the horizontal parallax of the moon, are given on the right-hand page. Where applicable, the quantities v and d are given to assist in interpolation. The quantity v is the differ-

ence between the actual change of GHA in one hour and a constant value used in the interpolation tables, while *d* is the change in declination in one hour. Both *v* and *d* are given to the nearest 0ʹ.1. To the right of the moon data is given the LMT (art. 1911) of sunrise, sunset, and beginning and ending of nautical and civil twilight for various latitudes from 72° N to 60° S. The LMT of moonrise and moonset at the same latitudes is given for each of the three days for which other information is given, and for the following day. Magnitude (art. 1405) of each planet at GMT 1200 of the middle day is given at the top of the column. The GMT (art. 1907) of transit across the celestial meridian of Greenwich is given as "Mer. Pass." The value for the first point of Aries for the middle of the three days is given to the nearest $0^m.1$ at the bottom of the Aries column. The time of transit of the planets for the middle day is given to the nearest whole minute, with SHA (at GMT 0000 of the middle day) to the nearest 0ʹ.1, below the list of stars. For the sun and moon, the time of transit to the nearest whole minute is given for each day. For the moon, both upper and lower transits are given. This information is tabulated below the rising, setting, and twilight information. Given there, also, are the equation of time for 0^h and 12^h, and the age and phase of the moon (art. 1423). Equation of time is given, without sign, to the nearest whole second. Age is given to the nearest whole day. Phase is given by symbol.

The main tabulation is preceded by a list of religious and civil holidays, phases of the moon, a calendar, information on eclipses occurring during the year, and notes and a diagram giving information on the planets.

The main tabulation is followed by explanation and examples. Next are four pages of standard times (zone descriptions) in use in various places in the world. Star charts are given next, followed by a list of 173 stars in order of increasing sidereal hour angle. This list includes the stars given on the daily pages. It gives the SHA and declination each month, and the magnitude. Stars are listed by Bayer's name and also by popular name where there is one. Following the star list are three pages of Polaris tables giving the azimuth and the corrections to be applied to the observed altitude to find the latitude. Next is a table for converting arc to time units. This is followed by a 30-page table called "Increments and Corrections," used for interpolation of Greenwich hour angle and declination. This table is printed on tinted paper, for quick location. Then come tables for interpolating for times of rising, setting, and twilight; followed by two indices of the 57 stars listed on the daily pages, one index being in alphabetical order, and the other in order of decreasing SHA.

Sextant altitude corrections are given at the front and back of the almanac. Tables for the sun, stars, and planets, and a dip table, are given on the inside front cover and facing page, with an additional correction for nonstandard temperature and atmospheric pressure on the following page. Tables for the moon, and an abbreviated dip table, are given on the inside back cover and facing page. Use of the altitude correction tables is explained in chapter XVI. Corrections for the sun, stars, and planets for altitudes greater than 10°, and the dip table, are repeated on one side of a loose bookmark. The star indices are repeated on the other side.

1803. *Air Almanac.*—As in the *Nautical Almanac,* the major portion of the *Air Almanac* is devoted to a tabulation of GHA and declination. However, in the *Air Almanac* values are given at intervals of ten minutes, to a precision of 1ʹ. Values are given for the sun, first point of Aries (GHA only), the three navigational planets most favorably located for observation, and the moon. The magnitude of each planet listed is given at the top of its column, and the phase of the moon is given at the top of its column. Values for the first 12 hours of the day are given on the right-hand page, and those for the second half of the day on the back. In addition, the right-hand page has a table of the moon's parallax in altitude, and below this the semidiameter of the sun,

and both the semidiameter and age of the moon (art. 1423). To the right of this is an ecliptic diagram, explained in article 2209. The afternoon side of each daily page includes the LMT of sunrise, sunset, moonrise, and moonset; duration of civil twilight; and a difference column for finding the time of moonrise and moonset at any longitude.

Critical tables for interpolation for GHA are given on the inside front cover, which also has an alphabetical listing of the stars, with the number, magnitude, SHA, and declination of each. The inside of the back cover has the same refraction table and Coriolis correction table given in H.O. Pub. No. 249. The outside back cover has a correction table for dip of the horizon, and a table of contents.

Following the daily pages are instructions for use of the almanac; a list of symbols and abbreviations in English, French, and Spanish; a list of time differences between Greenwich and various other places; a number of sky diagrams (art. 2212); information on setting the astrograph (art. 2123); polar sunlight, twilight, and moonlight diagrams; corrections to times of sunrise and sunset when observed from flight altitudes; a table for converting arc to time; interpolation tables for finding time of moonrise and moonset at any longitude; a star list similar to that given on the inside front cover, but in order of decreasing SHA; a list of the names and numbers of the stars used in H.O. Pub. No. 249, those in H.O. Pub. No. 218, and those which can be used by declination entry in H.O. Pub. No. 249, in addition to those listed by name; and an explanation of the navigational star chart, and the chart itself. The inside front cover page is repeated on the back of the star chart. Also given there are a single Polaris correction table, a standard aircraft astrodome refraction table, and a special refraction table for use with H.O. Pub. No. 218.

Minor modifications and changes to some of the foregoing items have been made in recent editions of the *Air Almanac*.

1804. Use of the almanacs.—The time used as an entering argument in the almanacs is GMT, and the information given is for the Greenwich meridian.

Tabulated information taken from the almanacs, as from any tables, should not be recorded to a greater precision than tabulated. The *units* in which values are given are shown at the tops of the columns.

The use of work forms, such as those shown in appendix Q, permits concentration on the extraction of data, with no need for dividing one's attention between this and thought regarding the order of work. It also simplifies the taking of all needed information from the double page to which the almanac is open, before turning to a different part of the almanac.

If the entering arguments are exactly those of any table, the desired value is taken directly from the table. Often, however, this is not the case, and the detailed instructions in the following articles relate principally to the method of interpolating in the various tables. Since Greenwich hour angle is measured in a westerly direction, the same direction as the apparent motion of celestial bodies, the tables are customarily entered with the next earlier tabulated time, with interpolation toward a later time. The correction to be applied for a fractional part of an hour is then always additive. If the sum exceeds 360°, this amount is subtracted.

In the *Nautical Almanac*, v is always positive unless a negative sign ($-$) is given. This can occur only in the case of Venus. For the sun, the tabulated values of GHA have been adjusted slightly to minimize the error of interpolation, so that no v value need be given. No sign is given for tabulated values of d, which can be considered positive if declination is increasing, and negative if it is decreasing. The sign of a v or d value is given also to the related correction.

In the *Air Almanac* the tabulated declination values are those for the *middle* of the interval between the time indicated and the next *following* time for which a value

is given. It is intended that declination be taken from the tables without interpolation.

1805. Finding GHA and declination of the sun.—*Nautical Almanac.* Enter the daily-page table with the whole hour next preceding the given GMT, unless this time is itself a whole hour, and take out the tabulated GHA and declination. Record, also, the *d* value given at the bottom of the declination column. Next, enter the increments and corrections table for the number of minutes of GMT. If there are seconds, use the next *earlier* whole minute. On the line corresponding to the seconds of GMT take the value from the sun-planets column. Add this to the value of GHA from the daily page to find GHA at the given time. Next, enter the correction table for the same minute with the *d* value, and take out the correction. Give this the sign of the *d* value, and apply it to the declination from the daily page. The result is the declination at the given time.

Example 1.—Find the GHA and declination of the sun at GMT $18^h24^m37^s$ on June 1, 1958, using the *Nautical Almanac.*

Solution.—

	Sun			Sun
GMT	$18^h24^m37^s$ June 1		GMT	$18^h24^m37^s$ June 1
18^h	90°34.′6		18^h	22°03.′4 N *d*
24^m37^s	6°09.′3		corr.	(+)0.′1 (+)0.′3
GHA	96°43.′9		d	22°03.′5 N

The correction table for GHA of the sun is based upon a rate of change of 15° per hour, the average rate during a year. At most times the rate differs slightly from this. The slight error thus introduced is minimized by the method of tabulation. The tabulated values are adjusted for *half* the error during the hour following the time of tabulation. Therefore, instead of increasing for an hour following the entry time, the error *decreases* for the first half hour, to zero, and then increases during the second half hour, and the maximum error is only about *half* what it would be if unadjusted values were used. The greatest error thus introduced is about 0.′1. An additional small error may be introduced by rounding off base and correction values to the nearest 0.′1. More exact values can be obtained by interpolating between the values for the half hours, or by referring to the ephemeris.

The *d* value is the amount that the declination changes between 1200 and 1300 on the middle day of the three shown.

Air Almanac. Enter the daily page with the whole 10^m next preceding the given GMT, unless this time is itself a whole 10^m, and extract the tabulated GHA and declination, without interpolation. The tabulated declination is correct for the time 30^m later than that tabulated, so that interpolation during the hour following tabulation is not needed for most purposes. Next, enter the "interpolation of GHA" table on the inside front cover, using the "sun, etc." entry column, and take out the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction given half a line *above* the entry time. Add this correction to the GHA taken from the daily page to find the GHA at the given time. No adjustment of declination is needed.

Example 2.—Find the GHA and declination of the sun at GMT $18^h24^m37^s$ on June 1, 1958, using the *Air Almanac.*

Solution.—

	Sun
GMT	$18^h24^m37^s$ June 1
18^h20^m	95°35′
4^m37^s	1°09′
GHA	96°44′
d	22°04′ N

1806. Finding GHA and declination of the moon.—*Nautical Almanac.* Enter the daily-page table with the whole hour next preceding the given GMT, unless this time is itself a whole hour, and take out the tabulated GHA and declination. Record, also, the corresponding v and d values tabulated on the same line, and determine the sign of the d value. The v value of the moon is always positive (+), and is not marked in the almanac. Next, enter the increments and corrections table for the minutes of GMT, and on the line for the seconds of GMT take the GHA correction from the moon column. Then, enter the correction table for the same minute with the v value, and extract the correction. Add both of these corrections to the GHA from the daily page to obtain the GHA at the given time. Then, enter the same correction table with the d value, and extract the correction. Give this correction the sign of the d value, and apply it to the declination from the daily page to find the declination at the given time.

Example 1.—Find the GHA and declination of the moon at GMT $21^h25^m44^s$ on June 1, 1958, using the *Nautical Almanac.*

Solution.—

	Moon	
GMT	$21^h25^m44^s$ June 1	
21^h	315°01′.6	
25^m44^s	6°08′.4	v
corr.	2′.4	(+) 5′.6
GHA	321°12′.4	

	Moon	
GMT	$21^h25^m44^s$ June 1	
21^h	18°46′.3S	d
corr.	(+) 1′.0	(+) 2′.4
d	18°47′.3S	

The correction table for GHA of the moon is based upon the *minimum* rate at which the moon's GHA increases, 14°19′.0 per hour. The v correction makes the adjustment for the actual rate. The v value itself is the difference between the minimum rate and the actual rate during the hour following the tabulated time. The d value is the amount that the declination changes during the hour following the tabulated time.

Air Almanac. Enter the daily page with the whole 10^m next preceding the given GMT, unless this time is itself a whole 10^m, and take out the tabulated GHA and the declination, without interpolation. Next, enter the "interpolation of GHA" table on the inside front cover, using the "moon" entry column, and take out the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction given half a line *above* the entry time. Add this correction to the GHA taken from the daily page to find the GHA at the given time. No adjustment of declination is needed.

Example 2.—Find the GHA and declination of the moon at GMT $21^h25^m44^s$ on June 1, 1958, using the *Air Almanac.*

Solution.—

	Moon
GMT	$21^h25^m44^s$ June 1
21^h20^m	319°49′
5^m44^s	1°23′
GHA	321°12′
d	18°47′S

The declination given in the table is correct for the time *five minutes later than tabulated,* so that it can be used for the ten-minute interval without interpolation, to an accuracy to meet most requirements. If greater accuracy is needed, it can be obtained by interpolation, remembering to allow for the five minutes indicated above.

1807. Finding GHA and declination of a planet.—*Nautical Almanac.* Enter the daily-page table with the whole hour next preceding the given GMT, unless the time itself is a whole hour, and take out the tabulated GHA and declination. Record, also,

the v value given at the bottom of each of these columns. Next, enter the increments and corrections table for the minutes of GMT, and on the line for the seconds of GMT take the GHA correction from the sun-planets column. Next, enter the correction table with the v value and extract the correction, giving it the sign of the v value. Add the first correction to the GHA from the daily page, and apply the second correction in accordance with its sign, to obtain the GHA at the given time. Then, enter the correction table for the same minute with the d value, and extract the correction. Give this correction the sign of the d value, and apply it to the declination from the daily page to find the declination at the given time.

Example 1.—Find the GHA and declination of Venus at GMT $5^h24^m07^s$ on June 2, 1958, using the *Nautical Almanac*.

Solution.—

	Venus	
GMT	$5^h24^m07^s$ June 2	
5^h	295°21′.8	
24^m07^s	6°01′.8	v
corr.	(−) 0′.1	(−) 0′.3
GHA	301°23′.5	

	Venus	
GMT	$5^h24^m07^s$ June 2	
5^h	9°53′.7 N	d
corr.	(+) 0′.4	(+) 1′.0
d	9°54′.1 N	

The correction table for GHA of planets is based upon the mean rate of the sun, 15° per hour. The v value is the difference between 15° and the change of GHA of the planet between 1200 and 1300 on the middle day of the three shown. The d value is the amount that the declination changes between 1200 and 1300 on the middle day.

Venus is the only body listed which ever has a negative v value.

Air Almanac.—Enter the daily page with the whole 10^m next preceding the given GMT, unless this time is itself a whole 10^m, and extract the tabulated GHA and declination, without interpolation. The tabulated declination is correct for the time 30^m later than tabulated, so that interpolation during the hour following tabulation is not needed for most purposes. Next, enter the "interpolation of GHA" table on the inside front cover, using the "sun, etc." column, and take out the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction half a line *above* the entry time. Add this correction to the GHA from the daily page to find the GHA at the given time. No adjustment of declination is needed.

Example 2.—Find the GHA and declination of Venus at GMT $5^h48^m45^s$ on June 2, 1958, using the *Air Almanac*.

Solution.—

	Venus
GMT	$5^h48^m45^s$ June 2
5^h40^m	305°22′
8^m45^s	2°11′
GHA	307°33′
d	9°54′ N

The declination is taken for the next *earlier* tabulated time, and is correct for GMT 5^h45^m.

1808. Finding GHA and declination of a star.—If the GHA and declination of each navigational star were tabulated separately, the almanacs would be several times their present size. But since the sidereal hour angle (art. 1426) and declination are nearly constant over several days (to the nearest 0′.1) or months (to the nearest 1′), separate tabulations are not needed. Instead, the GHA of the first point of Aries, from which SHA is measured, is tabulated on the daily pages, and a single listing of SHA and declination is given for each double page of the *Nautical Almanac*, and for an

entire volume of the *Air Almanac.* The finding of GHA ♈ is similar to finding GHA of the sun, moon, and planets.

Nautical Almanac. Enter the daily-page table with the whole hour next preceding the given GMT, unless this time is itself a whole hour, and take out the tabulated GHA ♈. Record, also, the tabulated SHA and declination of the star from the listing on the left-hand daily page. Next, enter the increments and corrections table for the minutes of GMT, and on the line for the seconds of GMT take the GHA correction from the Aries column. Add this correction and the SHA of the star to the GHA ♈ of the daily page to find the GHA of the star at the given time. No adjustment of declination is needed.

Example 1.—Find the GHA and declination of Canopus at GMT $3^h24^m33^s$ on June 2, 1958, using the *Nautical Almanac.*

Solution.—

	Canopus
GMT	$3^h24^m33^s$ June 2
3^h	295°04'.8
24^m33^s	6°09'.3
SHA	264°15'.0
GHA	205°29'.1
d	52°40'.6 S

The SHA and declination of 173 stars, including Polaris and the 57 listed on the daily pages, are given for the middle of each month, on almanac pages 268–273. For a star not listed on the daily pages this is the only almanac source of this information. Interpolation in this table is not necessary for ordinary purposes of navigation, but is sometimes needed for precise results. Thus, if the SHA and declination of *β Crucis* (Mimosa) are desired for March 1, 1958, they are found by simple eye interpolation to be SHA 168°40'.2 and d 59°27'.6 S.

If GHA ♈ is desired, it is found as indicated in example 1, but omitting the addition of SHA of a star. In the example GHA ♈ is 295°04'.8+6°09'.3=301°14'.1.

Air Almanac. Enter the daily page with the whole 10^m next preceding the given GMT, unless this is itself a whole 10^m, and extract the tabulated GHA ♈. Next, enter the "interpolation of GHA" table on the inside front cover, using the "sun, etc." entry column, and take out the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction given half a line *above* the entry time. From the tabulation at the left side of the same page, extract the SHA and declination of the star. Add the GHA from the daily page and the two values taken from the inside front cover to find the GHA at the given time. No adjustment of declination is needed.

Example 2.—Find the GHA and declination of Peacock at GMT $12^h17^m58^s$ on June 1, 1958, using the *Air Almanac.*

Solution.—

	Peacock
GMT	$12^h17^m58^s$ June 1
12^h10^m	71°58'
7^m58^s	2°00'
SHA	54°24'
GHA	128°22'
d	56°52' S

1809. Rising, setting, and twilight.—In both almanacs the times of sunrise, sunset, moonrise, moonset, and twilight information at various latitudes between 72° N and 60° S are given to the nearest whole minute. By definition, rising or setting occurs when the *upper* limb of the body is on the *visible* horizon, assuming standard refraction

for zero height of eye. Because of variations in refraction and height of eye, computation to a greater precision than 1^m is not justified. For high elevations, as those encountered on a mountain overlooking the sea, or at flight altitudes, a correction table is provided in the *Air Almanac*.

In high latitudes some of the phenomena do not occur during certain periods. The symbols used to indicate this condition are:

□ Sun or moon does not set, but remains continuously above the horizon.

■ Sun or moon does not rise, but remains continuously below the horizon.

//// Twilight lasts all night.

The *Nautical Almanac* makes no provision for finding the times of rising, setting, or twilight in polar regions. The *Air Almanac* has graphs for this purpose. The use of these, and other sources of such information, are explained in article 2536.

In the *Nautical Almanac*, sunrise, sunset, and twilight tables are given only once for the three days on each page opening, using average declination and equation of time. The results approximate the values for the middle day. For most purposes this information can be used for all three days. For more accurate results, the information can be taken from the *Air Almanac*, which has a table for each day. Both almanacs have moonrise and moonset tables for each day.

The tabulations are in local mean time (art. 1911). On the zone meridian, this is the zone time (ZT). For every 15′ of longitude that the observer's position differs from that of the zone meridian, the zone time of the phenomena differs by 1^m, being *later* if the observer is *west* of the zone meridian, and *earlier* if he is *east* of the zone meridian. The local mean time of the phenomena varies with latitude of the observer, declination of the body, and hour angle of the body relative to that of the mean sun.

Sunrise and sunset are also tabulated in the tide tables (from 76° N to 60° S) and in a supplement to the American ephemeris of 1946 entitled *Tables of Sunrise, Sunset, and Twilight* (from 75° N to 75° S). The meridian angle of any body at the time of its rising and setting can be computed by the formulas given in article 2536. The meridian angle of a body when its center is on the celestial horizon can be computed by the formula

$$\cos t = \tan L \tan d,$$

where t is the meridian angle, L is the latitude, and d is the declination. Solutions of this formula are given in table 25, if the table is entered with a latitude 90° from the given latitude. This table can be used for this purpose only when latitude and declination are of contrary name.

1810. Finding time of sunrise and sunset.—*Nautical Almanac.* Enter the table on the daily page, and extract the LMT for the tabulated latitude next *smaller* than the observer's latitude (unless this is an exact tabulated value). Apply a correction from table I on almanac page xxxii to interpolate for latitude, determining the sign of the correction by inspection. Then convert LMT to ZT by means of the difference in longitude (dλ) between the local and zone meridians.

Example.—Find the zone time of sunrise and sunset at lat. 43°31.′4 N, long. 36°14.′3 W on June 1, 1958.

Solution.—

L 43°31.′4 N June 1
λ 36°14.′3 W

	Sunrise		Sunset
40°	0433	40°	1922
T I	(−) 11	T I	(+) 11
LMT	0422	LMT	1933
dλ	(+) 25	dλ	(+) 25
ZT	0447	ZT	1958

Air Almanac. The procedure is the same as that for the *Nautical Almanac*, except that correction for latitude is made by linear interpolation directly from the tabulation, since no interpolation table is provided.

The tabulated times are for the Greenwich meridian. Except in high latitudes near the times of the equinoxes, the time of sunrise and sunset varies so little from day to day that no interpolation is needed for longitude. If such an interpolation is considered justified, it can be made in the same manner as for the moon (art. 1812).

In high latitudes, interpolation is not always possible. For instance, on June 1, 1958, sunrise at latitude 66° N occurs at 0114, but at latitude 68° N the sun does not set. Between these two latitudes the time of sunrise might be found from the graphs in the *Air Almanac*, or by computation, as explained in article 2536. However, in such a marginal situation, the time of sunrise itself is uncertain, being greatly affected by a relatively small change of refraction or height of eye.

1811. Finding time of twilight.—Morning twilight *ends* at sunrise, and evening twilight *begins* at sunset. The time of the darker limit can be found from the almanacs. The *time* of the darker limits of both *civil* and *nautical* twilights (center of the sun 6° and 12°, respectively, below the celestial horizon) is given in the *Nautical Almanac*. The *duration* (in minutes) of *civil* twilight is tabulated in the *Air Almanac*. The brightness of the sky at any given depression of the sun below the horizon may vary considerably from day to day, depending upon the amount of cloudiness and other atmospheric conditions. In general, however, the most effective period for observing stars and planets occurs when the center of the sun is between about 3° and 9° below the celestial horizon. Hence, the darker limit of civil twilight occurs at about the mid point of this period. At the darker limit of nautical twilight the horizon is generally too dark for good observations. At the darker limit of *astronomical twilight* (center of the sun 18° below the celestial horizon) full night has set in. The time of this twilight is given in the ephemeris. Its approximate value can be determined by extrapolation (art. P6) in the *Nautical Almanac*, noting that the duration of the different kinds of twilight is not proportional to the number of degrees of depression at the darker limit. More precise determination of the time at which the center of the sun is any given number of degrees below the celestial horizon can be determined by a large-scale diagram on the plane of the celestial meridian (art. 1432) or by computation (art. 2536). Duration of twilight at various angles of depression between 1°3 and 12° is given on pages A52 and A53 of the *Air Almanac* (not shown in appendix W).

Nautical Almanac. The method of finding the darker limit of twilight is the same as that for sunrise and sunset (art. 1810).

Example 1.—Find the zone time of beginning of morning nautical twilight and ending of evening nautical twilight at lat. 21°54′.7 S, long. 109°34′.2 E on June 1, 1958.

Solution.—

L 21°54′.7 S June 1
λ 109°34′.2 E

	Nautical twilight		Nautical twilight
20° S	0537	20° S	1819
T I	(+)3	T I	(−)3
LMT	0540	LMT	1816
dλ	(−) 18	dλ	(−) 18
ZT	0522	ZT	1758

Air Almanac. Find the ZT of sunrise and sunset as explained in article 1810, except that correction for latitude is made by linear interpolation, since no table is provided for this purpose. While taking the LMT from the almanac, extract, also, the duration of civil twilight, in minutes. *Subtract* this value from the time of sunrise

to find the time of beginning of morning civil twilight, and *add* it to the time of sunset to find the time of ending of evening civil twilight.

Example 2.—Find the zone time of beginning of morning civil twilight and ending of evening civil twilight at lat. 47°18.′8 S, long. 87°28.′3 W on June 1, 1958.

Solution.—

L 47°18.′8 S June 1
λ 87°28.′3 W

	Sunrise			Sunset	
45° S	0727		45° S	1628	
corr.	(+) 9		corr.	(−) 9	
LMT	0736		LMT	1619	
dλ	(−) 10		dλ	(−) 10	
ZT	0726	(sunrise)	ZT	1609	(sunset)
dur.	(−) 35		dur.	(+) 35	
ZT	0651	(twilight)	ZT	1644	(twilight)

Sometimes in high latitudes the sun does not rise but twilight occurs. This is indicated in the *Air Almanac* by the symbol ■ in the sunrise and sunset column. In this case the value in the twilight column indicates *half* the duration of twilight. To find the time of beginning of morning twilight, *subtract* this interval from the time of meridian transit of the sun; and for the time of ending of evening twilight, *add* it to the time of meridian transit. The LMT of meridian transit never differs by more than $16^{m}.4$ (approximately) from 1200. The actual time on any date can be determined from the almanac (art. 2104).

1812. Finding time of moonrise and moonset is similar to finding time of sunrise and sunset, with one important difference. Because of the moon's rapid change of declination, and its fast eastward motion relative to the sun, the time of moonrise and moonset varies considerably from day to day. These changes of position on the celestial sphere (art. 1403) are continuous, as moonrise and moonset occur successively at various longitudes around the earth. Therefore, the change in time is distributed over all longitudes. For precise results, it would be necessary to compute the time of the phenomena at any given place, by the method described in article 2536. For ordinary purposes of navigation, however, it is sufficiently accurate to interpolate between consecutive moonrises or moonsets at the Greenwich meridian. Since apparent motion of the moon is westward, relative to an observer on the earth, interpolation in west longitude is between the phenomenon on the given date and the *following* one. In east longitude it is between the phenomenon on the given date and the *preceding* one.

Nautical Almanac. For the given date, enter the daily-page table with latitude, and extract the LMT for the tabulated latitude next *smaller* than the observer's latitude (unless this is an exact tabulated value). Apply a correction from table I of the almanac "Tables for Interpolating Sunrise, Moonrise, etc." to interpolate for latitude, determining the sign of the correction by inspection. Repeat this procedure for the day following the given date, if in west longitude; or for the day preceding, if in east longitude. Using the difference between these two times, and the longitude, enter table II of the almanac "Tables for Interpolating Sunrise, Sunset, etc." and take out the correction. Apply this correction to the LMT of moonrise or moonset at the Greenwich meridian on the given date to find the LMT at the position of the observer. The sign to be given the correction is such as to make the corrected time fall between the times for the two dates between which interpolation is being made. This is nearly always positive (+) in west longitude and negative (−) in east longitude. Convert the corrected LMT to ZT.

Example 1.—Find the zone time of moonrise and moonset at lat. 58°23.′6 N, long. 144°07.′5 W on June 1, 1958, using the *Nautical Almanac.*

Solution.—

L 58°23'.6 N June 1
λ 144°07'.5 W

Moonrise			Moonset		
58° N	2011	June 1	58° N	0314	June 1
T I	(+)3		T I	(−)2	
LMT (G)	2014	June 1	LMT (G)	0312	June 1
58° N	2113	June 2	58° N	0401	June 2
T I	(+)3		T I	(−)3	
LMT (G)	2116	June 2	LMT (G)	0358	June 2
LMT (G)	2014	June 1	LMT (G)	0312	June 1
diff.	62		diff.	46	
T II	(+)25		T II	(+)18	
LMT (G)	2014	June 1	LMT (G)	0312	June 1
LMT	2039	June 1	LMT	0330	June 1
dλ	(−)24		dλ	(−)24	
ZT	2015	June 1	ZT	0306	

Air Almanac. For the given date, determine LMT for the observer's latitude at the Greenwich meridian, in the same manner as with the *Nautical Almanac,* except that linear interpolation is made directly from the main tabulation, since no interpolation table is provided. Extract, also, the value from the "Diff." column to the right of the moonrise and moonset column, interpolating if necessary. This "Diff." is the difference between the time of occurrence of the phenomenon at longitude 90° E and 90° W, and is intended for use in both east and west longitudes. The error introduced by this approximation is generally not more than a few minutes, although it increases with latitude. Using this difference, and the longitude, enter the "Interpolation of Moonrise, Moonset" table on page A54 of the *Air Almanac* and take out the correction. The *Air Almanac* recommends the taking of the correction from this table without interpolation. The results thus obtained are sufficiently accurate for ordinary purposes of navigation. If greater accuracy is desired, the correction can be taken by interpolation. However, since the "Diff." itself is an approximation, the *Nautical Almanac* or computation (art. 2536) should be used if accuracy is a consideration. Apply the correction to the LMT of moonrise or moonset at the Greenwich meridian on the given date to find the LMT at the position of the observer. The correction is positive (+) for west longitude, and negative (−) for east longitude, unless the "Diff." on the daily page is preceded by a negative sign (−), when the correction is negative (−) for west longitude, and positive (+) for east longitude. If the time is near midnight, record the date at each step, as in the *Nautical Almanac* solution.

Example 2.—Find the zone time of moonrise and moonset at lat. 58°23'.6 N, long. 144°07'.5 W on June 1, 1958, using the *Air Almanac.*

Solution.—

L 58°23'.6 N June 1
λ 144°07'.5 W

Moonrise		Moonset	
diff.	(+)34	diff.	(+)21
58° N	2011	58° N	0314
corr.	(+)3	corr.	(−)3
LMT (G)	2014	LMT (G)	0311
corr.	(+)29	corr.	(+)16
LMT	2043	LMT	0327
dλ	(−)24	dλ	(−)24
ZT	2019	ZT	0303

As with the sun, there are times in high latitudes when interpolation is inaccurate or impossible. At such periods, the times of the phenomena themselves are uncertain, but an approximate answer can be obtained by moonlight graph in the *Air Almanac* or by computation, as explained in article 2536. With the moon, this condition occurs when the moon rises or sets at one latitude, but not at the next higher tabulated latitude, as with the sun. It also occurs when the moon rises or sets on one day but not on the preceding or following day. This latter condition is indicated in the *Air Almanac* by the symbol ∗ in the "Diff." column.

Because of the eastward revolution of the moon around the earth, there is one day each synodical month (art. 1412) when the moon does not rise, and one day when it does not set. These occur near last quarter and first quarter, respectively. Since this day is not the same at all latitudes or at all longitudes, the time of moonrise or moonset found from the almanac may occasionally be the preceding or succeeding one to that desired. When interpolating near midnight, one should exercise caution to prevent an error.

Refer to the right-hand daily page of the *Nautical Almanac* for June 12, 13, 14 (app. V). On June 14 moonrise occurs at 0015 at latitude 70° N, and at 2326 at latitude 72° N. These are *not* the same moonrise, the one at 2326 occurring approximately one day *later* than the one occurring at 0015. This is indicated by the two times, which differ by nearly 24 hours. The table indicates that with increasing northerly latitude, moonrise occurs *earlier.* Between 70° N and 72° N the time crosses midnight *to the preceding day.* Hence, between these latitudes interpolation should be made between 0015 on June 14 and 2344 on June 13.

The effect of the revolution of the moon around the earth is to cause the moon to rise or set *later* from day to day. The daily retardation due to this effect does not differ greatly from 50^{m}. The change in declination of the moon may increase or decrease this effect. The effect due to change of declination increases with latitude, and in extreme conditions it may be greater than the effect due to revolution of the moon. Hence, the interval between successive moonrises or moonsets is more erratic in high latitudes than in low latitudes. When the two effects act in the same direction, daily differences can be quite large. Thus, at latitude 72° N the moon sets at 1834 on June 13, and at 2029 on June 14. When they act in opposite directions, they are small, and when the effect due to change in declination is larger than that due to revolution, the moon rises *earlier* on succeeding days. Thus, at latitude 72° N the moon rises at 2344 on June 13, and at 2326 on June 14. This condition is reflected in the *Air Almanac* by a negative "Diff." If this happens near last quarter or first quarter, two moonrises or moonsets might occur on the same day, one a few minutes after the day begins, and the other a few minutes before it ends. On June 12, 1958, for instance, at latitude 72° N, the moon rises at 0003, sets at 1649, and rises again at 2354 the same day. On those days on which no moonrise or no moonset occurs, the next succeeding one is shown with 24^{h} added to the time. Thus, at latitude 70° N the moon rises at 2358 on June 2, while the next moonrise occurs $24^{h}21^{m}$ later, at 0019 on June 4. This is listed both as 2419 on June 3 and as 0019 on June 4 (not shown in app. V).

Interpolation for longitude is always made between *consecutive* moonrises or moonsets, regardless of the days on which they fall.

Example 3.—Find the zone time of moonrise at lat. 71°38′.7 N, long. 56°21′.8 W during the night of June 12–13, 1958, using the *Nautical Almanac.*

Solution.—

L 71°38′.7 N June 12–13
λ 56°21′.8 W

	Moonrise	
70° N	0014	June 13
T I	(−) 16	
LMT (G)	2358	June 12
70° N	0015	June 14
T I	(−) 25	
LMT (G)	2350	June 13
LMT (G)	2358	June 12
diff.	8	
T II	(−) 2	
LMT (G)	2358	June 12
LMT	2356	June 12
dλ	(−) 15	
ZT	2341	June 12

Interpolation for the first entry is between 0014 on June 13 (lat. 70° N) and 2354 on June 12 (lat. 72° N); for the second entry, between 0015 on June 14 and 2344 on June 13. This solution might be more easily visualized by considering the 0014 moonrise of June 13 as occurring at 2414 on June 12, and that of 0015 on June 14 as occurring at 2415 on June 13.

1813. Rising, setting, and twilight at a moving craft.—Instructions given in the preceding three articles relate to a fixed position on the earth. Aboard a moving craft the problem is complicated somewhat by the fact that time of occurrence depends upon position of the craft, and vice versa. At ship speeds, it is generally sufficiently accurate to make an approximate mental solution, and use the position of the vessel at this time to make a more accurate solution. If higher accuracy is required, the position at the time indicated in the second solution can be used for a third solution. If desired, this process can be repeated until the same answer is obtained from two consecutive solutions. However, it is generally sufficient to alter the first solution by 1^m for each 15′ of longitude that the position of the craft differs from that used in the solution, adding if west of the estimated position, and subtracting if east of it. In applying this rule, use both longitudes to the nearest 15′.

1814. Miscellaneous.—*Sextant altitude corrections* are explained in chapter XVI.

Equation of time is given below the sunset and twilight information on the right-hand daily page of the *Nautical Almanac*, at intervals of twelve hours. Simple interpolation can be used for intervening values. By convention, the sign is positive (+) if the time in the sun's "Mer. Pass." column is earlier than 1200 (or if GHA indicates the sun has crossed the upper branch of the celestial meridian before 1200 or the lower branch before 0000), and negative (−) if later than 1200. In Great Britain, this convention is reversed. A heavy line is used to indicate a change of sign between consecutive entries, as shown between 00^h and 12^h on June 14, when the sign changes from positive to negative. The equation of time is not needed for ordinary purposes of modern navigation. Its use is explained in chapter XIX.

Time of transit. The GMT of transit of the sun across the upper branch of the celestial meridian of Greenwich is tabulated in the *Nautical Almanac*, to the right of the equation of time tables. Similar information is given for both the upper and lower transits of the moon in the "Mer. Pass." tables below the moonset tables. On a day when the moon does not transit the Greenwich meridian, the time for the next

day is given with 24^h added. Thus, on May 31 the upper transit is at 2308. The next transit is at 2407 on June 1, or at 0007 on June 2, as shown in appendix V. For the four navigational planets, time of upper transit for the middle day is given below the star list on the left-hand daily page. For all of these bodies the time is given to the nearest whole minute. Time of transit of the first point of Aries across the upper branch on the middle day is given to the nearest $0^m.1$, below the GHA Aries column. The tabulated times can be considered the LMT of transit at the Greenwich meridian, since LMT at 0° longitude is GMT. The LMT of transit at the observer's meridian can then be found by interpolation for longitude, as for moonrise and moonset (art. 1812). Eye interpolation is usually sufficient for bodies other than the moon. This information is of little value to the navigator. It is further discussed in chapter XIX.

Moon phases are indicated symbolically in the *Nautical Almanac* at the lower right-hand corner of the double-page opening. In the *Air Almanac* this information is given at the top of the GHA-declination column on each daily page. An open circle is used for full moon, and a solid black circle for new moon. The time of occurrence of each phase is given below the list of holidays, near the front of the *Nautical Almanac*. The *age* of the moon (art. 1423) is given between the "Mer. Pass." and phase listings in the *Nautical Almanac* and at the bottom of the right-hand daily page of the *Air Almanac*, near the right side.

Semidiameters of the sun and moon are given at the bottom of the sun and moon columns of the *Nautical Almanac*. In the *Air Almanac* they are given immediately above the age of the moon, on the right-hand daily page.

Magnitude of each planet listed is given at the top of its column on the daily pages of each almanac.

Ecliptic diagram. The diagram at the right of each right-hand daily page of the *Air Almanac* indicates the positions of the moon, navigational planets, the first point of Aries, and certain stars, relative to the sun. This diagram is discussed in article 2209. A single *Planet Location Diagram* has been substituted in recent editions.

GHA and declination for following year. If an almanac for the preceding year is available, but not for the current year, the approximate declination of the sun can be found by entering the almanac for the previous year with a time 5^h49^m *earlier*. The GHA of the sun can also be found in this way, if 87°15′ is *added* to the result. For stars, use the declination for the preceding year, and from the GHA subtract 15′.1. Every reference to the same date of the preceding year refers to the day 365 days earlier than the given date. When a February 29 has intervened, the day one day later should be used for the preceding year.

Right ascension (art. 1426), if required, can be found by subtracting SHA from 360°, and converting it from arc to time units (art. 1904). The SHA of the stars and planets is listed on the left-hand daily page. For the sun and moon, SHA can be found by subtracting the GHA of Aries from the GHA of the body.

Conversion of arc to time, or vice versa, can be made by the "Conversion of Arc to Time" tables of the almanacs, or mathematically as explained in article 1904.

Polaris tables are explained in article 2105.

Navigational star charts. These charts are explained in article 2204.

Sky diagrams of the *Air Almanac* are explained in article 2212.

Several additional items of general interest, such as a list of religious and civil holidays, a calendar, information on eclipses, planet notes for the year, and a list of standard times (zone descriptions) at various places throughout the world, are given in the *Nautical Almanac*. Items such as symbols and abbreviations in English with their French and Spanish equivalents, and a list of stars used in H.O. Pub. No. 249, are included in the *Air Almanac*.

Recent issues of the *Air Almanac* have been modified slightly. Beginning with the 1961 edition, references to H.O. Pub. No. 218, *Astronomical Navigation Tables*, have been omitted and an azimuth of Polaris table added. In the 1962 edition, both pages of tables of Corrections for Height and Depression were omitted. These two pages were replaced by five pages of Rising, Setting and Depression Graphs, and one page containing supplementary tables and an explanation of use of these graphs and tables.

Problems

1805a. Find the GHA and declination of the sun at GMT $7^h25^m54^s$ on June 2, 1958, using the *Nautical Almanac*.

Answers.—GHA 292°01′.9, d 22°07′.9 N.

1805b. Find the GHA and declination of the sun at GMT $23^h49^m04^s$ on June 1, 1958, using the *Air Almanac*.

Answers.—GHA 177°50′, d 22°05′ N.

1806a. Find the GHA and declination of the moon at GMT $0^h24^m18^s$ on June 1, 1958, using the *Nautical Almanac*.

Answers.—GHA 18°14′.8, d 17°28′.4 S.

1806b. Find the GHA and declination of the moon at GMT $12^h01^m22^s$ on June 1, 1958, using the *Air Almanac*.

Answers.—GHA 185°40′, d 18°19′ S.

1807a. Find the GHA and declination of Mars at GMT $2^h25^m39^s$ on May 31, 1958, using the *Nautical Almanac*.

Answers.—GHA 288°25′.5, d 3°53′.3 S.

1807b. Find the GHA and declination of Jupiter at GMT $21^h06^m21^s$ on June 1, 1958, using the *Air Almcnac*.

Answers.—GHA 5°22′, d 7°20′ S.

1808a. Find the GHA and declination of Procyon at GMT $4^h25^m18^s$ on June 1, 1958, using the *Nautical Almanac*.

Answers.—GHA 201°12′.0, d 5°19′.8 N.

1808b. Find the GHA and declination of γ *Velorum* at GMT $16^h24^m11^s$ on May 31, 1958, using the *Nautical Almanac*.

Answers.—GHA 12°38′.7, d 47°13′.2 S.

1808c. Find GHA ♈ at GMT $20^h25^m32^s$ on June 1, 1958, using the *Nautical Almanac*.

Answer.—GHA ♈ 196°11′.6.

1808d. Find the GHA and declination of Gienah at GMT $2^h53^m21^s$ on June 2, 1958, using the *Air Almanac*.

Answers.—GHA 109°59′, d 17°19′ S.

1810. Find the zone time of sunrise and sunset at lat. 52°18′.7 S, long. 58°43′.6 W on June 1, 1958.

Answers.—Sunrise, ZT 0751; sunset, ZT 1554.

1811a. Find the zone time of beginning of morning nautical twilight and ending of evening nautical twilight at lat. 16°22′.7 N, long. 163°19′.7 E on June 1, 1958.

Answers.—Morning twilight, ZT 0441; evening twilight, ZT 1928.

1811b. Find the zone time of beginning of morning civil twilight and ending of evening civil twilight at lat. 55°35′.6 N, long. 51°13′.7 W on June 1, 1958, using the *Air Almanac*.

Answers.—Morning twilight, ZT 0253; evening twilight, ZT 2153.

1812a. Find the zone time of moonrise and moonset at lat. 44°26′.3S, long. 172°29′.3E on June 3, 1958, using the *Nautical Almanac.*

Answers.—Moonrise, ZT 1732; moonset, ZT 0743.

1812b. Find the zone time of moonrise and moonset at lat. 3°27′.4S, long. 107°22′.8 W on June 1, 1958, using the *Air Almanac.*

Answers.—Moonrise, ZT 1815; moonset, ZT 0554.

1812c. Find the zone time of moonrise at lat. 71°44′.7N, long. 176°18′.1E on the night of June 13–14, 1958, using the *Nautical Almanac.*

Answer.—Moonrise, ZT 0007 June 14.

1813. The zone time of sunrise at a moving ship is desired. The first solution, based upon an estimate of the position, is 0537. The longitude used for the solution is 51°22′.2W. At 0537 the longitude will be 51°38′.8W.

Required.—The zone time of sunrise at the ship.

Answer.—Sunrise, ZT 0539.

1814a. Find the equation of time at GMT $16^h21^m04^s$ on June 1, 1958.

Answer.—Eq. T(+)2^m19^s.

1814b. Find the zone time of transit of the moon across the upper branch of the celestial meridian at long. 137°14′.4W on May 31, 1958.

Answer.—Transit, ZT 2340.

1814c. What are the phase and age of the moon on June 2, 1958?

Answers.—Phase, full moon; age, 15 days.

1814d. What are the semidiameters of the sun and moon on June 2, 1958?

Answers.—Sun, SD 15′.8; moon, SD 16′.1.

1814e. What is the magnitude of Jupiter on June 1, 1958?

Answer.—Mag.(−) 1.9.

1814f. Which of the navigational planets is nearest Aldebaran on June 2, 1958?

Answer.—Venus.

1814g. Find the GHA and declination of the sun at GMT $11^h14^m07^s$ on May 31, 1959, using the 1958 *Nautical Almanac.*

Answers.—GHA 349°09′.7, d 21°50′.5N.

1814h. Find the GHA and declination of Alioth at GMT $19^h25^m23^s$ on June 1, 1959, using the 1958 *Nautical Almanac.*

Answers.—GHA 347°48′.5, d 56°11′.3N.

CHAPTER XIX

TIME

1901. Introduction.—Time serves to regulate affairs aboard ship, as it does ashore. But to the navigator, it has additional significance. It is not enough to know *where* the ship is, was, or might be located in the future. The navigator wants to know *when* the various positions were or can reasonably be expected to be occupied. Time serves as a measure of progress. By considering the time at which a ship occupied various positions in the past, and by comparing the speed and various conditions it has encountered with those anticipated for the future, the skillful navigator can predict with reasonable accuracy the time of arrival at various future positions. Time can serve as a measure of safety, for it indicates when a light or other aid to navigation might be sighted, and if it is not seen by a certain time, the navigator knows he has cause for concern.

To the celestial navigator, time is of added significance, for it serves as a measure of the *phase* of the earth's rotation. That is, it indicates the position of the celestial bodies relative to meridians on the earth. Until an accurate *measure* of time became available at sea, longitude could not be found.

Very small *intervals* of time are used in certain electronic navigational aids, such as radar and loran.

Whatever the type of navigation, a thorough mastery of the subject of time is important to the navigator. The use of a time diagram (art. 1427) may help in understanding the principles or solution of the problems of this chapter.

1902. Kinds of time.—As a measure of part of a day, time can be stated in a number of different ways. At any given moment, the time depends upon (1) the point on the celestial sphere used as reference, (2) the reference meridian on the earth, and (3) the somewhat arbitrary starting point of the day.

When the sun is used as the celestial reference point, **solar time** results. If the actual sun observable in the sky is used, **apparent solar time** is involved, and if a fictitious **mean sun** is used to provide a time having an almost constant rate, **mean solar time** results. Time reckoned by use of the first point of Aries (♈) as the celestial reference point is called **sidereal time.** Use of the moon as the celestial reference point provides a variable-length **lunar day,** the basis of **lunar time,** which is useful in tide prediction and analysis. Because of its application, a lunar day is sometimes called a **tidal day.** It averages about 24^h50^m (mean solar units) in length.

If the meridian of the observer is used as the terrestrial reference, **local time** is involved. If a **zone** or **standard meridian** is used as the **time meridian** for mean solar time over an area, **zone** or **standard time** results. Use of a meridian farther east than would normally be used, so that the period of daylight is shifted later in the day, produces a form of zone time called **daylight saving** or **summer time.** Time based upon the Greenwich meridian is called **Greenwich time. Greenwich mean time** (**GMT**) is of particular interest to a navigator because it is the principal entering argument for the almanacs.

One complete revolution of the earth with respect to a celestial reference point is called a **day.** In modern usage every kind of solar time has its zero or starting point at **midnight,** when the celestial reference point is directly over the *lower branch* of the

terrestrial reference meridian. This has not always been so. Until January 1, 1925, the **astronomical day** began at *noon*, 12 hours *later* than the start of the calendar day of the same date. The **nautical day** began at *noon*, 12 hours *earlier* than the calendar day, or 24 hours earlier than the astronomical day of the same date. The **sidereal day** begins at **sidereal noon,** when the first point of Aries is over the *upper* branch of the reference meridian. There is no sidereal date.

1903. Expressing time.—Time is customarily expressed in time units, from 0^h through 24^h. To the nearest 1^m it is generally stated by navigators in a four-digit unit without punctuation. Thus, 0000 is midnight at the start of the day. One minute later the time is 0001. Half an hour after the start of the day the time is 0030, at one hour the time is 0100, at one hour and four minutes it is 0104, at 19 minutes after noon (solar time) it is 1219, at four hours and 23 minutes after (solar) noon it is 1623, etc. The term "hours" is sometimes used with the four-digit system to indicate that the number refers to the time or "hour" of the day. However, in those few occasions when any reasonable doubt may exist as to whether time is indicated, the fact can better be indicated in another way. Thus, the expression "1600 hours" to indicate "1600" or "16 hours" is not strictly correct, and is better avoided. **Watch time (W),** indicated by a watch or clock having a 12-hour dial, and **chronometer time** (C) are expressed on a 12-hour basis, with designations AM (ante meridian) and PM (post meridian), as in ordinary civil life ashore.

In contrast, a time interval is expressed as hours and minutes, as 5^h26^m. When either the time of day or a time interval is given to seconds, this same form is used, as $21^h15^m18^s$. The kind of time may be indicated, usually by abbreviation.

When a time interval is to be added to or subtracted from a time, the solution can be arranged conveniently in tabular form.

Example 1.—What is the time and date $14^h36^m53^s$ after $21^h14^m18^s$ on July 24?

Solution.—

$$\begin{array}{ll} 21^h14^m18^s & \text{July 24} \\ \underline{14^h36^m53^s} & \\ 35^h51^m11^s & \text{July 24} \\ =11^h51^m11^s & \text{July 25} \end{array}$$

The fact that the sum of hours exceeds 24 is an indication that the date increases by one. Similarly, in *subtracting* an interval, the date is one day earlier if 24^h must be added to the time before the subtraction can be made. That is, since 2400 of one day is 0000 of the following day, one might say that 2700 on one day is $2700-2400=0300$ on the following day. In the example above, $11^h51^m11^s$ on July 25 is the same as $11^h51^m11^s+24^h00^m00^s=35^h51^m11^s$ on July 24.

Date is sometimes expressed as an additional unit of the time sequence. Thus, $21^h14^m18^s$ on July 24 might be stated $24^d21^h14^m18^s$. This system is of particular value when an interval of several days is to be added or subtracted.

Example 2.—What is the time and date $9^d16^h35^m04^s$ before $5^h11^m33^s$ on September 15?

Solution.—

$$\begin{array}{l} 15^d05^h11^m33^s \\ \underline{9^d16^h35^m04^s} \\ 5^d12^h36^m29^s \text{ or } 12^h36^m29^s \text{ on Sept. 5.} \end{array}$$

By this method the month and day, if of significance, are recorded separately, or they, too, can be added to the sequence.

Example 3.—What is the time and date 3 years, 6 months, 25 days, 12 hours, 19 minutes, and 44 seconds after $7^h52^m24^s$ on November 14, 1958?

Solution.—

$$\begin{array}{r} 1958^y11^m14^d07^h52^m24^s \\ 3^y06^m25^d12^h19^m44^s \\ \hline 1962^y06^m08^d20^h12^m08^s \end{array} = 20^h12^m08^s \text{ on June 8, 1962.}$$

Since a month may contain a variable number of days, both the months and days should be solved together. Thus, in the example above, the answer would be 17 months, 39 days. If 12 months are converted to one year, this becomes five months, 39 days. Since the fifth month is May, this might be stated as May 39. Since there are 31 days in May, this is 39−31=8 days into the next month, or June 8.

A simpler method of determining the number of elapsed *days* between any two dates is to use the **Julian day** of each date, if the information is available. This also eliminates possible error due to change of calendar if long intervals are involved. The Julian day is the consecutive number of the day starting at 1200 on January 1, 4713 BC. Julian day is listed in the *American Ephemeris and Nautical Almanac.*

1904. Time and arc.—The time of day is an indication of the interval since the day began. One day represents one complete rotation of 360° of the earth with respect to a selected celestial point. Each day is divided into 24 **hours** of 60 **minutes,** each minute having 60 **seconds.** Thus, each day has $24\times60=1{,}440$ minutes or $1{,}440\times60=86{,}400$ seconds. This is time regardless of the celestial reference point used, and since the various references are in motion with respect to each other, as "seen" from the earth, apparent solar, mean solar, and sidereal days are of different lengths. Since they all have the same number and kind of fractional parts, these parts are themselves of different length in the different kinds of time. Mean solar units are customarily used to indicate time intervals. The smallest unit normally used in celestial navigation is the second, but in some electronic equipment the **millisecond** (one-thousandth of a second), **microsecond** (one-millionth of a second), and the **millimicrosecond** or **nanosecond** (one-billionth of a second) are used.

Time of day is an indication of the *phase* of rotation of the earth. That is, it indicates how much of a day has elapsed, or what part of a rotation has been completed. Thus, at zero hours the day begins. One hour later, the earth has turned through 1/24 of a day, or 1/24 of 360°, or $\frac{360°}{24}=15°$. Six hours after the day begins, it has turned through $6/24=1/4$ day, or $\frac{360°}{4}=90°$. Twelve hours after the start of the day, the day is half gone, having turned through 180°. Smaller intervals can also be stated in angular units, for since one hour or 60 minutes is equivalent to 15°, one minute of time is equivalent to $\frac{15°}{60}=0.°25=15'$, and one second of time is equivalent to $\frac{15'}{60}=0.'25=15''$. Thus,

$$\begin{array}{rrl} & \textit{Time} & \textit{Arc} \\ 1^d= & 24^h= & 360°=1 \text{ circle} \\ 60^m= & 1^h= & 15° \\ & 4^m= & 1°=60' \\ 60^s= & 1^m= & 15' \\ & 4^s= & 1'=60'' \\ & 1^s= & 15''=0.'25 \end{array}$$

Any time interval can be expressed as an angle of rotation, and vice versa. Interconversion of these units can be made by the relationships indicated above.

To convert time to arc:

1. Multiply the hours by 15 to obtain degrees.

2. Divide the minutes of time by four to obtain degrees, and multiply the remainder by 15 to obtain minutes of arc.

3. Divide the seconds of time by four to obtain minutes and tenths of minutes of arc, or multiply the remainder by 15 to obtain seconds of arc.

4. Add degrees, minutes, and tenths (or seconds).

Example 1.—Convert $14^h21^m39^s$ to arc units.

Solution.—

(1) $14^h \times 15 = 210°$
(2) $21^m \div 4 = \quad 5°15'$ (remainder $1^m \times 15 = 15'$)
(3) $39^s \div 4 = \quad 9'45''$ (remainder $3^s \times 15 = 45''$)
(4) $14^h21^m39^s = 215°24'45'' = 215°24.'8$ (to the nearest $0.'1$).

To convert arc to time:

1. Divide the degrees by 15 to obtain hours, and multiply the remainder by four to obtain minutes of time.

2. Divide the minutes of arc by 15 to obtain minutes of time, and multiply the remainder by four to obtain seconds of time.

3. Divide the seconds of arc by 15 to obtain seconds of time.

4. Add hours, minutes, and seconds.

Example 2.—Convert $215°24'45''$ to time units.

Solution.—

(1) $215° \div 15 = 14^h20^m$ (remainder $5° \times 4 = 20^m$)
(2) $24' \div 15 = \quad 1^m36^s$ (remainder $9' \times 4 = 36^s$)
(3) $45'' \div 15 = \quad 3^s$
(4) $215°24'45'' = 14^h21^m39^s$

Example 3.—Convert $161°53.'7$ to time units.

Solution.—

(1) $161° \div 15 = 10^h44^m$ (remainder $11° \times 4 = 44^m$)
(2) $53.'7 \div 15 = \quad 3^m34.^s8$ (remainder $8.'7 \times 4 = 34.^s8$)
(3) $161°53.'7 = 10^h47^m34.^s8 = 10^h47^m35^s$.

The navigator should be able to make these solutions mentally, writing only the answer. As a check, the answer can be converted back to the original value. Solution can also be made by means of arc to time tables in the almanacs. In the *Nautical Almanac* the table, given near the back of the volume (app. V), is in two parts, permitting separate entries with degrees, minutes, and quarter minutes of arc. The table is arranged in this manner because the navigator is confronted with the problem of converting arc to time more often than the reverse.

Example 4.—Convert $334°18'22''$ to time units, using the *Nautical Almanac* arc to time conversion table.

Solution.—

$$334° = 22^h16^m$$
$$18.'25 = \quad 1^m13^s$$
$$334°18'22'' = 22^h17^m13^s$$

The $22''$ are converted to the nearest quarter minute of arc for solution to the nearest second of time. Interpolation can be used if more precise results are required, since exact relationships are tabulated in the *Nautical Almanac* conversion table.

Example 5.—Convert 83°29′.6 to time units, using the *Nautical Almanac* arc to time conversion table.

Solution.—

$$\begin{aligned} 83° &= 5^h32^m \\ 29'.6 &= \underline{\quad 1^m58^s.4} \\ 83°29'.6 &= 5^h33^m58^s.4 \end{aligned}$$

In this solution, 58ˢ.4 was obtained by eye interpolation in the quarter-minute part of the table.

Example 6.—Convert $17^h09^m42^s$ to arc units, using the *Nautical Almanac* arc to time conversion table.

Solution.—

$$\begin{aligned} 17^h08^m &= 257° \\ 1^m42^s &= \underline{\quad 25'.5} \\ 17^h09^m42^s &= 257°25'.5 \end{aligned}$$

A similar table appears near the back of the *Air Almanac* (app. W), but values are given only to 180°, and quarter minutes of arc are not included. For angles greater than 180°, subtract 180° and add 12^h to the result.

Example 7.—Convert 334°47′.2 to time units, using the *Air Almanac* arc to time conversion table.

Solution.—

$$\begin{aligned} 334°-180°=154° &= 10^h16^m \\ 47'.2 &= \underline{\quad 3^m09^s} \\ 154°47'.2 &= 10^h19^m09^s \\ 334°47'.2 &= 22^h19^m09^s \end{aligned}$$

Example 8.—Convert $15^h13^m18^s$ to time units, using the *Air Almanac* arc to time conversion table.

Solution.—

$$\begin{aligned} 15^h12^m-12^h=3^h12^m &= 48° \\ 1^m18^s &= \underline{\quad 19'.5} \\ 3^h13^m18^s &= 48°19'.5 \\ 15^h13^m18^s &= 228°19'.5 \end{aligned}$$

Because the almanac conversion tables are exact relationships, interpolation in them can be carried to any degree of precision desired without introducing an error.

1905. Time and longitude.—As indicated in the preceding article, time is a measure of rotation of the earth, and any given time interval can be represented by a corresponding angle through which the earth turns. Suppose the celestial reference point were directly over a certain reference of the earth. An hour later the earth would have turned through 15°, and the celestial reference would be directly over a meridian 15° farther west. Any difference of longitude is a measure of the angle through which the earth must rotate for the local time at the western meridian to become what it was at the eastern meridian before the rotation took place. Therefore, places to the eastward of an observer have *later* time, and those to the westward have *earlier* time, and the difference is exactly equal to the difference in longitude, expressed in time units. When a meridian other than the local meridian is used as the time reference, the difference in time of two places is equal to the difference of longitude of their time reference meridians.

1906. The date line.—Since time becomes later toward the east, and earlier toward the west, time at the lower branch of one's meridian is 12 hours earlier or later depending upon the direction of reckoning. A traveler making a trip around the world gains or loses an entire day. To prevent the *date* from being in error, and to provide a starting place for each day, a **date line** is fixed by international agreement. This line coincides with the 180th meridian over most of its length. In crossing this line, one alters his

date by one day. In effect, this changes his time 24 hours to compensate for the slow change during a trip around the world. Therefore, it is applied in the opposite direction to the change of time. Thus, if a person is traveling *eastward* from east longitude to west longitude, time is becoming *later*, and when the date line is crossed, the date becomes one day *earlier*. That is, at any moment the date immediately to the *west* of the date line (east longitude) is one day *later* than the date immediately to the *east* of the line, except at GMT 1200, when the (mean time) date is the same all over the world. At any other time two dates occur, one boundary between dates being the date line, and the other being the midnight line along the lower branch of the meridian over which the mean sun is located. At GMT 1200 these two boundaries coincide. In the solution of problems, error can sometimes be avoided by converting local time to Greenwich time, and then converting this to local time on the opposite side of the date line. Examples are given in following articles.

1907. Zone time.—At sea, as well as ashore, watches and clocks are normally set approximately to some form of **zone time** (**ZT**). At sea the *nearest* meridian exactly divisible by 15° is usually used as the **time meridian** or **zone meridian.** Thus, within a **time zone** extending 7°5 on each side of each time meridian the time is the same, and time in consecutive zones differs by exactly one hour. The time is changed as convenient, usually at a whole hour, near the time of crossing the boundary between zones. Each time zone is identified by the number of times the longitude of its zone meridian is divisible by 15°, positive in west longitude and negative in east longitude. This number and its sign, called the **zone description** (**ZD**), is the number of whole *hours* that are added to or subtracted from the zone time to obtain **Greenwich mean time** (**GMT**), which is the zone time at the Greenwich (0°) meridian, and is sometimes called **universal time** (**UT**). The mean sun is the celestial reference point for zone time.

Example 1.—For an observer at long. 141°18′4 W the ZT is $6^h18^m24^s$.

Required.—(1) Zone description.

(2) GMT.

Solution.—(1) The nearest meridian exactly divisible by 15° is 135° W, into which 15° will go nine times. Since longitude is west, ZD is (+) 9.

	ZT	$6^h18^m24^s$
	ZD (+)	9
(2)	GMT	$15^h18^m24^s$

In converting GMT to ZT, a positive ZD is *subtracted*, and a negative one *added*, but its sign remains the same, being part of the description. The word "reversed" (rev.) is written to the right in the work form to indicate that the "reverse" process is to be performed.

Example 2.—The GMT is $15^h27^m09^s$.

Required.—(1) ZT at long. 156°24′4 W.

(2) ZT at 39°04′8 E.

Solution.—

(1) GMT	$15^h27^m09^s$			(2) GMT	$15^h27^m09^s$	
ZD (+)	10	(rev.)		ZD (−)	3	(rev.)
ZT	$5^h27^m09^s$			ZT	$18^h27^m09^s$	

When time at one place is converted to that at another, the date should be watched carefully. If a sum exceeds 24 hours, subtract this amount and add one day. If 24 hours are added before a subtraction is made, the date at the place is one day *earlier*.

Example 3.—At long. 73°29′2 W the ZT is $21^h12^m53^s$ on May 14.

Required.—(1) GMT and date.

(2) ZT and date at long. 107°15′7 W.

Solution.—

	ZT	$21^h12^m53^s$ May 14	
	ZD	(+) 5	
(1)	GMT	$2^h12^m53^s$ May 15	
	ZD	(+) 7	(rev.)
(2)	ZT	$19^h12^m53^s$ May 14	

The second part of this problem might have been solved by using the *difference* in zone description. Since the second place is two zones farther west, its time is two hours earlier. Problems involving zone times at various places generally involve nothing more than addition or subtraction of one small number, so solutions can generally be made mentally. However, when this forms part of a larger problem, or when a record of the solution is desired, the full solution should be recorded, including labels.

Example 4.—On November 30 the 1430 DR long. of a ship is 51°32′.4 W. Ten hours later the DR long. is 53°07′.2 W.

Required.—ZT and date of arrival at the second longitude.

Solution.—

ZT	1430 Nov. 30	
ZD	(+) 3	
GMT	1730 Nov. 30	
int.	10	
GMT	0330 Dec. 1	
ZD	(+) 4	(rev.)
ZT	2330 Nov. 30	

If a time zone boundary had not been crossed, there would have been no need to find GMT. It is particularly helpful to retain this step when the date line is crossed. This line is the *center* of a time zone, the western (east longitude) half being designated (−) 12, and the eastern (west longitude) half (+) 12.

Example 5.—On December 31 the 0800 DR long. of a ship is 177°23′.9 E. Forty hours later the DR long. is 171°53′.9 W.

Required.—ZT and date of arrival at the second longitude.

Solution.—

			Alternative solution		
ZT	0800 Dec. 31		ZT	$31^d08^h00^m$	
ZD	(−) 12		ZD	(−) 12	
GMT	2000 Dec. 30		GMT	$30^d20^h00^m$	
int.	40		int.	1^d16^h	
GMT	1200 Jan. 1		GMT	$1^d12^h00^m$	
ZD	(+) 11	(rev.)	ZD	(+) 11	(rev.)
ZT	0100 Jan. 1		ZT	$1^d01^h00^m$	

For certain communication purposes it is sometimes convenient to designate a time zone by a single letter. The system used is shown in figure 1907.

Use of time zones on land began in 1883, when railroads adopted four standard zones for the continental United States. The division of the United States into time zones was not officially adopted by Congress, however, until March 19, 1918, when a fifth zone was also established for Alaska. The system of time zones is now used almost universally throughout the world, although on land the zone boundaries are generally altered somewhat for convenience. In a few places, half-hour zones are used but these are not standard time zones.

On land, normal zone time is usually called **standard time,** often with an adjective to indicate the zone, as **eastern standard time.** In some areas timepieces are *advanced* one or more hours during the summer to provide greater use of daylight. This "fast" time is called **daylight saving time** in the United States, and **summer time** elsewhere.

FIGURE 1907. Time zone chart of the world (from H.O. Chart No. 5192, 1958 edition).
THE LATEST EDITION OF THIS CHART SHOULD BE CONSULTED.

When time is one hour fast, the zone description is (algebraically) one *less* than normal. When daylight saving or summer time is specified, an advance of one hour is understood unless a greater number is indicated.

Example 6.—What is the standard time and date at Tokyo, long. 140° E, when the daylight saving time at Washington, long. 77° W, is 1600 on Oct. 5?

Solution.—

$$\begin{array}{rll}
\text{ZT} & 1600 & \text{Oct. 5} \\
\text{ZD} & (+)\,4 & \\ \hline
\text{GMT} & 2000 & \text{Oct. 5} \\
\text{ZD} & (-)\,9 & \text{(rev.)} \\ \hline
\text{ZT} & 0500 & \text{Oct. 6}
\end{array}$$

During hostilities daylight saving time may be kept all year long throughout a nation, and designated **war time (WT).**

1908. Chronometer time (C) is time indicated by a chronometer. Since a chronometer is set approximately to GMT, and not reset until it is overhauled and cleaned, perhaps three years later (art. 1514), there is nearly always a **chronometer error** (CE), either *fast* (F) or *slow* (S). The change in chronometer error in 24 hours is called **chronometer rate,** or **daily rate,** and designated *gaining* or *losing.* With a consistent rate of 1^{s} per day for three years, the chronometer error would be approximately 18^{m}. Since chronometer error is subject to change, it should be determined from time to time, preferably daily at sea. Chronometer error is found by radio time signal (art. 1909), by comparison with another timepiece of known error, or by applying chronometer rate to previous readings of the same instrument. It is recorded to the nearest whole or half second. Chronometer rate is recorded to the nearest $0^{s}.1$.

Example 1.—At GMT 1200 on May 12 the chronometer reads $12^{h}04^{m}21^{s}$. At GMT 1600 on May 18 it reads $4^{h}04^{m}25^{s}$.

Required.—(1) Chronometer error at both comparisons.

(2) Chronometer rate.

(3) Chronometer error at GMT 0530 on May 27.

Solution.—

$$\begin{array}{rl}
\text{GMT} & 12^{h}00^{m}00^{s}\ \text{May 12} \\
\text{C} & 12^{h}04^{m}21^{s} \\ \hline
\text{(1) CE} & \text{(F)}\ 4^{m}21^{s}
\end{array}$$

$$\begin{array}{rl}
\text{GMT} & 16^{h}00^{m}00^{s}\ \text{May 18} \\
\text{C} & 4^{h}04^{m}25^{s} \\ \hline
\text{(1) CE} & \text{(F)}\ 4^{m}25^{s}
\end{array}$$

$$\begin{array}{rl}
\text{GMT} & 12^{d}12^{h} \\
\text{GMT} & 18^{d}16^{h} \\ \hline
\text{diff.} & 6^{d}04^{h}=6^{d}.2
\end{array}$$

$$\begin{array}{rll}
\text{CE} & \text{(F)}\ 4^{m}21^{s}\ 1200\ \text{May 12} & \\
\text{CE} & \text{(F)}\ 4^{m}25^{s}\ 1600\ \text{May 18} & \\ \hline
\text{diff.} & 4^{s}\ \text{gained} & \\
\text{(2) daily rate} & 0^{s}.6\ \text{per day, gaining.} & (4^{s} \div 6^{d}.2)
\end{array}$$

$$\begin{array}{rl}
\text{GMT} & 18^{d}16^{h}00^{m} \\
\text{GMT} & 27^{d}05^{h}30^{m} \\ \hline
\text{diff.} & 8^{d}13^{h}30^{m}=8^{d}.5
\end{array}$$

$$\begin{array}{rl}
\text{CE} & \text{(F)}\ 4^{m}25^{s}\ 1600\ \text{May 18} \\
\text{corr.} & (+)\ 5^{s}\ (8^{d}.5 \times 0^{s}.6\ \text{per day}) \\ \hline
\text{(3) CE} & \text{(F)}\ 4^{m}30^{s}\ 0530\ \text{May 27}
\end{array}$$

Because GMT is stated on a 24-hour basis, and chronometer time on a 12-hour basis, a 12-hour ambiguity exists. This is ignored in finding chronometer error. However, if chronometer error is applied to chronometer time to find GMT, a possible 12-hour error can result. This can be resolved by mentally applying zone description to local time to obtain approximate GMT. A time diagram can be used for resolving doubt as to approximate GMT and Greenwich date. If the sun for the kind of time used (mean or apparent) is between the *lower* branches of two time meridians (as the standard meridian for local time, and the Greenwich meridian for GMT), the date at the place farther *east* is one day *later* than at the place farther *west*.

Example 2.—On August 14 the DR long. of a ship is about 124° E, and the zone time is about 0500. Chronometer error is 12^m27^s slow.

Required.—GMT and date when the chronometer reads $8^h44^m22^s$.

Solution.—

approx.	ZT	0500 Aug. 14
	ZD	(−)8
approx.	GMT	2100 Aug. 13
	C	$8^h44^m22^s$
	CE	(S) 12^m27^s
	GMT	$20^h56^m49^s$ Aug. 13

The *A* chronometer, usually the best (having the most nearly uniform rate), is compared directly with the time signal (art. 1909). Other chronometers, designated *B*, *C*, etc., may then be compared with the *A* chronometer.

Example 3.—At GMT 1400 chronometer *A* is checked by time signal, and found to read $1^h57^m09^s$. A little later, when it reads $2^h05^m00^s$, chronometer *B* reads $2^h11^m38^s$.

Required.—(1) Error of chronometer *A*.

(2) Error of chronometer *B*.

Solution.—

GMT	$14^h00^m00^s$
C_A	$1^h57^m09^s$
(1) CE_A	(S) 2^m51^s
C_A	$2^h05^m00^s$
GMT	$14^h07^m51^s$
C_B	$2^h11^m38^s$
(2) CE_B	(F) 3^m47^s

If time signals are not available at the chronometer, a good comparing watch (art. 1515) should be compared with the radio signal, and this watch used to determine chronometer error, as indicated in example 3, substituting the watch for chronometer *A*.

1909. Time signals.—The usual method of determining chronometer error and daily rate is by radio **time signals,** popularly called **time ticks.** Most maritime nations broadcast time signals several times daily from one or more stations, and a vessel equipped with radio receiving equipment normally has no difficulty in obtaining a time tick anywhere in the world. The times of emission of signals transmitted by the U.S. are the same to about $0\overset{s}{.}001$ as those of Argentina, Australia, Canada, Japan, U.K., Republic of South Africa, and Switzerland. The time transmitted is maintained virtually uniform with respect to atomic clocks but follows GMT closely. The difference seldom amounts to $0\overset{s}{.}050$. The time, as received by a vessel, may be considered to be GMT to $0\overset{s}{.}1$. *Radio Navigational Aids*, H.O. Pubs. Nos. 117–A and 117–B, lists all time signals, together with their hours of transmission, system used, frequency, and other useful information.

At sea the chronometer should be checked daily by radio time signal, and in port daily checks should be maintained, or begun at least three days prior to departure, if conditions permit. Error and rate are entered in the chronometer record book (or record sheet) each time they are determined.

Prior to the development of radio time signals, chronometers were checked in port by **visual signals** which consisted of dropping a **time ball** or shape by telegraphic action, or firing a gun. Visual signals are still used in some ports. If a gun is used, the flash marks the correct time, as the report may not reach the observer until several seconds after the gun is fired.

The various time signal systems used throughout the world are explained in detail in H.O. Pubs. Nos. 117–A and B. Only the United States signals are discussed here.

The U. S. Naval Observatory at Washington, D. C., controls the transmissions of time signals from U. S. Naval radio stations. Beginning at 5 minutes before each even hour of GMT, dashes are transmitted on every second, except the 29th and certain others near the end of each minute, as shown in the following diagram:

Minutes	Seconds										
	50	51	52	53	54	55	56	57	58	59	60
55	—		—	—	—	—					—
56	—	—		—	—	—					—
57	—	—	—		—	—					—
58	—	—	—	—		—					—
59	—										——

The seconds marked "60" indicate the start of the next minute. The final dash, marking the hour, is considerably longer than any of the others. The number of dashes in the group near the end of any minute indicates the number of minutes before the hour. This is known as the **United States system.** In all cases the beginnings of the dashes indicate the beginning of the seconds, and the ends of the dashes are without significance.

Station WWV, near Washington, D. C., broadcasts continuous time signals obtained from the U. S. Naval Observatory time service. Station WWVH in Hawaii broadcasts the same signals, except for certain periods during which the station is off the air to compare its standards with those of WWV and to obtain ionospheric soundings. The signals broadcast by these stations are intended primarily for measurement of time *intervals*, and checking of frequencies of two standard audible tones, but signals can also be used for checking time. The system used is fully explained in H.O. Pubs. Nos. 117–A and 117–B.

1910. Watch time (**W**) is time indicated by a watch. This is usually an approximation of zone time, except that for timing celestial observations it is good practice to set a comparing watch (art. 1515) to GMT. If the watch has a second setting hand, the watch can be set exactly to ZT or GMT, and the time is so designated. If the watch is not set exactly to one of these times, the difference is known as **watch error** (**WE**), labeled *fast* (F) or *slow* (S) to indicate whether the watch is ahead of or behind the correct time, respectively.

If a watch is to be set exactly to ZT or GMT, it is set to some whole minute slightly ahead of the correct time, and stopped. When the set time arrives, the watch is started. It should then be checked for accuracy.

Example 1.—A chronometer 9^m46^s fast on GMT reads approximately 7^h23^m. At the next whole five minutes of GMT a comparing watch is to be set to GMT exactly.

Required.—(1) What should the watch read at the moment of starting?
(2) What should the chronometer read?
Solution.—

$$\begin{array}{lrl}
 & \text{C} & 7^h23^m00^s \\
 & \text{CE} & \text{(F) } 9^m46^s \\
 & \text{GMT} & 7^h13^m14^s \\
(1) & \text{GMT} & 7^h15^m00^s \text{ (next whole } 5^m) \\
 & \text{CE} & \text{(F) } 9^m46^s \\
(2) & \text{C} & 7^h24^m46^s
\end{array}$$

The GMT may be in error by 12^h, but if the watch is graduated to 12 hours, this will not be reflected. If a watch with a 24-hour dial is used, the actual GMT should be determined.

If watch error is to be determined, it is done by comparing the reading of the watch with that of the chronometer at a selected moment. This may be at some selected GMT, as in example 1.

Example 2.—If, in example 1, the watch had read $7^h14^m48^s$ at the moment the chronometer read $7^h24^m46^s$, what would be the watch error on GMT?

Solution.—

$$\begin{array}{rl}
\text{GMT} & 7^h15^m00^s \\
\text{W} & 7^h14^m48^s \\
\text{WE} & \text{(S) } 12^s
\end{array}$$

A more convenient chronometer time might be selected, as a whole minute. .

Example 3.—A watch is set to zone time approximately. The longitude is about 48° W. The watch is compared with a chronometer which is 19^m44^s fast on GMT. When the chronometer reads $5^h22^m00^s$, the watch reads $2^h01^m53^s$.

Required.—Watch error on zone time.

Solution.—

$$\begin{array}{rll}
\text{C} & 5^h22^m00^s & \\
\text{CE} & \text{(F) } 19^m44^s & \\
\text{GMT} & 5^h02^m16^s & \\
\text{ZD} & (+)\,3 & \text{(rev.)} \\
\text{ZT} & 2^h02^m16^s & \\
\text{W} & 2^h01^m53^s & \\
\text{WE} & \text{(S) } 23^s &
\end{array}$$

The possible 12^h error is not of significance. When such a watch is used for determining GMT, however, as for entering an almanac, the 12-hour ambiguity is important. Unless a watch is graduated to 24 hours, its time is designated AM before noon and PM after noon.

Example 4.—On January 3 the DR long. is 94°14.′7 E. An observation of the sun is made when the watch reads $12^h16^m23^s$ PM. The watch is 22^s fast on zone time.

Required.—GMT and date.

Solution.—

$$\begin{array}{rl}
\text{W} & 12^h16^m23^s \text{ PM Jan. 3} \\
\text{WE} & \text{(F) } 22^s \\
\text{ZT} & 12^h16^m01^s \\
\text{ZD} & (-)\,6 \\
\text{GMT} & 6^h16^m01^s \text{ Jan. 3}
\end{array}$$

Note that between 1200 and 1300 watch designations are PM. Between 0000 and 0100 they are AM.

Comparison of a watch and chronometer should be made carefully. If two observers are available, one can give a warning "stand-by" a few seconds before the selected time, and a "mark" at the appointed moment, while the other notes the time of the watch. A single observer can make a satisfactory comparison by counting with the chronometer. Chronometers beat in half seconds, with an audible "tick." Ten seconds before the selected time (perhaps a whole minute), the observer starts counting with the beats, as he watches the chronometer second hand, "50, and, 1, and, 2, and, 3, and, ... 9, and, mark." During the count the observer shifts his view from the chronometer to the second hand of the watch, continuing to count in cadence with the chronometer beats. At the "mark," the second, minute, and hour hands of the watch are read in that order, and the time recorded. A comparison of this time with the GMT or ZT corresponding to the selected chronometer time indicates the watch error.

Even though a watch is set to zone time approximately, its error on GMT can be determined and used for timing observations. In this case the 12-hour ambiguity in GMT should be resolved, and a time diagram used to avoid possible error. This method requires additional work, and presents a greater probability of error, without compensating advantages.

Still another method of determining GMT, generally used before zone time came into common use at sea, is to *subtract* watch time from chronometer time, to find C–W. This is then *added* to the watch time of an observation to obtain chronometer time (C−W+W=C). Chronometer error is then applied to the result to obtain GMT. A time diagram should *always* be used with this method, to resolve the 12-hour ambiguity and to be sure of the correct Greenwich date, unless an auxiliary solution is made using approximate ZT and ZD. This method has little to recommend it.

If a watch has a **watch rate** of more than a few seconds per day, watch error should be determined both before and after a round of sights, and any difference distributed proportionally among observations.

If a stop watch is used for timing observations, it should be started at some convenient GMT, as a whole 5^m or 10^m. The time of each observation is then this GMT *plus* the reading of the watch.

1911. Local mean time (**LMT**), like zone time, uses the mean sun as the celestial reference point. It differs from zone time in that the local meridian is used as the terrestrial reference, rather than a zone meridian. Thus, the local mean time at each meridian differs from that of every other meridian, the difference being equal to the difference of longitude, expressed in time units. At each zone meridian, including 0°, LMT and ZT are identical.

Example 1.—At long. 124°37′.2 W the LMT is $17^h24^m18^s$ on March 21.

Required.—(1) GMT and date.

(2) ZT and date at the place.

Solution.—

LMT	$17^h24^m18^s$	Mar. 21
λ	$8^h18^m29^s$ W	
(1) GMT	$1^h42^m47^s$	Mar. 22
ZD (+) 8		(rev.)
(2) ZT	$17^h42^m47^s$	Mar. 21

In navigation the principal use of LMT is in rising, setting, and twilight tables. The problem is usually one of converting the LMT taken from the table to ZT. At sea, the difference between these times is normally not more than 30^m, and the conver-

sion is made directly, without finding GMT as an intermediate step. This is done by applying a correction equal to the difference of longitude (dλ). If the observer is *west* of his time meridian, the correction is *added*, and if *east* of it, the correction is *subtracted*. If Greenwich time is desired, it is found from ZT.

Example 2.—At long. 63°24′.4 E the LMT is 0525 on January 2.

Required.—(1) ZT and date.

(2) GMT and date.

Solution.—

LMT	0525	Jan. 2
dλ	(−) 14	
(1) ZT	0511	Jan. 2
ZD	(−) 4	
(2) GMT	0111	Jan. 2

On land, with an irregular zone boundary, the longitude may differ by more than 7°.5 (30^m) from the time meridian.

If LMT is to be corrected to daylight saving time, the difference in longitude between the local and time meridian can be used, or the ZT can first be found and then increased by one hour.

Conversion of ZT (including GMT) to LMT is the same as conversion in the opposite direction, except that the sign of dλ is reversed. This problem is not normally encountered in navigation.

1912. Apparent time utilizes the **apparent** (real) **sun** as its celestial reference, and a meridian as the terrestrial reference. **Local apparent time** (**LAT**) uses the local meridian. The LAT at the 0° meridian is called **Greenwich apparent time** (**GAT**).

The LAT at one meridian differs from that at any other by the difference in longitude of the two places, the place to the eastward having the later time, and conversion is the same as converting LMT at one place to LMT at another.

Use of the apparent sun as a celestial reference point for time results in time of nonconstant rate for at least three reasons. First, revolution of the earth in its orbit is not constant. Second, motion of the apparent sun is along the ecliptic, which is tilted with respect to the celestial equator, along which time is measured. Third, rotation of the earth on its axis is not constant. The effect due by this third cause is extremely small.

For the various forms of mean time, the apparent sun is replaced by a fictitious **mean sun** conceived as moving eastward along the *celestial equator* at a uniform speed equal to the average speed of the apparent sun along the ecliptic, thus providing a nearly uniform measure of time equal to the approximate average apparent time. At any moment the accumulated difference between LAT and LMT is indicated by the **equation of time** (**Eq. T**), which reaches a maximum value of about $16^m.4$ in November. This quantity is tabulated at 12-hour intervals at the bottom of the right-hand daily page of the *Nautical Almanac*. In the United States, the sign is considered positive (+) if the time of sun's "Mer. Pass." is earlier than 1200, and negative (−) if later than 1200. If the "Mer. Pass." is given as 1200 (as on June 12–14, 1958), the sign is positive if the GHA at GMT 1200 is between 0° and 1°, and negative if it is greater than 359°. The sign is correct for conversion of GMT to GAT. In Great Britain, this convention is reversed. Since GMT is the entering argument for the almanacs, interconversion of apparent and mean time should preferably be made from Greenwich time, rather than from local time.

Example.—Find the LAT and date at ZT $15^h10^m40^s$ on May 31, 1958, for long. 73°18′.4 W.

Solution.—

$$\begin{array}{rl}
\text{ZT} & 15^h10^m40^s \text{ May } 31 \\
\text{ZD } (+) & \underline{5} \\
\text{GMT} & 20^h10^m40^s \text{ May } 31 \\
\text{Eq. T} & \underline{(+)2^m27^s} \\
\text{GAT} & 20^h13^m07^s \text{ May } 31 \\
\lambda & \underline{4^h53^m14^s \text{W}} \\
\text{LAT} & 15^h19^m53^s \text{ May } 31
\end{array}$$

In conversion from apparent to mean time, a second solution may be needed if the equation of time is large and changing rapidly, using the GAT for entering the almanac for the first solution, and using the GMT from this solution as the almanac entry value for the second solution.

Apparent time can also be found by converting hour angle to time units, and adding or subtracting 12 hours. If LAT is required, but not GAT, conversion of arc to time should be made from LHA, rather than GHA, to avoid the need for conversion of longitude to time units. Equation of time can be found by *subtracting* mean time from apparent time at the same meridian. This method of finding apparent time and equation of time is the only one available with the *Air Almanac*, which does not tabulate equation of time.

The navigator has little or no use for apparent time, as such. However, it can be used for finding the time of **local apparent noon (LAN),** when the apparent sun is on the celestial meridian.

The mean sun averages out the irregularities in time due to the variations of the speed of revolution of the earth in its orbit and the fact that the apparent sun moves in the ecliptic while hour angle is measured along the celestial equator. It does not eliminate the error due to slight variations in the *rotational* speed of the earth. When a correction for the accumulated error from this source is applied to mean time, **ephemeris time** results. This time is of interest to astronomers, but is not used directly by the navigator.

1913. Sidereal time uses the first point of Aries (vernal equinox) as the celestial reference point. Since the earth revolves around the sun, and since the direction of the earth's rotation and revolution are the same, it completes a rotation with respect to the stars in less time (about $3^m56\overset{s}{.}6$ of mean solar units) than with respect to the sun, and during one revolution about the sun (one year) it makes one complete rotation more with respect to the stars than with the sun. This accounts for the daily shift of the stars nearly 1° westward each night. Hence, sidereal days are shorter than solar days, and its hours, minutes, and seconds are correspondingly shorter. Because of nutation (art. 1417) sidereal time is not quite constant in rate. Time based upon the average rate is called **mean sidereal time,** when it is to be distinguished from the slightly irregular sidereal time. The ratio of mean solar time units to mean sidereal time units is 1:1.00273791.

The sidereal *day* begins when the first point of Aries is over the *upper* branch of the meridian, and extends through 24 hours of sidereal time. The sun is at the first point of Aries at the time of the vernal equinox, about March 21. However, since the solar day begins when the sun is over the *lower* branch of the meridian, apparent solar and sidereal times differ by 12 hours at the vernal equinox. Each month thereafter, sidereal time *gains* about two hours on solar time. By the time of the summer solstice, about June 21, sidereal time is 18 hours *ahead* or six hours *behind* solar time. By the time of the autumnal equinox, about September 23, the two times are together, and by the time of the winter solstice, about December 22, the sidereal time is six hours *ahead* of solar time. There need be no confusion of the date, for there is no sidereal date.

Local sidereal time (LST) uses the local meridian as the terrestrial reference. At the prime meridian this is called **Greenwich sidereal time (GST).** The difference between LST at two meridians is equal to the difference of longitude between them, the place to the eastward having the later time. Local sidereal time is LHA ♈ expressed in time units. To determine LST at any given moment, find GHA ♈ by means of an almanac, and then apply the longitude to convert it to LHA ♈. Then convert LHA ♈ in arc to LST in time units.

Example.—Find LST at ZT $8^h25^m51^s$ on May 31, 1958, for long. 103°16′.3 E.

Solution.—

ZT	$8^h25^m51^s$ May 31
ZD	(−) 7
GMT	$1^h25^m51^s$ May 31
1^h	263°01′.6
25^m51^s	6°28′.8
GHA ♈	269°30′.4
λ	103°16′.3 E
LHA ♈	12°46′.7
LST	$0^h51^m07^s$

Unless GST is required, conversion from arc to time units should be made from LHA ♈, rather than from GHA ♈, to avoid the need for converting longitude from arc to time units.

Conversion of sidereal to solar time is the reverse. Local sidereal time is converted to arc (LHA ♈), and the longitude is applied to find GHA ♈. This is used as an argument for entering the almanac to determine GMT, which can then be converted to any other kind of time desired. This is similar to one method of finding time of meridian transit, described in article 2104. Normally, the problem is not encountered by the navigator.

Sidereal time, as such, is little used by the navigator. It is the basis of star charts (art. 2204) and star finders (art. 2210), and certain sight reduction methods (notably H.O. Pub. No. 249), but generally in the form LHA ♈. This kind of time is used for these purposes because its celestial reference point remains almost fixed in relation to the stars. Sidereal time is used by astronomers to regulate mean time. Timepieces regulated to sidereal time can be purchased.

1914. Time and hour angle.—Both time and hour angle are a measure of the phase of rotation of the earth, since both indicate the angular distance of a celestial reference point *west* of a terrestrial reference meridian. Hour angle, however, applies to *any* point on the celestial sphere. Time might be used in this respect, but only the apparent sun, mean sun, the first point of Aries, and occasionally the moon are commonly used.

Hour angles are usually expressed in arc units, and are measured from the upper branch of the celestial meridian. Time is customarily expressed in time units. Sidereal time is measured from the upper branch of the celestial meridian, like hour angle, but solar time is measured from the lower branch. Thus, LMT = LHA mean sun plus or minus 180°, LAT = LHA apparent sun plus or minus 180°, and LST = LHA ♈.

As with time, **local hour angle (LHA),** based upon the local celestial meridian, at two places differs by the longitude between them, and LHA at longitude 0° is called **Greenwich hour angle (GHA).** In addition, it is often convenient to express hour angle in terms of the *shorter* arc between the local celestial meridian and the body. This is similar to measurement of longitude from the Greenwich meridian. Local hour angle measured in this way is called **meridian angle (t),** which is labeled *east* or *west*, like longitude, to indicate the direction of measurement. A westerly meridian angle is numerically equal to LHA, while an easterly meridian angle is equal to 360° − LHA; also, LHA = t (W), and LHA = 360° − t (E). Meridian angle is used in the solution of the navigational triangle (art. 1433).

Example 1.—Find LHA and t of the sun at GMT $3^h24^m16^s$ on June 1, 1958, for long. 118°48′.2 W.

Solution.—

GMT	$3^h24^m16^s$ June 1
3^h	225°36′.0
24^m16^s	6°04′.0
GHA	231°40′.0
λ	118°48′.2 W
LHA	112°51′.8
t	112°51′.8 W

Example 2.—Find LHA and t of Kochab at ZT $18^h24^m47^s$ on May 31, 1958, for long. 55°27′.3 W.

Solution.—

	Kochab	
ZT	$18^h24^m47^s$	May 31
ZD	(+) 4	
GMT	$22^h24^m47^s$	May 31
22^h	218°53′.4	
24^m47^s	6°12′.8	
SHA	137°17′.2	
GHA	2°23′.4	
λ	55°27′.3 W	
LHA	306°56′.1	
t	53°03′.9 E	

Problems

1903a. What is the time and date $9^h13^m29^s$ before $3^h16^m34^s$ May 9?

Answer.—T $18^h03^m05^s$ May 8.

1903b. What is the time and date $4^d19^h22^m50^s$ after $9^h31^m04^s$ on December 25?

Answer.—T $4^h53^m54^s$ on Dec. 30.

1903c. What is the time and date 2 years, 11 months, 16 days, 10 hours, 23 minutes, and 48 seconds before $2^h46^m17^s$ on October 4, 1958?

Answer.—T $16^h22^m29^s$ on Oct. 17, 1955.

1903d. What is the time and date 412 days, 15 hours, 6 minutes, and 56 seconds after $22^h27^m03^s$ on March 16, 1958?

Answer.—T $13^h33^m59^s$ on May 3, 1959.

1904a. Convert $6^h28^m31^s$ to arc units, without use of a conversion table.

Answer.—97°07′45″ or 97°07′.8.

1904b. Convert 217°28′.8 to time units, without use of a conversion table.

Answer.—$14^h29^m55^s.2$ or $14^h29^m55^s$.

1904c. Convert 196°21′46″ to time units, without use of a conversion table.

Answer.—$13^h05^m27^s.1$ or $13^h05^m27^s$.

1904d. Convert 107°49′44″ to time units, using appendix V.

Answer.—$7^h11^m19^s$.

1904e. Convert 211°37′.3 to time units, using appendix V.

Answer.—$14^h06^m29^s.2$.

1904f. Convert $8^h49^m33^s$ to arc units, using appendix V.

Answer.—132°23′.2.

1904g. Convert 251°09′.2 to time units, using appendix W.

Answer.—$16^h44^m37^s$.

1904h. Convert $23^h07^m38^s$ to time units, using appendix W.

Answer.—346°54′.5.

1907a. For an observer at long. 97°24′.6 E the ZT is $19^h10^m26^s$.

Required.—(1) Zone description.

(2) GMT.

Answers.—(1) ZD (−) 6, (2) GMT $13^h10^m26^s$.

1907b. The GMT is $11^h32^m07^s$.

Required.—(1) ZT at long. 133°24′.7 W.

(2) ZT at long. 111°43′.9 E.

Answers.—(1) ZT $2^h32^m07^s$, (2) ZT $18^h32^m07^s$.

1907c. At long. 165°18′.2 E the ZT is $17^h08^m51^s$ on July 11.

Required.—(1) GMT and date.

(2) ZT and date at long. 125°36′.7 W.

Answers.—(1) GMT $6^h08^m51^s$ on July 11, (2) ZT $22^h08^m51^s$ on July 10.

1907d. On January 26 the 0800 DR long. of a ship is 128°03′.2 E. Twenty-six hours later the EP long. is 125°01′.4 E.

Required.—ZT and date of arrival at the second longitude.

Answer.—ZT 0900 Jan. 27.

1907e. On April 1 the 1200 running fix long. of a ship is 179°55′.2 W. Eight hours later the DR long. is 178°48′.9 E.

Required.—ZT and date of arrival at the second longitude.

Answer.—ZT 2000 Apr. 2.

1907f. Inch'ŏn, long. 137° E, uses ZD (−) 8^h30^m for standard time. Find the standard time and date at San Francisco, long. 122° W, when the summer time at Inch'ŏn is 2000 on August 9.

Answer.—ZT 0230 Aug. 9.

1908a. At GMT 1400 on July 2 the chronometer reads $1^h42^m28^s$. At GMT 0800 on July 12 it reads $7^h42^m40^s$.

Required.—(1) Chronometer error at GMT 1400 on July 2.

(2) Chronometer error at GMT 0800 on July 12.

(3) Chronometer rate.

(4) Chronometer time at ZT 1800 July 20, at long. 153°21′.7 W.

Answers.—(1) CE 17^m32^s slow, (2) CE 17^m20^s slow, (3) rate $1^s.2$ gaining, (4) C $3^h42^m51^s$.

1908b. On March 5 the DR long. of a ship is about 151° E, and the zone time is about 1800. Chronometer error is 6^m40^s fast.

Required.—GMT and date when the chronometer reads $8^h02^m23^s$.

Answer.—GMT $7^h55^m43^s$ on Mar. 5.

1908c. On November 7 the EP long. of a ship is about 71° W, and the zone time is about 1900. Chronometer error is 1^m18^s slow.

Required.—GMT and date when the chronometer reads (1) $11^h55^m20^s$, (2) $11^h59^m50^s$.

Answers.—(1) GMT $23^h56^m38^s$ Nov. 7, (2) GMT $0^h01^m08^s$ Nov. 8.

1908d. At GMT 2200 a comparing watch is checked by time signal, and found to read $10^h00^m05^s$. The chronometer errors are then determined by means of the comparing watch. When the watch reads $10^h06^m00^s$, chronometer *A* reads $10^h11^m17^s$, and when the watch reads $10^h08^m00^s$, chronometer *B* reads $9^h59^m06^s$.

Required.—(1) Watch error.

(2) Error of chronometer *A*.

(3) Error of chronometer *B*.

Answers.—(1) WE 5^s fast on GMT, (2) CE_A 5^m22^s fast, (3) CE_B 8^m49^s slow.

1910a. A chronometer 7^m22^s slow on GMT reads approximately 3^h45^m. About two minutes later, when the GMT is a whole minute, a comparing watch will be set to GMT exactly.

Required.—(1) Reading of the watch at starting.

(2) Reading of the chronometer.

Answers.—(1) W $3^h54^m00^s$, (2) C $3^h46^m38^s$.

1910b. A chronometer 5^m10^s fast on GMT reads approximately 5^h50^m. About one minute later, when the GMT is a whole minute, a comparing watch with a 24-hour dial will be set to GMT exactly. The ZT is approximately 1145 and the long. 94° W.

Required.—(1) Reading of the watch at starting.

(2) Reading of the chronometer.

(3) Watch error if, instead of being set to GMT, the watch setting is unchanged and the watch reads $17^h45^m32^s$ at comparison.

Answers.—(1) W $17^h46^m00^s$, (2) C $5^h51^m10^s$, (3) WE 28^s slow on GMT.

1910c. A watch is set to zone time,approximately. The long. is about 160°E. The watch is compared with a chronometer which is 3^m16^s fast on GMT. When the chronometer reads $1^h48^m00^s$, the watch reads $12^h45^m02^s$.

Required.—Watch error on zone time.

Answer.—WE 18^s fast on ZT.

1910d. On February 14 the DR long. is 63°46′.1 W. An observation of Dubhe is made when the watch reads $6^h07^m30^s$ PM. The watch is 11^s slow on zone time.

Required.—GMT and date.

Answer.—GMT $22^h07^m41^s$ Feb. 14.

1910e. On December 11 a watch is set to zone time, approximately. The long. is 137° W. The chronometer is 3^m36^s fast on GMT. When the chronometer reads $4^h40^m00^s$, the watch reads $7^h36^m06^s$ PM.

Required.—(1) Watch error on GMT.

(2) GMT and date about 20 minutes later, when the watch reads $7^h55^m52^s$.

Answers.—(1) WE $2^h59^m42^s$ fast on GMT, (2) GMT $4^h56^m10^s$ Dec. 12.

1910f. Shortly before taking morning sights on January 17 the navigator compares his watch with the chronometer. When the chronometer reads $2^h30^m00^s$, the watch reads $6^h13^m12^s$ AM. The chronometer is 17^m15^s fast on GMT. The long. is 118° W.

Required.—(1) C–W.

(2) GMT and date a little later when Regulus is observed at W $6^h28^m47^s$ AM.

Answers.—(1) C–W $8^h16^m48^s$, (2) GMT $14^h28^m20^s$ Jan. 17.

1911a. At long. 138°09′.3 E the LMT is $0^h09^m57^s$ on April 23.

Required.—(1) GMT and date.

(2) ZT and date at the place.

Answers.—(1) GMT $14^h57^m20^s$ Apr. 22, (2) ZT $23^h57^m20^s$ Apr. 22.

1911b. At long. 157°18′.4 W the LMT is 1931 on June 29.

Required.—(1) ZT and date.

(2) GMT and date.

Answers.—(1) ZT 2000 June 29, (2) GMT 0600 June 30.

1911c. At long. 99°35′.7 W the daylight saving time is $21^h29^m45^s$ on August 31.

Required.—(1) Standard time and date.

(2) LMT and date.

Answers.—(1) Standard time $20^h29^m45^s$ Aug. 31, (2) LMT $20^h51^m22^s$ Aug. 31.

1912a. Find the LAT and date at ZT $5^h26^m13^s$ on June 12, 1958, for long. 9°28′.1 E.

Answer.—LAT $5^h04^m31^s$ June 12.

1912b. At long. 77°15′.5 W the LAT is 1500 on June 13, 1958.

Required.—(1) ZT.

(2) LMT.

Answers.—(1) ZT $15^h08^m56^s$, (2) LMT $14^h59^m54^s$.

1912c. Using the *Air Almanac*, find (1) LAT at long. 117°55′ W, and (2) the Eq. T, at ZT $20^h43^m09^s$ on June 1, 1958.

Answers.—(1) LAT $20^h53^m44^s$, (2) Eq. T (+) 2^m15^s.

1913a. Find LST at ZT $19^h24^m26^s$ on June 1, 1958, for long. 87°51′.2 E.

Answer.—LST $11^h53^m56^s$.

1913b.—Find the ZT at LST $21^h20^m07^s$ on May 31, 1958, for long. 54°21′.3 W.

Answer.—ZT $4^h24^m13^s$.

1914. Find LHA and t of the moon at GMT $9^h25^m07^s$ on May 31, 1958, (1) for long. 43°19′.0 W, and (2) for long. 43°19′.0 E. Use appendix V.

Answers.—(1) LHA 118°54′.8, t 118°54′.8 W; (2) LHA 205°32′.8, t 154°27′.2 E.

CHAPTER XX

SIGHT REDUCTION

2001. Introduction.—The process of deriving from a celestial observation the information needed for establishing a line of position is called **sight reduction.** The observation itself consists of measuring the altitude of a celestial body and noting the time. Although special methods may be used for finding certain coordinates such as latitude or longitude, the modern navigator generally thinks in terms of lines of position without regard to any special significance of these lines. The process of finding such a line of position may be divided into six steps:

1. Correction of sextant altitude (ch. XVI).
2. Determination of GHA and declination (ch. XVIII).
3. Selection of assumed position and finding meridian angle at that point.
4. Computation of altitude and azimuth.
5. Comparison of computed and observed altitudes (ch. XVII).
6. Plot of the line of position.

Broadly speaking, tables which assist in any of these steps can be considered **sight reduction tables.** However, the expression is generally limited to tables intended primarily for computation of altitude and azimuth. A great variety of such tables exists. In chapter XXI various methods of sight reduction, including graphical and mechanical solutions, are contrasted. All are based, directly or indirectly, upon solution of the navigational triangle (art. 1433). Thus, the process of sight reduction, in its limited sense, is one of converting coordinates of the celestial equator system (art. 1426) to those of the horizon system (art. 1428).

The U. S. Navy Hydrographic Office publishes a set of sight reduction tables giving tabulated solutions of the navigational triangle, intended primarily for use with the *Nautical Almanac* aboard ship. These *Tables of Computed Altitude and Azimuth*, popularly known by their publication number, "H.O. 214," are widely used among mariners of various nations. In addition to the United States printing, editions are published by Great Britain, Spain, and Italy. They are suitable for reduction of nearly all observations made aboard ship, and will be used to explain the principles of sight reduction as given in this chapter. Extracts from H.O. Pub. No. 214 are given in appendix AA.

2002. Preliminary computation.—Certain computations precede the use of sight reduction tables. The correction of the sextant altitude (hs) to find observed altitude (Ho), as explained in chapter XVI, is usually performed first, but not necessarily so. If any form of time other than GMT is used for timing the observation, it is first converted to GMT because this is the kind of time used for entering the almanacs. From the almanac, the GHA and declination are determined, as explained in chapter XVIII.

To enter H.O. Pub. No. 214 and most other sight reduction tables, the following variables are needed:

1. Latitude (L).
2. Declination (d).
3. Meridian angle (t).

The latitude to use in entering the tables is that of the **assumed position (AP).** This latitude is usually called the **assumed latitude (*a*L).** The assumed position should

be in the general vicinity of the actual position, which, of course, is usually unknown. Several methods of selecting an AP are in use. The dead reckoning position or estimated position might be used. However, when H.O. Pub. No. 214 and certain other methods of sight reduction are used, unnecessary interpolation can be avoided by selecting an AP that will result in two of the three variables being exact entry values. In H.O. Pub. No. 214, altitudes and azimuths are given for each whole degree of latitude. Therefore, it is customary to select an AP on the *nearest* whole degree of latitude to the DR or EP at the time of sight.

Declination is taken from the almanac, as explained in chapter XVIII. This value is used without adjustment to simplify the solution.

Meridian angle is the angular distance that the celestial body is east or west of the celestial meridian. It is found from local hour angle (LHA), which, in turn, is found from Greenwich hour angle by adding east longitude or subtracting west longitude. A time diagram (art. 1427) is useful in visualizing this relationship.

Example 1.—The GHA is 168°42.′6.

Required.—The LHA and t at (1) long. 137°24.′6 W, and (2) 158°24.′7 E.

Solution.—

(1)	GHA	168°42.′6	(2) GHA	168°42.′6
	λ	137°24.′6 W	λ	158°24.′7 E
	LHA	31°18.′0	LHA	327°07.′3
	t	31°18.′0 W	t	32°52.′7 E

In west longitude, if GHA is less than longitude, add 360° to GHA before subtracting. In east longitude, if the sum exceeds 360°, subtract this amount. If LHA is less than 180°, it is numerically equal to meridian angle, which is labeled W (west). If LHA is greater than 180°, t is 360°−LHA and is labeled E (east).

In H.O. Pub. No. 214, t (labeled "H.A.") is given at intervals of 1°. If t is to be a whole degree, the longitude of the assumed position, called **assumed longitude** (*a*λ), must be selected so that no minutes of arc will remain after it is applied to GHA. This means that in west longitude the minutes of *a*λ must be the same as those of GHA; while in east longitude the minutes of *a*λ must be equal to 60′ minus the minutes of GHA.

Example 2.—The GHA is 57°18.′9.

Required.—The LHA, t, and AP for use with H.O. Pub. No. 214 without interpolation for t or L, if the DR position is (1) lat. 11°48.′8 N, long. 151°53.′3 W; and (2) lat. 62°21.′7 N, long. 4°31.′3 E.

Solution.—

(1)	GHA	57°18.′9	(2) GHA	57°18.′9
	*a*λ	152°18.′9 W	*a*λ	4°41.′1 E
	LHA	265°00.′0	LHA	62°00.′0
	t	95°00.′0 E	t	62°00.′0 W
	*a*L	12°00.′0 N	*a*L	62°00.′0 N
	*a*λ	152°18.′9 W	*a*λ	4°41.′1 E

2003. *Tables of Computed Altitude and Azimuth* (H.O. Pub. No. 214).—These popular sight reduction tables are published by the U. S. Navy Hydrographic Office in nine volumes, each covering 10° of latitude in increments of 1°. For each degree of latitude there is a series of tables, with cutaway tabs providing quick reference to the first page of the tables for that latitude. Declination entries are given at intervals of 0.°5 from 0° to 29°. Beyond this, 37 selected declination entries are given to provide solutions for all of the stars listed on the daily pages of the almanacs, and most of the additional stars listed near the back of the *Nautical Almanac*. A total of 96 declination

entries are given for each latitude, arranged eight to a page. Each declination entry is given at the top of a column. The third variable, meridian angle, is given in the column at the extreme left and right sides of each page. These columns are labeled "H.A.", the abbreviation for "hour angle," the expression formerly used for meridian angle, but replaced because of confusion with *local* hour angle, *Greenwich* hour angle, etc. Meridian angle entries are given at intervals of 1° from 0° at the top of the page to the maximum value at which the altitude is 5° or greater.

At most page openings, separate tables are given for declination having the *same* name (N or S) as the latitude and those having *contrary* name (one N, the other S). That is, declination values on the left-hand page (same name) are duplicated on the right-hand page (contrary name). A maximum of ninety-one meridian angle entries (0°–90°) are given on the left-hand page (same name). As either the declination or the latitude increases, the number of same-name entries increases, and the number of contrary-name entries decreases. When the same-name entries exceed 90° of meridian angle, the additional ones are placed on the right-hand page, below the contrary-name entries. At extreme values of declination and latitude there are no contrary-name entries, the same-name entries occupying both pages.

In each declination column there are four sets of figures. The first, given in bold type, is the altitude (labeled "Alt.") to the nearest 0′.1. Following this is Δd in small type. This is the change of altitude for a unit change of declination. Except when the value is 1.0, entries are given in hundredths of a unit, although the position of the decimal point is not shown. Following Δd, and also in small type, is Δt, the change of altitude for a unit change of meridian angle. This is given in the same form as Δd. The last set of figures in the column is the azimuth angle (labeled "Az."), to the nearest 0°.1.

At latitude 0° the arrangement is modified because there is no "same" or "contrary" name of declination. Here a single set of declination entries is given. Declination replaces latitude as the prefix label for azimuth angle.

Following the altitude-azimuth section of each latitude is a two-page star identification table. The use of this table is explained in article 2213.

On the inside front cover and its facing page is a speed-time-distance table, which is useful in advancing or retiring lines of position, as well as for other purposes. This table contains information similar to that in table 19 of this publication, but in somewhat different form. Volume VIII and older printings of other volumes have sextant altitude correction tables on these pages.

Following the speed-time-distance table is an arc to time conversion table.

Following the title page and preface are given a description of the tables, and sample problems.

On the inside back cover and facing page is given a "multiplication table" to multiply Δd or Δt by the number of minutes between the declination or meridian angle and the value used for entering the main table. This is used in interpolating the altitude for declination or meridian angle.

On the two pages next preceding the multiplication table is given a somewhat similar table to provide easy interpolation for latitude.

The use of the various parts of H.O. Pub. No. 214 is explained in articles 2004–2007 and 2213. The primary purpose of H.O. Pub. No. 214 is to provide an easy method of sight reduction for use with the *Nautical Almanac* aboard ship. It may also be used with the *Air Almanac*, and for solution of any spherical triangle for which entry values are given. Therefore, it can be used in great-circle sailing for determining the initial course and the distance.

In the British edition (H.D. 486) the main tables are identical to those of H.O. Pub. No. 214, being a reproduction of the United States tables, but arranged with 15° of latitude in each of six volumes. The explanation has been rewritten to suit British usage. The Spanish edition is identical to H.O. Pub. No. 214, except that the explanation is in Spanish. The Italian edition is based on H.O. Pub. No. 214.

2004. H.O. Pub. No. 214 solution by Δd only.—If interpolation is made for all three variables—latitude, declination, and meridian angle—a triple interpolation is needed. A simpler solution, almost universally used with H.O. Pub. No. 214, is to select an assumed position that will eliminate interpolation for latitude and meridian angle, leaving a simple interpolation for declination.

Example.—Find computed altitude (Hc) and azimuth (Zn) if *a*L is 41°00′.0 N, d is 22°14′.3 N, and t is 36°00′.0 W.

Solution.—

t	36°00′.0 W		
d	22°14′.3 N	d diff. 14′.3	
*a*L	41°00′.0 N		
ht	54°16′.8	Δd (+)0.65	Z N111°.0W
corr.	(+) 9′.3		
Hc	54°26′.1		
Zn	249°.0		

The main table is entered with the three variables, t, d, and *a*L (being sure to note whether d and *a*L are of same or contrary name); and the values of ht (tabulated altitude, labeled "Alt." in H.O. Pub. No. 214), Δd, and Z are taken directly from the table, without interpolation. The tabulated altitude (ht) is the computed altitude (Hc) for the values used for entering the table. The designation ht is used to distinguish it from the Hc obtained by applying a correction to the value taken from the table.

The declination entry argument used should be the tabulated entry nearest the declination for which a solution is sought, normally differing by not more than half a degree. The difference between this value and the actual declination is recorded as "d diff." No sign (+ or −) is assigned to this value. It is good practice to show Δd as a decimal, even though it is not tabulated in this way. The sign of this value should be determined carefully by inspection of the main table of H.O. Pub. No. 214. Interpolation of altitude for declination is made between the base value taken from the table and the value given on the same line in the next column to the right or left. The choice of the second column depends upon the actual declination. If it is *greater* than the value used for entering the table, use the next column to the *right,* and if less, use the next column to the *left.* If the value in the second column is greater than the base value, the sign is plus (+), and if less, the sign is minus (−). The accuracy of this important step can be checked by comparing the computed altitude (Hc) with the altitudes given in the main table of H.O. Pub. No. 214. If Δd has been given the correct sign (and applied correctly), Hc should lie between the tabulated altitudes in the columns for tabulated declination next smaller and next larger than the actual declination.

The azimuth angle is given a prefix N or S to agree with the latitude, and a suffix E or W to agree with the meridian angle. For this reason it is good practice to label these values *when they are recorded.*

The next step is to multiply Δd by d diff., to interpolate between the altitude entries for consecutive declination columns. In most instances, the easiest way to do this is to use the multiplication table on the inside back cover of H.O. Pub. No. 214 and its facing page, entering separately with minutes and tenths of minutes of Δd and

adding the two parts. The correction, which is recorded below ht, is given the sign of Δd. The correction is then added or subtracted, in accordance with its sign, to ht. The answer is computed altitude (Hc).

Azimuth (Zn), which is recorded below Hc, is found by converting azimuth angle (Z) in accordance with its labels, as explained in article 1428. Usually, azimuth angle is found without interpolation. For exceptions to this practice, see article 2007.

If Δd is changing rapidly, or when it changes sign (at the maximum altitude for the given meridian angle and latitude), interpolation may be somewhat less accurate than in other parts of the tables, but this should not introduce a large error unless the celestial body is near the zenith, when the method of H.O. Pub. No. 214 is not recommended.

2005. H.O. Pub. No. 214 solution by Δd and Δt is similar to that using Δd only, but with the additional step of interpolating between the altitude entries for consecutive meridian angle entries, in a similar manner to interpolation for declination.

Example.—Find computed altitude and azimuth if *a*L is 41°00′.0 N, d is 20°48′.7 S, and t is 22°14′.0 E.

Solution.—

t	22°14′.0 E	t diff. 14′.0		t corr.	(−) 4′.2
d	20°48′.7 S	d diff. 11′.3		d corr.	(+) 10′.8
*a*L	41°00′.0 N			corr.	(+) 6′.6
ht	24°43′.1	Δd (+) 0.95	Δt (−) 0.30	Z	N 157°.4 E
corr.	(+) 6′.6				
Hc	24°49′.7				
Zn	157°.4				

In this solution, t diff. is the difference between the meridian angle and the nearest whole degree of t used for entering the table. The t corr. is t diff. × Δt, found by using the same multiplication table used for the d corr. The sign of Δt is found by inspection of the main table. In this example interpolation is between the base altitude for t 22°, and the altitude for t 23°. Since the altitude for t 23° is less than that for t 22°, the correction should be subtracted, so that the interpolated value will lie between the two values between which interpolation is being made. The sign of Δd is found by comparing the altitude for d 21°00′ with that for d 20°30′. The total correction is the algebraic sum of the t corr. and d corr.

The principal advantage of this solution is that a round of sights can be worked and plotted from the same assumed position. However, this advantage is offset by the additional length of the solution. The method is little used.

2006. H.O. Pub. No. 214 solution by Δd, Δt, and ΔL.—If the altitude and azimuth at a particular place are desired, interpolation should be made for all three variables, t, d, and L, if needed. The change in altitude for a change of latitude of 1′ (ΔL) is not tabulated. The table on the two pages *preceding* the multiplication table of H.O. Pub. No. 214 is used for finding the correction for latitude.

Example.—Find computed altitude and azimuth if *a*L is 41°12′.8 S, d is 21°32′.5 S, and t is 8°52′.3 W.

Solution.—

					+	−
t	8°52′.3 W	t diff. 7′.7		t corr.	2′.4	
d	21°32′.5 S	d diff. 2′.5		d corr.	2′.4	
*a*L	41°12′.8 S	L diff. 12′.8		L corr.		11′.7
ht	69°04′.0	Δd (+) 0.94	Δt (+) 0.32	sum	4′.8	11′.7
corr.	(−) 6′.9			corr.		(−) 6′.9
Hc	68°57′.1			Z		S 156°.0 W
Zn	336°.0					

The corrections for meridian angle and declination are found as explained in articles 2004 and 2005. The L corr. is found by entering the correction table with azimuth angle and L diff., the difference between the latitude of the assumed position and the nearest whole degree used for entering the main table. It is customary to interpolate in this table, where applicable, but the error introduced by not doing so is always less than 0'3 if the *nearest* whole degree of azimuth angle is used. The sign of the latitude correction is determined by the rules given at the bottom of the correction table. The total correction is the algebraic sum of the three individual corrections.

All of these are *altitude* corrections. The azimuth is that corresponding to the values used for entering the main table. If the exact value at the place is desired, it can be found by interpolation, as explained in article 2007. If such an interpolation is made, the interpolated value should be used for entering the latitude correction table for altitude interpolation.

2007. Interpolation for azimuth.—In sight reduction for plotting lines of position, it is not customary to interpolate for azimuth angle when H.O. Pub. No. 214 is used. However, if greater accuracy is desired, as for determining compass error, triple interpolation should be made. This is customarily accomplished by entering the main table of H.O. Pub. No. 214 with the *nearest* values of t, d, and L, and taking out the corresponding tabulated value. Simple eye interpolation is then used to determine separately the correction for each of the three variables. The algebraic sum of these is the correction applied to the base value. The Δd, Δt, and ΔL corrections are not used because these refer to the *altitude*, not the azimuth. Corrections are made to azimuth angle *before* it is converted to azimuth.

Example.—Find the azimuth by interpolation if L is 41°25'9 S, d is 22°19'6 N, and t is 17°22'4 E.

Solution.—

					+	−
t	17°4 E	t diff. 0°4	Z diff. (−) 1°0	t corr.		0°4
d	22°3 N	d diff. 0°2	Z diff. (−) 0°1	d corr.		—
L	41°4 S	L diff. 0°4	Z diff. (+) 0°2	L corr.	0°1	
tab.	162°7			sum	0°1	0°4
corr.	(−) 0°3			corr.		(−) 0°3
Z	S 162°4 E					
Zn	017°6					

The corrections are determined by inspection. In this example the tabulated value used as a base is compared with the value for the same d and L, but t 18°, to determine the t corr. Similarly, it is compared with the value for the same t and L, but d 22° to determine the d corr.; and with the value for the same t and d, but L 42° to determine the L corr. Interpolation is between whole degrees of t and L, but between half degrees of d. For declinations of more than 29°, the d interpolation interval varies.

If azimuth is needed to a greater precision than the nearest tenth of a degree, it should be determined by H.O. Pub. No. 260 or 261 (art. 2126), or by computation (art. 2125).

2008. Complete solution.—The complete solution includes all of the parts listed in article 2001. Because of the various alternatives available for the separate parts, a large number of variations might be used in the complete solution. The following examples combine some of the most commonly used variations.

Example 1.—On May 31, 1958, the 1425 DR position of a ship is lat. 40°39'6 N, long. 64°17'2 W. At watch time $2^h25^m51^s$ PM the navigator observes the lower limb of

the sun with a marine sextant having an IC of (−) 2'.0, from a height of eye of 36 feet. The watch is 23^s fast on zone time. The hs is 56°10'.9.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 214 (Δd only) and the *Nautical Almanac.*

Solution.—

	May 31	Sun		+	⊙	−
W	$2^h25^m51^s$ PM	18^h 21°55'.1 N *d*	IC			2'.0
WE	(F) 23^s	corr. (+) 0'.1 (+) 0'.3	D			5'.8
ZT	$14^h25^m28^s$	d 21°55'.2 N	⊙	15'.3		
ZD	(+) 4		sum	15'.3		7'.8
GMT	$18^h25^m28^s$ May 31		corr.			(+) 7'.5
18^h	90°36'.8		hs			56°10'.9
25^m28^s	6°22'.0		Ho			56°18'.4
GHA	96°58'.8					
*a*λ	63°58'.8 W					
LHA	33°00'.0					
t	33°00'.0 W					
d	21°55'.2 N	d diff. 4'.8				
*a*L	41°00'.0 N					
ht	56°22'.2	Δd (−) 0.67	Z N 114°.2 W			
corr.	(−) 3'.2					
Hc	56°19'.0					
Ho	56°18'.4					
a	0.6 A	*a*L 41°00'.0 N				
Zn	245°.8	*a*λ 63°58'.8 W				

It is good practice to have a standard work form. If this is not printed, or on a rubber stamp, it should be copied in its entirety before the solution is started. The first step should then be to fill in the known information. If the solution for observed altitude is made first, this value can then be copied in the main solution at the left of the form, so that it will be ready for comparison when Hc is determined. The best form to use is that which the individual navigator finds most logical and least likely to result in errors. Those shown in appendix Q, and used here, are slight modifications of forms developed at the United States Naval Academy, where they evolved as a result of long experience. Their use reduced materially the number of mistakes made in solutions. Some navigators include a time diagram (art. 1427) in the form, immediately below the name of the celestial body, as a check both on the time and meridian angle computation.

There is a growing tendency among navigators to keep the navigational watch set to GMT. This is particularly helpful when a number of observations are made, as during twilight, to eliminate the need for repeated application of watch error and zone description, and determination of Greenwich date. The use of a GMT watch is illustrated in the following example of a complete sight of the moon:

Example 2.—On June 2, 1958, the 0420 DR position of a ship is lat. 41°07'.6 N, long. 131°51'.2 W. At GMT $13^h24^m52^s$ the navigator observes the upper limb of the moon with a marine sextant having an IC of (+) 1'.5, from a height of eye of 29 feet. The hs is 7°40'.1.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 214 (Δd only) and the *Nautical Almanac.*

Solution.—

	June 2	
GMT	$13^h24^m52^s$ June 2	
13^h	185°38′.0	
24^m52^s	5°56′.0	*v*
corr.	2′.5	(+) 6′.0
GHA	191°36′.5	
*a*λ	131°36′.5 W	
LHA	60°00′.0	
t	60°00′.0 W	
d	19°09′.6 S	
*a*L	41°00′.0 N	
ht	8°14′.0	
corr.	(−) 7′.3	
Hc	8°06′.7	
Ho	8°12′.0	
a	5.3 T	
Zn	235°.8	

	Moon	
13^h	19°09′.5 S	*d*
corr.	(+) 0′.1	(+) 0′.3
d	19°09′.6 S	
d diff.	9′.6	
Δd	(−) 0.75	
*a*L	41°00′.0 N	
*a*λ	131°36′.5 W	

	+	☾ −
IC	1′.5	
D		5′.2
☾	61′.0	
U	4′.6	
add'l		30′.0
sum	67′.1	35′.2
corr.		(+) 31′.9
hs		7°40′.1
Ho		8°12′.0
Z		N 124°.2 W

Occasionally it is desired to solve an observation for the estimated position of the ship.

Example 3.—During morning twilight on June 1, 1958, the 0624 EP of a ship is lat. 41°12′.3 S, long. 178°39′.2 E. At ZT $6^h24^m57^s$ the navigator observes Saturn with a marine sextant having an IC of (−) 1′.0, from a height of eye of 53 feet. The hs is 20°52′.3.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 214 (Δd, Δt, ΔL) and the *Nautical Almanac*.

Solution.—

	June 1	
ZT	$6^h24^m57^s$	
ZD(−)12		
GMT	$18^h24^m57^s$ May 31	
18^h	255°51′.3	
24^m57^s	6°14′.3	*v*
corr.	(+) 1′.1	(+) 2′.7
GHA	262°06′.7	
*a*λ	178°39′.2 E	
LHA	80°45′.9	
t	80°45′.9 W	
d	21°50′.7 S	
*a*L	41°12′.3 S	
ht	20°48′.5	
corr.	(+) 7′.4	
Hc	20°55′.9	
Ho	20°41′.7	
a	14.2 A	
Zn	258°.4	

	Saturn	
18^h	21°50′.7 S	*d*
corr.	0′.0	0′.0
d	21°50′.7 S	
t diff.	14′.1	
d diff.	9′.3	
L diff.	12′.3	
Δd(−) 0.60	Δt(+) 0.74	
*a*L	41°12′.3 S	
*a*λ	178°39′.2 E	

	+	S −
IC		1′.0
D		7′.1
☆-P		2′.5
sum		10′.6
corr.		(−) 10′.6
hs		20°52′.3
Ho		20°41′.7

	+	−
t corr.	10′.5	
d corr.		5′.6
L corr.	2′.5	
sum	13′.0	5′.6
corr.		(+) 7′.4
Z		S 78°.4 W

The local date is used as the heading of the first column of the solution. The Greenwich date is recorded opposite GMT. In this example, the two dates are different. Even when they are the same, it is good practice to record both dates, to avoid possible error. It is desirable to show all closely related information on the same line, as t, t diff., and t corr. This is not always possible because of interference of other information. When this occurs, the form is changed in a way to cause the least upset to the usual form. Thus, in this example, it would be desirable to show ht, Δd, Δt, and Z on the same line and in the order these values are taken from H.O. Pub. No. 214. However, the sum of the t, d, and L corrections is at the logical place for Z, which is therefore kept in the same column, but moved *down* two spaces.

Example 4.—During evening twilight on June 1, 1958, the 1730 DR position of a ship is lat. 40°39.′2 S, long. 75°01.′2 E. At GMT $12^h31^m17^s$ the navigator observes Arcturus with a marine sextant having no IC, from a height of eye of 38 feet. The hs is 7°55.′2.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 214 (Δd only) and the *Air Almanac.*

Solution.—

	June 1	Arcturus		+	☆ −
GMT	$12^h31^m17^s$ June 1		IC	—	—
12^h30^m	76°59′		D		6′
1^m17^s	19′		R		7′
SHA	146°33′		sum	—	13′
GHA	223°51′		corr.		(−)13′
*a*λ	75°09′ E		hs		7°55′
LHA	299°00′		Ho		7°42′
t	61°00′ E				
d	19°24′ N	d diff. 6′			
*a*L	41°00′ S				
ht	7°14.′0	Δd (+)0.75	Z		S123.°8E
corr.	(+)4.′5				
Hc	7°18.′5				
Ho	7°42′				
a	24 T	*a*L 41°00′S			
Zn	056.°2	*a*λ 75°09′E			

Both ht and its correction are shown to tenths of a minute of arc, to avoid a possible error of 1′ in the algebraic sum, Hc.

There is no significance to the type of solution and almanac used with the various celestial bodies shown in the examples above. The combinations used for examples 1 (sun) and 2 (moon) are most commonly used by marine navigators, but the other two are shown to illustrate variations sometimes used at sea.

2009. Precomputation.—Sometimes it is desired to determine computed altitude *before* the observation, generally for the purpose of obtaining a line of position quickly after the observation has been completed. This is called **precomputation.** When it is done, sextant altitude corrections are generally applied *with reversed sign* to Hc to obtain **precomputed altitude (Hp)**, which is then compared directly with hs to obtain the altitude difference for plotting a line of position. Where altitude is needed for entering correction tables, the computed altitude (Hc) is used. The error introduced by this practice is negligible except at low altitudes, where the corrections should be adjusted by using the Hp to reenter the tables. If greater accuracy is required, limit precomputation to Hc and Zn, and apply corrections to the sextant altitude after observation.

Example.—On June 2, 1958, the 1025 DR position of a ship is lat. 42°21′.4 S, long. 118°47′.1 W. The navigator plans to observe the lower limb of the sun at this time with a marine sextant having an IC of (+)2′.5, from a height of eye of 25 feet.

Required.—(1) Precomputed altitude by H.O. Pub. No. 214, Δd only, (2) the *a*, Zn, and AP if hs is 22°23′.6. Use *Nautical Almanac.*

Solution.—

	June 2		Sun			+ $\odot$	−
ZT	$10^h25^m00^s$	18^h	22°11′.4 N	*d*	IC	2′.5	
ZD	(+) 8	corr.	(+) 0′.1	(+) 0′.3	D		4′.9
GMT	$18^h25^m00^s$ June 2	d	22°11′.5 N		$\odot$	13′.7	
18^h	90°32′.3				sum	16′.2	4′.9
25^m00^s	6°15′.0				corr.		(+) 11′.3
GHA	96°47′.3						
*a*λ	118°47′.3 W						
LHA	338°00′.0						
t	22°00′.0 E						
d	22°11′.5 N	d diff.	11′.5				
*a*L	42°00′.0 S						
ht	22°50′.5	Δd	(−) 0.95		Z	S 157°.9 E	
corr.	(−) 11′.0						
Hc	22°39′.5						
corr.	(+) 11′.3 (rev.)						
(1) Hp	22°28′.2						
hs	22°23′.6						
(2) *a*	4.6 A	*a*L	42°00′.0 S				
Zn	022°.1	*a*λ	118°47′.3 W				

At the AP used in the calculation, Hp is correct only for the time used. However, if the observation is made early or late, the same Hp and Zn can be used by moving the AP along the parallel of latitude, eastward for early observations, or westward for late observations, a distance equal to 0′.25 of longitude for each second (15′.0 for each minute) difference between actual and predicted times. This adjustment is based upon the assumptions that the apparent motion of the body is westward at the rate of 15° per hour, and the declination is constant. Over the seconds or minutes likely to be involved, these assumptions and the possible increased length of the plotted lines do not introduce a significant error, except possibly for the moon.

2010. Low altitudes.—When Hc is determined by inspection tables such as H.O. Pub. No. 214, a minimum tabulated altitude may be available. In H.O. Pub. No. 214, altitudes below 5° are not given. These tables can be used for low-altitude observations by selection of an AP that will result in Hc being 5° or greater. To do this, proceed as follows: At the time of observation, note the approximate azimuth of the celestial body. Plot the azimuth line through the DR or EP and measure off, toward the celestial body, a distance equal to 6°−hs (or 6°−Ho). Select the AP in relation to this point as if it were the DR or EP. Occasionally it may be necessary to use 7°−hs (or 7°−Ho). The increased length of the altitude difference line does not introduce a significant error over the distance that a rhumb line can be considered identical to a great circle. Only in high latitudes is this a problem, and here the error can be virtually eliminated by using a chart projection on which a great circle plots as a straight line or approximately so. The error introduced by using a rhumb line to represent the circle of equal altitude (the line of position) is not increased because the AP selected is near the azimuth line.

Example.—On June 1, 1958, the 1625 DR position of a ship is lat. 43°39.′7 S, long. 15°07.′0 W. At GMT $17^h24^m22^s$ the navigator observes the lower limb of the sun as it breaks out below an overcast, shortly before setting. He uses a marine sextant having no IC, and makes his observation from a height of eye of 52 feet. The hs is 1°00.′8, air temperature (art. 1614) 24°F, and the atmospheric pressure (art. 1615) 30.16 inches. The sun's azimuth is approximately 305°.

Required.—The *a*, Zn, and AP using H.O. Pub. No. 214 (Δd only), the *Nautical Almanac*, and tables 23 and 24.

Solution.—

	June 1	Sun		+	⊙	−
GMT	$17^h24^m22^s$ June 1	17^h 22°03.′1 N *d*	IC	—		—
17^h	75°34.′7	corr. (+) 0.′1 (+) 0.′3	D			7.′0
24^m22^s	6°05.′5	d 22°03.′2 N	sum	—		7.′0
GHA	81°40.′2		corr.			(−)7.′0
*a*λ	20°40.′2 W		hs			1°00.′8
LHA	61°00.′0		hr			0°53.′8
t	61°00.′0 W					
d	22°03.′2 N	d diff. 3.′2	⊙			9.′1
*a*L	41°00.′0 S		T			1.′3
ht	5°21.′8	Δd (−) 0.75 Z S 125.°5 W	B			0.′3
corr.	(−) 2.′5		sum	—		10.′7
Hc	5°19.′3		corr.			(−)10.′7
Ho	0°43.′1		hr			0°53.′8
a	276.2 A	*a*L 41°00.′0 S	Ho			0°43.′1
Zn	305.°5	*a*λ 20°40.′2 W				

Refer to figure 2010. From the 1625 DR position, the approximate azimuth of 305° is plotted, as shown by the broken line. Along this line a distance of 6°00.′0−0°43.′1=5°16.′9, or 316.9 miles, is measured, locating the point labeled *A*. The AP is selected with respect to this point as if it were the DR position. The sight is plotted from this AP as in any observation. If it makes the plot easier, record *a* in the solution in degrees and minutes of arc instead of in miles (5°19.′3−0°43.′1=4°36.′2=276.′2=276.2 miles).

Large altitude differences can be avoided by using a method of solution that provides for low altitudes. Among such methods are H.O. Pub. No. 249; nearly any trigonometric method such as the cosine-haversine formula, H.O. Pub. No. 208, or H.O. Pub. No. 211; and most graphical and mechanical methods. All of these methods are discussed in chapter XXI. If a trigonometric method is used, the *signs* of the various functions (or special rules) should be used if there is a possibility of Hc being negative. The rules needed for H.O. Pub. No. 208 and H.O. Pub. No. 211 are given in articles 2110 and 2111, respectively. The need for special care can be eliminated by using an assumed position about half a degree or more from the DR position or EP, in the direction of the celestial body, if the altitude is less than 0°30′.

By any method of solution, if *either* Hc or Ho (but not both) is negative, the altitude difference is found by numerically *adding* the two altitudes. Thus, if Hc is (+) 0°12.′6 and Ho is (−) 0°03.′2, the altitude difference, *a*, is 15.′8, or 15.8 miles. The positive altitude is greater than the negative one. Therefore, the *a* in this case is *away*. If *both* Hc and Ho are negative, the difference is found by subtraction, but in this case the one which is numerically smaller is the greater altitude. Thus, if Hc is (−) 0°09.′6 and Ho is (−) 0°04.′3, the altitude difference is 5.3 T.

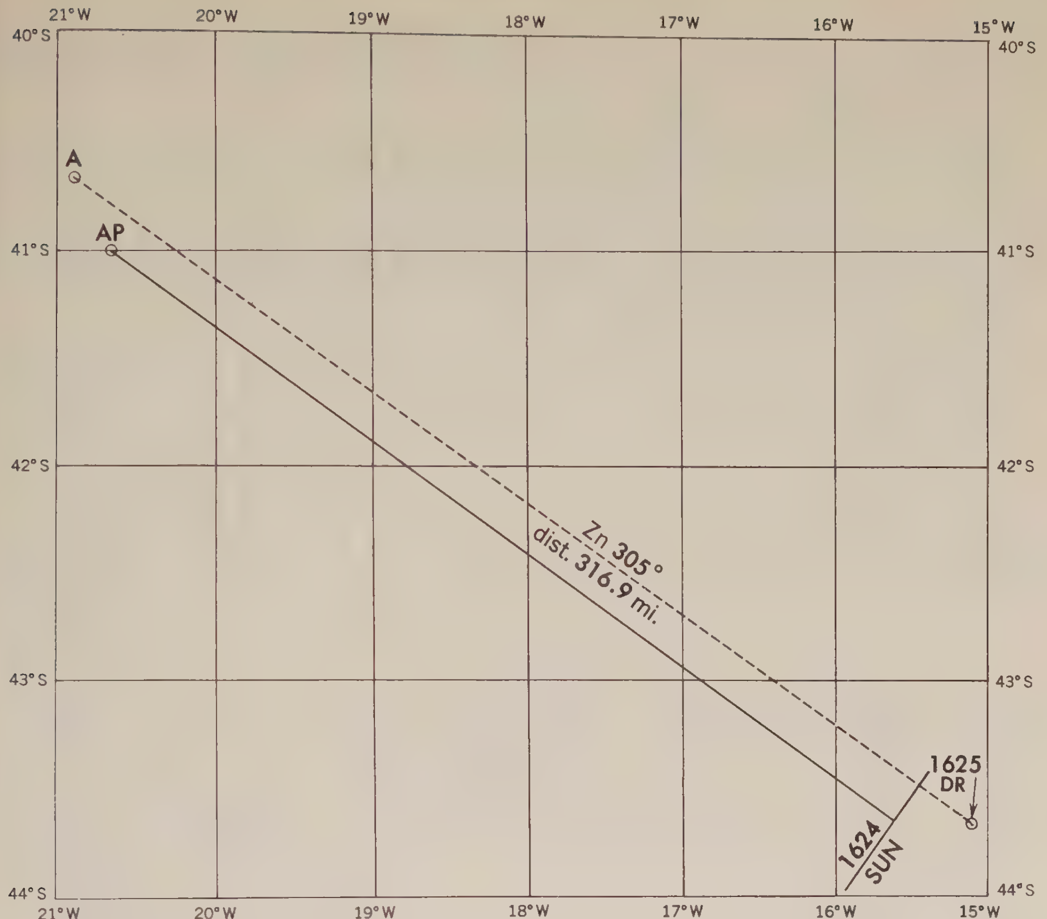


FIGURE 2010.—Selecting an AP for low-altitude solutions by H.O. Pub. No. 214.

2011. High altitudes are usually avoided for at least two reasons. First, bodies near the zenith are difficult to observe. A star or planet is difficult to "bring down" to the horizon. It is not always easy to determine the azimuth accurately, and when near the zenith, a body may be changing azimuth rapidly. On the other hand, such observations are little affected by astronomical refraction. The second reason for avoiding high altitudes is one of geometry. As the altitude increases, the radius of the circle of position decreases. For a body near the zenith, the radius is so small that the use of a straight line to approximate the circle may introduce serious error.

With higher altitudes, it is good practice to avoid use of lines of position extending a considerable distance from the azimuth line. Since the decrease in radius is gradual, there is no one altitude at which the curvature becomes excessive. However, a safe general rule, if one is needed, is to use the DR position or EP as the assumed position, and interpolate for azimuth angle, for all altitudes greater than 70°. The purpose of this is not primarily to decrease the altitude difference, as sometimes suggested, but to decrease the length of the line of position.

Within perhaps three degrees of the zenith, the curvature of the circle of position becomes so great that even for a short distance a straight line is not an adequate representation of the circle. At these altitudes, it is good practice to plot the line of position as a circle. This is done by using the geographical position (GP) of the celestial body as the center, and the zenith distance as the radius. Hence, no sight reduction tables

are needed. The same body can be used for obtaining a fix from two observations separated by several minutes. In celestial navigation, as in piloting, a circle of position is advanced or retired by moving its center.

Example.—On May 31, 1958, the 1224 DR position of a ship is lat. 20°17′.4 N, long. 50°07′.4 W. The ship is on course 127°, speed 18 knots. Using a marine sextant having no IC, the navigator observes the lower limb of the sun twice, from a height of eye of 65 feet. The first observation is made at GMT $15^h15^m15^s$, and hs is 88°01′.1. The second observation is made at GMT $15^h24^m13^s$, and hs is 87°34′.7. The GHA of the sun at GMT $15^h15^m15^s$ is 49°25′.9. Use the same declination as at GMT $15^h24^m13^s$.

Required.—The 1224 fix.

Solution.—

	May 31	
GMT	$15^h15^m15^s$	May 31
GHA	49°25′.9	
GP L_1	21°54′.1 N	
GP λ_1	49°25′.9 W	
radius	110.8 mi.	

	Sun
d	21°54′.1 N

	+	⊙	−
IC	—		—
D			7′.8
⊙	15′.9		
sum	15′.9		7′.8
corr.			(+) 8′.1
hs			88°01′.1
Ho			88°09′.2
z			1°50′.8

	May 31	
GMT	$15^h24^m13^s$	May 31
15^h	45°37′.1	
24^m13^s	6°03′.3	
GHA	51°40′.4	
GP L_2	21°54′.1 N	
GP λ_2	51°40′.4 W	
radius	137.2 mi.	

	Sun	
15^h	21°54′.0 N	*d*
corr.	(+) 0′.1	(+) 0′.3
d	21°54′.1 N	

	+	⊙	−
IC	—		—
D			7′.8
⊙	15′.9		
sum	15′.9		7′.8
corr.			(+) 8′.1
hs			87°34′.7
Ho			87°42′.8
z			2°17′.2

Answer.—1224 fix: L 20°09′.0 N, λ 50°06′.0 W.

The plot of this problem is shown in figure 2011. No significant error would be introduced by assuming the same declination and sextant altitude correction for both observations, and a change of GHA equal to the arc equivalent of the time difference between observations (art. 1904). In east longitude the GP longitude would be 360° − GHA.

Problems

2002a. The GHA is 51°47′.3.

Required.—The LHA and t at (1) long. 138°14′.1 W, and (2) 65°11′.7 E.

Answers.—(1) LHA 273°33′.2, t 86°26′.8 E; (2) LHA 116°59′.0, t 116°59′.0 W.

2002b. The GHA is 135°17′.3.

Required.—The LHA, t, and AP for use with H.O. Pub. No. 214 without interpolation for t or L, if the DR position is (1) lat. 71°36′.9 N, long. 137°25′.3 W, and (2) lat. 8°14′.1 S, long. 96°41′.7 E.

Answers.—(1) LHA 358°00′.0, t 2°00′.0 E, *a*L 72°00′.0 N, *a*λ 137°17′.3 W; (2) LHA 232°00′.0, t 128°00′.0 E, *a*L 8°00′.0 S, *a*λ 96°42′.7 E.

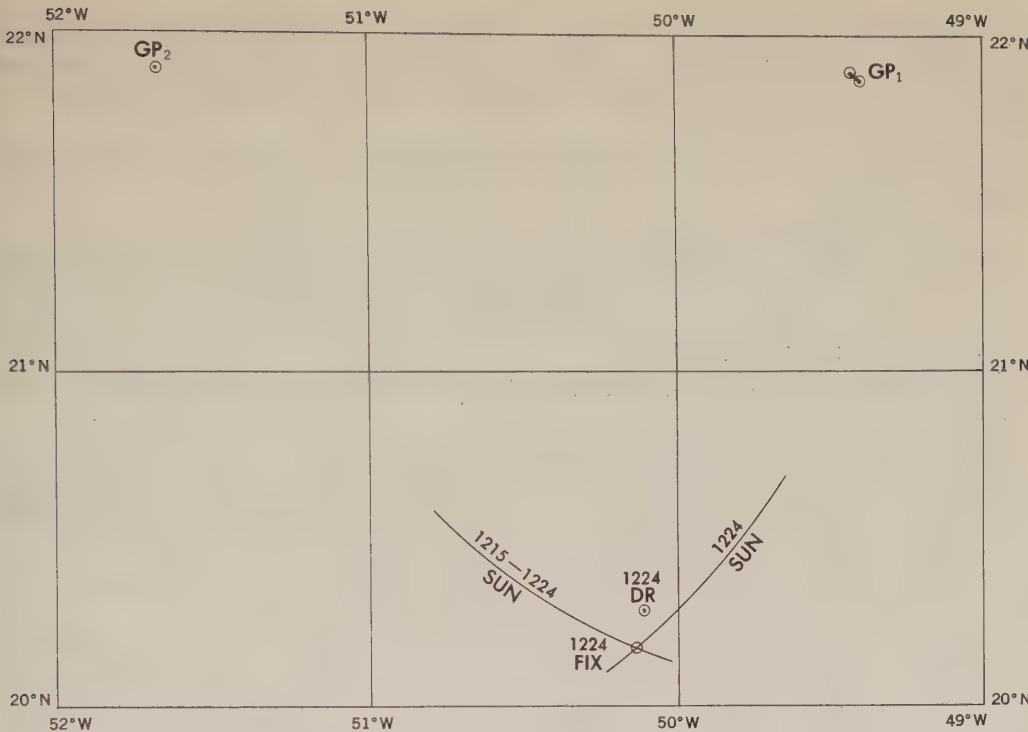


FIGURE 2011.—Plotting high-altitude observations.

2004. Find computed altitude (Hc) and azimuth (Zn) if *a*L is 42°00′.0 S, d is 21°09′.5 N, and t is 11°00′.0 E.

Answers.—Hc 26°01′.6, Zn 011°.4.

2005. Find computed altitude and azimuth if *a*L is 41°00′.0 S, d is 20°49′.1 S, and t is 37°11′.3 W.

Answers.—Hc 52°40′.1, Zn 291°.3.

2006. Find computed altitude and azimuth if *a*L is 41°53′.4 N, d is 19°45′.8 S, and t is 18°12′.7 W.

Answers.—Hc 26°05′.1, Zn 198°.8.

2007. Find the azimuth by interpolation if L is 41°33′.7 N, d is 20°18′.7 S, and t is 56°40′.5 E.

Answer.—Zn 127°.5.

2008a. On June 1, 1958, the 0425 DR position of a ship is lat. 41°07′.3 N, long. 153°03′.9 E. At GMT $18^h25^m16^s$ (May 31) the navigator observes Saturn with a marine sextant having no IC, from a height of eye of 30 feet. The hs is 9°13′.4.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 214 (Δd only) and the *Nautical Almanac.*

Answers.—*a* 2.0 A, Zn 230°.3, *a*L 41°00′N, *a*λ 152°48′.6 E.

2008b. On June 2, 1958, the 0625 DR position of a ship is lat. 40°38′.1 S, long. 24°08′.3 E. At watch time $6^h24^m48^s$ AM the navigator observes the upper limb of the moon with a marine sextant having an IC of (−) 2′.7, from a height of eye of 56 feet. The watch is 19^s slow on zone time. The hs is 14°38′.8.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 214 (Δd, Δt) and the *Nautical Almanac.*

Answers.—*a* 7.1 A, Zn 257°.9, *a*L 41°00′.0 S, *a*λ 24°08′.3 E.

2008c. During the morning of June 2, 1958, the 1025 DR position of a ship is lat. 41°12′.3 S, long. 13°45′.7 W. At GMT $11^h25^m42^s$ the navigator observes the lower limb of the sun with a marine sextant having an IC of (+) 1′.0, from a height of eye of 38 feet. The hs is 23°37′.3.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 214 (Δd, Δt, ΔL) and the *Nautical Almanac.*

Answers.—*a* 16.5 T, Zn 022°.3, *a*L 41°12′.3 S, *a*λ 13°45′.7 W.

2008d. During evening twilight on June 1, 1958, the 2100 DR position of a ship is lat. 40°47′.3N, long. 67°28′.7W. At ZT $21^h08^m01^s$ the navigator observes Arcturus through a break in the clouds, with a marine sextant having no IC, from a height of eye of 42 feet. The hs is 65°31′.8.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 214 (Δd only) and the *Air Almanac.*

Answers.—*a* 7 T, Zn 146°.8, *a*L 41°00′ N, *a*λ 67°33′ W.

2009. On June 3, 1958, the 0625 DR position of a ship is lat. 41°03′.7 S, long. 104°25′.6 E. The navigator plans to observe the upper limb of the moon at this time with a marine sextant having no IC, from a height of eye of 57 feet.

Required.—(1) Precomputed altitude by H.O. Pub. No. 214 (Δd, Δt, ΔL). (2) The *a*, Zn, and AP if hs is 19°09′.8. Use *Nautical Almanac.*

Answers.—(1) Hp 19°07′.1; (2) *a* 2.7 T, Zn 261°.6, *a*L 41°03′.7 S, *a*λ 104°25′.6 E.

2010a. On June 2, 1958, the 0725 DR position of a ship is lat. 45°07′.3 S, long. 48°05′.8 E. At GMT $4^h25^m21^s$ the navigator observes the lower limb of the sun shortly after the upper limb disappears behind an overcast, soon after sunrise. He uses a marine sextant having an IC of (+) 1′.2, and makes his observations from a height of eye of 35 feet. The hs is 0°45′.2, air temperature 30° F, and the atmospheric pressure 29.74 inches. The sun bears approximately 050°.

Required.—The *a*, Zn, and AP using H.O. Pub. No. 214 (Δd only), the *Nautical Almanac,* and tables 23 and 24.

Answers.—*a* 289.1A, Zn 053°.8, *a*L 42°00′.0 S, *a*λ 53°06′.0 E.

2010b. If Hc is 5°09′.2 and Ho is (−) 0°03′.4, find *a*.

Answer.—*a* 5°12′.6 A, or 312.6 A.

2010c. If Hc is (−) 0°18′.4 and Ho is (−) 0°01′.3, find *a*.

Answer.—*a* 17.1 T.

2011. On June 2, 1958, the 1225 DR position of a ship is lat. 23°47′.8 N, long. 130°13′.2 E. The ship is on course 200°, speed 20 knots. Using a marine sextant having an IC of (−) 2′.2, the navigator observes the lower limb of the sun twice, from a height of eye of 43 feet. The first observation is made at GMT $3^h10^m35^s$, and hs is 87°34′.8. The second observation is made at GMT $3^h25^m10^s$, and hs is 87°13′.4. The GHA of the sun at GMT $3^h10^m35^s$ is 228°12′.6. Assume no change in declination between observations.

Required.—The GP at the time of each observation, the radius of each circle of position, and the 1225 fix.

Answers.—GP L_1 22°06′.5 N, GP λ_1 131°47′.4 E, GP L_2 22°06′.5 N, GP λ_2 128°08′.7 E; radius$_1$ 137.9 mi., radius$_2$ 159.3 mi.; 1225 fix: L 23°52′.5 N, λ 130°17′.1 E.

CHAPTER XXI

COMPARISON OF VARIOUS METHODS OF SIGHT REDUCTION

2101. Introduction.—Before the development of a means of determining accurate time at sea (art. 127), longitude could not be found by celestial observation. Celestial bodies were used for determination of latitude, and as an indication of direction, often in a very general way. The development of the marine chronometer opened up a whole new vista to the navigator. Immediately, methods began to appear to utilize this new dimension of navigation. During the two centuries that have elapsed, many of the best minds have been directed to the problem of providing easier or more adequate methods of "reducing" the observations to a form suitable for determination of position.

2102. Kinds of methods.—Various "special" methods have been devised to take advantage of some unique relationship to provide a simplified solution. The most widely used are **latitude methods** for determination of latitude by meridian altitude or observation of Polaris, and **longitude methods** for determination of longitude by observation of a body near the prime vertical. Both latitude and longitude methods have now been largely superseded by the **altitude method,** based upon the discovery of the altitude difference, or intercept, by the Frenchman Marcq St.-Hilaire (art. 131). Most modern methods are of this type, although some latitude and longitude methods are still in use.

The most commonly used methods utilize computation for determining certain information which is then plotted as a line of position, two or more such lines being needed for a fix. The "method" might consist of one or more formulas to be solved by general mathematical tables, a set of special tables conveniently arranged for use with the formulas, or a set of tables constituting a list of computed answers. A method which determines latitude or longitude separately requires no plot. In fact, a plot would be misleading unless the celestial body were almost exactly on the celestial meridian or prime vertical, or unless the azimuth were considered. While a number of methods determine latitude and longitude by computation, without plotting, other methods substitute a graphical or mechanical solution for computation.

2103. Meridian altitudes.—If a celestial body is on the celestial meridian at the time of observation, a modification of the high-altitude method (art. 2011) can be used at any altitude, without plotting the GP. Both GP and observer are on the same meridian, and the difference of latitude between them is the zenith distance of the body ($90°-$Ho). The direction of the GP is the direction faced during observation (unless a backsight is made). The line of position is a latitude line.

Example 1.—At GMT $16^h24^m15^s$ on June 2, 1958, the navigator observes the lower limb of the sun on the celestial meridian, bearing north. He makes the observation with a marine sextant having an IC of (+) 2.′6, from a height of eye of 33 feet. The hs is 29°14.′6.

Required.—The latitude.

Solution.—

June 2
GMT $16^h24^m15^s$ June 2
GP L 22°10.′8N
l 60°34.′1S
L 38°23.′3S

Sun		
16^h 22°10.′7N	*d*	
corr.	(+)0.′1	(+)0.′3
d 22°10.′8N		

	+	⊙ −
IC	2.′6	
D		5.′6
⊙	14.′3	
sum	16.′9	5.′6
corr.		(+) 11.′3
hs		29°14.′6
Ho		29°25.′9
z		60°34.′1

Since the observer faces *north,* he is *south* of the GP. The GP latitude is equal to the declination, and the difference of latitude is equal to the zenith distance of the body. Since the body is known to be over the meridian, a knowledge of the longitude is not needed. Neither is a knowledge of the approximate latitude of the observer needed, but this information is useful, as it provides a check to prevent a gross error. Also, it can serve as the basis for precomputation, most of the solution being made before observation. The time is needed only for determining declination. If the body is observed at *lower* transit, the latitude is equal to observed altitude (Ho) plus polar distance (p=90°−d) of the body: L=Ho+p.

If an observation is made near but not exactly at meridian transit, it can be solved as a meridian altitude, with one modification. Enter table 29 with the approximate latitude of the observer and the declination of the body, and take out the **altitude factor** (*a*). This is the difference between meridian altitude and the altitude one minute of time later (or earlier). Next, enter table 30 with the altitude factor and the difference of time between meridian transit and the time of observation, and take out the correction. *Add* this value to Ho if near upper transit, or *subtract* it from Ho if near lower transit. Then proceed as for a meridian altitude, remembering that the value obtained is the latitude at the time of observation, not at the time of meridian transit. This method should not be used beyond the limits of table 30 unless reduced accuracy is acceptable. This process is called **reduction to the meridian,** the altitude before adjustment an **ex-meridian altitude,** and the observation an **ex-meridian observation.** It requires knowledge of the meridian angle, which depends upon knowledge of longitude. If reasonable doubt exists regarding the longitude, the azimuth of the body at the time of observation should be determined, and the line of position drawn perpendicular to it (through the point defined by the "observed" latitude and the assumed longitude), rather than as a latitude line. There are alternative methods available. A correction to latitude can be applied, using the factor f from table 26. In 1899 A. A. Vilkitskiy, a captain in the Russian Navy, developed a mechanical device for determining the correction to be applied for reduction to the meridian.

Several hundred years ago, when longitude could not be found accurately, and logarithms had not been invented, the finding of latitude furnished the only reliable navigation available on long sea voyages. Since most of these were in a generally easterly or westerly direction, it became common practice to sail first to the latitude of destination ("run down the latitude") and then to follow this parallel until landfall was made. The meridian observation of the sun at local apparent noon was the most important navigational event of the day, and became a well-established routine. On the basis of this observation at "high noon," clocks were reset, and a new day, the **nautical day,** began. Intentional meridian altitudes of other celestial bodies were not as widely used as those of the sun.

As accurate time became available at sea, and then more convenient tables and more accurate almanacs appeared, the noon sight lost its importance. Since the modern inspection table has been available, the use of meridian altitudes has decreased rapidly, and reduction to the meridian has all but disappeared. True, the solution of a meridian altitude is simple and quick, but this is more than offset by the need for determining the time at which to make the observation (art. 2104), the dislike of many mariners for having to make an observation at a predetermined time, the inconvenience sometimes experienced when local apparent noon occurs at a time when other activities conflict with observation, and the possibility of missing the observation because of overcast conditions. The practice of observing a body when a line of position is desired, and solving those which happen to have a meridian angle of 0° or 180° in the same manner as other observations, is a growing practice that eliminates the need for remembering a separate procedure for bodies on the celestial meridian. The modern navigator thinks primarily in terms of lines of position, rather than of latitude and longitude observations.

Example 2.—On June 2, 1958, the 1225 DR position of a ship is lat. 41°21′.2 S, long. 127°07′.3 W. At GMT $20^h25^m48^s$ the navigator observes the lower limb of the sun with a marine sextant having no IC, from a height of eye of 40 feet. The hs is 26°20′.1.

Required.—(1) The *a*, Zn, and AP, using H.O. Pub. No. 214 (Δd only) and the *Nautical Almanac.*

(2) The latitude at the time of observation.

Solution.—

	June 2		Sun		+ ☉	−
GMT	$20^h25^m48^s$ June 2	20^h	22°12′.0 N *d*	IC	—	—
20^h	120°32′.1	corr.	(+)0′.1 (+)0′.3	D		6′.1
25^m48^s	6°27′.0	d	22°12′.1 N	☉	14′.1	
GHA	126°59′.1			sum	14′.1	6′.1
*a*λ	126°59′.1 W			corr.		(+)8′.0
LHA	0°00′.0			hs		26°20′.1
t	0°00′.0			Ho		26°28′.1
d	22°12′.1 N	d diff.	12′.1			
*a*L	41°00′.0 S					
ht	27°00′.0	Δd	(−)1.0	Z	S180°.0 (E or W)	
corr.	(−) 12′.1					
Hc	26°47′.9					
Ho	26°28′.1					
(1) *a*	19.8 A	*a*L	41°00′.0 S			
Zn	000°.0	*a*λ	126°59′.1 W			

(2) L 41°19′.8 S

Since the azimuth is 000°, the line of position is a latitude line. It is 19′.8 south (*away* from 000°) of the assumed latitude of 41°00′.0, or latitude 41°19′.8 S.

When Δd is 1.0, the altitude changes one minute for each one minute change of declination. Therefore, the correction to ht is numerically equal to d diff. For this reason, no entry is given in the "multiplication table" for a Δ value of 100 (1.0). A Δ value of 1.0 should not be confused with one of 0.01 or 0.10.

2104. Finding time of meridian transit.—If a meridian altitude is to be observed, other than by chance, a knowledge of the time of transit of the body across the meridian is needed.

On a slow-moving vessel, or one traveling approximately east or west, the time need not be known with great accuracy. The right-hand daily page of the *Nautical*

Almanac gives the GMT of transit of the sun and moon across the Greenwich meridian (approximately LMT of transit across the local meridian) under the heading "Mer. Pass." In the case of the moon, an interpolation should be made for longitude. This is performed in the same manner as finding the LMT of moonrise and moonset (art. 1812). In the case of planets, the tabulated accuracy is normally sufficient without interpolation. The time of transit of the navigational planets is given at the lower right-hand corner of each left-hand daily page of the *Nautical Almanac*. The tabulated values are for the middle day of the page. These times are the GMT of transit across the Greenwich meridian, but are approximately correct for the LMT of transit across the local meridian. Observations are started several minutes in advance and continued until the altitude reaches a maximum and starts to decrease (a minimum and starts to increase for lower transit). The greatest altitude occurs at upper transit (and the least at lower transit). This method is not reliable if there is a large northerly or southerly component of the vessel's motion, because the altitude at meridian transit changes slowly, particularly at low altitudes. At this time the change due to the vessel's motion may be considerably greater than that due to apparent motion of the body (rotation of the earth), so that the highest altitude occurs several minutes before or after meridian transit.

If the moment at which the azimuth is 000° or 180° can be determined accurately, the observation can be made at this time. However, this generally does not provide a high order of accuracy.

If the longitude is known with sufficient accuracy, the time of transit can be computed. A number of methods of computation have been devised, but perhaps the simplest is to consider the GHA of the body equal to the longitude if west, or 360°–λ if east, and find the time at which this occurs.

Example.—Find the zone time of meridian transit of the sun at longitude 156° 44!2 W on May 31, 1958.

Solution.—

		May 31
λ		156°44!2 W
GHA		156°44!2
22^h		150°36!4
24^m31^s		6°07!8
GMT		$22^h24^m31^s$ May 31
ZD	(+)	10 (rev.)
ZT		$12^h24^m31^s$

This solution is the reverse of finding GHA. The largest tabulated value of GHA that does not exceed the desired GHA is found in the tabulation for the day, and recorded, with its time. The difference between this value and the desired GHA is then used to enter the "Increments and Corrections" table. The time interval corresponding to this value is added to the time taken from the daily page. If there is a *v* correction, it is subtracted from the GHA difference before the time interval is determined. For such bodies, the *Air Almanac* solution is easier, but not as precise by a fraction of a second of time. The GMT can be converted to any other kind of time desired. If the Greenwich date differs from the local date at the time of transit (for the sun this can occur only near the 180th meridian), a second solution may be needed. This possibility can often be avoided by making an approximate mental solution in advance. As the basis for this approximate solution, it is convenient to remember that the GMT of Greenwich transit (GHA 0°) is about the same as the LMT of local transit. To find the time of transit of a star, subtract its SHA from the desired GHA to find the

desired GHA♈. Determine the time corresponding to GHA♈, as explained above for the sun.

Aboard a moving vessel, the longitude at transit usually depends upon the time of transit. An approximate mental solution may provide a time sufficiently close. In the absence of better information, use ZT 1200 for the sun. Find the time of transit for the position at this time, and then make an adjustment, if necessary, for the sun between 1200 and the time found by computation. This adjustment is equal to four seconds for each minute of longitude involved. If the ship is *west* of the 1200 position at the computed time of transit, *add* the correction; and if *east*, *subtract* it. For high accuracy a second adjustment may occasionally be needed, but this is seldom justified because of the uncertainty of the vessel's position.

The time of transit of the sun can also be found by means of apparent time (art. 1912). Meridian transit occurs at LAT $12^h00^m00^s$. This can be converted to any other kind of time desired.

2105. Latitude by Polaris.—Another special method of finding latitude, available in most of the northern hemisphere, utilizes the fact that Polaris is less than 1° from the north celestial pole. As indicated in article 1432, the altitude of the elevated pole above the celestial horizon is equal to the latitude. Since Polaris is never far from the pole, its observed altitude (Ho), with suitable corrections, is the latitude.

Three corrections are commonly applied. The first, and principal one, is illustrated in figure 2105. In this figure, *P* is the north celestial pole, and the circle is the path followed by Polaris as it circles the pole daily. When Polaris is directly above the pole, at *A*, on the upper branch of the celestial meridian, the polar distance (90°−d) is subtracted from Ho to obtain latitude. At *B*, on the lower branch of the celestial meridian, this value is *added*. At *C* and *C′*, with meridian angle approximately 90°E or W (see below), there is no correction. At any other point, such as *D*, the correction is between these extreme values, being equal to the polar distance multiplied by the cosine of the meridian angle (approximately). This is shown at *D′P*.

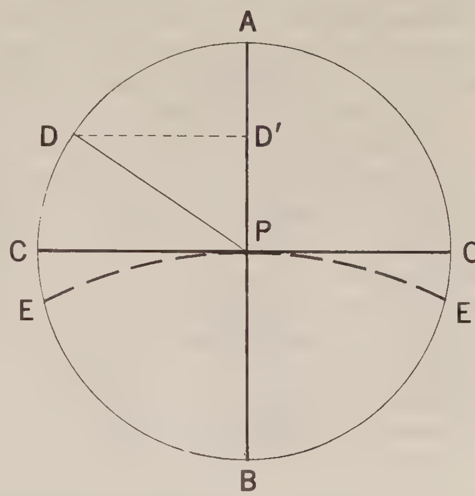


FIGURE 2105.—Latitude by Polaris.

The second correction corrects for the tilt of the diurnal circle of Polaris with respect to the vertical. Refer again to figure 2105. Zero correction occurs at *C* and *C′*, when Polaris is at the same altitude as the north celestial pole. Thus, point *C*, *P*, and *C′* are all on a parallel of altitude, which is a small circle everywhere the same angular distance above the celestial horizon. However, a meridian angle of 90°E or W occurs when Polaris is on an hour circle 90° from the celestial meridian. These 90° hour circles are not horizontal, like the circle of equal altitude, but are tilted *downward* toward the celestial horizon, which they cross at the prime vertical. Therefore, these 90° hour circles (*EPE′*) do not intersect the diurnal circle of Polaris at *C* and *C′* but at *E* and *E′*, which are at a lower altitude than the pole. The discrepancy between these points (*C–E* and *C′-E′*) increases as the latitude increases from zero at the equator to a maximum at the pole.

The third correction is needed because Polaris, being near the celestial pole, has a relatively large change in coordinates due to precession of the equinoxes (art. 1419) and aberration (art. 1417). The correction is the difference between actual coordinates and those used for the first correction.

In the Polaris correction tables of the *Nautical Almanac*, a constant is added to each correction so that all tabulated values are positive. The sum of the constants is 1°. Therefore, this value (1°) is subtracted from the result. The table at the top of each Polaris correction page is entered with LHA Aries, and the first correction (a_0) is taken out by single interpolation. The second and third corrections (a_1 and a_2, respectively) are taken from the double entry tables without interpolation, using the LHA Aries column with the latitude for the second correction and with the month for the third correction.

Example.—During morning twilight on June 2, 1958, the 0525 DR position of a ship is lat. 15°43′.6 N, long. 110°07′.3 W. At watch time $5^h24^m49^s$ AM the navigator observes Polaris with a marine sextant having an IC of (−)3′.0, from a height of eye of 44 feet. The watch is 23^s slow on zone time. The hs is 16°24′.0.

Required.—The latitude.

Solution.—

	June 2
W	$5^h24^m49^s$ AM
WE	(S) 23^s
ZT	$5^h25^m12^s$
ZD	(+)7
GMT	$12^h25^m12^s$ June 2
12^h	70°27′.0
25^m12^s	6°19′.0
GHA ♈	76°46′.0
λ	110°07′.3 W
LHA ♈	326°38′.7
Ho	16°11′.3
corr.	(−) 26′.2
L	15°45′.1 N

Polaris	+	−
a_0	33′.2	
a_1	0′.3	
a_2	0′.3	
add'l		60′.0
sum	33′.8	60′.0
corr.		(−) 26′.2

	+ ☆ −
IC	3′.0
D	6′.4
☆-P	3′.3
sum	(−) 12′.7
corr.	(−) 12′.7
hs	16°24′.0
Ho	16°11′.3

Since LHA ♈ is an entering value in all three correction tables, and since this is affected by the longitude, other observations, if available, should be solved and plotted first, to obtain a good longitude for the Polaris solution. For greater accuracy, particularly in higher latitudes, and especially if considerable doubt exists as to the longitude, it is good practice to find the azimuth of Polaris and draw the line of position perpendicular to it, through the point defined by the latitude found in the computation and the longitude used in the solution. The azimuth at various latitudes to 65° N is given below the Polaris corrections. This table can be extrapolated to higher latitudes, but Polaris would not ordinarily be used much beyond latitude 65°. In the example given above the azimuth is 000°.9.

The *Air Almanac* provides only the first correction, which it designates Q.

Polaris observations can be solved like those of other celestial bodies, using the declination and SHA given in the tabulation near the back of the *Nautical Almanac*, if a method of solution providing for such a high declination is available. H.O. Pub. No. 214 is not designed for solution of Polaris observations.

Like other special solutions, latitude by Polaris has lost much of its popularity since modern inspection tables have become available. Being of magnitude 2.1, Polaris is not a bright star. It is normally considered available to the mariner only during twilight, when the azimuths of various celestial bodies relative to each other are of

more interest than an "easy" solution which is little, if at all, simpler than the usual solution by inspection table. If provision were made for solution of Polaris sights by inspection table, the special method would no longer be needed for ordinary navigation.

2106. Longitude methods.—A celestial observation for a line of position, whether reduction is to be by longitude method or by latitude method, consists of measurement of the altitude of a body with the noting of the time. If sight reduction is to be by the longitude method, the latitude must be known, or the best estimate used. With altitude, latitude, and declination (from the almanac), one is able to solve the navigational triangle (art. 1433) for meridian angle. This is converted to local hour angle. The Greenwich hour angle at the time of observation is determined by means of the almanac. The difference between the GHA and LHA is the longitude. A time diagram (art. 1427) is useful in establishing the correct relationship.

Longitude can also be determined by establishing the exact time of meridian transit, at which time the GHA (or 360°−GHA) is the longitude (art. 2104).

If the latitude is known accurately, the longitude method provides a direct and relatively simple solution for position. However, since latitude is rarely known to the desired accuracy, a line of position is usually needed. This is obtained by either (1) solving for longitude at two or more assumed latitudes, and drawing a straight line through the points thus found (the Sumner method), or (2) solving for longitude at one point, determining the azimuth at this point, and drawing the line of position through the single point thus found, perpendicular to the azimuth of the body.

The error introduced in the computed longitude as a result of an inaccurate latitude used in the solution increases as the celestial body departs from the prime vertical. If it is learned that an incorrect latitude has been used in the solution, a correction can be applied, using the factor F from table 26. If the body is near the celestial meridian, a small error in the latitude introduces a large error in the longitude. At any location, the azimuth of the body can be determined by observation or computation, and a line of position drawn perpendicular to it, through the position defined by the latitude used in the computation, and the calculated longitude. Alternatively, solution can be made at two or more latitudes, and the line of position drawn through the two positions. It was the use of this second method in 1837 by Captain Thomas H. Sumner, when his latitude was in doubt, that led to the discovery of the line of position from celestial observation (art. 131).

No longitude method is more accurate than the GMT used for timing the observation. Before chronometers (art. 1514) and time signals (art. 1909) were available, relatively few navigators attempted to determine longitude, and it was never established reliably. The search for a method of "discovering" longitude at sea was primarily a search for a means of determining time at the Greenwich meridian (arts. 126, 127).

If the longitude is to be determined, most accurate results are obtained by observation of a body on the prime vertical. The observation having been made, sight reduction can be made by time sight or, more conveniently, by an ordinary solution for a line of position, using an inspection table such as H.O. Pub. No. 214. Any general method of sight reduction can be used, without need for a special solution.

Solution by the longitude method is usually called a **time sight.** The various longitude methods are all basically the same, differing only in choice of formulas and arrangement of tables. The basic formula is

$$\cos t = \frac{\sin h - \sin L \sin d}{\cos L \cos d},$$

in which t is the meridian angle, h is the altitude, L is the latitude, and d is the declination. Early tables for solution of meridian angle were called **horary tables.**

The time sight came into use following the development of the marine chronometer in 1763. Solution for meridian angle is usually by the formula

$$\text{hav } t = \sec L \csc p \cos s \sin (s-h),$$

in which $p=90°-d$ if L and d have same name, and $90°+d$ if L and d have contrary names; and $s=\frac{1}{2}(h+L+p)$.

When azimuth angle is used with the method, it is usually computed by one of the formulas

$$\sin Z = \sin t \cos d \sec h$$

or

$$\text{hav } (180°-Z) = \sec h \sec L \cos s \cos (s-p).$$

There are no rules with this method, but it is subject to possible large errors in high latitudes or if the body has a high declination. Various special tables have appeared for solution of the time sight:

Cassini. The first "inspection tables" were probably prepared by M. Cassini, a Frenchman, in 1770. These "horary tables" provided tabulated solutions for meridian angle.

Lalande. The horary tables prepared in 1793 by Jerome Lalande, a Frenchman, provided tabulated solution for meridian angle for the sun and stars for all latitudes to 61°.

Lynn Horary Tables, by Thomas Lynn, a commander of the East India Company Service, were published in 1827. These 242-page tables consisted of tabulated solutions of meridian angle computed by the time sight formula. Two years later they were followed by a volume of 364 pages of azimuth angle (*Lynn Azimuth Tables*) computed by the haversine azimuth formula. Entries are given for whole degrees of latitude to 60°, declination to 24°, and altitude to 60° (later 90°). The tables are accurate and well arranged, but the triple interpolation is tedious.

Hommey. Louis Hommey's *Table d'angles horaires* (horary tables), published in two volumes in France in 1863, contained more than 40,000 hour angles calculated for "all latitudes." These tables were an improvement on those of Cassini and Lalande.

Martelli. In 1873 a small volume of 49 pages by G. F. Martelli, an Italian, was published in New Orleans. This book, called simply *Tables of Logarithms*, provided a relatively short, fast solution for meridian angle, with very few rules and only one interpolation. Martelli abandoned the inspection table and provided five short tables for a four-place logarithmic solution by the formula

$$\text{hav } t = \frac{\cos (L \sim d) - \cos z}{2 \cos L \cos d}.$$

Solution required six book openings, six table entries, and four mathematical steps. Hour angles were given only to eight hours, and no provision was made for azimuth.

This method proved very popular, and is still used among navigators of several countries. A 1932 edition was published in Glasgow, Scotland, with explanations in French, Dutch, Italian, and Spanish, as well as in English. A 1944 edition added provision for finding azimuth angle, and for solution by the altitude method.

Thomson. A table of only nine pages by Sir William Thomson, better known as Lord Kelvin, was published in London in 1876 to provide a solution for the longitude method. This very thin volume, called *Tables for Facilitating Sumner's Method at Sea*, contains the first known solution by dividing the navigational triangle into two right triangles. In 1849 Towson (art. 2126) had divided the triangle in the same manner, but his solution was for reduction to the meridian. Lord Kelvin divided the triangle by dropping a perpendicular from the celestial body to the celestial meridian of the

observer, as shown in figure 2111. He used a for the length of the perpendicular v, b for x, and b' for w (of fig. 2111). His solution uses the formulas

$$\sin \mathrm{t}=\frac{\sin a}{\cos \mathrm{d}},$$

$$\cos b=\frac{\sin \mathrm{d}}{\cos a},$$

$$\sin \mathrm{h}=\cos a \cos b',$$

$$\sin \mathrm{z}=\frac{\sin a}{\cos \mathrm{h}}.$$

The tables are entered with *half* the colatitude (using colatitude to the nearest whole degree) in column b. With a pair of dividers, search is made in the "cohypo" column for two numbers, one agreeing with the altitude, and the other with the declination. The number in column A opposite the altitude in the cohypo column is the azimuth angle, and that opposite the declination is the meridian angle, interpolation being used if needed. The line of position is adjusted for the difference between the interpolated altitude and the observed altitude.

Although the tables are among the shortest of the various methods, their manipulation is difficult. In 1880 Kortazzi, a Russian astronomer, attempted to modify the tables to provide an easier solution, but without great success.

Davis' *Chronometer Tables,* providing a solution for the longitude method, were published in 1897 in London. They are similar to Lynn's tables, using his values but providing assistance in interpolation by adding values of change with latitude, declination, and altitude. As with Lynn, a separate volume is given for azimuth angle, in which there is no interpolation. Originally Davis' tables were limited to latitude 50° and declination 24°, but later tables were published for declinations 23° to 64°. A limited number of altitude entries is given.

Blackburne. Tables by H. S. Blackburne, a New Zealander, were published in London in 1914 under the title *The Excelsior Azimuth and Position Finding Table.* The tables, providing a solution by the longitude method, are similar to *Lynn Horary Tables* and Davis' *Chronometer Tables* but with a new determination of azimuth based upon the ratio of variation of latitude to variation of meridian angle. Azimuth angles (ten pages) are given in a separate tabulation in the same volume with meridian angles (242 pages).

Blackburne's arrangement is more modern than that of Davis. This was the first publication to include columns for variations of t for 1′ of declination, latitude, and altitude. Meridian angles are given to 0ˢ1. Latitude is limited to 30°, and declination to 23°.

Rust. In 1918 the *Practical Tables for Navigators and Aviators,* by Captain Armistead Rust, USN, were published in Philadelphia. This small volume of 37 pages of tables reverted to a logarithmic solution, as did Martelli's, using the following formula for determining meridian angle:

$$\log \mathrm{hav}\ \mathrm{t}=\log \sec \mathrm{L}+\log \sec \mathrm{d}+\log \tfrac{1}{2}\,[\cos (\mathrm{L}\sim\mathrm{d})-\sin \mathrm{h}].$$

The volume has three tables. Table A tabulates log secants for obtaining the first two terms of the formula. Table B is a double-entry table giving log ½[cos (L∼d) −sin h]. Table C gives log haversines. Values are given to four places.

Azimuth angle is obtained from an original diagram computed from the well-known formula

$$\sin \mathrm{Z}=\sin \mathrm{t} \cos \mathrm{d} \sec \mathrm{h}.$$

This diagram had been given in a volume of ex-meridian tables by Rust published in 1908. In the *Practical Tables* an auxiliary diagram was added to indicate the meridian angle when the celestial body is on the prime vertical. The purpose of the diagram is to resolve possible ambiguities when the azimuth angle is near 90°. The Rust azimuth diagram was used later by Goodwin and Weems, and in the Italian *Tavole H* (art. 2110).

Goodwin. *The Alpha, Beta, Gamma Navigation Tables* of H. B. Goodwin, an Englishman, were first published in London in 1921. This is a small volume having two tables with a total of 34 pages. Meridian angle is found from the formula

$$\text{ver } t = \frac{\cos (L \sim d) - \cos z}{\cos L \cos d}.$$

Table I has two values, α being the angle in seconds of arc, and β being four-place natural cosines multiplied by 1,000 to eliminate the decimal. Table II provides γ, the logarithms for the values of versine t.

The Rust diagram is used for determining azimuth angle.

Instructions are included for use of the tables for altitude method of solution, and for reduction to the meridian.

H.O. Pubs. Nos. 203 and 204 (Littlehales), *The Sumner Line of Position of Celestial Bodies*, were published by the U. S. Navy Hydrographic Office in 1923. These tables, prepared by George W. Littlehales, provide in two large volumes (847 and 675 pages, respectively) tabulated solutions of the meridian angle and azimuth angle, using the general time sight formulas. The arrangement is similar to that of Davis and Blackburne, but t and Z are tabulated together in consecutive columns. Latitude is limited to 60°, and declination to 27° in H.O. Pub. No. 203 and to 64° in H.O. Pub. No. 204. Interpolation for latitude is avoided by using the nearest whole degree, and shifting the line of position for the difference between the altitude at this latitude and the observed altitude.

These publications are no longer in print.

Soule and Dreisonstok. In 1932 these two Americans prepared a small volume providing a logarithmic solution of the longitude method, using the formulas

$$\frac{1}{\text{hav } t} = \frac{\sec s \csc (s-h)}{\sec L \csc p}$$

and

$$\frac{1}{\text{hav } (180° - Z)} = \frac{\sec s \sec (s-p)}{\sec L \sec h},$$

where $s = \frac{1}{2}(h+L+p)$ and $p = 90° \pm d$.

The "azimuth" determined in this way is the direction of the line of position ($Z \pm 90°$) rather than that of the celestial body.

Weems' ***Secant Time Sight*** was published in 1944 by Captain P. V. H. Weems, USN (Ret.), to provide a short solution based entirely upon secants and cosecants, using the formulas

$$\csc^2 \tfrac{1}{2} t = \frac{\sec s \csc (s-h)}{\sec L \sec d}$$

and

$$\csc Z = \frac{\csc t \sec d}{\sec h},$$

where $s = \frac{1}{2}(h+L+p)$. A Rust azimuth diagram is included for those who prefer a diagrammatic solution.

2107. Finding time on prime vertical.—Best results by time sight are obtained when the celestial body is on the prime vertical. As explained in article 1432, a celestial

body having a declination of opposite name to the latitude crosses the prime vertical below the horizon. Its nearest visible approach is at the time of rising and setting.

If a celestial body has a declination of the same name as the latitude, but is numerically *greater*, it does not cross the prime vertical. Its nearest approach (in azimuth) is at the point at which its azimuth angle is maximum. At this point the meridian angle is given by the formula

$$\sec t = \tan d \cot L,$$

and its altitude by the formula

$$\csc h = \sin d \csc L.$$

A celestial body having a declination of the same name as the latitude, and numerically *smaller*, crosses the prime vertical at some point before it reaches the celestial meridian, and again after meridian transit. At these two crossings of the prime vertical, the meridian angles are equal and are always less than 90°. They are given by the formula

$$\cos t = \tan d \cot L.$$

The altitudes are also equal, and are given by the formula

$$\sin h = \sin d \csc L.$$

Meridian angle and altitude of bodies on the prime vertical, and similar data for the nearest approach (in azimuth) of those bodies of same name which do not cross the prime vertical, are given in table 25 for various latitudes, and for declinations from 0° to 23°, inclusive. Similar information can be determined by means of H.O. Pub. No. 214, entering with latitude and declination, and finding the meridian angle and altitude corresponding to an azimuth angle of 90° (or the maximum azimuth angle for nearest approach). Since this information is generally not required to great accuracy, interpolation is not needed.

To find the *time* of crossing the prime vertical, convert t to LHA, and add west longitude or subtract east longitude to find GHA. The GMT at which this GHA occurs can be found, as explained in article 2104, and converted to any other time desired.

Example.—Determine (1) the approximate zone time, and (2) the approximate altitude of the sun when it crosses the prime vertical during the afternoon of May 30, 1958, at lat. 51°32′.3 N, long. 160°21′.7 W, using table 25 and the *Nautical Almanac.*

Solution.—

	May 30	
t	71°.6 W	(from table 25)
LHA	71°.6	
λ	160°.4 W	
GHA	232°.0	
3^h	225°.6	
26^m	6°.4	
GMT	0326	May 31
ZD (+)	11	(rev.)
(1) ZT	1626	May 30
(2) h	28°.4	(from table 25)

At the time of crossing the prime vertical, or at nearest approach (in azimuth), a celestial body is changing azimuth slowly, and therefore this is considered a good time to check compass deviation or to swing ship.

The prime vertical at any place is the celestial horizon of a point 90° away, on the same meridian. Therefore, a celestial body crosses the prime vertical at approximately

the same time it rises and sets at the point 90° away. Thus, if one is at latitude 35° N, the sun crosses his prime vertical at about the same time it rises or sets at latitude 55° S. If time of sunrise and sunset are to be obtained accurately by this method, corrections must be applied for semidiameter and refraction.

2108. Altitude methods, like longitude methods, require an accurately timed observed altitude of a celestial body. Usually, in both types of solution, the navigational triangle is solved, but in the altitude method, t, d, and L are used in solving for altitude. The method is based upon the concept of circles of equal altitude explained by Commander Marcq St.-Hilaire, a Frenchman, in 1875 (art. 131). For this reason it is often called the **Marcq St.-Hilaire method.** It may also be called the **altitude intercept method** because it uses the difference between computed and observed altitudes, a value sometimes called an **altitude intercept.**

The altitude method has largely replaced the latitude and longitude methods, although some navigators still prefer the older methods. The principal advantage of the altitude method is that it provides a universal solution that is equally reliable in all latitudes, with all values of declination, and with all values of meridian angle. Even for observations of celestial bodies near the zenith the altitude method is applicable, although in this case an arc of the circle itself is plotted, without the use of the altitude difference (art. 2011). However, the formulas selected for some of the "short methods" do impose some limitations when those methods are used.

For many years following introduction of the altitude method, the concept was termed the "new navigation," an expression now seldom heard. At first, an attempt was made to adapt existing tables to the altitude method. Some were more readily adaptable than others. In due course, various methods designed for use with the altitude method made their appearance. These methods may be grouped in six classifications: those which do not divide the triangle, those which divide it by dropping a perpendicular from each of the three vertices, those which do not use the navigational triangle, and the modern "inspection table." However, not all inspection tables are for altitude methods. In practice, the dropping of a perpendicular from the pole has not been used except in great-circle sailing (art. 822). This would result in dividing both the meridian angle and zenith distance into two parts, a condition that has not proved attractive.

2109. Altitude methods, triangle not divided.—The basic formula for solution of the undivided navigational triangle is

$$\sin h = \sin L \sin d + \cos L \cos d \cos t,$$

derived from the law of cosines. A number of special tables have been prepared for solution of the undivided triangle:

Davis' *Requisite Tables*, published in London in 1905, introduced the **cosine-haversine formula** to navigation, although it had been used previously by astronomers. This formula is

$$\text{hav } z = \text{hav } (L \sim d) + \cos L \cos d \text{ hav } t,$$

in which z is zenith distance $(90° - h)$. This is sometimes written

$$\text{hav } z = \text{hav } (L \sim d) + \text{hav } \theta,$$

in which hav $\theta = \cos L \cos d$ hav t. It might also be written entirely in haversines:

$$\text{hav } z = \text{hav } (d - L) + \text{hav } t \, [\text{hav } (180° - L - d) - \text{hav } (d - L)].$$

In this formula the sign of d is reversed if L and d are of contrary name. The haversine of an angle is positive whether the angle is positive or negative.

These tables were the first to give log haversines and natural haversines in one table. The method was little used at first, but later proved very popular, and as haversines became available from additional sources, the formula was used even more widely. Davis' original tables made no provision for azimuth.

Since the cosine-haversine formula can be used for solution with tables 33 and 34, an example is given below, with solution of azimuth by the formula

$$\sin Z = \sin t \cos d \sec h.$$

Example.—A celestial body a little to the south of west is observed, with the following results: t 80°45′.9 W, *a*L 41°12′.3 S, d 21°50′.7 S.

Required.—The Hc and Zn by the formulas given above.

Solution.—

t 80°45′.9 W	*l* hav 9.62300		*l* sin 9.99434
*a*L 41°12′.3 S	*l* cos 9.87643		
d 21°50′.7 S	*l* cos 9.96764		*l* cos 9.96764
θ —	*l* hav 9.46707	*n* hav 0.29313	
L∼d 19°21′.6		*n* hav 0.02827	
z 69°04′.3		*n* hav 0.32140	
Hc 20°55′.7			*l* sec 10.02964
Zn 258°.8		Z S 78°47′.0 W	*l* sin 9.99162

This is typical of logarithmic solutions, except that there are no "rules" for the altitude computation. In this example the coordinates are the same as those of example 3, article 2008, where solution is by H.O. Pub. No. 214, using Δd, Δt, ΔL. The slight differences are due to the interpolation and rounding off of different quantities. As pointed out in article 2125, the formula used for azimuth angle does not indicate whether the body is north or south of the prime vertical.

Ball. In 1907 Rev. Frederick Ball's *Altitude or Position Line Tables* were published in London. There are two volumes of 244 and 240 (later 313) pages, respectively, the first volume having tables for latitude 0° to 30°, and the second, 31° to 60° (later editions 24° to 60°). These were the first inspection tables for the altitude method. The tabulated altitudes were computed by the haversine formula. The assumed position is selected so that latitude and meridian angle are the nearest whole degree, but no assistance is given for interpolation of altitude for declination.

Azimuth angle is not tabulated, being found by the altitude tables, interchanging altitude and declination, and finding azimuth angle (in hours and minutes) in the meridian angle columns. Since declination is limited to 24° in the first edition and 60° in later editions, this method is not available for azimuth if altitude is greater than this amount. In this case azimuth angle is found by the formula

$$\sin Z = \frac{\sec L \times \Delta h \text{ (for } 8^{m})}{120}.$$

This formula had not previously been used in navigation.

Davis' ***Alt-Azimuth Tables*** were published in London in 1917. This volume lists both altitude and azimuth together for the first time. Latitudes included are from 30° to 64°, and declinations from 0° to 24°. In 1921 a second volume was published for latitudes 0° to 30°. Entries are for each whole degree of latitude and declination, and for each 4^{m} (1°) of meridian angle. However, for each meridian angle, altitude *or* azimuth angle is given alternately. Thus, azimuth angle is given for meridian angles of 0^{m}, 8^{m}, 16^{m}, 24^{m}, etc., and altitude for meridian angles of 4^{m}, 12^{m}, 20^{m}, 28^{m}, etc. Altitudes are given in bold type. All declination entries (0° to 24°) are given on facing pages. Tables for latitude and declination of the same name are

given in the first part of the book and those for contrary names in the last part, the two parts being separated by several auxiliary tables. Altitude is given to the nearest 1′, and azimuth angle to the nearest 0°1. Altitudes are carried down to the horizon, and the local apparent times of sunrise and sunset are also given, with the azimuth angle at these times. Because of the 8^m interval between altitude entries, and the use of an assumed position to avoid interpolation for change in meridian angle, large altitude differences sometimes arise.

H.O. Pub. No. 201, *Simultaneous Altitudes and Azimuths of Celestial Bodies*, was published by the U. S. Navy Hydrographic Office in 1919. In this volume of 606 pages, altitudes and azimuths were tabulated in parallel columns for the first time. Latitude is limited to 60° and declination to 24°. Virtually all altitudes above the horizon are included. The tables are well arranged and very legible, but no assistance is given for interpolation of altitude or azimuth angle for a change of declination. Meridian angles are given at intervals of 10^m (2°5). Since the assumed position is selected to avoid interpolation of altitude or azimuth for meridian angle, large altitude differences result in some instances. This publication is no longer in print.

Yonemura. In 1920 S. Yonemura's *Tables for Calculating Altitude and Azimuth of Celestial Bodies* were published in Japan. This small table of 39 pages contains logarithms of haversines and secants, arranged for convenient solution of the formulas

$$\text{hav }(90°-h)-\text{hav }(L\sim d)=\text{hav }\theta,$$

$$\log\frac{1}{\text{hav }\theta}-(\log\sec L+\log\sec d)=\log\frac{1}{\text{hav } t},$$

$$\log\csc Z=\log\csc t+\log\sec d-\log\sec h.$$

The method is similar to that of Davis' *Requisite Tables* but includes solution for azimuth angle. The table is included in the book of Ogura's tables (art. 2110).

Braga. The *Táboas de Alturas* by Romêo Braga, a Brazilian, were published in 1924 in Paris. This is a table of natural haversines arranged for solution of the formula

$$\text{hav } h=A+B,$$

in which $$A=\text{hav } t-[\text{hav }(L+d)\text{ hav } t]$$

and $$B=\text{hav }(L-d)-[\text{hav }(L-d)\text{ hav } t].$$

The first table of 108 pages is for solution for A and B. The second table of nine pages is for finding h.

The assumed latitude is selected so that (L+d) is a whole degree. The assumed longitude is selected so that t is a whole degree.

No provision is made for azimuth.

Japanese H.O. Pub. No. 601, *Celestial Navigation Computation Tables*, was published in 1942. The method is similar to that of Yonemura, the triangle not being divided and a modification of the cosine-haversine formula being used for altitude.

Waller. In 1946 George W. D. Waller, a naval officer on duty as a navigation instructor at the U. S. Naval Academy, proposed a solution by means of Gaussian logarithms, commonly called "addition and subtraction logs." The formula

$$\csc h=\frac{\csc d\csc L}{1+\dfrac{\csc d\csc L}{\sec d\sec L\sec t}}$$

was derived from the basic formula given above,

and $$\csc Z=\frac{\csc t\sec d}{\sec h}$$

was derived from the time and altitude azimuth formula given in article 2125.

A single table of 30 pages would contain in consecutive columns the following values:

$$\begin{aligned}
A\ &= \log \text{secant} \\
B\ &= \log \text{cosecant} \\
C- &= \log(\text{cosecant}+1) - \log \text{cosecant} \\
C+ &= \log \text{cosecant} - \log(\text{cosecant}-1).
\end{aligned}$$

All values would be multiplied by 100,000 to eliminate decimals. One additional page would contain A and B values for all whole degrees from 0° to 180°. The values C+ and C− are the Gaussian logs.

The method is reasonably short and simple. Its publication as the "A, B, C Method," with suitable explanation, was prevented by the untimely death of its originator.

Hugon. *Nouvelles Tables Pour le Calcul de la Droite de Hauteur a Partir du Point Estimé*, by the French hydrographic engineer, Professor P. Hugon, were published in 1947. This logarithmic solution is based upon the fundamental formula

$$\sin h = \sin L \sin d + \cos L \cos d \cos t,$$

from which the following is derived:

$$\text{hav } z = Xy + Yx$$

in which

$$\begin{aligned}
X &= \text{hav } (180° - t) = \text{cohav } t \\
y &= \text{hav } (d - L) \\
Y &= \text{hav } t \\
x &= \text{hav } [180° - (d + L)] = \text{cohav } (d + L).
\end{aligned}$$

The formula for z may then be written

$$\text{hav } z = A + B$$

in which

$$\begin{aligned}
\log A &= \log X + \log y \\
\log B &= \log Y + \log x.
\end{aligned}$$

Solution is by means of a table of 90 pages which lists in parallel columns values of log cohav, log hav, and natural hav for every minute of arc from 0° to 180°. The solution requires six book openings, seven table entries, and five mathematical steps.

Azimuth is found from a diagram in a pocket on the inside back cover. This diagram is designed to solve the formula

$$M = \alpha X + \beta Y,$$

in which

$$\begin{aligned}
M &= \cos h \cos Z \\
\alpha &= \sin (d - L) \\
X &= \cos^2 \frac{t}{2} \\
\beta &= \sin (d + L) \\
Y &= \sin^2 \frac{t}{2}.
\end{aligned}$$

Chiesa. About 1948 the *Tavole nautiche e Tavole dei Semisenoversi* of the Italian Stefano Chiesa were published in Genova, Italy. These include tables for computation of altitude by the cosine-haversine formula, and "A, B, C" tables for computation of azimuth angle by the formula

$$\text{hav } Z = [\text{hav } p - \text{hav}(L - h)] \sec L \sec h.$$

Rose. In 1952 the *Nautische Tafeln* of G. Rose were published in Germany. This volume has a convenient table for computation of the altitude by the cosine-haversine formula. It also includes the "A, B, C" azimuth tables of Lecky (art. 2126). Various other tables are included, a number of them having been taken from an earlier work of the same name by Dr. Otto Fulst, published in numerous editions since 1860.

Doniol. The *Miniature Navigation Table for Altitude and Azimuth*, by R. Doniol, a Frenchman, was published in 1955. This is undoubtedly the shortest of all sight reduction tables, consisting of only two pages. The formula for altitude was derived from the basic formula given above. The formula used is

$$\sin h = n - (n + m)\, a,$$

in which $n = \cos (L - d)$, $m = \cos (L + d)$, and $a = \text{hav } t$.
The formula for Z was derived from the formula

$$\cot Z = \frac{\tan d \cos L - \cos t \sin L}{\sin t}.$$

The formula used is

$$\tan Z = \frac{\cos d}{\gamma},$$

in which $\gamma = f\Delta_M + f'\Delta_N$. In this expression, $f = \frac{\tan \frac{1}{2} t}{2 \sin 1'}$, $f' = \frac{\cot \frac{1}{2} t}{2 \sin 1'}$, $\Delta_M = \sin (d + L) \sin 1'$, and $\Delta_N = \sin (d - L) \sin 1'$.

The first of the two tables gives sines and cosines for each half degree, and tangents for half degrees of 45° and more. Interpolation is performed by means of a tabulated Δ value which is the change of sine or cosine for 1′. Interpolation is minimized by selecting an assumed position so that t and either (L+d) or (L−d) are an exact half degree.

The second table gives the value of t in degrees, minutes, and seconds, and the values of f and f′ corresponding to selected values of *a* (natural haversines). The interval between consecutive tabulated values of haversine varies from 0.0002 to 0.005.

The solution is generally accurate to 0′.1 of altitude and 0°.1 of azimuth, but the method requires a number of relatively simple mathematical steps, making it somewhat longer than most "short" solutions.

2110. Altitude methods, perpendicular from zenith.—In figure 2110 the navigational triangle is shown in heavy lines on a diagram on the plane of the celestial meridian (art. 1432). The broken line is a perpendicular from the zenith to the hour circle of the celestial body. This perpendicular may fall outside the triangle. In figure 2110 it divides both the azimuth angle (at Z) and the codeclination side into two parts. The length of

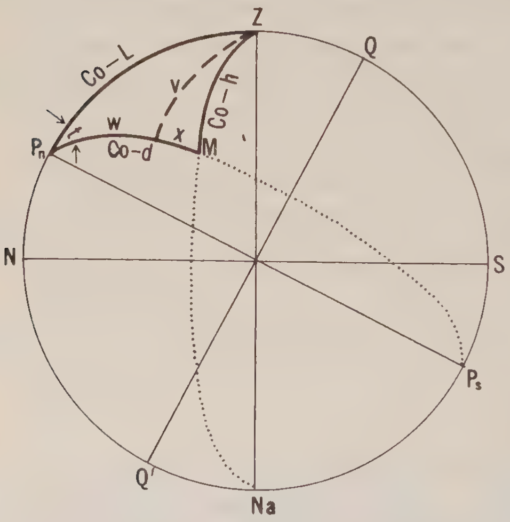


FIGURE 2110.—Navigational triangle with perpendicular from zenith to hour circle.

the perpendicular is designated v and the two parts of the codeclination are designated w and x. By means of Napier's rules (art. O42), the following basic formulas can be derived:

$$\sin v = \cos L \sin t \tag{1}$$

$$\cos w = \sin L \sec v, \text{ or } \tan w = \cot L \cos t \tag{2}$$

$$\sin Z' = \sin w \sec L, \text{ or } \cot Z' = \sin L \tan t \tag{3}$$

$$\sin h = \cos v \cos x \tag{4}$$

$$\sin Z'' = \sin x \sec h, \text{ or } \cos Z'' = \tan v \tan h, \tag{5}$$

in which $x = 90° - (d + w)$.

This basic method has been modified in a number of ways, having proved the most popular altitude method.

Souillagouet, a French professor of hydrography, was the first to divide the navigational triangle by dropping a perpendicular from the zenith. His *Tables du Point Auxiliare* were published in France in 1891. He designates various parts of the diagram of figure 2110 as follows:

v is designated a

w is designated b

x is designated $90° - (d \sim b)$.

His formulas for altitude are

$$\tan b = \cot L \cos t$$

$$\sin a = \cos L \sin t$$

$$\sin h = \cos a \sin (d \sim b).$$

For azimuth angle, the perpendicular is dropped from the celestial body to the celestial meridian, a being the perpendicular and b that part of the celestial meridian from the pole to the foot of the perpendicular. The following formulas are used:

$$\tan b = \cos t \cot d$$

$$\sin a = \sin t \cos d$$

$$\cot Z = \cos (L + b) \cot a.$$

The assumed position is selected so that latitude is the nearest 15′ and meridian angle is the nearest 1^m or 15′ (2^m or 30′ for latitudes greater than 60°). There are four separate tables with a total of 408 pages. The method requires five book openings, seven table entries, and six mathematical steps. Interpolation is not needed.

Bertin. A French professor of hydrography, Charles Bertin, devised tables similar to those of Souillagouet, which were published in Paris in 1919 under the title *Tablette de Point Sphérique*. Bertin used Souillagouet's formulas for altitude, but for azimuth angle he dropped the perpendicular from the zenith, as for altitude, and used the following formulas:

$$\sin Z_1 = \sin c \sec L$$

$$\cot Z_2 = \sin b \tan (c \sim d)$$

$$Z = Z_1 + Z_2.$$

In these formulas, b and c are substituted for the a and b, respectively, of Souillagouet. This is the first method in which Z is divided into two parts, found separately.

The assumed position is selected so that latitude and meridian angle are each to the nearest 20′. The tables are shorter than those of Souillagouet, having 324 pages, but still bulky for this type solution. The method has fewer steps and requires only two book openings, but interpolation is needed in two steps.

Ogura. In 1920 the *New Altitude and Azimuth Tables* by Sinkiti Ogura, Japanese hydrographic engineer, were published in Tokyo. The solution for altitude is generally similar to that of Souillagouet, a perpendicular being dropped from the zenith, but Ogura introduced secants and cosecants for the first time in this type solution. His solution for azimuth is similar to that of Blackburne (art. 2106) and Lecky (art. 2126).

The assumed position is selected so that latitude and meridian angle are each to the nearest whole degree. The altitude is determined by means of two tables (A and B-C) of a total of 27 pages, and azimuth by means of three additional tables (D, E, F) of a total of 29 pages. The altitude can be obtained to an accuracy of 0′.6 without interpolation. Latitude and declination are limited to 65°. The rules are numerous and complicated.

Both the Ogura and Yonemura (art. 2109) tables are given in the same book, the Japanese Hydrographic Office Pub. No. 225. An English edition, with slight modifications in the Ogura method, was published in 1924.

The Ogura tables have been widely copied.

Smart and Shearme's *Position Line Tables* were published in London in 1922, based upon a division of the triangle by a perpendicular from the zenith. The altitude formulas of Souillagouet were used, but the arrangement of the earlier tables was improved. It is somewhat similar to that of Ogura, but with the positions of meridian angle and latitude interchanged, providing a better arrangement when solutions of several observations are made simultaneously. Solution requires a log sine table which is not provided. There is no solution for azimuth. The assumed position is selected so that the meridian angle and latitude are each the nearest whole degree. No interpolation is needed.

Newton and Pinto. The *Navegação Moderna* of J. A. Newton and J. C. Pinto was published in Lisboa, Portugal, in 1924, providing a solution by dropping a perpendicular from the zenith. The method is based upon ideas expressed by Newton in 1912 and 1913. The formulas for altitude are almost the same as those of Souillagouet. The method of finding azimuth angle resembles somewhat the method of Bertin, but with the use of auxiliary angles. There are only two tables, the first occupying 120 pages, and the second two pages. The assumed position is selected so that latitude and meridian angle are each to the nearest 30′. No interpolation is needed, but the rules are somewhat complicated.

Weems' *Line of Position Book,* published in 1927, combines the Ogura altitude tables and the Rust azimuth diagram (art. 2106).

H.O. Pub. No. 208 (Dreisonstok), *Navigation Tables for Mariners and Aviators,* was published by the U. S. Navy Hydrographic Office in 1928 to provide a solution by the method of dropping a perpendicular from the zenith. The method, devised by Lieutenant Commander J. Y. Dreisonstok, USN, is similar to Ogura's. For altitude, it uses the Souillagouet formula inverted so as to be in secants and cosecants. For azimuth angle the formula is similar to that of Newton and Pinto, except that it does not use the parallactic angle at the celestial body. In the first edition the latitude was limited to 65°. There were two tables, one of 45 pages and the other of 18 pages. Later, a 23-page addition to the first table extended the coverage to the poles.

The assumed position is selected so that the latitude and meridian angle are each the nearest whole degree. The method requires four book openings, eight table entries, and six mathematical steps. Although values are usually obtained by relatively easy interpolation, altitude accuracy of 0′.5 can be obtained without interpolation.

As with H.O. Pub. No. 211 (art. 2111), the rules for this method were made on the assumption that only bodies above the celestial horizon would be observed. The rules may be restated to allow for both positive and negative altitudes, as follows:

If t is less than 90°, give b same name as latitude.

If t is greater than 90°, give b opposite name to latitude, and mark Z′ minus.

If (d+b) is numerically greater than 90°, mark Z″ minus.

If (d+b) is contrary name to latitude, the altitude is negative; use the supplement of Z″.

If Z is minus, subtract from 360° and mark plus.

The value labeled "t" in the tables is actually LHA. If t, east or west, is used, as in modern practice, the printed values greater than 180° can be ignored. The rules can be stated in abbreviated form on alternate pages, as follows:

At the top of each left-hand page of table I:

$$t < 90°, \text{ b same name as L.}$$

At the top of each right-hand page of table I:

$$t > 90°, \text{ b contrary name to L, Z}' \ (-).$$

At the top of each left-hand page of table II:

$$(d+b) > 90°, \ Z'' \ (-).$$

At the top of each right-hand page of table II:

$$Z \ (-), \text{ use } 360° - Z.$$

At the bottom of each page of table II (if desired):

$$(d+b) \text{ contrary name to L, Hc } (-)\text{: use } 180° - Z''.$$

If the D+B value used for finding Z″ exceeds 10,000, it is reduced by this amount, the remainder being used for entering table II. If desired, this can be stated in abbreviated form at the bottom of alternate pages of table II, as follows:

$$(C+D) > 10{,}000, \text{ use } (C+D) - 10{,}000.$$

Like H.O. Pub. No. 211 (art. 2111), H.O. Pub. No. 208 has been largely superseded by H.O. Pub. No. 214 (art. 2113).

Gingrich. The *Aerial and Marine Navigation Tables*, by Lieutenant John E. Gingrich, USN, were published in 1931 to provide another solution by the method of dropping a perpendicular from the zenith. The formulas for altitude are similar to those of Ogura, and the formulas for azimuth are similar to those of Perrin (art. 2126). The first two tables, of 31 and seven pages, respectively, are similar to those of Ogura. A single third table of 13 pages is given for azimuth. The general arrangement is in many respects similar to that of H.O. Pub. No. 208, and as with the earlier method, the assumed position is selected so that latitude and meridian angle are each to the nearest whole degree. The precision of tabulation of K, an auxiliary function, is not consistent. Consequently, if the tables are used without interpolation, errors as great as 0′.5 can arise in the computed altitude.

Weems' ***New Line of Position Tables*** are sometimes called the *Manuscript Tables* because they were in manuscript form from 1932 until they were published in 1943. They are similar to his earlier tables but arranged with the position of latitude and meridian angle values interchanged so that values for several observations can be taken from the tables with a single book opening. The latitude values are extended from the 65° given in earlier tables to 90°. As in the earlier edition, the Rust azimuth diagram (art. 2106) is included, but provision is also made for computation of azimuth angle. One part is found in terms of latitude and meridian angle, using the formula of H.O. Pub. No. 208, and the other part is found in terms of altitude and the perpendicular from the zenith. If the azimuth is required to a greater precision than 0°.5, interpolation is needed. The assumed position is selected so that the latitude and meridian angle are each the nearest whole degree.

Collins. The *I. C. S. Altitude and Azimuth Tables for Air and Sea Navigation*, by Elmer B. Collins, formerly of the U. S. Navy Hydrographic Office, were published by the International Correspondence Schools in 1934. The tables and method of solution are generally similar to those of H.O. Pub. No. 208.

F-Tafel, published by the German Oberkommandos der Kriegsmarine about 1937, divides the triangle by a perpendicular from the zenith. The formulas of Souillagouet are used for altitude. Azimuth is found by the familiar formula

$$\sin Z = \sin t \cos d \sec h.$$

There are four tables. Latitude, declination, and altitude are limited to 70°. The assumed position is selected so that latitude and meridian angle are each to the nearest whole degree.

Comrie. In 1938 the *Hughes' Tables for Sea and Air Navigation*, by L. J. Comrie, former Superintendent, H. M. Nautical Almanac Office, were published in London. These tables are similar to those of H.O. Pub. No. 208, but arranged with the positions of latitude and meridian angle interchanged as in the Weems' *New Line of Position Tables*.

Myerscough and Hamilton. The *Rapid Navigation Tables*, by W. Myerscough and W. Hamilton, were published in London in 1939. A perpendicular is dropped from the zenith to the hour circle of the celestial body. With slight modification, the altitude formulas of Souillagouet and the azimuth formula of Gingrich are used. Six quantities are tabulated in a single table of 90 pages. Both altitude and latitude are limited to 70°.

Ageton's ***Manual of Celestial Navigation,*** published in 1942, combines the first table of Weems' *New Line of Position Tables* as table I, and H.O. Pub. No. 211 (art. 2111) as table II. The basic formulas are restated in terms of secants and cosecants. The result is a short, easy solution without interpolation, involving four book openings, eight table entries, and four mathematical steps. Since the H.O. Pub. No. 211 table is included, the book can be used for Ageton's earlier method.

Benest and Timberlake. The *Astro-Navigation Tables for the Common Tangent Method* by two British professors, E. E. Benest and E. M. Timberlake, were published in 1945. In three tables of 61, 18, and 12 pages is given a logarithmic solution for altitude only, by dropping a perpendicular from the zenith. The formulas are slight modifications of those of Ogura.

The location of the line of position is somewhat similar to the method sometimes used in longitude method solutions such as H.O. Pubs. Nos. 203 and 204 (art. 2106). Two assumed positions are selected, usually 1° apart on the same meridian. The altitude difference at each position is determined, and a circle, or arc of a circle, is drawn with the assumed position as the center, and the altitude difference as the radius.

The line of position is the common tangent to the two circles. Since there are four common tangents, the general direction of the body is required. Where doubt exists as to which of two or more answers is the correct one, additional solutions from other assumed positions may resolve the ambiguity. If the celestial body is near the meridian, the two assumed positions are better taken on the same parallel of latitude. Even with these precautions, there is danger of selection of the wrong line.

Tavole H (*I. I. 3113*), published by the Istituto Idrografico della Marina of Italy in 1947, combines table I of Ogura and table II of Weems' *New Line of Position Tables*, including, also, the Rust azimuth diagram (art. 2106). This table is a modification of an earlier *Tavole F*.

2111. Altitude methods, perpendicular from body.—Figure 2111 is a diagram on the plane of the celestial meridian (art. 1432), with the navigational triangle shown in heavy lines. A perpendicular from the celestial body, *M*, to the celestial meridian divides the triangle into two right spherical triangles. In figure 2111 the length of the perpendicular is designated v and the two parts of the colatitude are designated w and x. By means of Napier's rules (art. O42), the following basic formulas can be derived:

$$\sin v = \cos \mathrm{d} \sin \mathrm{t} \qquad (1)$$
$$\cos w = \sin \mathrm{d} \sec v, \text{ or } \sin w = \cot \mathrm{t} \tan v \qquad (2)$$
$$\sin \mathrm{h} = \cos v \cos x \qquad (3)$$
$$\sin \mathrm{Z} = \sin v \sec \mathrm{h}, \text{ or } \cos \mathrm{Z} = \tan x \tan \mathrm{h} \qquad (4)$$

Since $x = 90° - (w + \mathrm{L})$, formula (3) can be written in terms of latitude, and w found from equation (2). Thus, both h and Z can be determined by means of t, d, and L and auxiliary functions found from them.

William Thomson (Lord Kelvin) was the first to divide the navigational triangle as shown in figure 2111 for sight reduction, but his method (art. 2106) was for determination of longitude. Various later methods made such a division for determination of altitude.

Fuss. The *Tables to Find Altitudes and Azimuths*, devised by V. E. Fuss, an astronomer at the Kronstadt (Russia) Naval Observatory, were published in 1901. In these tables a perpendicular is dropped from the celestial body, the following notation being used (fig. 2111):

v is designated a
w is designated 90°–b
x is designated B–90°
$\mathrm{B} = 90° - \mathrm{L} + \mathrm{b}$ (if v falls between Z and Q).

Solution is by the following formulas:

$$\sin \mathrm{a} = \cos \mathrm{d} \sin \mathrm{t}$$
$$\cot \mathrm{b} = \cot \mathrm{d} \cos \mathrm{t}$$
$$\sin \mathrm{h} = \cos \mathrm{a} \sin \mathrm{B}$$
$$\cot \mathrm{Z} = \cot \mathrm{a} \cos \mathrm{B}.$$

The assumed latitude is selected to provide the nearest 15′ value of B. The assumed longitude is selected so that t will be the nearest whole 1^m (0°25). The tables are entered twice, first with t and d to find a and b, interpolating for d, and then with B and a to find h and Z, interpolating for a. The method involves two book openings, eight table entries, four interpolations, and ten mathematical steps. There are 144 pages of tables.

Aquino. The *Altitude and Azimuth Tables* of Radler de Aquino, a Brazilian naval officer, were first published in 1909. These were followed the next year by his *Sea and*

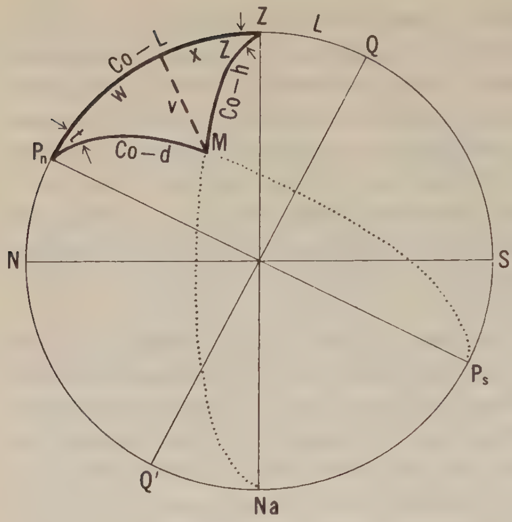


FIGURE 2111.—Navigational triangle with perpendicular from celestial body to celestial meridian.

Air Navigation Tables. Several later editions of both publications appeared with some modification, principally of the auxiliary material given. Aquino dropped a perpendicular from the celestial body to the celestial meridian, and used the same formulas as Fuss and generally the same arrangement, except that longitude is assumed so as to provide a meridian angle to the nearest whole degree.

H.O. Pub. No. 209 (Pierce), *Position Tables for Aerial and Surface Navigation,* was published by the U. S. Navy Hydrographic Office in 1930. These tables were devised by Commander M. R. Pierce, USN, in 1925, when he was navigator of the dirigible USS *Los Angeles.* The method is based upon a triangle divided by a perpendicular from the celestial body. It is generally similar to those designed by Fuss and Aquino, but the arrangement is somewhat different, requiring 206 pages of tables. This method was never widely used, and is now out of print.

H.O. Pub. No. 211 (Ageton), *Dead Reckoning Altitude and Azimuth Table,* was published by the U. S. Navy Hydrographic Office in 1931. This method, designed by Lieutenant Arthur A. Ageton, USN, while a student of the Post Graduate School, then at Annapolis, Maryland, is based upon a triangle divided by dropping a perpendicular from the celestial body. It is generally similar to those of Fuss and Aquino. However, Ageton modified the formulas so as to include only secants and cosecants. In terms of figure 2111, his notation is as follows:

v is designated R

w is designated 90°–K

x is designated K∼L

K=x+L.

Ageton's formulas are

$$\csc R = \csc t \sec d$$

$$\csc K = \frac{\csc d}{\sec R}$$

$$\csc h = \sec R \sec (K \sim L)$$

$$\csc Z = \frac{\csc R}{\sec h}.$$

A single table of 36 pages gives five-place log cosecants (labeled A) and log secants (labeled B), both multiplied by 100,000 to eliminate the decimal. These values are given in parallel columns for each 0'.5 of angle from 0° to 180°. The table is well arranged and indexed for quick reference. The rules are relatively simple and well presented. The method can be used for solution from the dead reckoning or any other assumed position. The method is intended for use without interpolation. These

features combined to make this a popular method, although solution is somewhat tedious, and large errors may be encountered if t is near 90°. The method has been largely superseded by H.O. Pub. No. 214 (art. 2113).

If a celestial body near the visible horizon is observed, it may be *below* the celestial horizon (zenith distance greater than 90°), because of refraction and dip. Under these conditions the *computed* altitude, Hc, is negative (art. 2010). In the solution by H.O. Pub. No. 211, Hc is negative if K is of the same name as L and greater than (90°+L), *or* if K is of contrary name to L and greater than (90°−L). Under the second of these conditions, Z is less than 90° and should be taken from the top of the table *if* K is greater than (180°−L).

Fontoura da Costa and Penteado's *Tabuas de Altura e Azimute* were published in Lisboa, Portugal, in 1936. These consist of 26 pages of log secant and log cosecant tables similar to those of H.O. Pub. No. 211. The method and formulas are slight modifications of those of H.O. Pub. No. 211.

Tillman. The *Altitude Tables for Mariners and Aviators*, by E. Tillman, were published in 1936 in Sweden. Solution is by three tables using the basic formulas given above.

USSR tables. About 1940 the USSR replaced the Fuss tables with a method that is similar but uses a much shorter table. However, the solution is about the same length as with the Fuss tables, requiring six book openings, nine table entries, and five mathematical steps. Visual interpolation is used.

Japanese H.O. Pub. No. 602, *Brief Celestial Navigation Table*, was published in 1942. A perpendicular is dropped from the celestial body, as in figure 2111. Side *w* is designated K, and the following formulas are derived from the basic formulas given above:

$$\log \tan K = \log \cot d + \log \cos t$$

$$\log \cot Z = \log \cot t + \log \csc K + \log \cos (K+L)$$

$$\log \cot h = \log \cot (K+L) + \log \sec Z.$$

These formulas result in a simple solution, at the expense of some duplication in the three tables of 49 pages.

Hickerson. In 1944 Thomas F. Hickerson, professor of applied mathematics at the University of North Carolina, published a small volume called *Navigational Handbook with Tables*, in which the tables of H.O. Pub. No. 211 are given with the interval between entries reduced to 0.′2. All values are given on 45 pages, by tabulating values only to 45° and interchanging the A and B values for angles between 45° and 90°. In 1947 a second edition was published under the title *Latitude, Longitude and Azimuth by the Sun or Stars.*

2112. Altitude methods without use of navigational triangle.—The navigational triangle is composed of arcs of three great circles: the celestial meridian of the observer, the hour circle of the celestial body, and the vertical circle of the celestial body. Arcs of other great circles might also be used in forming spherical triangles that can be solved to find altitude and azimuth.

Kotlarić. In 1956 Stjepo M. Kotlarić, technical assistant, Hydrographic Institute of the Yugoslavian Navy, proposed a method based upon the solution of three right spherical triangles composed of arcs of great circles, as follows:

triangle 1—celestial horizon, hour circle, and celestial equator;
triangle 2—celestial horizon, hour circle, and vertical circle;
triangle 3—celestial horizon, hour circle, and celestial meridian (lower branch).

The formulas are derived from Napier's rules:

$$\tan (Z+F) = -\sin L \tan t$$
$$\tan M = \cot L \cos t$$
$$\cos C = \cos L \sin t$$
$$\tan F = \cos C \tan (M+d)$$
$$\sin Hc = \sin C \sin (M+d)$$
$$Z = (Z+F) - F,$$

in which F, M, and C are auxiliary parts.

Four tables totaling about 200 pages would be needed with the method, although table III is not needed if the assumed position is selected so that latitude and meridian angle are the nearest whole or half degree. The size of the tables could be reduced considerably if half degrees were dropped. With an assumed position selected as indicated above, the method requires only four table entries and four mathematical steps. The rules are few and simple.

2113. Modern inspection tables may contain lists of altitude or azimuth, or both. Another type tabulates the information needed for finding longitudes. Values are taken directly from the tables, without the need for logarithms, auxiliary functions, or mathematical solutions (except interpolation). Inspection tables are not new, the horary tables of Cassini in 1770, Lalande in 1793, Lynn in 1827, and Hommey in 1863 (art. 2106) being of this type. Other inspection tables include Davis' *Chronometer Tables*, Blackburne, H.O. Pubs. Nos. 203 and 204, Ball, Davis' *Alt-Azimuth Tables*, and H.O. Pub. No. 201 (arts. 2106 and 2109). None of these tables is used to any extent today, largely because interpolation is difficult, and coverage is limited. A short logarithmic solution with wide coverage has often proved more popular.

In contrast, the modern inspection table, made practicable by recent developments in computation techniques, has largely replaced the trigonometric solution. The principal modern inspection tables are:

H.O. Pub. No. 214, *Tables of Computed Altitude and Azimuth*, were published by the U. S. Navy Hydrographic Office between 1936 and 1946, in nine volumes. Between 1951 and 1953 the British Admiralty published identical tables (H.D. 486) in six volumes, with altered explanation to suit British practice. The first volume of an identical Spanish edition was published in Spain in 1953, and the second volume in 1956. Several volumes of an Italian edition based on H.O. Pub. No. 214 have also been published. The H.O. Pub. No. 214 series is described in detail in chapter XX.

British Air Pub. 1618 (H.O. Pub. No. 218), *Astronomical Navigation Tables*, were published by the British Admiralty between 1938 and 1944 in 15 volumes (lat. 0°–79°). In 1941 the first 14 volumes (lat. 0°–69°) were republished by the U. S. Navy Hydrographic Office as H.O. Pub. No. 218. The tables are intended primarily for aviators.

These tables are similar to H.O. Pub. No. 214, but with several differences. In A.P. 1618 values are given to the nearest whole minute for altitude, and the nearest whole degree for azimuth. The altitude values include allowance for refraction at a height of 5,000 feet. The minimum altitude in most cases is 10°. Provision is made for interpolation for declination only, and this always from the *next smaller* whole degree, instead of from the *nearest* whole degree. Declination is given for each whole degree from 0° to 28° only. In addition, values of altitude and azimuth are given for the declination (in 1940) of 22 stars. This part of the table is entered with the star name (or an arbitrarily-assigned number), so the declination of the body need not be

known. An auxiliary table provides a correction for changes in declination during the years following 1940 (to the year 2000).

During World War II these tables were widely used by aviators. Some marine navigators also used them. Since publication of H.O. Pub. No. 249, their use has declined.

Japanese H.O. Pub. No. 351, *Celestial Navigation Observation Table,* was published in 1940–42, in seven volumes for latitudes 0°–70°. The original printing was classified "secret." The tables are similar to British Air Pub. 1618, with several differences. In H.O. Pub. No. 218 all star-name entry tables are given first, followed by all declination entry tables. In Pub. No. 351 the declination entry table for each degree of latitude is followed by the star-name entry table. Altitudes, including refraction at 4,000 meters (13,123 feet), are tabulated to a minimum value of 2°. Declination is extended to 29°. The latitude-declination contrary-name entries are inverted so that meridian angles increase *upward* on the page as in H.O. Pub. No. 260 (art. 2126), resulting in better utilization of space on the pages having both "same name" and "contrary name" entries. Twenty stars are used, the selection differing somewhat from that of H.O. Pub. No. 218. In H.O. Pub. No. 218 the stars are listed and numbered alphabetically. In Pub. No. 351 they are given in order of declination, from Dubhe, listed as 62°03′N, to Sirius, listed as 16°38′S.

Hoehne. In October 1941 George G. Hoehne, an American, proposed a set of tables similar to the star section of H.O. Pub. No. 218, except that a value approximating LHA♈ would replace meridian angle of the star as an entering argument, and a maximum of ten stars would be given in parallel columns for each whole degree of entering value. The value used for entering the tables would be determined by adjusting LHA♈ by an amount tabulated for each year for each star used. This would prevent the tables from becoming inaccurate because of precession of the equinoxes (art. 1419). Refraction at altitude 5,000 feet would be included as in H.O. Pub. No. 218. One volume of these tables (volume II, lat. 20°N to 39°N) was published in 1943.

Japanese H.O. Pub. No. 603, *Simplified Celestial Observation Table,* was published in 1943. This publication is virtually the same as Pub. No. 351, except that eight additional stars are given, all farther south than those of Pub. No. 351. This extends the list to *α Crucis* (Acrux), given as declination 62°48′S.

Altitude and Azimuth Almanac was published by the Japanese Hydrographic Office, beginning in 1944. Originally, this was a secret publication. Several different versions were printed, and there were some modifications after the first editions. In each, however, the functions of almanac and sight reduction tables were combined. For each of several specific locations, the altitude and azimuth of one or more celestial bodies are tabulated for the date and time, usually at ten-minute intervals. In the earlier editions, the locations selected were important points in the western Pacific. From this practice, these publications are sometimes called "destination tables." Later editions used positions differing in latitude by 5°. These tables provided a quick solution for observations made at the tabulated times. On a worldwide basis such a system would involve a very voluminous tabulation each year, or cumbersome corrections. The *Altitude and Azimuth Almanac* is no longer published.

Hohentafeln nach Sternzeit, an official German table, was published in 1944 as an experimental edition with a very limited range of latitude. The tables were similar to those of Hoehne, but with six stars listed for each minute of local sidereal time.

Méneclier and Chevalier. The *Cálculo del Punto* of Víctor Méneclier and Roberto Chevalier was published in 1945–49 by Aeronáutica Argentina. There are six volumes for latitudes 0° to 59° *south.* At intervals of 4^m LST (or 1° LHA♈) the altitude,

a correction factor, and azimuth (not azimuth angle) of selected stars are tabulated. Twelve columns are provided, but a number of blank areas appear, resulting in an average of about nine altitude-azimuth entries for each time entry. In most cases, altitudes are carried to a minimum value of 5°, and azimuth to the horizon. These tables are similar to those of Hoehne and volume I of H.O. Pub. No. 249.

H.O. Pub. No. 230 (Goetz), *High Latitude Celestial Navigation Tables,* designed in 1945 by Roy F. Goetz, was published by the U. S. Navy Hydrographic Office in 1946.

The first section, called "Star Tables," is entered with the latitude to the nearest 1° from 70°N to 89°N, the name of the star (for ten selected stars), and LHA♈ at intervals of 2° for latitude 70° to 79°, 5° for 80° to 84°, and 10° for 85° to 89°. Altitude is tabulated to the nearest 1′ and azimuth (not azimuth angle) to the nearest 0°1. A "ΔH" value is given for use with an auxiliary table to interpolate for precession of the equinoxes (art. 1419).

In the second section, called "Declination Tables," declination is substituted for the name of the star. A separate table is given for each 1° declination from 0° to 28°. For each degree a "same name" section is given first, followed by a "contrary name" section (to declination 19°). The minimum altitude is 1°. The declination tables give "d" in place of "ΔH" for use with an auxiliary table to interpolate for declination.

Only 400 of these tables were published. They were intended only for use in military aircraft operating beyond the latitude range of H.O. Pub. No. 218. After H.O. Pub. No. 249 became available, H.O. Pub. No. 230 was canceled.

H.O. Pub. No. 249, *Sight Reduction Tables for Air Navigation,* in three volumes, are published by the U. S. Navy Hydrographic Office. A preliminary edition of volume I for selected stars was published in 1947 under the title *Star Tables for Air Navigation,* using the principles and features of tables proposed previously by George G. Hoehne, Commander C. H. Hutchings, USN, and others. The altitudes of this edition were adjusted for refraction at a height of 10,000 feet. By the time the "first" edition was printed in 1951, for epoch 1955.0, more than 20,000 copies of the preliminary edition had been distributed. The 1951 edition dropped the refraction adjustment feature from the altitudes, and had an improved selection of stars. It was followed in 1952 with two volumes for declination entry at 1° intervals from 0° to 29°. In 1952 and 1953 a British edition was published with identical tables (A. P. 3270) but altered explanation. The tables have been accepted as standard by the air forces of Great Britain, Canada, and the United States. They are in limited use by mariners. Extracts from these tables (1957 edition, for epoch 1960.0) are given in appendix CC.

Volume I contains tabulations of altitude (to the nearest 1′) and azimuth (to the nearest 1°) in parallel columns. For each 1° of latitude a two-page table (one-page above 69°) is given. For each 1° (2° beyond latitude 69°) of LHA♈, altitude and azimuth are given for seven stars carefully selected with regard to azimuth, magnitude, altitude, and continuity. Stars of the first magnitude are shown in capital letters, and those of second and third magnitude in lower case with initial capital. After each 15 entries a break occurs and a new listing of stars is given, whether or not there are any changes from the previous list. Stars are listed in the order of increasing azimuth at the beginning of each period. A total of 41 stars is used, 19 of which are of the first magnitude, 17 of the second magnitude, and 5 of the third magnitude. The tables are intended for use with an assumed position selected so that latitude and LHA♈ are each the nearest whole degree (nearest *even* degree of LHA♈ at latitudes higher than 69°).

Tabulation by name of star eliminates the need for finding the declination, but for strict accuracy, a correction for precession of the equinoxes (art. 1419) and nutation (art. 1417) may be needed. This is given in an auxiliary table (tab. IV). Since it is

anticipated that the tables will be recomputed at five-year intervals, it will probably be possible for aviators to ignore this correction. However, it may reach a value of as much as 3′, and should not be neglected if the tables are used by mariners. This correction is applied to the *fix*, not to each *altitude*.

Tabulation of *azimuth* (not azimuth angle) eliminates the need for conversion.

Tabulation by LHA ♈ instead of meridian angle of the star eliminates the need for finding and applying SHA. It also makes of the tables a star finder for the seven stars given, since all values given for any entry of LHA ♈ are for the same *time*. In the air it is common practice, when H.O. Pub. No. 249 is used, to observe the stars at intervals of exactly four minutes. Solution is made for only one observation (usually the middle of three), altitude and azimuth entries being found on consecutive lines (neglecting the small difference between solar and sidereal time during a four-minute period), and *all are plotted from the same assumed position*, selected so that latitude and LHA ♈ are the nearest whole degree, and adjusted as necessary for the motion of the observer between observations. If the time selected for the observation to be solved is a whole 10^m of GMT, and the navigational watch is set to GMT, the GHA ♈ can be taken directly from the *Air Almanac* without interpolation. With addition or subtraction of only one longitude, a person has all the information needed for entering H.O. Pub. No. 249 for solving *three* observations. If the navigator had a watch set to read GHA ♈ in arc, the almanac would not be needed for solving an observation. Wing Commander E. W. Anderson and Dr. D. H. Sadler, both of Great Britain, have suggested a ruler for use with a Mercator chart of certain scales, and a circular computer for use with any projection and scale, to permit quick conversion of sidereal to solar units if observations are made at greater intervals.

Example 1.—During evening twilight on June 2, 1958, the 1724 DR position of a ship is lat. 40°39′.2 S, long. 128°01′.2 E. At GMT $8^h24^m03^s$ the navigator observes Canopus with a marine sextant having no IC, from a height of eye of 38 feet. The hs is 55°57′.1.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 249 (epoch 1960.0), vol. I, and the *Air Almanac*.

Solution.—

	June 2	Canopus
GMT	$8^h24^m03^s$ June 2	
8^h20^m	15°18′	
4^m03^s	1°01′	
GHA ♈	16°19′	
*a*λ	127°41′ E	
LHA ♈	144°00′	
*a*L	41°00′ S	
Hc	55°45′	
Ho	55°50′	
a	5 T	*a*L 41°00′ S
Zn	233°	*a*λ 127°41′ E

	+	☆ −
IC	—	—
D		6′
R		1′
sum	—	7′
corr.		(−) 7′
hs		55°57′
Ho		55°50′

This problem is similar to that of example 4, article 2008. A comparison of the two indicates that the H.O. Pub. No. 249 solution reduces the number of table entries over the number required by H.O. Pub. No. 214 solution by four, and the number of mathematical steps also by four. The use of a whole 10^m of GMT would eliminate one more table entry and one mathematical step. If three observations were made at 4^m intervals, two more table entries and two mathematical steps would be eliminated from the three solutions. Whether or not these "wrinkles" are used, all values needed for a fix are together on one page, and are extracted without interpolation.

Volumes II and III are somewhat similar in many respects to H.O. Pub. No. 214. Altitude and azimuth angle are given in parallel columns for every whole degree of latitude (0° to 89°), every whole degree of declination (0° to 29°), and every whole degree (2° beyond lat. 69°) of LHA for all values at which the altitude is greater than several degrees *below* the celestial horizon (to allow for large values of dip at aircraft heights, and for considerable difference between assumed position and the position of the craft at the time of observation). The values for latitude and declination contrary name are tabulated with values of meridian angle (LHA less than 180°) increasing *upward* on the page, as in some older tables such as H.O. Pub. No. 260 (art. 2126). This permits better utilization of space where same- and contrary-name tabulations are given on the same page. It also serves to emphasize the difference between the same- and contrary-name tabulations, the contrary-name tabulation being given in a "contrary" manner on the page. A more convenient arrangement of declination entries is provided by having the "top" of each page of the tables along the left side, requiring the turning of the page through 90°.

A "d" value is tabulated between the altitude and azimuth angle to facilitate interpolation of altitude for declination. No interpolation is needed for latitude and LHA because the assumed position is selected so that these are the nearest whole degree (nearest *even* degree of LHA beyond latitude 69°). The "d" value is the difference in minutes, with sign, between the accompanying altitude and that for declination 1° *greater*, at the same latitude and LHA. It is used for entering an auxiliary table (tab. III) for determining the correction to be applied to altitude for minutes of declination, in a manner similar to using Δd and the "multiplication table" of H.O. Pub. No. 214. Interpolation is normally made in the direction of increasing declination.

Volume II covers latitudes 0° to 39°, and volume III contains similar information for latitudes 40° to 89°. Since these tables are entered with LHA of the celestial body, they do not become inaccurate in succeeding years, and no correction is needed for precession and nutation, as in volume I. These volumes are intended for solution of observations of the sun, moon, planets, and any stars within the declination range.

Example 2.—During morning twilight on June 2, 1958, the 0724 DR position of a ship is lat. 40°39′.2 S, long. 131°01′.2 E. At GMT $22^h24^m03^s$ (June 1) the navigator observes Alpheratz with a marine sextant having no IC, from a height of eye of 38 feet. The hs is 20°15′.3.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 249, vol. III, and the *Air Almanac.*

Solution.—

	June 2		Alpheratz	
GMT	$22^h24^m03^s$	June 1		
22^h20^m	224°53′			
4^m03^s	1°01′			
SHA	358°26′			
GHA	224°20′			
*a*λ	130°40′ E			
LHA	355°00′			
d	28°52′ N		d diff. 52′	
*a*L	41°00′ S			
ht	20°51′		"d" (−)60	Z S175°E
corr.	(−)52′			
Hc	19°59′			
Ho	20°06′			
a	7 T		*a*L 41°00′ S	
Zn	005°		*a*λ 130°40′ E	

	+	☆ −
IC	—	—
D		6′
R		3′
sum	—	9′
corr.		(−) 9′
hs		20°15′
Ho		20°06′

This problem is similar to those of example 4, article 2008, and example 1 above. All volumes of H.O. Pub. No. 249 are intended for use with the *Air Almanac.*

Experimental Air Navigation Tables. During the early part of World War II the British Royal Air Force felt the need for an inspection table that would be faster than Air Pub. 1618 (H.O. Pub. No. 218), but free from the limitations of the astrograph (art. 2123). Wing Commander R. C. Alabaster suggested the addition of SHA to the hour angle (measured eastward) of the stars given in Air Pub. 1618, converted to time at the sidereal rate of 15°02′.5 per hour. This would give the time interval until the next meridian transit of the vernal equinox. Before observation, the time of passage of the vernal equinox across a convenient meridian would be marked on the chart or plotting sheet. After observation, the tables would be entered with assumed latitude and the nearest tabulated altitude. The (SHA+HA) corresponding to this altitude would be added to GMT at the time of observation. The result should be close to the time marked on the chart. The difference would be converted to arc units (or a time scale would be marked on the chart or plotting sheet) and the corresponding longitude determined. This point would serve as the assumed position. The difference between the observed altitude and that used for entering the table would be the altitude difference to be used with the azimuth for plotting the line of position.

Squadron Leaders A. Potter and A. J. Hagger suggested a method of printing a time scale on the chart or plotting sheet with an auxiliary table to assist in locating the assumed position.

Various modifications and conventions were later added to avoid negative values and other complications. As the method finally emerged, a quantity known as "scale time" was adopted. This value, designated T, would be equal to 26 hours plus the GMT of the next transit of the vernal equinox occurring after 0600 during the night of the flight. The GMT of observation would be designated t. The quantity $T-t$ would be the value tabulated.

Further attempts were made to simplify the conversion of mean to sidereal time so that the single setting might be used during an entire flight. One of these, called the "Astro-Scales," was suggested by Wing Commander E. W. Anderson in 1945. In 1953 he and D. H. Sadler suggested an improved version.

Although a considerable amount of thought was given to this method, and experimental tables were published for a limited band of latitude, the limitations of a longitude method and the inconvenience of converting mean time to sidereal time resulted in the method being discarded in favor of the less restrictive H.O. Pub. No. 249 method.

Ashton. In 1943 Philip Ashton proposed a set of tables called *Astrograph-time Star Tables for Air Navigation,* based upon the principle of the *Experimental Air Navigation Tables.* A permanent table would be entered with the name of the star, latitude, and "astrograph mean time" (art. 2123), and altitude and azimuth would be taken from the table. A set of tables issued each year would list the values to be used with GMT each night to determine the astrograph mean time. Before take-off, the chart or plotting sheet would be marked to agree with the astrograph mean time, and a metal tape would then be used to convert mean time to sidereal time for finding the assumed position.

Heard. About 1950 John F. Heard, associate professor of astronomy at the University of Toronto, prepared a modification of the *Experimental Air Navigation Tables.* The tabulation would be altered so that altitude would be given in the left-hand column at intervals of 20′. A delta ("diff.") value would be tabulated and this used with the difference between entering and observed altitudes to enter an auxiliary

table to determine a correction to be applied to $T-t$ so that the altitude difference need not be plotted. A correction for 60 minus seconds of T would also be applied. The "bearing" of the line of position (azimuth plus or minus 90°) would also be tabulated. The line of position would be plotted through the assumed position, in the direction indicated by the "bearing." For any given time three stars differing in azimuth by approximately 120° would be given. The part of the table to use would be determined by a rough computation of $T-t$.

2114. Azimuth methods.—Nearly all methods proposed for obtaining a line of position are based upon the use of altitudes. The azimuth might also be used if an instrument becomes available for measuring it to the required accuracy. The accuracy needed would depend upon the acceptable error of the line of position. The error would be proportional to the cosine of the altitude. For a celestial body on the celestial horizon an error of 1′ in the azimuth would introduce an error of one mile in the line of position, the same as it does in an altitude observation. For any altitude greater than 0°, the error would be less.

Each method of determining a line of position by altitude has its counterpart in the azimuth problem. Thus, if it can be determined that a celestial body is exactly on the celestial meridian, the west longitude is the same as the GHA of the body. If the body is exactly on the prime vertical, the latitude can be computed. As a more general case, two points on a given azimuth line can be computed and joined by a straight line, by assuming two latitudes or two longitudes. However, if one such position is known, the azimuth line of position can be drawn through it in the direction of the azimuth. If the celestial body is sufficiently high, or if a small scale is acceptable and allowance is made for chart distortion, the azimuth line can be plotted directly, just as the circle of position can be drawn if the altitude is known. The difference between the observed azimuth and that computed for an assumed position can be used in a manner similar to the altitude difference. The azimuth difference in minutes multiplied by the cosine of the altitude would be the "intercept" measured off from the assumed position in a direction perpendicular to the computed azimuth. Through the point thus determined, a line would be drawn in the direction of the observed azimuth. For small differences, the line could be drawn perpendicular to the line from the assumed position. The relative values of the observed and computed azimuths would indicate the direction (right or left) to draw the line from the assumed position.

If the altitude and azimuth were both known to sufficient accuracy, a single celestial body would suffice for determining position by any combination of altitude and azimuth methods or by direct computation of latitude and longitude. The two lines of position would always be perpendicular to each other.

Double altitudes. For a stationary observer the longitude can be determined by observing the altitude shortly before meridian transit (either upper or lower), and noting the time when the altitude has returned to exactly the same value after meridian transit. If there has been no change in declination between observations, the mid time represents the moment of meridian transit, at which time the azimuth is 000° or 180°. The GHA (or 360°−GHA for east longitude) is the longitude of the observer. This method might be considered as either a longitude or an azimuth method. A variation is to observe a number of altitudes shortly before and after meridian transit. These are then plotted against time on cross-section paper and a smooth curve plotted through them. The time corresponding to the maximum altitude (minimum altitude for lower transit) is the moment of meridian transit.

Quilter. In 1950 Commander E. S. Quilter, USN, suggested a method based upon azimuth difference. He would measure and compute azimuth to the nearest 0°.01 (when the means for doing so became available) and express the azimuth difference to the

same precision. A table would be provided to list values of K, a constant by which the azimuth difference would be multiplied for any given altitude to determine the "intercept" to measure off from the assumed position.

2115. Determination of latitude and longitude.—Most methods provide information needed for plotting a line of position. The fix is at the common intersection of two or more such lines. A line of position might be plotted in one of several ways. In the latitude and longitude methods, the lines are plotted at the computed coordinate. When one coordinate has been determined, the other can be computed without plotting. Thus, the longitude determined by time sight is generally correct only for the latitude used in its solution, and the plotting of a longitude line is misleading, unless the celestial body is on the prime vertical. A better procedure is to compute two points, using different latitudes (or longitudes, if latitude is being computed). These two points are on the line of position. A straight line connecting them is a good approximation of the circle of equal altitude. This was the method used by Captain Sumner when he discovered the line of position (art. 131), and the method of H.O. Pubs. Nos. 203 and 204 (art. 2106).

Another method is to compute one point and the azimuth (or Zn$\pm$90°), and plot the line of position through the point. This is the method used by Soule and Dreisonstok (art. 2106).

If only the altitude difference is computed for two points, the line of position is a common tangent of circles of radius equal to the altitude difference at these two points. This is the method of Benest and Timberlake (art. 2110).

The most common modern method of plotting the line of position is by means of the assumed position, altitude difference, and azimuth. If this information is available for two observations solved for the same assumed position, the position of the fix can be determined by computation instead of by plot, using the following formulas:

$$\tan B = \frac{a_2 - a_1 \cos A}{a_1 \sin A},$$

and

$$a' = a_1 \sec B,$$

in which A is the difference in azimuth of the two celestial bodies, B is the difference between the azimuth of the first celestial body and the direction of the "position vector" (the line connecting the common assumed position with the fix), a_1 is the altitude difference of the first celestial body, a_2 is the altitude difference of the second celestial body, and a' is the length of the position vector.

If A is greater than 90°, the minus sign in the numerator of the first formula is replaced with a plus sign. The common intersection of the two lines of position (the fix) is a' miles from the common assumed position, in a direction B degrees from the azimuth of the first observation. Since B is always between the azimuths of the two celestial bodies observed, it will always be *added* to the first azimuth if the left-hand body is considered the "first" one.

With the information a' and B one can find the latitude and longitude of the fix by (1) plot, (2) table 3, or (3) computation. If method (2) or (3) is used, the problem is the same as that encountered when course and distance from a known position is given, and point of arrival is desired. This can be solved by a combination of plane and parallel sailing, as explained in articles 813 and 815.

It is possible, too, to plot circles of position by using the geographical position of each body as the center of its circle, and the zenith distance as its radius. This is the method used for high-altitude observations (art. 2011), but is generally not practical for ordinary altitudes because of the small scale that would be needed, and the error that would be introduced by chart distortion, unless plotting were done on the surface of a sphere (art. 2124).

The use of a circle of equal altitude is similar to the use of a circle of position around a landmark of known range. The bearing of such a landmark also furnishes a line of position. Similarly, a line of position can be obtained by plotting the azimuth line of a celestial body, and a fix by plotting two such lines. This is generally not done because of the scale and chart limitations mentioned above, and also because the needed accuracy in observation is beyond the capability of equipment generally available to the navigator. Errors in both compass and measurement of azimuth are involved.

Various methods of determining position by computation from observations of two or more celestial bodies or four observations of a single celestial body are discussed in articles 2116 and 2117.

2116. Computed position from observation of two or more bodies.—Several methods have been proposed for computing the position directly from the observation of two or more celestial bodies. These generally consist of some combination of latitude and time sight methods. One form of automatic celestial navigation, proposed by Collins Radio Company, uses the principle of the planetarium in reverse, two bodies serving to position a horizontal-stabilized sphere (in principle) for latitude and local sidereal time. If the device is accurately set to Greenwich sidereal time, longitude is indicated.

Fox. In 1951 Charles Fox, associate professor of mathematics at McGill University, Montreal, proposed formulas for computing latitude and longitude if certain star pairs are observed, the two stars of each pair having almost the same SHA. Presumably, simultaneous observations would be needed. Five such star pairs are listed. Four of the stars in three of these pairs are dimmer than the third magnitude, and are not listed in the almanacs, either in the main tabulation or among the additional stars. More involved formulas are suggested for use of the method with any three celestial bodies.

de Jonge. In 1945 Joost H. Kiewiet de Jonge, a lieutenant in the Netherlands East Indies Army Air Force, proposed a method of determining position from the observation of three stars. The U. S. Navy Hydrographic Office published experimental tables for several star pairs for latitudes 20° to 30° under the title *Three Star Position Tables for Aerial Navigation.* It was anticipated that if the method proved popular, all possible three-star combinations (of the stars in the main tabulation of the almanacs) would be given, so that the navigator would not be limited in his selection.

No assumed position is needed with the method. Three stars are observed at intervals of three minutes, the stars being observed in the order of listing in the main table. Table I is entered with the three altitudes, h_1, h_2, and h_3, and for each a value is taken from the table. These values are labeled H_1, H_2, and H_3, respectively. They are combined to form $H_1+H_2=H_{12}$, and $H_2+H_3=H_{23}$. These combined values, H_{12} and H_{23}, are then used to enter the main table, from which local sidereal time (in arc units) and latitude are obtained. Greenwich sidereal time minus local sidereal time equals longitude (measured westward). Delta values and auxiliary tables provide corrections for motion of the observer and observation intervals differing from three minutes. Mean corrections for both atmospheric refraction and Coriolis are included in the tables, which are limited to altitudes between 20° and 75°, and azimuth difference between consecutive stars to 165°.

Dozier. In 1949 Charles T. Dozier proposed a method based upon the simultaneous observation of two celestial bodies and the solution of two spherical triangles, with vertices as follows:

triangle 1—the two celestial bodies and the elevated pole,
triangle 2—the two celestial bodies and the zenith.

The method involves the successive solution of seven formulas:

$$\cos D = \sin d_1 \sin d_2 + \cos d_1 \cos d_2 \cos S \tag{1}$$

$$\sin (X_1 \pm A_1) = \frac{\sin S \cos d_2}{\sin D} \tag{2}$$

$$\cos A_1 = \frac{\sin h_2}{\cos h_1 \sin D} \pm \frac{\tan h_1}{\tan D} \tag{3}$$

$$X_1 = (X_1 \pm A_1) \mp A_1 \tag{4}$$

$$\sin L = \sin d_1 \sin h_1 + \cos d_1 \cos h_1 \cos X_1 \tag{5}$$

$$\sin t_1 = \frac{\sin X_1 \cos h_1}{\cos L} \tag{6}$$

$$\lambda = GHA_1 \pm t_1 \tag{7}$$

in which D is the great-circle distance (angular) between the two celestial bodies, d_1 is the declination of the first body, d_2 is the declination of the second body, S is the difference of SHA of the two bodies, X_1 is the parallactic angle of the first body, A_1 is the angle at the first body between its vertical circle and the great circle between it and the second body, h_1 is the altitude of the first body (Ho is used), h_2 is the altitude (Ho) of the second body, L is the latitude of the observer, t_1 is the meridian angle of the first body, λ is the longitude of the observer, and GHA is the Greenwich hour angle of the first body.

If the great circle joining the two celestial bodies is on that side of the zenith opposite the elevated pole (if Z is within the angle formed by the vertical circle and hour circle of the first body), (X_1+A_1) is used in formulas (2) and (4), the sign in formula (3) is positive (+), and the sign of A_1 in formula (4) is negative (−). These signs are all reversed if the line adjoining the celestial bodies is on the opposite side of the zenith (Z outside the angle). If the great circle joining the two bodies passes almost through the zenith, an error might be made in the selection of the sign, and it is well to select another star pair. In formula (7) the sign is positive if the first celestial body is east of the observer's celestial meridian, and negative if it is west. The answer is in longitude measured westward from the Greenwich meridian. If the value exceeds 180°, it is subtracted from 360°, and the longitude is east.

If the quadrant of angle $(X_1 \pm A_1)$ or if t_1 is in doubt, the following formulas are suggested to replace (2) or (6):

$$\cot (X_1 \pm A_1) = \frac{\cos d_1 \tan d_2 - \sin d_1 \cos S}{\sin S} \tag{2A}$$

$$\cot t_1 = \frac{\cos d_1 \tan h_1 - \sin d_1 \cos X_1}{\sin X_1} \tag{6A}$$

In the presentation of the method it was suggested that simultaneous observations be obtained by a two-star tracker mounted on a stable platform, or by a double sextant. Several such sextants have been proposed, but none is in common use. Other possibilities would be to have two observers or to adjust the value of one observation for the change in altitude due to its apparent motion and the motion of the observer between observations.

It was proposed that values obtained by solution of formula (1) be published in a permanent table, since these values for various star pairs would be constant except for the very slight change due to proper motion (art. 1418). Since the values obtained by formula (2) change slowly with precession of the equinoxes (art. 1419), it was proposed that the angle (X_1+A_1) for a number of star pairs be published annually, perhaps in the almanacs. The other formulas would be solved after observation of the celestial bodies.

Kotlarić. In 1956 Stjepo M. Kotlarić (art. 2112), of Yugoslavia, proposed a method based upon computation of the same quantities suggested by Dozier. In the Kotlarić method most of the computation would be done in advance and published in tables divided into volumes for different latitude bands. This would generally eliminate the need for two answers for each set of altitudes, for the two intersections of the two circles of position would ordinarily be so far apart that only one solution would fall in the tabulated latitude band. Each volume would have a two-page index listing the stars used for each 5° latitude band and 15° LHA ♈ band, based upon the selection used in H.O. Pub. No. 249, Vol. I (art. 2113).

Similar tables were proposed for use with two observations of the sun or moon taken 45° of GHA apart. In this case, the first observation would be corrected (before the tables were entered) for the change in altitude due to motion of the craft between observations.

A separate table would be provided for each pair of observations. The tables would be entered with the altitudes, to the nearest 0°5, and the latitude of the observer; and the meridian angle of the second celestial body would be taken directly from the table. The meridian angle and GHA (from the almanac) would then be combined to find longitude. Delta values and a "multiplication table" would provide corrections for (1) the differences between observed and tabulated altitudes, (2) the difference between actual and tabulated declinations, and (3) the difference between the actual and tabulated SHA (or GHA) difference of the two celestial bodies. In the case of stars, the corrections for (2) and (3) are primarily due to precession of the equinoxes (art. 1419). If star observations were not taken simultaneously, a correction would be applied (before the tables were entered) to the first altitude to obtain the value it would have if made at the time of the second observation.

Uribe-White. A unique method of using two stars was suggested in 1952 by Enrique Uribe-White, of Colombia. A bubble sextant would be used to measure the altitude of one star, while a small, marine-type sextant attached to the bubble sextant would be used to measure simultaneously the angle at the star between the vertical circle and the great circle through this star and a second one. Prepared tables would give the great-circle distance between the two stars and also the angle between the great circle joining them and the hour circle of the first star. This angle, combined with the inclined angle which would be measured, constitute the parallactic angle (art. 1433). With this value, the observed altitude, and the declination of the first body, the latitude of the observer and the meridian angle of the first star could be computed by relatively simple formulas or by a mechanical computer proposed by the originator of the method. Meridian angle could be compared with GHA to determine longitude.

2117. Position from observation of single body.—If azimuth could be determined and plotted to sufficient accuracy, the altitude and azimuth of a single body could be used for establishing a fix. Any combination of altitude and azimuth methods (arts. 2108 and 2114) might be used, or the position could be computed without plotting. The following formulas might be used:

$$\sin t = \sin Z \cos h \sec d$$
$$\tan K_1 = \cos t \cot d$$
$$\tan K_2 = \cos Z \cot h$$
$$L = 90° - (K_1 \pm K_2) \qquad \text{(Approximate latitude must be known)}$$

A single body can be used for a running fix, of course, and if the body is near the zenith, a relatively short time might be needed. This is the case for high-altitude

observations (art. 2011) and has been used by a submarine measuring azimuth through its periscope when the sun is near the zenith (art. 2404).

Willis. Another method of determining position by a single body is by the use of altitude and rate of change of altitude. Three methods of doing this were suggested by Edward J. Willis in 1928.

Prime vertical observation. It can be shown by the use of differential calculus (art. O44) that

$$\cos L = \frac{dh}{dt} \csc Z \tag{1}$$

when $\frac{dh}{dt}$ is the rate of change of altitude with respect to time, specifically the change of altitude in minutes of arc during a one-minute-of-arc (four-seconds-of-time) change of hour angle of the body. However, to obtain latitude accurately in this way it is necessary to determine $\frac{dh}{dt}$ to an accuracy of perhaps four decimal places, and Z to an accuracy of perhaps one minute of arc. Two possible methods of obtaining $\frac{dh}{dt}$ are given below, but present instrument limitations do not permit measurement of azimuth to the required accuracy. However, the cosecant of 90° is unity, so that if the observation is made when the celestial body is on the prime vertical, the formula becomes

$$\cos L = \frac{dh}{dt} \tag{2}$$

Relatively little error is introduced if the body is within 1° of the prime vertical. The determination of position consists of the following steps:

1. Observe the altitude (h) and rate of change of altitude $\left(\frac{dh}{dt}\right)$ when the celestial body is within 1° of the prime vertical.
2. Compute latitude (L) by formula (2).
3. Determine longitude by any standard method, such as H.O. Pub. No. 214 or other line of position method, or by time sight (art. 2106).

Perpendicular lines of position. The great circle through the zenith and the celestial body (the vertical circle or azimuth line) furnishes an azimuth line of position that can be established if rate of change of altitude can be accurately determined. This line is perpendicular to the circle of equal altitude and therefore nearly perpendicular to the line of position determined in the usual manner. The intersection of the two lines is the position of the observer. The method involves the following steps:

1. Observe the altitude (h) and rate of change of altitude $\left(\frac{dh}{dt}\right)$.
2. Compute the direction of the great circle through the zenith and the celestial body (the vertical circle) at the point where the great circle crosses the celestial equator. This is the complement of the latitude of the vertex and so can be found from a modification of formula (2), which gives the latitude of the vertex:

$$\sin Z_0 = \frac{dh}{dt} \tag{3}$$

3. Compute the longitude (λ_0) at which the vertical circle crosses the celestial equator, using the formula

$$\sin (\lambda_0 \sim \lambda_b) = \tan Z_0 \tan d \tag{4}$$

The value λ_b is the longitude of the geographical position of the celestial body.

4. Solve for the latitude (L) at which the azimuth line of position crosses the meridian of the dead reckoning position, or for longitude (λ) at which the line crosses the parallel of latitude of the dead reckoning position, using one of the following formulas:

$$\tan L = \cot Z_0 \sin (\lambda_0 \sim \lambda_{DR}) \tag{5}$$

or

$$\sin (\lambda_0 \sim \lambda) = \tan Z_0 \tan L_{DR} \tag{6}$$

in which L_{DR} and λ_{DR} are the DR latitude and longitude, respectively. Any assumed position in the vicinity can be used in place of the DR. In general, it is preferable to use (5) if azimuth angle is between 45° and 135°, and (6) if it is outside these limits.

5. Solve for the direction (Z) of the azimuth line of position at the point determined in step (4), using the formula

$$\sin Z = \sin Z_0 \sec L \tag{7}$$

If the DR position or the AP is near the actual position, the azimuth can be considered the same at both without appreciable error.

6. Plot the azimuth line of position through the point found in step (4), in the direction found in step (5).

7. Compute *a* and Zn by any method and plot the resulting line of position. The intersection of the two lines of position is the fix.

Latitude and longitude by computation. This method is independent of a dead reckoning position, and requires no plotting. It is free from limitations except that observations near meridian transit should be avoided. At this time the rate of change of altitude decreases to zero and then reverses, introducing a possible error. The steps by this method are:

1. Observe the altitude (h) and rate of change of altitude $\left(\frac{dh}{dt}\right)$.
2. Compute Z_0, using formula (3).
3. Compute the latitude (L) of the observer by the formula

$$\sin L = \cos Z_0 \cos \left[h \pm \sin^{-1} \left(\frac{\sin d}{\cos Z_0} \right) \right] \tag{8}$$

In the solution of this equation, the angle whose sine is $\frac{\sin d}{\cos Z_0}$ is added to or subtracted from h. The cosine of this angle is then multiplied by $\cos Z_0$, and the result is the sine of the latitude of the observer. The sign is positive (+) unless L is greater than d and has the same name, when it is negative (−). However, if d is of the same name and greater, the angle to be added may be greater than 90°.

4. Compute the meridian angle of the observer by the formula

$$\sin t = \sin Z_0 \cos h \sec d \sec L \tag{9}$$

5. Determine GHA for the time of observation.
6. Convert t to LHA, and compute longitude (λ) by the formula

$$\lambda = GHA - LHA \tag{10}$$

If λ is greater than 180°, subtract it from 360° and label it E (east).

Formulas (8) and (10) yield a position on the circle of equal altitude regardless of the value of Z_0 used. The correct position is given only if the correct value of Z_0 is used.

Any of the three methods requires determination of $\frac{dh}{dt}$. Two methods are proposed:

In the first, the time needed for the sun (or moon) to change altitude an amount equal to its own diameter is measured. If the body is rising, the upper limb of the reflected image is brought a short distance below the horizon. As it makes contact with the horizon, a stop watch is started. When the lower limb makes contact with the

horizon (usually between 127.8 seconds, the minimum for a stationary observer, and ten minutes after the first contact) the watch is stopped, and the time is read to the nearest tenth of a second, if possible. If the body is setting, the lower limb of the reflected image is brought a short distance above the horizon and the watch started when the lower limb makes contact and stopped when the upper limb makes contact with the horizon. At sunrise or sunset no sextant is needed. Any lag in starting or stopping the watch will not affect the result if it is the same at both ends of the period. The diameter of the body, in minutes of arc, divided by one-fourth the number of seconds is $\frac{dh}{dt}$. Since semidiameter is tabulated, the most convenient procedure for determining $\frac{dh}{dt}$ is probably to solve the equation

$$\frac{dh}{dt}=\frac{8\,SD}{T},$$

where SD is the semidiameter of the body in minutes and T is the time interval in seconds. The semidiameter is given to the nearest 0'.1 in the *Nautical Almanac*. More accurate results will be obtained if the value is taken from the *Ephemeris*, where semidiameter is given to the nearest 0".01.

The motion of the observer introduces an error which can be corrected as follows: multiply *half* the run of the vessel between upper and lower limb contacts, expressed in nautical miles, by the cosine of the angle between the course of the vessel and the azimuth of the celestial body at the mid time of observation. If this angle is *less* than 90°, the correction is *added* to the tabulated semidiameter if the body is setting, and *subtracted* if it is rising. If the angle is *greater* than 90°, the correction is *added* if the body is rising and *subtracted* if it is setting.

Some practice may be needed to obtain an accurate measurement of the time interval. This practice might be obtained by making a number of observations at a known position and comparing these with values obtained by computation, using the formula

$$T=8\,SD\cos h\sec d\sec L\csc t,$$

using Hc for h.

The time of an observation is at the middle of the interval between contacts. In correcting hs, the reading of the sextant, to obtain Ho, omit the correction for semidiameter. This might be done by correcting in the usual manner, with an additional correction equal to the semidiameter. The additional correction is negative (−) if the lower limb correction is applied, and positive (+) if the upper limb correction is applied. Another way is to apply neither the lower nor upper limb correction, but a value equal to the algebraic average of both.

The second method of determining $\frac{dh}{dt}$ is given as the more accurate of the two. It consists of observing three altitudes of the celestial body at exactly equal intervals of from 15 to 30 minutes. A shorter interval may result in too great an error in rate, while a longer one increases the time without advantage. If h_1, h_2, and h_3 are the three altitudes and t_1 and t_3 are the meridian angles at the times of the first and third observations, respectively, $\frac{dh}{dt}$ can be computed by means of the formula

$$\frac{dh}{dt}=\sin \tfrac{1}{2}(h_1-h_3)\cos \tfrac{1}{2}(h_1+h_3)\csc \tfrac{1}{2}(t_1-t_3)\sec h_2.$$

If difficulty is experienced in making an accurate observation at a given time, better results might be obtained by computing the time for the third observation, by adding the interval between the first two observations to the time of the second observation, and then making several observations starting shortly before the computed

time. These can then be plotted on cross-section paper with altitude as one coordinate and time as the other. The altitude indicated by the intersection of the line representing the required time and a line faired through the plotted points is used as the third altitude. A similar procedure might increase the accuracy of the first two observations. A quicker but less accurate way of determining the third altitude is to take one observation shortly before the required time and another shortly after it, and interpolating to find the altitude at the required time. Another variation is to take an altitude at *about* the required time and adjust the second altitude to the corresponding value midway between the first and third observations, using the mean value found by interpolating from the first or third observation and extrapolating (art. P6) from the other. The time and altitude are those of the second observation.

This method assumes no change of declination between observations, and no change in the position of the observer. When the observer is not stationary, a correction is applied to h_1 and h_3 to convert them to the equivalent values at the position of the second observation. Assuming constant course and speed, this correction in minutes of arc is equal to the vessel's run between consecutive observations multiplied by the cosine of the angle between the course of the vessel and the average azimuth of the body. If the angle is *less* than 90°, the correction is *added* to h_1 and *subtracted* from h_3. If the angle is *greater* than 90°, the correction is *subtracted* from h_1 and *added* to h_3.

A possible variation of either method of determining $\frac{dh}{dt}$ would be to make a comparatively large number of observations (10 to 15) at short intervals and plot the altitudes versus time on cross-section paper. A point near each end of the line faired through the plotted points would then be corrected for the run of the vessel, as in the second method. Two points might then be selected, one near each end of the altitude-time line. The change in altitude, in minutes, divided by the number of seconds between the two points is $\frac{dh}{dt}$. If preferred, three points might be selected at equal intervals and the formula of the second method used.

Rate determined by two individual observations a few minutes apart would not be sufficiently accurate for practical navigation.

None of the methods employing rate of change of altitude have proved popular, probably because of the difficulty of obtaining an accurate value of $\frac{dh}{dt}$. The use of azimuth and rate of change of azimuth, altitude and rate of change of azimuth, or azimuth and rate of change of altitude have been even less attractive because of the even greater difficulty of obtaining accurate measurements of azimuth or rate of change of azimuth. With the further development of automatic devices for continuously measuring altitude or azimuth, with allowance for motion of the observer, such methods might prove more attractive.

2118. Use of unique situations.—Various unique situations might be used for determining position or a line of position. As a general rule these have not been attractive because they could be used only when the conditions were met. As an example, if a celestial body of known coordinates were known to be in the zenith, the declination of the body would be the same as the latitude of the observer. His longitude would be the same as GHA of the body (360°−GHA in east longitude).

Near the geographical poles, the poles can be used as the assumed position. Here the declination of the body is the same as the computed altitude, and GHA replaces azimuth.

Meridian altitudes (art. 2103) and latitude by Polaris (art. 2105) are examples of methods depending upon unique situations. These have both been used extensively,

but are decreasing in popularity because of their reliance upon unique conditions, without adequately compensating advantages.

Shchetkin. In 1899 N. O. Shchetkin proposed a method of computing latitude and meridian angle from measurement of the times at which two or more pairs of stars have the same altitude. Each star pair would provide, in effect, a single great-circle line of position. Variations of the method were proposed by Zinger, Pewzow, and W. W. Kawraisky, a Russian. The necessary tables for latitude 60° N to 80° N were published by the Astronomical Institute of Russia in 1936. A similar method was prepared by Simon Swahn in 1943.

Collins. In 1946 Oliver C. Collins, an astronomer at the University of Nebraska, proposed a variation of the method of Shchetkin, and extended it to include observations when two celestial bodies have the same azimuth.

McKee. In 1951 Lieutenant Merlin A. McKee, USMS, proposed a graphical solution of the same-altitude method of Collins.

Pierce. About 1951 Rear Admiral M. R. Pierce, USN (Ret.), suggested a method of establishing a line of position perpendicular to the course line when the altitude of a celestial body is observed at the moment it crosses the great circle through the observer and his destination.

2119. Graphical and mechanical solutions.—All of the methods described above require tables, either for a mathematical solution or to extract computed values of altitude and azimuth. The total number of possible tabular solutions must be very great. The number of graphical and mechanical solutions is almost endless. The ones selected for mention below are representative of the types that have been prepared or made available.

Graphical solutions are almost as old as tabular ones, having existed at least since 1790, when *Margetts' Horary Tables* appeared in graphical form. These were intended "for shewing by Inspection the Apparent Diurnal Motion of the Sun, Moon, and Stars, the Latitude of a Ship and the Azimuth, Time, or Altitude corresponding with any Celestial Object." They were intended primarily for use with the longitude method of laying down a line of position.

In general, graphical and mechanical solutions have not proved popular, for several reasons: First, they generally involve a small scale, yielding results of less accuracy than desired, even with careful work. Second, some of the methods must be used as a whole, and cannot be divided into parts to increase the scale. Third, such methods usually do not provide a record of the solution, and it is often difficult to check the results. Fourth, solutions requiring instruments are subject to errors due to lack of proper adjustment or mechanical damage which may not be apparent. Fifth, the required diagrams or instruments may be quite bulky, requiring considerable space for stowage and manipulation. Finally, in some cases the necessary instruments are expensive.

2120. Altitude and azimuth angle by graph.—One type of graphical solution is by means of a diagram that solves an equation.

d'Ocagne. Typical of such diagrams is that prepared by Maurice d'Ocagne, a Frenchman. Both altitude and azimuth angle can be found by means of this diagram, which is based upon the following formulas:

$$\text{hav } z = \text{hav } (L-d) + \{\text{hav } [180° - (L+d)] - \text{hav } (L-d)\} \text{ hav } t,$$

$$\text{hav } (90° \pm d) = \text{hav } (L-h) + \{\text{hav } [180° - (L+h)] - \text{hav } (L-h)\} \text{ hav } Z,$$

in which $z = 90° - h$.

The sides of a square are divided according to the haversines of angles, from 0° to 180°, and the corresponding graduations of opposite sides are connected with straight lines, forming a diagram as shown in figure 2120a. The graduations on the two sides run in opposite directions. To find the zenith distance, locate the value corresponding to (L−d) along the left of the diagram, and the value corresponding to (L+d) along the right of the diagram. Draw a straight line through these points. Locate the intersection of this line with the vertical line corresponding to meridian angle. A horizontal line from this intersection to the left edge indicates the zenith distance.

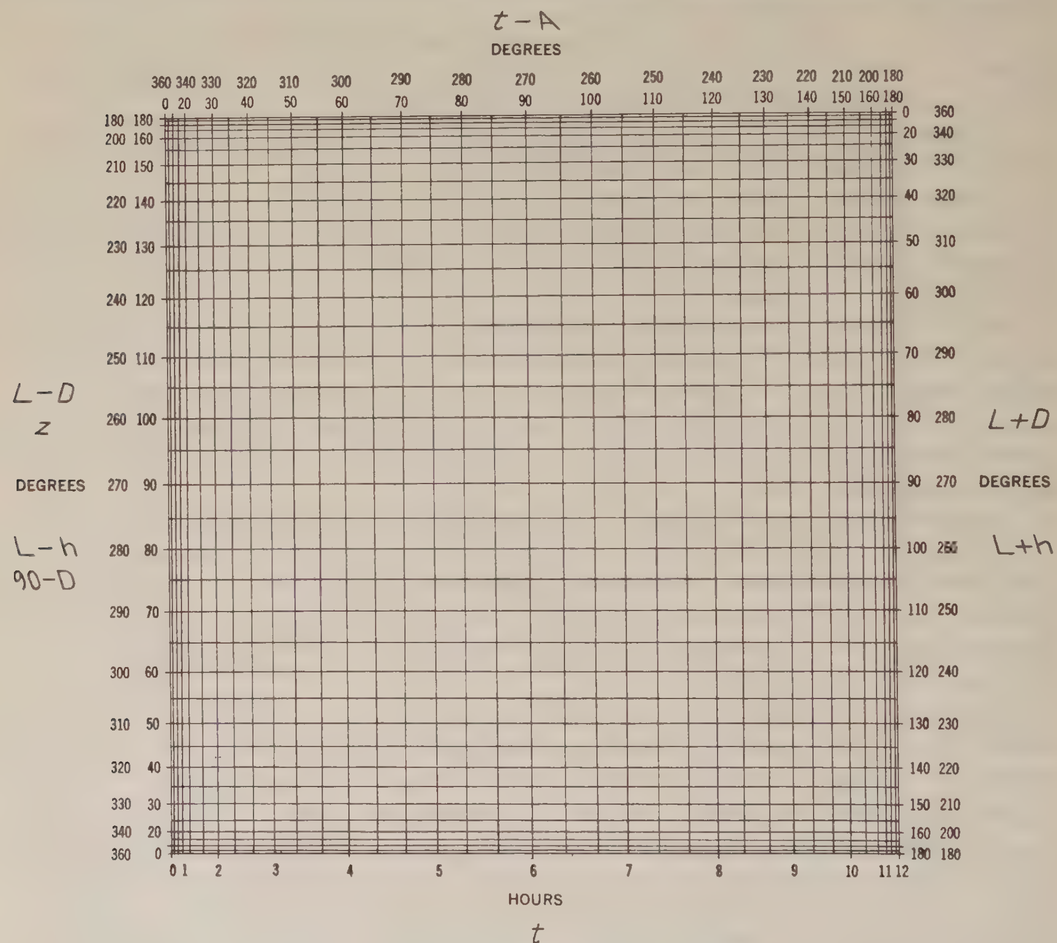


Figure 2120a.—The d'Ocagne diagram as H.O. Chart No. 2776.

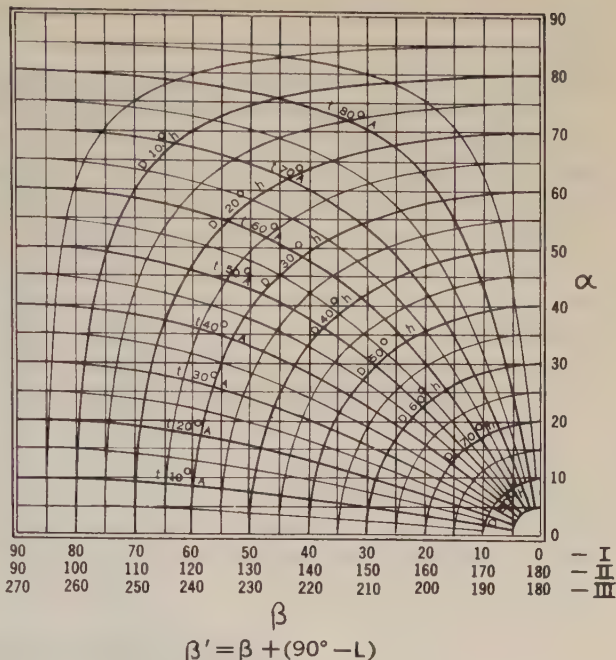
To find azimuth angle, draw a straight line between (L−h) at the left and (L+h) at the right. Locate the intersection of this line and the *horizontal* line corresponding to (90°−d). A *vertical* line from this intersection to the top of the diagram indicates the azimuth angle.

If the altitude, latitude, and declination are known, the first solution can be made in reverse for meridian angle, for a longitude method solution.

The diagram was first published in 1899 in *Traité de Nomographie* by d'Ocagne. Similar diagrams have since been published under the name *Spherical Triangle Nomo-*

gram by Wimperis, and under the title *Altitude, Azimuth, and Hour Angle Diagram* by Littlehales in 1906, and by the U. S. Navy Hydrographic Office in 1917.

Favé and Rollet de l'Isle.—If a perpendicular is dropped from the celestial body to the celestial meridian, a diagram can be prepared to solve the basic formulas given in article 2111, or others derived from these. Such a diagram is shown in figure 2120b. This diagram was devised by the French engineers Favé and Rollet de l'Isle in 1892. The diagram represents only one-eighth of a sphere, additional sections being needed. An alternative is to show additional labels, as in figure 2120b. This results in three "cases" and several rules similar to those used with some logarithmic solutions. Solutions for both altitude and azimuth angle are made in two steps, plus one addition or subtraction. This diagram was reproduced by the Frenchman M. E. Pereire in 1894 and by another Frenchman, P. Constan, in 1906 as a method of finding azimuth.



Case 1. L and D same Name —t<90°
Read β on scale II
Azimuth from upper pole, E or W as star is E or W of meridian

Case 2. L and D same name —t>90°
Read (180°—t) instead of t
Read β on scale I
Azimuth from upper pole, E or W as star is E or W of meridian

Case 3. L and D opposite names
Read β on scale I
Azimuth from lower pole, E or W as star is E or W of meridian

FIGURE 2120b.—The Favé diagram.

Jernæs.—In 1953 Leiv Jernæs, a Norwegian, invented a device he called a "Nauticator," which consists of various scales in a semicircle with radial scales on a plastic arm pivoted at the center of curvature of the semicircle. The device is used with a pair of dividers to solve various problems of spherical trigonometry to an accuracy of about 15′.

Bertin. In 1955 Rev. Maurice Bertin, a Frenchman, devised a graphical solution for the longitude method, using the formulas:

$$\tan^2 \tfrac{1}{2} t = \tan \tfrac{1}{2} (90° - \alpha) \tan \tfrac{1}{2} (90° - \beta) \quad (1)$$

and

$$\tan^2 \tfrac{1}{2} Z = \frac{\tan \tfrac{1}{2} (90° - \alpha)}{\tan \tfrac{1}{2} (90° - \beta)} \quad (2)$$

in which

$$\tan \tfrac{1}{2} \alpha = \tan \tfrac{1}{2} (h + d) \tan \tfrac{1}{2} (90° - L)$$

and

$$\tan \tfrac{1}{2} \beta = \frac{\tan \tfrac{1}{2} (h - d)}{\tan \tfrac{1}{2} (90° - L)}.$$

The diagram consists of three families of straight lines, one vertical, one horizontal, and the third at an angle of 45° to the others. The accuracy depends upon the scale of the diagram, but a large one is needed for navigational accuracy.

2121. Altitude and azimuth angle by computer.—Slide rules, like diagrams, have been devised to solve formulas. In the case of the navigational triangle, both suffer from the need for a scale that can be read to a subdivision at least as small as 1′. A number of such slide rules have been devised for use in reducing celestial observations.

Richer. In 1791 Jean Francisco Richer, a Frenchman, constructed a device composed of six arms, some hinged and some sliding, which won a prize offered by the Paris Academy of Science for a simple method of "clearing" lunar distances (art. 131) in the solution for longitude. The device solved a formula devised by the French mathematician Joseph Louis Lagrange, and was capable also of solving other problems involving spherical triangles, such as those related to time sight solution (art. 2106), computation of altitude, and great-circle sailing problems (art. 819).

Poor. A slide rule invented by Professor Charles L. Poor is shown in figure 2121a. This device, called the "Line of Position Computer," was designed to solve the cosine-haversine formula (art. 2109). Eight concentric circular scales are engraved on a metal disk about 15 inches in diameter. A plastic arm and circular sheet are pivoted at the center of the disk. The arm may be clamped to the plastic sheet. The seven

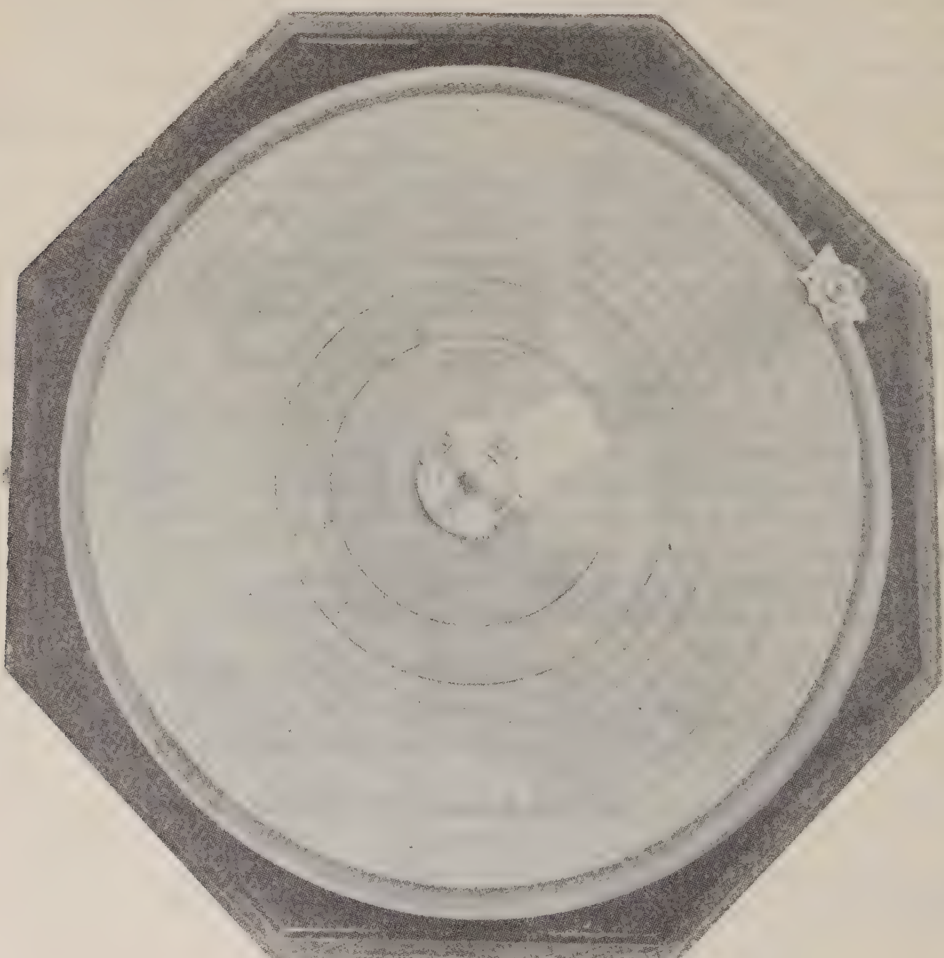


Figure 2121a.—The Poor Line of Position Computer.

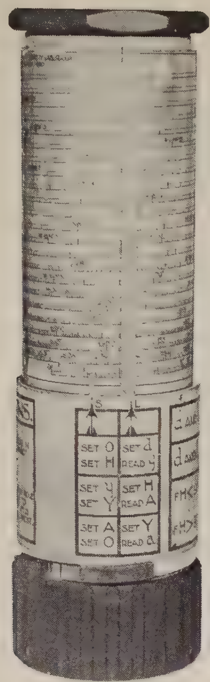


FIGURE 2121b.—The Bygrave slide rule.

outer scales are used in solving for altitude. The altitude scale is graduated at intervals of 10′, and further subdivisions can be estimated. The inner scale is used for determining azimuth angle. Several rules are needed, and the number of scales adds to the possibility of error.

Bygrave. A cylindrical slide rule was designed by the Englishman Bygrave to solve the navigational triangle divided by dropping a perpendicular from the celestial body to the celestial meridian (fig. 2111). This device, shown in figure 2121b, consists of three concentric tubes. The inner one has a spiral scale of logarithmic tangents, the middle one a spiral scale of logarithmic cosines, and the outer one a pointer for each scale. Solution is simple and relatively fast, but altered procedures are required if the azimuth angle is near 90°, or the meridian angle or declination is very small. The overall dimensions are about 2½ inches in diameter by nine inches long. An accuracy of about 1′ or 2′ is generally attainable.

Bertin. In 1955 Rev. Maurice Bertin devised an 18-inch slide rule to provide a solution of the longitude method to an accuracy of about 1°, using the formulas upon which his graphical solution (art. 2120) is based. He also devised a solution of the same formulas by a circular slide rule consisting essentially of two spirals. The inner one is on a disk 23 centimeters (9.2 inches) in diameter, and the outer one is on an annular ring 39 centimeters (15.6 inches) in outside diameter. The graduations are proportional to the log cotangents of half-angles. A window on a cover is provided with a radial line to serve as an index. Solution is facilitated if an approximation of the answer is known in advance. An accuracy of better than 3′ is claimed for this device. Still another solution proposed at the same time is by a computer consisting

of a strip four centimeters (1.6 inches) wide and 12 meters (nearly 40 feet) long, wound on two rollers and engraved with three sets of graduations. An accuracy of better than 1′ is claimed, but several arithmetical steps are required.

LeSort. A computing device based upon solution of formulas for a divided navigational triangle was designed by Commander LeSort of the French Navy. Logarithmic scales are placed on eight films wound on rollers. The films operate in pairs so arranged that the two films of any pair can be locked together at any point. Alternate films carry log cosine and log tangent scales. Although an accuracy of about 0′.2 can be obtained, the method is comparatively long and has no apparent advantage over modern inspection tables.

Desk computers. Several desk-type computers have been designed to solve the navigational triangle, but none has proved popular.

2122. Altitude and azimuth angle by map projection.—If the observer were to move along his meridian to the nearer pole, and the navigational triangle were to move with him without its proportions being changed, his zenith would coincide with the pole, and the vertical circle would coincide with some celestial meridian. Zenith distance or altitude could be read directly. Since both great circles forming the azimuth angle would now coincide with celestial meridians, the azimuth angle could also be determined directly.

Littlehales. To accomplish this with a sphere, to a useful accuracy, would require a sphere of impractical size for use by the navigator. However, the solution can be made by means of a map projection. George Littlehales, of the U. S. Navy Hydrographic Office, used the stereographic projection (art. 318) and a 12-foot sphere for this purpose. The projection is divided into 368 overlapping sheets which, with a key diagram, are bound together. An accuracy of about 1′ or 2′ can be obtained by a rapid and simple process, but the volume is bulky and not particularly convenient.

Veater. Commander Veater of the British Royal Navy used the transverse Mercator projection (art. 309), with the observer's meridian as the fictitious equator.

Hyatt. A similar principle is utilized in the diagram on the plane of the celestial meridian (art. 1432). A mechanical device based upon this diagram can be made by drawing a hemisphere by equatorial orthographic (art. 319) or stereographic projection and pivoting at its center an identical hemisphere on transparent material. If the top hemisphere is rotated until the arc between poles of the two hemispheres is equal to the colatitude of the observer, the lines of one hemisphere represent coordinates of the celestial equator system (art. 1426), and those of the other, coordinates of the horizon system (art. 1428). Thus, if a body is located by meridian angle and declination on one set of lines, its altitude and azimuth angle can be read from the other set. If altitude and declination are used to locate the body, meridian angle can be read from the diagram. In the United States such a device, on both the orthographic and stereographic projections, has been prepared by Commander Delwyn Hyatt, USN, under the titles "Celestial Coordinator" and "Coordinate Transformer." It has also been produced in other countries, notably in Germany, France, and Russia, where, in addition to such a device, precision instruments based upon the same principle have been constructed. The scale of the German instrument is so small that an accuracy of about 5′ is about the best that can be expected. The Bastien-Morin (French) and Kavroyskyy (Russian) instruments might yield results of slightly greater accuracy. The plastic device, if carefully made, might be generally accurate to half a degree. It has been used primarily for instructional purposes.

Brown-Nassau. The Brown-Nassau "Navigational Computer" utilizes the same principle, but uses the azimuthal equidistant projection (art. 320) and increases the

scale by limiting the device to an octant of the sphere, with separate solutions for altitude and azimuth, and various rules.

True. In his *Celestial Navigator for Aviators*, printed about 1943, Clarence H. True, of the Canal Zone, uses a single diagram on the orthographic projection. This serves as the basis for a solution by construction, claimed to be of sufficient accuracy for use in lifeboats. Various rules are needed.

Pierce. A series of diagrams on the azimuthal equidistant projection have been devised by Rear Admiral M. R. Pierce, USN (Ret.). The method is based upon the principle that angles are correctly represented at the point of tangency of this projection, and radial lines from this point represent great circles along which distances are represented by a uniform scale. A protractor is used for measuring the azimuth angle. Attached to the protractor is an arm with a linear scale graduated so that altitude can be read directly. The whole device is called a "Cadameter." The method is easy to use, and about as fast as modern inspection tables. With great care an accuracy of 1′ can be obtained. The method suffers from the need for a number of diagrams which are somewhat bulky and more susceptible to damage than a book.

2123. Latitude and longitude by diagram.—A number of graphical and mechanical solutions have been devised to yield latitude and longitude directly.

Beij. One proposed in 1924 by K. Hilding Beij, of the U. S. Bureau of Standards, was based upon the fact that latitude and local sidereal time are completely defined by the simultaneous altitudes of two celestial bodies whose declination and SHA are known. A page of his proposed diagrams is shown in figure 2123a, in which latitude is the abscissa, and LST is the ordinate. Position on the graph is located by the intersection of the curves representing the altitude of the two celestial bodies observed. The vertical line through the intersection indicates the latitude, and the horizontal line the LST. The difference between GST and LST is the longitude. If a timepiece keeping GST is available, not even an almanac is needed.

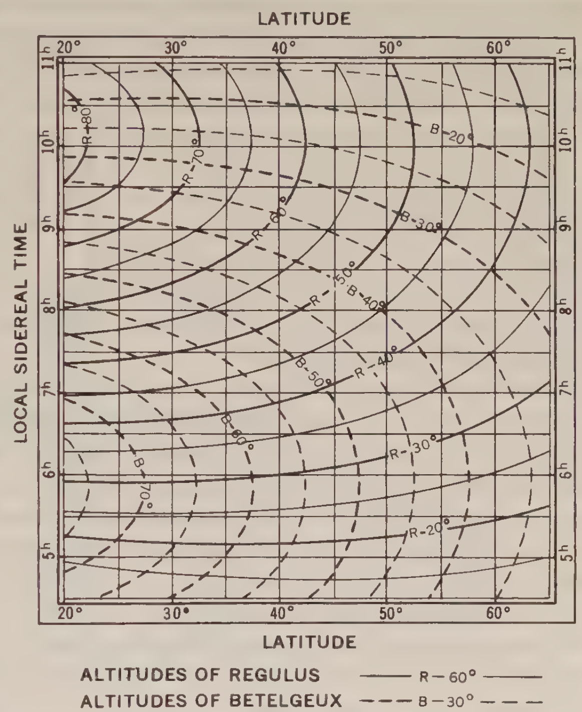


FIGURE 2123a.—The Beij two-star diagram.

The method is accurate, fast, and direct. The individual sheets can be drawn to any scale and cut to any size desired. For a large scale with sheets of a convenient size, a great many diagrams would be needed, but these might be bound together in convenient-size volumes, or placed on a tape wound around rollers, as originally proposed. A weakness of the method is the requirement for simultaneous observations. For nonsimultaneous observations a table might be provided to indicate the change in altitude during the interval between observations. Since the positions of the curves depend upon the declination and SHA of the body, the method is limited to celestial

bodies whose coordinates are nearly constant, unless the curves are intended only for a particular time. Even for stars, the diagrams become out-of-date in a few years. The method is limited to the particular bodies for which curves are shown, although the number of curves need not be limited to two. This is a form of precomputation, since the computation is performed in locating the curves, rather than by the navigator. In a sense, it might be considered a graphical form of H.O. Pub. No. 249 (art. 2113).

Weems. If the Beij diagram is rotated through 90°, the parallels of latitude become horizontal, as customary on a chart. If they are spaced according to the Mercator projection, azimuth is indicated by the normal to a curve. This is the arrangement used by Captain P. V. H. Weems, USN (Ret.), in his *Star Altitude Curves*, the first volume of which was published in 1928. Later he added a third star, using a different color for each star, and included a correction for refraction at sea level. A separate volume is used for each 10° of latitude, and a correction is provided for precession of the equinoxes. Coverage extends from latitude 50° S to 70° N, with a separate volume for latitude 70°–90° N. The curves for 80°–90° N are on the polar stereographic projection. Any orthomorphic projection (art. 302) could be used at any latitude.

Lines representing observations at different times can be advanced or retired as on any chart of the same projection. In addition to the adjustment due to motion of the craft between observations, the lines are shifted right or left for the elapsed time between observations. An accuracy of about 1′ is attainable by interpolation between curves for each 10′ of altitude.

The star altitude curves are undoubtedly the most widely used of all the graphical and mechanical methods. Two-star curves similar to Weems' first edition were published in Germany in 1940.

Pritchard and Lamplough. In 1940 H. C. Pritchard and F. E. Lamplough, of the British Royal Aircraft Establishment, devised a method of reducing the work involved in the adjustment for elapsed time between observations. They placed the star altitude curves on film which is used in a projector called an **astrograph.** The curves are projected onto a Mercator plotting sheet and can be moved across it to allow for rotation of the earth. The adjustment is critical, the setting of the projector somewhat involved (a special "astrograph mean time" being needed), and a bulky and expensive projector is needed to prevent distortion. Because of these disadvantages and the fact that any advantage over short tabular methods is slight, the astrograph decreased in popularity following World War II.

Longley. In 1943 Flight Lieutenant C. D. N. Longley, RAF, suggested a "Star Computer" based upon the principle of the astrograph. A circular disk serving as a base plate would have a mean time scale around its circumference. Altitude curves of a limited number of stars would be printed on a template for each latitude. The circumference of each template would also carry a mean time scale. A radial cursor would aid in reading the device, which is set by means of the GMT at which LHA ♈ is 0° at some convenient longitude, the time of observation, and observed altitude. Longitude is determined within a 10° band, the ambiguity being resolved by means of the dead reckoning position. With a modification of the procedure, the device can be used with the altitude method.

Baker. As early as 1919 Commander T. Y. Baker, RN, prepared altitude curves and their orthogonals (normals) on transparent tape which is wound on rollers in the "Baker Navigating Machine" (fig. 2123b). The transparent tape is moved across a Mercator plotting sheet, being oriented by means of a time scale set with respect to a meridian. The line of position is transferred to the plotting sheet by means of carbon paper. A single tape has curves for several stars, and a separate tape for each 4° of declination from 24° N to 24° S permits use of the device with the sun and other

bodies of the solar system. A rule attached to the machine (shown at the top of fig. 2123b) provides a correction for declination differing from that of the curves.

Davies. The principle of the Baker Navigating Machine was used by Commander T. D. Davies, USN, in 1947 in a device for use in the antarctic. A chart on the polar azimuthal equidistant projection (art. 320) is printed on plastic material. A set of altitude curves is printed on a second sheet and placed under the chart, being pivoted at the south pole. A slot in the material bearing the altitude curves permits adjustment for any declination between 8°S and 18°S, the values the sun was to have had during the original period of use. Additional sets of curves could be provided for other declination ranges, or the slot increased in length. In the use of the device, the curves are oriented for GHA and declination, and a short segment of the curve representing the observed altitude is traced on the chart.

Weems. In 1955 Captain P. V. H. Weems, USN (Ret.), prepared a somewhat similar device called a "Polar Computer," using his star altitude curves.

Leick. In 1911 Dr. A. Leick, a German, prepared a diagram by which latitude and LST could be obtained by altitudes of Polaris and one other star. The diagram can be used for finding the correction to apply to the altitude of Polaris to determine the latitude, and then to find the LST in a second step.

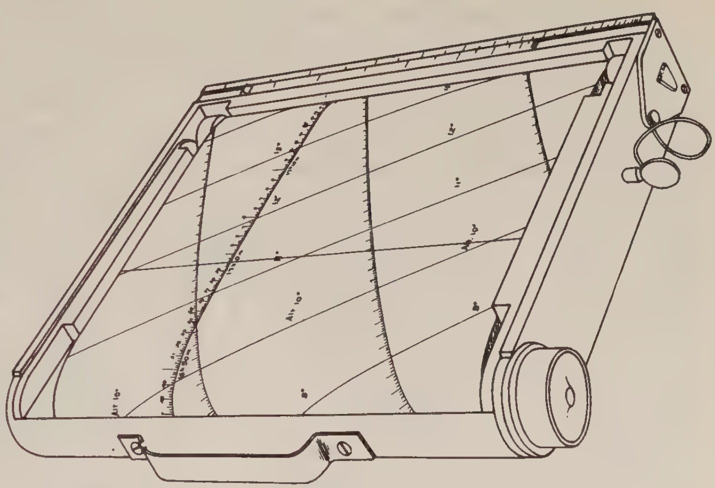


FIGURE 2123b.—The Baker Navigating Machine.

Favé. In 1901 Favé devised a graphical solution based upon the Marcq St.-Hilaire principle (art. 2108). A chart on the stereographic projection (art. 318) is used. Tables of computed altitude and azimuth for the point of tangency are needed. The chart is on transparent material. An additional sheet has a set of arcs of circles, with a straight azimuth line drawn normal to them. The chart is placed over the curves with the straight azimuth line through the point of tangency and oriented in the direction of the celestial body. A large circle on the chart assists in this orientation. The chart is then moved along the azimuth line until the curve representing the computed altitude at the point of tangency is under that point. The curve representing the observed altitude is then correctly placed and a segment of it can be traced on the chart. However, due to chart distortion, error is introduced in this way. It can be removed by means of a nomogram which indicates the correct curve to use. A mark is placed on the chart at the intersection of the azimuth line and the curve representing the observed altitude. The chart is then moved along the azimuth line a second time until the correct curve is in place, and the arc is traced. This process is repeated for each celestial body observed. For stars, a one-page set of curves can be used instead of tables for determining altitude and azimuth at the point of tangency. Favé recommended use of five separate charts with points of tangency at 0°, 30°, 45°, 75°, and 90°, respectively. Each chart could be used as a plotting sheet for any longitude at the same latitude, requiring computed altitude and azimuth for only five places. Favé

later put his method into instrumental form and used a special protractor and curved ruler.

Brill. In 1909 Dr. Alfred Brill, a German, invented a device based upon the same principle used by Favé, as shown in figure 2123c. In this device the plotting sheet is on the azimuthal equidistant projection (art. 320) and covers about 10° of latitude. Two sets of curves on separate sheets of tracing cloth are mounted below the plotting sheet. A handle turns the plotting sheet to the correct azimuth.

Voigt. The same principle used by Favé and Brill was used in the Voigt "Orion" instrument constructed in Germany in 1911. A plotting sheet on the azimuthal equidistant projection is engraved on aluminum. Each of the three plotting sheets, centered on latitudes 42°, 50°, and 55°, respectively, covers a spread of 10° of latitude. The line of position is drawn by means of a flexible ruler mounted on a bridge that can be clamped at any position over the plotting sheet. The curvature is controlled by means of gears, a scale being provided to indicate the correct value.

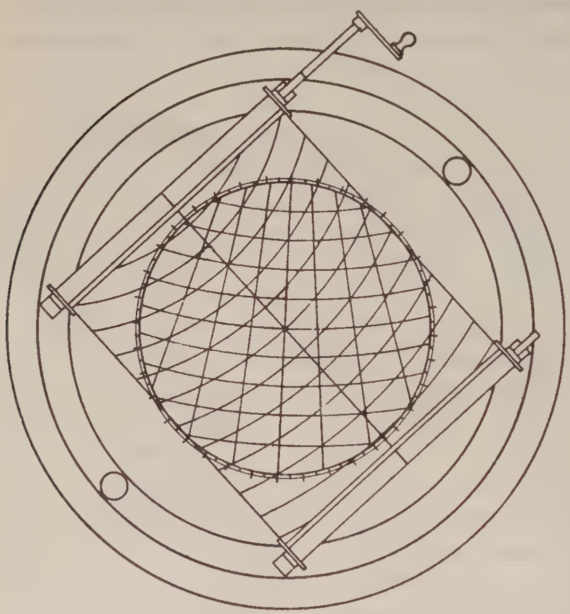


FIGURE 2123c.—The Brill device.

Vucetic. In 1921 a device called a "Toposcope" was prepared by Vucetic, a Frenchman. The device is identical with the Brill instrument except that a single set of curves is prepared and these are cut through the material as slots, and placed over the top of the plotting sheet.

Littlehales in 1918 suggested a method similar to that of Favé, but with a polyconic projection (art. 315).

Kahn. In 1928 Louis Kahn, a French naval architect, proposed that a set of navigational charts be prepared on the oblique Mercator projection (art. 310), a separate chart being provided for the great circle between various places on the earth. On each chart the small circles on the earth directly below the parallels of declination (that is, the daily paths of the geographical positions) of various navigational stars would be shown. These circles would be graduated in Greenwich sidereal time, so that the GP at any GST would be indicated. The distance from any assumed position to the GP at the instant of observation would be the zenith distance, and the direction of the line would be the azimuth. By comparing the observed zenith distance with that at the assumed position, the navigator could obtain the altitude difference, and plot the line of position. The common intersection of two or more such lines of position, advanced or retired to a common time if necessary, would define the position of the observer. The method would be limited to zenith distance within the range of the chart. A later version would produce greater accuracy, but with a little more trouble in making the measurements, by substituting the gnomonic projection (art. 317) for the oblique Mercator projection.

Dusinberre. In 1944 Lieutenant Commander H. W. Dusinberre, USN, suggested a method using star diagrams. A diagram for each 1° of latitude and 1° of LHA♈ would be provided. Each diagram would consist of a series of radial lines extending in the directions of the prominent stars favorable for observation. The 22 stars of

H.O. Pub. No. 218 (art. 2113) were suggested. Until changed by precession of the equinoxes (art. 1419) the common origin of these lines would represent a definite altitude for each star. The altitude at the next higher whole degree or half degree, adjusted for refraction, would be indicated by a tick on the appropriate azimuth line. After observation, a transparent plotting board would be properly oriented over the appropriate star diagram, using LHA ♈ and adjusting for the run between observations. The line of position would then be drawn at the correct point, perpendicular to the azimuth line, using the tick as a guide. An LHA ♈ computer was proposed for determining LHA ♈ at the time of each observation from a single LHA ♈ for a time near the start of each set of observations. When all lines of position were plotted, the fix would be transferred to the chart or plotting sheet.

2124. Solution by sphere.—Solution of a spherical triangle directly on a spherical surface, or by means of arcs representing great circles on the surface of an imaginary sphere, must have occurred to man quite early. Pictures of ancient navigators surrounded by their instruments and accessories invariably show a sphere. Solution by sphere is still suggested from time to time. Although this method is relatively simple and easy, the problem of scale is even more acute than in the graphical solutions.

Spherical methods can be classified in three groups: (1) those which solve the navigational triangle for a single line of position, (2) those which solve two or more observations for a fix, and (3) those which combine observation and solution for a fix.

The first group constructs the navigational triangle with arcs of great circles. Essentially, such a device consists of three arcs. The one representing the celestial meridian is usually fixed and a part of the frame. The base to which it is attached usually carries the azimuth scale. Movable arcs are provided for the vertical circle and the hour circle. If the latitude, meridian angle, and declination are properly set, the three arcs form the navigational triangle, and altitude and azimuth angle can be read from their scales. If altitude is used for constructing the triangle, meridian angle can be read from the instrument for a longitude solution.

Willis. A large number of teaching aids has been based upon this design or one of the many possible variations of it. Several precision instruments have been proposed or actually constructed. In 1932 such an instrument designed by Edward J. Willis, an American engineer, was constructed in Scotland. The marine version, weighing about 27 pounds, is graduated to 1′; and the aeronautical version, weighing between seven and eight pounds, is graduated to 5′. The longest dimension of either version is 11 inches.

Japanese Navy. During World War II, the Japanese Navy used an instrument virtually in the form described above. Results were accurate to approximately 1′.

McMillen. Of the various methods of determining a fix by sphere, the most obvious is that of providing an actual sphere as a plotting surface, with provision for striking arcs equal to the zenith distances, using the geographical positions of the celestial bodies as centers. In 1943 such a method was proposed by D. A. McMillen, a United States businessman in São Paulo, Brazil. His sphere, of a little more than 14 inches in diameter, had a scale of 8° (480 nautical miles) per inch along a great circle.

Hiltner. In 1945 Dr. W. F. Hiltner, a professor at Lehigh University, suggested a similar method using arcs of spheres and a billiard ball. This, in effect, sets up two navigational triangles, locating the observer at the common zenith of both triangles. Simultaneous observations are needed.

U. S. Navy Training Device Center. About the same time, the Training Device Center of the U. S. Navy prepared a device called the "Sphereman Craft Positioner," combining the functions of the devices of both McMillen and Hiltner, and providing a

plotting surface for dead reckoning. A line of position from a single observation can be drawn on the 17-inch aluminum globe, or the triangle of position from the observation of three stars can be mechanically set up. Provision is made for advancement or retirement of lines due to motion of the craft. The device was intended for training purposes.

Zerbee. In 1951 Louis J. Zerbee, of Bellfontaine, Ohio, proposed a device similar to that of Hiltner, but without the billiard ball. His instrument was called the "Zerbee Celestial Fix Finder." Like the Hiltner device, that of Zerbee makes no provision for nonsimultaneous observations (unless one of them is corrected to the value it would have if observed simultaneously with the other) or for a check by observation of additional bodies. Observations of bodies near the meridian or taken from high latitudes cannot be accommodated.

Combined sextant and computer. At least as early as 1895 an attempt was made to combine in a single instrument the functions of sextant and computer. Such instruments are fundamentally the same as those described above, except that they are set by alignment with one or more celestial bodies. If the instrument is level and accurately aligned with the meridian at the time of observation, the miniature sphere is oriented to the celestial sphere and the earth. If both the altitude and azimuth are used, a fix can be obtained by means of a single celestial body. If two bodies are observed simultaneously, accurate directional reference by compass is not needed.

The weakness of such methods is the need for a stable platform and either accurate directional reference or the need for observing two bodies simultaneously.

Beehler. In 1895 Lieutenant W. H. Beehler, USN, invented an instrument he called the "Solarometer," which was designed to furnish a position from observation of the sun. It requires a heavy cast iron base rigidly attached to the ship, with a bowl set in gimbals and filled with mercury. A float resting on the mercury carries the sighting instrument.

Hagner. In 1936 Fred Hagner, of San Antonio, Tex., invented a similar instrument he called the "Hagner Position Finder." This is a portable instrument operating on the same principle as the Solarometer, but obtaining the vertical by being hung from a suitable support, and therefore acting as a pendulum. This is reminiscent of the ancient astrolabe (art. 124).

Bedell. In 1953 A. L. Bedell, of St. Louis, Mo., proposed an instrument based upon simultaneous observation of two celestial bodies. The horizontal would be defined by spirit level.

Zenith photography. A number of suggestions have been made for eliminating a miniature sphere and locating the zenith among the stars. Several methods of doing this have been proposed, but the usual suggestion is to use a stabilized camera to photograph a portion of the sky in the vicinity of the zenith, which would be marked by a small cross within the camera. Use of a quick-developing method would reduce the delay. The position of the craft would be determined by comparison of the developed picture with a graduated star chart. Another suggestion is to reverse this process by comparing a previously made photograph with the actual sky.

Automatic celestial navigation. The principal weakness of methods requiring stabilization is the high order of accuracy needed. An error of 1′ introduces an error of one mile in the position. Such accuracy aboard a moving craft subject to various accelerations has been elusive. If stabilization of the required accuracy is available, it can be utilized with automatic star trackers to provide automatic celestial navigation. Such a system has been proposed. By means of the star trackers, the device would be continually oriented to two celestial bodies, and if the device were set for sidereal time, latitude and longitude would be indicated continuously on dials. The only setting

required would be the shifting from one star to another when the altitude of the first one became too low for convenient use of the body.

What would seem to be the "final" step in the development of such methods would be the scheduling of a voyage or flight in advance and the automatic comparison of preset values with automatically observed values, any discrepancy being used to actuate controls to change the heading or speed of the craft so that it would be automatically guided along the prescribed track on a preselected schedule. Several such methods, either singly or in combination with inertial or Doppler methods (art. 809), have been proposed for use in guided missiles. They could be adapted for use aboard ship, but are very expensive.

2125. Azimuth.—Most of the methods described above provide for determination of both altitude and azimuth angle. Several provide only for altitude. The number of tables, diagrams, and devices providing solution for azimuth only is very great, approaching the number providing solution for both altitude and azimuth. The reason for this is that azimuth is needed for other purposes than sight reduction. One common use is for checking the compass. Since modern inspection tables have provided parallel columns of computed altitude and azimuth or azimuth angle, separate azimuth tables have decreased in popularity.

Azimuth can be determined by computation or by amplitudes (tab. 27, 28), as well as by azimuth table. The method of computation depends somewhat upon the information available. There are three general approaches:

Time azimuth is the name given an azimuth or azimuth angle computed with meridian angle (a function of time), latitude, and polar distance (or declination) as the known quantities. Solution can be made by the following formula:

$$Z=X+Y,\ Z=X\sim Y,\ \text{or}\ Z=180°-(X\sim Y),$$

in which $\tan X=\sin D \csc S \cot \tfrac{1}{2} t$

and $\tan Y=\cos D \sec S \cot \tfrac{1}{2} t.$

Further, $D=\tfrac{1}{2}[p\sim(90°-L)]$

and $S=\tfrac{1}{2}[p+(90°-L)].$

If S is less than 90°, use $Z=X+Y$ if p is greater than $(90°-L)$, or $Z=X\sim Y$ if p is less than $(90°-L)$.

If S is greater than 90°, use $Z=180°-(X\sim Y)$.

To convert Z to Zn, label Z north or south to agree with the latitude, and east or west to agree with the meridian angle.

Altitude azimuth is an azimuth or azimuth angle computed with altitude, latitude, and polar distance as the known quantities. Solution can be made by the formula:

$$\text{hav } Z=\sin (s-L) \sin (s-h) \sec h \sec L,$$

in which $s=\tfrac{1}{2}(h+L+p).$

Azimuth angle is labeled N or S to agree with the latitude, and E or W as the celestial body is east or west of the celestial meridian.

Time and altitude azimuth is computed with meridian angle, declination, and altitude as the known quantities, the most common formula being

$$\sin Z=\sin t \cos d \sec h.$$

The weakness of this method is that it does not indicate whether the celestial body is north or south of the prime vertical. Usually there is no question on this point, but if Z is near 90°, the quadrant may be in doubt. If this occurs, either the meridian angle or altitude when on the prime vertical can be determined from table 25 or by computation, using the formula

$$\cos t = \tan d \cot L$$

or

$$\sin h = \sin d \csc L.$$

If the altitude is *less*, or the meridian angle is *greater* than the value when the body is on the prime vertical, the azimuth angle should be labeled N or S to agree with the latitude. If h is *greater* or t is *less* than when on the prime vertical, Z should be given the contrary name (N or S) to that of the latitude.

Amplitudes. For checking the compass, a low altitude is desirable because it can be measured easiest and most accurately. If a celestial body is observed when its center is on the *celestial* horizon, the amplitude (art. 1428) can be taken directly from table 27. It is given a prefix E (east) if rising or W (west) if setting. It is given a suffix N or S to agree with the *declination* of the body. When the center of the sun is on the celestial horizon, its *lower limb* is about two-thirds of a diameter above the visible horizon. When the center of the moon is on the celestial horizon, its *upper limb* is on the visible horizon. When planets and stars are on the celestial horizon, they are a little more than one sun diameter above the visible horizon.

If the body is observed when its center is on the *visible horizon*, the *observed* value should be corrected by the value from table 28, using the rules given with the table, before comparison with the value taken from table 27. If preferred, the correction can be applied with reversed sign to the value taken from table 27 and compared with the uncorrected observed value. This is the procedure used if amplitude or azimuth is desired when the celestial body is on the visible horizon.

Example.—The DR latitude of a ship is 51°24′.6 N, at a time when the declination of the sun is 19°40′.4 N.

Required.—(1) The amplitude (A) when the center of the setting sun is on the celestial horizon.

(2) The amplitude when the center of the setting sun is on the visible horizon.

(3) The azimuth when the center of the setting sun is on the visible horizon.

Solution.—

(1)	A	W 32°.6 N	(tab. 27)
	T 28	1°.1 S	(Rev.)—applied to *tabulated* amplitude
(2)	A	W 33°.7 N	
(3)	Zn	303°.7	

2126. Azimuth tables are numerous. Originally, they were designed primarily for use in determining compass error. Since the sun was the celestial body customarily used for this purpose, most of the tables were designed with the sun in mind. Meridian angle is commonly expressed in terms of local apparent time, in intervals varying from about one to 20 minutes. In many of the tables, meridian angle increases *upward* from the bottom of the page.

The following are some of the principal azimuth tables:

Wakeley. The first known azimuth tables for use of the navigator were *The Regiment of the Pole Star* by Andrew Wakeley. These tables were part of the author's *The Mariner's Compass Rectified*, published in London in 1665. These tables show the "true hour of the day" at which the sun is at the various points of the compass.

Lynn Azimuth Tables, by Thomas Lynn (art. 2106), were published in 1829. This 364-page table gives azimuth angle computed by the haversine formula of article 2106.

Towson and Atherton. The *Tables to Facilitate the Practice of Great Circle Sailing,* by the Englishmen John Thomas Towson and J. W. Atherton, were designed primarily for great-circle sailing, but since they indicate the course, they were easily adapted to finding azimuth angle. They were published in England in 1847.

Burdwood. The *Tables of Sun's True Bearing or Azimuth,* by Staff Commander John Burdwood, RN, were first published in 1852, with additional parts being added in 1858, 1862, 1864, and 1866. Captain John E. Davis, RN, and Percy L. H. Davis, of the British Nautical Almanac Office, later added to the tables, making them complete for all values of altitude and for declination between 64°N and 64°S. These tables were standard in Great Britain for more than a century. They have now been largely replaced by H.D. 486 (H.O. Pub. No. 214) for mariners and A.P. 3270 (H.O. Pub. No. 249) for aviators. Burdwood used modifications of the time azimuth formula.

Labrosse. Azimuth tables by the Frenchman F. Labrosse were published in London in 1868, and later in Paris. In 275 pages this *Table des Azimuts du Soleil* covers latitudes from 61°N to 61°S, and declinations from 0° to 30°N or S. The following formula was used:

$$\cot Z = \frac{\tan d \cos L}{\sin t} - \sin L \cot t.$$

Fifteen editions had been published by 1920.

Shortrede. In 1869 Captain Robert Shortrede's *Azimuth and Hour Angle for Latitude and Declination* and *Tables for Finding Azimuth at Sea* were published in London.

John E. Davis. The first azimuth tables by Captain John E. Davis were published in 1875. These were published as an extension of the Burdwood tables.

Perrin. In Paris the *Nouvelles Tables Destinées à Abréger les Calculs Nautiques,* by Ensign de Vaisseau E. Perrin, French Navy, were published first in 1876. These consist of three tables of nine, seven, and six pages, respectively, providing elements for determination of azimuth by a short computation. Several editions were published.

Kortazzi, a Russian, produced a volume appropriately called *Modification des Tables d'Azimuth de Thomson* (art. 2106). These were published in Paris in 1880.

H.O. Pub. No. 66 (Schroeder and Wainwright), *Arctic Azimuth Tables.* Lieutenants Seaton Schroeder and Richard Wainwright, USN, prepared azimuth tables for use of the USS *Rodgers* in her search for the arctic steamer *Jeanette.* These were published in 1881. Azimuths to the nearest 1′ are given for each 10^m meridian angle between 4^h and 7^h, for latitudes between 70° and 88°, declination 0° to 23°, same name.

Decante. In 1882 Lieutenant de Vaisseau E. Decante, of the French Navy, prepared *Table du Cadran Solaire Azimutal,* which was published in 1904, in eight volumes for latitudes 1° to 66° and declinations 0° to 48°.

H.O. Pub. No. 260 (Schroeder and Southerland). The *Azimuths of the Sun* were prepared in 1882 by Lieutenant Seaton Schroeder, USN, and Master W. H. H. Southerland, USN. These are popularly called "Red Azimuth Tables," because of the red binding used for most printings. This designation distinguishes them from the "Blue Azimuth Tables" (H.O. Pub. No. 261). After 15 editions, these tables are still in use. Azimuth angles are given to the nearest 1′, at 10^m intervals of local apparent time from "sunrise" to "sunset" (middle of the sun on the *celestial* horizon), with the LAT and the azimuth angle of these phenomena given at the bottom of each column. A separate table is given for each 1° of latitude from 0° to 70°. The first part of the

book is a table for latitude 0°. The second part is devoted to tables of latitude and declination "same name." The third part gives "contrary name" tables. Declination entries are given at 1° intervals from 0° to 23°, with the approximate dates on which this is the declination of the sun. Extracts from these tables are given in appendix Y. Values are customarily taken by triple interpolation, using the right-hand "PM" LAT column as meridian angle, as shown in the following example:

Example 1.—Find the azimuth of a celestial body when its meridian angle is 71°24.'3 W and its declination is 18°23.'2 N, if the latitude is 23°16.'1 N.

Solution.—

		diff. for	diff.	corr. for	+	−
t	$4^h45^m.6$ W	10^m	(+) 48′	$4^m.4$	21′	
d	18°.4 N	1°	(−) 62′	0°.4		25′
L	23°.3 N	1°	(+) 25′	0°.3	8′	
tab.	79°13′			sum	29′	25′
corr.	(+) 4′			corr.		(+) 4′
Z	N 79°17′ W					
Zn	280°.7					

In the solution, the meridian angle is expressed in time units to the nearest $0^m.1$, and the declination and latitude in arc to the nearest 0°.1. The "diff. for" is the unit of the entering argument. The "diff." is the difference in minutes of arc between the tabulated value for the *nearest* values of t, d, and L, and the next value for the t, d, or L on the opposite side of the actual value. The "corr. for" is the difference between the actual value of t, d, and L and that used for entering the table. The correction for each element is found from this tabulation. For instance, the correction for t is $\frac{48' \times 4^m.4}{10^m} = 21'$. The total net correction is applied to the tabulated value to find Z, labeled N or S to agree with the latitude, and E or W to agree with the meridian angle. In entering the table, one should keep in mind that values of t increase *upward* from the bottom of the page. Care should be used in locating meridian angle, for the manner of labeling the values can easily be misunderstood. For latitude 0°, Z is labeled N or S to agree with declination. Interpolation is made for t and d only, and the value converted to Zn. The Zn at latitude 1° is then computed, and interpolation for latitude is made between the two values of Zn.

Blackburne. The New Zealand nautical almanac for 1883 carried the 177-page "A and B" azimuth tables, by H. S. Blackburne. By 1911, after several modifications, these emerged as *"A, B, C" Tables for Azimuth, Great Circle Sailing, and Reduction to the Meridian.* The range of both the latitude and declination is from 90° N to 90° S.

Lecky. In 1892 Captain S. T. S. Lecky, an Englishman, modified the Blackburne tables and produced another set of "A, B, C" tables which have been widely used.

Ebsen. The *Azimut-Tabellen* of Julius Ebsen, published in Germany in 1896, uses the same formula as Labrosse, and is arranged like H.O. Pub. No. 260, except that azimuth angles are given to the nearest 0°.1, and the time and azimuth angle of sunrise and sunset are given at the top of the table, in place of the dates of H.O. Pub. No. 260. In two volumes, coverage is for latitudes 72° N to 72° S, and declinations 0° to 29°. Tables are for same name only, contrary-name situations being handled by using the supplement of meridian angle, and using the supplement of the value taken from the table, as in H.O. Pub. No. 261.

Johnson. A *Combined Time and Altitude Azimuth Table* for latitudes and declinations from 0° to 80°, by A. C. Johnson of the British Royal Navy, was published in London in 1900. In the same year, his *Short, Accurate, and Comprehensive Altitude-*

Azimuth Tables were published. This publication consists of three tables for computation of azimuth for each degree of latitude and altitude from 0° to 75°, and each degree of declination from 30° N to 30° S.

Zhdanko. The Russian *Tables of Azimuth of the Sun*, by M. Zhdanko, published in 1900, supplied computed azimuth angles for latitudes between 61° and 75°. These were later expanded by Yustchenko.

Percy L. H. Davis. In 1900 Percy L. H. Davis took over the work previously done by Burdwood and John E. Davis, continuing to improve and extend the tables.

H.O. Pub. No. 261, *Azimuths of Celestial Bodies*, published by the U. S. Navy Hydrographic Office in 1902, extend the H.O. Pub. No. 260 tables by providing information in similar form (but with meridian angle increasing *downward* on the page) for declinations 24° to 70°. These are popularly called "Blue Azimuth Tables," from their blue binding. Tables for "same name" only are given. If latitude and declination are of contrary name, the tables are entered with the supplement of the meridian angle. The value taken from the table is then the supplement of the azimuth angle, which is labeled N or S to agree with the latitude and E or W to agree with the meridian angle. Extracts from H.O. Pub. No. 261 are given in appendix Z.

Example 2.—Find the azimuth of a celestial body when its meridian angle is 49°31′.6 E and its declination is 57°41′.4 N, if the latitude is 51°25′.5 S.

Solution.—

		diff. for	diff.	corr. for		+	−
t	$3^h18^m.1$ E						
180°−t	$8^h41^m.9$	10^m	(−) 75′	$1^m.9$			14′
d	57°.7 N	1°	(+) 36′	0°.3		11′	
L	51°.4 S	1°	(+) 14′	0°.4		6′	
tab.	26°57′				sum	17′	14′
corr.	(+) 3′				corr.		(+) 3′
180°−Z	27°00′						
Z	S 153°00′ E						
Zn	027°.0						

One step can be eliminated by considering the corrected value Z instead of 180°−Z, and labeling it N or S to agree with the *declination*. In this example the body is below the horizon, showing that a solution is no assurance that a body is visible.

Symonds. The *Nautical Astronomy, with New Tables*, by W. P. Symonds, British Survey Commissioner, Bombay, includes azimuth tables. It was published in 1912.

Goodwin. *An Equatorial Azimuth-Table*, by H. B. Goodwin, was published in 1921.

Purey-Cust. *Azimuth by Logs*, by Admiral Sir H. E. Purey-Cust, RN, was published in England in 1929. It consists of a three-page table of the logarithms of the six principal trigonometric functions at 10′ intervals (5′ below 10°) for solution of the time azimuth and altitude azimuth formulas.

Yustchenko. In 1935 A. Yustchenko, a Russian, extended the Zhdanko tables to all latitudes, in the work entitled *Azimuty Svetil* (Azimuths of Celestial Bodies). For each 10° of latitude (5°, 15°, 25°, etc., to 85°) complete azimuth tables (to the nearest 0°.1) are given for each 1^m of meridian angle and each 30′ of declination from 0° to 30°. At the bottom of each page are given corrections for 1° of latitude. This value is multiplied by the number of degrees between the actual latitude and the latitude for which the table was computed.

Cugle. *Cugle's Two-Minute Azimuths*, by Charles H. Cugle, were printed in 1935 in two large volumes. Coverage is for latitude 0° to 65° and declination 0° to 23°. The arrangement is almost identical with that of H.O. Pub. No. 260, except that

meridian angle increases *downward* on the page. The number of entries is multiplied by *five*, values being given for each 2^m of meridian angle.

Table 902. *Azimuts*, published in Paris in 1953, with the concurrence of the Marine Hydrographic Service, contains azimuth angles to the nearest 0°1 for each whole degree of latitude from 70° N to 70° S, each whole degree of declination from 0° to 30°, and each 10^m of meridian angle. The arrangement is similar to that of H.O. Pub. No. 260, except that meridian angle increases *downward* on the page.

2127. Azimuth diagrams have appeared in various forms, in addition to the general graphical and mechanical solutions discussed above. A graphical solution is generally more acceptable for azimuth than for altitude, because the accuracy requirement for azimuth is usually less.

Godfrey. A graphical solution has been available at least since 1858 when the *Time Azimuth Diagram* of Hugh Godfrey was published in London.

Weir. The *Azimuth Diagram* devised by Captain Patrick Weir, of the British Merchant Navy, was published in London in 1890, and by the U. S. Navy Hydrographic Office in 1891, under the title *Time Azimuth Diagram*.

Molfino. In 1901 the *Nomograma degli Azimut del Sole* of Molfino was published.

Constan. In 1906 P. Constan's *Tables Graphiques d'Azimut* were published in Paris. This was a reproduction of the graph of Favé and Rollet de l'Isle (art. 2120).

Alessio. The *Diagrammi Altazimutali* of A. Alessio was published in 1908 in Italy.

Rust. In 1908 the diagram of Lieutenant Commander Armistead Rust, USN, (art. 2106) was published. This diagram was later used by Goodwin (art. 2106) and Weems (arts. 2106 and 2110), and in the Italian *Tavole H* (art. 2110).

Cornet. The *Graphique d'Azimut* of Cornet was published in 1927.

Romanovsky. About 1933 A. A. Romanovsky, a Russian, devised a simple nomogram for determining azimuth.

German Oberkommandos der Kriegsmarine. A large volume called *Azimutdiagramme*, containing sets of diagrams for each whole degree of latitude (2° beyond 80°) for all azimuth angles and for all altitudes to 80°, was published by the German Oberkommandos der Kriegsmarine in 1944.

Hugon. The azimuth diagram of Professor P. Hugon (art. 2109) was published in 1947.

Hilsenrath. About 1948 Joseph Hilsenrath, of the University of Maryland, produced a mechanical device for solving azimuth angle by the method of Weir's diagram.

2128. Summary.—The methods of sight reduction discussed in this chapter are undoubtedly only a small fraction of the number of methods that have been proposed. They are considered representative of the effort that has been made to reduce the work of the navigator. Individual preferences have largely dictated the use of the various methods. Presentation and description of a method have been important factors in the relative popularity of various methods.

There is no single "best" method for all circumstances and all navigators. The one which produces the desired results easiest and with least possibility of mistake is the one that should be selected. However, two practical precautions should be observed. First, one should be thoroughly familiar with the *limitations* or *weaknesses* of the method he selects. Second, a prudent navigator will never limit himself to a single method, particularly one requiring a special table that might some day be unavailable, or a device that is subject to mechanical damage or loss. The slight bending of an arc might be too insignificant to be noticed, yet might introduce intolerably large errors in the result. A wise practice is to memorize, or write on something always carried, fundamental formulas that can be used when no "special" tables are available.

Problems

2103a. At GMT $10^h25^m22^s$ on June 1, 1958, the navigator observes the lower limb of the sun on the celestial meridian, bearing south. He makes the observation with a marine sextant having no IC, from a height of eye of 50 feet. The hs is 41°58′.7.

Required.—The latitude by meridian altitude.

Answer.—L 69°54′.1 N.

2103b. At GMT $14^h15^m21^s$ on June 1, 1958, the sun is estimated to be on the upper branch of the celestial meridian. At this time the sun is obscured by clouds, but several minutes later it breaks out, and at GMT $14^h24^m22^s$ the navigator observes the lower limb, facing in a northerly direction. He makes the observation with a marine sextant having an IC of (−) 2′.5, from a height of eye of 33 feet. The hs is 70°46′.9. The approximate latitude is 3° N.

Required.—(1) The latitude by reduction to the meridian. (2) The latitude if the navigator learns that his ship was 1′.2 of longitude farther *west* than assumed for computation of the time of meridian transit, and the azimuth angle at the time of observation was N 6°.0 W. Use table 26.

Answers.—(1) L 3°04′.1 N, (2) L 3°04′.0 N.

2103c. On June 1, 1958, the 1225 DR position of a ship is lat. 40°45′.7 N, long. 142°01′.9 W. At GMT $21^h25^m36^s$ the navigator observes the lower limb of the sun with a marine sextant having an IC of (+) 3′.3, from a height of eye of 29 feet. The hs is 71°06′.8.

Required.—(1) The *a*, Zn, and AP, using H.O. Pub. No. 214 (Δd, Δt, ΔL, and interpolating for Z) and the *Nautical Almanac.*

(2) The approximate latitude at the time of observation.

Answers.—(1) *a* 1.8 T, Zn 179°.8, *a*L 40°45′.7 N, *a*λ 142°01′.9 W; (2) L 40°43′.9 N.

2104a. Find the watch time of meridian transit of the sun at longitude 68°08′.4 E on June 1, 1958, if the watch is 27^s fast on zone time.

Answer.—W $12^h25^m31^s$.

2104b. Find the zone time of meridian transit of the moon at longitude 166°23′.2 E on June 13, 1958, using the *Nautical Almanac,* GHA method.

Answer.—ZT $8^h25^m42^s$.

2104c. Find the GMT of meridian transit of Nunki at longitude 157°52′.2 W on June 1, 1958, using the *Air Almanac.*

Answer.—GMT $12^h46^m11^s$.

2104d. On June 1, 1958, the 1200 DR position of a ship is lat. 57°21′.9 N, long. 21°53′.2 W. The ship is on course 065°, speed 22 knots. Find the zone time of meridian transit of the sun at the ship.

Answer.—ZT $12^h24^m10^s$.

2104e. Find the zone time of transit of the sun at longitude 47°23′.4 E on June 2, 1958, using apparent time and the *Nautical Almanac.*

Answer.—ZT $11^h48^m13^s$.

2105. During evening twilight on May 31, 1958, the 2325 EP of a ship is lat. 58°38′.4 N, long. 165°34′.3 W. At watch time $11^h25^m01^s$ PM the navigator observes Polaris with a marine sextant having no IC, from a height of eye of 42 feet. The watch is 6^s fast on zone time. The hs is 57°54′.4.

Required.—(1) The latitude, (2) the azimuth of Polaris.

Answers.—(1) L 58°35′.6 N, (2) Zn 001°.0.

2106. On June 2, 1958, the 0725 EP of a ship is lat. 7°31′.2 S, long. 22°35′.7 W. At GMT $9^h24^m42^s$ the navigator observes the lower limb of the sun with a marine sextant having an IC of (+) 2′.0, from a height of eye of 47 feet. The hs is 23°15′.0.

Required.—(1) The longitude by time sight.

(2) The longitude if the navigator learns that his ship was 2.1 miles farther *north* than assumed for computation, and the azimuth at the time of observation was 062°0. Use table 26.

Answers.—(1) λ 22°38′1 W, (2) λ 22°39′2 W.

2107a. Determine (1) the approximate zone time, and (2) the approximate altitude of the sun at its nearest approach to the prime vertical during the morning of June 1, 1958, at lat. 12°14′7 N, long. 35°16′1 W, using table 25 and the *Nautical Almanac.*

Answers.—(1) ZT 0826, (2) h 33°7.

2107b. Determine (1) the approximate zone time, and (2) the approximate altitude of the sun when it crosses the prime vertical during the afternoon of May 31, 1958, at lat. 41°17′2 N, long. 154°37′4 W, using H.O. Pub. No. 214 and the *Nautical Almanac.*

Answers.—(1) ZT 1625, (2) h 34°8.

2109. The dead reckoning latitude of a vessel is 10°23′8 S. The navigator observes a star having a declination of 28°51′5 N and a meridian angle of 27°17′4 E. He notes that it is in the northeast quadrant of the sky.

Required.—The (1) Hc by cosine-haversine formula and (2) azimuth by the formula sin Z=sin t cos d sec h.

Answers.—(1) Hc 42°43′4, (2) 033°1.

2113a. During evening twilight on June 1, 1958, the 1740 DR position of a ship is lat. 41°28′5 S, long. 82°17′6 W. At ZT $17^h41^m08^s$ the navigator observes Rigel Kent. with a marine sextant having an IC of (−)1′2, from a height of eye of 55 feet. The hs is 44°05′2.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 249, vol. I (epoch 1960.0), and the *Air Almanac.*

Answers.—*a* 13 T, Zn 140°, *a*L 41°00′ S, *a*λ 82°11′ W.

2113b. During morning twilight on June 2, 1958, the DR position of a ship is lat. 41°08′2 N, long. 5°11′5 E. At GMT $3^h45^m11^s$ the navigator observes Fomalhaut with a marine sextant having an IC of (+)0′8, from a height of eye of 49 feet. The hs is 13°13′7.

Required.—The *a*, Zn, and AP, using H.O. Pub. No. 249, vol. III, and the *Air Almanac.*

Answers.—*a* 10 A, Zn 152°, *a*L 41°00′ N, *a*λ 5°27′ E.

2125. The DR latitude of a ship is 15°11′3 S when the declination of the sun is 18°36′5 N.

Required.—(1) The amplitude (A) when the center of the rising sun is on the celestial horizon.

(2) The amplitude and azimuth when the center of the rising sun is on the visible horizon.

Answers.—(1) A E 19°3 N; (2) A E 19°1 N, Zn 070°9.

2126a. The dead reckoning latitude of a vessel is 23°53′6 S. The navigator observes a celestial body having a declination of 20°26′2 S and a meridian angle of 18°37′4 E.

Required.—Azimuth by H.O. Pub. No. 260.

Answer.—Zn 082°2.

2126b. The dead reckoning latitude of a vessel is 51°46′5 N. The navigator observes a celestial body having a declination of 49°42′7 N and a meridian angle of 115°37′2 W.

Required. Azimuth by H.O. Pub. No. 261.

Answer.—Zn 319°9.

CHAPTER XXII

IDENTIFICATION OF CELESTIAL BODIES

2201. Introduction.—A basic requirement of celestial navigation is the ability to identify the bodies observed. This is not difficult because relatively few celestial bodies are commonly used for navigation, and various aids are available to assist in their identification, as explained in this chapter.

Many navigators consider it a matter of professional pride to have a more extensive acquaintance with the heavens than required by the relatively simple demands of navigation.

2202. Bodies of the solar system.—No problem is encountered in the identification of the sun and moon. However, the planets can be mistaken for stars. A person working continually with the night sky recognizes a planet by its changing position among the relatively fixed stars. He identifies the planets by noting their positions relative to each other, the sun, the moon, and the stars. He knows that they remain within the narrow limits of the zodiac (art. 1420) but are in almost constant motion relative to the stars. The magnitude and color may be helpful. The information he needs is found in the *Nautical Almanac*. The "Planet Notes" near the front of that volume are particularly useful.

Sometimes the light from a planet seems steadier than that from a star. This is because fluctuation of the unsteady atmosphere causes **scintillation** or **twinkling** of a star, which has no measurable diameter with even the most powerful telescopes. The navigational planets are less susceptible to twinkling because of the broader apparent area giving light.

Planets can also be identified by the *Air Almanac* ecliptic diagram (art. 2209), star finder (art. 2210), sky diagram (art. 2212), or by computation (art. 2213).

2203. Stars.—The average navigator regularly uses not more than perhaps 20 or 30 stars. The *Nautical Almanac* gives full navigational information on 19 first magnitude stars and 38 second magnitude stars, in addition to Polaris. Abbreviated information is given for 115 more. Additional stars are listed in *The American Ephemeris and Nautical Almanac* and in various star catalogs. About 6,000 stars of the sixth magnitude or brighter (on the entire celestial sphere) are visible to the unaided eye on a clear, dark night.

Stars are designated by one or more of the following:

Name. Most names of stars, as now used, were given by the ancient Arabs and some by the Greeks or Romans. One of the stars of the *Nautical Almanac*, Nunki, was named by the Babylonians. Only a relatively few stars have names. Several of the stars on the daily pages of the almanacs had no name prior to the 1953 edition, and were given coined names so that all stars listed on the daily pages might have names. The pronunciation, meaning, and other information of general interest regarding Polaris and the 57 stars listed on the daily pages of the *Nautical Almanac* are given in appendix H.

Bayer's name. Most bright stars, including those with names, have been given a designation consisting of a Greek letter followed by the possessive form of the name

of the constellation, as *α Cygni* (Deneb, the brightest star in the constellation *Cygnus*, the swan). Roman letters are used when there are not enough Greek letters. Usually, the letters are assigned in order of brightness within the constellation, but in some cases the letters are assigned in another order, where it seems logical to do so. An example is the big dipper, where the letters are assigned in order from the outer rim of the bowl to the end of the handle. This system of star designation was suggested by John Bayer of Augsburg, Germany, in 1603. All of the 173 stars included in the list near the back of the *Nautical Almanac* are given by Bayer's name as well as regular name, where there is one.

Flamsteed's number. A similar system, accommodating more stars, numbers them in each constellation, from west to east, the order in which they cross the celestial meridian. An example is 95 *Leonis*, the 95th star in the constellation *Leo*, the lion. This system was suggested by John Flamsteed (1646–1719), who was the first British Astronomer Royal.

Catalog number. Stars are sometimes designated by the name of a star catalog and the number of the star as given in that catalog, as A. G. Washington 632. In these catalogs stars are listed in order from west to east, without regard to constellation, starting with the hour circle of the vernal equinox. This system is used primarily for dimmer stars having no other designation. Navigators seldom have occasion to use this system.

The ability to identify stars by position relative to each other is useful to the navigator. A tabulation of the relative positions of the 57 stars given on the daily pages of the *Nautical Almanac*, and Polaris, is given in appendix G. A star chart (art. 2204) is helpful in locating these relationships and others which may be useful. This method is limited to periods of relatively clear, dark skies with little or no overcast. Stars can also be identified by the *Air Almanac* ecliptic diagram (art. 2209), star finder (art. 2210), H.O. Pub. No. 249 (art. 2211), sky diagram (art. 2212), or by computation (art. 2213).

2204. Star charts are based upon the celestial equator system of coordinates, using declination and sidereal hour angle (or right ascension). The zenith of the observer is at the intersection of the parallel of declination equal to his latitude, and the hour circle coinciding with his celestial meridian. This hour circle has an SHA equal to $360° - \text{LHA}\,♈$ (or $\text{RA} = \text{LHA}\,♈$). The horizon is everywhere 90° from the zenith. A **star globe** is similar to a terrestrial sphere, but with stars (and often constellations) shown instead of geographical positions. Star globes are used by British navigators, but not customarily by Americans. The combined *Nautical Almanac* includes instructions for using this device. On a star globe the celestial sphere is shown as it would appear to an observer *outside* the sphere. Constellations appear reversed. Star charts may show a similar view, but more often they are based upon the view from *inside* the sphere, as seen from the earth. On these charts, north is at the top, as with maps, but east is to the *left* and west to the *right*. The directions seem correct when the chart is held overhead, with the top toward the north, so that the relationship is similar to that in the sky. Any map projection (ch. III) can be used, but some are more suitable than others.

The *Nautical Almanac* has four star charts. The two principal ones are on the polar azimuthal equidistant projection (art. 320), one centered on each celestial pole. Each chart extends from its pole to declination 10° (same name as pole). Below each polar chart is an auxiliary chart on the Mercator projection, from 30° N to 30° S. On any of these charts, the zenith can be located as indicated above, to determine which stars are overhead. The horizon is 90° from the zenith. The charts can also be used

to determine the location of a star relative to surrounding stars. The *Air Almanac* has a fold-in chart at the back, on the rectangular projection (art. 311). This projection is suitable for indicating the coordinates of the stars, but excessive distortion occurs in regions of high declination. The celestial poles are represented by the top and bottom horizontal *lines* the same length as the celestial equator. To locate the horizon on this chart, first locate the zenith as indicated above, and then locate the four cardinal points. The north and south points are 90° from the zenith, along the celestial meridian. The distance to the elevated pole (having the same name as the latitude) is equal to the colatitude of the observer. The remainder of the 90° (the latitude) is measured *from* the same pole, along the *lower branch* of the celestial meridian, 180° from the upper branch containing the zenith. The east and west points are on the celestial equator at the hour circle 90° east and west (or 90° and 270° in the same direction) from the celestial meridian. The horizon is a sine curve (fig. O40b) through the four cardinal points. Directions on this projection are distorted.

The star charts shown in figures 2205–2208, on the transverse Mercator projection (art. 309), are designed to assist one in learning the stars listed on the daily pages of the *Nautical Almanac*, and Polaris. Each chart extends about 20° beyond each celestial pole, and about 60° (four hours) each side of the central hour circle (at the celestial equator). Therefore, they do not coincide exactly with that half of the celestial sphere above the horizon at any one time or place. The zenith, and hence the horizon, varies with the position of the observer on the earth, and also with the rotation of the earth (apparent rotation of the celestial sphere). The charts show all stars of fifth magnitude and brighter as they appear in the sky, but with some distortion toward the right and left edges.

The transparencies add certain information of use in locating the stars. Only Polaris and the 57 stars listed on the daily pages of the *Nautical Almanac* are named on the charts. The almanac star charts should be used for locating the additional stars given near the back of the *Nautical Almanac*. When a transparency is correctly placed over its accompanying chart, the information given is properly oriented to the chart. The broken lines connect stars of some of the more prominent constellations. The solid lines indicate the celestial equator and certain useful relationships among stars in different constellations. The celestial poles are marked by crosses, and labeled. By means of the celestial equator and the poles, one can locate his zenith approximately along the mid hour circle, when this coincides with his celestial meridian, as shown in the table below. At any time earlier than those shown in the table the zenith is to the *right* of center, and at a later time it is to the *left*, approximately one-quarter of the distance from the center to the outer edge (at the celestial equator) for each hour that the time differs from that shown. The stars in the vicinity of the north pole can be seen in proper perspective by inverting the chart, so that the zenith of an observer in the northern hemisphere is *up* from the pole.

	Fig. 2205	*Fig. 2206*	*Fig. 2207*	*Fig. 2208*
Local sidereal time	0000	0600	1200	1800
LMT 1800	Dec. 21	Mar. 22	June 22	Sept. 21
LMT 2000	Nov. 21	Feb. 20	May 22	Aug. 21
LMT 2200	Oct. 21	Jan. 20	Apr. 22	July 22
LMT 0000	Sept. 22	Dec. 22	Mar. 23	June 22
LMT 0200	Aug. 22	Nov. 22	Feb. 21	May 23
LMT 0400	July 23	Oct. 22	Jan. 21	Apr. 22
LMT 0600	June 22	Sept. 21	Dec. 22	Mar. 23

2205. Stars in the vicinity of *Pegasus* (fig. 2205).—In autumn the evening sky has few first magnitude stars. Most of these are near the southern horizon of an observer in the latitudes of the United States. A relatively large number of second and third magnitude stars seem conspicuous, perhaps because of the small number of brighter stars. High in the southern sky three third magnitude stars and one second magnitude star form a square with sides nearly 15° of arc in length. This is *Pegasus*, the winged horse, although to many modern men it more nearly resembles a baseball diamond, complete with catcher, pitcher, batter, umpire, base umpire near second base, infield and outfield; although there does seem to be a large number of outfielders. One may even see the next batter, bat boy, and coach.

Only Markab at the southwestern corner (third base) and Alpheratz at the northeastern corner (first base) are listed on the daily pages of the *Nautical Almanac*. Alpheratz is part of the constellation *Andromeda*, the princess, extending in an arc toward the northeast and terminating at Mirfak in *Perseus*, legendary rescuer of *Andromeda*.

A line extending northward through the eastern side (first-second base line) of the square of *Pegasus* passes through the leading (western) star of M-shaped (or W-shaped) *Cassiopeia*, the legendary mother of the princess *Andromeda*. The only star of this constellation listed on the daily pages of the *Nautical Almanac* is Schedar, the second star from the leading one as the configuration circles the pole in a counterclockwise direction. If the line through the eastern side of the square of *Pegasus* is continued on toward the north, it leads to second magnitude Polaris, the north star (less than 1° from the north celestial pole) and brightest star of *Ursa Minor*, the little bear. Kochab, a second magnitude star at the other end of the little dipper, is also listed in the almanacs. At this season the big dipper is low in the northern sky, below the celestial pole. A line extending from Kochab through Polaris leads to Mirfak, assisting in its identification when *Pegasus* and *Andromeda* are near or below the horizon.

Deneb, in *Cygnus*, the swan, and Vega are bright, first magnitude stars in the northwestern sky. They are discussed in article 2208. Capella, a bright star in the northeastern sky, is discussed in article 2206.

The line through the eastern side of the square of *Pegasus* (first-second base line) approximates the hour circle of the vernal equinox, shown at ♈ on the celestial equator to the south. The sun is at ♈ on or about March 21, when it crosses the celestial equator from south to north. If the line through the eastern side of *Pegasus* is extended southward and curved slightly toward the east, it leads to second magnitude Diphda. A longer and straighter line southward through the western side (home plate-third base line) of *Pegasus* leads to first magnitude Fomalhaut. A line extending northeasterly from Fomalhaut through Diphda leads to Menkar, a third magnitude star, but the brightest in its vicinity. Ankaa, Diphda, and Fomalhaut form an isosceles triangle, with the apex at Diphda. Ankaa is near or below the southern horizon of observers in latitudes of the United States. Four stars farther south than Ankaa may be visible when on the celestial meridian, just above the horizon of observers in latitudes of the extreme southern part of the United States. These are Acamar, Achernar, Al Na'ir, and Peacock. These stars, with each other and with Ankaa, Fomalhaut, and Diphda, form a series of triangles as shown in figure 2205. Almanac stars near the bottom of figure 2205 are discussed in succeeding articles.

Two other almanac stars can be located by their positions relative to *Pegasus*. These are Hamal in the constellation *Aries*, the ram, east of *Pegasus*, and Enif, west of the southern part of the square, identified as shown in figure 2205. The line leading to Hamal, if continued, leads to the *Pleiades*, not used by navigators for celestial observations, but a prominent figure in the sky, heralding the approach of the many conspicuous stars of the winter evening sky, figure 2206.

FIGURE 2205.—Stars in the vicinity of *Pegasus*.

2206. Stars in the vicinity of *Orion* (fig. 2206).—As *Pegasus* leaves the meridian and moves into the western sky, *Orion*, the mighty hunter, rises in the east. With the possible exception of the big dipper, no other configuration of stars in the entire sky is as well known as *Orion* and its immediate surroundings. In no other part are there so many first magnitude stars.

The belt of *Orion*, being nearly on the celestial equator, is visible by an observer in virtually any latitude, rising and setting almost on the prime vertical, and dividing equally its time above and below the horizon. Of the three second magnitude stars forming the belt, only Alnilam, the middle one, is listed on the daily pages of the *Nautical Almanac.*

Four conspicuous stars form a box around the belt. To the south is Rigel, one of the hottest and bluest of the stars, in contrast with relatively cool, red, variable Betelgeuse, at approximately an equal distance to the north. Bellatrix, bright for a second magnitude star but overshadowed by its more brilliant neighbors, is a few degrees west of Betelgeuse. Neither the second magnitude star forming the southeastern corner of the box, nor any star of the dagger, is listed on the daily pages of the *Nautical Almanac.*

A line extending eastward from the belt of *Orion* and curving toward the south leads to Sirius, the brightest star in the entire heavens, having a magnitude of (−) 1.6. Only Mars and Jupiter at or near their greatest brilliance, and the sun, moon, and Venus are brighter than Sirius. This is part of the constellation *Canis Major*, the large hunting dog of *Orion.* Starting at Sirius a curved line extends northward through first magnitude Procyon, in *Canis Minor*, the small hunting dog; first magnitude Pollux and second magnitude Castor (not listed on the daily pages of the *Nautical Almanac*), the twins of *Gemini;* brilliant Capella in *Auriga*, the charioteer; and back down to first magnitude Aldebaran, the follower, which trails the *Pleiades*, the seven sisters. Aldebaran, brightest star in the head of *Taurus*, the bull, may also be found by a curved line extending northwestward from the belt of *Orion.* The V-shaped figure forming the outline of the head and horns of *Taurus* points toward third magnitude Menkar. At the summer solstice the sun is between Pollux and Aldebaran.

If the curved line from *Orion's* belt southeastward to Sirius is continued, it leads to a conspicuous, small, nearly equilateral triangle of three bright second magnitude stars of nearly equal brilliancy. This is part of *Canis Major.* Only Adhara, the westernmost of the three stars, is listed on the daily pages of the *Nautical Almanac.* Continuing on with somewhat less curvature, the line leads to Canopus, second brightest star in the heavens and one of the two stars having a negative magnitude (−0.9). With Suhail and Miaplacidus, Canopus forms a large, equilateral triangle which partly encloses the false southern cross. The brightest star within this triangle is Avior, near its center. Canopus is also at one apex of a triangle formed with Adhara to the north and Suhail to the east, another triangle with Acamar to the west and Achernar to the southwest, and another with Achernar and Miaplacidus. Acamar, Achernar, and Ankaa form still another triangle toward the west. Because of chart distortion, these triangles do not appear in the sky in exactly the relationship shown on the star chart. Other daily-page almanac stars near the bottom of figure 2206 are discussed in succeeding articles.

During the winter evening sky the big dipper is east of Polaris, the little dipper is nearly below it, and *Cassiopeia* is west of it. Mirfak is northwest of Capella, nearly midway between it and *Cassiopeia.* Hamal is in the western sky. Regulus and Alphard are low in the eastern sky, heralding the approach of the configurations associated with the evening skies of spring.

FIGURE 2206.—Stars in the vicinity of *Orion*.

2207. Stars in the vicinity of *Ursa Major* (fig. 2207).—As if to enhance the splendor of the sky in the vicinity of *Orion*, the region toward the east, like that toward the west, has few bright stars, except in the vicinity of the south celestial pole. However, as *Orion* sets in the west, leaving Capella and Pollux in the northwestern sky, a number of good navigational stars move into favorable positions for observation.

The big dipper, part of *Ursa Major*, the great bear, appears prominently *above* the north celestial pole, directly opposite *Cassiopeia* (only partly shown in fig. 2207), which appears as a W just above the northern horizon of most observers in latitudes of the United States. Of the seven stars forming the big dipper, only Dubhe, Alioth, and Alkaid are listed on the daily pages of the *Nautical Almanac*.

The two second magnitude stars forming the outer part of the bowl of the big dipper are often called the *pointers* because a line extending northward (*down* in spring evenings) through them points to Polaris. The little dipper, with Polaris at one end and Kochab at the other, is part of *Ursa Minor*, the little bear. Relative to its bowl, the handle of the little dipper curves in the opposite direction to that of the big dipper. Other almanac stars near the top of figure 2207 are discussed elsewhere.

A line extending southward through the pointers, and curving somewhat toward the west, leads to first magnitude Regulus, brightest star in *Leo*, the lion. The head, shoulders, and front legs of this constellation form a sickle, with Regulus at the end of the handle. Toward the east is second magnitude Denebola, the tail of the lion. On toward the southwest from Regulus is second magnitude Alphard, brightest star in *Hydra*, the sea serpent. A dark sky and considerable imagination are needed to trace the long, winding body of this figure.

A curved line extending the arc of the handle of the big dipper leads to first magnitude Arcturus. With Alkaid and Alphecca, brightest star in *Corona Borealis*, the northern crown, Arcturus forms a large, inconspicuous triangle. If the arc through Arcturus is continued, it leads next to first magnitude Spica and then to *Corvus*, the crow, which appears most like a gaff mainsail of a schooner. The brightest star in this constellation is Gienah, but three others are nearly as bright. At autumnal equinox the sun is on the celestial equator, about midway between Regulus and Spica.

A long, slightly curved line from Regulus east-southeasterly through Spica leads to Zubenelgenubi (zo͞o·bĕn′ĕl·jĕ·nū′bē) at the southwestern corner of an inconspicuous box-like figure called *Libra*, the (weighing) scales.

Returning to *Corvus*, a line from Gienah, extending diagonally across the figure and then curving somewhat toward the east, leads to Menkent, just beyond *Hydra*.

Far to the south, below the horizon of most northern-hemisphere observers, a group of bright stars is a prominent feature of the spring sky of the southern hemisphere. *Crux*, the southern cross, is about 40° south of *Corvus*. This is a small figure and a poor cross, and hence disappointing to many who view it for the first time. The "false cross" to the west is a better but less conspicuous cross. Acrux at the southern end of the southern cross, and Gacrux at the northern end, are listed on the daily pages of the *Nautical Almanac*.

The triangles formed by Suhail, Miaplacidus, and Canopus, and by Suhail, Adhara, and Canopus, are west of the southern cross, Suhail being in line with the horizontal arm of the southern cross at this time. A line from Canopus, through Miaplacidus, curved slightly toward the north, leads to Acrux. A line through the east-west arm of *Crux*, eastward and then curving toward the south, leads first to Hadar and then to Rigil Kentaurus, two very bright stars. Continuing on, the curved line leads to small *Triangulum Australe*, the southern triangle, the easternmost star of which is Atria.

Scorpius, the scorpion, Kaus Australis, and Peacock, in the southeastern sky of the southern hemisphere, are discussed in article 2208.

FIGURE 2207.—Stars in the vicinity of *Ursa Major.*

Scale of magnitudes: 1st 2nd 3rd 4th 5th

2208. Stars in the vicinity of *Cygnus* (fig. 2208).—As the celestial sphere continues in its apparent westward rotation, the stars familiar to a spring evening observer sink low in the western sky. By midsummer the big dipper has moved to a position to the left of the north celestial pole, and the line from the pointers to Polaris is nearly horizontal. The little dipper is standing on its handle, with Kochab above and to the left of the celestial pole. *Cassiopeia* is at the right of Polaris, opposite the handle of the big dipper.

The only first magnitude star in the western sky is Arcturus, which forms a large, inconspicuous triangle with Alkaid, the end of the handle of the big dipper, and Alphecca, the brightest star in *Corona Borealis*, the northern crown.

The eastern sky is dominated by three very bright stars. The westernmost of these is Vega, the brightest star north of the celestial equator, and third brightest star in the heavens. Its magnitude is 0.1. Having a declination of a little less than 39° N, this star passes through the zenith along a path across the central part of the United States, from Washington in the east to San Francisco on the Pacific coast. Vega forms a large but conspicuous triangle with its two bright neighbors, Deneb to the northeast and Altair to the southeast. The angle at Vega is nearly a right angle. Deneb is at the end of the tail of *Cygnus*, the swan. This configuration is sometimes called the northern cross, with Deneb at the head. To modern youth it more nearly resembles a dive bomber while it is still well toward the east, with Deneb at the nose of the fuselage. Altair has two fainter stars close by, on opposite sides. The line formed by Altair and its two fainter companions, if extended in a northwesterly direction, passes through Vega, and on to second magnitude Eltanin. The angular distance from Vega to Eltanin is about half that from Altair to Vega. Vega and Altair, with second magnitude Rasalhague to the west, form a large equilateral triangle. This is less conspicuous than the Vega-Deneb-Altair triangle because the brilliance of Rasalhague is much less than that of the three first magnitude stars, and the triangle is overshadowed by the brighter one.

Far to the south of Rasalhague, and a little toward the west, is a striking configuration called *Scorpius*, the scorpion. The brightest star, forming the head, is red Antares. At the tail is Shaula.

Antares is at the southwestern corner of an approximate parallelogram formed by Antares, Sabik, Nunki, and Kaus Australis. With the exception of Antares, these stars are only slightly brighter than a number of others nearby, and so this parallelogram is not a striking figure. At winter solstice the sun is a short distance northwest of Nunki.

Northwest of *Scorpius* is the box-like *Libra*, the (weighing) scales, in which Zubenelgenubi marks the southwest corner.

With Menkent and Rigil Kentaurus to the southwest, Antares forms a large but unimpressive triangle. For most observers in the latitudes of the United States, Antares is low in the southern sky, and the other two stars of the triangle are below the horizon. To an observer in the southern hemisphere *Crux*, the southern cross, is to the right of the south celestial pole, which is not marked by a conspicuous star. A long, curved line starting with the now-vertical arm of the southern cross and extending northward and then eastward passes successively through Hadar, Rigil Kentaurus, Peacock, and Al Na'ir.

Fomalhaut is low in the southeastern sky of the southern hemisphere observer, and Enif is low in the eastern sky at nearly any latitude. With the appearance of these stars it is not long before *Pegasus* will appear over the eastern horizon during the evening, and as the winged horse climbs evening by evening to a position higher in the sky, a new annual cycle approaches.

FIGURE 2208.—Stars in the vicinity of *Cygnus*.

Scale of magnitudes: 1st 2nd 3rd 4th 5th

2209. Ecliptic diagram.—On each right-hand page of the daily tabulations of the *Air Almanac* (app. W) an ecliptic diagram shows a band of the sky 16° wide (the zodiac, art. 1420), with the sun at the center. Shown in correct position relative to the sun (except when very close to it) are the moon, selected planets and stars, and the vernal equinox. This diagram is useful for planning purposes and for locating the planets. That part of the diagram to the left of the sun is east of it, approximately coinciding with the visible part during evening twilight. That part to the right, or west, of the sun coincides approximately with the visible portion during morning twilight. The two ends are that point in the sky 180° from the sun. These diagrams were replaced in 1965 by a single *Planet Location Diagram.*

2210. Star finders.—Various devices have been invented to help an observer locate and identify individual stars. The most widely used is the Star Finder and Identifier published by the U. S. Navy Hydrographic Office. The current model, H.O. 2102–D, as well as the previous 2102–C model patented by E. B. Collins, formerly of that Office, employs the same basic principle as that used in the Rude Star Finder, which was patented by Captain G. T. Rude, USC&GS, and later sold to the Hydrographic Office. Successive models reflect various modifications to meet changing conditions and requirements.

The *star base* of H.O. 2102–D consists of a thin, white, opaque, plastic disk about 8½ inches in diameter, with a small peg in the center. On one side the north celestial pole is shown at the center, and on the opposite side the south celestial pole is at the center. All of the stars listed on the daily pages of the *Nautical Almanac* are shown on a polar azimuthal equidistant projection (art. 320) extending to the opposite pole. The south pole side is shown in figure 2210a. Many copies of an older edition, H.O. 2102–C, showing the stars listed in the almanacs prior to 1953, and having other minor differences, are still in use. These are not rendered obsolete by the newer edition, but should be corrected by means of the current almanac. The rim of each side is graduated to half a degree of LHA♈ (or 360°−SHA).

Ten transparent *templates* of the same diameter as the star base are provided. There is one template for each 10° of latitude, labeled 5°, 15°, 25°, etc., plus a tenth (printed in red) showing meridian angle and declination. The older edition (H.O. 2102–C) did not have the red meridian angle-declination template. Each template can be used on either side of the star base, being centered by placing a small center hole in the template over the center peg of the star base. Each latitude template has a family of altitude curves at 5° intervals from the horizon (from altitude 10° on the older H.O. 2102–C) to 80°. A second family of curves, also at 5° intervals, indicates azimuth. The north-south azimuth line is the celestial meridian. The star base, templates, and a set of instructions are housed in a circular leatherette container.

Since the sun, moon, and planets continually change apparent position relative to the "fixed" stars, they are not shown on the star base. However, their positions at any time, as well as the positions of additional stars, can be plotted. To do this, determine 360°−SHA of the body. For the stars and planets, SHA is listed in the *Nautical Almanac.* For the sun and moon, 360°−SHA is found by subtracting GHA of the body from GHA♈ at the same time. Locate 360°−SHA on the scale around the rim of the star base. A straight line from this point to the center represents the hour circle of the body. From the celestial equator, shown as a circle midway between the center and the outer edge, measure the declination (from the almanac) of the body *toward* the center if the pole and declination have the *same* name (*both* N or *both* S), and *away* from the center if they are of *contrary* name. Use the scale along the north-south azimuth line of any template as a declination scale. The meridian angle-declination template (the latitude 5° template of H.O. 2102–C) has an open slot

FIGURE 2210a.—The south pole side of the star base of H.O. 2102–D.

with declination graduations along one side, to assist in plotting positions, as shown in figure 2210b. In the illustration the celestial body being located has a 360°−SHA of 285°, and a declination of 14°.5 S. It is not practical to attempt to plot to greater precision than the nearest 0°.1. Positions of Venus, Mars, Jupiter, and Saturn on June 1, 1958, are shown plotted on the star base in figure 2210c. It is sometimes desirable to plot positions of the sun and moon, to assist in planning. Plotted positions of stars need not be changed. Plotted positions of bodies of the solar system should be replotted from time to time, the more rapidly moving ones oftener than others. The satisfactory interval for each body can be determined by experience. It is good practice to record the date of each plotted position of a body of the solar system, to serve later as an indication of the interval since it was plotted.

To orient the template properly for any given time, proceed as follows: enter the almanac with GMT, and determine GHA ♈ at this time. Apply the longitude to GHA ♈, subtracting if west or adding if east, to determine LHA ♈. If LMT is substituted for GMT in entering the almanac, LHA ♈ can be taken directly from the almanac, to sufficient accuracy for orienting the star finder template. Select the tem-

FIGURE 2210b.—Plotting a celestial body on the star base of H.O. 2102–D.

plate for the latitude nearest that of the observer, and center it over the star base, being careful that the correct sides (north or south to agree with the latitude) of both template and star base are used. Rotate the template relative to the star base until the arrow on the celestial meridian (the north-south azimuth line) is over LHA ♈ on the star base graduations. The small cross at the origin of both families of curves now represents the zenith of the observer. The approximate altitude and azimuth of the celestial bodies above the horizon can be read directly from the star finder, using eye interpolation. Consider Polaris, not shown, as at the north celestial pole. For more accurate results, the template can be lifted clear of the center peg of the star base, and shifted along the celestial meridian until the latitude, on the altitude scale, is over the pole. This refinement is not needed for normal use of the device. It should not be used for a latitude differing more than 5° from that for which the curves were drawn. If the altitude and azimuth of an identified body shown on the star base are known, the template can be oriented by rotating it until it is in correct position relative to that body.

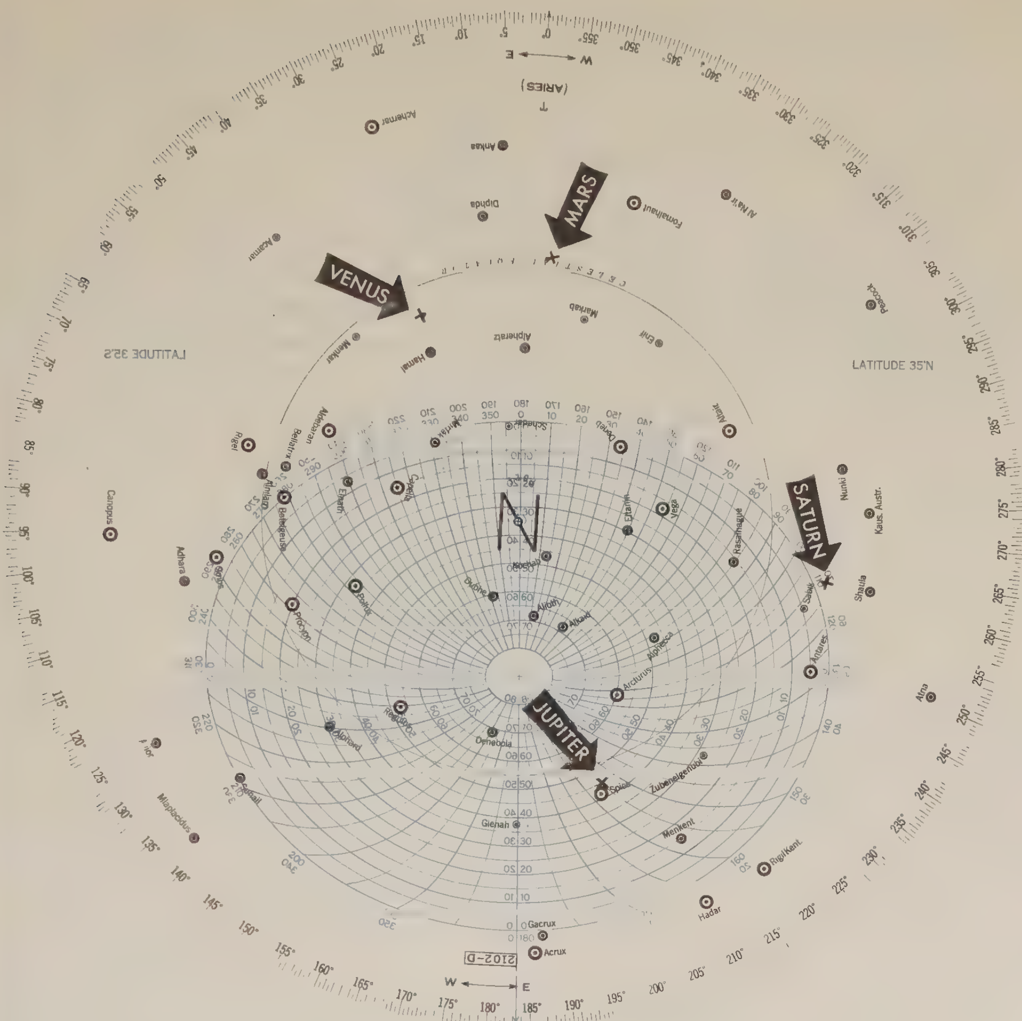


FIGURE 2210c.—A template in place over the star base of H.O. 2102–D.

Customarily, H.O. 2102–D is used in either of two ways:

1. To make an advance list of celestial bodies available for observation at a given time.

2. To identify an unknown celestial body which has been observed.

Example 1.—During evening twilight on June 1, 1958, the GMT 2324 DR position of a ship is lat. 34°12′.5 N, long. 57°46′.8 W.

Required.—The approximate altitude (h*a*) and azimuth of each first magnitude star, and any planets, between altitudes 15° and 75°.

Solution (fig. 2210c).—(1) Plot the positions of the planets, as shown. The values used are those for GMT 1200 on June 1, as follows:

Planet	*360° − SHA*	*Dec.*
Venus	28°.5	9°.6 N
Mars	356°.7	3°.5 S
Jupiter	201°.1	7°.3 S
Saturn	262°.8	21°.8 S

(2) Determine LHA ♈ by means of the *Nautical Almanac*, as follows:

GMT	2324	June 1
23^h	234°55′.0	
24^m	6°01′.0	
GHA ♈	240°56′.0	
λ	57°46′.8 W	
LHA ♈	183°09′.2	

(3) Select the template for latitude 35°, place it over the north side of the star base with "LATITUDE 35° N" appearing correctly, and orient it to 183°.2. It is customary to list the bodies in order of increasing azimuth, as follows:

Body	*ha*	*Zn*
Vega	17°	054°
Arcturus	59°	111°
Jupiter	45°	155°
Spica	42°	157°
Regulus	53°	240°
Procyon	20°	262°
Pollux	33°	284°
Capella	15°	316°

Example 2.—At the time and place of example 1, an unidentified celestial body is observed through a break in the clouds. Its sextant altitude is 15°27′.8, and its azimuth is 085°.

Required.—Identify the celestial body.

Solution (fig. 2210c).—Orient the template as in example 1. By means of its altitude and azimuth, identify the star as Rasalhague.

If no body appears at the measured altitude and azimuth, place the red meridian angle-declination template over the altitude-azimuth template and read off, by inspection, the declination and the 360°−SHA value of the body, and from this, determine its SHA. Using the SHA and declination, enter the list of stars near the back of the *Nautical Almanac*, and identify the body. If it is not found in this list, and no error has been made, one of the stars not listed in the almanac, or possibly the planet Mercury, has been observed. Unless a copy of *The American Ephemeris and Nautical Almanac* or another book containing the required information is available, the observation cannot be used. If right ascension (art. 1426) of the body is available, but not its SHA, the value taken from the star finder (360°−SHA) is converted to time units (art. 1904) and used directly, since RA=360°−SHA.

Example 3.—At the time and place of example 1 an unidentified celestial body is observed through a break in the clouds. Its altitude is 52°58′.9, and its azimuth is 170°.

Required.—Identify the celestial body.

Solution (fig. 2210c).—Orient the template as in example 1. Since no celestial body appears at the place indicated by its altitude and azimuth, the red meridian angle-declination template is placed over the altitude-azimuth template. The declination is found to be about 1° S. The 360°−SHA value is about 190°, and SHA is therefore about 170°. From the star list near the back of the *Nautical Almanac*, the star is identified as γ *Virginis*.

2211. Sight Reduction Tables for Air Navigation (H.O. Pub. No. 249).—Volume I of H.O. Pub. No. 249 can be used as a star finder for the stars tabulated at any given

time. For these bodies the altitude and azimuth are tabulated for each 1° of latitude and 1° of LHA♈ (2° beyond latitude 69°). The principal limitation is the small number of stars listed.

2212. Sky diagram.—Near the back of the *Air Almanac* are a number of **sky diagrams.** These are azimuthal equidistant projections (art. 320) of the celestial sphere on the plane of the horizon, at latitudes 70°N, 50°N, 30°N, 10°N, 10°S, and 30°S, at intervals of two hours of local mean time each month. A number of the brighter stars, the visible planets, and several positions of the moon are shown at their correct altitude and azimuth. These are of limited value because of their small scale; the large increments of latitude, time, and date; and the limited number of bodies shown. However, in the absence of other methods, particularly a star finder, these diagrams can be useful. Allowance can be made for variations from the conditions for which each diagram is constructed. Instructions for use of the diagrams are included in the *Air Almanac*.

2213. Identification by computation.—If the altitude and azimuth of the celestial body, and the approximate latitude of the observer, are known, the navigational triangle (art. 1433) can be solved for meridian angle and declination. The meridian angle can be converted to LHA, and this to GHA. With this and GHA of Aries at the time of observation, the SHA of the body can be determined. With SHA and declination, one can identify the body by reference to an almanac. Any method of solving a spherical triangle, with two sides and the included angle being given, is suitable for this purpose. "Short" methods such as H.O. Pubs. Nos. 208 and 211 include instructions for star identification by the tables provided. A large-scale, carefully-drawn diagram on the plane of the celestial meridian, using the refinement shown in figure 1432f, should yield satisfactory results. Perhaps the simplest method of actual computation is by H.O. Pub. No. 214. Following the tables of computed altitude and azimuth for each latitude, a two-page star identification table is given, as shown in appendix AA. The example given below is based upon this extract.

The steps in solution by H.O. Pub. No. 214 are:

1. Convert Zn to Z.

2. With Z and h*a* (usually the approximate value taken from the sextant, without correction) enter the H.O. Pub. No. 214 star identification pages for the nearest whole degree of latitude, and extract the declination and meridian angle, t (given as H.A. in the table). If the declination is given in roman type, above the heavy line, it has the *same* name as the latitude. If the declination is given in *italics*, below the heavy line, it has the *contrary* name to that of the latitude. When interpolating between roman and italic declinations, consider the italic value negative, using the arithmetical *sum* as the algebraic *difference* needed for interpolation. Extract values to the nearest whole degree.

3. Convert t to LHA.

4. Apply the longitude to LHA to find GHA, adding if in west longitude, and subtracting if in east longitude.

5. Enter the *Nautical Almanac* with GMT, and determine GHA♈.

6. Subtract GHA♈ from GHA☆ to find SHA (since GHA☆=GHA♈+SHA).

7. With the approximate SHA and d enter the *Nautical Almanac* star list and identify the body, checking first the SHA and then the declination. Do not overlook the possibility of having observed a planet or a star not listed in the almanac. For a planet, check first the declination. If this is approximately correct, check the GHA. It is not necessary to find the SHA of a planet.

Example.—On May 31, 1958, the 0425 DR position of a ship is lat. 41°13′.6 N, long. 140°41′.7 W. About this time the navigator observes an unknown star through a break in the clouds, as follows: GMT $13^h24^m46^s$, hs 15°01′.5, Zn 232°.

Required.—Identify the unknown celestial body, using H.O. Pub. No. 214.

Solution.—

	May 31				
GMT	$13^h24^m46^s$ May 31		Zn	232°	
13^h	83°31′.2		Z	N 128° W	
24^m46^s	6°12′.5		ha	15°	
GHA ♈	89°43′.7	(subtract)	d	16° S	(from H.O. Pub. No. 214)
GHA☆	193°		t	52° W	(from H.O. Pub. No. 214)
SHA☆	103°		LHA	52°	
d	16° S		λ	141° W	
Body	Sabik		GHA	193°	

Problems

2210a. During morning twilight on June 3, 1958, the GMT 1825 (June 2) DR position of a ship is lat. 26°21′.4 N, long. 157°10′.4 E.

Required.—The approximate altitude and azimuth of each first magnitude star, and any planets, between altitudes 10° and 80°, using H.O. 2102–D.

Answer.—

Body	*ha*	*Zn*
Venus	27°	092°
Mars	47°	128°
Fomalhaut	32°	160°
Saturn	14°	237°
Altair	59°	242°
Vega	50°	301°
Deneb	67°	334°

2210b. At the time and place of problem 2210a an unidentified celestial body is observed through a break in the clouds. Its sextant altitude is 21°04′.1 and its azimuth is 044°.

Required.—Identify the celestial body, using H.O. 2102–D.

Answer.—Mirfak.

2210c. The dead reckoning latitude of a ship is 25°06′.4 S. Two stars are observed in quick succession, as follows:

Star	*ha*	*Zn*
Antares	57°	100°
Unidentified	52°	337°

Required.—Identify the unknown celestial body, using H.O. 2102–D.

Answer.—*ε Virginis.*

2213. On June 2, 1958, the 1725 DR position of a ship is lat. 41°27′.3 S, long. 158°36′.9 E. About this time the navigator observes two unknown celestial bodies through breaks in the clouds, as follows: (1) GMT $6^h24^m15^s$, hs 16°04′.9, Zn 334°; (2) GMT $6^h25^m53^s$, hs 30°38′.1, Zn 071°. The second body appears to be of the first magnitude, with a bright but somewhat dimmer body above it and slightly to the right.

Required.—Identify the unknown celestial bodies, using H.O. Pub. No. 214.

Answers.—(1) Pollux, (2) Jupiter.

PART FIVE

THE PRACTICE OF NAVIGATION

PART FIVE

The Practice of Navigation

CHAPTER XXIII

THE PRACTICE OF MARINE NAVIGATION

2301. Introduction.—In the preceding 22 chapters, the various elements of navigation are discussed separately. In this chapter the interrelationship of the various parts is discussed. However, the most important element of successful navigation cannot be acquired from this book—nor from any book or instructor. The *science* of navigation can be taught, but the *art* of navigation must be acquired. Modern navigation is a blending of the two—a *scientific art*. The truly successful navigator is one who supplements his knowledge with judgment, utilizing every opportunity to improve his judgment through experience. Even with knowledge and judgment, the navigator cannot expect to be fully reliable unless he is alert, constantly evaluating the situation as it develops, avoiding dangerous situations before they arise, or recognizing them if they do occur, and always keeping "ahead of the vessel." The elements of successful navigation, then, are *knowledge*, *judgment*, and *alertness*. To the person possessing these, navigation can be a pleasure. A person who tries to navigate without them is at best a doubtful asset. He may be a menace to his vessel and shipmates.

It is not wise to attempt to reduce navigation to a series of steps that can be followed mechanically. The methods and techniques to be used are those which are applicable to the type of vessel, the equipment available, the training and experience of the navigator and any assistants, the local situation, etc. The navigation of a small craft proceeding up the Choptank River, for instance, might be quite different from that of an ocean liner entering New York harbor. Both might differ from the navigation of a naval vessel approaching an assigned anchorage. It is important that a navigator make an "estimate of the situation" and use the methods and techniques that are best adapted to the conditions at hand.

The discussion that follows is generally applicable to any vessel under average conditions, but is written primarily for an average ship which might be planning and executing an ocean voyage.

2302. Advance preparation.—Before starting a voyage, the navigator should familiarize himself with his equipment and the conditions to be encountered. Any defective or questionable instruments should be repaired or replaced. The necessary charts and publications should be on hand. If the voyage is to extend beyond the time range of any publication, such as an almanac or tide tables, the volume for the next period should be included, or provision should be made to acquire it before the expiration date of the current volume. Charts and light lists should be checked to see that they have been corrected through the latest *Notice to Mariners*.

When all equipment is on hand and in suitable condition, the navigator should study his charts and publications. He should determine which soundings are in feet, which in fathoms, and whether other units are used (app. L). It is good practice to underline or circle with a colored pencil the statement of units as given on each chart. The various notes on the chart should be read, and applicable ones marked. The latitude and longitude scales should be observed and the units noted. The channels, currents, shoals, aids to navigation, and natural landmarks should be studied so that the general arrangement is familiar. Useful natural ranges should be located and marked. Where needed, turning bearings, danger angles, and danger bearings should be determined.

The tides and currents to be encountered should be determined from the tables and charts. The advice and warnings given in coast pilots or sailing directions should be read and pertinent parts marked or copied out. The light list should be studied, and circles of visibility for the usual height of eye drawn in. Characteristics should be written on the chart, if not printed there, or in a notebook, to assist in identification. Useful radar targets, radiobeacons, loran rates, etc., should be noted if equipment to utilize them is available. If a danger sounding is useful, it should be drawn in. The bottom configuration should be studied for distinctive features that will prove helpful in locating the position of the vessel, or keeping it in safe water. If foreign charts are to be used, the symbols should be understood.

The extent of the preliminary study depends somewhat upon the navigator's previous knowledge of the area. But however familiar he may be with local conditions, the navigator should not overlook the need for checking his equipment to be sure it is complete and up-to-date, nor to refresh his memory regarding critical items of information. The prudent navigator leaves nothing to chance and *assumes nothing that can be verified.*

In pilot waters with limited maneuvering space, the desired track might well be plotted in advance, and the predicted time between buoys, turns, etc., determined. Where repeated runs are made over the same routes, the entire track may be plotted in ink. Courses, distances between lights, visibility circles, and other useful information might be prominently indicated. When this practice is followed, a positive routine should be set up to apply corrections and to bring these to the attention of all concerned.

2303. Getting underway.—Shortly before the ship gets underway the necessary charts, publications, and plotting equipment should be placed on the chart table. A check should be made to be sure that all marks (except those permanently plotted in ink or colored pencil) relating to a previous voyage have been erased from the charts. The navigator's binoculars should be checked to see that they are properly secured in their accustomed place on the bridge. The gyro compasses should be started sufficiently in advance to insure proper operation, and should then be compared with the repeaters and the magnetic compass on the bridge. A check should be made to see that the latest deviation tables are available, and that magnetic gear has not been left near the compass. Azimuth circles and peloruses should be in place and checked. The standard and emergency steering gear should be checked, as well as communication and signaling equipment. If practical, the mechanical log and electronic equipment such as radar, loran, radio direction finder, and echo sounder should be started and checked. The hand lead should be placed at a convenient location ready for immediate use. The anchor windlass should be tested. The sextant, chronometers, almanac, and tables should be checked to see that they are in their proper places. It is good practice for the navigator to prepare a check-off list to insure that nothing is overlooked. The checks should be made carefully by a responsible person.

Before getting underway the navigator should see that all navigational personnel are at their assigned stations and that each understands his duties. It is good practice to acquaint each person with the general plan of operation, for an informed person is less likely to make mistakes, and more likely to detect mistakes made by others.

2304. Leaving port.—In a harbor, the largest scale chart should be used for greatest accuracy and detail. The dead reckoning should be started as soon as the vessel steadies on its first course. If the desired track has not been plotted in advance, the dead reckoning is run ahead a short distance. In either event, the predicted time of arrival at the next turning bearing, or of passing the next aid to navigation is recorded on the chart. Predicted times of arrival at various points are of great importance in interpreting the information received and in avoiding dangerous situa-

tions. It is good practice to use *all* available information, and not rely solely upon a single aid. A good position should be maintained at all times. Fog may set in rapidly and without warning, obscuring landmarks before a round of bearings can be observed. Lights should be timed and identified by their characteristics. At a distance, the color and shape of buoys may not be apparent. Sometimes a sailboat can be mistaken for a buoy. Buoys may be out of position. Bearings and ranges on fixed objects are better than on floating aids which do not remain at fixed points. Soundings should be taken continuously in the vicinity of shoal water. It is good practice to check the compass at convenient opportunities, as when on a range. Ranges are of great value for checking position or keeping on the desired track, and should be used whenever available.

By skillful navigation, one may be able to save many miles of steaming. However, it is possible to allow insufficient margin of safety. The navigator should always keep in mind the possibility of failure of some item of equipment, unexpected fog, or the need for maneuvering room if another vessel approaches too close. He should remember, too, that in pilot waters currents may be strong and variable.

A detailed record should be kept in a notebook. Entries should be made showing bearings and ranges, important soundings, all changes of course and speed, the times of passing important aids to navigation, and other pertinent information. The record should leave nothing in doubt, indicating whether bearings are true or by magnetic compass, whether soundings are in feet or fathoms, etc. This record is useful in preparing the ship's log, providing guidance for future runs over the same area, establishing position if fog sets in, and in providing an acceptable record if the vessel experiences a mishap resulting in a later investigation.

The chart, also, should present a neat and intelligible record of the passage. Course lines and lines of position should be drawn lightly and neatly, and should be no longer than needed. Labels should be used wherever they contribute to an understanding of the plot. They should be so placed and worded that no doubt is left as to their applicability and meaning. If possible, lines and labels should not be drawn through chart symbols.

Outside the harbor, if the course is parallel to the coast, there may be advantages in remaining close enough to utilize major aids to navigation and other landmarks. However, a set toward the beach, particularly off the entrance to an estuary, can endanger the safety of a vessel. Many ships have grounded because a course was set too close to off-lying dangers.

2305. Taking departure.—When a vessel reaches the open sea and is about to leave the land astern, a last accurate position is obtained by means of landmarks available. This process is called **taking departure.** It marks the end of piloting and the beginning of the next phase of the navigation. The work of the navigator becomes less hurried, and fixes are obtained less frequently. Soundings become of less interest. The hand lead is secured. The position may be transferred from the chart to a plotting sheet. Courses and speeds will be maintained over relatively long periods. The sea routine begins. Even if the vessel is to follow the coast, it generally does so at such a distance that danger is some distance away, and navigation is an intermittent process rather than a continuous one.

2306. Navigation at sea, like piloting, varies somewhat from vessel to vessel depending upon the equipment available and the individual preferences of the navigator. A daily routine, called the **day's work,** is established by the navigator and carried out with such variations as dictated by circumstances. While details vary with the navigator, a typical minimum day's work is:

1. Plot of dead reckoning throughout the day.

2. Observation and reduction of celestial observations for a fix during morning twilight.

3. Winding of chronometers and determination of chronometer error.

4. Observation of the sun for a morning sun line (on or near the prime vertical if made at about the same time as 5).

5. Azimuth of the sun for a compass check. This may be an amplitude observation at sunrise, but is more commonly made at about the same time as a morning sun line observation.

6. Observation of the sun at or near noon. This is crossed with a morning sun line, advanced, or with an observation of the moon or Venus to obtain a noon (ZT 1200) position.

7. Computation of the day's run (noon to noon, or midnight to midnight).

8. Observation of the sun during the afternoon (on or near the prime vertical if made at about the same time as 9). This is primarily for use with the advanced noon sun line, or with a moon or Venus line, if the skies are overcast during evening twilight.

9. Azimuth of the sun for a compass check. This is commonly made at about the same time as an afternoon sun observation, but may be an amplitude observation at sunset.

10. Computation of the time of sunset, sunrise, and twilight, and preparation of a list of stars and any planets in favorable positions for observation during each twilight period, with the approximate altitude and azimuth of each body.

11. Observation and reduction of celestial observations for a fix during evening twilight.

12. Computation of the time of moonrise and moonset (if required).

13. Use of loran and any other available electronic aid on a regular schedule, as every hour.

The list of celestial bodies available for observation is customarily prepared with the aid of a star finder such as H.O. 2102–D (art. 2210). This list is particularly helpful during evening twilight, when one desires to know where to look for the brightest stars or planets before the general pattern of stars becomes visible. Some navigators list or make a simple plot of the *relative* azimuths of the bodies, to assist in locating them. The brightest bodies may be visible at about the time of sunset, or even a little before. In general, it is good practice to observe the brightest bodies as they appear in the evening, while the horizon is clear and sharp, and the dimmest first in the morning, before they fade from view. Several observations should be made of each body, each sight being taken quickly to avoid eye fatigue. In general, it is better to use one good observation than to average several of questionable accuracy. At least five or six bodies should be observed. If the three most favorably situated ones provide a good fix, additional sights need not be reduced, but if doubt remains, information for obtaining additional lines is available. It is better to observe bodies all around the horizon than in the same semicircle. Thus, three bodies separated by 120° are better than three separated by 60°, for in the former case any constant error in altitude will be neutralized.

If a comparing watch is used, it should be compared with the chronometer or a time tick every time celestial observations are made. The index correction should be determined each time the sextant is used. If the horizon is used for this purpose, the measurement should be made *before* evening twilight observations and *after* morning twilight observations, while the horizon is sharp. If the horizon is not equally sharp in all directions, the best part should be used.

When skies clear after a prolonged period of overcast, or when clouds threaten to obscure the heavens, additional observations should be made, if available. During

the day a series of sun lines might be obtained and advanced to a common time, or the moon or Venus might be available at a favorable azimuth. Sometimes observations can be made during the night, either by use of moonlight to illuminate the horizon, or by dark-adapting the eyes. At this time the moon, and bodies having an azimuth nearly the same as the moon, should be avoided because of the probability of false horizons on the illuminated water.

Sights may be reduced by any reliable method. The one most widely used by mariners is H.O. Pub. No. 214, used in conjunction with the *Nautical Almanac*. If a check is needed, a good practice is to use a different method and a different almanac, so that mistakes will not be repeated.

Before the development of modern sight reduction methods, celestial navigation was largely a matter of determining latitude by observation of bodies on or near the celestial meridian (including Polaris) and longitude by observation of bodies on or near the prime vertical. Longitude was computed by time sight. Frequently, this method of navigation was inconvenient. Often it produced misleading results, as when a north-south "longitude" line was used instead of the true line of position which might differ in direction by as much as 30° or more. Errors were introduced when an incorrect longitude was used for solving a reduction to the meridian, or an incorrect latitude for solving a time sight of a body some distance from the prime vertical. The use of azimuth with a time sight was an improvement, but was not well adapted to observations of celestial bodies near the celestial meridian. The modern navigator is freed from these restrictions. He is able to obtain a *line of position* extending in the correct direction almost any time a celestial body can be observed. He places no special significance upon latitude and longitude lines, and solves all sights by a common method of sight reduction.

It is good practice to use a workbook for the various solutions made at sea. This provides a valuable record which may be of inestimable value in the future. Entries should be neat, orderly, and intelligible to another navigator. All original data and computations should be included. The use of standard work forms is recommended. Those given in appendix Q are slight modifications of forms developed at the United States Naval Academy and used widely at sea. They are considered adequate but, for sight reduction of celestial observations, there is merit in using a form which uses a single column, so that several sights can be reduced in parallel columns. The *best* form for anyone to use is one he thoroughly understands and finds logical and least confusing. If an alteration in a work form reduces the number of errors made, it is a desirable change. Because of the difference of opinion among marine navigators, and the tendency to follow mechanically an established form without fully understanding the principles involved, a work form standardized for all navigators is probably undesirable, although such is widely used by air navigators, who use celestial navigation somewhat intermittently. When one has established the work forms he desires to use, he can have a rubber stamp made, or have the forms reproduced by printing. The former is probably preferable because it permits use of a bound workbook. However, printed forms can be punched for retention in a looseleaf binder.

At sea it is good practice to run the dead reckoning from fix to fix, determining set and drift of the current at each fix. The use of single lines of position and current to establish estimated positions is a matter of judgment. The ability to predict the difference between dead reckoning positions and fixes, which ability may be developed when the need is not apparent, can serve as a valuable asset when fixes are not available. In the U. S. Navy, the best position available is recorded in the log at 0800, 1200, and 2000. A typical plot of part of a day's run at sea (omitting possible loran fixes) is shown in figure 2306.

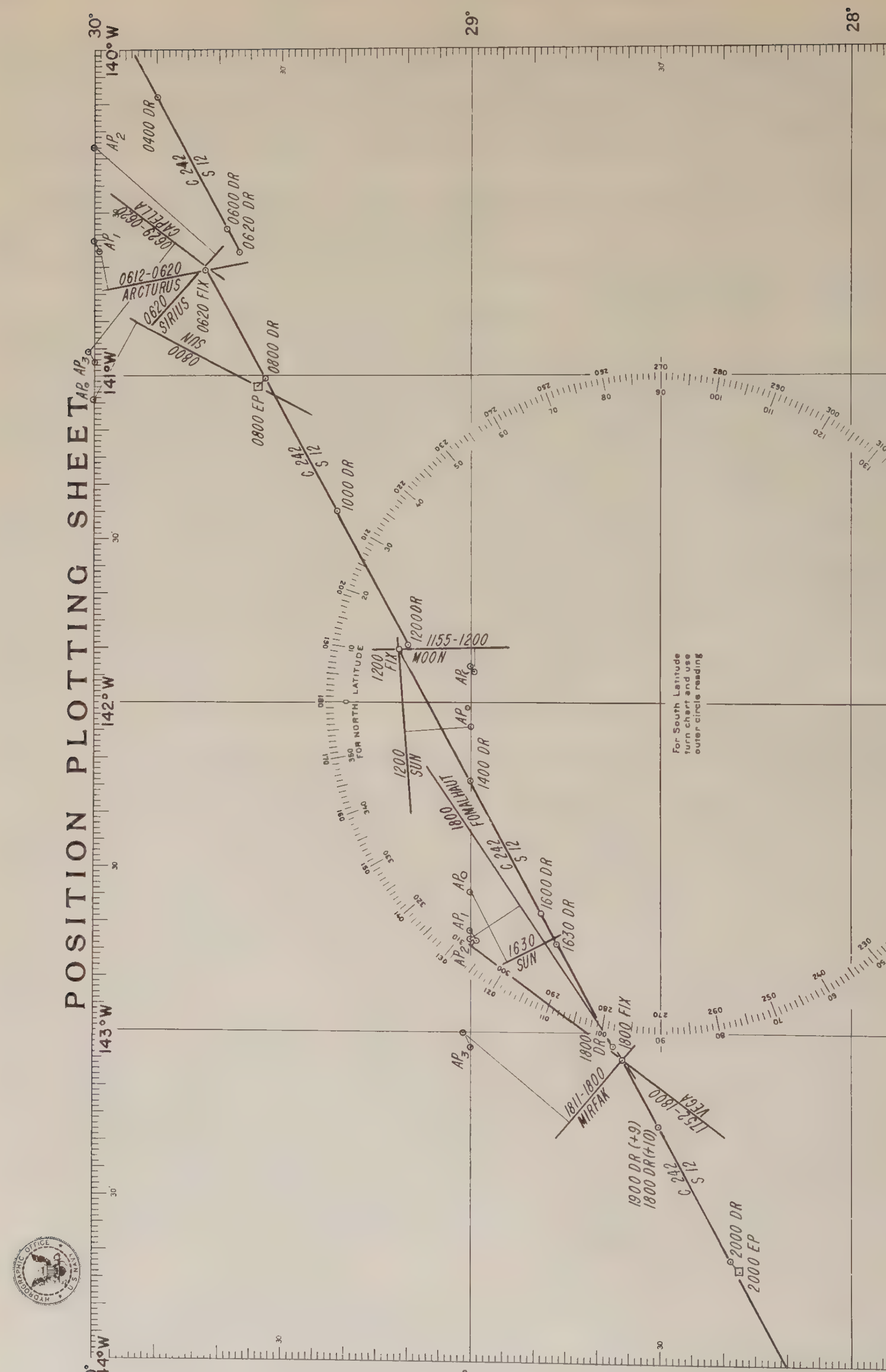


FIGURE 2306.—Typical minimum plot at sea (omitting possible loran fixes).

It is good practice to compare the gyro repeaters with the steering magnetic compass each half hour and after each change of course at sea, to detect any discrepancy which may arise through malfunction. In making the comparison, one should not overlook changes in variation and deviation. The master gyro compass should be compared with its repeaters from time to time.

One of the duties of the navigator is to inform the captain of the expected time of crossing time zone boundaries. The change of time is usually made at a convenient whole hour near the time of crossing a boundary, or during the night. Aboard some merchant ships the change is distributed equally through several watches, as 20 minutes during three consecutive watches.

It is common practice for the captain to maintain a **night order book.** Standing orders such as the conditions under which the captain is to be called, and the admonition to keep a sharp lookout, are usually given on the inside front cover. The orders for each night, if any, are recorded in order, over the captain's signature. They include items such as courses to be steered, speeds to be used, times and bearings of lights expected to be sighted, and any other pertinent navigational information. The navigator provides the captain with such information as he may require.

2307. Landfall.—After a voyage at sea, the first contact with land is of considerable importance. The accuracy with which one predicts the time and place of sighting land depends upon the accuracy of navigation. If consistent loran fixes have been obtained at frequent intervals, and these positions are confirmed by a recent fix from celestial observations or other information, the prediction should be highly accurate. But if no fix has been available for several days, considerable doubt may surround the landfall.

Often the approximate distance offshore, if not the position, can be determined by means of soundings. Along most of their coasts the continents have a **continental shelf** of relatively shoal water extending outward for a varying distance. A similar **insular shelf** extends outward from many island groups. At the outer edge, called the **continental talus** (or **insular talus**), a sharp increase in depth occurs. This is usually at about the 100-fathom curve. Therefore, the crossing of this curve is often quite abrupt, and gives information on the distance offshore. The position of this and other depth curves may be indicated on the chart.

The place of making landfall has a definite relationship to the safety of the vessel, particularly in an area where shoaling is not uniform along the beach. For some time before making a landfall in such an area, it may be advisable to maintain both a dead reckoning and estimated position plot. The best obtainable position should be determined. Methods which are acceptable a thousand miles from land may not provide sufficiently exact data when a landfall is expected.

Only judgment, based upon existing circumstances, can determine the existence of a dangerous situation. If the water has shoaled to a dangerous degree, for instance, and the position of the vessel is seriously in doubt, one may have no recourse but to stand off or anchor and await daylight, improved visibility, or better information.

When contact is made with land, the first step should be to identify the point of contact. The *anticipated* point of making contact should be of assistance, but one should be alert to the possibility of similarly appearing land at other points within a reasonable distance on each side. The position of the vessel relative to land might be established even before land is sighted. Soundings, radio bearings, and radar may be used for this purpose.

2308. Entering port.—Before entering port, the navigator should have reliable information regarding the draft of his vessel. He should also have a reliable position relative to the land. Preparations for entering are similar to those for getting underway. The tide and tidal current tables, light list, coast pilot or sailing directions, and

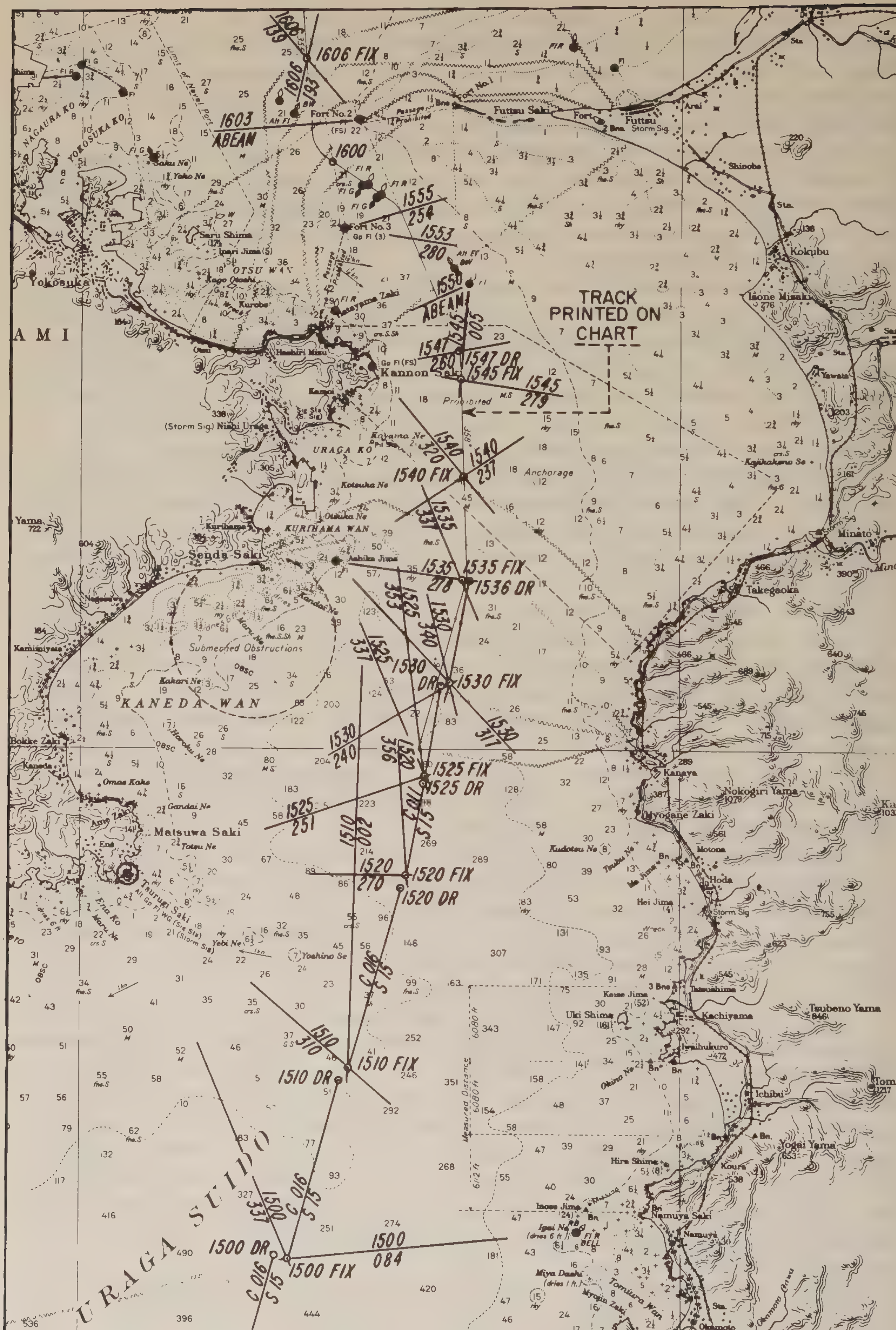


FIGURE 2308.—Typical plot in pilot waters.

charts should all be broken out and studied so that one is familiar with conditions to be encountered. The *time* of entering might be selected to take advantage of favorable currents, and to arrive at the assigned berth at slack water. One should have a mental picture of what to expect when approaching from seaward under the anticipated conditions of lighting and visibility. The characteristics of all aids to navigation by day or night, as appropriate, and fog signals should be known or immediately available. In entering a strange port the navigator should carefully select the most suitable aids to use, with substitutes if these prove inadequate, or if there is any doubt as to their identity. Useful ranges, natural or artificial, should be noted. Danger bearings and danger circles should be drawn in and labeled, if this has not already been done. A danger sounding should be selected and drawn on the chart, if needed. Any shoal areas, wrecks, areas of unusually swift current, etc., should be noted.

The courses to be steered and the distance on each should be determined and recorded, or drawn and labeled on the chart. The identification of each turning point should be indicated. Definite courses should be steered, and changes made only when established positions indicate a departure from the planned track, or when necessitated by traffic. Course changes should occur at preselected points having definite identification. The position should not be permitted to be in doubt at any time, even in ports which are familiar to the navigator and considered easy to enter. Most avoidable groundings are caused by erroneous assumptions which should have been verified. The position should be checked frequently, using the most reliable information available. This may seem to be an unnecessary refinement, but in an emergency a position might be needed at a time when it cannot be obtained. When changes of course are ordered, it is good practice to indicate the amount and direction of change, or the new course, to avoid the possibility of having one's attention diverted at the moment the order should be given to check the swing or steady on the new course. In general, course changes are best made when a given aid to navigation or other landmark is abeam, or when the ship is on a range.

If it becomes necessary to pass between visible dangers without suitable marks for obtaining fixes, a track midway between dangers can be followed by eye more accurately than one closer to either side. If a vessel is to pass near reefs or shoals, it is sometimes possible to observe these from a position aloft, particularly if the sun is astern.

The actual navigation while entering port is similar to that when leaving port. A typical plot in pilot waters is shown in figure 2308. The entering of pilot waters should be accompanied by a mental reorientation and an increased alertness. The use of a local pilot, unless this is a mandatory requirement, is a matter which should be decided in each case. Whether or not a pilot is used, local harbor regulations should be followed, for the presence of a pilot does not relieve the master of his responsibility. One should not forget to note the time of entering the area where local or inland rules of the road apply.

Speed in the vicinity of wharves, construction work, dredges, small boats, etc., should be carefully controlled to avoid damage to them.

If the vessel anchors, the anchorage should be selected carefully, considering local regulations as well as suitability and safety, including the holding qualities of the bottom. If there is any doubt as to the depth of water, a boat might be sent in ahead to take soundings. If space is limited, the approach to the anchorage should be planned and executed carefully. As soon as the anchor is let go, the position should be determined accurately, preferably by horizontal sextant angles. Bearings of a number of prominent landmarks and lights should be measured and recorded, as a guide in determining whether or not the vessel drags anchor.

2309. Fog.—During periods of reduced visibility, the navigator's work is more difficult. At sea he is prevented from making celestial observations. Even when the fog is so shallow that celestial bodies are visible, the horizon is not available as a reference. An artificial-horizon sextant may prove of some value at such a time, but unless the sea is almost a flat calm, the results are likely to be less reliable than the dead reckoning. Radio aids to navigation are affected little by fog. Unless the vessel is approaching land, there is generally no cause for concern regarding the navigation, the principal danger being one of collision with other vessels. Usually the navigator merely waits for the fog to lift.

When a coast is approached, however, a wait may be impractical. The safety of the vessel requires reliable positional information. Along a coast where the shoaling is gradual, the echo sounder or sounding machine can be of great assistance in indicating the distance off. But along a coast having abrupt shoaling, the first indication of shallow water may be obtained so close to the beach that action to avoid grounding is not possible. If radio aids such as loran, radio direction finder, and radar are available, they can provide useful information. If the vessel is near enough to a shore with steep cliffs, the echo of the vessel's whistle may provide indication of the distance off.

The decision of whether to enter a fogbound harbor should be made carefully. Once committed to the channel, the vessel may have no alternative but to continue on to the anchorage or wharf, for in some areas there is not room to turn back, and anchoring is unsafe. It is sometimes wiser to stand off or anchor for a few hours than to risk danger of grounding or collision.

If the decision is made to enter, one should be prepared for any reasonable eventuality. The proximity of danger and the presence of currents make necessary the maintenance of a good position at all times. Fog limits the number of objects that can be used for fixing position, and destroys the overall view of the area. The radio direction finder and radar, both shipborne and shore-based, have done much to reduce the hazard due to fog, but they have not eliminated it. The need for special precautions and increased vigilance is still present.

During periods of reduced visibility the practice of steering exact courses, with precise changes at definite points, is of great assistance in pilot waters. If the vessel is following a channel, each buoy should be located successively. If the fog is dense, this requires careful steering and attention to all details, such as indications of current, changes of wind, etc. If a single buoy is missed, consideration should be given to anchoring and waiting for improved visibility.

With the possible exception of radar, the most important navigational aid during fog in pilot waters is the echo sounder, sounding machine, or hand lead. Continuous soundings, compared with the chart, can provide valuable information on the position and safety of the vessel. The decision as to whether to plot a line of soundings on transparent material, or along the edge of a piece of paper, and compare this with the chart is a matter of judgment. In general, the procedure is valuable when approaching a harbor or proceeding in an open part of a large bay, but in a channel or other restricted waters the method is not needed and might prove distracting.

During fog one should keep a sharp lookout for any objects that might appear momentarily through thin places in the fog. It is well to have a lookout stationed aloft, and another in the bow, for the visibility may vary with height.

The lookouts and all persons on the bridge should listen intently for fog signals. As soon as such a signal is heard, an effort should be made to identify its source and determine its bearing. However, experience in the use of sound signals indicates that they are not wholly reliable. In particular, relative intensity of a sound is not a reliable indication of its distance, or whether the distance is increasing or decreasing. A signal

may be totally inaudible in certain areas close to its source. Neither is its apparent direction always a correct indication of its actual direction. A fog signal may not be in operation when fog is present a short distance from a station but is unobserved from it. Transmission of sound through water is subject to uncertainties due principally to differences in density in different parts of the sea, causing the sound to be deflected.

It is well to remember that at reduced speed the relative effect of current is correspondingly greater, since the effect of current is proportional to time, not to the speed of the vessel.

2310. Navigation of small craft.—In principle, the navigation of small craft is the same as that of a large ship, but because of the shallower draft, greater maneuverability, and possible limitations of equipment of small craft, there are important differences. Small craft spend most of their time within sight of land, where navigation is largely a matter of piloting. They generally skirt the beach close enough to be able to reach safety in case of storm or fog, and since most of them are used primarily for pleasure, there is a natural tendency for the navigation to be a less continuous process than in larger craft.

The equipment carried and the type of navigation employed depend primarily upon the use of the craft and the preference of the user. If a rowboat, canoe, or small sailboat is to be used only close to the shore in good weather, "seaman's eye" might be sufficient for all navigational purposes. But if there is any possibility of the craft being out in a fog, or proceeding to greater distances from shore, fog-signalling apparatus, a compass, and some means of taking soundings should be carried.

A wide variety of equipment is available for yachts, and from this, suitable items can be selected. A minimum list should include a compass, pelorus, charts, plotting equipment (many types are available), means for determining speed or distance, log book, tide and tidal current tables, light list, coast pilot or sailing directions, hand lead, binoculars, flashlight, and fog-signal apparatus. A barometer and thermometer are useful.

Several items of electronic equipment, some of which are relatively inexpensive, are available for use in small craft, to aid in navigation and increase safety. The principal item of radio equipment, from the standpoint of safety, is a marine radiotelephone, which in addition to providing normal communication to other boats and the shore, permits the boat carrying it to call for help in distress, and assists in the location of the distressed vessel. The radio direction finder is a simple device requiring little power, an important factor on small craft. A multiband direction finder may be used as a second receiver in the broadcast and radiotelephone bands. Portable broadcast receivers permit reception of weather information on even the smallest boats. For larger craft, where ample power is available, radar and loran may be good investments. In addition, every small craft should carry a corner reflector (art. 1209), so as more readily to reflect radar signals. In an emergency a metal bucket might be of some value as a reflector.

If the craft is to proceed out of sight of land for more than short intervals, celestial navigation equipment should be carried. This includes a sextant, an accurate timepiece, an almanac, sight reduction tables, and perhaps a star finder. If there is doubt as to advisability of including some item of equipment, the safer decision is to include it. It is better to have unused equipment than to risk danger of becoming lost because of lack of needed equipment.

The practice of navigation in small craft varies even more widely than the equipment carried. The variation extends from complete navigation similar to that of a large ocean steamer to no navigation other than by eye. The completeness of the navigation should fit the circumstances. There is an understandable tendency among small

craft navigators of limited experience to underestimate the need for thorough and complete navigation. In general, it is good practice for the navigator of a small craft to establish the routine of always following definite courses from buoy to buoy or from landmark to landmark, so that the sudden onset of low visibility will not find him unable to proceed to safety without delay. He should change course at established points, maintain knowledge of his position at all times, and have reliable information on the deviation of his compass. There is a place in small craft navigation for a complete, accurate, neat plot. Where this is impractical because of heavy weather or limited plotting space, a careful log and dead reckoning by table 3 should be substituted.

The accounts given in yachting magazines, and the large number of calls for assistance received by the Coast Guard, indicate an inadequacy of the navigation of many small craft. Part of this is due to a lack of appreciation of the need for careful navigation. Much of it is due to lack of knowledge on the part of the small craft owner. The decision to omit some part of navigation should stem from knowledge, not ignorance. To the adequately informed, navigation can be part of the pleasure of yachting.

CHAPTER XXIV

SUBMARINE NAVIGATION

2401. Introduction.—Submarines deserve special consideration, from a navigational viewpoint, because of their inherent or self-imposed limitations. Somewhat different techniques are used in each of four operating conditions, which will be considered separately. These are: (1) surfaced, (2) submerged by day and surfaced by night, (3) submerged at periscope depth, and (4) totally submerged.

2402. Surfaced.—The navigation of a submarine on the surface is essentially the same as that of other vessels, but there are some special considerations. The amount and type of equipment available is limited somewhat by space. Most of it is housed inside the hull, where it can be available for use when the vessel is submerged.

Careful dead reckoning by hand plot is important because of the lesser accuracy of mechanical equipment for this purpose. Speed or distance is measured as in other vessels. Direction measurement is dependent largely upon the gyro compass, because of the difficulty of adequately adjusting a magnetic compass heavily shielded by a steel hull. The areas of weak horizontal intensity of the earth's field, in which the magnetic compass is unreliable (art. 2513), are larger for submarines than for other vessels. Normally, leeway is negligible.

Piloting of submarines on the surface is carried on as in other vessels. However, the amount of exposed equipment is somewhat limited, as is the space for plotting and chart stowage. Because of the low height of eye, aids to navigation are not visible as far away as in other vessels of like size. At ten feet above the surface the horizon is only 3.6 miles away, while at 50 feet it is 8.1 miles distant. This may be an advantage when picking up a dark buoy, which may be more conspicuous against the background of a bright sky than against the darker water.

Electronic navigation is available to the submarines equipped to use it, but again space limitations are a consideration. Most submarines are equipped with radar, radio direction finders, and loran, as well as sonar and echo-sounding equipment. The low antenna height restricts the range at which signals can be received in some cases, particularly with radar.

With most of its hull under water, a submarine generally offers a steady platform for making celestial observations. However, if there is much of a sea, difficulty may be experienced in keeping the sextant mirrors dry, because the ship tends to go *through* the waves instead of riding over them. Because of the low height of eye, the state of the sea is an important consideration, and a correction for wave height (art. 1608) may be justified. If the sea is calm, excessive and somewhat unpredictable refraction may be encountered, particularly for heights of eye below about six feet.

2403. Submerged by day and surfaced by night.—This condition is not unusual during war patrols. By day the ship proceeds largely by dead reckoning, which becomes of even greater importance than on the surface, where ample means of checking its accuracy are usually available. Below the surface, where the ship is not buffeted by waves and wind, a steady course can be steered. The steadiness increases the reliability of the gyro compass. Speed is determined by log or shaft revolutions. If the latter is being used, it is well to remember that when the submarine is proceeding near the bottom, its actual speed may be somewhat less than indicated. Dead reckoning

should be kept up-to-date by a careful hand plot. The mechanical equipment for this purpose provides a useful check, but is less accurate. **Doppler** or **inertial navigation** (art. 809) may prove useful in submarines. The **ship's inertial navigation system (SINS)** is particularly promising.

Because of the very slow speeds normally used by submarines dependent upon batteries, current is an important consideration. Its drift may equal or even exceed the speed of the ship through the water. Pilot charts give helpful information on surface currents, but both the set and drift of the current below the surface may differ considerably. In relatively shallow water the drift may be greater for a short distance below the surface, but generally the drift decreases with depth. Near the bottom, the drift is noticeably reduced by friction. All available knowledge of subsurface currents should be used, but information on the subject is far from complete. A device called the **geomagnetic electrokinetograph,** or **GEK,** has been successfully used to determine ocean currents by a surface vessel towing two electrodes. If this device can be further perfected and adapted for use by submerged submarines, it will remove one of the principal uncertainties of underwater navigation.

It may not be advisable to take echo soundings because of the danger of revealing one's presence, but when they can be obtained, allowance should be made for the depth of the submarine below the surface.

Determination of sunset and the end of evening twilight has added significance to the crew of a submerged submarine in a war zone. In enemy waters it may not be safe to surface until full darkness has set in. By this time the visible horizon is gone, but if the sky is clear, there is no shortage of celestial bodies. If the moon is available, it may provide enough illumination to permit reasonably accurate altitude observations. However, false horizons frequently appear below a bright moon, and better results can usually be obtained by making back sights (art. 1633) of bodies near the moon's azimuth.

Some experiments have been made with night vision and star observations on a dark night. By thoroughly adapting their eyes to darkness and looking a little above or below the image of the body on the horizon, some navigators have reported acceptable results, using a relatively large number of observations. Most navigators using the method prefer a six-power telescope, as from a pair of binoculars, but others use no magnification and keep both eyes open.

When using this method, it is particularly desirable to observe stars all around the horizon, so that any constant error in estimating the position of the horizon will have minimum effect. When taking such observations, it is essential that nothing be done to disturb the dark-adaptation of the eyes, which must operate at peak performance. The observer and the assistant timing the observations should stand back to back. The timer's flashlight should be shielded so as to give a minimum of light needed for reading the watch. When the time has been noted and written down, the light should be turned off. The observer then hands the sextant to the assistant, who again faces away, turns on his light, and reads and records the sextant altitude. He then turns off the light, hands the sextant back to the observer, and the routine is repeated for the next observation. A dim red or blue light is preferable, and safer in a war zone. It is good practice to take and time several observations of each body, alternately increasing and decreasing the altitude setting of the sextant. If the results are plotted on cross-section paper, using altitude versus time, it should be possible to determine the best shots for solution.

In fairing a line through the plotted points, it may be helpful to know the correct *direction* of the line. One way of determining this direction is by means of H.O. Pub. No. 214. Since Δt is the unit change in altitude (to two decimal places), $15 \times \Delta t$ is the change for 15 minutes of arc (one minute of time). However, this result is for a sta-

tionary observer. To correct for his motion, multiply the distance run in one minute by the cosine of the relative azimuth of the body. If the body is forward of the beam, add the correction to $15 \times \Delta t$ if the body is rising, and subtract it if setting. For a body abaft the beam, reverse these signs. Having determined the change in altitude in one minute, draw a line at the slope indicated and move it parallel to itself until it best fits the plotted points. In fitting the line to the points, it is usually good practice to ignore the inconsistent shots. If Δt is changing rapidly, the change of altitude with time is not satisfactorily represented by a straight line, and should be plotted as a curve. This is most apparent for a body near the meridian, particularly one at a high altitude. Once the line is located, *any* point on it can be taken as the observation, whether or not it coincides with a plotted point, as long as the corresponding time is used. Thus, any convenient time or altitude might be selected.

Bubble or pendulum sextants (art. 1513) usually do not produce satisfactory results aboard ship because of the large acceleration errors produced by the motions of the vessel. However, it is desirable for a submarine to be provided with an artificial-horizon sextant, for there may be occasions when it affords the only available method of determining position, and the results may be of usable accuracy. With a reasonably smooth sea and a large number of observations, quite satisfactory results can be obtained. Since this instrument does not depend upon a visible horizon, the number of observations that can be obtained is limited only by the number of navigational bodies visible, and the time available to the observer. The assumed positions for the various sights should all be advanced or retired to a common time before the lines of positions are plotted, so that no confusion can result from the presence of the additional lines on the chart. For correction of artificial-horizon sextant altitudes, see article 1625. In using an artificial-horizon sextant, it may be desirable to make a number of observations of each body and plot them as explained above. Since acceleration error is due mostly to rolling, better observations can often be made over the bow or stern. If conditions warrant, the ship should be headed directly toward or away from each body as it is observed.

Whatever the method of observation, practice and some ingenuity are needed for best results.

2404. Submerged at periscope depth.—At periscope depth the view is seriously restricted, but not entirely lost. Reasonably accurate bearings of landmarks can be obtained through the periscope. Electronic navigation is available if an antenna can be surfaced, or, in some cases, kept submerged near the surface.

When the sun passes within a few degrees of the zenith, a number of azimuths can sometimes be measured by periscope before and after meridian transit. In using this information, it should be remembered that azimuth lines are great circles. Plot the geographical position at the time of each observation, and advance or retire it for the ship's run, so that all sights are for a common time. Apply the conversion angle correction from table 1 as for radio bearings, and plot the azimuth lines from the adjusted positions. Reasonably accurate results have been reported with this method when within about 400 miles of the geographical position of the sun. It can be used at greater distances if accurate azimuths can be obtained. The method requires a good level and cross-level of the periscope, and practice. For best results plot the azimuths against time on cross-section paper. Fair a curve through the plotted points, ignoring inconsistent observations. The points to use for plotting the azimuth lines on the chart or plotting sheet are taken at uniform intervals along the curve. If plotting is done on a gnomonic chart, conversion angle is not applied. However, because of the very small scale of these charts, plotting must be done carefully if accurate results are to be obtained. Lambert conformal charts can be used, but with some loss of

accuracy. The method can be used for similar observations of the moon, planets, and stars, if observations of these bodies can be made and the bodies identified. If a single body is used, the best times are just before and just after meridian transit, when the azimuth changes most rapidly and the ship is nearest the geographical position of the body. The principal limitation of the method is the accuracy with which azimuths can be observed. Unless the sea is almost a flat calm, the distance limitations are severe, and considerable skill is needed for good results under any conditions.

Some success has been obtained with altitude observations by means of the periscope, but this is difficult at best, and is limited to low altitudes. At low-power magnification, altitudes as high as 25° can be measured, but at high power the method is limited to about 7°. For good results the periscope should be accurately in the vertical, and steady. One method of obtaining the altitude is by counting the graduations of the vertical scale between the horizon and the body. Some navigators prefer the use of the periscope stadimeter, which may give satisfactory results in either of two ways. First, it may be offset so that the zero line of its vertical scale coincides with a selected line (as at 5° or 6°) of the periscope field. This causes a false horizon to appear at the selected periscope field line. As a body appears to cross this false horizon, it is at the selected altitude. A variation of this method is to offset the stadimeter zero line until the false horizon appears at the body, when the amount of the offset is the altitude. The second way of using the stadimeter is to read the distance corresponding to some arbitrarily chosen height. This height divided by the corresponding distance is the tangent of the altitude, which can be found in table 31. Better results might be obtained by noting the moment at which the lower or upper limb of the sun or moon is tangent to the horizon (or a planet or star crosses the horizon), and using the altitude as 0°00.′0, than by attempting to measure a low altitude through the periscope. Correction of low-altitude observations is explained in article 1632.

Solution of low-altitude observations can be made by virtually any method, including H.O. Pub. No 214. Low-altitude sight reduction is discussed in article 2010.

2405. Totally submerged.—The navigation of a totally submerged submarine is a problem which has not been fully solved. Some of the methods being developed or in use cannot be discussed because of security limitations. However, the following information should serve as a useful guide.

Dead reckoning, the basic form of navigation under virtually any conditions, is of increased importance to the totally submerged submarine. Submerged dead reckoning is discussed in article 2403.

Soundings can be used to help establish position in an area where a reliable chart of the bottom configuration is available, if allowance is made for the depth of the submarine below the surface. Sonar (art. 1108) can sometimes be useful in avoiding obstacles or even in locating position when there are identifiable, charted targets. Both sofar and rafos (art. 1313) can be used when available.

Electronic navigation has limited application unless an antenna is above water. However, very low frequency signals penetrate sea water to some extent, and if the submerged antenna can be placed close to the surface, usable signals can sometimes be obtained at great distances. These frequencies are little used for navigational information, but increased requirements for navigation below the surface will undoubtedly result in additional development in this part of the spectrum.

A useful method of submerged navigation might be based upon measurement of any quantity that varies from place to place in a known pattern, such as gravity or some element of the earth's magnetism, some form of radiation, or even water temperature or salinity. The use of such a method would require an instrument to provide

sufficiently accurate measurements, and an accurate chart showing the pattern at the time of observation. This would be similar to using bottom contours. Since a method of this type might often provide lines of position only, two or more methods may be needed to establish a fix.

Celestial navigation is not used by a totally submerged submarine.

The navigation of a submarine which remains below the surface for long periods, with nothing extending above water, presents a challenge that is partly met by making full use of every item of applicable information, and applying a generous amount of judgment and common sense.

CHAPTER XXV

POLAR NAVIGATION

Polar Regions

2501. Introduction.—No single definition of the limits of the polar regions satisfies the needs of all who are interested in these areas. Astronomically, the parallels of latitude at which the sun becomes circumpolar (the arctic and antarctic circles at about latitude 67°5) are considered the lower limits. Meteorologically, the limits are irregular lines which, in the arctic, coincides approximately with the tree line. For general purposes, the navigator may consider polar regions as extending from the geographical poles of the earth to latitude 70° (in the arctic coinciding approximately with the northern coast of Alaska), with transitional **sub-polar regions** extending for an additional 10° (in the northern hemisphere extending to the southern tip of Greenland).

This chapter deals primarily with marine navigation in high latitudes. Information relating to navigation ashore is given in chapter XXVII.

2502. Polar geography.—The north polar region, the **arctic,** consists of an elongated central water area a little smaller than the United States, almost completely surrounded by land (fig. 2502a). Some of this land is high and rugged with permanent **ice caps,** but part of it is low and marshy when thawed. Underlying **permafrost,** permanently frozen ground, prevents adequate drainage, resulting in large numbers of lakes and ponds and extensive areas of **muskeg,** soft spongy ground with characteristic growths of certain types of moss and tufts of grass or sedge. There are also large areas of **tundra,** low treeless plains with vegetation consisting of mosses, lichens, shrubs,

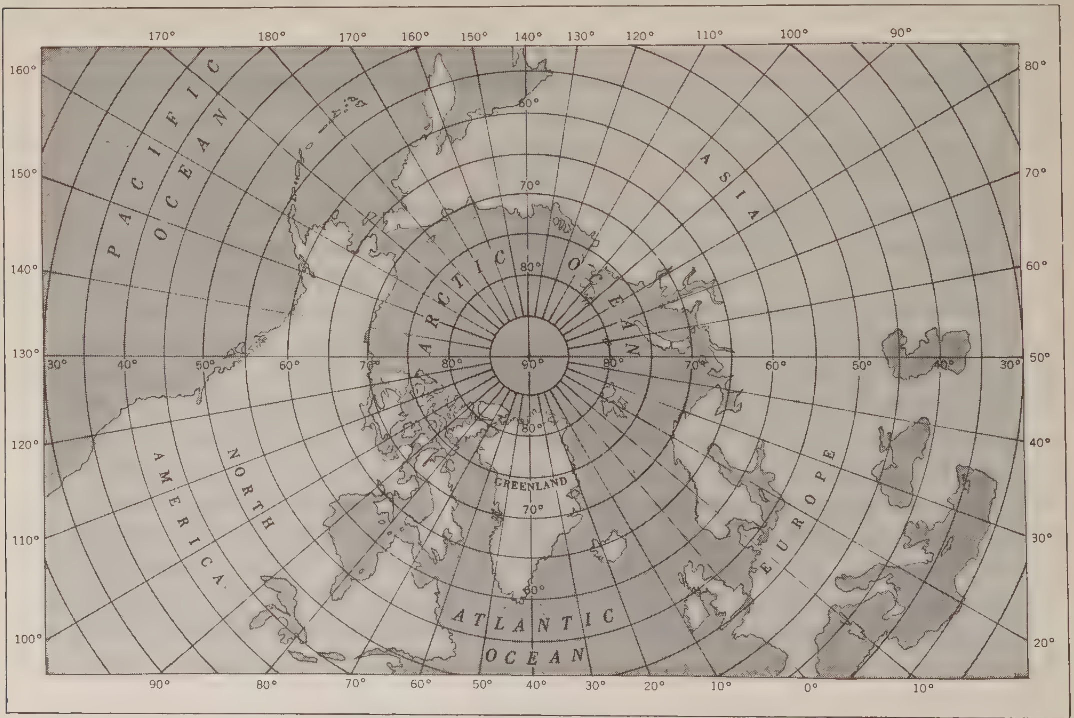


FIGURE 2502a.—The north polar region, or *arctic*.

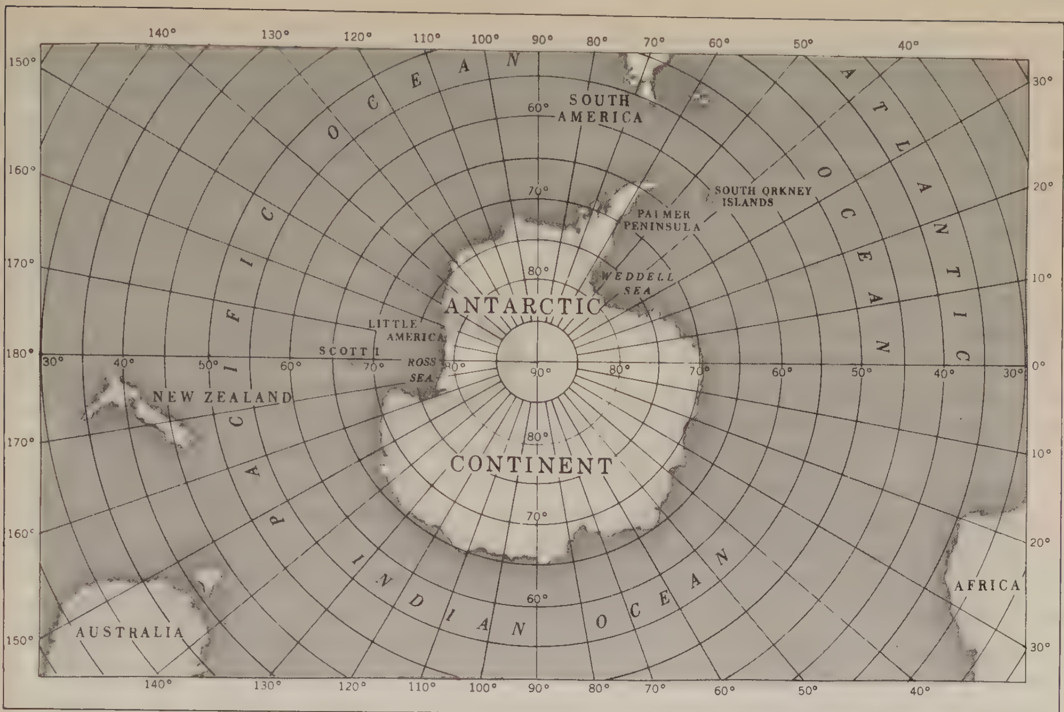


FIGURE 2502b.—The south polar region, or *antarctic.*

willows, etc., and usually having an underlying layer of permafrost. The northernmost point of land is Kap Morris Jesup, Greenland, about 380 nautical miles from the pole.

The central part of the **Arctic Ocean,** as the body of water is called, is a basin of about 12,000 feet average depth. However, the bottom is not level, having a number of seamounts and deeps. The greatest depth is probably a little more than 16,000 feet. At the north pole the depth is 14,150 feet. Surrounding the polar basin is an extensive continental shelf, broken only in the area between Greenland and Svalbard (Spitsbergen). The many islands of the Canadian archipelago are on this shelf. The Greenland Sea, east of Greenland; Baffin Bay, west of Greenland; and the Bering Sea, north of the Aleutians, each has its independent basin. In a sense, the Arctic Ocean is an arm of the Atlantic, as shown in figure 2502a.

The south polar region, the **antarctic,** is in marked contrast to the arctic in physiographical features. Here a high, mountainous land mass about twice the area of the United States is surrounded by water (fig. 2502b). An extensive polar plateau covered with snow and ice is about 10,000 feet high. There are several mountain ranges with peaks rising to heights of more than 13,000 feet. The average height of Antarctica is about 6,000 feet, which is higher than any other continent. The height at the south pole is about 9,500 feet. The barrier presented by land and tremendous **ice shelves** 500 to 1,000 feet thick prevent ships from reaching very high latitudes. Much of the coast of Antarctica is high and rugged, with few good harbors or anchorages.

2503. Navigation in polar regions.—Special techniques have been developed to adapt navigation to the unique conditions of polar regions. These conditions are largely the result of (1) high latitude, and (2) meteorological factors.

2504. High-latitude effects.—Much of the thinking of the marine navigator is in terms of the "rectangular" world of the Mercator projection, on which the meridians are equally spaced, vertical lines perpendicular to the horizontal parallels of latitude. Directions are measured relative to the meridians, and are maintained by means of a

magnetic or gyro compass. A straight line on the chart is a rhumb line, the line used for ordinary purposes of navigation. Celestial bodies rise above the eastern horizon, climb to a maximum altitude often high in the sky as they cross the celestial meridian, and set below the western horizon. By this motion the sun divides the day naturally into two roughly equal periods of daylight and darkness, separated by relatively short transitional periods of twilight. The hour of the day is associated with this daily motion of the sun.

In polar regions conditions are different. Meridians all converge at the poles, which are centers of series of concentric circles constituting the parallels of latitude. The rapid convergence of the meridians renders the usual convention of direction inadequate for some purposes. A rhumb line is a curve which differs noticeably from a great circle, even for short distances. Even visual bearings cannot adequately be represented as rhumb lines. At the pole all directions are south or north, depending upon the pole. Direction in the usual sense is replaced by longitude.

At the pole the zenith and celestial pole coincide. Hence, the celestial horizon and celestial equator also coincide, and declination and computed altitude are the same. Therefore, celestial bodies change computed altitude only by changing declination. Stars circle the sky without noticeable change in altitude. Planets rise and set once each sidereal period (12 years for Jupiter, 30 years for Saturn). At the north pole the sun rises about March 21, slowly spirals to a maximum altitude of about 23°27′ near June 21, as slowly spirals downward to the horizon about September 23, and then disappears for another six months. At the south pole a similar cycle takes place but during the opposite time of year. It requires about 32 hours for the sun to cross the horizon, during which time it circles the sky 1⅓ times. The twilight periods following sunset and preceding sunrise last for several *weeks*. The moon rises and sets about once each month. Half the sky is *always* visible and the other half is *never* seen.

The long polar night is not wholly dark. The full moon at this time rises relatively high in the sky. Light from the **aurora borealis** in the arctic and the **aurora australis** in the antarctic is often quite bright, occasionally exceeding that of the full moon. Even the planets and stars contribute an appreciable amount of light in this area where a snow cover provides an excellent reflecting surface.

All time zones, like all meridians, meet at the poles. *Local* time does not have its usual significance, since the hour of the day bears no relation to periods of light and darkness or to altitude of celestial bodies.

2505. Meteorological effects.—Polar regions are cold, but the temperature at sea is not as extreme as inland. The average winter temperature over the Arctic Ocean is (−) 30° F to (−) 40° F, with an extreme low value near (−) 60° F. Colder temperatures have been recorded in Yellowstone National Park. During the summer the temperature remains above freezing over the ocean. Inland, extreme values are sometimes reached. At least one point on the arctic circle has experienced a temperature of 100° F. Few points on the antarctic continent have recorded temperatures above freezing, and the interior is probably the coldest part of the world.

Fog and clouds are common in polar regions, yet there is less precipitation than in some desert regions, since the cold air has small capacity for holding moisture. Very cold air over open water sometimes produces steaming of the surface, occasionally to a height of several hundred feet. This is called **frost smoke** or **sea smoke** (fig. 2505). When there is no fog or frost smoke, the visibility is often excellent. Sounds can sometimes be heard at great distances.

Sharp discontinuities or inversions in the temperature lapse rate sometimes produce a variety of mirages and extreme values of refraction. The sun has been known to rise several *days* before it was expected in the spring. False horizons are not uncommon.

Strong winds are common in the sub-arctic and in both the antarctic and sub-antarctic. The belt of water surrounding Antarctica has been characterized as the stormiest in the world, being an area of high winds and high seas. Strong winds are not encountered over the Arctic Ocean.

In the polar and sub-polar regions the principal hazard to ships is ice, both that formed at sea and land ice which has flowed into the sea in the form of glaciers. Many low land areas are ice-free in summer. Ice is considered in more detail in chapter XXXVI.

When snow obliterates surface features, and the sky is covered with a uniform layer of cirrostratus or altostratus clouds, so that there are no shadows, the horizon

Figure 2505.—Frost smoke.

disappears and earth and sky blend together, forming an unbroken expanse of white, without features. Landmarks cannot be distinguished, and with complete lack of contrast, distance is virtually impossible to estimate. This is called **arctic** (or **antarctic**) **whiteout.** It is particularly prevalent in northern Alaska during late winter and early spring.

2506. Miscellaneous.—The cold surface water of the Arctic Ocean flows outward between Greenland and Svalbard and is replaced by warmer subsurface water from the Atlantic. The surface currents depend largely upon the winds, and are generally quite weak in the Arctic Ocean. However, there are a number of well-established currents flowing with considerable consistency throughout the year. The

general circulation in the arctic is clockwise on the American side and around islands, and counterclockwise on the Asian side. Tidal ranges in this area are generally small. In the restricted waters of the upper Canadian-Greenland area both tides and currents vary considerably from place to place. In the Baffin Bay-Davis Strait area the currents are strong and the tides are high, with a great difference between springs and neaps. In the antarctic, currents are strong, and the general circulation offshore is eastward or *clockwise* around the continent. Close to the shore, a weaker westerly or *counterclockwise* current may be encountered, but there are many local variations.

Since both magnetic poles are situated within the polar regions, the horizontal intensity of the earth's magnetic field is so low that the magnetic compass is of reduced value, and even useless in some areas. The magnetic storms centered in the auroral zones (art. 2526) disrupt radio communications and alter magnetic compass errors. The frozen ground in polar regions is a poor conductor of electricity, another factor adversely affecting radio wave propagation.

2507. Summary of conditions in polar regions.—The more prominent characteristic features associated with large portions of the polar regions may be summarized as follows:

1. High latitude.
2. Rapid convergence of meridians.
3. Nearly horizontal diurnal motion of celestial bodies.
4. Long periods of daylight, twilight, and semidarkness.
5. Low mean temperatures.
6. Short, cool summers and long, cold winters.
7. High wind-chill factor.
8. Low evaporation rate.
9. Scant precipitation.
10. Dry air (low absolute humidity).
11. Excellent sound-transmitting conditions.
12. Periods of excellent visibility.
13. Extensive fog and clouds.
14. Large number and variety of mirages.
15. Extreme refraction and false horizons.
16. Winter freezing of rivers, lakes, and part of the sea.
17. Areas of permanent land and sea ice.
18. Areas of permanently frozen ground.
19. Large areas of tundra (arctic).
20. Large areas of poor drainage, with many lakes and ponds (arctic).
21. Large areas of muskeg (a grassy marsh when thawed) (arctic).
22. Extensive auroral activity.
23. Large areas of low horizontal intensity of earth's magnetic field.
24. Intense magnetic storms.
25. Uncertain radio wave propagation.
26. Strong winds (antarctic).
27. Frequent blizzards (antarctic).
28. Large quantities of blowing snow.

Charts

2508. Projections.—In polar regions, as elsewhere, the chart is an important item of navigational equipment. The projections used for polar charts are considered in articles 321 and 322.

For ordinary navigation the Mercator projection has long been the overwhelming favorite of marine navigators, primarily because a rhumb line appears as a straight line on this projection. Even in high latitudes the mariner has exhibited an understandable partiality for Mercator charts, and these have been used virtually everywhere that ships have gone.

However, as the latitude increases, the superiority of the Mercator projection decreases, primarily because the value of the rhumb line becomes progressively less. At latitudes greater than 60° the decrease in utility begins to be noticeable, and beyond latitude 70° it becomes troublesome. In the clear polar atmosphere, visual bearings are observed at great distances, sometimes 50 miles or more. The use of a rhumb line to represent a bearing line introduces an error at any latitude, but at high latitudes this error becomes excessive.

Another objection to Mercator charts at high latitudes is the increasing rate of change of scale over a single chart. This results in distortion in the shape of land masses and errors in measuring distances.

At some latitude the disadvantages of the Mercator projection outweigh its advantages. The latitude at which this occurs depends upon the physical features of the area, the configuration and orientation of land and water areas, the nature of the operation, and, mostly, upon the previous experience and personal preference of the mariner. Because of differences of opinion in this matter, a transitional zone exists in which several projections may be encountered. The wise high-latitude navigator is prepared to use any of them, since coverage of his operating area may not be adequate on his favorite projection.

2509. Adequacy.—Charts of most polar areas are generally inferior to those of other regions in at least three respects:

1. *Lack of detail.* Polar regions have not been surveyed with the thoroughness needed to provide charts of the accustomed detail. Relatively few soundings are available and many of the coastal features are shown by their general outlines only. Large areas are perennially covered by ice, which presents a changing appearance as the amount, position, and the character of the ice change. Heavy covers of ice and snow prevent accurate determination of surface features of the earth beneath. Added to this is the similarity between adjacent land features where the hundreds of points and fiords in a rugged area or the extensive areas of treeless, flat coastal land in another look strikingly alike. The thousands of shallow lakes and ponds along a flat coastal plain lack distinctive features.

2. *Inaccuracy.* Polar charts are based upon incomplete surveys and reports of those who have been in the areas. These reports are less reliable than in other areas because icebergs are sometimes mistaken for islands, ice-covered islands are mistaken for grounded icebergs, shore lines are not easy to detect when snow covers both land and attached sea ice, inlets and sounds may be completely obscured by ice and snow, and meteorological conditions may introduce inaccuracy in determination of position. Consequently, many features are inaccurately shown in location, shape, and size, and there are numerous omissions. Isogonic lines, too, are based upon incomplete information, resulting in less than desired accuracy.

3. *Coverage.* Relatively few nautical charts of polar regions are available, and the limits of some of these are not convenient for some operations. As in other areas, charts have been made as the need has arisen. Hence, large-scale charts of some areas are completely lacking. Aeronautical charts are sometimes quite helpful, as they often show more detail of land areas than do the nautical charts. However, aeronautical charts do not show soundings.

2510. Polar grid.—Because of the rapid convergence of the meridians in polar regions, the true direction of an oblique line near the pole may vary considerably over a relatively few miles. The meridians are radial lines meeting at the poles, instead of being parallel, as they appear on the familiar Mercator chart.

Near the pole the convenience of parallel meridians is attained by means of a **polar grid.** On the chart a number of lines are printed parallel to a selected reference meridian, usually that of Greenwich. On transverse Mercator charts the fictitious meridians may serve this purpose. Any straight line on the chart makes the same angle with all grid lines. On the transverse Mercator projection it is therefore a **fictitious rhumb line.** On any polar projection it is a close approximation to a great circle. If north along the reference meridian is selected as the reference direction, all parallel grid lines can be considered extending in the same direction. The constant direction relative to the grid lines is called **grid direction.** North along the Greenwich meridian is usually taken as grid north in both the northern and southern hemispheres.

The value of grid directions is indicated in figure 2510. In this figure A and B are 400 miles apart. The true bearing of B from A is 023°, yet at B this bearing line, if continued, extends in true direction 163°, a change of 140° in 400 miles. The grid direction at any point along the bearing line is 103°.

When north along the Greenwich meridian is used as grid north, interconversion between grid and true directions is quite simple. Let G represent a grid direction and T the corresponding true direction. Then for the arctic,

$$\mathrm{G}=\mathrm{T}+\lambda\mathrm{W}.$$

That is, in the western hemisphere, in the arctic, grid direction is found by *adding* the longitude to the true direction. From this it follows that

$$\mathrm{T}=\mathrm{G}-\lambda\mathrm{W},$$

and in the eastern hemisphere

$$\mathrm{G}=\mathrm{T}-\lambda\mathrm{E},$$
$$\mathrm{T}=\mathrm{G}+\lambda\mathrm{E}.$$

In the southern hemisphere the signs (+ or −) of the longitude are reversed in all formulas.

If a magnetic compass is used to follow a grid direction, variation and convergency can be combined into a single correction called **grid variation** or **grivation.** It is customary to show lines of equal grivation on polar charts rather than lines of equal variation. Hydrographic Office chart number 1706 GN shows the **isogrivs** (lines of equal grivation) for the entire arctic.

With one modification the grid system of direction can be used in any latitude. Meridians 1° apart make an angle of 1° with each other where they meet at the pole. The **convergency** is one, and the 360° of longitude cover all 360° around the pole. At the equator the meridians are parallel and the convergency is zero. Between these two limits the convergency has some value between zero and one. On a sphere it is equal to the sine of the latitude. For practical navigation this relationship can be used on the spheroidal earth. On a simple conic or Lambert conformal chart a constant convergency is used over the entire chart, and is known as the **constant of the cone.** On a simple conic projection it is equal to the sine of the standard parallel. On a Lambert conformal projection it is equal (approximately) to the sine of the latitude midway between the two standard parallels. When convergency is printed on the chart, it is generally adjusted for ellipticity of the earth. If K is the constant of the cone,

$$K=\sin\tfrac{1}{2}\,(\mathrm{L}_1+\mathrm{L}_2),$$

where L_1 and L_2 are the latitudes of the two standard parallels. On such a chart, grid navigation is conducted as explained above, except that in each of the formulas the longitude is multiplied by K:

$$G = T + K\,\lambda W,$$
$$T = G - K\,\lambda W,$$
$$G = T - K\,\lambda E,$$
$$T = G + K\,\lambda E.$$

Thus, a straight line on such a chart changes its true direction, not by 1° for each degree of longitude, but by $K°$. As in higher latitudes, convergency and variation can be combined.

In using grid navigation one should keep clearly in mind the fact that the grid lines are parallel *on the chart.* Only on the transverse Mercator and polar gnomonic projections do the grid lines have geographical significance. On these projections, the grid lines are great circles which meet at "poles" on the equator, 90° from the meridian used as the fictitious equator. Since distortion varies on charts of different projections, and on charts of conic projections having different standard parallels, *the grid direction*

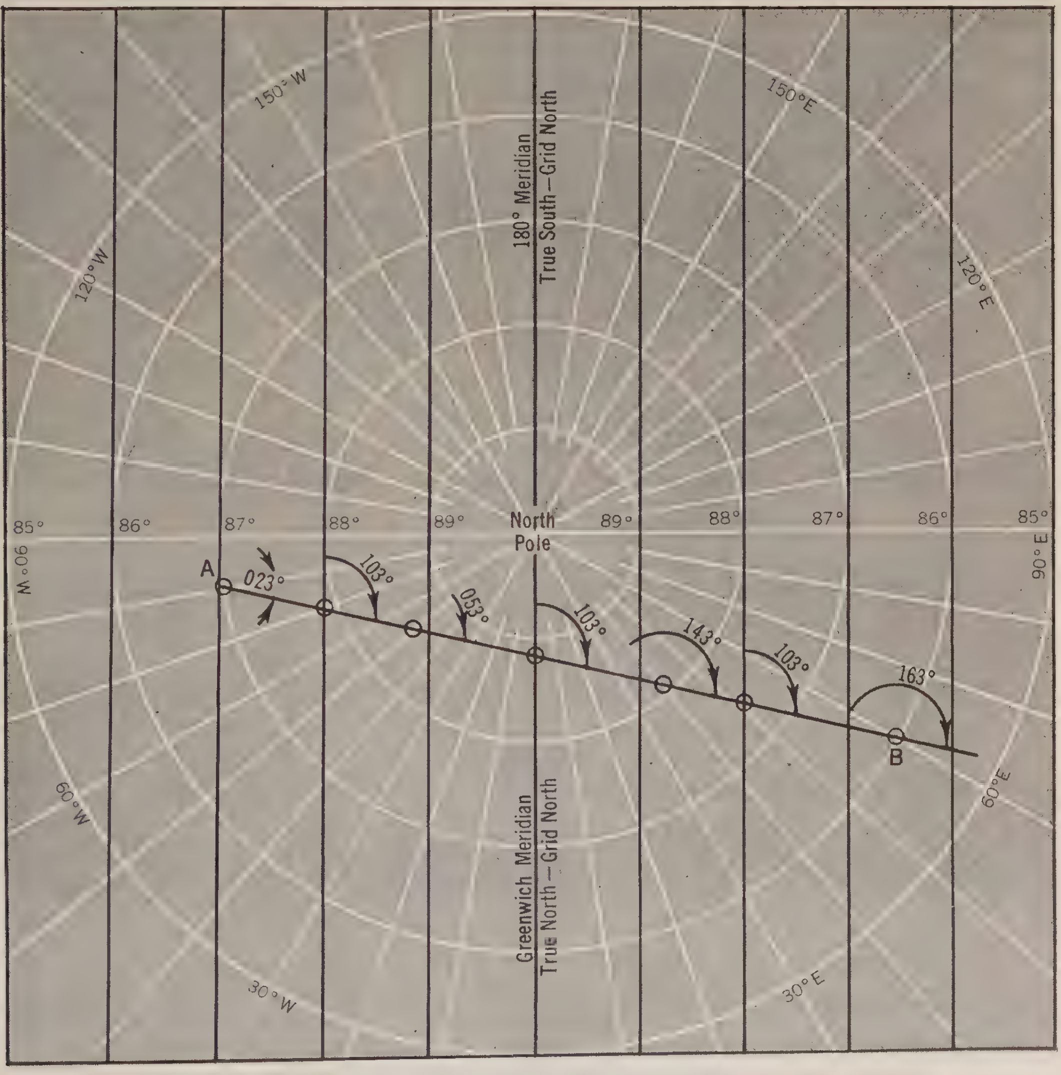


FIGURE 2510.—Polar grid navigation.

between any two given points is not the same on all charts. For operations which are to be coordinated by means of grid directions, *it is important that all charts showing the grid be on a single graticule.*

Except for nuclear powered submarines, ships seldom reach such high latitudes that grid navigation with full convergency of one is used. In the sub-polar regions in which most high-latitude surface ship navigation is conducted, a grid on a suitable projection should be available.

2511. Plotting on polar charts, as on other charts, involves the measurement of distance and direction. On a Mercator chart this is done as in lower latitudes. However, as latitude increases, expansion of the latitude scale increases at a more rapid rate. For accurate results, it is essential that distances be measured in relatively short steps and that an accurate mid latitude be used for each step, as shown in figure 2511a. As latitude increases, the departure of a rhumb line from a great circle becomes greater, and rhumb lines lose some of their value. If they are used to approximate a great circle,

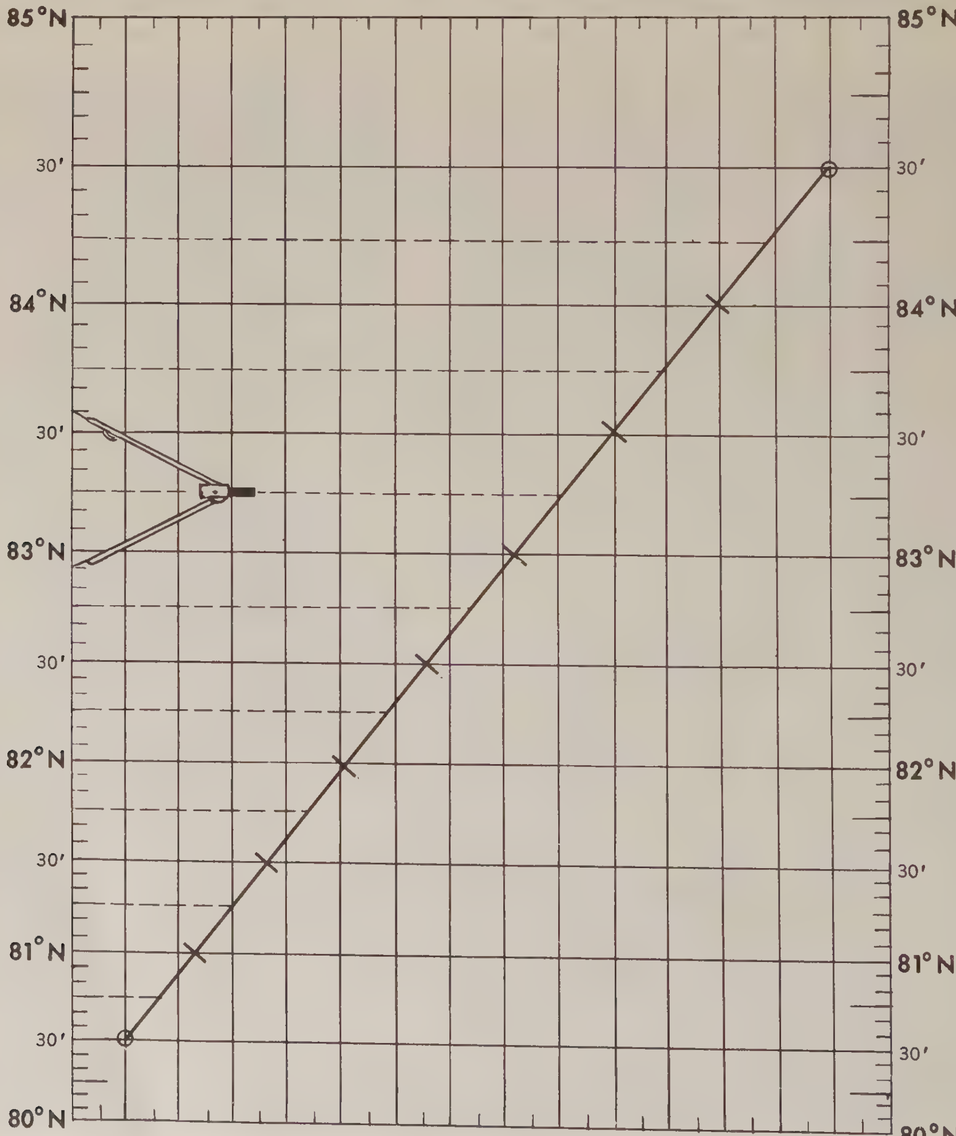


FIGURE 2511a.—Measuring distance on a high-latitude Mercator chart.

as in great-circle sailing, shorter legs are needed to retain a good approximation. Even visual bearing lines cannot accurately be represented by rhumb lines if the distance is great, unless a Mercator correction (tab. 1) is applied as in the case of radio bearings. Such reasons indicate using more suitable projections in high latitudes.

On a chart with converging meridians, as one on the Lambert conformal projection, distance is measured by means of the latitude scale, as on a Mercator chart, but this scale is so nearly constant that any part of it can be used without introducing a significant error. A mile scale is sometimes shown in or near the margin of such a chart, and can be used anywhere on that chart.

Since the meridians converge, a straight line makes a different angle with each meridian, as shown in figure 2510. For this reason, compass roses are not customarily shown on such a chart. If they do appear, *each one applies only to the meridian on which it is located.* The navigator accustomed to using a Mercator chart can easily forget this point, and hence will do well to ignore compass roses. If a drafting machine is used, it should be aligned with the correct meridian each time a measurement is made. Since this precaution can easily be overlooked, especially by a navigator accustomed to resetting his drafting machine only when the chart is moved, and since the resulting error may be too small to be apparent but too large to ignore, it is good practice to discard this instrument when the Mercator chart is replaced by one with converging meridians, unless positive steps are taken to prevent error.

The most nearly fool-proof and generally the most satisfactory method of measuring directions on a chart with converging meridians is to use a protractor, or some kind of plotter combining the features of a protractor and straightedge. One of the most popular is the type B–2 aircraft plotter (fig. 2511b) available to U. S. naval activities from the Aviation Supply Office at Philadelphia, or the AN plotter (or commercial counterpart) shown in figure 2511c.

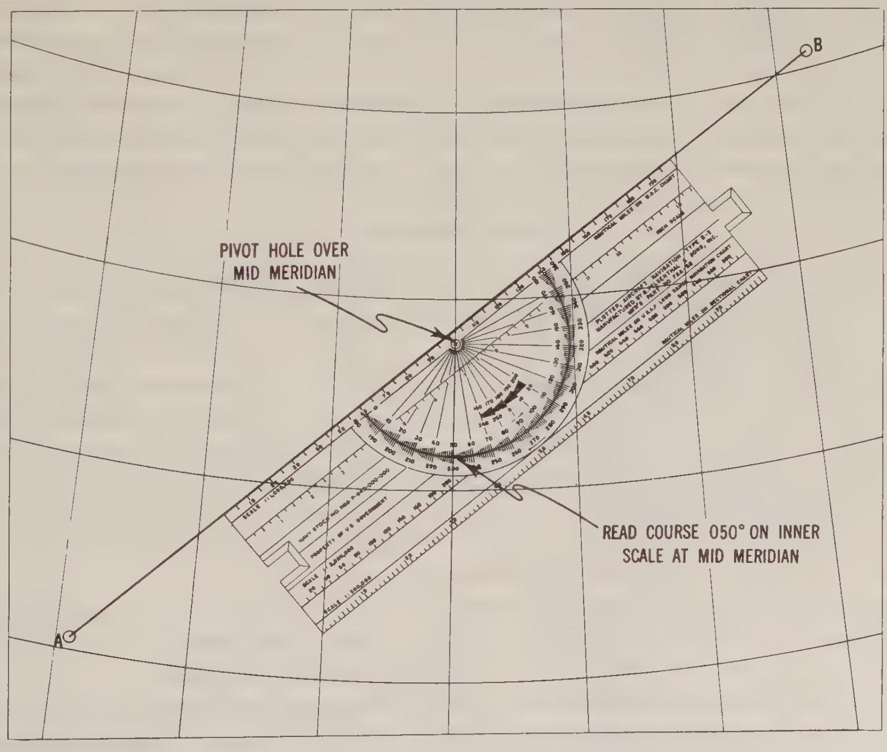


FIGURE 2511b.—Measuring a course on a Lambert conformal chart, by B–2 aircraft plotter. Note that measurement is made at the mid meridian.

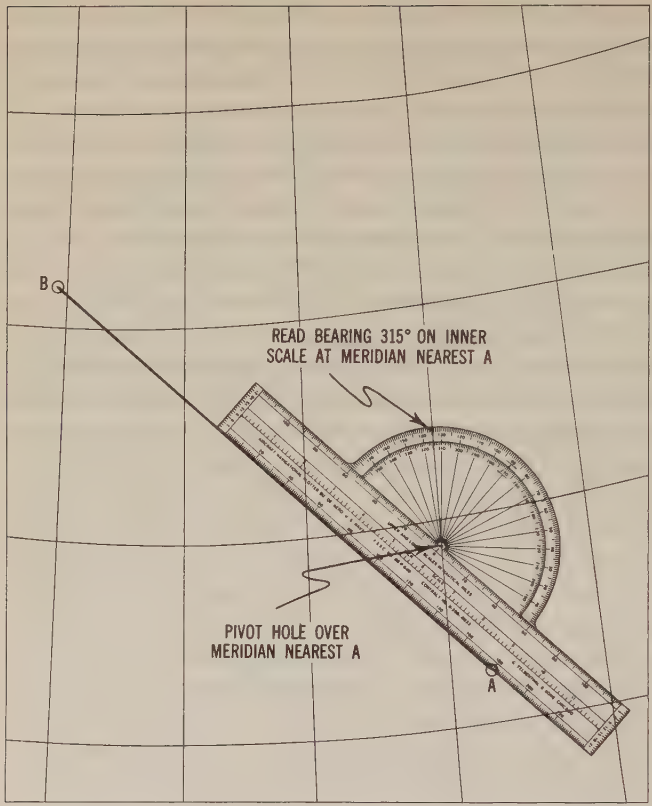


FIGURE 2511c.—Measuring a bearing on a Lambert conformal chart, by AN plotter. Note that measurement is made at the meridian nearest the ship.

If a course is to be measured, the mid meridian of each leg should be used, as shown in figure 2511b. If a bearing is to be measured, the meridian nearest the point at which the bearing was determined should be used, as shown in figure 2511c. Thus, in the usual case of determining the bearing of a landmark from a ship, the meridian nearest the ship should be used. In using either of the plotters shown in figures 2511b or 2511c, note that the center hole is placed over the meridian used, the straightedge part is placed along the line to be drawn or measured, and the angle is read on the protractor at the same meridian which passes under the center hole. It is sometimes more convenient to invert the plotter, so that the protractor part extends on the opposite side of the straightedge.

For plotting grid directions, angles are measured from grid north, using *any* grid meridian. Any convenient method can be used. If a protractor or plotter is being used for plotting grid directions, it is usually desirable to use the same instrument for plotting true directions. The distance is the same whether grid or true directions are used.

Dead Reckoning

2512. Polar dead reckoning.—In polar regions, as elsewhere, dead reckoning involves measurement of direction and distance traveled, and the use of this information for determination of position.

Direction is normally determined by a compass, but in polar regions both magnetic and gyro compasses are subject to certain limitations not encountered elsewhere. However, the navigator who thoroughly understands the use of these instruments in high latitudes can get much useful information from them. It is wise to carry, in addition, some form of *celestial compass*, discussed in article 2515. The polar navigator should not overlook the value of radar tracking or visual tracking for determining direction of motion. This is discussed in article 2516.

Speed or distance is normally measured by log or engine revolution counter, but these methods are not entirely suitable when the ship is operating in ice. The problem of determining speed or distance in ice is discussed in article 2516.

2513. The magnetic compass depends for its directive force upon the horizontal intensity of the magnetic field of the earth. As the magnetic poles are approached, this force becomes progressively weaker until at some point the magnetic compass

becomes useless as a direction-measuring device. In a marginal area it is good practice to keep the magnetic compass under almost constant scrutiny, as it is somewhat erratic in dependability and its errors may change rapidly. Frequent compass checks by celestial observation or any other method available are wise precautions. A log of compass comparisons and observations is useful in predicting future reliability.

The magnetic poles themselves are somewhat elusive, since they participate in the normal diurnal, annual, and secular changes in the earth's field, as well as the more erratic changes caused by magnetic storms. Measurements indicate that the north magnetic pole moves within an elongated area of perhaps 100 miles in a generally north-south direction and somewhat less in an east-west direction. Normally, it is at the southern end of its area of movement at local noon and at the northern extremity twelve hours later, but during a severe magnetic storm this motion is upset and becomes highly erratic. Because of the motions of the poles, they are sometimes regarded as *areas* rather than points. There is some evidence to support the belief that several secondary poles exist, although such alleged poles may be anomalies (local attractions), possibly of intermittent or temporary existence. Various severe anomalies have been located in polar areas and others may exist.

The continual motion of the poles may account, at least in part, for the large diurnal changes in variation encountered in high latitudes. Changes as large as 10° have been reported.

Measurements of the earth's magnetic field in polar regions are neither numerous nor frequent. The isogonic lines in these areas are close together, resulting in rapid change in short distances in some directions, and their locations are imperfectly known. As a result, charted variation in polar regions is not of the same order of accuracy as elsewhere.

The decrease in horizontal intensity encountered near the magnetic poles, as well as magnetic storms, affects the deviation. Any deviating magnetic influence remaining after adjustment, which is seldom perfect, exerts a greater influence as the directive force decreases. It is not uncommon for residual deviation determined in moderate latitudes to increase 10- or 20-fold in marginal areas. Interactions between correctors and compass magnets exert a deviating influence that may increase to a troublesome degree in high latitudes. The heeling magnet, correcting for both permanent and induced magnetism, is accurately located only for one magnetic latitude. Near the magnetic pole its position might be changed, but this may induce sufficient magnetism in the Flinders bar to more than offset the change in deviation due to the change in the position of the heeling magnet. The relatively strong vertical intensity may render the Flinders bar a stronger influence than the horizontal field of the earth. When this occurs, the compass reading remains nearly the same on any heading.

Another effect of the decrease in the directive force of the compass is a greater influence of frictional errors. This, combined with an increase in the period of the compass, results in greatly increased sluggishness in its return to the correct reading after being disturbed. For this reason the compass performs better in a smooth sea free from ice than in an ice-infested area where its equilibrium is frequently upset by impact of the vessel against ice.

Magnetic storms affect the magnetism of a ship as well as that of the earth. Changes in deviation of as much as 45° have been reported during severe magnetic storms, although it is possible that such large changes may be a combination of deviation and variation changes.

The area in which the magnetic compass is of reduced value cannot be stated in specific terms. In general, a remote-reading Flux Gate compass performs as well or better than a regular compass. A magnetic compass in an exposed position performs

better than one in a steel pilot house. The performance of the compass varies considerably with the type of compass, sensitiveness and period, thoroughness of adjustment, location on the vessel, and magnetic properties of the vessel. It also varies with local conditions.

In a very general sense the magnetic compass can be considered of reduced reliability when the horizontal intensity is less than 0.09 oersted, erratic when the field is less than 0.06 oersted, and useless when it is less than 0.03 oersted. The extent of these areas in the northern hemisphere is indicated in figure 2513. Similar areas extend

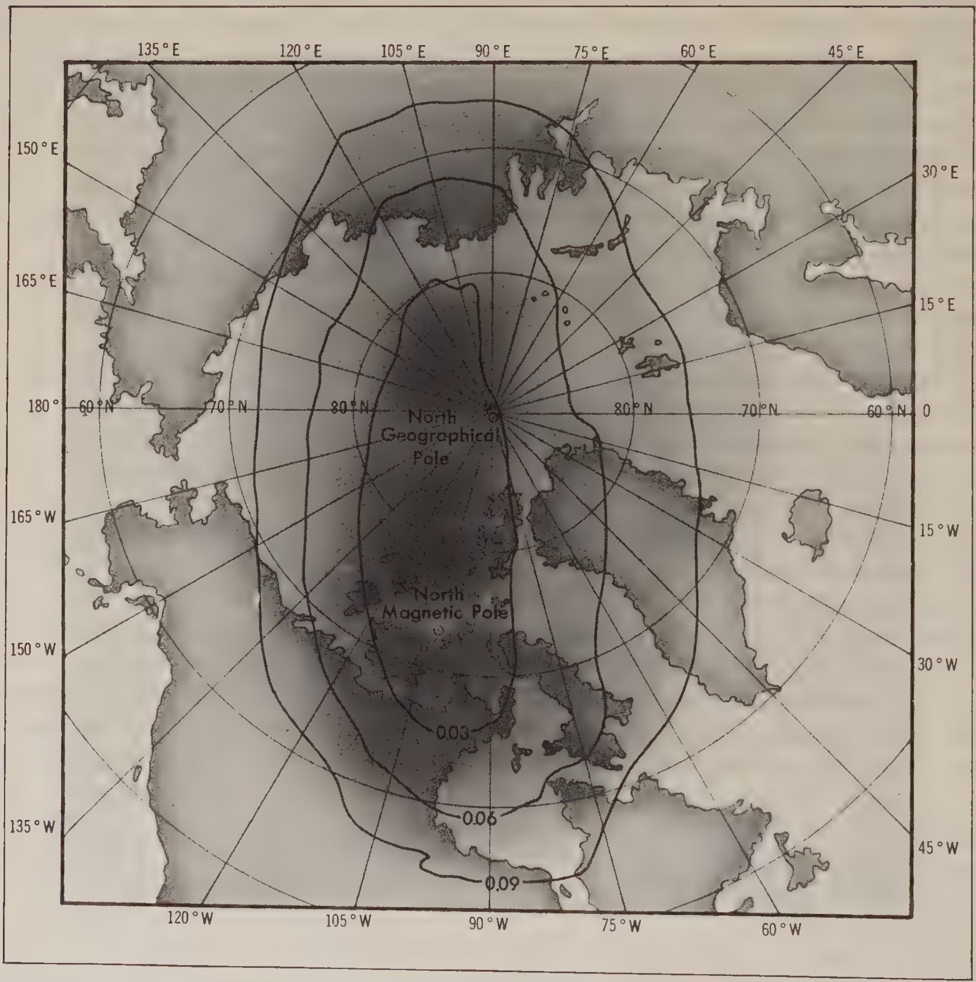


Figure 2513.—Arctic areas in which the magnetic compass is of reduced value. Inside the curves representing the 0.09, 0.06, and 0.03 oersted values of horizontal intensity the compass can be considered of reduced reliability, erratic, and useless, respectively.

around the south magnetic pole, which is located at latitude 68° S, longitude 144° E, not far from the eastern shore of the Ross Sea. Hydrographic Office charts 1701 N and 1701 S show lines of equal horizontal intensity in the north and south polar regions, respectively. However, the effectiveness of the magnetic compass is influenced also by local conditions. A compass on a vessel making a voyage through the islands of the Canadian archipelago has been reported to give fair indication of direction in certain small areas where the horizontal intensity is less than 0.02 oersted, yet to be useless at some places where the horizontal intensity is greater than 0.04 oersted.

Despite its various limitations, the magnetic compass is a valuable instrument in much of the polar regions, where the gyro compass is also of reduced reliability. With careful adjustment, frequent checks, and a record of previous behavior, the polar navigator can get much useful service from his instrument.

When a compass is subjected to extremely low temperatures, there is danger of the liquid freezing. Sufficient heat to prevent this can normally be obtained from the compass light, which should not be turned off during severe weather.

2514. The gyro compass depends for its operation upon the rotation of the earth about its axis. Its maximum directive force is at the equator, where the axis of the compass is parallel to the axis of the earth. As the latitude increases, the angle between these two axes increases. At the *geographical* poles the gyro compass has no directive force.

The gyro compass is generally reliable to latitude 70°. At higher latitudes the disturbing effect of imperfections in compass or adjustment is magnified. Latitude adjustment becomes critical. Speed error increases as the speed of the vessel approaches the rotational speed of the earth. Ballistic deflection error becomes large and the compass is slow to respond to correcting forces. Frequent changes of course and speed, often necessary when proceeding through ice, introduce errors which are slow to settle out. The impact of the vessel against ice deflects the gyro compass. which does not return quickly to the correct reading.

The error increases and becomes more erratic as the vessel proceeds to higher latitudes. Extreme errors as large as 27° have been reported at latitudes greater than 82°. The gyro compass probably becomes useless at about latitude 85°. At latitude 70° the gyro error should be determined frequently, perhaps every four hours, by means of celestial bodies when these are available. As the error increases and becomes more erratic, with higher latitude, it should be determined more frequently. In heavy ice at extreme latitudes an almost constant check is desirable. The gyro and magnetic compasses should be compared frequently and a log kept of the results of these comparisons and the gyro error determinations.

Most gyro compasses are not provided with a latitude correction setting above 70°. Beyond this, correction can be made by either of two methods: (1) set the latitude and speed correctors to zero and apply a correction from a table or diagram obtainable from the manufacturer of the compass, or constructed as explained in article 640; or (2) use an equivalent latitude and speed setting. Both of these methods have proved generally satisfactory, although the second is considered superior to the first because it at least partly corrects for errors introduced by a change in course. At least one gyro compass has been made with provision for setting the latitude corrector to 80°. As experience in high latitudes accumulates, improved gyro compass performance will undoubtedly become available. In certain later types of gyro compasses, facilities for their operation as directional gyros even to the poles is provided.

2515. Celestial compasses.—In some areas neither the magnetic nor gyro compass provides adequate directional reference. In all areas of reduced compass reliability frequent celestial checks are desirable. Several instruments are available for making the celestial observations needed for determining heading in this manner.

A pelorus, alidade, or azimuth circle can be used for measuring the relative or compass azimuth of a celestial body. Compass azimuth can then be compared with a computed true azimuth to determine compass error. However, this can become tedious and time-consuming when frequent heading checks are needed. Several instruments provide a quick mechanical solution.

A **sun compass** indicates direction by means of a shadow cast by a shadow pin exposed to sunlight. The course on a horizontal, graduated azimuth dial is set opposite a lubber's line aligned with the fore-and-aft axis of the vessel. By means of another dial adjusted by a latitude scale so as to be parallel to the plane of the equator, the shadow pin, perpendicular to the plane of this dial and hence parallel to the polar axis of the earth, is set to the local apparent time. When the vessel is on course, the shadow of the pin falls across the center of the local apparent time dial. In some models the local apparent time is maintained by clockwork; in others it is set frequently

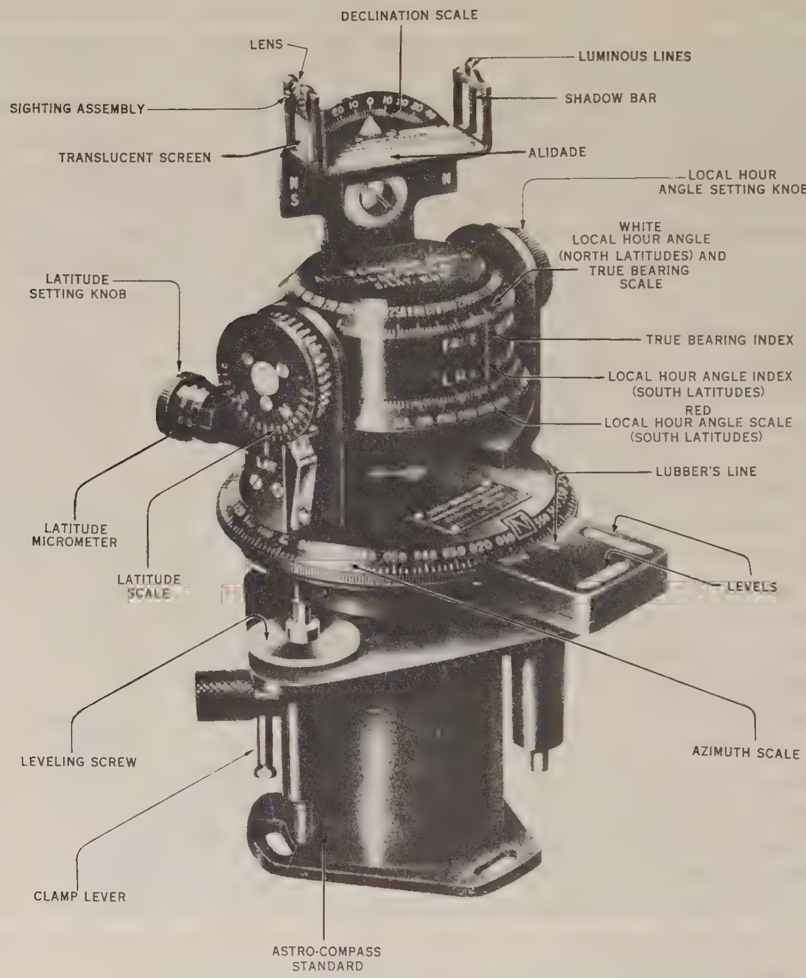


FIGURE 2515.—An astro compass.

by hand. The latitude and local apparent time (which varies with longitude) settings are adjusted from time to time to agree with the changing position of the vessel. The instrument is usable only when the sun is visible and when a knowledge of the position is available.

An **astro compass** is similar in principle to a sun compass, but is usable with any celestial body. When the device is set to the latitude of the observer and the local hour angle and declination of the body, and rotated until the sighting assembly points toward the body, the true heading is indicated at the lubber's line. This device is illustrated in figure 2515.

A **sky compass** indicates direction by means of the polarizing effect of the earth's atmosphere on sunlight. Unpolarized sunlight, upon entering the earth's atmosphere, is scattered and becomes *plane polarized*, its vibrations being in a plane perpendicular to the line from the sun to the observer. When the sun is on the horizon, this plane is vertical. By means of a suitable *analyzer* of Polaroid material and Cellophane, the sky compass detects this plane and the vertical plane which is perpendicular to it, or in the direction of the sun. A possible 90° or 180° ambiguity exists, but this is not of practical significance because the relative brightness of the sky indicates which of the four possible directions is toward the sun. The instrument is set to local apparent time, and a clock maintains this time. The analyzer is then rotated until dark and light portions are of equal brightness, and the heading is indicated at the lubber's line. Unlike the sun and astro compasses, the sky compass is maintained with its face in a level position, pointing at the zenith, which must be clear and unobstructed for an accurate reading. *The sun itself need not be visible*, and can even be several degrees *below* the horizon. Therefore, the compass can be used during twilight, when no other celestial bodies may be visible. For this reason it is sometimes called a **twilight compass.** It is most accurate when the zenith distance of the sun is 90°, and is seldom used when the sun is more than a few degrees from the horizon. Its usefulness arises principally from the fact that twilight periods in high latitudes are of several hours duration, during which time no celestial body is visible unless the moon or a bright planet is above the horizon.

Any celestial compass must be aligned with the fore-and-aft axis of the craft, and is limited in its usefulness to periods when the celestial body being observed (the zenith in the case of the sky compass) is visible. For accurate results certain parts must be kept level. Despite their limitations, these are useful instruments in high latitudes and a ship operating in these areas should be provided with one or more of them.

2516. Distance and direction in ice.—In ice-free waters, distance or speed is determined by some form of log or by engine revolution counter. In the presence of ice, however, most logs are inoperative or inaccurate due to clogging by the ice. Engine revolution counters are not accurate speed indicating devices when a ship is forcing its way through ice. With experience, one can estimate the speed in relation to ice, or a correction can be applied to speed by engine revolution counter. At best, however, these methods are seldom of the desired accuracy.

If ranges and bearings of a land feature can be determined either visually or by radar, course and speed of the vessel or distance traveled over the ground can be determined by tracking the landmark and plotting the results. The feature used need not be identified. Ice can be used if it is grounded or attached to the shore. Course and speed or distance through the water can be determined by tracking a floating iceberg or other prominent floating ice feature. However, an error may be introduced by this method if the effect of wind and current upon the floating feature is different than upon the ship.

Example 1.—The radar operator of a ship proceeding through ice measures the following bearings and ranges of a grounded iceberg:

Time	*Bearing*	*Range*
0835	028°	8,100 yds.
0840	037°	7,600 yds.
0845	047°	7,300 yds.
0850	057°	7,000 yds.
0855	066°	7,200 yds.

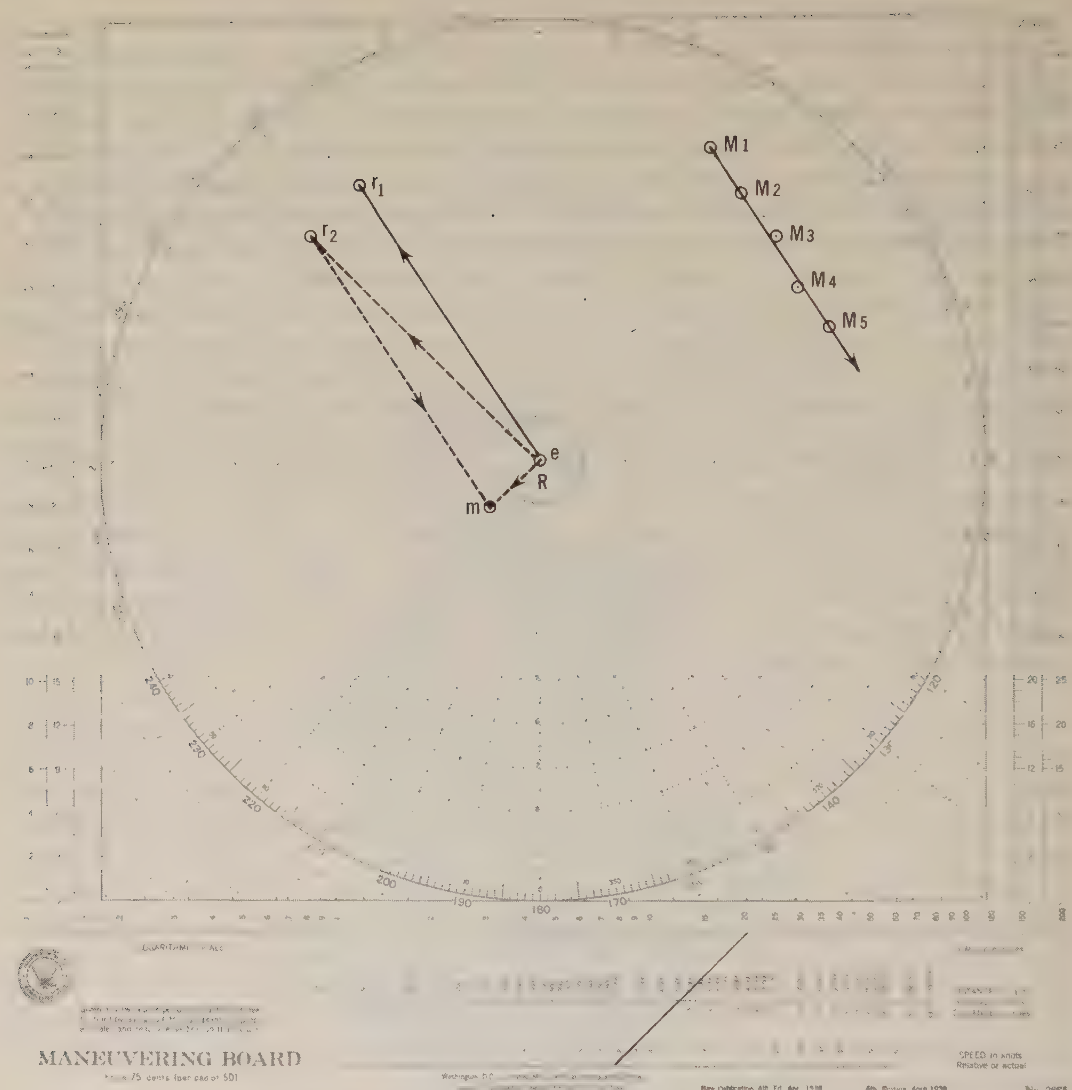


FIGURE 2516.—Determining course and speed by tracking an iceberg.

Required.—The course and speed of the ship.

Solution (fig. 2516).—The solution is conveniently made on a maneuvering board, H.O. 2665–10, but this form is not essential.

(1) From R, representing the ship, plot points M_1, M_2, M_3, M_4, and M_5 representing successive positions of the iceberg *relative to the ship*.

(2) Fair a straight line through the points thus determined.

(3) Measure the direction of line M_1M_5, 147°. This is the direction of the iceberg relative to the ship. The direction of the ship's motion relative to the iceberg is opposite, or 327°. Since the iceberg is stationary, this is the course of the ship.

(4) Measure the length of line M_1M_5, 5,000 yards or 2.5 miles. This is the distance of relative motion, and since the iceberg is stationary, the distance traveled by the ship, in 20 minutes. The distance traveled in 60 minutes, or the speed, is $3\times2.5=$ 7.5 knots. This solution is shown by nomogram at the bottom of the maneuvering board. Vector er_1 represents the course and speed of the ship.

Answers.—C 327°, S 7.5 kn.

Example 2.—Solve example 1 assuming the iceberg is estimated to be moving southwest at a speed of 1.5 knots.

Solution (fig. 2516).—(1) Plot R, M_1, M_2, M_3, M_4, and M_5 and fair a straight line as in example 1.

(2) From e, plot em, the course and speed vector of the iceberg, locating point m.

(3) Determine the direction and speed of the ship relative to the iceberg as in steps (3) and (4) of example 1. Lay this off from m, locating point r_2. The vector r_2m is the relative movement of the iceberg with respect to the ship.

(4) Draw er_2, the course and speed vector of the ship.

Answers.—C 315°, S 7.3 kn.

If speed only is required, the method can be simplified. Elapsed time of tracking and the relative distance covered can be determined by plot, as indicated above, or possibly from the radar scope directly. Speed may then be determined by nomogram or by the formula $S=\frac{60D}{T}$ in which S is the speed in knots, D the distance in miles, and T the time in minutes. If distance is given in yards, the formula is $S=\frac{3D}{100T}$, and if in feet, $S=\frac{D}{100T}$. If a standard distance or time is used, the formula can be further simplified because D or T becomes a constant. For instance, if a distance of five miles or 10,000 yards is used, $S=\frac{300}{T}$, or if a time interval of ten minutes is used, $S=6D$ if D is in miles $\left(\frac{3D}{1000}\text{ if D is in yards, or }\frac{D}{1000}\text{ if D is in feet}\right)$.

This is the basis of the **Dutchman's log,** which can be used without tracking. A ship traveling at one knot covers one mile or about 6076.1 feet in 60 minutes, or approximately 100 feet per minute. A length of 100 feet, or some fraction or multiple of this, can be measured off in a fore-and-aft direction along the ship, and the ends of the measured length marked. For maximum accuracy the longest possible line should be used. A man is then stationed at each mark to note the time of passing some object dead in the water, such as a prominent ice feature or an opening in pack ice. The elapsed time between marks is measured and the speed calculated, using $S=\frac{D}{100T}$ if D is measured in feet and T in minutes. In this application, T may be more conveniently measured in seconds, when $S=\frac{60D}{100T}=\frac{3D}{5T}$. For D=100 feet, $S=\frac{60}{T}$, or speed is found by dividing the number of seconds into 60. Thus, if T=15 seconds, the speed is $\frac{60}{15}=4$ knots. If D=200 feet, $S=\frac{60\times200}{100T}=\frac{120}{T}$; if D=50 feet, $S=\frac{30}{T}$, etc.

Speed over the ground can be determined by two fixes. However, fixes relative to land are not suitable for this purpose unless the land is accurately located on the chart, or the same land features are used for both fixes. High-latitude electronic and celestial fixes, too, are sometimes of less than usual accuracy (arts. 2526–2535).

2517. Tide, current, and wind.—Relatively little is known of tides and currents in the polar regions. The tables do not extend to these areas, but some information is given in the sailing directions. In general, tidal ranges are small, and the water in most anchorages is relatively deep.

Currents in many coastal areas are strong and somewhat variable. When a vessel is operating in ice, the current is often difficult to determine because of frequent changes

in course and speed of the vessel and inaccuracies in the measurement of direction and distance traveled.

In the vicinity of land, and in the whole antarctic area, winds are variable in direction, gusty, and often strong. Offshore, in the Arctic Ocean, the winds are not strong and are steadier, but ships rarely operate in this area. The wind in polar regions, as elsewhere, has two primary navigational effects upon vessels. First, its direct effect is to produce leeway. When a vessel is operating in ice, the leeway may be different from that in open water. It is well to determine this effect for one's own vessel. The second effect is to produce wind currents in the sea.

2518. Keeping the dead reckoning.—Because of the lack of facilities for fixing the position of a vessel in polar regions, accurate dead reckoning is even more important than elsewhere. The problem is complicated by the fact that the *elements* of dead reckoning, direction and distance, are usually known with less certainty than in lower latitudes. This only heightens the need for *keeping* the dead reckoning with all the accuracy obtainable. This may usually be accomplished by careful hand plotting on the available charts or plotting sheets.

Mechanical dead reckoning equipment is generally less accurate than a carefully-kept hand plot. Older models of such equipment cannot be set to a higher latitude than 70°. Newer equipment has provision for setting to latitude 80°. Dead reckoning equipment has been used beyond its maximum range by setting to a lower latitude and applying a correction, but this procedure is of questionable advisability because of the error introduced by a gyro compass also operating beyond its range. This equipment is intended for use with the Mercator projection. When a different projection is used, better results are generally obtainable by setting the equipment to latitude 0°, and letting its latitude indications represent change in latitude, and its difference of longitude indications represent miles in an east-west direction.

Piloting

2519. Piloting in high latitudes is basically no different from that elsewhere. However, in polar regions piloting is the primary method of marine navigation. As previously indicated, dead reckoning is difficult and generally less accurate than in lower latitudes. Celestial navigation has limited application. Electronic navigational aids are almost nonexistent.

Piloting is associated with proximity to land and shoal water. A ship in polar regions is seldom far from land, and the areas are not so accurately surveyed that the navigator can be sure that uncharted shoals are not nearby.

Piloting is characterized by an alertness not required when a vessel is far from danger of grounding. Nowhere is this alertness more necessary than in polar regions. Added to the usual reasons for constant vigilance are the uncertainties of charted information and the lack of detail, as discussed in article 2509.

2520. Aids to navigation are virtually nonexistent in polar regions. There are no lighthouses, few beacons, and very few buoys. Channels and shoals are not marked and may not even be indicated on the chart. A few radiobeacons are available, notably along the northern coast of Russia. Other radio transmitters are occasionally available for use as beacons.

2521. Natural landmarks are plentiful in some areas, but their usefulness is restricted by the difficulty in identifying them, or locating them on the chart. Along many of the coasts the various points and inlets bear a marked resemblance to each other. The appearance of a coast is often very different when many of its features are obliterated by a heavy covering of snow or ice than when it is ice-free.

2522. Bearings are useful, but have limitations. When bearings on more than two objects are taken, they may fail to intersect at a point because the objects may not be charted in their correct relation to each other. Even a point fix may be considerably in error geographically if all of the objects used are shown in correct relation to each other, but in the wrong position on the earth. However, in restricted waters it is usually more important to know the position of the vessel relative to nearby land and shoals than its latitude and longitude. The bearing and distance of even an unidentified or uncharted point are valuable.

When a position is established relative to nearby landmarks, it is good practice to use this to help establish the identity and location of some prominent feature a considerable distance ahead, so that this feature, in turn, can be used to establish future positions.

In high latitudes it is not unusual to make use of bearings on objects a considerable distance from the vessel. Because of the rapid convergence of the meridians in these areas, such bearings are not correctly represented by straight lines on a Mercator chart. If this projection is used, the bearings should be corrected in the same manner that radio bearings are corrected (using table 1), since both can be considered great circles. Neither visual nor radio bearings are corrected when plotted on a Lambert conformal chart.

2523. Soundings are so important in polar regions that echo sounders are customarily operated continuously while the vessel is under way. It is good practice to have at least two such instruments, preferably those of the recording type and having a wide flexibility in the range of the recorder. In few parts of the polar regions have enough soundings been obtained and made available to charting agencies to permit adequate portrayal of the bottom configuration. However, since depth of water is a primary consideration in avoiding an unwanted grounding, a constant watch should be maintained to avoid unobserved shoaling.

Polar regions have relatively few shoals, but in some areas, notably along the Labrador coast, a number of pinnacles and ledges rise abruptly from the bottom. These constitute a real danger to vessels, since they are generally not surrounded by any apparent shoaling. In such an area, or when entering an unknown harbor or any area of questionable safety, it is good practice to send one or more small craft ahead with portable sounding gear.

In very deep water, of the order of 1,000 fathoms or more, the echo returned from the bottom is sometimes masked by the sound of ice coming in contact with the hull, but this is generally not a problem when the bottom is close enough to be menacing.

The hand lead is of little value to a ship underway in ice, because the ice generally prevents its effective use unless the vessel is stopped.

If a ship becomes **beset** by ice, so that steerage way is lost and the vessel drifts with the ice, it may be in danger of grounding as the ice moves over a shoal. Hence, it is important that soundings be continued even when beset. If necessary, a hole should be made in the ice and a hand lead used. A vessel with limited means for freeing itself may prudently save such means for use only when there is danger of grounding.

Useful information on the depth of water in the vicinity of a ship can sometimes be obtained by watching the ice. A stream of ice moving faster than surrounding ice, or a stretch of open water in loose pack ice often marks the main channel through shoal water. A patch of stationary ice in the midst of moving ice often marks a shoal.

Knowledge of earth formations may also prove helpful. The slope of land is often an indication of the underwater gradient. Shoal water is often found off low islands, spits, etc., but seldom near a steep shore. Where glaciation has occurred, the moraine

deposits are likely to have formed a bar some distance offshore. Submerged rocks and pinnacles are more likely to be encountered off a rugged shore than near a low, sandy beach.

2524. Anchorages.—Because good anchorages are not plentiful in high latitudes, there is an understandable temptation to be less demanding in their selection. This is dangerous practice, for in polar regions some of the requirements are accentuated. The factors to be considered are:

1. *Holding quality of the bottom.* In polar regions a rocky bottom or one with only fair to poor holding qualities is not uncommon. Sometimes the bottom is steep or irregular. Since the nature of the bottom is seldom adequately shown on charts, a wise precaution is to sample the bottom, and sound in the vicinity before anchoring.

2. *Adequacy of room for swing.* Because high winds are frequent along polar shores, sometimes with little or no warning, long scopes of anchor chain are customarily used. Some harbors are otherwise suitable, but allow inadequate room for swing of the vessel at anchor, or even for its yaw in a high wind. If a vessel is to anchor in an unsurveyed area, the area should first be adequately covered by small boats with portable sounding gear to detect any obstructions.

3. *Protection from wind and sea.* In polar regions protection from wind is probably the most difficult requirement to meet. Generally, high land is accompanied by strong wind blowing directly down the side of the mountains. Polar winds are extremely variable, both in direction and speed. Shifts of 180° accompanied by an increase in speed of more than 50 knots in a few minutes have been reported. It is important that ground tackle be in good condition and that maximum-weight anchors be used. All available weather reports should be obtained and a continuous watch kept on the local weather. Whenever a heavy blow might reasonably be anticipated, the main engines should be kept in an operating condition and on a standby status. Heavy seas are seldom a problem.

4. *Availability of suitable exit in event of extreme weather.* In ice areas it is important that a continuous watch be kept to prevent blocking of the entrance by ice, or actual damage to the vessel by floating ice. However, in an unsurveyed area it may be dangerous to shift anchorage without first sounding the area. It is a wise precaution to do this in advance. Unless the vessel is immediately endangered by ice, it is generally safer to remain at anchor with optimum ground tackle and use of engines to assist in preventing dragging, than to proceed to sea in a high wind, especially in the presence of icebergs and growlers, and particularly during darkness.

5. *Availability of objects for position determination.* The familiar polar problem of establishing a position by inaccurately charted or inadequately surveyed landmarks is accentuated when an accurate position is desired to establish the position of an anchor. Sometimes a trial and error method is needed, and it may be necessary to add landmarks located by radar or visual observation. Because of chart inadequacy, the suitability of an anchorage, from the standpoint of availability of suitable landmarks, cannot always be adequately predicted before arrival.

An unsurveyed harbor should be entered with caution at slow speed, with both the pilot house and engine room force alerted to possible radical changes in speed or course with little or no warning. The anchor should be kept ready for letting go on short notice and should be adequately attended. An engine combination providing full backing power should be maintained.

2525. Sailing directions for high latitudes contain a wealth of valuable information acquired by those who have previously visited the areas. However, since high latitudes have not been visited with the frequency of other areas, and since they are inadequately surveyed, the sailing directions for polar areas are neither as complete

nor as accurate as for other areas, and information on unvisited areas is completely lacking. Until traffic in high latitudes increases and the sailing directions for these areas incorporate the additional information obtained, unusual caution should accompany their use. Each vessel that enters polar regions can help correct this condition by recording accurate information and sending it to the U. S. Navy Hydrographic Office or its counterpart in other countries.

Electronic Navigation

2526. Propagation.—In general, radio wave propagation in high latitudes follows the same principles that apply elsewhere, as described in chapter X. However, certain anomalous conditions occur, and although these are but imperfectly understood, and experience to date has not always seemed consistent, there is much that has been established. An understanding of these conditions is important if maximum effective use is to be made of electronics in high latitudes.

FIGURE 2526.—The auroral zone of the northern hemisphere.

Because of the influence of the ionosphere (art. 1008) upon radio wave propagation, the most disruptive effects are associated with **ionospheric disturbances,** one aspect of the familiar **magnetic storms.** These have been found to be related to sunspot activity, and this association provides a basis for their prediction. Warnings based upon such predictions are broadcast by radio station WWV, National Bureau of Standards, Washington, D. C., and by major U. S. Navy radio stations. Such warnings, usually broadcast several hours before the start of a disturbance, are confined to the expected occurrence. Predictions of intensity or duration have not been possible.

Severe ionospheric disturbances affect radio wave propagation throughout the world, but the most erratic and persistent effects occur in the **auroral zones.** The auroras (**aurora borealis** or "northern lights" in the northern hemisphere, and the **aurora australis** or "southern lights" in the southern hemisphere) are believed to be caused by emissions from the sun. When the emitted particles enter the earth's magnetic field, they tend to follow the earth's lines of force downward toward the geomagnetic poles (art. 706). When they encounter the ionosphere, they become luminous, constituting the aurora familiar to the night observer in high latitudes. The maximum auroral activity occurs in two belts, each about 600 miles wide and centered at about 1,200 miles from one of the geomagnetic poles, as shown in figure 2526. In the auroral zones, the aurora is a common occurrence, being visible on nearly any dark, clear night. Frequency of occurrence decreases with increased distance from the zones. During magnetic storms the auroral zones have a tendency to shift outward from the geomagnetic poles.

When an ionospheric disturbance occurs, fading and ionospheric absorption increase. The maximum usable frequency (art. 1008) decreases, and the minimum useful

high frequency increases. In extreme cases, the entire band of useful frequencies disappears, resulting in a radio **blackout** which may continue for any period from a few minutes to several days. In the auroral zones, higher frequencies used for communication have been known to be blacked out for as long as two weeks. The return to normal usually occurs first on lower frequencies.

During the early stages of an ionospheric disturbance, the *path* of propagation may deviate erratically from normal, resulting in erroneous direction finder bearings and consol readings.

Because of the shift of the auroral zones during a magnetic storm, radio propagation *within* the usual positions of the belts may improve. Transmission is usually of greater range *along* radial lines from the geomagnetic poles than *across* these lines.

Very low frequencies (10–30 kc) originating outside the auroral zone are not affected appreciably by ionospheric disturbances, and propagation between 30 and 200 kc may even improve. This is believed to be due to a great increase in the density of the lowest (D) layer of the ionosphere, which acts as a wave guide for lower frequencies, while absorbing higher frequency transmissions.

In polar regions, long-range, high frequency propagation is sometimes erratic even when conditions seem normal, and the usual procedure for selection of optimum working frequencies for communication is not always valid. The shielding effect of mountains seems to be greater than in lower latitudes.

2527. Radar.—In polar regions, where fog and long periods of continuous daylight or darkness reduce the effectiveness of both celestial navigation and visual piloting, and where other electronic aids are generally not available, radar is particularly valuable. Its value is further enhanced by the fact that polar seas are generally smooth, resulting in relatively little oscillation of the shipborne antenna. When ice is not present, relatively little sea return is encountered from the calm sea.

However, certain limitations attend the use of radar in polar regions. Similarity of detail along the polar shore is even more apparent by radar than by visual observation. Lack of accurate detail on charts adds to the difficulty of identification. Identification is even more of a problem when the shore line is beyond the radar horizon and accurate contours are not shown on the chart. When an extensive ice pack extends out from shore, accurate location of the shore line is extremely difficult.

Good training and extensive experience are needed to interpret accurately the returns in polar regions where ice may cover both land and sea. A number of icebergs close to a shore may be too close together to be resolved, giving an altered appearance to a shore line, or they may be mistaken for off-lying islands. The shadow of an iceberg or pressure ridge and the lack of return from an open lead in the ice may easily be confused. Smooth ice may look like open water. In making rendezvous, one might inadvertently close on an iceberg instead of a ship.

As with visual bearings, radar bearings need correction for convergency unless the objects observed are quite close to the ship.

2528. Loran is usable in polar regions, but the coverage is greatly restricted. As shown in the coverage diagram, figure 1302a, Loran-A groundwave coverage extends into the edge of the arctic in several places. The skywave coverage extends some distance beyond. Extensive areas in the arctic and all of the antarctic are without coverage.

2529. Other electronic aids are virtually nonexistent in polar regions.

The *radio direction finder* is useful when the few transmitting stations are within range. One of the principal uses of RDF in polar regions is to assist in locating other vessels, for rendezvous or other purposes. This is particularly true in an area of many icebergs, where radar may not distinguish between ships and icebergs.

Consol is available in the Norwegian Sea between Norway and Greenland.

The *echo sounder* is highly useful, as indicated in article 2523, and is operated continuously in high latitudes.

Sonar is useful primarily for detecting ice, particularly growlers. Since about ½ to ⅞ of the ice is under water, its presence can sometimes be detected by sonar when it is overlooked by radar or visual observation.

Celestial Navigation

2530. Celestial navigation in high latitudes.—Of the various types of navigation, celestial is perhaps least changed in polar regions. However, certain special considerations are applicable.

Because of the limitations of other forms of navigation, as discussed earlier in this chapter, celestial navigation provides the principal means of determining geographical position. However, as indicated in article 2522, position relative to nearby dangers is usually of more interest to the polar navigator than geographical position. Since ships in high latitudes are seldom far from land, and since celestial navigation is attended by several limitations, discussed in article 2531, its use in marine navigation is generally confined to the following applications:

1. Navigation while proceeding to and from polar regions.
2. Checking the accuracy of dead reckoning.
3. Checking the accuracy of charted positions of landmarks, shoals, etc.
4. Providing a directional reference, either by means of a celestial compass (art. 2515) or by providing a means of checking the magnetic or gyro compass.

Although its applications are limited, celestial navigation is important in high latitudes. Application 3 above, and application 4, even more so, can be of great value to the polar navigator.

2531. Celestial observations.—The best celestial fixes are usually obtained by star observations during twilight. As the latitude increases, these periods become longer, providing additional time for observation. But with this increase comes longer periods when the sun is just below the horizon and the stars have not yet appeared. During this period, which in the extreme condition at the pole lasts for several *days*, no celestial observations may be available. The moon is sometimes above the horizon during this period and bright planets, notably Venus and Jupiter, may be visible. With practice, the brighter stars can be observed when the sun is 2° to 3° below the horizon.

Beyond the polar circles the sun remains above the horizon without setting during part of the summer. The length of this period increases with latitude. At Thule, Greenland, about 10° inside the arctic circle, the sun remains above the horizon for four months. During this period of continuous daylight the sun circles the sky, changing azimuth about 15° each hour. A careful observation, or the average of several observations, each two hours provides a series of running fixes. An even better check on position is provided by making hourly observations and establishing the most probable position at each observation. Sometimes the moon is above the horizon, but within several days of the new or full phase it provides lines of position nearly parallel to the sun lines and hence of limited value in establishing fixes.

During the long polar night the sun is not available and the horizon is often indistinct. However, the long twilight, a bright aurora, and other sources of polar light (art. 2504) shorten this period. By adapting their eyes to darkness, some navigators can make reasonably accurate observations throughout the polar night. The full moon in winter remains above the horizon more than half the time and attains higher altitudes than at other seasons.

In addition to the long periods of darkness in high latitudes, other conditions are sometimes present to complicate the problem of locating the horizon. During daylight the horizon is frequently obscured by low fog, frost smoke, or blowing snow, yet the sun may be clearly visible. Hummocked sea ice is sometimes a problem, particularly at low heights of eye. Nearby land or an extensive ice foot can also be troublesome. Extreme conditions of abnormal refraction are not uncommon in high latitudes, sometimes producing false horizons and always affecting the refraction and dip corrections.

Because of these conditions, it is advisable to be provided with an artificial-horizon sextant (art. 1513). This instrument is generally not used aboard ship because of the excessive acceleration error encountered as the ship rolls and pitches. However, in polar regions there is generally little such motion and in the ice there may be virtually none. Some practice is needed to obtain good results with an artificial-horizon sextant, but these results are sometimes superior to those obtainable with a marine sextant, and when some of the conditions mentioned above prevail, the artificial-horizon sextant may provide the only means of making an observation. Better results with this instrument can generally be obtained if the instrument is hung from some support, as it generally is when used in aircraft.

An artificial horizon (art. 1512) can sometimes be used effectively, even an improvised one, as by placing heavy lubricating oil in a bucket.

It is sometimes possible to make better observations by artificial-horizon sextant or artificial horizon from a nearby cake of ice than from the ship.

Clouds and high fog are frequent in high latitudes, but it is not uncommon, particularly in the antarctic, for the fog to lift for brief periods, permitting an alert navigator to obtain observations.

As the latitude increases, an error of time has less effect upon altitude. At the equator an error of four seconds in time may result in an error in the location of the position line of as much as one mile. At latitude 60° a position error of this magnitude cannot occur unless the timing error is eight seconds. At 70° nearly 12 seconds are needed, and at 80° about 23 seconds are needed for such a position error.

Polaris is of diminished value in high northern latitudes because of its high altitude. At high latitudes the second correction to observed altitude (a_1) becomes greater. The almanac makes no provision for applying this beyond latitude 68°. Bodies at high altitudes are not desirable for azimuth determination, but if Polaris is used, the use of the actual azimuth given at the bottom of the Polaris tables of the *Nautical Almanac* is of increased importance because of its larger variation from 000° in high latitudes. No azimuth is provided beyond latitude 65°.

In applying a sextant altitude correction for dip of the horizon, one should use height of eye *above the ice at the horizon*, instead of height above water. The difference between ice and water levels at the horizon can often be estimated by observing ice near the vessel.

2532. Low-altitude observations.—Because of large and variable refraction at low altitudes, navigators customarily avoid observations below some minimum, usually 5° to 15°, if higher bodies can be observed. In polar regions low-altitude observations are often the only ones available. The sun, moon, and planets remain low in the sky for relatively long periods, their diurnal motion being nearly horizontal. The only lower limit is that imposed by the horizon itself. In fact, good observations can sometimes be made without a sextant by noting the time at which either the upper or lower limb is tangent to the horizon. To such an observation sextant altitude corrections are applied as for a marine sextant without an index correction.

Correction of low-altitude observations made by marine sextant is discussed in article 1632. If a bubble or other artificial-horizon sextant is used, corrections are made

as for higher altitudes, being careful to use the refraction value corrected for temperature, or to make a separate correction for air temperature. In addition, a correction for atmospheric pressure (tab. 24) is applied if of sufficient size to be of importance.

Solution of low-altitude observations is discussed in article 2010.

2533. Abnormal refraction and dip.—Tables of refraction correction are based upon a standard atmosphere. Variations in this atmosphere result in changes in the refraction, and since the atmosphere is seldom exactly standard, the mean refraction is seldom the same as shown in the tables. Variations from standard conditions are usually not great enough to be troublesome.

In polar regions, however, it is normal for the atmosphere to differ considerably from the standard, particularly near the surface. This affects both refraction and dip, as indicated in article 1606. Outside polar regions, variations in refraction seldom exceed 2′ or 3′, although extreme values of more than 30′ have been encountered In polar regions refraction variations of several minutes are not uncommon and an extreme value of about 5° has been reported. This would produce an error of 300 miles in a line of position. The sun has been known to rise as much as ten *days* before it was expected.

Most celestial observations in polar regions produce satisfactory results, but the high-latitude navigator should be on the alert for abnormal conditions, since they occur more often than elsewhere, and have greater extreme values. A wise precaution is to apply corrections for air temperature (tab. 23) and atmospheric pressure (tab. 24), particularly for altitudes of less than 5°.

Abnormal dip affects the accuracy of celestial observations equally at any altitude, if the visible horizon is used. Such errors may be avoided in any *one* of four ways:

1. The artificial-horizon sextant may be used, as indicated in article 2531.

2. When stars are available, three stars may be observed at azimuth intervals of approximately 120°, (or four at 90° intervals, five at 72°, etc.). Any error in dip *or refraction* will alter the size of the enclosed figure, but will not change the location of its center unless the dip or refraction error varies in different directions. The stars should preferably be at the same altitude.

3. The altitude of a single body may be observed twice, facing in opposite directions. The sum of the two readings differs from 180° by twice the sum of the index and dip corrections (also personal and instrument corrections, if present). This method assumes that dip is the same in both directions, an assumption that is usually approximately correct. Also, the method requires that the arc of the sextant be sufficiently long and the altitude of the body sufficiently great to permit observation of the back sight in the opposite direction. In making such observations, it is necessary that allowance be made for the change of altitude between readings. This may be done by taking a direct sight, a back sight, and then another direct sight at equal intervals of time, and using the average of the two direct sights.

4. A correction for the difference between air and sea temperatures (art. 1607) may be applied to the sextant altitude. This will often provide reasonably good results. However, there is considerable disagreement in the manner in which temperature is to be measured, and in the factor to use for any given difference. Therefore, the validity of this correction is not fully established.

There is still much to be learned regarding refraction and even with all known precautions, results may occasionally be unsatisfactory.

2534. Sight reduction in polar regions is virtually the same as elsewhere. Computation can be made by nearly any method. In H.O. Pub. No. 214, tabulations are not extended below an altitude of 5°, but this method can be used for lower altitudes, which are not uncommon in polar regions, by selecting an assumed position some distance away,

in the general direction of the body. Thus, if the altitude is 2°, an assumed position 3° (180 miles) nearer the body (4° is a better choice to allow for possible error in the dead reckoning and for adjustment for a convenient assumed position) should result in a computed altitude of 5° or more. This method will result in an unusually long altitude difference, but the error introduced will be negligible *if* the assumed position is in the direction of the body, and the chart used is one on which a straight line is a close approximation to a great circle. A Lambert conformal chart is satisfactory for this purpose. An example of such a solution is given in article 2010.

Some navigators prefer to use another method of sight reduction. Both H.O. Pub. No. 208 and H.O. Pub. No. 211 are suitable for this purpose, but perhaps the most satisfactory method is H.O. Pub. No. 249, which provides solutions down to the visible horizon for any height of eye to be anticipated aboard ship and for any reasonable altitude difference. However, except for certain specified stars, the method is limited to celestial bodies having declinations not exceeding 30°. This provides for the sun, moon, planets, and a number of good navigational stars. Having been designed for air navigation, H.O. Pub. No. 249 provides computed altitudes to the nearest whole minute, and azimuth to the nearest whole degree. For low altitudes this precision is realistic.

From latitude 70° to the pole, hour angles in H.O. Pub. No. 249 are tabulated at intervals of 2°. Near the pole this interval could be greatly increased because of the small diameter of the parallels of latitude. Based upon this and the fact that azimuth approaches LHA ±180° near the north pole (360°−LHA near the south pole), various special methods have been suggested for high latitudes, providing very short tables. However, there is considerable advantage in using familiar methods and avoiding special ones of limited application and often of little advantage.

One special method of considerable interest is conveniently applicable only within about 5° of the pole, a higher latitude than is usually attainable by ships. This is the method of using the pole as the assumed position. At this point the zenith and pole coincide and hence the celestial equator and celestial horizon also coincide, and the systems of coordinates based upon these two great circles of the celestial sphere become identical. The declination is computed altitude, and GHA replaces azimuth. A "toward" altitude difference is plotted along the upper branch of the meridian over which the body is located, and an "away" difference is plotted in the opposite direction, along the lower branch. Such a line or its AP is advanced or retired in the usual manner. This method is a special application of the meridian altitude sometimes used in lower latitudes. Beyond the limits of this method the meridian altitude can be used in the usual manner (art. 2103) without complications and with time of transit being less critical. However, table 29, for reduction to the meridian, extends only to latitude 60°. Tables providing a correction to permit use of the pole-assumed-position method with the polar stereographic projection at considerable distance from the pole have been prepared, but are rarely used, and never by mariners. These are known as the **Ellsworth Tables.**

2535. Plotting lines of position from celestial observations.—Lines of position from celestial observations in polar regions are plotted as elsewhere, using an assumed position, altitude difference, and azimuth. If a Mercator chart is used, the error introduced by using rhumb lines for the azimuth line (a great circle) and line of position (a small circle) is accentuated. This can be overcome by using a good dead reckoning or estimated position as the assumed position or by using a chart on a more favorable projection.

If a chart with nonparallel meridians, such as the Lambert conformal, is used, the true azimuth should be plotted by protractor or plotter and measured at the meridian

of the assumed position. On a chart having a grid overprint the true azimuth can be converted to grid azimuth, using the longitude of the assumed position, and the direction measured from any grid line. This method involves an additional step, with no real advantage.

Lines of position from high-altitude observations, to be plotted as circles with the geographical position as the center (art. 2011), should not be plotted on a Mercator chart because of the rapid change of scale, resulting in distortion of the circle as plotted on the chart.

Lines of position are advanced or retired as in any latitude. However, the movement of the line is no more accurate than the estimate of the direction and distance traveled, and in polar regions this estimate may be of less than usual accuracy. In addition to his problem of estimating direction of travel, the polar navigator may encounter difficulty in accurately plotting the direction determined. If an accurate gyro compass is used, the ship follows a rhumb line, which is accurately shown only on a Mercator chart. If a magnetic compass is used, the rapid change in variation may be a disturbing factor. If the ship is in ice, the course line may be far from straight.

Because of the various possible sources of error involved, it is good practice to avoid advancing or retiring lines for a period longer than about two hours. When the sun is the only body available, best results can sometimes be obtained by making an observation every hour, retiring the most recent line one hour and advancing for one hour the line obtained two hours previously. The present position is then obtained by dead reckoning from the running fix of an hour before. Another technique is to advance the one or two previous lines to the present time for a running fix. A third method is to drop a perpendicular from the dead reckoning or estimated position to the line of position to obtain a new estimated position, from which a new dead reckoning plot is carried forward to the time of the next observation. A variation of this method is to evaluate the relative accuracy of the new line of position and the dead reckoning or estimated position run up from the previous position and take some point *between* them, halfway if no information is available on which to evaluate the relative accuracies. None of these techniques is suitable for determining set and drift of the current.

2536. Rising, setting, and twilight data are tabulated in the almanacs to latitude 72° N and 60° S. Within these limits the times of these phenomena are determined as explained in chapter XVIII.

Beyond the northern limits of these tables the values can be obtained from a series of graphs given near the back of the *Air Almanac*. These graphs are shown in appendix W. For high latitudes, graphs are used instead of tables because graphs give a clearer picture of conditions, which may change radically with relatively little change in position or date. Under these conditions interpolation to practical precision is simpler by graph than by table. In those parts of the graph which are difficult to read, the times of the phenomena's occurrence are themselves uncertain, being altered considerably by a relatively small change in refraction or height of eye.

On all of these graphs any given latitude is represented by a horizontal line, and any given date by a vertical line. At the intersection of these two lines the duration is read from the curves, interpolating by eye between curves.

The "Semiduration of Sunlight" graph gives the number of hours between sunrise and meridian transit or between meridian transit and sunset. The dot scale near the top of the graph indicates the LMT of meridian transit, the time represented by the minute dot nearest the vertical date line being used. If the intersection occurs in the area marked "sun above horizon," the sun does not set; and if in the area marked "sun below horizon," the sun does not rise.

Example 1.—Find the zone time of sunrise and sunset at lat. 71°30′.0 N, long. 10°00′.0 W near Jan Mayen Island, on August 25, 1958.

Solution.—

	August 25	
LMT	1202	LAN, from top of graph
dλ	(−) 20	
ZT	1142	LAN
semidur.	840	from graph
ZT	0302	sunrise (− semidur.)
ZT	2022	sunset (+ semidur.)

A vertical line through August 25 passes nearest the dot representing LAN 1202 on the scale near the top of the graph. This is LMT; at longitude 10°00′.0 W the ZT is 20^m earlier, or at 1142. The intersection of the vertical date line with the horizontal latitude line occurs between the 8^h and 9^h curves, at approximately $8^h\ 40^m$. Hence, sunrise occurs at this interval before LAN and sunset at this interval after LAN.

The "Duration of Twilight" graph gives the number of hours between the beginning of morning *civil* twilight (center of sun 6° below the horizon) and sunrise, or between sunset and the end of evening civil twilight. If the sun does not rise, but twilight does occur, the time taken from the graph is half the total length of the single twilight period, or the number of hours from beginning of morning twilight to LAN, or from LAN to end of evening twilight. If the intersection occurs in the area marked "continuous twilight or sunlight," the center of the sun does not get more than 6° below the horizon; and if in the area marked "no twilight nor sunlight," the sun remains more than 6° below the horizon throughout the entire day.

Example 2.—Find the zone time of beginning of morning twilight and ending of evening twilight at the place and date of example 1.

Solution.—

	Twilight	
ZT	0302	sunrise, from example 1
dur.	153	from graph
ZT	0109	morning twilight

	Twilight	
ZT	2022	sunset, from example 1
dur.	153	from graph
ZT	2215	evening twilight

The intersection of the vertical date line and the horizontal latitude line occurs approximately one-sixth of the distance from the 2^h line toward the $1^h\ 20^m$ line; or at about $1^h\ 53^m$. Morning twilight begins at this interval before sunrise, and evening twilight ends at this interval after sunset.

The "Semiduration of Moonlight" graph gives the number of hours between moonrise and meridian transit or between meridian transit and moonset. The dot scale near the top of the graph indicates the LMT of meridian transit, each dot representing one hour. The phase symbols indicate the date on which the principal moon phases occur, the open circle indicating full moon and the dark circle indicating new moon. If the intersection of the vertical date line and the horizontal latitude line falls in the "moon above horizon" or "moon below horizon" area, the moon remains above or below the horizon, respectively, for the entire 24 hours of the day.

If approximations of the times of moonrise and moonset are sufficient, the values of semiduration taken from the graph can be used without adjustment. For more accurate results, the times on the required date and the adjacent date (the following date in west longitude and the preceding date in east longitude) should be determined, and an interpolation made for longitude, as in any latitude, since the intervals given are for the Greenwich meridian.

Example 3.—Find the zone time of moonrise and moonset at lat. 74°00′.0 N, long. 108°00′.0 W on May 16, 1958, and the phase of the moon on this date.

Solution.—

	May 16		May 17	
LMT	0952	LMT	1036	meridian transit, from graph
dλ	(+) 12	dλ	(+) 12	
ZT	1004	ZT	1048	meridian transit
semidur.	8^h48^m	semidur.	9^h55^m	from graph
ZT	0116	ZT	0053	(moonrise − semidur.)
ZT	1852	ZT	2043	(moonset + semidur.)

	Moonrise			Moonset	
ZT	0116	May 16	ZT	1852	May 16
ZT	0053	May 17	ZT	2043	May 17
diff.	(−) 23		diff.	(+) 111	
23×108.0/360	(−) 7		111×108.0/360	(+) 33	
ZT	0109		ZT	1925	

The phase is crescent, about three days before new moon. The LMT of meridian transits are found by noting the intersections of the vertical date lines with the dot scale near the top of the graph, interpolating by eye. At longitude 108°00′.0 W the ZT is 12^m later. The semiduration is found by noting the position, with respect to the semiduration curves, of the intersection of the vertical date line with the horizontal latitude line. This interval is subtracted from the time of meridian transit to obtain moonrise, and added to obtain moonset. These solutions are made for both May 16 and 17, and the difference determined in minutes. The adjustment to be applied to the ZT on May 16 at Greenwich is determined by multiplying this difference by the ratio λ/360. The phase is determined by noting the position of the vertical date line with respect to the phase symbols. If the answer indicates that the phenomenon occurs on a date differing from that desired, a new solution should be made, adjusting the starting date accordingly. The phenomenon may occur twice on the same day, or it may not occur at all. In high latitudes the effect on the time of moonrise and moonset of a relatively small change in declination is considerably greater than in lower latitudes, resulting in greater differences from day to day.

Sunlight, twilight, and moonlight graphs are not available for south latitudes. Beyond latitude 65° S, the northern hemisphere graphs can be used for determining the semiduration or duration, by using the vertical date line for a day when the declination has the same numerical value but opposite sign. The time of meridian transit and the phase of the moon are determined as explained above, using the correct date. Between latitudes 60° S and 65° S solution is made by interpolation between the tables and the graphs.

Several other methods of solution of these phenomena are available. The *Tide Tables* tabulate sunrise and sunset from latitude 76° N to 60° S. A supplement to the American Ephemeris of 1946, entitled *Tables of Sunrise, Sunset, and Twilight,* provides tabulations from latitude 75° N to 75° S and graphs for semiduration of sunlight and duration of twilight, with separate graphs for civil, nautical, and astronomical twilights. Semiduration or duration can be determined graphically by means of a diagram on the plane of the celestial meridian (art. 1432), or by computation. When computation is used, solution is made for the meridian angle at which the required negative altitude occurs. The meridian angle expressed in time units is the semiduration in the case of sunrise, sunset, moonrise, and moonset; and the semiduration

of the combined sunlight and twilight, or the time from meridian transit at which morning twilight begins or evening twilight ends. For sunrise and sunset the altitude used is (−) 50′. Allowance for height of eye can be made by algebraically subtracting (numerically adding) the dip correction from this altitude. The altitude used for twilight is (−) 6°, (−) 12°, or (−) 18° for civil, nautical, or astronomical twilight, respectively. The altitude used for moonrise and moonset is −34′−SD+HP, where SD is semidiameter and HP is horizontal parallax, from the daily pages of the *Nautical Almanac.* The time sight formula can be used for making the computation:

$$\text{hav } t = \sec L \csc p \cos s \sin (s-h),$$

where t=meridian angle, $s=\frac{1}{2}(h+L+p)$, h=altitude, L=latitude, and p=90°−d for L and d (declination) same name and 90°+d for L and d contrary name. Another formula which can be used is

$$\cos t = \sec L \sec d (\sin h - \sin L \sin d),$$

with the same notation as above.

General

2537. Ice.—Several references have been made to ice. The almost constant presence of large quantities of ice is one of the distinctive features of polar regions, and is one of the primary considerations in any operations in these areas. The subject of ice in the sea is covered in chapter XXXVI.

2538. Knowledge of polar regions.—Operations in polar regions are attended by hazards and problems not encountered elsewhere. Lack of knowledge, sometimes accompanied by fear of the unknown, has prevented navigation in these areas from being conducted with the same confidence with which it is pursued in more familiar areas. As experience in high latitudes has increased, much of the mystery surrounding these areas has been dispelled, and operations there have become more predictable.

Before entering polar regions, the navigator will do well to acquaint himself with the experience of those who have preceded him into the areas and under the conditions he anticipates. This information can be found in a growing literature composed of the accounts of explorers, reports of previous operations in high latitudes, articles in professional journals, and several books on operations in polar regions. Some of it is given in various volumes of sailing directions, particularly those for Antarctica (H.O. Pub. No. 27). Additional information is available at the U.S. Navy Hydrographic Office.

The search for knowledge should not be confined to navigation. The wise polar navigator will seek information on living conditions, survival, geography, ice, climate and weather, and operational experience of others who have been to the same area. As elsewhere, knowledge and experience are valuable.

2539. Planning, important in any operation, is vital to the success of polar navigation. The first step to adequate planning is the acquisition of full knowledge, as discussed in article 2538. No item, however trivial, should escape attention. The ship should be provided with all the needed charts, publications, and special navigational material. All available data and information from previous operations in the area should be studied. Key personnel should be adequately instructed in polar navigation prior to departure or while en route to the polar regions. Forecasts on anticipated ice and weather conditions should be obtained before departure and after getting under way. All equipment should be put in top operating condition. All material should be carefully inspected for completeness and condition. The navigator should make certain that all items of equipment are familiar to those who will use them. This is par-

ticularly true of items not generally used at sea, such as charts on an unfamiliar projection, or a bubble sextant. Do not *assume* anything that can be *known*. On the adequacy and thoroughness of the advanced planning and preparation, perhaps more than anything else, will depend the success of polar navigation.

Problems

2510a. Convert the following true directions to grid directions using (1) a convergency of one, (2) a convergency of 0.866. (Give answers to nearest whole degree.)

True	*Latitude*	*Longitude*
157°	N	27° W
353°	N	114° E
118°	S	63° E
042°	S	147° W

Answers.—(1) 184°, 239°, 181°, 255°; (2) 180°, 254°, 173°, 275°.

2510b. Convert the following grid directions to true directions using (1) a convergency of 0.629, (2) a convergency of one.

Grid	*Latitude*	*Longitude*
003°	N	174° W
148°	S	9° W
317°	N	64° E
256°	S	155° E

Answers.—(1) 254°, 154°, 357°, 159°; (2) 189°, 157°, 021°, 101°.

2516a. The radar operator of a ship proceeding through ice measures the following bearings and ranges of an iceberg:

Time	*Bearing*	*Range*
1430	110°	4,000 yds.
1435	121°	3,300 yds.
1440	139°	2,600 yds.
1445	163°	2,300 yds.
1450	188°	2,500 yds.
1455	206°	3,100 yds.

Required.—(1) The course and speed of the ship if the iceberg is stationary.

(2) The course and speed of the ship if the iceberg is moving north at two knots.

Answers.—(1) C 075°, S 6.5 kn.; (2) C 059°, S 7.3 kn.

2516b. A navigator measures off a distance of 300 feet in a fore-and-aft direction along the deck and stations a man at each end of this line. A stop watch is started when a prominent ice feature is opposite the forward man. When the after man reports that the same feature is opposite him, the watch is stopped, and the elapsed time is found to be 34 seconds.

Required.—Speed.

Answer.—S 5.3 kn.

2536a. Find the zone time of sunrise and sunset at lat. 79°20′.0 N, long. 33°00′.0 E, on August 31, 1958.

Answers.—Sunrise, ZT 0119; sunset, ZT 2219.

2536b. Find the zone time of beginning of morning civil twilight, sunrise, sunset, and ending of evening civil twilight at lat. 67°30′.0 N, long. 167°00′.0 W, on May 4, 1958.

Answers.—Morning twilight, ZT 0105; sunrise, ZT 0305; sunset, ZT 2105; evening twilight, ZT 2305.

2536c. Find the zone time of moonrise and moonset at lat. 82°30′.0 N, long. 56°15′.0 W, on June 23, 1958, and the phase of the moon on this date.

Answers.—Moonrise, ZT 0904; moonset, ZT 0315; phase, crescent, about one day before first quarter.

CHAPTER XXVI

LIFEBOAT NAVIGATION

Before Emergency Arises

2601. Introduction.—The methods and techniques used in lifeboat navigation are those available at the time. With full equipment, lifeboat navigation differs little from that aboard ship. More often, however, it is a matter of improvising equipment from available materials, and developing procedures from a knowledge of basic principles. Ingenuity is often essential. The officer who navigates by blindly "following the steps" may be of little more value in a lifeboat devoid of familiar navigational equipment than the man who has never set foot on the bridge of a ship. The wise officer becomes thoroughly familiar with the theory of navigation: the celestial triangle, the circle of equal altitude, and the other basic principles involved. He should be able to identify the most useful stars, and know how to solve his sights by any widely used method, because his favorite method may not be available. He should be able to construct a plotting sheet with a protractor, and use distress signaling equipment. Familiarity with the coordinates (latitude and longitude) of land points in the area of operations, ability to interpret wind and weather signs, knowledge of the ocean currents, and skill in handling a small boat are parts of the practical navigator's basic education which assume their greatest importance in an emergency. For the navigator prepared with such knowledge, and a determination to succeed, the situation is never hopeless. *Some* method of navigation is always available.

2602. Emergency navigation kit.—In time of national emergency, the prudent navigator will provide each lifeboat with a kit containing the equipment which it is practical to carry for emergency navigational purposes (art. 2603). Even in peacetime it is good practice to have one such kit permanently located in the chart house or the wheel house so that it can be quickly transferred to a lifeboat when needed.

The least preparation made should be a check-off list of items to be assembled if time permits, so that nothing will be overlooked. Such a list can be helpful even if one or more emergency kits have been provided. The list should be kept in a prominent place on the bridge or near the lifeboats, perhaps framed under glass. All officers should be familiar with its location and should be acquainted with the location and identity of each item listed.

Junior officers or reliable crew members should be assigned the duty of bringing to their stations, during abandon ship drill, emergency navigational equipment not permanently stowed in the boats. A senior officer should then check each item against the equipment check-off list to ascertain that nothing has been overlooked.

2603. Equipment.—If practicable, full navigational equipment should be provided. As many as possible of the items in the following list should be included. All of these except a timepiece, and possibly a sextant and radio, can be kept in the emergency navigation kit recommended in article 2602.

1. **Notebook** suitable for use as a deck log and for performing computations. Several items of information should be written in this notebook in advance, so as to be available when and if needed. Such items include the latitude and longitude of various places in the area of operation; any desired information on currents and weather;

declination and SHA of several widely scattered stars, with any needed information on identifying them; desired notes and tables from this chapter and elsewhere; any desired general information, such as a list of poisonous fish and those items which may prove useful for survival. This section of the notebook should be brief and the items limited to those most essential in time of emergency.

2. **Charts and other plotting materials.** A pilot chart is most suitable for lifeboat use, both for plotting and as a source of information on variation of the compass, shipping lanes, currents, winds, and weather. Charts for both the summer and winter seasons should be included. During World War II pilot charts were printed on waterproof material suitable for use in a lifeboat. **Plotting sheets** (art. 323) are useful but not essential if charts are available. The plotting sheets should cover the latitudes in which the ship operates. **Universal plotting sheets** (art. 324) may be preferred, particularly if the latitude coverage is large. Several **maneuvering boards,** H.O. 2665–10, (art. 1212) and several sheets of **cross-section paper** (preferably with ten squares per inch) should be included, as these have many uses.

3. **Plotting equipment.** Pencils, erasers, straightedge, protractor, dividers and compasses (not essential, but useful), and a knife or pencil sharpener should be included. Preferably, the straightedge and protractor should be combined in a single device constituting some kind of **plotter** (art. 605). A ruler graduated in inches and fractions may be useful.

4. **Timepiece.** A good watch is needed if longitude is to be determined astronomically. This watch should be waterproof or kept in a waterproof container which permits reading and winding of the watch without exposing it to the elements. The watch should be wound regularly and a record kept of its error and rate of change. Even if one or more such watches are available, the possibility of taking along the chronometers should not be overlooked.

5. **Sextant.** A marine sextant should be taken along if possible. However, since this may be impractical, a lifeboat sextant, or materials for constructing one, should be provided. Several commercially manufactured lifeboat sextants have been made available, particularly during wartime. A lifeboat sextant can be made of wood or other rigid material, two small mirrors, and a pivot. The graduations of the arc should be double those of a compass rose (an angle of 5° should be labeled 10°, etc.). It is not necessary to provide a vernier, or means of adjusting the sextant, since accuracy of 0.°1 is satisfactory for lifeboat use.

6. **Almanac.** A *Nautical Almanac* for the current year is desirable. In an emergency an almanac for another year can be used for stars and the sun without serious error by lifeboat standards, if suitable adjustment is made (art. 2617). Some form of long-term almanac, as that given in appendix X, might well be copied or pasted in the notebook suggested as item 1, above.

7. **Tables.** Some form of table will be needed for reducing celestial observations. The most suitable is one that does not require much space. If a table of trigonometric functions (either logarithmic or natural) is provided, formulas should be included with them. It is not wise to trust the memory for such vital information. A set of tables similar to H.O. Pub. No. 214 can be made at 5° intervals of t, d, and L. Only one page is needed for each latitude entry (5°) if declination is limited to about 30° (sufficient for bodies of the solar system and many stars), entries are given to the nearest 0.°1 for altitudes and 1° for azimuth, and the delta (Δ) values are omitted. **Traverse tables** and others given in this chapter are useful.

8. **Compass.** Each lifeboat is required to carry a magnetic compass. A deviation table for each compass should be made while in port, with magnetic material in its normal place. It would be well to check the accuracy of each table periodically.

9. **Flashlight.** A flashlight is required to be carried in each lifeboat. The batteries should be replaced from time to time, as necessary. Extra batteries and bulbs might well be carried.

10. **Portable radio.** If a portable radio is available, be sure it is included. Whether this is one of the transmitting-receiving sets approved by the Federal Communications Commission for lifeboat use, or merely a small receiver of limited range owned by a crew member, do not overlook it, as it may be used as a radio direction finder.

2604. Position of ship.—A knowledge of the position of the vessel at the time it is abandoned is of great importance. The officer on watch on the bridge should never permit himself to become careless in the matter of keeping a mental note of the approximate position of the vessel. During wartime, or whenever the possibility of abandoning ship might reasonably be anticipated, the radio operator should be provided with a list of advance dead reckoning positions.

Abandoning Ship

2605. Before lowering boats.—The period between the decision to abandon ship and the actual leaving of the vessel is a highly important one. It is also a period of mental strain and possible confusion. The degree to which the crew can be prepared for the ordeal ahead depends upon the amount of time available and the thoroughness of the preparation that has been made. If there has been advance warning of the possibility of the decision, certain preparations can be made before the decision is reached. If time permits, after the decision to abandon ship has been made, the radio operator should send a final distress message, giving the ship's position and any other pertinent information. It will be important later to know whether an acknowledgment of receipt of the message was received. Any available time can be wisely used to check the navigational equipment in each boat and assemble missing items. There may be time to make a last minute check of position of the ship, position of any nearby land, set and drift of current, present and forecast weather, watch error, and date. These items should be written down. Perhaps the chart can be taken along. Equipment should be properly secured before lowering the boats. In a rough sea it may be desirable to lower the sextant, chronometer, and radio into the boat after it is afloat.

2606. Establishing command.—The identity of the person in command of each boat, and the over-all commander, should be firmly established. Almost invariably this will be the senior officer present. In a lifeboat, perhaps more than in any other circumstances, strong leadership is required if the confidence of the crew is to be maintained. The officer whom the crew respects as a *man*, admires as a *seaman*, and recognizes as a *gentleman* will have little or no trouble with discipline and cooperation of all on board.

Morale is a prime consideration, and it grows in importance with the passage of time. The person in command should be recognized as the final authority in all matters, but it is important that he give to each person an opportunity to be heard, and that he keep all hands fully informed of the bad as well as of the good. Decisions will be more acceptable if the crew has been informed of each consideration as it arises, and so has been somewhat prepared. Complete fairness and impartiality are essential.

2607. Estimate of the situation.—Perhaps the first item which should engage the attention of the person in command, after the lifeboat has cleared the stricken vessel, is the questioning of each person aboard to collect all the useful information available. It is well to determine what is known regarding the position of the ship, ocean currents, weather, astronomy, navigation, seamanship, sailing, etc. Find out who owns watches and what each owner knows about the error and rate of his watch. Establish a routine for winding and comparing them. No useful skill or knowledge should be overlooked;

all should be fully considered in making the important decision of whether to remain in the vicinity of the disaster in the hope of rescue, or to attempt to reach land or a more heavily traveled shipping lane.

This decision of whether to stay or leave may be the most important one of the entire experience. Until comparatively recent times there was no problem. Because there was virtually no hope of assistance, the lifeboat crew had to rely upon itself. Since the development of modern communication and rescue facilities, however, it is often wiser to remain than to complicate the rescue problem by increasing the area to be searched.

The decision should not be made until careful consideration has been given to all factors, nor should it be delayed longer than necessary. Considerations vary with the circumstances, but certainly the following should be included:

Was a distress message sent before the ship was abandoned? Did it include the position of the ship? How accurate was the position? Is there any reasonable doubt that the message was received? If no message was sent, how soon will the ship be missed? What rescue facilities are available? How far away are they and how long will it be before help arrives? How conspicuous is the lifeboat? What facilities are available for attracting attention, either visually or by radar? How proficient is the crew in using such equipment? Is a radio transmitter available? What is the probable running time to the nearest land in several directions, considering the prevailing winds and currents, the motive power available, and the ability of the crew to use it? How long will the fresh water and rations last, and will they be sufficient to sustain the crew in the physical exertion required?

If the decision is to stay, how will the crew occupy its time, remembering the increased morale problem with an idle crew? How will position be maintained, or regained if the boat drifts? Would it be practical to wait two or three days, perhaps, in the hope of rescue, and then to set out for land if help does not come?

If the decision is to leave, where should the boat head? How soon can a well-traveled shipping lane be reached? In time of war, where is the enemy and where are friends? How large and conspicuous is the land in each direction, considering the low height of eye in a lifeboat? It may be better to head for conspicuous land 500 miles away than for a small, low island 200 miles away, particularly if the latter is in a direction of unfavorable winds or currents, or takes the boat farther away from shipping lanes.

Avoid, if possible, a hasty decision that will later be recognized as unwise. Discuss the matter thoroughly with the crew, and when the decision is made, inform them of the reason for it. Do this in a manner that will invite their confidence and support. Inform them of the best estimate of the situation.

2608. Selecting the route.—It is not always desirable to head directly for the objective. A longer route with favorable winds and currents may be quicker. A longer route by way of shipping lanes may enhance the possibility of rescue.

With clear skies, latitude can be found with relatively crude equipment. But unless accurate Greenwich time is available, longitude cannot be found astronomically, even with the best equipment; nor is a nonastronomical method likely to be available. In the absence of reliable longitude information, it is better to head for a point at the latitude of the destination but so far east or west of it that no reasonable doubt will exist as to the direction of land when that latitude is reached. The distance of the point from the destination depends upon the degree of uncertainty of the longitude, remembering that this uncertainty is likely to increase with time. This method of "parallel sailing" was used for centuries before a method of determining or "discovering" longitude at sea was developed.

If the objective has a considerable extent in a north-south direction, the need for a final east-west leg is less critical, and in attempting to reach a continent or very large island, one need not consider it at all. In the absence of better information, an east or west course should be selected from the outset, since most large land masses of the earth are oriented in a general north-south direction.

2609. Keeping boats together.—If more than one boat is launched, every effort should be made to keep them together. While the person in charge of each boat is responsible for decisions regarding his boat, considerable advantage is to be gained by keeping the boats together and recognizing one person, logically the senior officer present, as the over-all commander. Since navigational equipment and skill probably will differ widely from boat to boat, the benefits of any accurate navigation can be shared by all if the boats are close together. Other knowledge can be exchanged, equipment shared, and rations distributed equitably. It may be wise to shift some personnel among the boats, perhaps on a periodic basis, either to effect a better balance of skill and knowledge, or for morale purposes.

2610. Lookout.—Always there is the possibility of sighting another vessel. Hence, a lookout should be posted at all times. This becomes of even greater importance when approaching land, or if the location of all land along the route is not known. If it is possible to rig a metal object high in the boat, this should be done to enhance the possibility of detection by radar.

Dead Reckoning

2611. Importance of dead reckoning.—Of the various kinds of navigation, dead reckoning alone is always available in some form. It should never be neglected, but in a lifeboat it is of more than average importance. A close check should be kept on the direction and distance made good, and all disturbing elements such as wind and current should be carefully evaluated. Long voyages have been successfully completed by this method alone, and landfalls have been made with surprising accuracy. This is not meant to minimize the importance of other methods of determining position, but with the methods generally available in a lifeboat, one may well find that, during the first few days, his dead reckoning positions are more accurate than those determined by other methods. If the means of determining direction and distance—the elements of dead reckoning—are accurate, it might be well to make an adjustment to the dead reckoning only after consistent indication of the magnitude and direction of its error. The dropping of the dead reckoning at each uncertain "fix" is at best a questionable procedure. The conflicting information likely to be available calls for careful analysis and good judgment on the part of the navigator.

2612. Deck log.—From the beginning a careful log should be kept. The date and time of abandoning ship should be the first entry, followed by navigational information available, and the various important decisions and the reasons for them. Since the conservation of paper may be important, record only the essentials of the important items, but do not overlook the recording in considerable detail of the selection of a commanding officer, changes in command, deaths, missing persons, and navigational information.

The best determination of the position of abandoning ship should be recorded, followed by a full account of courses, distances, positions, winds, currents, and leeway. No important navigational information should be left to memory if it can be recorded.

2613. Direction.—As one of the elements of dead reckoning, direction is an important item. As indicated in article 2603, a deviation table for each lifeboat compass should be determined in port, and checked periodically. At the first convenient oppor-

tunity after abandoning ship the accuracy should be checked on the course to be followed.

If an almanac, accurate Greenwich time, and the necessary tables are available, the azimuth of any celestial body can be computed and this value compared with the azimuth as measured by the compass. If it is difficult to observe the compass azimuth, select a body dead ahead and note the compass heading. The difference between computed and observed azimuths is compass error. This is of more immediate value than deviation, but if the latter is desired, it can be determined by applying to the compass error the variation, from the pilot chart.

Several unique astronomical situations occur, permitting determination of azimuth without computation:

Polaris is always within 2° of true north for observers between the equator and latitude 60° N. When this star is directly above or below the celestial pole, its azimuth is exactly north at any latitude. This occurs approximately when the *trailing* star of either *Cassiopeia* (ϵ *Cassiopeiae*) or the big dipper (Alkaid) is directly above or directly below Polaris (fig. 2621). When a line through the trailing stars and Polaris is horizontal, the maximum correction should be applied. Below latitude 50° this can be considered 1°; and between 50° and 65°, 2°. If *Cassiopeia* is to the *right* of Polaris, the azimuth is 001° (or 002°), and if to the left, 359° (or 358°). The *south* celestial pole is located approximately at the intersection of a line through the longer axis of the southern cross with a line from the northernmost star of *Triangulum Australe* perpendicular to the line joining the other two stars of the triangle. No conspicuous star marks this spot (figs. 2205–2208).

Meridian transit. Any celestial body bears due north or south at meridian transit, either upper or lower. This is the moment of maximum (or minimum) altitude of the body. However, since the altitude at this time is nearly constant during a considerable change of azimuth, the instant of meridian transit may be difficult to determine. If time and an almanac are available, and the longitude is known, the time of transit can be computed.

Body on prime vertical. If any method is available for determining when a body is on the prime vertical (due east or west), the compass azimuth at this time can be observed. Table 25 provides this information. Any body on the celestial equator (declination 0°) is on the prime vertical at the time of rising or setting. For the sun this occurs at the time of the equinoxes (art. 1419). The star Mintaka (δ *Orionis*), the *leading* star of *Orion's* belt, has a declination of approximately 0°.3 S and can be considered on the celestial equator. For an observer near the equator, such a body is always nearly east or west. Because of refraction and dip, the azimuth should be noted when the center of the sun or a star is a little more than one sun diameter (half a degree) *above* the horizon. The moon should be observed when its *upper* limb is *on* the horizon.

Body at rising or setting. Except for the moon, the azimuth angle (art. 1428) of a body is almost the same at rising as at setting, except that the former is toward the *east* and the latter toward the *west.* If the azimuth is measured *both* at rising and setting, true south (or north) is midway between the two observed values, and the difference between this value and 180° (or 000°) is the compass error. Thus, if the compass azimuth of a body is 073° at rising, and 277° at setting, true south (180°) is at $\frac{073°+277°}{2}=175°$ by compass, and the compass error is 5° E. This method may be in error if the boat is moving rapidly in a north or south direction. If the declination and latitude are known, the true azimuth of any body at rising or setting can be deter-

mined by means of a diagram on the plane of the celestial meridian (art. 1432) or by computation (art. 2125). For this purpose the body (except the moon) should be considered as rising or setting when its center is a little more than one sun diameter (half a degree) above the horizon, because of refraction and dip.

The direction of the sun in relation to the hands of a watch is sometimes advocated, but the limitations of this method are too great to permit general application.

A simple nonastronomical method can be used for determining the *deviation*. An object that will float but not drift rapidly before the wind is thrown overboard. The boat is then steered as steadily as possible in the *opposite* direction to that desired. At a distance of perhaps half a mile, or more if the floating object is still clearly in view, the boat is turned around in the smallest practicable radius, and headed back toward the floating object. The *magnetic* course is midway between the course toward the object and the *reciprocal* of the course away from the object. Thus, if the boat is on compass course 151° while heading away from the object, and 337° while returning, the magnetic course is midway between 337° and $151°+180°=331°$, or $\frac{337°+331°}{2}=334°$. Since 334° magnetic is the same as 337° by compass, the deviation on this heading is 3° W.

If a compass is not available, any celestial body can be used to steer by, if its diurnal apparent motion is considered. A reasonably straight course can be steered by noting the direction of the wind, the movement of the clouds, the direction of the waves, or by watching the wake of the boat. A line can be secured to the side of the boat at a point amidships or forward. The line should tend parallel to the center line of the boat if on a straight course. The angle between the center line and the wake is an indication of the amount of leeway. The accuracy of the towed-object or wake method is affected adversely by a cross sea.

A body having a declination the same as the latitude of the destination is over the destination once each day, at the time when its hour angle is the same as the longitude, measured westward through 360°. At this time it should be dead ahead if the boat is following the great circle leading directly through the destination.

2614. Motive power.—A lifeboat is equipped with one or more of the following means of locomotion: oars, hand-operated propeller, motor, sail. Of these, only sail offers a practical means of travel over an extended period of time. Men living in an open boat, perhaps on reduced rations, should not attempt to expend their strength on hand locomotion, except for short periods. Likewise, the comparatively small fuel supply in a motorboat should be hoarded jealously. It may be desperately needed later, as for landing through a surf, preventing the boat from drifting onto a rocky coast, or making the land when a strong current is carrying the boat past an island.

A sail should be rigged, for in it lies the best hope of reaching distant land. If the standard lifeboat sail is not available, a substitute can usually be devised, using the boat cover, or even clothing, and oars.

2615. Distance can be determined directly between accurate fixes, but generally it is found by means of speed and elapsed time. A loaded lifeboat will not travel fast, under normal conditions. With fair wind and weather it may make good a speed of about two knots through the water. Hence the importance of wind and current. The navigator used to observing the sea from a high bridge usually overestimates his speed in a lifeboat, where he is only a few feet from the water. With practice, his ability should improve.

Speed may be determined by using a form of **chip log.** Attach a long line to a heavy, floating object. Put one knot in the line twelve or fifteen fathoms from the

object, and another just ten fathoms (or any convenient distance) from the first. Stream the device over the side and let the line run out freely, noting the elapsed time between passage of the two knots through the hand. A variation of this is the **Dutchman's log.** A floating object is thrown overboard at the bow, and the elapsed time required for a known length along the centerline to pass it is noted. If a line is attached to the object, it may be used many times. With either variation, it is well to tie the bitter end of the line to the boat, to minimize danger of losing the whole device overboard.

With either the chip or Dutchman's log, the speed is determined by the formula:

$$S=\frac{60 \text{ seconds per minute} \times 60 \text{ minutes per hour} \times \text{feet between marks}}{6{,}000 \text{ feet per mile} \times \text{ seconds of elapsed time}}.$$

This is equal to:

$$S=\frac{3{,}600 \times \text{feet between marks}}{6{,}000 \times \text{seconds of elapsed time}}=\frac{0.6 \times \text{feet between marks}}{\text{seconds of elapsed time}}.$$

Since the feet between marks is constant, a convenient number can be selected. Thus, if the length is 16⅔ feet, the formula becomes

$$S=\frac{10}{\text{seconds of elapsed time}}.$$

If the elapsed time is ten seconds, the boat is traveling at one knot; if five seconds, at two knots; if eight seconds, at 1¼ knots, etc.

If a watch is not available, a simple pendulum may be devised to time the interval. A piece of string with a weight attached, of a length of 9.8 inches (to the center of gravity of the weight), will, when suspended, make a complete swing (back *and* forth) once every second. For a pendulum 39.1 inches long the period is two seconds. With practice, time can be estimated with fair accuracy.

It is not always possible to head directly along the course to the destination, because of adverse winds. It is better to make good progress in the general direction desired than none at all, and much better on morale. However, at times conditions may be so adverse that it will be best to drop sail until the wind shifts or abates. At such a time a sea anchor should be streamed to minimize loss of precious mileage, and, in severe conditions, to keep the boat headed into the sea.

2616. Position by dead reckoning.—Plotting can be done directly on a pilot chart or plotting sheet. If this proves too difficult, or if an independent check is desired, some form of mathematical reckoning may be useful. Table 2616, a simplified traverse table, can be used for this purpose. This is a critical-type table, various factors being given for limiting values of certain angles. To find the difference or change of latitude, in minutes, enter the table with course angle, reckoned from north or south toward the east or west. Multiply the distance run, in miles, by the factor. To find the departure, in miles, enter the table with the *complement* of the course angle. Multiply the distance run, in miles, by the factor. To convert departure to difference of longitude, in minutes, enter the table with mid latitude. Divide the departure by the factor.

Angle	Factor
°	
0	
	1. 0
18	
	0. 9
31	
	0. 8
41	
	0. 7
49	
	0. 6
56	
	0. 5
63	
	0. 4
69	
	0. 3
75	
	0. 2
81	
	0. 1
87	
	0. 0
90	

TABLE 2616.—Simplified traverse table.

Example.—A lifeboat travels 26 miles on course 205°, from L 41°44′ N, λ 56°21′ W.

Required.—Latitude and longitude of the point of arrival.

Solution.—The course angle is $205°-180°=S\,25°\,W$, and the complement is $90°-25°=65°$. The factors corresponding to these angles are 0.9 and 0.4, respectively. The difference of latitude is $26\times0.9=23'$ (to the nearest minute) and the departure is $26\times0.4=10$ mi. Since the course is in the southwestern quadrant, in the northern hemisphere, the latitude of the point of arrival is $41°44'\,N-23'=41°21'\,N$. The factor corresponding to the mid latitude $41°32'\,N$ is 0.7. The difference of longitude is $10\div0.7=14'$. The longitude of the point of arrival is $56°21'\,W+14'=56°35'\,W$.

Answer.—L 41°21′ N, λ 56°35′ W.

Celestial Navigation

2617. Celestial coordinates.—Almanac information, particularly declination and Greenwich hour angle of bodies, is important to celestial navigation. If the current *Nautical Almanac* is available, there is no problem. If the only copy available is for a previous year, it can be used for the sun, Aries, and stars without serious error, by lifeboat standards. However, for greater accuracy, proceed as follows: For declination of the sun, enter the almanac with a time that is *earlier* than the correct time by 5^h49^m times the number of years between the date of the almanac and the correct date, adding 24^h for each February 29 that occurs between the dates. If the date is February 29, use March 1 and reduce by one the number of 24^h periods added. For GHA of the sun or Aries determine the value for the correct time, adjusting the minutes and tenths of arc to agree with that at the time for which the declination is determined. Since the adjustment never exceeds half a degree, care should be used when the value is near a whole degree, to prevent the value from being in error by 1°. Appendix X is a long-term almanac giving values of GHA ♈, and GHA and declination of the sun. Instructions for its use are included in the appendix. A reproduction of this almanac might profitably be included in the navigational kit mentioned in article 2602.

If no almanac is available, a rough approximation of the declination of the sun can be obtained as follows: Count the days from the given date to the *nearer* solstice (June 21 or December 22). Divide this by the number of days from that solstice to the equinox (March 21 or September 23), using the equinox that will result in the given date being between it and the solstice. Multiply the result by 90°. Enter table 2616 with the angle so found, and extract the factor. Multiply this by 23°.45 to find the declination.

Example 1.—The date is August 24.

Required.—The approximate declination of the sun.

Solution.—The number of days from the given date to the nearer solstice (June 21) is 64. There are 94 days between June 21 and September 23. Dividing and multiplying by 90°,

$$\frac{64}{94}\times90°=61.^\circ3.$$

The factor from table 2616 is 0.5. The declination is $23.^\circ45\times0.5=11.^\circ7$. It is known to be north because of the date.

Answer.—Dec. 11°.7 N.

The accuracy of this solution can be improved by considering the factor of table 2616 as the value for the mid angle beween the two limiting ones (except that 1.00 is correct for 0° and 0.00 is correct for 90°), and interpolating to one additional decimal. In this instance the interpolation would be between 0.50 at 59°.5 and 0.40 at 66°. The interpolated value is 0.47, giving a declination of 11°.0 N. Still greater accuracy can be obtained by using a table of natural cosines instead of table 2616. By natural cosine the value is 11°.3 N.

If the latitude is known, the declination of any body can be determined by observing a meridian altitude. In a lifeboat it is usually best to make a number of observations shortly before and after transit, plot the values on cross-section paper, letting the ordinate (vertical scale) represent altitude, and the abscissa (horizontal scale) the time. The altitude is found by fairing a curve or drawing an arc of a circle through the points, and taking the highest value. A meridian altitude problem is then solved in reverse.

Example 2.—The latitude of a lifeboat is 40°16′S. The sun is observed on the meridian, bearing north. The *observed* altitude is 36°29′.

Required.—Declination of the sun.

Solution.—The zenith distance is $90° - 36°29' = 53°31'$. The sun is 53°31′ north of the observer, or 13°15′ north of the equator. Hence, the declination is 13°15′N.

Answer.—Dec. 13°15′N.

The GHA♈ can be determined approximately by considering it equal to GMT (in angular units) on September 23. To find GHA♈ on any other date, add 1° for each day following September 23. The value is approximately 90° on December 22, 180° on March 21, and 270° on June 21. The values so found can be in error by as much as several *degrees*, and so should not be used if better information is available. An approximate check is provided by the great circle through Polaris, Caph (the leading star of *Cassiopeia*), and the eastern side of the square of *Pegasus*. When this great circle coincides with the meridian, LHA♈ is approximately 0°. The hour angle of a body is equal to its SHA plus the hour angle of Aries.

If an error of as much as 4°, or a little more, is acceptable, the GHA of the sun can be considered equal to GMT ± 180° (12^h). For more accurate results, one can make a table of the equation of time from the *Nautical Almanac* perhaps at five- or ten-day intervals, and include this in the emergency navigation kit mentioned in article 2602. The equation of time is applied according to its sign to GMT ± 180° to find GHA.

2618. Altitude measurement.—If a sextant is available, either one from the pilot house or an emergency-type instrument, altitudes are measured in the usual manner. The sextant should be shielded as much as possible from wind and spray. If the sea is rough, the observer should brace himself against the mast and make his observation when on the crest of a wave, when the horizon is least likely to be obscured by nearby waves. It is usually good practice to make a number of observations and average both the altitudes and times, or plot on cross-section paper the altitudes versus time, using any convenient time and the corresponding altitude for solving the observation.

The improvisations which may be made in the absence of a sextant are so varied that in virtually any circumstances the application of a little ingenuity and some effort will produce a device for measuring altitude. The results obtained with any improvised method will be approximate at best, but if a number of observations are averaged, the accuracy should be improved. Almost always a measurement, however approximate, is better than an estimate. Two general classes of improvisation are available:

1. *By circle.* Any circular scale, such as a maneuvering board (H.O. 2665–10), compass rose, protractor, or plotter can be used to measure altitude or zenith distance directly. This is the principle of the ancient astrolabe (art. 124). A maneuvering board or compass rose is usually handled best by mounting it on a flat board. A protractor or plotter may be so mounted or used directly. There are a number of variations of the technique of using such a device. Some of them are:

A peg or nail is placed at the center of the circle and perpendicular to it. A weight is hung from the 90° graduation, and a string for holding the device is attached at the 270° graduation. When it is held with the weight acting as a plumb bob, the 0°–180°

line is horizontal (fig. 2618a). In this position the board is turned in azimuth until it is in line with the sun. The intersection of the shadow of the center peg with the arc of the circle indicates the altitude of the center of the sun.

The weight and loop can be omitted and pegs placed at the 0° and 180° points of the circle. While one observer sights along the line of pegs to the horizon, an assistant notes the altitude.

The weight can be attached to the center pin, and the three pins (0°, center, 180°) aligned with the celestial body. The reading is made at the point where the string holding the weight crosses the scale. The reading thus obtained is the zenith distance unless the graduations are labeled to indicate altitude. This method, illustrated in figure 2618b, is used for bodies other than the sun.

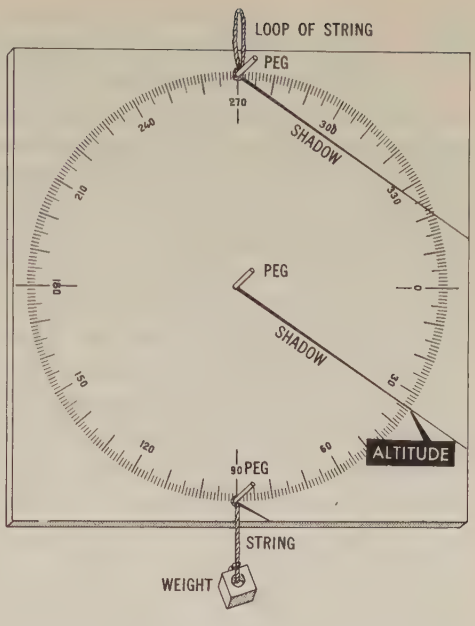


Figure 2618a.—Improvised astrolabe; shadow method. Pegs and board shown tilted for clarity.

Whatever the technique, it is good practice to reverse the device for half the readings of a series, to minimize errors of construction. Generally, the circle method produces more accurate results than the right triangle method, described below.

2. *By right triangle.* The principle of the ancient cross-staff can be used to establish one or more right triangles, which can be solved by measurement of the angle representing the altitude, either directly or by reconstructing the triangle. Another way of determining the altitude is to measure two of the sides of the triangle and divide one by the other to determine one of the trigonometric functions. This procedure, of course, requires a source of information on the values of trigonometric functions corresponding to various angles. If the cosine is found, table 2616 can be used. The tabulated factors can be considered correct to one additional decimal for the value midway between the limiting values (except that 1.00 is the correct value for 0° and 0.00 is the correct value for 90°) without serious error by lifeboat standards. Interpolation can then be made between such values. By either protractor or

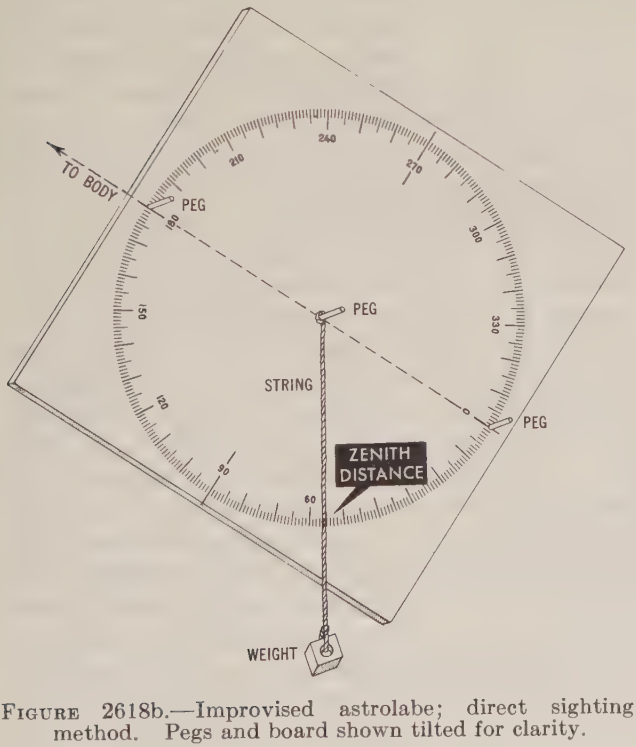


Figure 2618b.—Improvised astrolabe; direct sighting method. Pegs and board shown tilted for clarity.

table, most devices can be graduated in advance so that angles can be read directly. There are many variations of the right triangle method. Some of these are:

Two straight pieces of wood can be attached to each other in such a way that the shorter one can be moved along the longer, the two always being perpendicular to each other. The shorter piece is attached at its center. One end of the longer arm is held to the eye. The shorter arm is moved until its top edge is in line with the celestial body, and its bottom edge is in line with the horizon. Thus, two right triangles are used (the third sides being the slant distances between the ends of the arms) each representing half the altitude (fig. 2618c). For low altitudes, only one of the triangles is used, the long arm being held in line with the horizon. The length of half the short arm, divided by the length of that part of the long arm between the eye and the intersection with the short arm, is the tangent of half the altitude (the whole altitude if only one right triangle is used). The cosine can be found by dividing that part of the long arm between the eye and the intersection with the short arm by the slant distance from the eye to one end of the short arm. Graduations consist of a series of marks along the long arm indicating settings for various angles. The device should be inverted for alternate readings of a series.

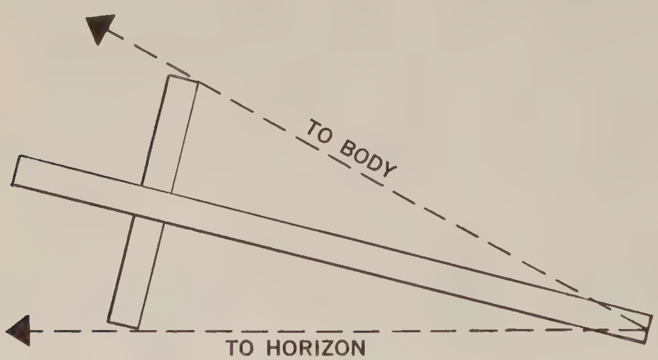


FIGURE 2618c.—Improvised cross-staff.

A rule or any stick can be held at arm's length. The top of the rule is placed in line with the celestial body being observed, and the top of the thumb is placed in line with the horizon. The rule is held vertical. The length of rule above the thumb, divided by the distance from the eye to the top of the thumb is the tangent of the angle observed. The cosine can be found by dividing the distance from the eye to the top of the thumb by the distance from the eye to the top of the rule. If the rule is tilted toward the eye until the minimum of rule is used, the distance from the eye to the middle of the rule is substituted for the distance from the eye to the top of the thumb, *half* the length of the rule above the thumb is used, and the angle found is multiplied by two. Graduations consist of marks on the rule or stick indicating various altitudes. For the average observer each inch of rule will subtend an angle of about 2°3, assuming an eye-to-ruler distance of 25 inches. This relationship is good to a maximum altitude of about 20°. The accuracy of this relationship for a specific observer can be checked by comparing the measurement against known angles in the sky. Angular distances between stars can be computed by sight reduction methods, including H.O. Pub. No. 214, by using the declination of one star as the latitude of the assumed position, and the difference between the hour angles (or SHA's) of the two bodies as the meridian angle. The angular distance is the complement of the computed altitude. The angular distances between some well-known star pairs are: end stars of *Orion's* belt, 2°7; pointers of the big dipper, 5°4; Rigel to *Orion's* belt, 9°0; eastern side of the great square of *Pegasus*, 14°0; Dubhe (the pointer nearer Polaris) and Mizar (the second star in the big dipper, counting from the end of the handle), 19°3.

The angle between the lines of sight from each eye is, at arm's length, about 6°. By holding a pencil or finger horizontal, and placing the head on its side, one can estimate an angle of about 6° by closing first one eye and then the other, and noting how much the pencil or finger appears to move in the sky.

The length of the shadow of a peg or nail mounted perpendicular to a horizontal board can be used as one side of an altitude triangle. The other sides are the height of the peg and the slant distance from the top of the peg to the end of the shadow. The height of the peg, divided by the length of the shadow, is the tangent of the altitude of the center of the sun. The length of the shadow divided by the slant distance is the cosine. Graduations consist of a series of concentric circles indicating various altitudes, the peg being at the common center. The device is kept horizontal by floating it in a bucket of water. Half the readings of a series are taken with the board turned 180° in azimuth.

Two pegs or nails can be mounted perpendicular to a board, with a weight hung from the one farther from the eye. The board is held perpendicular and the two pegs aligned with the body being observed. The finger is then placed over the string holding the weight, to keep it in position as the board is turned on its side. A perpendicular is dropped from the peg nearer the eye, to the string. The altitude is the acute angle nearer the eye. For alternate readings of a series, the board should be inverted. Graduations consist of a series of marks indicating the position of the string at various altitudes.

As the altitude decreases, the triangle becomes smaller. At the celestial horizon it becomes a straight line. No instrument is needed to measure the altitude when either the upper or lower limb is tangent to the horizon, as the "sextant" altitude is then 0°.

2619. Sextant altitude corrections.—If altitudes are measured by a marine sextant, the usual sextant altitude corrections apply (ch. XVI). If the center of the sun or moon is observed, either by sighting at the center or by shadow, the lower-limb corrections should be applied, as usual, and an additional correction of (−)16′ applied. If the upper limb is observed, use (−) 32′. If a weight is used as a plumb bob, or if the *length* of a shadow is measured, omit the dip (height of eye) correction.

If the almanac is not available for making corrections, each source of error can be corrected separately, as follows:

Index correction. If a sextant is used, the index correction should be determined and applied to all observations, or the sextant adjusted to eliminate index error.

Alt.	Refr.
°	′
5	
	9
6	
	8
7	
	7
8	
	6
10	
	5
12	
	4
15	
	3
21	
	2
33	
	1
63	
	0
90	

TABLE 2619.—Refraction.

Refraction is given to the nearest minute of arc in table 2619. The value for a horizon observation is 34′. If the nearest 0°1 is sufficiently accurate, as with an improvised method of observing altitude, a correction of 0°1 should be applied for altitudes between 5° and 18°, and no correction applied for greater altitudes. Refraction applies to all observations, and is always a minus (−) correction.

Dip, in minutes of arc, is approximately equal to the square root of the height of eye, in feet. The correction applies to all observations in which the horizon is used as the horizontal reference. It is always a minus (−) correction. If 0°1 accuracy is used, no dip correction is needed for lifeboat heights of eye.

Semidiameter. The semidiameter of either the sun or moon does not differ greatly from 16′. The correction does not apply to other bodies or to observations of the center of the sun and moon, by whatever method, including shadow. The correction is plus (+) if the lower limb is observed, and minus (−) if the upper limb is observed.

Parallax. For lifeboat accuracy, parallax is applied to observations of the moon only. An approximate value, in minutes of arc, can be found by multiplying 57′ by the factor from table 2616, entering that table with altitude. For more accurate results the factors can be considered correct to one additional decimal for the altitude

midway between the limiting values (except that 1.00 is correct for 0° and 0.00 is correct for 90°), and the values for other altitudes can be found by interpolation. This correction is always plus (+).

For observations of celestial bodies on the horizon, the total correction for zero height of eye is:

Sun. Lower limb: (−) 18′, upper limb: (−) 50′.
Moon. Lower limb: (+) 39′, upper limb: (+) 7′.
Planet or star. (−) 34′.

Dip should be added algebraically to these values.

Since the "sextant" altitude is zero, the "observed" altitude is equal to the total correction.

2620. Sight reduction.—If any tables designed for sight reduction, such as H.O. Pub. No. 214, are available, they should be safeguarded to prevent loss or damage. If trigonometric tables and the necessary formulas are available, they will serve the purpose. Speed in solution is seldom a factor in a lifeboat. A slow method might actually be an asset, from a morale standpoint, as it will provide occupation for a limited time for at least one crew member. If tables but no formulas are available, carefully determine the mathematical knowledge possessed by the crew. Someone may be able to provide the missing information. If the formulas are available, but no tables, approximate natural values of the various trigonometric functions can be obtained graphically by the method explained in article O39. Graphical solution of the navigational triangle can be made by the orthographic method explained in article 1432. A maneuvering board (H.O. 2665–10) might prove helpful in the graphical solution for either trigonometric functions or altitude and azimuth. Very careful work will be needed for useful results by either method.

Unless full navigational equipment is available, better results might be obtained by making separate determinations of latitude and longitude.

2621. Latitude determination.—Several methods are available for determining latitude, and in none of them is accurate time needed.

Meridian altitude. Latitude can be determined by means of a meridian altitude of any body, if its declination is known. The method is explained in article 2103. If accurate time, knowledge of the longitude, and an almanac are available, the observation can be made at the correct moment, as determined in advance. However, if any of these is lacking, or if an *accurate* altitude-measuring instrument is unavailable, better procedure is to make a number of altitude observations before and after meridian transit. A plot is then made of altitude versus time, if cross-section paper is available, and the highest (or lowest, for lower transit) altitude is scaled from a curve faired through the plotted points. At lifeboat speeds this procedure is not likely to introduce a significant error. The time used for plotting the observations need not be accurate, as *elapsed* time between observations is all that is needed, and this is not of critical accuracy. Thus, even a watch that has run down and then been rewound can be used without resetting. Any altitudes that are not consistent with others of the series should be discarded.

Polaris. Latitude by Polaris is explained in article 2105. In a lifeboat, only the first correction is of practical significance. If suitable tables are not available, this correction can be estimated. The trailing star of *Cassiopeia* (ϵ *Cassiopeiae*) and Polaris have almost exactly the same SHA. The trailing star of the big dipper (Alkaid) is nearly opposite Polaris and ϵ *Cassiopeiae*. These three stars, ϵ *Cassiopeiae*, Polaris, and Alkaid, form a line through the pole (approximately). When this line is horizontal, there is no correction. When it is vertical, the maximum correction of 56′ applies.

It should be added to the observed altitude if Alkaid is at the top, and subtracted if ϵ *Cassiopeiae* is at the top. For any other position, estimate the angle this line makes with the vertical (fig. 2621), and multiply the maximum correction (56′) by the factor from table 2616, adding if Alkaid is higher than ϵ *Cassiopeiae*, and subtracting if it is lower. For more accurate results, the factor from table 2616 can be considered accurate to one additional decimal for the mid value between those tabulated (except that 1.00 is correct for 0° and 0.00 for 90°). Other values can be found by interpolation.

Length of day. The length of the day varies with latitude. Hence, latitude can be determined if the elapsed time between sunrise and sunset can be observed. Correct the observed length of day by *adding* 1^m for each 15′ of longitude traveled toward the east and *subtracting* 1^m for each 15′ of longitude traveled toward the west. The latitude determined by length of day is the value for the time of meridian transit. Since meridian transit occurs approximately midway between sunrise and sunset, half the interval may be observed and doubled. If a sunrise and sunset table is not available, the length of daylight can be determined graphically by means of a diagram on the plane of the celestial meridian (art. 1432). A maneuvering board (H.O. 2665–10) is useful for this purpose. This method cannot be used near the time of the equinoxes, and is of little value near the equator. The moon can be used if moonrise and moonset tables are available, but with the moon the half-interval method is of insufficient accuracy, and allowance should be made for the longitude correction.

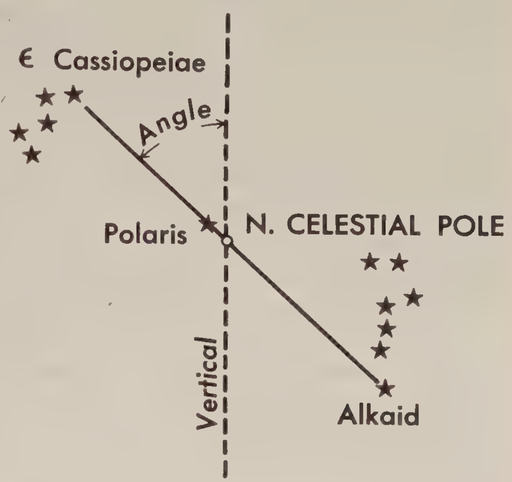


FIGURE 2621.—Relative positions of ϵ *Cassiopeiae*, Polaris, and Alkaid with respect to the north celestial pole.

Body in zenith. The declination of a body in the zenith is equal to the latitude of the observer. If no means are available for measuring the altitude, the position of the zenith may possibly be estimated in a calm sea by lying in the lifeboat and looking skyward. The accuracy of the results depends upon the ability to estimate the position of the zenith. Use of a plumb bob may help.

Variation of the compass can occasionally be used for determining latitude, as explained in article 2622.

2622. Longitude determination.—Unlike latitude, longitude requires accurate Greenwich time for its determination by astronomical means. All such methods consist of noting the Greenwich time at which a phenomenon occurs locally. In addition, a table indicating the time of occurrence of the same phenomenon at Greenwich, or equivalent information, is needed.

Time of transit. When a body is on the local celestial meridian, its GHA is the same as the longitude of the observer if in west longitude, or $360° - \lambda$ in east longitude. Thus, if the GMT of local transit is determined and a table of Greenwich hour angles (or time of transit of the Greenwich meridian) is available, longitude can be computed. If only the equation of time is available, the method can be used with the sun. This is the reverse of the problem of finding the time of transit of a body (art. 2104). The time of transit is not always apparent. If a curve is made of altitude versus time, as suggested in article 2621, the time corresponding to the highest altitude is used in the determination of longitude. Under some conditions it may be preferable to observe

an altitude before meridian transit and then again after meridian transit, when the body has returned to the same altitude as at the first observation. Meridian transit occurs midway between these two times. A body in the zenith is on the celestial meridian. If accurate azimuth measurement is available, note the time when the azimuth is 000° or 180°.

Sunrise and sunset. The difference between the observed GMT of sunrise or sunset and the LMT tabulated in the almanac is the longitude in time units, which can then be converted to angular measure. If the *Nautical Almanac* is used, this information is tabulated for each third day only. Greater accuracy can be obtained if interpolation is used for determining intermediate values. Moonrise or moonset can be used if the tabulated LMT is corrected for longitude (art. 1812). Planets and stars can be used if the means are available for determining the time of rising or setting. This can be determined by computation (art. 2536) or, approximately, by means of a diagram on the plane of the celestial meridian (art. 1432).

Either of these methods can be used in reverse to set a watch that has run down, or to check the accuracy of a watch, if the longitude is known. In the case of a meridian transit the time need not be determined at the instant of transit. The watch is started and the altitude is then measured several times before and after transit, or at equal altitudes. The times of these observations are noted and from them the time of meridian transit is determined. The difference between this time and the correct time of transit can then be used as a correction to reset the watch. If a watch runs down and cannot be reset from other timepieces, the correct time should be determined at the first opportunity, if the longitude accuracy is likely to deteriorate.

Variation of the compass. If the deviation of the compass is known accurately and an accurate azimuth can be observed, it is possible to determine the variation. If this is compared with the variation shown on the pilot chart, an approximate line of position can be determined. Since in many areas these lines run in a generally north-south direction, this may be an indication of the longitude. However, if the line has a large east-west component, it should be considered as any other such line of position, rather than as a longitude line. In some areas it is more nearly a latitude line. The accuracy of the method depends upon the accuracy with which the variation can be determined, and the spacing between adjacent isogonic lines.

Time sight. If altitude of a celestial body is available, including zero "sextant" altitude at rising or setting (art. 2619), longitude can be found by time sight (art. 2106).

Approaching Land

2623. Signs of land.—There are a number of signs which may indicate that the lifeboat is approaching land.

The sky will sometimes indicate a break in the open sea. A small fixed cloud, when surrounding ones are in motion or absent, will usually be over or close to land. At high latitudes, a light-colored reflection in the sky might be over an ice area; a light green reflection in the tropical sky might indicate a shallow lagoon. Such indications may be even more apparent on the under side of a uniform cloud layer.

Birds most often fly away from land at dawn and toward it at dusk. A large number of birds may indicate the nearness of land.

Swell, properly interpreted, may be used as a guide to land. Consecutive swells travel parallel until they reach an island and then "bend" around it. Eddies are formed where the distorted swell meets beyond the island. This eddy line may be used as a bearing to land, sometimes at a considerable distance.

The color of the sea may act as a guide in finding land as the open sea generally appears dark blue or dark green, and a lighter shade indicates shallow water, which may be near land.

The sound of the surf is often heard while still a considerable distance from land. Other sounds may also be heard at great distances.

Odors, as from burning wood, sometime carry a long way out to sea.

Sounds and odors may be particularly helpful in periods of reduced visibility.

2624. Distance off.—At sea in a lifeboat the navigator is handicapped by his limited range of visibility. Distance to the horizon, in nautical miles, is given approximately by the formula $1.15\sqrt{h}$, h being the height of eye in feet. Thus, distance in miles is approximately 1⅐ times the square root of the height in feet. At an eye height of nine feet, the horizon is about 3½ miles away. A loaded Victory ship, whose greatest mast height is about 81 feet above the water line, could be seen $1.15\sqrt{81}$ or 10.35 miles by an observer at zero height of eye. At a height of eye of nine feet the top of the mast should break the horizon when the ship is about 13.8 miles off.

If the height of an object above the horizon, or the distance between points on it, is known, a simple proportion can be solved to determine the distance off by use of the cross-staff (art. 2618) or a similar device. To do this, align the two ends of the crosspiece with top and bottom, or two ends, of the object. The ratio of the length of the crosspiece to the length from this piece to the eye is the same as the ratio of the

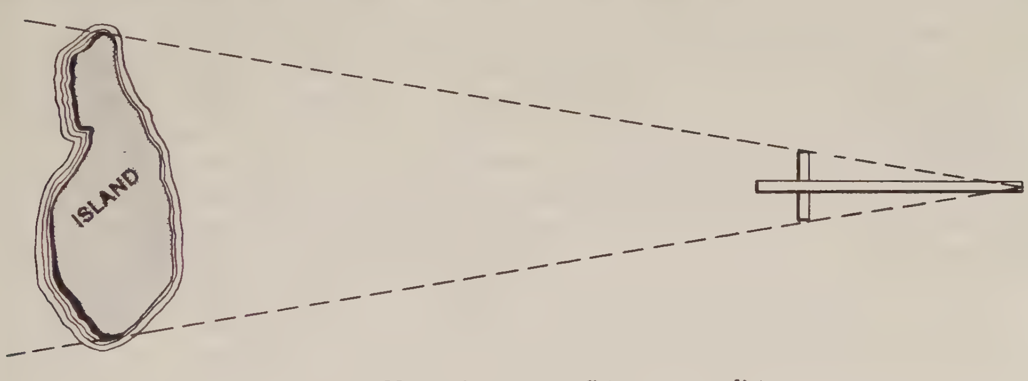


FIGURE 2624.—Using the cross-staff to measure distance.

height (or length) of the object to its distance from the observer (fig. 2624). Thus, if the crosspiece is 18 inches and the intercepted length of the long piece is 31 inches, the distance to an island 1½ miles wide in the line of sight is found from the proportion

$$\frac{18}{31}=\frac{1.5}{D}, \text{ or } \frac{D}{1.5}=\frac{31}{18}$$

$$D=\frac{1.5\times 31}{18}=2.6 \text{ miles.}$$

In this proportion the two parts of either fraction must be expressed in the same units if results are to be obtained without a conversion factor. Thus, both 18 and 31 are expressed in inches, and both 1.5 and 2.6 are in miles. For small or distant objects the crosspiece may be too long. In this case replace it with a shorter one, use half or less of it, or substitute some other device such as a rule held at arm's length. In the case of a height, only the visible part of the object is used if the horizon is between the observer and the object.

A variation of this method can produce approximate results rather quickly. Hold a pencil, stick, or finger vertical at arm's length. Close one eye and align the vertical member with one end of an object such as an island. Open the closed eye and close the other one. Estimate the distance the vertical member appears to move against the background. The distance of the background object is ten times the amount of apparent movement, in the same units. The actual ratio varies somewhat among individuals and can be determined by comparing the length of the outstretched arm with the distance between eyes—or by practice on objects of known size at known distances. For vertical objects hold the extended member horizontal and bend the head until it, also, is horizontal.

2625. Beaching the boat.—The beaching of a lifeboat may be one of the most dangerous parts of the entire experience. The approach to an island should be made on the lee side, if possible, and every effort should be made to attract the attention of any inhabitants so that advice on the best place to land, and perhaps assistance, might be obtained. If no help is available, sail parallel to the coast to study the terrain and determine the safest place to beach the boat. A lagoon or other sheltered area may be available. It may be necessary to delay the landing overnight to make a complete study of the terrain and to beach the boat by daylight. Surf appears less rough from the sea than from land. High spray indicates a rough surf.

If a steering oar is available, the rudder should be unshipped before the boat is brought in, as the steering oar will provide better control in the surf zone. The sea anchor should be used to lessen the possibility of broaching and capsizing. Storm oil should be used, if available, to reduce the roughness of the surf. It is possible that the course can be altered somewhat while heading in to the beach, to take advantage of a better opening, but care should be taken to avoid broaching. Additional information on handling a boat in a surf can be found in nearly any book on seamanship.

2626. Ashore.—Once the boat has been safely beached, the problem remains to lead the survivors to civilization. Perhaps the land will be heavily populated and the boat met by local people, or the way to safety may be indicated by a road or trail. But the boat may be beached at a deserted place where there are no signs of life.

Chapter XXVII, "Land Navigation," deals with this problem in detail, but remember that the choice of an initial course is almost as important in this case as it is at sea. Generally, it is good practice to follow the coast, but if the shore is obviously unsuitable for boat activity, a port is not likely to be nearby. The jungle should be avoided for travel, but it may be a plentiful source of food if nonpoisonous plants can be recognized. Often a stream can be followed to an inhabited place, for some source of water is essential to the maintenance of life. In an arid region, distant vegetation may be an indication of habitation.

Many of the methods used to determine position at sea may also be used ashore, and usually with greater accuracy due to the absence of motion. A distinctive method of determining the meridian while ashore is by a variation of the equal altitude method. Place a stick or rod upright on a level area, using a plumb bob to establish the perpendicularity of the stick. About an hour before noon mark the point where the tip of the stick's shadow falls, and draw a circle through this point, with the base of the stick as its center. Mark the point where the tip again falls exactly on the perimeter of the circle. Midway between the two points lies the meridian of the observer.

Problems

2613a. The compass azimuth of the sun is 126° at rising and 252° at setting.
Required.—Compass error.
Answer.—CE 9° W.

2613b. A life preserver is thrown overboard from a lifeboat and the boat headed away on course 355°. At a distance of half a mile the boat turns and heads back for the life preserver. The return course is 169°. The variation is 5° W.

Required.—(1) True course back to the life preserver.

(2) Deviation on this heading.

Answers.—(1) TC 167°, (2) D 3° E.

2615. The two knots in the log line of an improvised chip log of a lifeboat are 16⅔ feet apart. The elapsed time between passage of the knots through the hands of the observer is four seconds.

Required.—Speed of the lifeboat.

Answer.—S 2.5 kn.

2616. A lifeboat travels 18 miles on course 110°, from lat. 35°15′ S, long. 82°31′ W.

Required.—Latitude and longitude of the point of arrival.

Answer.—L 35°20′ S, λ 82°11′ W.

2617a. The date is November 15.

Required.—The approximate declination of the sun, without reference to an almanac.

Answer.—Dec. 18°.8 S.

2617b. The latitude of a lifeboat is 22°47′ N. A star is observed on the meridian, bearing north. The observed altitude is 66°50′.

Required.—Declination of the star.

Answer.—Dec. 45°57′ N.

2617c. The GMT is 1000, October 15.

Required.—Approximate GHA ♈, without reference to an almanac.

Answer.—GHA ♈ 172°.

2624. Approaching land, the navigator wishes to determine his distance from a lighthouse situated on the coast. He holds a rule at arm's length and finds that ⅝ inch of the rule appears the same height as the top of the lighthouse above water. He estimates the distance from his eye to the rule as 24 inches, and the height of the top of the lighthouse as 150 feet above water.

Required.—Distance to the lighthouse.

Answer.—D 0.9 mi.

CHAPTER XXVII

LAND NAVIGATION

2701. Introduction.—Land navigation is the process of directing movement across land or ice, from one point to another. When travel is along a well-marked system of highways, trails, railways, etc., a good map and distance-measuring device are all that are needed. But when travel is across unmarked areas, navigation may be more difficult. When the track leads across an open expanse of desert, tundra, or ice, the methods of navigation most nearly approach those used at sea or in the air.

The equipment used and the procedure followed should be suited to the circumstances. A high degree of common sense and adaptability is needed. It would be a waste of effort to measure accurately every change of course if one were following a stream whose general direction is known, but across an area without features, each change of course might be of great importance. Sometimes a considerable amount of ingenuity is needed to adapt available equipment or to improvise a suitable piece of equipment to meet a particular need. On land, as at sea or in the air, the navigator should use all available means to further his end. Even odors can be utilized to advantage in some instances. In any case, a preliminary estimate of the situation and advance planning are important, as is constant vigilance en route.

Basically, navigation on land combines the same elements as navigation at sea. Dead reckoning, piloting, electronic navigation, and celestial navigation all have their use. In general, the equipment should be simple, reliable, rugged, and capable of withstanding exposure to the weather. The mounting of equipment and the facilities for plotting may leave much to be desired. Plotting in a vehicle crossing rough terrain may be impossible while underway.

No trip across unfamiliar or desolate terrain should be attempted without provision for adequate navigation, whether travel is by wheeled vehicle, tank, dog sledge, or afoot. An adequately trained individual should have primary responsibility for navigation, and should be provided with the necessary navigational aids to suit the circumstances. Assistants or alternates should be provided when appropriate, as when a casualty is a reasonable possibility. During the trip the navigator should be given opportunity to perform his assignment. Sometimes this may involve stops that would not otherwise be scheduled.

2702. Charts.—The most useful charts for land navigation are topographic—those showing elevations and various features of the topography. With a large-scale map showing great detail, both the selecting and following of a route are relatively simple, if there is a sufficient number of identifiable landmarks. Over flat, open country the map is little more than a plotting sheet. The projection is not important, as long as it is conformal (art. 302) so that angles and small shapes are correctly represented. Since long, straight courses are rare, the form of representation of a great circle is seldom important.

2703. Dead reckoning on land, as on the sea or in the air, consists of determining position by means of the direction and distance traveled since leaving a known position. A careful log should be kept. If the track is across an unsurveyed area, and if there is a possibility of future passages over the same area, descriptions of the various landmarks encountered should be included in the log.

Plotting may or may not be desirable. Along a well-marked route, the track can be plotted in advance, and positions along the track can be marked as determined, either by dead reckoning or otherwise. Since it is usually difficult to plot accurately

while riding or walking, the usual procedure is to keep a log and stop at intervals to bring the plot up-to-date. Each item of the log should be recorded as it occurs, leaving nothing to memory.

Over rugged terrain it is seldom practical to proceed directly toward the objective, and the track selected is one which avoids the most difficult obstacles. Under these conditions, the directions and distances used are averages. With practice, one can become adept at estimating the course and distance made good along a track having many changes of direction. When traversing an unfamiliar or poorly mapped area, one may find it necessary to select the route as he proceeds, nearly always attempting to work closer to the destination, but frequently departing from the direct path to take advantage of features of the terrain.

Several types of mechanical dead reckoning equipment have been devised. One type, known as a **vehicle direction and position indicator,** is designed for vehicle installation and operates from the vehicle electrical system. With inputs from a gyro compass and the odometer drive, it automatically computes and continuously displays the vehicle position in map coordinates. It also computes and displays the distance and direction to a preselected destination. It is designed for the addition of a map plotter which can plot the course followed.

In any form of dead reckoning, it is well to keep in mind that if a heavy cross wind is blowing, a certain amount of leeway can occur.

2704. Direction.—Over flat, open country, a steady course is relatively easy to follow. In rugged or wooded country, many variations are needed. Under these circumstances the direction made good is more important than individual directions of motion. The determination of direction made good can often be accomplished by measuring the bearing of a distant feature such as a mountain peak, prominent tree or rock, a bend in a river or valley, etc., toward which one is steering. If the route follows a river, mountain ridge, etc., the general trend of the feature can usually be determined. The features which make necessary frequent changes of course can themselves sometimes be used for establishing average directions. Celestial bodies, too, can be used as a steering guide if their changing azimuth is considered. The sun and moon are invaluable aids. Polaris and bodies nearly east or west are particularly useful because of their relatively slow change of azimuth. Even a steady wind, sastrugi (windrows on snow), or clouds can be used, if one is careful to interpret them properly.

The type of compass used varies with the circumstances. Magnetic, gyro, and sun compasses are all used by land navigators.

FIGURE 2704a.—Pocket compass.

A **magnetic compass** used in land navigation is subject to the same errors as one used at sea or in the air. Seldom is a vehicle an ideal location for a compass. The deviating forces are large, and in some cases erratic. If a compass designed for the vehicle is not available, a boat compass might be used, but an aircraft-type compass may be more suitable. If the vehicle is a dog sledge, the compass might best be lashed to a convenient part. In a tank, truck, or jeep a permanent installation may be made. Frequent changes of compass from one vehicle to another should be avoided, but where this is necessary, some type of bracket or box should be provided to permit rapid installation and alignment.

Careful magnetic adjustment (compensation) is essential if accurate results are to be obtained. Usually, only permanent magnets are used, and in some compasses, notably those of the aircraft type, these are permanently mounted in the compass case, being controlled by a screw driver. If a compass is to be moved from vehicle to vehicle, the adjustment might be made in the vehicle itself. This is best done by attaching magnets to the box in which the compass is placed. With any type o adjustment, corrections may be needed if the magnetic latitude changes considerably

FIGURE 2704b.—Wrist compass.

Some experimentation may be necessary to establish the best location for the compass. In general, a position should be found as remote as convenient from the motor and electric wiring. A vertically graduated aircraft-type compass might be mounted near the top of the windshield with satisfactory results. The dashboard is probably the worst location in most instances.

Magnetic adjustment should be made with the motor running and with all magnetic equipment in its regular place. The effect on the compass of various electrical equipment, such as lights, windshield wiper, etc., should be noted. If needed, a deviation table should be made up, separate tables being made for as many conditions as necessary. In the determination of deviation, the vehicle can be pointed toward identifiable distant objects, or a hand-held compass used at a sufficient distance from the vehicle to permit accurate determination of direction and to preclude magnetic influence of the vehicle. A compass rose located on the surface of an aerodrome can be used satisfactorily, if available.

A **portable compass** has many uses, and if the party is proceeding on foot, it is the only type suitable. It may be a hand-held compass weighing several pounds, or, more often, a small pocket or wrist compass weighing but a few ounces. A pocket compass is shown in figure 2704a, and a wrist compass in figure 2704b. In figure 2704a note the

lubber's line and the sighting vanes for measuring bearings. No provision is made for adjusting a portable magnetic compass. When such an instrument is used, readings should be made a sufficient distance from a vehicle or other magnetic material and power lines to avoid any deviating influence. Magnetic material such as knives, keys, etc., should be removed from the observer's person during observation, unless it is determined that they have no effect on the compass in the position used. Magnetic material in the earth can cause deviation. The presence of such material is usually indicated by erratic operation of the compass when moved a short distance.

A vehicle **gyro compass** has been developed and is used where the need warrants. It is particularly useful in regions where the magnetic compass is not suitable, as near the magnetic poles and in areas of extensive deposits of magnetic material. It is also used in vehicles where satisfactory magnetic compass installations are difficult or impossible, as in some tanks.

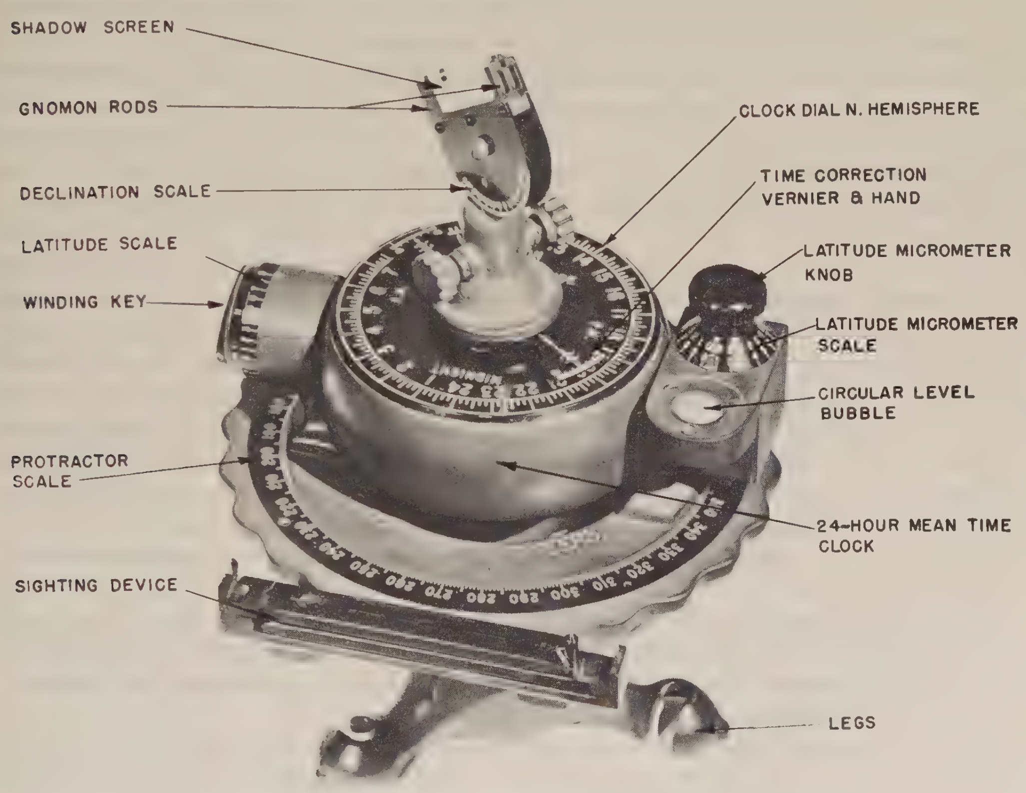


FIGURE 2704c.—Sun compass.

A **sun compass,** figure 2704c, is a mechanical device for determining true directions by means of celestial bodies, principally the sun. It is free from magnetic disturbances and gyro errors, and needs no source of power. However, an almanac or other source of celestial coordinates, accurate time, and a knowledge of the approximate position of the observer are needed, and use of the instrument is limited to periods when a celestial body is visible. This last limitation affects the choice of a mounting position for the instrument.

Like the astro compass used in aircraft (art. 2515), the sun compass is not a compass in the usual sense of *seeking* a reference direction. Both instruments consist of sighting

devices which are oriented with respect to the axis and equator of the earth, and the horizontal. When the device is so oriented and the sighting assembly or shadow is properly aligned with the celestial body, true directions are indicated on a circular scale. Generally, the device is not used as a continuous indication of direction, but as a means of checking direction at intervals. Each time an observation is made the setting of the instrument is changed to agree with the coordinates of the celestial body and observer at the time and place of observation. If the device is used for steering, it usually need not be reset oftener than every ten to 15 minutes.

Several emergency methods of establishing the approximate direction of north are available. These are discussed in chapter XXVI. The method of *double altitudes* is particularly applicable on land, where a plumb bob can be improvised and the length of the shadow measured. At the time the first altitude is observed, perhaps an hour before noon, an arc of a circle can be marked on the ground, with the center at the foot of the plumb line, and of radius equal to the length of the shadow. The position of the shadow tip on the circle is marked. The second altitude occurs when the end of the shadow is again on the arc of the circle. From the plumb line, north (or south if the sun is north of the observer) is midway between the two equal-length shadows, if the ground is level.

2705. Distance.—In land navigation, distance is usually determined directly, rather than by means of speed and time. Speed may be used if constant enough, but this is rarely the case.

For a vehicle with wheels, the obvious method is by **odometer,** the distance-measuring device associated with a speedometer. For accurate results such a device should be carefully calibrated. Size of tires, amount of tread left on tires, pressure, loading, speed of the vehicle, and nature of the surface over which the vehicle travels all affect the reading. However, except in extreme conditions, an average calibration should produce good results. If the odometer is attached to the drive shaft, a certain amount of slippage occurs, but allowance for this can be made in the calibration. Road slippage, as when traveling on a slippery surface or one of loose material, results in too great a reading. If the terrain is hilly, or if frequent minor changes in track are needed to avoid obstacles, the distance indicated by odometer is greater than that shown on the chart.

When one is traveling by sledge over snow or ice, a bicycle wheel can be attached and a metering device connected to it. Most accurate results can be obtained if the tire is used in a deflated condition, as it will then resist the tendency to increase in diameter by accumulation of snow.

When traveling on foot, one can use a **pedometer,** a small, watch-size instrument, usually attached to the belt, that records the number of steps taken. If this instrument is calibrated in distance, it should be adjusted to the length of step of the wearer. If such a device is not available, the number of steps or paces can be counted. If this is done, it is well to have some method of preventing loss of the count. This might be done by means of a counting device that registers each time a plunger is depressed. This may be done at each pace (two steps), or at each multiple such as ten or 100 paces. If no counting device is available, or if one of limited range is used, a small stone or other object transferred from one pocket to another at intervals, as each 1,000 paces, may prevent loss of count. With any method of counting steps or paces it is well to consider the nature of the terrain, and other factors affecting the length of the pace. Over soft ground, in high grass or weeds, and along inclined surfaces the length of the pace differs from that on firm, level ground, generally becoming shorter. It may also shorten as

the walker tires, and its effective length is reduced as he makes short detours to avoid obstacles. A strong wind may either shorten or lengthen the pace. Variations in the length of the pace can be handled by calibration over an area representative of the conditions encountered, or by dropping (or adding) one pace out of a certain number, as determined by measurement or estimate.

2706. Piloting.—In land navigation piloting is generally quite simple, consisting merely of the recognition of landmarks, and notation of the time and distance at which they are passed. It is somewhat similar to the passage of buoys as one proceeds along a channel. In open country with distant mountains, bearings of identifiable features can be plotted and ranges can be observed when two features are in line.

2707. Electronic navigation is seldom available on land, but should be used if one has access to it. The most common electronic aid used is some form of radio direction finder (directional characteristics of the loop antenna of a portable radio may be utilized), usually used in connection with a transmitter at the destination. In this case the direction finder is used as a homing device. If signals from other radio transmitters at known locations can be received, position can be determined by plotting two or more bearings. On land, radio reception is weak or nonexistent in certain "blind spots," and the accuracy of readings taken may be affected if the receiver is near man-made or natural obstacles. Power or telephone lines, fences, railroad tracks, buildings, or cliffs are particularly to be avoided if accurate bearings are required. Atmospheric and magnetic conditions also affect radio transmission.

2708. Celestial navigation.—In areas with an abundance of landmarks, or where an adequate method of homing is available, celestial navigation is not generally used, except, possibly, in relation to a sun compass (art. 2704). However, in open country without identifiable or stable features, as on the desert or in some parts of polar regions, celestial navigation may provide the only means, other than dead reckoning, of determining position.

A marine sextant is not suitable with the type of horizon generally encountered on land. However, if an artificial horizon (art. 1512) is available or can be improvised, the instrument can provide satisfactory results. An aircraft-type sextant with a built-in artificial horizon, such as a bubble (art. 1513), may be suitable. If greater precision is desired, a surveying instrument such as a theodolite (art. 4004) or astrolabe (art. 4002) can be used. For any of these instruments, accurate results can be expected only by stopping and making the observations from a stable position. Accurate timing of the observation is essential unless the observation is of a meridian altitude or Polaris, or unless the observer is near one of the geographic poles.

Sight reduction can be accomplished by any suitable method, several of which are described in chapters XX and XXI.

When one is operating in an area where celestial navigation is needed, it is good practice to obtain fixes twice daily, or oftener, if available. Celestial navigation may also be useful when the continuity of dead reckoning has been broken and identifiable landmarks are not available, as during a battle in time of war.

CHAPTER XXVIII

AIR NAVIGATION

2801. Introduction.—Air navigation is the navigation of all types of aircraft, including propeller-driven airplanes, jet airplanes, helicopters, dirigibles, blimps, balloons, and airborne guided missiles. The term **aircraft** includes any craft designed for transportation through the air, except (1) safety contrivances such as a parachute and (2) spacecraft designed for operation outside the earth's atmosphere.

The elements of air navigation are the same as those of marine and land navigation, but with some differences in methods, practices, and emphasis. These differences result primarily from unique conditions encountered in air navigation, as follows:

1. *Need for continued motion.* A ship or land vehicle can stop, if necessary, and resolve any uncertainty of position, awaiting more favorable conditions if necessary. Except to a limited extent, most aircraft must keep going.

2. *Limited endurance.* Most aircraft can remain aloft for a relatively short time, the period usually being a matter of hours.

3. *Greater speed.* The speed of propeller-driven aircraft is about 20 times that of ships. Jet aircraft are even faster. The speed of lighter-than-air craft and helicopters is about five to ten times that of ships. Quicker navigation methods are needed, even if some accuracy is sacrificed.

4. *Three-dimensional navigation.* In air navigation the third dimension is of considerably greater importance than in marine (even underwater) and land navigation.

5. *Effect of weather.* Except under extreme conditions, the weather element of greatest concern to the marine and land navigator is the visibility. In air navigation the visibility has a vital effect upon the ability to land and take-off, as well as the availability of landmarks. In addition, wind has a more direct effect upon the position of aircraft than upon that of ships or land vehicles. Changes of atmospheric pressure and temperature affect the height measurement of aircraft using barometric altimeters.

The importance of weather is reflected in the designations of different flight conditions, as follows:

Contact flight, when the surface of the earth is visible.

Visual flight, when the aircraft is more than a prescribed minimum distance above, below, or laterally from clouds.

Instrument flight, when the conditions of contact and visual navigation are not met.

A **closed aerodrome** is one at which the visibility (horizontal or vertical) is below certain prescribed minimums, rendering ordinary take-offs and landings unsafe.

Because of the conditions mentioned above, air navigation is closely regulated. In the United States, regulations are prescribed by the Civil Aeronautics Board (CAB) and the Federal Aviation Agency (FAA). Operating rules for military aircraft may differ from those governing commercial and private aircraft.

2802. Charts and publications.—Aeronautical charts, like nautical charts, show latitude and longitude scales, parallels, meridians, and aids to navigation. Unlike their marine counterparts, however, aeronautical charts do not show soundings. Contours and elevations are given more emphasis. Airways (art. 2805), control zones, airports, and radio aids are given considerable prominence.

Except for local, approach, and flight (strip) charts, many aeronautical charts are arranged in coordinated series with no (or uniform) overlapping between charts. In many cases this permits joining of adjacent charts to form a single large one (which

(THIS CHART SHOULD NOT BE USED FOR NAVIGATIONAL PURPOSES.)

ELEVATIONS IN FE

FIGURE 2802.—Part of a typical aeronautical chart.

may be wall mounted), or a strip chart for an individual flight. Because of the wider use of radio aids and great-circle "sailing" in air navigation, the Mercator projection is less commonly used in the air than aboard ship. Many aeronautical charts are on the Lambert conformal projection (art. 314). The Mercator projection may be used near the equator. Several projections are used in polar regions (arts. 321, 2508). Because of the greater speed of aircraft, smaller scales are used for en route navigation. For high-speed, high-altitude aircraft, charts are of even smaller scale, and show less detail. Part of a typical aeronautical chart is shown in figure 2802.

In the United States, aeronautical charts are published chiefly by the U.S. Air Force Aeronautical Chart and Information Center, St. Louis, Mo.; U.S. Coast and Geodetic Survey, Department of Commerce, Washington, D.C., and the U.S. Navy Hydrographic Office, Washington, D.C. Each of these three agencies publishes a catalog of its products. The Federal Aviation Agency, Washington, D.C., publishes regulatory material and other information.

Various publications are of assistance to the air navigator. Among these are the following:

The *Airman's Information Manual* (AIM), published by the FAA, provides information necessary for the planning and conduct of civil flights in the National Airspace System. The manual is divided into six sections each composed of a specific category of information consistent with the operational needs of aviation.

The *Alaska Airman's Guide and Chart Supplement* and the *Pacific Airman's Guide and Chart Supplement* provide civilian pilots with data required to supplement the navigational information on aeronautical charts in the Alaska and Pacific Areas.

The *International Flight Information Manual*, published by the FAA, is designed as a preflight and planning guide for use by U.S. non-scheduled operators, business, and private aviators contemplating flights outside of the United States.

Flight Information Publications (FLIP), published by the Department of Defense, contain textual and graphical information for military pilots. The series consists of publications for (1) flight planning, (2) enroute operations, and (3) terminal operations. Each publication may contain charts, tables, and text material.

Notice to Aviators, published every two weeks by the U. S. Navy Hydrographic Office, is the aviator's counterpart of *Notice to Mariners* (art. 425).

Notice to Airmen (NOTAM) contains urgent information requiring immediate dissemination. These notices are sent by teletype to airports throughout the United States.

Federal Aviation Regulations (FAR), published by the FAA.

A comprehensive book giving complete text and reference material on the principles and practices of air navigation is published by the U.S. Navy Hydrographic Office under the title *Air Navigation* (H.O. Pub. No. 216). A book, in three volumes, giving somewhat similar information on air navigation as practiced in the United States Air Force is published by the USAF under the title *Air Navigation* (Air Force Manual 51–40).

2803. Dead reckoning in the air, as aboard ship, comprises the elements of direction and distance. Several terms related to direction are used:

Heading, the horizontal direction in which an aircraft is pointed. This may be a momentary direction, an average, or the intended direction.

Heading line, a line extending in the direction of a heading.

Course, the intended horizontal direction of travel.

Course line, a line extending in the direction of a course.

Course made good, the direction from one established position to a later one.

Track, the horizontal component of the path followed or intended to be followed, and sometimes the direction of this path.

Drift angle, the angle between the heading line and the track, labeled "right" or "left" depending upon the direction of drift.

Drift correction angle, the angle between the heading line and the course line, or the anticipated drift angle.

As in marine navigation, the various directions can be stated relative to any of several reference directions, true, magnetic, compass, and grid being the usual ones. True directions are usually used for plotting, but magnetic directions are more widely used than in marine navigation because some form of magnetic compass is commonly used for measuring direction. With the development of better directional gyro compasses, grid directions are coming into wider use.

The effect of wind on aircraft is similar to that of current on ships. There is nearly always *some* wind, which varies from place to place and from time to time. The air navigator is alert to indications of changes, and has frequent occasion to solve the **wind triangle,** solutions similar to those for current (art. 807). The usual solution determines the heading to fly to make good the selected course. Because of the frequent, and often urgent, need for solution of the wind triangle, aviators customarily use some form of mechanical **computer.** If direction and speed of the wind are known, both heading (or track) and ground speed can be determined. If only drift angle is known, a quantity which usually can be measured in flight, the approximate heading (or track) can be determined without plot, but not the ground speed. With observed drift on two or more headings of considerable difference in direction, one can solve for wind speed and direction. Such observations might be made before and after a turn.

An **air plot** of heading and **air speed** (rate of motion relative to the air) provides a series of **no-wind positions,** sometimes called **air positions.** These are the successive positions an aircraft would occupy if there were no wind. A **dead reckoning plot** of course and **ground speed** (rate of motion relative to the surface of the earth) provides a series of dead reckoning positions.

Most aircraft compasses are magnetic, but those more commonly used are *remote indicating.* The active element is placed at a location relatively free from magnetic disturbances from the aircraft, such as in a wing or the tail, and provided with indicators at various locations, as needed. Because of the large errors introduced when a magnetic compass tilts, the better aircraft compasses are gyro stabilized. The most widely used aircraft compass is known as the **Gyro Flux Gate compass.** In general, aircraft magnetic compasses are compensated (adjusted) by means of flexible cams mounted within the case and controlled by a screw driver. No adjustment is made for vertical soft iron, for quadrantal deviation, or for heeling. Swinging for residual deviation may be done on the ground, by means of a hand-held compass or a compass rose located on the hard surface of an aerodrome; or in the air by means of celestial bodies or straight roads, power lines, etc. The **compass correction card** (deviation table) is usually made up on the basis of the compass direction to steer for a desired magnetic heading, the value of deviation not being given.

The north-seeking gyro compass commonly used aboard ship has not been practical in the air because of its weight and the fact that it would not work satisfactorily at modern aircraft speeds, which are comparable to or greater than the rotational speed of the earth. However, efforts have been made to overcome these obstacles, and it is possible that a suitable north-seeking gyro compass will be developed for use in aircraft. The **directional gyro** compass is used widely. Such an instrument is essentially a gyroscope pointed in a desired direction which it maintains over a period of several minutes. This instrument was devised primarily to provide directional guidance

during a turn of the aircraft, when the older magnetic compasses are erratic. More recent directional gyros require less frequent resetting, provide greater accuracy, and compensate for rotation of the earth (the gyroscope tends to maintain the same direction *in space*), and they may be monitored by a remote-indicating magnetic compass. With the best modern directional gyros an aircraft is able to follow a great circle with about the same accuracy that it can follow a rhumb line (using a magnetic compass). A directional gyro is checked from time to time by means of a magnetic compass or an astro compass (art. 2515).

Air speed is usually determined by measurement of the difference between static air pressure and the pressure exerted by the apparent wind, which, in the air, is always from dead ahead and equal to the speed of the aircraft through the air. This pressure difference is measured by a device called a "Pitot tube," and transmitted by tubes to the air speed indicator. Corrections are applied for nonstandard air temperature at the pressure altitude, compressibility, and heating effect. Higher speeds are sometimes stated in terms of a percentage of the speed of sound at the aircraft. On this basis the speed is called the *Mach* (mŏk) *number*. A Mach number of one is the speed of sound, which varies with the density of the air; Mach number 0.9 is 90 percent of the speed of sound, etc. A **Mach meter** is an instrument which measures Mach number directly.

Height is usually measured by means of a **barometric altimeter,** which is essentially an aneroid barometer graduated in feet above sea level. If the atmospheric pressure is not standard, the altimeter will not read the correct value at sea level unless adjusted to the existing pressure. A knob is provided for this purpose. If the decrease of pressure with height is not standard, additional error is introduced. Altitude separation of aircraft along airways is based upon indications of an accurate barometric altimeter set to *standard* conditions, so that all instruments at the same height should have the same reading. For landing and take-off the instrument is usually adjusted so that it will read the correct altitude when the aircraft is on the surface. For landings, the necessary information is supplied by radio from the control tower. An instrument which measures height above the surface (absolute altitude) is called an **absolute altimeter.** The usual absolute altimeter is a form of radar beamed vertically downward. It measures height in a manner similar to the measurement of water depth by an echo sounder (art. 619).

As in marine and land navigation, dead reckoning is the basis of navigation in the air, all other forms serving to correct positions so determined. However, because of the nature of air navigation and the aids available, dead reckoning in the air, particularly along airways, is often a mental process, or one in which the dead reckoning for the entire flight is plotted in advance, with DR position at frequent intervals, as every ten minutes, being marked on the plot. The problem is then one of maintaining the schedule or keeping a record of deviation from it.

Various automatic dead reckoning systems show promise of providing accurate means for determining position over long stretches of water or over terrain lacking in distinctive features as aids to navigation. Such systems are based upon accurate means of measuring direction and distance. These, in turn, require accurate directional and horizontal references. Examples of such systems are those based upon measurement of accelerations (**inertial systems**) and those based upon measurement of the Doppler shift (change of frequency) of echoes from radio or radar beams transmitted obliquely from the aircraft to the ground (**Doppler systems**). Additional information on this subject is given in article 809. Systems under development combine one or both of these principles with some other principle, such as automatic celestial navigation.

2804. Piloting, often called **pilotage** in air navigation, is similar in principle to that performed aboard ship. In practice it more nearly resembles land navigation. Lines of position from observed bearings or distances are rarely plotted by air navigators.

The more common practice is to compare observed features with the chart, keeping a record of one's progress as he proceeds. If this is not a *continuous* process, combined with mental or plotted dead reckoning, one can soon become lost, particularly in an area where many features are similar in appearance. One can learn to watch for distinctive features. Two towns may look very much alike, but the pattern of roads, railroads, and streams in the vicinity may be quite different. A race track, mine, or other distinctive feature may help one distinguish between otherwise similar areas. However, it is essential that the air navigator not be hasty in identification, for mistakes can easily be made. A position established by identifying a feature directly below the aircraft is called a **pinpoint.**

Over well-traveled areas, extensive use is made of various radio aids to navigation. These are discussed in article 2805. So complete is coverage across the United States that an experienced aviator with suitable publications can travel from coast to coast without an aeronautical chart, whether or not the surface is visible. Over such areas, navigational duties are customarily performed by the pilot and copilot, a separate navigator being carried only on flights requiring his services, as on long over-water flights or in polar regions.

2805. Electronic navigation is more widely used in the air than on land or sea, for several reasons. Because of the greater height of aircraft, there is less obstruction of radio signals, and higher frequency "line of sight" systems are available over greater ranges. Over land, aids can be placed at suitable intervals to provide essentially continuous, short-range guidance over long distances. The difficulty of observing bearings, celestial bodies, etc., from aircraft renders electronic methods more attractive. Decreased accuracy of other methods when used in the air enhance the value of electronics. The greater speed and adaptability of electronic methods are of higher value aboard a fast-moving aircraft. The electronic navigational equipment carried in aircraft is compact and especially adapted to use in the air. Airborne computers are being developed for use with advanced navigation systems.

The **automatic radio direction finder** commonly used in the air provides a continuous indication of direction toward the transmitter by means of a needle pivoted at the center of a compass rose. The navigator has only to tune to the correct frequency and watch the needle. Its steadiness is some indication of the reliability of the reading.

The first nationwide system of electronic navigational aids was composed of several hundred low frequency "four-course ranges." At each range station the international Morse code letter for *N* (▬ •) is transmitted in two opposite sectors called "quadrants." In the other two "quadrants" the letter *A* (• ▬) is transmitted. These signals are so related that along the boundaries between sectors, where the two signals are of equal strength, the dots and dashes interlock to form a continuous monotone. It is possible to control the direction of these monotones or "beams" so that they indicate desirable directions of travel. Along these "beams" a series of **airways** are established, somewhat resembling highways. The *magnetic* directions of these beams are indicated on the chart, as shown in figure 2802. To use these ranges the navigator has only to follow one leg to the station and another leg out until he picks up the next beam.

These ranges are being supplemented by a series of very high frequency **vortac** stations. Each vortac station provides two methods of establishing direction, and means for determining distance. The two direction systems are **tacan** (**tac**tical **air navigation**) for military aircraft, and **omnirange** (**VOR**) for commercial and private aircraft. The suitably equipped aircraft is thus provided an infinite number of "radials" (in practice 360 at 1° intervals around each station) by means of which an aviator can receive guidance along *any* radial line from the station. With direction and distance available at all times, a continuous fix is provided, whether or not the

aircraft is following a radial. If the aviator selects the radial he wishes to follow, a dial indicates when he is off the radial, and which direction he should turn to get back on it. A "TO-FROM" indicator tells the aviator whether the selected radial is to be measured toward or away from the station. If the aircraft is equipped with a **course computer,** he can fly toward or away from an offset **way point** with the same indications as though it were the range station. Thus, multilane airways are available.

Longer range aids used by aircraft, particularly over ocean areas, include loran (art. 1302), Decca (art. 1309), and consol (art. 1312). A number of other systems have been suggested, and the future will undoubtedly see an increase in the use of electronics in air navigation, particularly in dead reckoning and celestial systems.

Airborne radar is a valuable navigational aid. With practice one can learn to identify the echoes from different features of the terrain, and often to locate his position by piloting methods when the surface is obscured by an undercast.

Various beacons are designed primarily for use by aviators. A **racon** is a **ra**dar bea**con** which returns a coded signal when triggered by a signal from the aircraft's radar, thus providing identification as well as bearing and range. **Fan markers** transmit vertical fan- or bone-shaped patterns at selected points along an airway to indicate passage of those points. Nondirectional markers are placed at other points. Above each station of a four-course range an inverted **cone of silence** occurs, where little or no signal from the ranges is received. At some of these stations **Z markers** are installed to transmit distinctive signals upward to indicate location of the stations.

2806. Celestial navigation.—In the air, celestial navigation is used in polar regions and on long over-water flights. Observations are invariably made with some form of artificial-horizon sextant (art. 1513), usually one having a bubble or pendulum reference. In air navigation, positions are needed more often than on land or sea. An entire flight across the Atlantic may be made between evening twilight and dawn. A common practice over the oceans is to obtain fixes at intervals of one hour. Another reason for using an artificial-horizon sextant is that the natural horizon is often obscured by clouds or haze, while celestial bodies are clearly visible. If a periscopic sextant (fig. 1513a) is not available, observations are usually made through an astrodome.

Because of the speed of aircraft, time zones may be crossed at frequent intervals. It is customary to keep navigational timepieces set to GMT. For celestial navigation, a high-grade watch is carried. It may have a 24-hour dial, and in most instances it has a sweep second hand.

Rapid sight reduction is important at aircraft speeds. In ten minutes a modern plane may travel 100 miles. The method most commonly used is H.O. Pub. No. 249 (art. 2113), with the *Air Almanac*. By precomputation, a navigator can obtain a fix within two or three minutes after the observations. Observation at a selected time is not a problem, because the sight continues over a period, usually two minutes, during which an averaging device is in operation. This eliminates large acceleration errors that might arise from motions of the aircraft. Thus, ten minutes may be required for observation of three stars at four-minute intervals between the mid times of observation. Celestial observations in the air are inherently less accurate than good observations with a marine sextant aboard ship. In the air an error of five to ten miles is considered normal for favorable conditions.

As speeds increase, the need for faster observation and reduction becomes more urgent. This has lead to development of automatic celestial navigation (art. 2124).

2807. Pressure pattern navigation.—On a long over-water flight, the great circle is a good approximation of the shortest distance between point of departure and destination. However, it may not be the least-time route, because of unfavorable winds. At more than one or two thousand feet above the surface of the earth, winds tend to

blow along the isobars (except near the equator). If the pattern of isobars (the "pressure pattern") at flight altitude is known, an experienced air navigator can often select a route that may add miles to the flight but increase ground speed to such an extent that time is reduced. This is one form of **pressure pattern navigation.**

A pressure pattern flight is customarily made at a constant *pressure* altitude (for instance the 500-millibar level at a standard altitude of 18,281 feet). By means of barometric and absolute altimeters, the navigator is able to determine any increase or decrease in height of the constant pressure surface over a time interval. With this information, he is able to compute the cross component of the wind, or the lateral drift. This is of assistance in dead reckoning, and it serves as a check on predicted pressure and wind. It is the basis for alterations that may be needed in the original plan.

2808. Flight planning.—Before take-off, a careful study is made of weather conditions expected to be encountered en route and at the terminal. If a choice of route and altitude is available, the most favorable are selected. Wind triangles for various parts of the flight are solved, and heading and ground speed determined. From this, the flight *time* and the amount of fuel needed for the flight can be computed. A suitable **alternate aerodrome** is selected for use if weather makes landing at the scheduled destination hazardous. Fuel deemed sufficient for the flight to the destination and then to the alternate, plus the amount needed for warm-up and take-off, and an adequate reserve, is taken aboard.

During the flight a close check is kept upon the actual rate of fuel consumption, and if this exceeds the predicted rate to such an extent that there is danger of exhausting the fuel supply before reaching the destination, the aircraft returns or is diverted to another aerodrome.

An adequate flight plan, properly used, is vital to safe flight over long distances. The plan is filed at the aerodrome of departure or other designated place, which notifies the destination of the **estimated time of arrival** (**ETA**). During the flight, periodic reports by radio provide information on progress and deviations from the plan. These serve as the basis for search and rescue operations, should they become necessary.

2809. Space navigation.—Navigation of a spacecraft through the atmosphere of the earth and beyond left the realm of science fiction and became a reality with the first successful launching of an artificial earth satellite in 1957. The same basic principles that govern terrestrial navigation are involved in space navigation, but with some differences of technique and emphasis.

Space navigation is four-dimensional, in contrast to the essentially two-dimensional navigation on or near the surface of the earth. In addition to the obvious third dimension of space, time has increased significance. Progress toward a final point is, by itself, inadequate. One must arrive at the point at the right instant to effect satisfactory rendezvous with another moving object. Neither direction of motion nor speed is directly measurable to satisfactory accuracy for navigation, and both motion and speed are likely to be varying continually. With the tremendous speeds involved and serious power limitations, only minor corrections to either speed or direction are likely to be available. There is little probability of recovering from a serious mistake.

Because of the fantastic distances and the propulsion systems now considered feasible, flight times to the nearest stars will be measured in life spans. Consequently, meaningful space navigation in this century is likely to be limited to the solar system. The techniques expected to be used during this period differ somewhat in each of four phases of space flight: (1) escape, (2) in the near vicinity of a celestial body, (3) midcourse, and (4) terminal.

During escape, whether from the surface of the earth or an orbit around it, a carefully precalculated trajectory is followed. Tracking is performed from the earth. Acceler-

ometers control the cutoff at the desired speed. All space missions are so carefully planned in advance that the need for later corrections is dependent directly upon the accuracy with which the escape phase navigation is performed.

In the vicinity of a celestial body, as in the escape phase, position is determined with respect to the body involved. The coordinate system used is (1) distance from the body, and (2) latitude and longitude on the sphere thus identified. This might be determined from the celestial body, as is commonly done with artificial earth satellites and space probes from the earth, or it might be determined from aboard the spacecraft. Distance from the celestial body might be determined by radar or by measurement of the apparent diameter of the body. Position on the sphere can be established by observation of the position of the body among the background of stars, thus establishing celestial lines of position.

During the midcourse phase of a flight to another planet, navigation is primarily a matter of determining position and comparing it with the scheduled position at the time of fix. Thus, dead reckoning and position fixing serve the same functions as on earth. Position determined relative to other bodies of the solar system is a form of piloting. Celestial navigation involves use of the background of stars. Position is identified as distance from the sun and some form of "latitude" and "longitude" on the sphere thus identified. Distance from the sun can be determined directly by measurement of its apparent diameter. Optical measurement of the angle between lines of sight to a planet and star establishes position on a cone having its apex at the planet. Two such cones each with its apex at the same planet intersect in two lines, and a third cone will remove the ambiguity. Three-dimensional position can also be determined by means of cones referred to three bodies of the solar system or by means of celestial lines of position on two such bodies, noting their positions relative to the background of stars. A discrepancy in scheduled position might be corrected by (1) returning to the original schedule at a specific time, (2) establishing a new path to intercept the destination planet at the original time and place, or (3) determining a new optimum path to intercept the planet.

During the terminal phase, thrust is applied to place the spacecraft in an orbit around the destination planet or guide the craft to a soft landing. Position is determined with respect to the body being approached. It is important that the dead reckoning be advanced far enough ahead to allow timely alteration of path, if needed, to place the spacecraft in an appropriate position for carrying out terminal phase maneuvers.

When continuous thrust of relatively small power becomes available, a procedure which will greatly simplify the navigation will be to proceed first to the line connecting the sun and destination planet and then to apply continuous thrust to stay on this line, reaching the destination by homing techniques.

Use of some kind of physical phenomena has been suggested for either establishing lines or surfaces of position, or measuring either speed or direction of motion. This approach has not been promising.

The present state of the art seems adequate to develop a fully automatic system for navigation during any space mission possible. However, a considerable amount of engineering will be needed before a reliable system is available.

References

U.S. Department of the Air Force. *Air Navigation.* AFM 51–40. Washington, 1962.

U.S. Navy Hydrographic Office. *Air Navigation.* H.O. Pub. No. 216. Washington. U. S. Govt. Print. Off., 1955.

Moody, Alton B., "Space Navigation." Proceedings of the IRE, Vol. 50, No. 5 (May 1962).

Weems, P. V. H. *Air Navigation.* 4th ed. Annapolis, Weems, 1955.

CHAPTER XXIX

NAVIGATIONAL ERRORS

2901. Introduction.—As commonly practiced, navigation is not an exact science. A number of approximations which would be unacceptable in careful scientific work are used by the navigator, because greater accuracy may not be consistent with the requirements or time available, or because there is no alternative.

Thus, when the navigator uses his latitude graduations as a mile scale, or computes a great-circle course and distance, he neglects the flattening of the earth at the poles, a practice that is not acceptable to the geodetic surveyor. When the navigator plots a visual bearing, or an azimuth line for a celestial line of position, on a Mercator chart, he uses a rhumb line to represent a great circle. When he plots the celestial line of position, he substitutes a rhumb line for a small circle. When he interpolates in tables of logarithms or in loran tables, he assumes a linear (constant-rate) change between tabulated values. When he measures distance by radar, or depth by sonic depth finder, he assumes that the radio- or sound-wave has constant speed under all conditions. When he applies dip and refraction corrections to his sextant altitude, he generally assumes standard atmospheric conditions.

These are only a few of the approximations commonly applied by a navigator. There are so many that there is a natural tendency for some of them to cancel others. Thus, under favorable conditions, a position at sea, determined from celestial observation by an experienced observer, should seldom be in error by more than two miles. However, if the various small errors in a particular observation all have the same sign (all plus or all minus), the error might be several times this amount, without any mistake having been made by the navigator.

Greater accuracy could be attained, but at a price. The navigator is a practical individual. In the course of ordinary navigation, he would rather spend ten minutes determining a position having a probable error of plus or minus two miles, than to spend several hours learning where he *was* to an accuracy of a few yards. But if he can determine a recent or present position to greater accuracy, the decrease in error is attractive to him. The various navigational aids have been designed with this in mind. Greater accuracy in plotting could be achieved by increasing the scale of the chart or plotting sheet. This has been done for confined waters where a higher degree of accuracy is needed, but a large-scale plotting sheet would be a nuisance at sea. The hand-held marine sextant is not sufficiently accurate for use in determining an astronomical position in a geodetic survey. But it is much more satisfactory at sea than the surveyor's astrolabe or theodolite (arts. 4002, 4004), which require stable platforms if their potential accuracy is to be realized.

An understanding of the kinds of errors involved in navigation, and of the elementary principles of probability, should be of assistance to a navigator in interpreting his results.

2902. Definitions.—The following definitions apply to the discussions of this chapter:

Error is the difference between a specific value and the correct or standard value. As here used, it does not include mistakes, but is related to lack of perfection. Thus, an altitude determined by marine sextant is corrected for a standard amount of refraction,

but if the actual refraction at the time of observation varies from the standard, the value taken from the table is in error by the difference between standard and actual refraction. This error will be compounded with others in the observed altitude. Similarly, depth determined by echo sounder is in error, among other things, by the difference between the actual speed of sound waves in the water and the speed used for calibration of the instrument. It will also be in error if an echo is returned from a phantom bottom (art. 3504) instead of from the actual bottom. This chapter is concerned primarily with the deviation from standards. Thus, while variation of the compass is an error when referred to true directions, the difference between the assumed variation and that actually existing is an error with reference to magnetic direction. Corrections can be applied for standard values of error. It is the deviation from standard, as well as mistakes, that produce inaccurate results in navigation. Various kinds of error are discussed in the following articles.

Mistake is a blunder, such as an incorrect reading of an instrument, the taking of a wrong value from a table, or the plotting of a reciprocal bearing.

Standard is something established by custom, agreement, or authority as a basis for comparison. It is customary to use nautical miles for measuring distances between ports. By international agreement the nautical mile is defined as a certain number of meters. By authority of various countries which are parties to the agreement, this length is translated to the linear units adopted by that country. It is the fact of establishment or general acceptance that determines whether a given quantity or condition has become a standard of measure or quality. Thus, in 1960, the standard unit of length agreed upon at the Eleventh General (International) Conference on Weights and Measures to redefine the meter was 1,650,763.73 wavelengths of the orange-red radiation in vacuum of krypton 86 corresponding to the unperturbed transition between the $2p_{10}$ and $5d_5$ levels. Where accepted, this established standard of length now serves as a basis for measurement of any physical magnitude, as the length of the meridian, rather than the reverse, which was originally proposed. Multiples and submultiples of a standard are exact. In 1959, the U. S. adopted the exact relationships of one yard as equal to 0.9144 meter and one inch as equal to 2.54 centimeters. Hence, 39.37 U. S. inches are approximately equal to one meter. Because one foot equals 12 inches by definition, and the international nautical mile has been defined as 1852 meters, the international nautical mile is equal to 6,076.11549 U. S. feet (approximately). The previous U. S. foot (6,076.10333 . . . feet equals one nautical mile) has been redesignated as the U. S. survey foot. It will still be encountered frequently during the transitional period. The values and tables in this 1962 edition are based on those adopted by the United States in 1959.

Frequently, a standard is so chosen that it serves as a model which approximates a mean or average condition. However, the distinction between the standard value and the actual value at any time should not be forgotten. Thus, a standard atmosphere has been established in which the temperature, pressure, density, etc., are *precisely* specified for each altitude. Actual conditions, however, are generally different from those defined by the standard atmosphere. Similarly, the values for dip given in the almanacs are considered standard by those who use them, but actual dip may be appreciably different from that tabulated.

Accuracy is the degree of conformance with the correct value, while **precision** is the degree of refinement of a value. Thus, an altitude determined by marine sextant might be stated to the nearest 0!1, and yet be accurate only to the nearest 1′ if the horizon is indistinct. Accuracy and precision are further discussed in article O3.

2903. Systematic errors are those which follow some law by which they can be predicted. The accuracy with which a systematic error can be predicted depends

upon the accuracy with which the governing law is understood. An error which can be predicted can be eliminated, or compensation can be made for it.

The simplest form of systematic error is one of unchanging magnitude and sign. This is called a **constant error.** Examples are the index error of a marine sextant, watch error, or the error resulting from a lubber's line not being accurately aligned with the longitudinal axis of the craft. In each of these cases, all readings are in error by a constant amount *as long as the adjustment remains unchanged,* and can be removed by applying a correction of equal magnitude and opposite sign. Index error and watch error can be removed by adjustment of the instrument. Lubber's line error can be removed by aligning the lubber's line with the longitudinal axis of the craft.

Another type of systematic error results from a nonstandard rate. If a watch is gaining four seconds per day, its readings will be in error by one second after an interval of six hours, eight seconds at the end of two days, etc. This principle is used in establishing a chronometer rate (art. 1908) for determination of chronometer error between comparisons of the chronometer with time signals. It can be eliminated by adjusting the rate. If a current is running and no allowance for it is made in the dead reckoning, the DR position is in error by an amount proportional to elapsed time. The error introduced by maintaining heading by means of an inaccurate compass is proportional to distance, as is the lateral error in a position line plotted from an inaccurate bearing.

One of the causes of equation of time (art. 1912) is the fact that the ecliptic, around which annual motion occurs, is not parallel to the celestial equator, around or parallel to which apparent daily motion takes place. The same type systematic error is involved in other measurements. Consider the measurement of bearing with a tilted compass card. Bearing is measured by a system of uniform graduations (degrees) of a circle (such as a compass card) in the horizontal plane. If the card is tilted, and its graduations are projected onto the horizontal plane, the circle becomes an ellipse with the graduations unequally spaced. Along the axis of tilt and a line perpendicular to it, directions are correct. But near the axis of tilt the graduations are too close together, and near the perpendicular they are too widely spaced. The error thus introduced is similar to that which would arise if a watch face were tilted but the motion of the hands remained horizontal. If it were tilted around the "3–9" line, it would appear to run slow near the hour and half hour, and fast near the quarter and three-quarter hours. If the direction to be observed is of an object above or below the horizontal, as the azimuth of a celestial body, measurement is made to the foot of the perpendicular through the object. The sight vanes of a compass move in a plane perpendicular to the compass card. Hence, if the card is tilted, measurement is made to the foot of a perpendicular to the card, rather than to the foot of a perpendicular to the horizontal, introducing an error which increases with the angle of tilt and also with the angle of elevation (or depression) of the object. This error is greatest along the axis of tilt, and zero along the perpendicular to it. Both of these tilt errors can be corrected by leveling the compass card.

A different type of tilt error occurs when a reflection takes place from a tilted surface, such as the ionosphere (art. 1007), the error being proportional to the angle of tilt. In some respects, this error is similar to coastal refraction of a radio wave (art. 1006).

Additional examples of systematic error are uncorrected deviation of the compass (art. 709), polarization error (art. 1203), error due to a position in a pattern of hyperbolas (art. 1109), error due to incorrect location of a loran transmitter (art. 1306), uncorrected parallax (art. 1620), and uncorrected personal error (art. 1507).

2904. Random errors are chance errors, unpredictable in magnitude or sign. They are governed by the laws of probability. If the altitude of a celestial body is

Error	No. of obs.	Percent of obs.
−10′	0	0. 0
− 9′	1	0. 2
− 8′	2	0. 4
− 7′	4	0. 8
− 6′	9	1. 8
− 5′	17	3. 4
− 4′	28	5. 6
− 3′	40	8. 0
− 2′	53	10. 6
− 1′	63	12. 6
0	66	13. 2
+ 1′	63	12. 6
+ 2′	53	10. 6
+ 3′	40	8. 0
+ 4′	28	5. 6
+ 5′	17	3. 4
+ 6′	9	1. 8
+ 7′	4	0. 8
+ 8′	2	0. 4
+ 9′	1	0. 2
+10′	0	0. 0
0	500	100. 0

TABLE 2904.—Normal distribution of random errors.

observed, the reading may be (1) too great, (2) correct, or (3) too small. If a number of observations are made, and there is no systematic error, the probability of a positive error is exactly equal to the probability of a negative error. This does not mean that every second observation having an error will be too great. However, the greater the number of observations, the greater is the probability that the number of positive errors will equal the number of negative ones, and that their magnitudes will correspond.

Suppose that 500 observations are made, with the results shown in table 2904. A close approximation of the plot of these errors is shown in figure 2904a. The plot has been modified slightly to constitute the **normal curve** of random errors, which is the same as the actual curve except that the normal curve *approaches* zero as the error increases, while the actual curve *reaches* zero at (+)10′ and (−)10′. The height of the curve at any point represents the percentage of observations that can be expected to have the error indicated at that point. The probability of any similar observation having any given error is the proportion of the number of observations having this error to the total number of observations, or the percentage expressed as a decimal. Thus, the probability of an observation having an error of (−)3′ is $\frac{40}{500}=\frac{1}{12.5}=0.08$ (8%).

If the area under the curve represents 100 percent of the observations, half the area (the shaded portion of figure 2904a) represents 50 percent of the observations. The value of the error at the limits of this shaded portion is often called the "50 percent error," or **probable error,** meaning that 50 percent of the observations can be expected to have less error, and 50 percent greater error. Similarly, the limits which contain the central 95 percent of the area denote the 95 percent error. The percentage of error is found mathematically. For a normal curve, each error is squared, the sum of the squares is divided by one less than the number of observations, and the square root of the quotient is determined. This value is called the **standard deviation** (*s*) or **root mean square.** In the illustration, the standard deviation is the square root of $0\times(-)10^2+1\times(-)9^2+2\times(-)8^2+4\times(-)7^2+9\times(-)6^2$, etc., divided by 499 or $\sqrt{\frac{4474}{499}}=\sqrt{8.966}=2.99$ (about 3). The standard deviation is the 68 percent error. The probability of the occurrence of an error of or less than a specific magnitude may be determined by the following relationship (with the answers for the illustration given):

$$
\begin{aligned}
50\%\ \text{error} &= \tfrac{2}{3}\times s = 2' \text{ (approx.)}\\
68\%\ \text{error} &= 1 \times s = 3' \text{ (approx.)}\\
95\%\ \text{error} &= 2 \times s = 6' \text{ (approx.)}\\
99\%\ \text{error} &= 2\tfrac{2}{3}\times s = 8' \text{ (approx.)}\\
99.9\%\ \text{error} &= 3\tfrac{1}{3}\times s = 10' \text{ (approx.)}
\end{aligned}
$$

Many of the errors of navigation are not of the "normal" type. In H.O. Pub. No. 214 (art. 2003) values of altitude can be taken only to the nearest 0.′1. The error might

have any value from (+) 0.′05 to (−) 0.′05, and any value within these limits is as likely to occur as any other of the same precision. The same is true of a sextant that cannot be read closer than 0.′1, and of a loran receiver that cannot be read closer than 1μs. These values refer to the single errors indicated, and not to the total error that might be involved. This is a **rectangular error,** so called because of the shape of its plot, as shown in figure 2904b. The 100 percent error is half the difference between readings. The 50 percent error is half this amount, the 95 percent error is 0.95 times this amount, etc.

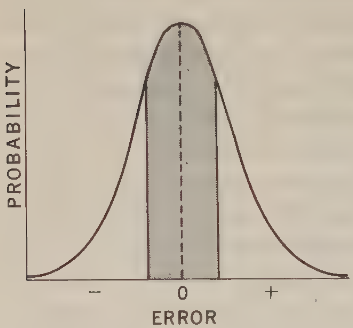


Figure 2904a—Normal curve of random error with 50 percent of area shaded. Limits of shaded area indicate probable error.

Still another type random error is encountered in navigation. If a compass is fluctuating periodically due to yaw of a ship, its motion slows as the end of a swing is approached, when the error approaches maximum value. If readings were taken continuously or at equal intervals of time, the interval being a small percentage of the total period of oscillation, the curve of errors would have a characteristic U-shape, as shown in figure 2904c. The same type error is involved in measurement of altitude of a celestial body from a wing of the bridge of a heavily rolling vessel, when the roll causes large changes in the height of eye. This type of error is called a **periodic error.** The effect is accentuated by the tendency of the observer to make readings near one of the extreme values because the instrument appears steadiest at this time. If it is impractical to make a reading at the center of the period, the error can be eliminated or reduced by averaging readings taken continuously or at short intervals, as indicated above. This is the method used in averaging type artificial-horizon sextants (art. 1513). Generally, better results can be obtained by taking maximum positive and maximum negative readings, and averaging the results.

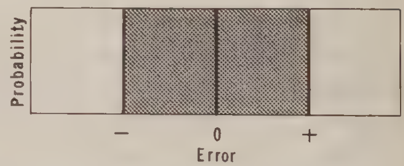


Figure 2904b.—Rectangular error, with 50 percent area shaded.

The curve of any type of random error is symmetrical about the line representing zero error. This means that in the ideal plot every point on one side of the curve is exactly matched by one on the other side, or for every positive error there is a negative error of the same magnitude. The average of all readings, considering signs, is zero. The larger the number of readings, the greater the probability of the errors fitting the ideal curve. Another way of stating this is that as the number of readings increases, the error of the average can be expected to decrease.

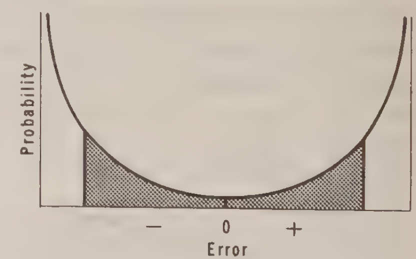


Figure 2904c.—Periodic error, with 50 percent area shaded.

2905. Combinations of errors.—Many of the results obtained in navigation are subject to more than one error. Chapter XVI lists 19 errors applicable to sextant altitudes. Some of these have several components. A number of possible errors are involved in the determination of computed altitude and azimuth. A rectangular error is possible in finding the altitude difference. Several additional errors may affect the accuracy of plotting. Thus, the line of position as finally plotted may include 30 errors or more. Corrections are applied for some of the larger ones, so that in each of these cases the applicable error is the difference

between the applied correction and the actual error. Thus, a dip correction may be applied for a height of eye of 30 feet, while the actual height at the moment of observation may be 31 feet 6 inches. Even if the height of eye is exactly 30 feet, a rectangular error may be involved in taking the dip correction from the table.

Corrections which might be random as far as an individual observation is concerned may be systematic for a series of observations. Thus, if the average or standard conditions upon which a correction is based do not exist at the time of observation the value at any given time is as likely to be greater as it is to be less than the standard amount. But if a number of observations are taken in quick succession, the error will be about the same for each.

If two or more errors are applicable to a given result, the total error is equal to the algebraic sum of all errors. Thus, if a given number is subject to errors of (+) 4, (−) 2, (−) 1, (+) 3, (+) 2, 0, and (−) 2, the total error is (+) 4. Systematic errors can be combined by adding the curves of individual errors. Thus, a magnetic compass may have a quadrantal error as shown by the top curve of figure 2905, and a semicircular error as shown by the second curve. The sum of these two errors is shown in the bottom curve. If, in addition, the compass has a constant error, the bottom curve is moved vertically upward or downward by the amount of the constant error, without undergoing a change of form. If the constant error is greater than the maximum value of the combined curves, all errors are positive or all are negative, but of varying magnitude.

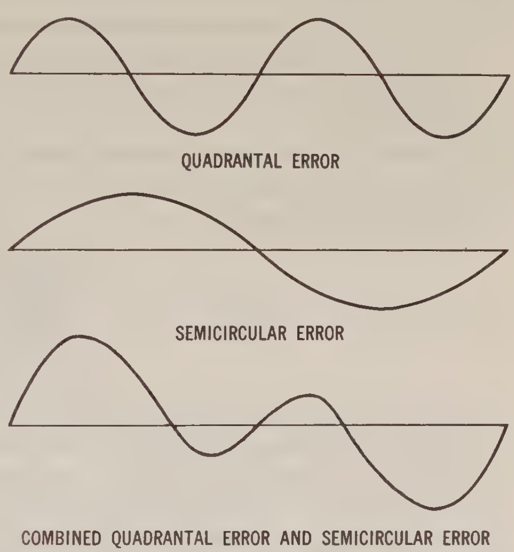


FIGURE 2905.—Combining systematic errors.

If a number of random errors are combined, the result tends to follow a normal curve regardless of the shape of the individual errors, and the greater the number, the more nearly the result can be expected to approach the normal curve (fig. 2904a). If a given result is subject to errors of plus or minus 3, 2, 1, 2, 4, 2, 1, 8, 1, and 2, the total error *could* be as much as 26 if all errors had the same sign. However, if these are truly random, the probability of them all having the same sign is only one in 1024. This is so because the chance of any one being positive (or negative) is $\frac{1}{2}$. That is, of a large number of results, approximately half will have any one particular correction positive (or negative). By the same reasoning, approximately half of the positive (or negative) results will have any one particular additional correction positive (or negative). Thus, the probability of any two particular corrections having the same sign is $\frac{1}{2}\times\frac{1}{2}=(\frac{1}{2})^2=\frac{1}{4}$. The probability of all ten corrections having the same sign is $(\frac{1}{2})^{10}=\frac{1}{1024}$. If there were 20 corrections, the probability of all having the same sign would be $(\frac{1}{2})^{20}=\frac{1}{1{,}048{,}576}$.

The standard deviation of the *sum* of such errors is found by squaring each error individually, *adding* the results and taking the square root of the sum. Thus, in the example, the following results are obtained:

Error	*Error Squared*
3	9
2	4
1	1
2	4
4	16
2	4
1	1
8	64
1	1
2	4
sum	108
square root	±10. 4

Thus, the standard deviation is ±10.4. Signs need not be considered because the square of either a positive or negative number is positive.

The individual errors have been treated as if they were fixed in amount. If they are 50 percent normal errors, the result is the standard deviations of the 50 percent errors; if 99 percent, the result is the standard deviations of the 99 percent errors, etc., *if* the individual errors are normal and independent. If they are of a different type, an adjustment is needed. Thus, the square of each rectangular error should be multiplied by the following factors:

50% error ⅔
95% error 1⅔
99% error 2½
99.9% error 4.

The information required to determine the standard deviation is usually not available to a navigator, because the probable magnitude of many of the individual errors has never been determined. However, the example given above reveals at least one interesting point which is highly practical. In the tabulation of errors, the largest has a value of eight. This single error accounts for less than one-third the total *possible* error, but its square is more than half the sum of squares. If this error could be eliminated, the standard deviation would be only 6.6. If it could be reduced to 5, a 37.5 percent reduction, the standard deviation would be reduced to 8.3. In contrast, if the next largest error, four, were reduced by three, a reduction of 75 percent, the standard deviation would be reduced by only 0.8, to 9.6. If the three errors of one each could be completely eliminated, the standard deviation would be reduced by only 0.2, to 10.2. In the reduction of total error, therefore, a relatively small reduction in a large error has a much greater effect upon the standard deviation than the same numerical reduction (larger percentage reduction) in a small error, because the result of a random error is proportional to its square.

Therefore, the perfection of one part of a process, sometimes at great expense or by the introduction of considerable inconvenience, may not be justified until larger errors are corrected. Thus, it would hardly be worth the effort and expense to build a loran receiver capable of making a reading to 0.1 μs (present receivers can be read to about 1 μs) as long as synchronization of signals may be in error as much as 2 μs or more. Conversely, the introduction of an additional small error may add considerably to the convenience of a process without materially affecting the accuracy. Thus, the use of some of the "short" methods of sight reduction (ch. XXI) without interpolation is justified if the interval of tabulation is small.

When both systematic and random errors are present in a process, both effects are present. An increase in the number of readings decreases the residual random error, but regardless of the number of readings, a systematic error is present in its entirety. Thus, if a number of Decca readings are made at a fixed point, the average should be a good approximation of the true value if there is no systematic error. But if the equipment is out of adjustment to the extent that the lane is incorrectly identified, no number of readings will correct this error. In this illustration, a constant error is combined with a normal random error. The normal curve has the correct shape, but is offset from the zero value.

Under some conditions, systematic errors can be eliminated from the results even when the magnitude is not determined. Thus, if two celestial bodies differ in azimuth by 180°, and the altitude of each is observed, the line midway between the lines of position resulting from these observations is free from any *constant* error in the *altitude* (such as abnormal refraction or dip, or incorrect IC). It would *not* be free from such a constant error as one in time (unless the bodies were on the celestial meridian). Similarly, a fix obtained by observations of three stars differing in azimuth by 120°, or four stars differing by 90° is free from constant error in the altitude, if the center of the figure made by the lines of position is used. The center of the figure formed by circles of position from distances of objects equally spaced in azimuth is free from a constant error in range. A constant error in bearing lines does not introduce an error in the fix if the objects are equally spaced in azimuth. In all of these examples, the correct position is *outside* the figure formed by the lines of position if all objects observed are on the same side of the observer (that is, if they lie within an arc of less than 180°).

2906. Most probable position.—Some navigators, particularly those of little experience, have been encouraged by the oversimplified definitions and explanations usually given in texts to conclude that the line of position is infallible, and that a fix is without error, overlooking the frequent incompatibility of these two notions. Too often the idea has prevailed that information is either all right or all wrong. An example is the practice of establishing an estimated position at the foot of the perpendicular from a dead reckoning position, or previous estimated position, to a line of position. The assumption is that the vessel *must* be somewhere on the line of position, and that the only value of the DR position is to locate which point on the line to use as the EP.

A more realistic concept is that of the **most probable position** (**MPP**), which recognizes the probability of error in *all* navigational information, and determines position by an evaluation of all available information, using the principles of errors.

Suppose a vessel were to start from a completely accurate position and proceed on dead reckoning. If course and speed over the bottom were of equal accuracy, the uncertainty of dead reckoning positions would increase equally in all directions with either distance or elapsed time (for any one speed these would be directly proportional and therefore either could be used). Therefore, a circle of uncertainty would grow around the dead reckoning position as the vessel proceeded. If the navigator had full knowledge of the distribution and nature of the errors of course and speed, and the necessary knowledge of statistical analysis, he could compute the radius of the circle of uncertainty, using the 50 percent, 95 percent, or other value of individual errors.

In ordinary navigation, this is not practicable, but based upon his experience and judgment, the navigator might estimate at any time the probable error of his dead reckoning or estimated position. With practice, he might acquire considerable skill in making this estimate. He would take into account, too, the fact that the area of uncertainty might better be represented by an ellipse than a circle, the major axis being along the course line if the probable error of the speed were greater than that of the course, and the minor axis being along the course line if the probable error of the

course were greater. He would recognize, too, that the size of the area of uncertainty would not grow in direct proportion to the distance or elapsed time, because disturbing factors such as wind and current could not be expected to remain of constant magnitude and direction. Also, he would know that the starting point of the dead reckoning would not be completely free from error.

At some future time additional positional information would be obtained. This might be a line of position from a celestial observation or by loran. This, too, would be accompanied by a probable error which might be computed if the necessary information and knowledge were available, but which in practice would be estimated. If the dead reckoning had started from a good position obtained by means of landmarks, the probable error of the initial position would be very small. At first the dead reckoning or estimated position would probably be more reliable than a line of position obtained by celestial observation or loran. But at *some* distance the two would be equal, and beyond this the line of position might be more accurate.

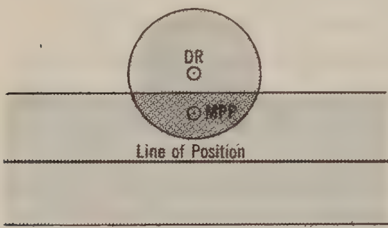


FIGURE 2906a.—A most probable position based upon a dead reckoning position and line of position having equal probable errors.

However, the determination of most probable position does not depend upon determination of *which* information is most accurate. In figure 2906a a dead reckoning position is shown surrounded by a circle of uncertainty. A line of position is also shown, with its area of uncertainty. The most probable position is within the overlapping area, and if the uncertainty of the dead reckoning position and that of the line of position are about equal, it might be taken at the center of the area. If the overall errors are considered normal, and they are probably approximately so, *the effect of each is proportional to its square* (art. 2905). Thus, if the probable error of a dead reckoning position is three miles, and that of a line of position is two miles, the most probable position is nearer the line of position, being at a distance equal to $\frac{2^2}{3^2}=\frac{4}{9}$ that from the dead reckoning position (or $\frac{9}{13}$ of the perpendicular distance from the dead reckoning position to the line of position).

If a fix is obtained from two lines of position, the area of uncertainty is a circle if the lines are perpendicular, have equal probable errors, and these errors can be considered normal. If one is considered more accurate than the other, the area is an ellipse, the two axes being proportional to the *squares* of the two errors. As shown in figure 2906b, it is also an ellipse if the probable error of each is equal and the lines cross at an oblique angle. If the errors are unequal, the major axis of the ellipse is more nearly in line with the line of position having the smaller probable error. If the angle between lines is very small, they are better considered a single line of position in the direction of the major axis of the ellipse.

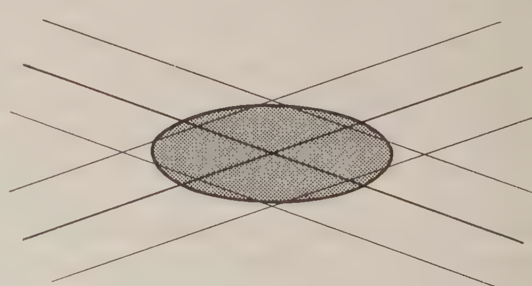


FIGURE 2906b.—Ellipse of uncertainty with lines of position of equal probable errors crossing at an oblique angle.

If a fix is obtained from three or more lines of position, and the error of each line is normal and equal to that of the others, the most probable position is the center of the figure. By "center" is meant that point within the figure which is equidistant from the sides. If the lines are of unequal probable error, the distance of the most

probable position from each line of position is proportional to the *square* of the probable error of that line. Thus, if three lines have probable errors of one, two, and three miles, respectively, the distances of the most probable position from the lines are in the ratio of one, four, and nine, respectively.

In the discussion of most probable position from lines of position, it has been assumed that no other positional information is available. Usually, this is an incorrect assumption, for there is nearly always a dead reckoning or estimated position. This can be considered in any of several ways. The square of its probable error can be used in the same manner as the square of the probable error of each line of position. A most probable position based upon the dead reckoning or estimated position and the most reliable line of position might be determined as explained above, and that line of position replaced with a new one parallel to it but passing through the most probable position just determined. This adjusted line of position can then be assigned a smaller probable error and used with the other lines of position to determine the overall most probable position. A third way is to establish a probable error for the fix, and consider the most probable position as that point along the straight line joining the fix and the dead reckoning or estimated position, the relative distances being equal to the square of the probable error of each position.

The value of the most probable position determined as suggested above depends upon the degree to which the various errors are in fact normal, and the accuracy with which the probable error of each is established. From a practical standpoint, the second factor is largely a matter of judgment based upon experience. It might seem that interpretation of results and establishment of most probable position is a matter of judgment anyway, and that the procedure outlined above is not needed. If a person will follow this procedure while gaining experience, and evaluate his results, the judgment he develops should be more reliable than if developed without benefit of a knowledge of the principles involved. The important point to remember is that the relative effects of normal random errors are proportional to their *squares*.

Systematic errors are treated differently. Generally, an attempt is made to discover the errors and eliminate them or compensate for them. In the case of a position determined by three or more lines of position resulting from readings with constant error, the error might be eliminated by finding and applying that correction (including sign) which will bring all lines through a common point.

2907. Mistakes.—The recognition of a mistake, as contrasted with an error (art. 2902), is not always easy, since a mistake may have any magnitude, and may be either positive or negative. A large mistake should be readily apparent if the navigator is alert and has an understanding of the size of error to be reasonably expected. A small mistake is usually not detected unless the work is checked.

If results by two methods are compared, as a dead reckoning position and a line of position, exact agreement is not to be expected. But if the discrepancy is unreasonably large, a mistake is logically suspected. The definition of "unreasonably large" is a matter of opinion. If the 99.9 percent areas of the two results just touch, it is *possible* that no mistake has been made. However, the *probability* of either one having so great an error is remote if the errors are normal. The probability of both having 99.9 percent error of opposite sign at the same instant is very small indeed. Perhaps a reasonable standard is that unless the most accurate result lies within the 95 percent area of the least accurate result, the possibility of a mistake should be investigated. Thus, if the areas of uncertainty shown in figure 2906a represent the 95 percent areas, it is probable that a mistake has been made.

As in other matters pertaining to navigation, judgment is important. The use to be made of the results is certainly a consideration. In the middle of an ocean pas-

sage a mistake is usually not serious, and will undoubtedly be corrected before it jeopardizes the safety of the vessel. But if landfall is soon to be made, or if search and rescue operations are to be based upon the position, almost any mistake is intolerable.

2908. Miscellaneous.—The correct identification of the nature of an error is important if the error is to be handled intelligently. Thus, the statement is sometimes made that a radio bearing need not be corrected if the receiver is within 50 miles of the transmitter. The need for a correction arises from the fact that radio waves are assumed to follow great circles, and if radio bearings are to be plotted on a Mercator chart, the equivalent rhumb line is needed. The statement regarding 50 miles implies that the size of the correction is proportional to distance only. It overlooks the fact that latitude and direction of the bearing line are also important factors, and is therefore a dangerous statement unless its limitations are understood.

The recognition of the type of error is also important. A systematic error has quite a different effect than a random error, and cannot be reduced by additional readings unless some method or procedure is instituted which will cause the errors to cancel each other. If a position is subject to a rectangular error only, its 100 percent circle has twice the radius and four times the area of the 50 percent circle. But if the error is normal, the 95 percent circle has approximately three times the radius and *nine* times the area of the 50 percent circle. It is not correct to suppose that a craft is as likely to be at one point within a circle of uncertainty as at any other point. If the error is normal, the probability might be represented by a three-dimensional figure formed by rotating the normal curve (fig. 2904a) around its axis of symmetry.

The probable error is usually of greater interest than the "average" value. The average of a large number of normal errors approaches zero, but the probable error might be quite large. An average or mean value determined by a number of observations is sometimes given with its probable error. Thus, a person might make a number of measurements of the speed of light and state his results as $299{,}792 \pm 2$ kilometers per second.

A person who understands the nature of errors avoids many pitfalls. Thus, the magnitude of the errors of individual lines of position is not a reliable indication of the size of the error of the fix obtained from them. The size of the triangle formed by three lines of position has often been used as a guide to the accuracy of the fix, although a large triangle might be the result of a large constant error if the objects observed are equally spaced in azimuth. On the other hand, two lines of position with small errors might produce a fix having a much larger error if the lines cross at a small angle.

The size of a triangle of position might be deceptive for another reason. A constant error in time shifts all lines of position from celestial observation an approximately equal amount (in minutes of arc) toward the east or toward the west. If all objects observed for a fix are on the same side of the observer, a constant error in measurement shifts all objects *and the fix*, so that if the constant error is larger than the random error, the actual position is outside the figure formed by the lines of position.

References

Croxton, F. E., and Cowden, D. J. *Applied General Statistics.* New York, Prentice-Hall, 1939.

Deming, W. E. *Statistical Adjustment of Data.* New York, Wiley, 1943.

Mills, F. C. *Statistical Methods.* New York, Holt, 1929.

PART SIX

OCEANOGRAPHY

PART SIX

OCEANOGRAPHY

CHAPTER XXX

THE OCEANS

3001. Introduction.—**Oceanography** is the application of the sciences to the phenomena of the oceans. It includes a study of their forms; physical, chemical, and biological features; and phenomena. Thus, it embraces the widely separated fields of geography, geology, chemistry, physics, and biology. Many subdivisions of these sciences, such as sedimentation, ecology (biological relationship between organisms and their environment), bacteriology, biochemistry, hydrodynamics, acoustics, and optics, have been extensively studied in the oceans.

The oceans cover 70.8 percent of the surface of the earth. The Atlantic covers 16.2 percent, the Pacific 32.4 percent (3.2 percent more than the land area of the entire earth), the Indian Ocean 14.4 percent, and marginal and adjacent areas (of which the largest is the Arctic Ocean) 7.8 percent. Their extent alone makes them an important subject for study. However, greater incentive lies in their use for transportation, their influence upon weather and climate, and their potentiality as a source of power, food, fresh water, and mineral and organic substances.

3002. History of oceanography.—The earliest studies of the oceans were concerned principally with problems of navigation. Information concerning tides, currents, soundings, ice, and distances between ports was needed as ocean commerce increased. According to Posidonius, a depth of 1,000 fathoms had been measured in the Sea of Sardinia as early as the second century BC. About the middle of the 19th century, the Darwinian theories of evolution gave a great impetus to the collection of marine organisms, since it is believed by some that all terrestrial forms have evolved from oceanic ancestors. Later, the serious depletion of many fisheries called for investigation of the relation of the economically valuable organisms to the physical characteristics of their environment, especially in northwestern Europe and off Japan. Still later, the growing use of the oceans in warfare, particularly after the development of the submarine, required that much effort be expended in problems of detection and attack, resulting in the study of many previously neglected scientific aspects of the sea.

Oceanographic exploration. Exploration of the seas was primarily geographical until the 19th century, although the accumulated observations of seafarers, as recorded in the early charts and sailing directions, often included data on tides, currents, and other oceanographic phenomena. The great voyages of discovery, particularly those beginning in 1768 with Captain Cook, and continued by such commanders as La Pérouse, Bellingshausen, and Wilkes, included scientists in their complements. However, scientific work on the oceans at this period was severely limited by lack of suitable instruments for probing conditions below the surface. Meanwhile, Lieutenant Matthew Fontaine Maury, USN, working in the forerunner of the U. S. Navy Hydrographic Office in Washington, developed to a high degree of perfection the analysis of log-book observations. His first results, published in 1848, were of great importance to ship operations in the recommendation of favorable sailing routes, and they stimulated international cooperation in the fields of oceanography and marine meteorology.

In the rapid advances in technology after 1850, oceanographic instrumentation problems were not neglected, with the result that the British Navy in 1872–76 was able to send HMS *Challenger* around the world on the first purely deep-sea oceano-

graphic expedition ever attempted. Her bottom samples, as analyzed by Sir John Murray, laid the foundation of geological oceanography, and 77 of her sea water samples, analyzed by C. R. Dittmar, proved for the first time that various constituents of the salts in sea water are everywhere in virtually the same proportions.

Since that time, the coastal waters and fishing banks of many nations have been extensively studied, and numerous vessels of various nationalities have conducted work on the high seas. Notable among these have been the American *Albatross* from 1882 to 1920; the Austrian *Pola* in the Mediterranean and Red Seas between 1890 and 1896; the Danish *Dana*, which during its voyages of 1920–22 discovered the breeding place of the European eels in the Sargasso Sea; the American *Carnegie* in 1927–29; the German *Meteor* in the Atlantic from 1928 to 1938; and the British *Discovery II* in the antarctic between 1930 and 1939. Notable also were the drifts of the Norwegian vessels *Fram* and *Maud* in the arctic ice pack from 1893 to 1896 and 1918 to 1925, respectively; the attempt by Sir George Hubert Wilkins to operate under the ice in the British submarine *Nautilus* in 1931; and the Russian station set up at the north pole in 1937, which made observations from the drifting pack ice.

At the same time, investigations pursued ashore provided the theoretical basis for the explanation of ocean currents, under the leadership of Helland-Hansen in Norway and Ekman and the Bjerknes in Sweden, while Martin Knudsen in Denmark worked out the precise details of the relationship between chlorinity, salinity, and density, enabling the theories to be verified by field observations.

During World War II, basic investigations were interrupted while work on purely military applications of oceanography was carried out. Deep-sea expeditions were renewed by the Swedish *Albatross* after the war, followed by the Danish *Galathea*, the second British *Challenger* (built in 1931) and *Discovery II* in the antarctic, and vessels of the American Scripps Institution in the Pacific. Oceanographic work was carried out by Americans and Russians in the arctic. By 1961, a total of ten Russian and three United States drifting ice stations had been established. Two United States stations were also established aboard floating ice islands.

Institutions. Among the leading oceanographic institutions in Europe are the Geophysical Institute of the University of Bergen in Norway; the Oceanographic Institute at Göteborg, Sweden; the National Institute of Oceanography in Great Britain; the German Hydrographic Institute in Hamburg; and the Museum of Oceanography at Monaco. The Marine Biological Station at Naples, Italy, has served as a model for others throughout the world.

In the Far East, the Hydrographic Division of the Maritime Safety Agency is perhaps the most prominent of a number of Japanese oceanographic activities. The Institute of Oceanology at Vladivostok is the foremost oceanographic establishment on the Asiatic mainland.

Canada maintains the Pacific Oceanographic Group at Nanaimo, B. C., and the Atlantic Oceanographic Group at St. Andrews, N. B. In the United States, the leading nongovernmental oceanographic institutions include the Scripps Institution of Oceanography of the University of California, La Jolla, Calif.; the Department of Oceanography of the University of Washington, Seattle, Wash.; Woods Hole Oceanographic Institution, Woods Hole, Mass.; the Marine Laboratory of the University of Miami, Coral Gables, Fla.; and the Department of Oceanography of Texas A. & M. College, College Station, Tex.

There exist also various international organizations in the field of oceanography, which coordinate and promote international cooperation. The International Council for the Exploration of the Sea, with headquarters in Copenhagen, which was established

to exchange data on fisheries problems in the waters of northwestern Europe, has been notably successful, and similar organizations have been established in other areas.

3003. Origin of the oceans.—Although many leading geologists still disagree with the conclusion that the structure of the continents is fundamentally different from that of the oceans, there is a growing body of evidence in support of the theory that the rocks underlying the ocean floors are more dense than those underlying the continents. According to this theory, all the earth's crust floats on a central liquid core, and the portions that make up the continents, being lighter, float with a higher freeboard. Thus, the thinner areas, composed of heavier rock, form natural basins where water has collected.

The origin of the water in the oceans is also controversial. Although some geologists have postulated that all the water existed as vapor in the atmosphere of the primeval earth, and that it fell in great torrents of rain as soon as the earth cooled sufficiently, another school holds that the atmosphere of the original hot earth was lost, and that the water gradually accumulated as it was given off in steam by volcanoes or worked to the surface in hot springs.

Most of the water on the earth's crust is now in the oceans—about 328,000,000 cubic statute miles, or about 85 percent of the total. The mean depth of the ocean is 2,075 fathoms, and the total area is 139,000,000 square statute miles.

3004. Oceanographic chemistry may be divided into three main parts: the chemistry of (1) sea water, (2) marine sediments, and (3) organisms living in the sea. The first is of particular interest to the navigator.

Chemical properties of sea water are determined by analyzing samples of water obtained at various places and depths. Samples from below the surface are obtained by means of metal bottles designed for this purpose. The open bottles are attached at suitable intervals to a wire lowered into the sea. When they reach the desired depths, a metal ring or **messenger** is dropped down the wire. When the messenger arrives at the first bottle, it causes the bottle to close, trapping a sample of the water at that depth, and releasing a second messenger which travels on down the wire. The process is repeated at each bottle until all are closed, when they are hauled up and each bottle detached as it comes within reach. Of the various types devised, the **Nansen bottle** is the most widely used. It is equipped with a removable frame for attaching a thermometer.

For centuries table salt has been produced from sea water by natural evaporation in countries with a suitable climate. More recently, practical industrial processes have been developed for recovering bromine and magnesium from the sea. Calcium carbonate, in the form of oyster shells or coral rock, is obtained after precipitation by living organisms.

Three elements in the sea, silicon, nitrogen, and phosphorus, are most significant in the growth of living organisms.

Certain of the elements, notably chlorine, bromine, sulfur, and boron, are much more abundant in the ocean than in the rest of the earth's crust. These elements are among the more volatile ones, and their abundance in the sea tends to confirm the hypothesis that volcanic action is largely responsible for the present oceans.

In many cases, chemical relationships influence the abundance of elements in the sea. Barium, for example, forms a sulfate of very limited solubility, and thus the high concentration of sulfate in sea water limits the possible amount of dissolved barium. Thus, the concentration of many elements is limited by the solubility of their most insoluble compounds. Table 3004 indicates the amounts of the various elements found in solution in the oceans.

In addition to dissolved solids, sea water contains in solution all of the gases found in the atmosphere (art. 1410), but not in the same proportions. The most abundant is nitrogen, which, however, because of its chemical inertness, does not enter into biological processes. Oxygen, produced in the surface layers by plant photosynthesis (art. 3024) or dissolved directly from the atmosphere, is of major importance for all forms of life. By biological activity, the oxygen concentration at depths below the surface is usually reduced to a fraction of the surface values, and under certain conditions, owing either to the presence of abundant oxidizable material, or a stagnant condition, or both, it may become completely exhausted. Under these conditions, sulfate-reducing bacteria produce hydrogen sulfide gas from the abundant sulfate in sea water. The existence of such conditions is often indicated to the mariner by the blackening of white lead paint, a well-known phenomenon in badly polluted estuaries.

Hydrogen sulfide may also be encountered at great depths in the ocean. The fiords of Norway, deep channels cut by former glaciers, are characterized in general by shallow sills at the entrances, where the terminal moraines of the glaciers were deposited. These sills serve as barriers to the mixing and renewing of the deeper waters within the fiords, and, as a result, conditions producing hydrogen sulfide are frequently encountered.

Element	Parts per million	Element	Parts per million
Aluminum	0. 01	Manganese	0. 001–0. 01
Arsenic	0. 003	Mercury	0. 00003
Barium	0. 006	Molybdenum	0. 01
Boron	4. 7	Nickel	0. 0005
Bromine	66	Nitrogen	0. 01–0. 7
Cadmium	0. 00006	Phosphorus	0. 001–0. 1
Calcium	408	Potassium	387
Carbon	28	Radium	0. 00000000003
Cerium	0. 0004	Rubidium	0. 3
Cesium	0. 001	Scandium	0. 00004
Chlorine	19, 353	Selenium	0. 004
Chromium	0. 00005	Silicon	0. 02–4. 0
Cobalt	0. 0005	Silver	0. 0003
Copper	0. 001–0. 01	Sodium	10, 769
Fluorine	1. 4	Strontium	10
Gallium	0. 0005	Sulfur	901
Gold	0. 000004	Thorium	0. 0007
Iodine	0. 05	Tin	0. 003
Iron	0. 002–0. 02	Titanium	0. 001
Lanthanum	0. 0003	Uranium	0. 0033
Lead	0. 003	Vanadium	0. 001
Lithium	0. 2	Yttrium	0. 0003
Magnesium	1, 297	Zinc	0. 01

TABLE 3004.—Elements found in solution in the ocean. The amounts are for a salinity of 35 parts per thousand. These values are based upon a tabulation by Professor E. D. Goldberg of the Scripps Institution of Oceanography.

A similar situation exists in the Black Sea. Here the Bosporus and Dardanelles act as sills, and all the deeper water of the Black Sea is cut off from contact with the surface waters, which, diluted by the runoff from the Danube and Don Rivers, have a salinity (art. 3006) of about 17.5 parts per thousand. The deeper water, renewed only by the bottom current through the Bosporus, has a salinity of 22 parts per thousand, and the great density difference between the surface layers and the deeper water effectively prevents mixing and the transfer of dissolved oxygen from the surface layers to greater depths. Below about 100 fathoms, therefore, the waters of the Black Sea are

completely devoid of dissolved oxygen, containing instead large concentrations of hydrogen sulfide.

No living creatures exist under these conditions except anaerobic bacteria, which comprise the only form of life in five-sixths of the waters of the Black Sea.

3005. Physical properties of sea water are dependent primarily upon salinity, temperature, and pressure. However, factors like motion of the water and the amount of suspended matter affect such properties as color and transparency, conduction of heat, absorption of radiation, etc.

3006. Salinity is the amount of dissolved solid material in the water, usually expressed as parts per thousand (by weight), under certain standard conditions. This is not the same as **chlorinity,** which is equal approximately to the amount of chlorine in the water. (Actually the chlorine content is about 1.00045 times the chlorinity as determined by standard procedures.) The two have been found to be related empirically by the formula

$$\text{salinity} = 0.03 + 1.805 \times \text{chlorinity}.$$

Since the determination of salinity is a slow and difficult process, while chlorinity can be determined easily and accurately by titration with silver nitrate, it is customary to determine chlorinity and compute salinity by the formula given above. By this process, salinity can be determined with an error not exceeding 0.02 parts per thousand. It generally varies between about 33 and 37 parts per thousand, the average being about 35 parts per thousand. However, when the water has been diluted, as near the mouth of a river or after a heavy rainfall, the salinity is somewhat less; and in areas of excessive evaporation, the salinity may be as high as 40 parts per thousand. In certain confined bodies of water, notably the Great Salt Lake in Utah, and the Dead Sea in Asia Minor, the salinity is several times this maximum. Chlorinity accounts for about 55 percent of salinity, the average being about 19 parts per thousand.

3007. Temperature in the ocean varies widely, both horizontally and with depth. Maximum values of about 90° F are encountered in the Persian Gulf in summer, and the lowest possible values of about 28° F (the usual minimum freezing point of sea water) occur in polar regions. H.O. Pub. No. 225, *World Atlas of Sea Surface Temperatures,* shows in detail the average sea surface temperatures for each month. The following tabulation gives the percentage distribution of temperatures for the world for the months of February and August, as derived from this source:

Surface temperature ° F	*Percentage of area of ocean February*	*August*
<35	12. 0	13. 1
35–40	6. 5	3. 3
40–45	4. 0	3. 0
45–50	4. 5	5. 0
50–55	4. 0	6. 5
55–60	5. 0	6. 0
60–65	5. 5	6. 3
65–70	8. 0	7. 0
70–75	10. 0	10. 4
75–80	17. 5	16. 5
80–85	23. 0	22. 7
85–90	0. 0	0. 2

The vertical distribution of temperature in the sea nearly everywhere shows a decrease of temperature with depth. Since colder water is denser, it sinks below warmer

water. This results in a temperature distribution just opposite to that in the earth's crust, where temperature increases with depth below the surface of the ground.

In general, in the sea there is usually a mixed layer of isothermal water below the surface, where the temperature is the same as that of the surface. This layer is best developed in the trade-wind belts, where it may extend to a depth of 100 fathoms; in temperate latitudes in the spring, it may disappear entirely. Below this layer is a zone of rapid temperature decrease, called the **thermocline,** to the temperature of the deep oceans. At a depth greater than 200 fathoms, the temperature everywhere is below 60°F, and in the deeper layers, fed by cooled waters that have sunk from the surface in the arctic and antarctic, temperatures as low as 33°F exist.

In the deepest ocean basins, the temperature increases slightly with depth, the increase being about 1°F at 3,000 fathoms. The warming is believed to be caused more by the slight compression of sea water than by heat from the earth's crust.

A typical curve of temperature at various depths is shown in figure 3503a. Temperature at any desired depth is determined by means of a **reversing thermometer** attached to a Nansen bottle (art. 3004). When the bottle closes, the thermometer measures the temperature to within 0°.04 F, thus providing a reading for a particular time and point. Within about 75 fathoms of the surface, where the principal changes occur, a continuous record of temperature can be obtained by an instrument called a **bathythermograph,** invented by Spilhaus in 1938.

3008. Pressure.—In oceanographic work, pressure is generally expressed in units of the centimeter-gram-second system. The basic unit of this system is one dyne per square centimeter. This is a very small unit, one million constituting a practical unit called a bar, which is nearly equal to one atmosphere. Atmospheric pressure is often expressed in terms of **millibars,** 1,000 of these being equal to one bar. In oceanographic work, water pressure is commonly expressed in terms of **decibars,** ten of these being equal to one bar. One decibar is equal to nearly 1½ pounds per square inch. This unit is convenient because it is very nearly the pressure exerted by one meter of water. Thus, the pressure in decibars is approximately the same as the depth in meters, the unit of depth customarily used in oceanographic research. In terms more familiar to the mariner, the pressure at various depths is as follows:

Depth in fathoms	*Pressure in pounds per square inch*
1, 000	2, 680
2, 000	5, 390
3, 000	8, 100
4, 000	10, 810
5, 000	13, 520

The increase in pressure with depth is nearly constant because water is only slightly compressible.

Although virtually all of the physical properties of sea water are affected to a measurable extent by pressure, the effect is not as great as those of salinity and temperature. Pressure is of particular importance to submarines, directly because of the stress it induces in the materials of the craft, and indirectly because of its effect upon buoyancy.

3009. Density is mass per unit volume. Oceanographers use the centimeter-gram-second system, in which density is expressed as grams per cubic centimeter. The ratio of the density of a substance to that of a standard substance under stated conditions is called **specific gravity.** By definition, the density of distilled water at 4°C (39°.2 F) is one gram per milliliter (approximately one gram per cubic centimeter).

Therefore, if this is used as the standard, as it is in oceanographic work, density and specific gravity are virtually identical numerically.

The density of sea water depends upon salinity, temperature, and pressure. At constant temperature and pressure, density varies with salinity or, because of the relationship between this and chlorinity, with the chlorinity. A temperature of 32° F and atmospheric pressure are considered standard for density determination. The effects of thermal expansion and compressibility are used to determine the density at other temperatures and pressures. The density at a particular pressure affects the buoyancy of submarines. It is also important in its relation to ocean currents.

The greatest changes in density of sea water occur at the surface, where the water is subject to influences not present at depths. Here density is decreased by precipitation, run-off from land, melting of ice, or heating. When the surface water becomes less dense, it tends to float on top of the more dense water below. There is little tendency for the water to mix, and so the condition is one of stability. The density of surface water is increased by evaporation, formation of sea ice, and by cooling. If the surface water becomes more dense than that below, it sinks to the level at which other water has the same density. Here it tends to spread out to form a layer, or to increase the thickness of the layer below it. The less dense water rises to make room for it, and the surface water moves in to replace that which has descended. Thus, a convective circulation is established. It continues until the density becomes uniform from the surface to the depth at which a greater density occurs. If the surface water becomes sufficiently dense, it sinks all the way to the bottom. If this occurs in an area where horizontal flow is unobstructed, the water which has descended spreads to other regions, creating a dense bottom layer. Since the greatest increase in density occurs in polar regions, where the air is cold and great quantities of ice form, the cold, dense polar water sinks to the bottom and then spreads to lower latitudes. This process has continued for a sufficiently long period of time that the entire ocean floor is covered with this dense polar water, thus explaining the layer of cold water at great depths in all the oceans.

In some respects, oceanographic processes are similar to those occurring in the atmosphere (ch. XXXVIII). The convective circulation in the ocean is somewhat similar to that in the atmosphere. Water masses having nearly uniform characteristics are analogous to air masses.

3010. Compressibility.—Sea water is nearly incompressible, its coefficient of compressibility being only 0.000046 per bar under standard conditions. This value changes slightly with changes of temperature or salinity. The effect of compression is to force the molecules of the substance closer together, causing it to become more dense. Even though the compressibility is low, its total effect is considerable because of the amount of water involved. If the compressibility of sea water were zero, sea level would be about 90 feet higher than it now is.

3011. Viscosity is resistance to flow. Sea water is slightly more viscous than fresh water. Its viscosity increases with greater salinity, but the effect is not nearly as marked as that occurring with decreasing temperature. The rate is not uniform, becoming greater as the temperature decreases. Because of the effect of temperature upon viscosity, an incompressible object might sink at a faster rate in warm surface water than in colder water below. However, for most objects, this effect may be more than offset by the compressibility of the object.

The actual relationships existing in the ocean are considerably more complicated than indicated by the simple explanation given above, because of turbulent motion within the sea. The disturbing effect is called **eddy viscosity.**

3012. Specific heat is the amount of heat required to raise the temperature of a unit mass of a substance a stated amount. In oceanographic work, specific heat is stated, in centimeter-gram-second units, as the number of calories needed to raise one gram of the substance 1°C. Specific heat at constant pressure is usually the quantity desired when liquids are involved, but occasionally the specific heat at constant volume is required. The ratio of these two quantities has a direct relationship to the speed of sound in sea water.

The specific heat of sea water decreases slightly as salinity increases. However, it is much greater than that of land. This accounts, in part, for the greater temperature range of land and the atmosphere above it, resulting in monsoons (art. 3810) and the familiar land and sea breezes of tropical and temperate regions (art. 3814).

3013. Thermal expansion.—The rate of expansion with increased temperature is greater in sea water than in fresh water. Thus, at temperature 15°C (59°F), and atmospheric pressure, the coefficient of thermal expansion is 0.000151 per degree Celsius for fresh water and 0.000214 per degree Celsius for water of 35 parts per thousand salinity. The coefficient of thermal expansion increases not only with greater salinity, but also with increased temperature and pressure. At 35 parts per thousand, the coefficient of surface water increases from 0.000051 per degree Celsius at 0°C (32°F) to 0.000334 per degree Celsius at 30°C (86°F). At a constant temperature of 0°C (32°F) and a salinity of 34.85 parts per thousand, the coefficient increases to 0.000276 per degree Celsius at a pressure of 10,000 decibars (at a depth of approximately 10,000 meters).

3014. Thermal conductivity.—In water, as in other substances, one method of heat transfer is by conduction. Fresh water is a poor conductor of heat, having a coefficient of thermal conductivity of 0.00139 calories per second per centimeter per degree Celsius. For sea water it is slightly less but increases with greater temperature or pressure.

However, if turbulence is present, which it nearly always is to some extent in the ocean, the processes of heat transfer are altered. The effect of turbulence is to increase greatly the rate of heat transfer. The "eddy" coefficient used in place of the still-water coefficient is so many times larger, and so dependent upon the degree of turbulence that the effects of temperature and pressure are not important.

3015. Electrical conductivity.—Water without impurities is a very poor conductor of electricity. However, when salt is in solution in water, the salt molecules are ionized (art. 1007) and therefore are carriers of electricity. Hence, the electrical conductivity of sea water is directly proportional to the number of salt molecules in the water. For any given salinity, the conductivity increases with an increase in temperature.

3016. Radioactivity.—Although the amount of radioactive material in sea water (tab. 3004) is very small, this material is present in marine sediments to a greater extent than in the rocks of the earth's crust. This is probably due to precipitation of radium or other radioactive material from the water. The radioactivity of the top layers of sediment is less than that of deeper layers. This may be due to absorption of radioactive material in the soft tissues of marine organisms.

3017. Refractive index (art. 1613) of sea water increases as salinity becomes greater, or as temperature decreases. Since it varies with frequency of the radiant energy, the "D line" of sodium is usually used as the standard for comparison.

3018. Surface tension of water in dynes per square centimeter is approximately equal to $75.64-0.144T+0.0399Cl$, where T is temperature in degrees Celsius (centigrade) and Cl is the chlorinity of the water in parts per thousand. As indicated by the last term, the surface tension increases with chlorinity, and is therefore a little

more for sea water than for fresh water. However, the presence of impurities causes it to be somewhat less than indicated by the formula.

3019. Transparency of sea water varies with the number, size, and nature of particles suspended in the water, as well as with the nature and intensity of illumination. The rate of decrease of light energy with depth is called the "extinction coefficient." The earliest method of measuring transparency was by means of a **Secchi disk,** a white disk 30 centimeters (a little less than one foot) in diameter. This was lowered into the sea, and the depth at which it disappeared was recorded. In coastal waters the depth varies from about 5 to 25 meters (16 to 82 feet). Offshore, the depth is usually about 45 to 60 meters (148 to 197 feet). The greatest recorded depth at which the disk has disappeared is 66 meters (217 feet), in the Sargasso Sea.

Although the Secchi disk still affords a simple method of measuring transparency, more exact methods have been devised.

3020. Color.—The color of sea water varies considerably. Water of the Gulf Stream is a deep indigo blue, while a similar current off Japan was named Kuroshio (Black Stream) because of the dark color of its water. Along many coasts the water is green. In certain localities a brown or brownish-red water has been observed.

Offshore, some shade of blue is common, particularly in tropical or sub-tropical regions. It is due to scattering of sunlight by minute particles suspended in the water, or by molecules of the water itself. Because of its short wave length, blue light is more effectively scattered than light of longer waves. Thus, the ocean appears blue for the same reason that the sky does (art. 3817). The green color often seen near the coast is a mixture of the blue due to scattering of light and a stable soluble yellow pigment associated with phytoplankton (art. 3024). Brown or brownish-red water receives its color from large quantities of certain types of **algae,** microscopic plants in the sea.

3021. Marine geology is a branch of oceanography dealing with bottom relief, particularly the characteristics of ocean basins and the geological processes that brought them into being and tend to alter them, as well as with marine sediments.

3022. Bottom relief.—Compared to land, relatively little is known of relief below the surface of the sea. It would be difficult to withhold knowledge of a major land feature in an area often visited by man, but the sea has until recent years proved an effective barrier to acquisition of knowledge of features below its surface. Although soundings of 1,000 fathoms were probably made as early as the second century BC (art. 3002), the number of deep sea soundings by means of a weight lowered to the bottom has been relatively few. The process is a time-consuming one requiring special equipment. Several hours are needed for a single sounding. Since the development of an effective echo sounder (art. 619) in 1922, the number of deep sea soundings has greatly increased. Later, a recording echo sounder was developed to permit the continuous tracing of a **bottom profile.** This has assisted materially in the acquisition of knowledge of bottom relief. By this means, many flat-topped seamounts (called **guyots**), mountain ranges, and other features have been discovered. Although the main features are becoming known, a great many details are yet to be learned.

Along most of the coasts of the continents, the bottom slopes gradually downward to a depth of about 100 fathoms or somewhat less, where it falls away more rapidly to greater depths. This **continental shelf** averages about 30 miles in width, but varies from nothing to about 800 miles, the widest part being off the Siberian arctic coast. A similar shelf extending outward from an island or group of islands is called an **insular shelf.** At the outer edge of the shelf, the steeper slope of 2° to 4° is called the **conti-**

nental talus or **continental slope,** or the **insular talus** or **insular slope,** according to whether it surrounds a continent or group of islands. The shelf itself is not uniform, but has numerous hills, ridges, terraces, and canyons, the largest being comparable in size to the Grand Canyon.

As a general rule, the slope of the deep sea bottom is gradual, averaging between 20′ and 40′, but there are many exceptions to this. Off a volcanic island it may be as much as 45°. The relief of the ocean floor is comparable to that of land. Both have steep, rugged mountains, deep canyons, rolling hills, plains, etc. Most of the ocean floor is considered to be made up of a number of more-or-less circular or oval depressions called **basins,** surrounded by walls of lesser depth.

The average depth of water in the oceans is 2,075 fathoms (12,450 feet), as compared to an average height of land above the sea of about 2,750 feet. The greatest known depth is 35,640 feet, in the Marianas Trench in the Pacific. The highest known land is Mount Everest, 29,002 feet. About 23 percent of the ocean is shallower than 10,000 feet, about 76 percent is between 10,000 and 20,000 feet, and a little more than one percent is deeper than 20,000 feet. A very deep part, generally that below 3,000 fathoms, is called a **deep.** A long, narrow depression with steep sides is called a **trench.**

3023. Marine sediments.—The ocean floor is composed of material deposited there through the years. This material consists principally of (1) earth and rocks washed into the sea by streams and waves, (2) volcanic ashes and lava, and (3) the remains of marine organisms. Lesser amounts of land material are carried into the sea by glaciers, or blown out to sea by wind. In the ocean, the material is transported by ocean currents, waves, and ice. Near shore the material is deposited at the rate of about three inches in 1,000 years, while in the deep water offshore the rate is only about half an inch in 1,000 years. Marine deposits in water deep enough to be relatively free from wave action are subject to little erosion. Because of this and the slow rate of deposit, marine sediments provide a better geological record than does the land.

Marine sediments are composed of individual particles of all sizes from the finest clay to large boulders. In general, the inorganic deposits near shore are relatively coarse (sand, gravel, shingle, etc.), while those in deep water are much finer (clay). In some areas the siliceous remains of marine organisms or the calcareous deposits (of either organic or inorganic origin) are sufficient to predominate on the ocean floor.

A wide range of colors is found in marine sediments. The lighter colors (white or a pale tint) are usually associated with coarse-grained quartz or limestone deposits. Darker colors (red, blue, green, etc.) are usually found in mud having a predominance of some mineral substance, such as an oxide of iron or manganese. Black mud is often found in an area that is little disturbed, such as at the bottom of an inlet or in a depression without free access to other areas.

Marine sediments are studied primarily by means of bottom samples. Samples of surface deposits are obtained by means of a **snapper** (for mud, sand, etc.) or "dredge" (usually for rocky material). If a sample of material below the bottom surface is desired, a "coring" device is used. This device consists essentially of a tube driven into the bottom by weights or explosives. A sample obtained in this way preserves the natural order of the various layers. Samples of more than 100 feet in depth have been obtained by means of coring devices. The bottom sample obtained by the mariner, by arming his lead with tallow or soap (art. 617), is an incomplete indication of bottom surface conditions.

3024. Marine biology.—Sea water has all of the chemical elements needed to sustain plant and animal life. Because of this, and the fact that the oceans contain about 300 times as much space for the existence of life as is available on land and in fresh water, organic material is present in vast quantities.

Marine life may be divided into three major groups: (1) **nekton** (strong-swimming animals such as fish), (2) **plankton** (tiny floating plants or feebly swimming or floating animals), and (3) **benthos** (plants and animals living on the bottom, such as seaweed, barnacles, and crabs). Plankton may be divided into: (a) the **phytoplankton**, consisting of microscopic floating plants; and (b) the **zooplankton**, consisting of feebly swimming or floating animals. Most plankton vary in size from microscopic units to those a small fraction of an inch in length.

Most organic material in the sea is in the form of plankton, which is carried by the ocean currents, not having sufficient strength to choose its environment. Either directly or indirectly, nearly all marine life depends upon these organisms. By means of **photosynthesis,** a process using sunlight, phytoplankton changes chemical nutrients (silicates, nitrates, phosphates) in the sea into primary food which is used by the zooplankton and, to some extent, by larger animals. However, most of the larger animals feed upon the zooplankton. The chemical nutrients are replaced by the excretion of animals and bacterial action in the decomposition of dead plants and animals. Thus, a food cycle is continually going on from chemical nutrient to phytoplankton to zooplankton to nekton and benthos to chemical nutrients.

As indicated above, growth of phytoplankton requires both sunlight and a supply of chemical nutrients. Sunlight in sufficient strength to permit photosynthesis penetrates to a maximum depth of about 500 feet or less. This upper layer in which the process occurs is called the **euphotic zone.** Within this zone, photosynthesis is limited primarily by the supply of chemical nutrients. Under favorable conditions, phytoplankton may increase by as much as 300 percent in a single day.

The abundance of marine life is directly related to the supply of phytoplankton. In shallow water, the chemical nutrients on the bottom are stirred up by motion of the water, and carried into the euphotic zone. This is why an area such as the Grand Banks is a good fishing ground. In polar regions the chemical nutrients are relatively abundant, being brought to the surface by convective currents as the cold surface water sinks and is replaced by the warmer water from the bottom. In the tropics, on the other hand, the sea is relatively stable, and the chemical nutrients have a tendency to sink below the euphotic zone. Even though the clear, blue water has the deepest euphotic zone, photosynthesis proceeds at a slow rate. For this reason blue is sometimes called the "desert color of the sea."

Ocean currents and marine life are so interrelated that currents can sometimes be traced by their supply of plankton. In general, the oceanic circulation helps sustain marine life by stirring up the chemical nutrients and carrying them, or the plankton formed from them, into regions which have an inadequate supply. However, the reverse effect can occur. A notable example occurs from time to time off the west coast of South America. At varying intervals averaging about 12 years, a well-developed stream of tropical water having a relatively small supply of chemical nutrients and plankton flows southward, close to the shore. This water replaces the colder water which is rich in chemical nutrients and plankton. The result is a wholesale destruction of fish which cannot obtain a sufficient supply of food. In some areas the dead fish are washed ashore in such quantities as to constitute a serious problem. With the destruction of so many fish, the supply of guano also decreases because of the death of large numbers of the birds which depend upon the fish for their food supply. Since it commonly occurs near Christmas, this phenomenon is called "El Niño." A strong current such as the Gulf Stream annually carries many fish to their deaths by transporting them from their normally warm habitat to areas where they encounter water which is too cold for them to endure.

References

Crease, J. "The Origin of Ocean Currents." *Journal of the Institute of Navigation* (British), vol. 5, no. 3 (July 1952).

Day, A., Rear Admiral. "Navigation and Hydrography." *Journal of the Institute of Navigation* (British), vol. 6, no. 1 (January 1953).

Deacon, G. E. R. "Oceanographical Research and Navigation." *Journal of the Institute of Navigation* (British), vol. 4, no. 3 (July 1951).

Defant, A. *Physical Oceanography.* (2 vols.) New York, Pergamon, 1961.

Marmer, H. A. *The Scope of Oceanography.* James Johnstone Memorial Volume. Liverpool, University Press of Liverpool, 1934.

National Research Council. *Physics of the Earth—Oceanography.* Bulletin no. 85, Chapter V. Washington, The National Academy of Sciences, 1932.

Satow, P. G. "Some Problems of Underwater Navigation." *Journal of the Institute of Navigation* (British), vol. 4, no. 3 (July 1951).

Shepard, F. P. *Submarine Geology.* New York, Harper, 1948.

Sverdrup, H. U., Johnson, M. W., and Fleming, R. H. *The Oceans, Their Physics, Chemistry and General Biology.* New York, Prentice-Hall, 1942.

CHAPTER XXXI

TIDES AND TIDAL CURRENTS

General

3101. The tidal phenomenon is the periodic motion of the waters of the sea due to differences in the attractive forces of various celestial bodies, principally the moon and sun, upon different parts of the rotating earth. It can be either a help or hindrance to the mariner—the water's rise and fall may at certain times provide enough depth to clear a bar and at others may prevent him from entering or leaving a harbor. The flow of the current may help his progress or hinder it, may set him toward dangers or away from them. By understanding this phenomenon and by making intelligent use of predictions published in tide and tidal current tables and of descriptions in sailing directions, the mariner can set his course and schedule his passage to make the tide serve him, or at least to avoid its dangers.

3102. Tide and current.—In its rise and fall, the tide is accompanied by a periodic horizontal movement of the water called **tidal current.** The two movements, tide and tidal current, are intimately related, forming parts of the same phenomenon brought about by the tide-producing forces of the sun and moon, principally.

It is necessary, however, to distinguish clearly between tide and tidal current, for the relation between them is not a simple one nor is it everywhere the same. For the sake of clearness and to avoid misunderstanding, it is desirable that the mariner adopt the technical usage: **tide** for the vertical rise and fall of the water, and **current** for the horizontal flow. The tide rises and falls, the tidal current floods and ebbs. In British usage, tidal current is called **tidal stream.**

3103. Cause.—Tides result from differences in the gravitational attraction of various celestial bodies, principally the moon and sun, upon different parts of the rotating earth. The gravity of the earth acts approximately toward the earth's center, and tends to hold the earth in the shape of a sphere. But the moon and sun provide disturbing, or tide-producing, forces. Consider the earth and moon. The moon appears to revolve about the earth, but actually the moon and earth revolve about their common center of mass. They are held together by gravitational attraction and kept apart by an equal and opposite centrifugal force. In this earth-moon system, the tide-producing force on the earth's hemisphere nearer the moon is in the direction of the moon's attraction, or toward the moon. On the hemisphere opposite the moon the tide-producing force is in the direction of the centrifugal force, or away from the moon.

At the sublunar point, and its antipode, the moon's attractive force is vertical, in the opposite direction to gravity. Along the great circle midway between these points, the force is horizontal, parallel to the earth's surface. At any other point, the moon's tide-producing force can be resolved into horizontal and vertical components. Both are very small compared to the earth's gravity. Since the horizontal component is not operating against gravity and can draw particles of water over the surface of the earth, it is the more effective in generating tides.

The tide-producing forces, then, tend to create high tides on the sides of the earth nearest to and farthest from the moon, with a low tide belt between them. As the

earth rotates, a point on earth passes through two high and two low areas each day if the moon is over the equator (fig. 3103, A). When the moon is north or south of the equator, the force pattern is as shown in figure 3103, B, and a point on the equator passes through two equal highs, but a point in higher latitudes passes through two unequal highs or only one high. Thus, due to changes in the moon's declination, there is introduced a diurnal inequality in the pattern of the tidal forces at a particular place. There are similar forces due to the sun, and the total tide producing force is the resultant of the two. Minute tidal effects are caused by other celestial bodies.

The mathematician develops his formulas by considering the difference in attraction between a point on the earth's surface and a point at the earth's center. In accordance with Newton's law, gravitational attraction of an astronomical body varies directly as its mass and inversely as the *square* of its distance. But the tide-producing (differential) force varies directly as the mass and inversely as the *cube* of the distance. As a consequence, only the moon and sun produce any appreciable tidal effect upon the earth. Further, although the moon's mass is but a fraction of the sun's, dividing such masses by the cube of their respective distances—$(238{,}862)^3$ statute miles and $(92{,}900{,}000)^3$ statute miles, respectively—reduces the sun's tide-producing force to only 0.46 that of the moon. It is because of this that the timing of the tides is identified so closely with the motions of the moon.

Though the tide-producing forces are distributed over the earth in a regular manner, the sizes and shapes of the ocean basins and the interference of the land masses prevent the tides of the oceans from assuming a simple, regular pattern. The way in which the waters in different parts of the oceans, as well as in the smaller waterways, respond to these known regular forces is dependent in large part upon the size, depth, and configuration of the basin or waterway.

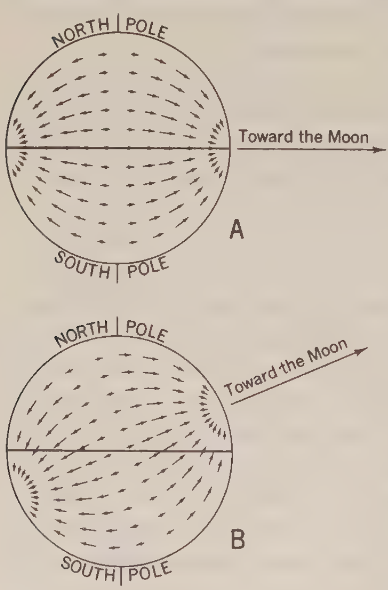


FIGURE 3103.—Tide-producing forces. The arrows represent the magnitude and direction of the horizontal component of the tide-producing force on the earth's surface. (A) When the moon is in the plane of the equator, the forces are equal in magnitude at the two points on the same parallel of latitude and 180° apart in longitude. (B) When the moon is at north (or south) declination, the forces are unequal at such points and tend to cause an inequality in the two high waters and the two low waters of a day.

Tide

3104. General features.—Tide is the periodic rise and fall of the water accompanying the tidal phenomenon. At most places it occurs twice daily. The tide rises until it reaches a maximum height, called **high tide** or **high water,** and then falls to a minimum level called **low tide** or **low water.**

The rate of rise and fall is not uniform. From low water, the tide begins to rise slowly at first but at an increasing rate until it is about halfway to high water. The rate of rise then decreases until high water is reached and the rise ceases. The falling tide behaves in a similar manner. The period at high or low water during which there is no sensible change of level is called **stand.** The difference in height between consecutive high and low waters is the **range.**

Figure 3104 is a graphical representation of the rise and fall of the tide at New York during a 24-hour period. The tide curve has the general form of a sine curve (fig. O40b).

3105. Types of tide.—A body of water has a natural period of oscillation that is dependent upon its dimensions. None of the oceans appears to be a

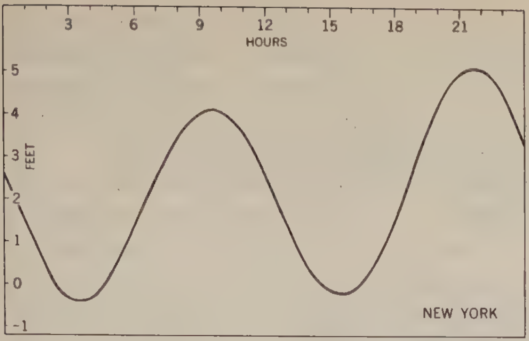


FIGURE 3104.—The rise and fall of the tide at New York, shown graphically.

single oscillating body, but rather each one is made up of a number of oscillating basins. As such basins are acted upon by the tide-producing forces, some respond more readily to daily or diurnal forces, others to semidiurnal forces, and others almost equally to both. Hence, tides at a place are classified as one of three types—**semidiurnal, diurnal,** or **mixed**—according to the characteristics of the tidal pattern occurring at the place.

In the **semidiurnal** type of tide, there are two high and two low waters each tidal day, with relatively small inequality in the high and low water heights. Tides on the Atlantic coast of the United States are representative of the semidiurnal type, which is illustrated in figure 3105a by the tide curve for Boston Harbor.

In the **diurnal** type of tide, only a single high and single low water occur each tidal day. Tides of the diurnal type occur along the northern shore of the Gulf of Mexico, in the Java Sea, the Gulf of Tonkin (off the Vietnam-China coast), and in a few other localities. The tide curve for Pakhoi, China, illustrated in figure 3105b, is an example of the diurnal type.

In the **mixed** type of tide, the diurnal and semidiurnal oscillations are both important factors and the tide is characterized by a large inequality in the high water heights, low water heights, or in both. There are usually two high and two low waters each day, but occasionally the tide may become diurnal. Such tides are prevalent along the Pacific coast of the United States and in many other parts of the world. Examples of mixed types of tide are shown in figure 3105c. At Los Angeles, it is typical that the inequalities in the high and low waters are about the same. At Seattle the greater inequalities are typically in the low waters, while at Honolulu it is the high waters that have the greater inequalities.

3106. Solar tide.—The natural period of oscillation of a body of water may accentuate either the solar or the lunar tidal oscillations. Though it is a general rule that the tides follow the moon, the relative importance of the solar effect varies in different areas. There are a few places, primarily in the South Pacific and the Indonesian areas, where the solar oscillation is the more important, and at those places the high and low waters occur at about the same time each day. At Port Adelaide, Australia (fig. 3106), the solar and lunar semidiurnal oscillations are equal and nullify one another at neaps (art. 3108).

3107. Special effects.—As a progressive wave enters shallow water, its speed is decreased. Since the trough is shallower than the crest, its retardation is greater, resulting in a steepening of the wave front. Therefore, in many rivers, the duration of rise is considerably less than the duration of fall. In a few estuaries, the advance of the low water trough is so much retarded that the crest of the

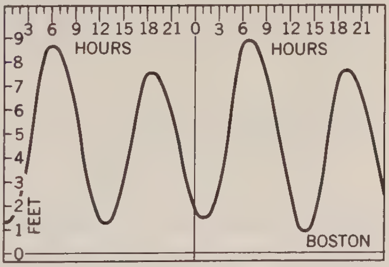


FIGURE 3105a.—Semidiurnal type of tide.

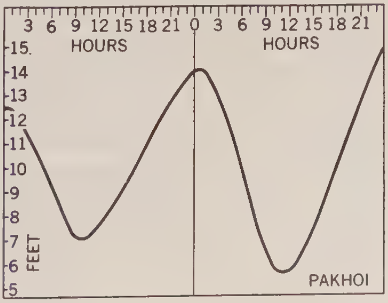


FIGURE 3105b.—Diurnal type of tide.

rising tide overtakes the low, and advances upstream as a churning, foaming wall of water called a **bore.** Bores that are large and dangerous at times of large tidal ranges may be mere ripples at those times of the month when the range is small. Examples occur in the Petitcodiac River in the Bay of Fundy and at Haining, China, in the Tsientang Kiang. The tide tables indicate where bores occur.

Other special features are the **double low water** (as at Hoek Van Holland) and the **double high water** (as at Southampton, England). At such places there is often a slight fall or rise in the middle of the high or low water period. The practical effect is to create a longer period of stand at high or low tide. The tide tables direct attention to these and other peculiarities where they occur.

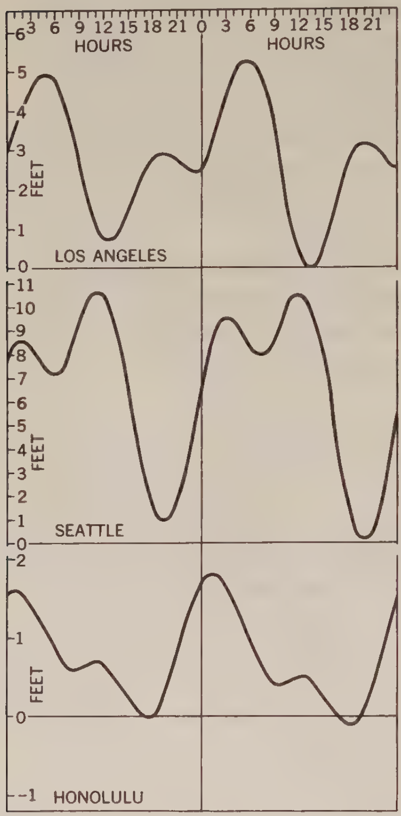


FIGURE 3105c.—Mixed types of tide.

3108. Variations in range.—Though the tide at a particular place can be classified as to type, it exhibits many variations during the month (fig. 3106). The range of the tide varies in accordance with the intensity of the tide-producing force, though there may be a lag of a day or two (**age of tide**) between a particular astronomic cause and the tidal effect.

Thus, when the moon is at the point in its orbit nearest the earth (at *perigee*), the lunar semidiurnal range is increased and **perigean** tides occur; when the moon is farthest from the earth (at *apogee*), the smaller **apogean** tides occur. When the moon and sun are in line and pulling together, as at new and full moon, **spring** tides occur (the term *spring* has nothing to do with the season of year); when the moon and sun oppose each other, as at the quadratures, the smaller **neap** tides occur.

When certain of these phenomena coincide, the great **perigean spring** tides, the small **apogean neap** tides, etc., occur.

These are variations in the semidiurnal portion of the tide. Variations in the diurnal portion occur as the moon and sun change declination. When the moon is at its maximum semi-monthly declination (either north or south), **tropic** tides occur in which the diurnal effect is at a maximum; when it crosses the equator, the diurnal effect is a minimum and **equatorial** tides occur.

It should be noted that when the range of tide is increased, as at spring tides, there is more water available only at *high* tide; at *low* tide there is less, for the high waters rise higher and the low waters fall lower at these times. There is more water at neap low water than at spring low water. With tropic tides, there is usually more depth at one low water during the day than at the other. While it is desirable to know the meanings of these terms, the best way of determining the height of the tide at any place and time is to examine the tide predictions for the place as given in the tide tables. Figure 3108 illustrates variations in the ranges and heights of tides in a locality where the water level always exceeds the charted depth.

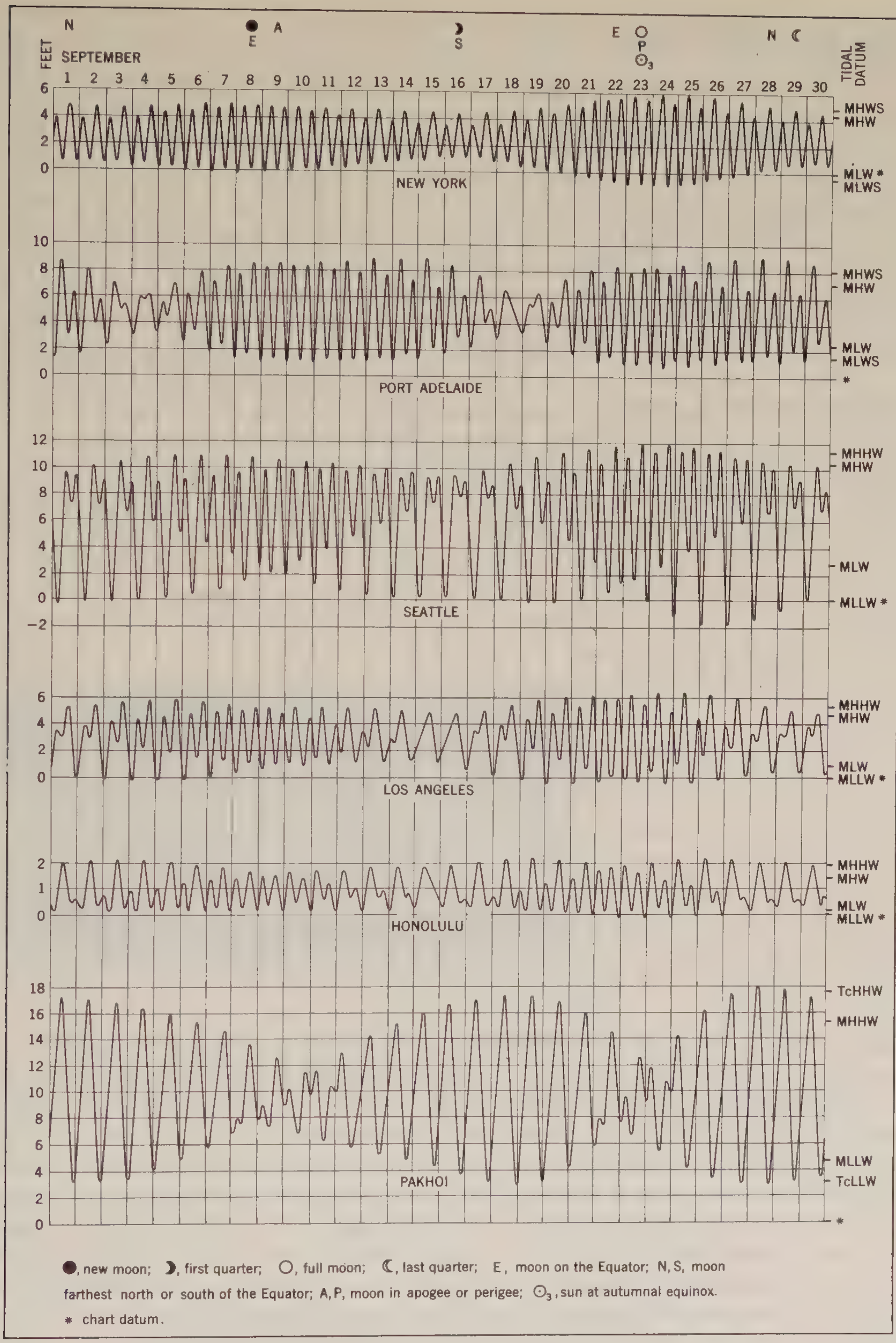


FIGURE 3106—Tidal variations at various places during a month.

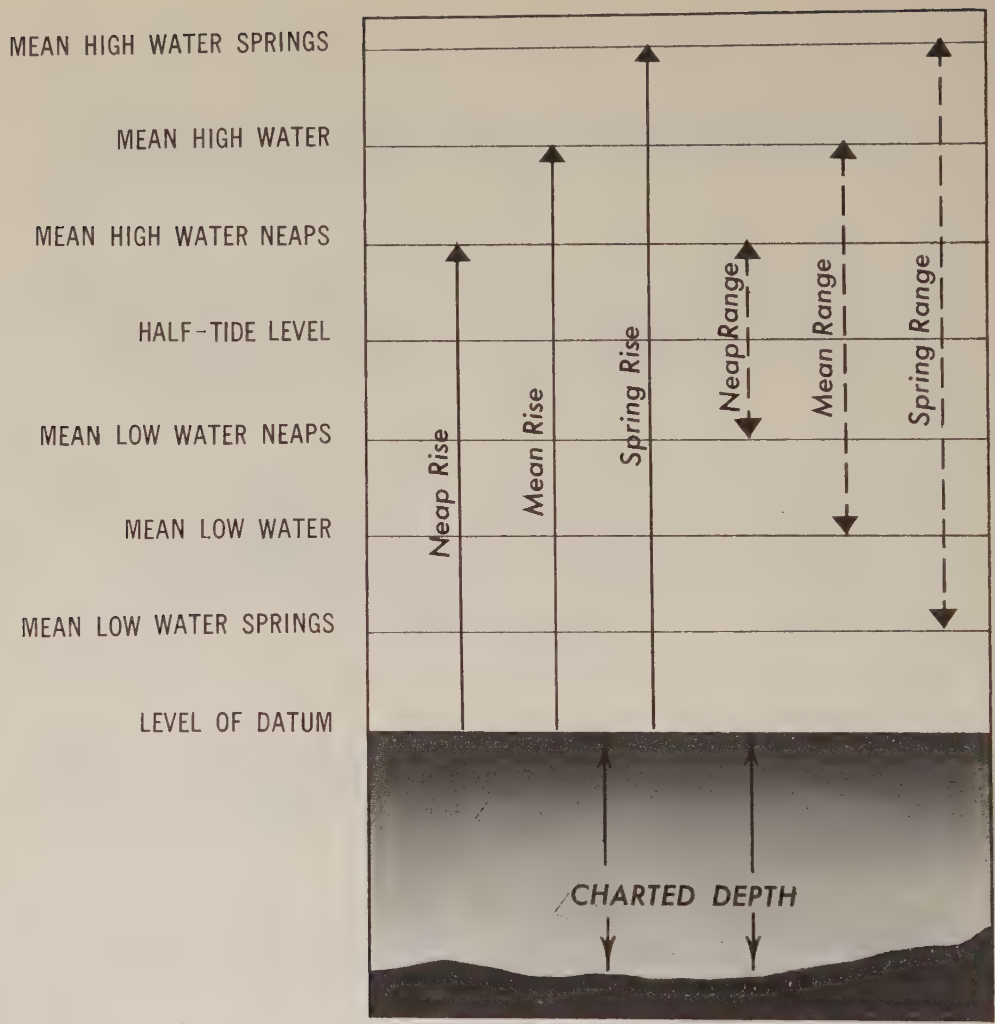


FIGURE 3108.—Variations in the ranges and heights of tide in a locality where the water level always exceeds the charted depth.

3109. Tidal cycles.—Tidal oscillations go through a number of cycles. The shortest cycle, completed in about 12 hours and 25 minutes for a semidiurnal tide, extends from any phase of the tide to the next recurrence of the same phase. During a **lunar day** (averaging 24 hours and 50 minutes) there are two highs and two lows (two of the shorter cycles) for a semidiurnal tide. The effect of the phase variation is completed in about two weeks as the moon varies from new to full or full to new. The effect of the moon's declination is also repeated about each two weeks. The cycle involving the moon's distance requires approximately a **lunar month** (a synodical month of about 29½ days). The sun's declination and distance cycles are respectively a half year and a year in length. An important lunar cycle, called the **nodal period,** is 18.6 years (usually expressed in round figures as 19 years). For a tidal value, particularly a range, to be considered a true mean, it must be either based upon observations extended over this period of time or adjusted to take account of variations known to occur during the cycle.

3110. Time of tide.—Since the lunar tide-producing force has the greater effect in producing tides at most places, the tides "follow the moon." Because of the rotation of the earth, high water lags behind meridian passage (upper and lower) of the moon. The **tidal day,** which is also the **lunar day,** is the time between consecutive transits

of the moon, or 24 hours and 50 minutes on the average. Where the tide is largely semidiurnal in type, the **lunitidal interval**—the interval between the moon's meridian transit and a particular phase of tide—is fairly constant throughout the month, varying somewhat with the tidal cycles. There are many places, however, where solar or diurnal oscillations are effective in upsetting this relationship, and the newer editions of charts of many countries now omit intervals because of the tendency to use them for prediction even though accurate predictions are available in tide tables. However, the lunitidal interval may be encountered. The interval generally given is the average elapsed time from the meridian transit (upper or lower) of the moon until the next high tide. This may may be called **mean high water lunitidal interval** or **establishment of the port.** The **high water full and change** (HWF&C) or **vulgar establishment,** sometimes given, is the average interval on days of full or new moon, and approximates the mean high water lunitidal interval.

In the ocean, the tide may be of the nature of a progressive wave with the crest moving forward, a stationary or standing wave which oscillates in a seesaw fashion, or a combination of the two. Consequently, caution should be used in inferring the time of tide at a place from tidal data for nearby places. In a river or estuary, the tide enters from the sea and is usually sent upstream as a progressive wave, so that the tide occurs progressively later at various places upstream.

3111. Tidal datums.—A **tidal datum** is a level from which heights and depths are measured. There are a number of such levels of reference that are important to the mariner. The relation of the tide each day during a month to these datums is shown, for certain places, in figure 3106.

The most important level of reference to the mariner is the datum of soundings on charts. Since the tide rises and falls continually while soundings are being taken during a hydrographic survey, the tide should be observed during the survey so that soundings taken at all stages of the tide can be reduced to a common datum. Soundings on charts show depths below a selected low water datum (occasionally mean sea level), and tide predictions in tide tables show heights above the same level. The depth of water available at any time is obtained by adding the height of the tide at the time in question to the charted depth, or by subtracting the predicted height if it is negative.

By international agreement, the level used as chart datum should be just low enough so that low waters do not go far below it. At most places, however, the level used is one determined from a mean of a number of low waters (usually over a 19-year period); therefore some low waters can be expected to fall below it. The following are some of the datums in general use.

The highest low water datum in considerable use is **mean low water (MLW),** which is the average height of all low waters at a place. About half of the low waters fall below it. **Mean low water springs** (MLWS), usually shortened to **low water springs,** is the average level of the low waters that occur at the times of spring tides. **Mean lower low water** (MLLW) is the average height of the lower low waters at a place. **Tropic lower low water** (TcLLW) is the average height of the lower low waters (or of the single daily low waters if the tide becomes diurnal) that occur when the moon is near maximum declination and the diurnal effect is most pronounced. This datum is not in common use as a tidal reference. **Indian spring low water** (ISLW) sometimes called **Indian tide plane** or **harmonic tide plane,** is a low datum that includes the spring effect of the semi-diurnal portion of the tide and the tropic effect of the diurnal portion. It is about the level of lower low water of mixed tides at the time that the moon's maximum declination coincides with the time of new or full moon. **Mean lower low water springs** is the aver-

age level of the lower of the two low waters on the days of spring tides. Some still lower datums used on charts are determined from tide observations and some are determined arbitrarily and later referred to the tide. Most of them fall close to one or the other of the following two datums. **Lowest normal low water** is a datum that approximates the average height of monthly lowest low waters, discarding any tides disturbed by storms. **Lowest low water** is an extremely low datum. It conforms generally to the lowest tide observed, or even somewhat lower. Once a tidal datum is established, it is generally retained for an indefinite period, even though it might differ slightly from a better determination from later observations. When this occurs, the established datum may be called **low water datum, lower low water datum,** etc.

In some areas where there is little or no tide, such as the Baltic Sea, **mean sea level** (MSL) is used as chart datum. This is the average height of the surface of the sea for all stages of the tide over a 19-year period. This may differ slightly from **half-tide level,** which is the level midway between mean high water and mean low water.

Inconsistencies of terminology are found among charts of different countries and between charts issued at different times. For example, the spring effect as defined here is a feature of only the semidiurnal tide, yet it is sometimes used synonymously with tropic effect to refer to times of increased range of a diurnal tide. Such inconsistencies are being reduced through increased international cooperation.

Large-scale charts usually specify the datum of soundings and may contain a tide note giving mean heights of the tide at one or more places on the chart. These heights are intended merely as a rough guide to the change in depth to be expected under the specified conditions. They should not be used for the prediction of heights on any particular day. Such predictions should be obtained from *tide tables* (arts. 921–924). The tidal datums used in various areas are listed in appendix M.

3112. High water datums.—Heights of land features are usually referred on nautical charts to a high water datum. The one used on charts of the United States, its territories, and possessions, and widely used elsewhere, is **mean high water (MHW),** which is the average height of all high waters over a 19-year period. Any other high water datum in use on charts is likely to be higher than this. Other high water datums are **mean high water springs (MHWS),** which is the average level of the high waters that occur at the time of spring tides; **mean higher high water (MHHW),** which is the average height of the higher high waters of each day; and **tropic higher high water (TcHHW),** which is the average height of the higher high waters (or the single daily high waters if the tide becomes diurnal) that occur when the moon is near maximum declination and the diurnal effect is most pronounced. A reference merely to "high water" leaves some doubt as to the specific level referred to, for the height of high water varies from day to day. Where the range is large, the variation during a two-week period may be considerable.

3113. Observations and predictions.—Since the tide at different places responds differently to the tide-producing forces, the nature of the tide at any place can be determined most accurately by actual observation. The predictions in tide tables and the tidal data on nautical charts are based upon such observations.

Tides are usually observed by means of a continuously recording gage. A year of observations is the minimum length desirable for determining the **harmonic constants** used in prediction. For establishing mean sea level and the long-time changes in the relative elevations of land and sea, as well as for other special uses, observations have been made over periods of 20, 30, and even 50 years at important locations. Observations for a month or less will establish the *type* of tide and suffice for comparison with a longer series of a similar type to determine tidal differences and constants.

Mathematically, the variations in the lunar and solar tide-producing forces, such as those due to changing phase, distance, and declination, are considered as separate constituent forces, and the **harmonic analysis** of observations reveals the response of each constituent of the tide to its corresponding force. At any one place this response remains constant and is shown for each constituent by **harmonic constants** which are in the form of a phase angle for the time relation and an amplitude for the height. Harmonic constants are used in making technical studies of the tide and predictions on the **tide predicting machine.** Most published tide predictions are made by machine.

3114. Tide tables are published annually by most of the maritime nations of the world. They consist primarily of two parts. One contains predictions of the time and height of each high and low water for every day of the year for many important ports called **reference stations.** The other part contains tidal differences for thousands of other places, called **subordinate stations,** and specifies the reference station to which the differences are to be applied in order to obtain time and height of tide for any day at the subordinate station. The type of tide at a subordinate station is the same as at its reference station. The use of tide tables is explained in articles 921–924.

3115. Meteorological effects.—The foregoing discussion of tide behavior assumes normal weather conditions. The level of the sea is affected by wind and atmospheric pressure. In general, onshore winds raise the level and offshore winds lower it, but the amount of change varies at different places. During periods of low atmospheric pressure, the water level tends to be higher than normal. For a stationary low, the increase in elevation can be found by the formula

$$R_0=0.0325\ (1010-P),$$

in which R_0 is the increase in elevation in feet, and P is the atmospheric pressure in millibars. This is equal approximately to one centimeter per millibar depression, or one foot (13.6 inches) per inch depression. For a moving low, the increase in elevation is given by the formula

$$R=\frac{R_0}{1-\frac{C^2}{gh}},$$

in which R is the increase in elevation in feet, R_0 is the increase in feet for a stationary low, C is the rate of motion of the low in feet per second, g is the acceleration due to gravity (32.2 feet per second per second), and h is the depth of water in feet.

Where the range of tide is very small, the meteorological effect may sometimes be greater than the normal tide.

Tidal Current

3116. Tidal and nontidal currents.—Horizontal movement of the water is **current.** It may be classified as "tidal" and "nontidal." **Tidal current** is the periodic horizontal flow of water accompanying the rise and fall of the tide, and results from the same cause. **Nontidal current** is any current not due to the tidal movement. Nontidal currents include the permanent currents in the general circulatory system of the oceans as well as temporary currents arising from meteorological conditions. The current experienced at any time is usually a combination of tidal and nontidal currents.

In navigation, the effect of the tidal current is often of more importance than the changing depth due to the tide, and many mariners speak of "the tide," when they have in mind the flow of the tidal current.

3117. General features.—Offshore, where the direction of flow is not restricted by any barriers, the tidal current is **rotary;** that is, it flows continuously, with the direction changing through all points of the compass during the tidal period. The tendency

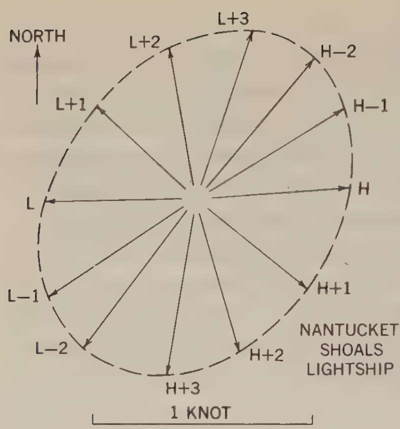


FIGURE 3117a.—Rotary tidal current. Times are hours before and after high and low tide at Nantucket Shoals Lightship. The bearing and length of each arrow represents the hourly direction and speed of the current. See figure 3120a.

for the rotation in direction has its origin in the deflecting force of the earth's rotation, and unless modified by local conditions, the change is clockwise in the northern hemisphere and counterclockwise in the southern hemisphere. The speed usually varies throughout the tidal cycle, passing through two maximums in approximately opposite directions, and two minimums about halfway between the maximums in time and direction. Rotary currents can be depicted as in figure 3117a, by a series of arrows representing the direction and speed of the current at each hour. This is sometimes called a **current rose.** Because of the elliptical pattern formed by the ends of the arrows, it is also referred to as a **current ellipse.**

In rivers or straits, or where the direction of flow is more or less restricted to certain channels, the tidal current is **reversing;** that is, it flows alternately in approximately opposite directions with an instant or short period of little or no current, called **slack water,** at each reversal of the current. During the flow in each direction, the speed varies from zero at the time of slack water to a maximum, called **strength of flood** or **ebb,** about midway between the slacks. Reversing currents can be indicated graphically, as in figure 3117b, by arrows that represent the speed of the current at each hour. The flood is usually depicted above the slack water line and the ebb below it. The tidal current curve formed by the ends of the arrows has the same characteristic sine form as the tide curve. (In illustrations for certain purposes, as in figures 3118b and 3120b, it is convenient to omit the arrows and show only the curve.)

A slight departure from the sine form is exhibited by the reversing current in a strait, such as East River, New York, that connects two tidal bodies of water. The tides at the two ends of a strait are seldom in phase or equal in range, and the current, called **hydraulic current,** is generated largely by the continuously changing difference in height of water at the two ends. The speed of a hydraulic current varies nearly as the square root of the difference in height. The speed reaches a maximum more quickly and remains at strength for a longer period than shown in figure 3117b, and the period of weak current near the time of slack is considerably shortened.

The current *direction* or **set** is the direction *toward* which the current flows. The *speed* is sometimes called the **drift.** The term "velocity" is often used as the equivalent of "speed" when referring to current, although strictly "velocity" implies direction as well as speed. The term "strength" is also used to refer to speed, but more often to greatest speed between consecutive slack waters. The movement toward shore or upstream is the **flood,** the movement away from shore or downstream is the **ebb.** In a purely semidiurnal type of current unaffected by nontidal flow, the flood and ebb each last about six hours and 13 minutes. But if there is either diurnal inequality or nontidal flow, the durations of flood and ebb may be quite unequal.

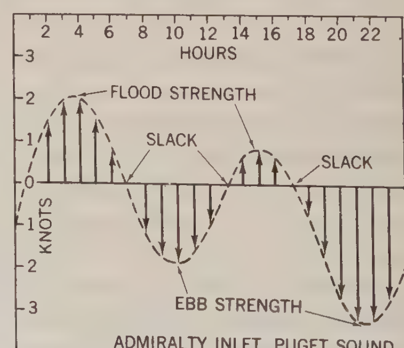


FIGURE 3117b.—Reversing tidal current. (Such graphs may show only the curved pattern without the arrows, as in figures 3118b and 3120b.) See figure 3120b.

3118. Types of tidal current.—Tidal currents may be of the **semidiurnal, diurnal,** or **mixed** type; corresponding to a considerable degree to the type of tide at the place, but often with a stronger semidiurnal tendency.

The tidal currents in tidal estuaries along the Atlantic coast of the United States are examples of the semidiurnal type of reversing current. At Mobile Bay entrance they are almost purely diurnal. At most places, however, the type is mixed to a greater or lesser degree. At Tampa and Galveston entrances there is only one flood and one ebb each day when the moon is near its maximum declination, and two floods and two ebbs each day when the moon is near the equator. Along the Pacific coast of the United States there are generally two floods and two ebbs every day, but one of the floods or ebbs has a greater speed and longer duration than the other, the inequality varying with the declination of the moon. The inequalities in the current often differ considerably from place to place even within limited areas, such as adjacent passages in Puget Sound and various passages between the Aleutian Islands. Figure 3118a shows several types of reversing current. Figure 3118b shows how the flood disappears as the diurnal inequality increases at one station.

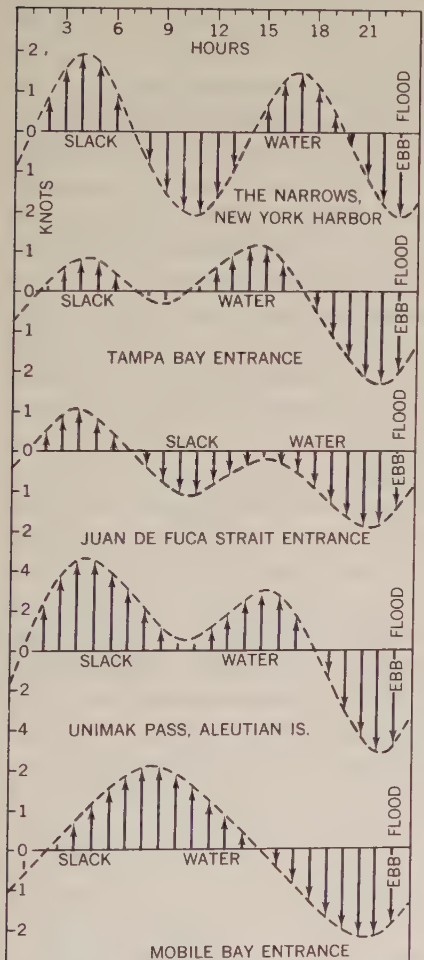


FIGURE 3118a.—Several types of reversing current. The pattern changes gradually from day to day, particularly for mixed types, passing through cycles somewhat similar to that shown for tides in figure 3106.

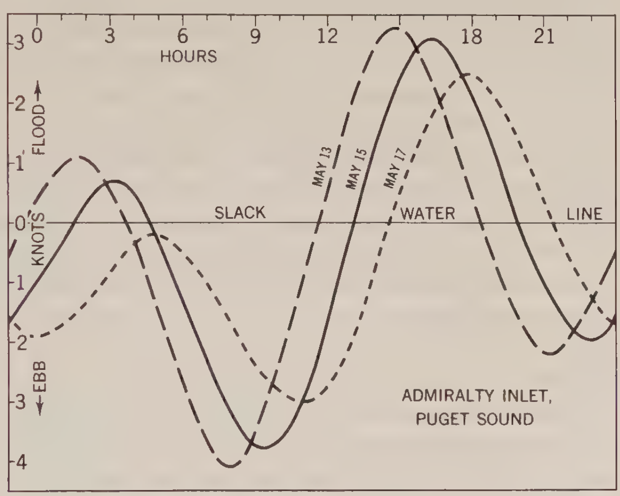


FIGURE 3118b.—Changes in a current of the mixed type. Note that each day as the inequality increases, the morning slacks draw together in time until on the 17th the morning flood disappears. On that day the current ebbs throughout the morning.

Offshore rotary currents that are purely semidiurnal repeat the elliptical pattern (fig. 3117a) each tidal cycle of 12 hours and 25 minutes. If there is considerable diurnal inequality, the plotted hourly current arrows describe a set of two ellipses of different sizes during a period of 24 hours and 50 minutes, as shown in figure 3118c, and the greater the diurnal inequality, the greater the difference between the sizes of the two ellipses. In a completely diurnal rotary current, the smaller ellipse disappears and only one ellipse is produced in 24 hours and 50 minutes.

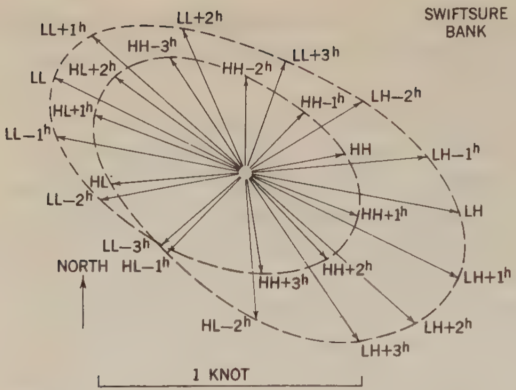


FIGURE 3118c.—Rotary tidal current with diurnal inequality. Times are in hours referred to tides (higher high, lower low, lower high, and higher low) at Swiftsure Bank.

3119. Variations and cycles.—Tidal currents have periods and cycles similar to those of the tides (art. 3109), and are subject to similar variations, but flood and ebb of the current do not necessarily occur at the same times as the rise and fall of the tide. The relationship is explained further in article 3121.

The speed at strength increases and decreases during the two-week period, month, and year with the variations in the range of tide. Thus, the stronger **spring** and **perigean currents** occur near the times of new and full moon and near the times of the moon's perigee, or at times of spring and perigean tides (art. 3108); the weaker **neap** and **apogean currents** occur at the times of neap and apogean tides; **tropic currents** with increased diurnal speeds or with larger diurnal inequalities in speed occur at times of tropic tides; and **equatorial currents** with a minimum diurnal effect occur at times of equatorial tides; etc.

As with the tide, a *mean value* represents an average obtained from a 19-year series. Since a series of current observations is usually limited to a day or two, and seldom covers more than a month or two, it is necessary to adjust the observed values, usually by comparison with tides at a nearby place, to obtain such a mean.

3120. Effect of nontidal flow.—The current existing at any time is seldom purely tidal, but usually includes also a nontidal current that is due to drainage, oceanic circulation, wind, or other cause. The method in which tidal and nontidal currents combine is best explained graphically, as in figures 3120a and 3120b. The pattern of the tidal current remains unchanged, but the curve is shifted from the point or line from which the currents are measured in the direction of the nontidal current and by an amount equal to it. It is sometimes more convenient graphically merely to move the line or point of origin in the opposite direction.

Thus, the speed of the current flowing in the direction of the nontidal current is increased by an amount equal to the magnitude of the nontidal current, and the speed of the current flowing in the opposite direction is decreased by an equal amount. In figure 3120a a nontidal current is represented both in direction and speed by the vector AO. Since this is greater than the speed of the tidal current in the opposite direction, the point A is outside the ellipse. The direction and speed of the combined tidal and nontidal currents at any time is represented by a vector from A to that point on the curve representing the given time, and can be scaled from the graph. The strongest and weakest currents may no longer be in the directions of the maximum and minimum of the tidal current. In a reversing current (fig. 3120b), the effect is to advance the time of one slack and to retard the following one. If the speed of

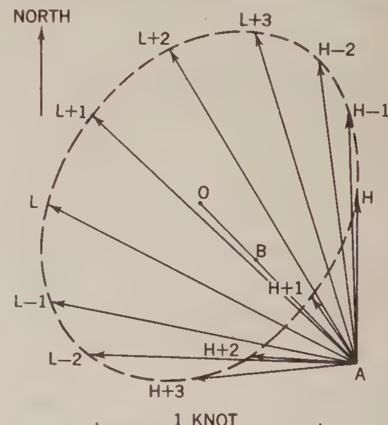


FIGURE 3120a.—Effect of nontidal current on the rotary tidal current of figure 3117a. If the nontidal current is northwest at 0.3 knot, it may be represented by BO, and all hourly directions and speeds will then be measured from B. If it is 1.0 knot, it will be represented by AO and the actual resultant hourly directions and speeds will be measured from A, as shown by the arrows.

the nontidal current exceeds that of the reversing tidal current, the resultant current flows continuously in one direction without coming to a slack. In this case, the speed varies from a maximum to a minimum and back to a maximum in each tidal cycle. In figure 3120b the horizontal line *A* represents slack water if only tidal currents are present. Line *B* represents the effect of a 0.5-knot nontidal ebb, and line *C* the effect of a 1.0-knot nontidal ebb. With the condition shown at *C* there is only one flood each tidal day. If the nontidal ebb were to increase to approximately two knots, there would be no flood, two maximum ebbs and two minimum ebbs occurring during a tidal day.

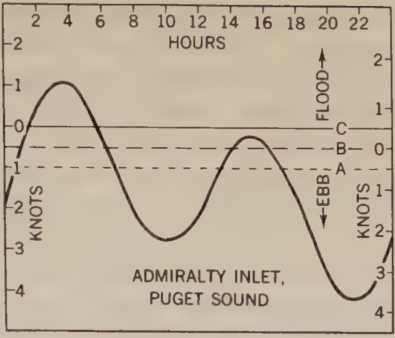


FIGURE 3120b.—Effect of nontidal current on the reversing tidal current of figure 3117b. If the nontidal current is 0.5 knot in the ebb direction, the ebb is increased by moving the slack water line from position *A* up 0.5 knot to position *B*. Speeds will then be measured from this broken line as shown by the scale on the right, and times of slack are changed. If the nontidal current is 1.0 knot in the ebb direction, as shown by line *C*, the speeds are as shown on the left, and the current will not reverse to a flood in the afternoon; it will merely slacken at about 1500.

3121. Relation between time of tidal current and time of tide.—At many places where current and tide are both semidiurnal, there is a definite relation between times of current and times of high and low water in the locality. Current atlases and notes on nautical charts often make use of this relationship by presenting for particular locations the direction and speed of the current at each succeeding hour after high and low water at a place for which tide predictions are available.

In localities where there is considerable diurnal inequality in tide or current, or where the type of current differs from the type of tide, the relationship is not constant, and it may be hazardous to try to predict the times of current from times of tide. Note the current curve for Unimak Pass in the Aleutians in figure 3118a. It shows the current as predicted in the tidal current tables. Predictions of high and low waters in the tide tables might have led one to expect the current to change from flood to ebb in the late morning, whereas actually the current continued to run flood with some strength at that time.

Since the relationship between times of tidal current and tide is not everywhere the same, and may be variable at the same place, one should exercise extreme caution in using general rules. The belief that slacks occur at local high and low tides and that the maximum flood and ebb occur when the tide is rising or falling most rapidly may be approximately true at the seaward entrance to, and in the upper reaches of, an inland tidal waterway. But generally this is not true in other parts of inland waterways. When an inland waterway is extensive or its entrance constricted, the slacks in some parts of the waterway often occur midway between the times of high and low tide. Usually in such waterways the relationship changes from place to place as one progresses upstream, slack water getting progressively later with respect to the local tide until at the head of tidewater (the inland limit of water affected by a tide) the slacks occur at the times of high and low tide.

3122. Relation between speed of current and range of tide.—The variation in the speed of the tidal current from place to place is not necessarily consistent with the range of tide. It may be the reverse. For example, currents are weak in the Gulf of Maine where the tides are large, and strong near Nantucket Island and in Nantucket Sound where the tides are small.

At any one place, however, the speed of the current at strength of flood and ebb varies during the month in about the same proportion as the range of tide, and

one can use this relationship to determine the relative strength of currents on any day.

3123. Variation across an estuary.—In inland tidal waterways the *time* of tidal current varies across the channel from shore to shore. On the average, the current turns earlier near shore than in midstream, where the speed is greater. Differences of half an hour to an hour are not uncommon, but the difference varies and the relationship may be nullified by the effect of nontidal flow.

The *speed* of the current also varies across the channel, usually being greater in midstream or midchannel than near shore, but in a winding river or channel the strongest currents occur near the concave shore. Near the opposite (convex) shore the currents are weak or may eddy.

3124. Variation with depth.—In tidal rivers the subsurface current acting on the lower portion of the hull may differ considerably from the surface current. An appreciable subsurface current may be present when the surface movement appears to be practically slack, and the subsurface current may even be flowing with appreciable speed in the opposite direction to the surface current.

In a tidal estuary, particularly in the lower reaches where there is considerable difference in density from top to bottom, flood usually begins earlier near the bottom than at the surface. The differences may be an hour or two or as little as a few minutes, depending upon the estuary, the location in the estuary, and freshet conditions. Even when the fresh water runoff becomes so great as to prevent the surface current from flooding, it may still flood below the surface. The difference in time of ebb from surface to bottom is normally small but subject to variation with time and location.

The ebb speed at strength usually decreases gradually from top to bottom, but the speed of flood at strength often is stronger at subsurface depths than at the surface.

3125. Observations.—Observations of the current are made by means of a current meter or current pole and log line. In the past, most successful meters required a vessel and observers in continual attendance, as is necessary with the pole and line. Because of the difficulty and expense of such observations, they usually covered only a period of a day or two at a place. Observations of a month are the exception, and longer series were obtained only where ship and observers were available because of other duties, such as at lightships, where observations have been continued over a number of years.

Newer meters have been and are being developed that are suspended from a buoy and that record either in the buoy or send speed and direction impulses by radio to a base station on ship or land. With them, the period of observation has been increased so that in some recent surveys of United States harbors, the minimum period of observation was four days, with observations at several stations being continued over a period of 15 to 29 days.

3126. Tidal current tables and other sources of information.—The navigator should not attempt to predict currents without specific information for the locality in which he is interested. Such information is contained in various forms in many navigational publications.

Tidal current tables, issued annually, list daily predictions of the times and strengths of flood and ebb currents, and of the times of intervening slacks. Due to lack of observational data, coverage is considerably more limited than for the tides. The tidal current tables do include supplemental data by which tidal current predictions can be determined for many places in addition to those for which daily predictions are given. The predictions are made by the tide-predicting machine, using current harmonic constants that are obtained by analyzing current observations in the same manner as for tides (art. 3113). The use of tidal current tables is explained in articles 925–929.

Sailing directions and **coast pilots** issued by maritime nations include general descriptions of current behavior in various localities throughout the world.

Tidal current charts. A number of important harbors and waterways are covered by sets of tidal current charts showing graphically the hourly current movement.

References

Doodson, A. T., and Warburg, H. D. *Admiralty Manual of Tides.* London, H. M. Stationery Office, 1941.

Marmer, H. A. *The Tide.* New York, Appleton, 1926.

Schureman, Paul. *Manual of the Harmonic Analysis and Prediction of Tides.* Rev. ed. U. S. Coast and Geodetic Survey Special Publication No. 98. Washington, U. S. Govt. Print. Off., 1940.

Schureman, Paul. *Tide and Current Glossary.* Rev. ed. U. S. Coast and Geodetic Survey Special Publication No. 228. Washington, U. S. Govt. Print. Off., 1949.

U. S. Coast and Geodetic Survey. *Manual of Current Observations.* Special Publication No. 215. Rev. ed. Washington, U. S. Govt. Print. Off., 1950.

U. S. Coast and Geodetic Survey. *Manual of Tide Observations.* Special Publication No. 196. Rev. ed. Washington, U. S. Govt. Print. Off., 1941.

U. S. Coast and Geodetic Survey. *Tidal Current Charts.* Washington, several published periodically.

U. S. Coast and Geodetic Survey. *Tide Tables* and *Tidal Current Tables.* Washington, U. S. Govt. Print. Off., several volumes of each published annually.

CHAPTER XXXII

OCEAN CURRENTS

3201. Introduction.—The movement of water comprising the oceans is one of the principal sources of discrepancy between dead reckoning and actual positions of vessels. Water in essentially horizontal motion is called a **current,** the direction *toward* which it moves being the **set,** and its speed the **drift.** A well-defined current extending over a considerable region of the ocean is called an **ocean current.**

A **periodic current** is one the speed or direction of which changes cyclically at somewhat regular intervals, as a tidal current. A **seasonal current** is one which has large changes in speed or direction due to seasonal winds. A **permanent current** is one which experiences relatively little periodic or seasonal change.

A **coastal current** flows roughly parallel to a coast, outside the surf zone, while a **longshore current** is one parallel to a shore, inside the surf zone, and generated by waves striking the beach at an angle. Any current some distance from the shore may be called an **offshore current,** and one close to the shore an **inshore current.**

A **surface current** is one present at the surface, particularly one that does not extend more than a relatively few feet below the surface. A **subsurface current** is one which is present below the surface only.

There is evidence to indicate that the strongest ocean currents consist of relatively narrow, high-speed streams that follow winding, shifting courses. Often associated with these currents are secondary **countercurrents** flowing adjacent to them but in the opposite direction, and somewhat local, roughly circular, **eddy currents.** A relatively narrow, deep, fast-moving current is sometimes called a **stream current,** and a broad, shallow, slow-moving one a **drift current.**

3202. Causes of ocean currents.—Although man's knowledge of the processes which produce and maintain ocean currents is far from complete, he does have a general understanding of the principal factors involved. The primary generating force is wind, and the chief secondary force is the density differences in the water. In addition, such factors as depth of water, underwater topography, shape of the basin in which the current is running, extent and location of land, and deflection by the rotation of the earth all affect the oceanic circulation.

3203. Wind currents.—The stress of wind blowing across the sea causes the surface layer of water to move. This motion is transmitted to each succeeding layer below the surface, but due to internal friction within the water, the rate of motion decreases with depth. The current thus set up is called a **wind current.** Although there are many variables, it is generally true that a steady wind for about 12 hours is needed to establish such a current.

A wind current does not flow in the direction of the wind, being deflected by Coriolis force (art. 1611), due to rotation of the earth. This deflection is toward the *right* in the northern hemisphere, and toward the *left* in the southern hemisphere. The Coriolis force is greater in higher latitudes, and is more effective in deep water. In general, the difference between wind direction and surface wind-current direction varies from about 15° along shallow coastal areas to a maximum of 45° in the deep oceans. The angle increases with depth. At several hundred fathoms the current may flow in the opposite direction to the surface current.

The speed of the current depends upon the speed of the wind, its constancy, the length of time it has blown, and other factors. In general, however, about two percent of the wind speed, or a little less, is a good average for deep water where the wind has been blowing steadily for at least 12 hours.

3204. Currents related to density differences.—As indicated in article 3009, the density of water varies with salinity, temperature, and pressure. At any given depth, the differences in density are due to differences in temperature and salinity. When suitable information is available, a map showing geographical density distribution at a certain depth could be drawn, with lines connecting points of equal density. These **isopycnic lines,** or lines connecting points at which a given density occurs at the same depth, would be similar to isobars on a weather map (art. 3827), and would serve an analogous purpose, showing areas of high density and those of low density. In an area of high density, the water surface is lower than in an area of low density, the maximum difference in height being of the order of one to two feet in 40 miles. Because of this difference, water tends to flow from an area of higher water (low density) to one of lower water (high density), but due to rotation of the earth, it is deflected toward the right in the northern hemisphere, and toward the left in the southern hemisphere. Thus, a circulation is set up similar to the cyclonic and anticyclonic circulation in the atmosphere. The greater the density gradient (rate of change with distance), the faster the related current.

3205. Oceanic circulation.—A number of ocean currents flow with great persistence, setting up a circulation that continues with relatively little change throughout the year. Because of the influence of wind in creating current (art. 3203), there is a relationship between this oceanic circulation and the general circulation of the atmosphere (art. 3804). The oceanic circulation is shown in figure 3205, with the names of the major ocean currents. Some differences in opinion exist regarding the names and limits of some of the currents, but those shown are representative. The spacing of the lines is a general indication of speed, but conditions vary somewhat with the season. This is particularly noticeable in the Indian Ocean and along the South China coast, where currents are influenced to a marked degree by the monsoons (art. 3810).

3206. Atlantic Ocean currents.—The trade winds (art. 3806), which blow with great persistence, set up a system of **equatorial currents** which at times extends over as much as 50° of latitude, or even more. There are two westerly flowing currents conforming generally with the areas of trade winds, separated by a weaker, easterly flowing countercurrent.

The **north equatorial current** originates to the northward of the Cape Verde Islands and flows almost due west at an average speed of about 0.7 knot.

The **south equatorial current** is more extensive. It starts off the west coast of Africa, south of the Gulf of Guinea, and flows in a generally westerly direction at an average speed of about 0.6 knot. However, the speed gradually increases until it may reach a value of 2.5 knots or more off the east coast of South America. As the current approaches Cabo de São Roque, the eastern extremity of South America, it divides, the southern part curving toward the south along the coast of Brazil, and the northern part being deflected by the continent of South America toward the north.

Between the north and south equatorial currents a weaker **equatorial countercurrent** sets toward the east in the general vicinity of the doldrums (art. 3805). This is fed by water from the two westerly flowing equatorial currents, particularly the south equatorial current. The extent and strength of the equatorial countercurrent changes with the seasonal variations of the wind. It reaches a maximum during July and August, when it extends from about 50° west longitude to the Gulf of Guinea.

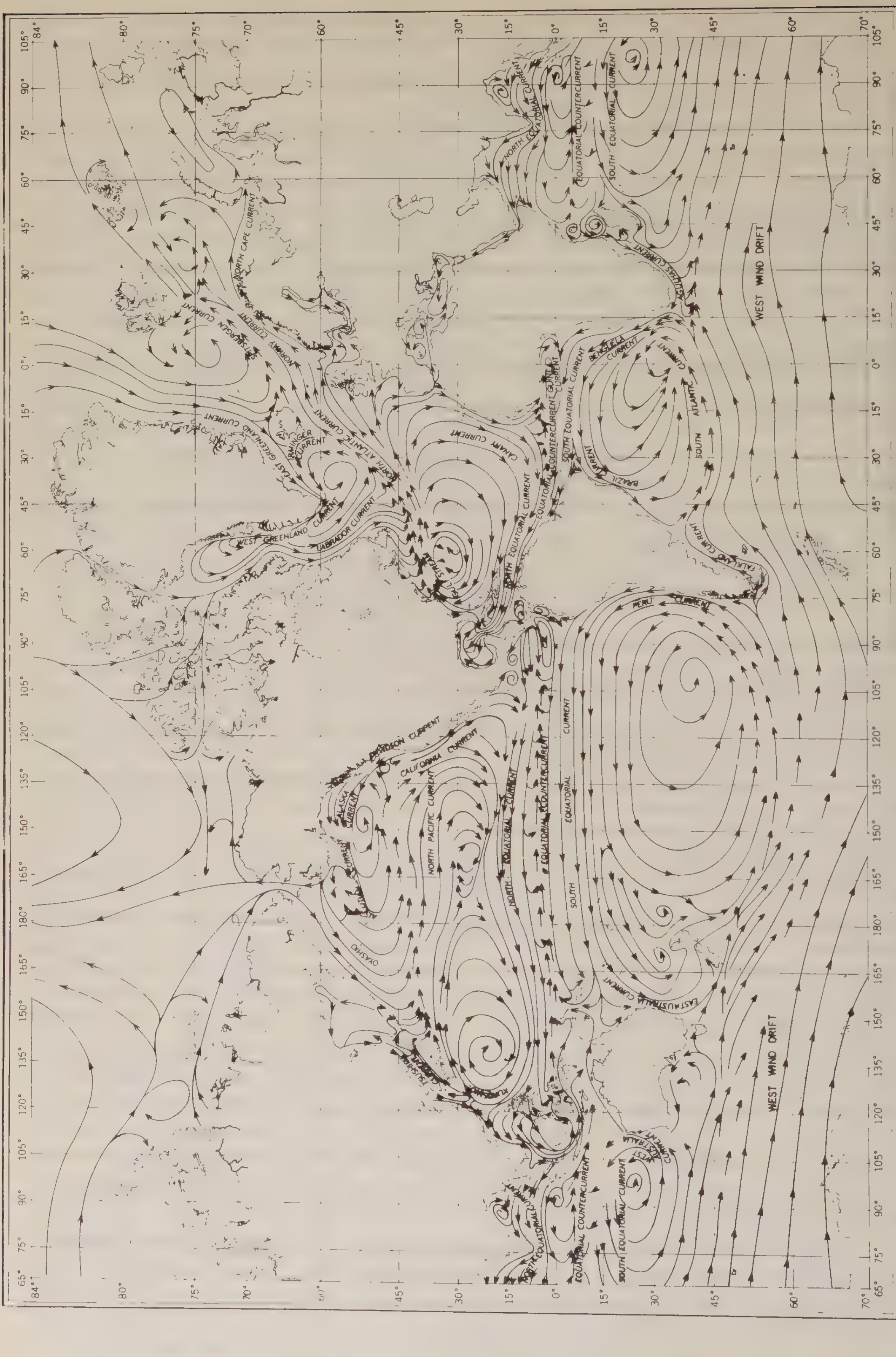


FIGURE 3205.—Major surface currents of the world (northern hemisphere winter).

During its minimum, in December and January, it is of very limited extent, the western portion disappearing altogether.

That part of the south equatorial current flowing along the northern coast of South America which does not feed the equatorial countercurrent unites with the north equatorial current at a point west of the equatorial countercurrent. A large part of the combined current flows through various passages between the Windward Islands, into the Caribbean Sea. It sets toward the west, and then somewhat north of west, finally arriving off the Yucatan peninsula. From here, some of the water curves toward the right, flowing some distance off the shore of the Gulf of Mexico, and part of it curves more sharply toward the east and flows directly toward the north coast of Cuba. These two parts reunite in the Straits of Florida to form the most remarkable of all ocean currents, the **Gulf Stream.** Off the southeast coast of Florida this current is augmented by a current flowing along the northern coasts of Puerto Rico, Hispaniola, and Cuba. Another current flowing eastward of the Bahamas joins the stream north of these islands.

The Gulf Stream follows generally along the east coast of North America, flowing around Florida, northward and then northeastward toward Cape Hatteras, and then curving toward the east and becoming broader and slower. After passing the Grand Banks, it turns more toward the north and becomes a broad drift current flowing across the North Atlantic. That part in the Straits of Florida is sometimes called the **Florida current.**

A tremendous volume of water flows northward in the Gulf Stream. It can be distinguished by its deep indigo-blue color, which contrasts sharply with the dull green of the surrounding water. It is accompanied by frequent squalls. When the Gulf Stream encounters the cold water of the Labrador current, principally in the vicinity of the Grand Banks, there is little mixing of the waters. Instead, the junction is marked by a sharp change in temperature. The line or surface along which this occurs is called the **cold wall.** When the warm Gulf Stream water encounters cold air, evaporation is so rapid that the rising vapor may be visible as frost smoke (art. 3815). The stream carries large quantities of gulfweed from the tropics to higher latitudes.

Recent investigations have shown that the current itself is much narrower and faster than previously supposed, and considerably more variable in its position and speed. The maximum current off Florida ranges from about two to four knots. To the northward the speed is generally less, and decreases further after the current passes Cape Hatteras. As the stream meanders and shifts position, eddies sometimes break off and continue as separate, circular flows until they dissipate. Boats in the Bermuda Race have been known to be within sight of each other and be carried in opposite directions by different parts of the same current. As the current shifts position, its extent does not always coincide with the area of warm, blue water. When the sea is relatively smooth, the edges of the current are marked by ripples.

Information is not yet available to permit prediction of the position and speed of the current at any future time, but it has been found that tidal forces apparently influence the current, which reaches its daily maximum speed about three hours after transit of the moon. The current generally is faster at the time of neap tides than at spring tides. When the moon is over the equator, the stream is narrower and faster than at maximum northerly or southerly declination. Variations in the trade winds (art. 3806) also affect the current.

As the Gulf Stream continues eastward and northeastward beyond the Grand Banks, it gradually widens and decreases speed until it becomes a vast, slow-moving drift current known as the **North Atlantic current,** in the general vicinity of the pre-

vailing westerlies (art. 3808). In the eastern part of the Atlantic it divides into the **northeast drift current** and the **southeast drift current.**

The northeast drift current continues in a generally northeasterly direction toward the Norwegian Sea. As it does so, it continues to widen and decrease speed. South of Iceland it branches to form the **Irminger current** and the **Norway current.** The Irminger current curves toward the north and northwest to join the East Greenland current southwest of Iceland. The Norway current continues in a northeasterly direction along the coast of Norway. Part of it, the **North Cape current,** rounds North Cape into the Barents Sea. The other part curves toward the north and becomes known as the **Spitzbergen current.** Before reaching Svalbard (Spitzbergen), it curves toward the west and joins the cold **east Greenland current** flowing southward in the Greenland Sea. As this current flows past Iceland, it is further augmented by the Irminger current.

Off Kap Farvel, at the southern tip of Greenland, the east Greenland current curves sharply to the northwest, following the coast line. As it does so, it becomes known as the **west Greenland current.** This current continues along the west coast of Greenland, through Davis Strait, and into Baffin Bay. Both east and west Greenland currents are sometimes known by the single name **Greenland current.**

In Baffin Bay the Greenland current follows generally the coast, curving westward off Kap York to form the southerly flowing **Labrador current.** This cold current flows southward off the coast of Baffin Island, through Davis Strait, along the coast of Labrador and Newfoundland, to the Grand Banks, carrying with it large quantities of ice (ch. XXXVI). Here it encounters the warm water of the Gulf Stream, creating the "cold wall." Some of the cold water flows southward along the east coast of North America, inshore of the Gulf Stream, as far as Cape Hatteras. The remainder curves toward the east and flows along the northern edge of the North Atlantic and northeast drift currents, gradually merging with them.

The southeast drift current curves toward the east, southeast, and then south as it is deflected by the coast of Europe. It flows past the Bay of Biscay, toward southeastern Europe and the Canary Islands, where it continues as the **Canary current.** In the vicinity of the Cape Verde Islands, this current divides, part of it curving toward the west to help form the north equatorial current, and part of it curving toward the east to follow the coast of Africa into the Gulf of Guinea, where it is known as the **Guinea current.** This current is augmented by the equatorial countercurrent and, in summer, it is strengthened by monsoon winds. It flows in close proximity to the south equatorial current, but in the opposite direction. As it curves toward the south, still following the African coast, it merges with the south equatorial current.

The clockwise circulation of the North Atlantic leaves a large central area having no well-defined currents. This area is known as the **Sargasso Sea,** from the large quantities of sargasso or gulfweed encountered there.

That branch of the south equatorial current which curves toward the south off the east coast of South America follows the coast as the warm, highly-saline **Brazil current,** which in some respects resembles the Gulf Stream. Off Uruguay, it encounters the colder, less-salty Falkland current and the two curve toward the east to form the broad, slow-moving **South Atlantic current,** in the general vicinity of the prevailing westerlies (art. 3808). This current flows eastward to a point west of the Cape of Good Hope, where it curves northward to follow the west coast of Africa as the strong **Benguela current,** augmented somewhat by part of the Agulhas current flowing around the southern part of Africa from the Indian Ocean. As it continues northward, the current gradually widens and slows. At a point east of St. Helena Island it curves westward to continue as part of the south equatorial current, thus completing the

counterclockwise circulation of the South Atlantic. The Benguela current is augmented somewhat by the **west wind drift,** a current which flows easterly around Antarctica. As the west wind drift flows past Cape Horn, that part in the immediate vicinity of the cape is called the **Cape Horn current.** This current rounds the cape and flows in a northerly and northeasterly direction along the coast of South America as the **Falkland current.**

3207. Pacific Ocean currents follow the general pattern of those in the Atlantic. The **north equatorial current** flows westward in the general area of the northeast trades, and the **south equatorial current** follows a similar path in the region of the southeast trades. Between these two, the weaker **equatorial countercurrent** sets toward the east, just north of the equator.

After passing the Mariana Islands, the major part of the north equatorial current curves somewhat toward the northwest, past the Philippines and Formosa. Here it is deflected further toward the north, where it becomes known as the **Kuroshio,** and then toward the northeast past the Nansei Shoto and Japan, and on in a more easterly direction. Part of the Kuroshio, called the **Tsushima current,** flows through Tsushima Strait, between Japan and Korea, and the Sea of Japan, following generally the northwest coast of Japan. North of Japan it curves eastward and then southeastward to rejoin the main part of the Kuroshio. The limits and volume of the Kuroshio are influenced by the monsoons (art. 3810), being augmented during the season of southwesterly winds, and diminished when the northeasterly winds are prevalent.

The Kuroshio (Japanese for "Black Stream") is so named because of the dark color of its water. It is sometimes called the **Japan Stream.** In many respects it is similar to the Gulf Stream of the Atlantic. Like that current, it carries large quantities of warm tropical water to higher latitudes, and then curves toward the east as a major part of the general clockwise circulation in the northern hemisphere. As it does so, it widens and slows. A small part of it curves to the right to form a weak clockwise circulation west of the Hawaiian Islands. The major portion continues on between the Aleutians and the Hawaiian Islands, where it becomes known as the **North Pacific current.**

As this current approaches the North American continent, most of it is deflected toward the right to form a clockwise circulation between the west coast of North America and the Hawaiian Islands. This part of the current has become so broad that the circulation is generally weak. A small part near the coast, however, joins the southern branch of the Aleutian current, and flows southeastward as the **California current.** The average speed of this current is about 0.8 knot. It is strongest near land. Near the southern end of Baja (Lower) California, this current curves sharply to the west and broadens to form the major portion of the north equatorial current.

During the winter, a weak countercurrent flows northwestward along the west coast of North America from Southern California to Vancouver Island, inshore of the southeasterly flowing California current. This is called the **Davidson current.**

Off the west coast of Mexico, south of Baja California, the current flows southeastward, as a continuation of part of the California current, during the winter. During the summer, the current in this area is northwestward, as a continuation of the equatorial countercurrent, before it turns westward to help form the north equatorial current.

As in the Atlantic, there is in the Pacific a counterclockwise circulation to the north of the clockwise circulation. Cold water flowing southward through the western part of Bering Strait between Alaska and Siberia is joined by water circulating counterclockwise in the Bering Sea to form the **Oyashio.** As the current leaves the strait, it curves toward the right and flows southwesterly along the coast of Siberia and the Kuril Islands. This current brings quantities of sea ice, but no icebergs. When it

encounters the Kuroshio, the Oyashio curves southward and then eastward, the greater portion joining the Kuroshio and North Pacific current. The northern portion continues eastward to join the curving Aleutian current.

As this current approaches the west coast of North America, west of Vancouver Island, part of it curves toward the right and is joined by water from the North Pacific current, to form the California current. The northern branch of the Aleutian current curves in a counterclockwise direction to form the **Alaska current,** which generally follows the coast of Canada and Alaska. When it arrives off the Aleutian Islands, it becomes known as the **Aleutian current.** Part of it flows along the southern side of these islands to about the 180th meridian, where it curves in a counterclockwise direction and becomes an easterly-flowing current, being augmented by the northern part of the Oyashio. The other part of the Aleutian current flows through various openings between the Aleutian Islands, into the Bering Sea. Here it flows in a general counterclockwise direction, most of it finally joining the southerly flowing Oyashio, and a small part of it flowing northward through the eastern side of the Bering Strait, into the Arctic Ocean.

The south equatorial current, extending in width between about 4° N latitude and 10° S, flows westward from South America to the western Pacific. After this current crosses the 180th meridian, the major part curves in a counterclockwise direction, entering the Coral Sea, and then curving more sharply toward the south along the east coast of Australia, where it is known as the **east Australia current.** In the Tasman Sea, northeast of Tasmania, it is augmented by water from the west wind drift, flowing eastward south of Australia. It curves toward the southeast and then the east, gradually merging with the easterly flowing west wind drift, a broad, slow-moving current that circles Antarctica.

Near the southern extremity of South America, most of this current flows eastward into the Atlantic, but part of it curves toward the left and flows generally northward along the west coast of South America as the **Peru current.** Occasionally a set directly toward land is encountered. At about Cabo Blanco, where the coast falls away to the right, the current curves toward the left, past the Galapagos Islands, where it takes a westerly set and constitutes the major portion of the south equatorial current, thus completing the counterclockwise circulation of the South Pacific.

During the northern hemisphere summer, a weak northern branch of the south equatorial current, known as the **Rossel current,** continues on toward the west and northwest along both the southern and northeastern coasts of New Guinea. The southern part flows through Torres Strait, between New Guinea and Australia, into the Arafura Sea. Here, it gradually loses its identity, part of it flowing on toward the west as part of the south equatorial current of the Indian Ocean, and part of it following the coast of Australia and finally joining the easterly flowing west wind drift. The northern part of the Rossel current curves in a clockwise direction to help form the Pacific equatorial countercurrent. During the northern hemisphere winter, the Rossel current is replaced by an easterly flowing current from the Indian Ocean.

3208. Indian Ocean currents follow generally the pattern of the Atlantic and Pacific, but with differences caused principally by the monsoons (art. 3810) and the more limited extent of water in the northern hemisphere. During the northern hemisphere winter, the **north equatorial current** and **south equatorial current** flow toward the west, with the weaker, easterly flowing **equatorial countercurrent** flowing between them, as in the Atlantic and Pacific (but somewhat south of the equator). But during the northern hemisphere summer, both the north equatorial current and the equatorial

countercurrent are replaced by the **monsoon current,** which flows eastward and southeastward across the Arabian Sea and the Bay of Bengal. Near Sumatra, this current curves in a clockwise direction and flows westward, augmenting the south equatorial current and setting up a clockwise circulation in the northern part of the Indian Ocean.

As the south equatorial current approaches the coast of Africa, it curves toward the southwest, part of it flowing through the Mozambique Channel between Madagascar and the mainland, and part flowing along the east coast of Madagascar. At the southern end of this island the two join to form the strong **Agulhas current,** which is analogous to the Gulf Stream.

A small part of the Agulhas current rounds the southern end of Africa and helps form the Benguela current. The major portion, however, curves sharply southward and then eastward to join the west wind drift. This junction is often marked by a broken and confused sea. During the northern hemisphere winter the northern part of this current curves in a counterclockwise direction to form the **west Australia current,** which flows northward along the west coast of Australia. As it passes Northwest Cape, it curves northwestward to help form the south equatorial current. During the northern hemisphere summer, the west Australia current is replaced by a weak current flowing around the western part of Australia as an extension of the southern branch of the Rossel current.

3209. Polar currents.—The waters of the North Atlantic enter the Arctic Ocean between Norway and Svalbard. The currents flow easterly north of Siberia to the region of the Novosibirskiye Ostrova, where they turn northerly across the north pole and continue down the Greenland coast to form the east Greenland current. On the American side of the arctic basin, there is a weak, continuous clockwise flow centered in the vicinity of 80° N, 150° W. A current north through Bering Strait along the American coast is balanced by an outward southerly flow along the Siberian coast, which eventually becomes part of the Oyashio. Each of the main islands or island groups in the arctic, as far as is known, seems to have a clockwise nearshore circulation around it. The Barents Sea, Kara Sea, and Laptev Sea each have a weak counterclockwise circulation. A similar but weaker counterclockwise current system appears to exist in the East Siberian Sea.

In the antarctic, the circulation is generally from west to east in a broad, slow-moving current extending completely around Antarctica. This is called the **west wind drift,** although it is formed partly by the strong westerly wind in this area and partly by density differences. This current is augmented by the Brazil and Falkland currents in the Atlantic, the east Australia current in the Pacific, and the Agulhas current in the Indian Ocean. In return, part of it curves northward to form the Cape Horn, Falkland, and most of the Benguela currents in the Atlantic, the Peru current in the Pacific, and west Australia current in the Indian Ocean.

3210. Ocean currents and climate.—Many of the ocean currents exert a marked influence upon the climate of the coastal regions along which they flow. Thus, warm water from the Gulf Stream, continuing as the North Atlantic, northeast drift, and Irminger currents, arrives off the southwest coast of Iceland, warming it to the extent that Reykjavík has a higher average winter temperature than New York City, far to the south. Great Britain and Labrador are at about the same latitude, but the climate of Great Britain is much milder because of the difference of temperature of currents. The West Coast of the United States is cooled in the summer by the California current, and warmed in the winter by the Davidson current. As a result of this condition, partly, the range of monthly average temperature is comparatively small.

Currents exercise other influences besides those on temperature. The pressure pattern is affected materially, as air over a cold current contracts as it is cooled, and

that over a warm current expands. As air cools above a cold ocean current, fog is likely to form. Frost smoke (art. 3815) is most prevalent over a warm current which flows into a colder region. Evaporation is greater from warm water than from cold water.

In these and other ways, the climate of the earth is closely associated with the ocean currents, although other factors, such as topography and prevailing winds, are also important.

References

Stream Drift Chart of the World—January. U. S. Navy Hydrographic Office Pilot Charts (various editions).

Stream Drift Chart of the World—July. U. S. Navy Hydrographic Office Pilot Charts (various editions).

Sverdrup, H. U., Johnson, M. W., and Fleming, R. H. *The Oceans, Their Physics, Chemistry and General Biology.* New York, Prentice-Hall, 1942.

CHAPTER XXXIII

OCEAN WAVES

3301. Introduction.—Undulations of the surface of the water, called **waves**, are perhaps the most widely observed phenomenon at sea, and possibly the least understood by the average seaman. The mariner equipped with a knowledge of the basic facts concerning waves is able to use them to his advantage, and either avoid hazardous conditions or operate with a minimum of danger if such conditions cannot be avoided.

3302. Causes of waves.—Waves on the surface of the sea are caused principally by wind, but other factors, such as submarine earthquakes, volcanic eruptions, and the tide, also cause waves. If a breeze of less than two knots starts to blow across smooth water, small wavelets called **ripples** form almost instantaneously. When the breeze dies, the ripples disappear as suddenly as they formed, the level surface being restored by surface tension of the water. If the wind speed exceeds two knots, more stable **gravity waves** gradually form, and progress with the wind.

While the generating wind blows, the resulting waves may be referred to as **sea**. When the wind stops or changes direction, the waves that continue on without relation to local winds are called **swell.**

Unlike wind and current, waves are not deflected appreciably by the rotation of the earth, but move in the direction in which the generating wind blows. When this wind ceases, friction and spreading cause the waves to be reduced in height, or **attenuated,** as they move across the surface. However, the reduction takes place so slowly that swell continues until it reaches some obstruction, such as a shore.

When sufficient data on wind conditions are available, the swell and state of the sea a day or more in advance can be predicted. Such forecasts have been found useful in wartime offshore unloading operations. The U. S. Navy Hydrographic Office forecasts sea and swell conditions.

3303. Wave characteristics.—Ocean waves are very nearly in the shape of an inverted **cycloid,** the figure formed by a point inside the rim of a wheel rolling along a level surface. This shape is shown in figure 3303a. The highest parts of waves are called **crests,** and the intervening lowest parts, **troughs.** Since the crests are steeper and narrower than the troughs, the mean or still water level is a little lower than halfway between the crests and troughs. The vertical distance between trough and crest is called **wave height,** labeled H in figure 3303a. The horizontal distance between successive crests, measured in the direction of travel, is called **wave length,** labeled L. The time interval between passage of successive crests at a stationary point is called **wave period (P).** Wave height, length, and period depend upon a number of factors, such as the wind speed, the length of time it has blown, and its **fetch** (the straight distance it has traveled over the surface). Table 3303 indicates the relationship between wind speed, fetch, length of time the wind blows, wave height, and wave period in deep water.

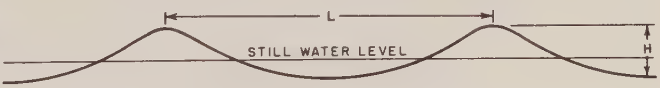


FIGURE 3303a.—A typical sea wave.

Fetch	Beaufort Number 3			4			5			6			7			8			9			10			11			Fetch
	T	H	P	T	H	P	T	H	P	T	H	P	T	H	P	T	H	P	T	H	P	T	H	P	T	H	P	
10	4.4	1.8	2.1	3.7	2.6	2.4	3.2	3.5	2.8	2.7	5.0	3.1	2.5	6.0	3.4	2.3	7.3	3.9	2.0	8.0	4.1	1.9	10.0	4.2	1.8	10.0	5.0	10
20	7.1	2.0	2.5	6.2	3.2	2.9	5.4	4.9	3.3	4.7	7.0	3.8	4.2	8.6	4.3	3.9	10.0	4.4	3.5	12.0	5.0	3.2	14.0	5.2	3.0	16.0	5.9	20
30	9.8	2.0	2.8	8.3	3.8	3.3	7.2	5.8	3.7	6.2	8.0	4.2	5.8	10.0	4.6	5.2	12.1	5.0	4.7	15.8	5.5	4.4	18.0	6.0	4.1	19.8	6.3	30
40	12.0	2.0	3.0	10.3	3.9	3.6	8.9	6.2	4.1	7.8	9.0	4.6	7.1	11.2	4.9	6.5	14.0	5.4	5.8	17.7	5.9	5.4	21.0	6.3	5.1	22.5	6.7	40
50	14.0	2.0	3.2	12.4	4.0	3.8	11.0	6.5	4.4	9.1	9.8	4.8	8.4	12.2	5.2	7.7	15.7	5.6	6.9	19.8	6.3	6.4	23.0	6.7	6.1	25.0	7.1	50
60	16.0	2.0	3.5	14.0	4.0	4.0	12.0	6.8	4.6	10.2	10.3	5.1	9.6	13.2	5.5	8.7	17.0	6.0	8.0	21.0	6.5	7.4	25.0	7.0	7.0	27.5	7.5	60
70	18.0	2.0	3.7	15.8	4.0	4.1	13.5	7.0	4.8	11.9	10.8	5.4	10.5	13.9	5.7	9.9	18.0	6.4	9.0	22.5	6.8	8.3	26.5	7.3	7.8	29.5	7.7	70
80	20.0	2.0	3.8	17.0	4.0	4.2	15.0	7.2	4.9	13.0	11.0	5.6	12.0	14.5	6.0	11.0	18.9	6.6	10.0	24.0	7.1	9.3	28.0	7.7	8.6	31.5	7.9	80
90	23.6	2.0	3.9	18.8	4.0	4.3	16.5	7.3	5.1	14.1	11.2	5.8	13.0	15.0	6.3	12.0	20.0	6.7	11.0	25.0	7.2	10.2	30.0	7.9	9.5	34.0	8.2	90
100	27.1	2.0	4.0	20.0	4.0	4.4	17.5	7.3	5.3	15.1	11.4	6.0	14.0	15.5	6.5	12.8	20.5	6.9	11.9	26.5	7.6	11.0	32.0	8.1	10.3	35.0	8.5	100
120	31.1	2.0	4.2	22.4	4.1	4.7	20.0	7.8	5.4	17.0	11.7	6.2	15.9	16.0	6.7	14.5	21.5	7.3	13.1	27.5	7.9	12.3	33.5	8.4	11.5	37.5	8.8	120
140	36.6	2.0	4.5	25.8	4.2	4.9	22.5	7.9	5.8	19.1	11.9	6.4	17.6	16.2	7.0	16.0	22.0	7.6	14.8	29.0	8.3	13.9	35.5	8.8	13.0	40.0	9.2	140
160	43.2	2.0	4.9	28.4	4.2	5.2	24.3	7.9	6.0	21.1	12.0	6.6	19.5	16.5	7.3	18.0	23.0	8.0	16.4	30.5	8.7	15.1	37.0	9.1	14.5	42.5	9.6	160
180	50.0	2.0	4.9	30.9	4.3	5.4	27.0	8.0	6.2	23.1	12.1	6.8	21.3	17.0	7.5	19.9	23.5	8.3	18.0	31.5	9.0	16.5	38.5	9.5	16.0	44.5	10.0	180
200				33.5	4.3	5.6	29.0	8.0	6.4	25.4	12.2	7.1	23.1	17.5	7.7	21.5	23.5	8.5	19.3	32.5	9.2	18.1	40.0	9.8	17.1	46.0	10.3	200
220				36.5	4.4	5.8	31.1	8.0	6.6	27.2	12.3	7.2	25.0	17.9	8.0	22.9	24.0	8.8	20.9	34.0	9.6	19.1	41.5	10.1	18.2	47.5	10.6	220
240				39.2	4.4	5.9	33.1	8.0	6.8	29.0	12.4	7.3	26.8	17.9	8.2	24.4	24.5	9.0	22.0	34.5	9.8	20.5	43.0	10.3	19.5	49.0	10.8	240
260				41.9	4.4	6.0	34.9	8.0	6.9	30.5	12.6	7.5	28.0	18.0	8.4	26.0	25.0	9.2	23.5	34.5	10.0	21.8	44.0	10.6	20.9	50.5	11.1	260
280				44.5	4.4	6.2	36.8	8.0	7.0	32.4	12.9	7.8	29.5	18.0	8.5	27.7	25.0	9.4	25.0	35.0	10.2	23.0	45.0	10.9	22.0	51.5	11.3	280
300				47.0	4.4	6.3	38.5	8.0	7.1	34.1	13.1	8.0	31.5	18.0	8.7	29.0	25.0	9.5	26.3	35.0	10.4	24.3	45.0	11.1	23.2	53.0	11.6	300
320							40.5	8.0	7.2	36.0	13.3	8.2	33.0	18.0	8.9	30.2	25.0	9.6	27.6	35.5	10.6	25.5	45.5	11.2	24.5	54.0	11.8	320
340							42.4	8.0	7.3	37.6	13.4	8.3	34.2	18.0	9.0	31.6	25.0	9.8	29.0	36.0	10.8	26.7	46.0	11.4	25.5	55.0	12.0	340
360							44.2	8.0	7.4	38.8	13.4	8.4	35.7	18.1	9.1	33.0	25.0	9.9	30.0	36.5	10.9	27.7	46.5	11.6	26.6	55.0	12.2	360
380							46.1	8.0	7.5	40.2	13.5	8.5	37.1	18.2	9.3	34.2	25.5	10.0	31.3	37.0	11.1	29.1	47.0	11.8	27.7	55.5	12.4	380
400							48.0	8.0	7.7	42.2	13.5	8.6	38.8	18.4	9.5	35.6	26.0	10.2	32.5	37.0	11.2	30.2	47.5	12.0	28.9	56.0	12.6	400
420							50.0	8.0	7.8	43.5	13.6	8.7	40.0	18.7	9.6	36.9	26.5	10.3	33.7	37.5	11.4	31.5	47.5	12.2	29.6	56.5	12.7	420
440							52.0	8.0	7.9	44.7	13.7	8.8	41.3	18.8	9.7	38.1	27.0	10.4	34.8	37.5	11.5	32.5	48.0	12.3	30.9	57.0	12.9	440
460							54.0	8.0	8.0	46.2	13.7	8.9	42.8	19.0	9.8	39.5	27.5	10.6	36.0	37.5	11.7	33.5	48.5	12.5	31.8	57.5	13.1	460
480							56.0	8.0	8.1	47.8	13.7	9.0	44.0	19.0	9.9	41.0	27.5	10.8	37.0	37.5	11.8	34.5	49.0	12.6	32.7	57.5	13.2	480
500							58.0	8.0	8.2	49.2	13.8	9.1	45.5	19.1	10.1	42.1	27.5	10.9	38.3	38.0	11.9	35.5	49.0	12.7	33.9	58.0	13.4	500
550										53.0	13.8	9.3	48.5	19.5	10.3	44.9	27.5	11.1	41.0	38.5	12.2	38.2	50.0	13.0	36.5	59.0	13.7	550
600										56.3	13.8	9.5	51.8	19.7	10.5	47.7	27.5	11.3	43.6	39.0	12.5	40.3	50.0	13.3	38.7	60.0	14.0	600
650													55.0	19.8	10.7	50.3	27.5	11.6	46.4	39.5	12.8	43.0	50.0	13.7	41.0	60.0	14.2	650
700													58.5	19.8	11.0	53.2	27.5	11.8	49.0	40.0	13.1	45.4	50.5	14.0	43.5	60.5	14.5	700
750																56.2	27.5	12.1	51.0	40.0	13.3	48.0	51.0	14.2	45.8	61.0	14.8	750
800																59.2	27.5	12.3	53.8	40.0	13.5	50.6	51.5	14.5	47.8	61.5	15.0	800
850																			56.2	40.0	13.8	52.5	52.0	14.6	50.0	62.0	15.2	850
900																			58.2	40.0	14.0	54.6	52.0	14.9	52.0	62.5	15.5	900
950																						57.2	52.0	15.1	54.0	63.0	15.7	950
1000																						59.3	52.0	15.3	56.3	63.0	16.0	1000

TABLE 3303.—Minimum Time (T) in hours that wind must blow to form waves of H significant height (in feet) and P period (in seconds). Fetch in nautical miles. Based upon the relationships given in H.O. Pub. No. 604, *Techniques for Forecasting Wind Waves and Swell.* See also H.O. Pub. No. 603, *Observing and Forecasting Ocean Waves.*

If the water is deeper than one-half the wave length (L), this length in feet is theoretically related to period (P) in seconds by the formula

$$L = 5.12P^2.$$

The actual value has been found to be a little less than this for swell, and about two-thirds the length determined by this formula for sea. When the waves leave the generating area and continue as free waves, the wave length and period continue to increase, while the height decreases. The rate of change gradually decreases.

The speed (S) of a free wave in deep water is nearly independent of its height or steepness. For swell, its relationship in knots to the period (P) in seconds is given by the formula

$$S = 3.03P.$$

The relationship for sea is not known.

The theoretical relationship between speed, wave length, and period is shown in figure 3303b. Thus, as waves continue on beyond the generating area, the period, length, and speed all increase, providing some indication of the distance of the gener-

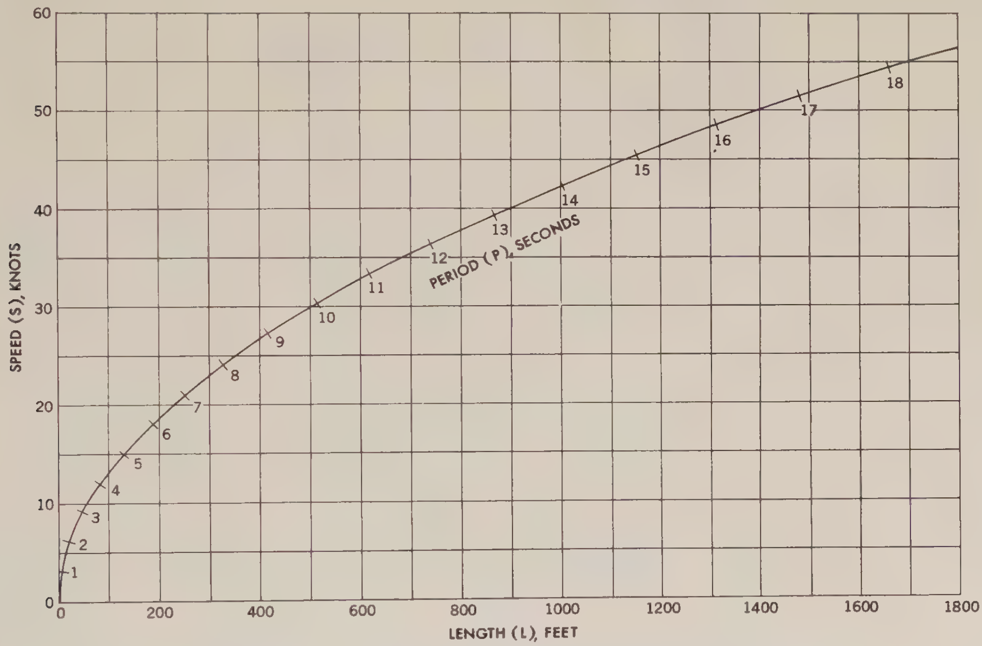


FIGURE 3303b.—Relationship between speed, length, and period of waves in deep water, based upon the theoretical relationship between period and length.

ating area. However, the time needed for a wave system to travel some distance is *double* that which would be indicated by the speed of individual waves. This is because the front wave gradually disappears and transfers its energy to succeeding waves. The process is followed by each front wave in succession, at such a rate that the wave *system* advances at a speed which is just *half* that of *individual* waves. This process can be seen in the bow wave of a vessel. The speed at which the wave system advances is called **group velocity.**

Because of the existence of many independent wave systems at the same time, the sea surface acquires a complex and irregular pattern. Also, since the longer waves outrun the shorter ones, the resulting interference adds to the complexity of the pattern.

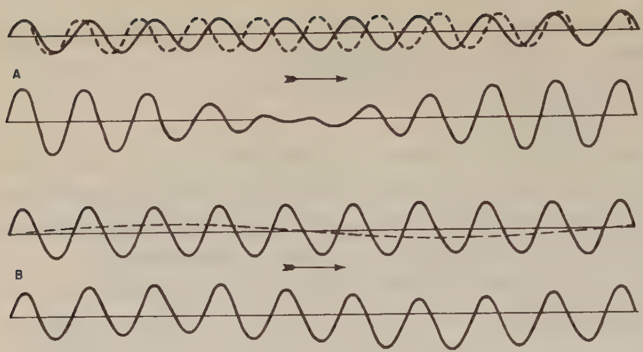


FIGURE 3303c.—Interference. The upper part of *A* shows two waves of equal height and nearly equal length traveling in the same direction. The lower part of *A* shows the resulting wave pattern. In *B* similar information is shown for short waves and long swell.

The process of interference, illustrated in figure 3303c, is duplicated many times in the sea, being the principal reason that successive waves are not of the same height. The irregularity of the surface may be further accentuated by the presence of wave systems crossing at an angle to each other, producing peak-like rises.

In reporting average wave heights, the mariner has a tendency to neglect the lower ones. It has been found that the reported value is about the average for the highest one-third. This is sometimes called the "significant" wave height. The approximate relationship between this height and others, is as follows:

Wave	*Relative height*
Average	0. 64
Significant	1. 00
Highest 10 percent	1. 29
Highest	1. 87

3304. Path of water particles in a wave.—As shown in figure 3304, a particle of water on the surface of the ocean follows a somewhat circular orbit as a wave passes, but moves very little in the direction of motion of the wave. The common wave producing this action is called an **oscillatory wave.** As the crest passes, the particle moves forward, giving the water the appearance of moving with the wave. As the trough passes, the motion is in the opposite direction. The radius of the circular orbit decreases with depth, approaching zero at a depth equal to about half the wave length. In shallower water the orbits become more elliptical, and in very shallow water, as at a beach, the vertical motion disappears almost completely.

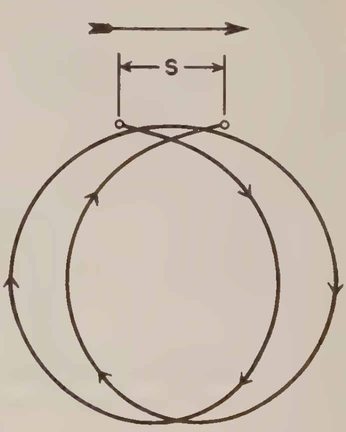


FIGURE 3304.—Orbital motion and displacement, *s*, of a particle on the surface of deep water during two wave periods.

Since the speed is greater at the top of the orbit than at the bottom, the particle is not at exactly its original point following passage of a wave, but has moved slightly in the direction of motion of the wave. However, since this advance is small in relation to the vertical displacement, a floating object is raised and lowered by passage of a wave, but moved little from its original position. If this were not so, a slow moving vessel might experience considerable difficulty in making way against a wave train. In figure 3304 the forward displacement is greatly exaggerated.

3305. Effects of currents on waves.—A following current increases wave lengths and decreases wave heights. An opposing current has the opposite effect, decreasing the length and increasing the height. A strong opposing current may cause the waves

to break. The extent of wave alteration is dependent upon the ratio of the still-water wave speed to the speed of the current.

Moderate ocean currents running at oblique angles to wave directions appear to have little effect, but strong tidal currents perpendicular to a system of waves have been observed to completely destroy them in a short period of time.

3306. The effect of ice on waves.—When ice crystals form in sea water, internal friction is greatly increased. This results in smoothing of the sea surface. The effect of pack ice is even more pronounced. A vessel following a lead through such ice may be in smooth water even when a gale is blowing and heavy seas are beating against the outer edge of the pack. Hail is also effective in flattening the sea, even in a high wind.

3307. Waves and shallow water.—When a wave encounters shallow water, the movement of the individual particles of water is restricted by the bottom, resulting in reduced wave speed. If the wave approaches the shoal at an angle, each part is

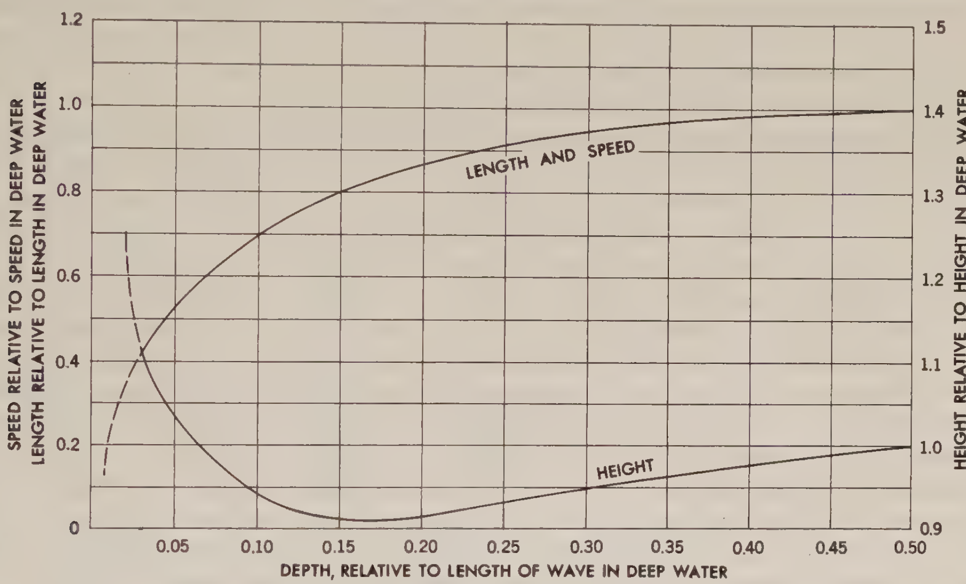


FIGURE 3307.—Alteration of the characteristics of waves as they cross a shoal.

slowed successively as the depth decreases. This causes a change in direction of motion or **refraction,** the wave tending to become parallel to the depth curves. The effect is similar to the refraction of light and other forms of radiant energy (art. 1613).

As each wave slows, the next wave behind it, in deeper water, tends to catch up. As the wave length decreases, the height generally becomes greater. The lower part of a wave, being nearest the bottom, is slowed more than the top. This may cause the wave to become unstable, the faster-moving top falling or **breaking.** Such a wave is called a **breaker,** and a series of breakers, **surf.** This subject is covered in greater detail in chapter XXXIV.

Swell passing over a shoal but not breaking undergoes a decrease in wave length and speed, and an increase in height. Such **ground swell** may cause heavy rolling if it is on the beam and its period is the same as the period of roll of a vessel, even though the sea may appear relatively calm. Figure 3307 illustrates the approximate alteration of the characteristics of waves as they cross a shoal.

3308. Energy of waves.—The potential energy of a wave is related to the vertical distance of each particle from its still-water position, and therefore moves with the

wave. In contrast, the kinetic energy of a wave is related to the speed of the particles, being distributed evenly along the entire wave.

The amount of kinetic energy in even a moderate wave is tremendous. A four-foot, ten-second wave striking a coast expends more than 35,000 horsepower per mile of beach. For each 56 miles of coast, the energy expended equals the power generated at Hoover Dam. An increase in temperature of the water in the relatively narrow **surf zone** in which this energy is expended would seem to be indicated, but no pronounced increase has been measured. Apparently, any heat that may be generated is dissipated to the deeper water beyond the surf zone.

3309. Wave measurement aboard ship.—With suitable equipment and adequate training, one can make reasonably reliable measurements of the height, length, period, and speed of waves. However, the mariner's estimates of height and length usually contain relatively large errors. There is a tendency to underestimate the heights of low waves, and overestimate the heights of high ones. There are numerous accounts of waves 75 to 80 feet high, or even higher, although waves more than 55 feet high are very rare. Wave length is usually underestimated. The motions of the vessel from which measurements are made perhaps contribute to such errors.

Height. Measurement of wave height is particularly difficult. A microbarograph (art. 3705) can be used if the wave is long enough to permit the vessel to ride up and down with it. If the waves are approaching from dead ahead or dead astern, this requires a wave length at least twice the length of the vessel. For most accurate results the instrument should be placed at the center of roll and pitch, to minimize the effects of these motions. Wave height can often be estimated with reasonable accuracy by comparing it with freeboard of the vessel. This is less accurate as wave height and vessel motion increase. If a point of observation can be found at which the top of a wave is in line with the horizon when the observer is in the trough, the wave height is equal to height of eye. However, if the vessel is rolling or pitching, this height at the moment of observation may be difficult to determine.

Length. The dimensions of the vessel can be used to determine wave length. Errors are introduced by perspective and disturbance of the wave pattern by the vessel. These errors are minimized if observations are made from maximum height. Best results are obtained if the sea is from dead ahead or dead astern.

Period. If allowance is made for the motion of the vessel, wave period can be determined by measuring the interval between passages of wave crests past the observer. The correction for the motion of the vessel can be eliminated by timing the passage of successive wave crests past a patch of foam or a floating object at some distance from the vessel. Accuracy of results can be improved by averaging several observations.

Speed can be determined by timing the passage of the wave between measured points along the side of the ship, if corrections are applied for the direction of travel of the wave and the speed of the ship.

More detailed instructions on making wave observations are given in H.O. Pub. No. 606–e, *Sea and Swell Observations*, and H.O. Spec. Pub. 44, *Visual Wave Observations*.

The length, period, and speed of waves in deep waters are interrelated by the relationships indicated in article 3303. However, these should be used as a general guide only, because exact mathematical relationships have not been established, as indicated. In the case of speed and period, there is evidence to indicate that for sea the relationship may be more nearly expressed by the formula $L=SP$ than by that given in article 3303, although there is considerable doubt as to the exact relationship. There is no definite mathematical relationship between wave height and length, period, or speed.

3310. Tsunamis are ocean waves produced by sudden, large-scale motion of a portion of the ocean floor or the shore, as by volcanic eruption, earthquake (sometimes called **seaquake** if it occurs at sea), or landslide. If they are caused by a submarine earthquake, they are usually called **seismic sea waves.** The point directly above the disturbance, at which the waves originate, is called the **epicenter.** Either a tsunami or a storm wave (art. 3311) that overflows the land is popularly called a **tidal wave,** although it bears no relation to the tide.

If a volcanic eruption occurs below the surface of the sea, the escaping gases cause a quantity of water to be pushed upward in the shape of a dome or mound. The same effect is caused by the sudden rising of a portion of the bottom. As this water settles back, it creates a wave which travels at high speed across the surface of the ocean.

Tsunamis usually occur in series, gradually increasing in height until a maximum is reached between about the third and eighth wave. Following the maximum, they again become smaller. Waves may continue to form for several hours, or even for days.

In deep water the wave height of a tsunami is probably never greater than two or three feet. Since the wave length is usually considerably more than 100 miles, the wave is not conspicuous at sea. In the Pacific, where most tsunamis occur, the wave period varies between about 15 and 60 *minutes,* and the speed in deep water is more than 400 knots. The approximate speed can be computed by the formula

$$S=0.6\sqrt{gd}=3.4\sqrt{d},$$

where S is the speed in knots, g is the acceleration due to gravity (32.2 feet per second per second), and d is the depth of water in feet. This formula is applicable to any wave in water having a depth of less than half the wave length. For most ocean waves it applies only in shallow water, because of the relatively short wave length.

When a tsunami enters shoal water, it undergoes the same changes as other waves. The formula indicates that speed is proportional to depth of water. Because of the great speed of a tsunami when it is in relatively deep water, the slowing is relatively much greater than that of an ordinary wave crested by wind. Therefore, the increase in height is also much greater. Tsunamis 50 feet in height or higher have reached the shore, inflicting widespread damage. On April 1, 1946, seismic sea waves originating at an epicenter near the Aleutians spread over the entire Pacific. Scotch Cap Light on Unimak Island, 57 feet above sea level, was completely destroyed. Traveling at an average speed of 490 miles per hour, the waves reached the Hawaiian Islands in four hours and 34 minutes, where they arrived as waves 50 feet above the high water level, and flooded a strip of coast more than 1,000 feet wide at some places. They left a death toll of 173, and property damage of $25,000,000. Less destructive waves reached the shores of North and South America, and Australia, 6,700 miles from the epicenter.

After this disaster, a tsunami warning system was set up in the Pacific, even though destructive waves are relatively rare (averaging about one in 20 years in the Hawaiian Islands). The system consists of three sections. First, a number of seismograph stations to provide information for establishing the time and epicenter of quakes. Second, a group of tide stations to report any evidence of a tsunami. These stations are alerted when a quake is recorded at the seismograph stations. Third, a communication system which gives tsunami warnings high priority because of their speed and possible destructiveness. A travel time chart centered upon the Hawaiian Islands is used to estimate time of arrival of the waves.

Fortunately, relatively few earthquakes produce seismic sea waves. The size of the waves that do form depends upon the nature and intensity of the disturbance. The height and destructiveness of the waves arriving at any place depend upon its distance from the epicenter, topography of the ocean floor, and the coast line itself.

The angle at which the wave arrives, the shape of the coast line, and the topography along the coast and offshore all have their effect. The position of the shore is also a factor, as it may be sheltered by intervening land, or be in a position where waves have a tendency to converge, either because of refraction or reflection, or both.

In addition to seismic sea waves, earthquakes below the surface of the sea may produce a longitudinal wave that travels upward toward the surface, at the speed of sound. When a ship encounters such a wave, it is felt as a sudden shock which may be of such severity that the crew thinks the vessel has struck bottom. Because of such reports, some older charts indicated shoal areas at places where the depth is now known to be a thousand fathoms or more.

3311. Storm waves.—In relatively tideless seas like the Baltic and Mediterranean, winds cause the chief fluctuations in sea level. Elsewhere, the astronomical tide usually masks these variations. However, under exceptional conditions, either severe extratropical storms or tropical cyclones can produce changes in sea level that exceed the normal range of tide. Low sea level is of little concern except to shipping, but a rise above ordinary high-water mark, particularly when it is accompanied by high waves, can result in a catastrophe.

Although, like tsunamis, these **storm waves** or **storm surges** are popularly called **tidal waves,** they are not associated with the tide. They consist of a single wave crest and hence have no period or wave length.

Three effects in a storm induce a rise in sea level. The first is wind stress on the sea surface, which results in a piling-up of water (sometimes called "wind set-up"). The second effect is the convergence of wind-driven currents, which elevates the sea surface along the convergence line. In shallow water, bottom friction and the effects of local topography cause this elevation to persist and may even intensify it. The low atmospheric pressure that accompanies severe storms causes the third effect, which is sometimes referred to as the "inverted barometer." An inch of mercury is equivalent to about 13.6 inches of water (art. 3115) and the adjustment of the sea surface to the reduced pressure can amount to several feet at equilibrium (art. 3911).

All three of these causes act independently, and if they happen to occur simultaneously, their effects are additive. In addition, the wave can be intensified or amplified by the effects of local topography. Storm waves may reach heights of 20 feet or more, and it is estimated that they cause three-fourths of the deaths attributed to hurricanes.

3312. Standing waves and microseisms.—Previous articles in this chapter have dealt with **progressive waves** which appear to move regularly with time. When two systems of progressive waves having the same period travel in opposite directions across the same area, a series of **standing waves** may form. These appear to remain stationary. Recent investigation has indicated that when this condition occurs, a pressure variation is exerted on the ocean bottom proportional to the product of the wave heights of the two wave systems. The period of these pressure variations is half that of the progressive waves. The magnitude and period of these variations are of the right order to cause a series of minute earth shocks of the magnitude of those recorded by very sensitive seismographs and known as **microseisms.** It is probable, therefore, that microseisms are generated by standing waves established in any manner, as by waves from independent sources, those in the wake of a moving circulation, waves at the center of a stationary circulation, or by reflection of waves striking a steep shore.

Another type of standing wave, called a **seiche** (sāsh), sometimes occurs in a confined body of water. It is a long wave, usually having its crest at one end of the confined space, and its trough at the other. Its period may be anything from a few

minutes to an hour or more, but somewhat less than the tidal period. Seiches are usually attributed to strong winds or differences in atmospheric pressure.

3313. Tide waves.—As indicated in chapter XXXI, there are, in general, two regions of high tide separated by two regions of low tide, and these regions move progressively westward around the earth as the moon revolves in its orbit. The high tides are the crests of these **tide waves,** and the low tides are the troughs. The wave is not noticeable at sea, but becomes apparent along the coasts, particularly in funnel-shaped estuaries. In certain river mouths or estuaries of particular configuration, the incoming wave of high water overtakes the preceding low tide, resulting in a high-crested, roaring wave which progresses upstream in one mighty surge called a **bore.**

3314. Internal waves.—Thus far, the discussion has been confined to waves on the surface of the sea, the boundary between air and water. **Internal waves,** or **boundary waves,** are created below the surface, at the boundaries between water strata of different densities. The density differences between adjacent water strata in the sea are considerably less than that between sea and air. Consequently, internal waves are much more easily formed than surface waves, and they are often much larger. The maximum height of wind waves on the surface is about 60 feet, but internal wave heights as great as 300 feet have been encountered.

Internal waves are detected by a number of observations of the vertical temperature distribution, using recording devices such as the bathythermograph (art. 3007). They have periods as short as a few minutes, and as long as 12 or 24 hours, these greater periods being associated with the tides.

A slow-moving ship operating in a fresh water layer having a depth approximating the draft of the vessel may produce short-period internal waves. This may occur off rivers emptying into the sea or in polar regions in the vicinity of melting ice. Under suitable conditions, the normal propulsion energy of the ship is expended in generating and maintaining these internal waves and the ship appears to "stick" in the water, becoming sluggish and making little headway. The phenomenon, known as **dead water,** disappears when speed is increased by a few knots.

The full significance of internal waves has not been determined, but it is known that they may cause submarines to rise and fall like a ship at the surface, and they may also affect sound transmission in the sea.

3315. Waves and ships.—The effects of waves on a ship vary considerably with the type ship, its course and speed, and the condition of the sea. A short vessel has a tendency to ride up one side of a wave and down the other side, while a larger vessel may tend to ride *through* the waves on an even keel. If the waves are of such length that the bow and stern of a vessel are alternately in successive crests and successive troughs, the vessel is subject to heavy sagging and hogging stresses, and under extreme conditions may break in two. A change of heading may reduce the danger. Because of the danger from sagging and hogging, a small vessel is sometimes better able to ride out a storm than a large one.

If successive waves strike the side of a vessel at the same phase of successive rolls, relatively small waves can cause heavy rolling. The effect is similar to that of swinging a child, where the strength of the push is not as important as its timing. The same effect, if applied to the bow or stern in time with the pitch, can cause heavy pitching. A change of either heading or speed can reduce the effect.

A wave having a length twice that of a ship places that ship in danger of falling off into the trough of the sea, particularly if it is a slow-moving vessel. The effect is especially pronounced if the sea is broad on the bow or broad on the quarter. An increase of speed reduces the hazard.

3316. Use of oil for modifying the effects of breaking waves.—Oil has proved effective in modifying the effects of breaking waves, and has proved useful to vessels at sea, whether making way or stopped, particularly when lowering or hoisting boats. Its effect is greatest in deep water, where a small quantity suffices if the oil can be made to spread to windward. In shallow water where the water is in motion over the bottom, oil is less effective but of some value.

The heaviest oils, notably animal and vegetable oils, are the most effective. Crude petroleum is useful, but its effectiveness can be improved by mixing it with animal and vegetable oils. Gasoline or kerosene are of little value. Oil spreads slowly. In cold weather it may need some thinning with petroleum to hasten the process and produce the desired spread before the vessel is too far away for the effect to be useful.

At sea, best results can be expected if the vessel drifts or runs slowly before the wind, with the oil being discharged on both sides from waste pipes or by other convenient method. If a sea anchor is used, oil can be distributed from a container inserted within it for this purpose. If such a container is not available, an oil bag can be fastened to an endless line rove through a block on the sea anchor. This permits distribution of oil to windward, and provides a means for hauling the bag aboard for refilling. If another vessel is being towed, the oil should be distributed from the towing vessel, forward and on both sides, so that both vessels will be benefited. If a drifting vessel is to be approached, the oil might be distributed from both sides of the drifting vessel or by the approaching vessel, which should distribute it to leeward of the drifting vessel so that that vessel will drift into it. If the vessel being approached is aground, the procedure best suiting the circumstances should be used.

If oil is needed in crossing a bar to enter a harbor, it can be floated in ahead of the vessel if a flood current is running. A considerable amount may be needed. During slack water a hose might be trailed over the bow and oil poured freely through it if no more convenient method is available. With an ebb current oil is of little use, unless it can be distributed from another vessel or in some other manner from the opposite side of the bar.

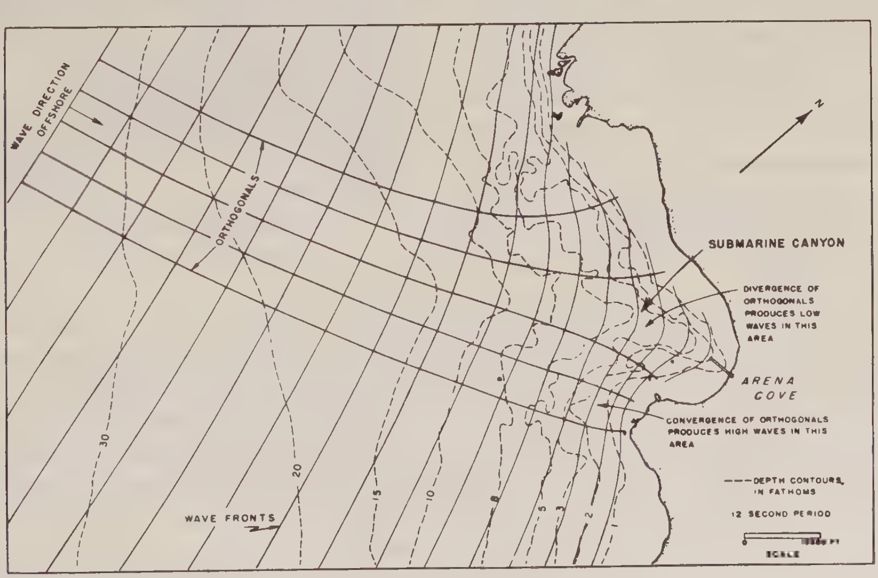
CHAPTER XXXIV

AMPHIBIOUS OPERATIONS

3401. Amphibious operations and the navigator.—Among the major problems in amphibious operations are the safe navigation of landing craft through the surf zone to the beach, trafficability of the beach, and movement inland. The purpose of this chapter is to acquaint the navigator with the first two of these and the oceanographic factors affecting them. The navigational aspects of the third problem are discussed in chapter XXVII, "Land Navigation."

3402. Refraction.—As explained in article 3307, wave speed is slowed in shallow water, causing **refraction** if the waves approach the beach at an angle. Along a perfectly straight beach, with uniform shoaling, the wave fronts tend to become parallel to the shore. Any irregularities in the coast line or bottom contours, however, affect the refraction, causing irregularity. In the case of a ridge perpendicular to the beach, for instance, the shoaling is more rapid, causing greater refraction. The waves tend to align themselves with the bottom contours. Waves on both sides of the ridge have a component of motion toward the ridge. This **convergence** of water toward the ridge causes an increase in wave or breaker height. A submarine canyon or valley perpendicular to the beach, on the other hand, produces **divergence,** with a decrease in wave or breaker height. These effects are illustrated in figure 3402. Bends in the coast line have a similar effect, convergence occuring at a *point,* and divergence if the coast is concave to the sea.

Under suitable conditions, currents also cause refraction. This is of particular importance at entrances of tidal estuaries. When waves encounter a current running in the opposite direction, they become higher and shorter. This results in a choppy



Courtesy of Robert L. Wiegel, Council on Wave Research, University of California.

FIGURE 3402.—The effect of bottom topography in causing wave convergence and wave divergence.

sea, often with breakers. When waves move in the same direction as current, they decrease in height, and become longer. Refraction occurs when waves encounter a current at an angle.

Refraction diagrams, useful in planning amphibious operations, can be prepared with the aid of nautical charts or aerial photographs. The method of doing so is explained in H.O. Pub. No. 605, *Graphical Construction of Wave Refraction Diagrams.*

3403. Breakers and surf.—In deep water, swell generally moves across the surface as somewhat regular, smooth undulations (ch. XXXIII). When shoal water is reached, the wave period remains the same, but the speed decreases. The amount of decrease is negligible until the depth of water becomes about one-half the wave length, when the waves begin to "feel" bottom. There is a slight decrease in wave height, followed by a rapid increase, if the waves are traveling perpendicular to a straight coast with a uniformly sloping bottom. As the waves become higher and shorter, they also become steeper, and the crest becomes narrower. When the speed of individual particles at the crest becomes greater than that of the wave, the front face of the wave becomes steeper than the rear face. This process continues at an accelerating rate as the depth of water decreases. At some point the wave may become unstable, toppling forward to form a **breaker.**

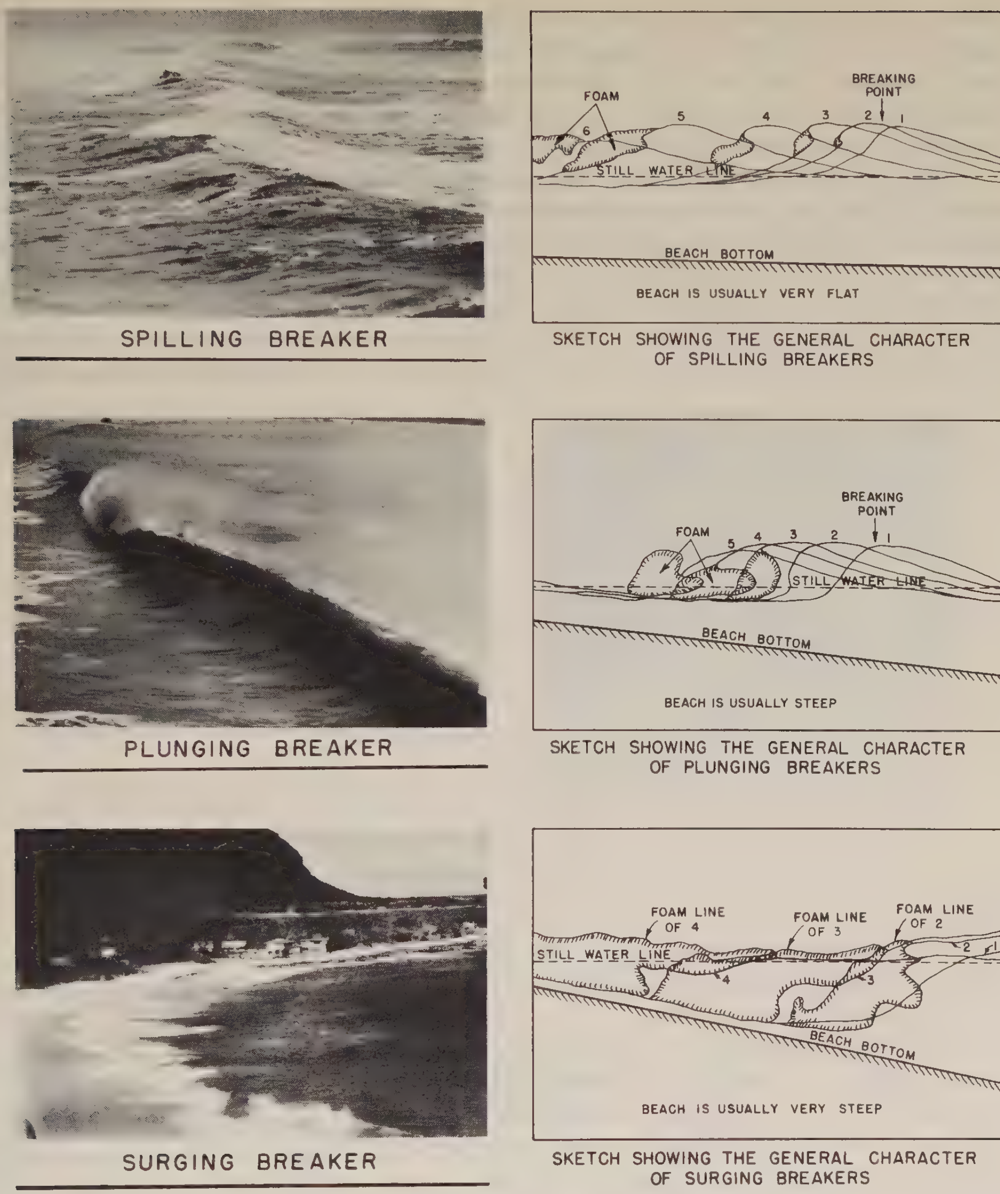
There are three general classes of breakers. A **spilling breaker** breaks gradually over a considerable distance. A **plunging breaker** tends to curl over and break with a single crash. A **surging breaker** peaks up, but surges up the beach without spilling or plunging. It is classed as a breaker even though it does not actually break. The type of breaker is determined by the steepness of the beach and the steepness of the wave before it reaches shallow water, as illustrated in figure 3403.

Longer waves break in deeper water, and have a greater breaker height. The effect of a steeper beach is also to increase breaker height. The height of breakers is less if the waves approach the beach at an acute angle. With a steeper beach slope there is greater tendency of the breakers to plunge or surge. Following the **uprush** of water onto a beach after the breaking of a wave, the seaward **backrush** occurs. The returning water is called **backwash.** It tends to further slow the bottom of a wave, thus increasing its tendency to break. This effect is greater as either the speed or depth of the backwash increases. The still water depth at the point of breaking is approximately 1.3 times the average breaker height.

Surf varies with both position along the beach and time. A change in position often means a change in bottom contour, with the refraction effects discussed in article 3402. At the same point, the height and period of waves vary considerably from wave to wave. A group of high waves is usually followed by several lower ones. Therefore, passage through surf can usually be made most easily immediately following a series of higher waves.

Since surf conditions are directly related to height of the waves approaching a beach, and the configuration of the bottom, the state of the surf at any time can be predicted if one has the necessary information and knowledge of the principles involved. Height of the sea and swell can be predicted from wind data, and information on bottom configuration can generally be obtained from the nautical chart. In addition, the area of lightest surf along a beach can be predicted if details of the bottom configuration are available. Detailed information on prediction of surf conditions is given in H.O. Pub. No. 234, *Breakers and Surf; Principles in Forecasting.*

3404. Currents in the surf zone.—In and adjacent to the surf zone, currents are generated by waves approaching the bottom contours at an angle, and by irregularities in the bottom.



Courtesy of Robert L. Weigel, Council on Wave Research, University of California.

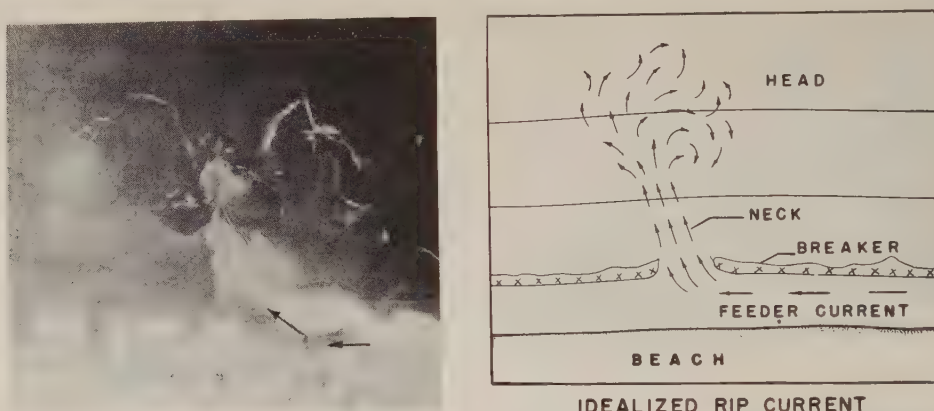
FIGURE 3403.—The three types of breakers.

Waves approaching at an angle produce a **longshore current** parallel to the beach, within the surf zone. Longshore currents are most common along straight beaches. Their speeds increase with increasing breaker height, decreasing wave period, increasing angle of breaker line with the beach, and increasing beach slope. Speed seldom exceeds one knot, but sustained speeds as high as three knots have been recorded. Longshore currents are usually constant in direction. They increase the danger of landing craft broaching to.

As explained in article 3402, wave fronts advancing over nonparallel bottom contours are refracted to cause convergence or divergence of the energy of the waves. Energy concentrations, in areas of convergence, form barriers to the returning back-

wash, which is deflected *along* the beach to areas of less resistance. Backwash accumulates at weak points, and returns seaward in concentrations, forming **rip currents** through the surf. At these points the large volume of returning water has a retarding effect upon the incoming waves, thus adding to the condition causing the rip current. The waves on one or both sides of the rip, having greater energy and not being retarded by the concentration of backwash, advance faster and farther up the beach. From here, they move *along* the beach as **feeder currents.** At some point of low resistance, the water flows seaward through the surf, forming the **neck** of the rip current. Outside the breaker line the current widens and slackens, forming the **head.** The various parts of a rip current are shown in figure 3404.

Rip currents may also be caused by irregularities in the beach face. If a beach indentation causes an uprush to advance farther than the average, the backrush is



Courtesy of Robert L. Weigel, Council on Wave Research, University of California.

FIGURE 3404.—A rip current (left) and a diagram of its parts (right).

delayed and this in turn retards the next incoming **foam line** (the front of a wave as it advances shoreward after breaking) at that point. The foam line on each side of the retarded point continues in its advance, however, and tends to fill in the retarded area, producing a rip current.

3405. Beach trafficability.—The trafficability of an area depends upon characteristics of both the area itself and the vehicles to be used in traversing the area.

In amphibious operations, landing craft must successively negotiate the submerged section of the beach between landing craft and shore, the moist "beach face" which is covered by water at high tide, the dry section which is never submerged, and the backshore. Several types and conditions of soils may be encountered in traversing the four sections.

Bearing capacity, traction capacity, and **rolling resistance** are the main factors in soil trafficability. Bearing capacity concerns the ability of the soil to support the vehicle; traction capacity deals with its ability to give the vehicle a forward thrust; rolling resistance refers to the tendency of the soil to oppose the forward thrust. These main factors are themselves dependent upon vehicle characteristics, traffic density, slope of the ground, nature of the soil, and certain soil characteristics which depend chiefly upon the water content of the soil.

Trafficability investigations have led to the following general conclusions: (1) Serious trafficability problems arise with mud. (2) The trafficability of sandy beaches composed of coarse-grained soils increases with increased flatness. The bearing and traction capacities of such soils are satisfactory if the soil is confined, as by the use of large low-pressure tires or wide tracks. (3) Wet sand offers less resistance than dry

sand. Therefore, vehicles are best discharged at low tide, onto the moist part of the beach face thus exposed. Vehicles landed after a heavy rain will encounter better trafficability than those landed when the beach is dry. (4) The landing of tracked vehicles on all but the flattest beaches disturbs the soil to such an extent that wheeled vehicles have great difficulty in operating. However, tracked vehicles can operate on many beaches where wheeled vehicles cannot be used under any circumstances.

When a beach is insufficiently stable to support sustained traffic from landing craft to backshore, artificial means may be used to increase the stability. The three general methods are (1) densification of the soil, (2) addition of cementing agents, and (3) elimination of excess moisture and the prevention of moisture accumulation. Another approach is to bring vehicles ashore by means of pontoon causeways, landing mats, or other types of temporary roadways.

CHAPTER XXXV

SOUND IN THE SEA

3501. Underwater sound and the navigator.—The clarity with which the noises associated with weighing anchor, propelling a ship, and other underwater motions are heard below the water line and near the skin of a vessel is an indication of the high sound-transmitting qualities of sea water. Water is a better conductor of sound than is air because it absorbs less energy from the sound. There are several ways in which underwater sound can be used in navigation.

The *direction* of travel of sound waves can be measured either by means of **binaural hearing** (hearing with two "ears"), or by equipment which has directional characteristics similar to those of a directional antenna used in radio (art. 1012). Either method can be used for determining the direction from which general noise is coming, but only the latter is used in sonar equipment (art. 1108) for determining direction and distance by reception of an echo from a directional signal, in a manner similar to radar (art. 1208).

Distance can be determined by (1) measuring the elapsed time between transmission of a signal and return of its echo, (2) measuring the elapsed time between transmission of a signal and its receipt at a second station, (3) measuring the time *difference* between reception of a signal transmitted through water and one transmitted through air, (4) measuring the difference in phase between two signals or change of phase of a signal when it returns as an echo, or (5) measuring the angle at which an echo is received from a signal produced at another place. The first method, used in sonar (art. 1108) and echo sounding equipment (art. 619), is similar in principle to radar (art. 1208). The second method is used primarily in RAR (art. 1205), in which underwater sound signals trigger a "sonobuoy," which transmits a radio signal to indicate the time of reception of the sound signal. The third method is used at distance finding stations (art. 1205). The fourth and fifth methods were used in early forms of echo sounders.

The *difference* in time of reception of the same signal at two or more points is used in sofar (art. 1313) in a manner which is similar but reversed to that of loran (art. 1302).

3502. Sources of sound in the ocean.—Underwater sounds intended for navigational use are produced in one of three basic ways: (1) by percussion, as the striking of a bell, gong, or the bottom of the vessel; (2) by oscillator, as the vibration of a diaphragm; (3) by explosion, as by small bomb or depth charge. Certain man-made noises ordinarily produced in water, such as those due to operation of the main engines of a vessel, can be detected by an appropriate listening device.

In addition, many noises are made by animals living in the ocean. Certain shrimp, great numbers of which inhabit some areas, make a snapping noise with their claws. Some fish make a noise by stridulating (scraping). When shellfish are being eaten, a sound is emitted as the shells are broken by the teeth of the fish which are feeding. Grunting noises are made by many kinds of fish, usually by means of their swim bladders. Porpoises produce sounds of a high pitch. Sounds of various frequency and amplitude are produced by other forms of marine life. Where sound-producing marine life is very abundant, it interferes with detection of man-made

sounds, requiring a high signal-to-noise ratio. The effect is similar to that of a high atmospheric noise level in radio.

3503. Speed of sound in sea water.—Three variables govern the speed (S) of sound in a fluid. They are density (ρ), compressibility (β), and the ratio between the specific heats of the fluid at constant pressure and at constant volume (γ). The following formula is sufficiently accurate for most navigational purposes:

$$S=\sqrt{\frac{\gamma}{\rho\beta}}.$$

Density and specific heat are discussed in articles 3009 and 3012, respectively. Compressibility refers to the relative change in volume for a given change in pressure. The compressibility of water is low, and consequently the speed of sound in water is high. The specific heat ratio enters the formula because the energy of a sound impulse is briefly transformed into heat, and then reconverted (with slight loss) into kinetic energy. The ratio rarely exceeds 1.02 in sea water and is commonly taken as unity.

For atmospheric pressure 29.92 inches of mercury, temperature 60° F, and salinity 34.85 parts per thousand, the density of sea water is 64 pounds per cubic foot and the compressibility approximately 0.0000435 per atmosphere (one atmosphere equals 14.696 pounds per square inch). Using these values and 32.174 feet per second per second (the acceleration of gravity at latitude 45°) and 144 square inches per square foot, and taking γ equal to unity, one obtains:

$$S=\sqrt{\frac{1.0\times 32.174\times 14.696\times 144}{64\times 0.0000435}}=4945 \text{ ft./sec.}$$

The same formula can be used to determine the speed of sound in air. For atmospheric pressure 29.92 and temperature 60° F, the density of air is 0.0764 pound per cubic foot and, since air is a gas, the compressibility is the reciprocal of the pressure. Taking γ equal to 1.4, one obtains:

$$S=\sqrt{\frac{1.4\times 32.174\times 14.696\times 144}{0.0764\times 1}}=1117 \text{ ft./sec.}$$

The speed of sound in water is approximately 4.5 times its speed in air.

An increase in temperature decreases both density and compressibility, resulting in an increase in the speed of sound. In sea water, an increase in pressure or salinity produces a slight increase in density and a larger decrease in compressibility, resulting in a net increase in the speed of sound. Thus, in sea water, an increase in temperature, pressure, or salinity results in greater speed of sound. Of the three, temperature has the greatest influence on the speed of sound in sea water in the upper layers. At depth, pressure, and in coastal areas, changes in salinity, may have the greatest effect.

Normally, the change of these three elements is much more rapid in a vertical direction than in a horizontal direction. The change with depth varies with location. With respect to temperature, much of the ocean is considered to consist of three layers, a **surface layer** influenced greatly by the temperature of the air above it, a **thermocline** of rapidly decreasing temperature, and a nearly uniform **deep-water layer.** Typical curves showing change of temperature and salinity with depth are shown in figure 3503a. The increase of pressure with depth is almost uniform, the pressure at 10,000 feet being approximately twice that at 5,000 feet, and ten times that at 1,000 feet. A typical curve of speed of sound with depth is shown in figure 3503b. The speeds for all temperature, pressure, and salinity conditions encountered in the sea are given in *Tables of*

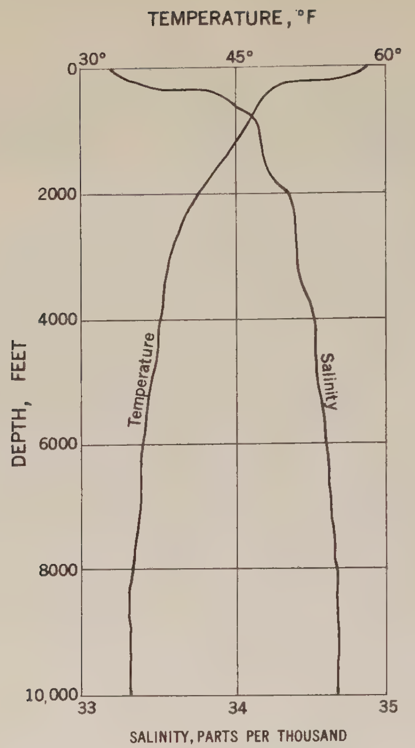


FIGURE 3503a.—Variation of temperature and salinity with depth at one locality.

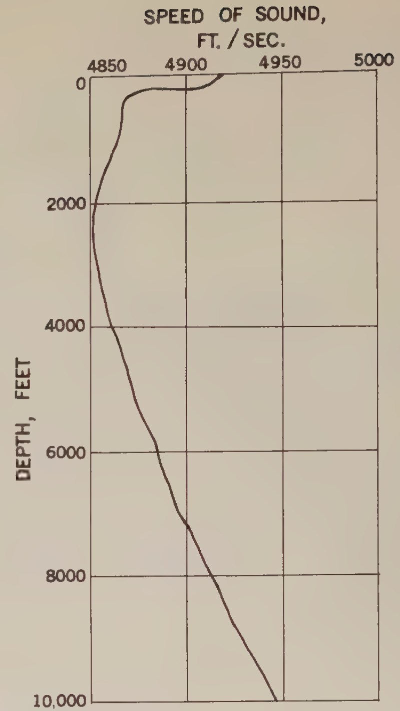


FIGURE 3503b.—Typical variation of speed of sound with depth in the ocean.

Sound Speed in Sea Water, SP 58, published by the U.S. Navy Hydrographic Office.

Study of transmission of sound from underwater explosions indicates that near the explosion the speed of sound may be somewhat higher than expected, probably due to increased pressure caused by the disturbance. This effect extends over such a short distance that it is insignificant in ordinary underwater sound transmission.

3504. Reflection of underwater sound waves.—In water, as in air, sound is reflected by obstructions in the form of solid objects or sharp discontinuities. Thus, sound is reflected from the bottom, the shore, hulls of ships, the surface of the water, etc. It is this reflecting energy that is used in echo sounders (art. 619) to determine depth, and in sonar equipment (art. 1108) used for echo ranging.

Reflecting properties of various substances differ markedly. Rock reflects almost all of the sound that strikes its surface, while soft mud absorbs or is penetrated by sound. Thus, in echo sounding, a layer of soft mud over rock may result in two echoes, indicating two depths.

Fish and even tiny sea animals also reflect sound. As a result, echo sounders are widely used among fishermen to locate schools of fish. In deep water it is not unusual for an echo sounder to receive an echo from a depth of about 200 fathoms, although the depth decreases somewhat at night. This **phantom bottom** or **deep scattering layer,** which is undoubtedly the source of many erroneous shoal sounding reports, is believed to be due to large numbers of tiny marine animals, or other marine life.

A sharp discontinuity within the water causes reflection of sound. Thus, an echo sounder may detect the boundary between a layer of fresh water overlying salt water, a condition which might occur near the mouth of a river.

Sharp, distinct echoes denoting precise depths are difficult to obtain over rough-surfaced bottoms. Therefore, considerable discretion should be exercised in evaluating soundings taken over bottoms possessing a high degree of relief.

3505. Refraction of underwater sound waves.—The laws of refraction as applied to light (art. 1613) and radio waves (art. 1006) apply also to sound. Because of differences of speed of sound in sea water, an advancing sound wave is refracted toward the area of slower speed. If sound is traveling vertically downward, as in echo sounding, the effect of refraction is relatively slight because the layers of water in which speed differs are approximately horizontal, and when the direction of travel of the sound is normal to the refracting surface or layer, there is no refraction.

When a *beam* of sound is directed in a horizontal direction, however, refraction is greatest. If the speed *decreases* with depth, the usual situation, the upper part of the beam travels faster than the lower part, and the beam is diverted downward, leaving a **shadow zone** near the surface in which the sound does not enter, except for a weak signal due to scattering. If the speed *increases* with depth, the lower part moves faster, and the beam is deflected upward toward the surface, where part of it is reflected, part moves along the surface with some scattering if the surface is not smooth, and part (less than 1%) is lost to the air.

With typical distribution of speed with depth, as shown in figure 3503b, speed decreases with depth until a minimum is reached at some level below the surface, and below this the speed increases. In figure 3503b minimum speed occurs at about 2,400 feet. In the tropics this level of minimum speed may be as deep at 6,000 feet, and in polar regions it may be at the surface. Sound produced at any level tends to be refracted to the level of minimum speed, and to remain there, for as it attempts to leave this level, it is refracted back toward it, as shown in figure 3505. This, of course, does not refer to sound traveling vertically. If a sound is produced at this level, as by the explosion of a bomb or depth charge, the sound waves start to move outward as expanding spheres, but most of the rays are refracted back toward the minimum speed level. Because of this effect, such a sound may travel great distances with relatively little decrease in intensity. Listening gear placed at this level has detected sounds produced thousands of miles away. This is the principle used in **sofar** (art. 1313).

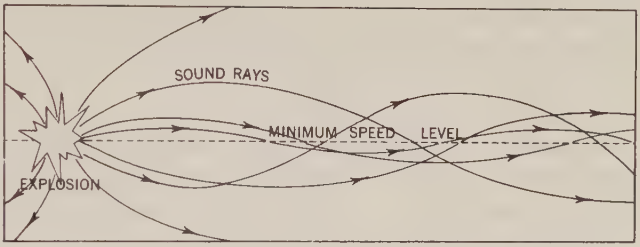


FIGURE 3505.—Transmission of sound rays along the minimum sound level.

CHAPTER XXXVI

ICE IN THE SEA

3601. Ice and the navigator.—The perpetually frozen Arctic Ocean and the solid sheet of ice beneath which Antarctica is buried offer evidence that the earth has not yet completely emerged from its most recent Ice Age. Each winter this polar ice increases and spreads toward more temperate latitudes, and each summer it contracts again as part of the ice melts. Some of the fragments are carried by ocean currents into shipping lanes, forming a major hazard to shipping. There is evidence to indicate that the polar regions are becoming warmer. Nearly all glaciers are receding; the ice shelves off northern Canada and Greenland are breaking up; shipping off the Siberian coast has become possible; cod are found ever farther north along the Greenland coast.

Ice is of direct concern to the navigator because it restricts and sometimes controls his movements, it affects his dead reckoning by forcing frequent and sometimes inaccurately determined changes of course and speed, it affects his piloting by altering the appearance or obliterating the features of landmarks and by rendering difficult the establishment and maintenance of aids to navigation, it affects his electronic navigation by its effect upon propagation of radio waves and the changes it produces both in surface features and radar returns from such features, it affects his celestial navigation by altering the refraction and obscuring his horizon and celestial bodies either directly or by the weather it influences, and it affects his charts by introducing various difficulties to the hydrographic surveyor.

Because of his direct concern with ice, the prospective polar navigator will do well to acquaint himself with its nature and extent in the area he expects to navigate. To this end he should consult the sailing directions for the area, and whatever other literature may be available to him, including reports of previous operations in the same area.

3602. Formation of ice.—As it cools, water contracts until the temperature of maximum density is reached. Further cooling results in expansion. The maximum density of fresh water occurs at a temperature of 39°2 F, and freezing takes place at 32° F. The addition of salt lowers both the temperature of maximum density and, to a lesser extent, that of freezing. The relationships are shown in figure 3602. The two lines meet at a salinity of 24.7 parts per thousand, at which maximum density occurs at the freezing temperature of 29°61 F. At this and greater salinities, the density increases right down to the freezing point. At a salinity of 35 parts per thousand, the approximate average for the oceans, the freezing point is 28°6 F.

Generally, ice forms first at the water surface. As it does, most of the dissolved solids remain in the water, beneath the ice, increasing the density of the water there. This lowers the freezing point, thus tending to retard the freezing process. It is further retarded by the fact that ice is a poor conductor of heat and therefore serves as an insulator to protect the water from colder air above.

In shoal water and streams, particularly where motion is sufficient to cause thorough mixing, the freezing temperature may extend from the surface to the bottom. When this occurs, ice crystals may form at any depth. Because of their decreased density, they tend to rise to the surface, unless they form at the bottom and attach themselves

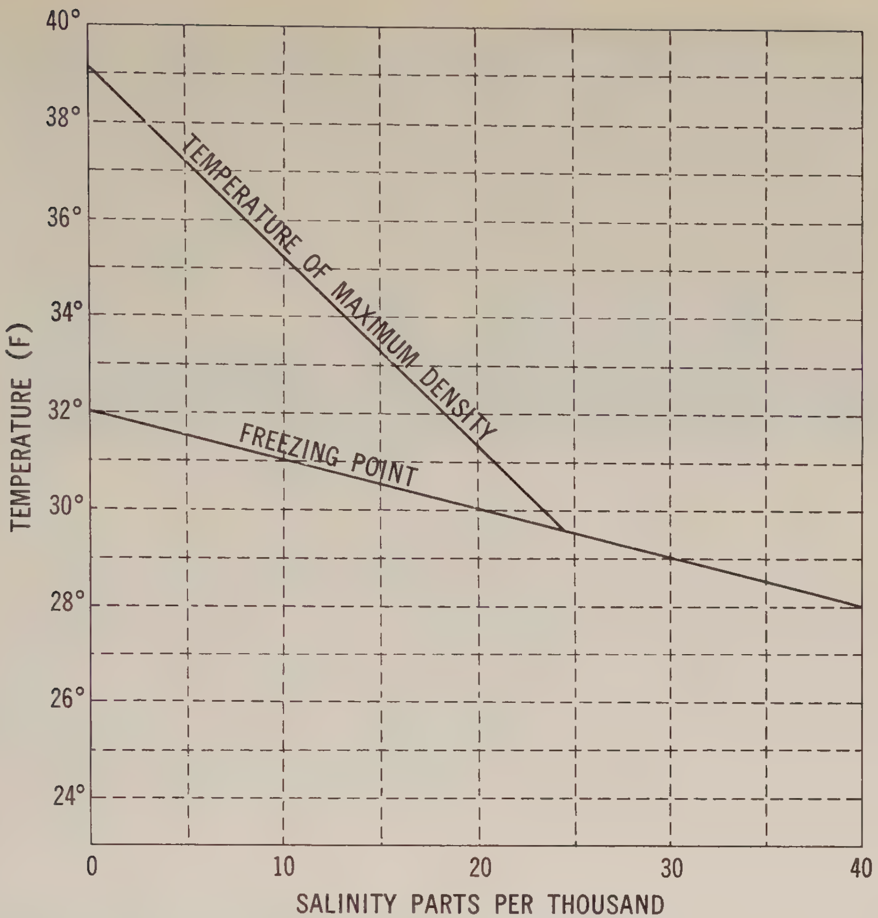


FIGURE 3602.—Relationship between temperature of maximum density and freezing point for water of varying salinity.

there. This **bottom ice,** sometimes called **anchor ice,** continues to grow as additional ice freezes to that already formed.

Ice may also be formed by the compacting of fallen snow, or by the freezing of a mixture of snow and sea water.

3603. Land ice is formed on land by the freezing of fresh water or the compacting of snow as layer upon layer adds to the pressure on that beneath. As snow becomes hardened by wind, temperature, and pressure, it reaches an intermediate stage when it is known as **névé** (nā′vā′).

Under great pressure ice becomes slightly plastic and is forced outward and downward along an inclined surface. If a large area is relatively flat, as on the antarctic plateau, or if the outward flow is obstructed, as on Greenland, an **ice cap** forms and remains winter and summer, in some places reaching depths of several thousand feet. Where ravines or mountain passes permit flow of the ice, a **glacier** is formed. This is a slow-moving river of ice that flows to lower levels, exhibiting many of the characteristics of rivers of water. The flow may be more than 100 feet per day, but is generally much less. When a glacier reaches a comparatively level area, it spreads out. When a glacier flows into the sea, the buoyant force of the water breaks off pieces from time to time, and these float away as **icebergs.**

An iceberg seldom melts uniformly because of lack of uniformity in the ice itself, differences in the temperature above and below the water line, exposure of one side to

the sun, strains, cracks, mechanical erosion, etc. The inclusion of rocks, silt, and other foreign matter further accentuates the differences. As a result, changes in equilibrium take place, which may cause the berg to tilt or capsize. Parts of it may break off or **calve,** forming separate, smaller bergs. A small berg about the size of a house is called a **bergy bit,** and one still smaller but large enough to inflict serious damage to a vessel is called a **growler** because of the noise it sometimes makes as it bobs up and down in the sea. Bergy bits and growlers are usually pieces calved from icebergs, but they may be formed by consolidation of sea ice or by the melting of an iceberg. The principal danger from icebergs is their tendency to break or shift position, and possible underwater extensions, called **rams.**

3604. Sea ice forms by the freezing of sea water. The first indication is a greasy or oily appearance of the surface, with a peculiar gray or leaden tint. The small individual particles of ice, called **spicules,** then become visible. As the number increases, the mixture of water and ice is soupy or mushy, having about the consistency of wet snow. At this stage it is called **slush.** The height of waves is noticeably reduced. As the individual particles freeze together, a thin layer of highly plastic ice forms. This bends easily and moves up and down with the waves. A layer of two inches of fresh-water ice is brittle but strong enough to support the weight of a heavy man. In contrast, the same thickness of newly formed sea ice will support not more than about ten percent of this weight, although its strength varies with the temperature at which it is formed, very cold ice supporting a greater weight than warmer ice. When snow falls into sea water which is near its freezing point, but colder than the melting point of snow, it does not melt, but floats on the surface, drifting with the wind into beds which may become several feet thick. If the temperature drops below the freezing point of the sea water, the mixture of snow and water freezes quickly into a soft ice similar to that formed when snow is not present. As it ages, sea ice becomes harder and more brittle.

Close to land the ice may be attached to the shore as an **ice foot.** The width of this **fast ice** varies considerably, but in an area with many irregularities in the coast line, especially if there are offshore islands or shoals, and relatively shallow water, it may extend for several miles to seaward. Although the width generally varies from two to 20 miles, a maximum of about 270 miles has been observed in the vicinity of Novosibirskiye Ostrova (New Siberian Islands). On an exposed, abrupt coast bordered by deep water there may be no ice foot at all.

In a bay or other sheltered area, ice formed on the surface of the sea, often augmented by snow and land ice, may build up a shelf which remains attached to the land for many years. In the Ross Sea in Antarctica this **shelf ice** attains a thickness of 500 to 1,000 feet. At the outer edge, large pieces eventually break away, forming **tabular icebergs** (fig. 3604a), with dimensions measured in *miles.* In 1854 and 1855 several ships in the South Atlantic reported a crescent-shaped iceberg with one horn 40 miles long, the other 60 miles long, and with an embayment 40 miles wide between the tips. In 1927 a berg 100 miles long, 100 miles wide, and 130 feet high above water was reported. The largest iceberg ever reported was sighted in 1956 by the USS *Glacier*, a U. S. Navy icebreaker, about 150 miles west of Scott Island. This berg was 60 miles wide and 208 miles long, more than twice the size of Connecticut. Icebergs ten miles or more in length have been seen on many occasions in the antarctic. In contrast, the largest iceberg reported in the northern hemisphere was seven miles long and three and a half miles wide. This berg was sighted off Baffin Island in 1882. In 1928 an iceberg four miles long was reported seen in the North Atlantic. The expression "tabular iceberg" is not applied to northern hemisphere bergs, but similar formations there are called **ice islands.** These are believed to originate when shelf ice breaks up north

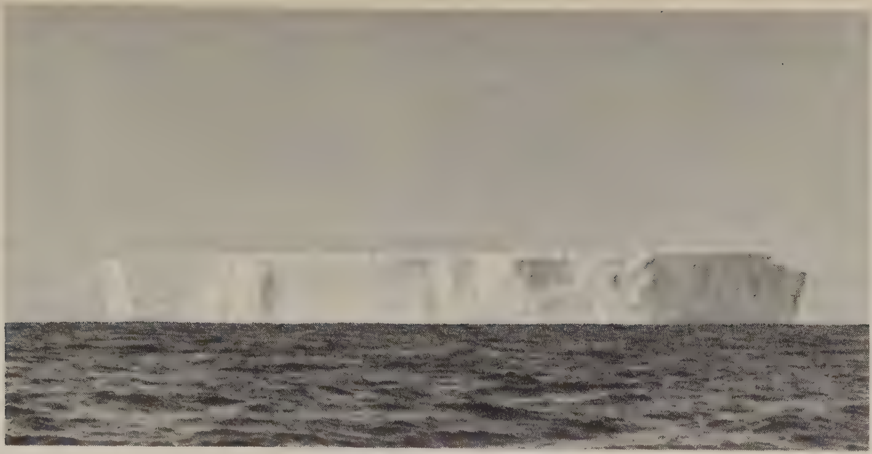


FIGURE 3604a.—A tabular iceberg.

of Canada and Greenland. Most of them remain in the Arctic Ocean and have not been encountered by ships, although the large icebergs sighted in 1882 and 1928 were possibly ice islands. For several years the United States maintained a weather station on one of the arctic ice islands.

Sea ice is exposed to several forces, including currents, wave motion, tides, wind, and temperature differences. In its early stages, its plasticity permits it to conform readily to virtually any shape required by the forces acting upon it. As it becomes older, thicker, and more brittle, exposed sea ice cracks and breaks under the strain. Under the influence of wind and current, the broken pieces may shift position relative to pieces around them.

A single piece of relatively flat sea ice is called an **ice cake.** When ice is formed in the presence of considerable wave motion, circular cakes several feet in diameter are formed, rather than a single large sheet. These circular cakes are called **pancakes,** and a collection of pancakes is called **pancake ice** (fig. 3604b). Wave motion may cause the pancakes to break into smaller pieces. With continued freezing, individual pieces unite into **floes,** and floes into **ice fields** which extend over many miles.

FIGURE 3604b.—Pancake ice, with an iceberg in the background.

When one floe encounters another, or the shore, the individual pieces may be forced closer together into a thickly compacted mass. If the force is sufficient, and the ice is sufficiently plastic, **bending** takes place, or **tenting** if the contacting edges of individual cakes force each other to rise above their surroundings. More frequently, however, **rafting** occurs as one cake overrides another. Sea ice having any readily observed roughness of the surface is called **pressure ice.** A line of ice piled haphazardly along the edge of two floes which have collided is called a **pressure ridge.** Pressure ice with numerous mounds or hillocks which have become somewhat rounded and smooth by weathering or the accumulation of snow is called **hummocked ice,** each mound being called a **hummock.**

The motion of adjacent floes is seldom equal. The rougher the surface, the greater the effect of wind, since each piece extending above the surface acts as a sail. Some floes are in rotary motion as they tend to trim themselves into the wind. Since ridges extend below as well as above the surface, the deeper ones are influenced more by deep-water currents. When a strong wind blows in the same direction for a considerable period, each floe exerts pressure on the next one, and as the distance increases, the pressure becomes tremendous. Near land the result is an almost unbelievably chaotic piling of ice. Individual ridges near the shore may extend as much as 60 or 70 feet above surrounding ice and have a total thickness of 150 to 200 feet in extreme cases. Far from land, the height and thickness seldom exceed half these figures.

The continual motion of various floes results in separation as well as consolidation. A long, narrow, jagged **crack** may appear and widen enough to permit passage of a ship, when it is called a **lead** (lēd). In winter, a thin coating of newly formed ice usually covers the water, but in summer the water remains ice-free until a shift in the movement forces the two sides together again. Before this occurs, lateral motion usually takes place between the floes, so that they no longer fit, and unless the pressure is extreme, numerous patches of open water remain. A large one is called a **polynya.**

A large mass of sea ice, consisting of various floes, pressure ridges, and openings, is called a **pack** (fig. 3604c). In the arctic the main pack extends over the entire Arctic Ocean and for a varying distance outward from it, the limits receding considerably during summer. Each year a large portion of the ice from the Arctic Ocean moves outward between Greenland and Norway, into the North Atlantic, and is replaced by new ice. Relatively little of the pack ice is more than ten years old. The **ice pole,** the approximate center of the arctic pack, is at latitude 83°.5 N, longitude 160° W, north of western Alaska and about 390 miles from the north pole. In the antarctic the pack exists as a relatively narrow strip between the continent of Antarctica and the notoriously stormy seas which hasten the pack's destruction.

The alternate melting and refreezing of the surface of the pack, producing **weathered ice,** combined with the various motions to which the pack is subjected, result in widely varying conditions within the pack itself. The extent to which it can be penetrated by a ship varies from place to place and with changing weather conditions. In some areas the limit of navigable water is abrupt and complete, as at the edge of shelf ice. Such ice is called a **barrier.**

3605. Thickness of sea ice.—The seasonal thickness of fast ice in two harbors of the northern hemisphere is shown in figure 3605, at the latitudes indicated. Pack ice in these latitudes undergoes a similar change. As ice thickens, it provides increased insulation to protect the sea water beneath from the colder air above, and the rate of freezing decreases. Sea ice rarely exceeds six feet in thickness during its first year. In a coastal area where the melting rate is less than the freezing rate, the thickness increases during succeeding winters, being augmented by compacted and frozen snow, until a maximum thickness of about 12 to 15 feet may eventually be reached. These

FIGURE 3604c.—Pack ice.

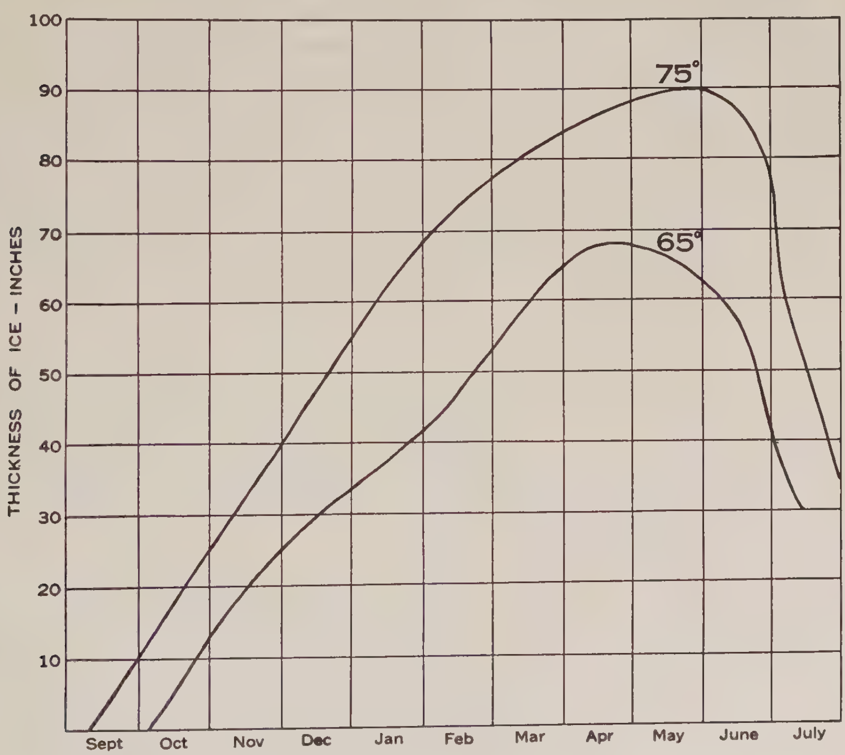


FIGURE 3605.—Thickness of ice in two typical sheltered harbors in the northern hemisphere, at the latitudes indicated.

values refer to single, unbroken pieces of floating ice. Shelf ice and pressure ice may be much thicker, as indicated previously (art. 3604).

During the summer, the sea ice insulates the sea water from warmer air above, so that melting is confined almost entirely to the upper portion. As the fresher melt water runs off into the sea, it tends to float on top of the heavier and colder salt water of the ocean. The temperature of the sea water may be lower than the freezing point of the fresher melt water, resulting in some refreezing as the melt water runs under the ice.

3606. Salinity of sea ice.—Sea ice forms first as salt-free crystals near the surface of the sea. As the process continues, these crystals are joined together and, as they do so, small quantities of brine are trapped within the ice. On the average, new ice six inches thick contains five to ten parts of salt per thousand. With lower temperature, freezing takes place faster. With faster freezing, a greater amount of salt is trapped in the ice.

Depending upon the temperature, the trapped brine may either freeze or remain liquid, but because its density is greater than that of the pure ice, it tends to settle down through the pure ice. As it does so, the ice gradually freshens, becoming clearer, stronger, and more brittle. At an age of one year sea ice is sufficiently fresh that its melt water, if found in **puddles** of sufficient size, and not contaminated by spray from the sea, can be used to replenish the fresh water supply of a ship. However, ponds of sufficient size to water ships are seldom found except in ice of great age, and then much of the melt water is from snow which has accumulated on the surface of the ice. When sea ice reaches an age of about two years, virtually all of the salt has been eliminated. Icebergs contain no salt, and uncontaminated melt water obtained from them is fresh.

The settling out of the brine gives sea ice a honeycomb structure which greatly hastens its disintegration when the temperature rises above freezing. In this state, when it is called **rotten ice,** much more surface is exposed to warm air and water, and the rate of melting is increased. In a day's time, a floe of apparently solid ice several inches thick may disappear completely.

3607. Density of ice.—The density of fresh-water ice at its freezing point is 0.917. Newly formed sea ice, due to its salt content, is more dense, 0.925 being a representative value. The density decreases as the ice freshens (art. 3606). By the time it has shed most of its salt, sea ice is less dense than fresh-water ice, because ice formed in the sea contains more air bubbles. Ice having no salt but containing air to the extent of eight percent by volume (an approximately maximum value for sea ice) has a density of 0.845.

The density of land ice varies over even wider limits. That formed by freezing of fresh water has a density of 0.917, as stated above. Much of the land ice, however, is formed by compacting of snow. This results in the entrapping of relatively large quantities of air. Névé, in the transitional stage between snow and ice, may have an air content of as much as 50 percent by volume. By the time the ice of a glacier reaches the sea, its density approaches that of fresh-water ice. A sample taken from an iceberg on the Grand Banks had a density of 0.899.

When ice floats, part of it is above water and part is below the surface. The percentage of the mass below the surface can be found by dividing the average density of the ice by the density of the water in which it floats. Thus, if an iceberg of density 0.920 floats in water of density 1.028 (corresponding to a salinity of 35 parts per thousand and a temperature of 30° F), 89.5 percent of its mass will be below the surface. That is, about nine-tenths of the mass will be below the surface, and only about one-tenth will be above the surface. If the ice is a perfectly uniform block, which some tabular

icebergs approach, the depth below the surface is about seven times the height above water, under the conditions stated above. However, most of the icebergs of the northern hemisphere are irregular in shape, the depth probably averaging about five times the height. Icebergs have been estimated to be as high as 1,000 feet above water, but the highest measured in the northern hemisphere was 447 feet. The largest tabular icebergs of the antarctic extend about 300 feet above the water.

3608. Drift of ice.—Although surface currents have some effect upon the drift of pack ice, the principal factor is wind. Due to Coriolis force (art. 1611), ice does not drift in the direction of the wind, but about 30° from this direction. In the northern hemisphere, this drift is to the *right* of the direction toward which the wind blows, and in the southern hemisphere it is toward the *left*. Since the surface wind is deflected about twice this amount from the direction of the pressure gradient, the total deflection of the ice is about 90° from the pressure gradient, or along the isobars, with the atmospheric low toward the left and the high toward the right in the northern hemisphere. In the southern hemisphere, these directions are reversed. The *rate* of drift is about one to seven percent of the wind speed, depending upon the roughness of the surface and the concentration of the ice.

Icebergs, which extend a considerable distance below the surface, and have a relatively small "sail area," are influenced more by surface currents than by wind. However, if a strong wind blows for a number of hours in a steady direction, the drift of icebergs will be materially affected. In this case the effect is two-fold. The wind acts directly against the iceberg, and also generates a surface current in about the same direction. Because of inertia, an iceberg may continue to move from the influence of wind for some time after the wind stops or changes direction.

3609. Extent of ice in the sea.—Several Hydrographic Office publications contain monthly charts showing average extent of various degrees of navigability in the northern and southern hemispheres throughout the year. A sample of the type of information given is shown in figure 3609. Similar information is shown on the various pilot charts (art. 414). Useful information on ice conditions in different localities is given in the sailing directions for those areas. The information given in H.O. Pub. No. 27, *Sailing Directions, Antarctica*, is particularly complete and of somewhat general application.

However, since formation of ice, in common with other meteorological and oceanographic phenomena, varies considerably from year to year, wide deviations from average conditions are not unusual. Most countries having vessels operating in ice maintain ice information services. Details of these services are given in the appropriate volumes of sailing directions. The ice bulletins broadcast by the U. S. Navy Hydrographic Office are discussed in article 3615. The latest bulletins, as well as information on average conditions, should be consulted when operating in ice.

3610. Ice in the North Atlantic.—Sea-level glaciers exist on a number of land masses bordering the northern seas, including Alaska, Greenland, Svalbard (Spitzbergen), Zemlya Frantsa-Iosifa (Franz Josef Land), Novaya Zemlya, and Severnaya Zemlya (Nicholas II Land). Except in Greenland, the rate of calving is relatively slow, and the few icebergs produced melt near their points of formation. Many of those produced along the coasts of Greenland, however, are eventually carried into the shipping lanes of the North Atlantic, where they constitute a major menace to ships. It is for this reason that more southerly lanes (art. 3611) are specified when icebergs are prevalent.

The icebergs produced along the east coast of Greenland are carried by the east Greenland current around Kap Farvel and northward by the west Greenland current toward Davis Strait. Relatively few of these icebergs menace shipping, but they have been encountered as far as 200 miles southeast of Kap Farvel.

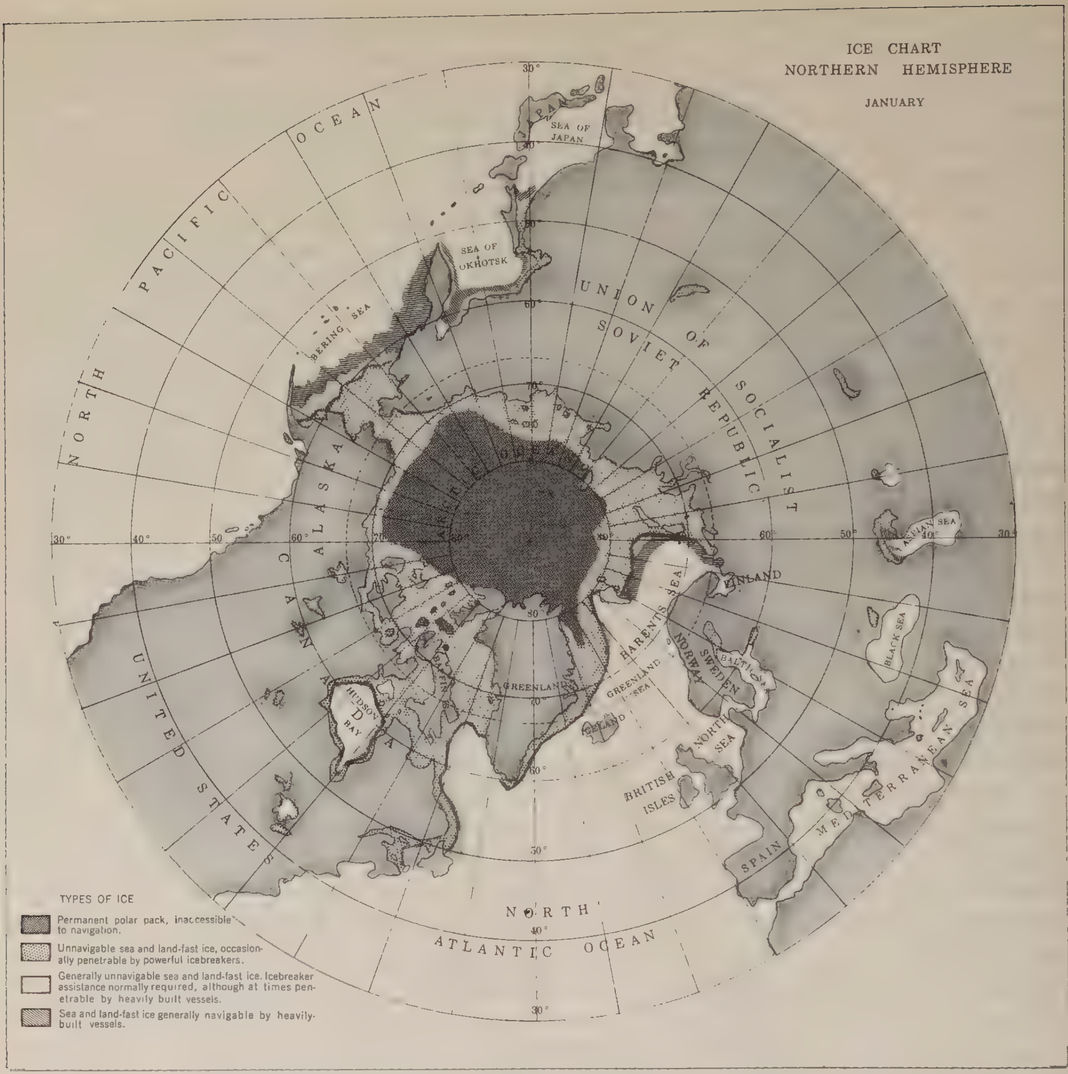


FIGURE 3609.—Average limits of various degrees of navigability of ice in the northern hemisphere in January.

The most prolific source of icebergs is the west coast of Greenland. In this area there are about 100 tidewater glaciers, 20 of them being the principal producers of icebergs. About 7,500 icebergs are formed here each year. The west Greenland current carries them northward and then westward until they encounter the south-flowing Labrador current. West Greenland icebergs generally spend their first winter in Baffin Bay. During the next summer they are carried southward by the Labrador current. In many cases, their second winter is spent in Davis Strait. When they are freed by the break-up of the pack ice, they drift southward. An average of about 400 per year reach latitude 48° N, and about 35 are carried south of the Grand Banks (latitude 43° N) before they melt. Icebergs have been encountered south of Bermuda, off the Azores, and within a few hundred miles of Great Britain.

The variation from average is considerable. More than 1,350 icebergs have been sighted south of latitude 48° N in a single year (1929), while in 1940 only two were encountered in this area. Although this variation has not been fully explained, it is

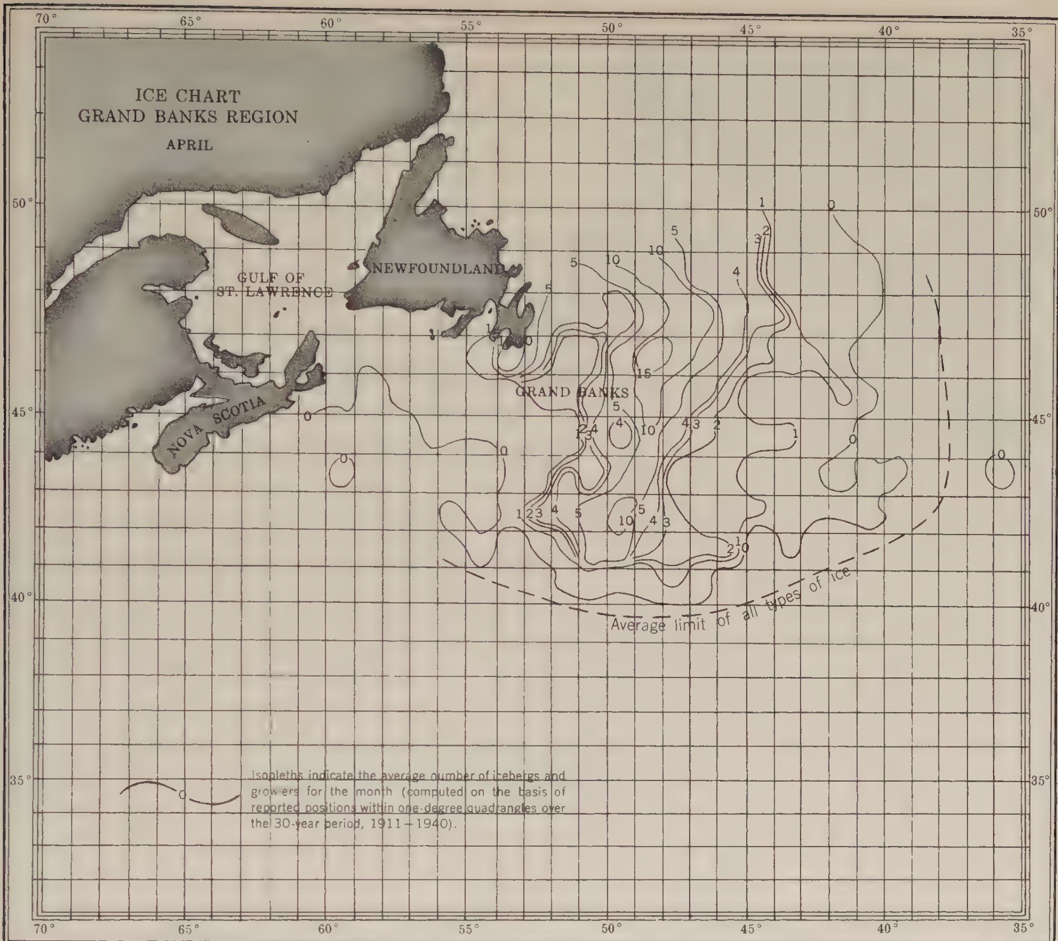


FIGURE 3610a.—Average iceberg conditions in the vicinity of the Grand Banks in April.

apparently related to wind conditions, the distribution of pack ice in Davis Strait, and to the amount of pack ice off Labrador. It has been suggested that the distribution of the Davis Strait-Labrador Sea pack ice influences the effectiveness of this ice in holding back the icebergs. According to this theory, when pack ice is heavy along the Labrador coast, the icebergs are forced well offshore, where warmer water causes them to melt before they reach the North Atlantic shipping lanes; but when the pack ice is not sufficient for this, the icebergs drift closer to shore, where there is colder water which prolongs their existence.

Icebergs may be encountered during any part of the year, but in the Grand Banks area they are most numerous during the spring. Average iceberg and pack ice conditions in this area during April, May, and June are shown in figures 3610a, 3610b, and 3610c. Off Newfoundland, part of the pack ice is brought south by the Labrador current, and part of it comes through Cabot Strait, having originated in the Gulf of St. Lawrence.

3611. The North Atlantic lane routes.—In his 1855 sailing directions, Matthew Fontaine Maury included a section on "Steam Lanes Across the Atlantic." Maury was inspired by the collision and sinking the previous year of the French *Vesta* and American *Arctic*, in which about 300 lives were lost, and he recommended separate

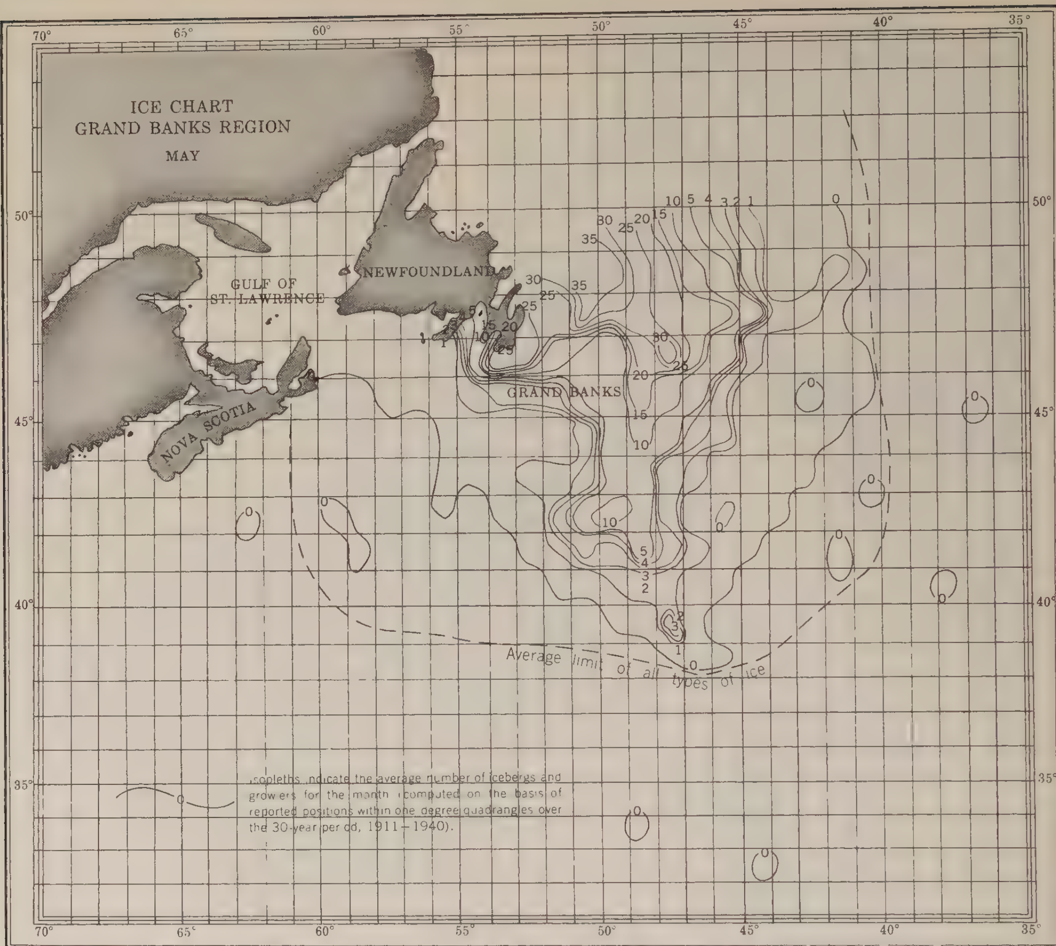


FIGURE 3610b.—Average iceberg conditions in the vicinity of the Grand Banks in May.

routes for eastbound and westbound vessels to avoid the risks due to fog. The U. S. Navy Hydrographic Office continued to advocate the use of lane routes during the next 35 years, ultimately designating different routes for different times of the year to avoid ice dangers. In 1889 representatives of 26 maritime nations, meeting at the International Marine Conference in Washington, ruled against establishing steamer lanes by international agreement of the governments involved, but recommended that companies engaged in the North Atlantic trade establish such routes for their own vessels. Two years later a group of steamship companies operating passenger liners in the North Atlantic, led by the Cunard Line, agreed to follow designated tracks which were essentially the ones proposed by the U. S. Navy Hydrographic Office. The lanes have been altered somewhat from time to time. The principal ones now in use are shown in figure 3611. Each lane is composed of two tracks separated by a safe distance, the southern track being used by eastbound vessels, and the northern one by westbound vessels.

Routes *A*, *B*, and *C* connect the United States and Europe, while routes *D*, *E*, *F*, and *G* run between Canada and Europe. Normally, route *B* is used between April 11 and June 30, and route *C* during the remainder of the year. However, when icebergs are numerous south of the Grand Banks, the use of lane *A* is specified. This route

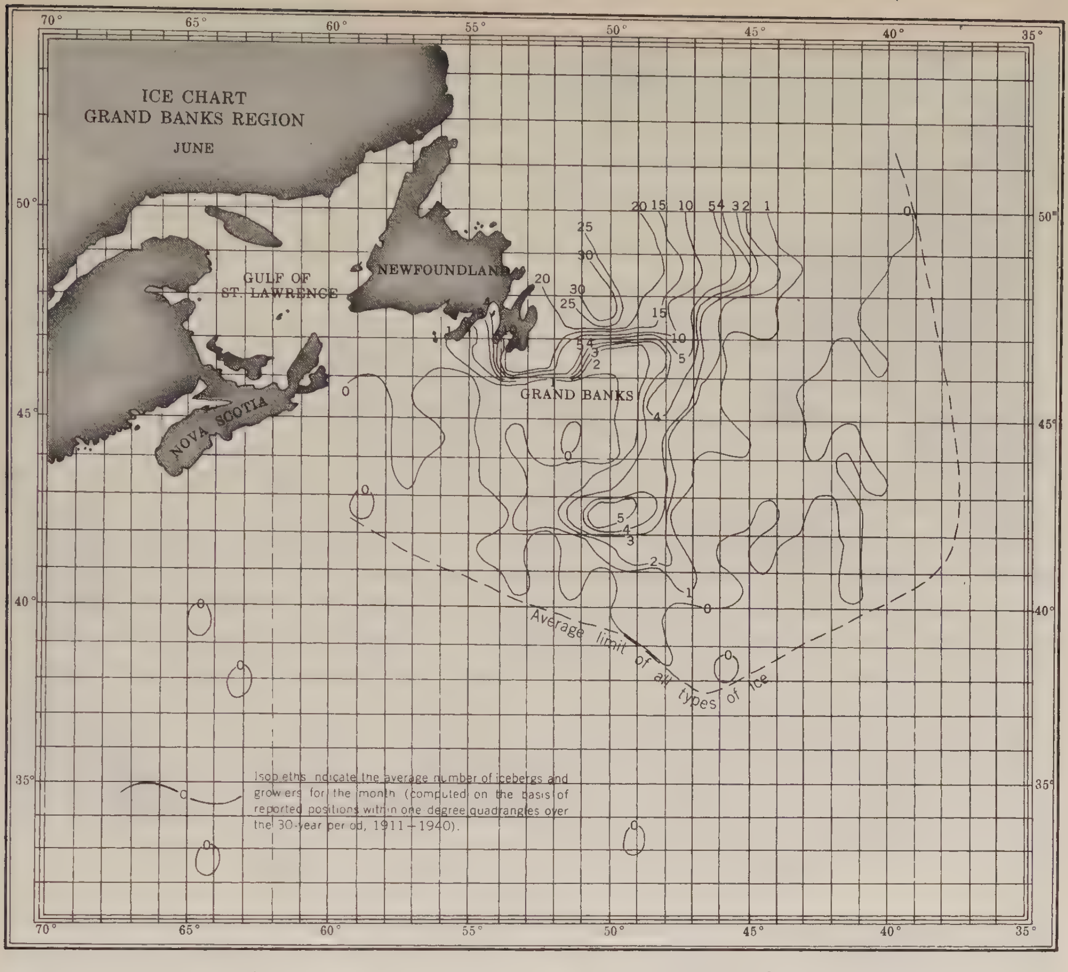


FIGURE 3610c.—Average iceberg conditions in the vicinity of the Grand Banks in June.

adds 150 to 200 miles to the great-circle track, but the increased distance is acceptable because it improves the safety and reduces the possibility of delays due to pack ice. Normally, route *D* is used between February 15 and April 10; route *E* from April 11 through May 15, and from December 1 through February 14; route *F* from May 16 until route *G* is clear, about July 1, or through November 30 if route *G* is not used; and route *G* from the opening of Belle Isle Strait, about July 1, through November 14. Specified lanes are shown on pilot charts for the North Atlantic (H.O. Chart No. 1400).

Variations in this schedule are specified by a designated official of the Cunard Line, acting upon advice from the Hydrographer of the U. S. Navy. The Hydrographer makes his recommendation after consultation with the Commandant of the U. S. Coast Guard, taking into account the information provided by the International Ice Patrol (art. 3612). Virtually all passenger liners and most freight vessels use these routes.

3612. The International Ice Patrol was established in 1913 by the International Convention for the Safety of Life at Sea held that year as a result of the sinking of the SS *Titanic* the previous year. On its maiden voyage this vessel struck an iceberg and sank with the loss of 1,513 lives. In accordance with the agreement reached at the convention, this patrol is conducted by the U. S. Coast Guard, which each year

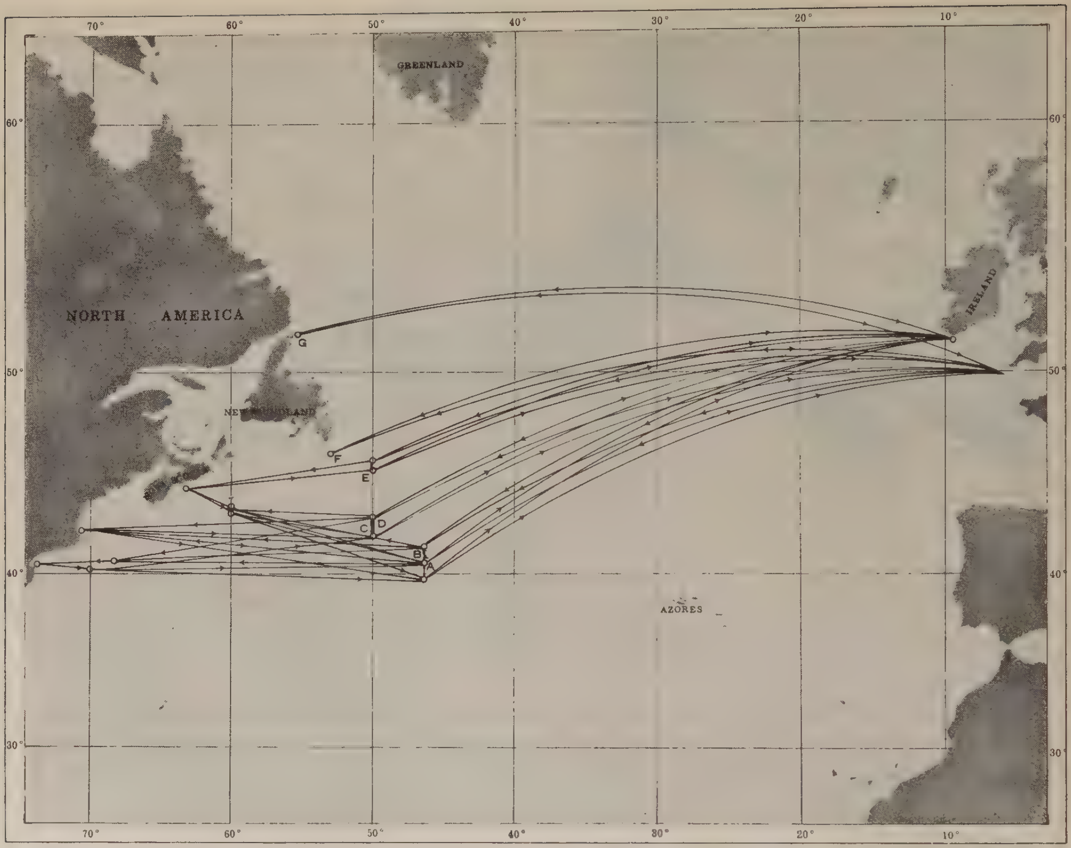


FIGURE 3611.—Principal North Atlantic steamer lanes.

assigns vessels to remain in the vicinity of the Grand Banks during the ice season to observe and report ice conditions.

During the war years of 1916–18 and 1941–45 the patrol was suspended. Following World War II, aircraft were added to the patrol force, and Argentia, Newfoundland, was established as the base of operations. Aircraft have played an increasing role in ice reconnaissance each year since then, and today they perform most of the work. Twice each day during the iceberg season an ice bulletin is broadcast from Argentia and printed in the *Daily Memorandum* of the U. S. Navy Hydrographic Office. Ice patrol vessels copy the broadcasts when on station and make them available to other ships upon request. In return for this service, all vessels in the area are requested to report to the patrol vessels any ice observed, and to send weather data and surface sea water temperature every four hours.

When engaged in patrolling ice areas, the vessels conduct oceanographic surveys and maintain an up-to-date map of the currents, for use in predicting future drift of icebergs. Recommendations for changes in the use of lane routes (art. 3611) are based upon information gathered by the International Ice Patrol.

As used by the U. S. Coast Guard, the expression "ice observation service" means that a continuous surface vessel patrol is *not* in effect, ice reconnaissance being accomplished chiefly by aircraft. When a continuous surface patrol is used to augment the ice observation service, the expression "ice patrol service" is used.

3613. Ice detection.—As a ship proceeds into higher latitudes, the first ice it encounters is likely to be in the form of icebergs, because such large pieces require a longer time to disintegrate. Icebergs can easily be avoided if detected soon enough.

The distance at which an iceberg can be seen depends upon the visibility, height of the berg, source and condition of lighting, and the observer. On a clear day with excellent visibility a large berg might be sighted at a distance of 18 miles. With a low-lying haze around the horizon this may be reduced to ten miles. In light fog or drizzling rain this is further reduced to one to three miles. There is a tendency to over-estimate the distance.

In a dense fog a berg may not be visible until it is close aboard, when it appears as a luminous, white mass if the sun is shining; or as a dark, sombre mass if the sun is not shining. If the layer of fog is not thick, an iceberg may be sighted from aloft sooner than from a point lower in the vessel, but this fact should not be considered justification for omitting a bow lookout.

On a clear, dark night an iceberg will seldom be picked up visually at a distance greater than one-fourth of a mile, but if its bearing is known, an observer with binoculars can occasionally observe a light spot where a wave breaks against it at a distance of a mile.

A moon may either help or hinder, depending upon its phase and position relative to ship and berg. A full moon in the direction of the berg interferes with its detection, while light from one in the opposite direction produces a "blink" which renders the iceberg visible for a greater distance, possibly as much as three miles. Clouds, particularly broken clouds, with intermittent moonlight, add to the difficulty of detecting ice.

If an iceberg is in the process of disintegration, its presence may be detected by the cracking sound as a piece breaks off, or by the thunderous roar as a large piece falls into the water. The appearance of smaller pieces of ice in the water often indicates the presence of an iceberg nearby. In calm weather such pieces may form a curved line with the parent iceberg on the concave side. Some of the pieces broken from an iceberg are themselves large enough to be a menace to ships.

As the ship proceeds to higher latitudes, it eventually encounters pack ice. If the ice is approached from leeward, it is likely to be loose and somewhat scattered, often in long, narrow arms. If it is approached from windward, it is usually compact and the edge is sharply defined.

One of the most reliable signs of the approach to pack ice, especially from leeward, is the somewhat abrupt smoothing of the sea in a fresh breeze, and the more gradual lessening of the swell. Abrupt changes in air or sea temperature or sea-water salinity are not reliable signs of the approach to either icebergs or pack ice, but if the sea temperature gradually drops below 32° F, the ship *may* be nearing an ice field.

Another reliable sign of the approach to pack ice is the appearance of the horizon or sky. A yellowish glare or **ice blink** appears in the sky above an ice field. If clouds are present, the blink is whiter. Reflection of light from snow, whether on land or sea ice, is white and is called **snow blink.** In contrast, the sky above open water is dark. This is called **water sky.** Somewhat similar **land sky** above ice- and snow-free land is grayer. The combination of these various effects in the sky is called a **sky map.** One experienced in reading the sky map finds it very useful in avoiding ice or searching out openings which may permit his vessel to make progress while proceeding through an ice field.

The presence of seals or certain types of birds may indicate the presence of ice nearby. It is well to observe the habits of the various species encountered.

If aircraft or other vessels can be contacted by radio, much useful information can sometimes be obtained from them. Some ships, particularly icebreakers, proceeding into high latitudes carry helicopters, which are invaluable in locating ice and determining the relative navigability of different portions of it.

Echoes from the ship's whistle or horn will sometimes reveal the presence of icebergs, but are useless against pack ice. Such echoes can give an indication of direction, and if the time interval between the sound and its echo is measured, the distance in feet can be determined by multiplying the number of seconds by 550. However, echoes are not a reliable indication because only those pieces of ice with large vertical areas facing the ship return enough echo to be heard, and also because echoes might be received from land or a fog bank.

At relatively short ranges, sonar is sometimes helpful in locating ice. The first contact with icebergs may be when as much as three miles or more off, but is usually considerably less. Growlers may be picked up at one-half to one mile and even smaller pieces may be detected in time to avoid them. Since one-half to seven-eighths of the mass of ice is below the surface, the underwater portion presents a better target than the portion above water.

Radar is highly useful in detecting ice, but is by no means infallible. Ice is a relatively poor radar target, and much depends upon the nature of the exposed surface. Icebergs with sides sloping gently toward the vessel can be seen visually long before they are picked up by radar, if the day is clear. One iceberg 700 feet long and 200 feet high was reported to have been approached to within three miles before it appeared on the radar screen. However, the average berg is picked up at a range of eight to ten miles, and the large vertical-sided tabular icebergs of the antarctic are usually detected at ranges of 15 to 30 miles, with an extreme range of 37 miles having been reported. Growlers are the chief concern. While a large iceberg is almost always detected in time to be avoided, a growler large enough to be a serious menace may be lost in the sea return and escape detection altogether. If an iceberg or growler is detected, tracking is sometimes necessary to distinguish it from a rock, islet, or ship.

Against sea ice, radar can be of great assistance to one experienced in interpreting the scope picture. Smooth sea ice, like smooth water, returns little or no echo, but rough, hummocky sea ice can be detected at a range of two to three miles. The return is similar to sea return, but the same echoes appear at each sweep. A lead in smooth ice broken by a preceding vessel is clearly visible, even though a thin coating of new ice has formed in the opening. A light covering of snow obliterating many of the features to the eye has little effect upon a radar return.

The ranges at which ice can be detected by radar are somewhat dependent upon refraction, which is sometimes quite abnormal in polar regions. Adequate training and experience are essential if full benefit is to be realized from radar.

No method yet devised to detect the presence of ice is infallible, and all should be regarded with suspicion, although none should be overlooked. In ice, as elsewhere, *there is no substitute for constant vigilance.*

3614. Operations in ice.—For operations in ice it is preferable to have a vessel designed for this purpose. Such a vessel has a heavily reinforced bow, reinforced plating along the water line, absence of vertical sides, deep screws, blunt bow, and other desirable features. The full list depends upon the area of operations, kinds of ice to be encountered, length of stay in the vicinity of ice, anticipated assistance by icebreakers, and possibly other factors. Any vessel expecting to penetrate the pack ice should as a minimum have reinforcement along the water line, particularly at the bow, which should be strengthened both inside and outside.

Whatever the nature of the vessel, it will be subjected to various hazards which may cause damage. Its safety depends largely upon the thoroughness of advance preparations, the alertness and skill of its crew, and their ability to make repairs if damage is sustained. Before the ice is entered, the ship should be trimmed so as to be down by the stern slightly (not more than two or three feet).

In the vicinity of icebergs, a sharp lookout should be kept and all bergs given a wide berth. It is dangerous to approach close to them because of the possibility of encountering underwater extensions and because bergs that are disintegrating may suddenly capsize or readjust their masses to new positions of equilibrium. In periods of low visibility the utmost caution is needed. The speed should be reduced and the watch prepared for quick maneuvering.

Upon the approach to pack ice, a careful decision is needed to determine the best action. Often it is possible to go around the ice, rather than through it. Unless the pack is quite loose, this action usually gains rather than loses time. When skirting a field of ice or an iceberg, do so to windward, if a choice is available, to avoid projecting tongues of ice or individual pieces that have been blown away from the main body of ice.

When it is considered necessary to enter pack ice, select the point of entry with great care. Get all available information on the nature and extent of ice and open water. Seek the weakest part of the ice and particularly avoid ice under pressure. If an offshore wind is blowing, a relatively ice-free shore lead may be available. Enter ice from leeward if possible, at slow speed. Enter on a course perpendicular to the ice edge, avoiding projecting tongues of ice.

Having entered the pack, always *work with the ice, not against it*, and *keep moving*, but do not rush the work of negotiating the pack. Patience may pay big dividends. Respect the ice but do not fear it. Stay in open water or areas of weak ice if possible, remembering that it is better to make good progress in the *general* direction desired than to fight heavy floes in the *exact* direction to be made good. However, avoid the temptation to proceed far to one side of the course. It is sometimes better to back out and seek a more penetrable area, being careful not to damage the screws while backing. Keep clear of corners and projecting points of ice. Never hit a large piece of ice if it can be avoided, but if it cannot be avoided, hit it head-on. Keep a sharp watch on the screws and rudder, fending off pieces of ice which might damage these vital parts, or stopping the propellers if the ice cannot be avoided. Back with extreme caution. Aircraft, particularly helicopters, are of great value in determining the nature and distribution of ice ahead. Since ice is continually shifting its position, the changing situation should be kept under observation and all forms of pressure avoided if possible. The windward side of icebergs within pack ice should be avoided because the pack ice usually moves with the wind, while the berg does not do so to the same extent, resulting in pressure on the windward side and open water to leeward. Because of its poor maneuverability in ice, a vessel may even be set down upon the iceberg.

If a narrow strait or a bay is entered, an alert watch should be maintained, because if the wind blows directly into the confined space, drifting ice may be forced down upon the vessel. An increase in wind on the windward side of a prominent point, grounded iceberg, or land ice tongue extending into the sea may similarly endanger a vessel.

While a ship is in pack ice, it is always in danger of being **beset,** or so closely surrounded by ice that steering control is lost. It may then be carried into shallow water or heavy ice with dangerous underwater projections. If pressure is exerted against the hull, the vessel is said to be **nipped.** When this occurs, it is in danger of being crushed. A ship in the ice is in constant danger of colliding with sharp pieces of ice, and while in the ice sharp turns to avoid such collisions may throw the stern against the ice, resulting in a bent or broken screw blade or propeller shaft. If a ship cannot free itself by maneuvering, an explosive charge or ice saws may have to be used. Dynamite is the explosive usually used. If detonated while the engines are going full astern and a strain is taken on an **ice anchor** (a stockless, single-fluked hook imbedded in the ice),

a 2½ pound charge placed in a hole cut nearly to the bottom of the ice about 35–40 feet off the beam may help to free a beset ship.

Attempts to clear ice jams in navigable channels by use of explosives and with the heat generated by thermite charges have met with some success, but the same tactics used against icebergs at sea have proven of little value. Icebergs in the southern Grand Banks area have disintegrated more rapidly when depth charges have been detonated against their underwater portions, but destruction by this method, as well as by other types of ordnance attempted, has been resisted by icebergs in higher latitudes.

If an icebreaker is in the vicinity, its instructions should be followed carefully in all ice operations.

Underice submarine operations require information on the thickness of ice below the surface as well as the extent of water openings between ice floes. While icebergs are believed to extend to depths of nearly 1,000 feet, ice hummocks between floes of polar ice may extend 150 feet below sea level. Submarine navigation thus involves vertical positioning as well as dead reckoning considerations, plus the necessity of finding open water or thin ice for surfacing purposes.

Only the basic principles of operating in ice have been given. Before entering areas of ice, those responsible for the maneuvering of a ship should become well acquainted with the experience of others who have operated in ice, especially those who have been in the same area. Some of this information is to be found in various volumes of sailing directions, particularly those for Antarctica (H.O. Pub. No. 27), and additional information is available at the U.S. Navy Hydrographic Office.

3615. Ice observing and forecasting.—Advance knowledge of ice conditions to be encountered is valuable in both planning and operational phases of any program to be conducted in high latitudes. Through the cooperation of observers aboard ship, in the air, and on land, the U. S. Navy Hydrographic Office collects and analyzes ice data in the arctic, and distributes ice information in the form of ice bulletins as part of regularly scheduled broadcasts.

For this program to be fully effective, it is essential that all vessels and air units operating in ice areas cooperate by submitting reports. To assist in this program, and to provide uniformity in reporting procedure, the U. S. Navy Hydrographic Office has published an observer's manual, H.O. Pub. No. 606–d, *Ice Observations;* H.O. Pub. No. 609, *A Functional Glossary of Ice Terminology;* and convenient ice log forms for recording the observations. When filled in, the log sheets are mailed to the U. S. Navy Hydrographic Office, Washington, D. C., and certain reports are sent by radio. The mariner who regularly sends complete reports can contribute to an increase in knowledge of ice conditions and to the accuracy and completeness of ice bulletins.

In addition to its ice bulletins, the U. S. Navy Hydrographic Office is developing techniques for forecasting ice growth and thickness, movement and concentration, and melting and break-up.

PART SEVEN

WEATHER

PART SEVEN

WEATHER

CHAPTER XXXVII

WEATHER OBSERVATIONS

3701. Introduction.—Weather forecasts are generally based upon information acquired by observations made at a large number of stations. Ashore, these stations are located so as to provide adequate coverage of the area of interest. Most observations at sea are made by mariners, wherever they happen to be. Since the number of observations at sea is small compared to the number ashore, marine observations are of importance in areas where little or no information is available from other sources. Results of these observations are recorded in the deck log (art. 3726), or other appropriate form. Data recorded by designated vessels are sent by radio to centers ashore, where they are plotted, along with other observations, to provide data for drawing synoptic charts (art. 3827). These charts are used to make forecasts. Complete weather information gathered at sea is mailed to the appropriate meteorological services for use in the preparation of weather atlases and in marine climatological studies.

The analysis of the weather map can be no better than the weather reports used for making the map. A knowledge of weather elements and the instruments used to measure them is therefore of importance to the mariner who hopes to benefit from weather forecasts.

Instruments of various types have been developed to aid in making weather observations. Some have been in use for many years, while others have been developed only recently. Electronic devices have aided materially, but the full impact of electronics upon meteorology has not yet been felt. Several new types of electronic weather instruments are in various stages of development.

3702. Atmospheric pressure measurement.—The sea of air surrounding the earth exerts a pressure of about 14.7 pounds per square inch on the surface of the earth. This **atmospheric pressure,** sometimes called **barometric pressure,** varies from place to place, and at the same place it varies with time.

Atmospheric pressure is one of the basic elements of a meteorological observation. When the pressure at each station is plotted on a synoptic chart, lines of equal atmospheric pressure, called **isobars,** are drawn to indicate the areas of high and low pressure and their centers. These are useful in making weather predictions, because certain types of weather are characteristic of each type area, and often the wind patterns over large areas are deduced from the isobars.

Atmospheric pressure is measured by means of a **barometer.** A **mercurial barometer** does this by balancing the weight of a column of air against that of a column of mercury. The **aneroid barometer** has a partly evacuated, thin-metal cell which is compressed by atmospheric pressure, the amount of the compression being related to the pressure.

Early mercurial barometers were calibrated to indicate the height, usually in inches or millimeters, of the column of mercury needed to balance the column of air above the point of measurement. While the units **inches of mercury** and **millimeters of mercury** are still widely used, many modern barometers are calibrated to indicate the centimeter-gram-second unit of pressure, the **millibar,** which is equal to 1,000 dynes per square centimeter. A **dyne** is the force required to accelerate a mass of one gram at the rate of one centimeter per second per second. A reading in any of the three units

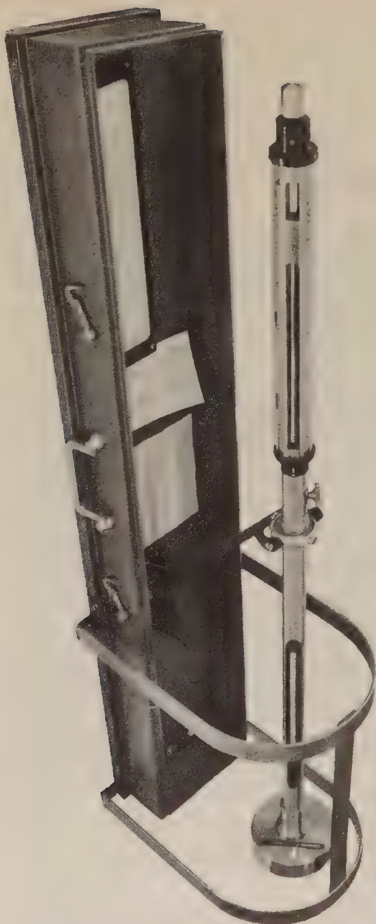


FIGURE 3703.—A shipboard-type mercurial barometer.

of measurement can be converted to the equivalent reading in either of the other units by means of table 14, or the conversion factors given in appendix D.

3703. The mercurial barometer was invented by Evangelista Torricelli in 1643. In its simplest form it consists of a glass tube a little more than 30 inches in length and of uniform internal diameter; one end being closed, the tube is filled with mercury, and inverted into a cup of mercury. The mercury in the tube falls until the column is just supported by the pressure of the atmosphere on the open cup, leaving a vacuum at the upper end of the tube. The height of the column indicates atmospheric pressure, greater pressures supporting higher columns of mercury. A shipboard type is shown in figure 3703.

The mercurial barometer is subject to rapid variations in height, called **pumping,** due to pitch and roll of the vessel and temporary changes in atmospheric pressure in the vicinity of the barometer. Because of this, the care required in the reading of the instrument, its bulkiness, and its vulnerability to physical damage, the mercurial barometer has been largely replaced at sea by the aneroid barometer.

3704. The aneroid barometer (fig. 3704) measures atmospheric pressure by means of the force exerted by the pressure on a partly evacuated, thin-metal element called a **sylphon cell.** A small spring is used, either internally or externally, to partly counteract the tendency of the atmospheric pressure to crush the cell. Atmospheric pressure is indicated directly by a scale and a pointer connected to the cell by a combination of levers. The linkage provides considerable magnification of the slight motion of the cell, to permit readings to higher precision than could be obtained without it.

An aneroid barometer should be mounted permanently. Prior to installation, the barometer should be carefully set to station pressure (art. 3706). An adjustment screw is provided for this purpose. The error in the reading of the instrument is determined by comparsion with a mercurial barometer or a standard precision aneroid barometer. If a qualified meteorologist is not available to make this adjustment, it is good practice to remove only one-half the apparent error. The case should then be tapped gently to assist the linkage to adjust itself, and the process repeated. If the remaining error is not more than half a millibar (0.015 inch), no attempt should be made to remove it by further adjustment. Instead, a correction should be applied to the readings. The accuracy of this correction should be checked from time to time.

A **precision aneroid barometer** used at weather stations ashore, and for comparison of shipboard instruments, is constructed and tested to more exacting tolerances than the ordinary barometer, and provides readings to greater accuracy.

3705. The barograph (fig. 3705) is a recording barometer. Basically, it is the same as a nonrecording aneroid barometer except that the pointer carries a pen at its

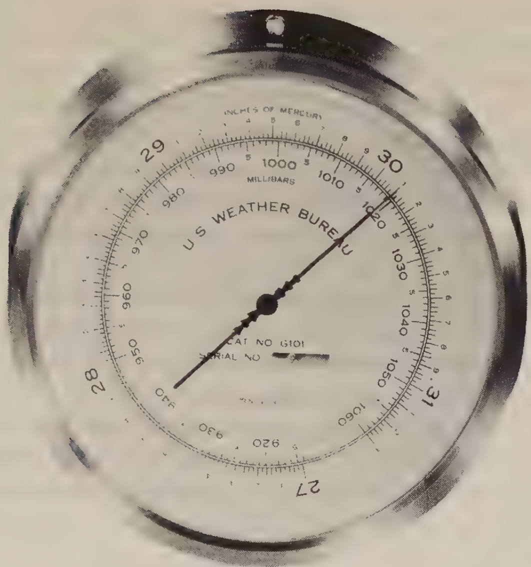


FIGURE 3704.—An aneroid barometer.

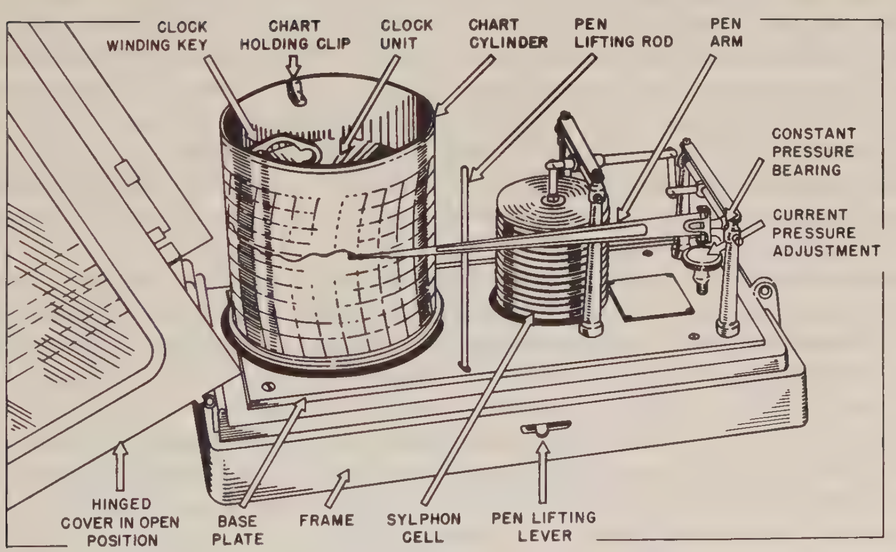


FIGURE 3705.—A barograph.

outer end, and the scale is replaced by a slowly rotating cylinder around which a prepared chart is wrapped. A clock mechanism inside the cylinder rotates the cylinder so that a continuous line is traced on the chart to indicate the pressure at any time.

A **microbarograph** is a precise barograph with greater magnification of deformations due to pressure changes, and a correspondingly expanded chart. Two sylphon cells are used, one being mounted over the other in tandem. Minor fluctuations due to shocks or vibrations are eliminated by damping. Since oil-filled dashpots are used for this purpose, the instrument should not be inverted.

The barograph is usually mounted on a shelf or desk in a room open to the atmosphere, and in a location which minimizes the effect of the ship's vibration. Shock-absorbing material such as sponge rubber is placed under the instrument to minimize the transmission of shocks.

The pen should be checked and the inkwell filled each time the chart is changed, every week in the case of the barograph, and each four days in the case of the microbarograph. The dashpots of the microbarograph should be kept filled with dashpot oil to within three-eighths inch of the top.

Both instruments require checking from time to time to insure correct indication of pressure. The position of the pen is adjusted by a small knob provided for this purpose. The adjustment should be made in stages, eliminating half the apparent error, tapping the case to insure linkage adjustment to the new setting, and then repeating the process.

3706. Adjustment of barometer readings.—Atmospheric pressure as indicated by a barometer or barograph may be subject to several errors, as follows:

Instrument error. Any inaccuracy due to imperfection or incorrect adjustment of the instrument can be determined by comparison with a standard instrument. The U. S. Weather Bureau provides a comparison service. In certain ports a representative brings a standard barometer on board ships which participate in the cooperative observation program of that Bureau. If a barometer is taken to the Weather Bureau, comparison can be made there. The correct sea-level pressure can be obtained by telephone. The shipboard barometer should be corrected for height, as explained below, before comparison with this telephoned value. If there is reason to believe that the barometer is in error, it should be compared with a standard, and if an error is found, the barometer should be adjusted to the correct reading, or a correction applied to all readings.

Height error. Since atmospheric pressure is caused by the weight of air above the place, the pressure decreases as height increases. The correct value at the barometer is called **station pressure.** Isobars adequately reflect wind conditions and geographic distribution of pressure only when they are drawn for pressure at constant height (or the varying height at which a constant pressure exists). On synoptic charts it is customary to show the equivalent pressure at sea level, called **sea level pressure.** This is found by applying a correction to station pressure. The correction, given in table 11, depends upon the height of the barometer and the average temperature of the air between this height and the surface. The outside air temperature taken aboard ship is sufficiently accurate for this purpose. *This is an important correction which should be applied to all readings of any type barometer.*

Gravity error. Mercurial barometers are calibrated for standard sea-level gravity at latitude 45°32′40″. If the gravity differs from this amount, an error is introduced. The correction to be applied to readings at various latitudes is given in table 12. *This correction does not apply to readings of an aneroid barometer.* Gravity also changes with height above sea level, but the effect is negligible for the first few hundred feet, and so is not needed for readings taken aboard ship.

Temperature error. Barometers are calibrated at a standard temperature of 32° F. The liquid of a mercurial barometer expands as the temperature of the mercury rises, and contracts as it decreases. The correction to adjust the reading of the instrument to the true value is given in table 13. *This correction is to be applied to readings of mercurial barometers only.* Modern aneroid barometers are compensated for temperature changes by the use of different metals having unequal coefficients of linear expansion.

3707. Determination of height by barometer.—Since atmospheric pressure is related to height, a barometer can be used to determine height. This is the principle of the **barometric altimeter** commonly used in aircraft.

Ordinary barometers can be used for determination of height *difference,* a problem which often arises in surveying. Simultaneous pressure and temperature readings should be made at both places (heights), if practicable. If this cannot be done, and more than a few minutes will elapse between readings, better values can be obtained by making a reading at the first height, then at the second, and then returning and making another reading at the first station, with approximately equal time intervals between readings. The average of the two readings at the first station is used. All appropriate corrections should be applied except that for height. If P_1 and P_2 are the atmospheric pressures at the two heights, the difference in height can be computed by Babinet's formula:

$$\text{Diff. in height} = C \times \frac{P_1 \sim P_2}{P_1 + P_2}.$$

If T_1 and T_2 are the air temperatures at the two places in degrees Fahrenheit, and difference in height is in feet,

$$C = 52{,}494\left(1 + \frac{T_1 + T_2 - 64}{900}\right).$$

If temperature is in degrees Celsius (centigrade), and difference in height is in meters,

$$C = 16{,}000\left[1 + \frac{2(T_1 + T_2)}{1000}\right].$$

For differences of not more than a few hundred feet, approximate results can be obtained by dividing the pressure difference in inches by 0.0011 inch, to obtain the answer in feet. This is almost the same as multiplying the pressure difference in hundredths of an inch by nine. For large differences, Babinet's formula is not strictly accurate, although the results should meet most requirements.

3708. Wind measurement consists of determination of the direction *from* which the wind is blowing, and the speed of the wind. Wind direction is measured by a **wind vane,** and wind speed by an **anemometer.**

A wind vane consists of a device pivoted on a vertical shaft, with more surface area on one side of the pivot than on the other, so that the wind exerts more force on one side, causing the smaller end to point into the wind. An indicator may be connected to the shaft to provide continuous measurement of wind direction.

In its simplest form, an anemometer consists of a number of cups mounted on short horizontal arms attached to a longer vertical shaft which rotates as the wind blows against the cups. The speed at which the shaft rotates is directly proportional to the wind speed. The number of rotations may be indicated by a counter or by marks on a revolving drum, or the speed may be indicated directly by a device similar to an automobile speedometer. Still another method is to connect a buzzer or flashing light so calibrated that the number of signals per unit time is the speed in knots or miles per hour.

The standard anemometer used aboard ship has three cups. Some anemometers have four cups, and certain naval vessels use a type called the **bridled cup anemometer,** which has a large number of cups mounted on a shaft which does not rotate freely. An anemometer which uses a propeller as the rotor to measure wind speed, and has a streamlined, tail-type vane to indicate direction, is being installed on some ships. Similar equipment is used ashore, customarily mounted on a guyed mast 13 feet high. Wind direction is transmitted to an indicator or recorder by a synchronous motor, while wind speed is transmitted as a voltage generated by a direct-current magneto driven by the propeller. A synchro system is connected to some wind-measuring equipment to provide remote indication of the velocity (both direction and speed). Lightweight, portable, hand-held instruments for measuring and indicating wind speed in knots are used on some ships, principally aircraft carriers.

Several types of wind speed and direction **recorders** are available. Each instrument is normally supplied with a description and complete operating instructions.

If no anemometer is available, wind speed can be estimated by its effect upon the sea and objects in its path, as explained in article 3710.

Measurement of winds aloft is discussed in articles 3717–3722.

3709. True and apparent wind.—An observer aboard a vessel proceeding through still air experiences an **apparent wind** which is from dead ahead and has an apparent speed equal to the speed of the vessel. Thus, if the actual or **true wind** is zero and the speed of the vessel is ten knots, the apparent wind is from dead ahead at ten knots. If the true wind is from dead ahead at 15 knots, and the speed of the vessel is ten knots, the apparent wind is $15+10=25$ knots from dead ahead. If the vessel makes a 180° turn, the apparent wind is $15-10=5$ knots from dead astern.

In any case, the apparent wind is the vector sum (art. O18) of the true wind and the *reciprocal* of the vessel's course and speed vector. Since wind vanes and anemometers measure *apparent* wind, the usual problem aboard a vessel equipped with an anemometer is to convert this to true wind. There are several ways of doing this. Perhaps the simplest is by the graphical solution illustrated in the following example:

Example 1.—A ship is proceeding on course 150° at a speed of 17 knots. The apparent wind is from 40° off the starboard bow, speed 15 knots.

Required.—The relative direction, true direction, and speed of the true wind.

Solution (fig. 3709a).—Starting at the center of a maneuvering board (art. 1212) or other suitable form, draw a line in the relative direction *from* which the apparent wind is blowing. Locate point 1 on this line, at a distance from the center equal to the speed of the apparent wind (2:1 scale is used in figure 3709a). From point 1, draw a line vertically *downward.* Locate point 2 on this line at a distance from point 1 equal to the speed of the vessel in knots, to the same scale as the first line. The relative direction of the true wind is *from* point 2 (120°) toward the center, and the speed of the true wind is the distance of point 2 from the center, to the same scale used previously (11 kn.). The true direction of the wind is the relative direction plus the true heading, or $120°+150°=270°$.

Answers.—True wind from 120° relative, 270° true, at 11 knots.

A quick solution can be made without an actual plot, in the following manner: On a maneuvering board (H.O. 2665–10), label the circles 5, 10, 15, 20, etc., from the center, and draw vertical lines tangent to these circles. Cut out the 5:1 scale and discard that part having graduations greater than the maximum speed of the vessel. Keep this equipment for all solutions. (For durability, the two parts can be mounted on cardboard or other suitable material.) To find true wind, spot in point 1 by eye. Place the zero of the 5:1 scale on this point and align the scale (inverted) by means of the vertical lines. Locate point 2 at the speed of the vessel as indicated on the 5:1

FIGURE 3709a.—Finding true wind by maneuvering board.

scale. It is always vertically *below* point 1. Read the relative direction and the speed of the true wind using eye interpolation if needed. The U.S. Weather Bureau distributes a wind vector computer called a *Shipboard Wind Plotter* (fig. 3709b). Solution by means of this plotter is illustrated in the following example:

Example 2.—A ship is proceeding on course 270° at a speed of 14.5 knots. The apparent wind is from 40° off the starboard bow, speed 20 knots.

Required.—The relative direction, true direction, and speed of the true wind by U.S. Weather Bureau Shipboard Wind Plotter.

Solution (fig. 3709b).—The true direction of the apparent wind is determined by adding the apparent wind direction to the ship's heading if the wind is from off the starboard bow and subtracting the apparent wind direction if the wind is from off the port bow. In this example, the true direction of the apparent wind is 310°. In this solution the red arrowhead is considered the top of the plotter. Set ship's course, 270°, to the top of the plotter by rotating the protractor disk to set 270° at the red arrow. Using a convenient linear scale, measure vertically downward from the center peg of the

plotting board a distance equivalent to 14.5 knots. Mark this point "S" for ship. Rotate the protractor disk of the plotting board until 310° is at the red arrowhead at the top of the plotting board. Using the same linear scale as for ship's speed, plot vertically downward from the center peg of the plotting board a distance equivalent to 20 knots. Mark this point "W". Rotate the protractor disk until the "S" is vertically above the "W", using the vertical lines on the plotting board to line up the two points. Read the true wind direction at the top of the plotting board. The distance between points "S" and "W" is the true wind speed, using the same scale as in plotting points "S" and "W".

Answers.—True wind direction is 357°, true wind speed is 13 knots.

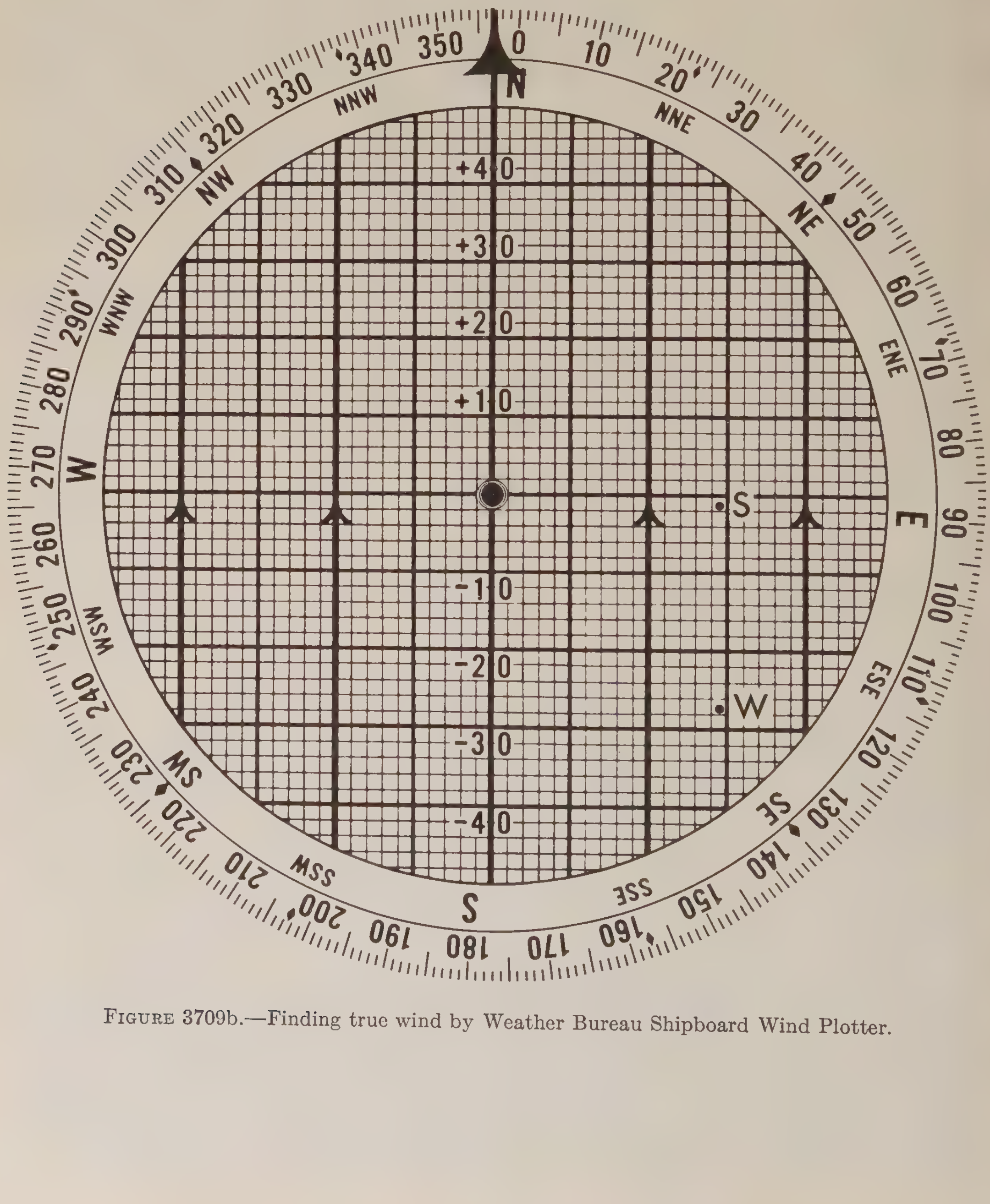


FIGURE 3709b.—Finding true wind by Weather Bureau Shipboard Wind Plotter.

Such problems can be solved by the use of true directions and a regular vector solution, but the use of relative directions simplifies the plot because that component of the apparent wind due to the vessel's motion is always parallel (but reversed) to the vessel's motion, and the apparent wind is always *forward* of the true wind.

A tabular solution based upon the same principle can be made by means of table 10. The entering values for this table are the apparent wind speed *in units of ship's speed,* and the difference between the heading and the apparent wind direction. The values taken from the table are the relative direction (right or left) of the true wind, and the speed of the true wind *in units of ship's speed.* If a vessel is proceeding at 12 knots, six knots constitutes one-half (0.5) unit, 12 knots one unit, 18 knots 1.5 units, 24 knots two units, etc.

Example 3.—A ship is proceeding on course 270° at a speed of ten knots. The apparent wind is from 10° off the port bow, speed 30 knots.

Required.—The relative direction, true direction, and speed of the true wind by table 10.

Solution.—The apparent wind speed is $\frac{30}{10}=3.0$ ship's speed units. Enter table 10 with 3.0 and 10° and find the relative direction of the true wind to be 15° off the port bow (345° relative), and the speed to be 2.02 times the ship's speed, or $2.02\times10=20$ knots, approximately. The true direction is $345°+270°=255°$.

Answers.—True wind from 345° relative, 255° true, at 20 knots.

By variations of this problem, one can find the apparent wind from the true wind, the course or speed required to produce an apparent wind from a given direction or speed, or the course and speed to produce an apparent wind of a given speed from a given direction. Such problems arise in aircraft carrier operations.

Wind speed determined by appearance of the sea (art. 3710) is the speed of the true wind. The sea also provides an indication of the direction of the true wind, because waves move in the same direction as the generating wind, not being deflected by earth rotation (art. 3302). If a wind vane is used, the direction of the apparent wind thus determined can be used with the speed of the true wind to determine the direction of the true wind by vector diagram. If a maneuvering board is used, draw a circle about the center equal to the speed of the true wind. From the center, plot the ship's vector (true course and speed). From the end of this vector draw a line in the direction in which the apparent wind is blowing (reciprocal of the direction from which it is blowing) until it intersects the speed circle. This line is the apparent wind vector, its length denotes the speed. A line from the center of the board to the end of the apparent wind vector is the true wind vector. The reciprocal of this vector is the direction from which the true wind is blowing. If the true wind speed is less than the speed of the vessel, two solutions are possible. If solution is by table 10, the true speed, in units of ship's speed, is found in the column for the direction of the apparent wind. The number to the left is the relative direction of the true wind. The number on the same line in the side columns is the speed of the apparent wind in units of ship's speed. Again, two solutions are possible if true wind speed is less than ship's speed.

3710. Wind and the sea.—The action of the wind in creating ocean currents and waves is discussed in chapters XXXII and XXXIII, respectively. There is a relationship between the speed of the wind and the state of the sea in the immediate vicinity of the wind. This is useful in predicting the sea conditions to be anticipated when future wind speed forecasts are available. It can also be used to estimate the speed of the wind, which may be desirable when an anemometer is not available.

Wind speeds are usually grouped in accordance with the **Beaufort scale** named after Admiral Sir Francis Beaufort, who devised it in 1806. As adopted in 1838, Beaufort numbers ranged from 0, calm, to 12, hurricane. They have now been extended to 17. The Beaufort scale, with certain other pertinent information, is given in appendix R. The appearance of the sea at different Beaufort scale numbers from 0 through 12 is shown in figures 3710a through 3710m.

FIGURE 3710a.—Beaufort scale 0.

FIGURE 3710b.—Beaufort scale 1.

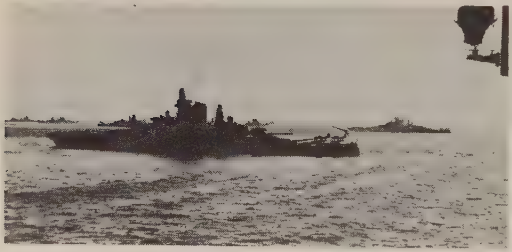


FIGURE 3710c.—Beaufort scale 2.

FIGURE 3710d.—Beaufort scale 3.

FIGURE 3710e.—Beaufort scale 4.

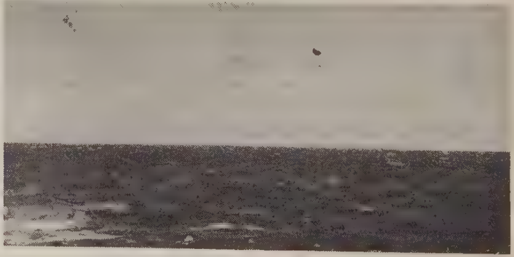


FIGURE 3710f.—Beaufort scale 5.

FIGURE 3710g.—Beaufort scale 6.

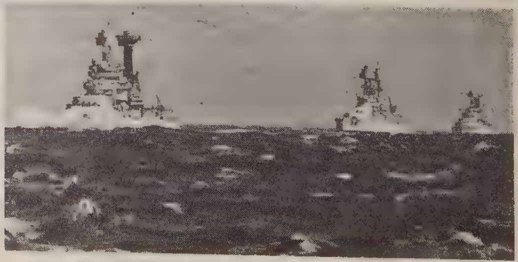


FIGURE 3710h.—Beaufort scale 7.

FIGURE 3710i.—Beaufort scale 8.

FIGURE 3710j.—Beaufort scale 9.

FIGURE 3710k.—Beaufort scale 10.

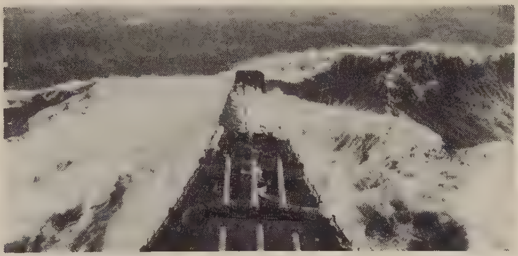


FIGURE 3710l.—Beaufort scale 11.

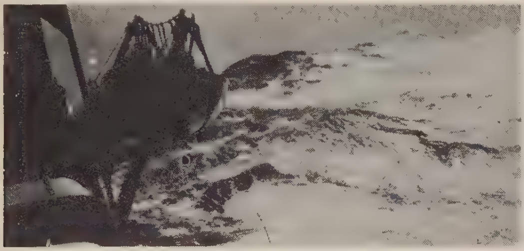


FIGURE 3710m.—Beaufort scale 12.

3711. Temperature is the intensity or degree of heat. It is measured in degrees. Several different temperature scales are in use.

On the **Fahrenheit** (**F**) scale commonly used in the United States and other English-speaking countries, pure water freezes at 32° and boils at 212°.

On the **Celsius** (**C**) scale commonly used with the metric system, the freezing point of pure water is 0° and the boiling point is 100°. This scale has been known by various names in different countries. In the United States it was formerly called the **centigrade** scale. The Ninth General Conference of Weights and Measures, held in France in 1948, adopted the name Celsius to be consistent with the naming of other temperature scales after their inventors, and to avoid the use of different names in different countries. On the original Celsius scale, invented in 1742 by a Swedish astronomer named Anders Celsius, the numbering was the reverse of the modern scale, 0° representing the boiling point of water, and 100° its freezing point.

Réaumur temperature is based upon a scale in which water freezes at 0° and boils at 80°.

Absolute zero is considered to be the lowest possible temperature, at which there is no molecular motion and a body has no heat. For some purposes, it is convenient to express temperature by a scale at which 0° is absolute zero. This is called **absolute**

temperature. If Fahrenheit degrees are used, it may be called **Rankine (R)** temperature; and if Celsius, **Kelvin (K)** temperature. The Kelvin scale is more widely used than the Rankine. Absolute zero is at (−) 459°67 F or (−) 273°15 C.

Temperature by one scale can be converted to that at another by means of the relationship that exists between the scales. Thus,

$$C=\frac{5}{9}(F-32)$$

and

$$F=\frac{9}{5}C+32.$$

A temperature of (−) 40° is the same by either the Celsius or Fahrenheit scale. Similar formulas can be made for conversion of other temperature scale readings. Table 15 gives the equivalent values of Fahrenheit, Celsius, and Kelvin temperatures.

The intensity or degree of heat (temperature) should not be confused with the *amount* of heat. If the temperature of air or some other substance is to be increased (the substance made hotter) by a given number of degrees, the amount of heat that must be added is dependent upon the amount of the substance to be heated. Also, equal amounts of different substances require the addition of unequal amounts of heat to effect equal increase in temperature because of their difference of specific heat (art. 3012). Units used for measurement of amount of heat are the **British thermal unit (BTU)**, the amount of heat needed to raise the temperature of one pound of water one degree Fahrenheit; and the **calorie**, the amount of heat needed to raise the temperature of one gram of water one degree Celsius.

3712. Temperature measurement is made by means of a **thermometer.** Most thermometers are based upon the principle that materials expand with increase of temperature, and contract as temperature decreases. In its most usual form (fig. 3712a) a thermometer consists of a bulb filled with mercury and connected to a tube of very small cross-sectional area. The mercury only partly fills the tube. In the remainder is a vacuum created during construction of the instrument. The air is driven out by boiling the mercury, and the top of the tube is then sealed by a flame. As the mercury expands or contracts with changing temperature, the length of the mercury column in the tube changes. Temperature is indicated by the position of the top of the column of mercury with respect to a scale etched on the glass tube or placed on the thermometer support.

A **maximum thermometer** has a constriction in the tube, near the bulb. As temperature increases, the expanding mercury is forced past the constriction, but will not return as temperature decreases. Thus, it indicates the highest temperature which has occurred since the last setting. This principle is utilized in clinical thermometers, used for measuring body temperature. The mercury can be forced back into the bulb by centrifugal force applied by swinging the arm rapidly. Meteorologists have a device called a "Townsend support" for accomplishing this with less effort and less possibility of breakage.

A **minimum thermometer** (fig. 3712b) uses alcohol instead of mercury. The upper part of the tube contains air under slight pressure, to prevent evaporation of the alcohol with resultant "breaks" in the column as the alcohol later condenses. The thermometer contains an index which is so constructed as to allow alcohol to flow past it up the tube with rising temperatures, but which moves downward in the tube if the temperature falls below it, being drawn down by the effect of surface tension exerted by the bottom of the meniscus (curved upper surface) of the column of alcohol as it reaches the index. Due to this effect, the index remains at the lowest temperature

Figure 3712a.—A thermometer for measuring air temperature.

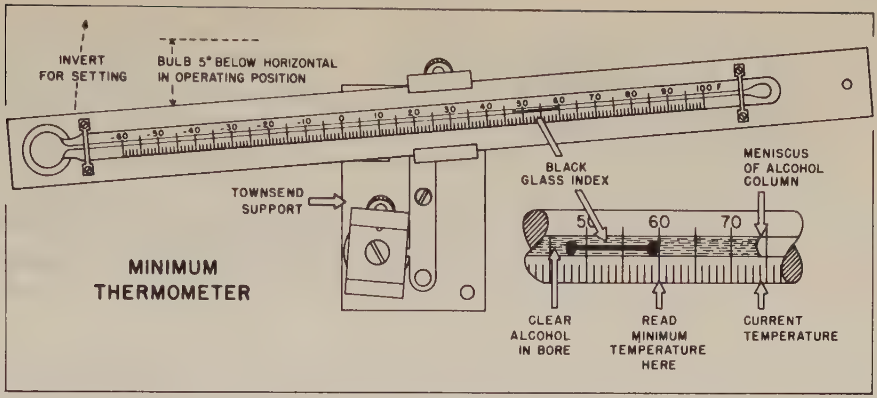


Figure 3712b.—A minimum thermometer.

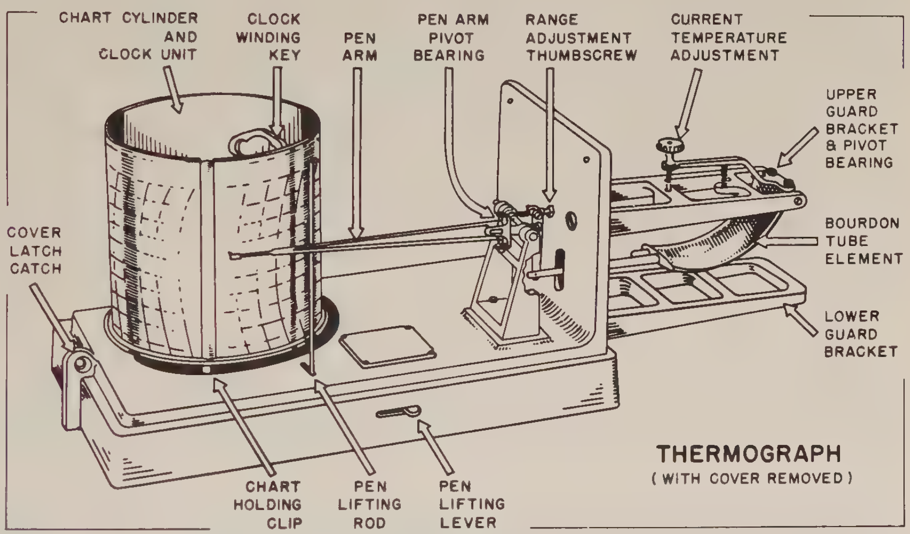


Figure 3712c.—A thermograph with cover removed.

which has occurred since the last setting. Setting is accomplished by tilting the thermometer until the bulb is uppermost, when the index returns to the current temperature. The thermometer is normally maintained at an angle of about 5° to the horizontal, with the bulb at the lower end. A Townsend support is used for this purpose.

Temperature can be measured by means of a **thermograph** (fig. 3712c), which is a recording thermometer. In its outward appearance this instrument is similar to a barograph (fig. 3705). The pen arm is connected, through a linkage, to the thermometric element, which usually consists of a metal tube shaped in the form of an arc and containing alcohol. As the alcohol expands with temperature increase, it tends to straighten the tube; and as the temperature decreases, the contracting alcohol permits the tube to resume its curved shape. The linkage magnifies these variations and transmits them to the pen, which records the temperature on a chart placed around a clock-driven, revolving cylinder.

The freezing point of mercury is about (−) 38° F. Various substances are used to measure lower temperatures, the most common being some form of alcohol, which has a freezing point well below (−) 100° F. For even lower temperatures, below those

ever recorded in the atmosphere, gas may be used instead of a liquid. Thermometers based upon other principles, such as unequal expansion of dissimilar metals, melting point of a substance, color, etc., are sometimes used, particularly for temperatures considerably higher or much lower than those occurring in the atmosphere.

Temperature measuring equipment should be placed in a shelter which protects it from mechanical damage and direct rays of the sun. The shelter should have louvered sides to permit free access of air. Aboard ship, the shelter should be placed in an exposed position as far as practicable from metal bulkheads. On vessels where shelters are not available, the temperature measurement should be made in shade at an exposed position on the windward side.

Sea water temperature is normally measured at the condenser intake. Although this is not a true measure of surface water temperature, the error is generally small. Measurement should be made near the entrance of the intake.

If the temperature of the water at the surface is desired, a sample should be obtained by bucket, preferably a canvas bucket, from a forward position well clear of any discharge lines. The sample should be taken immediately to a place where it is sheltered from wind and sun. The water should then be stirred with the thermometer, keeping the bulb submerged, until an essentially constant reading is obtained.

3713. Humidity is the condition of the atmosphere with reference to its water vapor content. **Absolute humidity** is a measure of the mass of vapor per unit volume of air. **Relative humidity** is the ratio (stated as a percentage) of the existing vapor pressure to the vapor pressure corresponding to saturation at the prevailing temperature and atmospheric pressure. This is very nearly the ratio of the amount of water vapor present to the amount that the air could hold at the same temperature and pressure if it were saturated.

As air cools, its capacity for holding water vapor decreases. Therefore, as air temperature decreases, the relative humidity increases. At some point, saturation takes place, and any further cooling results in condensation of some of the moisture. The temperature at which this occurs is called the **dew point,** and the moisture deposited upon natural objects is called **dew** if it forms in the liquid state, or **frost** if it forms in the frozen state.

The same process causes moisture to form on the outside of a container of cold liquid, the liquid cooling the air in the immediate vicinity of the container until it reaches the dew point. When moisture is deposited on man-made objects, it is usually called **sweat.** It occurs whenever the temperature of a surface is lower than the dew point of the air in contact with it. It is of particular concern to the mariner because of its effect upon his instruments, and possible damage to his ship or its cargo. Lenses of optical instruments may sweat, usually with such small droplets that the surface has a "frosted" appearance. When this occurs, the instrument is said to "fog" or "fog up," and is useless until the moisture is removed. Damage is often caused by corrosion or direct water damage when pipes sweat and drip, or when the inside of the shell plates of a vessel sweat. Cargo may sweat if it is cooler than the dew point of the air. One of the principal problems of preserving ships of the reserve fleet is the protection against moisture. An important step is the draining of all water, sealing of compartments, and drying of the air.

Clouds and fog form by "sweating" of minute particles of dust, salt, etc., in the air. Each particle forms a nucleus around which a droplet of water forms. If air is completely free from solid particles on which water vapor may condense, the extra moisture remains in the vapor state, and the air is said to be **supersaturated.**

Relative humidity and dew point are measured by means of a **hygrometer.** The most common type, called a **psychrometer,** consists of two thermometers mounted

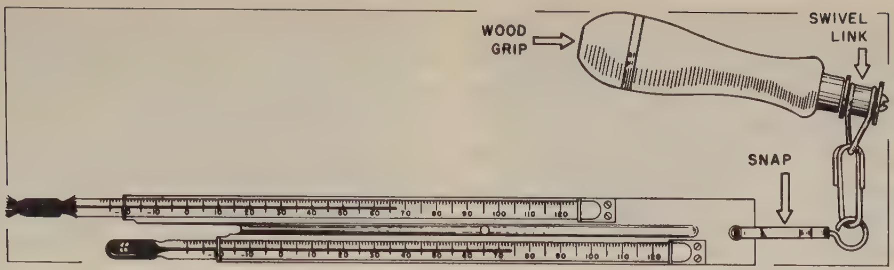


FIGURE 3713.—A sling psychrometer.

together on a single strip of material, as shown in figure 3713. One of the thermometers is mounted a little lower than the other, and has its bulb covered with muslin. When the muslin covering is thoroughly moistened and the thermometer well ventilated, evaporation cools the bulb of the thermometer, causing it to indicate a lower reading than the other. A **sling psychrometer,** illustrated in figure 3713, is ventilated by whirling the thermometers. Some psychrometers use a fan. **Dry-bulb temperature** is indicated by the uncovered **dry-bulb thermometer,** and **wet-bulb temperature** is indicated by the muslin-covered **wet-bulb thermometer.** The difference between these two temperatures, and the dry-bulb temperature, are used to enter **psychrometric tables** to find the relative humidity (tab. 16) and dew point (tab. 17). If the wet-bulb temperature is above freezing, reasonably accurate results can be obtained by a psychrometer consisting of wet- and dry-bulb thermometers mounted so that air can circulate freely around them without special ventilation. This type of installation is common aboard ship.

Example.—The dry-bulb temperature is 65°F and the wet-bulb temperature is 61°F.

Required.—(1) Relative humidity, (2) dew point.

Solution.—The difference between readings is 4°. Entering table 16 with this value and a dry-bulb temperature of 65°, the relative humidity is found to be 80 percent. From table 17 the dew point is found to be 58°.

Answers.—(1) Relative humidity 80 percent, (2) dew point 58°.

A recording hygrometer, called a **hygrograph,** provides a continuous record of relative humidity. In outward appearance this instrument is similar to a barograph (fig. 3705) and a thermograph (fig. 3712c), using the same clock movement and chart cylinder. The measuring element, however, generally consists of a number of strands of human hair separated into groups kept apart by a spreader device. The hairs are kept taut by a counterbalance. As the relative humidity rises, the hairs increase in length, and as the relative humidity falls, they decrease in length. A linkage magnifies these changes and transmits them to a pen which records the relative humidity on a chart placed around the clock-driven, revolving cylinder. The hygrograph is a convenient device, but lacks accuracy, lags considerably behind changes in relative humidity, and is not reliable at low temperatures. It requires frequent calibration.

A **hygrothermograph** combines the features of both the hygrograph and the thermograph, providing a continuous record of both relative humidity and air temperature for seven days on a single chart. It has the same limitations as the hygrograph and the thermograph and its indications should be checked daily by psychrometer and thermometer.

3714. Clouds are visible assemblages of numerous tiny droplets of water, or ice crystals, formed by condensation of water vapor in the air, with the bases of the assemblages above the surface of the earth. **Fog** is a similar assemblage in contact with the surface of the earth.

The shape, size, height, thickness, and nature of a cloud depend upon the conditions under which it is formed. Therefore, clouds are indicators of various processes occurring in the atmosphere. The ability to recognize different types and a knowledge of the conditions associated with them are useful in predicting future weather.

Although the variety of clouds is virtually endless, they may be classified according to general type. Clouds are grouped generally into four "families" according to some common characteristic. **High clouds** are those having a mean lower level above 20,000 feet. They are composed principally of ice crystals. **Middle clouds** have a mean level between 6,500 and 20,000 feet. They are composed largely of water droplets, although the higher ones have a tendency toward ice particles. **Low clouds** have a mean upper level of less than 6,500 feet. These clouds are composed entirely of water droplets. **Clouds with vertical development** are a distinctive group formed by rising air which is cooled as it reaches greater heights. When it reaches the height of the dew point, some of its water vapor condenses. Therefore, the bottoms of such clouds are usually flat. Clouds with vertical development may begin at almost any level, but generally within the low cloud range. They may extend to great heights, well above the lower limit of high clouds. They form as water droplets, but toward the top they may freeze.

Within these four families are ten principal cloud types. The names of these are composed of various combinations and forms of the following basic words, all from Latin:

Cirrus, meaning "curl."
Cumulus, meaning "heap."
Stratus, meaning "layer."
Alto, meaning "high."
Nimbus, meaning "rain."

The first three are the basic cloud types. Individual cloud types recognize certain characteristics, variations, or combinations of these. The ten principal cloud types are:

High clouds. **Cirrus** (**Ci**) are detached high clouds of delicate and fibrous appearance, without shading, generally white in color, and often of a silky appearance (figs. 3714a and 3714d). Their fibrous and feathery appearance is due to the fact that they are composed entirely of ice crystals. Cirrus appear in varied forms such as isolated tufts; long, thin lines across the sky; branching, feather-like plumes; curved wisps which may end in tufts, etc. These clouds may be arranged in parallel bands which cross the sky in great circles and appear to converge toward a point on the horizon. This may indicate, in a general way, the direction of a low pressure area. Cirrus may be brilliantly colored at sunrise and sunset. Because of their height, they become illuminated before other clouds in the morning, and remain lighted after others at sunset. Cirrus are generally associated with fair weather, but if they are followed by lower and thicker clouds, they are often the forerunner of rain or snow.

Cirrocumulus (**Cc**) are high clouds composed of small white flakes or scales, or of very small globular masses, usually without shadows and arranged in groups or lines, or more often in ripples resembling those of sand on the seashore (fig. 3714b). One form of cirrocumulus is popularly known as "mackerel sky" because the pattern resembles the scales on the back of a mackerel. Like cirrus, cirrocumulus are composed of ice crystals and are generally associated with fair weather, but may precede a storm if they thicken and lower. They may turn gray and appear hard before thickening.

Cirrostratus (**Cs**) are thin, whitish, high clouds (fig. 3714c) sometimes covering the sky completely and giving it a milky appearance and at other times presenting, more or less distinctly, a formation like a tangled web. The thin veil is not sufficiently dense to blur the outline of sun or moon. However, the ice crystals of which the cloud is

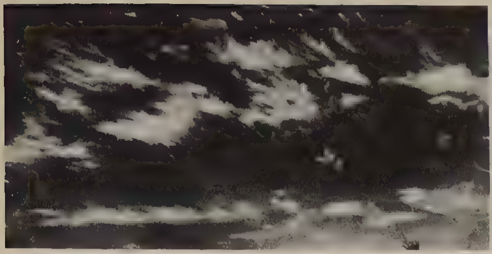


FIGURE 3714a.—Cirrus.

FIGURE 3714b.—Cirrocumulus.

FIGURE 3714c.—Cirrostratus.

FIGURE 3714d.—Cirrus and cirrostratus.

composed refract the light passing through in such a way that halos (art. 3819) may form with the sun or moon at the center. Figure 3714d shows cirrus thickening and changing into cirrostratus. In this form it is popularly known as "mares' tails." If it continues to thicken and lower, the ice crystals melting to form water droplets, the cloud formation is known as altostratus. When this occurs, rain may normally be expected within 24 hours. The more brushlike the cirrus when the sky appears as in figure 3714d, the stronger the wind at the level of the cloud.

Middle clouds. **Altocumulus** (**Ac**) are middle clouds consisting of a layer of large, ball-like masses that tend to merge together. The balls or patches may vary in thickness and color from dazzling white to dark gray, but they are more or less regularly arranged. They may appear as distinct patches (fig. 3714e) similar to cirrocumulus (fig. 3714b) but can be distinguished by the fact that individual patches are generally larger, and show distinct shadows in some places. They are often mistaken for stratocumulus (fig. 3714i). If this form thickens and lowers, it may produce thundery weather and showers, but it does not bring prolonged bad weather. Sometimes the patches merge to form a series of big rolls that resemble ocean waves, but with streaks of blue sky (fig. 3714f). Because of perspective, the rolls appear to run together near the horizon. These regular parallel bands differ from cirrocumulus in that they occur in larger masses with shadows. These clouds move in the direction of the short dimension of the rolls, as do ocean waves. Sometimes altocumulus appear briefly in the form shown in figure 3714g, usually before a thunderstorm. They are generally arranged in

FIGURE 3714e.—Altocumulus in patches.

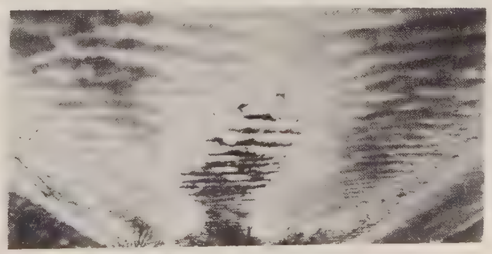


FIGURE 3714f.—Altocumulus in bands.

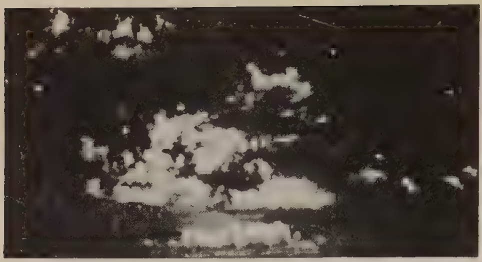


FIGURE 3714g.—Turreted altocumulus.

FIGURE 3714h.—Altostratus.

a line with a flat horizontal base, giving the impression of turrets on a castle. The turreted tops may look like miniature cumulus and possess considerable depth and great length. These clouds usually indicate a change to chaotic, thundery skies.

Altostratus (**As**) are middle clouds having the appearance of a grayish or bluish, fibrous veil or sheet (fig. 3714h). The sun or moon, when seen through these clouds, appears as if it were shining through ground glass, with a corona (art. 3820) around it. Halos are not formed. If these clouds thicken and lower, or if low, ragged "scud" or rain clouds (nimbostratus) form below them, continuous rain or snow may be expected within a few hours.

Low clouds. **Stratocumulus** (**Sc**) are low clouds composed of soft, gray, roll-shaped masses (fig. 3714i). They may be shaped in long, parallel rolls similar to altocumulus (fig. 3714f), moving forward with the wind. The motion is in the direction of their short dimension, like ocean waves. These clouds, which vary greatly in altitude, are the final product of the characteristic daily change that takes place in cumulus clouds. They are usually followed by clear skies during the night.

Stratus (**St**) is a low cloud in a uniform layer (fig. 3714j) resembling fog. Often the base is not more than 1,000 feet high. A veil of thin stratus gives the sky a hazy appearance. Stratus is often quite thick, permitting so little sunlight to penetrate that it appears dark to an observer below it. From above, it looks white. Light mist may descend from stratus. Strong wind sometimes breaks stratus into shreds called "fractostratus."

Nimbostratus (**Ns**) is a low, dark, shapeless cloud layer, usually nearly uniform, but sometimes with ragged, wet-looking bases. Nimbostratus is the typical rain cloud. The precipitation which falls from this cloud is steady or intermittent, but not showery.

Clouds with vertical development. **Cumulus** (**Cu**) are dense clouds with vertical development. They have a horizontal base and dome-shaped upper surface, with protuberances extending above the dome. Cumulus appear in small patches, and never cover the entire sky. When the vertical development is not great, the clouds appear in patches resembling tufts of cotton or wool, being popularly called "woolpack" clouds (fig. 3714k). The horizontal bases of such clouds may not be noticeable. These are

FIGURE 3714i.—Stratocumulus.

FIGURE 3714j.—Stratus.

FIGURE 3714k.—Cumulus.

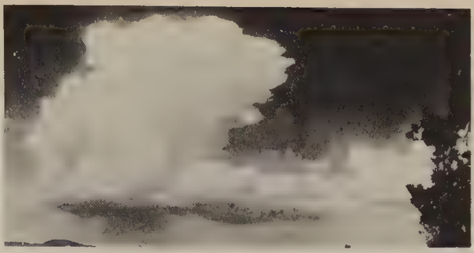


FIGURE 3714l.—Cumulonimbus.

called "fair weather" cumulus because they always accompany good weather. However, they may merge with altocumulus, or may grow to cumulonimbus before a thunderstorm. Since cumulus are formed by updrafts, they are accompanied by turbulence, causing "bumpiness" in the air. The extent of turbulence is proportional to the vertical extent of the clouds. Cumulus are marked by strong contrasts of light and dark.

Cumulonimbus (**Cb**) is a massive cloud with great vertical development, rising in mountainous towers to great heights (fig. 3714l). The upper part consists of ice crystals, and often spreads out in the shape of an anvil which may be seen at such distances that the base may be below the horizon. Cumulonimbus often produces showers of rain, snow, or hail, frequently accompanied by thunder. Because of this, the cloud is often popularly called a "thundercloud" or "thunderhead." The base is horizontal, but as showers occur it lowers and becomes ragged.

3715. Cloud height measurement.—At sea, cloud heights are often determined by estimate. This is a difficult task, particularly at night. A searchlight may be of some assistance. Radar operating at the higher frequencies, particularly three-centimeter radar, indicates returns from some clouds. Certain models permit measurement of height.

Ceiling balloons can be used to determine height of low clouds with reasonable accuracy. Any type balloon having a known rate of ascent is suitable. The following balloons are in use:

1. A 10-gram spherical balloon with 40 grams of hydrogen or 43 grams of helium. Used at Navy and Air Force stations.
2. A 10-gram spherical balloon with 45 grams of helium. Used at civil stations.
3. A 30-gram balloon with 125 grams of hydrogen or 139 grams of helium. Used at civil and Navy stations.
4. A 30-gram balloon with 132 grams of hydrogen or 147 grams of helium. Used at Air Force stations.

The ascent of these four balloons, in feet, is

Minutes	*1*	*2*	*3*	*4*
1	480	500	710	720
2	850	960	1, 360	1, 380
3	1, 210	1, 420	2, 010	2, 040
4	1, 570	1, 880	2, 630	2, 670
5	1, 930	2, 300	3, 250	3, 300
6	2, 290	2, 720	3, 840	3, 900
7	2, 650	3, 140	4, 430	4, 500
8	3, 010	3, 560	5, 020	5, 100.

For elapsed times greater than eight minutes, the rate is 360 feet per minute for balloon 1, 420 feet per minute for balloon 2, 590 feet per minute for balloon 3, and 600 feet per minute for balloon 4.

Cloud height is determined by measurement of the elapsed time from release of the balloon until it disappears in the clouds. Horizontal motion due to wind generally has negligible effect upon the rate of ascent.

The height of the base of clouds formed by vertical development (any form of cumulus), if formed in air that has risen from the surface of the earth, can be determined by psychrometer, because the height to which the air must rise before condensation takes place is proportional to the difference between surface air temperature and the dew point. At sea, this difference multiplied by 236 gives the height in feet. That is, for every degree difference between surface air temperature and the dew point, the air must rise 236 feet before condensation will take place. Thus, if the dry-bulb temperature is 80°F, and the wet-bulb temperature is 77°F, the dew point (from tab. 17) is 76°F, or four degrees lower than the surface air temperature. The height of the cloud base is $4 \times 236 = 944$ feet.

Ashore, cloud height measurement can be made at night by means of a **ceiling light projector** and **clinometer** (fig. 3715). The projector throws a beam of light vertically upward, casting a spot of light on the clouds. An observer at a known distance from the projector measures the angle of elevation of the spot of light. This is usually done by means of a clinometer, a single hand-held sighting device with a pointer

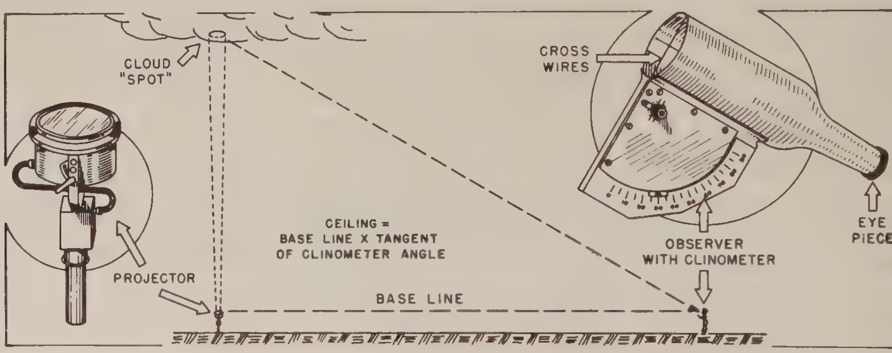


FIGURE 3715.—Ceiling light projector and clinometer for measuring cloud height at night.

which hangs freely and indicates elevation angle on an arc. The height of the cloud base is equal to the tangent of the elevation angle times the distance between the projector and observer, the curvature of the earth being neglected. Thus, if the observer is 400 feet from the projector, and the angle of elevation is 73°, the height of the base of the clouds is $400 \times 3.2709 = 1{,}308$ feet. If two ships are in company, an approximation of cloud height might be determined in this way, using a searchlight on one ship and a clinometer (a sextant can be used if the horizon is distinguishable) on the other. For low clouds, this might be performed on a single vessel, mounting the searchlight at one end of the vessel, and placing the observer at the other end. Reasonably accurate results might be obtained at sea if the searchlight can be stabilized in the vertical.

Measurements can be made both day and night by means of a **ceilometer.** This device consists of a projector, detector, and recorder to provide a continuous record of cloud height above the observing station, both day and night. The ceilometer uses a beam of light that is pulse modulated (art. 1016) like radar signals. The projector and detector are some distance apart, height being determined by the same principle used with the ceiling light projector and clinometer. Either the projector or detector continuously scans a 90° arc from the vertical to the horizontal, and back, in line with

the other instrument, which remains vertical. When the spot of light is in the line of sight of the detector, a photoelectric cell detects it, and actuates the recorder, which reads directly in height. The ceilometer is not suitable for use at sea.

3716. Visibility measurement.—**Visibility** is the extreme horizontal distance at which prominent objects can be seen and identified by the unaided eye. It is usually measured directly by the human eye. Ashore, the distances of various buildings, trees, lights, and other objects are measured and used as a guide in estimating the visibility. At sea, however, such an estimate is difficult to make with accuracy. Other ships and the horizon may be of some assistance.

Visibility is sometimes measured by a **transmissometer,** a device which measures the transparency of the atmosphere by passing a beam of light over a known short distance, and comparing it with a reference light.

3717. Upper air observations.—Upper air information provides the third dimension to the weather map. Unfortunately, the equipment necessary to obtain such information is quite expensive, and the observations are time consuming. Consequently, the network of observing stations is quite sparse compared to that for surface observations, particularly over the oceans and in isolated land areas. Where facilities exist, upper air observations are made by means of unmanned balloons in conjunction with theodolites, radiosondes, radar, and radio direction finders. Observations are sometimes made by aircraft.

3718. Pilot balloons are free balloons released at the surface of the earth and followed by optical means to determine their movement in relation to the point from which released. They are of neoprene latex (occasionally of natural rubber latex) a few thousandths of an inch thick, and have a nominal weight of either 30 or 100 grams. The balloons are inflated with helium or hydrogen to a definite free-lift capacity for which ascensional rate tables have been prepared. The neck of each balloon is then securely fastened to prevent leakage of the gas, and the balloon is released. A theodolite is trained on the balloon, which is kept in the field of vision of the instrument throughout the observation.

By means of a buzzer signal the observer is warned five seconds prior to the end of each minute after release. The cross hairs of the theodolite are then brought to bear on the balloon at the end of each minute (also signalled by the buzzer), and the horizontal and vertical angles are read to the nearest tenth of a degree. These data are then plotted on polar coordinate paper similar to a maneuvering board (art. 1212), and the wind speed and direction at each selected level (each 1,000-foot level) are determined.

An observation of winds aloft made in this manner is called a **pibal,** from **pil**ot **bal**loon observation. If the same procedure is used with a sounding balloon (art. 3720), the observation is called a **rabal,** from **ra**dio **bal**loon observation.

3719. The theodolite.—Survey theodolites are discussed in article 4004. The instrument by the same name used for pilot balloon observations is constructed on the same principle, but with some differences to suit the use for which it is intended.

The shore-type theodolite used by the meteorologist is essentially a telescope so mounted that the horizontal and vertical angles of its axis can be measured. The telescope is mounted in a yoke secured to a base plate. The base plate is mounted on a tripod or pipe support, with provision for accurate leveling. By means of a 45° prism, the line of sight is bent through an angle of 90°. The eyepiece is mounted on the horizontal axis of the theodolite. Tangent-screw controls permit adjustment in both the horizontal and vertical directions.

The shipboard-type theodolite (fig. 3719) differs considerably from the shore type, being mounted on gimbals atop a tripod. A counterbalance is provided to serve

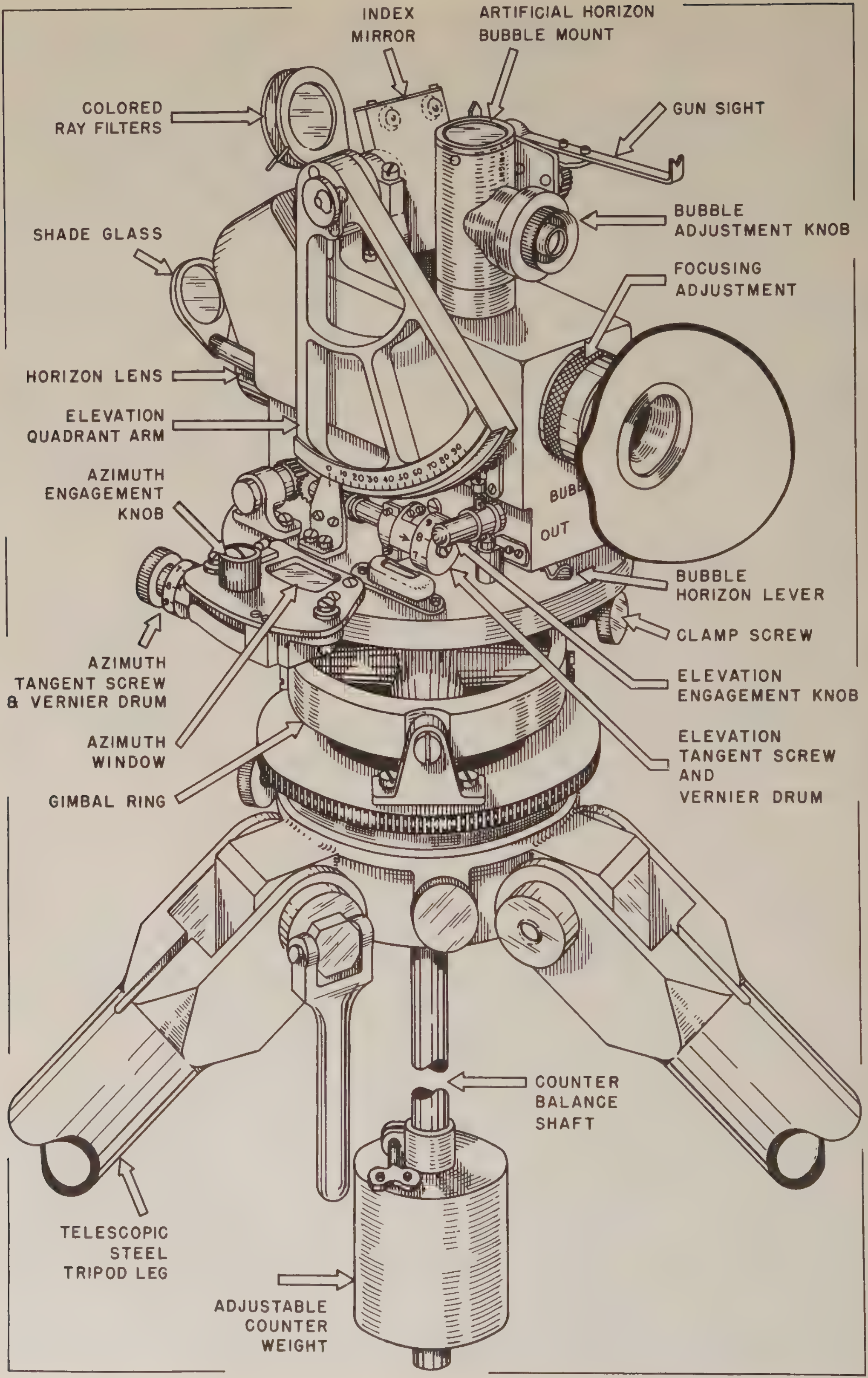


FIGURE 3719.—A shipboard-type meteorological theodolite.

as a pendulum in maintaining the instrument approximately horizontal. The instrument is aligned with the longitudinal axis of the craft, so that *relative* bearings are observed. Elevation angles are measured in a manner similar to the measurement of altitudes of celestial bodies, an image of the balloon being brought into coincidence with the direct view of the horizon. A bubble artificial horizon is also provided.

3720. Radiosondes are miniature radio transmitters carried aloft by **sounding balloons** which ascend at the rate of about 1,000 feet per minute, to a height of nearly 100,000 feet. The transmitter, powered by a compact battery, transmits on a frequency of 72, 403, or 1,680 megacycles per second. In the United States the 72-megacycle instruments have been replaced by 403-megacycle radiosondes.

As the radiosonde ascends, it transmits a continuous-wave radio signal on its assigned frequency. This signal is modulated (art. 1016) by pressure, temperature, and relative humidity in turn.

The transmitted radio signals are received by an antenna and radio receiver at the surface. They are fed through an electronic frequency meter, and then recorded. By this means a continuous record is made to the height at which the balloon bursts or its signals can no longer be received.

An observation made in this way is called a **raob,** from **ra**diosonde **ob**servation.

3721. Electronic measurement of winds aloft.—If either a pilot balloon (art. 3718) or sounding balloon (art. 3720) is fitted with a metal target and tracked by radar, height, slant distance, and bearing are available, permitting determination of wind speed and direction. Radio direction finder equipment which permits measurement of both horizontal and vertical directions has been developed and is in use ashore for tracking radiosondes. Similar equipment for use aboard ship is under development. An observation made by tracking with either radar or radio direction finder is called a **rawin,** from **ra**dio **win**ds-aloft observation. A combined raob (art. 3720) and rawin is called a **rawinsonde.**

3722. Observations by aircraft.—Reports from aircraft are helpful in making upper air observations. By this means, winds, heights of clouds, visibility, etc., can be determined. An aircraft flying over the ocean and equipped with both absolute and barometric altimeters can supply valuable information on the height of the pressure level at which it is flying. Such reports are used in connection with pressure pattern navigation (art. 2807). They are also useful in establishing positions of high and low pressure centers.

The Air Weather Service of the U. S. Department of Defense makes regular flights to collect weather information. These flights are made along established routes over the oceans and in the arctic where adequate coverage is not otherwise available. In addition, the U. S. Navy and U. S. Air Force, in cooperation with the Weather Bureau, make flights into tropical cyclones (ch. XXXIX) to collect useful information.

Prior to the advent of the radiosonde (art. 3720), an instrument known as the **aerograph** or **aerometeorograph** was widely used by most weather services. In effect, this instrument is a combination barograph (art. 3705), thermograph (art. 3712), and hygrograph (art. 3713). The instrument is attached to an aircraft, and during flight it makes a continuous trace of pressure, temperature, and relative humidity on a chart or "smoked sheet" attached to the drum of a clock-driven cylinder. By means of electrical connections to the pens, the pilot of the airplane indicates the time at which he enters and leaves phenomena such as haze, fog, clouds, rain, snow, etc. Since the heights reached are restricted by the ceiling of the aircraft, they are generally less than those attained by radiosondes. The use of the aerograph is now limited principally to storm reconnaissance.

3723. Storm detection radar.—During World War II, it was found that certain radar equipment gave an indication of weather fronts (art. 3812) and precipitation areas. It was of particular value near hurricanes and typhoons. Since the close of that war a great amount of work has been done in perfecting radar equipment for use in weather observation. It has proved of immense value in detecting, tracking, and interpreting weather activity out to a distance of as much as 400 miles from the observing station.

3724. Precipitation measurement.—Any type of condensed water vapor that falls to the earth's surface is called **precipitation.** It may be liquid, freezing, or frozen when it arrives at the surface. Measurement of precipitation normally includes only the determination of the amount of rain or snow that has fallen in a given period of time. For purposes of comparison, snow measurement is obtained by melting the snow to its water equivalent. Depth of snow is also measured to determine the amount of snowfall.

The usual type of **nonrecording precipitation gage** consists of a collector ring, funnel, and measuring cylinder set within a receiver. All precipitation falling on the area encompassed by the collector ring descends through the funnel into the measuring cylinder, where it is measured directly by means of a rod graduated in tenths of an inch. Since the cross-sectional area of the measuring cylinder is exactly one-tenth that of the collector ring, each 0.1 inch collected is a measure of 0.01 inch of precipitation. When precipitation is in the form of snow, the measuring tube is removed, permitting the snow to collect in the larger receiver. The receiver is placed in a container of warm water until the snow melts. The resulting liquid is then poured into the measuring tube and measured.

The most representative measurement of precipitation from snow is obtained by removing the collector ring and funnel, and using a slat screen to reduce the effect of wind.

One type of **recording rain gage** is known as the "tipping-bucket rain gage." The rainfall from a funnel-shaped collector is directed into one of two small buckets so arranged that when 0.01 inch of rain is collected, the bucket is forced downward, causing the other bucket to move into the collecting position. When a bucket is in the "down" position, its water runs into the base of the collector, where it can be measured later. As each bucket lowers in its turn, it causes a small cam to rotate into contact position and close a battery-powered electric circuit. This causes a magnetic relay at the recorder to operate a pen arm, which marks the additional 0.01 inch of rainfall on a chart secured to a clock-driven drum.

Another type of recording rain gage, used principally at locations which are not continuously attended, employs a weighing device which actuates a pen arm, causing it to trace measurements on a chart secured to a clock-driven drum.

The precipitation gage, whatever its form, should be placed in an exposed position as far as practicable from obstructions. Precipitation measurement is not ordinarily made aboard ship because the motions of the vessel, and the possibility of collecting salt spray, introduce errors into the measurement.

3725. Automatic weather stations provide regularly scheduled transmissions of meteorological measurements by radio. They are used at isolated and relatively inaccessible locations from which weather data are of great importance to the weather forecaster. The measurements usually obtained are of wind speed and direction, atmospheric pressure, temperature, and relative humidity.

3726. Recording observations.—Aboard ship, weather observations are recorded on the *Ship Weather Observation Sheet* (fig. 3726). Instructions for using this sheet are given in OPNAV Instruction 3140.37C, *Manual for Ship's Surface Weather Observations.*

OPNAV FORM 3144-1 (9-64)

DEPARTMENT OF THE NAVY

SHIP WEATHER OBSERVATION SHEET

USS ______________________ DATE (GMT) ______________ 19 ______

AT/PASSAGE FROM ______________________ TO ______________________

TABLE I

TIME (GMT)	WINDS ☐ √ IF ESTIMATED Direction (True)	Force (Knots)	VISI-BIL-ITY (Miles)	WEATHER (Symbols)	BAROMETER (Inches)	TEMPERATURE (Degrees and tenths) Dry Bulb	Wet Bulb	CLOUDS Amount (Tenths)	Height	Type	SEA WATER TEMP. (Degrees and tenths)	SEA WAVES Direction (True)	Period (Seconds)	Height (Feet)	SWELL WAVES Direction (True)	Period (Seconds)	Height (Feet)
00																	
01																	
02																	
03																	
04																	
05																	
06																	
07																	
08																	
09																	
10																	
11																	
12																	
13																	
14																	
15																	
16																	
17																	
18																	
19																	
20																	
21																	
22																	
23																	

TABLE II
SYNOPTIC OBSERVATIONS

FIRST GROUP OF MESSAGE	Day of Week (1-7) (GMT)	POSITION OF SHIP Octant (0-3) (5-8)	Latitude (Degrees and tenths)	Longitude (Degrees and tenths)	TIME (GMT)	Total Cloud Amt. (Coded)	WIND Direction (True) (00-36)	Speed (True) (Knots)	Visibility (90-99)	WEATHER Present (00-99)	Past (0-9)	PRESSURE Barometer Corrected (Mb)	AIR TEMP. (°C)	CLOUDS Amount of Low Cloud	Type of C_L (0-9)	Height of Low Cloud	Type of C_M (0-9)	Type of C_H (0-9)	Course of Ship (0-9)	Speed of Ship (0-9)	3-HOUR PRESSURE TENDENCY Characteristic (0-8)	Amount of Change (Mb and tenths)	SIGNIFICANT CLOUD Indicator	Amount (Eights)	Type	Height
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
	Y	Q	$L_a L_a L_a$	$L_o L_o L_o$	GG	N	dd	ff	VV	ww	W	ppp	TT	N_h	C_L	h	C_M	C_H	D_s	V_s	a	pp	8	N_s	C	$h_s h_s$
SHIP					00																		8			
SHIP					06																		8			
SHIP					12																		8			
SHIP					18																		8			

Indicator	AIR-SEA DIFF. (Coded)	DEW POINT (°C)	SEA WAVES Indicator	Direction (Coded)	Period (Coded)	Height (Coded)	SWELL WAVES Indicator	Direction (Coded)	Period (Coded)	Height (Coded)	ICE ACCRETION Indicator	Source	Thickness	Rate	SEA ICE Indicator	Kind	Effect	Bearing	Distance	Orientation
28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
0	$T_s T_s$	$T_d T_d$	1	$d_w d_w$	P_w	H_w	1	$d_w d_w$	P_w	H_w	2	I_s	$E_s E_s$	R_s	ICE	C_2	K	D_i	r	e
0			1				1				2				ICE					
0			1				1				2				ICE					
0			1				1				2				ICE					
0			1				1				2				ICE					

DO NOT TRANSMIT Dry Bulb (Degrees and tenths)	Wet Bulb (Degrees and tenths)	Sea Water Temp. (Degrees and tenths)
A_1	A_2	A_3
Celsius	Celsius	Celsius

REMARKS ______________________ EXAMINED ______________________ USN, NAVIGATOR

D-11814

FIGURE 3726.—Ship Weather Observation Sheet.

To assist in the preparation of synoptic observations for transmission, Table II (which is similar to WB Form 615–5, *Ship's Weather Observations*, for merchant ships which participate in the cooperative observation program of the U.S. Weather Bureau) is arranged in the correct code form with the five-digit groups separated by heavy lines. Two wave groups are given because two separate wave systems are sometimes persent.

The symbols to use in the "weather" column of Table I are as follows:

CLR	Clear or a few clouds
SCT	Scattered clouds—0.1 to 0.5 clouds
BKN	Broken clouds—0.6 to 0.9 clouds
OVC	Overcast—more than 0.9 clouds
T	Thunderstorm
R	Rain
RW	Rain showers
L	Drizzle
ZR	Freezing rain
ZL	Freezing drizzle
E	Sleet
F	Fog
GF	Shallow fog (ground fog)
EW	Sleet showers
S	Snow
SW	Snow showers
IC	Ice crystals
A	Hail
IF	Ice fog
H	Haze
K	Smoke
D	Dust
BY	Blowing spray

Problems

3709a. A ship is proceeding on course 180° at a speed of 22 knots. The apparent wind is from 70° off the port bow, speed 20 knots.

Required.—The relative direction, true direction, and speed of the true wind by maneuvering board or Weather Bureau plotter.

Answers.—True wind from 231° relative, 051° true, at 24.3 knots.

3709b. A ship is proceeding on course 050° at a speed of 13.5 knots. The apparent wind is from broad on the starboard bow, speed 20 knots.

Required.—The relative direction, true direction, and speed of the true wind by table 10.

Answers.—True wind from 086° relative, 136° true, at 14.3 knots.

3709c. A ship is proceeding on course 020° at a speed of 16 knots. The true wind is estimated to be from 110° on the port bow, speed 10 knots.

Required.—The relative direction, true direction, and speed of the apparent wind by maneuvering board or Weather Bureau plotter.

Answers.—Apparent wind from 323° relative, 343° true, at 15.6 knots.

3709d. A ship is proceeding on course 190° at a speed of 14 knots. The true wind is estimated to be from broad on the starboard quarter, speed 20 knots.

Required.—The relative direction, true direction, and speed of the apparent wind by table 10.

Answers.—Apparent wind from 090° relative, 280° true, at 14.0 knots.

3709e. The true wind has been determined to be from 210°, speed 12 knots. The captain of an aircraft carrier desires an apparent wind of 30 knots from 10° on the port bow for launching aircraft.

Required.—The course and speed of the aircraft carrier.

Answers.—C 235°, S 18.6 kn. (The required apparent wind could also be produced by C 005°, S 40.5 kn.)

3709f. A ship is proceeding on course 255° at a speed of 15 knots. The wind vane indicates the apparent wind is broad on the starboard beam. From the appearance of the sea the navigator estimates the speed of the true wind as Beaufort 5 (19 knots).

Required.—(1) Relative and true directions of the true wind, (2) speed of the apparent wind. Use the maneuvering board.

Answers.—(1) True wind from 142° relative, 037° true; (2) apparent wind speed 11.6 knots.

3709g. A ship is proceeding on course 135° at a speed of 18 knots. The wind vane indicates the apparent wind is 40° on the starboard bow. From the appearance of the sea the navigator estimates the speed of the true wind as Beaufort 6 (24.5 knots).

Required.—(1) Relative and true directions of the true wind, (2) speed of the apparent wind. Use table 10.

Answers.—(1) True wind from 069° relative, 204° true; (2) apparent wind speed 36 knots.

3709h. A ship is proceeding on course 330° at a speed of 20 knots. The wind vane indicates the apparent wind is 30° on the port bow. From the appearance of the sea the navigator estimates the speed of the true wind as Beaufort 4 (13.5 knots).

Required.—(1) Relative and true directions of the true wind, (2) speed of the apparent wind. Solve first by maneuvering board and then by table 10.

Answers.—Graphical solution: (1) true wind from 199° relative, 169° true or from 282° relative, 252° true; (2) apparent wind speed 8.5 knots or 26.3 knots. Table 10 solution: (1) true wind from 197° relative, 167° true or from 283° relative, 253° true; (2) apparent wind speed 8.0 knots or 26.0 knots.

3713. The dry-bulb temperature is 41° F and the wet-bulb temperature is 35° F.

Required.—(1) Relative humidity, (2) dew point.

Answers.—(1) Relative humidity 53 percent, (2) dew point 26°.

3715a. A 30-gram balloon with 139 grams of helium is released, and 10^m12^s later it disappears in the clouds.

Required.—Height of the base of the clouds.

Answer.—Height 6,318 feet.

3715b. The dry-bulb temperature is 72° F and the wet-bulb temperature is 58° F.

Required.—Height of the base of cumulonimbus clouds formed in air which has risen from the surface of the sea.

Answer.—Height 5,900 feet.

3715c. An observer 1,000 feet from a ceiling light projector measures the elevation angle of the spot of light on the base of the clouds as 68°.

Required.—Height of the base of the clouds.

Answer.—Height 2,475 feet.

References

Berry, F. A., Bollay, E., and Beers, N. R. *Handbook of Meteorology.* New York, McGraw-Hill, 1945.

Burgess, C. R. *Meteorology for Seamen.* Glasgow, Brown, 1950.

Byers, H. R. *General Meteorology.* 2nd ed. New York, McGraw-Hill, 1944.

Haynes, B. C. *Techniques of Observing the Weather.* New York, Wiley, 1947.

Knight, A. M. *Modern Seamanship.* 12th ed. New York, Van Nostrand, 1953.

Middleton, W. E. K., and Spilhaus, A. F. *Meteorological Instruments.* Toronto, U. of Toronto, 1953.

Smithsonian Institution. *Smithsonian Meteorological Tables.* 6th ed. Washington, Smithsonian Inst., 1951.

U.S. Weather Bureau. *Manual of Barometry.* Vol. 1. 1st ed. Washington, U.S. Govt. Print. Off., 1963

U.S. Weather Bureau. *Manual of Marine Meteorological Observations.* Circular M. 12th ed. Washington, U.S. Govt. Print. Off., 1964.

U.S. Weather Bureau. *Weather Surveillance Radar Manual.* Washington, U.S. Govt. Print. Off., 1960.

CHAPTER XXXVIII

WEATHER AND WEATHER FORECASTS

3801. Introduction.—**Weather** is the state of the earth's atmosphere with respect to temperature, humidity, precipitation, visibility, cloudiness, etc. In contrast, the term **climate** refers to the prevalent or characteristic meteorological conditions of a place or region.

All weather may be traced ultimately to the effect of the sun on the earth, including the lower portions of the atmosphere. Most changes in weather involve large-scale, approximately horizontal, motion of air. Air in such motion is called **wind.** This motion is produced by differences of atmospheric pressure, which are largely attributable to differences of temperature.

The weather is of considerable interest to the mariner. The wind and state of the sea affect dead reckoning. Reduced horizontal visibility limits piloting. The state of the atmosphere affects electronic navigation and radio communication. If the skies are overcast, visual celestial observations are not available; and under certain conditions refraction and dip are disturbed. When wind was the primary motive power, knowledge of the areas of favorable winds was of great importance. This consideration led Matthew Fontaine Maury, more than a century ago, to seek information from ships' logs to establish speed and direction of prevailing winds over the various trade routes of the world. The information thus gathered was shown on pilot charts. By means of these charts, the mariner could select a suitable route for a favorable passage. Even power vessels are affected considerably by wind and sea. Less fuel consumption and a more comfortable passage are to be expected if wind and sea are moderate and favorable. Pilot charts are useful in selecting suitable routes. Since longer range forecasts have become possible, some experimental work has been done in routing ocean vessels to take advantage of anticipated conditions during passage.

3802. The atmosphere is a relatively thin shell of air, water vapor, dust, smoke, etc., surrounding the earth. The air is a mixture of transparent gases (art. 1410) and, like any gas, is elastic and highly compressible. Although extremely light, it has a definite weight which can be measured. A cubic foot of air at standard sea-level temperature and pressure weighs 1.22 ounces, or about 1/817th part of the weight of an equal volume of water. Because of this weight, the atmosphere exerts a pressure upon the surface of the earth, amounting to about 15 pounds per square inch.

As altitude increases, less atmosphere extends upward, and pressure decreases. With less pressure, the density decreases. More than three-fourths of the air is concentrated within a layer averaging about seven statute miles thick, called the **troposphere.** This is the region of most "weather," as the term is commonly understood.

The top of the troposphere is marked by a thin transition zone called the **tropopause.** Beyond this lie several other layers having distinctive characteristics, as listed in article 1410, and shown in figure 1410. The average height of the tropopause ranges from about five miles or less over the poles to about 11 miles over the equator.

The **standard atmosphere** is a conventional vertical structure of the atmosphere characterized by standard sea level pressure of 29.92 inches of mercury (1013.25 millibars), sea level temperature of 59°F (15°C), and a uniform decrease of temperature

and moisture content of the air with height, the rate of temperature decrease being 3°6 F (2°C) per thousand feet to 11 kilometers (36,089 feet) and thereafter a constant temperature of (−)69°7 F (−56°5 C). The rate of temperature decrease with height in the standard atmosphere is called the **standard temperature lapse rate.**

Meteorologists are continually learning more of the characteristics of atmospheric processes above the lowest portions of the atmosphere. In recent years, greatly increased attention has been directed to such features as the **jet stream,** a meandering stream of air which circles the globe at speeds of 100 to more than 250 knots at heights of about 20,000 to 40,000 feet. Some similarity has been noted between major wind streams such as the jet stream, and ocean currents such as the Gulf Stream (art. 3206).

3803. Wind.—When air is not confined, changes in temperature produce changes in volume, heated air expanding and cooled air contracting. If a large volume of air near the surface of the earth is cooled, it contracts, causing a downdraft. Air from neighboring regions aloft moves horizontally to fill the void. This results in a greater mass of air over the region, and the pressure is correspondingly increased. By a similar process in reverse, heating of air near the surface causes expansion and an updraft, resulting in decreased pressure over the heated area. Near the surface of the earth, the air tends to move from an area of high pressure to one of low pressure. Thus, a circulation is set up, air moving across the surface of the earth from an area of high pressure and low temperature to one of low pressure and high temperature, then vertically upward, then horizontally at high altitudes from the area of low pressure to that of high pressure, where it moves vertically downward to complete the circuit. The actual circulation is much more complex than this, due to such factors as rotation of the earth and continual changes in temperature and pressure.

If there were no heating and cooling, the temperature at any given altitude remaining everywhere the same, there would be no tendency for the air to move from one place to another. Air would lie sluggish and at rest on the earth's surface. There would be no wind and no variation in weather.

As a result of the position and motion of the earth in relation to the sun, and the physical processes involving radiation and absorption of energy, certain regions of the earth are always warmer than others. For similar reasons, the air over some parts of the earth is seasonally warmer than that over other parts. This general pattern is modified to a varying degree by the local heating and cooling which is continually taking place. Consequently, winds in some areas are relatively steady in both direction and speed, others are seasonal, and this general circulation is continually being modified by local conditions.

3804. General circulation of the atmosphere.—The heat required for warming the air is supplied originally by the sun. As radiant energy from the sun arrives at the earth, about 43 percent is reflected back into space by the atmosphere, about 17 percent is absorbed in the lower portions of the atmosphere, and the remaining 40 percent (approximately) reaches the surface of the earth and much of it is reradiated into space. This earth radiation is in comparatively long waves relative to the short-wave radiation from the sun, since it emanates from a cooler body. Long-wave radiation, being readily absorbed by the water vapor in the air, is primarily responsible for the warmth of the atmosphere near the earth's surface. Thus, the atmosphere acts much like the glass on the roof of a greenhouse. It allows part of the incoming solar radiation to reach the surface of the earth, but is heated by the terrestrial radiation passing outward. Over the entire earth and for long periods of time, the total outgoing energy must be equivalent to the incoming energy (minus any converted to another form and retained), or the temperature of the earth, including its atmosphere, would steadily increase or decrease. In local areas, or over relatively short periods of time,

such a balance is not required, and in fact does not exist, resulting in changes such as those occurring in the different seasons, and in different parts of the day.

As shown in figure 1419b, the more nearly perpendicular the rays of the sun strike the surface of the earth, the more heat energy per unit area is received at that place. Physical measurements show that in the tropics more heat per unit area is received than is radiated away, and that in polar regions the opposite is true. Unless there were some process to transfer heat from the tropics to polar regions, the tropics would be much warmer than they are, and the polar regions would be much colder. The process which brings about the required transfer of heat is the general circulation of the atmosphere.

If the earth had a uniform surface, did not rotate on its axis (but received sunlight equally all around the equator), and did not revolve around the sun (with its axis tilted), a simple circulation would result, as shown in figure 3804a. However, the surface of the earth is far from uniform, being covered with an irregular distribution of land of various heights, and water; the earth rotates about its axis once in approximately 24 hours, so that the portion heated by the sun continually changes; and the axis of rotation is tilted so that as the earth moves along its orbit about the sun, seasonal changes occur in the exposure of specific areas to the sun's rays, resulting in variations in the heat balance of these areas. These factors, coupled with others, result in constantly changing large-scale movements of air. Based upon averages over long periods, however, a general circulation is discernible. Figures 3804b and 3804c give a generalized picture of the world's pressure distribution and wind systems as actually observed. A simplified diagram of the general pattern is shown in figure 3804d.

The rotation of the earth diverts the air from a direct path between high and low pressure areas, the diversion being toward the *right* in the northern hemisphere and toward the *left* in the southern hemisphere. At some distance above the surface of the

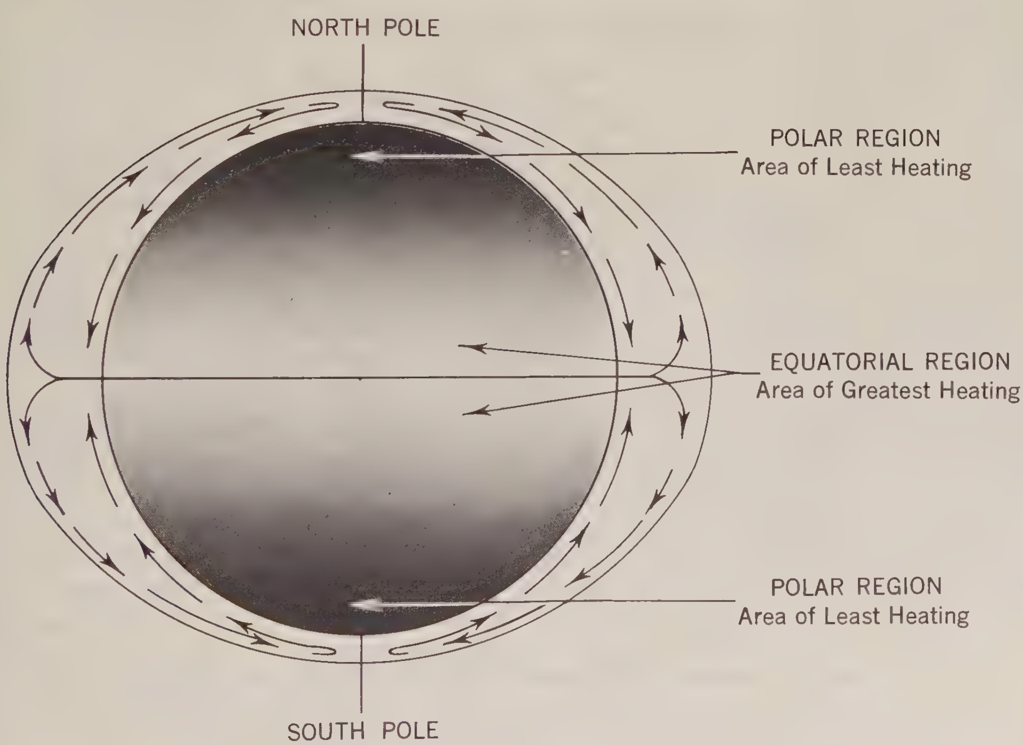


FIGURE 3804a.—Ideal atmospheric circulation for a uniform, nonrotating, nonrevolving earth.

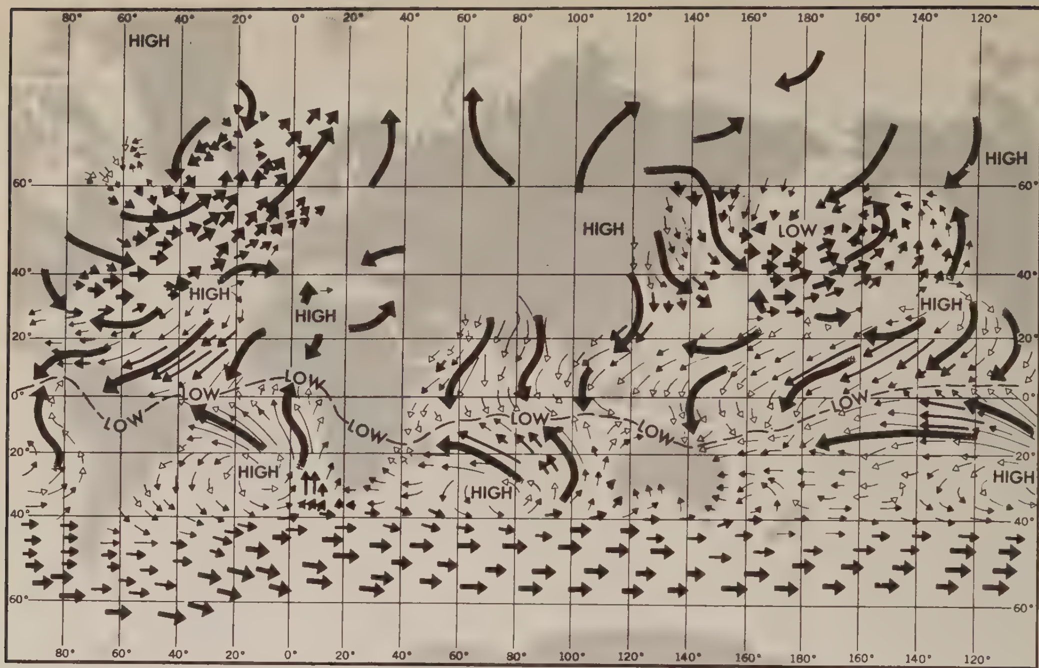


FIGURE 3804b.—Generalized pattern of actual surface winds in January and February.

KEY

PREVAILING WINDS

LENGTH of arrow indicates generalized degree of CONSTANCY OF WIND DIRECTION

WIDTH of arrow indicates average FORCE OF WIND

= 20+ Knots

= 15—20 Knots

= 10—15 Knots

= 10— Knots

= DIRECTION OF MOVEMENT OF AIR MASS

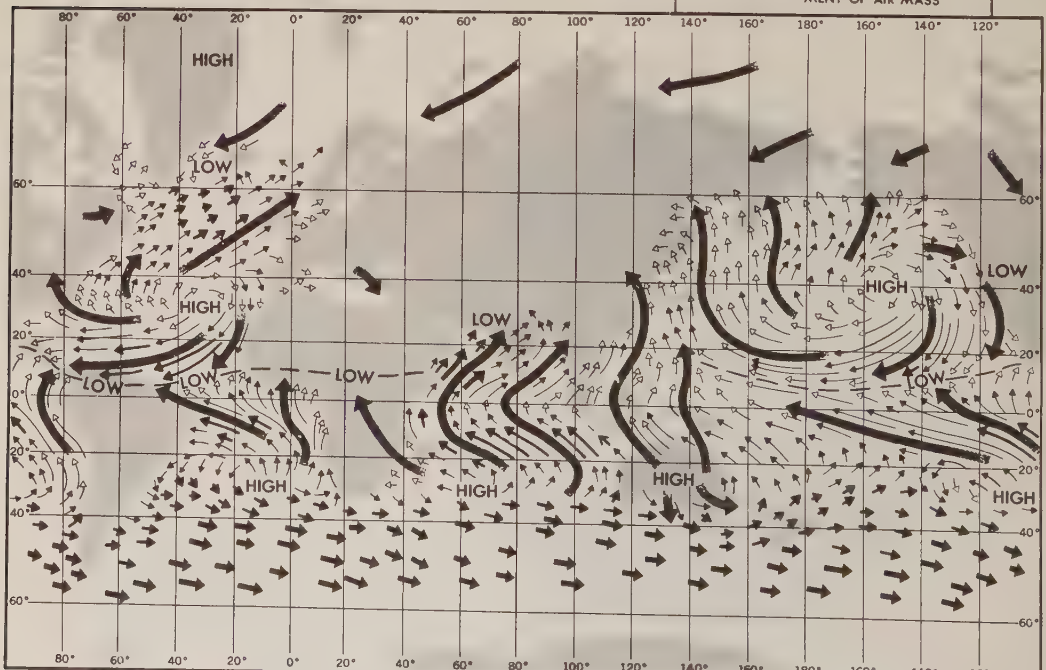


FIGURE 3804c.—Generalized pattern of actual surface winds in July and August.

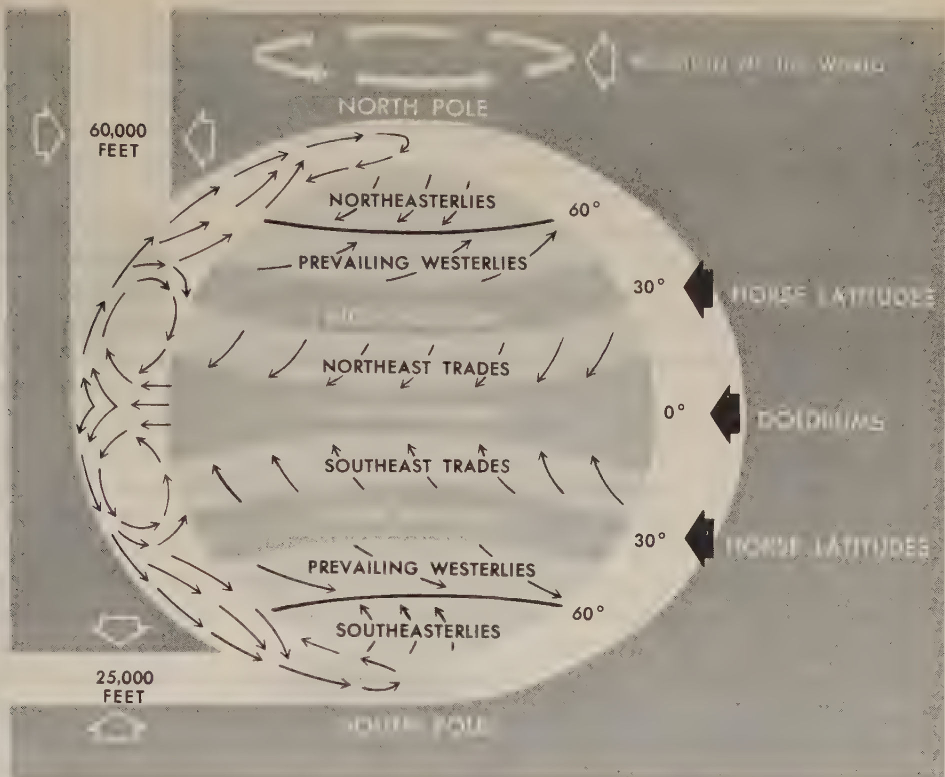


FIGURE 3804d.—Simplified diagram of the general circulation of the atmosphere.

earth, the wind tends to blow along the isobars, being called the **geostrophic wind** if the isobars are straight (great circles), and **gradient wind** if they are curved. Near the surface of the earth, friction tends to divert the wind from the isobars toward the center of low pressure. At sea, where friction is less than on land, the wind follows the isobars more closely.

The decrease of pressure with distance is called the **pressure gradient.** It is maximum along a normal (perpendicular) to the isobars, decreasing to zero along the isobars. Speed of the wind is directly proportional to the maximum pressure gradient.

3805. The doldrums.—The belt of low pressure near the equator occupies a position approximately midway between high pressure belts at about latitude 30° to 35° on each side. Except for slight diurnal changes, the atmospheric pressure along the equatorial low is almost uniform. With almost no pressure gradient, wind is practically nonexistent. The light breezes that do blow are variable in direction. Hot, sultry days are common. The sky is often overcast, and showers and thundershowers are relatively frequent.

The area involved is a thin belt near the equator, the eastern part in both the Atlantic and Pacific being wider than the western part. However, both the position and extent of the belt vary somewhat with the season. During February and March it lies immediately to the north of the equator and is so narrow that it may be considered virtually nonexistent. In July and August the belt is centered on about latitude 7° N, and is several degrees in width, even at the narrowest point.

3806. The trade winds blow from the belts of high pressure, toward the equatorial belt of low pressure. Because of the rotation of the earth, the moving air is deflected

toward the west. Therefore, the trade winds in the northern hemisphere are from the northeast and are called the **northeast trades,** while those in the southern hemisphere are from the southeast and are called the **southeast trades.** Over the eastern part of both the Atlantic and Pacific these winds extend considerably farther from the equator, and their original direction is more nearly along the meridians, than in the western part of each ocean.

The trade winds are generally considered among the most constant of winds. Although they sometimes blow for days or even weeks with little change of direction or speed, their constancy is sometimes exaggerated. At times they weaken or shift direction, and there are regions where the general pattern is disrupted. A notable example is the island groups of the South Pacific, where they are practically nonexistent during January and February. Their highest development is attained in the South Atlantic and in the South Indian Ocean. Everywhere they are fresher during the winter than during the summer.

In July and August, when the belt of equatorial low pressure moves to a position some distance north of the equator, the southeast trades blow across the equator, into the northern hemisphere, where the earth's rotation diverts them toward the right, causing them to be southerly and southwesterly winds. The "southwest monsoons" of the African and Central American coasts have their origin partly in such diverted southeast trades.

Cyclonic storms generally do not enter the regions of the trade winds, although hurricanes and typhoons (ch. XXXIX) may originate within these areas.

3807. The horse latitudes.—Along the poleward side of each trade-wind belt, and corresponding approximately with the belt of high pressure in each hemisphere, is another region with weak pressure gradients and correspondingly light, variable winds. These are called the **horse latitudes.** The weather is generally clear and fresh, unlike that in the doldrums, and periods of stagnation are less persistent, being of a more intermittent nature. The difference is due primarily to the fact that rising currents of warm air in the equatorial low carry large amounts of moisture which condenses as the air cools at higher levels, while in the horse latitudes the air is apparently descending and becoming less humid as it is warmed at lower heights.

3808. The prevailing westerlies.—On the poleward side of the high pressure belt in each hemisphere the atmospheric pressure again diminishes. The currents of air set in motion along these gradients toward the poles are diverted by the earth's rotation toward the east, becoming southwesterly winds in the northern hemisphere and northwesterly in the southern hemisphere. These two wind systems are known as the **prevailing westerlies** of the temperate zones.

In the northern hemisphere this relatively simple pattern is distorted considerably by secondary wind circulations, due primarily to the presence of large land masses. In the North Atlantic, between latitudes 40° and 50°, winds blow from some direction between south and northwest during 74 percent of the time, being somewhat more persistent in winter than in summer. They are stronger in winter, too, averaging about 25 knots (Beaufort 6) as compared with 14 knots (Beaufort 4) in the summer.

In the southern hemisphere the westerlies blow throughout the year with a steadiness approaching that of the trade winds (art. 3806). The speed, though variable, is generally between 17 and 27 knots (Beaufort 5 and 6). Latitudes 40°S to 50°S (or 55°S) where these boisterous winds occur, are called the **roaring forties.** These winds are strongest at about latitude 50°S.

The greater speed and persistence of the westerlies in the southern hemisphere are due to the difference in the atmospheric pressure pattern, and its variations, from that of the northern hemisphere. In the comparatively landless southern hemisphere,

the average yearly atmospheric pressure diminishes much more rapidly on the poleward side of the high pressure belt, and has fewer irregularities due to continental interference, than in the northern hemisphere.

3809. Winds of polar regions.—Because of the low temperatures near the geographical poles of the earth, the pressure tends to remain higher than in surrounding regions. Consequently, the winds blow outward from the poles, and are deflected westward by the rotation of the earth, to become **northeasterlies** in the arctic, and **southeasterlies** in the antarctic. Where these meet the prevailing westerlies, the winds are variable.

In the arctic, the general circulation is greatly modified by surrounding land masses. Winds over the Arctic Ocean are somewhat variable, and strong surface winds are rarely encountered.

In the antarctic, on the other hand, a high central land mass is surrounded by water, a condition which augments, rather than diminishes, the general circulation. The high pressure, although weaker than in some areas, is stronger than in the arctic, and of great persistence near the south pole. The upper air descends over the high continent, where it becomes intensely cold. As it moves outward and downward toward the sea, it is deflected toward the west by the earth's rotation. The winds remain strong throughout the year, frequently attaining hurricane force, and sometimes reaching speeds of 100 to 200 knots at the surface. These are the strongest surface winds encountered anywhere in the world, with the possible exception of those in well-developed tropical cyclones (ch. XXXIX).

3810. Modifications of the general circulation.—The general circulation of the atmosphere as described in articles 3804–3809 is greatly modified by various conditions.

The high pressure in the horse latitudes is not uniformly distributed around the belts, but tends to be accentuated at several points, as shown in figures 3804b and 3804c. These **semipermanent highs** remain at about the same places with great persistence.

Semipermanent lows also occur in various places, the most prominent ones being west of Iceland, and over the Aleutians (winter only) in the northern hemisphere, and at the Ross Sea and Weddell Sea in the antarctic. The areas occupied by these semipermanent lows are sometimes called the graveyards of the lows, since many lows move directly into these areas and lose their identity as they merge with and reinforce the semipermanent lows. The low pressure in these areas is maintained largely by the migratory lows which stall there, but partly by the sharp temperature difference between polar regions and warmer ocean areas.

Another modifying influence is land, which undergoes greater temperature changes than does the sea. During the summer, a continent is warmer than its adjacent oceans. Therefore, low pressures tend to prevail over the land. If a belt of high pressure encounters such a continent, its pattern is distorted or interrupted. A belt of low pressure is intensified. The winds associated with belts of high and low pressure are distorted accordingly. In winter, the opposite effect takes place, belts of high pressure being intensified over land and those of low pressure being interrupted.

The most striking example of a wind system produced by the alternate heating and cooling of a land mass is the **monsoons** of the China Sea and Indian Ocean. A portion of this effect is shown in figures 3810a and 3810b. In the summer (fig. 3810a), low pressure prevails over the warm continent of Asia, and high pressure over the adjacent sea. Between these two systems the wind blows in a nearly steady direction. The lower portion of the pattern is in the southern hemisphere, extending to about 10° south latitude. Here the rotation of the earth causes a deflection to the left, resulting in southeasterly winds. As they cross the equator, the deflection is in the opposite direction, causing them to curve toward the right, becoming southwesterly

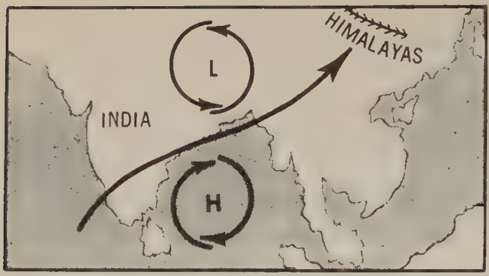


FIGURE 3810a.—The summer monsoon.

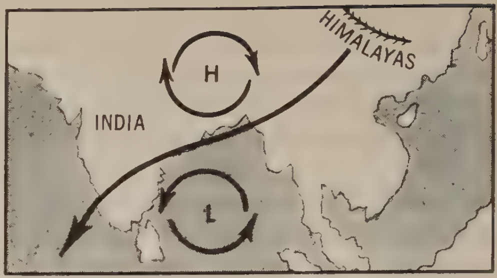


FIGURE 3810b.—The winter monsoon.

winds. In the winter (fig. 3810b), the positions of high and low pressure areas are interchanged, and the direction of flow is reversed.

In the China Sea the summer monsoon blows from the southwest, usually from May to September. The strong winds are accompanied by heavy squalls and thunderstorms, the rainfall being much heavier than during the winter monsoon. As the season advances, squalls and rain become less frequent. In some places the wind becomes a light breeze which is unsteady in direction, or stops altogether, while in other places it continues almost undiminished, with changes in direction or calms being infrequent. The winter monsoon blows from the northeast, usually from October to April. It blows with a steadiness similar to that of the trade winds, often attaining the speed of a moderate gale (28–33 knots). Skies are generally clear during this season, and there is relatively little rain.

The general circulation is further modified by winds of cyclonic origin (art. 3813), and various local winds (art. 3814).

3811. Air masses.—Because of large differences in physical characteristics of the earth's surface, particularly the oceanic and continental contrasts, the air overlying these surfaces acquires differing values of temperature, moisture, etc. The processes of radiation and convection in the lower portions of the troposphere act in differing, characteristic manners for a number of well-defined regions of the earth. The air overlying these regions acquires characteristics common to the particular area, but contrasting to those of other areas. Each distinctive part of the atmosphere, within which common characteristics prevail over a reasonably large area, is called an **air mass.**

Air masses are named according to their source regions. Four such regions are generally recognized: (1) *equatorial* (E), the doldrum area between the north and south trades; (2) *tropical* (T), the trade wind and lower temperate regions; (3) *polar* (P), the higher temperate latitudes; and (4) *arctic* (A), the north polar region of ice and snow (or, by extension, the antarctic). This classification is a general indication of relative temperature, as well as latitude of origin.

Tropical and polar air masses are further classified as *maritime* (m) or *continental* (c), depending upon whether they form over water or land. This classification is an indication of the relative moisture content of the air mass. Since the moisture content of equatorial and arctic air is essentially independent of the surface over which they form, these sub-classifications are not applied to them. Tropical air, then, might be designated *maritime tropical* (mT) or *continental tropical* (cT). Similarly, polar air may be either *maritime polar* (mP) or *continental polar* (cP).

A third classification sometimes applied to tropical and polar air masses indicates whether the air mass is *warm* (w) or *cold* (k) relative to the underlying surface. Thus, the symbol mTw indicates maritime tropical air which is warmer than the under-

lying surface, and *cPk* indicates continental polar air which is colder than the underlying surface. The *w* and *k* classifications are primarily indications of stability. If the air is cold relative to the surface, the lower portion of the air mass is being heated, resulting in instability as the warmer air tends to rise by convection. Conversely, if the air is warm relative to the surface, the lower portion of the air mass is cooled, tending to remain close to the surface. This is a stable condition.

Two other types of air masses are sometimes recognized. These are *monsoon* (*M*), a transitional form between *cP* and *E*; and *superior* (*S*), a special type formed in the free atmosphere by the sinking and consequent warming of air aloft.

3812. Fronts.—As air masses move within the general circulation, they travel from their source regions and invade other areas dominated by air having different characteristics. There is little tendency for adjacent air masses to mix. Instead, they are separated by a thin zone in which air mass characteristics exhibit such sharp gradients as to appear as discontinuities. This is called a **frontal surface.** The intersection of a frontal surface and a horizontal plane is called a **front,** although the term "front" is commonly used as a short expression for "frontal surface" when this will not introduce an ambiguity.

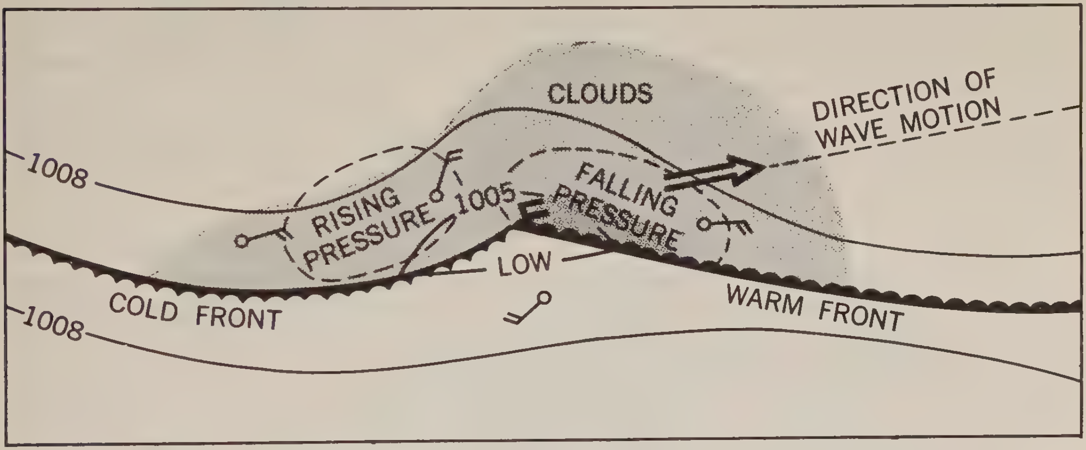


FIGURE 3812a.—First stage in the development of a frontal wave (top view).

Because of differences in the motion of adjacent air masses, "waves" form along the frontal surface between them.

Before the formation of frontal waves, the isobars (lines of equal atmospheric pressure) tend to run parallel to the fronts. As a wave is formed, the pattern is distorted somewhat, as shown in figure 3812a. In this illustration, colder air is north of warmer air. Isobars are shown at intervals of three millibars. The wave tends to travel in the direction of the general circulation, which in the temperate latitudes is usually in a general easterly and slightly poleward direction.

Along the leading edge of the wave, warmer air is replacing colder air. This is called the **warm front.** The trailing edge is the **cold front,** where colder air is replacing warmer air.

The warm air, being less dense, tends to ride up over the colder air it is replacing, causing the warm front to be tilted in the direction of motion. The slope is gentle, varying between 1:100 and 1:300. Because of the replacement of cold, dense air with warm, light air, the pressure decreases. Since the slope is gentle, the upper part of a warm frontal surface may be many hundreds of miles ahead of the surface portion. The decreasing pressure, indicated by a "falling barometer," is often an indication of

the approach of such a wave. In a slow-moving, well-developed wave, the barometer may begin to fall several *days* before the wave arrives. Thus, the amount and nature of the change of atmospheric pressure between observations, called **pressure tendency,** is of assistance in predicting the approach of such a system.

The advancing cold air, being more dense, tends to cut under the warmer air at the cold front, lifting it to greater heights. The slope here is in the opposite direction, at a rate of about 1:25 to 1:100, being steeper than the warm front. Therefore, after a cold front has passed, the pressure increases—a "rising barometer."

In the first stages, these effects are not marked, but as the wave continues to grow, they become more pronounced, as shown in figure 3812b. As the amplitude of the wave increases, pressure near the center usually decreases, and the "low" is said to "deepen." As it deepens, its forward speed generally decreases.

The approach of a well-developed warm front is usually heralded not only by falling pressure, but also by a more-or-less regular sequence of clouds. First, cirrus appear. These give way successively to cirrostratus, altostratus, altocumulus, and nimbostratus. Brief showers may precede the steady rain accompanying the nimbostratus.

As the warm front passes, the temperature rises, the wind shifts to the right (in the northern hemisphere), and the steady rain stops. Drizzle may fall from low-lying stratus clouds, or there may be fog for some time after the wind shift. During passage of the **warm sector** between the warm front and the cold front, there is little change in temperature or pressure. However, if the wave is still growing and the low deepening, the pressure might slowly decrease. In the warm sector the skies are generally clear or partly cloudy, with cumulus or stratocumulus clouds most frequent. The warm air is usually moist, and haze or fog may often be present.

As the faster moving, steeper cold front passes, the wind shifts abruptly to the right (in the northern hemisphere), the temperature falls rapidly, and there are often brief and sometimes violent showers, frequently accompanied by thunder and lightning. Clouds are usually of the convective type. A cold front usually coincides with a well-defined **wind-shift line** (a line along which the wind shifts abruptly from southerly or southwesterly to northerly or northwesterly in the northern hemisphere and from northerly or northwesterly to southerly or southwesterly in the southern hemisphere). At sea a series of brief showers accompanied by strong, shifting winds may occur along or some distance (up to 200 miles) ahead of a cold front. These are called **squalls,** and the line along which they occur is called a **squall line.** Because of its greater speed and steeper slope, which may approach or even exceed the vertical near the earth's surface (due to friction), a cold front and its associated weather passes more quickly than a warm front. After a cold front passes, the pressure rises, often quite rapidly, the visibility usually improves, and the clouds tend to diminish.

As the wave progresses and the cold front approaches the slower moving warm front, the low becomes deeper and the warm sector becomes smaller. This is shown in figure 3812c.

Finally, the faster moving cold front overtakes the warm front (fig. 3812d), resulting in an **occluded front** at the surface, and an **upper front** aloft (fig. 3812e). When the two parts of the cold air mass meet, the warmer portion tends to rise above the colder part. The warm air continues to rise until the entire system dissipates. As the warmer air is replaced by colder air, the pressure gradually rises, a process called "filling." This usually occurs within a few days after an occluded front forms, but the process is sometimes delayed by a slowing of the forward motion of the wave. In general, however, a filling low increases in speed.

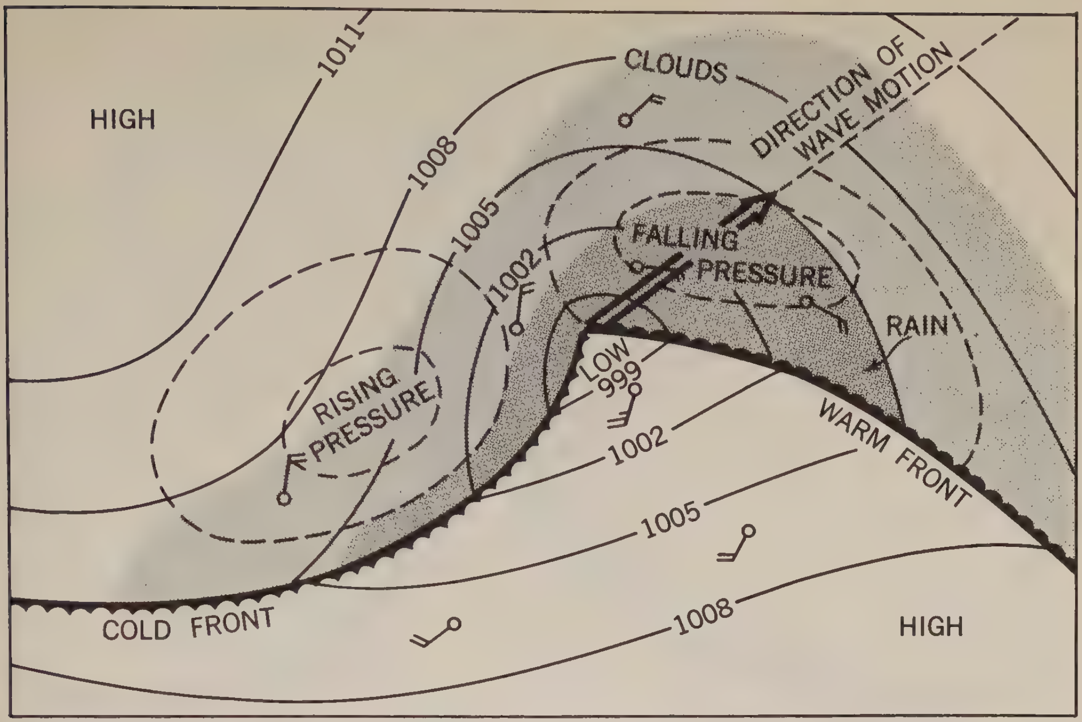


FIGURE 3812b.—A fully developed frontal wave (top view).

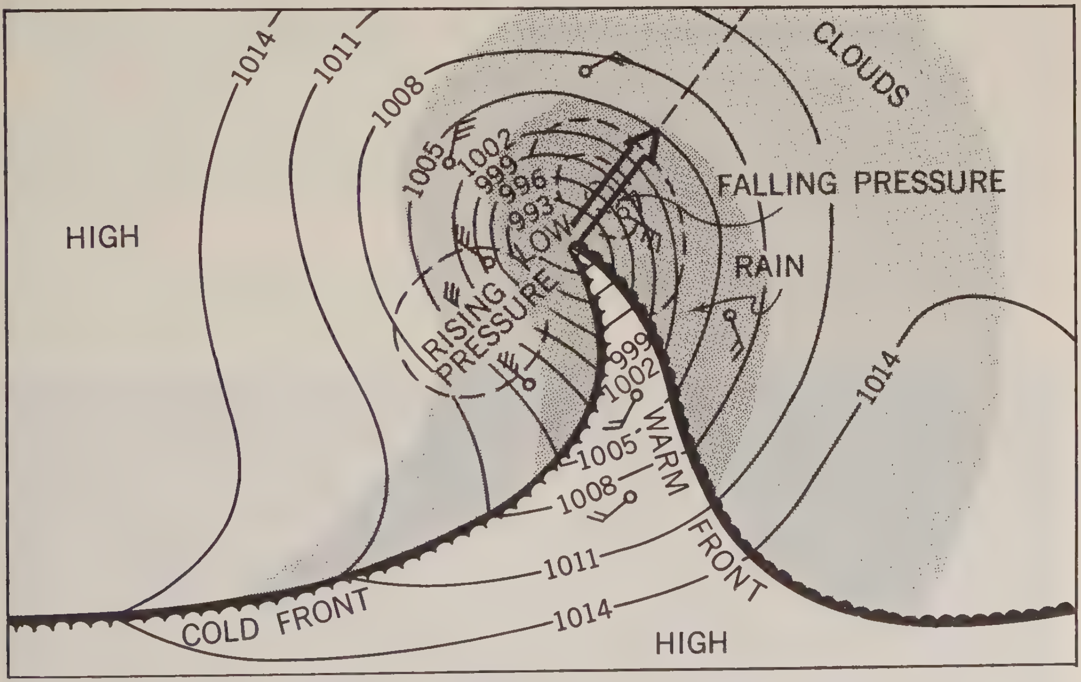


FIGURE 3812c.—A frontal wave nearing occlusion (top view).

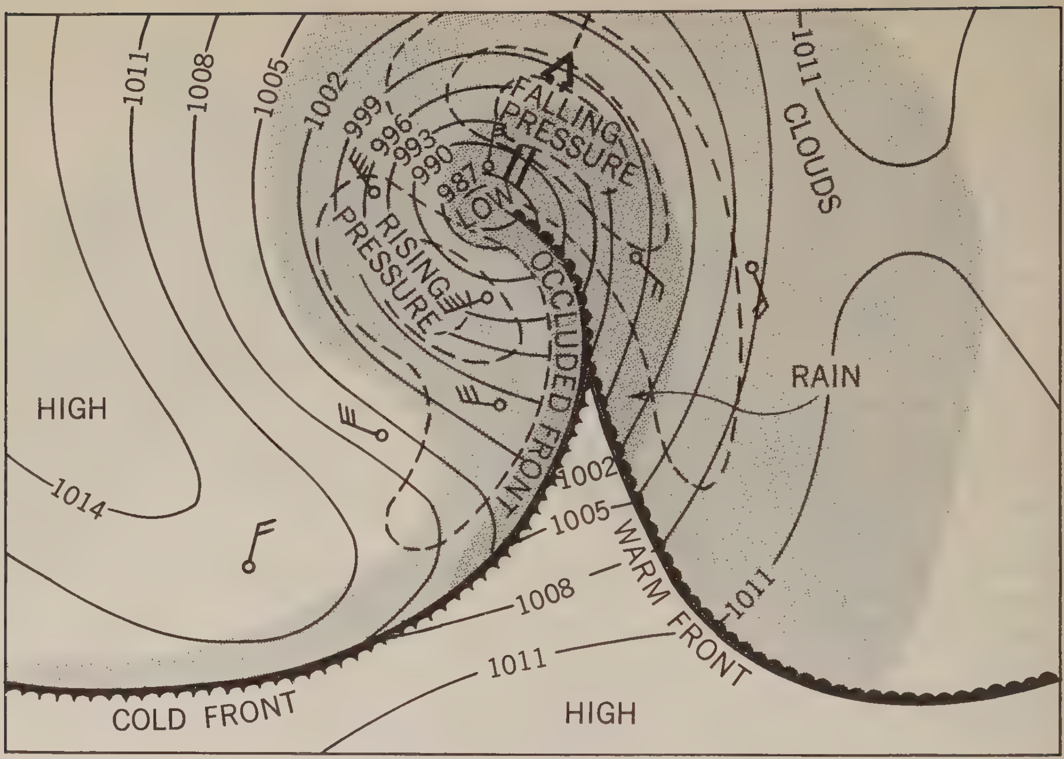


FIGURE 3812d.—An occluded front (top view).

The sequence of weather associated with a low depends greatly upon location with respect to the path of the center. That described above assumes that the observer is so located that he encounters each part of the system. If he is poleward of the path of the center of the low, the abrupt weather changes associated with the passage of fronts are not experienced. Instead, the change from the weather characteristically found ahead of a warm front to that behind a cold front takes place gradually, the exact sequence being dictated somewhat by distance from the center, as well as severity and age of the low.

Although each low follows generally the pattern given above, no two are ever exactly alike. Other centers of low pressure and high pressure and the air masses associated with them, even though they may be 1,000 miles or more away, influence

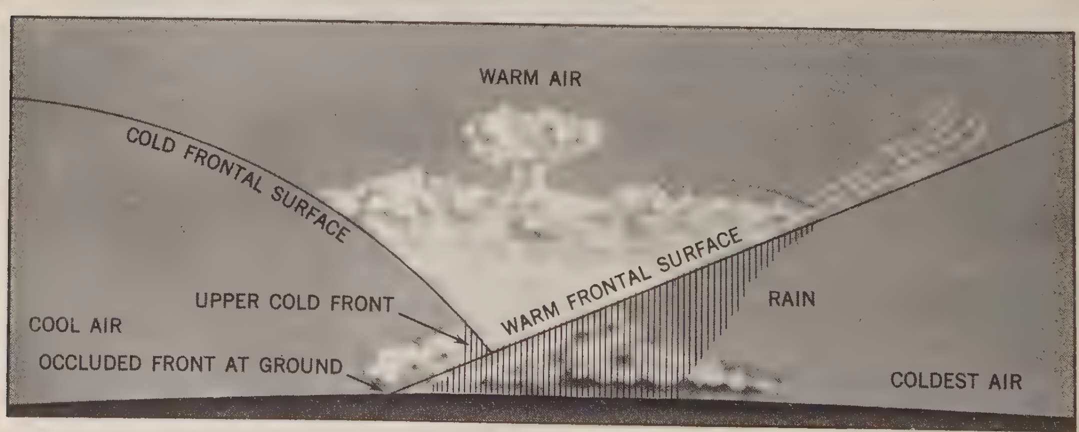


FIGURE 3812e.—An occluded front (cross section).

the formation and motion of individual low centers and their accompanying weather. Particularly, a high stalls or diverts a low. This is true of temporary highs as well as semipermanent highs.

3813. Cyclones and anticyclones.—An approximately circular portion of the atmosphere in the vicinity of a low pressure area is called a **cyclone.** A similar portion in the vicinity of an atmospheric high is called an **anticyclone.** These terms are used particularly in connection with the winds associated with such centers. Wind tends to blow from an area of high pressure to one of low pressure, but due to rotation of the earth, they are deflected toward the right in the northern hemisphere and toward the left in the southern hemisphere (art. 3804).

Because of the rotation of the earth, therefore, the circulation tends to be counterclockwise around areas of low pressure in the northern hemisphere (figs. 3812c and 3812d), and clockwise around areas of high pressure, the speed being proportional to the spacing of isobars. In the southern hemisphere, the direction of circulation is reversed. Based upon this condition, a general rule (**Buys Ballot's Law**) can be stated thus:

If an observer in the northern hemisphere faces the wind, the center of low pressure is toward his right, somewhat behind him; and the center of high pressure is toward his left and somewhat in front of him.

If an observer in the southern hemisphere faces the wind, the center of low pressure is toward his left and somewhat behind him; and the center of high pressure is toward his right and somewhat in front of him.

In a general way, these relationships apply in the case of the general distribution of pressure, as well as to temporary local pressure systems.

The reason for the wind shift along a front is that the isobars have an abrupt change of direction along these lines, as shown in figures 3812a–3812d. Since the direction of the wind is directly related to the direction of isobars, any change in the latter results in a shift in the wind direction.

In the northern hemisphere, the wind shifts toward the *right* when either a warm or cold front passes. In the southern hemisphere, the shift is toward the *left.* When the wind shifts in this direction (clockwise in the northern hemisphere and counterclockwise in the southern hemisphere), it is said to **veer.** If it shifts in the opposite direction, as when an observer is on the poleward side of the path of a frontal wave, it is said to **back.**

In an anticyclone, successive isobars are relatively far apart, resulting in light winds. In a cyclone, the isobars are more closely spaced. With a steeper pressure gradient, the winds are stronger.

Since an anticyclonic area is a region of outflowing winds, air is drawn into it from aloft. Descending air is warmed, and as air becomes warmer, its capacity for holding uncondensed moisture increases. Therefore, clouds tend to dissipate. Clear skies are characteristic of an anticyclone, although scattered clouds and showers are sometimes encountered.

In contrast, a cyclonic area is one of converging winds. The resulting upward movement of air results in cooling, a condition favorable to the formation of clouds and precipitation. More or less continuous rain and generally stormy weather are usually associated with a cyclone.

Between the two belts of high pressure associated with the horse latitudes (art. 3807), cyclones form only occasionally, generally in certain seasons, and always in certain areas at sea. These **tropical cyclones** are usually quite violent, being known under various names according to their location. They are discussed in chapter XXXIX.

In the areas of the prevailing westerlies (art. 3808), cyclones are a common occurrence, the cyclonic and anticyclonic circulation being a prominent feature of temperate latitudes. These are sometimes called **extratropical cyclones** to distinguish them from the more violent tropical cyclones. Although most of them are formed at sea, their formation over land is not unusual. As a general rule, they decrease in intensity when they encounter land, and increase when they move from the land to a water area. In their early stages, cyclones are elongated, as shown in figure 3812a, but as their life cycle proceeds, they become more nearly circular (figs. 3812b–3812d).

3814. Local winds.—In addition to the winds of the general circulation (arts. 3804–3809) and those associated with cyclones and anticyclones (art. 3813), there are numerous local winds which influence the weather in various places.

The most common of these are the **land** and **sea breezes,** caused by alternate heating and cooling of land adjacent to water. The effect is similar to that which causes the monsoons (art. 3810), but on a much smaller scale, and over shorter periods. By day the land is warmer than the water, and by night it is cooler. This effect occurs along many coasts during the summer. Between about 0900 and 1100 the temperature of the land becomes greater than that of the adjacent water. The lower levels of air over the land are warmed, and the air rises, drawing in cooler air from the sea. This is the **sea breeze.** Late in the afternoon, when the sun is low in the sky, the temperature of the two surfaces equalizes and the breeze stops. After sunset, as the land cools below the sea temperature, the air above it is also cooled. The contracting cool air becomes more dense, increasing the pressure. This results in an outflow of winds to the sea. This is the **land breeze,** which blows during the night and dies away near sunrise. Since the atmospheric pressure changes associated with this cycle are not great, the accompanying winds do not exceed gentle breezes. The circulation is generally of limited extent, reaching a distance of perhaps 20 miles inland, and not more than five or six miles offshore, and to a height of a few hundred feet. In the tropics, this process is repeated with great regularity throughout most of the year. As the latitude increases, it becomes less prominent, being masked by winds of cyclonic origin (art. 3813). However, the effect may often be present to reinforce, retard, or deflect stronger prevailing winds.

Varying conditions of topography produce a large variety of local winds throughout the world. In light airs, winds tend to follow valleys, and to be deflected from high banks and shores. Many local winds have been given distinctive names. An **anabatic wind** is one which blows up an incline, as one which blows up a hillside due to surface heating. A **katabatic wind** is one which blows down an incline due to cooling of the air. The cooler air becomes heavier than surrounding air and flows downward along the incline under the force of gravity.

A dry wind with a downward component, warm for the season, is called a **foehn.** The foehn occurs when horizontally moving air encounters a mountain barrier. As it blows upward to clear the barrier, it is cooled below the dew point, resulting in loss of moisture by cloud formation and perhaps rain. As the air continues to rise, its rate of cooling is reduced because the condensing water vapor gives off heat to the surrounding atmosphere. After crossing the mountain barrier, the air flows downward along the leeward slope, being warmed by compression as it descends to lower levels. Thus, since it loses less heat on the ascent than it gains during descent, and since it loses moisture during ascent, it arrives at the bottom of the mountains as very warm, dry air. This accounts for the warm, arid regions along the eastern side of the Rocky Mountains and in similar areas. In the Rocky Mountain region this wind is known by the name **chinook.** It may occur at any season of the year, at any hour of the day or night, and have any speed from a gentle breeze to a gale. It may last for several days, or for a very

short period. Its effect is most marked in winter, when it may cause the temperature to rise as much as 20°F to 30°F within 15 minutes, and cause snow and ice to melt within a few hours. On the west coast of the United States, the name "chinook" is given to a moist southwesterly wind from the Pacific Ocean, warm in winter and cool in summer. Cloudy weather and rain may accompany or follow this wind, which is thus quite different from the other chinook mentioned above. A foehn given the name **Santa Ana** blows through a pass and down a valley by that name in Southern California. This wind usually starts suddenly, without warning, and blows with such force that it may capsize small craft off the coast.

A cold wind blowing down an incline is called a **fall wind.** Although it is warmed somewhat during descent, as is the foehn, it is cold relative to the surrounding air. It occurs when cold air is dammed up in great quantity on the windward side of a mountain and then spills over suddenly, usually as an overwhelming surge down the other side. It is usually quite violent, sometimes reaching hurricane force. A different name for this type wind is given at each place where it is common. The **williwaw** of the Aleutian coast, the **tehuantepecer** of the Mexican and Central American coast, the **pampero** of the Argentine coast, the **mistral** of the western Mediterranean, and the **bora** of the eastern Mediterranean are examples of this type wind.

Many other local winds common to certain areas have been given distinctive names.

A **blizzard** is a violent, intensely cold wind laden with snow mostly or entirely picked up from the ground, although the term is often used popularly to refer to any heavy snowfall accompanied by strong wind. A **dust whirl** is a rotating column of air about 100 to 300 feet in height, carrying dust, leaves, and other light material. This wind, which is similar to a waterspout at sea (art. 3825), is given various local names such as **dust devil** in southwestern United States and **desert devil** in South Africa. A **gust** is a sudden, brief increase in wind speed followed by a slackening, or the violent wind or squall that accompanies a thunderstorm. A puff of wind or a light breeze affecting a small area, such as would cause patches of ripples on the surface of water, is called a **cat's paw.**

3815. Fog, like a cloud (art. 3714), is a visible assemblage of numerous tiny droplets of water, or ice crystals, formed by condensation of water vapor in the air. However, the base of a cloud is above the surface of the earth, while fog is in contact with the surface.

Radiation fog forms over low-lying land on clear, calm nights. As the land radiates heat and becomes cooler, it cools the air immediately above the surface. This causes a **temperature inversion** to form, the temperature for some distance upward *increasing* with height. If the air is cooled to its dew point (art. 3713), fog forms. Often, cooler and more dense air drains down surrounding slopes to heighten the effect. Radiation fog is often quite shallow, and is usually thickest at the surface. After sunrise the fog may "lift," as shown in figure 3815, and gradually dissipate, usually being entirely gone by noon. At sea the temperature of the water undergoes little change between day and night, and so radiation fog is seldom encountered more than ten miles from shore.

Advection fog forms when warm, moist air blows over a colder surface and is cooled below its dew point. This type, most commonly encountered at sea, may be quite thick and often persists over relatively long periods. The maximum density might be at nearly any height. Advection fog is common over cold ocean currents. If the wind is strong enough to thoroughly mix the air, condensation may take place at some distance above the surface of the earth, forming low stratus clouds (art. 3714) rather than fog.

Off the coast of California, winds create an offshore current which displaces the warm surface water, causing an upwelling of colder water. Moist air being transported along the coast in the same wind system is cooled, and advection fog results. In the coastal valleys, fog is sometimes formed when moist air blown inland during the afternoon is cooled by radiation during the night. Both of these are called **California fog** because they are peculiar to California and its coastal valleys.

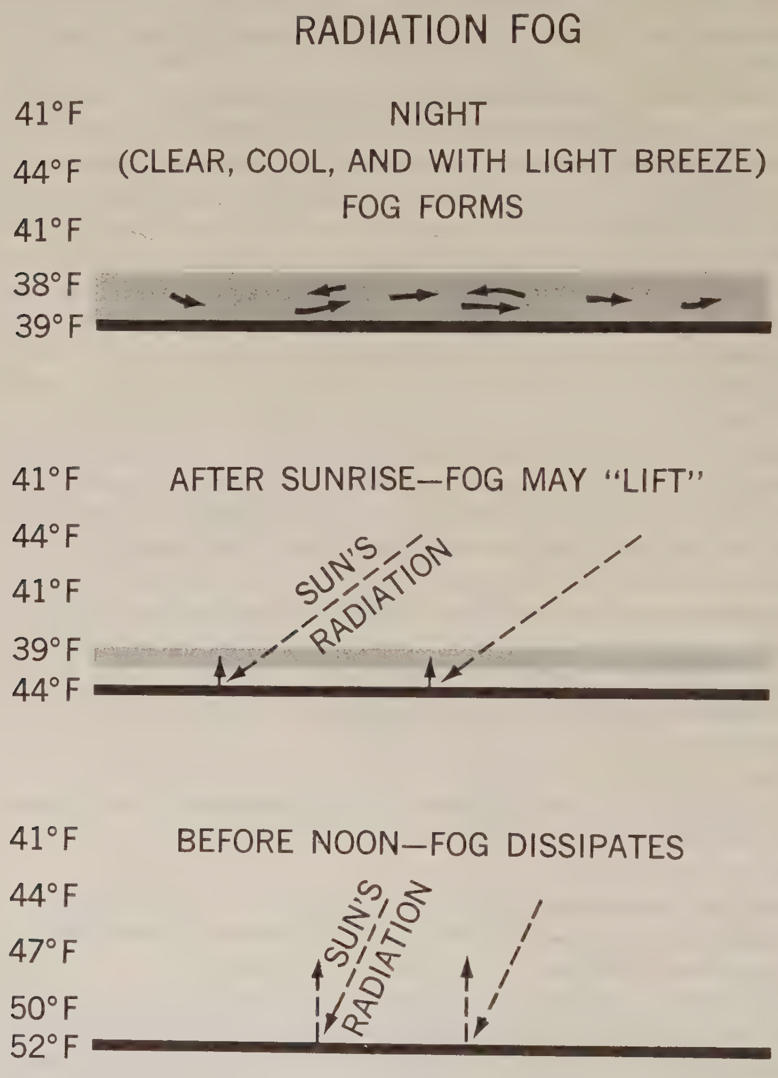


FIGURE 3815.—Formation and dissipation of radiation fog.

When very cold air moves over warmer water, wisps of visible water vapor may rise from the surface as the water "steams," as shown in figure 2505. In extreme cases this **frost smoke,** or **arctic sea smoke,** may rise to a height of several hundred feet, the portion near the surface constituting a dense fog which obscures the horizon and surface objects, but usually leaves the sky relatively clear.

Fog consisting of ice crystals is called **ice fog,** or **pogonip** by Western American Indians. Thin fog of relatively large particles, or very fine rain lighter than drizzle, is called **mist.** A mixture of smoke and fog is called **smog.**

Haze consists of fine dust or salt particles in the air, too small to be individually apparent, but in sufficient number to reduce horizontal visibility and cast a bluish or yellowish veil over the landscape, subduing its colors and making objects appear indistinct. This is sometimes called **dry haze** to distinguish it from **damp haze,** which consists of small water droplets or moist particles in the air, smaller and more scattered than light fog. In international meteorological practice, the term "haze" is used to refer to a condition of atmospheric obscurity caused by dust and smoke.

3816. Mirage.—As explained in article 1613, light is refracted as it passes through the atmosphere. When refraction is normal, objects appear slightly elevated, and the visible horizon is farther from the observer than it otherwise would be. Since the effects are uniformly progressive, they are not apparent to the observer. When refraction is not normal, some form of **mirage** may occur. A mirage is an optical phenomenon in which objects appear distorted, displaced (raised or lowered), magnified, multiplied, or inverted due to varying atmospheric refraction which occurs when a layer of air near the earth's surface differs greatly in density from surrounding air. This may occur when there is a rapid and sometimes irregular change of temperature or humidity with height.

If there is a temperature inversion (increase of temperature with height), particularly if accompanied by a rapid decrease in humidity, the refraction is greater than normal. Objects appear elevated, and the visible horizon is farther away. Objects which are normally below the horizon become visible. This is called **looming.** If the upper portion of an object is raised much more than the bottom part, the object appears taller than usual, an effect called **towering.** If the lower part of an object is raised more than the upper part, the object appears shorter, an effect called **stooping.** When the refraction is greater than normal, a **superior mirage** may occur. An inverted image is seen above the object, and sometimes an erect image appears over the inverted one, with the bases of the two images touching. Greater than normal refraction usually occurs when the water is much colder than the air above it.

If the temperature decrease with height is much greater than normal, refraction is less than normal, or may even cause bending in the opposite direction. Objects appear lower than normal, and the visible horizon is closer to the observer. This is called **sinking.** Towering or stooping may occur if conditions are suitable. When the refraction is reversed, an **inferior mirage** may occur. A ship or an island appears to be floating in the air above a shimmering horizon, possibly with an inverted image beneath it. Conditions suitable to the formation of an inferior mirage occur when the surface is much warmer than the air above it. This usually requires a heated land mass, and therefore is more common near the coast than at sea.

When refraction is not uniformly progressive, objects may appear distorted, taking an almost endless variety of shapes. The sun when near the horizon is one of the objects most noticeably affected. A **fata morgana** is a complex mirage characterized by marked distortion, generally in the vertical. It may cause objects to appear towering, magnified, and at times even multiplied.

3817. Sky coloring.—White light is composed of light of all colors. Color is related to wave length, the visible spectrum varying from about 0.000038 to 0.000076 centimeters (art. 1003). The characteristics of each color are related to its wave length (or frequency). Thus, the shorter the wave length, the greater the amount of bending when light is refracted. It is this principle that permits the separation of light from celestial bodies into a **spectrum** ranging from red, through orange, yellow, green, and blue, to violet, with long-wave infrared (black light) being slightly outside the visible range at one end and short-wave ultraviolet being slightly outside the visible

range at the other end. Light of shorter wave length is scattered and diffracted more than that of longer wave length.

Light from the sun and moon is white, containing all colors. As it enters the earth's atmosphere, a certain amount of it is scattered. The blue and violet, being of shorter wave length than other colors, are scattered most. Most of the violet light is absorbed in the atmosphere. Thus, the scattered blue light is most apparent, and the sky appears blue. At great heights, above most of the atmosphere, it appears black.

When the sun is near the horizon, its light passes through more of the atmosphere than when higher in the sky, resulting in greater scattering and absorption of blue and green light, so that a larger percentage of the red and orange light penetrates to the observer. For this reason the sun and moon appear redder at this time, and when this light falls upon clouds, they appear colored. This accounts for the colors at sunset and sunrise. As the setting sun approaches the horizon, the sunset colors first appear as faint tints of yellow and orange. As the sun continues to set, the colors deepen. Contrasts occur, due principally to difference in height of clouds. As the sun sets, the clouds become a deeper red, first the lower clouds and then the higher ones, and finally they fade to a gray.

When there is a large quantity of smoke, dust, or other material in the sky, unusual effects may be observed. If the material in the atmosphere is of suitable substance and quantity to absorb the longer wave red, orange, and yellow radiations, the sky may have a greenish tint, and even the sun or moon may appear green. If the green light, too, is absorbed, the sun or moon may appear blue. A **green moon** or **blue moon** is most likely to occur when the sun is slightly below the horizon and the longer wave length light from the sun is absorbed, resulting in green or blue light being cast upon the atmosphere in front of the moon. The effect is most apparent if the moon is on the same side of the sky as the sun.

3818. Rainbows.—The familiar arc of concentric colored bands seen when the sun shines on rain, mist, spray, etc., is caused by refraction, internal reflection, and diffraction of sunlight by the drops of water. The center of the arc is a point 180° from the sun, in the direction of a line from the sun, through the observer. The radius of the brightest rainbow is 42°. The colors are visible because of the difference in the amount of refraction of the different colors making up white light, the light being spread out to form a spectrum (art. 3817). Red is on the outer side and blue and violet on the inner side, with orange, yellow, and green between, in that order from red.

Sometimes a secondary rainbow is seen outside the primary one, at a radius of about 50°. The order of colors of this rainbow is reversed. On rare occasions a faint rainbow is seen on the same side as the sun. The radius of this rainbow and the order of colors are the same as those of the primary rainbow.

A similar arc formed by light from the moon (a lunar rainbow) is called a **moonbow.** The colors are usually very faint. A faint, white arc of about 39° radius is occasionally seen in fog opposite the sun. This is called a **fogbow,** although its origin is controversial, some considering it a halo (art. 3819).

3819. Halos.—Refraction, or a combination of refraction and reflection, of light by ice crystals in the atmosphere (cirrostratus clouds, art. 3714) may cause a **halo** to appear. The most common form is a ring of light of radius 22° or 46° with the sun or moon at the center. Occasionally a faint, white circle with a radius of 90° appears around the sun. This is called a **Hevelian halo.** It is probably caused by refraction and internal reflection of the sun's light by bipyramidal ice crystals. A halo formed by refraction is usually faintly colored like a rainbow (art. 3818), with red nearest the celestial body, and blue farthest from it.

A brilliant rainbow-colored arc of about a quarter of a circle with its center at the zenith, and the bottom of the arc about 46° above the sun, is called a **circumzenithal arc.** Red is on the outside of the arc, nearest the sun. It is produced by the refraction and dispersion of the sun's light striking the top of prismatic ice crystals in the atmosphere. It usually lasts for only about five minutes, but may be so brilliant as to be mistaken for an unusually bright rainbow. A similar arc formed 46° *below* the sun, with red on the upper side, is called a **circumhorizontal arc.** Any arc tangent to a heliocentric halo (one surrounding the sun) is called a **tangent arc.** As the sun increases in elevation, such arcs tangent to the halo of 22° gradually bend their ends toward each other. If they meet, the elongated curve enclosing the circular halo is called a **circumscribed halo.** The inner edge is red.

A halo consisting of a faint, white circle through the sun and parallel to the horizon is called a **parhelic circle.** A similar one through the moon is called a **paraselenic circle.** They are produced by reflection of sunlight or moonlight from vertical faces of ice crystals.

A **parhelion** (plural *parhelia*) is a form of halo consisting of an image of the sun at the same altitude and some distance from it, usually 22°, but occasionally 46°. A similar phenomenon occurring at an angular distance of 120° (sometimes 90° or 140°) from the sun is called a **paranthelion.** One at an angular distance of 180°, a rare occurrence, is called an **anthelion,** although this term is also used to refer to a luminous, colored ring or **glory** sometimes seen around the shadow of one's head on a cloud or fog bank. A parhelion is popularly called a **mock sun** or **sun dog.** Similar phenomena in relation to the moon are called **paraselene** (popularly a **mock moon** or **moon dog**), **parantiselene,** and **antiselene.** The term *parhelion* should not be confused with *perihelion,* that orbital point nearest the sun when the sun is the center of attraction (art. 1407).

A **sun pillar** is a glittering shaft of white or reddish light occasionally seen extending above and below the sun, usually when the sun is near the horizon. A phenomenon similar to a sun pillar, but observed in connection with the moon, is called a **moon pillar.** A rare form of halo in which horizontal and vertical shafts of light intersect at the sun is called a **sun cross.** It is probably due to the simultaneous occurrence of a sun pillar and a parhelic circle.

3820. Corona.—When the sun or moon is seen through altostratus clouds (art. 3714), its outline is indistinct, and it appears surrounded by a glow of light called a **corona.** This is somewhat similar in appearance to the corona seen around the sun during a solar eclipse (art. 1424). When the effect is due to clouds, however, the glow may be accompanied by one or more rainbow-colored rings of small radii, with the celestial body at the center. These can be distinguished from a halo by their much smaller radii and also by the fact that the order of the colors is reversed, red being on the inside, nearest the body, in the case of the halo, and on the outside, away from the body, in the case of the corona.

A corona is caused by diffraction of light by tiny droplets of water. The radius of a corona is inversely proportional to the size of the water droplets. A large corona indicates small droplets. If a corona decreases in size, the water droplets are becoming larger and the air more humid. This may be an indication of an approaching rainstorm.

The glow portion of a corona is called an **aureole.**

3821. The green flash.—As light from the sun passes through the atmosphere, it is refracted. Since the amount of bending is slightly different for each color, separate images of the sun are formed in each color of the spectrum. The effect is similar to that

of imperfect color printing in which the various colors are slightly out of register. However, the difference is so slight that the effect is not usually noticeable. At the horizon, where refraction is maximum, the greatest difference, which occurs between violet at one end of the spectrum and red at the other, is about ten seconds of arc. At latitudes of the United States, about 0.7 second of time is needed for the sun to change altitude by this amount when it is near the horizon. The red image, being bent least by refraction, is first to set and last to rise. The shorter wave blue and violet colors are scattered most by the atmosphere, giving it its characteristic blue color (art. 3817). Thus, as the sun sets, the green image may be the last of the colored images to drop out of sight. If the red, orange, and yellow images are below the horizon, and the blue and violet light is scattered and absorbed, the upper rim of the green image is the only part seen, and the sun appears green. This is the **green flash.** The shade of green varies, and occasionally the blue image is seen, either separately or following the green flash (at sunset). On rare occasions the violet image is also seen. These colors may also be seen at sunrise, but in reverse order. They are occasionally seen when the sun disappears behind a cloud or other obstruction.

The phenomenon is not observed at each sunrise or sunset, but under suitable conditions is far more common than generally supposed. Conditions favorable to observation of the green flash are a sharp horizon, clear atmosphere, a temperature inversion (art. 3815), and an attentive observer. Since these conditions are more frequently met when the horizon is formed by the sea than by land, the phenomenon is more common at sea. With a sharp sea horizon and clear atmosphere, an attentive observer may see the green flash at as many as 50 percent of sunsets and sunrises, although a telescope may be needed for some of the observations.

Duration of the green flash (including the time of blue and violet flashes) of as long as ten seconds has been reported, but such length is rare. Usually it lasts for a period of about half a second to two and one-half seconds with about one and a quarter seconds being average. This variability is probably due primarily to changes in the index of refraction (art. 1613) of the air near the horizon.

Under favorable conditions, a momentary green flash has been observed at the setting of Venus and Jupiter. A telescope improves the chances of seeing such a flash from a planet, but is not a necessity.

3822. Crepuscular rays are beams of light from the sun passing through openings in the clouds, and made visible by illumination of dust in the atmosphere along their paths. Actually, the rays are virtually parallel, but because of perspective appear to diverge. Those appearing to extend downward are popularly called **backstays of the sun,** or **sun drawing water.** Those extending upward and across the sky, appearing to converge toward a point 180° from the sun, are called **anticrepuscular rays.**

3823. The atmosphere and radio waves.—Radio waves traveling through the atmosphere exhibit many of the properties of light, being refracted, reflected, diffracted, and scattered. These and other effects are discussed in chapter X.

3824. Atmospheric electricity.—Various conditions induce the formation of electrical charges in the atmosphere. When this occurs, there is often a difference of electron charge between various parts of the atmosphere, and between the atmosphere and earth or terrestrial objects. When this difference exceeds a certain minimum value depending upon the conditions, the static electricity is discharged, resulting in phenomena such as lightning or St. Elmo's fire.

Lightning is the discharge of electricity from one part of a thundercloud (art. 3714) to another, from one such cloud to another, or between such a cloud and the earth or a terrestrial object.

Enormous electrical stresses build up within thunderclouds and between such clouds and the earth. At some point the resistance of the intervening air is overcome. At first the process is a progressive one, probably starting as a brush discharge (St. Elmo's fire) and growing by ionization. The breakdown follows an irregular path along the line of least resistance. A hundred or more individual discharges may be necessary to complete the path between points of opposite polarity. When this "leader stroke" reaches its destination, a heavy "main stroke" immediately follows in the opposite direction. This main stroke is the visible lightning, which may be tinted any color, depending upon the nature of the gases through which it passes. The illumination is due to the high degree of ionization of the air, which causes many of the atoms to be in excited states and emit radiation.

Thunder, the noise that often accompanies lightning, is caused by the heating and ionizing of the air by lightning, which results in rapid expansion of the air along its path and the sending out of a compression wave. Thunder may be heard at a distance of as much as 15 miles, but generally does not carry that far. The elapsed time between the flash of lightning and reception of the accompanying sound of thunder is an indication of the distance, because of the difference in travel time of light and sound. Since the former is comparatively instantaneous, and the speed of sound is about 1,117 feet per second, the approximate distance in nautical miles is equal to the elapsed time in seconds, divided by 5.5. If there is no accompanying thunder, the flash is called **heat lightning.**

St. Elmo's fire is a luminous discharge of electricity from pointed objects such as the masts and yardarms of ships, lightning rods, steeples, mountain tops, blades of grass, human hair, arms, etc., when there is a considerable difference in the electrical charge between the object and the air. It appears most frequently during a storm. An object from which St. Elmo's fire emanates is in danger of being struck by lightning, since this type discharge may be the initial phase of the leader stroke. Throughout history those who have not understood St. Elmo's fire have regarded it with superstitious awe, considering it a supernatural manifestation. This view is reflected in the name **corposant** (from "corpo santo," meaning "body of a saint") sometimes given this phenomenon.

The **aurora** is a luminous glow appearing in varied forms in the thin atmosphere high above the earth, due to radiation from the sun. This phenomenon is discussed in article 2526.

3825. Waterspouts.—A waterspout is a small, whirling storm over the ocean or inland waters. Its chief characteristic is a funnel-shaped cloud extending, in a fully developed spout, from the surface of the water to the base of a cumulus type cloud (fig. 3825). The water in a spout is mostly confined to its lower portion, and may be either salt spray drawn up by the sea surface, or fresh water resulting from condensation due to the lowered pressure in the center of the vortex creating the spout. Waterspouts usually rotate in the same direction as cyclones (counterclockwise in the northern hemisphere and clockwise in the southern hemisphere), but the opposite rotation is occasionally observed. They are found most frequently in tropical regions, but are not uncommon in higher latitudes.

Waterspouts may be divided into two classes, according to their different origins and appearances. In the true waterspout, the vortex is formed in clouds by the interaction of air currents flowing in opposite directions. This type occurs mainly in the vicinity of a squall line (art. 3812). A similar disturbance over land is called a **tornado.** The second type, which may be considered a pseudo waterspout, originates just above the water surface, in unstable air, and builds upward, frequently under clear skies.

FIGURE 3825.—Waterspouts.

This type is identical to the whirling pillars of sand and dust often seen on deserts (art. 3814) and usually occurs only over very warm water surfaces.

Waterspouts vary in diameter from a few feet to several hundred feet, and in height from a few hundred feet to several thousand feet. Sometimes they assume fantastic shapes and may even seem to coil about themselves. Since a waterspout is often inclined to the vertical, its actual length may be much greater than indicated by its height. The highest waterspout on record was one of 5,014 feet observed near New South Wales, Australia, on May 16, 1898.

3826. Deck ice.—Ships traveling through regions where the air temperature is below freezing may acquire thick deposits of ice as a result of salt spray freezing on

the rigging or deck areas (fig. 3826). Also, precipitation may freeze to the superstructure and exposed areas of the vessel, increasing the load of ice.

On small vessels in heavy seas and freezing weather, deck ice may accumulate very rapidly and increase the topside weight to such an extent as to reduce seriously the stability of the vessel.

3827. Forecasting weather.—The prediction of weather at some future time is based upon an understanding of weather processes, and observations of present conditions. Thus, one learns that when there is a certain sequence of cloud types (art. 3714), rain can usually be expected to follow within a certain period. If the sky is cloudless, more heat will be received from the sun by day, and more heat will be radiated outward from the warm earth by night than if the sky is overcast. If the wind is in such a direction that warm, moist air will be transported to a colder surface, fog can be

FIGURE 3826.—Deck ice.

expected. A falling barometer indicates the approach of a "low," probably accompanied by stormy weather. Thus, before the science of meteorology was developed, many individuals learned to interpret certain phenomena in terms of future weather, and to make reasonably accurate forecasts for short periods into the future.

With the establishment of weather observation stations, additional information became available. As such observations expanded, and communication facilities improved, knowledge of simultaneous conditions over wider areas became available. This made possible the collection of these "synoptic" reports at civilian forecast centers and Navy Fleet Weather Centrals.

The individual observations are made at government-operated stations on shore, and aboard vessels at sea. Observations aboard merchant ships at sea are made and transmitted on a voluntary and cooperative basis. The various national meteorological services supply shipmasters with blank forms, printed instructions, and other materials essential to the making, recording, and interpreting of observations. Any shipmaster can render a particularly valuable service by reporting all contacts with tropical cyclones (ch. XXXIX).

Symbols and numbers are used to indicate on a **synoptic chart,** popularly called a **weather map,** the conditions at each observation station. Isobars are drawn through lines of equal atmospheric pressure, fronts are located and marked by symbol (fig. 3827), areas of precipitation and fog are indicated, etc.

Ordinarily, surface charts are prepared every six hours, but at a few centers they are drawn every three hours. In addition, synoptic charts for selected heights are

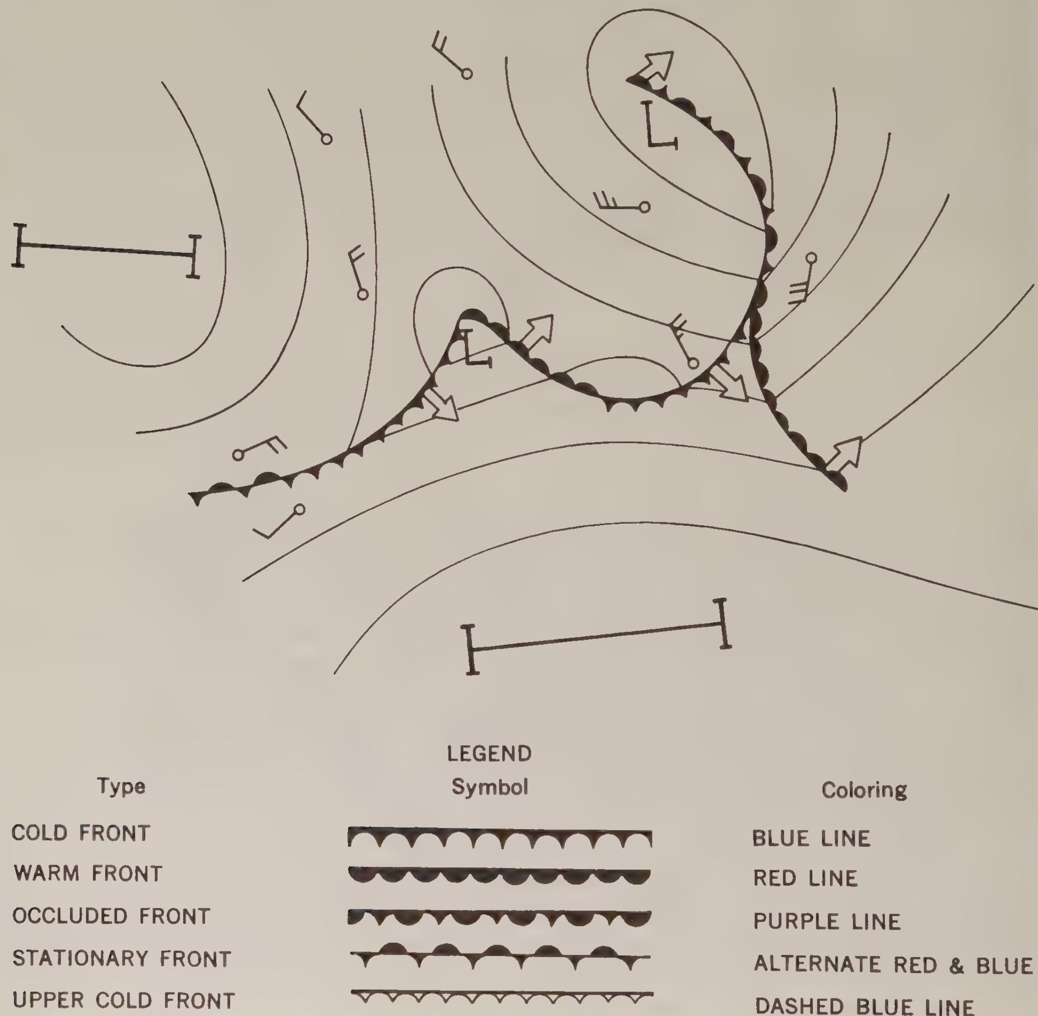


FIGURE 3827.—Designation of fronts on weather maps.

prepared two to four times per day. Knowledge of conditions aloft is of value in establishing the three-dimensional structure of the atmosphere at any time, and the motions upon which forecasts are based.

By studying the latest synoptic weather chart and comparing it with previous charts, a trained meteorologist having a knowledge of local weather peculiarities can draw certain inferences regarding future weather, and issue a forecast. Weather forecasts are essentially a form of extrapolation (art. P6). Past changes and present trends are used to predict future events. In areas where certain sequences follow with great certainty, the probability of an accurate forecast is very high. In transitional areas, or areas where an inadequate number of synoptic reports is available, the fore-

casts are less reliable. Forecasts, then, are based upon the principles of probability (ch. XXIX), and where nature provides low probability, high reliability should not be expected. In any area, the probability of a given event occurring decreases with the lead time. Thus, a forecast for six hours after a synoptic chart is drawn should be more reliable than one for 24 hours ahead. Long-term forecasts for two weeks or a month in advance are limited to general statements. For example, a prediction is made as to which areas will have temperatures above or below normal, and how precipitation will compare with normal, but no attempt is made to state that rainfall will occur at a certain time and place.

Forecasts are issued for various areas. The national meteorological services of most maritime nations, including the United States, issue forecasts for ocean areas and warnings of the approach of storms. The efforts of the various nations are coordinated through the World Meteorological Organization.

3828. Dissemination of weather information is carried out in a number of ways. Forecasts are widely broadcast by commercial and government radio stations, and printed in newspapers. Shipping authorities on land are kept informed by telegraph and telephone. Visual storm warnings are displayed in various ports, and storm warnings are broadcast by radio.

Through the use of codes, a simplified version of synoptic weather charts is transmitted to various stations ashore and afloat. Rapid transmission of completed maps has been made possible by the development of facsimile transmitters and receivers. This system is based upon detailed scanning, by a photoelectric detector, of properly illuminated black and white copy. The varying degrees of light intensity are converted to electric energy which is transmitted to the receiver and converted back to a black and white presentation.

Complete information on dissemination of weather information by radio is given in H.O. Pubs. Nos. 118–A and 118–B, *Radio Weather Aids*. This publication lists broadcast schedules and weather codes. Information on day and night visual storm warnings is given in the various volumes of sailing directions and coast pilots.

3829. Interpreting the weather.—The factors which determine weather are numerous and varied. Ever-increasing knowledge regarding them makes possible a continually improving weather service. However, the ability to forecast is acquired through study and long practice, and therefore the services of a trained meteorologist should be utilized whenever available.

The value of a forecast is increased if one has access to the information upon which it is based, and understands the principles and processes involved. It is sometimes as important to know the various types of weather that *might* be experienced as it is to know which of several possibilities is *most likely* to occur.

At sea, reporting stations are unevenly distributed, sometimes leaving relatively large areas with incomplete reports, or none at all. Under these conditions, the locations of highs, lows, fronts, etc., are imperfectly known, and their very existence may even be in doubt. At such times the mariner who can interpret the observations made from his own vessel may be able to predict weather during the next 24 hours more reliably than a trained meteorologist some distance away with incomplete information.

Knowledge of the various relationships given in chapters XXXVII, XXXVIII, and XXXIX is of value, but only the more elementary principles are presented. Further information can be obtained from meteorological publications such as those listed at the ends of the weather chapters. The information obtained from these references will provide a background for proper interpretation of individual experience. If one uses every opportunity to observe and interpret weather sequences, he can develop knowledge and skill that will serve as a valuable supplement to information

given in weather broadcasts, or to supply information for areas not covered by such broadcasts.

3830. Influencing the weather.—Meteorological activities are devoted primarily to understanding weather processes, and predicting future weather. However, as knowledge regarding cause-and-effect relationships increases, the possibility of being able to induce certain results by artificially producing the necessary conditions becomes greater. The most promising results to date have been in the encouraging of precipitation by "seeding" supercooled clouds with powdered dry ice or silver iodide smoke. The effectiveness of this procedure is controversial. Various methods of decreasing the intensity of tropical cyclones, or of diverting their courses, have been suggested, but a satisfactory method has not been devised.

If a way is found to influence weather on a major scale, legal and possibly moral problems will be created due to conflicting interests.

References

American Meteorological Society. *Compendium of Meteorology.* Boston, American Meteorological Society, 1951.

American Meteorological Society. *Glossary of Meteorology.* Boston, American Meteorological Society, 1959.

Burgess, C. R. *Meteorology for Seamen.* Glasgow, Brown, 1950.

Byers, H. R. *General Meteorology.* 2nd ed. New York, McGraw-Hill, 1944.

Halpine, C. G. *A Pilot's Meteorology.* 2nd ed. New York, Van Nostrand, 1953.

Knight, A. M. *Modern Seamanship.* Revised by Capt. John V. Noel, Jr., U.S. Navy. 13th ed. New York, Van Nostrand, 1960.

Neuberger, Hans. *Introduction to Physical Meteorology.* State College, Pennsylvania State University, 1951.

Petterssen, Sverre. *Introduction to Meteorology.* 2nd ed. New York, McGraw-Hill, 1958.

Shaw, Napier. *The Drama of Weather.* 2nd ed. [London] Cambridge University Press, 1939.

Tannehill, I. R. *Weather Around the World.* 2nd ed. Princeton, Princeton University Press, 1951.

Trewartha, G. T. *An Introduction to Climate.* 3rd ed. New York, McGraw-Hill, 1953.

U. S. Department of the Navy. *Meteorology for Naval Aviators.* NAVAER 00–80U–24. Washington, Chief of Naval Operations, 1958.

U. S. Department of the Navy. *U. S. Navy Marine Climatic Atlas of the World,* vol. I, North Atlantic Ocean, NAVAER 50–1C–528; vol. II, North Pacific Ocean, NAVAER 50–1C–529.

U.S. Weather Bureau. *Preparation and Use of Weather Maps at Sea.* Circular R. 4th ed. Washington, U.S. Govt. Print. Off., 1961.

CHAPTER XXXIX

TROPICAL CYCLONES

3901. Introduction.—A **tropical cyclone** is a violent cyclone (art. 3813) originating in the tropics. Although it generally resembles the extratropical cyclone originating in higher latitudes, there are important differences, the principal one being the concentration of a large amount of energy into a relatively small area. Tropical cyclones are infrequent in comparison with middle- and high-latitude storms, but they have a record of destruction far exceeding that of any other type of storm. Because of their fury, and the fact that they are predominantly oceanic, they merit the special attention of all mariners, whether professional or amateur.

Rarely does the mariner who has experienced a fully developed tropical cyclone at sea wish to encounter a second one. He has learned the wisdom of avoiding them if possible. The uninitiated may be misled by the deceptively small size of a tropical cyclone as it appears on a weather map, and by the fine weather experienced only a few hundred miles from the reported center of such a storm. The rapidity with which the weather can deteriorate with approach of the storm, and the violence of the fully developed tropical cyclone, are difficult to visualize if they have not been experienced.

3902. Areas of occurrence.—Tropical cyclones occur almost entirely in six rather distinct regions, four in the northern hemisphere and two in the southern hemisphere, as shown in figure 3902. The name by which such a disturbance is commonly known varies somewhat with the locality, as follows:

Region I. North Atlantic (West Indies, Caribbean Sea, Gulf of Mexico, and waters off the East Coast of the United States). A tropical cyclone with winds of 64 knots or greater is called a **hurricane.**

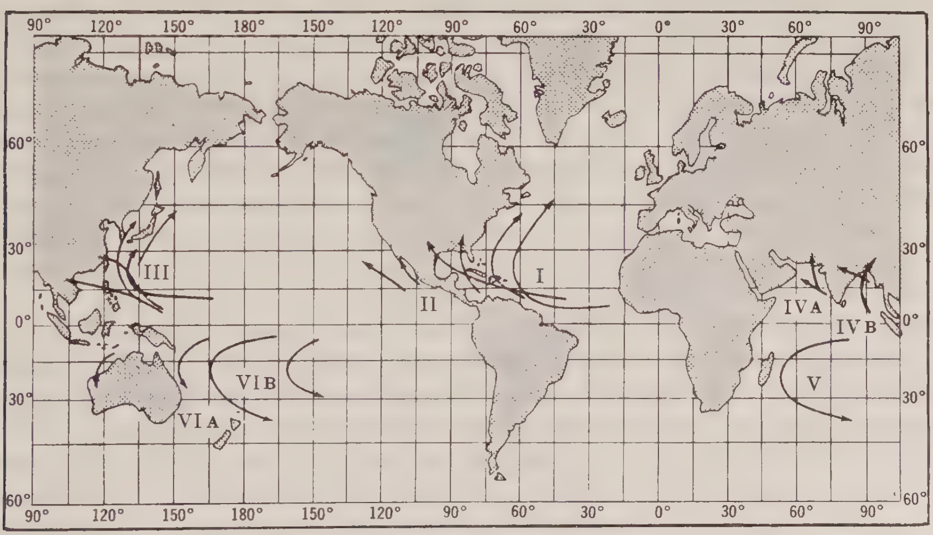


FIGURE 3902.—Areas in which tropical cyclones occur, and their approximate tracks.

Region II. Southeastern North Pacific (waters off west coast of Mexico and Central America). The name **hurricane** is applied, as in Region I.

Region III. Far East (the entire area west of the Mariana and Caroline Islands, across the Philippines and the China Sea, and northeastward to China and Japan). A fully developed storm with winds of 60 knots or greater is called a **typhoon** or, locally in the Philippine Islands, a **baguio.**

Region IV. *A.* Arabian Sea. *B.* Bay of Bengal. In these areas the storms are called **cyclones.**

Region V. South Indian Ocean (in the vicinity and to the east of Madagascar). As in Region IV, the tropical cyclone is called a **cyclone.**

Region VI. *A.* Australian waters (to longitude 160° E). *B.* South Pacific (the western portion, east of longitude 160° E). Several names are applied in this area, **cyclone** being the most common. One originating in the Timor Sea and moving southwest and then southeast across the interior of northwestern Australia is called a **willy-willy.** One to the east of Australia may be called a **hurricane.**

The only tropical ocean area in which tropical cyclones have not been encountered at some time is the South Atlantic.

As a tropical cyclone moves out of the tropics to higher latitudes, it normally loses energy slowly, expanding in area until it gradually dissipates or acquires the characteristics of extratropical cyclones. At any stage, a tropical cyclone normally loses energy at a much faster rate if it moves over land.

3903. Season and frequency of occurrence.—In Region III tropical cyclones may be encountered in any month of the year, though less frequently in winter than in summer. In the other regions, they occur only in the summer or autumn of that area, as shown in figure 3903. The total number for the northern hemisphere reaches a sharp peak in September. In general, this is the month of greatest frequency in each of the first four regions, although the Far East reaches its maximum in August, and in the Arabian Sea there are two peaks, one in June, and the other in late October. In the southern hemisphere, the maximum number is not as sharply peaked, being distributed nearly equally over January, February, and March, the summer season of that hemisphere.

The occurrence of tropical cyclones in an area is not as regular as might be inferred from a curve such as any of those of figure 3903, which are averages over a great many years. Even near the peak of a tropical cyclone season in any area there are periods when no tropical storms are observed. At the other extreme, as many as three hurricanes have been in progress at the same time in the North Atlantic, and as many as four typhoons in the Far East. The *average* total number of tropical cyclones occurring per year is 43 in the Northern Hemisphere and 13 in the Southern Hemisphere, or 56 throughout the world. However, the actual number in an area varies greatly from year to year. In the North Atlantic, where the greatest irregularity occurs, there have been as few as two and as many as 21 in a year, although the average number is seven. In the Far East, the number has varied from 13 to 25.

3904. Storm tracks.—Tropical cyclones form over the ocean, in low latitudes. As one forms, it drifts slowly westward with the current of free air in which it forms. As it reaches the edge of a subtropical anticyclone, the storm, together with the general mass of air, drifts farther from the equator, in many instances curving poleward and then eastward with the winds of the general circulation (art. 3804). In general, a tropical cyclone moves very slowly at first, its speed varying from about five to 20 knots. The speed gradually increases as the storm progresses, and may, in a few instances, reach a value of 50 knots or more when the storm reaches temperate latitudes.

The average track varies somewhat as the season progresses, and individual storm tracks may differ widely from the average. Region I, the North Atlantic, is typical of the changes. In August, about 80 percent originate in the southern North Atlantic and the eastern Caribbean, and about 20 percent in the western Caribbean and Gulf of Mexico. About 60 percent curve toward the right, roughly paralleling the coast of North America, and about 40 percent continue on westward, as shown in figure 3904a.

By the peak of the season, in September, the number forming in the southern North Atlantic and eastern Caribbean has dropped to 70 percent, but the number curving to-

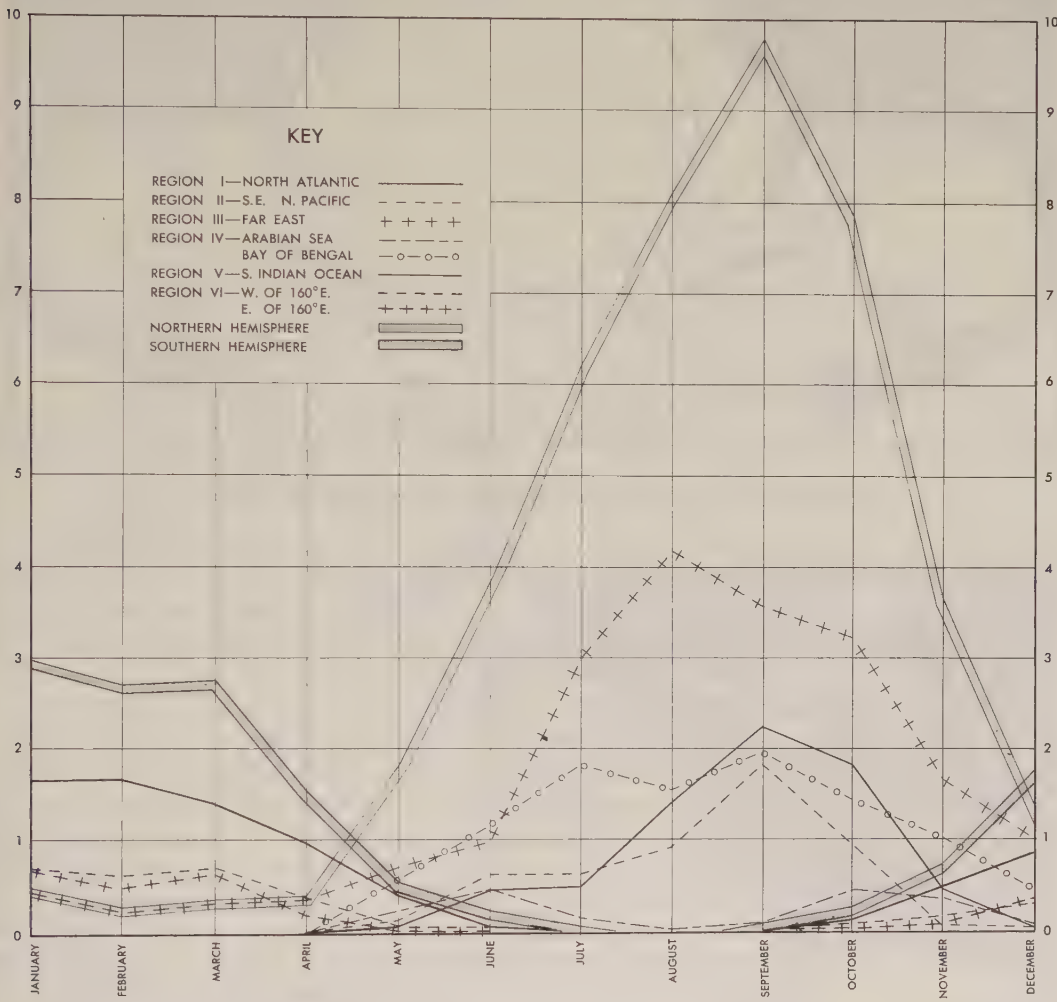


FIGURE 3903.—Average number of tropical disturbances per month in the various regions.

ward the right has increased to about the same percentage. The normal track has moved a little farther offshore in the lower latitudes, but has straightened somewhat so as to pass over eastern Newfoundland. This is shown in figure 3904b.

By October, the number originating in the southern North Atlantic and eastern Caribbean has dropped to 50 percent; and 80 percent of them curve, but at a point farther west, and more sharply, as shown in figure 3904c. By November, the change has been somewhat back toward the condition in September. As the season progresses, the deviation from average becomes greater and more common.

FIGURE 3904a.—Average North Atlantic storm tracks in August.

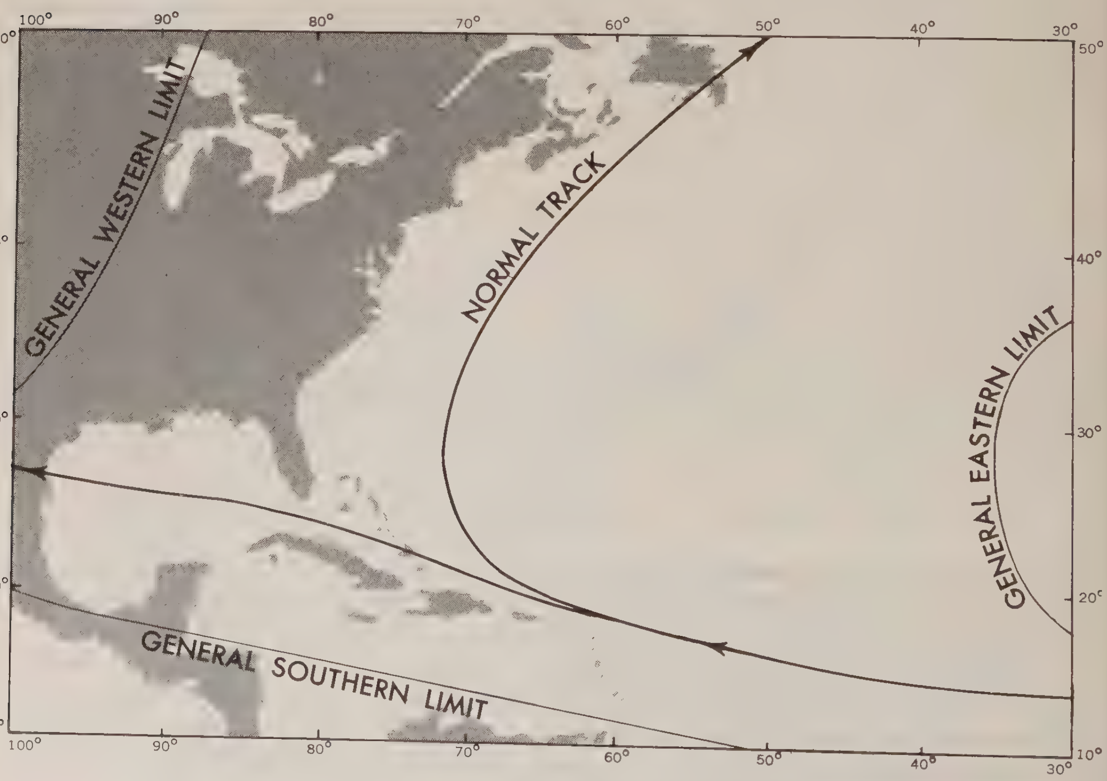


FIGURE 3904b.—Average North Atlantic storm tracks in September.

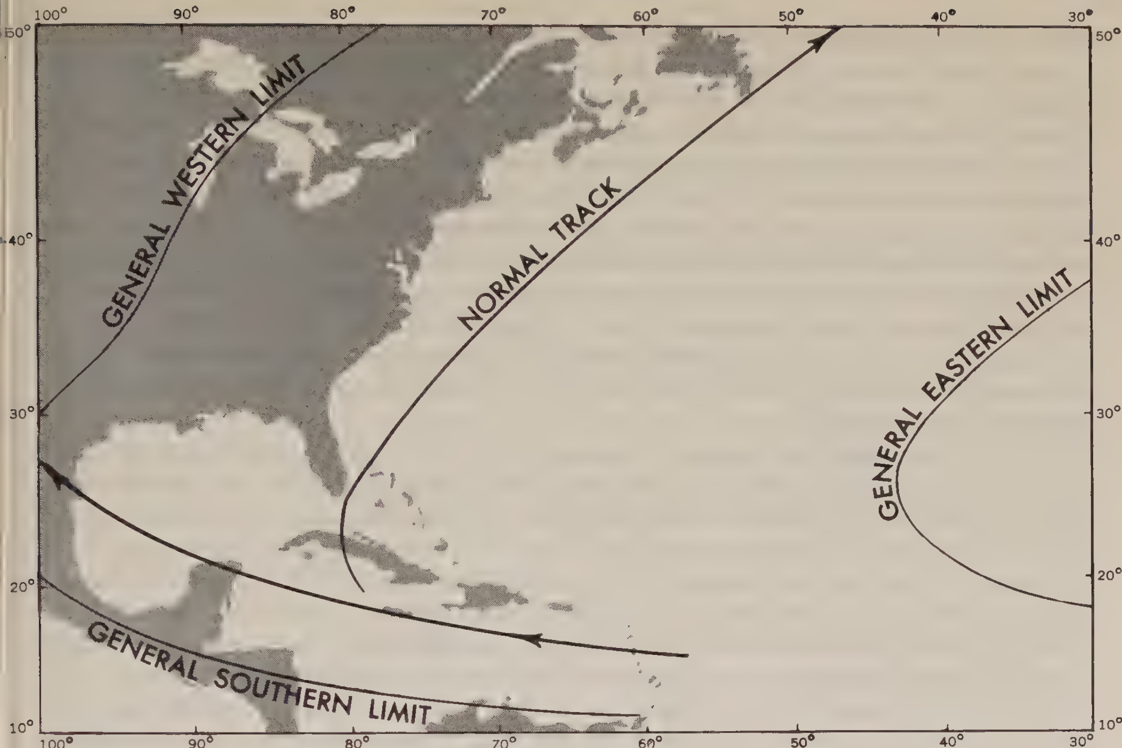


FIGURE 3904c.—Average North Atlantic storm tracks in October.

The differences, both in the averages and in individual tracks, are due to differences in the pressure pattern, particularly the location and movement of highs, which any cyclone tends to avoid.

3905. Life cycle.—The life cycle of a tropical cyclone may be considered to consist of four rather distinct stages, as follows:

Formation. A cyclonic circulation (art. 3813) develops, and wind speed increases to hurricane force (64 knots) over a restricted area near the center. Atmospheric pressure drops to about 1,000 millibars (29.53 inches). This stage may occupy several days, or may be completed in a period of 12 hours or less.

Immaturity. The pressure at the center continues to fall (the storm "deepens") and the wind speed increases, but the storm is still confined to a small area.

Maturity. The pressure at the center remains about the same, but the area of hurricane winds expands, to a radius of perhaps 150 to 200 miles, with winds of gale force (app. R) extending to perhaps 300 miles. Individual storms may differ considerably from these averages.

Decay. The area continues to increase, the pressure at the center rises, and the wind speed decreases. The storm loses its tropical characteristics and gradually dissipates, a process that may require several days over an ocean area. Over land the decay is more rapid.

3906. Origin and development.—Tropical cyclones originate between the doldrums (art. 3805) and the zones of the strongest trade winds. This accounts, at least partly, for the absence of such storms in the South Atlantic, for the Atlantic doldrums remain several degrees north of the equator except for occasional brief periods.

Some of the details regarding the formation of tropical cyclones are not understood, but the fact that such storms form only over water, and dissipate rapidly if they en-

counter land, probably indicates the need for a supply of water vapor. Over the tropical ocean this is abundantly available in the lower portion of the atmosphere. When a low develops over tropical oceans, hot, vapor-laden air flows in from adjacent regions. This air ascends near the center of the low, and condensation occurs. Each pound of water vapor that condenses into cloud or rain liberates approximately 970 British thermal units (art. 3711) of heat. This heat warms the surrounding air, thus increasing further the instability, and hastening the ascent of the air. Thus, the pressure continues to drop and the winds increase in speed, bringing in an increasing quantity of warm, moist air from the regions surrounding the low. At some height, the ascending air flows outward from the center of the cyclonic circulation. This process of inward flow, rising air current, condensation, warming, and high-level outflow causes the low to deepen and the wind speed to increase. Thus, as long as conditions remain suitable, the storm grows more intense.

While the actual mechanics of tropical cyclone formation are somewhat more involved than just described, the essential steps are given. Several theories exist regarding the details of initial formation of the low pressure area. Dropping pressure at the surface due to disturbances at high levels of the atmosphere; interaction of two air streams to produce a cyclonic eddy, causing convergence of surface air and the resulting ascent; and the joining of minor disturbances in the wind and pressure patterns in the atmosphere are all considered possibilities. The process is probably begun by several factors which combine in just the right relationship.

When it becomes fully developed, a tropical cyclone covers a well-defined area, more or less circular in shape, within which the atmospheric pressure decreases rapidly toward the center. This decrease of pressure may amount to a maximum of 0.01 or even 0.02 inch per mile. Because of the rapid decrease of pressure with distance, the wind speed is high, being greatest at the regions of steepest pressure gradient.

At the center of the storm, there is normally an area five to 30 miles in diameter (most often ten to 15 miles) within which the wind speed drops to a relative calm, usually ten to 15 knots or less. This is the **eye of the storm.** Ascending air causes the dense cover of clouds to give way to a thin layer of low clouds with holes through which the sun may shine. Around the edge of the eye, the wind speed increases from the relative calm to the full fury of maximum speed within a distance of a few feet. Here the heavy cloud seems thickest, and the torrential rains surrounding the central area appear concentrated. This is the **wall of the eye.** When a tropical cyclone moves to higher latitude, its eye becomes less clearly defined as the maximum wind moves outward from the center, the wall of the eye becomes more indistinct, and its cloud cover increases.

3907. Locating and tracking a tropical cyclone.—By means of radio, organized meteorological services collect weather observations daily from island stations, ships at sea, and aircraft. When a tropical cyclone is located, usually in its early formative stage, it is followed closely. In the North Atlantic, aircraft of the U. S. Navy and U. S. Air Force, in cooperation with the Weather Bureau, make frequent flights to the vicinity of such storms to provide information needed for tracking the hurricane and determining its intensity. Bulletins are broadcast to ships several times daily, giving information on each storm's location, intensity, and movement. As a further aid, the mariner may obtain weather reports by radio directly from other ships in the vicinity of a tropical cyclone. Radar may be used to follow the movements of the precipitation areas when they are within range.

Although these aids normally prove adequate for locating and avoiding a tropical cyclone, knowledge of the appearance of the sea and sky in the vicinity of such a storm is useful to the mariner. This information is given in article 3908.

3908. The passage of a tropical cyclone at sea is an experience not soon to be forgotten.

An early indication of the approach of such a storm is the presence of a long swell. In the absence of a tropical cyclone, the crests of swell in the deep waters of the Atlantic pass at the rate of perhaps eight per minute. Swell generated by a hurricane is about twice as long, the crests passing at the rate of perhaps four per minute. Swell may be observed several days before arrival of the storm.

When the storm center is 500 to 1,000 miles away, the barometer usually rises a little, and the skies are relatively clear. Cumulus clouds, if present at all, are few in number and their vertical development appears suppressed. The barometer usually appears restless, **pumping** up and down a few hundredths of an inch.

As the tropical cyclone comes nearer, a cloud sequence begins which resembles that associated with the approach of a warm front in middle latitudes (art. 3812). Snow-white, fibrous "mare's tails" (cirrus) appear when the storm is about 300 to 600 miles away. Usually these seem to converge, more or less, in the direction from which the storm is approaching. This convergence is particularly apparent at about the time of sunrise and sunset.

Shortly after the cirrus appears, but sometimes before, the barometer starts a long, slow fall. At first the fall is so gradual that it only appears to alter somewhat the normal daily cycle (two maxima and two minima in the tropics). As the rate of fall increases, the daily pattern is completely lost in the more or less steady fall.

The cirrus becomes more confused and tangled, and then gradually gives way to a continuous veil of cirrostratus. Below this veil, altostratus forms, and then stratocumulus (art. 3714). These clouds gradually become more dense, and as they do so, the weather becomes unsettled. A fine, mist-like rain begins to fall, interrupted from time to time by showers. The barometer has fallen perhaps a tenth of an inch.

As the fall becomes more rapid, the wind increases in gustiness, and its speed becomes greater, reaching a value of perhaps 22 to 40 knots (Beaufort 6–8). On the horizon appears a dark wall of heavy cumulonimbus (art. 3714), the **bar** of the storm. Portions of this heavy cloud become detached from time to time and drift across the sky, accompanied by rain squalls and wind of increasing speed. Between squalls, the cirrostratus can be seen through breaks in the stratocumulus.

As the bar approaches, the barometer falls more rapidly and wind speed increases. The seas, which have been gradually mounting, become tempestuous. Squall lines, one after the other, sweep past in ever increasing number and intensity.

With the arrival of the bar, the day becomes very dark, squalls become virtually continuous, and the barometer falls precipitously, with a rapid increase in wind speed. The center may still be 100 to 200 miles away in a fully developed tropical cyclone. As the center of the storm comes closer, the ever-stronger wind shrieks through the rigging and about the superstructure of the vessel. As the center approaches, rain falls in torrents. The wind fury increases. The seas become mountainous. The tops of huge waves are blown off to mingle with the rain and fill the air with water. Objects at a short distance are not visible. Even the largest and most seaworthy vessels become virtually unmanageable, and may sustain heavy damage. Less sturdy vessels do not survive. Navigation virtually stops as safety of the vessel becomes the prime consideration. The awesome fury of this condition can only be experienced. Words are inadequate to describe it.

If the eye of the storm passes over the vessel, the winds suddenly drop to a breeze as the wall of the eye passes. The rain stops, and the skies clear sufficiently to permit the sun to shine through holes in the comparatively thin cloud cover. Visibility improves. Mountainous seas approach from all sides, apparently in complete confu-

sion. The barometer reaches its lowest point, which may be an inch and a half or two inches below normal in fully developed tropical cyclones. As the wall on the opposite side of the eye arrives, the full fury of the wind strikes as suddenly as it ceased, but from the opposite direction. The sequence of conditions that occurred during approach of the storm is reversed, and pass more quickly, as the various parts of the storm are not as wide in the rear of a storm as on its forward side.

Typical cloud formations associated with a hurricane are shown in figure 3908.

FIGURE 3908.—Typical hurricane cloud formations.

3909. Locating the center of a tropical cyclone.—If intelligent action is to be taken to avoid the full fury of a tropical cyclone, early determination of its location and direction of travel relative to the vessel is essential. The bulletins and forecasts are an excellent general guide, but they are not infallible and may be sufficiently in error to induce a mariner in a critical position to alter course so as to unwittingly increase the danger to his vessel. Often it is possible, using only those observations made aboard ship, to obtain a sufficiently close approximation to enable the vessel to maneuver to the best advantage.

As stated in article 3908, the presence of an exceptionally long swell is usually the first visible indication of the existence of a tropical cyclone. In deep water it approaches from the general direction of origin (the position of the storm center *when the swell was generated*). However, in shoaling water this is a less reliable indication because the direction is changed by refraction, the crests being more nearly parallel to the bottom contours (art. 3307).

When the cirrus clouds appear, their point of convergence provides an indication of the direction of the storm center. If the storm is to pass well to one side of the observer, the point of convergence shifts slowly in the direction of storm movement. If the storm center will pass near the observer, this point remains steady. When the bar (art. 3908) becomes visible, it appears to rest upon the horizon for several hours. The darkest part of this cloud is in the direction of the storm center. If the storm is to pass to one side, the bar appears to drift slowly along the horizon. If the storm is heading directly toward the observer, the position of the bar remains fixed. Once within the area of the dense, low clouds, one should observe their direction of move-

ment, which is almost exactly along the isobars, with the center of the storm being 90° from the direction of cloud movement (left of direction of movement in the northern hemisphere, and right in the southern hemisphere).

The winds are probably the best guide to the direction of the center of a tropical cyclone. The circulation is cyclonic (art. 3813), but because of the steep pressure gradient near the center, the winds there blow with greater violence and are more nearly circular than in extratropical cyclones.

According to Buys Ballot's law (art. 3813) an observer who faces into the wind has the center of the low pressure on his right in the northern hemisphere, and on his left in the southern hemisphere, and in each case somewhat behind him. If the wind followed circular isobars exactly, the center would be exactly eight points, or 90°, from dead ahead when facing into the wind. However, the track of the wind is usually inclined somewhat toward the center, so that the angle from dead ahead varies between perhaps 8 and 12 points (90° to 135°). The inclination varies in different parts of the same storm. It is least in front of the storm, and greatest in the rear, since the actual wind is the vector sum of that due to the pressure gradient and the motion of

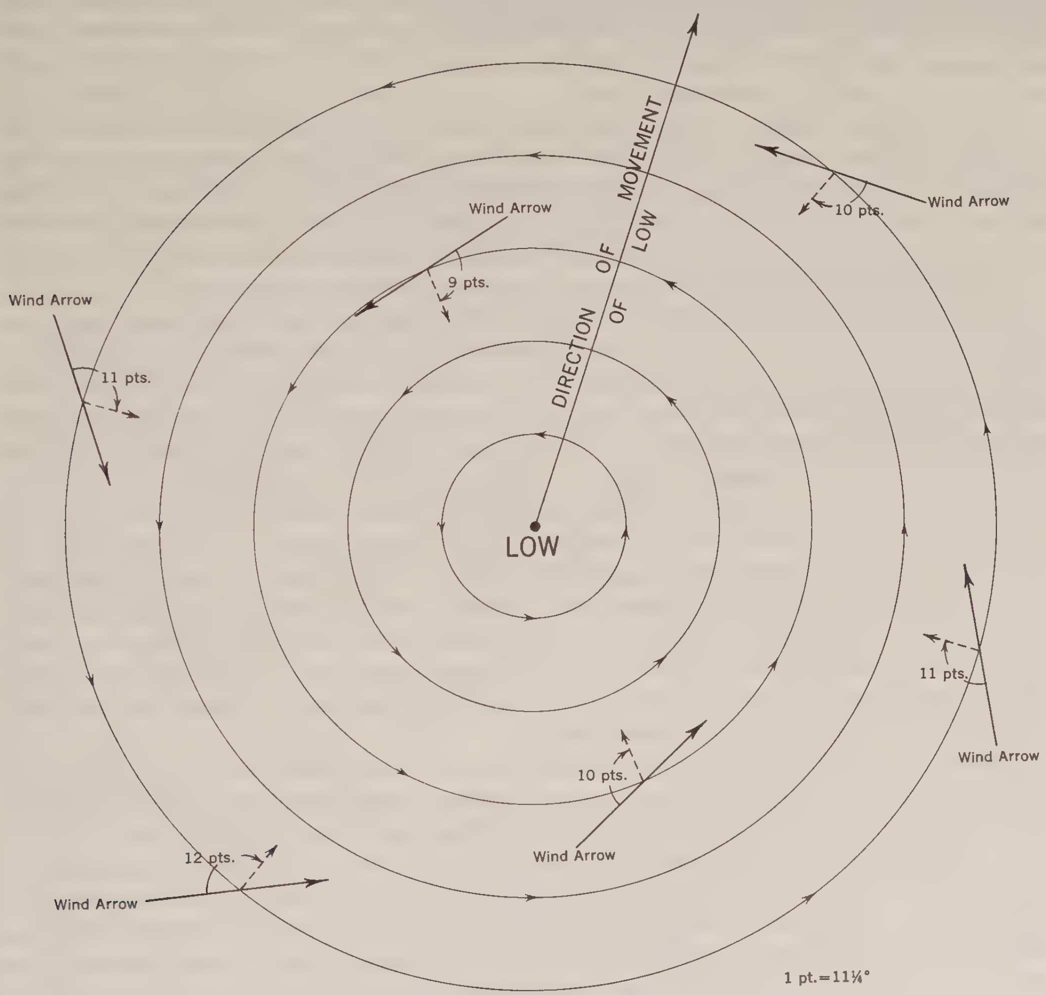


FIGURE 3909a.—Approximate relationship of wind to isobars and storm center in the northern hemisphere.

the storm along the track. A good average is perhaps ten points in front, and 11 or 12 points in the rear. These values apply when the storm center is still several hundred miles away. Closer to the center, the wind blows more nearly along the isobars, the inclination being reduced by one or two points at the wall of the eye. Since wind direction usually shifts temporarily during a squall, its direction at this time should not be used for determining the position of the center. The approximate relationship of wind to isobars and storm center in the northern hemisphere is shown in figure 3909a.

When the center is within radar range, it might be located by this equipment. However, since the radar return is predominantly from the rain, results can be deceptive, and other indications should not be neglected. Figure 3909b shows a typical radar PPI presentation of a tropical cyclone.

Distance from the storm center is more difficult to determine than direction. Radar is perhaps the best guide. However, the rate of fall of the barometer is some indication. If a vessel is hove-to in front of a storm which is advancing directly toward it, the fall of pressure per hour might be about as shown in figure 3909c. However, this is an imperfect indication, for the rate of fall may be quite erratic, and will vary somewhat with the depth of the low at the center, the speed of the storm center along its track, and the stage in the life cycle of the storm. The usefulness of this information is further reduced by the fact that a vessel would not normally remain hove-to in the path of a tropical cyclone.

3910. Maneuvering to avoid the storm center.—The safest procedure with respect to tropical cyclones is to avoid them. If action is taken sufficiently early, this is simply a matter of setting a course that will take the vessel well to one side of the probable track of the storm, and then continuing to plot the positions of the storm center, as given in the weather bulletins, revising the course as needed.

However, such action is not always possible. If one finds himself within the storm area, the proper action to take depends in part upon his position relative to the storm center and its direction of travel. It is customary to divide the circular area of the storm into two parts. In the northern hemisphere, that part to the *right* of the storm track (facing in the direction *toward* which the storm is moving) is called the **dangerous semicircle.** It is considered dangerous because (1) the actual wind *speed* is greater than that due to the pressure gradient alone, since it is augmented by the forward motion of the storm, and (2) the *direction* of the wind and sea is such as to carry a vessel into the path of the storm (in the forward part of the semicircle). The part to the left of the storm track is called the **navigable semicircle.** In this part, the wind is decreased by the forward motion of the storm, and the wind blows vessels away from the storm track (in the forward part). Because of the greater wind speed in the dangerous semicircle, the seas are higher here than in the navigable semicircle. In the southern hemisphere, the dangerous semicircle is to the left of the storm track, and the navigable semicircle is to the right of the storm track.

A plot of successive positions of the storm center should indicate the semicircle in which a vessel is located. However, if this is based upon weather bulletins, it is not a reliable guide because of the lag between the observations upon which the bulletin is based and the time of reception of the bulletin, with the ever present possibility of a change in the direction of motion of the storm. The use of one's radar eliminates this lag, but the return is not always a true indication of the center. Perhaps the most reliable guide is the wind. Within the cyclonic circulation, a *veering* wind (one changing direction to the right in the northern hemisphere and to the left in the southern hemisphere) indicates a position in the dangerous semicircle, and a *backing* wind (one changing in a direction opposite to a veering wind) indicates a position in the navigable semicircle. However, if a vessel is underway, its motion should be considered. If it is

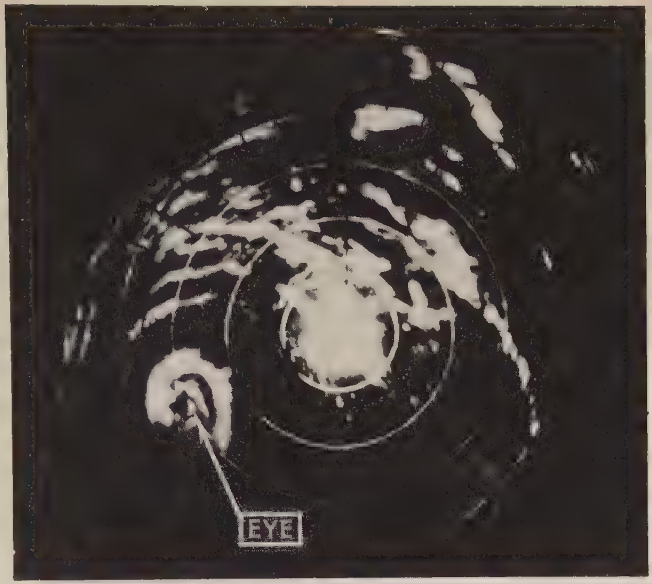


FIGURE 3909b.—Typical radar PPI presentation of a tropical cyclone.

outrunning the storm or pulling rapidly toward one side (which is not difficult during the early stages of a storm, when its speed is low), the opposite effect occurs. This should usually be accompanied by a rise in atmospheric pressure, but if motion of the vessel is nearly along an isobar, this may not be a reliable indication. If in doubt, the safest action is usually to stop long enough to determine definitely the semicircle. The loss in valuable time may be more than offset by the minimizing of the possibility of taking the wrong action and increasing the danger to the vessel. If the wind direction remains steady (for a vessel which is stopped), with increasing speed and falling barometer, the vessel is in or near the path of the storm. If it remains steady with decreasing speed and rising barometer, the vessel is on the storm track, behind the center.

The first action to take if one finds himself within the cyclonic circulation, is to determine the position of his vessel with respect to the storm center. While the vessel can still make considerable way through the water, a course should be selected to take it as far as possible from the center. If the vessel can move faster than the storm, it is a relatively simple matter to outrun the storm if sea room permits. But when the storm is faster, the solution is not as simple. In this case, the vessel, if ahead of the storm, will approach nearer to the center. The problem is to select a course that will produce the greatest possible minimum distance. This is best determined by means of a relative movement plot, as shown in the following example solved on a maneuvering board (art. 1212):

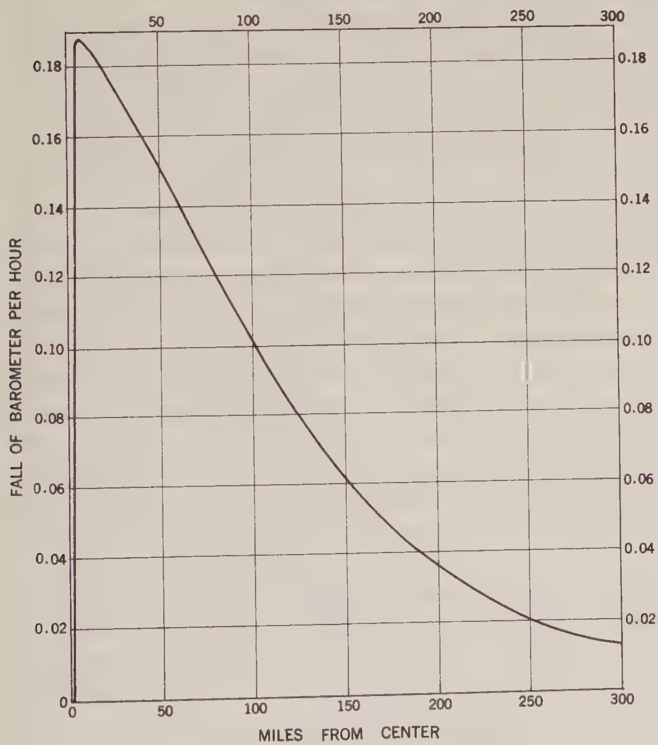


FIGURE 3909c.—Typical average pressure drop as tropical cyclone approaches.

Example.—A tropical cyclone is estimated to be moving

in direction 320° at 19 knots. Its center bears 170°, at an estimated distance of 200 miles from a vessel which has a maximum speed of 12 knots.

Required.—(1) The course to steer at 12 knots to produce the greatest possible minimum distance between the vessel and the storm center.

(2) The distance of the storm center at nearest approach.

(3) Elapsed time until nearest approach.

Solution (fig. 3910).—Consider the vessel remaining at the center of the plot throughout the solution, as on a radar PPI.

(1) Plot point *C* at a distance of 200 miles (scale 20:1) in direction 170° from the center of the diagram, to locate the position of the storm center relative to the vessel. From the center of the diagram, draw *RA*, the speed vector of the storm center, in direction 320°, speed 19 knots (scale 2:1). From *A* draw a line tangent to the 12-knot speed circle (labeled 6 at scale 2:1) *on the side opposite the storm center.* From the

FIGURE 3910.—Solution to determine course for avoiding storm center.

center of the diagram draw a perpendicular to this tangent line, locating point B. The line RB is the required speed vector for the vessel. Its direction, 011°, is the required course.

(2) The path of the storm center *relative to the vessel*, will be along a line from C in the direction BA, if both storm and vessel maintain course and speed. The point of nearest approach will be at D, the foot of a perpendicular from the center of the diagram. This distance, at scale 20:1, is 187 miles.

(3) The length of the vector BA (14.8 knots) is the speed of the storm with respect to the vessel. Mark this on the lowest scale of the nomogram at the bottom of the diagram. The relative distance CD is 72 miles, by measurement. Mark this (scale 10:1) on the middle scale at the bottom of the diagram. Draw a line between the two points and extend it to intersect the top scale at 29.2 (292 at 10:1 scale). The elapsed time is therefore 292 minutes, or 4 hours 52 minutes, or 5 hours approximately.

Answers.—(1) C 011°, (2) D 187 mi., (3) t 5^h (approximately).

The storm center will be dead astern at its nearest approach.

As a very general rule, for a vessel in the northern hemisphere, safety lies in placing the wind on the starboard bow in the dangerous semicircle and on the starboard quarter in the navigable semicircle. If on the storm track ahead of the storm, the wind should be put about two points on the starboard quarter until the vessel is well within the navigable semicircle, and the rule for that semicircle then followed. A study of figure 3909a should indicate why these headings are desirable. In the southern hemisphere the same rules hold, but with respect to the port side. With a faster than average vessel, the wind can be brought a little farther aft in each case. However, as the speed of the storm increases along its track, the wind should be brought farther forward. If land interferes with what would otherwise be the best maneuver, the solution should be altered to fit the circumstances. If the speed of a vessel is greater than that of the storm, it is possible for the vessel, if behind the storm, to overtake it. In this case, the only action usually needed is to slow enough to let the storm pull ahead.

In all cases, one should be alert to changes in the direction of movement of the storm center, particularly in the area where the track normally curves toward the pole. If the storm maintains its direction and speed, the ship's course should be maintained as the wind shifts.

If it becomes necessary for a vessel to heave to, the characteristics of the vessel should be considered. A power vessel is concerned primarily with damage by direct action of the sea. A good general rule is to heave to with head to the sea in the dangerous semicircle or stern to the sea in the navigable semicircle. This will result in greatest amount of headway away from the storm center, and least amount of leeway toward it. If a vessel handles better with the sea astern or on the quarter, it may be placed in this position in the navigable semicircle or in the rear half of the dangerous semicircle, but *never* in the forward half of the dangerous semicircle. It has been reported that when the wind reaches hurricane speed and the seas become confused, some ships ride out the storm best if the engines are stopped, and the vessel is permitted to seek its own position. In this way, it is said, the ship rides *with* the storm instead of fighting *against* it.

In a sailing vessel, while attempting to avoid a storm center, one should steer courses as near as possible to those prescribed above for power vessels. However, if it becomes necessary for such a vessel to heave to, the wind is of greater concern than the sea. A good general rule always is to heave to on whichever tack permits the shifting wind to draw aft. In the northern hemisphere this is the starboard tack in

the dangerous semicircle and the port tack in the navigable semicircle. In the southern hemisphere these are reversed.

While each storm requires its own analysis, and frequent or continual resurvey of the situation, the general rules for a steamer may be summarized as follows:

NORTHERN HEMISPHERE

Right or dangerous semicircle.—Bring the wind on the starboard bow (045° relative), hold course and make as much way as possible. If obliged to heave to, do so with head to the sea.

Left or navigable semicircle.—Bring the wind on the starboard quarter (135° relative), hold course and make as much way as possible. If obliged to heave to, do so with stern to the sea.

On storm track, ahead of center.—Bring the wind two points on the starboard quarter (157°5 relative), hold course and make as much way as possible. When well within the navigable semicircle, maneuver as indicated above.

On storm track, behind center.—Avoid the center by the best practicable course, keeping in mind the tendency of tropical cyclones to curve northward and eastward.

SOUTHERN HEMISPHERE

Left or dangerous semicircle.—Bring the wind on the port bow (315° relative), hold course and make as much way as possible. If obliged to heave to, do so with head to the sea.

Right or navigable semicircle.—Bring the wind on the port quarter (225° relative), hold course and make as much way as possible. If obliged to heave to, do so with stern to the sea.

On storm track, ahead of center.—Bring the wind two points on the port quarter (202°5 relative), hold course and make as much way as possible. When well within the navigable semicircle, maneuver as indicated above.

On storm track, behind center.—Avoid the center by the best practicable course, keeping in mind the tendency of tropical cyclones to curve southward and eastward.

Whenever a tropical cyclone is encountered, the wise procedure is to begin preparing the vessel for heavy weather in sufficient time to permit thorough preparation, so that damage may be minimized. One should be particularly careful to keep free surfaces of liquids to a minimum.

3911. Coastal effects.—The high winds of a tropical cyclone inflict widespread damage when such a storm leaves the ocean and crosses land. Aids to navigation may be blown out of position or destroyed. Craft in harbors, unless they are properly secured, drag anchor or are blown against obstructions. Ashore, trees are blown over, houses are damaged, power lines are blown down, etc. The greatest damage usually occurs in the dangerous semicircle a short distance from the center, where the strongest winds occur. As the storm continues on across land, its fury subsides faster than it would if it had remained over water.

Along the coast, particularly, greater damage may be inflicted by water than by the wind. There are at least four sources of water damage. First, the unusually high seas generated by the storm winds pound against shore installations and craft in their way. Second, the continued blowing of the wind toward land causes the water level to increase perhaps three to ten feet above its normal level. This **storm tide,** which may begin when the storm center is 500 miles or even farther from the shore, gradually increases until the storm passes. The highest storm tides are caused by a slow-moving tropical cyclone of large diameter, because both of these effects result in greater duration of wind in the same direction. The effect is greatest in a partly enclosed body

of water, such as the Gulf of Mexico, where the concave coastline does not readily permit the escape of water. It is least on small islands, which present little obstruction to the flow of water. Third, the furious winds which blow around the wall of the eye create a ridge of water called a **storm wave,** which strikes the coast and often inflicts heavy damage. The effect is similar to that of a **seismic sea wave,** caused by an earthquake in the ocean floor. Both of these waves are popularly called **tidal waves.** Storm waves of 20 feet or more have occurred. About three or four feet of this is due to the decrease of atmospheric pressure, and the rest to winds. Like the damage caused by wind, that due to high seas, the storm tide, and the storm wave is greatest in the dangerous semicircle, near the center. The fourth source of water damage is the heavy rain that accompanies a tropical cyclone. This causes floods that add to the damage caused in other ways.

When proceeding along a shore recently visited by a tropical cyclone, a navigator should remember that time is required to restore aids to navigation which have been blown out of position or destroyed. In some instances the aid may remain but its light, sound apparatus, or radiobeacon may be inoperative. Landmarks may have been damaged or destroyed.

References

Burgess, C. R. *Meteorology for Seamen.* Glasgow, Brown, 1950.

Byers, H. R. *General Meteorology.* 2nd ed. New York, McGraw-Hill, 1944.

Colon, J. A. "A Study of Hurricane Tracks for Forecasting Purposes." *Monthly Weather Review.* March 1953. Washington, U. S. Govt. Print. Off.

McDonald, W. F. "On a Hypothesis Concerning the Normal Development and Disintegration of Tropical Hurricanes." *Monthly Weather Review.* January 1942. Washington, U. S. Govt. Print. Off.

Nimitz, Chester W., et. al. "Typhoon Doctrine." *U. S. Naval Institute Proceedings,* vol. 82, No. 1 (January 1956), pp. 83–93.

Riehl, Herbert. *Tropical Meteorology.* New York, McGraw-Hill, 1954.

Tannehill, I. R. *Hurricanes.* 9th ed. Princeton, Princeton University Press, 1956.

U. S. Weather Bureau. *The Hurricane.* Washington, U. S. Govt. Print. Off., 1956.

PART EIGHT

HYDROGRAPHY

PART EIGHT

HYDROGRAPHY

CHAPTER XL

INSTRUMENTS FOR HYDROGRAPHIC SURVEYING

4001. Introduction.—Although the expression "hydrographic surveying" denotes investigation of water areas to obtain information for use in making nautical charts, land surveying methods are frequently used to establish points on shore from which positions of all hydrographic observations such as soundings, currents, etc., can be related. Therefore, this chapter describes such instruments as the astrolabe, theodolite, level, and special drafting equipment, as well as tide gages, current meters, and other instruments directly associated with hydrography. Instruments used in hydrographic surveying, but described elsewhere, include the sextant (ch. XV), echo sounder (art. 619), and electronic equipment (ch. XIII). In general, surveying instruments are characterized by a high order of accuracy, as compared with navigation instruments.

4002. Astrolabe.—Unlike the earlier instrument of the same name (art. 124), the modern **astrolabe** is used in hydrographic surveying to determine the instant at which various celestial bodies arrive at a preselected altitude. From a number of such

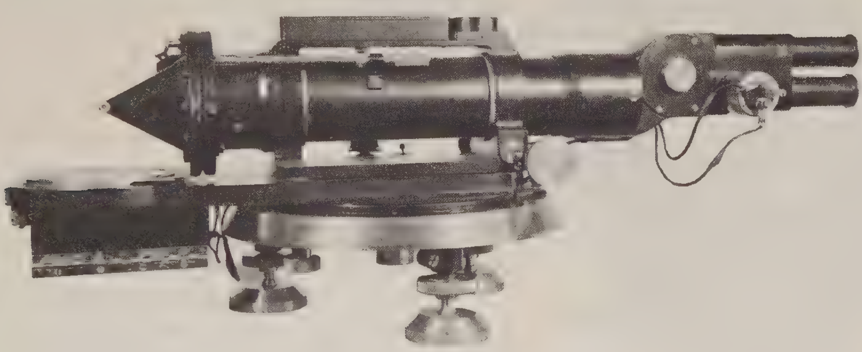


FIGURE 4002a.—A prismatic astrolabe.

observations, the position of the instrument can be calculated. Astrolabes now available use an altitude of 45° or 60°.

As its name implies, the **prismatic astrolabe** (fig. 4002a) depends upon an accurately-ground prism to maintain the fixed angle of observation. In addition, the prism permits the observation of a second image of a star by reflection from an artificial horizon, which is a small pan of mercury placed below the prism. In figure 4002b, a light ray (R) from a star enters directly the upper surface of the prism and is reflected through the horizontal observing telescope. A parallel ray (R′) from the same star is reflected from the mercury surface, enters the lower surface of the prism, and is then reflected through the telescope. Since the latter image is a doubly reflected one, its apparent motion will be opposite that of the former (fig. 4002b, rays (a) and (a′) and inset (1)). When the direct and reflected light rays are perpendicular to the upper and lower surfaces of the prism, respectively, the two images are coincident and the star is at the altitude fixed by the angle between the surfaces of the prism.

In practice, the prism is turned slightly on an axis coincident with the telescope axis so that the images will, at the fixed altitude, be side by side on a horizontal line

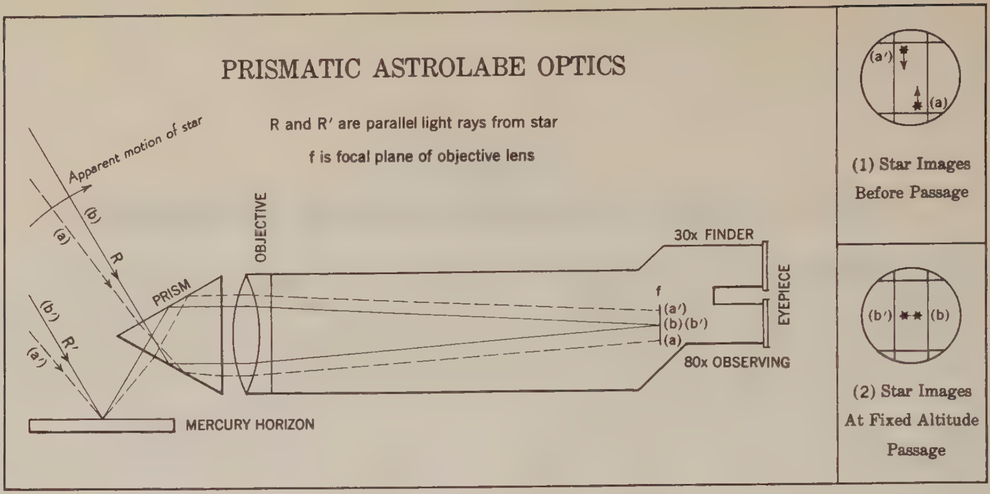


FIGURE 4002b.—Optics of the prismatic astrolabe.

(fig. 4002b, rays (b) and (b′) and insert (2)) rather than coincident. This permits easier, more accurate observation.

In addition to the basic parts already mentioned, the prismatic astrolabe is provided with level bubbles and three leveling screws to adjust it to the horizontal. For orienting to north and setting at desired azimuths, it is equipped with a magnetic compass and an azimuth circle. Adjusting screws are provided for collimation; that is, making the vertical surface of the prism perpendicular to the axis of the telescope. There is also an erecting screw which rotates the prism about the axis of the telescope. Flashlight batteries supply power to illuminate the azimuth circle and reticle, the intensity of the latter being controlled by a rheostat.

A prism actuated by a lever deflects the light rays upward to a 30-power, wide field-of-view eyepiece to facilitate finding the star. Once the star is located, final observations are made through the 80-power observing eyepiece. Accessories for screening the mercury from wind and dust, and equipment for cleaning the surface of the mercury are included with the instrument.

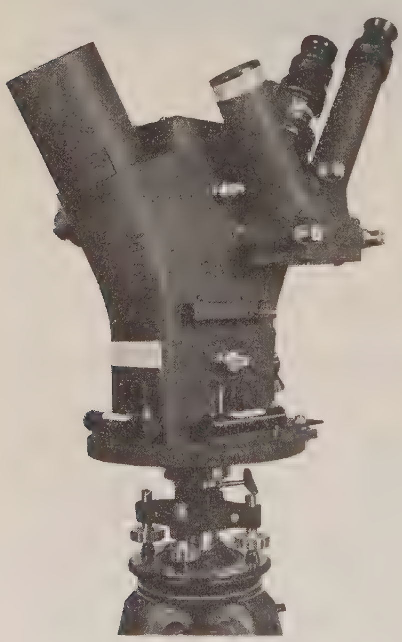


FIGURE 4002c.—A pendulum astrolabe.

The **pendulum astrolabe** (fig. 4002c) fixes the observation altitude by directing the light rays from a star down a 60° objective tube to a pendulum-supported horizontal mirror, from which it is reflected up a 60° eyepiece tube. Thus, in a sense, it is a telescope bent at 60° with a pendulum mirror to reflect the light rays accordingly (fig. 4002d). Since only one star image is seen, the exact time of star passage is noted as being the mean of the times at which it passes a set of horizontal cross hairs.

In addition to the 80-power observing telescope, it has an 11-power finder telescope. This instrument also has internal illumination and is provided with means for leveling and setting in azimuth.

4003. Timing equipment.—The timing equipment used in conjunction with astrolabes consists of a **chronometer,** a **radio,** a **chronograph,** and a break-circuit **key** (fig. 4003). The chronometer is set to run on sidereal time and is connected through an amplifier to the chronograph, which records a tick mark on moving paper at each one-second break except the 59th second of every minute. This is omitted so that the beginning of each minute can be easily identified on the record.

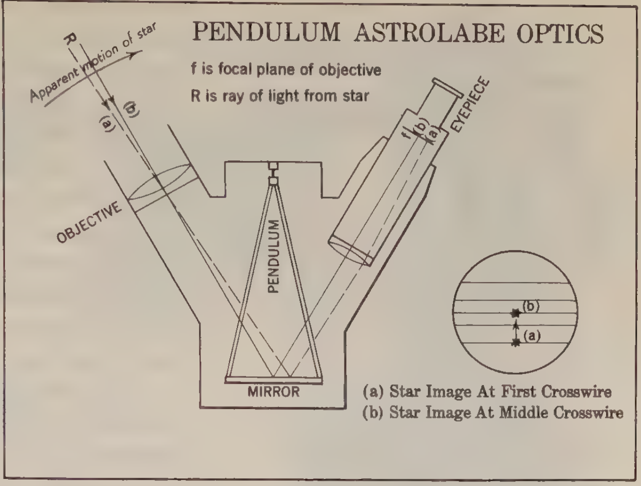


Figure 4002d.—Optics of the pendulum astrolabe.

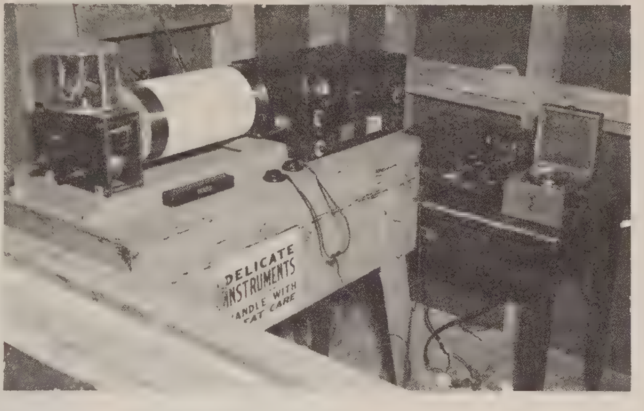


Figure 4003.—Timing equipment.

The radio, also, is connected to the chronograph recorder through the amplifier, and is used to receive standard time signals for determining chronometer correction and rate. The break-circuit key is tapped by the observer at the instant of star passage, making a tick mark on the chronograph paper. The chronometer time of star passage can be scaled off the chronograph record. By application of chronometer correction and rate, one can determine the GMT of the observation.

4004. Theodolite.—A **theodolite** is an instrument designed to measure precise horizontal and vertical angles. Thus, it can be used for determining the bearing (called "azimuth" by surveyors) of a line by observing the angle between that line and the azimuth line of a star. It can be used to measure the angles in a triangulation net, and to measure vertical angles for the trigonometric computation of elevations.

The **direction theodolite** (fig. 4004a) consists essentially of two graduated circles (one horizontal and one vertical), equipment for leveling and centering the instrument, an observing telescope, and an eyepiece for reading the circles. Horizontal angles can be read directly to the nearest 0.2 second of arc on the instrument illustrated. A smaller, lighter model can be read directly to the nearest second of arc, and tenths can be estimated.

Both circles are completely enclosed, and are read at one eyepiece through a system of prisms within the instrument. As the upper portion of the instrument turns about a vertical axis, two sets of prisms scan diametrically opposite sides of the horizontal circle. This upper portion can be clamped and final pointing on target can be made with a slow-motion tangent screw. The vertical cross hair is brought exactly on target, and the instrument is collimated and read. The horizontal circle can be set at any desired initial reading.

When the telescope is turned about its horizontal axis, two sets of prisms scan

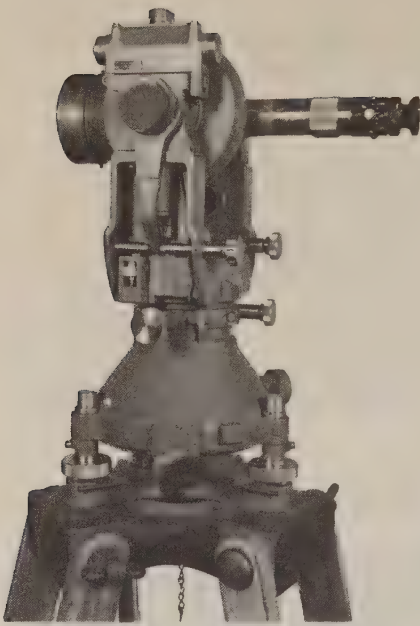


FIGURE 4004a.—A direction theodolite.

diametrically opposite sides of the vertical circle. This motion also has a clamp and slow-motion tangent screw. Collimation is effected before reading. The vertical circle is read through the same eyepiece as the horizontal circle, by turning a knob near the bottom of the rear face of the right-hand standard. When the line on this knob is horizontal, the horizontal circle is seen, and when the line is vertical, the vertical circle is seen. Since the vertical circle reads zero when the telescope is pointed directly toward the zenith, the angles read are zenith distances.

Accessory equipment for this type instrument includes an optical centering device, internal illumination for night observations, prismatic eyepieces, and accurate levels. Complete instructions for use of the instrument are furnished by the manufacturer.

The **repeating theodolite** (fig. 4004b), unlike the direction theodolite, has a lower motion clamp screw and slow-motion tangent screw. Thus, with the upper portion clamped and the lower motion free, the horizontal circle and upper portion of the instrument carrying the telescope and two verniers rotate as a unit about the vertical axis. With the lower motion clamped and the upper motion free, the circle remains fixed and the upper portion only rotates, indicating angles by the position of the verniers relative to the horizontal circle. Also, the circle is exposed at the verniers for reading through a magnifying glass. These verniers can be read to the nearest ten seconds of arc.

Since the vertical circle is graduated to read zero when the telescope is horizontal, other readings are either elevation or depression angles. The verniers of the vertical circle read directly to the nearest 15 seconds of arc.

4005. Transit.—The **surveyor's transit** (fig. 4005a) is similar to the repeating theodolite, except that it is smaller, lighter, and less precise. It rests on four leveling screws rather than three, and some models read only to the nearest minute of arc.

The **camera transit** (fig. 4005b) consists essentially of a surveyor's transit with a camera mounted between widely separated standards. A transit telescope is mounted on top of the camera. In use, the instrument is pointed on a known con-

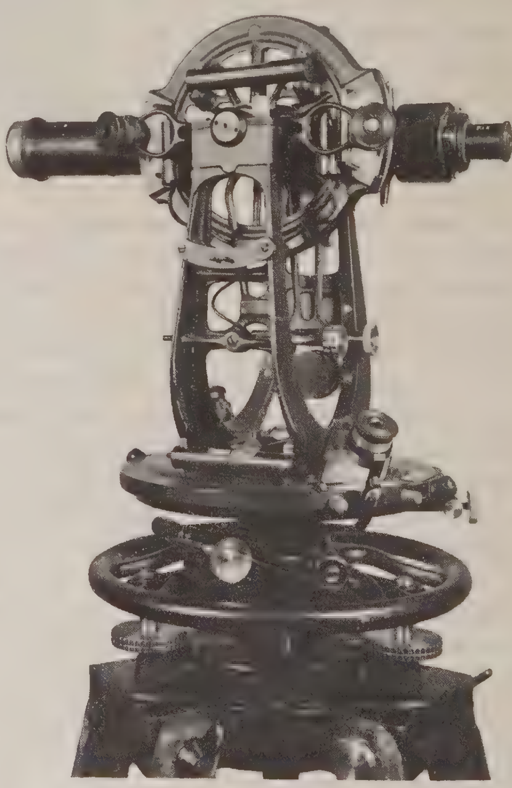


FIGURE 4004b.—A repeating theodolite.

FIGURE 4005a.—A surveyor's transit.

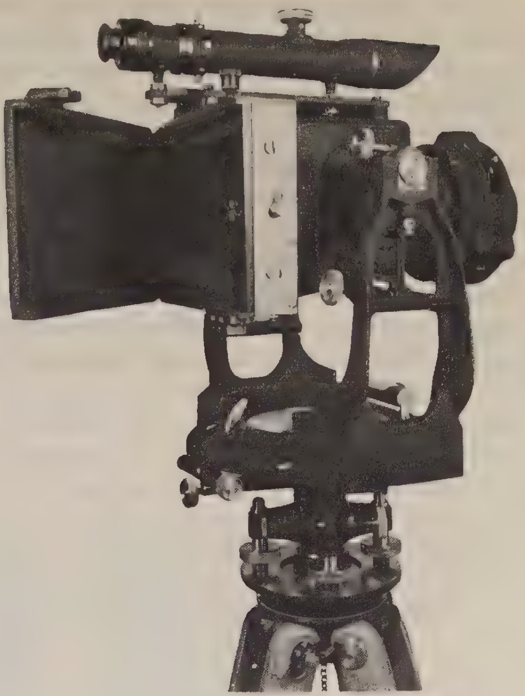


FIGURE 4005b.—A camera transit.

trol point as if it were an ordinary transit. A photograph is taken, then the instrument is turned in azimuth and another picture is obtained. This is repeated until a complete panorama of overlapping pictures is obtained around the observation point. During this procedure, the instrument may be pointed on other known points to obtain additional control. Prints of these pictures can be used for determining angles to additional points for supplementary horizontal and vertical control. A somewhat similar instrument, consisting of a combination camera and theodolite, is called a **phototheodolite.**

4006. Level.—The **precise level** (fig. 4006) is used for determining precise elevation differences between two points. The instrument illustrated has generally supplanted the "Wye" level formerly used in most hydrographic surveys. The split image of a sensitive level bubble is seen through an eyepiece adjacent to the telescope eyepiece, and as long as the two parts remain matched, the telescope line of sight is in the horizontal.

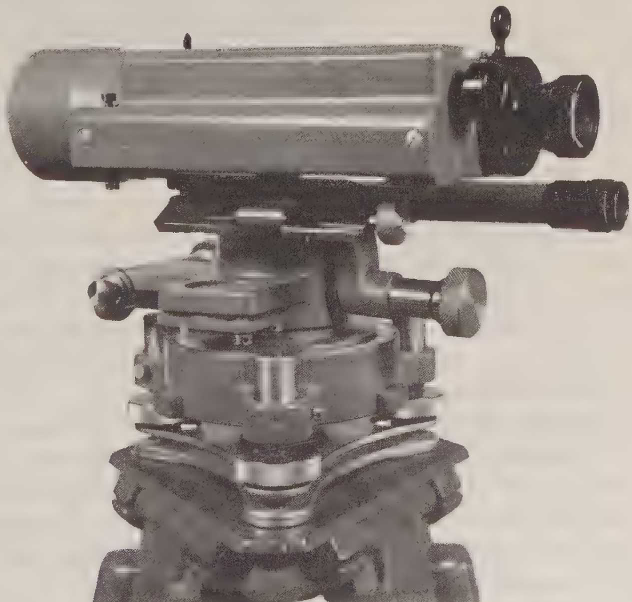


FIGURE 4006.—A precise level.

For reconnaissance or other rough work, a small hand level may be used. The **Locke type hand level** determines only a horizontal line of sight. The **Abney type** adds a small vertical arc which can be used for observing elevation angles. Neither should be considered a precision instrument.

4007. Distance measurement can be accomplished by any of several different methods and procedures. For precise work, special **tapes** which have a low coefficient of expansion and have been calibrated by the National Bureau of Standards under controlled temperature, tension, and support conditions are used. They are used in the field under standard tension and support, and the temperatures and support elevations are observed and recorded so that corrections can be applied to the measured distances to adjust them to the corrected horizontal distances. If such precision is not required, a surveyor's steel tape may be used.

Slightly less precise, but more rapid, is the measurement of distance by means of a **subtense bar** (fig. 4007). In this method, a direction theodolite is used to measure the angle between the end points of a distant Invar bar. The bar is mounted horizontally on a tripod and is oriented to be perpendicular to the line between its center and the theodolite by a small telescope mounted on it for this purpose. The size of the angle subtended by this bar is a measure of the distance from theodolite to bar. Tables

FIGURE 4007.—A subtense bar.

of angles and corresponding distances are available from the manufacturer, or can be computed.

Another still less precise method of measuring distance is by a **stadia,** a graduated rod. In addition to the cross hairs used in angle measurement, a transit is equipped with two other horizontal cross hairs so spaced that they will subtend one foot on a vertical stadia rod at 100 feet distance. At any distance the stadia cross hairs will intercept on the rod a length of about $\frac{1}{100}$th of that distance. If the ratio is other than 1:100, a stadia constant is furnished with the instrument or can be determined by comparing a stadia measurement of distance with the value of that same distance as carefully measured with a tape.

4008. Bottom samplers.—Samples of the bottom are obtained by means of **snapper** or **scoopfish** type **bottom samplers.** The former is secured to the base of a sounding lead and is used while the craft is lying to. Two clamshell-shaped castings are snapped together by a heavy spring when triggered by hitting the bottom, and a handful-size sample is thus obtained. The scoopfish is designed for use with the vessel underway. It is essentially a hollow tube with diving fins aft, and a special towing bridle. When properly set and towed, it dives to the bottom. When it strikes the bottom, a sample is forced into the tube, the bridle suspension point is automatically shifted forward so

that the instrument will no longer dive but can be hoisted up, and a cover flips into position over the forward end of the tube to retain the sample.

4009. Tide gage.—It is necessary to obtain as complete a record as possible of the tide in the survey area during operations. The information furnished by this record is used to determine the reference plane, or datum, for all heights and depths. It is also used for adjusting all original observations to that datum, and from it is computed the tidal data which is printed on the chart, such as mean sea level, spring rise, neap rise, lunitidal interval, etc.

The **portable automatic recording tide gage** (fig. 4009) is a light, compact instrument which records on single sheets of special paper a graph of the tide. The paper is clipped onto a drum which is rotated at one-half revolution per day by an eight-day clock movement contained inside. A float is suspended by a wire inside a pipe-float well

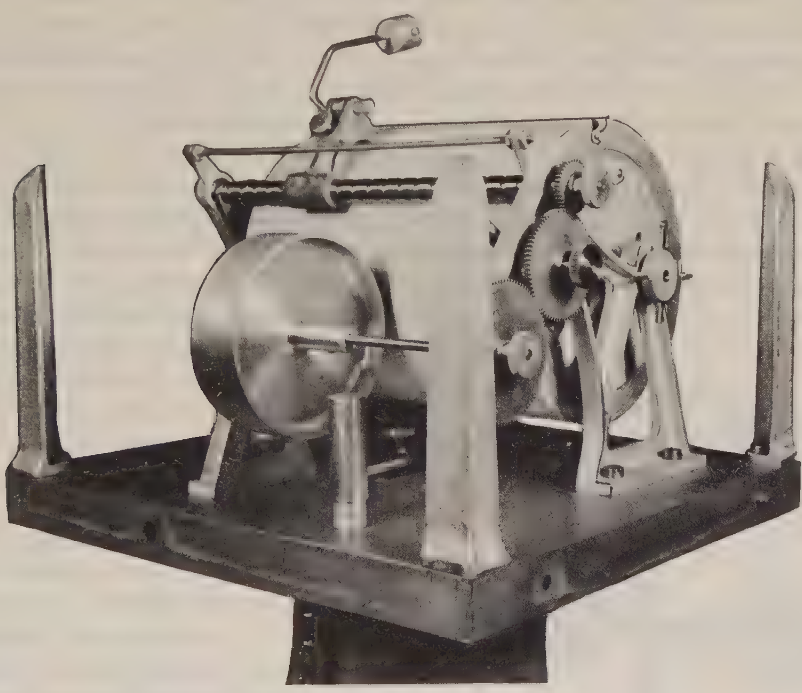


FIGURE 4009.—A portable automatic recording tide gage.

which protects it from wave and current action, yet permits long-period water level variations through a small aperture in the bottom of the pipe.

The float wire is guided up to the gage over an idler pulley centered over the top of the pipe. At the gage, the wire is wound around a grooved drum. Inside this drum is a spring which keeps tension on the wire so that as the float rises with the water, the drum rotates and takes up the slack. When the water level falls, the weight of the float overcomes the tension of the spring and the drum rotates in the opposite direction. The axle of the float-wire drum is geared to a long-pitch screw on which rides a stylus. This screw moves the stylus across the paper parallel to the axis of the record drum. Thus, while the record drum is rotated by its clock, the stylus is moved back and forth across the drum as the water level rises and falls. The record paper is black or red and coated with white wax. The stylus scratches the wax, drawing a graph of the water level variations with time as abscissa around the drum and height as ordinate across the drum.

Record paper is furnished in different height scales and corresponding sets of gears are used to vary the stylus motion. Thus, the instrument can be adapted for use in accordance with the range of the tide.

The **standard automatic recording tide gage** operates in a similar manner, with the following exceptions: the paper, fed from a supply roller, passes over a main roller across which a pencil rides, and is taken up by a receiving roller. The paper is long enough to accommodate a continuous one-month tide record. Two clocks are used; one to advance the paper by rotating the main roller, and the other to strike hour marks on the paper. The receiving roller, which winds the paper, is actuated by a weight suspended by a cord which is wound around a drum. Friction springs retard the supply roll so it will not unwind too fast. A counterpoise weight is suspended by a wire which runs over a drum secured to the same shaft as the float-wire drum. This keeps tension on the float wire and takes up the slack as the tide rises. In addition to the tide-marking pencil, this instrument is equipped with a datum-marking pencil. This is set to draw a straight line at the datum height. Scale changes to accommodate various ranges of tide are accomplished by using float-wire drums of varying circumference, different pencil screw pitch, and corresponding counterpoise weights.

A number of other methods may be used for observing tidal data. Most important is the **tide staff.** This is a graduated board from which the water height is read at regular time intervals. It can be installed vertically or, with properly exaggerated graduations, inclined. The latter installation is best used in calm water with small range of tide. Other devices for measuring tide include the **float gage, tape gage,** and **pipe gage.** These are all nonregistering. A number of gages have been designed to operate on the bottom. Their mechanisms are actuated by pressure changes caused by variations in the depth of water. This type of gage is not in general use.

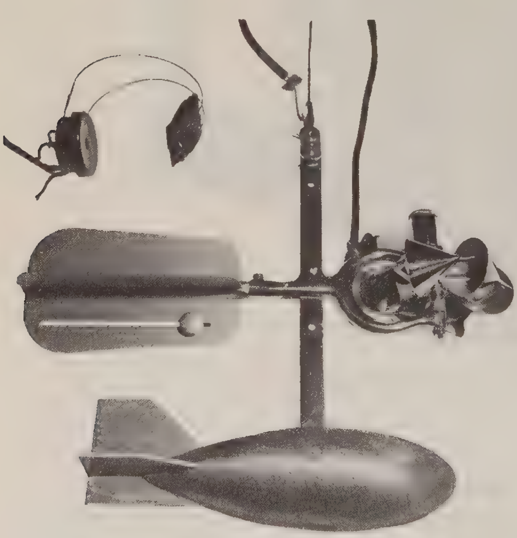


Figure 4010.—A device for measuring current speed.

4010. Current observations are also an integral part of hydrographic surveying. When printed on the chart, the information is valuable to navigators, especially in channels and other areas of limited maneuvering space. Types and designs of **current meters** are so varied that only those features which are common to most of them will be presented here.

In general, current speed is determined by counting the number of revolutions of a propeller per unit time. Propeller turns are counted in many ways. One type is illustrated in figure 4010.

Direction of the current is indicated by one of many methods of determining the heading of the meter relative to a compass magnet. The meters are kept headed into the current by fins.

4011. Drafting instruments.—Certain drafting instruments are of special value in plotting the information obtained in hydrographic surveys. Distance measurements in chart drafting are taken from a metal **diagonal metric scale** direct to the nearest 0.0001 meter. This device consists of a flat metal bar a little more than one meter long. Vertical lines (fig. 4011a) are spaced at intervals of one centimeter (0.01 meter) and graduated 0 to 100. To the left of the meter is an additional centimeter with diagonal

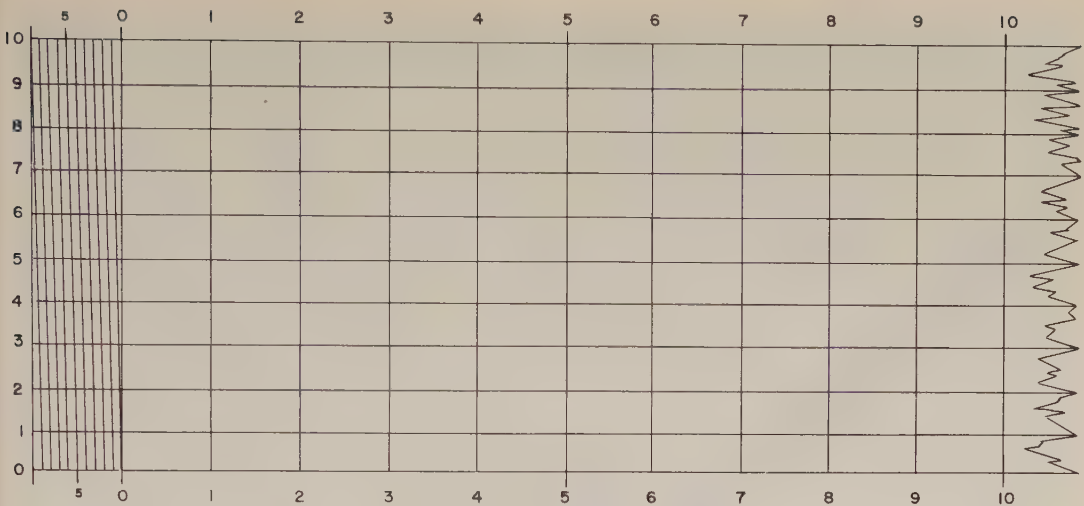


FIGURE 4011a.—A diagonal metric scale.

lines permitting measurement to two additional decimal places. Along the bottom horizontal line this additional centimeter is divided into ten equal parts. Hundredths are obtained by making the measurement on the corresponding horizontal line. Thus, a distance of 7.43 centimeters is measured horizontally along line 3, from vertical line 7 on the right to diagonal line 4 on the left. When the distance is greater than can be accommodated by the usual dividers or compasses, **beam compasses** (fig. 4011b) are used. This consists of a wood or metal bar on which slide two beam heads carrying steel points, exchangeable for an inking pen or pencil point. A thumb screw on each point clamps it in position on the bar, and one is equipped with a slow-motion screw for fine adjustment.

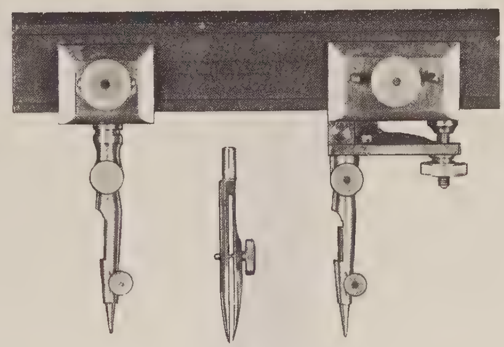


FIGURE 4011b.—Beam compasses.

A **three-arm protractor** is used for rapid plotting of three-point fixes. It can also be used for plotting angles to secondary survey stations. The center arm of a three-arm protractor is secured to, or a part of, a graduated circle. The left and right arms are pivoted about the center of this circle and are equipped with clamping devices. A plastic type has a two-minute vernier on each movable arm, and angles can be set to the nearest estimated one minute. This type is easy and rapid when plotting three-point fixes on signals which are comparatively near the boat position. For more precise plotting and for fixes on distant signals, the metal protractor (fig. 4011c) is used. The verniers of this instrument read to the nearest one minute. A magnifying glass is attached to facilitate reading the verniers, and slow motion screws are provided to permit fine adjustments. Detachable extension arms are furnished for use on distant objects. Since only one arm can be set to small angles down to zero, this instrument is manufactured in left- and right-hand models, on which the left and right angles, respectively, may be set to zero.

Proportional dividers (fig. 4011d) are an aid to transferring measurements between charts or other drawings which are not to the same scale. This device consists of two

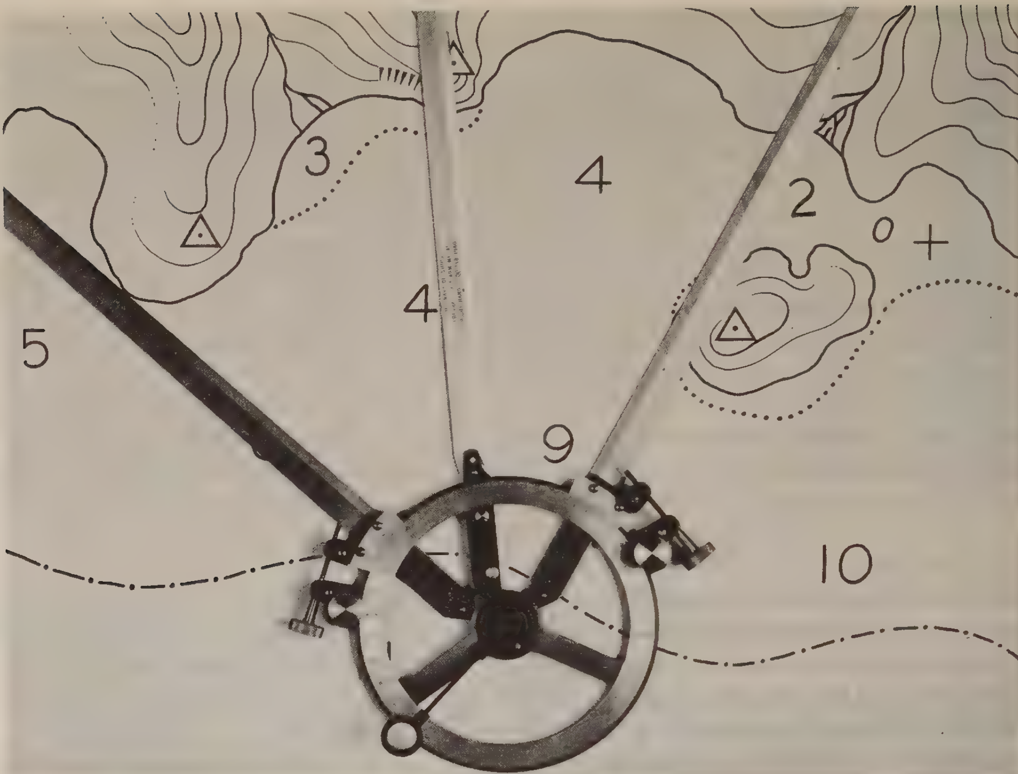


FIGURE 4011c.—Three-arm protractor.

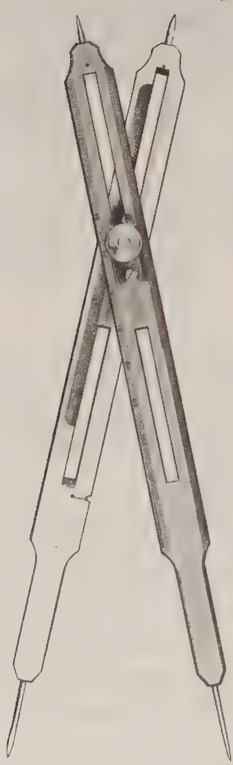


FIGURE 4011d.—Proportional dividers.

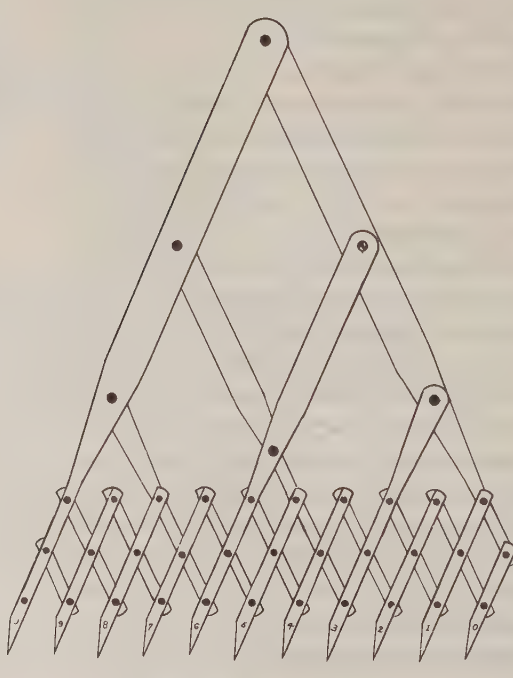


FIGURE 4011e.—Spacing dividers.

legs, each with a point at each end, and a movable pivot. When the pivot is at the middle, the two leg openings are equal. If the pivot is moved toward one end, the leg opening at that end is less than at the other, at a fixed ratio. The path of travel of the pivot is graduated so that a predetermined ratio can be set on the instrument.

Spacing dividers (fig. 4011e) are useful in subdividing distances into equal parts, such as spacing soundings along a line between two boat positions. They are designed so that as they are opened, the spaces between points are equal.

Other, commonly used drawing instruments such as scales, triangles, etc., are also used in chart work.

CHAPTER XLI

HYDROGRAPHIC SURVEYING

Introduction

4101. Hydrography is that science which deals with the measurement and description of the physical features of the oceans, seas, lakes, rivers, and other waters, and their adjoining coastal areas, with particular reference to their use for navigational purposes. The process of making the measurements upon which the description depends is called **surveying.** Precise determination and marking of positions on land, and accurate measurement of a reference direction and distance, taking into account the earth's curvature, constitute a **geodetic control survey.** Measurement of details of water areas and appropriate details of adjoining coastal areas is called a **hydrographic survey.** In addition to delineation of coast lines and location and measurement of submerged features, a hydrographic survey usually includes measurement of magnetic declination (variation) and dip, tides, currents, and meteorological elements. Limited surveys may be conducted to satisfy particular requirements. Before a hydrographic survey can be conducted, a geodetic control survey may be needed if available information does not provide adequate control of positions.

The principal objective of most hydrographic surveys is to obtain information on water areas and adjacent coastal regions, to serve as source material for nautical charts, sailing directions or coast pilots, and other nautical publications of value to the mariner. The results of the surveys are also used for planning harbor improvements and seaplane anchorages; for studies of silting and erosion, oceanographic features, and earth sciences; and for military defense projects.

Nearly 71 percent of the earth's surface is covered by water. Only a small part of this area has been adequately surveyed, and much of the land area has not been accurately measured. The changes caused by nature and man, and the continual increase in requirements of more precise and more nearly automatic systems and methods of navigation, render obsolete the charts or surveys once considered adequate. Consequently, the need for ever more accurate, more complete surveys continues, with no end in sight.

Surveys are usually conducted by personnel who have been given specialized training, and are provided with complete equipment. A modern survey ship is shown in figure 4101. Detailed information on the conducting of such a survey is given in Special Publication No. SP–4, *Hydrographic Office Technical Specifications for U.S. Naval Surveys and Supplementary Data;* and in U.S. Coast and Geodetic Survey Pub. No. 20–2, *Hydrographic Manual.* The purpose of the present chapter is to acquaint the mariner with the principles of surveying, to provide him with sufficient knowledge to conduct an exploratory survey (art. 4127) of a previously uncharted area, and to obtain and record in suitable form new data for the correction and improvement of existing charts.

4102. Planning a survey.—As in other operations, an efficient and adequate survey requires advance preparation. In addition to a knowledge of surveying, one should acquaint himself with the available information on the area. A study of all available charts, aerial photographs, and written material should be the first step. Work sheets or planning charts can be prepared showing the location of principal

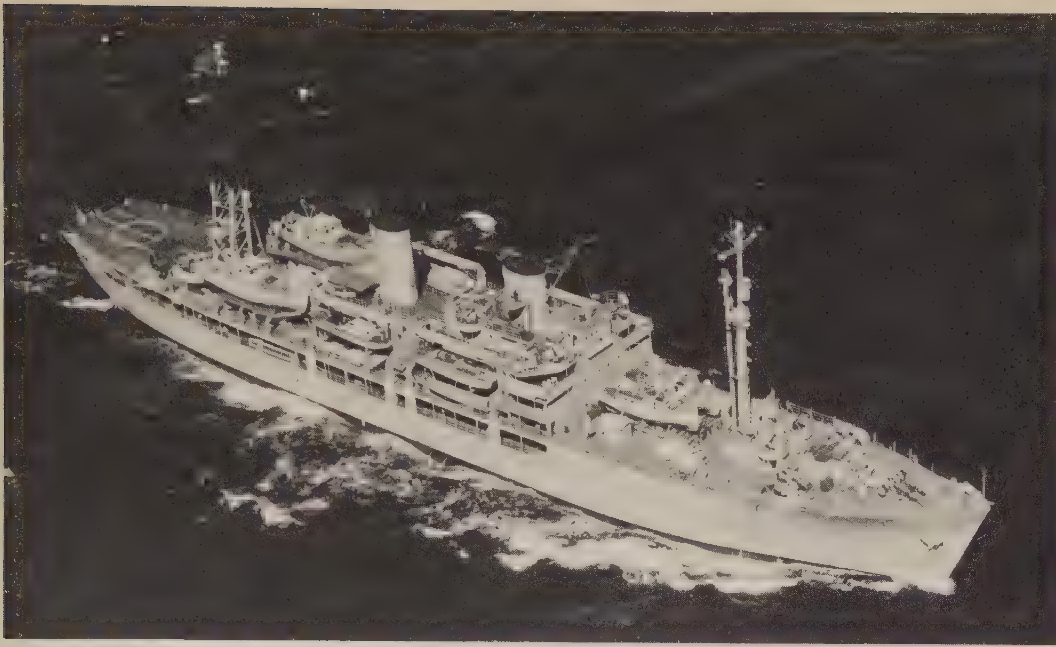


FIGURE 4101.—A modern survey ship, the USS *Maury* (AGS16).

landmarks; general configuration of coast lines; tentative anchorage areas, landing beaches, and camp sites; probable sites for the erection of signals to mark the various stations; and possible tide gage locations. Several copies should be made.

When the survey craft arrive at the area to be surveyed, several reconnaissance parties should be sent out to verify the information on the work sheets or planning charts. Tentative anchorage areas should be the first investigated. Protection during foul weather is essential to the safety as well as the comfort of the personnel who will man the small craft used for inshore soundings.

In the selection of landing beaches, safety of personnel and equipment is the primary consideration. Surf conditions, beach gradients, and bottom characteristics should be observed. The convenience of the locations and access to good routes of travel on shore are also important.

The selection of sites for signal marks should be made by an experienced surveyor, if one is available. Visibility from the sea is the most important consideration, but intervisibility among stations, or with those used for a geodetic control survey, are important to the accurate establishment of the positions of the stations. High points and conspicuous features of the terrain should be utilized to the fullest extent, not only for hydrographic control, but also as aids in controlling aerial photography. A helicopter is a valuable aid in making such a study, permitting rapid reconnaissance of the whole area.

Particular care should be exercised in the selection of a tide station. It should be located in a protected area where there is little wave action, but where access of sea water is adequate and representative of the area. A depth of at least five feet below the predicted lowest tide is desirable. The tide gage should be installed on a rigid structure, which may have to be constructed if one is not already available. Wharves are most frequently used, but the gage should be so located that it will not be damaged by operations in the area.

A decision will have to be made as to whether or not a geodetic control survey is needed and, if so, its extent.

Geodetic Control Survey

4103. Origin of survey.—An important requirement of a survey is to establish the position of each feature measured. The first step is to determine carefully the location of one reference point. This should be established as accurately as circumstances permit, for all other positions are located relative to this one point, called the **origin.** Any error in the origin is carried over to the entire survey.

If an accurate land survey has been made in the vicinity, the origin of the new survey might be determined with respect to the earlier survey, so that there will be no discontinuity between surveys. If this method is not available, an astronomical position is customarily obtained.

4104. Astronomical observations are made by the best method available. The most accurate available in the field is generally by astrolabe (art. 4002). Both latitude and longitude are determined by a single set of observations. If a direction theodolite (art. 4004) is used, latitude and longitude are determined separately and somewhat less accurately. If neither of these methods is available, position is determined by the best available means. However careful the measurement, all astronomical positions are subject to a possible error due to deflection of the vertical (art. 1610).

Surveyors generally time observations by means of a chronometer rated to sidereal time, and set approximately to GST. The chronometer error on GST is determined by finding the GST at the time of comparison (using the *American Ephemeris and Nautical Almanac*) and comparing this with the reading of the chronometer. Usually, the longitude of an assumed position is converted to time units and combined algebraically with the chronometer error (or correction) on GST to find chronometer error (or correction) on LST. Thus, if the chronometer is $8^m15^s.2$ *fast* on GST and the longitude is $5^h06^m23^s.4$ *east* (76°35′51″.0E), the chronometer is $4^h58^m08^s.2$ *slow* on LST.

For survey accuracy, the *Nautical Almanac* does not provide sufficiently precise data. The *American Ephemeris and Nautical Almanac* or other source should be used. If observations are timed by sidereal time, local hour angle is found by subtracting right ascension (art. 1426) from LST. Similarly, the sight reduction methods commonly used by navigators are not sufficiently precise for use in surveying. In general, only those computations necessary to the conducting of the survey are made in the field. All of the data should be sent to the appropriate government charting agency, where detailed computations are made to check and perhaps refine those already made, and to supply the additional answers needed for interpreting and utilizing the information. The important part of the field work is to make all measurements carefully and accurately, and supply all needed data, suitably labeled, so that the end products will be reliable. All measurements should be made to a higher order of precision than in ordinary navigation.

4105. Observation by astrolabe.—The astrolabe is set up and carefully leveled. Star lists are available to indicate the name and constellation, right ascension, magnitude, azimuth, and local sidereal time at which various stars will have the desired altitude, neglecting refraction. A body near the prime vertical should be observed first to provide a check on the local sidereal time, which is used for timing observations. As each additional star approaches the fixed altitude of the astrolabe, the observer picks it up and centers it in the lower power eyepiece. He then shifts to the higher-power eyepiece, and as the body reaches the fixed altitude, he presses a telegraph key, recording a mark on a chronometer-chronograph record tape. If available, about 100 celestial bodies are thus measured.

About ten or 12 of these stars, well distributed in azimuth around the entire horizon, are selected. The zenith distance of each star is computed for the time of observation. Computation is customarily made by means of the navigational triangle; assumed latitude, meridian angle, and declination of the body being known. With a good assumed position, the computed zenith distance of most bodies will be greater than that indicated by the astrolabe used (30° for a 60° astrolabe; 45° for a 45° astrolabe) because of refraction. The differences in zenith distances, computed minus assumed (30° or 45°), are plotted as distances in seconds of arc along the azimuth line of the body, from the assumed position. A negative difference is plotted along the reciprocal of the azimuth line. At the points so determined, lines are drawn perpendicular to the azimuth lines. These are lines of position. If they are accurate, they

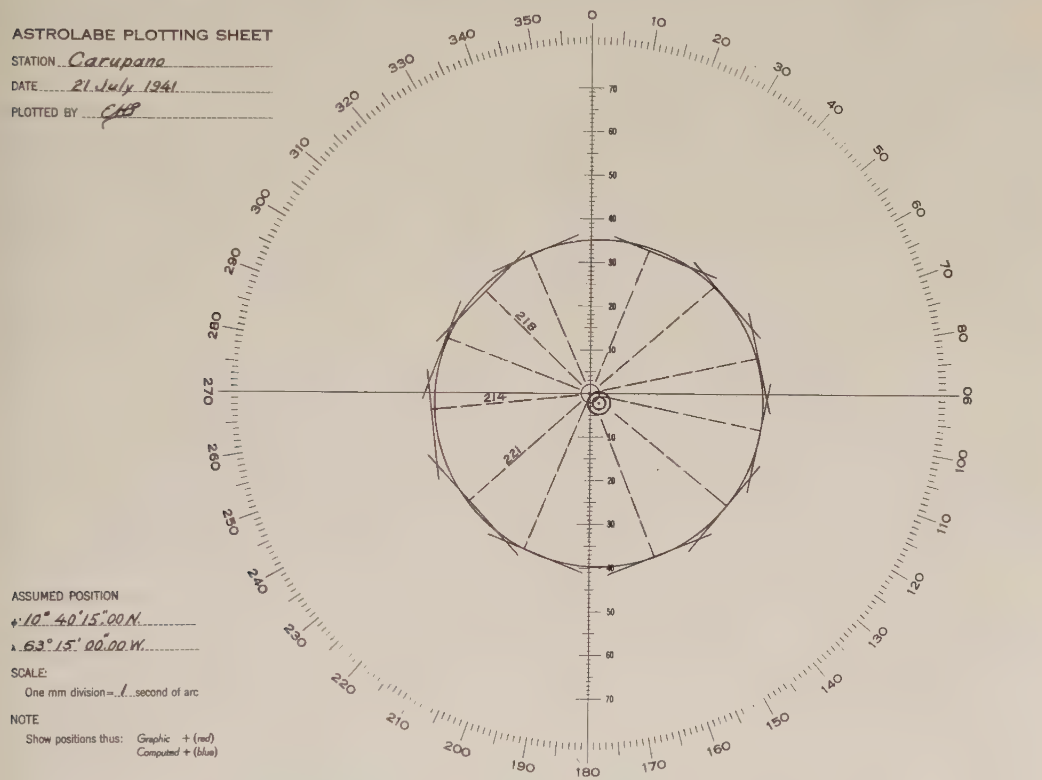


FIGURE 4105.—A typical plot of astrolabe observations.

are tangent to a circle which can be drawn within the figure formed by them. The radius of the circle is the constant error of observation, the principal component of which is refraction. The center of the circle is the position of the observer. Its latitude and longitude can be found by applying corrections to the assumed position. For highly accurate results, a correction is applied for convergency of the meridians. A typical plot of astrolabe observations is shown in figure 4105.

4106. Observation by direction theodolite.—When accuracy requirements are less exacting, as for a magnetic observation station, a position can be obtained in considerably less time by means of a direction theodolite.

Zenith distances of two stars are measured when they are within 5° of the prime vertical (one east, the other west) and their altitudes do not differ by more than 1°. If convenient, a minimum altitude of 30° should be used. Two sets of such observa-

tions are made, each observation being carefully timed. The meridian angle at the time of observation can be computed by time sight formula (art. 2106). This can be converted to LHA, which can then be compared with GHA to determine longitude. Surveyors generally compute local sidereal time and compare this with the Greenwich sidereal time of the observation.

The selection of stars near the prime vertical can be made by measuring the horizontal angle from Polaris or any other star near the meridian.

Latitude is determined from theodolite observations of northerly and southerly stars near the celestial meridian. In north latitude, Polaris is usually used for the northerly star. The two stars should have altitudes which do not differ by more than 1°, and should be within about 2° (azimuth) of the meridian.

4107. High-latitude observations.—The methods described in articles 4105 and 4106 are unsuitable in high latitudes because of the nearly-horizontal apparent motion of celestial bodies, and the continuous daylight during the summer, when surveys are customarily conducted.

The method usually employed is to set up a direction theodolite and observe the zenith distance of the sun at approximately hourly intervals. Timing is probably best done by means of a stop watch, which is started at the moment of observation and stopped at a convenient chronometer time shortly thereafter. Chronometer time at comparison minus the interval recorded by stop watch is the chronometer time of the observation. The chronometer should be checked by radio time tick immediately before and after observations, and the difference in chronometer error distributed evenly over the period of observation.

Local hour angle is determined and converted to meridian angle. With assumed latitude, declination, and meridian angle, the zenith distance of the sun is computed. This is compared with the observed value *corrected for refraction*. The difference is used to plot the line of position as in ordinary navigation. Since zenith distances are used, a greater computed value results in a *toward* situation. The navigator may find it less confusing to convert all zenith distances to the more familiar altitudes. If all observations and computations are completely accurate and the actual refraction does not vary from that used in the computation, all lines of position will intersect at a common point. However, this is rarely the case, and the center of the plotted figure is used, as in ordinary navigation.

4108. Direction.—A reference direction for a survey is established by carefully determining the angle between a meridian and the straight line connecting two prominently marked points. The angle is measured clockwise from *south*, as astronomers usually measure azimuth. This direction is determined at one of the marks by observation of the azimuth of a celestial body. The reference direction, which a navigator would call a "bearing" (measured from south), of the second mark from the observer is customarily referred to by surveyors as an "azimuth."

Azimuth is established by observation of a celestial body. A body having a nearly constant azimuth during the period of observation is the most desirable to use. In the northern hemisphere (except in very low or high latitudes) Polaris is ideal for this purpose. For maximum accuracy, it should be observed when it is at its greatest horizontal distance from the north celestial pole, for at this time its movement is most nearly vertical and the least change in azimuth occurs. If another body is used, it should be observed when its motion is most nearly vertical. A body that crosses the prime vertical should be observed when its meridian angle is 90°. For a body that does not cross the prime vertical (declination greater than the latitude of the observer, and of the same name), the desired condition occurs when the body is

nearest the prime vertical. This time can be determined by means of table 25, or by an inspection table such as H.O. Pub. No. 214. When H.O. Pub. No. 214 is used, the desired condition is indicated by a number of identical, or nearly identical, azimuth angles in consecutive entries in a column. The meridian angle at the most favorable moment can be converted to LHA, and this to GHA, which can be used with an almanac or the ephemeris to determine the time to make the observation (art. 2107).

Azimuth is determined by measuring the horizontal angle between the celestial body and the mark indicating the second place, using a theodolite. The azimuth of the celestial body is computed to the nearest 0″.1, and the horizontal angle is added (subtracted if measured counterclockwise from the body) to determine the desired azimuth. For best results, celestial bodies having low altitudes should be selected, if such are available.

A direction measured at the same point at which an astronomical longitude is determined is called a **Laplace azimuth.** Positions and directions at a second place, determined by a series of measurements of direction or distance, or both, do not, in general, coincide with values obtained by astronomical observation at the second station because of a difference in deflection of the vertical at the two places. The position as determined by a series of measurements from a "known" position is called a **geodetic position.** The geodetic azimuth differs from the astronomical azimuth by the amount of the **Laplace correction,** which is equal to $(\lambda_A - \lambda_G) \sin L_G$, where λ_A is the astronomical longitude, λ_G is the geodetic longitude, and L_G is the geodetic latitude. The accumulated error in a series of measurements is far greater in azimuth than in longitude. The Laplace correction, which assumes all the error is in the computed geodetic azimuth and none in the computed geodetic longitude, is applied to the astronomical azimuth at the second place to find the corrected geodetic azimuth.

4109. The base line.—Following the determination of a single accurate position and a reference direction, the length of a **base line** is measured, to serve as the basis for other measurements of distance. The length of the base line should be at least one-fifth that of the average side of the principal network of lines of the survey.

The length of the base line should be determined as accurately as equipment and conditions permit. For field surveys conducted by the U. S. Navy Hydrographic Office, the maximum error is specified as one part in 150,000. This is one foot in about 25 nautical miles, or about half an inch per nautical mile. The probable error (art. 2904) specified is one part in 500,000.

For such accuracy a carefully calibrated, low-expansion-coefficient, Invar tape is used under a standard tension which allows for sag and stretching. Corrections are applied for temperature, height above sea level, and inclination (if the ground is not level). With standard professional methods this accuracy can be obtained over moderately rough terrain with slopes up to 20°. The base line is divided into sections about one kilometer (a little more than half a nautical mile) in length. Each section is measured in each direction, using separate tapes, if available. It may be necessary to clear the line of brush or other growth to provide an unobstructed view. Stakes are driven at each tape length, and the distance between stakes is measured. A precise level (art. 4006) is used to determine the inclination. In making the measurement, care should be exercised to prevent kinks, abrasion, and tension greater than that prescribed.

The measurement of a base line can require a considerable amount of time. An approximate length can be determined quickly by means of a subtense bar (art. 4007) or even by a stadia, making the measurements in lengths of about 300 feet. Results obtained in this way are sufficiently accurate for graphic plotting done in the craft

running sounding lines. If later measurement indicates the need for adjustment, this can be provided by multiplying all distances by the ratio of accurate distance to preliminary distance. For a plot, only the *scale* need be changed.

If the survey covers a limited area, as a harbor, a single base line is sufficient. However, if the survey is to extend over a considerable area, additional base lines are needed as checks.

4110. Triangulation.—A network or chain of triangles with vertices at selected points on the ground is established in the area to be surveyed. In the selection of these points, which are commonly called **stations,** consideration is given to both topographic features and geometric factors. By carefully measuring the angles at the vertices of each triangle and using these measurements together with the position of the origin of the survey, and the length and direction of the base line, one is able to compute the position of each station. This forms a **triangulation net** (fig. 4110) covering the area and serving as the framework or skeleton to which all other positions are referred. The stations should be selected carefully to provide a strong net. This net, with its reference to the origin and observed azimuth, is called the **geodetic control** of the survey. Those control points which are used for fixing position during survey of the water area constitute the **hydrographic control.**

4111. Trilateration.—If a network of lines similar to a triangulation net is established by measuring the *length* of each line instead of the angles between lines, the process is called **trilateration,** and the system of lines is called a **trilateration net.** If lengths are measured by tape, as explained in article 4109, this is a time-consuming and often difficult or impossible process. However, with the development of electronic methods of measuring distance, trilateration became a practical method. In periods of low visibility, or where lines are too long for visual observation (as between off-lying islands and the shore), it may be the only method available.

4112. Traverse.—In some areas the best, or only available, method is by measuring both direction and distance of a series of lines. This is a **traverse.** It might be used where a clear view of one station is not available from others, or along an irregular beach. The application is usually limited, so that a traverse is generally somewhat subsidiary to the main triangulation or trilateration.

In planning a traverse, one should select a route affording reasonably long legs as free as possible from obstacles. When the traverse is used to connect two parts of a net, reciprocal directions should be measured at the two ends, to provide a check. This provides two routes by which a line of the triangulation or trilateration can be carried through to a line of the traverse. Where the traverse follows a route with many curves, as along a stream or beach, it should be connected to the main triangulation or trilateration net at several points. If this is not practicable, an azimuth line should be established between points several legs apart, so that directions can be carried forward with greater accuracy. All main traverse stations should be permanently marked, but intermediate stations need not be permanently established.

4113. Signals.—As each station is established, a conspicuous structure or **signal** is constructed or designated to mark the site. It is essential that the signal be accurately centered over the station, which for principal stations is marked by a bronze marker set in concrete. The signals take different forms depending upon the distance over which they are to be seen, obstructions, the need for identification, background, and the availability of existing structures. The shape, target area, and color are factors to be considered. The color is selected with particular reference to the background. The three types of signal most used for principal stations are:

Tower. A tower is used when needed to clear obstructions, where the distance is so great that the curvature of the earth is a consideration, or where a smaller target

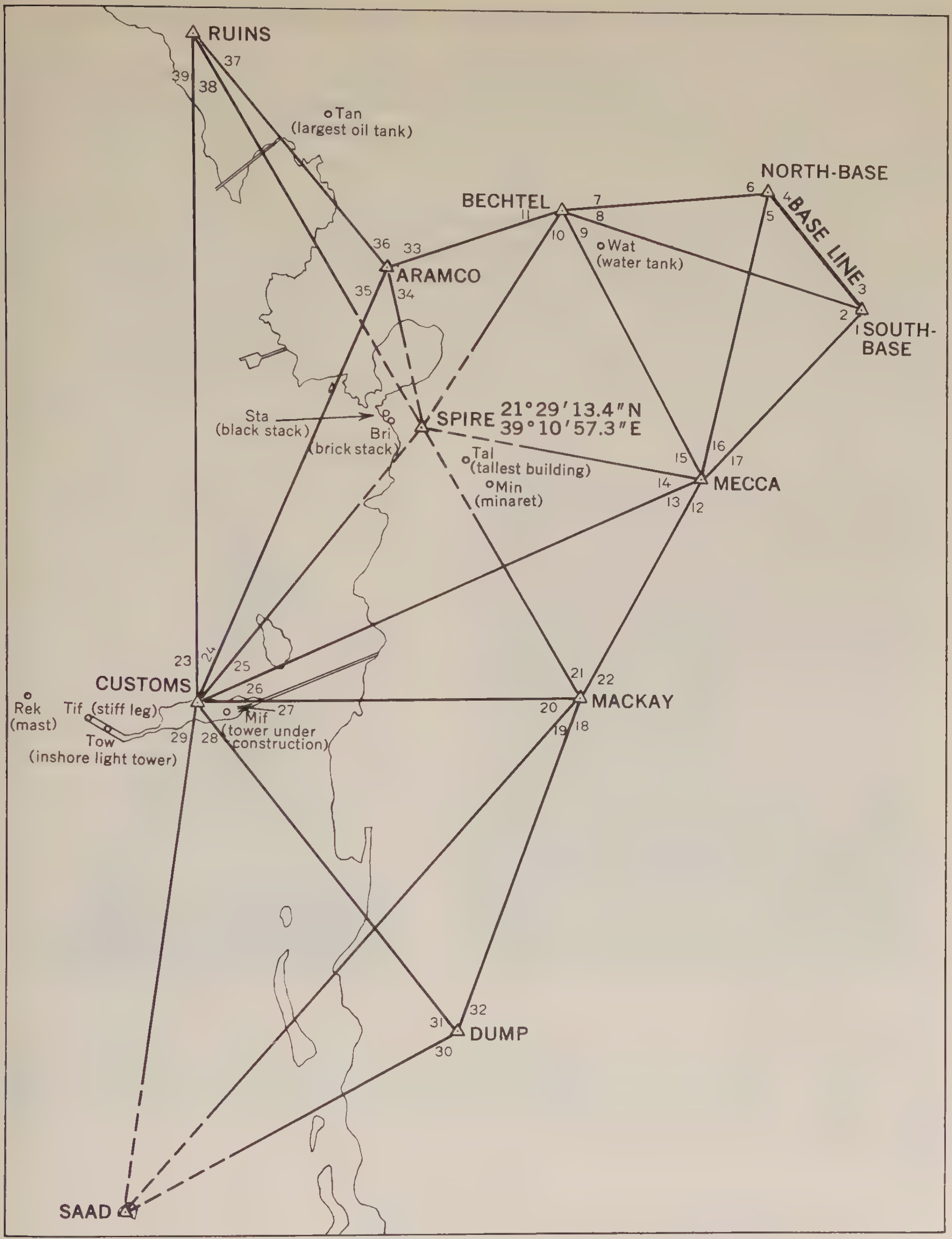


FIGURE 4110.—A typical triangulation net.

might not be sufficiently conspicuous. The type of tower generally used is an open-framework, prefabricated structure of galvanized steel, which can be assembled rapidly by an experienced crew (fig. 4113a). It consists of an inner tower to furnish instrument support, and an outer independent structure to support a platform for the observer. The tower is constructed in sections, to a maximum height of 113 feet. It can be made

smaller by omitting one or more of the bottom sections. Part of the outside may be covered with cloth to make it more conspicuous.

Tripod. A wooden tripod (fig. 4113b) is the signal used most frequently when the observer has good visibility from the ground. Usually, the lumber is cut aboard ship and assembled at the site. It is generally about 25 to 30 feet high and covered with cloth or provided with latticed lumber to make it more conspicuous. It is firmly anchored and guyed by wires as necessary.

Existing structures. Conspicuous church spires, chimneys, flagpoles, lighthouses, etc., can be used. For a complex structure, the specific part used should be specified. If there is any reasonable possibility of confusion, as when several chimneys are close together, the structure should be avoided. When an existing structure is used, a permanent marker is not installed. Usually it is necessary to observe from some point nearby, called an **eccentric point,** and provide a correction to the observations made from the station. This is done by measuring (1) the distance (D) between the structure

FIGURE 4113a.—A tower survey signal.

FIGURE 4113b.—A tripod survey signal.

and the point of observation; and (2) the angle (α), at the point of observation, between a line to the structure and one to another signal, at an approximate distance (s) from the site. The correction (C) in seconds of arc to be applied to the direction of the distant station observed is then

$$C=\frac{D \sin \alpha}{s \sin 1''}.$$

Secondary signals for intermediate stations may be improvised from any suitable material available. Examples are a single pole with cross-lattice work or a flag, a whitewashed tree trunk or rock, a whitewashed box or barrel filled with stones or earth and surmounted by a flag, a piece of sheeting wrapped around a bush, etc.

It is common practice to give each station a short name, for easy identification.

4114. Records.—It is of importance that measurements be made carefully, and that complete records be kept. Each observer should be provided with a notebook in which to make notes as the survey progresses. At the end of each period of observa-

tion these notes should be converted to good form for the permanent record of the survey.

All information should be evaluated as it is received. In many instances this requires at least preliminary computations to determine whether the work is of acceptable accuracy. Surveyors usually measure distances in meters to the nearest 0.01 meter and angles to the nearest 0.″1, and compute geodetic positions to the nearest 0.″01. In a triangle formed by survey lines, the three angles should equal 180° plus the **spherical excess** due to curvature of the earth. This amounts to about 0.″0175 per square nautical mile of area. If facilities are not available for computation, large-scale, carefully drawn plots may suffice. As the various positions are determined, they are plotted on a polyconic projection (art. 315). Any results that seem inconsistent with others should be measured again.

A complete description of each station, preferably with a sketch, should be prepared. It is generally desirable that preliminary descriptions be prepared independently by two observers, who should then collaborate in preparing the final description. This information may be needed if the station is to be reoccupied, perhaps many years later.

When a regular survey party is sent out, it is provided with standard computation forms, tables, and blank books for recording observations, as well as the instruments and other equipment needed to do the work. Others make the best use of whatever is available.

When the survey is completed, all of the records are forwarded to the government agency responsible for charting the area. For United States personnel, this is the U. S. Coast and Geodetic Survey for United States territory, and the U. S. Navy Hydrographic Office for foreign areas.

Hydrographic Survey

4115. Control.—Hydrographic surveys differ in several respects from geodetic control surveys. The surface of the water is relatively flat, and the water obscures vision of the relief of the bottom. As a result, sharp discontinuities in the bottom level, such as pinnacles, might escape detection. Permanent stations are not established at sea, and the lack of a stable platform precludes precision measurement of angles with the type of equipment used ashore. Measurement of distances by tape is impractical over water.

The principal function of a hydrographic survey is to determine depths of water. The positions at which soundings are obtained are determined by reference to established points on shore. In addition to locating the points at their correct geographical positions, this practice results in the land and marine features being in correct relationship to each other. This is an important consideration because the marine navigator near a coast also locates himself relative to the land, in many instances using the same landmarks used by the surveyor.

The means used for determining the position of the sounding craft is called **control.** The two kinds of hydrographic control in common use are **visual** and **electronic.** At great distances from the shore, celestial navigation might be used.

4116. Visual control is the determination of position by visual reference to conspicuous landmarks. The most commonly used method is to obtain horizontal sextant angles and plot the position by means of a three-arm protractor (art. 4011). This is called the **three-point fix method.**

Any conspicuous object which has been accurately located can be used. Geodetic control survey signals might be available. Natural objects such as prominent

trees or sharp mountain peaks are often used. Existing structures such as lighthouses and church steeples make satisfactory marks. When marks are not available at desired locations, a signal might be constructed. The type most commonly used is a single mast 20 to 30 feet high, to which three triangular skirts are attached at angles of 120° to each other (fig. 4116). A flag of distinctive color may be attached to the top to aid in identification. Floating signals may be used in shoal areas to extend control beyond the limits of shore visibility. The signal generally used consists of a mast provided with skirts or wooden slats, and supported on an anchored floating structure. The location of such a signal should be checked frequently, as it might be displaced by wind and wave action.

FIGURE 4116.—A single mast survey signal.

The distance between signals depends upon the scale of the survey, general contour, and visibility. In general, signals should be one-half to one mile apart for harbor and anchorage surveys, and one to two miles apart for coastal surveys.

The signals used for hydrographic surveys are normally positioned by reference to geodetic control survey stations. This is usually done by one of the following methods:

Intersection of bearing lines from three or more stations, the position being determined either by computation or plotting.

Resection by observing the bearing of three or more stations from the position to be determined.

Traverse from established stations.

A "ship-shore" method is occasionally used. Horizontal sextant angles between the signal to be located and an established point are observed aboard ship at the same instant that the shore party measures angles from known points to locate the ship.

4117. Electronic control is used in periods of low visibility, and beyond the range of normal visibility from shore. Any electronic positioning system meeting the accuracy requirements might be used. Those which have been extensively used are radio acoustic ranging (art. 1205); radar (art. 1208), usually with transponder beacons (art. 1108); shoran (art. 1213); electronic position indicator (art. 1213); Lorac (art. 1310); Decca (art. 1309); and Raydist (arts. 1214, 1311). Radio acoustic ranging, electronic position indicator, and radar are no longer in common use for control.

To provide survey accuracy, the electronic equipment should be accurately tuned and calibrated, and should be operated within the closest practicable tolerances.

The direct ranging methods (radio acoustic ranging, radar, shoran, and electronic position indicator) provide results that can be used without special equipment. Ranges are usually plotted by means of a number of concentric circular arcs drawn in advance on the plotting sheet, or in some cases by means of a beam compass and a diagonal metric scale (art. 4011). The accuracy of such readings varies with conditions, but about the best that can be expected for single readings is 15 yards for shoran, 75 yards for electronic position indicator, and 150 yards for radar with transponder beacons.

The hyperbolic systems (Lorac, Decca, and Raydist) require location of the hyperbolas. These are plotted at intervals, and intermediate values are obtained by interpolation. The accuracy of these systems varies with position relative to the transmitters, but is sufficient for offshore surveys.

4118. Plotting sheets.—When the approximate extent of the area to be surveyed has been determined, a master plotting sheet is prepared, usually on the polyconic projection (art. 315). The scale depends upon the contour and the amount of detail to be shown. As the various items of information are determined, they are plotted on this survey sheet. Smaller sheets are prepared for use of the parties conducting the survey. Close to the beach these are of the same scale as the master sheet, but farther out they may be of smaller scale. These smaller sheets may be called **boat sheets, ship sheets, shore line sheets,** etc., as appropriate.

4119. Topography.—The positions of the shore line, streams, mountains, hills, etc., may be available from a land survey. If this information is not available, it is determined as part of the hydrographic survey.

The position of the shore line is best determined by means of vertical aerial photography (ch. XLIII). If this method is not available, positions are obtained, usually by horizontal sextant angles, at short intervals along the beach. The beach line is sketched in through the established points.

Inland features are located by horizontal sextant angles or by transit angles from triangulation stations. The heights of hills near the shore can be determined by vertical angle measurement and table 9 (or more accurately by computation), or by difference in the reading of a barometer, using table 11 or the formula of article 3707. If enough information is available, contours should be sketched in. The location of a summit should be indicated by a dot, and the height indicated by a number.

4120. Hydrographic features.—Depth is determined by running a series of parallel **sounding lines.** Usually, these are run normal to the general trend of the beach, but in areas of shoals or other dangers, they should be run in such direction as to provide the best indication of the bottom features. The sounding lines should be spaced at intervals of two-tenths of an inch on the plotting sheet. Check lines should be run perpendicular to the main group, at intervals of perhaps two inches on the plot. When these sheets are prepared, the desired lines should be drawn lightly in pencil to serve as a guide to the sounding craft. Bottom samples should be taken at intervals of not more than two inches on the plotting sheet, except in depths greater than 50 fathoms, where bottom samples normally are not taken unless required for the oceanographic aspects of the survey.

In shoal water and sheltered areas, sounding lines are run by small craft. Farther from shore larger craft, including the survey ship itself, run the lines.

For harbor and anchorage surveys, the scale of the plotting sheets is generally 1:5,000, 1:10,000, or occasionally 1:25,000. A fix should be obtained every two minutes. Soundings should preferably be obtained by a recording echo sounder, to provide a bottom profile. Every 15 seconds the sounding should be recorded in a **sounding book** provided for this purpose. If the depth is not greater than 11 fathoms, soundings should be recorded to the nearest foot. For greater depths, the nearest one-half fathom is sufficient.

For channel surveys, the scale of the plotting sheet may be 1:10,000, 1:25,000, or even 1:50,000 in some cases. Fixes should be obtained at intervals of two minutes, and soundings recorded every 15 seconds unless the scale of the survey is 1:50,000, when every 30 seconds should suffice.

For coastal surveys, the scale should be about 1:50,000 to a depth of 20 fathoms, 1:100,000 between 20 and 100 fathoms, and 1:250,000 for greater depths. The interval between fixes should be about three, five, or ten minutes, respectively, for the three scales. Soundings should be recorded every 30 seconds for a 1:50,000 plot, and every minute for smaller scales.

Sometimes it is necessary to sound an area well offshore, as a bank in the open sea. The individual circumstances govern the choice of technique to use. Control is provided by the best means available. If the area is beyond the range of the electronic position indicator, celestial navigation or loran might be used. If the water is sufficiently shoal to permit anchoring, a relatively large number of observations might be made to establish one position from which others can be determined. Open ocean surveys are further discussed in chapter XLII.

In any hydrographic survey, an area in which the existence of a shoal or other obstruction is suspected should be sounded thoroughly by a number of closely spaced lines, to be reasonably certain that the least depths have been found and their positions accurately determined. The surest way of determining that the least depth has been found is to use a **wire drag.** This is particularly important in rocky or coral areas, where individual pinnacles may not be found by sounding, however thorough. Basically, a drag consists of a submerged horizontal "ground wire" suspended by upright wires from buoys and held at a constant depth by weights and submerged floats. The ground wire is towed over the area between two vessels, and will strike or *hang* on obstructions extending above the depth at which it is towed. If the ground wire rides up over the obstruction, the fact is indicated by the falling over of the supporting buoys. The depth at which the ground wire is towed can be varied by altering the length of the upright wires. The depth usually used is 42 feet. The wire drag was developed by the U. S. Coast and Geodetic Survey. A detailed description of the construction and use of the device is given in Publication No. 20–1 of that organization. Since wire drag surveys are costly and time-consuming, they are normally used only in critical areas such as important harbors, anchorages, and channels.

A pier and its surrounding area should be surveyed carefully. Its direction and dimensions should be established accurately. Hand lead soundings should be taken every 20 feet along the face of the pier. Additional sounding lines should be run parallel to the pier at distances of 20, 40, and 60 feet.

In general, a small stream is sufficiently surveyed for chart purposes if a few lines of soundings are run in the navigable part, parallel to the principal reaches, with an estimate of the distances to each shore. However, individual circumstances should govern.

4121. Tide and tidal current observations.—Tide observations should begin as soon as practicable, using the appropriate equipment (art. 4009). A permanent tide station may be installed, but more often temporary stations are used in surveying. If the area to be surveyed is extensive, or if local conditions indicate a possible wide variation in tidal conditions at different points in the area, several stations should be established at representative points. Observations should continue throughout the period of the survey, or longer if practicable. It is desirable that the period of observation extend over an entire synodical month (29½ days).

A sufficient number of tidal current observations should be made to establish the current pattern for the area, with particular reference to the direction and maximum speed in the principal channels, and the times of all maximum speeds and slacks.

If a current meter (art. 4010) is not available, observations can be made by an improvised method. A **current pole** consists of a pole weighted so as to float vertically, and having a **log line** attached. The pole is placed in the water from an anchored vessel or fixed point, and permitted to drift with the current. The amount of drift in a timed interval can be determined by measuring the length of line paid out. A simple computation can be used to convert this to speed. The direction can be determined by noting the direction the log line tends. This method is particularly adapted to current measurement at the anchorage of the survey ship. In the channels, a

launch can be permitted to drift with the current, its position being determined at short intervals by horizontal sextant angles and three-arm protractor, or other suitable method. Occasionally, current can be measured by water-soluble dyes.

4122. Magnetic measurements.—If specialized magnetic instruments are available, a magnetic observatory can be set up ashore to determine all magnetic elements for which equipment is available. Observations might continue throughout the period of the survey. If such equipment is not available, a magnetic compass might be taken ashore, free from the deviating influence of the vessel, and the variation determined by carefully measuring the magnetic direction of any accurately measured line of the survey. If no such line is available, magnetic azimuths of the sun or other celestial body can be measured and compared with the computed true azimuth at the same instants. A dip needle might be available to measure the magnetic dip.

It is desirable to take readings at a number of places, to check for anomalies. In the water areas, anomalies which affect variation can be detected by steering a steady course and measuring the compass bearings of established shore points from a series of known positions as the vessel proceeds. These can then be compared with computed or measured true directions to determine compass error, which should remain essentially constant as long as the course remains unchanged.

At the principal shore station, observations should preferably continue over the period of the survey, to eliminate the effects of any magnetic disturbances. Because of possible diurnal change, readings should be taken at different times during the day. If this is not practicable, readings are best made at about noon.

4123. Geographic names.—The correct names and spellings of all named places and features in the area covered by the survey should be determined from reliable local sources, noting any established variations. Full information on names should be submitted with the survey records.

4124. Aids to navigation.—The location of each aid to navigation should be determined carefully. A description of the aid should be prepared and, if lighted, its characteristics should be timed. Any discrepancies between actual conditions and information given in the light lists or sailing directions should be noted. The prominence of the aids with respect to their backgrounds should be observed, and any advisable precautions with respect to the aids should be recorded. Lines of demarkation between color sectors should be measured carefully. The directions and lengths of ranges should be measured. If aids are moved from time to time because of changes in hydrographic features or seasonal ice or weather conditions, detailed information should be recorded. Signal stations and other prominent landmarks which might be useful to a navigator should be located and described.

4125. Miscellaneous information.—In addition to the various measurements, descriptive information forms an important part of a hydrographic survey. This is useful in the interpretation of the measurements, and it provides a major source of information for notes on the charts, and for compilation of sailing directions or coast pilots. The amount and detail of the information to be collected varies with individual circumstances. The surveyor should be alert to note any items that should be included, recording the appropriate details as they come to his attention. Even negative information is helpful when it answers a question that might logically come to the mind of the mariner. Examples of the items that might be included are:

Errors, omissions, or ambiguous statements in publications such as sailing directions or light lists.

A description of the general trend, features, and aspect of the coast as it is approached. This description might well be supplemented with pictures or radar scope photographs from stated positions and heights.

The color and extent of discolored water.

The nature and extent of meteorological and seasonal influences.

The kind and type of ice.

The location of fishing stakes, nets, and fishing boat operations.

The location, landing places, and shore markers of submarine cables; and the location and height of overhead cables.

The location of ferry crossings and other areas where local traffic may be heavy.

The location of restricted or military operating areas, with a statement of the regulations pertaining to them.

The location and extent of overfalls, rips, etc.

A description of the various awash dangers at various stages of the tide.

The location and nature of all wrecks, with all pertinent information regarding their visibility, depth, markers, etc.

Whether or not channels are dredged, and the probability of their filling with sediment.

Safe speed to use through channels, confined waters, etc.

Suitability of anchorages with respect to holding qualities, availability of mooring buoys, freedom from obstructions, direction and speed of wind and currents, amount and direction of swell, etc.

Location and description of special anchorages, with the regulations concerning their use.

Prevalence of fog and other visibility-limiting phenomena.

Any needed explanatory information on tides and currents.

The appearance and effect of mirages, abnormal refraction, phosphorescent seas, etc.

Local harbor regulations.

Port and aerodrome facilities.

Pertinent information regarding shore settlements.

4126. Records.—As information is collected, it should be evaluated and incorporated in the one master record. Each item should be verified as it is recorded. When the survey has been completed, the smooth copy of the completed information should be sent to the appropriate government charting agency. When an acknowledgement of the receipt of this information is received, the additional records such as sounding books, angle books, etc., should be forwarded.

Limited Surveys

4127. Exploratory survey.—When time or lack of equipment does not permit, or where desired results do not justify the carrying out of a standard geodetic control or hydrographic survey, a limited exploratory survey may be conducted. This might be an advance investigation to determine the desirability of making a full detailed survey, an operation to make a preliminary chart of an anchorage, an investigation of a reported shoal, etc. The principles and techniques in general conform to those described earlier in this chapter, but are adapted to meet the requirements, instrument limitations, and training of personnel. This is the type of survey that might well be assigned to a nonsurvey vessel.

When the area to be surveyed is covered by maps or charts of reasonable reliability, it is customary to establish the origin of the survey by scaling the position of one landmark from the chart. When this source of information is not available, the origin might be determined by careful measurement of electronic or celestial information needed for a position. If the position is determined from celestial observations ashore,

a theodolite or transit should be used if available. If it is not, an artificial horizon might be used with a sextant. It is desirable to observe at least 12 bodies well distributed around the horizon. At sea, it is desirable to anchor, or remain in the vicinity of an anchored aid to navigation.

A reference direction is best determined by astronomical means. If a theodolite or transit is not available, a sextant might be used with a body at low altitude, measurement being made of the horizontal angle. If visibility limitations or available time does not permit, a gyro compass might be used, as follows: With the ship at anchor, bearings of an observer on shore are observed. The observer measures the horizontal angle between the ship's gyro repeater used for the observation, and a landmark, using a theodolite, transit, or sextant. The reciprocal of the gyro repeater bearing, with the measured angle applied, is the direction from the observer to the landmark.

If the length of the base line cannot be measured by one of the methods described in article 4109, an approximation of sufficient accuracy for some purposes might be determined by measurement from the ship, using any available means, such as radar. Either of two methods might be used. A base line across navigable water might be selected. As the ship steams across the base line the distance to each shore station is measured. The least sum of the two distances is the length of the base line. The average of several such determinations should be used. By the second method, the distance to a single shore station is measured. At the same moment, an observer at the shore station measures the angle between the ship's radar antenna and a second shore station. At the second station an observer measures the angle between the antenna and the first shore station. With this information, the length of the line between the two shore stations can be computed.

If triangulation is needed, it is carried out as accurately as time and equipment permit. If horizontal sextant angles are used, the stations should be at nearly the same height if practicable.

An essential part of an exploratory survey of a harbor or anchorage is to delineate the shore line and coastal topography as accurately and completely as time permits. The quickest and best method is to use vertical aerial photography, if available. If this is used, established control points should be marked and described. If this method is not available, a reasonably accurate method is to run a traverse along the beach. A quick method of obtaining a rough approximation consists of determining radar bearings and distances to a number of shore points, from an anchored ship, and sketching in the shore line. A photograph or trace of a radar PPI presentation is another possibility.

Heights might be determined by transit or sextant angles, or by air search radar, with table 9.

Sounding lines are run as close together as conditions and time permit. Fixes are obtained by horizontal sextant angles, cross bearings, or radar at such intervals as warranted by the requirements of the survey and available time and equipment. Fixes at three-minute intervals are commonly used in harbor areas. The position of a sounding boat might be determined relative to the anchored survey ship, using radar. The sounding lines should be run in a systematic manner, with shoal areas being given extra attention. Soundings should be plotted directly on the work sheet, and fathom curves sketched in as the information becomes available.

Tide and current observations should be made as completely as time and conditions permit. A tide staff is usually used with half-hourly readings of the height. Current is usually measured by an improvised current pole (art. 4121).

A complete and accurate record should be made and forwarded to the appropriate government authority upon completion of the work.

4128. Running survey.—A limited survey can be conducted as a ship steams along a coast. The position at the beginning of the run is determined as accurately as conditions permit. If accurately charted landmarks are not available, it may be possible to send a landing party ashore to establish a good astronomical position.

As the survey progresses, the ship steams at a safe distance from the shore, determining its courses and speeds as accurately as practicable. Natural ranges may be available from time to time to provide good courses. If charted shore objects are available, frequent fixes can be determined and the dead reckoning between them adjusted to avoid gaps in the plot. As the ship proceeds, continuous soundings are taken, preferably by a recording echo sounder. Bottom samples are taken at frequent intervals if conditions permit, and if required.

If the shore has not been accurately surveyed and charted, positions of various prominent landmarks are established by a series of horizontal sextant angles or bearings, or by radar, as the vessel proceeds. A minimum of three readings should be made on each object, so that its plotted position will be reasonably accurate, and to be sure of identification. Since errors in this method are cumulative—the positions of landmarks being established from the ship, and then future positions of the ship established by means of the same landmarks—it is desirable to make all measurements as accurately as practicable. Generally it is best to steam at moderate and constant speed, stopping only if this contributes to the establishment of better positions.

If available, one or more launches might proceed along parallel courses between the ship and the shore to obtain additional lines of soundings. Their positions might be determined by a series of bearing and distance measurements, as by radar, or whatever means are available. These launches can collect additional information regarding the shore line and beach topography. Under some conditions a launch might contribute most to the survey by proceeding at will, obtaining angles and making sketches and notes, rather than taking soundings. With a recording echo sounder it might serve both functions.

All observations should be recorded, and all positions plotted as soon as received, so that apparent errors might be corrected while the landmarks are still visible. Because of the approximate nature of the survey, a large scale plotting sheet is not justified, a scale of 1:100,000 usually being adequate. Sketches and descriptions of various details along the coast can serve useful purposes later. If shore parties are landed, distinguishing marks might be established at some points. The amount of detail recorded depends primarily upon the time available, and perhaps upon the requirements of the survey. Discrepancies are certain to occur. These are resolved as accurately and completely as available information permits.

4129. Beach survey.—The most common purpose of a beach survey is to provide preliminary data for use in planning the constructions of piers, docks, or other harbor facilities. Another common purpose is to obtain data useful for landing supplies, equipment, and personnel directly on the beach from landing craft or amphibious vehicles.

Since a beach survey seeks detailed information about a relatively small area, accurate control (position) is essential. Several methods are in use:

Range and distance. Several ranges are established on shore, accurately measured by transit and tape, and marked by suitable markers. These ranges are established perpendicular to the general trend of the coast, and numbered for identification. The sounding boat runs lines of soundings in line with the ranges, determining distance offshore by stadia (with the rod in the boat and the observer on shore), or by attaching a line to an object on the beach and streaming out the line as the boat proceeds along the range, away from the shore.

Two ranges. Where the trend of the beach permits, two series of ranges can be established nearly perpendicular to each other, so that distance measurements are not needed.

Two transits can be set up at accurately determined positions on shore. Angles to the sounding boat are observed simultaneously at frequent intervals. This method is precise, but does not provide guidance to the sounding boat, since positions are determined on shore at a later time.

Horizontal sextant angles can be determined simultaneously by two observers in the boat, the results being plotted by three-arm protractor. This method is not attractive unless the area to be sounded is extensive.

Depth of water may be determined by echo sounder or pressure gage where practicable, but generally soundings close to the beach are made by hand lead or sounding pole. If the bottom is very soft, the lower end of the lead or pole should be fitted with a disk to prevent excessive penetration.

Chart projections are seldom required for a beach survey. A plane coordinate grid oriented with respect to an origin on shore, or the ranges, is sufficient for most purposes. The scale should be appropriate to the area and accuracy requirements, 1 inch=100 feet being commonly used. Soundings should be recorded to the nearest one-half foot out to the depth considered critical for the project.

Tide observations should be recorded continuously during the survey and, if practicable, should be extended to cover a synodical month (29½ days). One or more permanent bench marks should be established, and the height of the water level determined relative to the nearest foot mark if a tide staff is used.

Current measurements should be made to determine any current along the shore, and also maximum ebb and flood.

If the beach is to be used for landing vehicles, the suitability of the beach and backshore for landing and operating the type vehicles involved should be determined by inspection and also by penetration and other tests, as practicable. Wind, sea and swell, coastal currents, and character of beach materials are factors which govern possible sedimentation or erosion, and so should receive attention in the survey.

If the sea approaches have not been surveyed, these areas should be given attention in connection with the beach survey.

4130. Bathymetric survey.—Sounding lines run at sea are of assistance in adding detail to existing charts, or in constructing special charts to serve particular purposes. Most of the required information is obtained by ships proceeding between ports, either singly or in company with other ships. The important factors are accurate depths and accurate positions. These operations are given more detailed attention in chapter XLII.

4131. Checking accuracy of existing charts.—A mariner can perform a real service to himself and others by being continually alert to detect errors on the charts or in sailing directions. When such errors are suspected, an *opinion* that an error exists, or the submission of a chart with corrections shown, is of relatively little value to a charting agency. Chart requirements demand a higher order of accuracy than that of a vessel fixing its position by normal methods of navigation.

Positions of shoals, aids to navigation, landmarks, etc., should be determined carefully by whatever method is available. The average ship is provided with means for determining position to accepted accuracy. For instance, if an uncharted shoal is found, a launch should be sent to investigate. A sounding lead, two sextants, a chart, a three-arm protractor (improvised if necessary), and plotting board may be all that is needed. As the launch moves back and forth across the shoal, soundings are taken and simultaneous horizontal sextant angles between conspicuous charted objects are meas-

ured. If a single sextant is available, angles may be measured in quick succession, followed by a second measurement of the first angle, at approximately the same time interval as that between the first two measurements. If the objects are at a considerable distance, so that angles change slowly, the average of the two readings of the first angle can be used without significant error. Positions are then plotted and soundings recorded. The height of tide should be noted by tide table or other available means. The investigation should be continued over a sufficient area, and with enough thoroughness, to obtain an accurate indication of the nature and extent of the feature. If time permits, the surrounding area should be investigated to determine other possible shoals.

Points on land might be located by a number of bearing or distance measurements from different accurately located positions of the ship, or by measurements of direction made on land. A new structure might be located by information obtainable ashore, or by reference to other nearby structures. If a charted landmark is missing or has been moved, information should be sought ashore to determine the permanency of the change, and perhaps precise information regarding position, height, etc.

The number of variations is almost limitless, but the important thing to remember is the need to be alert to detect possible errors in the chart, and to obtain as complete and accurate information as practicable, submitting all details and an evaluation of the reliability of the data submitted. If complete information is not available, send what can be obtained, to at least alert the charting agency of the need for a correction. In the case of man-made changes, a possible source of complete information is helpful if the data itself cannot be obtained.

The mariner himself is one of the most valuable sources of information. By providing reliable data, he can help keep his charts and sailing directions accurate, current, and complete.

References

Bilby, J. S. *Signal Building.* U.S. Coast and Geodetic Survey Special Publication No. 234. Washington, U.S. Govt. Print. Off., 1943.

Breed, C. B. and Hosmer, G. L. *The Principles and Practice of Surveying.* Vol. I, *Elementary Surveying.* 8th ed., 1945. Vol. II, *Higher Surveying.* 6th ed., 1947. New York, Wiley.

Gossett, F. R. *Manual of Geodetic Triangulation.* U.S. Coast and Geodetic Survey Special Publication No. 247. Rev. ed. Washington, U.S. Govt. Print. Off., 1959.

Hoskinson, A. J. and J. A. Duerkson. *Manual of Geodetic Astronomy.* U.S. Coast and Geodetic Survey Special Publication No. 237 (1952 reprint). Washington, U.S. Govt. Print. Off., 1952.

Hosmer, G. L. *Geodesy.* 2d ed. New York, Wiley, 1930.

Jeffers, K. B. *Hydrographic Manual.* U.S. Coast and Geodetic Survey Publication 20–2. Washington, U.S. Govt. Print. Off., 1960.

Mitchell, H. C. *Definition of Terms Used in Geodetic and Other Surveys.* U.S. Coast and Geodetic Survey Special Publication No. 242. Washington, U.S. Govt. Print. Off., 1948.

Mussetter, William. *Manual of Reconnaissance for Triangulation.* U.S. Coast and Geodetic Survey Special Publication No. 225. Rev. ed. Washington, U.S. Govt. Print. Off., 1959.

Reynolds, W. F. *Manual of Triangulation Computation and Adjustment.* U.S. Coast and Geodetic Survey Special Publication No. 138 (1955 reprint). Washington, U.S. Govt. Print. Off., 1934.

Ulm, K. S. *Wire Drag Manual.* U.S. Coast and Geodetic Survey Publication 20–1. Washington, U.S. Govt. Print. Off., 1959.

U.S. Coast and Geodetic Survey. *Formulas and Tables for the Computation of Geodetic Positions.* U.S. Coast and Geodetic Survey Special Publication No. 8. 7th ed. (1963 reprint). Washington, U.S. Govt. Print. Off., 1963.

U.S. Coast and Geodetic Survey. *Horizontal Control Data.* U.S. Coast and Geodetic Survey Special Publication No. 227 (Revised 1957. Reprinted in 1961 with minor corrections). Washington, U.S. Govt. Print. Off., 1961.

U.S. Coast and Geodetic Survey. *Manual of Current Observations.* U.S. Coast and Geodetic Survey Special Publication No. 215. Rev. ed. Washington, U.S. Govt. Print. Off., 1950.

U.S. Coast and Geodetic Survey. *Manual of Tide Observations.* U.S. Coast and Geodetic Survey Publication 30–1. Washington, U.S. Govt. Print. Off., 1965.

U.S. Department of the Army. *Special Surveys.* Technical Manual TM 5–235. Washington, 1964.

U.S. Navy Hydrographic Office. *Hydrographic Office Technical Specifications for U.S. Naval Surveys and Supplementary Data.* U.S. Navy Hydrographic Office Special Publication No. SP–4. Washington, 1961.

CHAPTER XLII

OCEANIC SOUNDINGS

4201. Introduction.—Relatively little is known of the surface features of the nearly 71 percent of the earth covered by water. However, enough has been learned to indicate that the unseen topography beneath the oceans has all the features common to that above water. It is known that there are submerged mountains extending to greater heights above their surroundings than do the Rockies, and depressions deeper than the Grand Canyon.

While many of the general features are known, details are lacking. A very large number of accurately located soundings are needed to provide sufficient information to

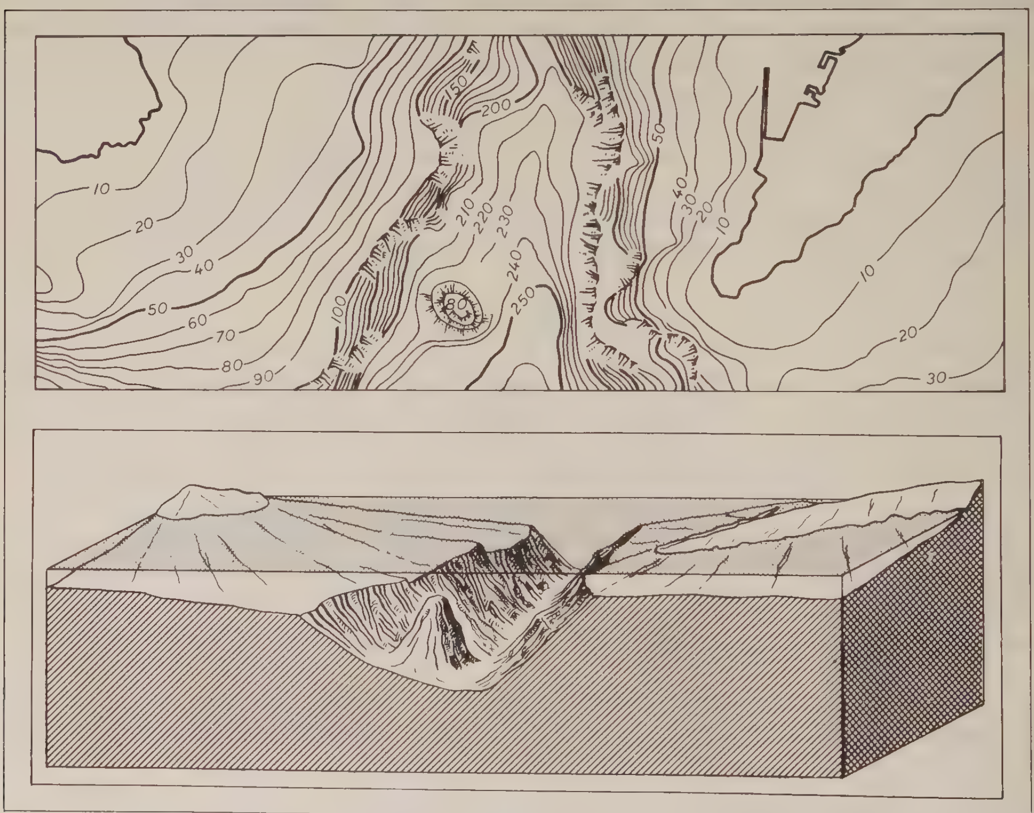


FIGURE 4201.—Contour lines and hachures (top) may be used to show underwater relief (bottom).

describe adequately the underwater relief. If sufficient information is available, such relief can be delineated on nautical charts by means of contours and hachures. A simplified chart of this type is shown in the upper part of figure 4201. The lower part of the figure is a block diagram of the area shown on the chart. Only a small part of the oceans has been sounded sufficiently to provide the detailed information needed for such a chart. Even in narrow strips along many coasts, along the route of the North Atlantic cable, and along a strip of the Pacific from California to the Carolines, where

soundings have been most numerous, the underwater relief is not known with the desired completeness and accuracy.

As long as oceanic soundings could be made only by a vessel stopping and lowering a weight, a process which might require several hours for a single sounding in very deep water, it was impractical for most vessels to obtain very much depth information at sea. With the development of the echo sounder, however, this situation has changed. With a recording echo sounder, a ship can obtain a profile along its track from continent to continent without slowing, using about a yard of recording paper per day. Such information, if reliable, is of great assistance to charting agencies in preparing more adequate charts of the ocean areas.

4202. Sounding equipment.—While lead lines and sounding machines have been used at sea, almost all deep-sea soundings are now taken by echo sounder (art. 619). If a depth recording device is available, it should be used, as the profile thus produced is a better indication of the bottom than even the most closely spaced visual readings.

All echo sounding equipment is subject to certain errors unless the operator has a clear understanding of the operating characteristics and limitations of the instrument. The routine checks recommended by the manufacturer should be made at every change of the watch, or oftener. In addition, the operator should be alert for certain possible errors peculiar to his instrument. A close watch should be kept on the proper functioning of the stylus, recorder speed, the zero adjustment, and the frequency of the electric current. The percentage error in the recorded depth is the same as that of the electric current frequency. Thus, at 3,000 fathoms, the error of a 60-cycle echo sounder is 100 fathoms if the actual frequency is in error by two cycles.

4203. Evaluating results.—Inaccurate results may be worse than no information at all. Therefore, every effort should be made to obtain reliable data. Particularly, soundings which conflict with known or charted depths should be carefully analyzed. Even when the equipment is operating correctly, false returns might be received due to sources external to the vessel. A shoal "phantom bottom" may be due to marine life, there may be multiple echoes or interference, or no return may be received because of aeration of the water or suspended matter in it. Such errors are further discussed in article 3504. Unusual local conditions may be a source of error. If an error is believed probable, but no source is detected, full information should be submitted with the soundings, for the charting agency may be able to interpret the results. This action is particularly important where the measured depths are less than those shown on the chart. If no error can be found, the charting agency may have no alternative but to enter the shoal soundings upon the charts affected, and take the first opportunity to send a survey vessel to verify or disprove them.

The speed at which sound travels in water varies with the salinity, temperature, and pressure. When these are known, corrections can be applied to obtain more accurate results. However, this is normally done only for scientific purposes Those soundings submitted to a charting agency should be the uncorrected values obtained by using an assumed standard speed of 4,800 feet per second.

4204. Deep sea sounding lines.—Most deep sea soundings are obtained by ships proceeding between ports. Soundings should be taken at every opportunity. Those taken in well-surveyed areas can be of assistance to the navigator in locating his position. If they conflict with values shown on the chart, and no error is found, they should be sent to the appropriate charting agency, with full particulars. All soundings in areas for which little depth information is shown on the chart should be submitted.

In addition to reliable soundings, accurate positions are needed. Navigation should be in accordance with standard practice, using every practicable means to reduce error and provide frequent checks on position.

When two or more ships are operating together, they should steam on parallel courses about five miles apart, maintaining stations abeam of each other by continuous monitoring by radar and pelorus, or other available means. Only one ship should perform the navigation used for controlling the survey.

4205. Investigating small areas.—If a feature of particular interest, such as an isolated shoal or a seamount, is found or reported in the vicinity of the vessel, a service can be rendered by conducting a further investigation in the vicinity of the feature. Two methods are in common use for this purpose:

Radial. A system of radial lines 20° apart are laid out from a central control point, preferably at the center of the feature to be investigated. These are extended outward for a distance of about 30 miles, and the ends of alternate ones are connected, as shown in figure 4205a. These form a series of course lines as shown.

Parallel. A north-south, east-west square is laid out with perhaps 60-mile sides, the center of the feature of interest being at the center of the square. A series of course lines are drawn parallel to one side of the square, at intervals of about five miles. The ends of alternate parallel course lines are connected, as shown in figure 4205b.

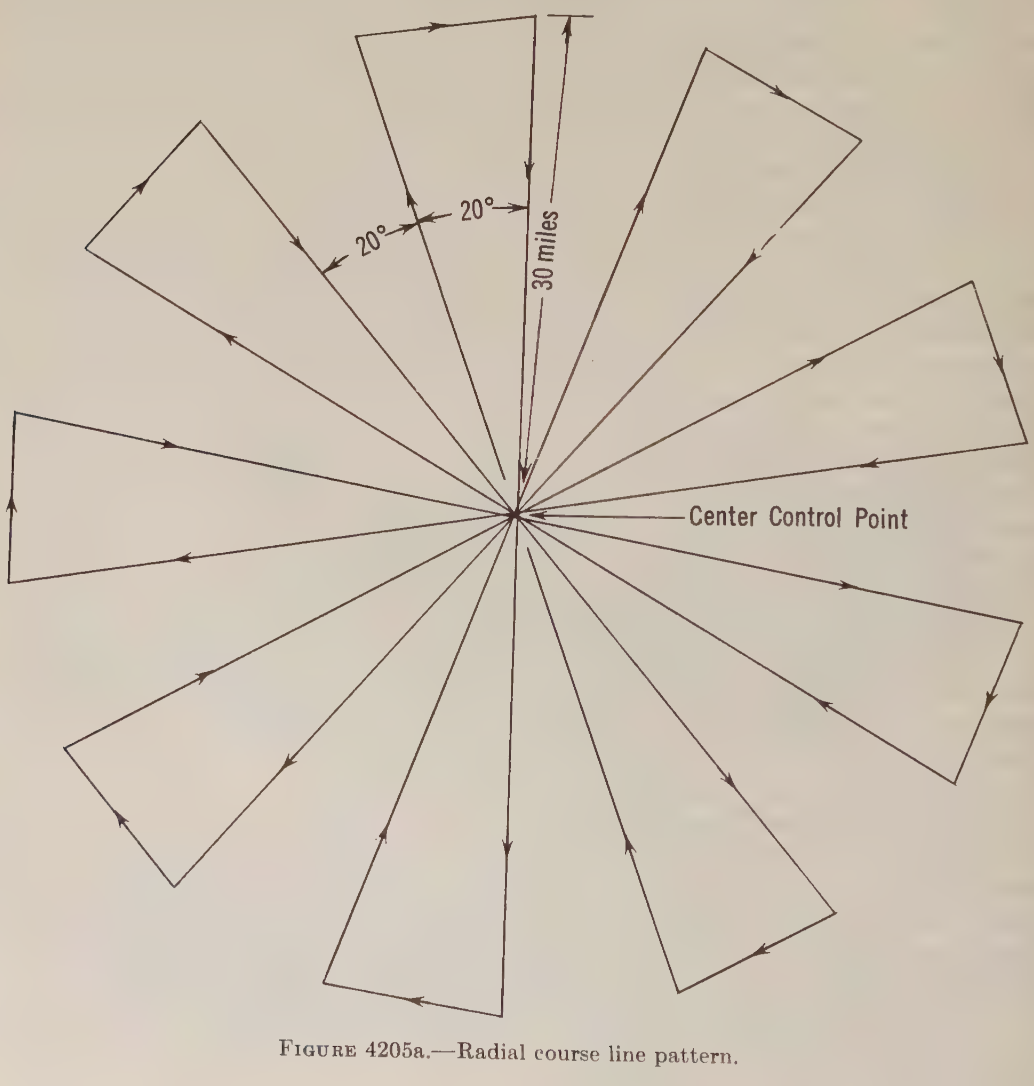


FIGURE 4205a.—Radial course line pattern.

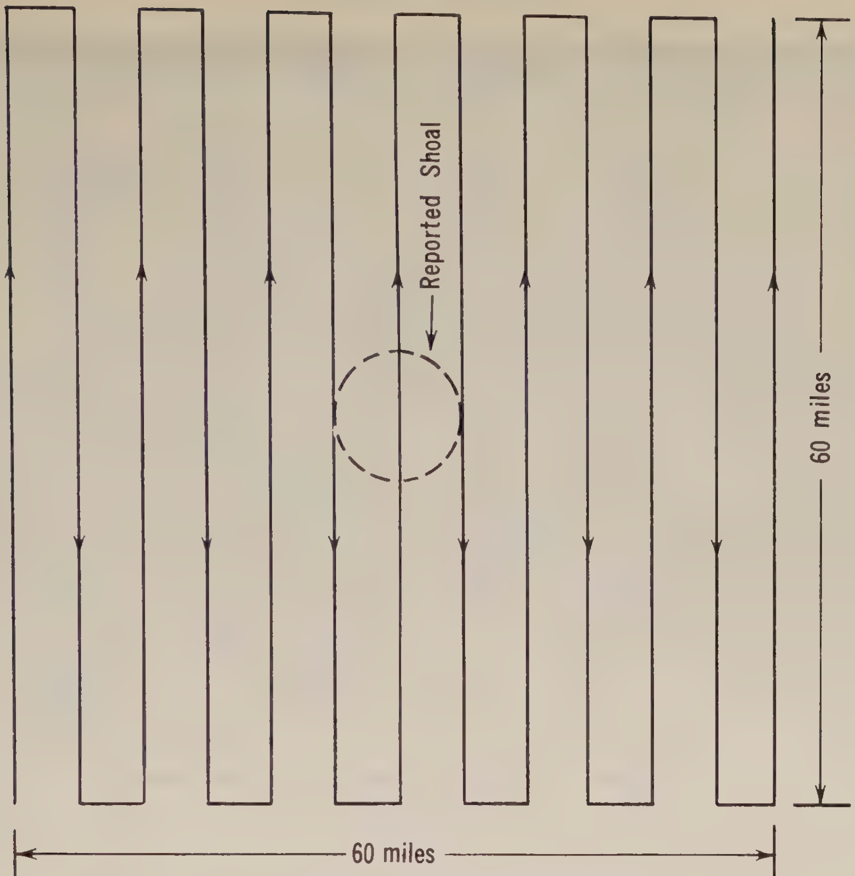


FIGURE 4205b.—Parallel course line pattern.

During such an investigation, by either method, the best control of position can usually be obtained by anchoring a buoy, if practicable, at the center of the area. In some instances, several buoys might be used. Any rig having buoyancy adequate to support the necessary length of anchor cable is satisfactory. The type generally used consists of a steel drum or mooring buoy with a weight attached to a cable, in the case of a large buoy, or piano wire if the buoy is small and of insufficient buoyancy to support a cable. A chain is not generally used. Buoys of this type have been successfully anchored in depths to 2,500 fathoms. The position of the buoy is determined as accurately as practicable, using celestial navigation, loran, or whatever means are available. Position of the vessel is determined relative to the buoy or buoys, using visual or radar bearings and ranges at intervals of half an hour or less. Beyond this range, the best available means are used. A balloon with a suspended radar reflector might be attached to the buoy to extend its range of usefulness. The securing line of the balloon should be at least 400 feet long, if practicable.

Sonar ranging, if available, should be used to assist in the location of shoal areas.

4206. Records.—While a reliable trace of the bottom is being obtained, the recorder should be operated continuously. Each hour, preferably on the hour, the time should be written on the graph, with an arrow pointing to the correct position on the trace. A fix marker may be used if the recorder is provided with one. In addition, the time of sharp changes in depth and other interesting features should be recorded. The date should be entered each watch, and the ship's name given at each end of the graph. Other pertinent information should be recorded. Figure 4206a illustrates a properly marked

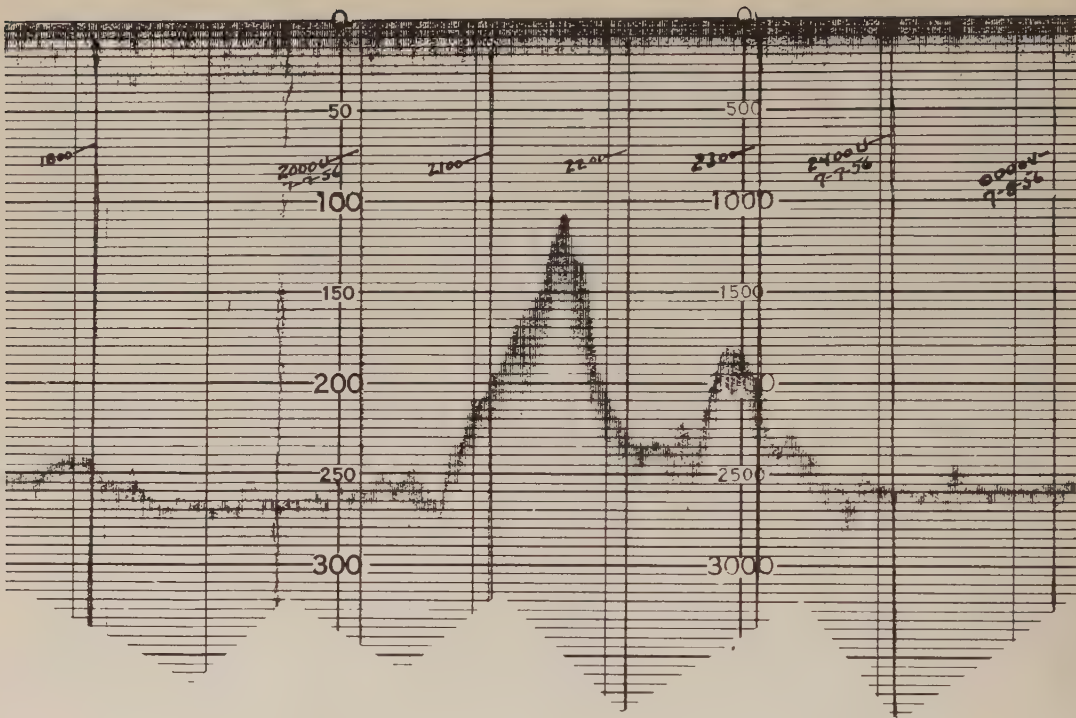


FIGURE 4206a.—A properly marked depth recorder graph.

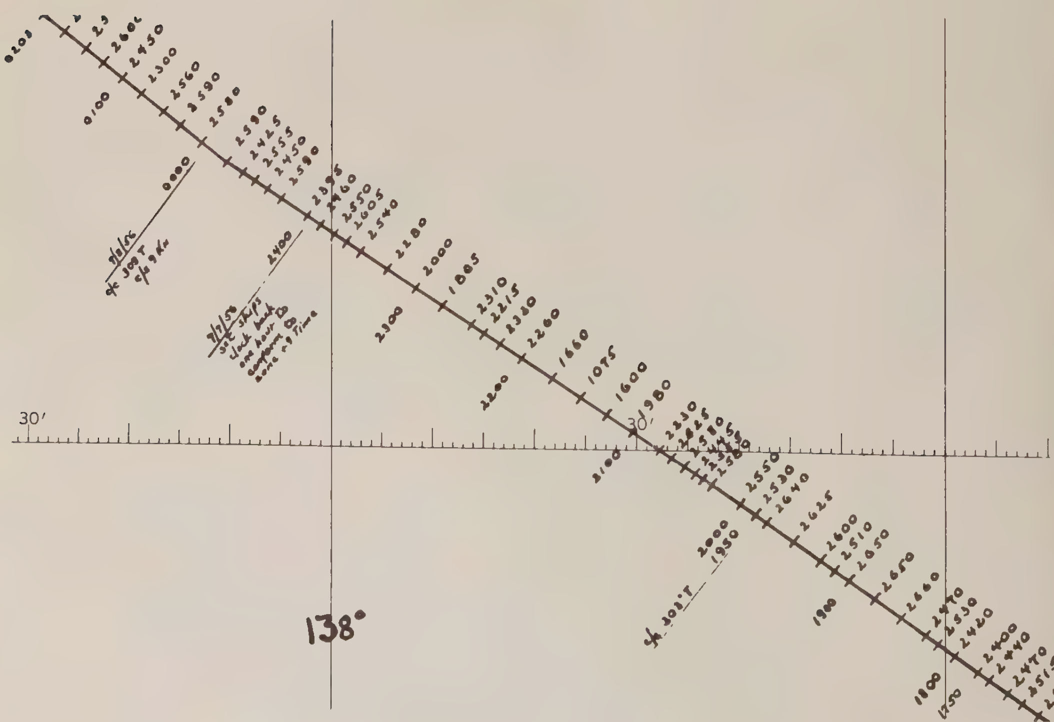


FIGURE 4206b.—An adjusted plot with soundings.

graph. When a reliable trace is not recorded, visible or audible soundings should be recorded in a log at least every five minutes. The time and date of the entries should be included in the log.

A plot of the track should be made on a plotting sheet. On this plot the dead reckoning lines and times should be adjusted to provide a continuous run, without gaps. Pertinent navigational data should be included, but no extraneous information should be given. Soundings should be spaced about one-quarter inch apart on the plot. The position of each sounding should be indicated by a tick with the sounding in fathoms given alongside, at an angle to the track line. Most plotting sheets have printed labels for the parallels of latitude. It is necessary that labels for the meridians be recorded. an item that should not be overlooked. The date of the soundings and the name of the vessel should also be recorded. Accuracy is essential, and neatness is desirable. Figure 4206b illustrates a typical adjusted plot.

When the work has been completed, all necessary information should be sent to the appropriate charting agency, usually the U. S. Navy Hydrographic Office. This should include the unadjusted plot used for navigation, the adjusted plot; data for each fix, with the navigator's evaluation of its reliability, and appropriate comments on weather conditions, etc.; depth recorder trace; and sounding log (if kept). The commanding officer's or captain's forwarding letter should indicate where the soundings were taken, type of sounding equipment, any difficulties encountered, and pertinent remarks regarding estimated reliability of the data. Additional information of value to cooperating observers is given in H.O. Pub. No. 606–b.

CHAPTER XLIII

PHOTOGRAMMETRY

4301. Introduction.—One of the most significant contributions to modern map making has been the development of the precision aerial camera and the techniques for interpreting and utilizing the information appearing on the photographs made by it. Such photographs constitute a detailed and permanent record of all unobscured natural and man-made features of a given section of the earth's surface, and as such, furnish more completely than any other means the information required for making maps. However, all photographs, whether aerial or terrestrial, are perspective views, and it is necessary to change these to orthographic views in order to obtain reliable map information. Although an aerial photograph is often map-like in appearance, there are many errors, both systematic and random, which prevent the photograph from being a true map. The science of photogrammetry is used to eliminate or correct these errors and also to properly record all the photographed information into a true map presentation. Its development into a complex and exact science has made photogrammetry the most efficient, accurate, and economical method for mapping large areas.

It is not the purpose of this chapter to present the detailed theory or working procedure of photogrammetric instruments and methods, but rather to acquaint the reader with the fact that such methods and instruments do exist and also to present simple formulas and techniques that a nonphotogrammetrist can utilize to obtain valuable, map-like information from aerial and ground (or shipboard) photographs.

4302. General photography classifications.—Photography used in map making is of three general classifications:

Vertical (aerial) photography, made with the optical axis of the camera vertical to the earth, or approximately so.

Oblique (aerial) photography, made with the optical axis of the camera at an angle to the vertical.

Terrestrial (ground) photography, made with the camera's optical axis in a generally horizontal position.

Each of these types of photography has its own particular field of usefulness, but the vertical aerial photograph is the type most widely used for mapping, since it most nearly resembles a map.

4303. Vertical photography.—The vertical photograph is not a map, but a perspective projection of three-dimensional terrain onto a two-dimensional photograph. This results in the photographic images being displaced from their true horizontal relationships due to the relief of the terrain features and any tilt of the aerial camera. This in turn results in a photograph which does not have a uniform scale. These displacements of image positions prevent the accurate determination of either distance or direction directly from the photograph. Figure 4303a illustrates the principle of image displacements due to the relief of terrain features.

For regular photogrammetric mapping purposes, vertical aerial photography is accomplished with less than 3° of tilt and in such a manner that there is approximately 60 percent overlap between photographs in line-of-flight, and approximately 20 percent sidelap between adjacent strips of photographs. The 60 percent overlap provides at

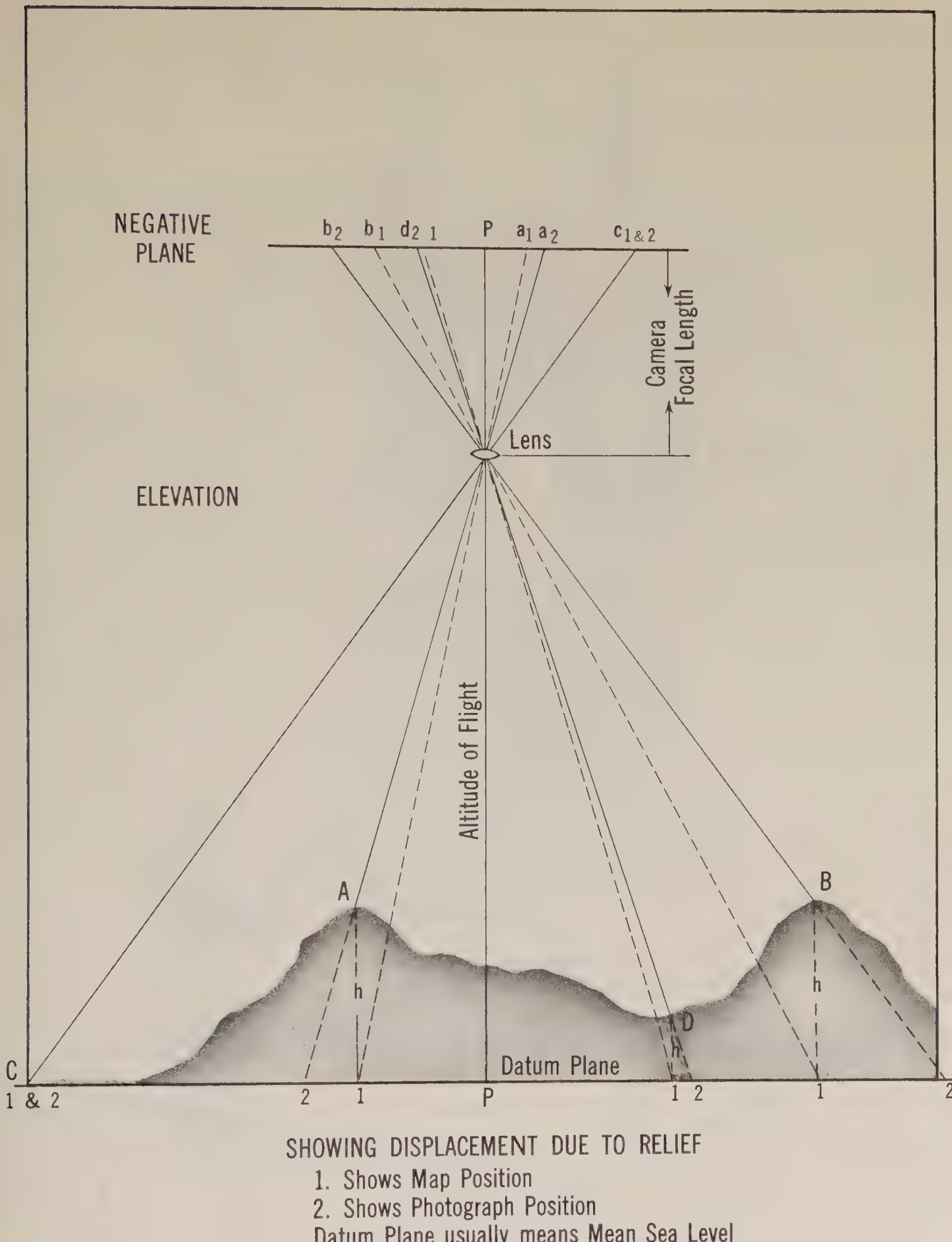


FIGURE 4303a.—Displacement due to relief.

least two different views of all features photographed. This is necessary to achieve the stereoscopic effect by which interpretation and measurements can be accomplished. By understanding and utilizing the geometric properties of this photography, one can obtain the information required to make a map; and accurate vertical, as well as horizontal, measurements can be made.

The **radial line plot** is one method of compiling a **planimetric map** (one showing horizontal position only) from overlapping vertical aerial photographs. Displacement due to relief and small amounts of tilt is corrected graphically by the **radial-line intersection method.** The center of each photograph is located, and any ground control points are identified and marked on the photographs. Auxiliary control points are selected on all photographs to strengthen the network. Radial-line intersections are taken from successive photograph centers to all control points. These intersections define the distances and directions of the points on the photographs.

The ground control points are plotted on a grid or map projection for the purpose of orienting the photographs. The intersecting of the radial lines and the orienting of the photographs to the plotted control network can be done graphically or mechanically through the use of arms or templates. After all the auxiliary control points have been correctly plotted on the manuscript, planimetric detail can be traced from the photographs by aligning corresponding points on the manuscript and photographs.

There are photogrammetric instruments capable of utilizing the basic photographic information to plot a standard topographic **map manuscript.** These instruments encompass a tremendous range from relatively simple monocular devices to highly complex optical and mechanical instruments. In basic theory, they recreate the three-dimensional view as photographed from aircraft, and permit the plotting of these terrain features onto a map manuscript.

Figure 4303b shows the working principle of one of these instruments, the multiplex. Nine-inch square aerial negatives are reduced by the use of a reduction printer to small transparencies called **diapositives.** These are placed in projectors held by brackets above a flat table. The brackets have adjustments by which the relationship that existed between the pictures at the time of exposure can be reestablished to scale with the two projectors. As many as 24 projectors can be placed on a frame. Thus, control established at a known point on the earth can be extended many miles.

The projectors are oriented in such a way that corresponding rays from the adjacent diapositives intersect at the image space above the table. By this means a stereoscopic model of the terrain appearing on the original negatives is precisely created. The adjacent diapositives are projected in complementary colors (red and green) onto a **tracing table** at any level of the stereoscopic model as a two-color **anaglyph,** and the operator views this model through spectacles in which one glass is red and the other green. Briefly, the tracing table consists of a round white disk called a **platen,** in the center of which is a very small illuminated hole. The platen is supported by two columns, and by means of a knurled screw it can be raised and lowered. In this way, the point of light can be made to appear on the surface of the ground, and differences in elevations can be read on a counter mounted on one of the columns. A pencil holder is mounted directly beneath the illuminated point, and the whole device is mounted on agate foot pads so that it can be easily moved over a piece of suitable drawing material. In this way, the operator can trace the horizontal position of selected data, including relief features, in an orthographic presentation of the earth's surface corresponding to the photographic coverage.

4304. Scale of vertical photograph.—The scale of the vertical aerial photograph, usually expressed as a representative fraction (e.g. 1:40,000), denotes only the average

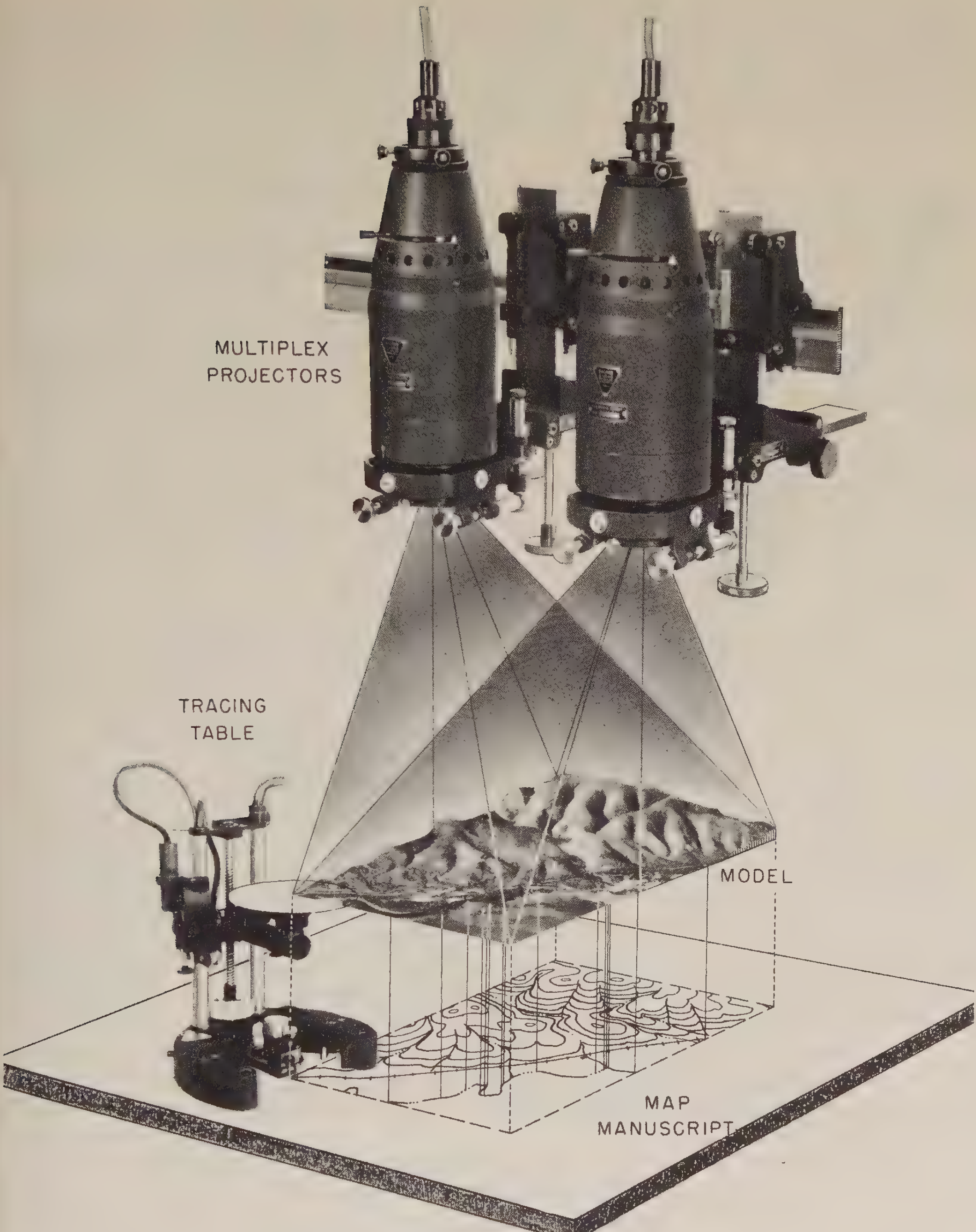


FIGURE 4303b.—Principles of the multiplex.

or approximate scale, for even on an untilted vertical photograph there can be no uniform scale due to the varying elevations to the terrain. If the scale of an aerial photograph is to be determined, two items of basic information must be known: (1) the focal length of the aerial camera, and (2) the flight altitude of the photographic aircraft above the terrain. The scale is determined by the ratio of these factors. For example: If the focal length (f)=6 inches=0.5 feet, and the flight altitude (H)=20,000 feet, then:

$$\text{photo scale}=\frac{f}{H}=\frac{0.5}{20{,}000}=\frac{1}{40{,}000}=1:40{,}000.$$

The approximate scale of the vertical aerial photograph can also be established by comparison of distances on the photograph with corresponding distances on a map of known scale.

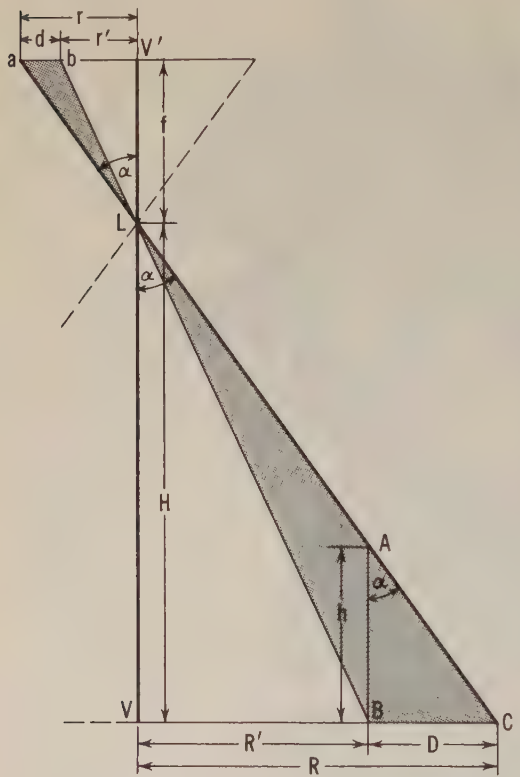


Figure 4305.—Determination of height by image displacement.

4305. Height determination.—If the foreshortened side of an object of considerable height, such as a tower or lighthouse, appears on a vertical aerial photograph, a simple application of the geometric properties of a photograph can determine the true height. The factors which must be known are: (1) the flight altitude of the aircraft above the terrain, (2) the length of the foreshortened side of the object on the photograph, and (3) the distance between the top of the object as it appears on the photograph and the center of the photograph. Refer to figure 4305 for the development of the height-determination formula. In the figure, H is the flight altitude above terrain, h is the height of the object, r is the distance from the top of the object image to photograph center, d is the length of the foreshortened image of the object, and f is the focal length of the aerial camera.

By the geometry of the figure:

(1) triangles LVC, ABC, and $LV'a$ are similar, with angles α equal.

(2) triangles LBC and Lba are similar.

(A) From (1):

$$\tan \alpha=\frac{R}{H}=\frac{D}{h}=\frac{r}{f}, \text{ or } D=\frac{rh}{f}.$$

(B) From (1) and (2):

$$\frac{D}{d}=\frac{H}{f}, \text{ or } D=\frac{dH}{f}.$$

From (A) and (B):

$$D=\frac{rh}{f}=\frac{dH}{f}.$$

Therefore: rh=dH

and

$$h=\frac{dH}{r}.$$

Thus, if the foreshortened image of a lighthouse appearing on a vertical aerial photograph (d) is measured as 0.08 inch, the distance on the photograph from the lighthouse top to the center of the photograph (r) is measured as 3.75 inches, and the flight altitude above terrain (H) is 3,750 feet, the height of the lighthouse is $h=\frac{dH}{r}=0.08\times\frac{3750}{3.75}=80$ feet.

4306. Oblique photography.—Oblique aerial photographs are obtained by tilting the optical axis of the camera from the vertical. The oblique camera has the advantage of being able to photograph vast areas in a single exposure. Oblique mapping photographs are of two types: the **high oblique,** which shows the horizon line; and the **low oblique,** which does not show the horizon. Sometimes two high obliques are exposed in opposite directions simultaneously with a vertical, in order to provide photographic coverage from horizon to horizon, perpendicular to the line of flight. Although this method (named **trimetrogon** after the three metrogon lenses used in the camera system) of employing two obliques and one vertical photograph enables relatively rapid map compilation of large areas, the type of information obtained from obliques is adequate for only small-scale maps of a reconnaissance nature. The use of high oblique photos for mapping is more limited than that of verticals because the methods entailed are more time consuming and the results obtained are ordinarily less accurate. This is due to the perspective distortion of the oblique, and the masking of distant features by closer ones.

Twin low-oblique photography is obtained with a twin-camera arrangement consisting of a pair of wide-angle precision aerial cameras coupled rigidly together. The optical axes of the two cameras lie in a common vertical plane and form an angle of 20° with a plumb line (assuming no tilt) and 40° with each other.

4307. Maps from oblique photography.—High oblique photographs can be used for reconnaissance mapping, particularly in areas where the relief of the terrain is slight and the required map is essentially planimetric. Methods and instruments for use by trained personnel are available for extracting elevations from high obliques, but the procedure is involved, and will not be covered here. Transfer of planimetric detail from the oblique photograph to a map manuscript may be simply and effectively accomplished by a graphical process known as the **perspective grid method.** In basic principle, this method permits sketching of the linear detail (shore line, lakes, rivers, town lines, etc.) from a perspective grid at the picture plane to a rectangular grid at the map plane. For the reconnaissance study of unmapped flat areas, it is feasible for a nonphotogrammetrist to prepare his own base maps by this method, even using pictures taken by himself. It is required only that a distinct and regular horizon line should be visible on the photograph, and that certain basic factors be known, as explained in articles 4308 and 4309.

4308. Construction of the perspective grid.—The perspective grid is drawn on a transparent medium which is positioned securely over the photograph. In the construction of the perspective grid, three factors are required:

1. The altitude of the photographic aircraft.
2. The focal length of the camera lens.
3. The true, as distinguished from the apparent, depression angle of the photograph. This factor can be computed from measurements, as explained in step (7) below.

The first consideration in construction of the perspective grid is the scale to use for the drawing; that is, the size the squares of the map grid are to be, and what distance on the ground each side of the square will represent. Suppose that the perspective

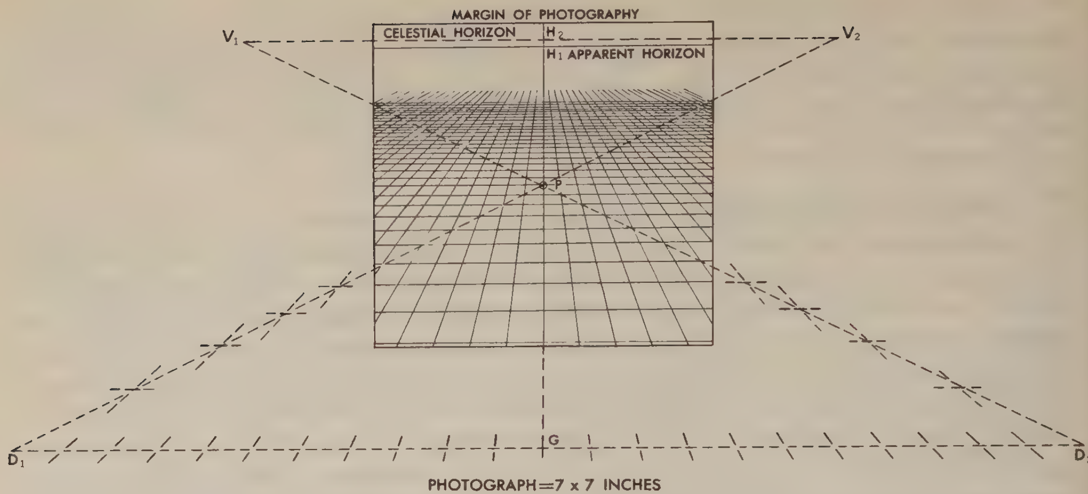


FIGURE 4308.—Construction of a perspective grid for a high oblique aerial photograph.

grid shown in figure 4308 is to be drawn, and it is decided that the map grids are to be one inch square, and that each side of the square will represent 660 feet on the ground. Suppose, further, that H, the flight altitude, was 5,000 feet; that f, the focal length of the camera lens, is 5.216 inches; and that the photograph is seven inches by seven inches. The grid construction follows (shown to reduced scale in fig. 4308):

(1) *On the photograph,* draw a line along the apparent horizon.

(2) *On the photograph,* draw a line perpendicular to the apparent horizon and through the **principal point** (the center or point of intersection of the two diagonals, designated P), extending this line from one edge of the photograph to the opposite edge.

These two lines are the only ones to be drawn on the photograph.

(3) Lay the material to be used for the perspective grid over the photograph, and on it trace the two lines drawn on the photograph (steps (1) and (2)).

(4) Measure the distance PH_1. Suppose this is three inches.

(5) Compute the apparent depression angle by the formula: tangent of the apparent depression angle$=\frac{PH_1}{\text{f}}=\frac{3}{5.216}=0.57515$. The angle having this tangent is 29°54′.3, which is the angle of apparent depression.

(6) Compute the dip (D) by: D in seconds$=58.82\sqrt{\text{H in feet}}$. In the example, $58.82\sqrt{5{,}000}=4{,}159$ seconds, or 1°09′.3. (In the *Nautical Almanac,* dip is now determined by using 58.2 for the constant 58.82. The value 58.8 was formerly used for almanac dip tables. In this example, the use of 58.2 gives a dip of 1°08′.6. With either value of D the answer to step (8) is the same.

(7) Add the dip to the apparent depression angle, to obtain θ, the true depression angle: 29°54′.3+1°09′.3=31°03′.6.

(8) Compute the distance from the principal point to the celestial horizon by: $PH_2=\text{f}\tan\theta=5.216\times0.60229=3.14$ inches.

(9) Scale this distance (3.14 inches) from P, along the principal line to H_2, and draw the line $V_1H_2V_2$ through it, parallel to the apparent horizon.

(10) The scale of the map grid has already been decided upon as being one inch= 660 feet. Compute the distance from the celestial horizon to the front ground line at G, where one inch on the front ground line subtends 660 feet on the ground trace of the photograph, by the formula: $H_2G=\frac{\text{H}\sec\theta}{660}=\frac{5{,}000\times1.16737}{660}=8.84$ inches.

(11) Scale this distance (8.84 inches) from H_2, along the principal line to G, and draw D_1GD_2 parallel to $V_1H_2V_2$.

(12) From G, in both directions along D_1GD_2, measure off one-inch divisions, and from these points draw lines through the vanishing point H_2. These are the meridian lines *of the perspective grid* (not geographic meridians).

(13) Determine the vanishing points V_1, V_2 for the diagonals, by the formula: $H_2V_1=H_2V_2=\text{f} \sec \theta=5.216\times1.16737=6.09$ inches.

(14) Determine D_1, D_2, by the formula: $GD_1=GD_2=\frac{H_2V_1}{PH_2}\times PG=\frac{6.09}{3.14}\times 5.70=$ 11.06 inches.

(15) Draw the diagonals V_1D_2 and V_2D_1. These lines will cross at P if the drawing and computations have been carefully done.

(16) Draw a horizontal line through each intersection of a diagonal and a meridian line, as shown in figure 4308.

(17) Lay out the limits of the photograph, and ink all lines within the photograph limits except the construction lines.

4309. Use of the perspective grid.—Prepare a grid of one inch squares on a separate sheet of paper large enough to take the entire plottable area covered by the photograph. Superimpose the transparent perspective grid over the photograph in its proper position. By inspection, transfer the detail from the photograph to the square grid. This is illustrated in figures 4309a and 4309b. Figure 4309a represents a transparent perspective grid, as if superimposed on a photograph showing a main shore line and offlying islands. Figure 4309b shows the transfer of the shore line and islands from the photograph to the plotting or map grid. The entire area of figure 4309a is not included, to permit a larger scale illustration. The scale of the map grid is one-third the scale of the perspective grid.

4310. Terrestrial (ground) photography is comparable to shipboard photography, and both may be utilized in a similar manner. The principles of the perspective grid method previously described can be adapted for use with terrestrial photography. Another method of determining horizontal positions is to obtain two photographs of the same area taken from different camera stations. The positions of the camera stations must be known and the horizontal directions of the camera axis at the time of each exposure must be determined. The focal length of the camera must also be known. Where the reproduction is not at the same scale as the negative, the reduction or enlargement must be known so that the focal length value may be altered proportionately. Preferably, the celestial horizon should run through the principal point of the photographs and be parallel to the top and bottom margins.

Place a sheet of transparent material over each photograph. Select the points (A, B, C, D, fig. 4310a) for which the positions are desired, making certain that all points are included in both photographs. Through each of these points and also P, the principal point (located by diagonals, as shown), draw a line perpendicular to the horizon. Near the bottom of each photograph draw a line (JK) parallel to the horizon, intersecting all vertical lines at right angles (A', B', C', D', and P'). Extend vertical line PP' (through the principal point), and from P' measure off the focal length, $P'S$, locating the camera station, S. Connect point S with points A', B', C', and D'. Next, place each overlay on the chart with its point S over the correct camera position, and line SP in the direction the camera was facing when the picture was taken. The location of point A on the chart is at the intersection of the two SA' lines, extended if necessary. The other points are located similarly.

If the position of the camera station for any photograph is unknown, it can be determined if there are several correctly positioned points on a chart that are readily

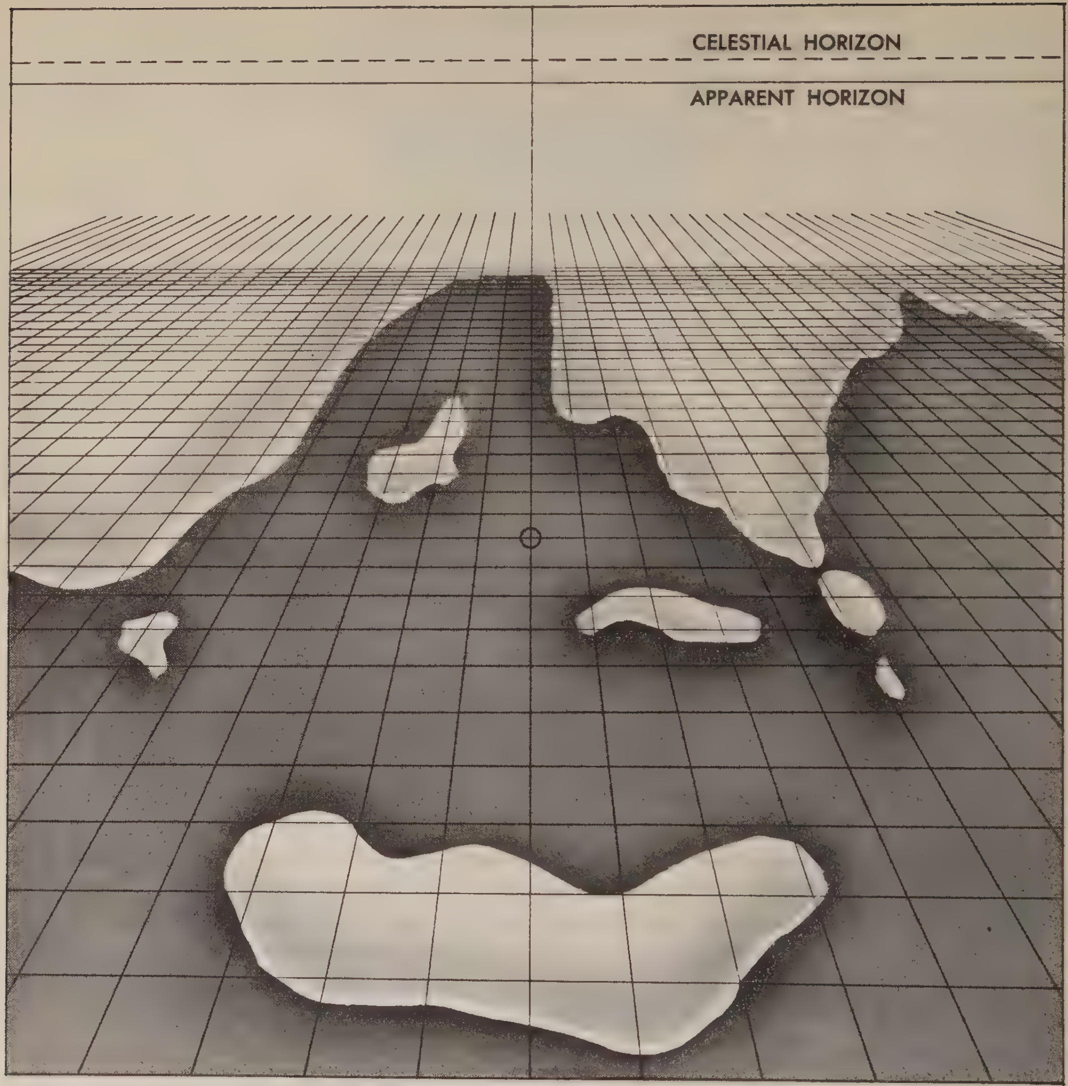


FIGURE 4309a.—Perspective grid as if superimposed upon a photograph.

identifiable on the photograph. The procedure described above is applied to the single photograph, but using the identifiable points (fig. 4310b). The transparent overlay is positioned on the chart so that the radial lines pass through the corresponding points on the chart. The position of the camera station is thus determined by resection (art. 4116).

4311. Photo-interpretation is the examination of photographic images of objects for the purpose of identifying the objects and deducing their significance. In identifying objects from their photographic images, the following characteristics should be considered:

Shape relates to the general form, configuration, or outline of an individual object. The shape of an object appearing on a vertical photograph may differ widely from its shape when viewed from the ground. Because of this, a certain amount of practice and experience is necessary in order to make a reliable identification.

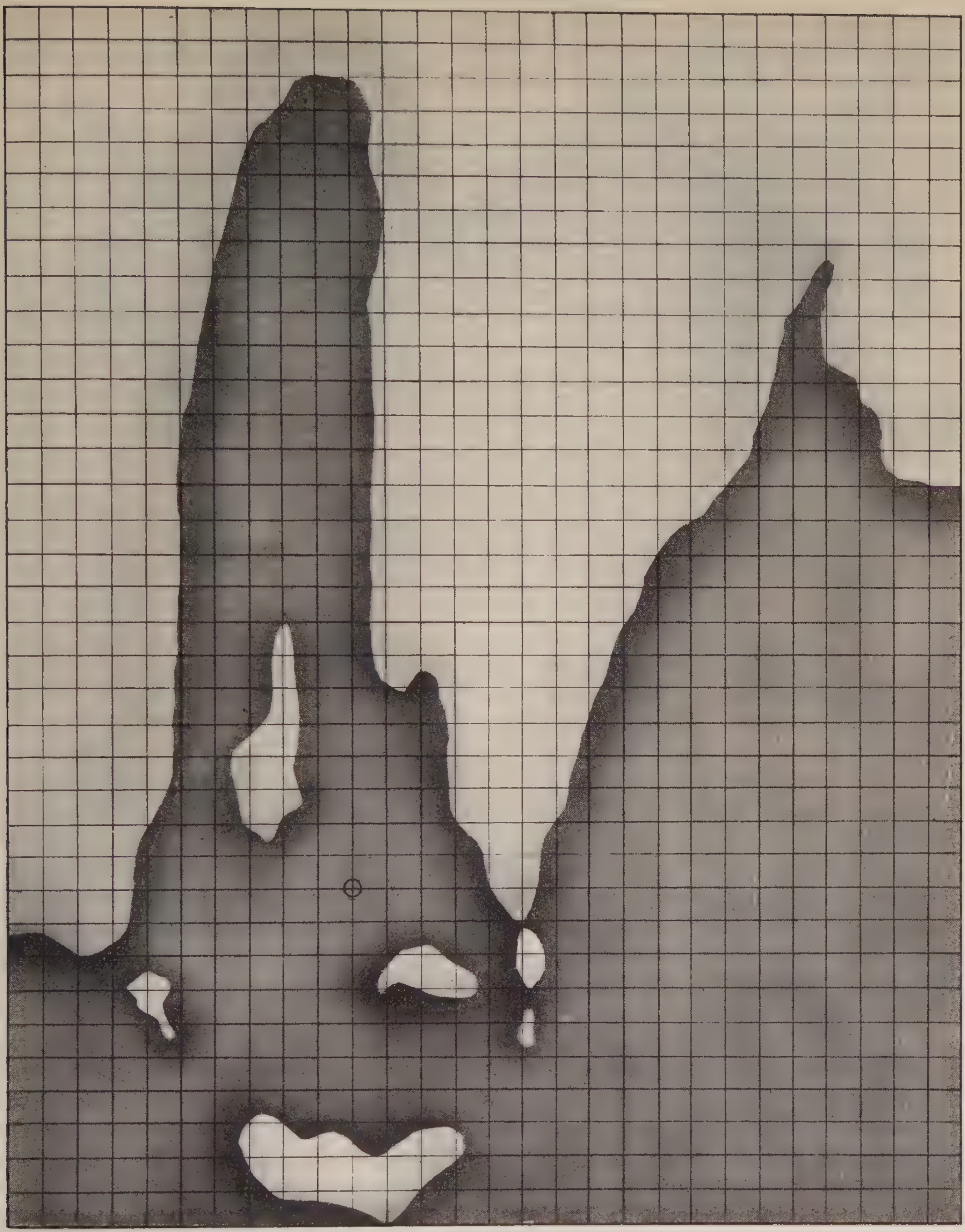


FIGURE 4309b.—Section of a map grid from the perspective grid of figure 4309a.

The *size* of an object as compared with another object can be of invaluable assistance in determining the size of the second object. A truck on a road gives an idea of the width of the road.

Pattern refers to the spatial arrangements of objects. Many objects, both natural and man-made, conform to certain repetitious patterns. For example, streams which

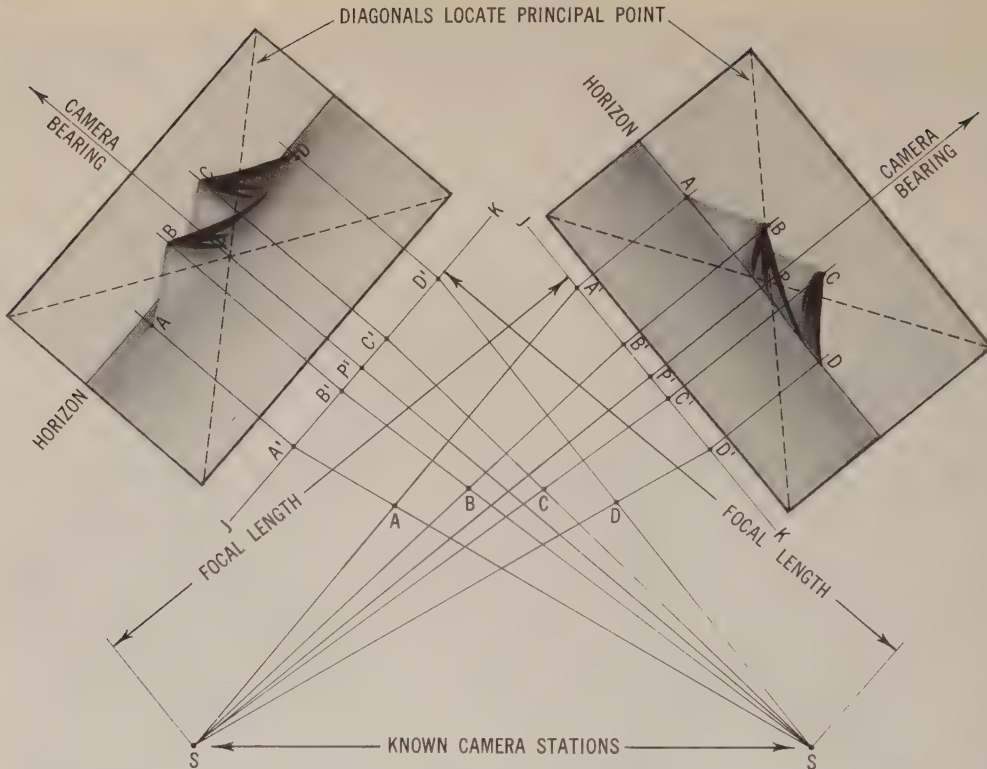


FIGURE 4310a.—Determination of horizontal positions from two photographs.

branch and reunite in a braided pattern are readily distinguished from simple meandering streams.

Tone refers to the shade of gray in which an object appears on a photograph. The tone of an object may differ on two adjacent photographs because the light is reflected back at different angles in different amounts. The image of smooth water may appear light on one photograph and dark on the next. Most roads are good reflectors of light over a wide angle and almost invariably appear as light lines on the photograph.

Texture is the nature of the surface photographed, with particular reference to size and arrangement of individual units. For example, the photographic texture of the beach area on which an amphibious landing is contemplated may indicate the coarseness of particles composing the beach and, thereby, the ability of the beach to support military vehicles.

Shadows frequently give the best indication of the profile view of an object, thus facilitating the identification of the object. Shadows will give an indication as to heights of trees and buildings, types of bridges and towers, and depth of cuts, quarries, etc. Frequently, it is possible to obtain some idea of the character of the relief from the shadows on a single photograph.

Site is the location of an object in relation to its surroundings. It is important in the interpretation of both man-made and natural features. Many types of vegetation are confined to specific topographic sites such as swamps, stream banks, sandy flats, or rock knolls.

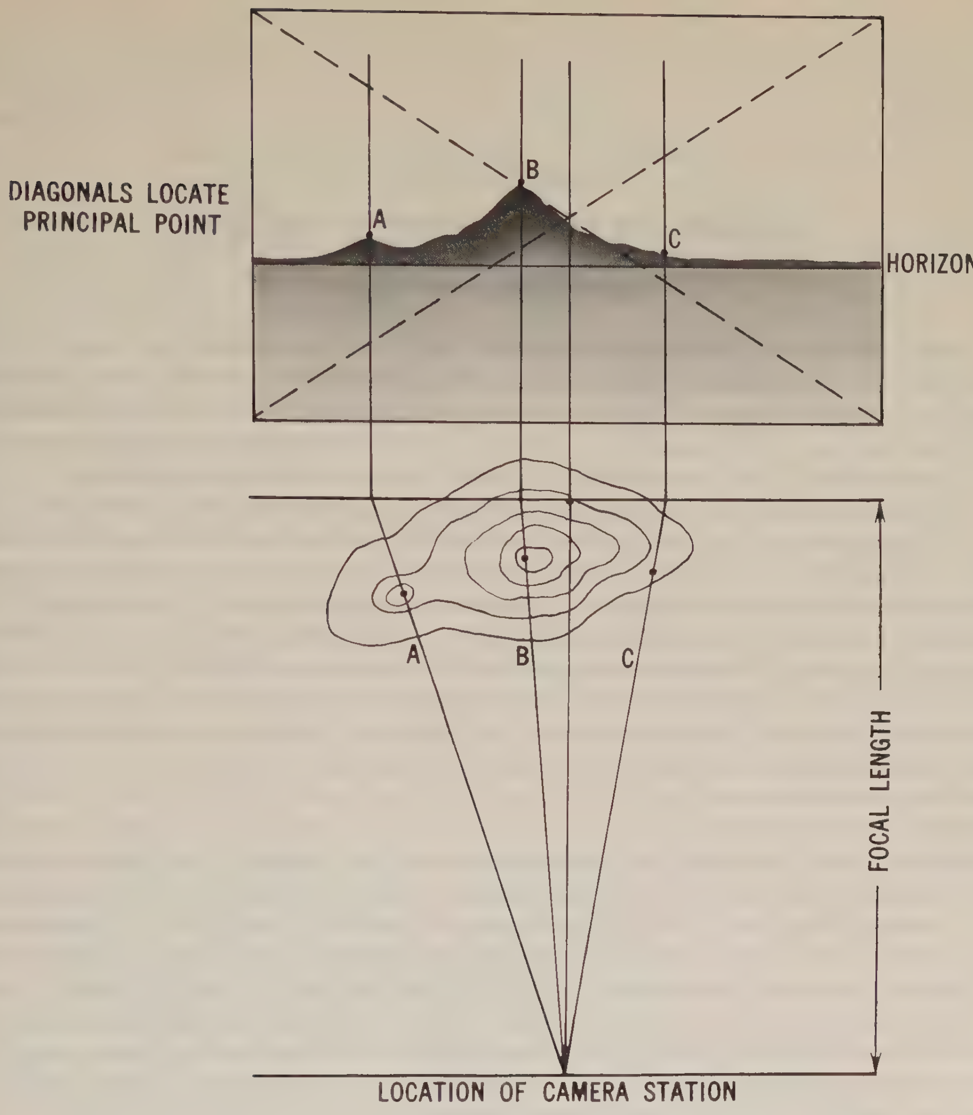


FIGURE 4310b.—Determination of camera station from one photograph.

References

American Society of Photogrammetry. *Manual of Photogrammetry.* 2nd ed. [Washington, 1952.]

McNeil, G. T. *Photographic Measurements, Problems, and Solutions.* New York, Pitman, 1954.

U.S. Department of the Army. *Map Reading.* Field Manual FM 21–26. Washington, 1965.

U. S. Departments of the Army, Navy, and Air Force. *Cartographic Aerial Photography.* Department of the Army Technical Manual TM 5–243. Department of the Navy Hydrographic Office Publication H.O. Pub. No. 595. Department of the Air Force Technical Order AFM 95–6. [Washington, U.S. Govt. Print. Off.] 1964.

U. S. Departments of the Army, Navy, and Air Force. *Photographic Interpretation Handbook.* Department of the Army Technical Manual TM 30–245. Department of the Navy NAVAER 10–35–610. Department of the Air Force Manual AFM 200–50. Washington, U.S. Govt. Print. Off., 1954.

CHAPTER XLIV

PRODUCTION OF NAUTICAL CHARTS

The Production of Charts

4401. Introduction.—The nautical chart has become so reliable and readily available that one unacquainted with the tremendous amount of material, labor, and time involved in its production might easily fail to appreciate the contribution made by charting agencies. So dependent has the mariner become upon this commonplace but important aid, that he generally considers it essential to safe navigation.

The four preceding chapters describe briefly the collection of survey data. This chapter sets forth the basic techniques used in the production of nautical charts from survey data and other source material. The information is given so that the mariner might have a better understanding of his chart and its limitations. Thus equipped, he is capable of more effective, reliable navigation. Further, the mariner is an important source of data used in the production and correction of nautical charts. If he is familiar with the use made of these data, he can better evaluate the information which comes to his attention, and can forward it in form that will be of value to the charting agencies. With such information, these agencies can produce more accurate charts of greater usefulness to the mariner.

4402. Federal charting agencies.—The U. S. Navy Hydrographic Office has responsibility for producing and maintaining nautical charts of any area of interest to the United States Navy or merchant marine which is not the responsibility of other U. S. charting agencies. The U. S. Coast and Geodetic Survey, of the Department of Commerce, has responsibility for charting the coastal waters of the United States and its territories and possessions. The U. S. Lake Survey, of the U. S. Army Corps of Engineers, has similar responsibilities for the Great Lakes. Consequently, the Hydrographic Office limits its activities to foreign waters and the high seas, except in time of war or national emergency. Under these circumstances, charting activities would be carried out under a coordinated program.

The charting activities of these agencies seek to provide maximum navigational safety and facility to vessels of the United States Navy and merchant marine. They keep informed of new navigational requirements, and utilize the latest developments in the production of charts. Many new methods and instruments originate with these agencies.

While the information given in this chapter relates primarily to the methods, techniques, etc., of the U. S. Navy Hydrographic Office, it applies, with minor variations, to those in use by the U. S. Coast and Geodetic Survey and other charting organizations, both domestic and foreign.

4403. Improvements in charts.—The nautical chart has kept pace with the latest developments in the field of marine transportation and navigation. Special-purpose nautical charts have been produced to satisfy specific navigational requirements, such as those of loran and radar. New or improved developments in electronic positioning systems, magnetic observations, and aerial photographic coastal and topographic delineations are some of the factors instrumental in improved chart accuracy.

Presentation of data is planned to conform with standard specifications which have been established by U. S. Government charting agencies and the International Hydrographic Bureau, located in the Principality of Monaco. This organization, established in 1921, is composed of more than 30 of the leading maritime nations of the world. Its purpose is to promote international agreement in the form of nautical charts and publications, and to effect collaboration in the common task of collecting and disseminating hydrographic information. Adoption of chart standards and the practice of exchanging cartographic information and techniques among various agencies and scientific and professional societies have aided materially in improving the nautical chart. Some of the prominent innovations which have improved the appearance and utility of the nautical chart are the following: use of colors to accentuate navigational hazards, revision and simplification of typography for improved legibility, use of tint shading in combination with contours to give a pictorial presentation of relief. The transition in presentation has been gradual, with convenience to the chart user being constantly kept in mind.

4404. Sources of chart data.—The preferred source of data used in the construction of nautical charts is the hydrographic survey (ch. XLI). The completed survey usually covers more extensive areas than the charts because of chart size limitations and the absence of chart requirements in areas without special naval or economic significance. However, hydrographic survey plans generally include a proposed chart layout to insure sufficient detail at suitable scale in potential chart areas.

In addition to surveys, chart data are collected from many other sources. Foreign data, the main source for U. S. Navy Hydrographic Office nautical charts, are collected in published chart form on a continuing basis under exchange agreements with member states of the International Hydrographic Bureau and with other maritime countries. Various government agencies, engineering firms, educational institutions, and private individual sources furnish documented information which is incorporated into nautical charts. Each week, the Hydrographic Office receives more than 400 documents which are evaluated for use in preparing charts and publications. The published chart does not outwardly reflect the amount of information which is screened during its construction. The following list of source material is representative of the type from which useful information is obtained:

Surveys and charts of the United States and foreign countries (hydrographic data).

Maps (transportation, topography, planimetry, and communications data).

Periodicals, travel folders, atlases, and gazetteers (place names, town plans, and descriptive data).

Photographs (aerial mapping and others).

Mariners' reports of port facilities and other miscellaneous information (harbor and dock construction work, conspicuous aids and dangers to navigation, etc.).

Sketches (used to verify and amplify other information).

Notice to Mariners (navigational data).

4405. Chart production methods.—Four common methods of producing nautical charts are in use at the U. S. Navy Hydrographic Office: construction of the chart from modern U. S. Navy surveys; production of the chart from a compilation mosaic (art. 4416), based upon numerous foreign charts; redraft of an existing foreign chart; and facsimile reproduction of an existing foreign chart. The last method is employed only in emergencies, and the facsimile is not sold to the public. A fifth method of increasing popularity is the modified facsimile reproduction of foreign charts (based upon bilateral agreement), which are sold as H.O. charts.

Copper engraving of nautical charts is no longer carried on by the U. S. Navy

Hydrographic Office, having been replaced by methods considered more economical. Plastic engraving (engraving on plastic) of the chart original, recently introduced, is a promising source of further reduction in time and cost.

Photolithography is the principal chart reproduction method used. Negatives are made on glass, film, or plastic, with reproduction plates of zinc or aluminum.

4406. Chart terminology.—The following terminology is in use at the U. S. Navy Hydrographic Office:

New chart. A chart published for the first time, covering an area not previously charted at the same scale by the same organization.

New edition. A chart incorporating corrections too numerous or extensive to be reported in *Notice to Mariners.* A new edition makes previous printings obsolete.

Corrected (New) print. A chart incorporating corrections which have been published in *Notice to Mariners,* and other information of insufficient importance to justify a new edition. Chart dates are discussed in article 506.

Reprint. A chart reprinted without any corrections or changes.

Field chart. A chart published on board a survey vessel, and reprinted by the Hydrographic Office. It is identified by the letter "F" preceding the chart number.

Chartlet. A graphic supplement to *Notice to Mariners* of corrections which are too extensive for the narrative version.

Proof. A copy of a chart made prior to release for final processing. **Photo proofs** are made from negatives. **Litho proofs** are made from the printing plates, in any desired color. A **composite proof** is a combined proof of more than one color plate. A **watercote** is a composite proof, in color, from negatives.

Withdrawal. A chart removed temporarily from issue with the intention of replacement or reissue at a future date. All copies are destroyed, except standards for record purposes. The printing plates may either be retained or destroyed.

Cancellation. A chart permanently removed from issue. All copies and printing plates are destroyed, except standard copies for historical purposes.

Discontinuance. Removal from issue of a reproduction of a foreign chart.

Elements of Chart Construction

4407. Drafting instruments.—The ordinary drafting instruments are employed in making charts. In addition, the following have proved useful in compilation and drafting of a nautical chart:

Pantograph (fig. 4407a), a parallelogram-linkage device for reproducing a drawing at a different scale. A **pantograver,** a pantograph which corrects for distortion due to unequal expansion or contraction of the paper of the source material, is used by the U. S. Navy Hydrographic Office to make corrections on wet (glass) plates.

Projection-ruling machine (fig. 4407b), a device for drawing the meridians and parallels of a map projection.

Vertical projector (fig. 4407c), a device for projecting a vertical aerial photograph (arts. 4302–4305), drawing, or other chart onto a work table at any scale between specified limits, so that details can be traced onto a map manuscript (art. 4303). The projector shown is portable, stands 74 inches high when the movable carriage is at the top of its travel, and provides for any scale from ⅓ to 3½ times the original.

Light table, a table with a glass top over a light source, to facilitate inspection by direct comparison of different charts of the same scale, and the tracing of a drawing at the same scale. Light tables are available in a variety of sizes and shapes. They may be part of another device, as shown in figure 4407b, or separate. A portable version requiring a table or desk for support is called a **light box.**

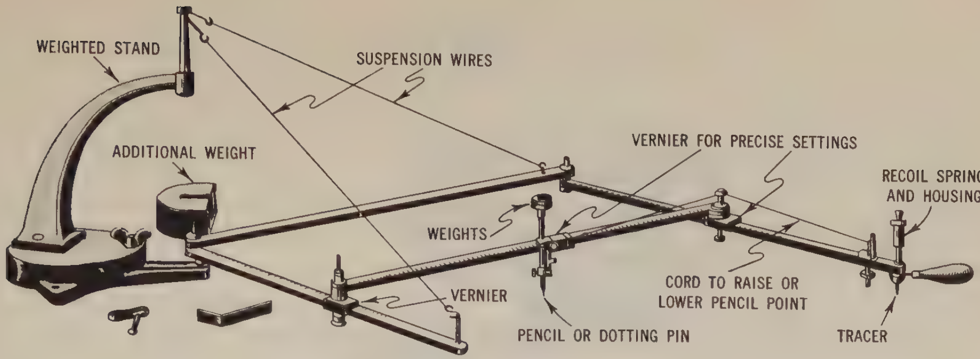


FIGURE 4407a.—A suspended pantograph.

Three-arm protractor (fig. 4011c), a device for plotting a position from horizontal sextant angles.

Proportional dividers (fig. 4011d), a device for transferring distances at a different scale.

Spacing dividers (fig. 4011e), a device for dividing a length into equal segments.

Beam compasses (fig. 4011b), a device for drawing circles of large radius, or measuring distances too great for ordinary compasses or dividers.

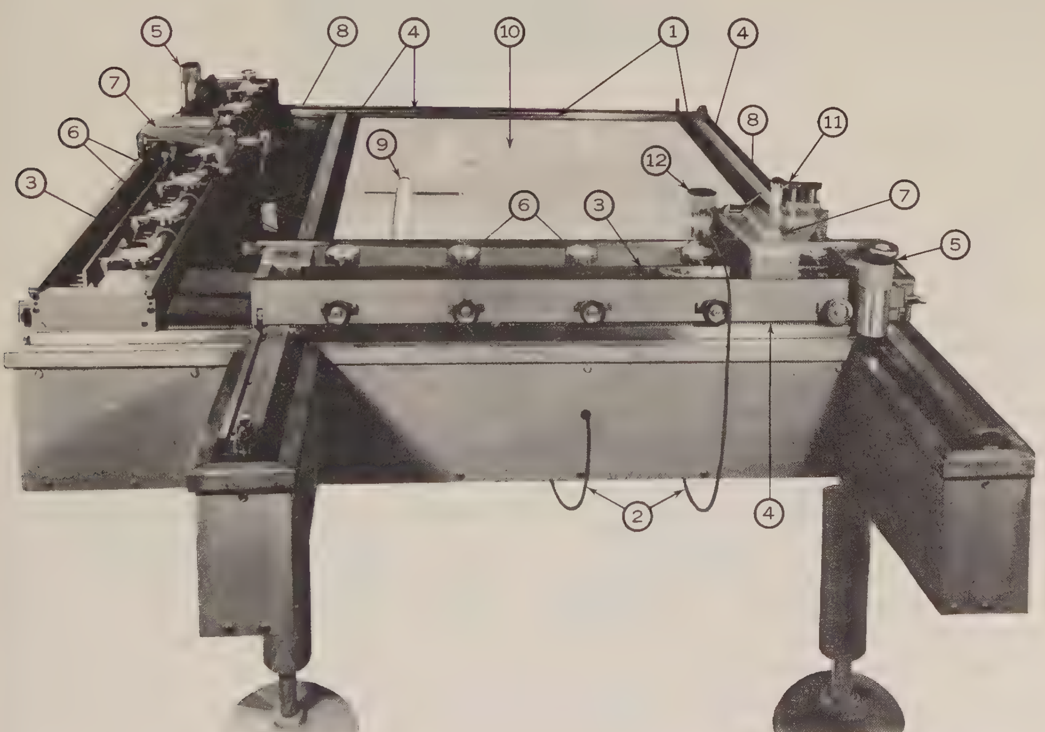


FIGURE 4407b.—Projection-ruling machine. 1. Metric bar scale(s), subdivisions in millimeters. 2. Power supply to illuminate light table and microscopes for reading metric bar scales. 3. Metal spline—can be set to rule straight or curved lines. 4. Gear tracks for carriage(s) to travel on. 5. Microscope for reading metric bar scales. 6. Dials for setting curvature in metal spline. 7. Ruling pen carriage. 8. Ruling pen. 9. Fluorescent light. 10. Transparent glass table top (light table). 11. Rectangular (X−Y) coordinate plotting device. 12. Microscope for reading scale on movable plate.

FIGURE 4407c.—Portable vertical projector.

Road pen (fig. 4407d), a device with a double pen, to permit drawing of two curved lines with constant distance between them.

Plastic splines, long narrow strips of plastic material, which can be bent into a large variety of curves, to form a pen guide for drawing curved lines.

Border-scale subdividing device (fig. 4407e), a mechanical device for drawing equal subdivisions of a border (latitude and longitude) or similar scale. The illustration shows: *A*, a steel straightedge 60 inches long which is secured, by weights or clamps, parallel to the inside border or neat line of the chart; *B*, steel divider plate, one of 12 plates measuring $9 \times 4 \times \frac{1}{16}$ inches and cut with deep, diverging grooves; *C*, steel triangle to be placed with its base against the divider plate; *D*, clamp screws for securing the divider plate to the straightedge; *E*, knob for controlling a screw to move the divider point toward or away from the straightedge; *F*, divider point, which rests in the grooves of the divider plate; and *G*, a set screw to prevent creeping of the divider point. When the device is set up, the divider point is set so that the distance between consecutive grooves is the required distance between graduations of the scale. For a variable scale such as that for latitude on a Mercator chart, this setting must be changed periodically for accurate results.

Diagonal metric scale (fig. 4011a), a device for measuring distances in metric scale units.

4408. Drawing material.—The drawings for a nautical chart should be made on a dimensionally stable drafting medium for maximum accuracy. Plastics, "Duco" plate, and metal-laminated paper, all have minimum distortion qualities and good inking surfaces. The vinyl resins, introduced in the United States in 1928, are among the more recent materials receiving widespread use. Plastics, known under many trade names, are produced in opaque, transparent, and translucent sheets. The "Duco" plate is a grained zinc lithographic printing plate, sprayed with a good grade of enamel paint, which produces a good drafting surface. Metal-laminated paper is produced by mounting a good grade of drawing paper (Bristol board) on a metal plate, using an adhesive and pressure. The metal plate used is normally a zinc or aluminum lithographic printing plate. Use of this type of drawing medium has lost much of its popularity since the introduction of plastics.

FIGURE 4407d.—Swivel road pen in contour penholder.

4409. Reduction methods.—Source material at the same or larger scale as the compilation manuscript is frequently employed in cartography. This practice avoids inaccuracies which might be introduced in enlarging smaller scale data to the scale of the compilation. The four principal methods of reduction are by camera, projector,

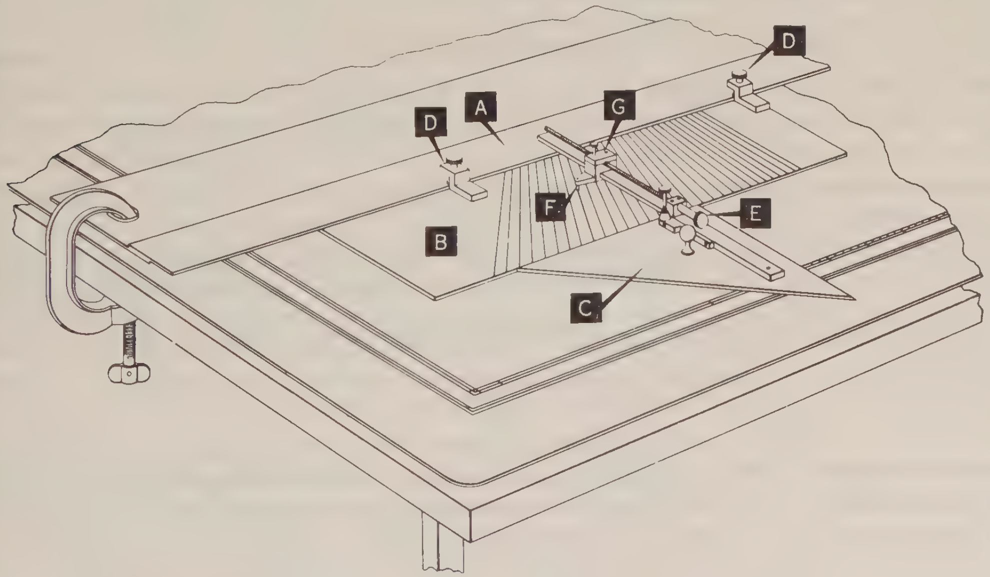


FIGURE 4407e.—Border-scale subdividing device.

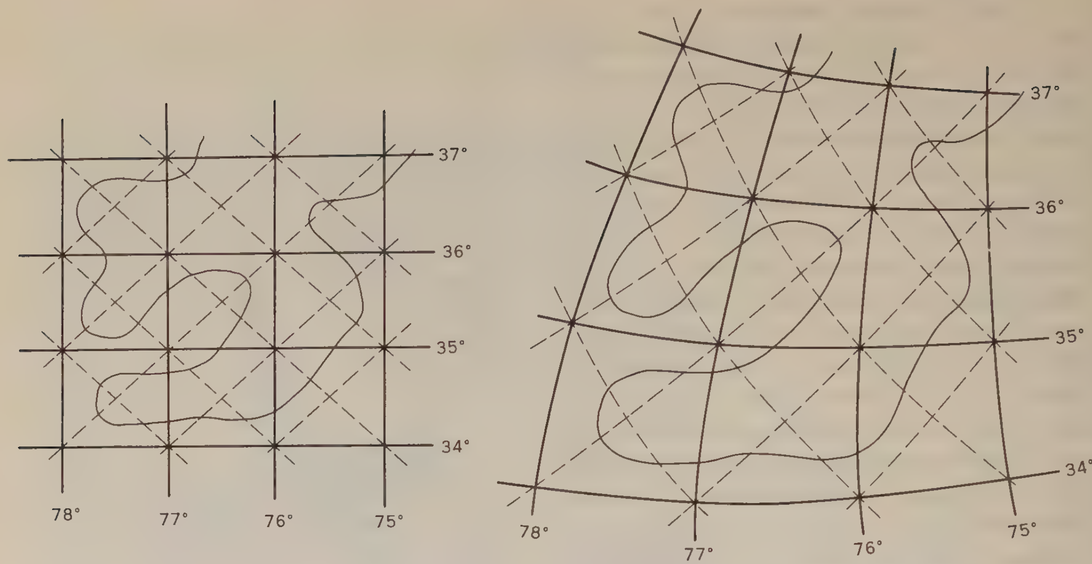


FIGURE 4409.—The "diagonal square" method of sketch plotting.

pantograph, or "diagonal squares." The camera method is the most accurate and is preferred when reduction is the only consideration. In this method the source material is photographed to the desired scale. The projector (fig. 4407c) and pantograph (fig. 4407a) are used when the work is not extensive, and some alterations are to be made. The "diagonal square" method is used to "sketch-plot" data from one scale or projection to another, for example shore line data from a Mercator chart replotted onto a gnomonic or an azimuthal equidistant projection. This method, similar to the perspective grid method of photogrammetry (art. 4309), is illustrated in figure 4409.

4410. Datums.—As explained in chapter XLI, each survey has an "origin," and geographic coordinates of other points are determined with reference to this control point. In some regions, a single origin has been used as control for an extensive area. A system of control points established with reference to a single origin is called a **datum** (plural in this usage is *datums*, not *data*), and given an identifying name. Examples are the North American Datum of 1927, Tokyo Datum, Nahrwan Datum, and the Kelienphur Datum. Since each origin was determined independently, coordinates determined with reference to one datum will not agree with coordinates of the same point determined with reference to a second datum. Normally, there is no confusion because various points are generally defined with reference to a single datum, which is used for the entire area. The limits of a given datum are usually defined by some natural barrier such as an ocean, uninhabited area, etc., so that transitions do not generally occur at a troublesome place. Occasionally, political boundaries of adjacent countries prevent the establishment of geographic coordinate agreement at common stations.

A datum which controls geographic positions of points on the earth is called a **horizontal datum** to distinguish it from a reference level for heights and depths, called a **vertical datum.** For depths, the reference level found to be most realistic for measuring the height of tide is used for charts. This varies in different parts of the world, as indicated in appendix M. This reference level is sometimes referred to as the **chart datum.** In nearly all instances it is some form of *low* water. Heights are nearly always indicated with reference to *mean* sea level or *high* water. Therefore, two different vertical datums are generally used for the same chart.

When the information is available, both horizontal and vertical datums are indicated in the chart legend. If a chart extends over more than one datum, it is normally of such small scale that differences between datums are not of significance.

4411. Borders and scales.—Selection of the border style to use depends primarily upon the scale of the chart, but consistency with other charts in the vicinity is a factor. In general, some form of **plan border** (fig. 4411a) is used for charts of scale larger than 1:50,000; and a **scale border** (fig. 4411a) for charts of smaller scale.

On large-scale charts, **graphic scales** are usually added. On recent charts with plan borders, yard scales are placed along the right and left sides, and both kilometer and nautical mile scales at the top. On charts with scale borders, and older plan border charts, graphic scales are shown at a convenient location inside the border. When this

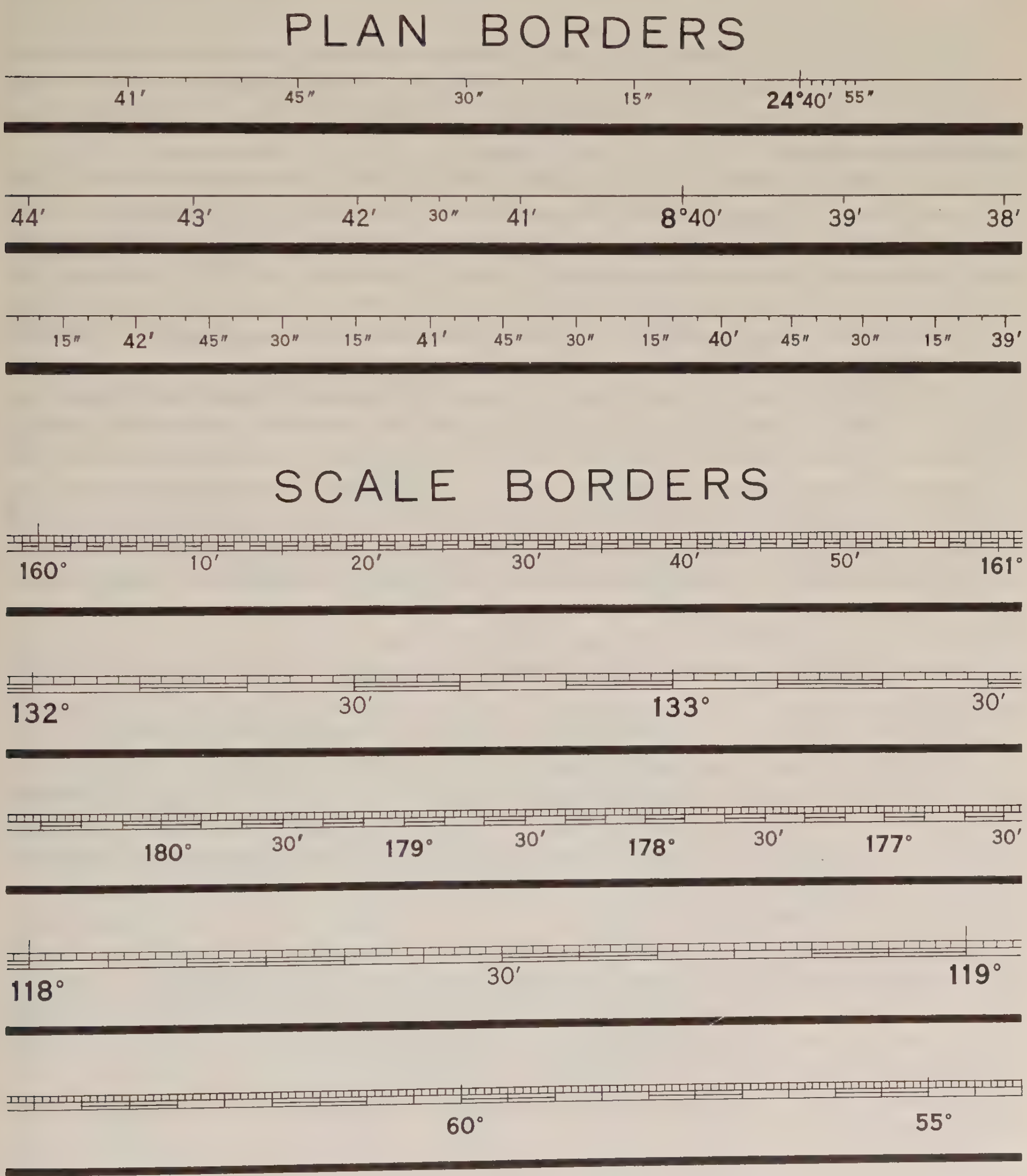


Figure 4411a.—Border styles.

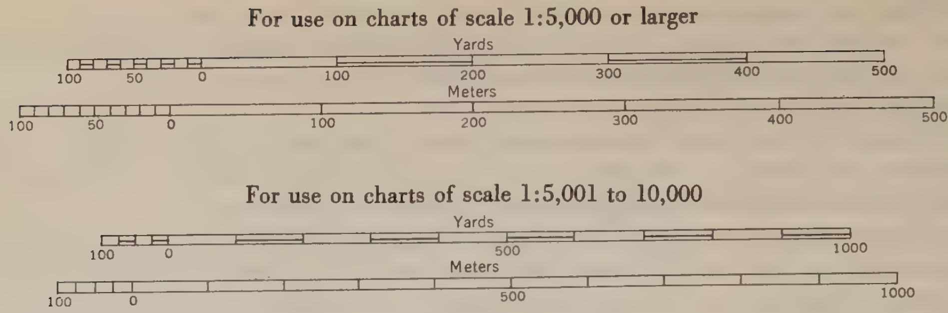


FIGURE 4411b.—Typical graphic scales.

is done, a nautical mile scale is given first, followed by a yard scale and a meter scale. The number of yards and meters shown is dependent upon the amount of space available, the scale of the chart, and the overall size of the chart. Figure 4411b illustrates typical graphic scales.

4412. Charted details.—The standard symbols and abbreviations which have been approved for use on nautical charts published by the United States of America are shown in appendix K. These are in substantial agreement with the recommendations of the International Hydrographic Bureau. From time to time changes in the standards are made to keep pace with changing requirements.

Topographic data are normally obtained from aerial photogrammetric compilations, surveys, and existing maps and charts.

Depths are indicated by soundings or explanatory notes. Only a small percentage of the soundings obtained in a hydrographic survey can be shown on a nautical chart. The least depths are generally selected first, and a pattern built around them to provide a representative indication of bottom relief. In shallow water, soundings may be spaced 0.2 to 0.4 inch apart. The spacing is gradually increased as water deepens, until a spacing of 0.8 to 1.0 inch is reached in deeper waters offshore. Where a sufficient number of soundings are available to permit adequate interpretation, depth curves are drawn in at 1-, 3-, 6-, 10-, 20-, 30-, 50-, and 100-fathom depths. Other features are shown as indicated in chapter V and appendix K.

Aids to navigation are shown by symbol and legend, as indicated in chapter V and appendix K.

Place names are given according to sources and decisions recommended by the United States Board on Geographic Names. The general policy is to use the local source. Foreign names are used in foreign areas. Generally, international features such as the Gulf of Mexico or a river that flows through several countries (for instance, the Danube) are given the commonly accepted English name. The names of countries are also given in English. When appropriate, the English equivalents of foreign terms are shown in a glossary on the chart.

All letters and numbers shown on a chart are printed on sheets of cellulose acetate or white paper, backed with a suitable adhesive which adheres to the drafting material when pressed into place. This practice promotes uniformity and legibility. Various type styles are used to distinguish between different kinds of features and to provide a pleasing appearance.

Notes are used to convey information which does not lend itself to convenient symbolization.

Compass roses are placed where they are readily available for use, yet obscure a minimum of chart information. The number, size, and location are suited to the individual chart.

The chart title is placed at a location that will result in minimum loss of chart information, preferably on land and in one of the corners.

The official seal of the charting agency is generally placed above the chart title.

The chart number is placed at several convenient locations in the margin. When a chart is cancelled, its number is not reassigned to another chart for several years. When a series of charts is planned, a block of numbers is reserved to provide continuity throughout the series.

When a chart is prepared by a government charting agency, and printed by a different government agency or commercial printing establishment; or if a government charting agency prints a chart for another office or department of the government, a suitable imprint note is placed in the bottom margin.

From Requirement to Printed Chart

4413. Requirement.—A new or altered chart comes into being as a result of recognition of a requirement. This requirement may be established by the charting agency itself, from its continual review of existing charts, the receipt of new information, and the needs of operating units; or it may originate with the operating forces or military planners. Whatever its source, the requirement precedes all other steps in the production of the chart.

4414. Estimate of the situation.—Having recognized the requirement, the charting agency having jurisdiction then studies the situation to determine priority and availability of source information. The intended use, required scale, and urgency are considered in selecting the chart production method (art. 4405).

If a survey is needed, the type, extent, and thoroughness are determined according to the availability of survey vessels, personnel, and other pertinent factors. A thorough hydrographic survey is a slow and costly process. If it must be preceded by geodetic control or aerial photographic surveys, the time and expense are increased. Several weeks may be needed to thoroughly survey a single harbor of moderate size. Survey operations are generally planned on a long-range basis to provide adequate coverage for an entire area once operations have begun. In time of war or national emergency, such plans may have to be abandoned and survey ships sent into forward areas. During World War II, survey ships went into the Pacific with the fleet, and it was not uncommon for survey parties to be in operation on shore before fighting had ceased. To meet urgent requirements, the larger survey ships were provided with drafting and printing facilities so that charts could be produced almost as soon as the data were collected. In time of peace such urgency is not generally required.

4415. Research and planning.—Before construction of a chart begins, all available data are investigated and evaluated. All details of the chart are planned, and specifications and procedures are prepared. The best sources of data are recommended. The area is selected to provide maximum usefulness consistent with limitations imposed by scale, size of sheets that can be accommodated by the printing press, land and water configuration, etc. The use expected to be made of the chart is an important consideration. Port and harbor charts normally portray the most important hydrographic region centered on the sheet, while approach charts embrace maximum sea room and only a limited amount of land area, sufficient to include the prominent features of navigational value. Care is exercised to avoid omission of important aids to navigation, river entrances, channels, etc. This often results in some overlapping of adjoining charts. The extent of adequate survey and other compilation data are also considerations.

The chart projection is selected to meet the requirements. Nearly all nautical charts are on the Mercator projection, but the gnomonic, Lambert conformal, and other projections are used for special-purpose charts. This subject is further discussed in chapters III, V, and XXV.

4416. Compilation.—From the recommended sources, the compiler selects the data to be used. The task of determining what to include and what to omit may be of considerable magnitude, particularly when some of the information is inconsistent or of questionable reliability. The skill and wisdom with which this assignment is filled has a direct bearing upon the value of the completed chart.

Various methods of compilation are in use depending upon the amount and nature of the data. When most of it comes from published charts, the compiler may prepare a film positive **compilation mosaic.** To do this he computes and drafts the **graticule** (latitude and longitude lines for the map projection used) and plots the control points at their correct positions. The source material is photographed to the scale of the chart, and film positives are provided. These are secured to the graticule in their proper geographic positions. If necessary, the film positives are cut into small pieces, so that errors in positions of distorted features can be proportionately distributed between control points. Deletions and corrections are then made, and broken lines are connected.

The compilation is carefully reviewed for completeness and accuracy before a contact negative is made. Black-line paper prints are made for specification sheets, and light blue-line plastic prints for drafting. The blue-line prints are made on stable plastic material so that drafting can be done directly on the print. Since light blue does not photograph, only those features inked in black will appear on the negative to be made of each finished drawing for chart reproduction. The black-line prints are used to indicate which items to include on the finished drawing. Generally, separate prints are used for indicating topographic and cultural features, hydrographic features, type faces and sizes, and approved geographic names.

4417. Drafting of the **chart original** is done on a medium having minimum distortion qualities. The graticule and borders are drawn first, followed by planimetric (horizontal) detail, and then relief. Next, the lettering is added. The chart title, notes, etc., are added last, so they will not interfere with charted features.

A separate drawing is made for black and for each color to be shown on the chart. The use of several draftsmen on the same chart permits the work on the various drawings to go forward simultaneously and makes better use of the difference in experience and skill of the various draftsmen. The preparation of the chart originals for a single chart may require several months of continuous drafting.

4418. Review and edit.—When drafting has been completed, the chart original is reviewed by an experienced cartographer. He carefully checks every detail for accuracy, and consults the latest *Notice to Mariners* and all other sources, to be certain that nothing that should be charted has been omitted, and that the latest data have been used. Any corrections to be made are indicated on a transparent overlay which is returned with the chart original to the draftsman for action.

When all corrections have been made and checked, the chart original goes to a chart editor. Here it is checked to see that the line work is sharp and clear, the type is securely fastened, and that all work is in accordance with established standards. Another check is made to be sure that the latest and most complete information has been used.

When the chart editor is satisfied that the chart original meets the foregoing requirements and is safe for navigation, he releases it for reproduction. The various steps in preparing a chart original by the mosaic process are shown in figure 4418.

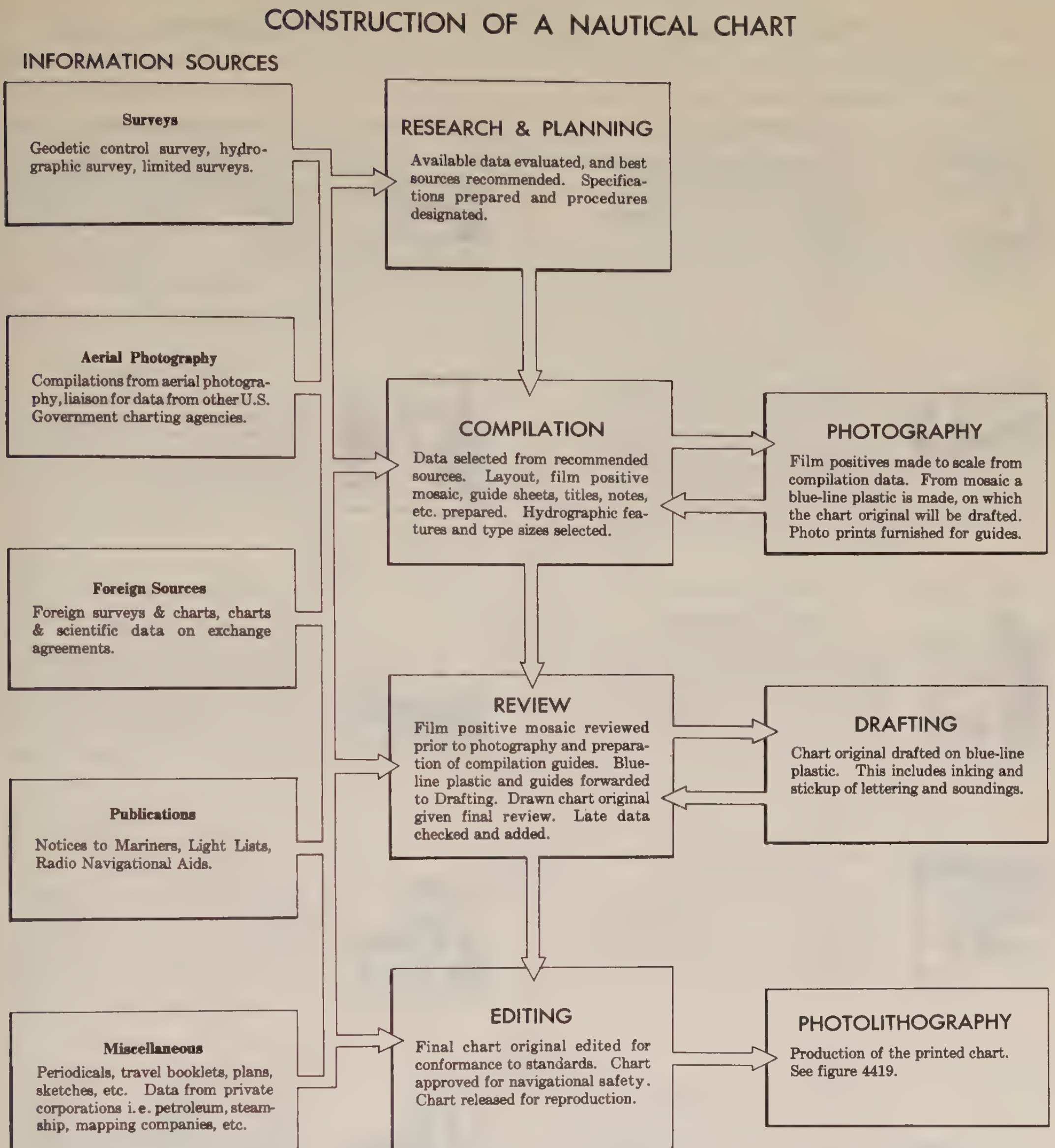


FIGURE 4418.—Flow of work in the construction of a nautical chart by the mosaic method.

4419. Reproduction.—Three basic processes of reproduction (printing) are in use commercially: **letterpress** prints directly from raised type or other image; **gravure** prints directly from a depressed image; and **lithography** prints indirectly (by **offset**) from a surface that is neither raised nor depressed, operating on the principle of the mutual repulsion of grease and water. The lithographic printing plate has a grease image which is receptive to greasy ink, and a nonprinting portion which is receptive to water. Charts are usually reproduced by **photolithography,** which uses photography in the preparation of the lithographic plates. The essential steps in the reproduction of a chart by this process are shown in figure 4419.

In this process, the results of each step are checked carefully. A chart editor edits black photo proofs made from the negatives, and color litho proofs made from the plates. During printing, a continual check is made by the pressman to insure uni-

FIGURE 4419.—Flow of work in the photolithographic processing of a nautical chart.

formity of impressions and accurate "register" (each color being printed at the correct place on the chart). The highest standards possible are maintained throughout the entire operation.

When printing is completed, the plates are removed from the cylinders, cleaned, and the image side given a protective coating of soluble gum arabic and asphaltum. The plates are then stored in vertical racks. The glass negatives have already been stored. A photographic duplicate of the chart original may have been made, and the original and a copy stored at separate locations for possible future use.

4420. Chart record.—A record of each chart is maintained to provide a history, from the authorization of its first edition to the date of the last action, perhaps many editions and printings later. The record accompanies the chart original throughout production, with appropriate entries being made as it progresses through the various steps. Included in the record are such data as sources of information used, method of construction, the map projection graticule computation, geodetic control used, and decisions rendered.

A well-written chart record is valuable in the prevention of duplicative effort. Also, it is the medium through which research is conducted when questions arise as to why certain data were shown on or removed from any issue of the chart.

APPENDICES

APPENDICES

APPENDIX A

ABBREVIATIONS AND SYMBOLS

A more complete listing of abbreviations and symbols is given in H.O. Pub. No. 220, *Navigation Dictionary*, of which this is an abridgment.

A, amplitude; augmentation; away (altitude difference).

a, altitude difference (Ho$\sim$Hc); altitude factor (change of altitude in one minute of time from meridian transit); assumed.

a_0, first Polaris correction.

a_1, second Polaris correction.

a_2, third Polaris correction.

AC, alternating current.

add'l, additional.

AF, audio frequency.

***a*L,** assumed latitude.

AM, amplitude modulation.

AM, ante meridian (before noon).

antilog, antilogarithm.

AP, assumed position.

approx., approximate, approximately.

AU, astronomical unit.

$a\lambda$, assumed longitude.

B, atmospheric pressure correction (altitude); bearing, bearing angle.

B_A, difference between heading and apparent wind direction.

Bn, bearing (as distinguished from bearing angle).

Bpgc, bearing per gyro compass.

B_T, difference between heading and true wind direction.

C, acceleration correction (altitude); Celsius (centigrade); chronometer time; compass (direction); correction; course, course angle.

CB, compass bearing.

CC, compass course.

CCU, chart comparison unit.

CE, chronometer error; compass error.

CH, compass heading.

cm, centimeter, centimeters.

Cn, course (as distinguished from course angle).

co-, the complement of (90° minus).

colog, cologarithm.

corr., correction.

cos, cosine.

cot, cotangent.

cov, coversine.

Cpgc, course per gyro compass.

cps, cycles per second.

csc, cosecant.

CW, continuous wave.

CZn, compass azimuth.

D, deviation; dip (of horizon); distance.

d, declination (astronomical); difference.

d, declination change in one hour.

DC, direct current.

dec., declination.

Dev., deviation.

DG, degaussing.

diff., difference.

dist., distance.

DLo, difference of longitude (arc units).

DR, dead reckoning, dead reckoning position.

DRE, dead reckoning equipment.

DRM, direction of relative movement.

DRT, dead reckoning tracer.

D_s, dip short of horizon.

dur., duration.

dλ, difference of longitude (time units).

E, east.

e, base of Naperian logarithms.

e, eccentricity.

EHF, extremely high frequency.

EP, estimated position.

EPI, electronic position indicator.

Eq.T, equation of time.

ETA, estimated time of arrival.

F, Fahrenheit; fast; longitude factor; phase correction (altitude).

f, latitude factor.

FM, frequency modulation.
ft., foot, feet.
G, Greenwich, Greenwich meridian (upper branch); grid (direction).
g, acceleration due to gravity; Greenwich meridian (lower branch).
GAT, Greenwich apparent time.
GB, grid bearing.
GC, grid course.
GE, gyro error.
GH, grid heading.
GHA, Greenwich hour angle.
GMT, Greenwich mean time.
GP, geographical position.
Gr., Greenwich.
GST, Greenwich sidereal time.
GV, grid variation.
GZn, grid azimuth.
H, high (loran PRR); horizontal intensity of earth's magnetic field; sea tilt correction (altitude).
h, altitude (astronomical); height above sea level.
***ha*,** approximate altitude.
hav, haversine.
Hc, computed altitude.
Hdg., heading.
HE, heeling error; height of eye.
HF, high frequency.
HHW, higher high water.
HLW, higher low water.
H.O., Hydrographic Office.
Ho, observed altitude.
HP, horizontal parallax.
Hp, precomputed altitude.
Hpgc, heading per gyro compass.
hr, rectified altitude.
hr., hour.
hrs., hours.
hs, sextant altitude.
ht, tabulated altitude.
HW, high water.
I, dip (magnetic); instrument correction.
IC, index correction.
in., inch, inches.
int., interval.
ISLW, Indian spring low water.
J, irradiation correction (altitude).
K, Kelvin (temperature); constant of the cone; constant proportional to required length of Flinders bar.
kc, kilocycle, kilocycles; kilocycles per second.
km, kilometer, kilometers.
kmc, kilomegacycle, kilomegacycles; kilomegacycles per second.
kn., knot, knots.
L, latitude; low (loran PRR); lower limb correction for moon (from *Nautical Almanac*); wave length (water).
***l*,** difference of latitude; logarithm, logarithmic.
LAN, local apparent noon.
LAT, local apparent time.
lat., latitude.
LF, low frequency.
LHA, local hour angle.
LHW, lower high water.
LL, lower limb.
LLW, lower low water.
Lm, middle latitude.
LMT, local mean time.
log, logarithm, logarithmic.
$\log_e$, natural logarithm (to the base e).
$\log_{10}$, common logarithm (to the base 10).
long., longitude.
LST, local sidereal time.
LW, low water.
M, celestial body; meridian (upper branch); magnetic (direction); meridional parts; nautical mile, miles.
m, meridian (lower branch); meridional difference ($M_1 \sim M_2$); meter, meters; statute mile, miles.
mag., magnetic; magnitude.
MB, magnetic bearing.
mb, millibar, millibars.
MC, magnetic course.
mc, megacycle, megacycles; megacycles per second.
MF, medium frequency.
MH, magnetic heading.
MHHW, mean higher high water.
MHW, mean high water.
MHWN, mean high water neaps.
MHWS, mean high water springs.
mi., mile, miles.
mid, middle.
min., minute, minutes.
MLLW, mean lower low water.
MLW, mean low water.
MLWN, mean low water neaps.

MLWS, mean low water springs.
mm, millimeter.
mo., month.
mos., months.
mph, miles (statute) per hour.
MPP, most probable position.
MSL, mean sea level.
MZn, magnetic azimuth.
N, north; tilt correction (altitude).
***n*,** natural (trigonometric function).
Na, nadir.
P, atmospheric pressure; parallax; planet; pole; wave period (water).
p, departure, polar distance.
PC, personal correction.
pgc, per gyro compass.
P in A, parallax in altitude.
PM, pulse modulation.
PM, post meridian (after noon).
Pn, north pole; north celestial pole.
PPI, plan position indicator.
PRR, pulse repetition rate.
Ps, south pole; south celestial pole.
psc, per standard compass.
p stg c, per steering compass.
Pub., publication.
PV, prime vertical.
Q, Polaris correction (*Air Almanac*).
QQ′, celestial equator.
R, Rankine (temperature); refraction.
RA, right ascension.
rad, radian, radians.
RAR, radio acoustic ranging.
Ra. Ref., radar reflector.
RB, relative bearing.
R Bn, radiobeacon.
RDF, radio direction finder.
rev., reversed.
RF, radio frequency.
R Fix, running fix.
RZn, relative azimuth.
S, sea-air temperature difference correction (altitude); slow; south; special (loran PRR); speed.
s, ½ (h+L+p); standard deviation.
S_A, speed of apparent wind in units of ship's speed.
SD, semidiameter.
sec, secant.
sec., second, seconds.
semidur., semiduration.
SH, ship's head (heading).
SHA, sidereal hour angle.
SHF, super high frequency.
sin, sine.
SRM, speed of relative movement.
S_T, speed of true wind in units of ship's speed.
T, air temperature correction (altitude); table; temperature; time; toward (altitude difference); true (direction).
t, dry-bulb temperature; elapsed time; meridian angle.
t′, wet-bulb temperature.
tab., table.
tan, tangent.
TB, true bearing; air temperature-atmospheric pressure correction (altitude).
TC, true course.
TcHHW, tropic higher high water.
TcHLW, tropic higher low water.
TcLHW, tropic lower high water.
TcLLW, tropic lower low water.
T_G, ground-wave reading (loran).
T_{GS}, ground-wave–sky-wave reading (loran).
TH, true heading.
TR, track.
Tr., transit.
T_S, sky-wave reading (loran).
T_{SG}, sky-wave–ground-wave reading (loran).
TZn, true azimuth.
U, upper limb correction for moon (from *Nautical Almanac*).
UHF, ultra high frequency.
UL, upper limb.
UT, universal time.
V, deflection of the vertical; variation; vertex.
***v*,** excess of GHA change from adopted value for one hour.
Var., variation.
ver, versine.
VHF, very high frequency.
VLF, very low frequency.
VPR, virtual PPI reflectoscope.
W, watch time; wave-height correction (altitude); west.
WE, watch error.
WT, war time.
X, parallactic angle.

yd., yard.
yds., yards.
yr., year.
yrs., years.
Z, azimuth angle; Coriolis correction (altitude); vertical intensity of earth's magnetic field; zenith.
z, zenith distance.
ZD, zone description.
Zn, azimuth (as distinguished from azimuth angle).
Znpgc, azimuth per gyro compass.
ZT, zone time.
α, damping error (gyro compass).
Δ, a small increment, or the change in one quantity corresponding to unit change in another.
δ, speed error (gyro compass).
λ, longitude; shielding factor; wave length (radiant energy).
μ, index of refraction; permeability.
μs, microsecond.
π, ratio of circumference of circle to diameter$=3.14159+$.

Positions

⊙ Dead reckoning position; fix, running fix.
⊡ Estimated position.

Mathematical Symbols

$+$ Plus (addition)
$-$ Minus (subtraction)
$\pm$ Plus or minus
$\sim$ Absolute difference
$\times$ Times (multiplication)
$\div$ Divided by (division)
$\sqrt{\ }$ Square root
$\sqrt[n]{\ }$ nth root
$=$ Equals
$>$ Is greater than
$<$ Is less than
$\int$ Integral sign
∞ Infinity
. . . Repeating decimal

Celestial Bodies

☉ Sun
☾ Moon
☿ Mercury
♀ Venus
⊕ Earth
♂ Mars
♃ Jupiter
♄ Saturn
♅ Uranus
♆ Neptune
♇ Pluto
☆ Star
☆-P Star-planet altitude correction (altitude)
$\underline{☉}$ $\underline{☾}$ Lower limb
$\ominus$ ☾ Center
$\overline{☉}$ $\overline{☾}$ Upper limb
● New moon
● Crescent moon
◐ First quarter
○ Gibbous moon
○ Full moon
○ Gibbous moon
◑ Last quarter
● Crescent moon

Signs of the Zodiac

♈ Aries (vernal equinox)
♉ Taurus
♊ Gemini
♋ Cancer (summer solstice)
♌ Leo
♍ Virgo
♎ Libra (autumnal equinox)
♏ Scorpius
♐ Sagittarius
♑ Capricornus (winter solstice)
♒ Aquarius
♓ Pisces

Miscellaneous Symbols

^{y}	Years
^{m}	Months
^{d}	Days
^{h}	Hours
^{m}	Minutes of time
^{s}	Seconds of time
▬	Remains below horizon
▭	Remains above horizon
////	Twilight all night
⚹	Interpolation impractical
$^{\circ}$	Degrees
$'$	Minutes of arc
$''$	Seconds of arc
☌	Conjunction
☍	Opposition
□	Quadrature
☊	Ascending node
☋	Descending node

APPENDIX B

GREEK ALPHABET

Α α a	Alpha	Ν ν	Nu
Β β ϐ	Beta	Ξ ξ	Xi
Γ γ	Gamma	Ο ο	Omicron
Δ δ	Delta	Π π ϖ	Pi
Ε ε	Epsilon	Ρ ρ	Rho
Ζ ζ	Zeta	Σ σ ς	Sigma
Η η	Eta	Τ τ	Tau
Θ θ ϑ	Theta	Υ υ	Upsilon
Ι ι	Iota	Φ φ ϕ	Phi
Κ κ	Kappa	Χ χ	Chi
Λ λ	Lambda	Ψ ψ	Psi
Μ μ	Mu	Ω ω	Omega

APPENDIX C

GLOSSARY

This appendix is an abridgment of the definitions of H.O. Pub. No. 220, *Navigation Dictionary*, to which reference should be made for complete definitions.

abeam. Bearing approximately 090° relative (*abeam to starboard*) or 270° relative (*abeam to port*).

aberration. The apparent displacement of a celestial body in the direction of orbital motion of the earth.

abscissa. The horizontal coordinate of a set of rectangular coordinates.

absolute humidity. The mass of water vapor per unit of volume of air.

absolute zero. The lowest possible temperature, about (−) 459°.67 F or (−) 273°.15 C.

aclinic line. The magnetic equator.

acoustic navigation. Sonic navigation.

acute angle. An angle less than 90°.

advance. The distance a vessel moves in its original direction in making a turn.

advanced line of position. A line of position which has been moved forward to allow for the run since the line was established.

advection. Horizontal movement of part of the atmosphere.

age of the moon. The elapsed time, usually expressed in days, since the last new moon.

agonic line. A line connecting points of no magnetic variation.

aground. Touching, resting, or lodged on the bottom.

ahead. Bearing approximately 000° relative.

aid to navigation. A device external to a craft, designed to assist in determination of position of the craft, or of a safe course, or to warn of dangers.

air almanac. A periodical publication of astronomical data designed primarily for air navigation.

air mass. An extensive body of air within which the conditions of temperature and moisture in a horizontal plane are essentially uniform.

air navigation. The navigation of aircraft.

air temperature correction. That sextant altitude correction due to changes in refraction caused by difference between the actual temperature and the standard temperature used in the computation of the refraction table.

alidade. A telescope or other device mounted over a compass, compass repeater, or compass rose, for measuring direction.

alignment. Adjustment of the tuned circuits of electronic equipment for optimum performance, or synchronization of two or more components of an electronic system.

almanac. A periodical publication of astronomical data useful to a navigator.

Alnico. The trade name for an alloy composed principally of aluminum, nickel, cobalt, and iron; used for permanent magnets.

alternating current. An electric current that continually changes in magnitude and periodically reverses polarity.

alternating fixed and flashing light. A fixed light varied at regular intervals by one or more flashes of greater brilliance, with color variations in either the fixed light or flashes, or both.

alternating fixed and group flashing light. A fixed light varied at regular intervals by a group of two or more flashes of greater brilliance, with color variations in either the fixed light or flashes, or both.

alternating flashing light. A light showing one or more flashes with color variations at regular intervals, the duration of light being less than that of darkness.

alternating group flashing light. A light showing groups of flashes with color variations at regular intervals, the duration of light being less than that of darkness.

alternating group occulting light. A light having groups of total eclipses at regular intervals and having color variations, the duration of light being equal to or greater than that of darkness.

alternating light. A light having periodic color variations, particularly one with constant luminous intensity.

alternating occulting light. A light having one or more total eclipses at regular intervals and having color variations, the duration of light being equal to or greater than that of darkness.

altitude. Angular distance above the horizon; the arc of a vertical circle between the horizon and a point on the celestial sphere, measured upward from the horizon.

altitude azimuth. An azimuth determined when altitude, declination, and latitude are known.

altitude circle. Parallel of altitude.

altitude difference. The difference between computed and observed altitudes, or between precomputed and sextant altitudes.

altitude intercept. Altitude difference.

altocumulus. A cloud layer (or patches) within the middle level (mean height 6,500–20,000 ft.) composed of rather flattened globular masses, the smallest elements of the regularly arranged layers being fairly thin, with or without shading. These elements are arranged in groups, in lines or in waves, following one or two directions, and are sometimes so close together that their edges join.

altostratus. A sheet of gray or bluish cloud within the middle level (mean height 6,500–20,000 ft.). Sometimes the sheet is composed of a compact mass of dark, thick, gray clouds of fibrous structure; at other times the sheet is thin and through it the sun or moon can be seen dimly as though gleaming through ground glass.

amplitude. **1.** Angular distance north or south of the prime vertical; the arc of the horizon or the angle at the zenith between the prime vertical and a vertical circle, measured north or south from the prime vertical to the vertical circle. **2.** The maximum value of the displacement of a wave or other periodic phenomenon from the zero position.

amplitude balance. Equality in the amplitude of two or more signals.

amplitude modulation. The process of changing the amplitude of a carrier wave in accordance with the variations of a modulating wave.

anchorage. An area where a vessel anchors or may anchor, either because of suitability or designation.

anchorage buoy. One of a series of buoys marking the limits of an anchorage.

anchorage chart. A nautical chart showing prescribed or recommended anchorages.

anchor buoy. A buoy marking the position of an anchor.

anemometer. An instrument for measuring the speed of the wind. Some instruments also indicate the direction from which it is blowing.

aneroid barometer. An instrument which determines atmospheric pressure by the effect of such pressure on a thin-metal cylinder from which the air has been partly exhausted.

angle. The inclination to each other of two intersecting lines, measured by the arc of a circle intercepted between the two lines forming the angle, the center of the circle being the point of intersection.

angular distance. The angular difference between two directions, or the arc of the great circle joining two points.

anneal. To heat (metal) to a high temperature and then allow to cool slowly, for the purpose of softening, making less brittle, or removing permanent magnetism.

annular. Ring-shaped.

annular eclipse. An eclipse in which a thin ring of the source of light appears around the obscuring body.

anode. The positive pole or electrode of an electron tube or an electric cell.

ante meridian. Before noon.

antenna. A conductor or system of conductors for radiating or receiving radio waves.

antenna array. A group of antennas arranged so as to obtain directional characteristics.

anticyclone. An approximately circular portion of the atmosphere, having relatively high atmospheric pressure and winds which blow clockwise around the center in the northern hemisphere and counterclockwise in the southern hemisphere.

antilogarithm. The number corresponding to a given logarithm.

aperiodic compass. A compass that, after being deflected, returns by one direct movement to its proper reading, without oscillation.

aphelion. That orbital point farthest from the sun when the sun is the center of attraction (as in the case of a planet).

apogean tides. Tides of decreased range occurring when the moon is near apogee.

apogee. That orbital point farthest from the earth when the earth is the center of attraction (as in the case of the moon).

apparent altitude. Rectified altitude.

apparent horizon. Visible horizon.

apparent motion. Motion relative to a specified or implied reference point which may itself be in motion.

apparent sun. The actual sun as it appears in the sky.

apparent time. Time based upon the rotation of the earth relative to the apparent (true) sun.

apparent wind. Wind relative to a moving point, such as a vessel.

approximate altitude. An altitude determined by inexact means, as by estimation or by a star finder or star chart.

arc. **1.** Part of a curved line, as a circle. **2.** The graduated scale of an instrument for measuring angles, as a marine sextant.

arc of visibility. The arc of a light sector, designated by its limiting bearings as observed at points other than the light.

argument. One of the values used for entering a table or diagram.

arm. **1.** An inlet. **2.** A slender part of an instrument, device, or machine.

Armco. The trade name for a high-purity, low-carbon iron, used for Flinders bars, quadrantal correctors, etc.

arming. Tallow or other substance placed in the recess at the lower end of a sounding lead, for obtaining a sample of the bottom.

artificial horizon. A device for indicating the horizontal.

artificial-horizon sextant. A sextant having an artificial horizon built in.

A-scope. A cathode ray scope on which the trace appears as a horizontal or vertical range scale and the signals appear as perpendicular deflections.

assumed latitude. The latitude at which an observer is assumed to be located for an observation or computation.

assumed longitude. The longitude at which an observer is assumed to be located for an observation or computation.

assumed position. A point at which a craft is assumed to be located, particularly one used as a preliminary to establishing certain navigational data, as that point on the surface of the earth for which the computed altitude is determined in the solution of a celestial observation.

astern. Bearing approximately 180° relative.

astrolabe. An instrument used for determining an accurate astronomical position ashore, as in survey work.

astronomical latitude. Angular distance between the direction of gravity and the plane of the equator.

astronomical longitude. The angle between the plane of the reference meridian and the plane of the celestial meridian.

astronomical position. A point on the earth determined by celestial observation.

astronomical refraction. Atmospheric refraction of a ray of radiant energy from outer space.

astronomical tide. Tide related to the attractions of celestial bodies, particularly the sun and moon.

astronomical triangle. The navigational triangle, either terrestrial or celestial, used in the solution of celestial observations.

astronomical twilight. The period of incomplete darkness when the upper limb of the sun is below the visible horizon, and the center of the sun is not more than 18° below the celestial horizon.

astronomical unit. The mean distance between the earth and the sun, approximately 92,900,000 statute miles, used as a unit of measurement of distance within the solar system.

atmosphere. The envelope of air surrounding the earth or other celestial body.

atmospheric absorption. The loss of power in transmission of radiant energy by dissipation in the atmosphere.

atmospheric noise. Static.

atmospheric pressure. The pressure exerted by the weight of the earth's atmosphere. Its standard value at sea level is about 14.7 pounds per square inch.

atmospheric pressure correction. That sextant altitude correction due to changes in refraction caused by nonstandard atmospheric pressure.

atmospheric refraction. Refraction of a ray of radiant energy passing obliquely through the atmosphere.

A trace. The first trace of a scope having more than one, as the upper trace of a loran indicator.

attenuation. A lessening in amount, particularly the reduction of the amplitude of a wave with distance from the origin.

audio frequency. A frequency within the audible range, about 20 to 20,000 cycles per second.

augmentation. The apparent increase in the semidiameter of a celestial body as its altitude increases, due to the decreased distance from the observer.

aural null. A null detected by listening for a minimum or the complete absence of an audible signal.

aurora. A luminous phenomena due to electrical discharge in the upper atmosphere, most commonly seen in high latitudes.

aurora australis. The aurora in the southern hemisphere.

aurora borealis. The aurora in the northern hemisphere.

auroral zone. The area of maximum auroral activity.

automatic celestial navigation. Automatic and continuous indication of position by a device which tracks celestial bodies and solves for geographical coordinates.

automatic radio direction finder. A radio direction finder which indicates automatically and continuously the great-circle direction of the transmitter to which it is tuned.

autumnal equinox. That point of intersection of the ecliptic and the celestial equator occupied by the sun as it changes from north to south declination, on or about September 23, or the instant this occurs.

awash. Situated so that the top is intermittently washed by waves or tidal action.

azimuth. The horizontal direction of a celestial point from a terrestrial point. It is usually measured from 000° at the reference direction clockwise through 360°.

azimuthal equidistant projection. An azimuthal map projection in which distances from the point of tangency are accurately represented according to a uniform scale.

azimuthal projection. A map projection in which the surface of a sphere or spheroid, such as the earth, is conceived as developed on a tangent plane, with the result that azimuths or bearings of any point from the center are correctly represented.

azimuth angle. Azimuth measured from 0° at the north or south reference direction clockwise or counterclockwise through 90° or 180°.

azimuth bar. A slender bar with a vane at each end, designed to fit over a central pivot in the glass cover of a magnetic compass for measurement of compass azimuths.

azimuth circle. A ring designed to fit snugly over a compass or compass repeater, and provided with means for observing compass bearings and azimuths.

azimuth instrument. An instrument for measuring azimuths, particularly a device which fits snugly over a central pivot in the glass cover of a magnetic compass.

azimuth tables. Publications providing tabulated azimuths or azimuth angles.

back. Of the wind, to change direction counterclockwise in the northern hemisphere and clockwise in the southern hemisphere.

back sight. An observation of a celestial body made by facing 180° from the azimuth of the body.

ballistic damping error. That error introduced in a nonpendulous gyro compass as a result of the method used to damp the oscillations of the gyro spin axis.

ballistic deflection error. A temporary error introduced in a gyro compass by the accelerating force acting upon the damping mechanism when a vessel changes course or speed.

bandwidth. The number of units (cycles, kilocycles, etc.) of frequency required for transmission.

barograph. A recording barometer.

barometer. An instrument for measuring atmospheric pressure.

barometric pressure. Atmospheric pressure as indicated by a barometer.

barometric tendency. The change of barometric pressure within a specified time (usually three hours) before an observation, together with the direction of change and the characteristics of the rise or fall.

bar scale. A line or series of lines on a chart, subdivided and labeled with the distances represented on the chart.

base line. **1.** The line between two transmitters operating together to provide a line of position, as in loran. **2.** Any line serving as the basis for measurement of other lines, as in surveying. **3.** The trace of a cathode ray tube.

base line delay. The time interval needed for the signal from a loran master station to travel the length of the base line, introduced as a delay between transmission of the master and slave signals.

base line extension. The extension of a base line beyond the transmitters.

basic pulse repetition rate. The lowest pulse repetition rate of a group differing only slightly from each other.

bathymetric chart. A topographic chart of the bed of a body of water.

bathythermograph. A recording thermometer for determining temperature of the sea at various depths.

Bayer's name. The Greek (or Roman) letter and the possessive form of the Latin name of a constellation, used as a star name.

beacon. **1.** A fixed aid to navigation. **2.** An unlighted aid to navigation. **3.** Anything serving as a signal or conspicuous indication, either for guidance or warning.

beam width. The angular width of a beam of radiant energy between half-power intensities.

bearing. The horizontal direction of one terrestrial point from another. It is usually measured from 000° at the reference direction clockwise through 360°.

bearing angle. Bearing measured from 0° at the north or south reference direction clockwise or counterclockwise through 90° or 180°.

bearing bar. A slender bar with a vane at each end, designed to fit over a central pivot in the glass cover of a magnetic compass, for measurement of compass bearings.

bearing circle. A ring designed to fit snugly over a compass or compass repeater, and provided with vanes for observing compass bearings.

bearing line. A line extending in the direction of a bearing.

bearing repeater. A compass repeater used primarily for observing bearings.

Beaufort scale. A numerical scale for indicating wind speed, named after Admiral Sir Francis Beaufort, who devised it in 1806.

beset. Surrounded so closely by sea ice that steering control is lost.

binnacle. The stand in which a compass is mounted.

blinking. Regular shifting right and left of a loran signal to indicate that the signals are out of synchronization.

blue azimuth tables. H.O. Pub. No. 261, *Azimuths of Celestial Bodies*.

blue magnetism. The magnetism of the south-seeking end of a freely suspended magnet.

boat compass. A small compass mounted in a box for convenient use in small craft.

bobbing a light. Quickly lowering the height of eye several feet and then raising it again when a light is first sighted, to determine whether the observer is at the geographical range of the light.

bottom sample. A portion of the material forming the bottom, brought up for inspection.

bow and beam bearings. Successive relative bearings (right or left) of 45° and 90° of a fixed object.

boxing the compass. Stating in order the names of the points (and sometimes fractional points) of the compass.

broad on the beam. Bearing 090° relative ("broad on the starboard beam") or 270° relative ("broad on the port beam").

broad on the bow. Bearing 045° relative ("broad on the starboard bow") or 315° relative ("broad on the port bow").

broad on the quarter. Bearing 135° relative ("broad on the starboard quarter") or 225° relative ("broad on the port quarter").

B trace. The second trace of a scope having more than one, as the lower trace of a loran indicator.

bubble sextant. A sextant with a bubble to indicate the horizontal.

buoy. A floating object, other than a lightship, moored or anchored to the bottom as an aid to navigation.

buoyage. A system of buoys.

cable. **1.** A unit of distance equal to 720 feet in the U. S. Navy. **2.** A chain or strong fiber or wire rope used to anchor or moor vessels or buoys. **3.** A stranded electric conductor or several conductors laid up together.

cage. To erect a gyro or lock it in place.

calculated altitude. Computed altitude.

calibrate. To determine or rectify the scale graduations of an instrument.

calibration table. A table of calibration corrections or calibrated values.

calving. The breaking away of a mass of ice from a parent iceberg, glacier, or ice shelf.

can buoy. A buoy the above-water part of which is in the shape of a cylinder.

candela. The United States and international unit of luminous intensity.

cardinal point. North, east, south, or west.

carrier wave. A radio wave used as a vehicle for conveying intelligence, generally by modulation.

Cartesian coordinates. Magnitudes defining a point relative to two intersecting lines or *axes*.

cartography. The art and science of making charts or maps.

cathode. The negative pole or electrode of an electron tube or an electric cell.

cathode ray tube. The "picture" tube of radar, loran, television, etc.

C-band. A radio-frequency band of 3,900 to 6,200 megacycles.

celestial body. Any aggregation of matter in space constituting a unit, such as the sun, a planet, etc.

celestial coordinates. Any set of coordinates used to define a point on the celestial sphere.

celestial equator. The intersection of the celestial sphere and the extended plane of the equator.

celestial equator system of coordinates. Declination and hour angle or declination and sidereal hour angle.

celestial fix. A fix established by observation of celestial bodies.

celestial horizon. That great circle of the celestial sphere formed by the intersection of the celestial sphere and a plane through the center of the earth and perpendicular to the zenith-nadir line.

celestial latitude. Angular distance north or south of the ecliptic; the arc of a circle of latitude between the ecliptic and a point on the celestial sphere, measured northward or southward from the ecliptic through 90°, and labeled N or S to indicate the direction of measurement.

celestial line of position. A line of position established by observation of a celestial body.

celestial longitude. Angular distance east of the vernal equinox, along the ecliptic; the arc of the ecliptic or the angle at the ecliptic pole between the circle of latitude of the vernal equinox and the circle of latitude of a point on the celestial sphere, measured eastward from the circle of latitude of the vernal equinox, through 360°.

celestial meridian. A great circle of the celestial sphere, through the celestial poles and the zenith.

celestial navigation. Navigation with the aid of celestial bodies.

celestial observation. Observation of celestial phenomena.

celestial poles. The intersection of the celestial sphere and the extended axis of the earth.

celestial sphere. An imaginary sphere of infinite radius concentric with the earth, on which all celestial bodies except the earth are imagined to be projected.

celestial triangle. A spherical triangle on the celestial sphere, especially the navigational triangle.

Celsius temperature. Temperature based upon a scale in which, under standard atmospheric pressure, water freezes at 0° and boils at 100°. Called "centigrade temperature" before 1948.

centering control. A control used to center the image on a cathode ray tube.

centering error. That instrumental error due to inaccurate pivoting of a moving part.

centigrade temperature. Celsius temperature.

change of tide. A reversal of the direction of motion (rising or falling) of a tide.

characteristics of a light. The sequence and length of light and dark periods and the color or colors by which a navigational light is identified.

character of the bottom. The type of material of which the bottom is composed.

chart. A map intended primarily for navigational use.

chart comparison unit. A device which provides simultaneous, superimposed views of a chart and radar scope.

chart datum. The tidal datum to which soundings on a chart are referred.

charted depth. The vertical distance from the chart datum to the bottom.

charted visibility. The extreme distance, shown in numbers on a chart, at which a navigational light can be seen under standard conditions.

chartlet. **1.** A small chart. **2.** A graphic supplement to *Notice to Mariners*.

chart projection. A map projection used for a chart.

chart reading. Interpretation of the symbols, lines, abbreviations, and terms appearing on charts.

chronometer. A timepiece with a nearly constant rate.

chronometer error. The amount by which chronometer time differs from the correct time to which it was set, usually Greenwich mean time.

chronometer rate. The amount gained or lost by a chronometer in unit time, usually seconds per day.

chronometer time. Time as indicated by a chronometer.

chronometer watch. A small chronometer, especially one with an enlarged watch-type movement.

circle of declination. Hour circle.

circle of equal altitude. A circle on the surface of the earth, on every point of which the altitude of a given celestial body is the same at a given instant.

circle of equal declination. Parallel of declination.

circle of latitude. **1.** A great circle of the celestial sphere, perpendicular to the ecliptic. **2.** A meridian of the earth.

circle of longitude. **1.** A circle of the celestial sphere, parallel to the ecliptic. **2.** A parallel of latitude on the earth.

circle of position. A circular line of position.

circle of right ascension. Hour circle.

circle of uncertainty. A circle within which a craft is considered to be located.

circle of visibility. That circle surrounding an aid to navigation in which the aid is visible.

circumpolar. Revolving about the elevated pole without setting.

cirrocumulus. High clouds (mean lower level above 20,000 ft.) composed of small white flakes or of very small globular masses, usually without shadows, which are arranged in groups or lines, or more often in ripples resembling those of sand on the seashore.

cirrostratus. Thin, whitish, high clouds (mean lower level above 20,000 ft.) sometimes covering the sky completely and giving it a milky appearance and at other times presenting, more or less distinctly, a formation like a tangled web.

cirrus. Detached high clouds (mean lower level above 20,000 ft.) of delicate and fibrous appearance, without shading, generally white in color, and often of a silky appearance.

civil twilight. The period of incomplete darkness when the upper limb of the sun is below the visible horizon, and the center of the sun is not more than 6° below the celestial horizon.

clamp screw. A screw for holding a moving part in place, as during an observation or reading, particularly such a device used in connection with the tangent screw of a marine sextant.

clamp screw sextant. A marine sextant having a clamp screw for controlling the position of the tangent screw.

cloud. A visible assemblage of numerous tiny droplets of water or ice crystals formed by condensation of water vapor in the air, with the base above the surface of the earth.

clutter. Atmospheric noise, extraneous signals, etc., which tend to obscure the reception of a desired signal in a radio receiver, on a radar scope, etc.

coaltitude. Ninety degrees minus the altitude.

coarse delay. On a loran indicator, a dial for controlling relatively large changes in the position of the B trace pedestal.

coastal current. An ocean current flowing roughly parallel to a coast, outside the surf zone.

coastal refraction. A small change in the direction of travel of a radio signal when it crosses a shore line obliquely.

coast chart. A nautical chart intended for use near a shore, as in entering and leaving harbors.

coasting. Proceeding approximately parallel to a coast line and near enough to be in pilot waters most of the time.

coast pilot. A descriptive book for the use of mariners, containing detailed information of coastal waters, harbor facilities, etc., of an area, particularly along the coasts of the United States.

coast piloting. The directing of the movements of a vessel near a coast, by means of terrestrial reference points.

coastwise navigation. Navigation in the vicinity of a coast.

codeclination. Ninety degrees minus the declination.

coding delay. An arbitrary time delay in the transmission of pulse signals.

colatitude. Ninety degrees minus the latitude.

cold air mass. An air mass that is colder than surrounding air, and usually colder than the surface over which it is moving.

cold front. That line of discontinuity, at the earth's surface or at a horizontal plane aloft, along which an advancing cold air mass is undermining and displacing a warmer air mass.

collision bearing. A constant bearing maintained while the distance between two craft is decreasing.

cologarithm. The logarithm of the reciprocal of a number.

combination buoy. A buoy having more than one means of conveying intelligence, as by light and sound.

comparing watch. A hack watch having its error determined by comparison with a chronometer.

compass. An instrument for determining a horizontal reference direction relative to the earth.

compass adjustment. The process of neutralizing the magnetic effect a vessel exerts on a magnetic compass.

compass amplitude. Amplitude relative to compass east or west.

compass azimuth. Azimuth relative to compass north.

compass bearing. Bearing relative to compass north.

compass bowl. That part of a compass in which the compass card is mounted.

compass card. That part of a compass on which the direction graduations are placed.

compass compensation. The process of neutralizing the effects which degaussing currents exert on a marine magnetic compass.

compass course. Course relative to compass north.

compass error. The angular difference between a compass direction and the corresponding true direction.

compasses. An instrument for drawing circles.

compass heading. Heading relative to compass north.

compass north. The direction north as indicated by a magnetic compass.

compass points. The 32 divisions of a compass, at intervals of $11\frac{1}{4}°$.

compass repeater. That part of a remote-indicating compass system which repeats at a distance the indications of the master compass.

compass rose. A circle graduated in degrees, clockwise from 0° at the reference direction to 360°, or in compass points, or in both degrees and points.

complement. An angle equal to 90° minus the given angle.

composite sailing. A modification of great-circle sailing used when it is desired to limit the highest latitude.

computed altitude. Altitude of the center of a celestial body above the celestial horizon at a given time and place, as determined by computation, table, mechanical device, or graphics.

computed point. The foot of a perpendicular from a dead reckoning position to a celestial line of position.

conformal projection. A map projection in which all angles around any point are correctly represented.

conic projection. A map projection in which the surface of a sphere or spheroid, such as the earth, is conceived as developed on one or more cones which are then spread out to form a plane.

conjunction. The situation of two celestial bodies having either the same celestial longitude or the same sidereal hour angle.

consol. An electronic navigational system providing a number of rotating equisignal zones that permit determination of bearings from a transmitting station by counting a series of dots and dashes and referring to a table or special chart.

constant error. A systematic error of unchanging magnitude and sign.

constellation. Originally, a conspicuous configuration of stars; now, a region of the celestial sphere marked by arbitrary boundary lines.

continuous wave. A series of waves of like amplitude and frequency.

contour. A line connecting points of equal elevation or equal depth.

contrary name. A name (such as north or south) opposite or contrary to that of something else. Usually used in connection with declination and latitude.

controlling depth. The least depth in the approach or channel to an area, such as a port, governing the maximum draft of vessels that can enter.

convergence constant. The angle at a given latitude between meridians 1° apart.

conversion angle. The angle between the rhumb line and the great circle between two points.

coordinate. One of a set of magnitudes defining a point in space.

Coriolis force. An apparent force acting on a body in motion, due to rotation of the earth, causing deflection to the right in the northern hemisphere and to the left in the southern hemisphere.

corner reflector. A combination of mutually intersecting, conducting surfaces designed to return electromagnetic radiations toward their sources, and used primarily to render objects more conspicuous to radar observations.

correcting. The process of applying corrections, particularly compass corrections.

corrector. A magnet, piece of soft iron, or device used in the adjustment or compensation of a magnetic compass.

countercurrent. A secondary current flowing adjacent and in the opposite direction to another current.

course. The intended horizontal direction of travel. It is usually measured from 000° at the reference direction clockwise through 360°.

course angle. Course measured from 0° at the reference direction clockwise or counterclockwise through 90° or 180°.

course error. Angular difference between the course and the course made good.

course line. **1.** A line extending in the direction of a given course. **2.** A line of position approximately parallel to the course.

course made good. The direction of a point of arrival from a point of departure.

course of advance. The course expected to be made good over the ground.

course over the ground. The course actually made good over the ground.

course recorder. A device which records the headings of a vessel.

critical range. The spread of ranges in which there is an element of uncertainty in interpretation, as in the case of ground waves and sky waves of loran.

critical table. A table in which values of the quantity to be found are tabulated for limiting values of the entering argument.

cross bearings. Two or more bearings used as intersecting lines of position for fixing the position of a vessel.

culmination. Meridian transit.

culture. Map details which represent cultural features, such as cities, railroads, aids to navigation, latitude and longitude lines, etc., as contrasted with natural features.

cumulonimbus. A massive cloud with great vertical development, the summits of which rise in the form of mountains or towers, the upper parts often spreading out in the form of an anvil.

cumulus. A dense cloud with vertical development, having a horizontal base and dome-shaped upper surface, exhibiting protuberances.

current. **1.** Water in essentially horizontal motion. **2.** A hypothetical horizontal motion of such set and drift as to account for the difference between a dead reckoning position and a fix at the same time. **3.** Air in essentially vertical motion. **4.** Electricity flowing along a conductor.

current chart. A chart on which current data are graphically depicted.

current diagram. A graph showing the average speeds of flood and ebb currents throughout the current cycle for a considerable part of a tidal waterway.

current difference. The difference between the time of slack water or strength of current at a subordinate station and at its reference station.

current direction. The direction *toward* which a current is flowing.

current meter. An instrument for measuring the speed of a current, and sometimes the direction of flow, also.

current rips. Small waves formed on the surface of water by the meeting of opposing ocean currents.

cursor. A device used with an instrument, to provide a movable reference.

cut. The intersection of lines of position, constituting a fix, with particular reference to the angle of intersection.

cyclone. An approximately circular portion of the atmosphere, having relatively low atmospheric pressure and winds which blow counterclockwise around the center in the northern hemisphere and clockwise in the southern hemisphere.

cylindrical buoy. Can buoy.

cylindrical projection. A map projection in which the surface of a sphere or spheroid, such as the earth, is conceived as developed on a tangent cylinder, which is then spread out to form a plane.

daily rate. The change in chronometer error or watch error in 24 hours.

damping. The progressive diminishing of amplitude of oscillations, waves, etc.

dan buoy. A buoy consisting of a ballasted float carrying a staff which supports a flag or light.

danger angle. The maximum or minimum angle between two points (separated either horizontally or vertically), as observed from a vessel, indicating the limit of safe approach to an off-lying danger.

danger bearing. The maximum or minimum bearing of a point for safe passage past an off-lying danger.

danger buoy. A buoy marking an isolated danger to navigation.

danger line. A line drawn on a chart, to indicate the limits of safe navigation for a vessel of specific draft.

dangerous semicircle. That half of a cyclonic storm area to the *right* of the storm track in the northern hemisphere, and to the *left* of the storm track in the southern hemisphere. In this semicircle the winds are stronger and tend to blow a vessel into the path of the storm.

danger sounding. A minimum sounding chosen for a vessel of specific draft in a given area to indicate the limit of safe navigation.

date line. The boundary between the (−)12 and (+)12 time zones, corresponding approximately with the 180th meridian.

datum. The base value, level, direction, or position from which any quantity is measured.

daybeacon. An unlighted beacon.

daylight saving time. A variation of zone time, usually one hour later than standard time.

daymark. A distinctive structure serving as an aid to navigation during daylight, whether or not the structure has a light.

day's run. The distance traveled by a vessel in one day, usually reckoned from noon to noon.

day's work. The daily routine of the navigation of a vessel at sea.

dead ahead. Bearing 000° relative.

dead astern. Bearing 180° relative.

deadbeat compass. Aperiodic compass.

dead reckoning. Determination of position by advancing a previous position for courses and distances.

dead reckoning equipment. A device that continuously indicates the dead reckoning position of a vessel.

dead reckoning plot. A plot of the movements of a craft as determined by dead reckoning.

dead reckoning position. A position determined by dead reckoning.

dead reckoning tracer. A device that automatically provides a graphical record of the dead reckoning track.

dead reckoning track. A line representing successive dead reckoning positions of a craft.

Decca. An electronic navigational system by which hyperbolic lines of position are determined by measuring the phase difference of synchronized continuous wave signals.

decibel. A unit for expressing the loudness of sounds, one decibel being approximately the least change detectable by the average human ear.

deck log. A written record of the movements of a vessel with regard to courses, speeds, positions, and other navigational information, and important events aboard the vessel.

declination. Angular distance north or south of the celestial equator; the arc of an hour circle between the celestial equator and a point on the celestial sphere, measured northward or southward from the celestial equator through 90°, and labeled N or S to indicate the direction of measurement.

deep. An unmarked fathom point on a lead line.

deep sea lead (lĕd). A heavy sounding lead (about 30 to 100 pounds), usually having a line 100 fathoms or more in length.

deflection of the vertical. The angular difference between the direction of a plumb line (the vertical) and the perpendicular (the normal) to the reference spheroid.

deflector. An instrument for measuring the relative directive force acting on a magnetic compass on different headings, for use in compass adjustment.

degaussing. Neutralization of the strength of the magnetic field of a vessel, by means of suitably arranged electric coils permanently installed in the vessel.

demodulation. The process of obtaining a modulating wave from a modulated wave.

departure. **1.** The distance between two meridians at any given parallel of latitude, expressed in linear units, or the distance to the east or west made good by a vessel in proceeding from one point to another. **2.** Act of departing or leaving.

deperming. The process of changing the magnetic condition of a vessel by wrapping a large conductor around it a number of times in a vertical plane, athwartships, and energizing the coil thus formed.

depressed pole. That celestial pole below the horizon, of contrary name to the latitude.

depth. Vertical distance from a given water level to the bottom.

depth contour. A contour connecting points of equal depth.

destination. The point of intended arrival.

deviation. The angle between the magnetic meridian and the axis of a compass card.

deviation table. A table of the deviation of a magnetic compass on various headings.

dew point. The temperature to which air must be cooled at constant pressure and constant water vapor content to reach saturation.

diagram on the plane of the celestial meridian. A diagram in which the local celestial meridian appears as a circle with the zenith at the top, and the horizon as a horizontal diameter.

diaphone. A device for producing a distinctive fog signal by means of a slotted reciprocating piston actuated by compressed air.

difference of latitude. The shorter arc of any meridian between the parallels of two places, expressed in angular measure.

difference of longitude. The smaller angle at the pole or the shorter arc of a parallel between the meridians of two places, expressed in angular measure.

dip. 1. The vertical angle, at the eye of an observer, between the horizontal and the line of sight to the visible horizon. **2.** The angle between the horizontal and the lines of force of the earth's magnetic field. **3.** The first detectable decrease in the altitude of a celestial body after reaching its maximum at or near meridian transit.

dip circle. An instrument for measuring magnetic dip.

dip correction. That correction to a sextant altitude due to dip of the horizon.

dip needle. A magnetized needle mounted so as to indicate magnetic dip.

dip of the horizon. Dip, definition 1.

direct current. An electric current which flows continuously in the same direction.

direction. The position of one point in space relative to another without reference to the distance between them.

direction finder deviation. Error in the reading of a radio direction finder due to its environment.

direction of current. The direction *toward* which a current is flowing.

direction of waves or swell. The direction *from* which waves or swell are moving.

direction of wind. The direction *from* which a wind is blowing.

directive force. The force tending to cause the directive element of a compass to line up with the reference direction.

direct wave. A radio wave which travels from transmitter to receiver without an abrupt change due to refraction or reflection.

disposition of lights. The arrangement, order, etc., of navigational lights in an area.

distance finding station. A radiobeacon with a synchronized sound signal.

distance marker. A device indicating distance, particularly one on a radar indicator.

diurnal. Having a period of, occurring in, or related to a day.

diurnal circle. The apparent daily path of a celestial body.

diurnal current. Tidal current having one flood current and one ebb current each tidal day.

diurnal inequality. The difference between the heights of the two high tides or two low tides during the tidal day, or the difference in speed between the two flood currents or the two ebb currents during a tidal day.

diurnal motion. The apparent daily motion of a celestial body.

diurnal tide. Tide having one high tide and one low tide each tidal day.

dividers. An instrument consisting in its simple form of two pointed legs joined by a pivot, used principally for measuring distances or coordinates.

dock. The space between two piers, or a basin or enclosure for reception of vessels and controlling the water level.

Doppler effect. The apparent change in frequency of radiant energy when the distance between the source and the observer or receiver is changing.

double. To travel around with a near reversal of course, as to *double a cape*.

double pulsing. The transmitting of loran signals of two rates by a single station.

double tide. A high tide consisting of two maxima of nearly the same height separated by a relatively small depression, or a low tide consisting of two minima separated by a relatively small elevation.

doubling the angle on the bow. A method of obtaining a running fix by measuring the distance a vessel travels while the relative bearing (right or left) of a fixed object doubles.

draft. The depth to which a vessel is submerged.

drafting machine. An instrument consisting essentially of a protractor and one or more arms attached to a parallel motion device.

drift. **1.** The speed of a current. **2.** The distance a vessel is moved by current and wind. **3.** Downwind or downcurrent motion due to wind or current.

drift current. Any broad, shallow, slow-moving ocean current.

drift lead. A lead placed on the bottom to indicate movement of a vessel.

drogue. Sea anchor.

dry compass. A compass without a liquid-filled bowl.

dumb compass. Pelorus.

earth inductor compass. A compass depending for its indications upon the current generated in a coil revolving in the earth's magnetic field.

easting. The distance a craft makes good to the east.

ebb current. Tidal current moving away from land or down a tidal stream.

echo ranging. Determination of distance by measuring the time interval between transmission of a radiant energy signal, usually sound, and the return of its echo.

echo sounder. An instrument used for echo sounding.

echo sounding. Determination of the depth of water by measuring the time interval between emission of a sonic or ultrasonic signal and the return of its echo from the bottom.

eclipse. The obscuration of a source of light by the intervention of an object.

ecliptic. The apparent annual path of the sun among the stars.

ecliptic diagram. A diagram of the zodiac, indicating the positions of certain celestial bodies in this region.

ecliptic pole. On the celestial sphere, either of the two points 90° from the ecliptic.

ecliptic system of coordinates. Celestial latitude and celestial longitude.

electrode. A terminal at which electricity passes from one medium into another.

electromagnetic energy. Radiant energy in radio waves, light waves, X-rays, heat waves, etc.

electronic navigation. Navigation by means of electronic equipment.

electronics. The science and technology relating to the emission, flow, and effects of electrons in a vacuum or through a semiconductor such as a gas, and to systems using devices in which this action takes place.

elevated pole. That celestial pole above the horizon, of the same name as the latitude.

***E*-link.** A bracket attached to one of the arms of a binnacle to permit the mounting of a quadrantal corrector in an intermediate position between the fore-and-aft and athwartship lines through a magnetic compass.

ellipsoid. A surface whose cross-sections are all ellipses or circles, or the solid enclosed by such a surface.

endless tangent screw. A tangent screw which can be moved over the entire range of its arc without resetting.

engine revolution counter. An instrument for registering the number of revolutions of a propeller shaft.

ephemeris. An almanac for astronomers.

epoch. A particular instant for which certain data are given.

equal altitudes. Two altitudes numerically the same.

equation of time. Apparent time minus mean time (U. S. usage).

equator. The primary great circle of the earth, or a similar body, perpendicular to the polar axis.

equatorial chart. A chart of equatorial areas or one on an equatorial projection.

equatorial projection. A map projection centered on the equator.

equatorial tides. The tides that occur when the moon is near the celestial equator, when the difference in height between consecutive high or low tides is a minimum.

equinoctial. Celestial equator.

equinoctial tides. The tides that occur at or about the time of the equinoxes, when the spring range is greater than average.

equinox. One of the two points of intersection of the ecliptic and the celestial equator, or the instant the sun occupies one of these points, when its declination is 0°.

error of perpendicularity. That error in the reading of a marine sextant due to nonperpendicularity of the index mirror to the frame.

establishment. The interval of time between the transit (upper or lower) of the moon and the next high water.

estimated position. The most probable position of a craft, determined from incomplete data or data of questionable accuracy.

excess of arc. That part of a sextant arc indicating negative readings.

ex-meridian observation. Measurement of the altitude of a celestial body near the celestial meridian, for conversion to an equivalent meridian altitude; or the altitude so measured.

explement. An angle equal to 360° minus the given angle.

extrapolation. The process of estimating the value of a quantity beyond the limits of known values by assuming that the rate or system of change continues.

extremely high frequency. Radio frequency of 30,000 to 300,000 megacycles per second.

eye of the storm. The center of a tropical cyclone.

fade. Of a radiant energy signal, to decrease, often temporarily, in strength without a change of receiver controls.

Fahrenheit temperature. Temperature based upon a scale in which, under standard atmospheric pressure, water freezes at 32° and boils at 212°.

fair tide. A tidal current which increases the speed of a vessel.

fair wind. A wind which aids a craft in making progress in a desired direction.

falling tide. A tide in which the depth of water is decreasing.

false horizon. A line resembling the visible horizon but above or below it.

far vane. That instrument sighting vane on the opposite side of the instrument from the observer's eye.

fata morgana. A complex mirage, characterized by marked distortion, generally in the vertical.

fathom. A unit of length equal to six feet.

fathom curve, fathom line. A depth contour with depth measured in fathoms.

Fathometer. The trade name for a widely used echo sounder.

favorable current. A current which increases the speed of a vessel over the ground.

favorable wind. A wind which helps a craft make progress in a desired direction.

feel the bottom. The action of a vessel in shoal water, when its speed is reduced and it sometimes becomes hard to steer.

fictitious craft. An imaginary craft used in the solution of certain maneuvering problems.

fictitious latitude, fictitious longitude. Coordinates based upon a set of fictitious parallels and fictitious meridians similar to the geographical graticule, but offset from it. These are usually used with a transverse or oblique map projection, or with a navigational grid.

fictitious rhumb line. A line making the same oblique angle with all fictitious meridians.

final great-circle course. The great-circle course at the destination.

fine delay. A dial on a loran indicator, for controlling relatively small changes in the position of the B trace pedestal.

first estimate-second estimate method. The process of determining the value of a variable quantity by trial and error. Used particularly for finding the time of meridian transit at a moving craft.

first point of Aries. Vernal equinox.

fish stakes. Poles or stakes placed in shallow water to outline fishing areas, or to support fish nets.

fix. A relatively accurate position determined without reference to any former position.

fixed and flashing light. A fixed light varied at regular intervals by one or more flashes of greater brilliance.

fixed and group flashing light. A fixed light varied at regular intervals by a group of two or more flashes of greater brilliance.

fixed light. A light having constant luminous intensity.

flashing. The process of reducing the amount of permanent magnetism in a vessel by placing a single coil horizontally around the vessel and energizing the coil.

flashing light. A light showing one or more flashes at regular intervals, the duration of light being less than that of darkness.

Flinders bar. A bar of soft unmagnetized iron placed in a vertical position near a magnetic compass to counteract deviation caused by magnetic induction in vertical soft iron of the craft.

float chamber. A sealed, hollow part attached to the compass card of a magnetic compass as part of the compass card assembly.

floe. Sea ice, either a single unbroken piece or many individual pieces, covering an area of water.

floeberg. A mass of heavily hummocked sea ice resembling an iceberg in appearance.

flood current. Tidal current moving toward land or up a tidal stream.

focal length. The distance between the optical center of a lens, or the surface of a mirror, and its focus.

focal point. Focus.

focus (*pl. foci*). That point at which parallel rays of light meet after being refracted by a lens or reflected by a mirror.

fog. A visible assemblage of numerous tiny droplets of water, or ice crystals formed by condensation of water vapor in the air, with the base at the surface of the earth.

fog signal. A warning signal transmitted by a vessel or aid to navigation during periods of low visibility.

form line. An approximation of a contour without a definite elevation value.

foul berth. A berth in which a vessel at anchor is in danger of striking or fouling another vessel, the ground, or an obstruction.

four-point bearing. A relative bearing of 045° or 315°.

frequency. The rate at which a cycle is repeated.

frequency modulation. The process of changing the frequency of a carrier wave in accordance with the variations of a modulating wave.

front. The intersection of a frontal surface and a horizontal plane.

frontal surface. The thin zone of discontinuity between two air masses.

frost smoke. Fog produced by apparent steaming of a relatively warm sea in the presence of much colder air.

gain. The ratio of output voltage, current, or power to input voltage, current, or power in electronic equipment.

galaxy. A vast assemblage of stars, nebulae, etc., composing an island universe.

gas buoy. A buoy having a gas light.

gauss. The centimeter-gram-second electromagnetic unit of magnetic induction.

Gaussin error. Deviation of a magnetic compass due to transient magnetism which remains in a vessel's structure for short periods after the inducing force has been removed.

gee. An electronic navigation system providing hyperbolic lines of position similar to those of loran.

general chart. A nautical chart intended for offshore coastwise navigation.

geocentric latitude. The angle between the plane of the equator and a line from a point on the surface of the earth to the center of the earth.

geocentric parallax. The difference in the apparent direction or position of a celestial body as observed from the center of the earth and a point on its surface.

geodesic line. The shortest line, on a mathematically derived surface, between two points on that surface.

geodesy. That science which deals mathematically with the size and shape of the earth, and with surveys in which this is considered.

geodetic latitude. The angle between the plane of the equator and a normal to the spheroid.

geodetic line. A geodesic line on the spheroidal earth.

geodetic longitude. The angle between the plane of the prime meridian and the plane through the polar axis and a normal to the spheroid.

geodetic survey. A survey which takes into account the size and shape of the earth.

geographical mile. The length of one minute of arc of the equator, or 6087.090 feet (on the Clarke spheroid of 1866).

geographical position. **1.** That point on the earth at which a given celestial body is in the zenith at a specified time. **2.** Any position on the earth defined by means of its geographical coordinates.

geographic latitude. Geodetic latitude.

geographic longitude. Geodetic longitude.

geographic range. The extreme distance at which an object or light can be seen when limited by the curvature of the earth and the heights of the object and the observer.

geoid. The figure of the earth as defined by mean sea level over the entire surface of the earth.

geoidal horizon. That circle of the celestial sphere formed by the intersection of the celestial sphere and a plane through a point on the sea-level surface of the earth, and perpendicular to the zenith-nadir line.

geomagnetic electrokinetograph. A device for measurement of the lateral component of the speed of an ocean current by means of two pairs of electrodes towed astern.

geomagnetic equator. That terrestrial great circle everywhere 90° from the geomagnetic poles.

geomagnetic pole. Either of two points marking the intersection of the earth's surface with the extended axis of a hypothetical bar magnet at the center of the earth and approximating the source of the actual magnetic field of the earth.

geomagnetism. The magnetism of the earth.

geometrical dip. The vertical angle, at the eye of an observer, between the horizontal and a straight line tangent to the surface of the earth.

geometrical horizon. Originally, the celestial horizon; now more commonly the intersection of the celestial sphere and a cone tangent to the surface of the earth and with its apex at the eye of the observer.

geometric projection. Perspective projection.

ghost. **1.** A radar signal the origin of which cannot readily be determined. **2.** A signal, on a scope, which is not repeated each time a trace is made.

gibbous. Bounded by convex curves.

gimballing error. That error introduced in a gyro compass by the tilting of the gimbal mounting system due to horizontal acceleration, as during a roll.

gimbals. A device for supporting anything, such as an instrument, in such a manner that it remains essentially horizontal when the support tilts.

glacier. A field or stream of ice which moves or has moved slowly down an incline.

gnomonic projection. A map projection in which points on the surface of a sphere or spheroid, such as the earth, are conceived as projected by radials from the center to a tangent plane.

goniometer. An instrument for measuring angles.

gradient. The change of any quantity with distance in any given direction.

gradient tints. A series of color tints used on some charts to indicate relative heights or depths.

graduation error. Inaccuracy in the graduations of the scale of an instrument.

graph. A diagram indicating the relationship between two or more variables.

grass. Sharp, closely spaced deflections of the trace of a cathode ray tube, produced by random interference.

graticule. The network of lines representing parallels and meridians on a map, chart, or plotting sheet.

great circle. The intersection of a sphere and a plane through its center.

great-circle bearing. The initial direction of a great circle through two terrestrial points.

great-circle chart. A chart on which a great circle appears as a straight line or approximately so, particularly a chart on the gnomonic projection.

great-circle course. The direction of the great circle through the point of departure and the destination.

great-circle distance. The length of the shorter arc of the great circle joining two points.

great-circle sailing. Any method of solving the various problems involving courses, distances, etc., as they relate to a great-circle track.

great-circle track. The track of a craft following a great circle, or a great circle which it is intended a craft will follow approximately.

greater ebb. The stronger of two ebb currents occurring during a tidal day.

greater flood. The stronger of two flood currents occurring during a tidal day.

greatest elongation. The maximum angular distance of a body of the solar system from the sun, as observed from the earth.

Greenwich apparent time. Local apparent time at the Greenwich meridian; the arc of the celestial equator, or the angle at the celestial pole, between the lower branch of the Greenwich celestial meridian and the hour circle of the apparent (true) sun, measured westward from the lower branch of the Greenwich celestial meridian through 24 hours; Greenwich hour angle of the apparent or true sun, expressed in time units, plus 12 hours.

Greenwich civil time. Greenwich mean time.

Greenwich hour angle. Local hour angle at the Greenwich meridian; angular distance west of the Greenwich celestial meridian; the arc of the celestial equator, or the angle at the celestial pole, between the upper branch of the Greenwich celestial meridian and the hour circle of a point on the celestial sphere, measured westward from the Greenwich celestial meridian through 360°.

Greenwich mean time. Local mean time at the Greenwich meridian; the arc of the celestial equator, or the angle at the celestial pole, between the lower branch of the Greenwich celestial meridian and the hour circle of the mean sun, measured westward from the lower branch of the Greenwich celestial meridian through 24 hours; Greenwich hour angle of the mean sun, expressed in time units, plus 12 hours.

Greenwich meridian. The meridian through Greenwich, England, serving as the prime meridian and the reference meridan for Greenwich time.

Greenwich sidereal time. Local sidereal time at the Greenwich meridian; the arc of the celestial equator, or the angle at the celestial pole, between the upper branch of the Greenwich celestial meridian and the hour circle of the vernal equinox, measured westward from the upper branch of the Greenwich celestial meridian through 24 hours; Greenwich hour angle of the vernal equinox, expressed in time units.

grid. **1.** A series of lines, usually straight and parallel, superimposed on a chart or plotting sheet to serve as a directional reference for navigation. **2.** Two sets of mutually perpendicular lines dividing a map or chart into squares or rectangles to permit location of any point by a system of rectangular coordinates.

grid amplitude. Amplitude relative to grid east or grid west.

grid azimuth. Azimuth relative to grid north.

grid bearing. Bearing relative to grid north.

grid course. Course relative to grid north.

grid declination. The angular difference between grid north and true north.

grid heading. Heading relative to grid north.

grid latitude. Fictitious latitude on a navigational grid.

grid longitude. Fictitious longitude on a navigational grid.

grid navigation. Navigation by the use of grid directions.

grid north. An arbitrary reference direction used with grid navigation.

grid variation. The angular difference between magnetic north and grid north.

grivation. Grid variation.

grounding. The touching of the bottom by a vessel.

ground swell. A long ocean wave, or series of waves, in shoal water, at a considerable distance from its origin.

ground tackle. The anchors, anchor chains, fittings, etc., used for anchoring a vessel.

ground wave. That portion of a radio wave in proximity to and affected by the ground.

group flashing light. A light showing groups of flashes at regular intervals, the duration of light being less than that of darkness.

group occulting light. A light having groups of eclipses at regular intervals, the duration of light being equal to or greater than that of darkness.

growler. A small iceberg, piece broken from an iceberg, or detached piece of sea ice, large enough to be a hazard to shipping but small enough that it may escape detection.

gyro compass. A compass having one or more gyroscopes as the directive element, and tending to indicate true north.

gyro error. The error in the reading of the gyro compass.

gyro pilot. An automatic device for steering a vessel by means of control signals from a gyro compass.

gyro repeater. That part of a remote-indicating gyro compass system which repeats at a distance the indications of the master gyro compass.

gyro sextant. A sextant provided with a gyroscope to indicate the horizontal.

hachures. Short lines on maps or charts, to indicate the slope of the ground.

hack watch. A watch used for timing observations of celestial bodies, regulating ship's clocks, etc.

half pulse repetition rate delay. An interval of time equal to half the pulse repetition rate of a pair of loran transmitters, introduced as a delay between transmission of the master and slave signals.

half-tide level. The level midway between mean high water and mean low water.

hand lead (lĕd). A light sounding lead (7 to 14 pounds), usually having a line of not more than 25 fathoms.

harbor chart. A nautical chart intended for navigation and anchorage in harbors and smaller waterways.

hard iron. Iron or steel which is not readily magnetized by induction, but which retains a high percentage of the magnetism acquired.

haul. Of the wind, to shift in a counterclockwise direction, or to shift forward of a vessel.

haversine. Half of the versine, or $\frac{1-\cos}{2}$.

haze. Fine dust or salt particles in the air, too small to be individually apparent but in sufficient number to reduce visibility and cast a bluish or yellowish veil over the landscape, subduing its colors.

heading. The horizontal direction in which a craft is pointed. It is usually measured from 000° at the reference direction clockwise through 360°.

heading angle. Heading measured from 0° at the reference direction clockwise or counterclockwise through 90° or 180°.

heading line. A line extending in the direction of a heading.

heading-upward plan position indicator. A plan position indicator with the heading of the craft maintained at the top of the indicator.

headway. Motion in a forward direction.

heel. Lateral inclination, as of a vessel during a roll or when listed.

heeling adjuster. A dip needle with a sliding weight that can be moved along one of its arms to balance the magnetic force, used to determine the correct position of a heeling magnet.

heeling error. The change in the deviation of a magnetic compass when a craft heels.

heeling magnet. A permanent magnet placed vertically in a tube under the center of a magnetic compass, to correct for heeling error.

height of eye correction. That correction to sextant altitude due to dip of the horizon.

height of tide. Vertical distance from the tidal datum to the level of the water at any time.

heliocentric parallax. The difference in the apparent positions of a celestial body outside the solar system, as observed from the earth and sun.

high altitude method. The establishing of a circular line of position from the observation of the altitude of a celestial body by means of the geographical position and zenith distance of the body.

higher high water. The higher of two high tides occurring during a tidal day.

higher low water. The higher of two low tides occurring during a tidal day.

high frequency. Radio frequency of three to 30 megacycles per second.

high tide. The maximum height reached by a rising tide.

high water. High tide.

high water full and change. The average interval of time between the transit (upper or lower) of the full or new moon and the next high water.

high water inequality. The difference between the height of the two high tides during a tidal day.

high water lunitidal interval. The interval of time between the transit (upper or lower) of the moon and the next high water at a place.

hiran. High precision shoran.

homing. Navigation toward a point by maintaining constant some navigational coordinate(s), usually bearing.

hop. Travel of a radio wave to the ionosphere and back to earth.

horizon. That great circle of the celestial sphere midway between the zenith and nadir, or a line resembling or approximating such a circle.

horizon glass. That glass of a marine sextant attached to the frame, through which the horizon is observed.

horizon system of coordinates. Altitude and azimuth or altitude and azimuth angle.

horizontal parallax. The geocentric parallax of a celestial body on the celestial horizon.

hour angle. Angular distance west of a celestial meridian or hour circle; the arc of the celestial equator, or the angle at the celestial pole, between the upper branch of a celestial meridian or hour circle and the hour circle of a point on the celestial sphere, measured westward through 360°.

hour circle. On the celestial sphere, a great circle through the celestial poles.

humidity. The amount of water vapor in the air.

hummock. A mound or hill in pressure ice.

hunting. Fluctuation about a mid-point, due to instability, as oscillation of the needle of an instrument about the zero point.

hydrographic survey. A survey of a water area.

hydrography. That science which deals with the measurement of the physical features of waters and their marginal land areas, with special reference to the elements that affect safe navigation, and the publication of such information in a form suitable for use of navigators.

hydrolant. An urgent notice of dangers to navigation in the Atlantic.

hydrometeor. Any product from the condensation of atmospheric water vapor, whether formed in the free atmosphere or at the earth's surface.

hydropac. An urgent notice of dangers to navigation in the Pacific.

hydrophone. A listening device for receiving underwater sounds.

hygrometer. An instrument for measuring the humidity of the air.

hyperbolic line of position. A line of position determined by measuring the difference in distance to two fixed points.

hypsometric tints. Gradient tints.

ice anchor. An anchor used for securing a vessel to ice.

ice barrier. Impenetrable ice.

iceberg. A mass of land ice which has broken away from its parent formation on the coast and either floats in the sea or is stranded.

ice buoy. A sturdy buoy, usually a metal spar, used to replace a more easily damaged buoy during a period when heavy ice is anticipated.

ice chart. A chart showing prevalence of ice, usually with reference to navigable waters.

ice field. Sea ice covering an area greater than five miles across.

ice jam. An accumulation of broken ice caught in a narrow part of a stream or blown against the shore of a lake.

ice shelf. A thick ice formation with level surface extending over the sea but attached to the land.

ice tongue. A narrow peninsula of ice.

index chart. An outline chart showing the limits and identifying designations of charts, volumes of sailing directions, etc.

index correction. That correction due to index error.

index error. That error in the reading of an instrument equal to the difference between the zero of the scale and the zero of the index.

index mirror. That mirror attached to the index arm of a marine sextant.

indirect wave. Any wave which arrives by an indirect path, having undergone an abrupt change of direction by refraction or reflection.

induced magnetism. Magnetism acquired by a piece of magnetic material while it is in a magnetic field.

inertial navigation. Dead reckoning performed automatically by a device which gives a continuous indication of position by double integration of accelerations since leaving a starting point.

infrared. Having a frequency immediately beyond the red end of the visible spectrum.

initial great-circle course. The great-circle course at the point of departure.

inshore. In or near the shore.

installation error. That error of an instrument reading due to incorrect installation of the instrument.

instrument error. The inaccuracy of an instrument due to imperfections within the instrument.

instrument shelter. A cage or screen in which a thermometer and sometimes other instruments are placed to shield them from conditions that would interfere with registration of true conditions.

intercardinal point. Northeast, southeast, southwest, or northwest.

intercardinal rolling error. Quadrantal error of a gyro compass.

intercept. Altitude difference.

international nautical mile. The nautical mile, of 1,852 meters.

interpolation. The process of determining intermediate values between given values in accordance with some known or assumed rate or system of change.

interrupted quick flashing light. A light showing quick flashes for several seconds, followed by a period of darkness.

inverse Mercator projection. Transverse Mercator projection.

inversion. A condition of the atmosphere in which temperature increases with height.

ionosphere. That part of the earth's atmosphere composed of several layers of ionized gas, at a height of about 50 to 250 miles, which bend certain radio waves back toward the surface of the earth.

irradiation. The apparent enlargement of a bright surface against a darker background.

isobar. A line connecting points having the same atmospheric pressure reduced to a common datum.

isoclinal. A line connecting points of equal magnetic dip.

isoclinal chart. A chart showing isoclinals.

isogonic. A line connecting points of equal magnetic variation.

isogonic chart. A chart showing isogonics.

isogriv. A line connecting points of equal grid variation.

isogriv chart. A chart showing isogrivs.

isomagnetic. A line connecting points of equality in some magnetic element.

isomagnetic chart. A chart showing isomagnetics.

isopor. A line connecting points of equal rate of change of any magnetic element.

isoporic chart. A chart showing isopors.

isotherm. A line connecting points of equal temperature.

junction buoy. A buoy marking the junction of two channels or two parts of a channel, when proceeding from seaward.

K-band. A radio-frequency band of 10,900 to 36,000 megacycles.

Kelvin temperature. Temperature based upon a scale starting at absolute zero (−273°15 C) and using Celsius degrees.

kilocycle. One thousand cycles.

kilometer. One thousand meters (about 0.54 nautical mile).

knot. A unit of speed equal to one nautical mile per hour.

Lambert conformal projection. A conformal conic map projection in which the surface of a sphere or spheroid, such as the earth, is conceived as developed on a cone which intersects the sphere or spheroid at two standard parallels.

land effect. Coastal refraction.

landfall. The first sighting of land when approached from seaward.

land ice. All ice formed on land.

landmark. A conspicuous object on land, serving as an indicator for guidance or warning.

land mile. Statute mile.

land navigation. Navigation across the surface of land or ice.

lane. **1.** An established route. **2.** One of the sections of the coverage area for a phase comparison system, such as Decca, in which every phase relationship may be measured.

lapse rate. The rate of decrease of temperature in the atmosphere with height.

large scale. A scale involving a relatively small reduction in size.

latitude. Angular distance north or south of the equator; the arc of a meridian between the equator and a point on the surface of the earth, measured northward or southward from the equator through 90°, and labeled N or S to indicate the direction of measurement.

latitude factor. The change in latitude along a celestial line of position for a 1′ change in longitude.

latitude line. A line of position extending in a generally east-west direction.

lattice. A pattern formed by two or more families of intersecting lines, such as loran lines of two or more rates of overlapping coverage.

L-band. A radio-frequency band of 390 to 1,550 megacycles.

lead (lĕd). A weight attached to a line.

lead (lēd). A long, narrow, navigable passage through pack ice, between rocks or shoals, etc.

leader cable. A cable carrying an electric current, signals from or the magnetic influence of which indicate the path to be followed by a craft equipped with suitable instruments.

leading light(s). A light or lights arranged to indicate the path to be followed.

lead line. The line attached to a sounding lead.

lee. That side toward which the wind blows.

leeway. The leeward motion of a vessel, due to wind, expressed as distance, speed, or an angle.

leg. One part of a track, consisting of a single course line.

legend. A title or explanation on a chart, diagram, illustration, etc.

lesser ebb. The weaker of two ebb currents occurring during a tidal day.

lesser flood. The weaker of two flood currents occurring during a tidal day.

light. A lighted aid to navigation, or its luminous energy.

lighthouse. A distinctive structure exhibiting a major navigational light.

light list. A publication tabulating navigational lights and related information.

light sector. A sector in which a navigational light is visible or has a distinctive color.

lightship. A distinctively marked vessel anchored or moored at a charted point, to serve as an aid to navigation. It has a characteristic light or lights, and usually other aids.

light vessel. Lightship.

limb. **1.** The graduated curved part of an instrument for measuring angles. **2.** The circular outer edge of a celestial body, particularly with respect to the top (upper limb) or bottom (lower limb).

line of position. A line on some point of which a vessel may be presumed to be located, as a result of observation or measurement.

line of soundings. A series of soundings obtained by a vessel underway, usually at regular intervals.

liquid compass. A compass having a bowl completely filled with liquid in which the compass card is mounted.

local apparent noon. The instant at which the apparent (true) sun is over the upper branch of the local meridian.

local apparent time. The arc of the celestial equator, or the angle at the celestial pole, between the lower branch of the local celestial meridian and the hour circle of the apparent (true) sun, measured westward from the lower branch of the local celestial meridian through 24 hours; local hour angle of the apparent (true) sun, expressed in time units, plus 12 hours.

local attraction. Local magnetic disturbance.

local civil time. Local mean time.

local hour angle. Angular distance west of the local celestial meridian; the arc of the celestial equator, or the angle at the celestial pole, between the upper branch of the local celestial meridian and the hour circle of a point on the celestial sphere, measured westward from the local celestial meridian through 360°.

local magnetic disturbance. An anomaly of the magnetic field of the earth, extending over a relatively small area, due to local magnetic influences.

local mean time. The arc of the celestial equator, or the angle at the celestial pole, between the lower branch of the local celestial meridian and the hour circle of the mean sun, measured westward from the lower branch of the local celestial meridian through 24 hours; local hour angle of the mean sun, expressed in time units, plus 12 hours.

local meridian. The meridian through any particular place or observer.

local sidereal time. The arc of the celestial equator, or the angle at the celestial pole, between the upper branch of the local celestial meridian and the hour circle of the vernal equinox, measured westward from the upper branch of the local celestial meridian through 24 hours; local hour angle of the vernal equinox, expressed in time units.

log. **1.** An instrument for measuring the speed or distance, or both, traveled by a vessel. **2.** Deck log.

long-distance navigation. **1.** Navigation requiring only aids usable at long range, relatively few of which could provide world coverage. **2.** Navigation on a long trip, as a voyage across an ocean.

longitude. Angular distance east or west of the prime meridian; the arc of a parallel, or the angle at the pole, between the prime meridian and the meridian of a point on the earth, measured eastward or westward from the prime meridian through 180°, and labeled E or W to indicate the direction of measurement.

longitude factor. The change in longitude along a celestial line of position for a 1′ change in latitude.

longitude line. A line of position extending in a generally north-south direction.

long-range navigation. Long-distance navigation, definition 1.

loom. The glow of a light which is below the horizon, caused by reflection by solid particles in the air.

loran. An electronic navigational system by which hyperbolic lines of position are determined by measuring the difference in the time of reception of synchronized pulse signals.

loran rate. The frequency channel and pulse repetition rate by which a pair of loran stations is identified.

loran tables. Publications containing tabular data for plotting loran lines of position.

lower branch. That half of a meridian or celestial meridian from pole to pole which passes through the antipode or nadir of a place.

lower high water. The lower of two high tides occurring during a tidal day.

lower limb. That half of the limb (of a celestial body) having the lesser altitude.

lower low water. The lower of two low tides occurring during a tidal day.

lower transit. Transit across the lower branch of the celestial meridian.

low frequency. Radio frequency of 30 to 300 kilocycles per second.

low tide. The minimum height reached by a falling tide.

low water. Low tide.

low water inequality. The difference between the heights of the two low tides during a tidal day.

low water lunitidal interval. The interval of time between the transit (upper or lower) of the moon and the next low water at a place.

loxodrome. Rhumb line.

lubber's line. A reference line on any direction-indicating instrument, marking the reading which coincides with the heading.

luminous range. The extreme distance at which a light can be seen when limited only by the intensity of the light, clearness of the atmosphere, and sensitiveness of the observer's eyes.

lunar tide. That part of the tide due solely to the tide-producing force of the moon.

lunitidal interval. The interval of time between the transit (upper or lower) of the moon and the next high water or low water at a place.

magnetic amplitude. Amplitude relative to magnetic east or west.

magnetic azimuth. Azimuth relative to magnetic north.

magnetic bearing. Bearing relative to magnetic north.

magnetic chart. A chart showing magnetic information.

magnetic compass. A compass depending for its directive force upon the attraction of the magnetism of the earth for a magnet free to turn in any horizontal direction.

magnetic compass table. Deviation table.

magnetic course. Course relative to magnetic north.

magnetic declination. Variation.

magnetic dip. The angle between the horizontal and lines of force of the earth's magnetic field.

magnetic equator. That line on the surface of the earth connecting all points at which the magnetic dip is zero.

magnetic field. The space in which a magnetic influence exists.

magnetic heading. Heading relative to magnetic north.

magnetic latitude. The angle having a tangent equal to half that of the magnetic dip at the place.

magnetic lines of force. Closed lines indicating by their direction the direction of magnetic influence.

magnetic meridian. A line of horizontal magnetic force of the earth.

magnetic north. The direction north as indicated by the earth's magnetic lines of force.

magnetic pole. Either of the two places on the surface of the earth where the magnetic dip is 90°.

magnetic storm. Violent, prolonged disturbance of the magnetic characteristics of the earth.

magnetic track. The direction of the track relative to magnetic north.

magnetic variation. Variation.

magnitude. Relative brightness of a celestial body.

maneuvering board. A polar coordinate plotting sheet devised to facilitate solution of problems involving relative movement.

manual radio direction finder. A radio direction finder which requires manual operation.

map. A representation, usually on a plane surface, of all or part of the surface of the earth, celestial sphere, or other area; showing relative size and position, according to a given projection, of the various features represented.

map projection. A representation or method of representing all or part of the surface of a sphere or spheroid, such as the earth, upon a plane surface.

March equinox. Vernal equinox.

marine navigation. The navigation of water craft.

marine sextant. A sextant designed primarily for marine navigation.

master compass. That part of a remote-indicating compass system which determines direction for transmission to various repeaters.

master station. The governing station of two or more synchronized transmitting stations.

maximum ebb. The greatest speed of an ebb current.

maximum flood. The greatest speed of a flood current.

mean sea level. The average height of the surface of the sea for all stages of the tide, usually determined from hourly readings.

mean sun. A fictitious sun conceived to move eastward along the celestial equator at a uniform rate equal to the average rate of the apparent sun along the ecliptic.

mean tide level. Half-tide level.

mean time. Time based upon rotation of the earth relative to the mean sun.

measured mile. A length of one nautical mile, the limits of which have been accurately measured and are indicated by ranges ashore.

medium frequency. Radio frequency of 300 to 3,000 kilocycles per second.

megacycle. One million cycles.

Mercator projection. A conformal cylindrical map projection in which the surface of a sphere or spheroid, such as the earth, is conceived as developed on a cylinder tangent along the equator, with the expansion of the meridians being equal to that of the parallels.

Mercator sailing. A method of solving the various problems involving course, distance, difference of latitude, difference of longitude, and departure by considering them in the relation in which they are plotted on a Mercator chart.

mercurial barometer. An instrument which determines atmospheric pressure by measuring the height of a column of mercury which the atmosphere will support.

meridian. A great circle through the geographical poles of the earth or a similar body.

meridian altitude. The altitude of a celestial body when it is on the celestial meridian.

meridian angle. Angular distance east or west of the local celestial meridian; the arc of the celestial equator, or the angle at the celestial pole, between the upper branch of the local celestial meridian and the hour circle of a point on the celestial sphere, measured eastward or westward from the local celestial meridian through 180°, and labeled E or W to indicate the direction of measurement.

meridian observation. Measurement of meridian altitude, or the altitude so measured.

meridian passage. Meridian transit.

meridian sailing. Following a true course of 000° or 180°.

meridian transit. The passage of a celestial body across a celestial meridian.

meridional difference. The difference between the meridional parts of any two given parallels.

meridional parts. The length of the arc of a meridian between the equator and a given parallel on a Mercator chart, expressed in units of 1′ of longitude at the equator.

meteorological tide. A change in water level due to meteorological conditions.

meteorology. The science of the atmosphere.

micrometer drum. A cylinder having a vernier for precise measurement, as on certain type sextants.

micrometer drum sextant. A marine sextant providing a precise reading by means of a micrometer drum attached to the index arm, and having an endless tangent screw for controlling the position of the index arm.

microsecond. One-millionth of a second.

microwave. A very short radio wave, usually one shorter than one meter.

middle ground. A shoal with channels on both sides.

middle latitude. Half the arithmetical sum of the latitudes of two places on the same side of the equator.

middle-latitude sailing. A method of converting departure into difference of longitude, or vice versa, when the course is not 090° or 270°, by assuming that such a course is steered at the middle latitude.

mid latitude. Middle latitude.

millibar. A unit of pressure equal to 1,000 dynes per square centimeter.

millisecond. One-thousandth of a second.

mist. Thin fog of relatively large particles, or very fine rain.

mixed current. A type of tidal current characterized by a conspicuous difference in speed between the two flood currents or two ebb currents usually occurring each tidal day.

mixed tide. A type of tide having a large inequality in the heights of either the two high tides or the two low tides usually occurring each tidal day.

modified Lambert conformal projection. A modification of the Lambert conformal projection for use in polar regions, the higher standard parallel being almost at the pole, and the parallels being expanded slightly to form complete concentric circles.

modulation. Variation of some characteristic of a wave, called the carrier wave, in accordance with instantaneous values of another wave, called the modulating wave.

most probable position. That position of a craft judged to be most accurate when the exact position is not known.

Mumetal. The trade name for an alloy of nickel and iron used for temporary magnets.

nadir. That point on the celestial sphere vertically below the observer, or 180° from the zenith.

name. The label of a numerical value, particularly the N (north) or S (south) label of latitude and declination.

natural scale. The ratio between the linear dimensions of a chart, drawing, etc., and the actual dimensions represented, expressed as a proportion.

nautical almanac. A periodical publication of astronomical data designed primarily for marine navigation.

nautical astronomy. Navigational astronomy.

nautical chart. A chart intended primarily for marine navigation.

nautical mile. A unit of distance equal to 1,852 meters (6,076.11549 U. S. feet, approximately). This is equal approximately to the length of 1′ of latitude.

nautical twilight. The period of incomplete darkness when the upper limb of the sun is below the visible horizon, and the center of the sun is not more than 12° below the celestial horizon.

naveam. An urgent notice of dangers to navigation in Eastern Atlantic or Mediterranean waters.

navigable semicircle. That half of a cyclonic storm area to the *left* of the storm track in the northern hemisphere, and to the *right* of the storm track in the southern hemisphere. In this semicircle the winds are weaker and tend to blow a vessel away from the path of the storm.

navigation. The process of directing the movement of a craft from one point to another.

navigational aid. An instrument, device, chart, method, etc., intended to assist in the navigation of a craft.

navigational astronomy. That part of astronomy of direct use to a navigator, comprising principally celestial coordinates, time, and the apparent motions of celestial bodies.

navigational planets. The four planets commonly observed in celestial navigation; Venus, Mars, Jupiter, and Saturn.

navigational triangle. The spherical triangle solved in computing altitude and azimuth and great-circle sailing problems.

neap tides. The tides occurring near the times of first and last quarter of the moon, when the range of tide tends to decrease.

Ney's projection. Modified Lambert conformal projection.

night effect. A radio bearing error occurring chiefly at night.

nightmark. An object of distinctive characteristics serving as an aid to navigation during darkness.

night order book. A notebook in which the commanding officer of a vessel writes, as a guide to deck watch officers, various memoranda and orders relating to the navigation of the vessel during the night.

nimbostratus. A dark, low, shapeless cloud layer (mean upper level below 6,500 ft.) usually nearly uniform; the typical rain cloud.

nimbus. A characteristic rain cloud.

noise. Random interference which appears as extraneous signals in radio receivers or on the scope of electronic instruments.

noon constant. A predetermined value added to a meridian or ex-meridian sextant altitude to determine the latitude.

noon sight. Measurement of the altitude of the sun at local apparent noon, or the altitude so measured.

northing. The distance a craft makes good to the north.

north-upward plan position indicator. A plan position indicator with north at the top of the indicator regardless of heading.

null. Minimum or zero signal.

nun buoy. A buoy the above water part of which is in the shape of a cone or a truncated cone.

nutation. Irregularities in the precessional motion of the equinoxes.

oblique Mercator projection. A conformal cylindrical map projection in which points on the surface of a sphere or spheroid, such as the earth, are conceived as developed by Mercator principles on a cylinder tangent along an oblique great circle.

observed altitude. Corrected sextant altitude.

observed latitude. Latitude determined by means of an observation.

observed longitude. Longitude determined by means of an observation.

occluded front. The front formed when a cold front overtakes a warm front.

occulting light. A light totally eclipsed at intervals, the duration of light being equal to or greater than that of darkness.

oceanography. The application of the sciences to the phenomena of the oceans.

ocean station vessel. A ship which remains close to an assigned position at sea to take weather observations, assist aircraft, etc.

oersted. The centimeter-gram-second electromagnetic unit of magnetic intensity.

offshore. Away from the shore.

off soundings. In an area where the depth of water cannot be measured by an ordinary sounding lead, generally considered to be beyond the 100-fathom line.

omnidirectional. In all directions.

on soundings. In an area where the depth of water can be measured by an ordinary sounding lead, generally considered to be within the 100-fathom line.

on the beam. Bearing approximately 090° relative ("on the starboard beam") or 270° relative ("on the port beam").

on the bow. Bearing approximately 045° relative ("on the starboard bow") or 315° relative ("on the port bow").

on the quarter. Bearing approximately 135° relative ("on the starboard quarter") or 225° relative ("on the port quarter").

opposition. The situation of two celestial bodies having either celestial longitudes or sidereal hour angles differing by 180°.

ordinate. The vertical coordinate of a set of rectangular coordinates.

orthographic projection. A perspective azimuthal projection in which the projecting lines, emanating from a point at infinity, are perpendicular to a tangent plane.

orthomorphic projection. A projection in which very small shapes are correctly represented.

overfalls. Short, breaking waves occurring when a current passes over a shoal or other submarine obstruction or meets a contrary current or wind.

pack. A large field of floating pieces of sea ice which have drifted together.

parallactic angle. That angle of the navigational triangle at the celestial body.

parallax. The difference in the apparent direction or position of an object when viewed from different points.

parallax in altitude. Geocentric parallax of a celestial body at any given altitude.

parallel. A circle on the surface of the earth, or a similar body, parallel to the plane of the equator and connecting all points of equal latitude, or a closed curve resembling or approximating such a circle.

parallel of altitude. A circle of the celestial sphere parallel to the horizon, connecting all points of equal altitude.

parallel of declination. A circle of the celestial sphere parallel to the celestial equator.

parallel of latitude. 1. Parallel. **2.** A circle of the celestial sphere, parallel to the ecliptic, and connecting points of equal celestial latitude.

parallel rulers. An instrument for transferring a line parallel to itself.

parallel sailing. A method of converting departure into difference of longitude, or vice versa, when the true course is 090° or 270°.

patent log. Any mechanical log, particularly a taffrail log.

P-band. A radio-frequency band of 225 to 390 megacycles.

pelorus. A dumb compass, or a compass card without a directive element, suitably mounted to provide means for measuring bearings.

per gyro compass. Relating to the gyro compass.

perigean tides. Tides of increased range occurring when the moon is near perigee.

perigee. That orbital point nearest the earth when the earth is the center of attraction (as in the case of the moon).

perihelion. That orbital point nearest the sun when the sun is the center of attraction (as in the case of a planet).

Permalloy. The trade name for an alloy of nickel and iron, which is easily magnetized and demagnetized.

permanent magnetism. Magnetism which is retained for long periods without appreciable reduction, unless the magnet is subjected to a demagnetizing force.

personal error. A systematic error in observations due to the characteristics of the observer.

perspective projection. The representation of a figure on a surface by means of projecting lines emanating from a single point.

per standard compass. Relating to the standard magnetic compass.

per steering compass. Relating to the magnetic steering compass.

phase correction. That correction to sextant altitude due to offset of the apparent center of a body because of its phase.

photogrammetry. The art or science of surveying by photography.

pilot chart. A chart giving information on ocean currents, weather, and other items of interest to a navigator.

piloting. Navigation involving frequent or continuous determination of position or a line of position relative to geographical points, to a high order of accuracy.

pilot station. The place where the services of a pilot may be obtained.

pilot waters. 1. Areas in which the services of a pilot are desirable. **2.** Waters in which navigation is by piloting.

Pitot tube. A tube with an open end pointed toward a moving stream of fluid. It is usually associated with a coaxial or nearly parallel tube having holes in its side to permit measurement of static pressure.

plane sailing. A method of solving the various problems involving course, distance, difference of latitude, and departure, in which the earth or a small part of it is considered a plane.

plan position indicator. A radar scope which provides a maplike presentation of the surrounding area.

plot. A drawing consisting of lines and points graphically representing certain conditions, as the progress of a craft.

plotter. An instrument for plotting lines and measuring angles on a chart or plotting sheet.

plotting chart. A chart designed primarily for plotting dead reckoning, lines of position from celestial observations, or radio aids, etc.

plotting sheet. A blank chart showing only the graticule and one or more compass roses, so that the plotting sheet can be used for any longitude.

point of arrival. The position a craft is assumed to have reached or will reach after following specified courses for specified distances from a specified point.

point of departure. The point from which the initial course to reach the destination begins.

point of destination. The point at which the final course from the point of departure ends, exclusive of the courses needed to reach a berth.

polar distance. Angular distance from a celestial pole, usually the elevated pole.

Polaris correction. A correction to be applied to the observed altitude of Polaris to obtain the latitude.

polarization error. That radio bearing error due to horizontally polarized components of the electric field under certain transmission conditions.

polar navigation. Navigation in polar regions.

polar projection. A map projection centered on a pole.

pole. **1.** Either of the two points of intersection of the surface of the earth or similar body and its axis. **2.** A magnetic pole.

polyconic projection. A conic map projection in which the surface of a sphere or spheroid, such as the earth, is conceived as developed on a series of tangent cones, which are then spread out to form a plane.

position. A point defined by stated or implied coordinates, particularly one on the surface of the earth.

position angle. Parallactic angle.

post meridian. After noon.

precession. Change in the direction of the axis of rotation of a spinning body, as a gyroscope, when acted upon by a torque.

precession of the equinoxes. The conical motion of the earth's axis about the vertical to the plane of the ecliptic, caused by the attractive force of the sun, moon, and other planets on the equatorial protuberance of the earth. It produces a slow change in declination and sidereal hour angle of stars.

precomputed altitude. The altitude of a celestial body computed before observation, and with the sextant altitude corrections applied with reversed sign.

pressure ice. Sea ice having any readily observed roughness of the surface.

primary radar. Radar using only reflection for indication of targets.

primary tide station. A place at which continuous tide observations are made over a number of years to obtain basic tidal data for the locality.

prime meridian. The meridian of longitude 0°, used as the origin for the measurement of longitude.

prime vertical. Prime vertical circle.

prime vertical circle. That vertical circle through the east and west points of the horizon.

principal vertical circle. That vertical circle through the north and south points of the horizon, coinciding with the celestial meridian.

prismatic error. That error due to lack of parallelism of the two faces of an optical element, such as a mirror or a shade glass.

profile. A graph showing elevation or distribution of some property along a line; as the graphic record made by a recording echo sounder while a vessel is underway.

proper motion. That component of the space motion of a celestial body perpendicular to the line of sight, resulting in the change of a star's apparent position relative to other stars.

proportional parts. Numbers in the same proportion as a set of given numbers, used as an aid to interpolation.

protractor. An instrument for measuring angles on a surface; an angular scale.

psychrometer. An instrument consisting of suitably mounted dry-bulb and wet-bulb thermometers for determining relative humidity and dew point.

pulse. A very short burst of electromagnetic energy.

pulse duration. The time interval during which the amplitude of a pulse is at or greater than a specified fraction of the maximum value.

pulse interval. The time interval between corresponding parts of successive pulses in a sequence characterized by uniform spacing.

pulse length. Pulse duration.

pulse modulation. The process of forming very short bursts of a carrier wave, separated by relatively long periods during which no carrier wave is transmitted.

pulse recurrence rate. Pulse repetition rate.

pulse repetition rate. The rate at which recurrent pulses are transmitted, usually expressed in pulses per second.

pulse separation. The time interval between the trailing edge of one pulse and the leading edge of the next pulse.

pulse train. A group of related pulses, constituting a series.

pulse width. Pulse duration.

pumping. Unsteadiness in the height of the mercury column of a barometer.

Q-band. A radio-frequency band of 36,000 to 46,000 megacycles.

quadrant. An instrument similar to a sextant but having a range of 180°. Usually called a "sextant."

quadrantal correctors. Masses of soft iron placed near a magnetic compass to correct for quadrantal deviation.

quadrantal deviation. Deviation which changes its sign (E or W) approximately each 90° change of heading.

quadrantal error. An error which changes sign (plus or minus) each 90°.

quadrantal spheres. Spherical quadrantal correctors.

quick flashing light. A light showing short flashes at the rate of not less than 60 per minute.

quintant. An instrument similar to a sextant but having a range of 144°. Usually called a "sextant."

race. A rapid current or a constricted channel in which such a current flows.

racon. A nondirectional radar beacon which returns a coded signal when triggered by a radar signal.

radar. A system of determining distance of an object by measuring the time interval between transmission of a pulse signal and reception of a signal returned as an echo or by a transmitter triggered by the outgoing signal. The bearing of the object can be determined by noting the orientation of the directional antenna.

radar beacon. A radiobeacon transmitting a characteristic signal on radar frequency, permitting a craft to determine the bearing and with some types the distance of the beacon.

radar bearing. A bearing obtained by radar.

radar conspicuous object. An object which returns a strong radar echo.

radar horizon. The radio horizon of a radar antenna.

radar reflector. A device capable of or intended for reflecting radar signals.

radar shadow. A condition in which radar signals do not reach a region because of an intervening obstruction.

radar target. An object which reflects a sufficient amount of a radar signal to produce an echo signal on the radar screen.

radian. The angle subtended at the center of a circle by an arc equal in length to a radius of the circle. It is equal to $180° \div \pi$, or approximately $57°17'44''.8$.

radiant energy. Energy transmitted by radiation, as sound, heat, light, etc.

radiation. The emission, transmission, and absorption of radiant energy by emanation through space.

radio. Communication by electromagnetic waves, without a connecting wire.

radio acoustic ranging. Determining distance by a combination of radio and sound, the radio being used to indicate the instant of transmission or reception of the sound, and distance being determined by the time of transit of sound, usually in water.

radio aid to navigation. An aid to navigation transmitting information by radio waves.

radio astronomy. The science which deals with radio and thermal radiation from extraterrestrial sources.

radiobeacon. A radio transmitter emitting a characteristic signal to permit a craft with suitable equipment to determine its direction, distance, or position relative to the beacon.

radio bearing. The bearing of a radio transmitter from a receiver, as determined by a radio direction finder.

radio compass. Obsolete expression for radio direction finder.

radio direction finder. Radio receiving equipment which determines the direction of arrival of a signal by measuring the orientation of the wave front, using a loop antenna.

radio direction finder station. A radio station provided with equipment for obtaining radio bearings, particularly such a station on the shore.

radio frequency. Any frequency at which electromagnetic radiation of energy is useful for communication.

radio horizon. The line at which direct rays from a transmitting antenna become tangent to the earth's surface.

radio navigation. Navigation by means of radio.

radio range. A radio station providing course guidance, or the courses so provided.

radiosonde. An instrument carried aloft by a free, unmanned balloon and equipped with elements for determining temperature, pressure, and relative humidity and automatically transmitting the measurements by radio.

radio time signal. A time signal sent by radio.

radio waves. Waves produced by oscillation of an electric charge at a frequency useful for radio communication.

radius of visibility. The radius of a circle limiting the area in which an objective can be seen under specified conditions.

radome. A radio-transparent housing for a radar antenna assembly.

ramark. A radar beacon which continuously transmits a signal appearing as a radial line on the PPI, the line indicating the direction of the beacon.

random error. A chance error, unpredictable in magnitude or sign.

range. **1.** Two or more objects in line. **2.** Distance in a single direction or along a great circle. **3.** The extreme distance at which an object or light can be seen, or a radio signal can be used. **4.** A radio station providing course guidance, or the courses so provided. **5.** A predetermined line along which a craft moves while certain data are recorded, or the station at which this takes place.

range finder. An optical instrument for measuring the distance to an object.

range lights. Two or more lights in the same horizontal direction, particularly those lights so placed as navigational aids to mark any line of importance to vessels, as a channel.

range of tide. The difference in height between consecutive high and low tides at a place.

range of visibility. The extreme distance at which an object or light can be seen.

Rankine temperature. Temperature based upon a scale starting at absolute zero (−459°67 F) and using Fahrenheit degrees.

rational horizon. Celestial horizon.

ratio of ranges. The ratio of the ranges of tide at two places.

ratio of rise. The ratio of the height of tide at two places.

Réaumur temperature. Temperature based upon a scale in which, under standard atmospheric pressure, water freezes at 0° and boils at 80° above zero.

rectangular projection. A cylindrical map projection with uniform spacing of the parallels.

rectified altitude. Sextant altitude corrected for inaccuracies in the reading (instrument, index, and personal errors, as applicable) and inaccuracies in the reference level (principally dip or Coriolis), but not for other errors. This is the altitude a celestial body appears to be above the celestial horizon, the value measured at an observatory, and for this reason is called "apparent altitude" by astronomers.

red azimuth tables. H.O. Pub. No. 260, *Azimuths of the Sun.*

red magnetism. The magnetism of the north-seeking end of a freely suspended magnet.

red sector. A sector of the circle of visibility of a navigational light in which a red light is exhibited.

reduction. The process of substituting for an observed value one derived therefrom.

reduction to the meridian. The process of applying a correction to an altitude observed when a celestial body is near the celestial meridian, to find the equivalent meridian altitude.

reference station. A place for which independent daily predictions are given in the tide or tidal current tables, from which corresponding predictions are obtained for other stations by means of differences or factors.

refraction. The change in direction of motion of a ray of radiant energy as it passes obliquely from one medium into another in which the speed of propagation is different.

relative azimuth. Azimuth relative to heading.

relative bearing. Bearing relative to heading or to the craft.

relative humidity. The percentage of saturation of the air.

relative movement. Motion of one object or body relative to another.

relief. Inequalities in the elevations of the terrain, or their representation on a chart.

remote-indicating compass. A compass equipped with one or more indicators to repeat at a distance the readings of the master compass.

repeater. A device for repeating at a distance the indications of an instrument or device.

residual deviation. Deviation of a magnetic compass after adjustment or compensation.

resolution. The separation, by a radar or optical system, of parts of an object or of two or more objects close together, or the degree of ability to make such a separation.

retired line of position. A line of position which has been moved backward to correspond with a time previous to that for which the line was established.

retrace. The path of the visible dot from the end of one sweep to the start of the next sweep across the face of a cathode ray tube.

retrograde motion. The apparent motion of a planet westward among the stars.

rhumb bearing. The direction of a rhumb line through two terrestrial points.

rhumb course. The direction of the rhumb line from the point of departure to the destination.

rhumb line. A line on the surface of the earth making the same oblique angle with all meridians.

rhumb line distance. Distance along a rhumb line.

right ascension. Angular distance east of the vernal equinox; the arc of the celestial equator, or the angle at the celestial pole, between the hour circle of the vernal equinox and the hour circle of a point on the celestial sphere, measured eastward from the hour circle of the vernal equinox through 24^h.

rise of tide. Vertical distance from the chart datum to a high water datum, such as mean high water.

rocking the sextant. Swinging the arc.

rotary current. A tidal current which changes direction progressively through 360° during a tidal-day cycle, without coming to slack water.

round of sights. A group of sights made over a short period of time.

running fix. A position determined by crossing lines of position with an appreciable time difference between them and advanced or retired to a common time.

sailing. A method of solving the various problems involving course, distance, difference of latitude, difference of longitude, and departure.

sailing chart. A small-scale nautical chart for offshore navigation.

sailing directions. A descriptive book for the use of mariners, containing detailed information of coastal waters, harbor facilities, etc., of an area, particularly along coasts other than those of the United States.

St.-Hilaire method. The establishing of a line of position from the observation of the altitude of a celestial body by the use of an altitude difference and azimuth.

same name. A name (such as north or south) the same as that of something else. Usually used in connection with declination and latitude.

S-band. A radio-frequency band of 1,550 to 5,200 megacycles.

scalar. A quantity having magnitude only.

scale. **1.** A series of marks or graduations at definite intervals. **2.** The ratio between the linear dimensions of a chart, map, drawing, etc., and the actual dimensions represented.

scope. The face of a cathode ray tube.

sea-air temperature difference correction. That sextant altitude correction resulting from abnormal refraction occurring when there is a difference in the temperature of the water and air at the surface.

sea anchor. An object towed by a vessel to keep it end-on to a heavy sea or surf or to reduce the drift.

sea buoy. The outermost buoy marking the entrance to a channel or harbor.

sea ice. Ice formed by the freezing of sea water.

sea level. The height of the surface of the sea.

seamark. A conspicuous object in the water, serving as an indicator for guidance or warning of a craft.

sea mile. Nautical mile.

seamount. An elevation of relatively small horizontal extent rising from the bottom of the sea.

sea return. Radar echoes reflected from the sea.

sea room. Space in which to maneuver without grounding or colliding.

sea tilt correction. That altitude correction due to tilting of the surface of the sea.

seaway. A moderately rough sea.

secondary radar. Radar using automatic retransmission when triggered by a radar signal.

secondary tide station. A place at which tide observations are made over a short period to obtain data for a specific purpose.

sector. Part of a circle bounded by two radii and an arc.

sectored light. A light having sectors of different colors or the same color in specific sectors separated by dark sectors.

secular. Of or pertaining to a long period of time.

seismic sea wave. One of a series of ocean waves propagated outward from the epicenter of a submarine earthquake.

semicircular deviation. Deviation which changes sign (E or W) approximately each 180° change of heading.

semidiurnal. Having a period of, occurring in, or related to approximately half a day.

semidiurnal current. Tidal current having two flood currents and two ebb currents each tidal day.

semidiurnal tide. Tide having two high tides and two low tides each tidal day.

sense. The general direction from which a radio signal arrives.

sense antenna. An antenna used to resolve a 180° ambiguity in a directional antenna.

sensible horizon. That circle of the celestial sphere formed by the intersection of the celestial sphere and a plane through the eye of the observer and perpendicular to the zenith-nadir line.

set. The direction toward which a current flows.

seven-eighths rule. A rule of thumb which states that the approximate distance to an object broad on the beam equals ⅞ of the distance traveled while the relative bearing (right or left) changes from 30° to 60° or from 120° to 150°.

seven-tenths rule. A rule of thumb which states that the approximate distance to an object broad on the beam equals 7/10 of the distance traveled while the relative bearing (right or left) changes from 22°.5 to 45° or from 135° to 157°.5.

seven-thirds rule. A rule of thumb which states that the approximate distance to an object broad on the beam equals 7/3 of the distance traveled while the relative bearing (right or left) changes from 22°.5 to 26°.5, 67°.5 to 90°, 90° to 112°.5, or 153°.5 to 157°.5.

sextant. A double-reflecting instrument for measuring angles, primarily altitudes of celestial bodies. Originally, the term was applied only to such instruments having an arc of 60°, but the term is now generally applied to all such instruments regardless of the length of arc.

sextant adjustment. The process of checking the accuracy of a sextant and removing or reducing its error.

sextant altitude. Altitude as indicated by sextant, before corrections are applied.

sextant altitude correction. Any of several corrections applied to a sextant altitude in the process of converting it to observed altitude.

sextant error. The error in the reading of a sextant, due either to lack of proper adjustment or imperfection of manufacture.

shade. Shade glass.

shade error. That error of an optical instrument due to refraction in the shade glasses.

shade glass. A darkened transparency that can be moved into the line of sight of an optical instrument, such as a sextant, to reduce the intensity of light reaching the eye.

shielding factor. The ratio of the strength of the magnetic field at a compass to the strength if there were no disturbing material nearby.

ship heading marker. A mark indicating the position or direction of the ship's head.

ship's head. Heading of a vessel.

shoran. A precision electronic position fixing system using a pulse transmitter and receiver and two transponder beacons at fixed points.

short-distance navigation. **1.** Navigation employing aids usable at short ranges only. **2.** Navigation on a short trip.

short-long flashing light. A light showing a short flash of about 0.4 second, and a long flash of four times that duration, this combination recurring about six to eight times per minute.

short-range navigation. Short-distance navigation, definition 1.

side error. That error in the reading of a marine sextant due to nonperpendicularity of the horizon glass to the frame.

sidereal. Of or pertaining to the stars.

sidereal day. The duration of one rotation of the earth on its axis, with respect to the vernal equinox.

sidereal hour angle. Angular distance west of the vernal equinox; the arc of the celestial equator, or the angle at the celestial pole, between the hour circle of the vernal equinox and the hour circle of a point on the celestial sphere, measured westward from the hour circle of the vernal equinox through 360°.

sidereal time. Time based upon the rotation of the earth relative to the vernal equinox.

sight. Observation of the altitude, and sometimes also the azimuth, of a celestial body for a line of position; or the data obtained by such an observation.

sight reduction. The process of deriving from a sight the information needed for establishing a line of position.

sight reduction tables. Tables for performing sight reduction, particularly those for determining computed altitude.

signal-to-noise ratio. The ratio of the amplitude of a desired radio signal at any point to the amplitude of noise at the same point.

signature. The graphic record of the magnetic properties of a vessel traced as the vessel passes over a recording instrument.

skip distance. The least distance from a transmitting antenna at which a sky wave can normally be received.

skip zone. The area between the outer limit of reception of ground waves and the inner limit of reception of sky waves, where no signal is received.

sky compass. An instrument for determining azimuth of the sun by utilizing the polarization of sunlight in the sky.

sky wave. An indirect radio wave which travels from the transmitting antenna into the sky, where the ionosphere bends it back toward the earth.

sky-wave correction. A correction to be applied to the reading of the indicator of an electronic instrument when sky waves are used, to obtain the equivalent ground-wave reading.

slack water. The condition when the speed of a tidal current is zero.

slave station. A transmitting station the emissions of which are controlled by a master station.

small circle. The intersection of a sphere and a plane which does not pass through its center.

small scale. A scale involving a relatively large reduction in size.

smog. A mixture of smoke and fog.

sofar. A navigational system by which hyperbolic lines of position are determined by measuring, at shore listening stations, the difference in the time of reception of sound signals produced in a sound channel in the sea, under the vessel.

soft iron. Iron or steel which is easily magnetized by induction, but loses its magnetism when the magnetic field is removed.

solar day. The duration of one rotation of the earth on its axis, with respect to the sun.

solar tide. That part of the tide due solely to the tide-producing force of the sun.

solar time. Time based upon the rotation of the earth relative to the sun.

solstice. One of the two points of the ecliptic farthest from the celestial equator, or the instant the sun occupies one of these points, when its declination is maximum.

solstitial tides. Tides occurring near the times of the solstices, when the tropic range is especially large.

sonar. A system of determining distance of an underwater object by measuring the interval of time between transmission of an underwater sonic or ultrasonic signal and return of its echo.

sonic depth finder. An echo sounder operating in the audible range of signals.

sonic navigation. Navigation by means of sound waves whether or not they are within the audible range.

sonne. A German forerunner of the British consol.

sonobuoy. A buoy with equipment for automatically transmitting a radio signal when triggered by an underwater sound signal.

sound buoy. A buoy equipped with a characteristic sound signal.

sounding. Measured or charted depth of water, or the measurement of such depth.

sounding lead (lĕd). A lead used for determining depth of water.

sounding line. The line attached to a sounding lead.

sounding machine. An instrument for measuring depth of water by lowering a recording device.

sounding wire. The wire attached to the recording device of a sounding machine.

sound wave. An audible disturbance in any material medium or, by extension, a similar disturbance outside the audible range.

southing. The distance a craft makes good to the south.

space motion. Motion of a celestial body through space.

spar buoy. A buoy made of a tapered log or of metal similarly shaped.

specific pulse repetition rate. The pulse repetition rate of a pair of transmitting stations using a group of rates differing only slightly from each other.

speed error. That error introduced in a gyro compass by the north-south component of the craft's motion.

speed line. A line of position approximately perpendicular to the course.

speed of advance. The speed expected to be made good over the ground.

speed over the ground. The speed actually made good over the ground.

spherical sailing. Any of the sailings that takes into account the spherical or spheroidal shape of the earth.

spherical triangle. A closed figure having arcs of three great circles as sides.

spheroid. An ellipsoid.

spillover. The receiving of a radio signal of a frequency differing from that to which the receiver is tuned.

splitting. The dividing of a sky-wave signal into two or more peaks.

spring range. The mean semidiurnal range of tide when spring tides are occurring.

spring tides. The tides occurring near the times of full moon and new moon, when the range of tide tends to increase.

SS loran. Sky-wave synchronized loran.

stadimeter. An instrument for determining the distance to an object of known height by measuring the angle subtended at the observer by the object.

stand. The condition at high tide or low tide when there is no change in the height of the water.

standard compass. A compass designated as the standard for a vessel.

standard parallel. A parallel on a map projection, along which the scale is as stated.

standard time. A variation of zone time used on or near land, with somewhat irregular but defined zone limits.

star finder. A device to facilitate the recognition of stars.

star globe. A globe representing the celestial sphere, on which the apparent positions of the stars are indicated.

static. Radio noise caused by natural electrical discharges in the atmosphere.

station buoy. A buoy used to mark the approximate station of an important buoy or a lightship.

station error. The difference between the direction of gravity and the perpendicular (normal) to the reference spheroid representing the earth.

station pointer. Three-arm protractor.

statute mile. A unit of distance equal to 5,280 feet in the United States.

steam fog. Frost smoke.

steering compass. A compass by which a craft is steered.

steering repeater. A compass repeater by which a craft is steered.

stereographic projection. A perspective, conformal, azimuthal map projection in which points on the surface of a sphere or spheroid, such as the earth, are conceived as projected by radial lines from any point on the surface to a plane tangent to that point opposite the point of projection.

storm tide. Increased water level due to a storm.

storm wave. A high tide caused by wind.

stranding. A serious grounding.

stratocumulus. Low clouds (mean upper level below 6,500 ft.) composed of a layer or patches of globular masses or rolls.

stratus. A low cloud (mean upper level below 6,500 ft.) in a uniform layer, resembling fog but not resting on the surface.

stream current. A relatively narrow, deep, fast-moving ocean current.

strength of current. The phase of a tidal current at which the speed is a maximum, or the speed at this time.

submarine bell. A bell whose signal is transmitted through the water.

submarine navigation. **1.** Navigation of a submarine, whether or not submerged. **2.** Underwater navigation.

submarine oscillator. A large, electrically operated diaphragm horn which produces a powerful sound for transmission through water.

submarine sound signal. A sound signal transmitted through water.

subordinate station. A place for which tide or tidal current predictions are determined by applying a correction to the predictions of a reference station.

summer solstice. That point on the ecliptic occupied by the sun at maximum northerly declination, or the instant the sun occupies this position, about June 21.

Sumner line. A celestial line of position, particularly one established by the Sumner method.

Sumner method. The establishing of a celestial line of position by computing two points on the line and connecting these with a straight line.

super high frequency. Radio frequency of 3,000 to 30,000 megacycles per second.

supplement. An angle equal to 180° minus the given angle.

surface navigation. Navigation of a vessel on the surface of the earth.

surveying sextant. A sextant intended primarily for use in hydrographic surveying.

sweep. The motion of the visible dot across the face of a cathode ray tube, as a result of deflections of the electron beam.

sweeping. The process of towing a submerged line or object to locate any submerged dangers or determine the least depth of an area; or the process of clearing an area of such dangers.

swell. A relatively long wind wave, or series of waves, that have traveled a considerable distance from the generating area.

swell direction. The direction *from* which swell is moving.

swinging ship. Placing a vessel on various headings to determine deviation.

swinging the arc. The process of rotating a sextant during observation, to determine the foot of the vertical circle through the body being observed.

swirl error. The additional error in the reading of a magnetic compass during a turn, due to friction in the compass liquid.

synoptic chart. A chart showing the distribution of meteorological conditions over an area at a given time. Popularly called a "weather map."

systematic error. An error due to some law by which it might be predicted.

tabulated altitude. Altitude taken directly from a table, before interpolation.

taffrail log. A log consisting essentially of a rotor towed through the water by a line attached to a distance-registering device secured at the taffrail.

tangent screw. A screw providing tangential movement along an arc, as that of a marine sextant.

telegraph buoy. A buoy used to mark the position of a submarine telegraph cable.

telemeter. The equipment for measuring any quantity, transmitting the results electrically to a distant point, and there recording the values measured.

temperature error. That instrument error due to nonstandard temperature.

terrestrial refraction. Atmospheric refraction of a ray of radiant energy from a point on or near the surface of the earth.

terrestrial triangle. A triangle on the surface of the earth, especially the navigational triangle.

theodolite. An optical surveying instrument for accurately measuring horizontal and vertical angles.

thermometer. An instrument for measuring temperature.

three-arm protractor. An instrument consisting of a circle graduated in degrees, to which is attached one fixed arm and two movable arms which can be clamped at any angle to the fixed arm, within the limits of the instrument.

tidal current. Current due to tidal action.

tidal current tables. Tables listing predictions of the times and speeds of tidal currents at various places, and other pertinent information.

tidal datum. A level of the sea, defined by some phase of the tide, from which water depths and heights of tide are reckoned.

tidal day. The period of the daily cycle of the tides, averaging about 24^h50^m in length.

tidal difference. The difference between the time or height of tides at a subordinate station and its reference station.

tidal wave. The ridge of water raised by tidal action, resulting in tides at various places. The expression is popularly but incorrectly used to refer to a tsunami or storm wave which overflows the land.

tide. The periodic rise and fall of the water surfaces of the earth due principally to the gravitational attraction of the moon and sun.

tide correction. That altitude correction due to tilting of the surface of the sea, as by a tide wave.

tide gage. An instrument for measuring the height of tide.

tide rips. Small waves formed by the meeting of opposing tidal currents or by a tidal current crossing an irregular bottom.

tide station. A place at which tide observations are made.

tide tables. Tables listing predictions of the times and heights of tides.

tide wave. The ridge of water raised by tidal action.

tilt error. That error introduced in the reading of an instrument due to tilt.

time. **1.** The hour of the day reckoned by the position of a celestial reference point relative to a reference celestial meridian. **2.** An elapsed interval.

time and altitude azimuth. An azimuth determined when meridian angle, declination, and altitude are known.

time azimuth. An azimuth determined when meridian angle, polar distance (or declination), and latitude are known.

time base. The sweep of a cathode ray tube, used for measuring time intervals.

time diagram. A diagram in which the celestial equator appears as a circle, and celestial meridians and hour circles appear as radial lines.

time meridian. Any meridian used as a reference for reckoning time, particularly zone time.

time sight. An observation of the altitude of a celestial body, made for the purpose of determining longitude, or the method of reducing such an observation.

time signal. A signal marking a specified time.

time tick. A time signal consisting of one or more short audible sounds.

time zone. An area in all parts of which the same time is kept.

topmark. A characteristic shape secured at the top of a buoy or beacon to aid in its identification.

trace. The line appearing on the face of a cathode ray tube when the visible dot repeatedly sweeps across the face of the tube.

track. The horizontal component of the path followed or expected to be followed by a vessel or a storm center.

track chart. A chart showing recommended, required, or established tracks, and usually indicating turning points, courses, and distances.

tracking. The process of following the movements of an object.

transfer. The distance a vessel moves perpendicular to its initial direction of motion in making a turn.

transit. 1. Meridian transit. **2.** A theodolite that can be reversed in its supports without being lifted from them.

transponder. A combined receiver and transmitter which transmits signals automatically when triggered by an incoming signal.

transverse Mercator projection. A map projection similar to a Mercator projection but with the cylinder rotated through 90°, so that it is tangent along a meridian.

traverse. A series of directions and distances, as the courses and speeds of a vessel zigzagging.

traverse sailing. A method of determining the equivalent course and distance made good by a vessel following a track consisting of a series of rhumb lines.

traverse table. A table giving relative values of various parts of plane right triangles, for use in solving such triangles.

tropical cyclone. A violent cyclone originating in the tropics.

tropic range. The difference in height between tropic higher high water and tropic lower low water.

tropic tides. The tides that occur when the moon is near its maximum declination, when the diurnal range tends to increase.

true amplitude. Amplitude relative to true east or west.

true azimuth. Azimuth relative to true north.

true bearing. Bearing relative to true north.

true course. Course relative to true north.

true heading. Heading relative to true north.

true north. The direction of the north geographical pole.

true wind. Wind relative to a fixed point on the earth.

tsunami. An ocean wave produced by a submarine earthquake, landslide, or volcanic action. Popularly called a "tidal wave" when it overflows the land.

turning buoy. A buoy marking a turn, as in a channel.

twilight. The periods of incomplete darkness following sunset or preceding sunrise.

twilight compass. A compass for indicating direction during twilight, particularly a sky compass.

ultra high frequency. Radio frequency of 300 to 3,000 megacycles per second.

ultrasonic depth finder. An echo sounder operating at a frequency above the audible range.

ultraviolet. Having a frequency immediately beyond the violet end of the visible spectrum.

uncorrecting. The process of converting true direction to magnetic, compass, or gyro direction, or magnetic direction to compass direction.

undercurrent. A current below the surface.

underwater navigation. Navigation of a submerged vessel.

unfavorable current. A current which decreases the speed of a vessel over the ground.

unfavorable wind. A wind which delays the progress of a craft in a desired direction.

unidirectional. In one direction only.

universal plotting sheet. A plotting sheet that can be used at various latitudes and any longitude.

universal time. Greenwich mean time.

upper air sounding. Determination of the characteristics of the upper air.

upper branch. That half of a meridian or celestial meridian from pole to pole which passes through a place or its zenith.

upper limb. That half of the limb (of a celestial body) having the greatest altitude.

upper transit. Transit across the upper branch of the celestial meridian.

variation. The angle between the magnetic and geographical meridians.

V-band. A radio-frequency band of 46,000 to 56,000 megacycles.

vector. A straight line representing both direction and magnitude.

vector diagram. A diagram of more than one vector drawn to the same scale and reference direction, and in correct position relative to each other.

vector quantity. A quantity having both magnitude and direction.

veer. Of the wind, (a) to change direction clockwise in the northern hemisphere and counterclockwise in the southern hemisphere, or (b) to shift aft.

velocity. Rate of motion in a given direction.

velocity ratio. The ratio of the speed of tidal currents at a subordinate station and its reference station.

vernal equinox. That point of intersection of the ecliptic and the celestial equator, occupied by the sun as it changes from south to north declination, on or about March 21, or the instant this occurs.

vernier. A scale or control used for interpolation in the reading of an instrument or for closer adjustment of any equipment.

vernier sextant. A marine sextant having a vernier used directly with the arc.

versine. One minus the cosine (1—cos).

vertical circle. A great circle of the celestial sphere, through the zenith and nadir, and hence perpendicular to the horizon.

very high frequency. Radio frequency of 30 to 300 megacycles per second.

very low frequency. Radio frequency of less than 30 kilocycles per second.

vigia. A rock or shoal the existence or position of which is doubtful.

visibility. The extreme horizontal distance at which prominent objects can be seen and identified by the unaided eye.

visible horizon. That line where earth and sky appear to meet.

vulgar establishment. The average interval of time between the transit (upper or lower) of the full or new moon and the next high water.

warm air mass. An air mass that is warmer than surrounding air, and usually warmer than the surface over which it is moving.

warm front. That line of discontinuity, at the earth's surface or at a horizontal plane aloft, where the forward edge of an advancing warm air mass is replacing a colder air mass.

warm sector. An area at the earth's surface bounded by the warm and cold fronts of a cyclone.

war time. Daylight saving time kept throughout the year during a war.

watch buoy. Station buoy.

watch error. The amount by which watch time differs from the correct time.

watch rate. The amount gained or lost by a watch or clock in unit time, usually seconds per day.

watch time. The hour of the day as indicated by a watch or clock.

wave. 1. An undulation or ridge on the surface of a liquid, or anything resembling this. 2. A disturbance propagated in such a manner that it may progress from point to point.

wave crest. The highest part of a wave.

wave direction. The direction *from* which waves are moving.

wave height. The distance from the trough to the crest of a wave, measured perpendicular to the direction of advance.

wave height correction. That altitude correction due to elevation of the visible horizon by waves.

wave length. The distance in the direction of advance between the same phase of consecutive waves.

wave period. The time interval between passage of successive wave crests at a fixed point.

wave train. A group of related waves, constituting a series.

wave trough. The lowest part of a wave, between two crests.

weather map. Synoptic chart.

weather signal. A visual signal displayed to indicate a weather forecast.

weather vane. A device to indicate the direction from which the wind blows.

westing. The distance a craft makes good to the west.

wind. Moving air, especially a mass of air having a common direction of motion.

wind current. A current created by the action of wind.

wind direction. The direction *from* which wind blows.

wind rose. A diagram showing the relative frequency and sometimes the average speed of the winds blowing from different directions in a specified region.

wind vane. A device to indicate wind direction.

wind wave. A wave generated by friction between wind and a fluid surface.

winter solstice. That point on the ecliptic occupied by the sun at maximum southerly declination, or the instant the sun occupies this position, about December 22.

wiping. The process of reducing the amount of permanent magnetism in a vessel by placing a single coil horizontally around the vessel and moving it, while energized, up and down along the sides of the vessel.

wire drag. A buoyed wire towed at a given depth to determine whether any isolated rocks, small shoals, etc., extend above that depth, or for determining the least depth of an area.

X-band. A radio-frequency band of 5,200 to 10,900 megacycles.

young ice. Newly formed ice.

zenith. That point of the celestial sphere vertically overhead.

zenithal projection. Azimuthal projection.

zenith distance. Angular distance from the zenith.

zodiac. That band of the sky extending 8° either side of the ecliptic.

zone description. The number, with its sign, that must be added to or subtracted from zone time to obtain Greenwich mean time.

zone time. The local mean time of a reference or zone meridian whose time is kept throughout a designated zone.

APPENDIX D

MISCELLANEOUS DATA

Exact relationships shown by asterisk (*). See footnote on page 962.

Area

1 square inch	=6.4516 square centimeters*
1 square foot	=144 square inches*
	=0.09290304 square meter*
	=0.00002296 acre
1 square yard	=9 square feet*
	=0.83612736 square meter
1 square (statute) mile	=27,878,400 square feet*
	=640 acres*
	=2.589988110336 square kilometers*
1 square centimeter	=0.15500031 square inch
	=0.00107639 square foot
1 square meter	=10.76391045 square feet
	=1.19599005 square yards
1 square kilometer	=247.1053815 acres
	=0.38610216 square statute mile
	=0.29155335 square nautical mile

Astronomy

1 mean solar unit	=1.00273791 sidereal units
1 sidereal unit	=0.99726957 mean solar unit
1 microsecond	=0.000001 second*
1 second	=1,000,000 microseconds*
	=0.01666667 minute
	=0.00027778 hour
	=0.00001157 day
1 minute	=60 seconds*
	=0.01666667 hour
	=0.00069444 day
1 hour	=3,600 seconds*
	=60 minutes*
	=0.04166667 day
1 mean solar day	=$24^h03^m56^s.55536$ of mean sidereal time
	=1 rotation of earth with respect to sun (mean)*
	=1.00273791 rotations of earth with respect to vernal equinox (mean)
	=1.0027378118868 rotations of earth with respect to stars (mean)
1 mean sidereal day	=$23^h56^m04^s.09054$ of mean solar time
1 sidereal month	=27.321661 days
	=$27^d07^h43^m11^s.5$
1 synodical month	=29.530588 days
	=$29^d12^h44^m02^s.8$
1 tropical (ordinary) year	=31,556,925.975 seconds
	=525,948.766 minutes
	=8,765.8128 hours
	=$365^d.24219879-0^d.0000000614(t-1900)$, where t = the year (date)
	=$365^d05^h48^m46^s$
1 sidereal year	=$365^d.25636042+0.0000000011(t-1900)$, where t = the year (date)
	=$365^d06^h09^m09^s.5$

Astronomy—Continued

1 calendar year (common)	=31,536,000 seconds*
	=525,600 minutes*
	=8,760 hours *
	=365 days *
1 calendar year (leap)	=31,622,400 seconds*
	=527,040 minutes*
	=8,784 hours*
	=366 days*
1 light-year	=9,460,000,000,000 kilometers
	=5,880,000,000,000 statute miles
	=5,110,000,000,000 nautical miles
	=63,300 astronomical units
1 parsec	=31,000,000,000,000 kilometers
	=19,300,000,000,000 statute miles
	=16,700,000,000,000 nautical miles
	=206,265 astronomical units
	=3.26 light years
1 astronomical unit	=149,500,000 kilometers
	=92,900,000 statute miles
	=80,700,000 nautical miles
	=mean distance, earth to sun
Mean distance, earth to moon	=384,411 kilometers
	=238,862 statute miles
	=207,565 nautical miles
Mean distance, earth to sun	=149,500,000 kilometers
	=92,900,000 statute miles
	=80,700,000 nautical miles
	=1 astronomical unit
Sun's diameter	=1,393,000 kilometers
	=866,000 statute miles
	=752,000 nautical miles
Sun's mass	=1,987,000,000,000,000,000,000,000,000,000,000 grams
	=2,200,000,000,000,000,000,000,000,000 short tons
	=2,000,000,000,000,000,000,000,000,000 long tons
Speed of sun relative to neighboring stars	=19.6 kilometers per second
	=12.2 statute miles per second
	=10.6 nautical miles per second
Orbital speed of earth	=29.8 kilometers per second
	=18.5 statute miles per second
	=16.1 nautical miles per second
Obliquity of the ecliptic	$=23°27'08''.26-0''.4684(t-1900)$, where t= the year (date)
General precession of the equinoxes	$=50''.2564+0''.000222\ (t-1900)$ per year, where t= the year (date)
Precession of the equinoxes in right ascension	$=46''.0850+0''.000279\ (t-1900)$ per year, where t= the year (date)
Precession of the equinoxes in declination	$=20''.0468-0''.000085\ (t-1900)$ per year, where t= the year (date)
Magnitude ratio	=2.512
	$=\sqrt[5]{100}$*

Charts

Nautical miles per inch	=reciprocal of natural scale÷72,913.39
Statute miles per inch	=reciprocal of natural scale÷63,360*
Inches per nautical mile	=72.913.39×natural scale
Inches per statute mile	=63,360×natural scale*
Natural scale	=1:72,913.39×nautical miles per inch
	=1:63,360×statute miles per inch*

Earth

Acceleration due to gravity (standard)	=980.665 centimeters per second per second
	=32.1740 feet per second per second
Mass	=5,980,000,000,000,000,000,000,000,000 grams
	=6,600,000,000,000,000,000,000 short tons
	=5,900,000,000,000,000,000,000 long tons
Mean density	=5.517 grams per cubic centimeter
Velocity of escape	=6.94 statute miles per second
Curvature of surface	=0.8 foot per nautical mile
Clarke spheroid of 1866	
Equatorial radius (a)	=20,925,874.05 feet
	=6,975,291.35 yards
	=6,378,206.4 meters
	=3,963.234 statute miles
	=3,443.957 nautical miles
Polar radius (b)	=20,854,933.76 feet
	=6,951,644.59 yards
	=6,356,583.8 meters
	=3,949.798 statute miles
	=3,432.282 nautical miles
Mean radius $\left(\frac{2a+b}{3}\right)$	=20,902,227.28 feet
	=6,967,409.09 yards
	=6,370,998.9 meters
	=3,958.755 statute miles
	=3,440.065 nautical miles
1′ of equator	=6,087.090 feet
	=2,029.030 yards
	=1,855.345 meters
	=1.153 statute miles
	=1.002 nautical miles
1′ of latitude at equator	=6,045.889 feet
	=2,015.296 yards
	=1,842.787 meters
	=1.145 statute miles
	=0.995 nautical mile
1′ of latitude at pole	=6,107.795 feet
	=2,035.932 yards
	=1,861.656 meters
	=1.157 statute miles
	=1.005 nautical miles
Flattening or ellipticity $\left(f=\frac{a-b}{a}\right)$	$=\frac{1}{294.98}$
	=0.00339007530
Eccentricity ($e=\sqrt{2f-f^2}$)	=0.08227185422
Eccentricity squared (e^2)	=0.00676865800
Clarke spheroid of 1880	
Equatorial radius (a)	=20,926,014.29 feet
	=6,975,338.10 yards
	=6,378,249.145 meters
	=3,963.260 statute miles
	=3,443.980 nautical miles
Polar radius (b)	=20,854,707.61 feet
	=6,951,569.20 yards
	=6,356,514.870 meters
	=3,949.755 statute miles
	=3,432,245 nautical miles

Earth—Continued

Clarke spheroid of 1880—Continued

Mean radius $\left(\frac{2a+b}{3}\right)$	=20,902,245.39 feet
	=6,967,415.13 yards
	=6,371,004.387 meters
	=3,958.759 statute miles
	=3,440.068 nautical miles
1′ of equator	=6,087.129 feet
	=2,029.043 yards
	=1,855.357 meters
	=1.153 statute miles
	=1.002 nautical miles
1′ of latitude at equator	=6,045.719 feet
	=2,015.240 yards
	=1,842.735 meters
	=1.145 statute miles
	=0.995 nautical mile
1′ of latitude at pole	=6,107.943 feet
	=2,035.981 yards
	=1,861.701 meters
	=1.157 statute miles
	=1.005 nautical miles
Flattening or ellipticity $\left(f=\frac{a-b}{a}\right)$	$=\frac{1}{293.465}$
	=0.00340756138
Eccentricity $(e=\sqrt{2f-f^2})$	=0.08248339904
Eccentricity squared (e^2)	=0.00680351112

International spheroid

Equatorial radius (a)	=20,926,469.85 feet
	=6,975,489.95 yards
	=6,378,388 meters
	=3,963.347 statute miles
	=3,444.055 nautical miles
Polar radius (b)	=20,856,010.35 feet
	=6,952,003.45 yards
	=6,356,911.946 meters
	=3,950.002 statute miles
	=3,432.459 nautical miles
Mean radius $\left(\frac{2a+b}{3}\right)$	=20,902,983.35 feet
	=6,967,661.12 yards
	=6,371,229.315 meters
	=3,958.8987 statute miles
	=3,440.190 nautical miles
1′ of equator	=6,087.264 feet
	=2,029.088 yards
	=1,855.398 meters
	=1.153 statute miles
	=1.002 nautical miles
1′ of latitute at equator	=6,046.342 feet
	=2,015.447 yards
	=1,842.925 meters
	=1.145 statute miles
	=0.995 nautical mile
1′ of latitude at pole	=6,107.828 feet
	=2,035.943 yards
	=1,861.666 meters
	=1.157 statute miles
	=1.005 nautical miles

Earth—Continued

International spheroid—Continued

Flattening or ellipticity $\left(f=\frac{a-b}{a}\right)$	$=\frac{1}{297}$
	$=0.00336700337$
Eccentricity $(e=\sqrt{2f-f^2})$	$=0.08199188997$
Eccentricity squared (e^2)	$=0.00672267002$

Length

1 inch	=25.4 millimeters*
	=2.54 centimeters*
1 foot (U.S.)	=12 inches*
	=1 British foot
	=⅓ yard*
	=0.3048 meter*
	=⅙ fathom*
1 foot (U.S. Survey)	=0.30480061 meter
1 yard	=36 inches*
	=3 feet*
	=0.9144 meter*
1 fathom	=6 feet*
	=2 yards*
	=1.8288 meters*
1 cable	=720 feet*
	=240 yards*
	=219.4560 meters*
1 cable (British)	=0.1 nautical mile
1 statute mile	=5,280 feet*
	=1,760 yards*
	=1,609.344 meters*
	=1.609344 kilometers*
	=0.86897624 nautical mile
1 nautical mile	=6,076.11548556 feet
	=2,025.37182852 yards
	=1,852 meters*
	=1.852 kilometers*
	=1.150779448 statute miles
1 meter	=100 centimeters*
	=39.370079 inches
	=3.28083990 feet
	=1.09361330 yards
	=0.54680665 fathom
	=0.00062137 statute mile
	=0.00053996 nautical mile
1 kilometer	=3,280.83990 feet
	=1,093.61330 yards
	=1,000 meters*
	=0.62137119 statute mile
	=0.53995680 nautical mile

Mass

1 ounce	=437.5 grains*
	=28.349523125 grams*
	=0.0625 pound*
	=0.028349523125 kilogram*
1 pound	=7,000 grains*
	=16 ounces*
	=0.45359237 kilogram*
1 short ton	=2,000 pounds*
	=907.18474 kilograms*
	=0.90718474 metric ton*
	=0.89285714 long ton

Mass—Continued

1 long ton	=2,240 pounds*
	=1,016.0469088 kilograms*
	=1.12 short tons*
	=1.0160469088 metric tons*
1 kilogram	=2.204622622 pounds
	=0.00110231 short ton
	=0.00098421 long ton
1 metric ton	=2,204.6226218 pounds
	=1,000 kilograms*
	=1.10231131 short tons
	=0.98420653 long ton

Mathematics

π	=3.1415926535897932384626433832795028841971
π^2	=9.8696044011
$\sqrt{\pi}$	=1.7724538509
Base of Naperian logarithms (e)	=2.718281828459
Modulus of common logarithms ($\log_{10}e$)	=0.4342944819032518
1 radian	=206,264″.80625
	=3,437′.7467707849
	=57°.2957795131
	=57°17′44″.80625
1 circle	=1,296,000″*
	=21,600′*
	=360°*
	=2π radians*
180°	=π radians*
1°	=3600″*
	=60′*
	=0.0174532925199432957666 radian
1′	=60″*
	=0.000290888208665721596 radian
1″	=0.000004848136811095359933 radian
Sine of 1′	=0.00029088820456342460
Sine of 1″	=0.00000484813681107637

Meteorology

Atmosphere (dry air)		
Nitrogen	=78.08%	
Oxygen	=20.95%	99.99%
Argon	= 0.93%	
Carbon dioxide	= 0.03%	
Neon	= 0.0018%	
Helium	= 0.000524%	
Krypton	= 0.0001%	
Hydrogen	= 0.00005%	
Xenon	= 0.0000087%	
Ozone	= 0 to 0.000007% (increasing with altitude)	
Radon	= 0.000000000000000006% (decreasing with altitude)	
Standard atmospheric pressure at sea level	=1,013.250 dynes per square centimeter*	
	=1,033.227 grams per square centimeter	
	=1,033.227 centimeters of water	
	=1,013.250 millibars*	
	=760 millimeters of mercury	
	=76 centimeters of mercury	
	=33.8985 feet of water	
	=29.92126 inches of mercury	
	=14.6960 pounds per square inch	
	=1.033227 kilograms per square centimeter	
	=1.013250 bars*	
Absolute zero	=(−) 273°.15 C	
	=(−) 459°.67 F	

Pressure

1 dyne per square centimeter	=0.001 millibar*
	=0.000001 bar*
1 gram per square centimeter	=1 centimeter of water
	=0.980665 millibar*
	=0.07355592 centimeter of mercury
	=0.0289590 inch of mercury
	=0.0142233 pound per square inch
	=0.001 kilogram per square centimeter*
	=0.000967841 atmosphere
1 millibar	=1,000 dynes per square centimeter*
	=1.01971621 grams per square centimeter
	=0.7500617 millimeter of mercury
	=0.03345526 foot of water
	=0.02952998 inch of mercury
	=0.01450377 pound per square inch
	=0.001 bar*
	=0.00098692 atmosphere
1 millimeter of mercury	=1.35951 grams per square centimeter
	=1.3332237 millibars
	=0.1 centimeter of mercury*
	=0.04460334 foot of water
	=0.039370079 inch of mercury
	=0.01933677 pound per square inch
	=0.001315790 atmosphere
1 centimeter of mercury	=10 millimeters of mercury*
1 inch of mercury	=34.53155 grams per square centimeter
	=33.86389 millibars
	=25.4 millimeters of mercury*
	=1.132925 feet of water
	=0.4911541 pound per square inch
	=0.03342106 atmosphere
1 centimeter of water	=1 gram per square centimeter
	=0.001 kilogram per square centimeter
1 foot of water	=30.48000 grams per square centimeter
	=29.89067 millibars
	=2.241985 centimeters of mercury
	=0.882671 inch of mercury
	=0.4335275 pound per square inch
	=0.02949980 atmosphere
1 pound per square inch	=68,947.57 dynes per square centimeter
	=70.30696 grams per square centimeter
	=70.30696 centimeters of water
	=68.94757 millibars
	=51.71493 millimeters of mercury
	=5.171493 centimeters of mercury
	=2.306659 feet of water
	=2.036021 inches of mercury
	=0.07030696 kilogram per square centimeter
	=0.06894757 bar
	=0.06804596 atmosphere
1 kilogram per square centimeter	=1,000 grams per square centimeter*
	=1,000 centimeters of water
1 bar	=1,000,000 dynes per square centimeter*
	=1,000 millibars*

Speed

1 foot per minute	=0.01666667 foot per second
	=0.00508 meter per second*

Speed—Continued

1 yard per minute	=3 feet per minute*
	=0.05 foot per second*
	=0.03409091 statute mile per hour
	=0.02962419 knot
	=0.01524 meter per second*
1 foot per second	=60 feet per minute*
	=20 yards per minute*
	=1.09728 kilometers per hour*
	=0.68181818 statute mile per hour
	=0.59248380 knot
	=0.3048 meter per second*
1 statute mile per hour	=88 feet per minute*
	=29.33333333 yards per minute
	=1.609344 kilometers per hour*
	=1.46666667 feet per second
	=0.86897624 knot
	=0.44704 meter per second*
1 knot	=101.26859143 feet per minute
	=33.75619714 yards per minute
	=1.852 kilometers per hour*
	=1.68780986 feet per second
	=1.15077945 statute miles per hour
	=0.51444444 meter per second
1 kilometer per hour	=0.62137119 statute mile per hour
	=0.53995680 knot
1 meter per second	=196.85039340 feet per minute
	=65.6167978 yards per minute
	=3.6 kilometers per hour*
	=3.28083990 feet per second
	=2.23693632 statute miles per hour
	=1.94384449 knots
Light in vacuo	=299,792 kilometers per second
	=186,282 statute miles per second
	=161,875 nautical miles per second
	=983.570 feet per microsecond
Light in air	=299,708 kilometers per second
	=186,230 statute miles per second
	=161,829 nautical miles per second
	=983.294 feet per microsecond
Sound in dry air at 60° F and standard sea level pressure	=1,116.99 feet per second
	=761.59 statute miles per hour
	=661.80 knots
	=340.46 meters per second
Sound in 3.485 percent salt water at 60° F	=4,945.37 feet per second
	=3,371.85 statute miles per hour
	=2,930.05 knots
	=1,507.35 meters per second

Volume

1 cubic inch	=16.387064 cubic centimeters*
	=0.01638661 liter
	=0.00432900 gallon
1 cubic foot	=1,728 cubic inches*
	=28.31605503 liters
	=7.48051946 U.S. gallons
	=6.22883522 imperial (British) gallons
	=0.028316846592 cubic meter*

Volume—Continued

1 cubic yard	=46,656 cubic inches*
	=764.53367616 liters
	=201.974010624 U.S. gallons
	=168.17859283 imperial (British) gallons
	=27 cubic feet*
	=0.764554857984 cubic meter*
1 cubic centimeter	=0.06102374 cubic inch
	=0.00026417 U.S. gallon
	=0.00021997 imperial (British) gallon
1 cubic meter	=264.17203187 U.S. gallons
	=219.96923879 imperial (British) gallons
	=35.31466655 cubic feet
	=1.30795059 cubic yards
1 quart (U.S.)	=57.75 cubic inches*
	=32 fluid ounces*
	=2 pints*
	=0.94632645 liter
	=0.25 gallon*
1 gallon (U.S.)	=3,785.3984784 cubic centimeters*
	=231 cubic inches*
	=0.13368056 cubic foot
	=4 quarts*
	=3.7853058 liters
	=0.83267412 imperial (British) gallon
1 liter	=1,000.028 cubic centimeters
	=61.02545 cubic inches
	=1.05671780 quarts
	=0.26417945 gallon
1 register ton	=100 cubic feet*
	=2.8316846592 cubic meters*
1 measurement ton	=40 cubic feet*
	=1 freight ton*
1 freight ton	=40 cubic feet*
	=1 measurement ton*

Volume-mass

1 cubic foot of sea water	=64 pounds
1 cubic foot of fresh water	=62.428 pounds at temperature of maximum density (4° C=39°.2 F)
1 cubic foot of ice	=56 pounds
1 displacement ton	=35 cubic feet of sea water*
	=1 long ton

NOTE:–All values in this appendix are based on the following relationships:
1 inch=2.54 centimeters*
1 yard=0.9144 meter*
1 pound (avoirdupois)=0.45359237 kilogram*
1 nautical mile=1852 meters*
Absolute zero=(–) 273°.15 C=(–) 459°.67 F.

APPENDIX E

NAVIGATIONAL COORDINATES

Coordinate	Symbol	Measured from	Measured along	Direction	Measured to	Units	Precision	Maximum value	Labels
latitude	L, lat.	equator	meridian	N, S	parallel	°,′	0′.1	90°	N, S
colatitude	colat.	poles	meridian	S, N	parallel	°,′	0′.1	90°	—
longitude	λ, long.	prime meridian	parallel	E, W	local meridian	°,′	0′.1	180°	E, W
declination	d, dec.	celestial equator	hour circle	N, S	parallel of declination	°,′	0′.1	90°	N, S
polar distance	p	elevated pole	hour circle	S, N	parallel of declination	°,′	0′.1	180°	—
altitude	h	horizon	vertical circle	up	parallel of altitude	°,′	0′.1	90°*	—
zenith distance	z	zenith	vertical circle	down	parallel of altitude	°,′	0′.1	180°	—
azimuth	Zn	north	horizon	E	vertical circle	°	0°.1	360°	—
azimuth angle	Z	north, south	horizon	E, W	vertical circle	°	0°.1	180° or 90°	N, S . . . E, W
amplitude	A	east, west	horizon	N, S	body	°	0°.1	90°	E, W . . . N, S
Greenwich hour angle	GHA	Greenwich celestial meridian	parallel of declination	W	hour circle	°,′	0′.1	360°	—
local hour angle	LHA	local celestial meridian	parallel of declination	W	hour circle	°,′	0′.1	360°	—
meridian angle	t	local celestial meridian	parallel of declination	E, W	hour circle	°,′	0′.1	180°	E, W
sidereal hour angle	SHA	hour circle of vernal equinox	parallel of declination	W	hour circle	°,′	0′.1	360°	—
right ascension	RA	hour circle of vernal equinox	parallel of declination	E	hour circle	^{h}, ^{m}, ^{s}	1^s	24^h	—
Greenwich mean time	GMT	lower branch Greenwich celestial meridian	parallel of declination	W	hour circle mean sun	^{h}, ^{m}, ^{s}	1^s	24^h	—
local mean time	LMT	lower branch local celestial meridian	parallel of declination	W	hour circle mean sun	^{h}, ^{m}, ^{s}	1^s	24^h	—
zone time	ZT	lower branch zone celestial meridian	parallel of declination	W	hour circle mean sun	^{h}, ^{m}, ^{s}	1^s	24^h	—
Greenwich apparent time	GAT	lower branch Greenwich celestial meridian	parallel of declination	W	hour circle apparent sun	^{h}, ^{m}, ^{s}	1^s	24^h	—
local apparent time	LAT	lower branch local celestial meridian	parallel of declination	W	hour circle apparent sun	^{h}, ^{m}, ^{s}	1^s	24^h	—
Greenwich sidereal time	GST	Greenwich celestial meridian	parallel of declination	W	hour circle vernal equinox	^{h}, ^{m}, ^{s}	1^s	24^h	—
local sidereal time	LST	local celestial meridian	parallel of declination	W	hour circle vernal equinox	^{h}, ^{m}, ^{s}	1^s	24^h	—

*When measured from celestial horizon.

APPENDIX F

PLANETS

Item	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Symbol	☿	♀	⊕	♂	♃	♄	⛢	♆	♇
Known satellites	0	0	1	2	12	9	5	2	0
Mean distance from sun in astronomical units	0. 39	0. 72	1. 00	1. 52	5. 20	9. 54	19. 19	30. 07	39. 52
Mean distance from sun in millions of miles (statute)	36. 0	67. 2	92. 9	141. 5	483. 3	886. 2	1, 782. 0	2, 792. 4	3, 671. 2
Mean diameter in miles (statute)	3, 100	7, 700	7, 917	4, 200	86, 740	71, 500	32, 000	31, 700	3, 550
Volume (earth=1)	0. 06	0. 92	1. 00	0. 15	1, 318	736	64	60	0. 09
Mass (earth=1)	0. 04	0. 81	1. 00	0. 11	316. 94	94. 9	14. 7	17. 2	0. 1
Density (water=1)	5. 46	4. 96	5. 52	4. 12	1. 33	0. 71	1. 56	2. 47	5. 5
Mean surface gravity (earth=1)	0. 29	0. 86	1. 00	0. 37	2. 64	1. 17	0. 91	1. 12	<0.5
Oblateness	0. 00	0. 00	$\frac{1}{297}$	$\frac{1}{192}$	$\frac{1}{15.4}$	$\frac{1}{9.5}$	$\frac{1}{14}$	$\frac{1}{45}$	?
Period of axial rotation	$88^{d}_{.}0$	?	$23^{h}56^{m}$	$24^{h}37^{m}$	$9^{h}50^{m}$	$10^{h}14^{m}$	$10^{h}45^{m}$	$15^{h}48^{m}$	$6^{d}_{.}390$
Mean orbital velocity in statute miles per second	29. 76	21. 78	18. 52	15. 00	8. 12	6. 00	4. 23	3. 37	2. 95
Sidereal period of revolution	$88^{d}_{.}0$	$224^{d}_{.}7$	$365^{d}_{.}2$	$687^{d}_{.}0$	$11^{y}_{.}9$	$29^{y}_{.}5$	$84^{y}_{.}0$	$164^{y}_{.}8$	$248^{y}_{.}4$
Eccentricity of orbit	0. 206	0. 007	0. 017	0. 093	0. 048	0. 056	0. 047	0. 009	0. 249
Inclination of equator to orbit	?	?	23°27′	25°10′	3°07′	26°45′	98°	29°	?
Inclination of orbit to ecliptic	7°00′	3°24′	0°00′	1°51′	1°18′	2°29′	0°46′	1°47′	17°09′
Stellar magnitude	−1. 2 to 1. 1	−4. 3 to −3. 3	—	−2. 8 to 1. 6	−2. 5 to −1. 4	−0. 4 to 0. 9	5. 7 to 5. 9	7. 7	14. 5

APPENDIX G

IDENTIFICATION OF NAVIGATIONAL STARS

Introduction.—The following summary is not intended as a substitute for a star finder such as H.O. 2102–D, or of a knowledge of the heavens, but is given as a supplementary reference to assist in locating the 57 stars included in the main listing in the *Nautical Almanac*, plus Polaris. The observer is assumed to be at about the average latitude of the United States, unless another latitude is indicated. If a celestial body is said to be *east* of another, it is lower in the sky if both are rising and higher if both are setting. A body *north* of another is nearer the north celestial pole. Directions refer to great circles on the celestial sphere. Figures referred to are the star charts of chapter XXII, which should be of assistance in interpreting the descriptions given. It is assumed the reader is familiar with such well-known configurations as the big dipper and *Orion*. Constellation names are given in *italics*.

Acamar crosses the celestial meridian near the southern horizon during evening twilight in February, and during morning twilight in August. It is part of the constellation *Eridanus*, the river, which is not a striking configuration. It is the faintest star listed among the 57 in the almanac, but is the brightest in its immediate vicinity. The nearest bright star is Achernar, about 20° away in a generally southwesterly direction. Dec. 40° S, SHA 316°, mag. 3.1. Fig. 2205.

Achernar, at the southern end of the inconspicuous constellation *Eridanus*, the river, is one of the brightest stars of the southern hemisphere. It is not visible north of latitude 33° N. It crosses the celestial meridian during evening twilight in January, and during morning twilight in early August. Nearly a straight line is formed by Fomalhaut, about 40° WNW; Achernar; and Canopus, about the same distance in the opposite direction. However, since these stars are widely separated, the relationship is not striking. Achernar forms large triangles with Acamar and Ankaa, Ankaa and Al Na'ir, and with Al Na'ir and Peacock. Dec. 57° S, SHA 336°, mag. 0.6. Fig. 2205.

Acrux is the brightest and most southerly star in the famed southern cross. It is not visible north of latitude 27° N. It crosses the celestial meridian during evening twilight in early June and during morning twilight in January. It is about 15° WSW of first magnitude Hadar and Rigil Kentaurus. Dec. 63° S, SHA 174°, mag. 1.1. Fig. 2207.

Adhara. About 10° S and a little to the east of Sirius is a small, approximately equilateral triangle of three second magnitude stars. Adhara is the westernmost and brightest of the three. It crosses the celestial meridian to the south during evening twilight in March, and during morning twilight in October. Dec. 29° S, SHA 256°, mag. 1.6. Fig. 2206.

Aldebaran. If the line formed by the belt of *Orion*, the hunter, is extended about 20° to the northwestward, and curved somewhat toward the north, it leads to first magnitude Aldebaran in *Taurus*, the bull. This is a group of stars forming a V. A long, curving line starting at Sirius extends through Procyon, Pollux, Capella, and Aldebaran. Dec. 16° N, SHA 292°, mag. 1.1. Fig. 2206.

Alioth is the third star from the outer end of the handle of the big dipper, and the brightest star of the group. Dec. 56°N, SHA 167°, mag. 1.7. Fig. 2207.

Alkaid is the star at the outer end of the handle of the big dipper, farthest from the bowl. It is the second brightest star of the group. Dec. 50°N, SHA 154°, mag. 1.9. Fig. 2207.

Al Na'ir is the westernmost of two second magnitude stars of nearly equal brightness about midway between first magnitude Fomalhaut, approximately 20° to the northeast, and second magnitude Peacock, about the same distance in the opposite direction. A curved line extending eastward from the southern cross passes through Hadar and Rigil Kentaurus and, if extended with less curvature, leads first to Peacock and then to Al Na'ir. This star forms triangles with Fomalhaut and Ankaa, Ankaa and Achernar, and with Achernar and Peacock. It is not visible north of latitude 43°N. It crosses the celestial meridian during evening twilight early in December, and during morning twilight in June. Dec. 47°S, SHA 29°, mag. 2.2. Figs. 2205, 2208.

Alnilam is the middle star of the belt of *Orion*, the hunter. Dec. 1°S, SHA 277°, mag. 1.8. Fig. 2206.

Alphard, a second magnitude star, is the brightest in the inconspicuous constellation *Hydra*, the water monster. The nearest bright star is first magnitude Regulus, about 20° NNE. It is about midway between the horizon and zenith when it crosses the celestial meridian to the southward during evening twilight in late April, and during morning twilight in November. Dec. 8°S, SHA 219°, mag. 2.2. Fig. 2207.

Alphecca is the brightest star of *Corona Borealis*, the northern crown, about 20°ENE of first magnitude Arcturus. It forms a triangle with Arcturus and Alkaid. It crosses the celestial meridian near the zenith during evening twilight in July, and during morning twilight in February. Dec. 27°N, SHA 127°, mag. 2.3. Figs. 2207, 2208.

Alpheratz, a second magnitude star, is at the northeast corner of the great square of *Pegasus*, the winged horse, and is the brightest of the four stars forming the square. It crosses the celestial meridian near the zenith during evening twilight early in January, and during morning twilight in July. Dec. 29°N, SHA 359°, mag. 2.2. Fig. 2205.

Altair is at the southern vertex of a large, nearly right triangle which is a conspicuous feature of the evening sky in late summer and in autumn. The right angle is at Vega and the northern vertex is at Deneb. All three are first magnitude stars. Two fainter stars close to Altair, one on each side in a line through Vega, form a characteristic pattern making Altair one of the easiest stars to identify. It crosses the celestial meridian during evening twilight in October, and during morning twilight in May. Dec. 9°N, SHA 63°, mag. 0.9. Fig. 2208.

Ankaa, a second magnitude star, is the brightest star in inconspicuous *Phoenix*. It is surrounded by and forms a series of triangles with Diphda, Fomalhaut, Al Na'ir, Achernar, and Acamar. It crosses the celestial meridian low in the southern sky in January, and during morning twilight in July. Dec. 42°S, SHA 354°, mag. 2.4. Fig. 2205.

Antares is the brightest star in the conspicuous constellation *Scorpio*, the scorpion, which is low in the southern sky during evening twilight in late July, and morning twilight in late February. No other first magnitude star is within 40° of Antares and none toward the north is within 60°. It has a noticeable reddish hue and in appearance somewhat resembles Mars, which is occasionally near it in the sky. Dec. 26°S, SHA 113°, mag. 1.2. Fig. 2208.

Arcturus. The curved line along the stars forming the handle of the big dipper, if continued in a direction away from the bowl, passes through brilliant, first magnitude Arcturus. The distance from Alkaid, at the end of the big dipper, to Arcturus is a little more than the length of the dipper. Arcturus forms a large triangle with Alkaid and Alphecca. Dec. 19° N, SHA 147°, mag. 0.2. Figs. 2207, 2208.

Atria is the brightest of three stars forming a small triangle called *Triangulum Australe*, the southern triangle, not far from the south celestial pole. It is not seen north of latitude 21° N. A line through the east-west arm of the southern cross, if continued toward the east and curved somewhat toward the south, leads first to Hadar, then to Rigil Kentaurus, then, by curving more sharply, to the northernmost star of the triangle, and finally to Atria, only about 21° from the south celestial pole. Dec. 69° S, SHA 109°, mag. 1.9. Fig. 2207.

Avior is the westernmost star of *Vela*, the sails, or false southern cross, about 30° WNW of the true southern cross, about 15° ESE of the brilliant Canopus, and nearly enclosed within a large triangle formed by Canopus, Suhail, and Miaplacidus. It is not visible north of latitude 31° N. Below this, it crosses the celestial meridian low in the southern sky during evening twilight in April, and morning twilight in early November. Dec. 59°S, SHA 235°, mag. 1.7. Figs. 2206, 2207.

Bellatrix is a second magnitude star north and a little west of the belt of *Orion*, the hunter. It is about equidistant from the belt and first magnitude, red Betelgeuse. Bellatrix is at the northwest corner of a box surrounding the belt of *Orion*. Dec. 6° N, SHA 279°, mag. 1.7. Fig. 2206.

Betelgeuse is a conspicuous, reddish star of variable brightness about 10° north and a little east of the belt of *Orion*, the hunter. A line through the center of the belt and perpendicular to it passes close to red Betelgeuse to the north and blue Rigel about the same distance south of the belt. Betelgeuse and Rigel are at opposite corners of a box surrounding the belt of *Orion*. Dec. 7° N, SHA 272°, mag. 0.1–1.2 (variable). Fig. 2206.

Canopus, second brightest star in the sky, is about 35° south of Sirius. A line extending eastward through the belt of *Orion* and curving toward the south passes first through Sirius, then through the small triangle of which Adhara is the brightest star, and finally to Canopus, which forms a large, almost equilateral triangle with Suhail and Miaplacidus. This triangle nearly encloses *Vela*, the sails or false southern cross, about 20°ESE of Canopus. Canopus is not visible north of latitude 37°N. It is on the edge of the Milky Way and while many relatively bright stars are nearby, none in the immediate vicinity of Canopus approaches it in brightness. Dec. 53° S, SHA 264°, mag. (—)0.9. Fig. 2206.

Capella is a brilliant star about 45° north of the belt of *Orion*, the hunter. A curved line starting at Sirius and extending through Procyon, Pollux, Capella, Aldebaran, the belt of *Orion*, and back to Sirius forms an inverted tear-drop figure with Capella at the top and the various parts being about equally spaced along the curve. Capella crosses the celestial meridian near the zenith during evening twilight in early March, and during morning twilight in late September. Dec. 46° N, SHA 282°, mag. 0.2. Fig. 2206.

Deneb is a bright star at the northeastern vertex of a large, nearly right triangle formed by Altair, Vega, and Deneb, the right angle being at Vega. These three stars are the brightest in the eastern sky during summer evenings. Deneb is not as bright as the other two, but is the brightest star in the constellation *Cygnus*, the swan. It crosses the celestial meridian near the zenith during evening twilight in November, and during morning twilight in late May. Dec. 45° N, SHA 50°, mag. 1.3. Fig. 2208.

Denebola, in *Leo*, the lion, is a second magnitude star at the opposite end of the constellation from Regulus. A straight line from Regulus, on the west, to Arcturus, on the east, passes close to Denebola, which is somewhat nearer Regulus. Denebola crosses the celestial meridian to the south during evening twilight in May, and during morning twilight in December. Dec. 15° N, SHA 183°, mag. 2.2. Fig. 2207.

Diphda. A line extending southward through the eastern side of the great square of *Pegasus*, the winged horse, and curving slightly toward the east, leads to second magnitude Diphda. The distance from the southern star of *Pegasus* to Diphda is about twice the length of one side of the square. Diphda is part of the inconspicuous constellation *Cetus*, the whale. The only nearby first magnitude star is Fomalhaut, about 25° in a generally southwest direction. Diphda, Fomalhaut, and Ankaa form a nearly equilateral triangle. Dec. 18° S, SHA 350°, mag. 2.2. Fig. 2205.

Dubhe forms the outer rim of the bowl of the big dipper. It and Merak (not one of the 57 navigational stars) are the two "pointers" used to locate Polaris, Dubhe being the one nearer the pole star. Dec. 62° N, SHA 195°, mag. 2.0. Fig. 2207.

Elnath is a second magnitude star between Capella, about 15° to the north, and Betelgeuse, about 20° to the south. It is a little north of a line connecting Aldebaran and Pollux. It is at the end of the northern fork of V-shaped *Taurus*, the bull. Aldebaran is the principal star at the closed end of the V. This constellation is approximately 25° NNW of *Orion*, the hunter. Dec. 29° N, SHA 279°, mag. 1.8. Fig. 2206.

Eltanin is the southernmost and brightest star in the inconspicuous constellation *Draco*, the dragon, south and somewhat east of the little dipper. A straight line extending northwestward through Altair and its two fainter companions passes first through brilliant Vega, and, about 15° beyond, to second magnitude Eltanin. Eltanin crosses the celestial meridian high in the sky toward the north during evening twilight in early September, and during morning twilight in late March. Dec. 51°N, SHA 91°, mag. 2.4. Fig. 2208.

Enif is a third magnitude star approximately midway between Altair, about 25° west, and Markab, about 20° ENE. From Markab, at the southwestern corner of the great square of *Pegasus*, the winged horse, a line extending in a generally west-southwesterly direction passes through two almost equally spaced fourth magnitude stars. From the second of these, a line about 5° long extending in a northwesterly direction leads to Enif. Enif crosses the celestial meridian to the south during evening twilight in November, and during morning twilight in June. Dec. 10° N, SHA 35°, mag. 2.5. Figs. 2205, 2208.

Fomalhaut is a first magnitude star well separated from stars of comparable brightness and from conspicuous configurations. A line through the western side of the great square of *Pegasus*, the winged horse, and extended about 45° toward the south passes close to Fomalhaut, which forms two large, nearly equilateral triangles with Diphda and Ankaa and with Ankaa and Al Na'ir. Dec. 30°S, SHA 16°, mag. 1.3. Fig. 2205.

Gacrux is the northernmost star of the southern cross. It is bright for a second magnitude star, but its brilliance is overshadowed by the brighter β *Crucis* (not listed among the 57 navigational stars) and Acrux, the two brightest stars of the southern cross, and by Hadar and Rigil Kentaurus, about 15° ESE. Gacrux crosses the celestial meridian during evening twilight in early June, and during morning twilight in late December, but is not visible north of latitude 33° N. Dec. 57° S, SHA 173°, mag. 1.6. Fig. 2207.

Gienah is a third magnitude star, the brightest in the constellation *Corvus*, the crow. A long, sweeping arc starting with the handle of the big dipper and extending

successively through Arcturus and Spica leads to this relatively small, four-sided figure made up of third magnitude stars. Gienah is at the northwest corner. It crosses the celestial meridian during evening twilight in late May, and during morning twilight in December. Dec. 17° S, SHA 177°, mag. 2.8. Fig. 2207.

Hadar is a first magnitude star about 10° east of the southern cross, and about 5° west of Rigil Kentaurus, the brightest of several bright stars in this part of the sky. Dec. 60° S, SHA 150°, mag. 0.9. Fig. 2207.

Hamal is the brightest star of the inconspicuous constellation *Aries*, the ram. A line through the center of the great square of *Pegasus*, the winged horse, extended about 25° east, and curved slightly toward the north, leads to Hamal. It is over the meridian to the south during evening twilight in January, and during morning twilight in August. Dec. 23° N, SHA 329°, mag. 2.2. Fig. 2205.

Kaus Australis is near the southern end of a group of second and third magnitude stars forming the constellation *Sagittarius*, the archer, about 25° ESE of Antares, in *Scorpio*, the scorpion. It is about 10° SW of Nunki, also in *Sagittarius*, and about the same distance ENE of Shaula, in *Scorpio*. With Antares, Sabik, and Nunki, it forms a large, poorly defined box. It is over the meridian to the south during evening twilight in September and during morning twilight in April. Dec. 34° S, SHA 85°, mag. 2.0. Fig. 2208.

Kochab forms the outer rim of the bowl of the little dipper, at the opposite end from Polaris, about 15° north. It is directly above the pole during evening twilight in early July and during morning twilight in January; and directly below the pole, low in the northern sky, during evening twilight of early February and morning twilight of late August. Dec. 74° N, SHA 137°, mag. 2.2. Fig. 2208.

Markab is the star at the southwest corner of the great square of *Pegasus*, the winged horse, at the opposite corner from Alpheratz. It is over the celestial meridian to the south during evening twilight in December, and during morning twilight late in June. Dec. 15° N, SHA 14°, mag. 2.6. Fig. 2205.

Menkar is a third magnitude star at the eastern end of the inconspicuous constellation *Cetus*, the whale. No bright stars are nearby. A straight line from Aldebaran extending about 25° in the direction indicated by the point of the V of *Taurus*, the bull, leads to Menkar. A long, straight line from Fomalhaut east-northeastward through Diphda, and extended about 40°, leads to Menkar. It crosses the celestial meridian during evening twilight in February, and during morning twilight in August. Dec. 4° N, SHA 315°, mag. 2.8. Figs. 2205, 2206.

Menkent is a second magnitude star about 25° north of Hadar and about 30° northeast of the southern cross. A line from Gienah across the opposite corner of the small, four-sided *Corvus*, the crow, and then curving a little toward the east, leads to Menkent. A number of third magnitude stars are nearby, but they do not form a conspicuous configuration. With Antares and Rigil Kentaurus, Menkent forms a large triangle. It crosses the celestial meridian low in the southern sky during evening twilight in late June and during morning twilight in early January. Dec. 36° S, SHA 149°, mag. 2.3. Figs. 2207, 2208.

Miaplacidus is a second magnitude star about 10° south of the false southern cross. It is the nearest of the 57 navigational stars to the south celestial pole, about 20° away, and is not visible north of latitude 20° N. With Suhail and brilliant Canopus it forms a large, nearly equilateral triangle almost enclosing the false southern cross. South of latitude 20° S, it does not set, but circles the south celestial pole in a clockwise direction, reaching its maximum altitude above the pole during evening twilight in early May

and during morning twilight in November. Dec. 70°S, SHA 222°, mag. 1.8. Figs. 2206, 2207.

Mirfak is a second magnitude star at the northeastern end of a gently curving line extending in a northeasterly direction from Alpheratz at the northeastern corner of the great square of *Pegasus*, the winged horse, through two other second magnitude stars, Mirach and Almach, not included among the 57 navigational stars. Mirfak is about 25° east and a little south of *Cassiopeia*, and about 20° WNW of Capella. A line from Kochab through Polaris, and curved slightly toward the east, leads to Mirfak. Dec. 50°N, SHA 310°, mag. 1.9. Figs. 2205, 2206.

Nunki is the more northerly of the two brightest stars of a group of second and third magnitude stars forming the constellation *Sagittarius*, the archer, about 30°E of Antares. It is about 10° NE of Kaus Australis, also in *Sagittarius*. With Sabik, Antares, and Kaus Australis, it forms a large, poorly defined box. It is over the meridian to the south during evening twilight in early October and during morning twilight in April. Dec. 26°S, SHA 77°, mag. 2.1. Fig. 2208.

Peacock, the brightest star in the southern constellation of the same name, is not a part of a conspicuous configuration of stars. A curved line extending eastward from the southern cross passes through Hadar and Rigil Kentaurus and, if extended with less curvature, leads to Peacock, about 30° southeast of *Scorpio*, the scorpion, and about 20° southwest of Al Na'ir. With Al Na'ir and Achernar it forms a large, poorly defined triangle. It crosses the celestial meridian during evening twilight in early November, and during morning twilight in late May, but is not visible north of latitude 33°N. Dec. 57°S, SHA 54°, mag. 2.1. Figs. 2205, 2208.

Polaris is not listed among the 57 navigational stars, but is treated separately because it is less than 1° from the north celestial pole. It is about midway between the big dipper and *Cassiopeia*. A line through Dubhe and Merak (not one of the 57 navigational stars), the pointers forming the outer side of the bowl of the big dipper, if extended northward for about 30°, leads almost directly to Polaris. A line extending north from Alpheratz at the northwest corner of the great square of *Pegasus*, the winged horse, passes through Caph (not one of the 57 navigational stars) in *Cassiopeia* and then Polaris at about equal intervals. Dec. 89°N, SHA 332°, mag. 2.1. Figs. 2205–2208.

Pollux is the brighter of the "twins of *Gemini*," two relatively bright stars about 45°NE of *Orion*, the hunter, and about 45°ENE of Aldebaran. A curved line starting at Sirius extends through Procyon, Pollux, and Capella, all first magnitude stars. Dec. 28°N, SHA 244°, mag. 1.2. Fig. 2206.

Procyon is a bright star about 30° east of *Orion*, the hunter. A curved line starting at Sirius extends through Procyon, Pollux, and Capella, all first magnitude stars. Dec. 5°N, SHA 246°, mag. 0.5. Fig. 2206.

Rasalhague forms a large, nearly equilateral triangle with Altair and Vega, Rasalhague being at the western vertex. Both of the other stars are considerably brighter than Rasalhague. It crosses the celestial meridian to the south during evening twilight in early September, and during morning twilight in late March. Dec. 13°N, SHA 97°, mag. 2.1. Fig. 2208.

Regulus is at the opposite end of *Leo*, the lion, from Denebola, and is the brightest star of the constellation. A line through Dubhe and Merak (not one of the 57 navigational stars), the pointers by which Polaris is usually identified, extended about 45° *southward*, and curved slightly toward the west, leads to Regulus, which forms the southern end of the handle of the sickle, part of *Leo*. Dec. 12° N, SHA 209°, mag. 1.3. Fig. 2207.

Rigel is a brilliant bluish star about 10°S and a little to the west of the belt of *Orion*, the hunter. A line through the center of the belt and perpendicular to it passes close to blue Rigel to the south and red Betelgeuse about the same distance north of the belt. Rigel and Betelgeuse are at opposite corners of a box surrounding the belt of *Orion*. Dec. 8°S, SHA 282°, mag. 0.3. Fig. 2206.

Rigil Kentaurus is the brighter and more easterly of two first magnitude stars about 15° east of the southern cross. It is over the meridian during evening twilight in early July, and during morning twilight in late January, but is not visible north of latitude 29°N. Dec. 61°S, SHA 141°, mag. 0.1. Figs. 2207, 2208.

Sabik is part of the inconspicuous constellation *Ophiuchus*, the serpent holder, about 20° north of *Scorpio*, the scorpion. With Antares, Kaus Australis, and Nunki, it forms a large, poorly defined box in the southern sky on summer evenings. Sabik crosses the celestial meridian during evening twilight in August, and during morning twilight in March. Dec. 16°S, SHA 103°, mag. 2.6. Fig. 2208.

Schedar is the southernmost star of the W (or M) of *Cassiopeia*, on the opposite side of Polaris from the big dipper. It is the second star from the leading edge of this configuration as it circles the north celestial pole. Dec. 56°N, SHA 351°, mag. 2.5. Figs. 2205, 2206, 2208.

Shaula is a second magnitude star marking the end of the tail of *Scorpio*, the scorpion, at the opposite end from Antares. This constellation is low in the southern sky on summer evenings. Shaula is about 15° southeast of Antares and about 10° WSW of Kaus Australis. It crosses the celestial meridian during evening twilight in early September, and during morning twilight in March. Dec. 37°S, SHA 97°, mag. 1.7. Fig. 2208.

Sirius, the brightest star in the heavens, is in the constellation *Canis Major*, the "large dog" of *Orion*, the hunter. The line formed by the belt of *Orion*, if extended about 20° to the eastward and curved toward the south, leads to Sirius. Dec. 17°S, SHA 259°, mag. (−)1.6. Fig. 2206.

Spica is the brightest star of *Virgo*, the virgin, an inconspicuous constellation on the celestial equator to the south during evening twilight in early summer. The curved line along the stars forming the handle of the big dipper, if continued in a direction away from the pointers, passes through Arcturus and then Spica. The distance between Alkaid, at the end of the big dipper, and Arcturus is about the same as that between Arcturus and Spica, and is a little more than the length of the big dipper. Spica crosses the celestial meridian during evening twilight in June, and during morning twilight late in December. Dec. 11°S, SHA 159°, mag. 1.2. Fig. 2207.

Suhail is one of a number of second magnitude stars extending along the Milky Way between Sirius and the southern cross. It is about 10° north of the false southern cross, which is nearly enclosed by a large, nearly equilateral triangle formed by Suhail, Canopus, and Miaplacidus. Canopus and Suhail are on opposite edges of the Milky Way, with a number of second magnitude stars between them. A straight line extending eastward through the east-west arm of the southern cross leads to Suhail, about 35° away. In the southern United States, Suhail crosses the celestial meridian near the southern horizon during evening twilight in April, and during morning twilight in November. Dec. 43° S, SHA 223°, mag. 2.2. Figs. 2206, 2207.

Vega is the brightest star north of the celestial equator, and the third brightest in the entire sky. It is at the western vertex and the nearly right angle of a large triangle which is a conspicuous feature of the evening sky in late summer and in autumn. The other two stars of the triangle are Altair and Deneb, both of the first magnitude.

Vega passes through the zenith approximately at latitude 38°45′N during evening twilight in September and during morning twilight in April. Dec. 39°N, SHA 81°, mag. 0.1. Fig. 2208.

Zubenelgenubi, a third magnitude star, is the southern (or western) basket of *Libra*, the balance. The boxlike *Libra* is about 25° WNW of Antares, in *Scorpio*, the scorpion. A long line extending eastward from Alphard, between Gienah and Spica, leads to Zubenelgenubi. Dec. 16°S, SHA 138°, mag. 2.9. Figs. 2207, 2208.

APPENDIX H

NAVIGATIONAL STARS AND THE PLANETS

Name	Pronunciation	Bayer name	Origin of name	Meaning of name	Distance*
Acamar	ā'kȧ·mär	θ Eridani	Arabic	another form of Achernar	120
Achernar	ā'kēr·när	α Eridani	Arabic	end of the river (Eridanus)	72
Acrux	ā'krŭks	α Crucis	Modern	coined from Bayer name	220
Adhara	ȧ·dā'rȧ	ε Canis Majoris	Arabic	the virgin(s)	350
Aldebaran	ăl dĕb'ȧ·răn	α Tauri	Arabic	follower (of the Pleiades)	64
Alioth	ăl'ĭ·ŏth	ε Ursa Majoris	Arabic	another form of Capella	49
Alkaid	ăl·kād'	η Ursa Majoris	Arabic	leader of the daughters of the bier	190
Al Na'ir	ăl·nār'	α Gruis	Arabic	bright one (of the fish's tail)	90
Alnilam	ăl'nĭ·lăm	ε Orionis	Arabic	string of pearls	410
Alphard	ăl'färd	α Hydrae	Arabic	solitary star of the serpent	200
Alphecca	ăl·fĕk'ȧ	α Corona Borealis	Arabic	feeble one (in the crown)	76
Alpheratz	ăl·fē'răts	α Andromeda	Arabic	the horse's navel	120
Altair	ăl·tār'	α Aquilae	Arabic	flying eagle or vulture	16
Ankaa	än'kä	α Phoenicis	Arabic	coined name	93
Antares	ăn·tā'rēz	α Scorpii	Greek	rival of Mars (in color)	250
Arcturus	ärk·tū'rŭs	α Bootis	Greek	the bear's guard	37
Atria	ăt'rĭ·ȧ	α Trianguli Australis	Modern	coined from Bayer name	130
Avior	ā'vĭ·ôr	ε Carinae	Modern	coined name	350
Bellatrix	bĕ·lā'trĭks	γ Orionis	Latin	female warrior	250
Betelgeuse	bĕt'ĕl·jūz	α Orionis	Arabic	the arm pit (of Orion)	300
Canopus	kȧ·nō'pŭs	α Carinae	Greek	city of ancient Egypt	230
Capella	kȧ·pĕl'ȧ	α Aurigae	Latin	little she-goat	46
Deneb	dĕn'ĕb	α Cygni	Arabic	tail of the hen	600
Denebola	dĕ·nĕb'ō·lȧ	β Leonis	Arabic	tail of the lion	42
Diphda	dĭf'dȧ	β Ceti	Arabic	the second frog (Fomalhaut was once the first)	57
Dubhe	dŭb'ē	α Ursa Majoris	Arabic	the bear's back	100
Elnath	ĕl'năth	β Tauri	Arabic	one butting with horns	130
Eltanin	ĕl·tā'nĭn	γ Draconis	Arabic	head of the dragon	150
Enif	ĕn'ĭf	ε Pegasi	Arabic	nose of the horse	250
Fomalhaut	fō'măl·ôt	α Piscis Austrini	Arabic	mouth of the southern fish	23
Gacrux	gā'krŭks	γ Crucis	Modern	coined from Bayer name	72
Gienah	jē'nȧ	γ Corvi	Arabic	right wing of the raven	136
Hadar	hā'där	β Centauri	Modern	leg of the centaur	200
Hamal	hăm'ȧl	α Arietis	Arabic	full-grown lamb	76
Kaus Australis	kôs ôs·trā'lĭs	ε Sagittarii	Ar., L.	southern part of the bow	163
Kochab	kō'kăb	β Ursa Minoris	Arabic	shortened form of "north star" (named when it was that, c. 1500 BC–AD 300)	100
Markab	mär'kăb	α Pegasi	Arabic	saddle (of Pegasus)	100
Menkar	mĕn'kär	α Ceti	Arabic	nose (of the whale)	1,100
Menkent	mĕn'kĕnt	θ Centauri	Modern	shoulder of the centaur	55
Miaplacidus	mī'ȧ·plăs'ĭ·dŭs	β Carinae	Ar., L.	quiet or still waters	86
Mirfak	mĭr'făk	α Persei	Arabic	elbow of the Pleiades	130
Nunki	nŭn'kē	σ Sagittarii	Bab.	constellation of the holy city (Eridu)	150
Peacock	pē'kŏk	α Pavonis	Modern	coined from English name of constellation	250
Polaris	pō·lā'rĭs	α Ursa Minoris	Latin	the pole (star)	450
Pollux	pŏl'ŭks	β Geminorum	Latin	Zeus' other twin son (Castor, α Geminorum, is first twin)	33
Procyon	prō'sĭ·ŏn	α Canis Minoris	Greek	before the dog (rising before the dog star, Sirius)	11
Rasalhague	răs'ăl·hā'gwē	α Ophiuchi	Arabic	head of the serpent charmer	67
Regulus	rĕg'ū·lŭs	α Leonis	Latin	the prince	67
Rigel	rī'jĕl	β Orionis	Arabic	foot (left foot of Orion)	500
Rigil Kentaurus	rī'jĭl kĕn·tô'rŭs	α Centauri	Arabic	foot of the centaur	4.3
Sabik	sā'bĭk	η Ophiuchi	Arabic	second winner or conqueror	69
Schedar	shĕd'ȧr	α Cassiopeiae	Arabic	the breast (of Cassiopeia)	360
Shaula	shô'lȧ	λ Scorpii	Arabic	cocked-up part of the scorpion's tail	200
Sirius	sĭr'ĭ·ŭs	α Canis Majoris	Greek	the scorching one (popularly, the dog star)	8.6
Spica	spī'kȧ	α Virginis	Latin	the ear of corn	155
Suhail	sōō·hāl'	λ Velorum	Arabic	shortened form of Al Suhail, one Arabic name for Canopus	200
Vega	vē'gȧ	α Lyrae	Arabic	the falling eagle or vulture	27
Zubenelgenubi	zōō·bĕn'ĕl·jē·nū'bē	α Librae	Arabic	southern claw (of the scorpion)	66

PLANETS

Name	Pronunciation	Origin of name	Meaning of name
Mercury	mûr'kū·rĭ	Latin	god of commerce and gain
Venus	vē'nŭs	Latin	goddess of love
Earth	ûrth	Mid. Eng.	—
Mars	märz	Latin	god of war
Jupiter	jōō'pĭ·tēr	Latin	god of the heavens, identified with the Greek Zeus, chief of the Olympian gods
Saturn	săt'ērn	Latin	god of seed-sowing
Uranus	ū'rȧ·nŭs	Greek	the personification of heaven
Neptune	nĕp'tūn	Latin	god of the sea
Pluto	plōō'tō	Greek	god of the lower world (Hades)

Guide to pronunciations:
fāte, ădd, finăl, làst, ȧbound, ärm; bē, ĕnd, camĕl, readēr; īce, bĭt, anĭmal; ōver, pōetic, hŏt, lôrd, mōōn; cūbe, ūnite, tŭb, circŭs, ûrn

*Distances in light-years. One light-year equals approximately 63,300 AU, or 5,880,000,000,000 miles. Authorities differ on distances of the stars; the values given are representative.

APPENDIX I

CONSTELLATIONS

Name	Pronunciation	Genitive	Pronunciation	Meaning	Navigational stars or approximate position
Andromeda*	ăn·drŏm′·ḗ·dȧ	Andromedae	ăn·drŏm′ḗ·dē	Andromeda [the chained woman]†	Alpheratz
Antlia	ănt′lĭ·ȧ	Antliae	ănt′lĭ·ē	(air) pump††	d 35°S, SHA 210°
Apus	ā′pŭs	Apodis	ăp′ŏ·dĭs	bird of paradise	d 75°S, SHA 120°
Aquarius (♒)*	ȧ·kwâr′ĭ·ŭs	Aquarii	ȧ·kwâr′ĭ·ī	water carrier	d 5°S, SHA 25°
Aquila*	ăk′wĭ·lȧ	Aquilae	ăk′wĭ·lē	eagle	Altair
Ara*	ā′rȧ	Arae	ā′rē	altar	d 55°S, SHA 100°
Aries (♈)*	ā′rĭ·ēz	Arietis	ȧ·rī′ḗ·tĭs	ram	Hamal
Auriga*	ô·rī′gȧ	Aurigae	ô·rī′jē	charioteer	Capella
Bootes*	bô·ō′tēz	Bootis	bȯ·ō′tĭs	herdsman	Arcturus
Caelum	sē′lŭm	Caeli	sē′lī	graving tool	d 40°S, SHA 290°
Camelopardalis	kȧ·mĕl′ȯ·pär′dȧ·lĭs	Camelopardalis	kȧ·mĕl′ȯ·pär′dȧ·lĭs	giraffe	d 70°N, SHA 275°
Cancer (♋)*	kăn′sẽr	Cancri	kăng′krī	crab	d 20°N, SHA 230°
Canes Venatici	kā′nēz vḗ·năt′ĭ·sī	Canum Venaticorum	kā′nŭm vḗ·năt′ĭ·kō′rŭm	hunting dogs	d 40°N, SHA 165°
Canis Major*	kā′nĭs mā′jẽr	Canis Majoris	kā′nĭs mȧ·jō′rĭs	larger dog	Adhara, Sirius
Canis Minor*	kā′nĭs mī′nẽr	Canis Minoris	kā′nĭs mĭ·nō′rĭs	smaller dog	Procyon
Capricornus (♑)	kăp′rĭ·kôr′nŭs	Capricorni	kăp′rĭ·kôr′nī	horned goat	d 20°S, SHA 45°
Carina**	kȧ·rī′nȧ	Carinae	kȧ·rī′nē	keel	Avior, Canopus, Miaplacidus
Cassiopeia*	kăs′ĭ·ȯ·pē′yȧ	Cassiopeiae	kăs′ĭ·ȯ·pē′yē	Cassiopeia [the lady in the chair]†	Schedar
Centaurus*	sĕn·tô′rŭs	Centauri	sĕn·tô′rī	centaur	Hadar, Menkent, Rigil Kentaurus
Cepheus*	sē′fūs	Cephei	sē′fḗ·ī	Cepheus [the shepherd]†	d 75°N, SHA 15°
Cetus*	sē′tŭs	Ceti	sē′tī	whale	Diphda, Menkar
Chamaeleon	kȧ·mē′lḗ·ŭn	Chamaeleontis	kȧ·mē′lḗ·ŏn′tĭs	chameleon	d 80°S, SHA 200°
Circinus	sûr′sĭ·nŭs	Circini	sûr′sĭ·nī	pair of compasses	d 65°S, SHA 140°
Columba	kȯ·lŭm′bȧ	Columbae	kȯ·lŭm′bē	dove	d 35°S, SHA 275°
Coma Berenices	kō′mȧ bẽr·ḗ·nī′sēz	Comae Berenices	kō′mē bẽr′ḗ·nī′sēz	Berenice's hair	d 25°N, SHA 170°
Corona Australis*	kȯ·rō′nȧ ôs·trā′lĭs	Coronae Australis	kȯ·rō′nē ôs·trā′lĭs	southern crown	d 40°S, SHA 80°
Corona Borealis*	kȯ·rō′nȧ bō′rḗ·ā′lĭs	Coronae Borealis	kȯ·rō′nē bō′rḗ·ā′lĭs	northern crown	Alphecca
Corvus*	kôr′vŭs	Corvi	kôr′vī	crow	Gienah
Crater*	krā′tẽr	Crateris	krȧ·tē′rĭs	cup	d 15°S, SHA 190°
Crux	krŭks	Crucis	kro͞o′sĭs	cross	Acrux, Gacrux
Cygnus*	sĭg′nŭs	Cygni	sĭg′nī	swan	Deneb
Delphinus*	dĕl·fī′nŭs	Delphini	dĕl·fī′nī	dolphin	d 15°N, SHA 50°
Dorado	dȯ·rä′dō	Doradus	dȯ·rä′dŭs	dorado [a fish]†	d 60°S, SHA 285°
Draco*	drā′kō	Draconis	drȧ·kō′nĭs	dragon	Eltanin
Equuleus*	ḗ·kwo͞o′lḗ·ŭs	Equulei	ḗ·kwo͞o′lḗ·ī	colt	d 10°N, SHA 40°
Eridanus*	ḗ·rĭd′ȧ·nŭs	Eridani	ḗ·rĭd′ȧ·nī	Eridanus [a river]†	Acamar, Achernar
Fornax	fôr′năks	Fornacis	fôr·nā′sĭs	furnace	d 30°S, SHA 320°
Gemini (♊)*	jĕm′ĭ·nī	Geminorum	jĕm′ĭ·nō′rŭm	twins	Pollux
Grus	grŭs	Gruis	gro͞o′ĭs	crane [a bird]†	Al Na'ir
Hercules*	hûr′kû·lēz	Herculis	hûr′kû·lĭs	Hercules [mythological hero]†	d 30°N, SHA 100°
Horologium	hŏr′ȯ·lō′jĭ·ŭm	Horologii	hŏr′ȯ·lō′jĭ·ī	clock	d 50°S, SHA 310°
Hydra*	hī′drȧ	Hydrae	hī′drē	water monster	Alphard
Hydrus	hī′drŭs	Hydri	hī′drī	water snake	d 70°S, SHA 320°
Indus	ĭn′dŭs	Indi	ĭn′dī	Indian	d 60°S, SHA 35°

Zodiacal constellations are given in bold type, with their symbols.
*One of the original constellations of Ptolemy.
**Part of the single constellation Argo Navis of Ptolemy.
†Parts within brackets are amplifications of the meanings of constellation names.
††Parts within parentheses are the meanings of words deleted from former, more complete constellation names.

Guide to pronunciations:

fāte, cȧre, hăt, finȧl, ȧbound, sofȧ, ärm; bē, crḗate, ĕnd, readẽr; īce, bĭt; ōver, pȯetic, hŏt, cŏnnect, lôrd, mo͞on; tūbe, ŭnite, tŭb, circŭs, ûrn.

APPENDIX I
CONSTELLATIONS

Name	Pronunciation	Genitive	Pronunciation	Meaning	Navigational stars or approximate position
Lacerta	lȧ·sûr'tȧ	Lacertae	lȧ·sûr'tē	lizard	d 45°N, SHA 25°
Leo (♌)*	lē'ō	Leonis	lē·ō'nĭs	lion	Denebola, Regulus
Leo Minor	lē'ō mī'nẽr	Leonis Minoris	lē·ō'nĭs mĭ·nō'rĭs	smaller lion	d 35°N, SHA 205°
Lepus*	lē'pŭs	Leporis	lĕp'ŏ·rĭs	hare	d 20°S, SHA 275°
Libra (♎)*	lī'brȧ	Librae	lī'brē	balance [scales]†	Zubenelgenubi
Lupus*	lū'pŭs	Lupi	lū'pī	wolf	d 45°S, SHA 130°
Lynx	lĭngks	Lyncis	lĭn'sĭs	lynx	d 50°N, SHA 240°
Lyra*	lī'rȧ	Lyrae	lī'rē	lyre	Vega
Mensa	mĕn'sȧ	Mensae	mĕn'sē	table (mountain)††	d 75°S, SHA 275°
Microscopium	mī'krȯ·skō'pĭ·ŭm	Microscopii	mī'krȯ·skō'pĭ·ī	microscope	d 35°S, SHA 45°
Monoceros	mȯ·nŏs'ẽr·ŏs	Monocerotis	mȯ·nŏs'ẽr·ō'tĭs	unicorn	d 0°, SHA 255°
Musca	mŭs'kȧ	Muscae	mŭs'sē	fly	d 70°S, SHA 175°
Norma	nôr'mȧ	Normae	nôr'mē	square (and rule)††	d 50°S, SHA 120°
Octans	ŏk'tănz	Octantis	ŏk·tăn'tĭs	octant	d 85°S, SHA 40°
Ophiuchus*	ŏf'ĭ·ū'kŭs	Ophiuchi	ŏf'ĭ·ū'kī	serpent holder	Rasalhague, Sabik
Orion*	ō·rī'ŏn	Orionis	ō'rĭ·ō'nĭs	Orion [the hunter]†	Alnilam, Bellatrix, Betelgeuse, Rigel
Pavo	pā'vō	Pavonis	pȧ·vō'nĭs	peacock	Peacock
Pegasus*	pĕg'ȧ·sŭs	Pegasi	pĕg'ȧ·sī	Pegasus [winged horse]†	Enif, Markab
Perseus*	pûr'sūs	Persei	pûr'sȇ·ī	Perseus [mythological character]†	Mirfak
Phoenix	fē'nĭks	Phoenicis	fȇ·nī'sĭs	phoenix [the immortal bird]†	Ankaa
Pictor	pĭk'tẽr	Pictoris	pĭk·tō'rĭs	painter (easel of)††	d 55°S, SHA 275°
Pisces (♓)*	pĭs'ēz	Piscium	pĭsh'ĭ·ŭm	fishes	d 15°N, SHA 355°
Piscis Austrinus*	pĭs'ĭs ôs·trī'nŭs	Piscis Austrini	pĭs'ĭs ôs·trī'nī	southern fish	Fomalhaut
Puppis**	pŭp'ĭs	Puppis	pŭp'ĭs	stern [of ship]†	d 30°S, SHA 245°
Pyxis**	pĭk'sĭs	Pyxidis	pĭk'sĭ·dĭs	mariner's compass	d 25°S, SHA 230°
Reticulum	rȇ·tĭk'ů·lŭm	Reticuli	rȇ·tĭk'ů·lī	net	d 60°S, SHA 300°
Sagitta*	sȧ·jĭt'ȧ	Sagittae	sȧ·jĭt'ē	arrow	d 20°N, SHA 65°
Sagittarius (♐)*	săj'ĭ·tā'rĭ·ŭs	Sagittarii	săj'ĭ·tā'rĭ·ī	archer	Kaus Australis, Nunki
Scorpius (♏)*	skôr'pĭ·ŭs	Scorpii	skôr'pĭ·ī	scorpion	Antares, Shaula
Sculptor	skŭlp'tẽr	Sculptoris	skŭlp·tō'rĭs	sculptor (workshop of)††	d 30°S, SHA 355°
Scutum	skū'tŭm	Scuti	skū'tī	shield	d 10°S, SHA 80°
Serpens*	sûr'pĕnz	Serpentis	sẽr·pĕn'tĭs	serpent	d 10°N, SHA 125°
Sextans	sĕks'tănz	Sextantis	sĕks·tăn'tĭs	sextant	d 0°, SHA 205°
Taurus (♉)*	tô'rŭs	Tauri	tô'rī	bull	Aldebaran, Elnath
Telescopium	tĕl'ȇ·skō'pĭ·ŭm	Telescopii	tĕl'ȇ·skō'pĭ·ī	telescope	d 50°S, SHA 75°
Triangulum*	trī·ăng'gů·lŭm	Trianguli	trī·ăng'gů·lī	triangle	d 30°N, SHA 330°
Triangulum Australe	trī·ăng'gů·lŭm ôs·trā'lē	Trianguli Australis	trī·ăng'gů·lī ôs·trā'lĭs	southern triangle	Atria
Tucana	tů·kā'nȧ	Tucanae	tů·kā'nē	toucan [a bird]†	d 65°S, SHA 5°
Ursa Major*	ûr'sȧ mā'jẽr	Ursae Majoris	ûr'sē mȧ·jō'rĭs	larger bear	Alioth, Alkaid, Dubhe
Ursa Minor*	ûr'sȧ mī'nẽr	Ursae Minoris	ûr'sē mĭ·nō'rĭs	smaller bear	Kochab, Polaris
Vela**	vē'lȧ	Velorum	vȇ·lō'rŭm	sails	Suhail
Virgo (♍)*	vûr'gō	Virginis	vûr'jĭ·nĭs	virgin	Spica
Volans	vō'lănz	Volantis	vȯ·lăn'tĭs	flying (fish)††	d 70°S, SHA 240°
Vulpecula	vŭl·pĕk'ů·lȧ	Vulpeculae	vŭl·pĕk'ů·lē	little fox	d 25°N, SHA 60°

Zodiacal constellations are given in bold type, with their symbols.
*One of the original constellations of Ptolemy.
**Part of the single constellation Argo Navis of Ptolemy.
†Parts within brackets are amplifications of the meanings of constellation names.
††Parts within parentheses are the meanings of words deleted from former, more complete constellation names.

Guide to pronunciations:

fāte, câre, hăt, finȧl, ȧbound, sofȧ, ärm; bē, crȇate, ĕnd, readẽr; īce, bĭt; ōver, pȯetic, hŏt, cŏnnect, lôrd, mōōn; tūbe, ůnite, tŭb, circŭs, ûrn.

APPENDIX J

BUOYAGE SYSTEMS

With modifications, two systems of buoyage are in general use throughout the world. These are the **lateral system** and the **cardinal system.**

The lateral system is best suited for well-defined channels. The location of each buoy indicates the direction of the danger it marks relative to the course which should normally be followed. Thus, a buoy which should be kept on the port hand lies between the vessel and the danger when the buoy is abeam to port, approximately.

In principle, the positions of marks in the lateral system are determined by the general direction taken by the mariner when approaching a harbor, river, estuary, or other waterway from seaward, and may also be determined with reference to the main stream of flood current. The application of this principle is defined, as required, by nautical documents such as sailing directions.

The cardinal system is best suited for coasts with numerous rocks, shoals, and islands, and for dangers in the open sea. The location of each buoy indicates the approximate true bearing of the danger it marks. Thus, an eastern quadrant buoy marks a danger, such as a shoal, which lies to the west of the buoy, approximately.

Although almost all of the major maritime nations have used either the lateral or the cardinal system for many years, details such as the shapes and colors of the buoys and the characteristics and colors of lighted aids generally have varied from country to country. With the passage of time and the increase in maritime communication between countries, the desirability of a uniform system of buoyage has become increasingly apparent. Consequently, over the past century a number of attempts have been made to standardize the various systems of buoyage. International conferences have been held on the subject and recommendations have been made. These recommendations have often been conflicting, however, and although the differences in the various methods as applied to the cardinal system are comparatively slight, two distinct methods of applying the lateral system have evolved. The major discrepancy has been in the colors of the buoys and of their lights.

In 1889 the *International Marine Conference* held in Washington, D. C., recommended that in the lateral system starboard hand buoys be painted red and port hand buoys black. With the introduction of lighted aids to navigation, these recommendations logically led to the use, by nations which had accepted the recommendation, of red or white lights on the starboard side and green or white lights on the port side.

In 1936 in the most recent international pronouncement on the subject, a League of Nations subcommittee recommended a coloring system diametrically opposed to the 1889 proposal. This is part of the **Uniform System,** and it provides for black buoys with green or white lights on the starboard side and red buoys with red or white lights on the port side.

Most maritime countries using the lateral system have adopted one of these two systems, usually with small variations. It may be said that, *very generally,* European countries follow the Uniform System of 1936 and most other countries follow the system proposed in 1889. Special Publication No. 38 of the International Hydrographic Bureau, *Systems of Maritime Buoyage and Beaconage Adopted by Various Countries,* contains discussions and illustrations of the systems actually used by 39 maritime

countries, as well as the Uniform System. When actually piloting, *the navigator should in every case consult the latest nautical literature of the country in question.* The following is an abridgement of parts of IHB Special Publication No. 38:

United States System

The waters of the United States are marked by the lateral system of buoyage recommended by the International Marine Conference of 1889. As all channels do not lead from seaward, arbitrary assumptions are at times made in order that the system may be consistently applied. Along the sea coasts of the United States, the characteristics are based upon the assumption that proceeding "from seaward" constitutes a *clockwise* direction: a southerly direction along the Atlantic coast, a northerly and westerly direction along the Gulf coast, and a northerly direction along the Pacific coast. On the Great Lakes, a westerly and northerly direction is taken as being "from seaward" (except on Lake Michigan, where a southerly direction is used). On the Mississippi and Ohio Rivers and their tributaries, the characteristics of aids to navigation are determined as proceeding from sea toward the head of navigation. On the Intracoastal Waterway, proceeding in a generally southerly direction along the Atlantic coast and in a generally westerly direction along the Gulf coast is considered as proceeding "from seaward."

The continuation of the lateral system along the coasts in the order indicated refers only to the side of the vessel on which buoys are to be kept, as indicated by color, shape, and light, if any; there is no numerical continuity between coast buoys. In fairways and channels, however, buoys are numbered consecutively from seaward.

In the United States System, lighted buoys, bell buoys, whistle buoys, and combination buoys differ in shape (fig. 917) from the unlighted buoys shown in this appendix, but not in color or marking.

In the Mississippi River, the numbering and lighting of buoys differ from that shown under "Fairways and Channels."

Uniform System

As recommended by the League of Nations in 1936, a country uses the *Uniform Lateral System* or the *Uniform Cardinal System*, or both, according to its requirements or preference. When both are used, the transition from one to the other must be clearly indicated in appropriate publications, such as sailing directions, or by suitable buoyage marks.

Both the Uniform Lateral System and the Uniform Cardinal System employ **topmarks** as an additional means of identification. Unless otherwise stated in this appendix, a topmark is painted the darker of the colors used on the buoy. They are optional in every case except on wreck buoys in the Uniform Cardinal System. Topmarks are not used in the United States System.

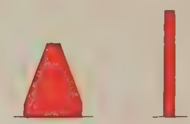
In both the Uniform Lateral System and the Uniform Cardinal System, lighted buoys have the same shape as the unlighted buoys shown. This differs from the United States System, in which distinctively shaped buoys are used for lighted aids.

In both the Uniform Lateral System and the Uniform Cardinal System, a quick flashing light is regarded as a single flashing light.

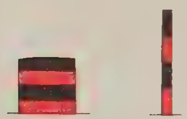
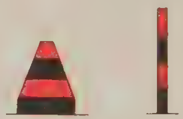
The numbering or lettering of fairway and channel buoys is an optional feature of the Uniform Lateral System. In the United States System these buoys are always numbered, commencing from seaward.

UNITED STATES SYSTEM

Fairways and Channels

	PORT HAND	STARBOARD HAND
BUOY:		
MARKING:	Odd numbers, commencing from seaward.	Even numbers, commencing from seaward.
LIGHTED BUOY:	*White* or *green*, flashing or occulting; or, when marking important turns, quick flashing.	*White* or *red*, flashing or occulting; or, when marking important turns, quick flashing.

Middle Grounds

	MAIN CHANNEL TO RIGHT	MAIN CHANNEL TO LEFT
BUOY:		
MARKING:	May be lettered.	May be lettered.
LIGHTED BUOY:	*White* or *green*, interrupted quick flashing.	*White* or *red*, interrupted quick flashing.

Where channels are of equal importance, either of the above buoys is used, without regard to the uppermost band.

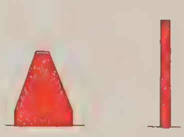
Mid Channel

BUOY: 

MARKING: May be lettered.

LIGHTED BUOY: *White*, short-long flashing.

Wrecks or Other Obstructions

	TO BE PASSED ON PORT HAND	TO BE PASSED ON STARBOARD HAND
BUOY:		
MARKING:	Usually lettered "WR."	Usually lettered "WR."
LIGHTED BUOY:	*White* or *green*, quick flashing.	*White* or *red*, quick flashing.

Where wrecks or other obstructions may be passed on either hand, either Middle Ground *buoy is used, without regard to the uppermost band.*

UNITED STATES SYSTEM

Miscellaneous

SHAPE: Optional.

COLOR: **Quarantine**—*Yellow.*
Anchorage—*White.*
Fish Nets—*Black-and-white* horizontal bands.
Dredging—*White* with *green* top.
Seadromes—*Yellow-and-black* vertical stripes.
Special Purpose—*White-and-international orange* horizontal or vertical bands.

MARKING: May be lettered.

LIGHTED BUOY: Any color except *red* or *green*; fixed, occulting, or slow flashing.

UNIFORM LATERAL SYSTEM

Fairways and Channels

	PORT HAND	STARBOARD HAND
TOPMARK:		

"T"-shaped topmark not used at channel entrance.

Diamond-shaped top-mark not used at channel entrance.

BUOY:

In secondary channels only, yellow *may be substituted for* white *in checkered buoys.*

MARKING:

- Port Hand: Even numbers, commencing from seaward.
- Starboard Hand: Odd numbers, commencing from seaward.

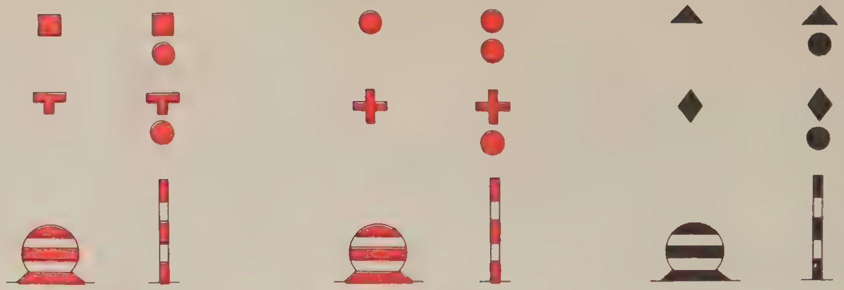
LIGHT:

- Port Hand: *Red,* single flashing or occulting or group flashing or occulting, with a number of flashes or occultations up to four; or *white,* group flashing or occulting (2 or 4); both *red and white* with above characteristics.
- Starboard Hand: *White,* single flashing or occulting, or group flashing or occulting (3); or *green,* of a different character from wreck markings; or both *white and green* with the above characteristics.

UNIFORM LATERAL SYSTEM

Middle Grounds

	MAIN CHANNEL TO RIGHT	CHANNELS OF EQUAL IMPORTANCE	MAIN CHANNEL TO LEFT
TOPMARK: *Bifurcation*			
Junction			
BUOY:			



LIGHT:	Distinctive where possible.	Distinctive where possible.	Distinctive where possible.

Mid Channel

TOPMARK: Shape optional, but not conical, cylindrical, or spherical.

BUOY: Shape optional, but not conical, cylindrical, or spherical.

COLOR: *Red-and-white* or *black-and-white* vertical stripes; topmark *red* or *black* to conform with buoy.

LIGHT: Different from neighboring lights.

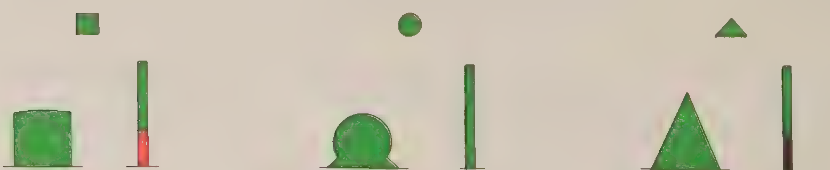
Marking of Wrecks

	TO BE PASSFD ON PORT HAND	TO BE PASSED ON EITHER HAND	TO BE PASSED ON STARBOARD HAND

By Buoys

TOPMARK:

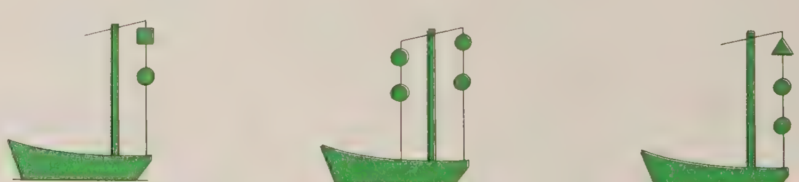
BUOY:



MARKING:	"W" in *white*.	"W" in *white*.	"W" in *white*.
LIGHT:	*Green*, group flashing (2).	*Green*, single occulting.	*Green*, group flashing (3).

By Vessels

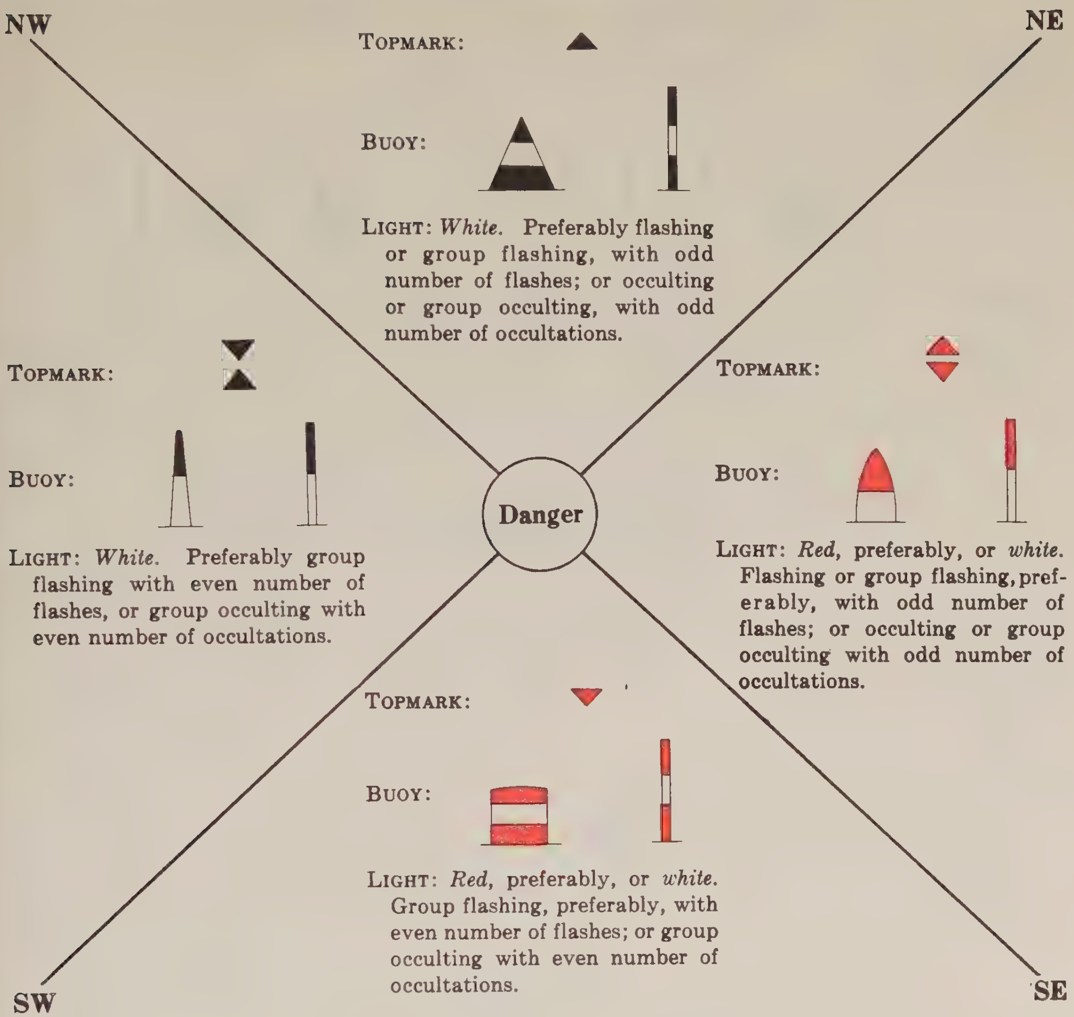
VESSEL:



MARKING:	"W" or "WRECK" in *white*.	"W" or "WRECK" in *white*.	"W" or "WRECK" in *white*.
LIGHT:	Fixed *green*, corresponding in number and arrangement to shapes displayed by day.		
BELL:	Two strokes at intervals of not more than 30 seconds.	Four strokes at intervals of not more than 30 seconds.	Three strokes at intervals of not more than 30 seconds.

UNIFORM CARDINAL SYSTEM

Danger Markings



Variations in Danger Markings

Note: The number of characteristic shapes employed for the buoy itself may be limited to two, the conical shape being employed in the northern and eastern quadrants and the cylindrical shape in the southern and western quadrants, as shown above.

Note: When spars only are used, it may be advantageous in the northern and eastern quadrants to reverse the positions of the dark colors, as shown above.

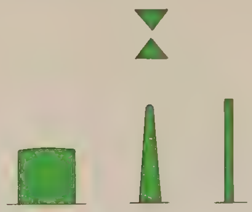
UNIFORM CARDINAL SYSTEM

Marking of Wrecks

Western Quadrant

Topmark:

Buoy:



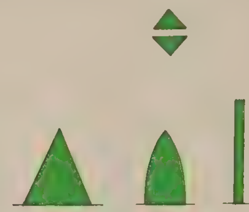
Marking: "W" in *white*, if possible.

Light: *Green*, quick flashing.

Eastern Quadrant

Topmark:

Buoy:



Marking: "W" in *white*, if possible.

Light: *Green*, interrupted quick flashing.

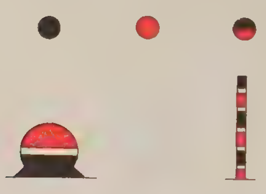
In the Uniform Cardinal System, wreck buoys are not used in the northern or southern quadrants.

UNIFORM SYSTEM—LATERAL AND CARDINAL
(Common To Both)

Isolated Dangers

Topmark:

Buoy:



Light: *White* or *red*, rhythmic.

Miscellaneous

Topmark: **Landfall**—Shape optional, but not misleading.
Transition—Shape optional, but not misleading.
Others—None.

Buoy: Shape optional, but not misleading.

Color: **Landfall**—*Black-and-white* or *red-and-white* vertical stripes.
Transition—*Red-and-white* or *black-and-white* spiral bands.
Quarantine—*Yellow*.
Outfall—*Yellow* above and *black* below.
Military Practice Area—*White*, with two *blue* stripes rising from the waterline and intersecting at right angles on top of the buoy, and, optionally, lettering in the national language indicating a danger area (*e.g.*, in English, "D.A.").

Light: **Landfall**—Rhythmic.
Outfall—Optional, with due regard to other lights in the area.
Others—None.

APPENDIX K

CHART SYMBOLS

(Extracts from Chart No. 1, September 1963)

GENERAL REMARKS

Chart No. 1 contains the standard symbols and abbreviations which have been approved for use of nautical charts published by the United States of America.

Symbols and abbreviations shown on Chart No. 1 apply to the regular nautical charts and may differ from those shown on certain reproductions and special charts.

Terms, symbols and abbreviations are numbered in accordance with a standard form approved by a Resolution of the Sixth International Hydrographic Conference, 1952.

Vertical figures indicate those items where the symbol and abbreviation are in accordance with the Resolutions of the International Hydrographic Conferences.

Slanting figures indicate those items where the symbol and/or abbreviation differ from the Resolutions of the Conferences, or for which Resolutions do not yet exist.

(*Those items which differ from the Resolutions are underlined.*)

Slanting letters in parentheses indicate that the items are in addition to those shown on the approved standard form.

Colors are optional for characterizing various features and areas on the charts.

Lettering styles and capitalization as used on Chart No. 1 are not always rigidly adhered to on the charts.

Longitudes are referred to the Meridian of Greenwich.

Scales are computed on the middle latitude of each chart, or on the middle latitude of a series of charts.

Buildings—A conspicuous feature on a building may be shown by a landmark symbol with descriptive note (See L–63 & I–n). Prominent buildings that are of assistance to the mariner are crosshatched (See I–3a, 5, 47 & 66).

Shoreline is the line of Mean High Water, except in marsh or mangrove areas, where the outer edge of vegetation (berm line) is used. A heavy line (A–9) is used to represent a firm shoreline. A light line (A–7) represents a berm line.

Heights of land and conspicuous objects are given in feet above Mean High Water, unless otherwise stated in the title of the chart.

Depth Contours and Soundings may be shown in meters on charts of foreign waters.

Visibility of a light is in nautical miles for an observer's eye 15 feet above water level.

Buoys and Beacons—On entering a channel from seaward, buoys on starboard side are red with even numbers, on port side black with odd numbers. Lights on buoys on starboard side of channel are red or white, on port side white or green. Mid-channel buoys have black-and-white vertical stripes. Junction or obstruction buoys, which may be passed on either side, have red-and-black horizontal bands. This system does not always apply to foreign waters. The dot of the buoy symbol, the small circle of the light vessel and mooring buoy symbols, and the center of the beacon symbol indicate their positions.

Improved channels are shown by limiting dashed lines, the depth, month, and the year of latest examination being placed adjacent to the channel, except when tabulated.

U.S. Coast Pilots, Sailing Directions, Light Lists, Radio Aids, and related publications furnish information required by the navigator that cannot be shown conveniently on the nautical chart.

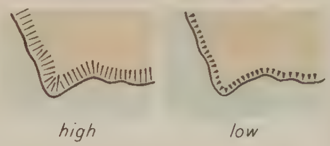
U.S. Nautical Chart Catalogs and Indexes list nautical charts, auxiliary maps, and related publications, and include general information (marginal notes, etc.) relative to the charts.

A glossary of foreign terms and abbreviations is generally given on the charts on which they are used, as well as in the Sailing Directions.

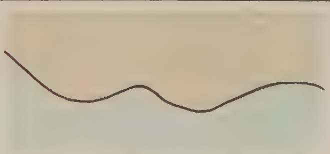
Charts already on issue will be brought into conformity as soon as opportunity affords.

A. The Coastline (Nature of the Coast)

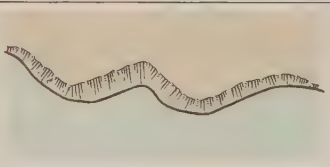
1 *Shoreline unsurveyed*



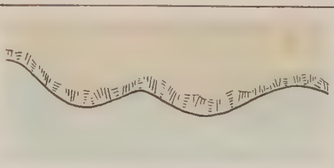
2 *Steep coast (Bluff)*



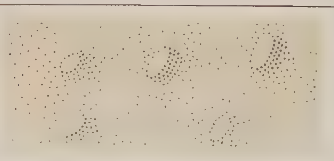
2a *Flat coast*



3 *Cliffy coast*

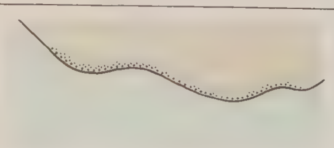


3a *Rocky coast*

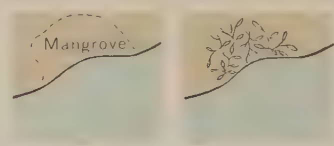


4 *Sandhills; Dunes*

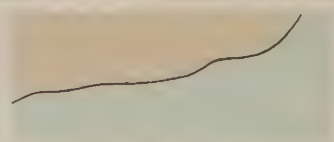
5 *Stony or Shingly shore*



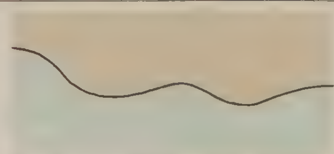
6 *Sandy shore*



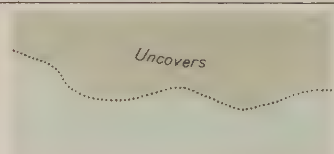
7 *Mangrove*



8 *Surveyed coastline*

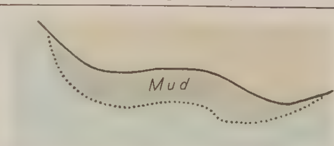


9 *High water line*

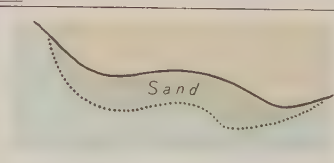


10 *Low water line*

11 **Foreshore**
(Strand in general)

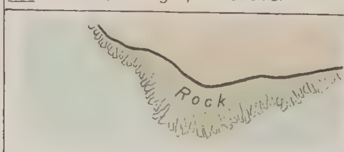


11a *Mud*



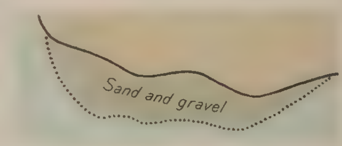
11b *Sand*

11c *Stones; Shingle; or Gravel*

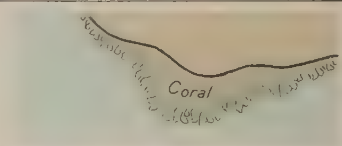


11d *Rock, uncovers at sounding datum (See A-11g)*

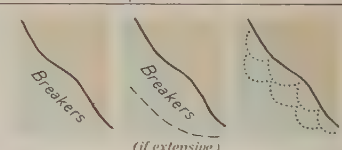
11e *Sand and mud*



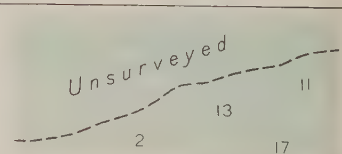
11f *Sand and gravel*



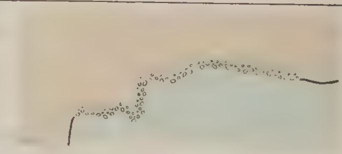
11g *Coral, uncovers at sounding datum (See O-10)*



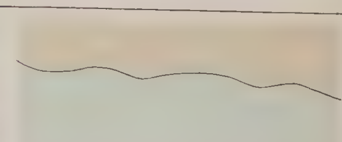
12 *Breakers along a shore (See O-25)*



14 *Limit of unsurveyed areas*



(Aa) *Rubble*

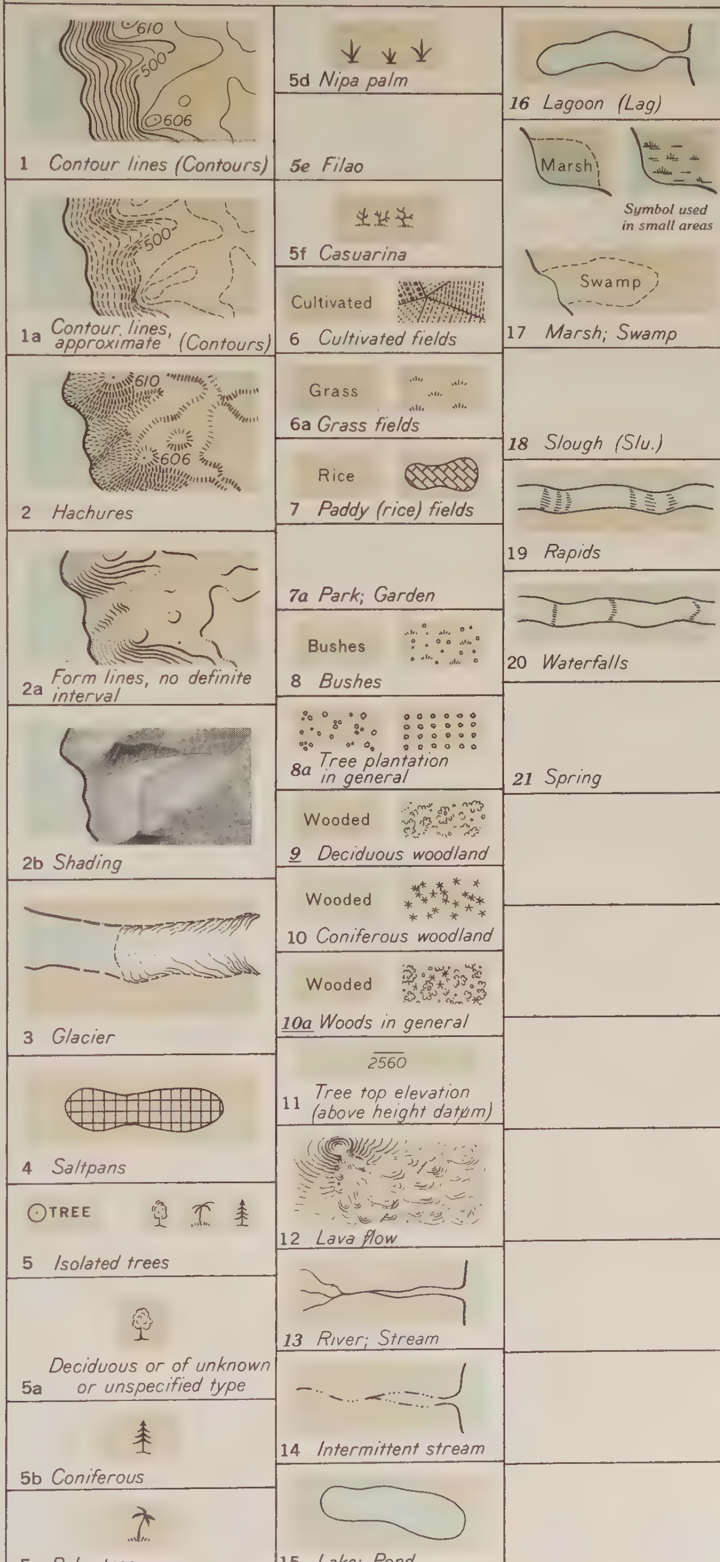


(Ab) *Shoreline from older surveys or small-scale charts*

B. Coast Features

1	G	Gulf
2	B	Bay
(Ba)	B	Bayou
3	Fd	Fjord
4	L	Loch; Lough; Lake
5	Cr	Creek
5a	C	Cove
6	In	Inlet
7	Str	Strait
8	Sd	Sound
9	Pass Thoro	Passage; Pass Thorofare
10	Chan	Channel
10a		Narrows
11	Entr	Entrance
12	Est	Estuary
12a		Delta
13	Mth	Mouth
14	Rd	Road; Roadstead
15	Anch	Anchorage
16	Hbr	Harbor
16a	Hn	Haven
17	P	Port
(Bb)	P	Pond
18	I	Island
19	It	Islet
20	Arch	Archipelago
21	Pen	Peninsula
22	C	Cape
23	Prom	Promontory
24	Hd	Head; Headland
25	Pt	Point
26	Mt	Mountain; Mount
27	Rge	Range
27a		Valley
28		Summit
29	Pk	Peak
30	Vol	Volcano
31		Hill
32	Bld	Boulder
33	Ldg	Landing
34		Table-land (Plateau)
35	Rk	Rock
36		Isolated rock
(Bc)	Str	Stream
(Bd)	R	River
(Be)	Slu	Slough
(Bf)	Lag	Lagoon
(Bg)	Apprs	Approaches
(Bh)	Rky	Rocky

C. The Land (Natural Features)



D. Control Points

1	△		*Triangulation point (station)*
2	⊙		*Fixed point (landmark) (See L-63)*
3	· 256		*Summit of height (Peak) (when not a landmark)*
(Da)			*Peak, accentuated by contours*
(Db)			*Peak, accentuated by hachures*
(Dc)			*Peak, elevation not determined*
(Dd)	⊙ 256		*Peak, when a landmark*
4	⊕	Obs Spot	*Observation spot*
5		BM	*Bench mark*
6	∘	See View	*View point*
7			*Datum point for grid of a plan*
8			*Graphical triangulation point*
9		Astro	*Astronomical*
10		Tri	*Triangulation*
(De)		C of E	*Corps of Engineers*
12			*Great trigonometrical survey station*
13			*Traverse station*
14		Bdy. Mon	*Boundary monument*
(Df)	◇		*International boundary monument*

E. Units

1	hr	*Hour*
2	m; min	*Minute (of time)*
3	sec	*Second (of time)*
4	m	*Meter*
4a	dm	*Decimeter*
4b	cm	*Centimeter*
4c	mm	*Millimeter*
4d	m^2	*Square meter*
4e	m^3	*Cubic meter*
5	km	*Kilometer*
6	in	*Inch*
7	ft	*Foot*
8	yd	*Yard*
9	fm	*Fathom*
10	cbl	*Cable length*
11	M	*Nautical mile*
12	kn	*Knot*
12a	t	*Ton*
12b	cd	*Candela (new candle)*
13	lat	*Latitude*
14	long	*Longitude*
15	pub	*Publication*
16	Ed	*Edition*
17	corr	*Correction*
18	alt	*Altitude*
19	ht; elev	*Height; Elevation*
20	°	*Degree*
21	′	*Minute (of arc)*
22	″	*Second (of arc)*
23	No	*Number*
(Ea)	St. M	*Statute mile*
(Eb)	Msec	*Microsecond*

F. Adjectives, Adverbs and other abbreviations

1	gt	*Great*
2	lit	*Little*
3	lrg	*Large*
4	sml	*Small*
5		*Outer*
6		*Inner*
7	mid	*Middle*
8		*Old*
9	anc	*Ancient*
10		*New*
11	St	*Saint*
12	conspic	*Conspicuous*
13		*Remarkable*
14	D., Destr	*Destroyed*
15		*Projected*
16	dist	*Distant*
17	abt	*About*
18		*See chart*
18a		*See plan*
19		*Lighted; Luminous*
20	sub	*Submarine*
21		*Eventual*
22	AERO	*Aeronautical*
23		*Higher*
24	exper	*Experimental*
25	discontd	*Discontinued*
26	prohib	*Prohibited*
27	explos	*Explosive*
28	estab	*Established*
29	elec	*Electric*
30	priv	*Private, Privately*
31	prom	*Prominent*
32	std	*Standard*
33	subm	*Submerged*
34	approx	*Approximate*
(Fa)	unverd	*Unverified*
(Fb)	AUTH	*Authorized*
(Fc)	CL	*Clearance*
(Fd)	maintd	*Maintained*
(Fe)	aband	*Abandoned*
(Ff)	cor	*Corner*
(Fg)	concr	*Concrete*
(Fh)	fl	*Flood*
(Fi)	extr	*Extreme*
(Fj)	mod	*Moderate*
(Fk)	bet	*Between*
(Fl)	1st	*First*
(Fm)	2nd	*Second*
(Fn)	3rd	*Third*
(Fo)	4th	*Fourth*

G. Ports and Harbors

No.	Symbol	Abbreviation	Description
1	⚓	*Anch*	*Anchorage (large vessels)*
2	⚓	*Anch*	*Anchorage (small vessels)*
3		*Hbr*	*Harbor*
4		*Hn*	*Haven*
5		*P*	*Port*
6		Bkw	*Breakwater*
6a			*Dike*
7			*Mole*
8			*Jetty (partly below MHW)*
8a			*Submerged jetty*
(Ga)			*Jetty (small scale)*
9		Pier	*Pier*
10			*Spit*
11			*Groin (partly below MHW)*
12	ANCHORAGE PROHIBITED	ANCH PROHIB	*Anchorage prohibited (See P-25)*
13	*Spoil Area*		*Spoil ground*
(Gb)	*Dumping Ground*		*Dumping ground*
(Gc)	80 85 83 *Disposal Area* *Depths from survey of June 1963* 95 90 98		*Disposal area*
14		Fsh stks	*Fisheries; Fishing stakes*
14a			*Fish trap; Fish weirs (actual shape charted)*
14b			*Duck blind*
15			*Tunny nets (See G-14a)*
15a	*Oys*	*Oys*	*Oyster bed*
16		Ldg	*Landing place*
17			*Watering place*
18		Whf	*Wharf*
19			*Quay*

No.	Symbol	Abbreviation	Description
20			*Berth*
20a	14		*Anchoring berth*
20b	3		*Berth number*
21	∘	Dol	*Dolphin*
22			*Bollard*
23			*Mooring ring*
24			*Crane*
25			*Landing stage*
25a			*Landing stairs*
26		Quar	*Quarantine*
27			*Lazaret*
28		Harbor Master	*Harbor master's office*
29		Cus Ho	*Customhouse*
30			*Fishing harbor*
31			*Winter harbor*
32			*Refuge harbor*
33		*B Hbr*	*Boat harbor*
34			*Stranding harbor (uncovers at LW)*
35			*Dock*
36			*Dry dock (actual shape on large-scale charts)*
37			*Floating dock (actual shape on large-scale charts)*
38			*Gridiron; Careening grid*
39			*Patent slip; Slipway; Marine railway*
39a		Ramp	*Ramp*
40	Lock		*Lock (point upstream) (See H-13)*
41			*Wet dock*
42			*Shipyard*
43			*Lumber yard*
44		Health Office	*Health officer's office*
45		*Hk*	*Hulk (actual shape on lrg. scale charts) (See O-11)*
46	PROHIBITED AREA	PROHIB AREA	*Prohibited area*
47			*Anchorage for seaplanes*
48			*Seaplane landing area*
49	*Under construction*		*Work in progress*
50			*Under construction*
(Gd)	*Subm ruins*		*Submerged ruins*

H. Topography (Artificial Features)

Small-scale chart

1 *Road* (Rd) *or Highway* (Hy)

2 *Track, Footpath, or Trail*

M & L S RR

Same grade *Ry. above* *Ry. below*

3 *Railway* (Ry) *(single or double track); Railroad* (RR)

3a *Tramway*

3b *Railway station*

3c *Tunnel (railroad or road)*

3d *Embankment, Levee*

3e *Cutting*

OVERHEAD POWER CABLE AUTHORIZED CL. 140 FT

TOWER TOWER

4 *Overhead power cable* (OVHD. PWR. CAB.)

5 *Power transmission line*

5a *Power transmission mast*

Tel

6 *Prominent telegraph or telephone line*

7 *Aqueduct; Water pipe*

Viaduct

8 *Viaduct*

8a *Oil pipeline* Pipeline

Pile Piling Post

9 *Pile; Piling; Post (above MHW) (See L-59, O-30)*

9a *Mast* MAST Mast

10 *Highway (See H-1)*

11 *Sewer* Sewer

12 *Culvert*

Canal Lock

Ditch Sluice (Tidegate, Floodgate)

13 *Canal; Ditch; Lock, Sluice (point upstream)*

14 *Bridge* (BR) *in general*

14a *Stone, concrete bridge (Same as H-14)*

14b *Wooden bridge (Same as H-14)*

14c *Iron bridge (Same as H-14)*

14d *Suspension bridge (Same as H-14)*

15 *Drawbridge (in general)*

16 *Swing bridge (Same as H-15)*

16a *Lift bridge*

16b *Weighbridge or Bascule bridge*

17 *Pontoon bridge*

17a *Footbridge*

18 *Transporter bridge (Same as H-14)*

VERT. CL. 6 FT

18a *Bridge clearance, vertical*

HOR CL 28 FT

18b *Bridge clearance, horizontal*

Ferry Ferry

19 *Ferry* (Fy) *On small-scale chart*

20 *Ford*

21 *Dam*

22 *Fence*

23 *Training wall*

Log boom

(Ha) *Log boom*

I. Buildings and Structures (see General Remarks)

No.	Symbol	Abbreviation	Meaning
1			City or Town (large scale)
(Ia)			City or Town (small scale)
2			Suburb
3		Vil	Village
3a			Buildings in general
4		Cas	Castle
5			House
6			Villa
7			Farm
8		Ch	Church
8a		Cath	Cathedral
8b	SPIRE	Spire	Spire; Steeple
8c			Christian Shrine
9			Roman Catholic Church
10			Temple
11			Chapel
12			Mosque; Minaret
(Ib)			Moslem Shrine
13			Marabout
14		Pag	Pagoda
15			Buddhist Temple; Joss-House
15a			Shintō Shrine
16			Monastery; Convent
17			Calvary; Cross
17a	Cem		Cemetery, Non-Christian
18	Cem		Cemetery, Christian
18a			Tomb
19			Fort (actual shape charted)
20			Battery (Same as I-19)
21			Barracks
22			Powder magazine
23	Airport		Airplane landing field
24	Airport		Airport, large scale (See P-13)
(Ic)			Airport, military (small scale)
(Id)			Airport, civil (small scale)
25			Mooring mast
26	King St	St	Street

No.	Symbol	Abbreviation	Meaning
26a	Locust Ave	Ave	Avenue
(Ie)	Grand Blvd	Blvd	Boulevard
27		Tel	Telegraph
28		Tel.Off	Telegraph office
29		P.O	Post office
30		Govt. Ho	Government house
31			Town hall
32		Hosp	Hospital
33			Slaughterhouse
34		Magz	Magazine
34a			Warehouse; Storehouse
35	MON	Mon	Monument
36	CUP	Cup	Cupola
37	ELEV	Elev.	Elevator; Lift
(If)		Elev	Elevation; Elevated
38			Shed
39			Zinc roof
40	Ruins	Ru	Ruins
41	TR	Tr	Tower
42	WINDMILL		Windmill
43			Watermill
43a	WINDMOTOR		Windmotor
44	CHY	Chy	Chimney; Stack
45	S'PIPE	S'pipe	Water tower; Standpipe
46			Oil tank
47		Facty	Factory
48			Saw mill
49			Brick kiln
50			Mine; Quarry
51	Well		Well
52			Cistern
53	TANK	Tk	Tank
54			Noria
55			Fountain

I. Buildings and Structures (continued)

61		Inst	*Institute*
62			*Establishment*
63			*Bathing establishment*
64		Ct Ho	*Courthouse*
65		Sch	*School*
(Ig)		H. S	*High school*
(Ih)		Univ	*University*
66		Bldg	*Building*
67		Pav	*Pavilion*
68			*Hut*
69			*Stadium*
70		T	*Telephone*
71			*Gas tank; Gasometer*
72	GAB Gab		*Gable*
73			*Wall*
(Ii)		Ltd	*Limited*
(Ij)		Apt	*Apartment*
(Ik)		Cap	*Capitol*
(Il)		Co	*Company*
(Im)		Corp	*Corporation*
(In)			*Landmark (conspicuous object)*
(Io)			*Landmark (position approx.)*

J. Miscellaneous Stations

1		Sta	*Any kind of station*
2		Sta	*Station*
3	C.G		*Coast Guard station (Similar to LS. S.)*
(Ja)	C.G WALLIS SANDS		*Coast Guard station (when landmark)*
4	LOOK.TR		*Lookout station; Watch tower*
5			*Lifeboat station*
6	LS. S		*Lifesaving station (See J-3)*
7		Rkt. Sta	*Rocket station*
8	PIL. STA		*Pilot station*
9		Sig. Sta	*Signal station*
10		Sem	*Semaphore*
11		S. Sig Sta	*Storm signal station*
12			*Weather signal station*
(Jb)	W.B. SIG. STA		*Weather Bureau signal station*
13			*Tide signal station*
14			*Stream signal station*
15			*Ice signal station*
16			*Time signal station*
17			*Time ball*
18			*Signal mast*
19	FS FP FS FP		*Flagstaff; Flagpole*
(Jc)	F. TR F.Tr		*Flag tower*
20			*Signal*
21		Obsy	*Observatory*
22		Off	*Office*
(Jd)	BELL		*Bell (on land)*
(Je)	HECP		*Harbor entrance control post*

K. Lights

No.	Symbol / Abbreviation	Description
1		*Position of light*
2	Lt	*Light*
(Ka)		*Riprap surrounding light*
3	Lt. Ho	*Lighthouse*
4	AERO	*Aeronautical light (See F-22)*
4a		*Marine and air navigation light*
5	Bn	*Light beacon*
6		*Light vessel; Lightship*
8		*Lantern*
9		*Street lamp*
10	REF	*Reflector*
11	Ldg. Lt	*Leading light*
12	RED	*Sector light*
13	RED GREEN	*Directional light*
14		*Harbor light*
15		*Fishing light*
16		*Tidal light*
17	Priv maintd	*Private light (maintained by private interests; to be used with caution)*
21	F	*Fixed light*
22	Occ	*Occulting light*
23	Fl	*Flashing light*
24	Qk Fl	*Quick flashing (scintillating) light*
24a	I Qk Fl Int Qk F	*Interrupted quick flashing light*
(Kb)	E Int	*Equal interval (isophase) light*
25a	S Fl	*Short flashing light*
26	Alt	*Alternating light*
27	Gp Occ	*Group occulting light*
28	Gp Fl	*Group flashing light*
28a	S-L Fl	*Short-long flashing light*
28b		*Group short flashing light*

No.	Abbreviation	Description
29	F Fl	*Fixed and flashing light*
30	F Gp Fl	*Fixed and group flashing light*
31	Rot	*Revolving or Rotating light*
(Khb)	Mo	*Morse code*
41		*Period*
42		*Every*
43		*With*
44		*Visible (range)*
(Kc)	M	*Nautical mile (See E-11)*
(Kd)	m. min	*Minutes (See E-2)*
(Ke)	sec	*Seconds (See E-3)*
45	Fl	*Flash*
46	Occ	*Occultation*
46a		*Eclipse*
47	Gp	*Group*
48	Occ	*Intermittent light*
49	*SEC*	*Sector*
50		*Color of sector*
51	Aux	*Auxiliary light*
52		*Varied*
61	Vi	*Violet*
62		*Purple*
63	Bu	*Blue*
64	G	*Green*
65	Or	*Orange*
66	R	*Red*
67	W	*White*
67a	Am	*Amber*
68	*OBSC*	*Obscured light*
(Kf)	*Fog Det Lt*	*Fog detector light (See Nb)*

K. Lights (continued)

69		*Unwatched light*
70	Occas	*Occasional light*
71	Irreg	*Irregular light*
72	Prov	*Provisional light*
73	Temp	*Temporary light*
(Kg)	D; Destr	*Destroyed*
74	Exting	*Extinguished light*
75		*Faint light*
76		*Upper light*
77		*Lower light*
78		*Rear light*
79		*Front light*
80	Vert	*Vertical lights*
81	Hor	*Horizontal lights*
(Kh)	VB	*Vertical beam*
(Ki)	*RGE*	*Range*
(Kj)	Exper	*Experimental light*
(Kk)	TRLB	*Temporarily replaced by lighted buoy showing the same characteristics*
(Kl)	TRUB	*Temporarily replaced by unlighted buoy*
(Km)	TLB	*Temporary lighted buoy*
(Kn)	TUB	*Temporary unlighted buoy*

L. Buoys and Beacons (see General Remarks)

1		*Position of buoy*
2		*Light buoy*
3	*BELL*	*Bell buoy*
3a	*GONG*	*Gong buoy*
4	*WHIS*	*Whistle buoy*
5	*C*	*Can or Cylindrical buoy*
6	*N*	*Nun or Conical buoy*
7	*SP*	*Spherical buoy*
8	*S*	*Spar buoy*
8a	*P*	*Pillar buoy*
9		*Buoy with topmark (ball) (See L-70)*
10		*Barrel or Ton buoy*
(La)		*Color unknown*
(Lb)	*FLOAT*	*Float*
12	*FLOAT*	*Lightfloat*
13		*Outer or Landfall buoy*
14	*BW*	*Fairway buoy (BWVS)*
14a	*BW*	*Mid-channel buoy (BWVS)*
15	*R "2"*	*Starboard-hand buoy (entering from seaward)*
16	*"1"*	*Port-hand buoy (entering from seaward)*
17	*RB* *RB*	*Bifurcation buoy (RBHB)*
18	*RB* *RB*	*Junction buoy (RBHB)*
19	*RB* *RB*	*Isolated danger buoy (RBHB)*
20	*RB* *G*	*Wreck buoy (RBHB or G)*
20a	*RB* *G*	*Obstruction buoy (RBHB or G)*
21	*Tel*	*Telegraph-cable buoy*
22		*Mooring buoy (colors of mooring buoys never carried)*
22a		*Mooring*
22b	*Tel*	*Mooring buoy with telegraphic communications*
22c	*T*	*Mooring buoy with telephonic communications*
23		*Warping buoy*
24	*Y*	*Quarantine buoy*
25	*Explos Anch*	*Explosive anchorage buoy*
25a	*AERO*	*Aeronautical anchorage buoy*
26	*Deviation*	*Compass adjustment buoy*
27	*BW*	*Fish trap buoy (BWHB)*
27a		*Spoil ground buoy*
28		*Anchorage buoy (marks limits)*
29	*Priv maintd*	*Private buoy (maintained by private interests, use with caution)*

L. Buoys and Beacons (continued)

No.	Symbol	Abbreviation	Description
30			*Temporary buoy (See Kk, l, m, n)*
30a			*Winter buoy*
31		HB	*Horizontal stripes or bands*
32		VS	*Vertical stripes*
33		Chec	*Checkered*
(Lc)		Diag	*Diagonal buoy*
41		W	*White*
42		B	*Black*
43		R	*Red*
44		Y	*Yellow*
45		G	*Green*
46		Br	*Brown*
47		Gy	*Gray*
48		Bu	*Blue*
(Ld)		Am	*Amber*
(Le)		Or	*Orange*
51			*Floating beacon*
52	△RW Bn △W Bn ▲R Bn		*Fixed beacon (unlighted or daybeacon)*
	▲ Bn		*Black beacon*
	△ Bn		*Color unknown*
(Lf)	⊙ MARKER		*Private aid to navigation*
53		Bn	*Beacon, in general (See L-52)*
54			*Tower beacon*

No.	Symbol	Abbreviation	Description
55			*Cardinal marking system*
56	△ Deviation Bn		*Compass adjustment beacon*
57			*Topmarks (See L-9, 70)*
58			*Telegraph-cable (landing) beacon*
59	Piles		*Piles (See O-30, H-9)*
	⊥ ⊥		*Stakes*
	Stumps		*Stumps (See O-30)*
	⊥ ⊥		*Perches*
61	⊙ CAIRN ∘ Cairn		*Cairn*
62			*Painted patches*
63	⊙		*Landmark (conspicuous object) (See D-2)*
(Lg)	∘		*Landmark (position approximate)*
64		REF	*Reflector*
65	⊙ MARKER		*Range targets, markers*
(Lh)	W Or W Or		*Special-purpose buoys*
70			Note: *TOPMARKS on buoys and beacons may be shown on charts of foreign waters. The abbreviation for black is not shown adjacent to buoys or beacons.*
(Li)		Ra Ref	*Radar reflector (See M-13)*

M. Radio and Radar Stations

No.	Symbol	Description
1	°R. Sta	*Radio telegraph station*
2	°R. T	*Radio telephone station*
3	R. Bn	*Radiobeacon*
4	R. Bn	*Circular radiobeacon*
5	R.D	*Directional radiobeacon; Radio range*
6		*Rotating loop radiobeacon*
7	R.D.F	*Radio direction finding station*
(Ma)	TELEM ANT	*Telemetry antenna*
9	R. MAST	*Radio mast*
	R. TR	*Radio tower*
(Mb)	TV TR	*Television tower*
10	R. TR. (WBAL) 1090 Kc	*Radio broadcasting station (commercial)*
10a	°R. Sta	*Q.T.G. Radio station*
11	Ra	*Radar station*

No.	Symbol	Description
12	Racon	*Radar responder beacon*
13	Ra Ref	*Radar reflector (See Li)*
14	Ra (conspic)	*Radar conspicuous object*
14a		*Ramark*
15	D.F.S	*Distance finding station (synchronized signals)*
(Mc)	AERO R. Bn 302	*Aeronautical radiobeacon*
(Md)	AERO R. Rge 342	*Aeronautical radio range*
(Me)	Ra Ref Calibration Bn	*Radar calibration beacon*
(Mf)	CONSOL Bn 190 Kc MMF	*Consol (Consolan) station*
(Mg)	Loran Sta Venice	*Loran station (name)*
(Mh)	LORAN TR SPRING ISLAND	*Loran tower (name)*
(Mi)	10	*Radio calling-in point for traffic control*

N. Fog Signals

No.	Symbol	Description
1	Fog Sig	*Fog-signal station*
2		*Radio fog-signal station*
3	*GUN*	*Explosive fog signal*
4		*Submarine fog signal*
5	*SUB-BELL*	*Submarine fog bell (action of waves)*
6	*SUB-BELL*	*Submarine fog bell (mechanical)*
7	*SUB-OSC*	*Submarine oscillator*
8	*NAUTO*	*Nautophone*
9	*DIA*	*Diaphone*
10	*GUN*	*Fog gun*
11	*SIREN*	*Fog siren*

No.	Symbol	Description
12	*HORN*	*Fog trumpet*
13	*HORN*	*Fog horn*
14	*BELL*	*Fog bell*
15	*WHIS*	*Fog whistle*
16	*HORN*	*Reed horn*
17	*GONG*	*Fog gong*
18		*Submarine sound signal not connected to the shore (See N-5,6,7)*
18a		*Submarine sound signal connected to the shore (See N-5,6,7)*
(Na)	*HORN*	*Typhon*
(Nb)	*Fog Det Lt*	*Fog detector light (See Kf)*

O. Dangers

No.	Symbol	Description
1	(25)	Rock which does not cover (elevation above MHW) (See general remarks)
2	Uncov 2 ft; Uncov 2 ft; (2); (2)	Rock which covers and uncovers, with height in feet above chart (sounding) datum
3		Rock awash at the level of chart (sounding) datum
		When rock of O-2 or O-3 is considered a danger to navigation
4		Sunken rock with less than 6 feet of water over it (Same as O-26)
5		Sunken rock with between 6 and 33 ft. of water over it (Same as O-26)
5a	5 Rk	Shoal sounding on isolated rock (replaces symbol)
6		Sunken rock with more than 66 feet of water over it (Same as O-26)
6a	21 Rk; 21 Wk; 21 Obstr	Sunken danger with depth cleared by wire drag (in feet or fathoms)
7	Reef	Reef of unknown extent
8	Sub Vol	Submarine volcano
9	Discol Water	Discolored water
10	Coral; Co; Co; Co	Coral reef, detached (uncovers at sounding datum)
	Co 3; Reef Line	Coral or Rocky reef, covered at sounding datum (See A-lld, llg)
11		Wreck showing any portion of hull or superstructure above sounding datum
12	Masts	Wreck with only masts visible above sounding datum
13		Old symbols for wrecks
13a		Wreck always partially submerged
14		Sunken wreck which may be dangerous to surface navigation (See O-6a)
15	5½ Wk	Wreck over which depth is known
16		Sunken wreck, not dangerous to surface navigation
17	Foul	Foul ground
18	Tide Rips	Overfalls or Tide rips (Symbol used only in small areas)
19	Eddies	Eddies (Symbol used only in small areas)
20	Kelp	Kelp, Seaweed (Symbol used only in small areas)
21	Bk	Bank
22	Shl	Shoal
23	Rf	Reef (See A-lld, llg; O-10)
23a		Ridge
24	Le	Ledge
25		Breakers (See A-12)
26	+	Sunken rock (depth unknown)
		When rock is considered a danger to navigation
27	5½ Obstr	Obstruction
28		Wreck (See O-11 to 16)
29	Wreckage; Wks	Wreckage
29a		Wreck remains (dangerous only for anchoring)
30	Subm piles	Submerged piling (See H-9, L-59)
30a	Snags; Stumps	Snags; Submerged stumps (See L-59)
31		Lesser depth, possible
32	Uncov	Dries (See A-10; O-2, 10)
33	Cov	Covers (See O-2, 10)
34	Uncov	Uncovers (See A-10; O-2, 10)
	3 Rep (1958)	Reported (with date)
35	Eagle Rk (rep 1958)	Reported (with name and date)
36	Discol	Discolored (See O-9)
37		Isolated danger
38		Limiting danger line
39	rky	Limit of rocky area
41	PA	Position approximate
42	PD	Position doubtful
43	ED	Existence doubtful
44	P Pos	Position
45	D	Doubtful
(Oa)	Subm Crib; Crib (above water)	Crib
(Ob)	Platform (lighted) HORN	Offshore platform (unnamed)
(Oc)	Hazel (lighted) HORN	Offshore platform (named)

P. Various Limits, etc.

1		Leading line; Range line
2		Transit
3		In line with
4		Limit of sector
5		Channel, Course, Track recommended (marked by buoys or beacons)(See P-21)
(Pa)		Alternate course
6		Leader cable
7		Submarine cable (power, telegraph, telephone, etc.)
7a	Cable Area	Submarine cable area
8	Pipeline	Submarine pipeline
8a	Pipeline Area	Submarine pipeline area
9		Maritime limit in general
9a	RESTRICTED AREA	Limit of restricted area
10		Limit of fishing zone (fish trap areas)
11		Limit of dumping ground, spoil ground (See P-9, G-13)
12		Ancnorage limit
13		Limit of airport (See I-23, 24)
14		Limit of sovereignty (Territorial waters)
15		Customs boundary
16		International boundary (also State boundary)
17		Stream limit
18		Ice limit
19		Limit of tide
20		Limit of navigation
21		Course recommended (not marked by buoys or beacons)(See P-5)
22		District or province limit
23		Reservation line
24	COURSE 053°00′ TRUE MARKERS MARKERS	Measured distance
25	PROHIBITED AREA	Prohibited area (See G-12)

Q. Soundings

1	SD		Doubtful sounding
2	65		No bottom found
3			Out of position
4			Least depth in narrow channel
5	30 FEET APR 1958		Dredged channel (with controlling depth indicated)
6	24 FEET MAY 1958		Dredged area
7			Swept channel (See Q-9)
8	6		Drying or uncovering height in feet above chart (sounding) datum
9	17 119		Swept area, not adequately sounded (shown by green tint)
9a	29 23 3 22 8 30 8 21 7		Swept area adequately sounded (swept by wire drag to depth indicated)
10			Hair-line depths
10a	8_2	19	Figures for ordinary soundings
11	8_2	19	Soundings taken from foreign charts
12	8_2	19	Soundings taken from older surveys or smaller scale charts
13	8_2	19	Soundings taken by echo
14	8_2	19	Sloping figures (See Q-12)
15	8_2	19	Upright figures (See Q-10a)
16	(25)	(2)	Bracketed figures (See O-1, 2)
17	6		Underlined sounding figures (See Q-8)
18	3_2	6_1	Soundings expressed in fathoms and feet
(Qa)	6 5 2ft		Stream

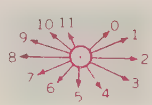
R. Depth Contours and Tints (see General Remarks)

Feet	Fathoms		Feet	Fathoms	
0	0		300	50	
6	1		600	100	
12	2		1,200	200	
18	3		1,800	300	
24	4		2,400	400	
30	5		3,000	500	
36	6		6,000	1,000	
60	10		12,000	2,000	
120	20		18,000	3,000	
180	30		Or continuous lines, with values		5 (blue or black) 100
240	40				

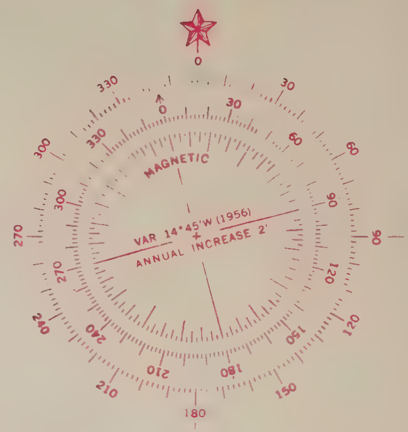
S. Quality of the Bottom

1		Ground	25	Ms	Mussels	50	spk	Speckled
2	S	Sand	26	Spg	Sponge	51	gty	Gritty
3	M	Mud; Muddy	27		Kelp	52		Decayed
4	Oz	Ooze	28	Wd	Seaweed	53	fly	Flinty
5	Ml	Marl		Grs	Grass	54	glac	Glacial
6	Cl	Clay	29		Seatangle	55		Tenacious
7	G	Gravel				56	wh	White
8	Sn	Shingle	31		Spicules	57	bk	Black
9	P	Pebbles	32	Fr	Foraminifera	58	vi	Violet
10	St	Stones	33	Gl	Globigerina	59	bu	Blue
11	Rk; rky	Rock; Rocky	34	Di	Diatoms	60	gn	Green
11a	Blds	Boulders	35	Rd	Radiolaria	61	yl	Yellow
12	Ck	Chalk	36	Pt	Pteropods	62	or	Orange
12a	Ca	Calcareous	37	Po	Polyzoa	63	rd	Red
13	Qz	Quartz	38		Cirripeda	64	br	Brown
13a		Schist	38a		Fucus	65	ch	Chocolate
14	Co	Coral	38b		Mattes	66	gy	Gray
(Sa)	Co Hd	Coral head	39	fne	Fine	67	lt	Light
15	Mds	Madrepores	40	crs	Coarse	68	dk	Dark
16	Vol	Volcanic	41	sft	Soft			
(Sb)	Vol Ash	Volcanic ash	42	hrd	Hard	70		Varied
17	La	Lava	43	stf	Stiff	71		Uneven
18	Pm	Pumice	44	sml	Small			
19	T	Tufa	45	lrg	Large			
20	Sc	Scoriae	46	stk	Sticky			
21	Cn	Cinders	47	brk	Broken			
22	Mn	Manganese	47a	grd	Ground	76		Fresh water springs in sea-bed
23	Sh	Shells	48		Rotten			
24	Oys	Oysters	49		Streaky			

T. Tides and Currents

1	HW	High water
1a	HHW	Higher high water
2	LW	Low water
(Ta)	LWD	Low water datum
2a	LLW	Lower low water
3	MTL	Mean tide level
4	MSL	Mean sea level
4a		Elevation of mean sea level above chart (sounding) datum
5		Chart datum (datum for sounding reduction)
6	Sp	Spring tide
7	Np	Neap tide
8	MHWS	Mean high water springs
8a	MHWN	Mean high water neaps
8b	MHHW	Mean higher high water
(Tb)	MHW	Mean high water
9	MLWS	Mean low water springs
9a	MLWN	Mean low water neaps
9b	MLLW	Mean lower low water
(Tc)	MLW	Mean low water
10	ISLW	Indian spring low water
11		High water full and change (vulgar establishment of the port)
12		Low water full and change
13		Mean establishment of the port
13a		Establishment of the port
14		Unit of height
15		Equinoctial
16		Quarter; Quadrature
17	Str.	Stream
18	2 kn	Current, general, with rate
19	2 kn	Flood stream (current) with rate
20	2 kn	Ebb stream (current) with rate
21	○ Tide gauge	Tide gauge; Tidepole; Automatic tide gauge
23	vel.	Velocity; Rate
24	kn.	Knots
25	ht.	Height
26		Tide
27		New moon
28		Full moon
29		Ordinary
30		Syzygy
31	fl.	Flood
32		Ebb
33	A B	Tidal stream diagram
34		Place for which tabulated tidal stream data are given
35		Range (of tide)
36		Phase lag
(Td)		Current diagram, with explanatory note

U. Compass



Compass Rose

The outer circle is in degrees with zero at true north. The inner circles are in points and degrees with the arrow indicating magnetic north.

1	N	North
2	E	East
3	S	South
4	W	West
5	NE	Northeast
6	SE	Southeast
7	SW	Southwest
8	NW	Northwest
9	N	Northern
10	E	Eastern
11	S	Southern
12	W	Western
21	brg	Bearing
22		True
23	mag	Magnetic
24	var	Variation
25		Annual change
25a		Annual change nil
26		Abnormal variation; Magnetic attraction
27	deg	Degrees (See E-20)
28	dev	Deviation

APPENDIX L

UNITS OF DEPTH MEASUREMENT ON CHARTS OF VARIOUS NATIONS

Nation	Unit of depth measurement	Equivalent in United States units	
		Feet	Fathoms
Argentina	Braza	6.000	1.000
Australia	Fathom	6.000	1.000
Belgium	Metre	3.281	0.547
Brazil	Metro	3.281	0.547
Canada	Fathom	6.000	1.000
Chile	Metro	3.281	0.547
Denmark	Favn	6.176	1.029
	Meter	3.281	0.547
Finland	Metre	3.281	0.547
France	Metre	3.281	0.547
Germany	Meter	3.281	0.547
Great Britain	Fathom	6.000	1.000
Greece	Metre (Metpa)	3.281	0.547
Italy	Metre	3.281	0.547
Japan	Metre	3.281	0.547
Netherlands	Vadem	5.905	0.984
	Meter	3.281	0.547
Norway	Favn	6.176	1.029
	Meter	3.281	0.547
Portugal	Metro	3.281	0.547
Russia (USSR)	Sazhen'	6.000	1.000
	Metre	3.281	0.547
Thailand	Metre	3.281	0.547
Spain	Metro	3.281	0.547
Sweden	Famn	5.844	0.974
	Meter	3.281	0.547
Turkey	Fathom (Kulac)	6.000	1.000
Uruguay	Metro	3.281	0.547
Yugoslavia	Metar	3.281	0.547

APPENDIX M

TIDAL DATUMS IN USE IN VARIOUS AREAS

Area	Datum	Area	Datum
Admiralty Islands	Lowest normal LW	Egypt (Mediterranean)	Mean LW springs
Alaska	Mean lower LW	El Salvador	Mean LW springs
Algeria	Lowest LW	Estonia	Mean sea level
Angola	Lowest normal LW	Ethiopia	Indian spring LW*
Argentina	Lowest normal LW	Finland	Mean sea level
Australia	Lowest normal LW**	France	Lowest LW
Azores	Lowest normal LW	French Guiana	Lowest LW
Bahama Islands	Mean lower LW springs	French Somaliland	Mean LW springs*
Belgium	Mean lower LW springs	Gabon	Lowest LW
Bermuda	Mean lower LW springs	Gambia	Mean lower LW springs
Bismarck Archipelago	Lowest normal LW	Germany (Baltic)	Mean sea level
Brazil	Indian spring LW*	Germany (North Sea)	Mean LW springs
British Guiana	Lowest normal LW	Ghana	Mean lower LW springs
British Honduras	Mean LW springs	Gilbert Islands	Mean LW springs
Bulgaria	Mean sea level	Great Britain	Mean LW springs*
Burma	Lowest normal LW	Greece	Mean LW springs*
Cambodia	Lowest LW	Greenland	Mean LW springs
Cameroon	Lowest LW	Guadeloupe	Lowest LW
Canada	Lowest normal LW	Guam	Mean lower LW
Canal Zone (Atlantic)	Mean LW	Guatemala	Mean LW springs
Canal Zone (Pacific)	Mean LW springs	Guinea	Lowest LW
Canary Islands	Lowest normal LW	Haiti	Mean LW
Caroline Islands	Lowest normal LW	Hawaiian Islands	Mean lower LW
Chile	Lowest normal LW	Honduras (Atlantic)	Mean lower LW
China	Lowest normal LW	Honduras (Pacific)	Mean LW springs
Colombia (Atlantic)	Mean LW	Iceland	Mean LW springs
Colombia (Pacific)	Mean LW springs	India	Indian spring LW*
Congo	Lowest LW	Indonesia	Lowest normal LW
Costa Rica (Atlantic)	Mean LW	Iran	Indian spring LW
Costa Rica (Pacific)	Mean LW springs	Iraq	Indian spring LW
Cuba	Mean LW	Israel	Mean LW springs
Denmark (Baltic)	Mean sea level	Italy	Mean LW springs
Denmark (North Sea)	Mean LW springs	Ivory Coast	Lowest LW
Dominican Republic	Mean LW	Jamaica	Mean lower LW springs
Ecuador	Mean LW springs	Japan	Indian spring LW
Egypt (Red Sea)	Indian spring LW*	Kenya	Indian spring LW

*The chart datum is somewhat lower than the datum indicated.

**A chart datum approximating mean LW springs or Indian spring LW is used for a number of places on the north and northwest coasts of Australia.

APPENDIX M

TIDAL DATUMS IN USE IN VARIOUS AREAS

Area	Datum	Area	Datum
Korea	Indian spring LW	Saudi Arabia	Indian spring LW*
Latvia	Mean sea level	Senegal	Lowest LW
Liberia	Mean lower LW springs	Sierra Leone	Mean lower LW springs
Libya	Mean LW springs	Solomon Islands	Lowest normal LW
Lithuania	Mean sea level	Somali Republic	Mean LW springs*
Loyalty Islands	Lowest normal LW	South Africa	Mean LW springs
Madagascar	Lowest LW	South-West Africa	Mean LW springs*
Madeira Islands	Lowest normal LW	Spain	Lowest normal LW
Malaya	Lowest normal LW	Spanish Sahara	Lowest normal LW
Mariana Islands	Lowest normal LW**	Sudan	Indian spring LW*
Marshall Islands	Lowest normal LW	Surinam	Mean lower LW springs
Martinique	Lowest LW	Sweden	Mean sea level
Mauritania	Lowest LW	Syria	Mean LW springs*
Mexico (part of Atlantic)	Mean LW	Tanganyika	Mean LW springs
Mexico (Pacific and part of Atlantic)	Mean lower LW	Thailand	Lowest normal LW
Morocco	Lowest LW	Tinian	Mean lower LW
Mozambique	Lowest LW	Togo	Lowest LW
Netherlands	Mean lower LW springs	Trinidad	Mean lower LW springs
New Caledonia	Lowest LW	Tuamoto Archipelago	Mean LW springs
New Hebrides	Lowest normal LW	Tunisia	Lowest LW
New Zealand	Lowest normal LW	Turkey (Black Sea)	Mean sea level
Nicaragua	Mean LW springs	Turkey (Mediterranean)	Mean LW springs
Nigeria	Mean LW springs	United Arab Republic (Egypt) (Mediterranean)	Mean LW springs
Norway	Equatorial spring LW	United Arab Republic (Egypt) (Red Sea)	Indian spring LW*
Pakistan	Lowest normal LW	USA (Atlantic)	Mean LW
Panama (Atlantic)	Mean LW	USA (Pacific)	Mean lower LW
Panama (Pacific)	Mean LW springs	USSR (Baltic and Black Sea)	Mean sea level
Papua	Lowest LW	USSR (Arctic and Pacific)	Lowest normal LW
Peru	Mean LW springs	Uruguay	Lowest normal LW
Philippines	Mean lower LW	Venezuela	Mean LW springs
Poland	Mean sea level	Vietnam	Lowest LW
Portugal	Lowest normal LW	Virgin Islands	Mean LW
Portuguese Guinea	Lowest normal LW	Yugoslavia	Mean lower LW springs
Puerto Rico	Mean LW		
Rio Muni	Lowest normal LW		
Rumania	Mean sea level		
Saipan	Mean lower LW		
Samoa	Mean LW springs		

*The chart datum is somewhat lower than the datum indicated.
**At Guam, Saipan, and Tinian the chart datum is mean lower LW.

APPENDIX N

SOURCES OF CHARTS AND PUBLICATIONS

Certain types of charts and publications, listed below, can be purchased from the places indicated. Many of the publications listed are also available from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402.

Orders for charts or publications, when addressed to Government agencies, should be accompanied by a check or post office money order made payable to the Treasurer of the United States. Postage stamps are not accepted, and cash is sent at the sender's risk of loss.

Government agencies make no charge for postage to addresses in the United States and possessions, and none to Canada, Cuba, Mexico, and Panama if the total weight of the shipment does not exceed four pounds. In all other cases, postage is required at the rates for printed matter. Remittance should accompany the order.

Orders for charts and publications should be as specific as possible, citing the numbers assigned by the publishing agency as identification of the items desired. To facilitate selection, the U.S. Navy Hydrographic Office, Washington, D.C. 20390 and U.S. Coast and Geodetic Survey, Washington, D.C. 20230, distribute free of charge, catalogs of salable material. More detailed information on U.S. Navy Hydrographic Office nautical charts and publications is given in Pub. No. 1–N, Introduction Part I. Aeronautical charts and publications are listed in the *Catalog of U.S. Navy Aeronautical Charts and Related Publications*, the *Coast and Geodetic Survey Catalog of Aeronautical Charts and Related Publications* and the *DOD Catalog of Aeronautical Charts and Flight Information Publications*. The DOD catalog is available only to military users.

Nautical Charts

Coasts of the United States and its territories and possessions.	U.S. Coast and Geodetic Survey and sales agents.
Mississippi River from the Head of Passes to Cairo, Ill.	Mississippi River Commission, Vicksburg, Miss.
Illinois waterway system (Great Lakes to Gulf of Mexico).	District Engineer, Chicago District, Chicago, Ill.
Various United States rivers.	District Engineer Offices.
Great Lakes, Lake Champlain, and the St. Lawrence River above St. Regis and Cornwall, Canada.	U.S. Lake Survey, Detroit, Mich., and District Engineer, Buffalo District, Buffalo, N.Y.
New York State canals.	U.S. Lake Survey, Detroit, Mich.; Superintendent of Public Works, Albany, N.Y.; and District Engineer, Buffalo District, Buffalo, N.Y.
Coasts of foreign countries.	U.S. Navy Hydrographic Office sales agents.

Oceanographic Charts and Publications

Tide and tidal current tables.	U.S. Coast and Geodetic Survey and sales agents.
Tidal current charts, certain United States harbors.	U.S. Coast and Geodetic Survey and sales agents.
Current charts of the oceans.	U.S. Navy Hydrographic Office sales agents.
Pilot charts.	U.S. Navy Hydrographic Office sales agents.
Bottom sediment charts.	U.S. Navy Hydrographic Office sales agents.
Surface temperature charts.	U.S. Navy Hydrographic Office sales agents.
Sea and swell charts.	U.S. Navy Hydrographic Office sales agents.
Water temperature and density tables.	U.S. Coast and Geodetic Survey and sales agents.
Oceanographic Atlas of the Polar Seas (Pub. No. 705).	U.S. Navy Hydrographic Office sales agents.
Sonic Soundings (H.O. Pub. No. 606–b).	U.S. Navy Hydrographic Office sales agents.
Bathythermograph Observations (H.O. Pub. No. 606–c).	U.S. Navy Hydrographic Office sales agents.
Ice Observations (H.O. Pub. No. 606–d).	U.S. Navy Hydrographic Office sales agents.
Sea and Swell Observations (H.O. Pub. No. 606–e).	U.S. Navy Hydrographic Office sales agents.
Miscellaneous oceanographic publications.	U.S. Navy Hydrographic Office sales agents and U.S. Coast and Geodetic Survey and sales agents.

Electronic Navigation

Loran charts.	U.S. Navy Hydrographic Office sales agents; U.S. Coast and Geodetic Survey and sales agents; and U.S. Air Force Aeronautical Chart and Information Center.
Loran tables (H.O. Pub. No. 221, various rates).	U.S. Navy Hydrographic Office sales agents.
Radio Navigational Aids (H.O. Pub. No. 117).	U.S. Navy Hydrographic Office sales agents.
Radio Weather Aids (H.O. Pub. No. 118).	U.S. Navy Hydrographic Office sales agents.
Weather Station Index (H.O. Pub. No. 119).	U.S. Navy Hydrographic Office sales agents.
Radio circulars giving schedules, frequencies, and data included in weather broadcasts.	Chief, U.S. Weather Bureau. (Also in H.O. Pub. No. 117).
International Code of Signals, Vol. II, radio (H.O. Pub. No. 104).	U.S. Navy Hydrographic Office sales agents.
Federal Communications Commission Rules and Regulations, Vol. IV, July 1964.	Superintendent of Documents.
Communications Act of 1934, Revised 1960.	Superintendent of Documents.
International Convention for the Safety of Life at Sea, 1960.	Intergovernmental Maritime Consultative Organization, London, England.
International Publications:	International Telecommunication Union, Geneva, Switzerland.
List of Frequencies.	
List of Coast Stations.	
List of Ship Stations.	
List of Broadcasting Stations.	
List of Radio Determination and Special Service Stations.	
List of Call Signs of Stations Used by the Maritime Mobile Service.	
List of Fixed Stations Operating International Circuits.	
Radio Aids to Maritime Navigation and Hydrography. (IHB Special Pub. No. 39).	International Hydrographic Bureau, Monaco.

Navigational Publications

Coast pilots (sailing directions), coasts of the United States and its territories and possessions.	U.S. Coast and Geodetic Survey and sales agents.
Sailing directions (coast pilots), foreign coasts.	U.S. Navy Hydrographic Office sales agents.
Light lists, United States waters.	Published by U.S. Coast Guard, distributed by Superintendent of Documents and sales agents.
Light lists, foreign coasts.	U.S. Navy Hydrographic Office sales agents.
Navigational tables:	U.S. Navy Hydrographic Office sales agents.
Table of Distances Between Ports (H.O. Pub. No. 151).	
Tables of Computed Altitude and Azimuth (H.O. Pub. No. 214, nine vols.).	
Sight Reduction Tables for Air Navigation (H.O. Pub. No. 249, three vols.).	
Various sight reduction tables (H.O. Pubs. Nos. 208, 211, 218).	
Azimuths of the Sun (H.O. Pub. No. 260).	
Azimuths of Celestial Bodies (H.O. Pub. No. 261).	
Distances Between United States Ports.	U.S. Coast and Geodetic Survey and sales agents.
Almanacs:	Published by U.S. Naval Observatory, distributed by Superintendent of Documents and sales agents.
The Air Almanac.	
The American Ephemeris and Nautical Almanac.	
The Nautical Almanac.	

Periodical Publications

Notice to Mariners.	U.S. Navy Hydrographic Office and branches.
Daily Memorandum.	U.S. Navy Hydrographic Office and branches.

Miscellaneous

Isomagnetic charts.	U.S. Navy Hydrographic Office sales agents.
Magnetic variation charts of the United States, Caribbean, and Alaska.	U.S. Coast and Geodetic Survey and sales agents.
Great-circle charts.	U.S. Navy Hydrographic Office sales agents.
Charts of polar regions.	U.S. Navy Hydrographic Office sales agents and U.S. Air Force Aeronautical Chart and Information Center.
Aeronautical charts, United States.	U.S. Coast and Geodetic Survey and sales agents.
Aeronautical charts, world coverage.	U.S. Hydrographic Office sales agents and U.S. Air Force Aeronautical Chart and Information Center.
Aeronautical publications.	U.S. Navy Hydrographic Office sales agents and Federal Aviation Agency.
Plotting charts and plotting sheets.	U.S. Navy Hydrographic Office sales agents.
Special charts.	U.S. Navy Hydrographic Office sales agents and U.S. Coast and Geodetic Survey and sales agents.
International Code of Signals, Vol. I, visual (H.O. Pub. No. 103).	U.S. Navy Hydrographic Office sales agents.
Merchant Marine House Flags and Stack Insignia (H.O. Pub. No. 100).	U.S. Navy Hydrographic Office sales agents.
Navigational Observations (H.O. Pub. No. 606–a).	U.S. Navy Hydrographic Office sales agents.
Shipboard Wind Plotter.	U.S. Weather Bureau.
Weather maps and reports.	U.S. Weather Bureau.
Mariners Weather Log.	Published by U.S. Weather Bureau, distributed by Superintendent of Documents.
World Port Index (H.O. Pub. No. 150).	U.S. Navy Hydrographic Office sales agents.
Eskimo Place Names and Aids to Conversation (H.O. Misc. 10578).	U.S. Navy Hydrographic Office sales agents.
Star Finder and Identifier (H.O. 2102–D).	U.S. Navy Hydrographic Office sales agents.
Rules of the Road—International—Inland (CG–169).	Published by U.S. Coast Guard, distributed by Superintendent of Documents and sales agents.
Rules of the Road—Great Lakes (CG–172).	Published by U.S. Coast Guard, distributed by Superintendent of Documents and sales agents.
Rules of the Road—Western Rivers (CG–184).	Published by U.S. Coast Guard, distributed by Superintendent of Documents and sales agents.
Systems of Maritime Buoyage and Beaconage Adopted by Various Countries. (IHB Special Pub. No. 38).	International Hydrographic Bureau, Monaco.
Aids to Marine Navigation of the United States (CG–193).	Published by U.S. Coast Guard, distributed by Superintendent of Documents and sales agents.
Radar Plotting Manual (H.O. Pub. No. 257).	U.S. Navy Hydrographic Office sales agents.
Maneuvering Board Manual (H.O. Pub. No. 217).	U.S. Navy Hydrographic Office sales agents.
Navigation Dictionary (H.O. Pub. No. 220).	U.S. Navy Hydrographic Office sales agents.
Handbook of Magnetic Compass Adjustment and Compensation (H.O. Pub. No. 226).	U.S. Navy Hydrographic Office sales agents.
Laws Governing Marine Inspection (CG–227).	Published by U.S. Coast Guard, distributed by Superintendent of Documents and sales agents.
Instrument instruction pamphlets.	Manufacturer of equipment.
Great Lakes Pilot.	U.S. Lake Survey, Detroit, Michigan.

APPENDIX O

MATHEMATICS

Arithmetic

O1. Definitions.—**Arithmetic** is that branch of mathematics dealing with computation by numbers. The principal processes involved are addition, subtraction, multiplication, and division. A number consisting of a single symbol (1, 2, 3, etc.) is a **digit.** Any number that can be stated or indicated, however large or small, is called a **finite** number; one too large to be stated or indicated is called an **infinite** number; and one too small to be stated or indicated is called an **infinitesimal** number.

The **sign** of a number is the indication of whether it is positive (+) or negative (−). This may sometimes be indicated in another way. Thus, latitude is usually indicated as *north* (N) or *south* (S), but if north is considered positive, south is then negative with respect to north. In navigation, the north or south designation of latitude and declination is often called the "name" of the latitude or declination. A **positive number** is one having a positive sign (+); a **negative number** is one having a negative sign (−). The **absolute value** of a number is that number without regard to sign. Thus, the absolute value of both (+) 8 and (−) 8 is 8. Generally, a number without a sign can be considered positive.

O2. Expressing numbers.—In navigation, fractions are usually expressed as decimals. Thus, $\frac{1}{4}$ is expressed as 0.25 and $\frac{1}{3}$ as 0.33. To determine the decimal equivalent of a fraction, divide the **numerator** (the number above the line) by the **denominator** (the number below the line). When a decimal is less than 1, as in the examples above, it is good practice to show the zero at the left of the decimal point (0.25, not .25).

A number should not be expressed to a greater precision than justified. The precision of a decimal is indicated by the number of digits shown to the right of the decimal point. Thus, the expression "14 miles" indicates a precision to the nearest whole mile, or any value between 13.5 and 14.5 miles. The expression "14.0 miles" indicates a precision of a tenth of a mile, or any value between 13.95 and 14.05 miles.

In a number without a decimal there is sometimes doubt as to the degree of precision indicated. For example, the number 186,000 may indicate a precision to three, four, five, or six places. This ambiguity is sometimes avoided by expressing numbers as powers of 10 (art. O8). Thus, 18.6×10^4 ($18.6 \times 10{,}000$) indicates a precision to the nearest thousand (three places), 18.60×10^4 to the nearest hundred (four places), 18.600×10^4 to the nearest ten (five places), and 18.6000×10^4 to the nearest unit (six places). The position of the decimal is not important if the correct power of 10 is given. For example, 18.6×10^4 is the same as 1.86×10^5, 186×10^3, etc.

The small number above and to the right of 10 (the **exponent**) indicates the number of places the decimal point is to be moved to the *right.* If the exponent is negative, it indicates a reciprocal, and the decimal point is moved to the *left.* Thus, $1.86 \times 10^{-6} = 0.00000186$. This system is sometimes used to avoid long numbers.

Another way of indicating degree of precision is to state the number of **significant digits.** These are the digits in a number, excluding zeros at the left and sometimes those at the right. Thus, 1,325, 1,001, 1.408, 0.00005926, 625.0, and 0.04000 have four significant digits each. But in the number 312,600 there may be four, five, or six

significant digits. Any doubt may be removed by expressing the number times a power of 10, as explained above.

If there are no more significant digits, regardless of how far a computation is carried, this may be indicated by use of the word "exactly." Thus, $12 \div 4 = 3$ exactly, and one nautical mile=1,852 meters exactly; but $12 \div 7 = 1.7$ approximately, the word "approximately" indicating that additional decimal places might be computed. Another way of indicating an approximate relationship is by placing a positive or negative sign after the number. Thus, $12 \div 7 = 1.7+$, and $11 \div 7 = 1.6-$. This system has the advantage of showing whether the approximation is too great or too small.

In any arithmetical computation the answer is no more accurate than the least precise value used. Thus, if it is desired to add 16.4 and 1.88, the answer might be given as 18.28, but since the first term might be anything from 16.35 to 16.45, the answer is anything from 18.23 to 18.33. Hence, to retain the second decimal place in the answer is to give a false indication of accuracy, for the number 18.28 indicates a value between 18.275 and 18.285. However, additional places are sometimes retained until the end of a computation to avoid an accumulation of small errors due to rounding off (art. O4). In marine navigation it is customary to give most values to a precision of 0.1, even though some uncertainty may exist as to the accuracy of the last place. Examples are the dip and refraction corrections of sextant altitudes (arts. 1606, 1613).

In general, a value obtained by interpolation in a table should not be expressed to more decimal places than given in the table.

O3. Precision and accuracy.—The word "precision" as used above is not the same as "accuracy," although the two are sometimes confused. A quantity may be expressed to a greater precision than is justified by the accuracy of the information from which the quantity is derived. For instance, if a ship steams one mile in 3^m21^s, its speed is $60^m \div 3^m21^s = 60 \div 3.35 = 17.910447761194$ knots, approximately. The division can be carried to as many places as desired, but if the time is measured only to the nearest second, the speed is accurate only to one decimal place in this example, because an error of 0.5 second introduces an error of more than 0.05 knot in the speed. Hence, the additional places are meaningless and possibly misleading, unless more accurate time is available. In general, it is not good practice to state a quantity to greater precision than justified by its accuracy. However, in marine navigation the accuracy of information is often unknown, and it is customary to give positions to a precision of $0'.1$ of latitude and longitude, although they *may* not be accurate even to the nearest whole minute.

The **absolute precision** of a number is indicated by its number of decimal places; its **relative precision** by its number of significant digits. Although this is an indication of precision, it may also be a measure of accuracy, and the expressions **absolute accuracy** and **relative accuracy** used. However, the term "accuracy" should not be used when "precision" only is intended. Thus, the values 186,000 and 0.00000186 may each have three significant digits, or "be correct to three digits," although the first value may be accurate ("absolute accuracy") only to the nearest 1,000, and the second to the nearest 0.00000001. If the numbers are accurate to the number of significant digits shown, each has an error ("relative accuracy") of less than "one part in 186."

Unless all numbers are exact, doubt exists as to the accuracy of the last digit in a computation. Thus, $12.3+9.4+4.6=26.3$. But if the three terms to be added have been rounded off from 12.26, 9.38, and 4.57, the correct answer is 26.2, obtained by rounding off the answer of 26.21 found by retaining the second decimal place until the end. It is good practice to work with one more place than needed in the answer,

when the information is available. In computations involving a large number of terms, or if great accuracy is desired, it is sometimes advisable to retain two or more additional places until the end.

O4. Rounding off.—In rounding off numbers to the number of places desired, one should take the nearest value. Thus, the number 6.5049 is rounded to 6.505, 6.50, 6.5, or 7, depending upon the number of places desired. If the number to be rounded off ends in 5, the nearer *even* number is taken. Thus, 1.55 and 1.65 are both rounded to 1.6. Likewise, 12.750 is rounded to 12.8 if only one decimal place is desired. However, 12.749 is rounded to 12.7. That is, 12.749 is not first rounded to 12.75 and then to 12.8, but the entire number is rounded in one operation. When a number ends in 5, the computation can sometimes be carried to additional places to determine whether the correct value is more or less than 5.

O5. Reciprocals.—The reciprocal of a number is 1 divided by that number. The reciprocal of a fraction is obtained by interchanging the numerator and denominator. Thus, the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. A whole number may be considered a fraction with 1 as the denominator. Thus, 54 is the same as $\frac{54}{1}$, and its reciprocal is $\frac{1}{54}$. Division by a number produces the same result as multiplying by its reciprocal, or vice versa. Thus, $12 \div 2 = 12 \times \frac{1}{2} = 6$, and $12 \times 2 = 12 \div \frac{1}{2} = 24$.

O6. Addition.—When two or more numbers are to be added, it is generally most convenient to write them in a column, with the decimal points in line. Thus, if 31.2, 0.8874, and 168.14 are to be added, this may be indicated by means of the addition sign (+): $31.2 + 0.8874 + 168.14 = 200.2$. But the addition can be performed more conveniently by arranging the numbers as follows:

$$\begin{array}{r@{.}l} 31 & 2 \\ 0 & 8874 \\ 168 & 14 \\ \hline 200 & 2. \end{array}$$

The answer is given only to the first decimal place, because the answer is no more accurate than the least precise number among those to be added, as indicated previously. Often it is preferable to state all numbers in a problem to the same precision before starting the addition, although this may introduce a small error, as indicated in article O3:

$$\begin{array}{r@{.}l} 31 & 2 \\ 0 & 9 \\ 168 & 1 \\ \hline 200 & 2. \end{array}$$

If there are no decimals, the last digit to the right is aligned:

$$\begin{array}{r} 166 \\ 2 \\ 96{,}758 \\ \hline 96{,}926. \end{array}$$

Numbers to be added should be given to the same absolute accuracy, when available, to avoid a false impression of accuracy in the result. Consider the following:

$$\begin{array}{r} 186{,}000 \\ 71{,}832 \\ 9{,}614 \\ 728 \\ \hline 268{,}174. \end{array}$$

The answer would imply an accuracy to six places. If the first number given is accurate to only three places, or to the nearest 1,000, the answer is not more accurate, and hence the answer should be given as 268,000. Approximately the same answer would be obtained by rounding off at the start:

$$\begin{array}{r} 186{,}000 \\ 72{,}000 \\ 10{,}000 \\ \underline{1{,}000} \\ 269{,}000. \end{array}$$

If numbers are **added arithmetically,** their absolute values are added without regard to signs; but if they are **added algebraically,** due regard is given to signs. If two numbers to be added algebraically have the same sign, their absolute values are added and given their common sign. If two numbers to be added algebraically have unlike signs, the smaller absolute value is subtracted from the larger, and the sign of the value having the larger absolute value is given to the result. Thus, if $+8$ and -7 are added arithmetically, the answer is 15, but if they are added algebraically, the answer is $+1$.

An answer obtained by addition is called a **sum.**

O7. Subtraction is the inverse of addition. Stated differently, the *addition* of a *negative* number is the same as the *subtraction* of a *positive* number. That is, if a number is to be subtracted from another, the sign (+ or −) of the **subtrahend** (the number to be subtracted) is reversed and the result added algebraically to the **minuend** (the number from which the subtrahend is to be subtracted). Thus, $6-4=2$. This may be written $+6-(+4)=+2$, which yields the same result as $+6+(-4)$. For solution, larger numbers are often conveniently arranged in a column with decimal points in a vertical column, as in addition. Thus, $3{,}728.41-1{,}861.16$ may be written

$$\begin{array}{ll} (+)3{,}728.41 & \\ (+)\underline{1{,}861.16} & \text{(subtract)} \\ (+)1{,}867.25 & \end{array}$$

This is the same as

$$\begin{array}{ll} (+)3{,}728.41 & \\ (-)\underline{1{,}861.16} & \text{(add algebraically)} \\ (+)1{,}867.25 & \end{array}$$

The rule of sign reversal applies likewise to negative numbers. Thus, if -3 is to be *subtracted* from $+5$, this may be written $+5-(-3)=5+3=8$.

In the algebraic addition of two numbers of opposite sign (numerical subtraction), the smaller number is subtracted from the larger and the result is given the sign of the larger number. Thus, $+7-4=+3$, and $-7+4=-3$, which is the same as $+4-7=-3$.

In navigation, numbers to be numerically subtracted are usually marked (−), and those to be numerically added are marked (+) or the sign is not indicated. However, when a sign is part of a designation, and the reverse process is to be used, the word "reversed" (rev.) is written after the number. Thus, if GMT is known and ZT in the (+)5 zone is to be found (by subtraction), the problem may be written:

$$\begin{array}{lr} \text{GMT} & 1754 \\ \text{ZD} & (+)\underline{5} \ \text{(rev.)} \\ \text{ZT} & 1254 \end{array}$$

The symbol $\sim$ indicates that an absolute difference is required without regard to sign of the *answer*. Thus, $28\sim13=15$, and $13\sim28=15$. In both of these solutions 13 and 28 are positive and 15 is an absolute value without sign. If the signs or names of both numbers are the same, either positive or negative, the smaller is subtracted from the larger, but if they are of opposite sign or name, they are numerically added. Thus, $(+)16\sim(+)21=5$ and $(-)16\sim(-)21=5$, but $(+)16\sim(-)21=37$ and $(-)16\sim(+)21=37$. Similarly, the difference of latitude between 15° N and 20° N, or between 15° S and 20° S, is 5°, but the difference of latitude between 15° N and 20° S, or between 15° S and 20° N, is 35°. If motion from one latitude to another is involved, the difference may be given a sign to indicate the direction of travel, or the location of one place with respect to another. Thus, if B is 50 miles west of A, and C is 125 miles west of A, B and C are 75 miles apart regardless of the direction of travel. However, B is 75 miles *east* of C, and C is 75 miles *west* of B. When direction is indicated, an algebraic difference is given, rather than an absolute difference, and the symbol $\sim$ is not appropriate.

It is sometimes desirable to consider all addition and subtraction problems as addition, with negative signs (−) given before those numbers to be subtracted, so that there can be no question of which process is intended. The words "add" and "subtract" may be used instead of signs. In navigation, "names" (usually north, south, east, and west) are often used, and the relationship involved in a certain problem may need to be understood to determine whether to add or subtract. Thus, $LHA=GHA-\lambda$(west) and $LHA=GHA+\lambda$(east). This is the same as saying $LHA=GHA-\lambda$ if west longitude is considered positive, for in this case, $LHA=GHA-(-\lambda)$ or $LHA=GHA+\lambda$ in east longitude, the same as before.

If numbers are **subtracted arithmetically,** they are subtracted without regard to sign; but if they are **subtracted algebraically,** positive (+) numbers are *subtracted* and negative (−) numbers are *added.*

An answer obtained by subtraction is called a **difference.**

O8. Multiplication may be indicated by the multiplication sign (×), as $154\times28=4{,}312$. For solution, the problem is conveniently arranged thus:

$$\begin{array}{r} 154 \\ (\times)\ 28 \\ \hline 1232 \\ 308 \\ \hline 4312. \end{array}$$

Either number may be given first, but it is generally more convenient to perform the multiplication if the larger number is placed on top, as shown. In this problem, 154 is first multiplied by 8 and then by 2. The second answer is placed under the first, but set one **place** to the left, so that the right-hand digit is directly below the 2. These steps might be reversed, multiplication by 2 being performed first. This procedure is sometimes used in estimating.

When one number is placed below another for multiplication, as shown above, it is usually best to align the right-hand digits without regard for the position of the

decimal point. The number of decimal places in the answer is the sum of the decimal places in the **multiplicand** (the number to be multiplied) and the **multiplier** (the second number):

$$\begin{array}{r} 163.27 \\ (\times)\ 263.9 \\ \hline 146943 \\ 48981 \\ 97962 \\ 32654 \\ \hline 43086.953 \end{array}$$

However, when a number ends in one or more zeros, these may be ignored until the end and then added on to the number:

$$\begin{array}{r} 1924 \\ (\times)\ 1800 \\ \hline 15392 \\ 1924 \\ \hline 3463200. \end{array}$$

This is also true if both multiplicand and multiplier end in zeros:

$$\begin{array}{r} 1924000 \\ (\times)\ 1800 \\ \hline 15392 \\ 1924 \\ \hline 3463200000. \end{array}$$

When negative values are to be multiplied, the sign of the answer is positive if an *even* number of negative signs appear, and negative if there are an *odd* number. Thus, $2\times3=6$, $2\times(-3)=-6$, $-2\times3=-6$, $-2\times(-3)=(+)6$. Also, $2\times3\times8\times(-2)\times5=-480$, $2\times(-3)\times8\times(-2)\times5=480$, $2\times(-3)\times(-8)\times(-2)\times5=-480$, $2\times(-3)\times(-8)\times(-2)\times(-5)=480$, and $(-2)\times(-3)\times(-8)\times(-2)\times(-5)=-480$.

An answer obtained by multiplication is called a **product.** Any number multiplied by 1 is the number itself. Thus, $125\times1=125$. Any number multiplied by 0 is 0. Thus, $125\times0=0$ and $1\times0=0$.

To multiply a number by itself is to **square** the number. This may be indicated by the **exponent** 2 placed to the right of the number and above the line as a **superior.** Thus, 15×15 may be written 15^2. Similarly, $15\times15\times15=15^3$, and $15\times15\times15\times15=15^4$, etc. The exponent (2, 3, 4, etc.) indicates the **power** to which a number is to be **raised,** or how many times the number is to be used in multiplication. The expression 15^2 is usually read "15 squared," 15^3 is read "15 cubed" or "15 to the third power," 15^4 (or higher power) is read "15 to the fourth (or higher) power." The answer obtained by **raising to a power** is called the "square," "cube," etc., or the " . . . power" of the number. Thus, 225 is the "square of 15," 3,375 is the "cube of 15" or the "third power of 15," etc. The zero power of any number except zero (if zero is considered a number) is 1. The zero power of zero is zero. Thus, $15^0=1$ and $0^0=0$.

Parentheses may be used to eliminate doubt as to what part of an expression is to be raised to a power. Thus, -3^2 may mean either $-(3\times3)=-9$ or $-3\times-3=(+)9$. To remove the ambiguity, the expression may be written $-(3)^2$ if the first meaning is intended, and $(-3)^2$ if the second meaning is intended.

O9. Division is the inverse of multiplication. It may be indicated by the division sign ($\div$), as $376 \div 21 = 18$ approximately; or by placing the number to be divided, called the **dividend** (376), over the other number, called the **divisor** (21), as $\frac{376}{21} = 18$ approximately. The expression $\frac{376}{21}$ may be written 376/21 with the same meaning. Such a problem is conveniently arranged for solution as follows:

$$
\begin{array}{r|r}
 & 17 \\ \hline
21 & 376 \\
 & 21 \\ \cline{2-2}
 & 166 \\
 & 147 \\ \cline{2-2}
 & 19.
\end{array}
$$

Since the **remainder** is 19, or more than half of the divisor (21), the answer is 18 to the nearest whole number.

An answer obtained by division is called a **quotient.** Any number divided by 1 is the number itself. Thus, $65 \div 1 = 65$. A number cannot be divided by 0.

If the numbers involved are accurate only to the number of places given, the answer should not be carried to additional places. However, if the numbers are exact, the answer might be carried to as many decimal places as desired. Thus, $374 \div 21 = 17.809523809523809523809523809523 \ldots$ When a series of digits repeat themselves with the same remainder, as 809523 (with remainder 17) in the example given above, an exact answer will not be obtained regardless of the number of places to which the division is carried. The series of dots (. . .) indicates a **repeating decimal.** In a nonrepeating decimal, a plus sign (+) may be given to indicate a remainder, and a minus sign (−) to indicate that the last digit has been rounded to the next higher value. Thus, 18.68761 may be written 18.6876+ or 18.688−. If the last digit given is rounded off, the word "approximately" may be used instead of dots or a plus or minus sign.

If the divisor is a whole number, the decimal point in the quotient is directly above that of the dividend when the work form shown above is used. Thus, in the example given above, if the dividend had been 37.6 instead of 376, the quotient would have been 1.8 approximately. If the divisor is a decimal, both it and the dividend are multiplied by the power of 10 having an exponent equal to the number of decimal places in the divisor, and the division is then carried out as explained above. Thus, if there are two decimal places in the divisor, both divisor and dividend are multiplied by $10^2 = 100$. This is done by moving the decimal to the right until the divisor is a whole number. If necessary, zeros are added to the dividend. Thus, if 3.7 is to be divided by 2.11, both quantities are first multiplied by 10^2, and 370 is divided by 211. This is usually performed as follows:

$$
\begin{array}{r|r}
 & 1.75 \\ \hline
2_{\wedge}11 & 3_{\wedge}70.00 \\
 & 211 \\ \cline{2-2}
 & 1590 \\
 & 1477 \\ \cline{2-2}
 & 1130 \\
 & 1055 \\ \cline{2-2}
 & 75.
\end{array}
$$

If *both* the dividend and divisor are positive, or if *both* are negative, the quotient is positive; but if *either* is negative, the quotient is negative. Thus, $6 \div 3 = 2$, $(-6) \div (-3) = +2$, $(-6) \div 3 = -2$, and $6 \div (-3) = -2$.

The **square root** of a number is that number which, multiplied by itself, equals the given number. Thus, $15 \times 15 = 15^2 = 225$, and $\sqrt{225} = 225^{1/2} = 15$. Either the symbol $\sqrt{\ }$, called the **radical sign,** or the exponent ½ indicates square root. Also, $\sqrt[3]{\ }$, or ⅓ as an exponent, indicates **cube root.** Fourth, fifth, or any root is indicated similarly, using the appropriate number. Nearly any arithmetic book explains the process of extracting roots, but this process is most easily performed by table, logarithms (art. O12), or slide rule (art. O15). If no other means are available, it can be done by trial and error. The process of finding a root of a number is called **extracting a root.**

O10. Logarithms ("logs") provide an easy way to multiply, divide, raise numbers to powers, and extract roots. The logarithm of a number is the power to which a fixed number, called the base, must be raised to produce the value to which the logarithm corresponds. The base of **common logarithms,** (given in tables 32 and 33) is 10. Hence, since $10^{1.8} = 63$ approximately, 1.8 is the logarithm, approximately, of 63 to the base 10. In table 32 logarithms of numbers are given to five decimal **places.** This is sufficient for most purposes of the navigator. For greater precision, a table having additional places should be used. In general, the number of *significant digits* which are correct in an answer obtained by logarithms is the same as the number of *places* in the logarithms used.

A logarithm is composed of two parts. That part to the left of the decimal point is called the **characteristic.** That part to the right of the decimal point is called the **mantissa.** The principal advantage of using 10 as the base is that any given combination of digits has the same mantissa regardless of the position of the decimal point. Hence, only the mantissa is given in the main tabulation of table 32. Thus, the logarithm (mantissa) of 2,374 is given as 37548. This is correct for 2,374,000,000; 2,374; 23.74; 2.374; 0.2374; 0.000002374; or for any other position of the decimal point.

The position of the decimal point determines the characteristic, which is not affected by the actual digits involved. The characteristic of a whole number is one less than the number of digits. The characteristic of a **mixed decimal** (one greater than 1) is one less than the number of digits to the left of the decimal point. Thus, in the example given above, the characteristic of the logarithm of 2,374,000,000 is 9; that of 2,374 is 3; that of 23.74 is 1; and that of 2.374 is 0. The complete logarithms of these numbers are:

$$\begin{aligned}
\log 2{,}374{,}000{,}000 &= 9.37548 \\
\log 2{,}374 &= 3.37548 \\
\log 23.74 &= 1.37548 \\
\log 2.374 &= 0.37548.
\end{aligned}$$

Since the mantissa of the logarithm of any multiple of ten is zero, the main table starts with 1,000. This can be considered 100, 10, 1, etc. Since the mantissa of these logarithms is zero, the logarithms consist of the characteristic only, and are whole numbers. Hence, the logarithm of 1 is 0 (0.00000), that of 10 is 1 (1.00000), that of 100 is 2 (2.00000), that of 1,000 is 3 (3.00000), etc.

The characteristic of the logarithm of a number less than 1 is negative. However, it is usually more conveniently indicated in a positive form, as follows: the characteristic is found by subtracting the number of zeros immediately to the right of the decimal point from 9 (or 19, 29, etc.) and following this by −10 (or −20, −30, etc.). Thus, the

characteristic of the logarithm of 0.2374 is 9−10; that of 0.000002374 is 4−10; and that of 0.000000000002374 is 8−20. The complete logarithms of these numbers are:

$$\begin{aligned}\log 0.2374 &= 9.37548-10\\ \log 0.000002374 &= 4.37548-10\\ \log 0.000000000002374 &= 8.37548-20.\end{aligned}$$

When there is no question of the meaning, the −10 may be omitted. This is usually done when using logarithms of trigonometric functions, as shown in table 33. Thus, if there is no reasonable possibility of confusion, the logarithm of 0.2374 may be written 9.37548.

Occasionally, the logarithm of a number less than 1 is shown by giving the negative characteristic with a minus sign above it (since only the characteristic is negative, the mantissa being positive). Thus, the logarithms of the numbers given above might be shown thus:

$$\begin{aligned}\log 0.2374 &= \bar{1}.37548\\ \log 0.000002374 &= \bar{6}.37548\\ \log 0.000000000002374 &= \overline{12}.37548.\end{aligned}$$

In each case, the negative characteristic is one *more* than the number of zeros immediately to the right of the decimal point.

There is no real logarithm of 0, since there is no *finite* power to which *any* number can be raised to produce 0. As numbers approach 0, their logarithms approach negative infinity.

To find the number corresponding to a given logarithm, called finding the **antilogarithm** ("antilog"), enter the table with the mantissa of the given logarithm and determine the corresponding number, interpolating if necessary. Locate the position of the decimal point by means of the characteristic of the logarithm, in accordance with the rules given above.

O11. Multiplication by logarithms.—To *multiply* one number by another, *add* their logarithms and find the antilogarithm of the sum. Thus, to multiply 1,635.8 by 0.0362 by logarithms:

$$\begin{aligned}\log 1635.8 &= 3.21373\\ \log 0.0362 &= \underline{8.55871-10}\ \text{(add)}\\ \log 59.216 &= 11.77244-10 \text{ or } 1.77244.\end{aligned}$$

Thus, $1{,}635.8 \times 0.0362 = 59.216$. In navigation it is customary to use a slightly modified form, and to omit the −10 where there is no reasonable possibility of confusion, as follows:

$$\begin{array}{ll} 1635.8 & \log 3.21373\\ 0.0362 & \log \underline{8.55871}\\ 59.216 & \log 1.77244. \end{array}$$

To *raise a number to a power*, multiply the logarithm of that number by the power indicated, and find the antilogarithm of the product. Thus, to find 13.156^3 by logarithms, using the navigational form:

$$\begin{array}{lr} 13.156 & \log 1.11913\\ & \times\ \underline{3}\ \text{(multiply)}\\ 2277.2 & \log 3.35739. \end{array}$$

O12. Division by logarithms.—To *divide* one number by another, subtract the logarithm of the divisor from that of the dividend, and find the antilogarithm of the remainder. Thus, to find 0.4637÷28.03 by logarithms, using the navigational form:

$$\begin{array}{rll} 0.4637 & \log & 9.66624 \\ 28.03 & \log\ (-) & \underline{1.44762}\ \text{(subtract)} \\ 0.016543 & \log & 8.21862. \end{array}$$

It is sometimes necessary to modify the first logarithm before the subtraction can be made. This would occur in the example given above, for instance, if the divisor and dividend were reversed, so that the problem became 28.03÷0.4637. In this case 10−10 would be added to the logarithm of the dividend, becoming 11.44762−10:

$$\begin{array}{rll} 28.03 & \log & 11.44762-10 \\ 0.4637 & \log\ (-) & \underline{9.66624-10} \\ 60.448 & \log & 1.78138. \end{array}$$

One experienced in the use of logarithms usually carries this change mentally, without showing it in his work form:

$$\begin{array}{rll} 28.03 & \log & 1.44762 \\ 0.4637 & \log\ (-) & \underline{9.66624} \\ 60.448 & \log & 1.78138. \end{array}$$

Any number can be added to the characteristic as long as that same number is also subtracted. Conversely, any number can be subtracted from the characteristic as long as that same number is also added.

To *extract a root* of a number, divide the logarithm of that number by the root indicated, and find the antilogarithm of the quotient. Thus, to find $\sqrt{7}$ by logarithms:

$$\begin{array}{rl} 7 & \log\ \underline{0.84510}\ (\div 2) \\ 2.6458 & \log\ 0.42255. \end{array}$$

To divide a negative logarithm by the root indicated, first modify the logarithm so that the quotient will have a −10. Thus, to find $\sqrt[3]{0.7}$ by logarithms:

$$\begin{array}{rl} 0.7 & \log\ \underline{29.84510-30}\ (\div 3) \\ 0.88792 & \log\ \ 9.94837-10 \end{array}$$

or, carrying the −30 and −10 mentally,

$$\begin{array}{rl} 0.7 & \log\ \underline{29.84510}\ (\div 3) \\ 0.88792 & \log\ \ 9.94837. \end{array}$$

O13. Cologarithms.—The **cologarithm** ("colog") of a number is the value obtained by subtracting the logarithm of that number from zero, usually in the form 10−10. Thus, the logarithm of 18.615 is 1.26987. The cologarithm is:

$$\begin{array}{r} 10.00000-10 \\ (-)\underline{1.26987\qquad} \\ 8.73013-10. \end{array}$$

Similarly, the logarithm of 0.0018615 is 7.26987−10, and its cologarithm is:

$$\begin{array}{r} 10.00000-10 \\ (-)\underline{7.26987-10} \\ 2.73013. \end{array}$$

The *cologarithm* of a number is the *logarithm* of the reciprocal of that number. Thus, the cologarithm of 2 is the logarithm of ½. Since division by a number is the same as multiplication by its reciprocal, the use of cologarithms permits division problems to be converted to problems of multiplication, eliminating the need for subtraction of logarithms. This is particularly useful when both multiplication and division are involved in the same problem. Thus, to find $\frac{92.732\times 0.0137\times 724.3}{0.516\times 3941.1}$ by logarithms, one might *add* the logarithms of the three numbers in the numerator, and *subtract* the logarithms of the two numbers in the denominator. If cologarithms are used for the numbers in the denominator, all logarithmic values are added. Thus, the solution might be made as follows:

92.732		log 1.96723
0.0137		log 8.13672
724.3		log 2.85992
0.516	log 9.71265	colog 0.28735
3941.1	log 3.59562	colog 6.40438
0.45248		log 9.65560.

O14. Various kinds of logarithms.—As indicated above, **common logarithms** use 10 as the base. These are also called **Briggs' logarithms.** For some purposes, it is convenient to use 2.7182818 approximately (designated e) as the base for logarithms. These are called **natural logarithms** or **Naperian logarithms** ($\log_e$). Common logarithms are shown as $\log_{10}$ when the base might otherwise be in doubt.

Addition and subtraction logarithms are logarithms of the sum and difference of two numbers. They are used when the logarithms of two numbers to be added or subtracted are known, making it unnecessary to find the numbers themselves.

O15. Slide rule.—A **slide rule** is a convenient device for making logarithmic solutions mechanically. There are many types and sizes of slide rule, some designed for specific purposes. The most common form consists of an outer "body" or "frame" with grooves to permit a "slide" to be moved back and forth between the two outer parts, so that any graduation of a scale on the slide can be brought opposite any graduation of a scale on the body. A cursor called an "indicator" or "runner" is provided to assist in aligning the desired graduations. In a **circular slide rule** the "slide" is an inner disk surrounded by a larger one, both pivoted at their common center. The scales of a slide rule are *logarithmic*. That is, they increase proportionally to the logarithms of the numbers indicated, rather than to the numbers themselves. This permits addition and subtraction of logarithms by simply measuring off part of the length of the slide from a graduated point on the body, or vice versa. Two or three complete scales within the length of the rule may be provided for finding squares, cubes, square roots, and cube roots.

Full instructions for use of a slide rule are provided with each rule, and given in some mathematical texts. Properly used, a slide rule can provide quick answers to many of the problems of navigation. However, its precision is usually limited to from two to four significant digits, and should not be used if greater precision is desired. It is frequently used to provide a quick, approximate check on answers obtained by a more laborious method.

Great care should be used in placing the decimal point in an answer obtained by slide rule, as the correct location often is not immediately apparent. Its position is usually determined by making a very rough mental solution. Thus, 2.93×8.3 is *about* $3\times 8=24$. Hence, when the answer by slide rule is determined to be "243," it is known that the correct value is 24.3, not 2.43 or 243.

O16. Mental arithmetic.—Many of the problems of the navigator can be solved mentally. The following are a few examples.

If the speed is a number divisible into 60 a whole number of times, distance problems can be solved by a simple relationship. Thus, at 10 knots a ship steams 1 mile in $\frac{60}{10}=6$ minutes. At 12 knots it requires 5 minutes, at 15 knots 4 minutes, etc. As an example of the use of such a relationship, a vessel steaming at 12 knots travels 5.6 miles in 28 minutes, since $\frac{28}{5}=5\frac{3}{5}=5.6$, or 0.1 mile every half minute.

For relatively short distances, one nautical mile can be considered equal to 6,000 feet. Since one hour has 60 minutes, the speed in hundreds of feet per minute is equal to the speed in knots. Thus, a vessel steaming at 15 knots is moving at the rate of 1,500 feet per minute.

With respect to time, 6 minutes $=0.1$ hour, and 3 minutes $=0.05$ hour. Hence, a ship steaming at 13 knots travels 3.9 miles in 18 minutes (13×0.3), and 5.8 miles in 27 minutes (13×0.45).

In arc units, $6'=0\overset{\circ}{.}1$ and $6''=0\overset{'}{.}1$. This relationship is useful in rounding off values given in arc units. Thus, $17°23'44''=17°23\overset{'}{.}7$ to the nearest $0\overset{'}{.}1$, and $17\overset{\circ}{.}4$ to the nearest $0\overset{\circ}{.}1$. A thorough knowledge of the six multiplication table is valuable. The 15 multiplication table is also useful, since $15°=1^h$. Hence, $16^h=16\times15=240°$. This is particularly helpful in quick determination of zone description. Pencil and paper or a table should not be needed, for instance, to decide that a ship at sea in longitude $157°18\overset{'}{.}4$ W is in the (+)10 zone.

It is also helpful to remember that $1°=4^m$ and $1'=4^s$. In converting the LMT of sunset to ZT, for instance, a quick mental solution can be made without reference to a table. Since this correction is usually desired only to the nearest whole minute, it is necessary only to multiply the longitude difference in degrees (to the nearest quarter degree) by four.

Vectors

O17. Scalars and vector quantities.—A **scalar** is a quantity which has *magnitude* only; a **vector quantity** has both *magnitude* and *direction*. If a vessel is said to have a tank of 5,000 gallons capacity, the number 5,000 is a scalar. As used in this book, *speed* alone is considered a scalar, while *speed* and *direction* are considered to constitute *velocity*, a vector quantity. Thus, if a vessel is said to be steaming at 18 knots, without regard to direction, the number 18 is considered a scalar; but if the vessel is said to be steaming at 18 knots on course 157°, the combination of 18 knots and 157° constitutes a vector quantity. *Distance* and *direction* also constitute a vector quantity.

A *scalar* can be represented fully by a number. A *vector quantity* requires, in addition, an indication of direction. This is conveniently done graphically by means of a straight line, the length of which indicates the *magnitude*, and the direction of which indicates the *direction* of application of the magnitude. Such a line is called a **vector.** Since a straight line has two directions, reciprocals of each other, an arrowhead is placed along or at one end of a vector to indicate the direction represented, unless this is apparent or indicated in some other manner.

O18. Addition and subtraction of vectors.—Two vectors can be *added* by *starting* the second at the *termination* (rather than the origin) of the first. A common navigational use of vectors is the dead reckoning plot of a vessel. Refer to figure O18. If a ship starts at A and steams 18 miles on course 090° and then 12 miles on course 060°, it arrives by dead reckoning at C. The line AB is the vector for the first run, and

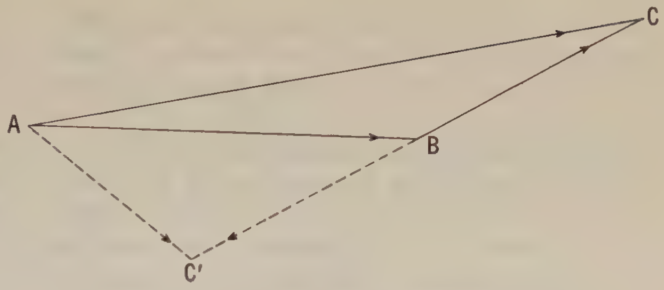


FIGURE O18. Addition and subtraction of vectors.

BC is the vector for the second. Point C is the position found by *adding* vectors AB and BC. The vector AC, in this case the *course and distance made good*, is the **resultant.** Its value, both in direction and amount, can be determined by measurement. Lines AB, BC, and AC are all **distance vectors. Velocity vectors** are used when determining the effect of, or allowing for, current (art. 807) or interconverting true and apparent wind (art. 3709).

The **reciprocal** of a vector has the same magnitude but opposite direction of the vector. To *subtract* a vector, *add* its reciprocal. This is indicated by the broken lines in figure O18, in which the vector BC' is drawn in the opposite direction to BC. In this case the resultant is AC'. Subtraction of vectors is involved in some current and wind problems.

Algebra

O19. Definitions.—**Algebra** is that branch of mathematics dealing with computation by letters and symbols. It permits the mathematical statement of certain relationships between variables. When numbers are substituted for the letters, algebra becomes arithmetic. Thus if $a=2b$, any value may be assigned to b, and a can be found by multiplying the assigned value by 2. Any statement of equality (as $a=2b$) is an **equation.** Any combination of numbers, letters, and symbols (as $2b$) is a **mathematical expression.**

O20. Symbols.—As in arithmetic, plus (+) and minus (−) signs are used, and with the same meaning. Multiplication (×) and division (÷) signs are seldom used. In algebra, $a \times b$ is usually written ab, or sometimes $a \cdot b$. For division $a \div b$ is usually written $\frac{a}{b}$ or a/b. The symbol $>$ means "greater than" and $<$ means "less than." Thus, $a>b$ means "a is greater than b," and $a \geqq b$ or $a \geq b$ means "a is equal to or greater than b."

The order of performing the operations indicated in an equation should be observed carefully. Consider the equation $a=b+cd-e/f$. If the equation is to be solved for a, the value cd should be determined by multiplication and e/f by division *before* the addition and subtraction, as each of these is to be considered a single quantity in making the addition and subtraction. Thus, if $cd=g$ and $e/f=h$, the formula can be written $a=b+g-h$.

If an equation including both multiplication and division between plus or minus signs is not carefully written, some doubt may arise as to which process to perform first. Thus, $a \div b \times c$ or $a/b \times c$ may be interpreted to mean either that a/b is to be multiplied by c or that a is to be divided by $b \times c$. Such an equation is better written ac/b if the first meaning is intended, or a/bc if the second meaning is intended. **Parentheses,** ( ), may be used for the same purpose or to indicate any group of quantities that is to be considered a single quantity. Thus, $a(b+c)$ is an indication that the sum of b and c is to be multiplied by a. Similarly, $a+(b-c)^2$ indicates that c is first to be subtracted from b, and then the result is to be squared and the value thus obtained added to a. When an expression within parentheses is part of a larger expression

which should also be in parentheses, **brackets,** [ ], are used in place of the outer parentheses. If yet another set is needed, **braces,** { }, are used.

A quantity written $\sqrt{3}\, ab$ is better written $ab\sqrt{3}$ to remove any suggestion that the square root of $3ab$ is to be found.

O21. Addition and subtraction.—A plus sign before an expression in parentheses means that each term retains its sign as given. Thus, $a+(b+c-d)$ is the same as $a+b+c-d$. A minus sign preceding the parentheses means that each sign within the parentheses is to be reversed. For example, $a-(b+c-d)=a-b-c+d$.

In any equation involving addition and subtraction, similar terms can be combined. Thus, $a+b+c+b-2c-d=a+2b-c-d$. Also, $a+3ab+a^2-b-ab=a+2ab+a^2-b$. That is, to be combined, the terms must be truly alike, for a cannot be combined with ab, or with a^2.

Equal quantities can be added to or subtracted from both members of an equation without disturbing the equality. Thus, if $a=b$, $a+2=b+2$, or $a+x=b+x$. If $x=y$, then $a+x=b+y$.

O22. Multiplication and division.—When an expression in parentheses is to be multiplied by a quantity outside the parentheses, each quantity separated by a plus or minus sign within the parentheses should be multiplied separately. Thus, $a(b+cd-e/f)$ may be written $ab+acd-ae/f$. Any quantity appearing in *every* term of one member of an equation can be separated out by **factoring,** or dividing each term by the common quantity. Thus, if $a=bc+\frac{bd}{e}-b^2+b$, the equation may be written $a=b\left(c+\frac{d}{e}-b+1\right)$.

Note that $\frac{b}{b}=1$ and $\frac{b^2}{b}=b$. This is the inverse of multiplication: $a\times 1=a$, but $a\times a=a^2$. Also, $a^2\times a^3=a^5$; and $\frac{a^7}{a^2}=a^5$. Thus, in multiplying a power of a number by a power of the same number, the powers are added, or, stated mathematically, $a^m\times a^n=a^{m+n}$. In division, $\frac{a^m}{a^n}=a^{m-n}$, or the exponents are subtracted. If n is greater than m, a *negative* exponent results. A value with a negative exponent is equal to the reciprocal of the same value with a positive exponent. Thus, $a^{-n}=\frac{1}{a^n}$ and $\frac{a^2\, b^{-3}}{c}=\frac{a^2}{b^3c}$.

In raising to a power a number with an exponent, the two exponents are multiplied. Thus, $(a^2)^3=a^{2\times 3}=a^6$, or $(a^n)^m=a^{nm}$. The inverse is true in extracting a root. Thus, $\sqrt[3]{a^2}=a^{\frac{2}{3}}=a^{0.667}$, or $\sqrt[m]{a^n}=a^{\frac{n}{m}}$.

Both members of an equation can be multiplied or divided by equal quantities without disturbing the equality, excluding division by zero or some expression equal to zero. Thus, if $a=b+c$, $2a=2(b+c)$, or if $x=y$, $ax=y(b+c)$ and $\frac{a}{x}=\frac{b+c}{y}$. Sometimes there is more than one answer to an equation. Division by one of the unknowns may eliminate one of the answers.

Both members of an equation can be raised to the same power, and like roots of both members can be taken, without disturbing the equality. Thus, if $a=b+c$, $a^2=(b+c)^2$, or if $x=y$, $a^x=(b+c)^y$. This is *not* the same as $a^x=b^y+c^y$. Similarly, if $a=b+c$, $\sqrt{a}=\sqrt{b+c}$, or if $x=y$, $\sqrt[x]{a}=\sqrt[y]{b+c}$. Again, $\sqrt[y]{b+c}$ is *not* equal to $\sqrt[y]{b}+\sqrt[y]{c}$, as a numerical example will indicate: $\sqrt{100}=\sqrt{64+36}$, but $\sqrt{100}$ does not equal $\sqrt{64}+\sqrt{36}$.

If two quantities to be multiplied or divided are *both* positive or *both* negative, the result is positive. Thus, $(+a)\times(+b)=ab$ and $\frac{-a}{-b}=+\frac{a}{b}$. But if the signs are opposite,

the answer is negative. Thus, $(+a)\times(-b)=-ab$, and $\frac{-a}{+b}=-\frac{a}{b}$; also, $(-a)\times(+b)=-ab$, and $\frac{+a}{-b}=-\frac{a}{b}$.

In expressions containing both parentheses and brackets, or both of these and braces, the innermost symbols are removed first. Thus, $-\left\{6z-\frac{[x(x+4)-5y]}{y}\right\}=-\left\{6z-\frac{[x^2+4x-5y]}{y}\right\}=-\left\{6z-\frac{x^2}{y}-\frac{4x}{y}+5\right\}=-6z+\frac{x^2}{y}+\frac{4x}{y}-5$.

O23. Fractions.—To add or subtract two or more fractions, convert each to an expression having the same denominator, and then add the numerators. Thus, $\frac{a}{b}+\frac{c}{d}+\frac{e}{f}=\frac{adf}{bdf}+\frac{cbf}{bdf}+\frac{ebd}{bdf}=\frac{adf+cbf+ebd}{bdf}$. That is, both numerator and denominator of each fraction are multiplied by the denominator of the other remaining fractions.

To multiply two or more fractions, multiply the numerators by each other, and also multiply the denominators by each other. Thus, $\frac{a}{b}\times\frac{c}{d}\times\frac{e}{f}=\frac{ace}{bdf}$.

To divide two fractions, invert the divisor and multiply. Thus, $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$.

If the same factor appears in all terms of a fraction, it can be factored out without changing the value of the fraction. Thus, $\frac{ab+ac+ad}{ae-af}=\frac{b+c+d}{e-f}$. This is the same as factoring a from the numerator and denominator separately. That is, $\frac{ab+ac+ad}{ae-af}=\frac{a(b+c+d)}{a(e-f)}$, but since $\frac{a}{a}=1$, this part can be removed, and the fraction appears as above.

O24. Transposition.—It is sometimes desirable to move terms of an expression from one side of the equals sign (=) to the other. This is called **transposition,** and to move one term is to **transpose** it. If the term to be moved is preceded by a plus or a minus sign, this sign is reversed when the term is transposed. Thus, if $a=b+c$, then $a-b=c$, $a-c=b$, $-b=c-a$, $-b-c=-a$, etc. Note that the signs of *all* terms can be reversed without destroying the equality, for if $a=b$, $b=a$. Thus, if *all* terms to the left of the equals sign are exchanged for *all* those to the right, no change in sign need take place, yet if each is moved individually, the signs reverse. For instance, if $a=b+c$, $-b-c=-a$. If each term is multiplied by -1, this becomes $b+c=a$.

A term which is to be multiplied or divided by *all* other terms on its side of the equation can be transposed if it is also moved from the numerator to the denominator, or vice versa. Thus, if $a=\frac{b}{c}$, then $ac=b$, $c=\frac{b}{a}$, $\frac{1}{b}=\frac{1}{ac}$, $\frac{c}{b}=\frac{1}{a}$, etc. $\left(\text{Note that } a=\frac{a}{1}.\right)$ The same result could be obtained by multiplying both sides of an equation by the same quantity. For instance, if both sides of $a=\frac{b}{c}$ are multiplied by c, the equation becomes $ac=\frac{bc}{c}$ and since any number (except zero) divided by itself is unity, $\frac{c}{c}=1$, and the equation becomes $ac=b$, as given above. Note, also, that *both* sides of an equation can be *inverted* without destroying the relationship, for if $a=b$, $\frac{a}{1}=\frac{b}{1}$, and $\frac{1}{b}=\frac{1}{a}$ or $\frac{1}{a}=\frac{1}{b}$. This is accomplished by transposing *all* terms of an equation.

Note that in the case of transposition by changing the plus or minus sign, an entire expression must be changed, and not a part of it. Thus, if $a=bc+d$, $a-bc=d$, but it

is not true that $a+b=c+d$. Similarly, a term to be transposed by reversing its multiplication-division relationship must bear that relationship to *all* other terms on its side of the equation. That is, if $a=bc+d$, it is *not* true that $\frac{a}{b}=c+d$, or that $\frac{a}{bc}=d$, but $\frac{a}{bc+d}=1$. If $a=b(cd+e)$, then $\frac{a}{b}=cd+e$.

O25. Ratio and proportion.—If the relationship of a to b is the same as that of c to d, this fact can be written $a : b :: c : d$, or $\frac{a}{b}=\frac{c}{d}$. Either side of this equation, $\frac{a}{b}$ or $\frac{c}{d}$ is called a **ratio** and the whole equation is called a **proportion.** When a ratio is given a numerical value, it is often expressed as a decimal or as a percentage. Thus, if $\frac{a}{b}=\frac{1}{4}$ (that is, $a=1$, $b=4$), the ratio might be expressed as 0.25 or as 25 percent.

Since a ratio is a fraction, it can be handled as any other fraction.

Geometry

O26. Definitions.—**Geometry** is that branch of mathematics dealing with the properties, relations, and measurement of lines, surfaces, solids, and angles. **Plane geometry** deals with plane figures, and **solid geometry** deals with three-dimensional figures.

A **point,** considered mathematically, is a place having position but no extent. It has no length, breadth, or thickness. A point in motion produces a **line,** which has length, but neither breadth nor thickness. A **straight** or **right line** is the shortest distance between two points in space. A line in motion in any direction except along itself produces a **surface,** which has length and breadth, but not thickness. A **plane surface** or **plane** is a surface without curvature. A straight line connecting any two of its points lies wholly within the plane. A plane surface in motion in any direction except within its plane produces a **solid,** which has length, breadth, and thickness. **Parallel** lines or surfaces are those which are everywhere equidistant. **Perpendicular** lines or surfaces are those which meet at right angles. A perpendicular may be called a **normal,** particularly when it is perpendicular to the tangent to a curved line or surface at the point of tangency. All points equidistant from the ends of a straight line are on the perpendicular bisector of that line. The distance from a point to a line is the length of the perpendicular between them, unless some other distance is indicated.

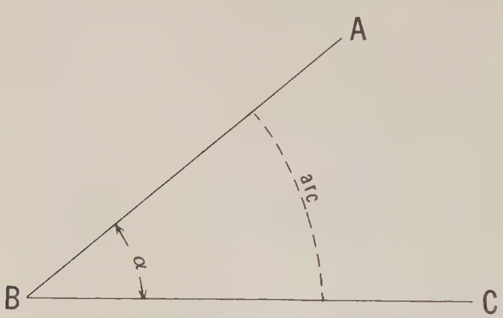


FIGURE O27a. An angle.

O27. Angles.—An **angle** is the inclination to each other of two straight lines which meet at a point. It is measured by the arc of a circle intercepted between the two lines forming the angle, the center of the circle being at the point of intersection. Referring to figure O27a, the angle formed by lines AB and BC, measured by the arc shown, may be designated "angle B," "angle ABC," or "angle CBA"; or by Greek letter (app. B), as "angle α." The first method should not be used if there is more than one angle at the point, as at G in figure O27b. When three letters are used, the middle one should always be that at the **vertex** of the angle, as G in figure O27b.

An **acute angle** is one less than a right angle (90°). In figure O27b, angles AGB, BGC, CGD, DGE, and EGF are all acute angles.

A **right angle** is one whose sides are perpendicular (90°). In figure O27b, angles *AGC*, *BGD*, *CGE*, and *DGF* are right angles.

An **obtuse angle** is one greater than a right angle (90°) but less than a straight angle (180°). In figure O27b, angles *AGD*, *BGE*, and *CGF* are obtuse angles. Angle *AGF* is also obtuse if measured counterclockwise from *AG* to *FG*.

A **straight angle** is one whose sides form a continuous straight line (180°). In figure O27b, angles *AGE* and *BGF* are straight angles.

A **reflex angle** is one greater than a straight angle (180°) but less than a circle (360°). In figure O27b, angle *AGF* is reflex if measured clockwise from *AG* to *FG*. Actually, any two lines meeting at a point form two angles, one less than a straight angle of 180° (unless exactly a straight angle) and the other greater than a straight angle (180°).

An **oblique angle** is any angle not a multiple of 90°.

Two angles whose sum is a right angle (90°) are **complementary angles,** and either is the **complement** of the other. In figure O27b, angles *AGB* and *BGC*, *BGC* and *CGD*, *CGD* and *DGE*, and *DGE* and *EGF* are complementary. The angles need not be adjacent. Angles *AGB* and *DGE*, and angles *BGC* and *EGF* are complementary.

Two angles whose sum is a straight angle (180°) are **supplementary angles,** and either is the **supplement** of the other. In figure O27b, angles *AGB* and *BGE*, *AGC* and *CGE*, *AGD* and *DGE*, *BGC* and *CGF*, *BGD* and *DGF*, *BGE* and *EGF*, and *AGC* and *DGF* are supplementary.

Two angles whose sum is a circle (360°) are **explementary angles,** and either is the **explement** of the other. The two angles formed when any two lines terminate at a common point are explementary.

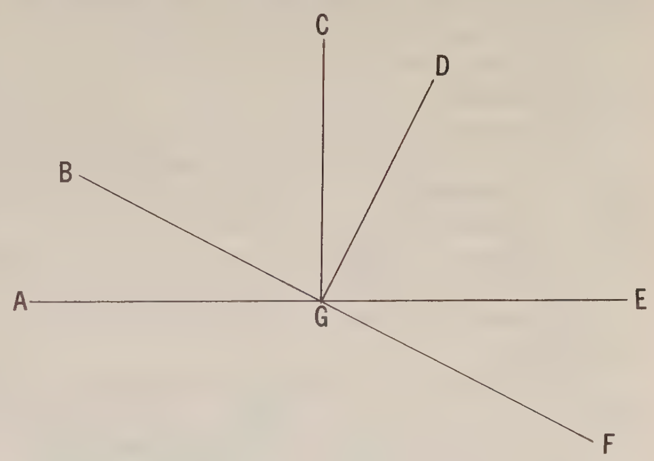


FIGURE O27b. Acute, right, and obtuse angles.

Since angles *AGB* and *CGD* (fig. O27b) are each complementary to angle *BGC*, angles *AGB* and *CGD* are equal. Similarly, it can be shown that angle *EGF* is also equal to angle *CGD* (and therefore also equal to angle *AGB*) and also that angles *BGC* and *DGE* are equal to each other. Since *AGC* and *CGE* are both right angles with a common side, *CG* is perpendicular to *AE*. Similarly, *DG* is perpendicular to *BF*. If the sides of one angle are perpendicular to those of another, the two angles are either equal or supplementary. Also, if the sides of one angle are parallel to those of another, the two angles are either equal or supplementary.

When two straight lines intersect, forming four angles, the two opposite angles, called **vertical angles,** are equal. Thus, in figure O27b, lines *AE* and *BF* intersect at *G*. Angles *AGB* and *EGF* form a pair of equal acute vertical angles, and *BGE* and *AGF* form a pair of equal obtuse vertical angles. Angles which have the same vertex and lie on opposite sides of a common side are **adjacent angles.** Adjacent angles formed by intersecting lines are supplementary, since each pair of adjacent angles forms a straight angle (fig. O27b).

A **transversal** is a line that intersects two or more other lines. If two or more parallel lines are cut by a transversal, groups of adjacent and vertical angles are formed,

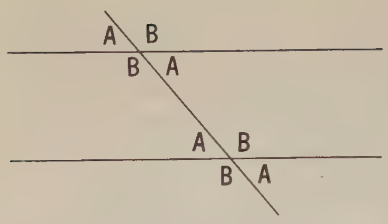


FIGURE O27c. Angles formed by a transversal.

as shown in figure O27c. In this situation, all acute angles (A) are equal, all obtuse angles (B) are equal, and each acute angle is supplementary to each obtuse angle.

A **dihedral angle** is the angle between two intersecting planes.

O28. Triangles.—A **plane triangle** is a closed figure formed by three straight lines, called **sides,** which meet at three points called **vertices** (singular **vertex**). The vertices are usually labeled with capital letters, and the sides with lowercase letters, as shown in figure O28a.

An **equilateral triangle** is one with its three sides equal. An **equiangular triangle** is one with its three angles equal. When either of these conditions is present, the other always is, so that a triangle which is equilateral is also equiangular, and vice versa.

An **isosceles triangle** is one with two equal sides, called **legs.** The angles opposite the legs are equal. A line which bisects (divides into two equal parts) the *unequal* angle of an isosceles triangle is the perpendicular bisector of the opposite side, and divides the triangle into two equal right triangles.

A **scalene triangle** is one with no two sides equal. In such a triangle, no two angles are equal.

An **acute triangle** is one with three acute angles.

A **right triangle** is one with a right angle. The side opposite the right angle is called the **hypotenuse.** The other two sides may be called **legs.** A plane triangle can have only one right angle.

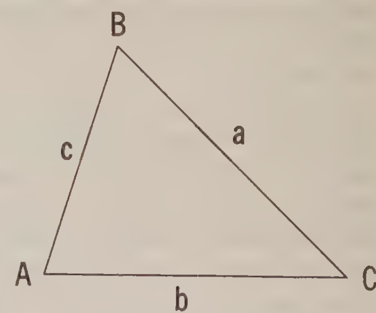


FIGURE O28a. A triangle.

An **obtuse triangle** is one with an obtuse angle. A plane triangle can have only one obtuse angle.

An **oblique triangle** is one which does not contain a right angle.

The **altitude** of a triangle is a perpendicular line from any vertex to the opposite side, extended if necessary, or the length of this perpendicular line.

A **median** of a triangle is a line from any vertex to the center of the opposite side. The three medians of a triangle meet at a point called the **centroid** of the triangle. This point divides each median into two parts, that part between the centroid and the vertex being twice as long as the other part.

Lines bisecting the three angles of a triangle meet at a point which is equidistant from the three sides, and is the center of the **inscribed circle,** as shown in figure O28b. This point is of particular interest to navigators because it is the point taken as the fix when three lines of position of equal weight and having only random errors do not meet at a common point.

The perpendicular bisectors of the three sides of a triangle meet at a point which is equidistant from the three vertices, and is the center of the **circumscribed circle,** the circle through the three vertices and therefore the smallest circle which can be drawn enclosing the triangle. The center of a circumscribed circle is within an acute triangle, on the hypotenuse of a right triangle, and outside an obtuse triangle.

A line connecting the mid points of two sides of a triangle is parallel to the third side and half as long. Also, a line parallel to one side of a triangle and intersecting the other two sides divides these sides proportionally. This principle can be used to divide a line into any number of equal or proportional parts. Refer to figure O28c. Suppose

it is desired to divide line AB into four equal parts. From A draw any line AC. Along C measure four equal parts of any convenient lengths (AD, DE, EF, and FG). Draw GB, and through F, E, and D draw lines parallel to GB and intersecting AB. Then AD', $D'E'$, $E'F'$, and $F'B$ are equal and AB is divided into four equal parts.

The sum of the angles of a plane triangle is 180°. Therefore, the sum of the acute angles of a right triangle is 90°, and the angles are complementary. If one side of a triangle is extended, the **exterior angle** thus formed is supplementary to the adjacent **interior angle** and, therefore, equal to the sum of the two nonadjacent angles. If two angles of one triangle are equal to two angles of another triangle, the third angles are also equal, and the triangles are **similar.** If the area of one triangle is equal to the area of another, the triangles are **equal.** Triangles having equal bases and altitudes have equal areas. Two figures are **congruent** if one can be placed over the other to make an exact fit. Congruent figures are both similar and equal. If any side of one triangle is equal to any side of a similar triangle, the triangles are congruent. For example, if two right triangles have equal sides, they are congruent; if two right triangles have two corresponding sides equal, they are congruent. Triangles are congruent only if the sides and angles are equal.

The sum of two sides of a plane triangle is always greater than the third side; their difference is always less than the third side.

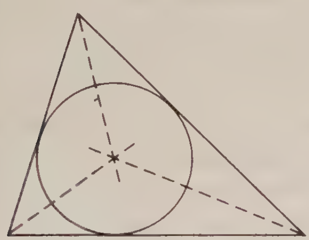


FIGURE O28b. A circle inscribed in a triangle.

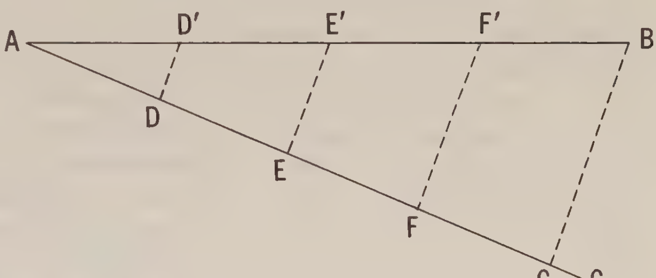


FIGURE O28c. Dividing a line into equal parts.

If A=area, b=one of the legs of a right triangle or the base of any plane triangle, h=altitude, c=the hypotenuse of a right triangle, a=the other leg of a right triangle, and S=the sum of the interior angles:

Area of plane triangle: $A=\frac{bh}{2}$

Length of hypotenuse of plane right triangle: $c=\sqrt{a^2+b^2}$

Sum of interior angles of plane triangle: $S=180°$.

O29. Polygons.—A **polygon** is a closed plane figure made up of three or more straight lines called **sides.** A polygon with three sides is a **triangle,** one with four sides is a **quadrilateral,** one with five sides is a **pentagon,** one with six sides is a **hexagon,** and one with eight sides is an **octagon.** An **equilateral polygon** has equal sides. An **equiangular polygon** has equal interior angles. A **regular polygon** is both equilateral and equiangular. As the number of sides of a regular polygon increases, the figure approaches a circle.

A **trapezoid** is a quadrilateral with one pair of opposite sides parallel and the other pair not parallel. A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram, or either of the parallel sides of a trapezoid, is the **base** of the figure. The perpendicular distance from the base to the opposite

side is the altitude. A **rectangle** is a parallelogram with four right angles. (If any one is a right angle, the other three must be, also.) A **square** is a rectangle with equal sides. A **rhomboid** is a parallelogram with oblique angles. A **rhombus** is a rhomboid with equal sides.

The sum of the exterior angles of a convex polygon (one having no interior reflex angles), made by extending each side in one direction only (consistently), is 360°.

A **diagonal** of a polygon is a straight line connecting any two vertices which are not adjacent. The diagonals of a parallelogram bisect each other.

The **perimeter** of a polygon is the sum of the lengths of its sides.

If A=area, s=the side of a square, a=that side of a rectangle adjacent to the base or that side of a trapezoid parallel to the base, b=the base of a quadrilateral, h=the altitude of a parallelogram or trapezoid, S=the sum of the angles of a polygon, and n=the number of sides of a polygon:

Area of square: $A=s^2$
Area of rectangle: $A=ab$
Area of parallelogram: $A=bh$
Area of trapezoid: $A=\frac{(a+b)h}{2}$
Sum of angles in convex polygon: $S=(n-2)180°$.

O30. Circles.—A **circle** is a plane, closed curve, all points of which are equidistant from a point within, called the **center** (C, fig. O30); or the figure formed by such a curve. The line forming the circle is called the **circumference.** The length of this line is the **perimeter,** although the term "circumference" is often used with this meaning. An **arc** is part of a circumference. A **major arc** is more than a semicircle (180°), a **minor arc** is less than a semicircle (180°). A **semicircle** is half a circle (180°), a **quadrant** is a quarter of a circle (90°), a **quintant** is a fifth of a circle (72°), a **sextant** is a sixth of a circle (60°), an **octant** is an eighth of a circle (45°). Some of these names have been applied to instruments used by navigators for measuring altitudes of celestial bodies because of the part of a circle originally used for the length of the arc of the instrument.

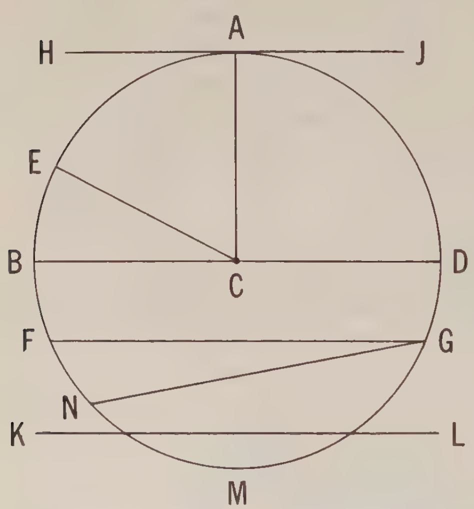


FIGURE O30. Elements of a circle.

Concentric circles have a common center.

A **radius** (plural **radii**) or **semidiameter** is a straight line connecting the center of a circle with any point on its circumference. In figure O30, CA, CB, CD, and CE are radii.

A **diameter** of a circle is a straight line passing through its center and terminating at opposite sides of the circumference, or two radii in opposite directions (BCD, fig. O30). It divides a circle into two equal parts. The ratio of the length of the circumference of any circle to the length of its diameter is 3.14159+, or π (the Greek letter *pi*), a relationship that has many useful applications.

A **sector** is that part of a circle bounded by two radii and an arc. In figure O30, BCE, ECA, ACD, BCA, and ECD are sectors. The angle formed by two radii is called a **central angle.** Any pair of radii divides a circle into sectors, one less than a semicircle (180°) and the other greater than a semicircle (unless the two radii form a diameter).

A **chord** is a straight line connecting any two points on the circumference of a circle (*FG*, *GN* in fig. O30). Chords equidistant from the center of a circle are equal in length.

A **segment** is that part of a circle bounded by a chord and the intercepted arc (*FGMF*, *NGMN* in fig. O30). A chord divides a circle into two segments, one less than a semicircle (180°), and the other greater than a semicircle (unless the chord is a diameter). A diameter perpendicular to a chord bisects it, its arc, and its segments. Either pair of vertical angles formed by intersecting chords has a combined number of degrees equal to the sum of the number of degrees in the two arcs intercepted by the two angles.

An **inscribed angle** is one whose vertex is on the circumference of a circle and whose sides are chords (*FGN* in fig. O30). It has half as many degrees as the arc it intercepts. Hence, an angle inscribed in a semicircle is a right angle if its sides terminate at the ends of the diameter forming the semicircle.

A **secant** of a circle is a line intersecting the circle, or a chord extended beyond the circumference (*KL* in fig. O30).

A **tangent** to a circle is a straight line, in the plane of the circle, which has only one point in common with the circumference (*HJ* in fig. O30). A tangent is perpendicular to the radius at the **point of tangency** (*A* in fig. O30). The two tangents from a point to opposite sides of a circle are equal in length, and a line from the point to the center of the circle bisects the angle formed by the two tangents. An angle formed outside a circle by the intersection of two tangents, a tangent and a secant, or two secants has *half* as many degrees as the *difference* between the two intercepted arcs. An angle formed by a tangent and a chord, with the apex at the point of tangency, has half as many degrees as the arc it intercepts. A **common tangent** is one tangent to more than one circle. Two circles are tangent to each other if they touch at one point only. If of different sizes, the smaller circle may be either inside or outside the larger one.

Parallel lines intersecting a circle intercept equal arcs.

If A=area; r=radius; d=diameter; C=circumference; s=linear length of an arc; α=angular length of an arc, or the angle it subtends at the center of a circle, in degrees; β=angular length of an arc, or the angle it subtends at the center of a circle, in radians; rad=radians (art. O38), and sin=sine (art. O39):

Area of circle: $A=\pi r^2=\dfrac{\pi d^2}{4}$

Circumference of circle: $C=2\pi r=\pi d=2\pi \text{ rad}$

Area of sector: $A=\dfrac{\pi r^2\alpha}{360}=\dfrac{r^2\beta}{2}=\dfrac{rs}{2}$

Area of segment: $A=\dfrac{r^2(\beta-\sin\alpha)}{2}$

O31. Polyhedrons.—A **polyhedron** is a solid having plane sides or **faces.**

A **cube** is a polyhedron having six square sides.

A **prism** is a solid having parallel, similar, equal, plane geometric figures as bases, and parallelograms as sides. By extension, the term is also applied to a similar solid having nonparallel bases, and trapezoids or a combination of trapezoids and parallelograms as sides. The **axis** of a prism is the straight line connecting the centers of its bases. A **right prism** is one having bases perpendicular to the axis. The sides of a right prism are rectangles. A **regular prism** is a right prism having regular polygons as bases. The **altitude** of a prism is the perpendicular distance between the planes of its bases. In the case of a right prism, it is measured along the axis.

A **pyramid** is a polyhedron having a polygon as one end, the **base;** and a point, the **apex,** as the other; the two ends being connected by a number of triangular sides or **faces.** The **axis** of a pyramid is the straight line connecting the apex and the center of the base. A **right pyramid** is one having its base perpendicular to its axis. A **regular pyramid** is a right pyramid having a regular polygon as its base. The **altitude** of a pyramid is the perpendicular distance from its apex to the plane of its base. A **truncated pyramid** is that portion of a pyramid between its base and a plane intersecting all of the faces of the pyramid.

If A=area, s=edge of a cube or slant height of a regular pyramid (from the center of one side of its base to the apex), V=volume, a=side of a polygon, h=altitude, P=perimeter of base, n=number of sides of polygon, B=area of base, and r=perpendicular distance from the center of a side of a polygon to the center of the polygon:

Cube:

Area of each face: $A=s^2$
Total area of all faces: $A=6s^2$
Volume: $V=s^3$

Regular prism:

Area of each face: $A=ah$
Total area of all faces: $A=Ph=nah$
Area of each base: $B=\frac{nar}{2}$
Total area of both bases: $A=nar$
Volume: $V=Bh=\frac{narh}{2}$

Regular pyramid:

Area of each face: $A=\frac{as}{2}$

Total area of all faces: $A=\frac{nas}{2}$

Area of base: $B=\frac{nar}{2}$

Volume: $V=\frac{Bh}{3}=\frac{narh}{6}$.

O32. Cylinders.—A **cylinder** is a solid having two parallel plane **bases** bounded by closed congruent curves, and a surface formed by an infinite number of parallel lines, called **elements,** connecting similar points on the two curves. A cylinder is similar to a prism, but with a curved lateral surface, instead of a number of flat sides connecting the bases. The **axis** of a cylinder is the straight line connecting the centers of the bases. A **right cylinder** is one having bases perpendicular to the axis. A **circular cylinder** is one having circular bases. The **altitude** of a cylinder is the perpendicular distance between the planes of its bases. The **perimeter** of a base is the length of the curve bounding it.

If A=area, P=perimeter of base, h=altitude, r=radius of a circular base, B=area of base, and V=volume, then for a right circular cylinder:

Lateral area: $A=Ph=2\pi rh$
Area of each base: $B=\pi r^2$
Total area, both bases: $A=2\pi r^2$
Volume: $V=Bh=\pi r^2h$.

O33. Cones.—A **cone** is a solid having a plane **base** bounded by a closed curve, and a surface formed by lines, called **elements,** from every point on the curve to a common point called the **apex.** A cone is similar to a pyramid, but with a curved surface connecting the base and apex, instead of a number of flat sides. The **axis** of a cone is the straight line connecting the apex and the center of the base. A **right cone** is one having its base perpendicular to its axis. A **circular cone** is one having a circular base. The **altitude** of a cone is the perpendicular distance from its apex to the plane of its base. A **frustum** of a cone is that portion of the cone between its base and any parallel plane intersecting all elements of the cone. A **truncated cone** is that portion of a cone between its base and any nonparallel plane which intersects all elements of the cone but does not intersect the base.

If A=area, r=radius of base, s=slant height or length of element, B=area of base, h=altitude, and V=volume, then for a right circular cone:

Lateral area: $A=\pi rs$
Area of base: $B=\pi r^2$
Slant height: $s=\sqrt{r^2+h^2}$
Volume: $V=\frac{Bh}{3}=\frac{\pi r^2 h}{3}$.

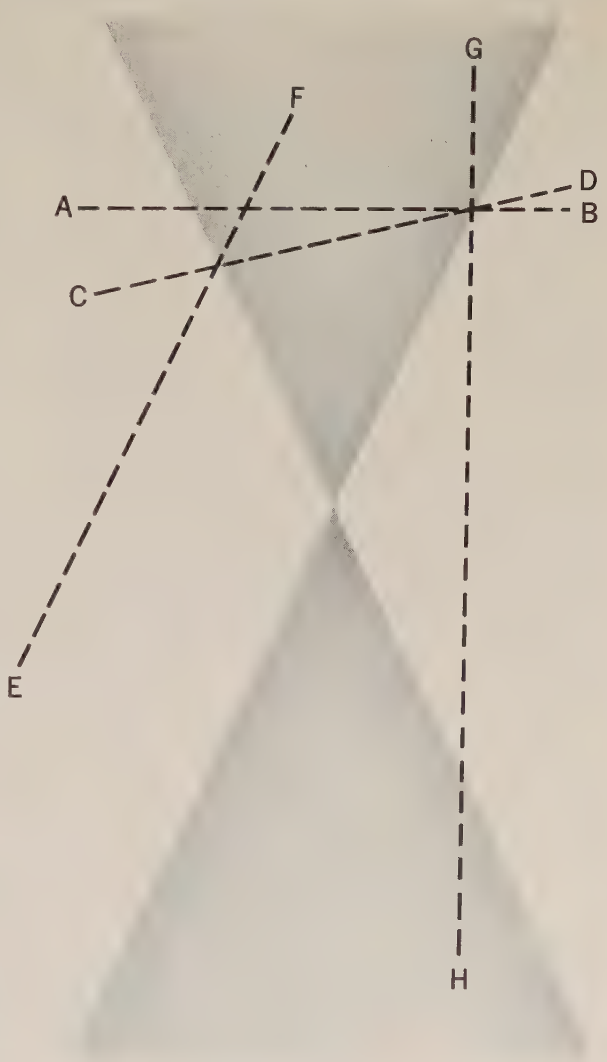


FIGURE O34a. Conic sections.

O34. Conic sections.—If a right circular cone of indefinite extent is intersected by a plane perpendicular to the axis of the cone (AB, fig. O34a), the line of intersection of the plane and the surface of the cone is a **circle,** discussed in article O30.

If the intersecting plane of figure O34a is tilted to some position such as CD, the intersection is an **ellipse** or flattened circle, figure O34b. The longest diameter of an ellipse is called its **major axis,** and half of this is its **semimajor axis,** $\boldsymbol{a}$. The shortest diameter of an ellipse is called its **minor axis,** and half of this is its **semiminor axis,** $\boldsymbol{b}$. Two points, F and F', called **foci** (singular **focus**) or **focal points,** on the major axis are so located that the sum of their distances from any point P on the curve is equal to the length of the major axis. That is, $PF+PF'=2a$ (fig. O34b). The **eccentricity** (e) of an ellipse is equal to $\frac{c}{a}$, where c is the distance from the center to one of the foci ($c=CF=CF'$). It is always greater than 0 but less than 1.

If the intersecting plane of figure O34a is parallel to one element of the cone, as at EF, the intersection is a **parabola,** figure O34c. Any point P on a parabola is equi-

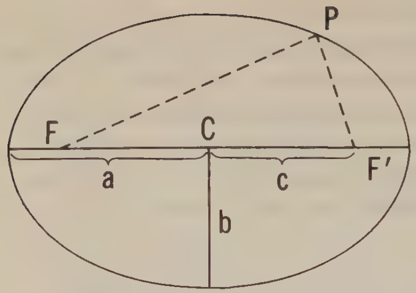


FIGURE O34b. An ellipse.

distant from a fixed point F, called the **focus** or **focal point,** and a fixed straight line, AB, called the **directrix.** Thus, for any point P, $PF=PE$. The point midway between the focus F and the directrix AB is called the **vertex,** V. The straight line through F and V is called the **axis,** CD. This line is perpendicular to the directrix AB. The **eccentricity** (*e*) of a parabola is 1.

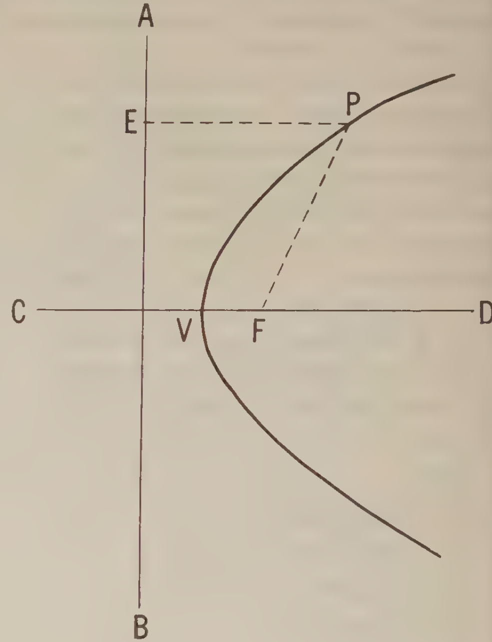


FIGURE O34c. A parabola.

If the elements of the cone of figure O34a are extended to form a second cone having the same axis and apex but extending in the opposite direction, and the intersecting plane is tilted beyond the position forming a parabola, so that it intersects both curves, as at GH, the intersections of the plane with the cones is a **hyperbola,** figure O34d. There are two intersections or branches of a hyperbola, as shown. At any point P on either branch, the *difference* in the distance from two fixed points called **foci** or **focal points,** F and F', is constant and equal to the short-

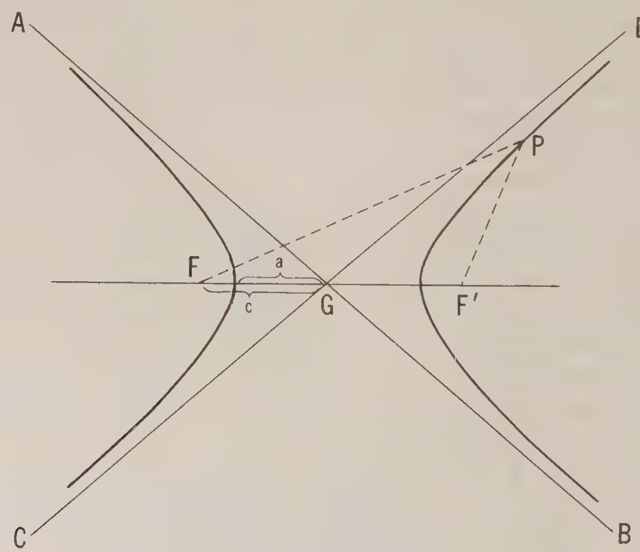


FIGURE O34d. A hyperbola.

est distance between the two branches. That is, $PF-PF'=2a$ (fig. O34d). The straight line through F and F' is called the **axis.** The **eccentricity** (*e*) of a hyperbola

is the ratio $\frac{c}{a}$ (fig. O34d). It is always greater than 1. Each branch of a hyperbola approaches ever closer to, but never reaches, a pair of intersecting straight lines, AB and CD, called **asymptotes.** These intersect at G.

The various conic sections bear an eccentricity relationship to each other. The eccentricity of a circle is 0, that of an ellipse is greater than 0 but less than 1, that of a parabola or straight line (a limiting case of a parabola) is 1, and that of a hyperbola is greater than 1.

If e=eccentricity, A=area, a=semimajor axis of an ellipse or half the shortest distance between the two branches of a hyperbola, b=the semiminor axis of an ellipse, and c=the distance between the center of an ellipse and one of its focal points or the distance between the focal point of a hyperbola and the intersection of its asymptotes:

Circle:
Eccentricity: $e=0$
Other relationships given in article O30.

Ellipse:
Area: $A=\pi ab$
Eccentricity: $e=\frac{c}{a}$, greater than 0, but less than 1.

Parabola:
Eccentricity: $e=1$.

Hyperbola:
Eccentricity: $e=\frac{c}{a}$, greater than 1.

When cones are intersected by some surface other than a plane, as the curved surface of the earth, the resulting sections do not follow the relationships given above, the amount of divergence therefrom depending upon the individual circumstances. Thus, a "hyperbolic" line of position (art. 1109) is not a true hyperbola.

O35. Spheres.—A **sphere** is a solid bounded by a surface every point of which is equidistant from a point within, called the **center.** It may be formed by rotating a circle about any diameter.

A **radius** or **semidiameter** of a sphere is a straight line connecting its center with any point on its surface. A **diameter** of a sphere is a straight line through its center and terminated at both ends by the surface of the sphere. The poles of a sphere are the ends of a diameter.

The intersection of a plane and the surface of a sphere is a circle, a **great circle** if the plane passes through the center of the sphere, and a **small circle** if it does not. The shorter arc of the great circle between two points on the surface of a sphere is the shortest distance, on the surface of the sphere, between the points. Every great circle of a sphere bisects every other great circle of that sphere. The **poles** of a circle on a sphere are the extremities of the sphere's diameter which is perpendicular to the plane of the circle. All points on the circumference of the circle are equidistant from either of its poles. In the case of a great circle, *both* poles are 90° from any point on the circumference of the circle. Any great circle may be considered a **primary,** particularly when it serves as the origin of measurement of a coordinate. The great circles through its poles are called **secondaries.** Secondaries are perpendicular to their primary.

A **spherical triangle** is the figure formed on the surface of a sphere by the intersection of three great circles. The lengths of the sides of a spherical triangle are measured in degrees, minutes, and seconds, as the angular lengths of the arcs forming them. The

sum of the three sides is always less than 360°. The sum of the three angles is always *more* than 180° and *less* than 540°.

A **lune** is that part of the surface of a sphere bounded by halves of two great circles.

A **spheroid** is a flattened sphere, which may be formed by rotating an ellipse about one of its axes. An **oblate spheroid,** such as the earth, is formed when an ellipse is rotated about its minor axis. In this case the diameter along the axis of rotation is less than the major axis. A **prolate spheroid** is formed when an ellipse is rotated about its major axis. In this case the diameter along the axis of rotation is greater than the minor axis.

If A=area, r=radius, d=diameter, and V=volume of a sphere:

$$\text{Area: } A=4\pi r^2=\pi d^2$$

$$\text{Volume: } V=\frac{4\pi r^3}{3}=\frac{\pi d^3}{6}$$

If A=area, a=semimajor axis, b=semiminor axis, e=eccentricity, and V=volume of an oblate spheroid:

$$\text{Area: } A=4\pi a^2\left(1-\frac{e^2}{3}-\frac{e^4}{15}-\frac{e^6}{35}-\cdots\right)$$

$$\text{Eccentricity: } e=\sqrt{\frac{a^2-b^2}{a^2}}$$

$$\text{Volume: } V=\frac{4\pi a^2 b}{3}$$

O36. Coordinates are magnitudes used to define a position. Many different types of coordinates are used.

If a position is known to be at a stated point, no magnitudes are needed to identify the position, although they may be required to locate the point. Thus, if a vessel is at port A, its position is known if the location of port A is known, but latitude and longitude may be needed to locate port A.

If a position is known to be on a given line, a single magnitude (coordinate) is needed to identify the position if an origin is stated or understood. Thus, if a vessel is known to be *south* of port B, it is known to be on a line extending southward from port B. If its distance from port B is known, and the position of port B is known, the position of the vessel is uniquely defined.

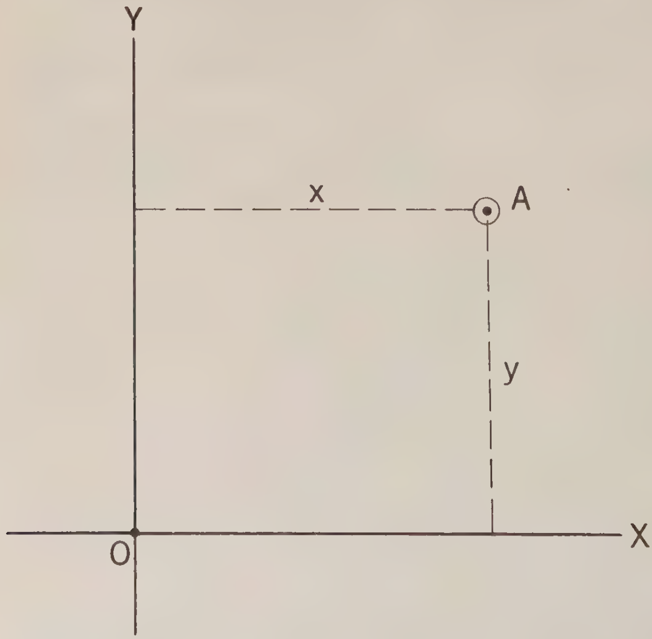


FIGURE O36a. Rectangular coordinates.

If a position is known to be on a given surface, two magnitudes (coordinates) are needed to define the position. Thus, if a vessel is known to be on the surface of the earth, its position can be identified by means of latitude and longitude. Latitude indicates its angular distance north or south of the equator, and longitude its angular distance east or west of the prime meridian.

If nothing is known regarding a position other than that

it exists in space, three magnitudes (coordinates) are needed to define its position. Thus, the position of a submarine may be defined by means of latitude, longitude, and depth below the surface.

Each coordinate requires an origin, either stated or implied. If a position is known to be on a given plane, it might be defined by means of its distance from each of two intersecting lines, called **axes.** Thus, in figure O36a the position of point A can be defined by stating that it is x units to the right of line OY and y units upward from line OX. These are called **rectangular coordinates.** The coordinate along OY is called the **ordinate,** and the coordinate along OX is called the **abscissa.** Point O is the **origin,** and lines OX and OY the **axes** (called the X and Y axes, respectively). Point A is at position x, y. If the axes are not perpendicular but the lines x and y are drawn parallel to the axes, **oblique coordinates** result. Either type are **Cartesian coordinates.** A three-dimensional system of Cartesian coordinates, with X, Y, and Z axes, is called **space coordinates.**

Another system of plane coordinates in common usage consists of the *direction* and *distance* from the origin (called the **pole**), as shown in figure O36b. A line extending in the direction indicated is called a **radius vector.** Direction and distance from a fixed point constitute **polar coordinates,** sometimes called the rho- (the Greek ρ, to indicate distance) theta (the Greek θ, to indicate direction) system. Navigators more commonly call it the "bearing-distance" system. An example of its use is with respect to a radar PPI (art. 1208).

Spherical coordinates are used to define a position on the surface of a sphere or spheroid by indicating angular distance from a primary great circle and a reference secondary great circle. Familiar examples are latitude and longitude, altitude and azimuth, and declination and hour angle.

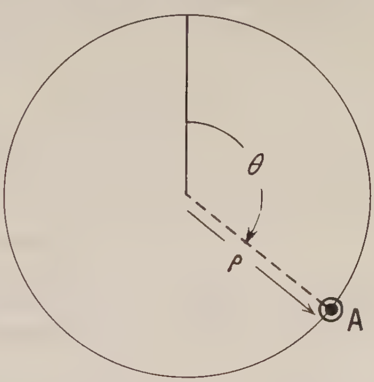


FIGURE O36b. Polar coordinates.

Trigonometry

O37. Definitions.—Trigonometry is that branch of mathematics dealing with the relations among the angles and sides of triangles. **Plane trigonometry** is that branch dealing with plane triangles, and **spherical trigonometry** is that branch dealing with spherical triangles.

O38. Angular measure.—A circle may be divided into 360 **degrees** (°), which is the **angular length** of its circumference. Each degree may be divided into 60 **minutes** (′), and each minute into 60 **seconds** (″). The angular length of an arc is usually expressed in these units. By this system a right angle or quadrant has 90° and a straight angle or semicircle 180°. In marine navigation, altitudes, latitudes, and longitudes are usually expressed in degrees, minutes, and tenths (27°14′.4). Azimuths are usually expressed in degrees and tenths (164°.7). The system of degrees, minutes, and seconds indicated above is the **sexagesimal system.** In the **centesimal system,** used chiefly in France, the circle is divided into 400 **centesimal degrees** (sometimes called **grades**) each of which is divided into 100 **centesimal minutes** of 100 **centesimal seconds** each.

A **radian** is the angle subtended at the center of a circle by an arc having a linear length equal to the radius of the circle. A radian is equal to 57°.2957795131 approximately, or 57°17′44″.80625 approximately. The radian is sometimes used as a unit of angular measure. A circle (360°) $=2\pi$ radians, a semicircle (180°) $=\pi$ radians, a right angle (90°) $=\frac{\pi}{2}$ radians, 1° $=0.0174532925$ radians approximately, 1′ $=0.0002908882$ radians approximately, and 1″ $=0.0000048481$ radians approximately.

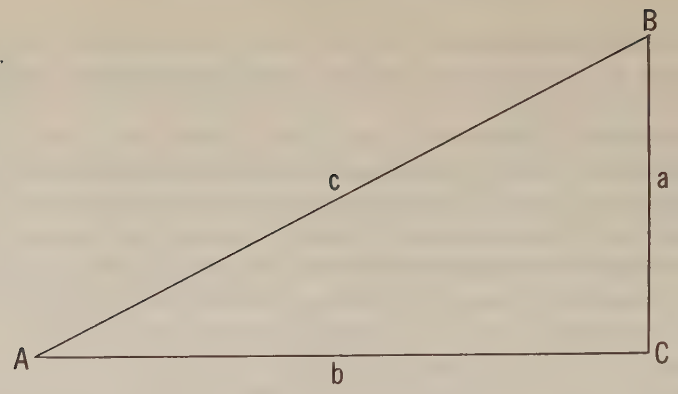


FIGURE O39a. A right triangle.

O39. Trigonometric functions are the various proportions or ratios of the sides of a plane right triangle, defined in relation to one of the acute angles. In figure O39a, A, B, and C are the angles of a plane right triangle, the right angle being at C. The sides are a, b, c, as shown. The six principal trigonometric functions of angle A are:

$$\text{sine } A = \sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\text{cosine } A = \cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\text{tangent } A = \tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b}$$

$$\text{cotangent } A = \cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{b}{a}$$

$$\text{secant } A = \sec A = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{b}$$

$$\text{cosecant } A = \csc A = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{a}.$$

Certain additional relations are also classed as trigonometric functions:

versed sine A=versine A=vers A=ver $A=1-\cos A$

versed cosine A=coversed sine A=coversine A=covers A=cov $A=1-\sin A$

haversine A=hav $A=\frac{1}{2}$ ver $A=\frac{1}{2}(1-\cos A)$.

The numerical value of a trigonometric function is sometimes called the **natural function** to distinguish it from the logarithm of the function, called the **logarithmic function.** Numerical values of the six principal functions are given at 1′ intervals in table 31. Logarithms are given at the same intervals in table 33. Both natural and logarithmic haversines are given in table 34.

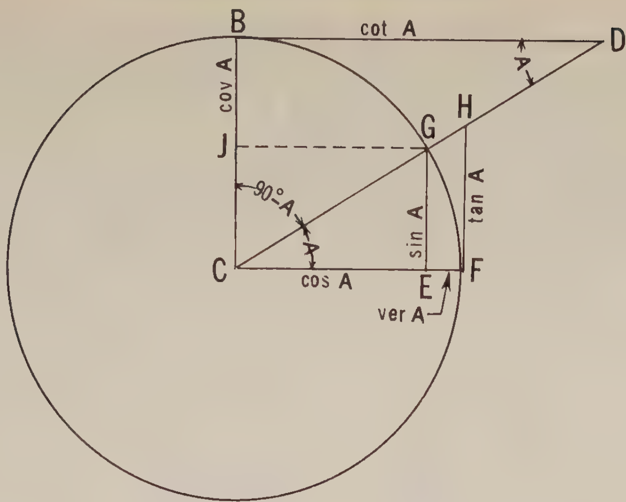


FIGURE O39b. Line definitions of trigonometric functions.

Various functions may be represented by lines associated with a circle, as shown in figure O39b. The radius of the circle is considered 1. Angle BDC=angle ECG=angle A.

$\sin A=GE$	$\cot A=BD$
$\cos A=CE$	$\sec A=CH$
$\tan A=HF$	$\csc A=CD$
$\text{ver } A=EF$	$\text{cov } A=BJ.$

Some relationships apply only to plane trigonometry and others to both plane and spherical trigonometry. Those which apply to both are called **fundamental identities.** Examples are given below.

Of the six principal functions, the second three are the reciprocals of the first three. Thus,

$$\sin A=\frac{1}{\csc A} \qquad \csc A=\frac{1}{\sin A}$$

$$\cos A=\frac{1}{\sec A} \qquad \sec A=\frac{1}{\cos A}$$

$$\tan A=\frac{1}{\cot A} \qquad \cot A=\frac{1}{\tan A}$$

From figure O39a:

$$\sin B=\frac{b}{c}=\cos A \qquad \cot B=\frac{a}{b}=\tan A$$

$$\cos B=\frac{a}{c}=\sin A \qquad \sec B=\frac{c}{a}=\csc A$$

$$\tan B=\frac{b}{a}=\cot A \qquad \csc B=\frac{c}{b}=\sec A.$$

Since A and B are *complementary*, these relations show that the sine of an angle is the cosine of its complement, the tangent of an angle is the cotangent of its complement, and the secant of an angle is the cosecant of its complement. Thus, the **co** function of an angle is the function of its **co**mplement.

O40. The functions in various quadrants.—The sign (+ or −) of the functions varies with the quadrant of an angle. This is shown in figure O40a. In the left-hand diagram a radius is imagined to rotate in a counterclockwise direction through 360°

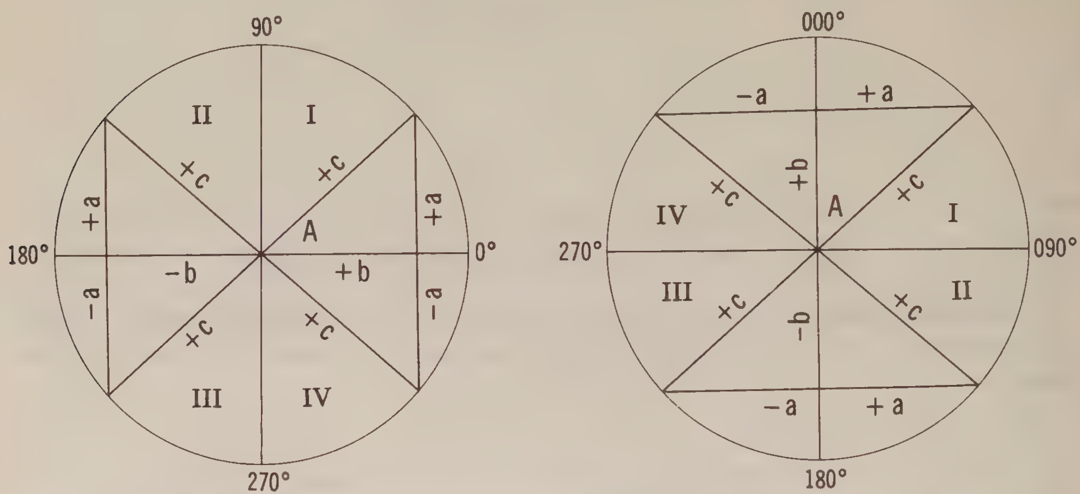


FIGURE O40a. Trigonometric functions in the four quadrants. *Left*, mathematical convention; *right*, navigational convention.

from the horizontal position at 0°. This is the mathematical convention. In the right-hand figure this concept is shown in the usual navigational convention of a compass rose, starting with 000° at the top and rotating clockwise. In either diagram the angle A between the original position of the radius and its position at any time increases from 0° to 90° in the *first quadrant* (I), 90° to 180° in the *second quadrant* (II), 180° to 270° in the *third quadrant* (III), and 270° to 360° in the *fourth quadrant* (IV). If the values of a and b are considered positive in the directions they extend in the first quadrant (*upward* and to the *right*) and negative in the opposite directions, and if c is regarded as always positive, the signs of the functions can be determined by considering the signs of the sides involved, as shown in the following table:

Functions	I	II	III	IV
sine and cosecant	+	+	−	−
cosine and secant	+	−	−	+
tangent and cotangent	+	−	+	−
versine, coversine, and haversine	+	+	+	+

TABLE O40a. Signs of trigonometric functions by quadrant.

The numerical values vary as shown in the following table and in figure O40b:

Functions	I	II	III	IV
sine cosecant	0 to +1 $+\infty$ to +1	+1 to 0 +1 to $+\infty$	0 to −1 $-\infty$ to −1	−1 to 0 −1 to $-\infty$
cosine secant	+1 to 0 +1 to $+\infty$	0 to −1 $-\infty$ to −1	−1 to 0 −1 to $-\infty$	0 to +1 $+\infty$ to +1
tangent cotangent	0 to $+\infty$ $+\infty$ to 0	$-\infty$ to 0 0 to $-\infty$	0 to $+\infty$ $+\infty$ to 0	$-\infty$ to 0 0 to $-\infty$
versine coversine haversine	0 to +1 +1 to 0 0 to $+\frac{1}{2}$	+1 to +2 0 to +1 $+\frac{1}{2}$ to +1	+2 to +1 +1 to +2 +1 to $+\frac{1}{2}$	+1 to 0 +2 to +1 $+\frac{1}{2}$ to 0

TABLE O40b. Values of trigonometric functions in various quadrants. These relationships are shown graphically in figure O40b.

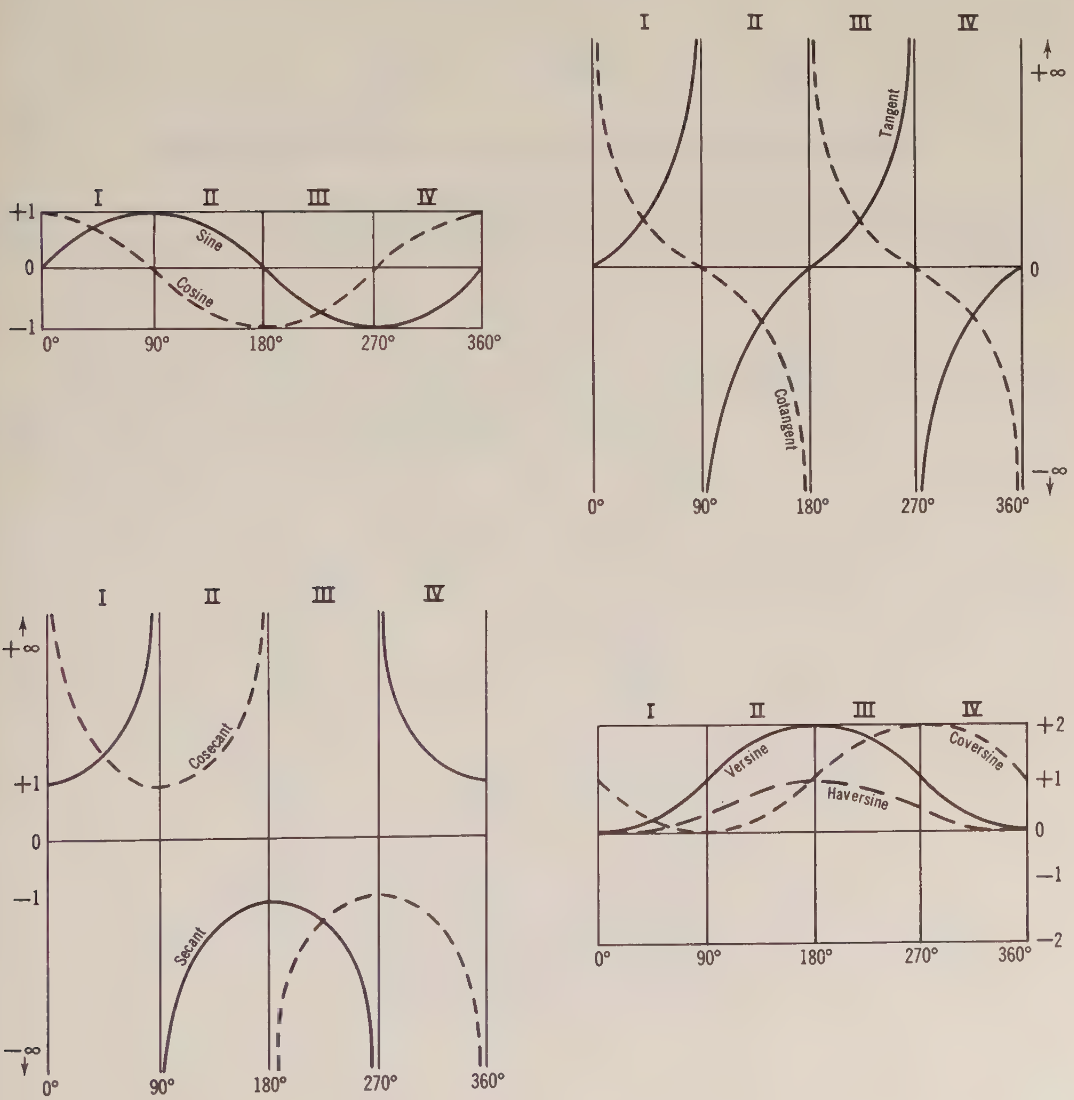


FIGURE O40b. Graphic representation of values of trigonometric functions in various quadrants.

The functions of any angle in the second, third, and fourth quadrants are numerically equal to the same functions of some angle in the first quadrant, as follows:

Quadrant	*Corresponding angle in first quadrant*
II	180°—angle
III	angle—180°
IV	360°—angle.

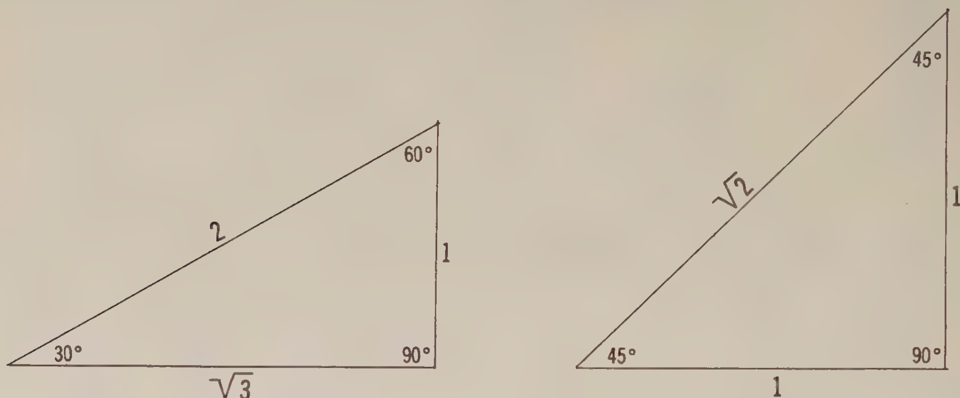


FIGURE O40c. Numerical relationship of sides of 30°-60° and 45° triangles.

Since the relationships of 30°-60° and 45° right triangles are as shown in figure O40c, certain values of the basic functions can be stated exactly as shown in the following table:

Function	*30°*	*45°*	*60°*
sine	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}=\frac{1}{2}\sqrt{2}$	$\frac{\sqrt{3}}{2}=\frac{1}{2}\sqrt{3}$
cosine	$\frac{\sqrt{3}}{2}=\frac{1}{2}\sqrt{3}$	$\frac{1}{\sqrt{2}}=\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$
tangent	$\frac{1}{\sqrt{3}}=\frac{1}{3}\sqrt{3}$	$\frac{1}{1}=1$	$\frac{\sqrt{3}}{1}=\sqrt{3}$
cotangent	$\frac{\sqrt{3}}{1}=\sqrt{3}$	$\frac{1}{1}=1$	$\frac{1}{\sqrt{3}}=\frac{1}{3}\sqrt{3}$
secant	$\frac{2}{\sqrt{3}}=\frac{2}{3}\sqrt{3}$	$\frac{\sqrt{2}}{1}=\sqrt{2}$	$\frac{2}{1}=2$
cosecant	$\frac{2}{1}=2$	$\frac{\sqrt{2}}{1}=\sqrt{2}$	$\frac{2}{\sqrt{3}}=\frac{2}{3}\sqrt{3}$

TABLE O40c. Values of various trigonometric functions for angles of 30°, 45°, and 60°.

O41. Inverse trigonometric functions.—The angle having a given trigonometric function may be indicated in any of several ways. Thus, sin $y=x$, $y=\text{arc sin } x$, and $y=\sin^{-1} x$ have the same meaning. The superior -1 is not an exponent in this case. In each case, y is "the angle whose sine is x." In this case, y is the **inverse sine** of x. Similar relationships hold for all trigonometric functions.

O42. Solution of triangles.—A triangle is composed of six parts: three angles and three sides. The angles may be designated A, B, and C; and the sides opposite these angles as a, b, and c, respectively. In general, when three parts are known, the other three parts can be found, unless the known parts are the three angles of a plane triangle.

Right plane triangles.—In a right plane triangle it is only necessary to substitute numerical values in the appropriate formulas representing the basic trigonometric functions (art. O39) and solve. Thus, if a and b are known:

$$\tan A=\frac{a}{b}$$

$$B=90°-A$$

$$c=a \csc A.$$

Similarly, if c and B are given:

$$A=90°-B$$

$$a=c \sin A$$

$$b=c \cos A.$$

Oblique plane triangles.—In solving an oblique plane triangle, it is often desirable to draw a rough sketch of the triangle approximately to scale, as shown in figure O42a. The following laws are helpful in solving such triangles:

Law of sines: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

Law of cosines: $a^2=b^2+c^2-2\ bc \cos A.$

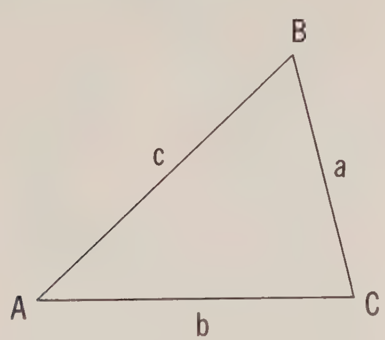


FIGURE O42a. A plane oblique triangle.

The unknown parts of oblique plane triangles can be computed by the formulas of table O42a, among others. By reassignment of letters to sides and angles, these formulas can be used to solve for all unknown parts of oblique plane triangles.

Known	*To find*	*Formula*	*Comments*
a, b, c	A	$\cos A = \frac{c^2+b^2-a^2}{2\,bc}$	Cosine law
a, b, A	B	$\sin B = \frac{b \sin A}{a}$	Sine law. Two solutions if $b>a$
	C	$C=180°-(A+B)$	$A+B+C=180°$
	c	$c=\frac{a \sin C}{\sin A}$	Sine law
a, b, C	A	$\tan A=\frac{a \sin C}{b-a \cos C}$	
	B	$B=180°-(A+C)$	$A+B+C=180°$
	c	$c=\frac{a \sin C}{\sin A}$	Sine law
a, A, B	b	$b=\frac{a \sin B}{\sin A}$	Sine law
	C	$C=180°-(A+B)$	$A+B+C=180°$
	c	$c=\frac{a \sin C}{\sin A}$	Sine law

TABLE O42a. Formulas for solving oblique plane triangles.

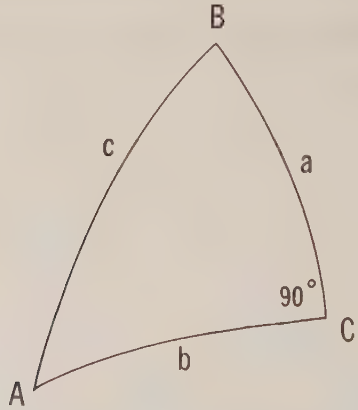


FIGURE O42b. Parts of a right spherical triangle as used in Napier's rules.

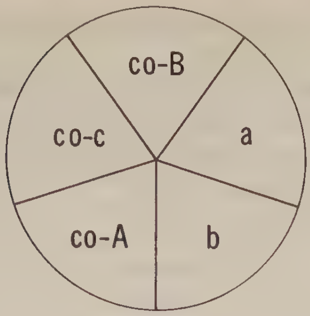


FIGURE O42c. Diagram for Napier's rules.

Right spherical triangles can be solved with the aid of **Napier's rules,** devised by John Napier. If the right angle is omitted, the triangle has five parts: two angles and three sides, as shown in figure O42b. The triangle can be solved if any two parts are known. If the two sides forming the right angle, and the *complements* of the other three parts are used, these elements (called "parts" in the rules) can be arranged in five sectors of a circle in the same order in which they occur in the triangle, as shown in figure O42c. Considering *any* part as the *middle* part, the two parts nearest it in the diagram are considered the *adjacent* parts, and the two farthest from it the *opposite* parts. The rules are:

The sine of a middle part equals the product of (1) the tangents of the adjacent parts or (2) the cosines of the opposite parts.

In the use of these rules, the *co* function of a complement can be given as the function of the element. Thus, the cosine of co-A is the same as the sine of A. From these rules the following formulas can be derived:

$$\sin a = \tan b \cot B = \sin c \sin A$$
$$\sin b = \tan a \cot A = \sin c \sin B$$
$$\cos c = \cot A \cot B = \cos a \cos b$$
$$\cos A = \tan b \cot c = \cos a \sin B$$
$$\cos B = \tan a \cot c = \cos b \sin A.$$

The following rules apply:

1. An oblique angle and the side opposite are in the same quadrant.
2. Side c (the hypotenuse) is less than 90° when a and b are in the same quadrant, and more than 90° when a and b are in different quadrants.

If the known parts are an angle and its opposite side, two solutions are possible.

A **quadrantal spherical triangle** is one having one side of 90°. A **biquadrantal spherical triangle** has two sides of 90°. A **triquadrantal spherical triangle** has three sides of 90°. A biquadrantal spherical triangle is isosceles and has two right angles opposite the 90° sides. A triquadrantal spherical triangle is equilateral, has three right angles, and bounds an octant (one-eighth) of the surface of the sphere. A quadrantal spherical triangle can be solved by Napier's rules provided any two elements in addition to the 90° side are known. The 90° side is omitted and the other parts are arranged in order in a five-sectored circle, using the complements of the three parts farthest from the 90° side. In the case of a quadrantal triangle, rule 1 above is used, and rule 2 restated: angle C (the angle opposite the side of 90°) is *more* than 90° when A and B are in the same quadrant, and *less* than 90° when A and B are in different quadrants. If the

rule requires an angle of more than 90° and the solution produces an angle of less than 90°, subtract the solved angle from 180°.

Oblique spherical triangles. An oblique spherical triangle can be solved by dropping a perpendicular from one of the apexes to the opposite side, extended if necessary, to form two right spherical triangles. It can also be solved by the following formulas, reassigning the letters as necessary.

Known	*To find*	*Formula*	*Comments*
a, b, c	A	$\text{hav } A = \dfrac{\text{hav } a - \text{hav }(b-c)}{\sin b \sin c}$	
A, B, C	a	$\text{hav } a = \dfrac{-\cos S \cos (S-A)}{\sin B \sin C}$	$S = \frac{1}{2}(A+B+C)$
a, b, C	c	$\text{hav } c = \text{hav }(a \sim b) + \sin a \sin b \text{ hav } C$	
	A	$\tan A = \dfrac{\sin D \tan C}{\sin (b-D)}$	$\tan D = \tan a \cos C$
	B	$\sin B = \dfrac{\sin C \sin b}{\sin c}$	
c, A, B	C	$\cos C = \sin A \sin B \cos c - \cos A \cos B$	
	a	$\tan a = \dfrac{\tan c \sin E}{\sin (B+E)}$	$\tan E = \tan A \cos c$
	b	$\tan b = \dfrac{\tan c \sin F}{\sin (A+F)}$	$\tan F = \tan B \cos c$
a, b, A	c	$\sin (c+G) = \dfrac{\cos a \sin G}{\cos b}$	$\cot G = \cos A \tan b$ Two solutions
	B	$\sin B = \dfrac{\sin A \sin b}{\sin a}$	Two solutions
	C	$\sin (C+H) = \sin H \tan b \cot a$	$\tan H = \tan A \cos b$ Two solutions
a, A, B	C	$\sin (C-K) = \dfrac{\cos A \sin K}{\cos B}$	$\cot K = \tan B \cos a$ Two solutions
	b	$\sin b = \dfrac{\sin a \sin B}{\sin A}$	Two solutions
	c	$\sin (c-M) = \cot A \tan B \sin M$	$\tan M = \cos B \tan a$ Two solutions

TABLE O42b. Formulas for solving oblique spherical triangles.

Calculus

O43. Definitions.—**Calculus** is that branch of mathematics dealing with the rate of change of one quantity with respect to another.

A **constant** is a quantity which does not change. If a vessel is making good a course of 090°, the latitude does not change and is therefore a constant.

A **variable,** where continuous, is a quantity which can have an infinite number of values, although there may be limits to the maximum and minimum. Thus, from latitude 30° to latitude 31° there are an infinite number of latitudes, if infinitesimally small units are taken, but no value is less than 30° nor more than 31°. If two variables are so related that for every value of one there is a corresponding value of the other, one of the values is known as a **function** of the other. Thus, if speed is constant, the distance a vessel steams depends upon the elapsed time. Since elapsed time does not depend upon any other quantity, it is called an **independent variable.** The distance depends upon the elapsed time, and therefore is called a **dependent variable.** If it is required to find the time needed to travel any given distance at constant speed, distance is the independent variable and time is the dependent variable.

The principal processes of calculus are differentiation and integration.

O44. Differentiation is the process of finding the rate of change of one variable with respect to another. If x is an independent variable, y is a dependent variable, and y is a function of x, this relationship may be written $y=f(x)$. Since for every value of x there is a corresponding value of y, the relationship can be plotted as a curve, figure O44. In this figure, A and B are any two points on the curve, a short distance apart.

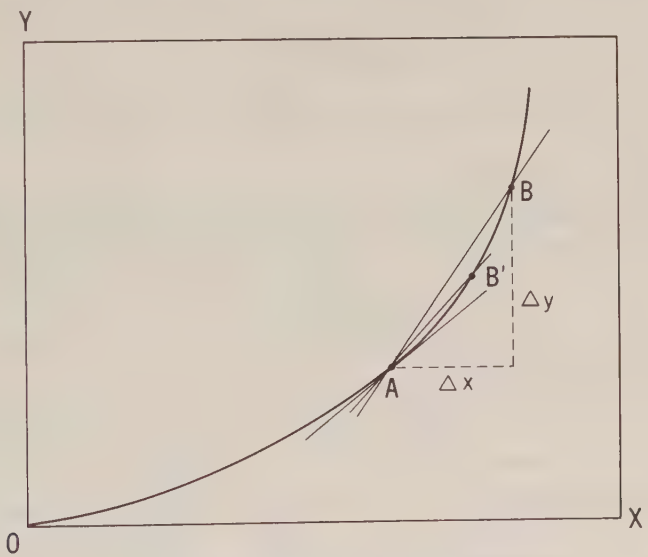


FIGURE O44. Differentiation.

The difference between the value of x at A and at B is Δx (delta x), and the corresponding difference in the value of y is Δy (delta y). The straight line through points A and B is a **secant** of the curve (art. O30). It represents the rate of change between A and B, for anywhere along this line the change of y is proportional to the change of x.

As B moves closer to A, as shown at B', both Δx and Δy become smaller, but at a different rate, and $\frac{\Delta y}{\Delta x}$ changes. This is indicated by the difference in the slope of the secant. Also, that part of the secant between A and B moves closer to the curve and becomes a better approximation of it. The limiting case occurs when B reaches A or is at an infinitesimal distance from it. As the distance becomes infinitesimal, both Δy and Δx become infinitely small, and are designated dy and dx, respectively. The straight line becomes tangent to the curve, and represents the rate of change, or slope, of the curve at that point. This is indicated by the expression $\frac{dy}{dx}$, called the **derivative** of y with respect to x.

The process of finding the value of the derivative is called **differentiation.** It depends upon the ability to connect x and y by an equation. For instance, if $y=x^n$, $\frac{dy}{dx}=nx^{n-1}$. If $n=2$, $y=x^2$, and $\frac{dy}{dx}=2x$. This is derived as follows: If point A on the curve is x, y; point B can be considered $x+\Delta x, y+\Delta y$. Since the relation $y=x^2$ is true anywhere on the curve, at B:

$$y+\Delta y=(x+\Delta x)^2=x^2+2x\Delta x+(\Delta x)^2.$$

Since $y=x^2$, and equal quantities can be subtracted from both sides of an equation without destroying the equality:

$$\Delta y=2x\Delta x+(\Delta x)^2.$$

Dividing by Δx:

$$\frac{\Delta y}{\Delta x}=2x+\Delta x.$$

As B approaches A, Δx becomes infinitesimally small, approaching 0 as a limit. Therefore $\frac{\Delta y}{\Delta x}$ approaches $2x$ as a limit.

This can be demonstrated by means of a numerical example. Let $y=x^2$. Suppose at A, $x=2$ and $y=4$, and at B, $x=2.1$ and $y=4.41$. In this case $\Delta x=0.1$ and $\Delta y=0.41$, and

$$\frac{\Delta y}{\Delta x}=\frac{0.41}{0.1}=4.1.$$

From the other side of the equation:

$$2x+\Delta x=2\times 2+0.1=4.1.$$

If Δx is 0.01 and Δy is 0.0401, $\frac{\Delta y}{\Delta x}=4.01$. If Δx is 0.001, $\frac{\Delta y}{\Delta x}=4.001$; and if Δx is 0.0001, $\frac{\Delta y}{\Delta x}=4.0001$. As Δx approaches 0 as a limit, $\frac{\Delta y}{\Delta x}$ approaches 4, which is therefore the value $\frac{dy}{dx}$. Therefore, at point A the *rate* of change of y with respect to x is 4, or y is increasing in value 4 times as fast as x.

An example of the use of differentiation in navigation is the Δd value in H.O. Pub. No. 214. This is the change of altitude for a change of 1′ of declination. In this case, declination is the independent variable, altitude is the dependent variable, and both meridian angle (H.A.) and latitude are constants. The rate of change at the tabulated value is desired, so that the table can be entered with the *nearest* tabulated

value of declination, and interpolation performed in either direction (either larger or smaller values of declination).

O45. Integration is the inverse of differentiation. Unlike the latter, however, it is not a direct process, but involves the recognition of a mathematical expression as the differential of a known function. The function sought is the **integral** of the given expression. Most functions can be differentiated, but many cannot be integrated.

Integration can be considered the summation of an infinite number of infinitesimally small quantities, between specified limits. Consider, for instance, the problem of finding an area below a specified part of a curve for which a mathematical expression can be

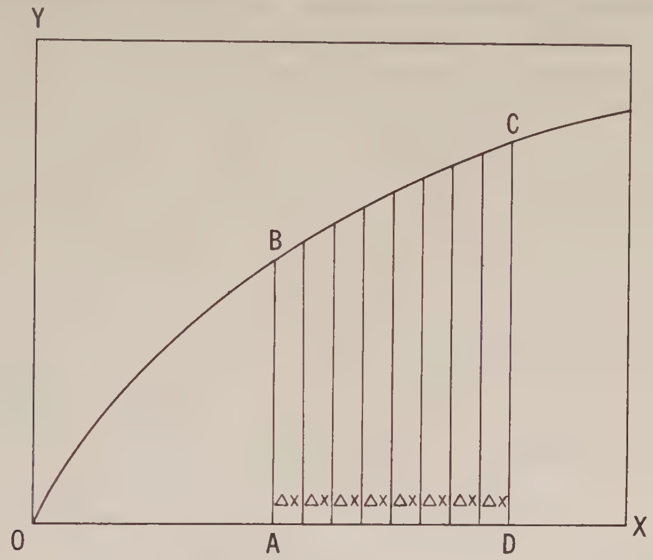


FIGURE O45. Integration.

written. Suppose it is desired to find the area *ABCD* of figure O45. If vertical lines are drawn dividing the area into a number of vertical strips, each Δx wide, and if y is the height of each strip at the midpoint of Δx, the area of each strip is approximately $y\Delta x$; and the approximate total area of all strips is the sum of the areas of the individual strips. This may be written $\sum_{x_1}^{x_2} y\Delta x$, meaning the sum of all $y\Delta x$ values between x_1 and x_2. The symbol $\sum$ is the Greek letter *sigma*, the equivalent of the English *S*. If Δx is made progressively smaller, the sum of the small areas becomes ever closer to the true total area. If Δx becomes infinitely small, the summation expression is written $\int_{x_1}^{x_2} ydx$, the symbol dx denoting an infinitely small Δx. The symbol $\int$, called the "integral sign," is a distorted *S*.

An expression such as $\int_{x_1}^{x_2} ydx$ is called a **definite integral** because limits are specified (x_1 and x_2). If limits are not specified, as in $\int ydx$, the expression is called an **indefinite integral.**

A navigational application of integration is the finding of meridional parts, table 5. The *rate* of change of meridional parts with respect to latitude changes progressively. The formula given in the explanation of the table is the equivalent of an integral representing the sum of the meridional parts from the equator to any given latitude.

O46. Differential equations.—An expression such as dy or dx is called a **differential.** An equation involving a differential or a derivative is called a **differential equation.**

As shown in article O44, if $y=x^2$, $\frac{dy}{dx}=2x$. Neither dy nor dx is a finite quantity, but both are *limits* to which Δy and Δx approach as they are made progressively smaller. Therefore $\frac{dy}{dx}$ is merely a ratio, the limiting value of $\frac{\Delta y}{\Delta x}$, and not one finite number divided by another. However, since the ratio is the same as would be obtained by using finite quantities, it is possible to use the two differentials dy and dx independently in certain relationships. Differential equations involve such relationships.

APPENDIX P

INTERPOLATION

P1. Introduction.—If one quantity varies with changing values of a second quantity, and the mathematical relationship of the two is known, a curve can be drawn to represent the values of one corresponding to various values of the other. To find the value of either quantity corresponding to a given value of the other, one finds that point on the curve defined by the given value, and reads the answer on the scale relating to the other quantity. This assumes, of course, that for each value of one quantity, there is only one value of the other quantity.

Information of this kind can also be tabulated. Each entry represents one point on the curve. The finding of a value *between* tabulated entries is called **interpolation.** The extending of tabulated values to find values *beyond* the limits of the table is called **extrapolation.**

Thus, the *Nautical Almanac* tabulates values of declination of the sun for each hour of Greenwich mean time. The finding of declination for a time between two whole hours requires interpolation. Since there is only one entering **argument** (in this case GMT), **single interpolation** is involved.

Table 19 gives the distance traveled in various times at certain speeds. In this table there are two entering arguments. If both given values are between tabulated values, **double interpolation** is needed.

In H.O. Pub. No. 214, azimuth angle varies with a change in any of the three variables latitude, declination, and meridian angle. With intermediate values of all three, **triple interpolation** is needed.

Interpolation can sometimes be avoided. A table having a single entering argument can be arranged as a **critical table.** An example is the dip (height of eye) correction on the inside front cover of the *Nautical Almanac*. In such a table limiting values of the entering argument are given. Another way of avoiding interpolation would be to include every possible entering argument. If this were done for H.O. Pub. No. 214, interpolation being eliminated for declination only, and assuming declination values to 0.′1, the number of volumes would be increased from nine to more than 5,000. If interpolation for meridian angle and latitude, to 0.′1, were also to be avoided, a total of more than 1,800,000,000 volumes would be needed. A more practical method is to select an assumed position to avoid the need for interpolation for two of the variables. For stars, which change declination slowly, interpolation for the third argument can be avoided by using values for the declination of each body in the preparation of the table, as in H.O. Pub. No. 249, volume I. Another way of avoiding interpolation is to portray the information graphically. Still another way is to solve the appropriate equation each time a value is needed.

Notwithstanding all these available devices, the need for interpolation is frequently encountered in navigation. The person who thoroughly understands it is least likely to make mistakes in its use.

P2. Single interpolation.—The accurate determination of intermediate values requires knowledge of the nature of the change between tabulated values. The simplest relationship is **linear,** the change in the tabulated value being directly proportional to

the change in the entering argument. Thus, if a vessel is proceeding at 15 knots, the distance traveled is directly proportional to the time, as shown in figure P2a. The same information might be given in tabular form, as shown in table P2a. Mathematically, this relationship is written $D=\frac{15t}{60}=\frac{t}{4}$, where D is distance in nautical miles, and t is time in minutes.

In such a table, interpolation can be accomplished by simple proportion. Suppose, for example, that the distance is desired for a time of 15 minutes. It will be some

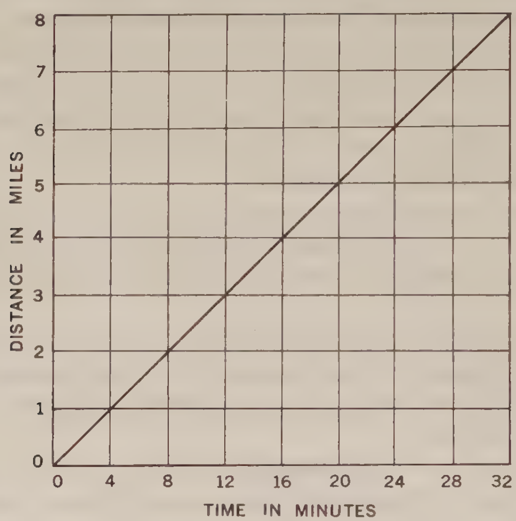


FIGURE P2a. Plot of $D=\frac{t}{4}$.

Minutes	*Miles*
0	0. 0
4	1. 0
8	2. 0
12	3. 0
16	4. 0
20	5. 0
24	6. 0
28	7. 0
32	8. 0

TABLE P2a. Table of $D=\frac{t}{4}$.

value between 3.0 and 4.0 miles, because these are the distances for 12 and 16 minutes, respectively, the tabulated times on each side of the desired time. The proportion might be formed as follows:

$$\begin{array}{lll} & 12 & \\ 3 & & \\ & 15 & 4 \\ & 16 & \end{array} \qquad\qquad \begin{array}{lll} & 3.0 & \\ x & & \\ & y & 1.0 \\ & 4.0 & \end{array}$$

$$\frac{3}{4}=\frac{x}{1.0}$$

$$x=\frac{3\times 1.0}{4}=0.75 \text{ (0.8 to nearest 0.1 mi.)}$$

$$y=3.0+x=3.0+0.8=3.8 \text{ mi.}$$

A simple interpolation such as this should be performed mentally. During the four-minute interval between 12 and 16 minutes, the distance *increases* 1.0 mile from 3.0 to 4.0 miles. At 15 minutes, ¾ of the interval has elapsed, and so the distance increases ¾ of 1.0 mile, or 0.75 mile, and is therefore 3.0+0.8=3.8, to the nearest 0.1 mile.

This might also have been performed by starting with 16 minutes, as follows:

$$1\left[\begin{array}{c|c} 12 & \\ 15 & 4 \\ 16 & \end{array}\right] \qquad\qquad (-)x\left[\begin{array}{c|c} 3.0 & \\ y & 1.0 \\ 4.0 & \end{array}\right]$$

$$\frac{1}{4}=\frac{(-)x}{1.0}$$

$x=(-)0.25$ (-0.2 to the nearest 0.1 mi.)

$y=4.0-0.2=3.8$

Mentally, 15 is one quarter of the way from 16 to 12, and therefore the distance is ¼ the way between 4.0 and 3.0, or 3.8.

This interpolation might have been performed by noting that if distance changes 1.0 mile in four minutes, it must change $\frac{1.0}{10}=0.1$ mile in $\frac{4}{10}=0.4$ minute, or 24 seconds. This relationship can be used for mental interpolation in situations which might seem to require pencil and paper. Thus, if distance to the nearest 0.1 mile is desired for $13^{m}15^{s}$, the answer is 3.3 miles, determined as follows: The time $13^{m}15^{s}$ is $1^{m}15^{s}$ ($1^{m}.2$ approx.) more than 12^{m}. If 1.2 is divided by 0.4, the quotient is 3, to the nearest whole number. Therefore, $3\times0.1=0.3$ is added to 3, the tabulated value for 12 minutes. Alternatively, $13^{m}15^{s}$ is $2^{m}45^{s}$ ($2^{m}.8$ approx.) less than 16^{m}, and $2.8\div0.4=7$, and therefore the interpolated value is $7\times0.1=0.7$ *less* than 4, the tabulated value for 16^{m}. In either case, the interpolated value is 3.3 miles.

A common mistake in single interpolation is to apply the correction (x) with the wrong sign, particularly when it should be negative ($-$). This mistake can be avoided by always checking to be certain that the interpolated value lies between the two values used in the interpolation.

When the curve representing the values of a table is a straight line, as in figure P2a, the process of finding intermediate values in the manner described above is called **linear interpolation.** If tabulated values of such a line are exact (not approximations), as in table P2a, the interpolation can be carried to any degree of precision without sacrificing accuracy. Thus, in 21.5 minutes the distance is $5.0+\frac{1.5}{4}\times1.0=5.375$ miles. Similarly, for 29.9364 minutes the distance is $7.0+\frac{1.9364}{4}\times1.0=7.4841$ miles, a value which has little or no significance in practical navigation. If one had occasion to find such a value, it could most easily be done by dividing the time, in minutes, by 4, since the distance increases at the rate of one mile each four minutes. This would be a case of avoiding interpolation by solving the equation connecting the two quantities. For a simple relationship such as that involved here, such a solution might be easier than interpolation.

Many of the tables of navigation are not linear. Consider figure P2b. From table 29 it is found that for latitude 25° and declination 8°, same name, the variation of altitude in one minute of time from meridian transit (the altitude factor) is $6''.0$ ($0'.1$). For a limited angular distance on each side of the celestial meridian, the change in altitude is approximately equal to at^2, where a is the altitude factor (from table 29)

and t is the time in minutes from meridian transit. Figure P2b is the plot of change in altitude against time. The same information is shown in tabular form in table P2b.

To be strictly accurate in interpolating in such a table, one should consider the curvature of the line. However, in most navigational tables the points on the curve selected for tabulation are sufficiently close that the portion of the curve between entries can be considered a straight line without introducing a significant error. This is similar to considering the line of position from a celestial observation as a part of the circle of equal altitude. Thus, to the nearest 0.′1, the change of altitude for 3.4 minutes is $0.'9+(0.4\times0.'7)=0.'9+0.'3=1.'2$. The correct value by solution of the formula is 1.′156. The value for 6.8 minutes is 4.′6 by interpolation and 4.′624 by computation.

If the *direction* of curvature of the curve changes between entering arguments, an erroneous result might be obtained. Thus, in H.O. Pub. No. 214, the tabulated altitude for latitude 53° and meridian angle 0° is 89°30.′0 for declination 52°30′ same name, and 89°00.′0 for declination 54°00′ same name, the next entry. By

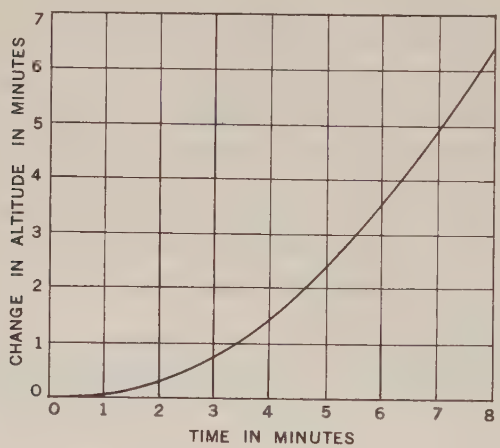


FIGURE P2b. Plot of altitude change=at^2.

Minutes	*Altitude change*
0	0. 0
1	0. 1
2	0. 4
3	0. 9
4	1. 6
5	2. 5
6	3. 6
7	4. 9
8	6. 4

TABLE P2b. Table of altitude change=at^2, where $a=0.'1$

linear interpolation for declination 53° same name the altitude is 89°20.′0. The correct value is 90°00.′0. Between declination 52°30′ and 53°00′ the altitude increases to 90°00.′0 and then decreases as declination increases. Such instances are infrequent in navigation, and generally occur at a part of the table that is not commonly used, or for which special provisions are made.

P3. Double interpolation.—In a double-entry table it may be necessary to interpolate for each entering argument. Table P3a is an extract from table 27 (amplitudes). If one entering argument is an exact tabulated value, the amplitude can be found by single interpolation. For instance, if latitude is 45° and declination is 21.°8, amplitude is $31.^\circ2+\left(\frac{3}{5}\times0.^\circ8\right)=31.^\circ2+0.^\circ5=31.^\circ7$. However, if neither entering argument is a tabulated value, double interpolation is needed. This may be accomplished in any of several ways:

Lat.	*Declination*	
	21.°5	22.°0
°	°	°
45	31. 2	32. 0
46	31. 8	32. 6

TABLE P3a. Excerpts from amplitude table.

1. *"Horizontal" method.* Use single interpolation for declination for each tabulated value of latitude, followed by single interpolation for latitude. Suppose latitude is 45°7 and declination is 21°8. First, find the amplitude for latitude 45°, declination 21°8, as above, 31°7. Next, repeat the process for latitude 46°: $31°8+\left(\frac{3}{5}\times 0°8\right)=32°3$. Finally, interpolate between 31°7 and 32°3 for latitude 45°7: $31°7+(0.7\times 0°6)=32°1$. This is the equivalent of first inserting a new column for declination 21°8, followed by single interpolation in this column, as shown in table P3b.

Lat.	*Declination*		
	21°5	*21°8*	22°0
°	°	°	°
45	31.2	*31.7*	32.0
45.7		***32.1***	
46	31.8	*32.3*	32.6

TABLE P3b. "Horizontal" method of double interpolation.

Lat.	*Declination*		
	21°5	*21°8*	22°0
°	°	°	°
45	31.2		32.0
45.7	*31.6*	***32.1***	*32.4*
46	31.8		32.6

TABLE P3c. "Vertical" method of double interpolation.

2. *"Vertical" method.* Use single interpolation for latitude for each tabulated value of declination, followed by single interpolation for declination. Consider the same example as above. First, find the amplitude for declination 21°5, latitude 45°7: $31°2+(0°7\times 0°6)=31°6$. Next, repeat the process for declination 22°0: $32°0+(0°7\times 0°6)=32°4$. Finally, interpolate between 31°6 and 32°4 for declination 21°8: $31°6+\left(\frac{3}{5}\times 0°8\right)=32°1$. This is the equivalent of first inserting a new line for latitude 45°7, followed by single interpolation in this line, as shown in table P3c.

3. *Combined method.* Select a tabulated "base" value, preferably that nearest the given tabulated entering arguments. Next, find the correction to be applied, with its sign, for single interpolation of this base value both horizontally and vertically. Finally, add these two corrections algebraically and apply the result, in accordance with its sign, to the base value. In the example given above, the base value is 32°6, for declination 22°0 (21°8 is nearer 22°0 than 21°5) and latitude 46° (45°7 is nearer 46° than 45°). The correction for declination is $\frac{2}{5}\times(-)\,0°8=(-)\,0°3$. The correction for latitude is $0°3\times(-)\,0°6=(-)\,0°2$. The algebraic sum is $(-)\,0°3+(-)\,0°2=(-)\,0°5$. The interpolated value is then $32°6-0°5=32°1$. This is the method customarily used by navigators.

P4. Triple interpolation.—With three entering arguments, the process is similar to that for double interpolation. It would be possible to perform double interpolation for the tabulated value on each side of the given value of one argument, and then interpolate for that argument, but the method would be tedious. The only method commonly used by navigators is that of selecting a base value and applying corrections. Suppose, for instance, that the azimuth angle is desired for latitude 41°3, declination 21°9 contrary name, meridian angle 16°6 using H.O. Pub. No. 214. The base value (lat. 41°, dec. 22°, t 17°) is 162°6. The corrections are

lat. 41°3:	0.3 × (+) 0°1=	0°0
dec. 21°9:	0.2 × (−) 0°1=	0°0
t 16°6:	0.4 × (−) 1°0=(−)	0°4
	Total (−)	0°4.

The triple interpolated value is 162°6−0°4=162°2. A convenient navigational form for solving this problem is shown in article 2007 and appendix Q.

P5. Interpolation tables.—A number of frequently used navigation tables are provided with auxiliary tables to assist in interpolation. Table 32 (Logarithms of Numbers) provides columns of "d" (difference between consecutive entries) and auxiliary "proportional parts" tables. The auxiliary table for the applicable difference "d" is selected and entered with the digit of the additional place in the entering argument. The value taken from the auxiliary table is *added* to the base value for the next *smaller* number from the main table. Suppose the logarithm (mantissa) for 32747 is desired. The base value for 3274 is 51508, and "d" is 13. The auxiliary table for 13 is entered with 7, and the correction is found to be 9. If this is added to 51508, the interpolated value is found to be 51517. This is the same result that would be obtained by subtracting 51508 from 51521 (the logarithm for 3275) to obtain 13, multiplying this by 0.7, and adding the result (9) to 51508.

Tables 31 and 33 provide the difference between consecutive entries, but no proportional parts tables.

In H.O. Pub. No. 214, Δd and Δt values are given, with "multiplication tables" to make the correction. The use of these tables is explained in chapter XX. The Δd of H.O. Pub. No. 249 (vols. II, III) is similar, except that a sign is given and interpolation is always made from the tabulated value of declination next *smaller* than the given value. This table is explained in chapter XXI.

The *Nautical Almanac* "Increments and Corrections" are interpolation tables for the hourly entries of GHA and declination. The use of these tables and the interpolation tables of the *Air Almanac* is explained in chapter XVIII.

The method of table 3 of using additional decimal places is still another form of interpolation.

P6. Extrapolation.—The extending of a table is usually performed by assuming that the difference between the last few tabulated entries will continue at the same rate. This assumption is strictly correct only if the change is truly linear, but in most tables the assumption provides satisfactory results for a *slight* extension beyond tabulated values. The extent to which the assumption can be used reliably can often be determined by noting the last few differences. If the "second differences" (differences between consecutive differences) are nearly zero, the curve is nearly a straight line, for a short distance. But if consecutive second differences are appreciable, extrapolation is not reliable. For examples of linear and nonlinear relationships, refer to the first page of table 33 and compare the tabulated differences of the logarithms of secant (approximately linear on this page) and sine (nonlinear on this page).

As an example of extrapolation, consider table 27. Suppose the amplitude for latitude 45°, declination 24°3 is desired. The last declination entry is 24°0. The amplitude for declination 23°5 is 34°3, and for declination 24°0 it is 35°1. The difference is (+) 0°8. Assuming this same difference between declinations 24°0 and 24°5, one finds the value for 24°3 is $35^\circ1+\left(\frac{3}{5}\times 0^\circ8\right)=35^\circ6$. Below latitude 50° this table is so nearly linear that extrapolation can be carried to declination 30° without serious error.

For double or triple extrapolation, differences are found as in single interpolation.

P7. General comments.—As a general rule, the final answer should not be given to greater precision than tabulated values. A notable exception to this rule is the case where tabulated values are known to be exact, as in table P2a. A slight increase in accuracy can sometimes be attained by retaining one additional place in the solution until the final answer. Suppose, for instance, that the corrections for triple interpolation are (+)0.2, (+)0.3, and (−)0.3. The total correction is (+)0.2. If the total

correction, rounded to tenths, had been obtained from the sum of (+)0.17, (+)0.26, and (−)0.34, the correct total would have been (+)0.09=(+)0.1. The retaining of one additional place may be critical if the correction factors end in 0.5. Thus, in double interpolation, one correction value might be (+)0.15, and the other (−)0.25. The correct total is (−)0.1. But if the individual differences are rounded to (+)0.2 and (−)0.2, the total is 0.0.

The difference used for establishing the proportion is also a matter subject to some judgment. Thus, if the latitude is 17°14.′6, it might be rounded to 17.°2 for many purposes. Slightly more accurate results can sometimes be obtained by retaining the minutes, using $\frac{14.6}{60}$ instead of 0.2. If the difference to be multiplied by this proportion is small, the increase in accuracy gained by using the more exact value is small, but if the difference is large, the gain might be considerable. Thus, if the difference is 0.°2, the correction by using either $\frac{14.6}{60}$ or 0.2 is less than 0.°05, or 0.°0 to the nearest 0.°1. But if the difference is 3.°2, the value by $\frac{14.6}{60}$ is 0.°8, and the value by 0.2 is 0.°6.

If the tabulated entries involved in an interpolation are all positive or all negative, the interpolation can be carried out on either a numerical or an algebraic basis. Most navigators prefer the former, carrying out the interpolation as if all entries were positive, and giving to the interpolated value the common sign of all entries. When both positive and negative entries are involved, all differences and corrections should be on an algebraic basis, and careful attention should be given to signs. Thus, if single interpolation is to be performed between values of (+)0.9 and (−)0.4, the difference is 0.9−(−0.4)=0.9+0.4=1.3. If the correction is 0.2 of this difference, it is (−)0.3 if applied to (+)0.9, and (+)0.3 if applied to (−)0.4. In the first case, the interpolated value is (+)0.9−0.3=(+)0.6. In the second case, it is (−)0.4+0.3=(−)0.1. If the correction had been 0.4 of the difference, it would have been (−)0.5 in the first case, and (+)0.5 in the second. The interpolated value would have been (+)0.9−0.5=(+)0.4, or (−)0.4+0.5=(+)0.1, respectively.

With practice, much of the interpolation used in navigation can be performed mentally, and is not customarily shown in the work forms. Notable exceptions are the interpolation for GHA and declination in the almanacs, and interpolation for declination (and meridian angle and latitude, if used) in H.O. Pub. No. 214.

Because of the variety in methods of interpolation used, solutions by different persons may differ slightly.

APPENDIX Q

WORK FORMS

The use of standard work forms reduces the probability of mistakes, by relieving the mind of details taken care of in the forms. It also provides a permanent record that can be checked, and that improves the appearance of the navigator's work book. The best forms to use are those which seem easiest, and provide a solution with the least probability of mistakes. The forms used throughout this book have been found effective in teaching navigation. The more commonly used ones are repeated on the following pages.

The individual navigator may wish to develop his own forms to reflect his own personal preferences. If the addition of a line or label, or the shifting of position of some part of a form assists in the avoidance of mistakes, or makes the solution seem easier, it serves a useful purpose. The mere changing for the sake of change, on the other hand, may encourage mistakes. The forms of this book are the result of considerable thought and the application of logic. The new navigator would do well to start with them, making changes only as the need arises.

The principal change sometimes made is the placing of a sight reduction form in a single column so that several observations can be solved in parallel columns. Methods such as H.O. Pub. No. 214 (Δd only) and H.O. Pub. No. 249 lend themselves readily to this type solution. A solution by cosine-haversine formula and some of the "short" methods of chapter XXI do not.

In these forms, and throughout the book, the standard abbreviations and symbols of appendix A are used.

The best use of a form is to first copy the entire form, then fill in *all* given or known information, and then proceed with the solution.

In the forms, entries such as "NS" or "TA" are given to indicate that a label is needed. Only the applicable label should be used. Where "Local date" or "Gr. date" appears, the actual date should be used. Zeros are used to indicate the units to use and, in general, the number of places that should be used. The numbers given in parentheses are article numbers where an example is given of the solution of a problem by the use of the form or forms.

Mercator Sailing (art. 817)

Course and distance by computation:

L_1 00°00′.0 NS	M_1 0000.0	λ_1 000°00′.0 EW
L_2 00°00′.0 NS	M_2 0000.0	λ_2 00°00′.0 EW
l 0°00′.0 NS	m 000.0	DLo 00°00′.0 EW
l 000′.0 NS		DLo 0000′.0 EW
DLo 0000′.0 EW	log 0.00000	
m 000.0	log (−)0.00000	
C NS 00°00′.0 EW	l tan 0.00000	l sec 0.00000
l 000′.0 NS		log 0.00000
D 0000.0 mi.		log 0.00000
Cn 000°.0		

Course and distance by traverse table:

L_1 00°00′.0 NS	M_1 0000.0	λ_1 000°00′.0 EW		
L_2 00°00′.0 NS	M_2 0000.0	λ_2 00°00′.0 EW		
l 0°00′.0 NS	m 000.0	DLo 00°00′.0 EW		
l 000′.0 NS		DLo 0000′.0 EW		
DLo 0000′.0 EW	log 0.00000	l	D (000°)	D (000°)
m 000.0	log (−)0.00000	000.0	000.0	000.0
DLo÷m 0.000	log 0.00000	00.0	000.0	000.0
C NS 00°.0 EW		0.0	00.0	00.0
Cn 000°.0		0.0	0.0	0.0
D 0000.0 mi.		000.0	000.0	000.0

Great-circle Sailing (art. 822)

Course and distance:

λ_1 000°00′.0 EW			D 000°00′.0
λ_2 000°00′.0 EW			coL_1 00°00′.0
DLo 000°00′.0 EW	l hav 0.00000		D∼coL_1 00°00′.0
L_1 00°00′.0 NS	l cos 0.00000		l sec 0.00000
L_2 00°00′.0 NS	l cos 0.00000		
θ	l hav 0.00000	n hav 0.00000	
l 00°00′.0 NS		n hav 0.00000	
D 000°00′.0		n hav 0.00000	l csc 0.00000
coL_2 00°00′.0	n hav 0.00000		
D∼coL_1 00°00′.0	n hav (−) 0.00000		
	n hav 0.00000		l hav 0.00000
Cn 000°.0		C NS 00°00′.0 EW	l hav 0.00000
D 0000.0 mi.			

Vertex:

			l cos 0.00000
L_1 00°00′.0 NS	l cos 0.00000		
C NS 00°00′.0 EW	l sin 0.00000	l cos 0.00000	
L_v 00°00′.0 NS	l cos 0.00000	l csc 0.00000	
λ_v 000°00′.0 EW	DLo_v 00°00′.0 EW	l sin 0.00000	l sin 0.00000
D_v 00°00′.0			l sin 0.00000
D_v 0000.0 mi.			

Points along the great circle:

DLo_{vx}	00°00′.0	00°00′.0	00°00′.0	00°00′.0	00°00′.0
$l \cos DLo_{vx}$	0.00000	0.00000	0.00000	0.00000	0.00000
$l \tan L_v$	0.00000	0.00000	0.00000	0.00000	0.00000
$l \tan L_x$	0.00000	0.00000	0.00000	0.00000	0.00000
L_x	00°00′.0NS	00°00′.0NS	00°00′.0NS	00°00′.0NS	00°00′.0NS
λ_x	000°00′.0EW	000°00′.0EW	000°00′.0EW	000°00′.0EW	000°00′.0EW
λ_x	000°00′.0EW	000°00′.0EW	000°00′.0EW	000°00′.0EW	000°00′.0EW

Correction of Sextant Altitude (arts. 1628–1632)

Solution by Nautical Almanac:

Sun LL

	+	⊙	−
IC	0′.0		
D			0′.0
⊙	00′.0		
sum	00′.0		0′.0
corr.			(±)0′.0
hs			00°00′.0
Ho			00°00′.0

Moon UL

	+	☾	−
IC			0′.0
D			0′.0
☾	00′.0		
U	0′.0		
add'l			30′.0
sum	00′.0		00′.0
corr.			(±)00′.0
hs			00°00′.0
Ho			00°00′.0

Venus, Mars

	+	V, M	−
IC			0′.0
D			0′.0
☆-P			0′.0
add'l	0′.0		
sum	0′.0		0′.0
corr.			(±)0′.0
hs			00°00′.0
Ho			00°00′.0

Jupiter, Saturn, Star

	+	J, S, ☆	−
IC	0′.0		
D			0′.0
☆-P			0′.0
sum	0′.0		0′.0
corr.			(±)0′.0
hs			00°00′.0
Ho			00°00′.0

Solution by Air Almanac:

Sun UL

	+	⊙	−
IC	0′		
D			0′
R			0′
SD			00′
sum	0′		00′
corr.			(±)00′
hs			00°00′
Ho			00°00′

Moon LL

	+	☾	−
IC			0′
D			0′
R			0′
SD	00′		
P	00′		
sum	00′		0′
corr.			(±)00′
hs			00°00′
Ho			00°00′

Planet, Star

	+	P, ☆	−
IC	0′		
D			0′
R			0′
sum	0′		0′
corr.			(±)0′
hs			00°00′
Ho			00°00′

Low altitude observation, sun:

Nautical Almanac

	+	⊙	−
IC	0′.0		
D			0′.0
sum	0′.0		0′.0
corr.			(±)0′.0
hs			0°00′.0
hr			0°00′.0
⊙			00′.0
TB	0′.0		
sum	0′.0		0′.0
corr.			(±)0′.0
hr			0°00′.0
Ho			0°00′.0

Tables 23, 24

	+	⊙	−
IC	0′.0		
D			0′.0
sum	0′.0		0′.0
corr.			(±)0′.0
hs			0°00′.0
hr			0°00′.0
⊙			00′.0
T	0′.0		
B			0′.0
sum	00′.0		00′.0
corr.			(±)00′.0
hr			0°00′.0
Ho			0°00′.0

Air Almanac

	+	⊙	−
IC	0′		
D			0′
sum	0′		0′
corr.			(±)0′
hs			0°00′
hr			0°00′
R			00′
B	0′		
SD	00′		
sum	00′		00′
corr.			(±)00′
hr			0°00′
Ho			0°00′

Note: Some corrections may be either (+) or (−). See text if in doubt.

Sight Reduction by H.O. Pub. No. 214 (art. 2008)

Solution by Δd only and Nautical Almanac, *sun observation:*

	Local date		Sun		+ ⊙	−
GMT	$00^{h}00^{m}00^{s}$ Gr. date	00^{h}	00°00′.0 NS *d*	IC		0′.0
00^{h}	00°00′.0	corr.	(±)0′.0 (±)0	D		0′.0
$00^{m}00^{s}$	0°00′.0	d	00°00′.0 NS	⊙	00′.0	
GHA	00°00′.0			sum	00′.0	0′.0
*a*λ	00°00′.0 EW			corr.		(±)00′.0
LHA	00°00′.0			hs		00°00′.0
t	00°00′.0 EW			Ho		00°00′.0
d	00°00′.0 NS	d diff.	0′.0			
*a*L	00°00′.0 NS			Z	NS 000°.0 EW	
ht	00°00′.0	Δd	(±)0.00			
corr.	(±)0′.0					
Hc	00°00′.0					
Ho	00°00′.0					
a	0.0 TA	*a*L	00°00′.0 NS			
Zn	000°.0	*a*λ	00°00′.0 EW			

Sight Reduction by H.O. Pub. No. 249 (art. 2113)

Solution by volume I and Air Almanac:

	Local date		Name of star		+ ☆	−
GMT	$00^{h}00^{m}00^{s}$ Gr. date			IC	0′	
$00^{h}00^{m}$	000°00′			D		0′
$0^{m}00^{s}$	0°00′			R		0′
GHA ♈	000°00′			sum	0′	0′
*a*λ	000°00′ EW			corr.		(±)0′
LHA ♈	000°00′			hs		00°00′
*a*L	00°00′ NS			Ho		00°00′
Hc	00°00′					
Ho	00°00′					
a	00 TA	*a*L	00°00′ NS			
Zn	000°	*a*λ	00°00′ EW			

Solution by volume II or III and Air Almanac:

	Local date		Name of body		+ P	−
GMT	$00^{h}00^{m}00^{s}$ Gr. date			IC	0′	
$00^{h}00^{m}$	000°00′			D		0′
$0^{m}00^{s}$	0°00′			R		0′
GHA	000°00′			sum	0′	0′
*a*λ	000°00′ EW			corr.		(±)0′
LHA	000°00′			hs		00°00′
d	00°00′ NS	d diff.	00′	Ho		00°00′
*a*L	00°00′ NS					
ht	00°00′	"d"	(±)00	Z	NS 000° EW	
corr.	(±)00′					
Hc	00°00′					
Ho	00°00′					
a	0 TA	*a*L	00°00′ NS			
Zn	000°	*a*λ	000°00′ EW			

Sight Reduction by Cosine-Haversine Formula (art. 2109)

	Local date		Name of body				+ ☆	−
GMT	$00^h00^m00^s$ Gr. date					IC	0′.0	
00^h	000°00′.0					D		0′.0
00^m00^s	0°00′.0					☆-P		0′.0
SHA	000°00′.0					sum	0′.0	0′.0
GHA	000°00′.0					corr.		(±) 0′.0
$a\lambda$	00°00′.0 EW					hs		00°00′.0
LHA	00°00′.0					Ho		00°00′.0
t	00°00′.0 EW	l hav	0.00000			l sin	0.00000	
aL	00°00′.0 NS	l cos	0.00000					
d	00°00′.0 NS	l cos	0.00000			l cos	0.00000	
θ		l hav	0.00000	n hav	0.00000			
L∼d	00°00′.0			n hav	0.00000			
z	00°00′.0			n hav	0.00000			
Hc	00°00′.0					l sec	0.00000	
Ho	00°00′.0			Z	NS 00°00′.0 EW	l sin	0.00000	
a	0.0 TA	aL	00°00′.0 NS					
Zn	000°.0	$a\lambda$	00°00′.0 EW					

Sight Reduction of Polaris Observation (art. 2105)

Solution by Nautical Almanac

	Local date		Polaris			+ ☆	−
GMT	$00^h00^m00^s$ Gr. date		+	−	IC		0′.0
00^h	000°00′.0	a_0	00′.0		D		0′.0
00^m00^s	0°00′.0	a_1	0′.0		☆-P		0′.0
GHA ♈	000°00′.0	a_2	0′.0		sum	—	0′.0
λ	00°00′.0 EW	add'l		60′.0	corr.		(±) 0′.0
LHA ♈	000°00′.0	sum	00′.0	00′.0	hs		00°00′.0
Ho	00°00′.0	corr.		(±) 00′.0	Ho		00°00′.0
corr.	(±) 00′.0						
L	00°00′.0 N						

Azimuth

Solution by H.O. Pub. No. 214 (art. 2007):

	Local date						
GMT	$00^h00^m00^s$ Gr. date						
00^h	000°00′.0						
00^m00^s	0°00′.0						
GHA	000°00′.0						
$a\lambda$	00°00′.0 EW						
LHA	000°00′.0					+	−
t	00°.0 EW	t diff.	0°.0	Z diff. (±) 0°.0	t corr.	0°.0	
d	00°.0 NS	d diff.	0°.0	Z diff. (±) 0°.0	d corr.		0°.0
L	00°.0 NS	L diff.	0°.0	Z diff. (±) 0°.0	L corr.	0°.0	
tab.	000°.0				sum	0°.0	0°.0
corr.	(±) 0°.0				corr.		(±) 0°.0
Z	NS 000°.0 EW						
Zn	000°.0						

Solution by H.O. Pub. No. 260 or H.O. Pub. No. 261 (art. 2126):

	Local date					
GMT	$00^h00^m00^s$ Gr. date					
00^h	000°00.′0					
00^m00^s	0°00.′0					
GHA	000°00.′0					
aλ	000°00.′0 EW					
LHA	000°00.′0					
t	00°00.′0 EW	diff. for	diff.	corr. for	+	−
t	$0^h00^m.0$ EW	10^m	(±) 00′	$0^m.0$		00′
d	0°.0 NS	1°	(±) 00′	0°.0	00′	
L	00°.0 NS	1°	(±) 00′	0°.0	00′	
tab.	000°00′			sum	00′	00′
corr.	(±) 0°00′			corr.		(±) 00′
Z	NS 000°00′ EW					
Zn	000°.0					

Sunrise, Sunset, Twilight

Solution by Nautical Almanac *(arts. 1810, 1811):*

L 00°00.′0 NS Local date
λ 000°00.′0 EW

	Sunrise		Sunset
00°NS	0000	00°NS	0000
T I	(±) 00	T I	(±) 00
LMT	0000	LMT	0000
dλ	(±) 00	dλ	(±) 00
ZT	0000	ZT	0000
	Twilight		Twilight
00°NS	0000	00°NS	0000
T I	(±) 00	T I	(±) 00
LMT	0000	LMT	0000
dλ	(±) 00	dλ	(±) 00
ZT	0000	ZT	0000

Solution by Air Almanac *(art. 1811):*

L 00°00.′0 NS Local date
λ 00°00.′0 EW

	Sunrise		Sunset
00°NS	0000	00°NS	0000
corr.	(±) 00	corr.	(±) 00
LMT	0000	LMT	0000
dλ	(±) 00	dλ	(±) 00
ZT	0000 (sunrise)	ZT	0000 (sunset)
dur.	(−) 00	dur.	(+) 00
ZT	0000 (twilight)	ZT	0000 (twilight)

Moonrise, Moonset (art. 1812)

Solution by Nautical Almanac:

L 00°00′.0 NS Local date
λ 00°00′.0 EW

	Moonrise			Moonset	
00° NS	0000	Date	00° NS	0000	Date
T I	(±)00		T I	(±)00	
LMT (G)	0000	Date	LMT (G)	0000	Date
00° NS	0000	Date	00° NS	0000	Date
T I	(±)00		T I	(±)00	
LMT (G)	0000	Date	LMT (G)	0000	Date
LMT (G)	0000	Date	LMT (G)	0000	Date
diff.	00		diff.	00	
T II	(±)00		T II	(±)00	
LMT (G)	0000	Date	LMT (G)	0000	Date
LMT	0000	Date	LMT	0000	Date
dλ	(±)00		dλ	(±)00	
ZT	0000	Date	ZT	0000	Date

Solution by Air Almanac:

L 00°00′.0 NS Local date
λ 000°00′.0 EW

	Moonrise		Moonset
diff.	(±)00	diff.	(±)00
00°NS	0000	00°NS	0000
corr.	(±)00	corr.	(±)00
LMT (G)	0000	LMT (G)	0000
corr.	(±)00	corr.	(±)00
LMT	0000	LMT	0000
dλ	(±)00	dλ	(±)00
ZT	0000	ZT	0000

APPENDIX R

BEAUFORT SCALE

WITH CORRESPONDING SEA STATE CODES

Beaufort number	Wind speed: knots	Wind speed: mph	Wind speed: meters per second	Wind speed: km per hour	Seaman's term	World Meteorological Organization (1964)	Estimating wind speed: Effects observed at sea	Estimating wind speed: Effects observed on land	Hydrographic Office: Term and height of waves, in feet	Hydrographic Office: Code	World Meteorological Organization: Term and height of waves, in feet	World Meteorological Organization: Code
0	under 1	under 1	0.0–0.2	under 1	Calm	Calm	Sea like mirror.	Calm; smoke rises vertically.	Calm, 0	0	Calm, glassy, 0	0
1	1–3	1–3	0.3–1.5	1–5	Light air	Light air	Ripples with appearance of scales; no foam crests.	Smoke drift indicates wind direction; vanes do not move.	Smooth, less than 1	1		
2	4–6	4–7	1.6–3.3	6–11	Light breeze	Light breeze	Small wavelets; crests of glassy appearance, not breaking.	Wind felt on face; leaves rustle; vanes begin to move.	Slight, 1–3	2	Calm, rippled, 0–⅓	1
3	7–10	8–12	3.4–5.4	12–19	Gentle breeze	Gentle breeze	Large wavelets; crests begin to break; scattered whitecaps.	Leaves, small twigs in constant motion; light flags extended.	Moderate, 3–5	3	Smooth, wavelets, ⅓–1⅔	2
4	11–16	13–18	5.5–7.9	20–28	Moderate breeze	Moderate breeze	Small waves, becoming longer; numerous whitecaps.	Dust, leaves, and loose paper raised up; small branches move.			Slight, 2–4	3
5	17–21	19–24	8.0–10.7	29–38	Fresh breeze	Fresh breeze	Moderate waves, taking longer form; many whitecaps; some spray.	Small trees in leaf begin to sway.	Rough, 5–8	4	Moderate, 4–8	4
6	22–27	25–31	10.8–13.8	39–49	Strong breeze	Strong breeze	Larger waves forming; whitecaps everywhere; more spray.	Larger branches of trees in motion; whistling heard in wires.			Rough, 8–13	5
7	28–33	32–38	13.9–17.1	50–61	Moderate gale	Near gale	Sea heaps up; white foam from breaking waves begins to be blown in streaks.	Whole trees in motion; resistance felt in walking against wind.				
8	34–40	39–46	17.2–20.7	62–74	Fresh gale	Gale	Moderately high waves of greater length; edges of crests begin to break into spindrift; foam is blown in well-marked streaks.	Twigs and small branches broken off trees; progress generally impeded.	Very rough, 8–12	5	Very rough, 13–20	6
9	41–47	47–54	20.8–24.4	75–88	Strong gale	Strong gale	High waves; sea begins to roll; dense streaks of foam; spray may reduce visibility.	Slight structural damage occurs; slate blown from roofs.	High, 12–20	6		
10	48–55	55–63	24.5–28.4	89–102	Whole gale	Storm	Very high waves with overhanging crests; sea takes white appearance as foam is blown in very dense streaks; rolling is heavy and visibility reduced.	Seldom experienced on land; trees broken or uprooted; considerable structural damage occurs.	Very high, 20–40	7	High, 20–30	7
11	56–63	64–72	28.5–32.6	103–117	Storm	Violent storm	Exceptionally high waves; sea covered with white foam patches; visibility still more reduced.		Mountainous, 40 and higher	8	Very high, 30–45	8
12	64–71	73–82	32.7–36.9	118–133	Hurricane	Hurricane	Air filled with foam; sea completely white with driving spray; visibility greatly reduced.	Very rarely experienced on land; usually accompanied by widespread damage.	Confused	9	Phenomenal, over 45	9
13	72–80	83–92	37.0–41.4	134–149								
14	81–89	93–103	41.5–46.1	150–166								
15	90–99	104–114	46.2–50.9	167–183								
16	100–108	115–125	51.0–56.0	184–201								
17	109–118	126–136	56.1–61.2	202–220								

Note: Since January 1, 1955, weather map symbols have been based upon wind speed in knots, at five-knot intervals, rather than upon Beaufort number.

APPENDIX S

MARITIME POSITIONS

With the 1958 edition, this appendix has been completely revised to reflect changes in United States Government approved names and to provide ready reference to sources of additional information about the individual entries. With very few exceptions, the newly approved names are native names. This revision presented an opportunity to provide a world-wide listing of such names. Well-known secondary names from older charts and publications are listed in parentheses, following the approved name. Abbreviations have been avoided wherever possible, except for the frequently used Island (I. or I), Islands (Is. or Is), and Light (Lt), which are usually abbreviated, except when used as headings.

The appendix contains 25 major headings (**ARCTIC REGIONS**). Lesser headings, given in boldface type (**Greenland**), have been chosen from both political and geographical subdivisions. Where desirable, a further subdivision is provided by the use of headings given in capitals and small capitals (STREYMOY (STROMO ISLAND)). Entries preceded by dashes follow each such capitalized heading, the dashes indicating that such entries are integral parts of that heading.

All entries appearing in italics (*Thule*) are listed in H.O. Pub. No. 150, *World Port Index*. All entries followed by the abbreviation Lt (Kajartalik: Lt) are listed in the appropriate H.O. light list for that area. Coordinates for these two types of entries were obtained from the above-mentioned publications. The remaining entries and their coordinates were selected from charts and sailing directions.

Where the larger political subdivision (State or nation) of a port (italic entry) is not otherwise apparent, this information is included as part of the entry. Because of changing political boundaries, geographic names or their spellings do not necessarily reflect recognition by the United States Government of the political status of an entry.

Because some of the newly approved names may not be familiar to all users, both the approved names and the secondary names are listed in the alphabetical index. For this reason, the alphabetical index should be used only as an aid to locating entries in the main listing. *In all cases* the main listing should be referred to for positive identification of the desired position and the approved name.

The index has been extensively cross-indexed, particularly in the case of transliterated and hyphenated names. In some instances strict alphabetization has been sacrificed in the interest of logic. For example, Spain, east coast and Spain, north coast are followed by Spain, south coast, rather than by Spain, Port-of-.

Each entry in the alphabetical index is identified by a four- or five-digit number corresponding to the numerical sequence in the main listing. Where two or more alphabetical entries possess identical or similar names, the general location is provided in parentheses, following the entry. Thus the user may easily distinguish between Aberdeen (Scotland) and Aberdeen (Washington).

Entries are listed in the following geographical arrangement.

APPENDIX S

MARITIME POSITIONS

ARCTIC REGIONS

Index No.	Place	Lat.	Long.
1000	**Greenland**	° ′	° ′
1010	Kap Morris Jesup	83 39 N	34 18 W
020	Dragon Point	82 18 N	53 00 W
030	Kap Stanton	82 13 N	57 10 W
040	Kap Brevoort	81 58 N	60 15 W
050	Thank God Harbor	81 38 N	61 44 W
060	Kap Bryan	81 06 N	64 05 W
070	Kap Jackson	80 00 N	67 28 W
080	Etah	78 19 N	72 45 W
090	Kap Alexander	78 10 N	73 09 W
1100	Kap Parry	77 01 N	71 10 W
110	*Thule*	76 32 N	68 50 W
120	Kap Atholl	76 23 N	69 38 W
130	Kap York	75 53 N	66 25 W
140	Kap Melville	76 01 N	63 40 W
150	*Upernivik*	72 46 N	56 09 W
160	*Prøven*	72 22 N	55 34 W
170	Svartenhuk	71 41 N	55 53 W
180	*Umanak*	70 41 N	52 09 W
190	*Godhavn, Disko*	69 14 N	53 32 W
1200	*Jakobshavn*	69 13 N	51 06 W
210	*Christianshaab*	68 49 N	51 11 W
220	*Egedesminde*	68 42 N	52 52 W
230	*Holsteinsborg*	66 56 N	53 42 W
240	Camp Lloyd, Sondre Stromfjord	66 58 N	50 57 W
250	*Kangâmiut*	65 49 N	53 18 W
260	*Sukkertoppen*	65 25 N	52 56 W
270	*Godthaab*	64 11 N	51 39 W
280	*Færingehavn*	63 42 N	51 33 W
290	*Fiskenæsset*	63 05 N	50 42 W
1300	Ravns Storø Havns	62 43 N	50 25 W
310	*Frederikshaab*	62 00 N	49 43 W
320	Kajartalik: Lt	61 10 N	48 32 W
330	*Ivigtut*	61 12 N	48 11 W
340	Arsuk Ø	61 08 N	48 20 W
350	*Narsarssuak*	61 08 N	45 25 W
360	*Julianehaab*	60 43 N	46 02 W
370	*Sydprøven (South Prøven)*	60 28 N	45 34 W
380	*Nanortalik*	60 08 N	45 15 W
390	*Frederiksdal*	60 00 N	44 40 W
1400	Kap Farvel (Cape Farewell)	59 45 N	43 53 W
410	Igalalik I., Prince Christian Sound	60 02 N	43 06 W
420	Kap Tordenskjold	61 24 N	42 20 W
430	Kap Bille	62 10 N	42 03 W
440	Kap Juel: Peak	63 16 N	41 07 W
450	Kap Løvenørn	64 31 N	40 07 W
460	Dannebrogs Ø (Kivdlak I.): Cairn	65 19 N	39 28 W
470	*Angmagssalik*	65 36 N	37 38 W
480	Kap Irminger	68 04 N	30 56 W
490	Rignys Bjerg (Mount Rigney)	69 00 N	26 14 W
1500	Kap Brewster	70 09 N	22 03 W
510	*Scoresbysund*	70 29 N	21 58 W
520	Kap Wardlaw	71 44 N	21 52 W
530	Bontekoe Ø	73 07 N	21 20 W
540	*Eskimonæs*	74 05 N	21 16 W
550	Lille Pendulum I	74 40 N	18 27 W
560	*Nanok, Fangst Station*	75 09 N	19 47 W
570	Kap Philip Broke	74 56 N	17 37 W
580	Kap Bismarck	76 42 N	18 36 W
590	Nordostrundingen (Northeast Foreland)	81 22 N	11 45 W
1600	**Jan Mayen Island**		
1610	Sørkapp (South Cape)	70 50 N	8 59 W
620	Nordaustkapp (Northeast Cape)	71 10 N	7 58 W
1700	**Iceland**		
1710	Dyrhólaey (Portland): Lt	63 24 N	19 08 W
720	Vestmannaeyjar, Stórhofdhi: Lt	63 24 N	20 18 W
730	*Eyrarbakki*	63 52 N	21 09 W
740	Reykjanes: Lt	63 49 N	22 42 W
750	Gardhskagi (Skagi): Lt	64 05 N	22 42 W
760	*Keflavík*	64 00 N	22 33 W
770	*Reykjavík*	64 09 N	21 57 W
780	Malarrif: Lt	64 44 N	23 48 W
790	Svörtuloft: Lt	64 52 N	24 03 W
1800	Bjargtangar: Lt	65 30 N	24 32 W
810	Svalvogar: Lt	65 55 N	23 51 W
820	Straumnes: Lt	66 26 N	23 08 W

Index No.	Place	Lat.	Long.
	Iceland—Continued	° ′	° ′
1830	Hornbjarg: Lt	66 25 N	22 23 W
840	Gjögur: Lt	66 00 N	21 19 W
850	Skagatá: Lt	66 07 N	20 06 W
860	Saudhanes: Lt	66 11 N	18 57 W
870	*Hrísey*	65 59 N	18 22 W
880	Flatey: Lt	66 10 N	17 51 W
890	Raudhinúpur: Lt	66 30 N	16 33 W
1900	Raufarhöfn: Lt	66 27 N	15 56 W
910	Langanes: Lt	66 23 N	14 32 W
920	Bjarnarey: Lt	65 47 N	14 19 W
930	*Seydhisfjördhur*	65 16 N	13 55 W
940	Dalatangi: Lt	65 16 N	13 35 W
950	Kambanes: Lt	64 48 N	13 50 W
960	Papey: Lt	64 35 N	14 11 W
970	Stokksnes: Lt	64 14 N	14 58 W
980	Ingólfshöfdhi: Lt	63 48 N	16 38 W
2000	**Faeroe Islands**		
2010	Sudhuroy (Sydero), Sunnböur (Sumbo): Lt	61 24 N	6 40 W
020	Nólsoy, Kabelen: Lt	61 57 N	6 36 W
2100	STREYMOY (STROMO ISLAND)		
110	—*Tórshavn*	62 00 N	6 45 W
120	—*Vestmanna*	62 09 N	7 10 W
2200	*Kongshavn, Eysturoy (Ostero I.)*	62 07 N	6 44 W
210	Kallsoy, Sydhradalur: Lt	62 14 N	6 39 W
220	Mykines (Myggenaes), Mykinesbygd: Lt	62 06 N	7 40 W
2300	**Svalbard**		
2400	VESTSPITSBERGEN (WEST SPITSBERGEN)		
410	—*Sveagruva*	77 53 N	16 46 E
420	—Kapp Martin, Bellsund: Lt	77 43 N	13 59 E
430	—Isfjord, Kapp Linné: Lt	78 04 N	13 38 E
440	—*Barentsburg*	78 04 N	14 14 E
450	—*Longyearbyen*	78 14 N	15 35 E
460	—*Ny-Ålesund*	78 55 N	11 56 E
2500	Prins Karls Forland, Fuglehuken: Lt	78 53 N	10 31 E
510	*Austervåg, Bjørnøya (Bear I.)*	74 29 N	19 14 E
2600	**USSR**		
2610	*Pechenga (Petsamonvuono)*	69 39 N	31 10 E
620	Mys Nemetskiy (Majakkaniemi): Lt	69 58 N	31 56 E
630	Mys Tsyp-Navolok: Lt	69 44 N	33 06 E
640	Mys Set'-Navolok: Lt	69 24 N	33 30 E
650	*Murmansk*	68 59 N	33 03 E
660	Ostrov Kil'din: Lt	69 23 N	34 09 E
670	Mys Teriberskiy: Lt	69 15 N	35 10 E
680	Ostrov Bol'shoy Oleniy: Lt	69 05 N	36 21 E
690	Semi-Ostrovov, Ostrov Kharlov: Lt	68 49 N	37 20 E
2700	Mys Chernyy: Lt	68 22 N	38 39 E
710	*Yokan'ga (Iokanka)*	68 03 N	39 31 E
720	Mys Svyatoy Nos: Lt	68 08 N	39 46 E
730	Mys Malyy Gorodetskiy: Lt	67 42 N	40 59 E
740	Mys Orlov: Lt	67 12 N	41 19 E
750	Ostrov Sosnovets: Lt	66 30 N	40 44 E
760	*Kandalaksha*	67 08 N	32 25 E
770	*Kovda*	66 41 N	32 52 E
780	*Keret'*	66 17 N	33 34 E
790	*Arkhangel'sk (Archangel)*	64 32 N	40 31 E
2800	Mys Zimnegorskiy (Zimnie): Lt	65 29 N	39 43 E
810	Mys Intsy: Lt	65 58 N	40 43 E
820	Mys Voronov: Lt	66 31 N	42 15 E
830	Ostrov Morzhovets: Lt	66 44 N	42 29 E
840	Mys Tolstik (Konushin): Lt	67 14 N	43 49 E
850	Mys Kanin Nos: Lt	68 39 N	43 18 E
860	Ostrov Kolguyev (Kolguey I.), N. extremity: Lt	69 32 N	49 08 E
870	Zemlya Frantsa-Iosifa (Franz Josef Land), Ostrov Vil'cheka	79 55 N	58 30 E
2900	NOVAYA ZEMLYA		
910	—Mys Chernyy Nos: Lt	70 51 N	53 20 E
920	—Mys Severnyy Gusinyy Nos: Lt	72 08 N	51 51 E
930	—Mys Stolbovoy: Lt	73 18 N	53 56 E

APPENDIX S

MARITIME POSITIONS

ARCTIC REGIONS—Continued

Index No.	Place	Lat.	Long.
	USSR—Continued	° ′	° ′
2940	—Mys Zhelaniya: Lt	76 59 N	68 30 E
950	—Ostrova Pakhtusova: Lt	74 24 N	59 08 E
960	—Mys Vykhodnoy (Cape Vuikhodnoi): Lt	73 14 N	56 42 E
970	—Mys Men'shikova: Lt	70 42 N	57 36 E
3000	Khodovarikha Sopka, Mys Russki Zavarot: Lt	68 55 N	53 39 E
010	Mys Greben', Proliv Yugorskiy Shar (Yugorski Strait): Lt	69 40 N	59 59 E
020	Ostrov Vaygach, Ostrov Kolyubakin: Lt	70 15 N	58 20 E
030	Bolvanskiy Nos: Lt	70 27 N	59 04 E
040	Ostrov Belyy (Byeli I.): Lt	73 22 N	70 03 E
050	Ostrov Shokal'skogo, Obskaya Guba (Gulf of Ob): Lt	72 59 N	74 22 E
060	*Ostrov Dikson (Dickson)*	73 30 N	80 25 E
070	Ostrov Sverdrup: Lt	74 31 N	79 32 E
080	Ostrova Izvestiy Tsik: Lt	75 53 N	83 17 E
090	Ostrov Russkiy: Lt	77 10 N	96 26 E
3100	Mys Chelyuskin: Lt	77 42 N	104 38 E
110	Ostrov Kotel'nyy	75 30 N	139 00 E
120	Mys Shelagskiy: Lt	70 06 N	170 24 E
130	Mys Billingsa: Lt	69 52 N	176 08 E
140	Ostrov Vrangelya (Wrangel I.)	71 15 N	179 00 W
150	Mys Uelen: Lt	66 09 N	169 41 W
160	Mys Dezhneva, Bering Strait: Lt	66 01 N	169 43 W
170	Ostrov Ratmanova (Big Diomede I.)	65 50 N	169 06 W
3200	**Alaska**		
3210	Cape Prince of Wales: Lt	65 36 N	168 05 W
220	Cape Espenberg: Lt	66 35 N	163 40 W
230	*Deering*	66 05 N	162 44 W
240	*Kotzebue*	66 55 N	162 35 W
250	Point Hope: Lt	68 22 N	166 46 W
260	*Wainwright*	70 39 N	160 00 W
270	*Barrow*	71 17 N	156 47 W
3300	**Yukon Territory**		
3310	Herschel I	69 35 N	139 05 W
3400	**Northwest Territories**		
3410	Kittigazuit	69 21 N	133 43 W
420	Port Brabant (Tuktoyatuk)	69 26 N	133 02 W
430	Cape Bathurst	70 36 N	128 00 W
440	Cape Parry	70 12 N	124 33 W
450	Pearce Point	69 55 N	122 45 W
460	Cape Bexley	69 03 N	116 00 W
470	*Coppermine*	67 48 N	115 03 W
480	Cape Alexander	68 56 N	106 11 W
3500	BANKS ISLAND		
510	—Cape Kellett	72 02 N	125 40 W
520	—Sachs Harbor	71 56 N	124 40 W
530	—Nelson Head	71 03 N	122 30 W
3600	Victoria I., Cambridge Bay: Weather Station	69 07 N	105 01 W
610	King William I., Gjoa Haven	68 38 N	95 55 W
620	Prince of Wales I	72 30 N	99 00 W
630	Melville I., Winter Harbor	74 47 N	110 48 W
640	Prince Patrick I., Mould Bay: Weather Station	76 17 N	119 28 W
3700	RINGNES ISLANDS		
710	—Ellef Ringnes I	78 30 N	101 00 W
720	—Isachsen: Weather Station	78 47 N	103 32 W
3800	Bathurst I., Cape Cockburn	75 04 N	100 22 W
810	Cornwallis I., Resolute Bay: Weather Station	74 43 N	94 59 W

Index No.	Place	Lat.	Long.
	Northwest Territories—Cont.	° ′	° ′
3820	Somerset I., Port Leopold	73 50 N	90 17 W
3900	DEVON ISLAND		
910	—Graham Harbor	74 30 N	88 18 W
920	—Dundas Harbor	74 32 N	82 30 W
4000	Axel Heiberg I., Hyperite Point	78 08 N	89 15 W
4100	ELLESMERE ISLAND		
110	—Slidre Bay	79 59 N	85 57 W
120	—Eureka: Weather Station	80 00 N	85 56 W
130	—Ward Hunt I	83 05 N	75 00 W
140	—Alert: Weather Station	82 30 N	62 20 W
150	—Cape Sheridan	82 28 N	61 25 W
160	—Fort Conger, Discovery Harbor	81 44 N	64 45 W
170	—Cape Baird	81 31 N	64 30 W
180	—Cape Sabine	78 44 N	74 25 W
190	—Craig Harbor	76 12 N	81 05 W
4200	Coburg I., Cape Spencer	75 54 N	79 58 W
210	Bylot I., Cape Hay	73 51 N	79 50 W
4300	BAFFIN ISLAND		
310	—Arctic Bay: Weather Station	73 00 N	85 18 W
320	—*Pond Inlet*	72 45 N	76 45 W
330	—*River Clyde*	70 22 N	67 49 W
340	—Padloping I.: Weather Station	67 06 N	62 21 W
350	—Cape Dyer	66 40 N	61 17 W
360	—Cape Mercy	65 02 N	63 20 W
370	—*Pangnirtung*	66 06 N	65 40 W
380	—Cape Murchison	63 18 N	64 07 W
390	—Frobisher Bay, Koojesse Inlet	63 44 N	68 32 W
4400	HUDSON STRAIT		
410	—Resolution I.: Lt	61 18 N	64 53 W
420	—*Lake Harbor*	62 48 N	69 50 W
430	—Big I., Rabbit I.: Lt	62 32 N	70 33 W
440	—*Dorset I*	64 15 N	76 40 W
450	—King Charles Cape	64 14 N	77 19 W
4500	BAFFIN ISLAND		
510	—Cape Dorchester	65 26 N	77 24 W
520	—Cape Hallowell	70 00 N	85 15 W
530	—Cape Kater	71 54 N	90 10 W
4600	Fort Ross	72 00 N	94 15 W
610	Cape Margaret	70 08 N	91 15 W
620	Pelly Bay: Mission	68 27 N	89 38 W
630	Cape Englefield	69 50 N	85 48 W
640	Igloolik: Mission	69 22 N	81 48 W
650	Cape Penrhyn	67 23 N	81 08 W
4700	**Hudson Bay**		
4800	SOUTHAMPTON ISLAND		
810	—Seahorse Point	63 46 N	80 12 W
820	—*Coral Harbor, N. W. T*	64 06 N	83 24 W
4900	Chesterfield Inlet: Lt	63 20 N	90 43 W
910	*Churchill, Manitoba*	58 47 N	94 11 W
920	*Port Nelson, Manitoba*	57 03 N	92 35 W
930	*Moosonee, Ontario*	51 17 N	80 37 W
940	*Charlton Depot, N. W. T*	52 02 N	79 17 W
950	*Port Harrison, N. W. T*	58 28 N	78 08 W
960	*Smith I., N. W. T*	60 44 N	78 30 W
970	Coats I.: Lt	62 10 N	83 08 W
980	Mansel I.: Lt	62 27 N	79 38 W
5000	**Hudson Strait**		
5010	Digges Is.: Lt	62 35 N	78 07 W
020	Nottingham I.: Lt	63 06 N	77 57 W
030	Charles I., Cape Moses Oates: Lt	62 36 N	73 56 W
040	*Wakeham Bay, Quebec*	61 42 N	71 58 W
050	Cape Hopes Advance: Lt	61 05 N	69 33 W
060	*Fort Chimo, Quebec*	58 09 N	68 18 W
070	Cape Chidley	60 26 N	64 26 W

APPENDIX S

MARITIME POSITIONS

EAST COAST OF NORTH AMERICA

Index No.	Place	Lat.	Long.
6000	**Labrador**	° ′	° ′
6010	*Hebron*	58 12 N	62 38 W
020	Nutak	57 29 N	61 47 W
030	*Nain*	56 33 N	61 41 W
040	*Ford Harbor*	56 28 N	61 11 W
050	Cape Harrigan: Lt	55 51 N	60 19 W
060	Napakataktalik (Manuel I.): Lt	55 32 N	60 14 W
070	*Hopedale*	55 27 N	60 12 W
080	Tikkerasuk I.: Lt	55 20 N	59 43 W
090	Cut Throat I.: Lt	54 29 N	57 06 W
6100	Goose Bay Narrows: Lt	53 27 N	59 57 W
110	*Terrington*	53 21 N	60 24 W
120	Packs Harbor: Lt	53 52 N	56 59 W
130	*Cartwright*	53 42 N	57 02 W
140	Cape North: Lt	53 46 N	56 26 W
150	White Point: Lt	53 35 N	56 01 W
160	Domino Point: Lt	53 28 N	55 44 W
170	Double I., Battle Harbor: Lt	52 15 N	55 33 W
180	Amour Point: Lt	51 27 N	56 51 W
6200	**Newfoundland**		
6210	Belle Isle, South Point: Lt	51 53 N	55 22 W
220	Flowers I.: Lt	51 18 N	56 45 W
230	Cape Norman: Lt	51 38 N	55 54 W
240	Cape Bauld: Lt	51 38 N	55 25 W
250	*Saint Anthony*	51 22 N	55 35 W
260	Fox Point: Lt	51 21 N	55 33 W
270	Cape Fox: Lt	50 52 N	55 54 W
280	Bell I., Grey I.: Lt	50 42 N	55 37 W
290	Canada Bay, White Point: Lt	50 43 N	56 06 W
6300	Orange Bay (Great Harbor Deep): Lt	50 23 N	56 23 W
310	*Western Arm (Western Cove)*	49 47 N	56 37 W
320	Partridge Point: Lt	50 10 N	56 09 W
330	Baie (Bay) Verte	49 57 N	56 10 W
340	Saint Barbe (Horse) Is.: Lt	50 12 N	55 44 W
350	La Scie: Lt	49 58 N	55 36 W
360	Gull I.: Lt	50 00 N	55 22 W
370	Nippers Is.: Lt	49 47 N	55 50 W
380	Little Bay I.: Lt	49 38 N	55 46 W
390	Gull Rock: Lt	49 41 N	55 42 W
6400	Long I., Southern Head: Lt	49 36 N	55 35 W
410	Leading Tickles: Lt	49 30 N	55 24 W
420	Fortune Harbor: Lt	49 32 N	55 14 W
430	*Botwood*	49 09 N	55 20 W
440	Surgeon Cove Point: Lt	49 31 N	55 07 W
450	*Lewisporte*	49 15 N	55 03 W
460	*Twillingate (Toulinguet)*	49 40 N	54 47 W
470	Bacalhao (Bacchalhao) I.: Lt	49 41 N	54 34 W
480	Fogo, Rag's I.: Lt	49 44 N	54 16 W
490	Brooks Point, Joe Batts Point: Lt	49 45 N	54 09 W
6500	Little Fogo I.: Lt	49 49 N	54 05 W
510	Cann I.: Lt	49 35 N	54 11 W
520	Gander I	49 27 N	54 23 W
530	Offer Wadham I.: Lt	49 36 N	53 46 W
540	Peckford I.: Lt	49 32 N	53 51 W
550	Penguin I.: Lt	49 27 N	53 49 W
560	Cabot (Stinking) I.: Lt	49 10 N	53 22 W
570	Puffin I.: Lt	49 04 N	53 33 W
580	Little Denier I.: Lt	48 41 N	53 35 W
590	King's Cove: Lt	48 35 N	53 19 W
6600	Cape Bonavista: Lt	48 42 N	53 05 W
610	*Catalina*	48 30 N	53 03 W
620	Green I.: Lt	48 30 N	53 03 W
630	*Trinity*	48 22 N	53 22 W
640	Ragged Is.: Lt	48 14 N	53 27 W
650	Random I., Motion I.: Lt	48 06 N	53 33 W
660	Hopeall Head: Lt	47 38 N	53 34 W
670	Jeans Head: Lt	47 56 N	53 22 W
680	Perlican I.: Lt	48 05 N	53 01 W
690	Baccalieu I., North Point: Lt	48 09 N	52 48 W
6700	Carbonear I.: Lt	47 44 N	53 10 W
710	*Harbor Grace*	47 41 N	53 13 W
720	*Bay Roberts*	47 36 N	53 15 W
730	Brigus Bay, North Head: Lt	47 33 N	53 12 W
740	Salmon Cove Point: Lt	47 28 N	53 10 W
750	*Wabana, Bell I.*	47 37 N	52 56 W
760	Cape St. Francis: Lt	47 48 N	52 48 W
770	*Saint John's*	47 34 N	52 42 W
780	Cape Spear: Lt	47 31 N	52 38 W
790	Bull Head, Bay Bulls: Lt	47 19 N	52 45 W

Index No.	Place	Lat.	Long.
	Newfoundland—Continued	° ′	° ′
6800	Ferryland Head: Lt	47 01 N	52 52 W
810	Cape Race: Lt	46 39 N	53 05 W
820	*Trepassey*	46 44 N	53 24 W
830	Cape Pine: Lt	46 37 N	53 33 W
840	Point la Haye: Lt	46 54 N	53 37 W
850	Cape St. Marys: Lt	46 49 N	54 12 W
860	Point Verde: Lt	47 14 N	54 01 W
870	*Placentia*	47 15 N	53 58 W
880	Latine Point: Lt	47 19 N	54 01 W
890	*Argentia*	47 18 N	53 59 W
6900	Fox I.: Lt	47 21 N	54 00 W
910	Marticot I.: Lt	47 19 N	54 35 W
920	Iron I.: Lt	47 03 N	55 08 W
930	Dodding Head, Burin I.: Lt	47 00 N	55 09 W
940	*Burin*	47 03 N	55 10 W
950	St. Lawrence Harbors, Middle Head: Lt	46 54 N	55 21 W
960	Lamaline Bay, Allan I.: Lt	46 51 N	55 48 W
970	Green I.: Lt	46 52 N	56 05 W
7000	**St. Pierre and Miquelon Islands**		
7010	*St. Pierre*	46 47 N	56 11 W
020	Gallantry Head: Lt	46 46 N	56 10 W
030	Platte Point: Lt	46 49 N	56 25 W
040	Cape Blanc: Lt	47 06 N	56 25 W
7100	**Newfoundland**		
7110	*Grand Bank*	47 06 N	55 45 W
120	Garnish: Lt	47 14 N	55 22 W
130	Long Harbor Point: Lt	47 34 N	55 08 W
140	St. Jacques I.: Lt	47 28 N	55 25 W
150	Brunette I., Mercer Head: Lt	47 15 N	55 53 W
160	Pass I.: Lt	47 29 N	56 12 W
170	Cape La Hune: Lt	47 32 N	56 52 W
180	Penguin Is.: Lt	47 23 N	56 59 W
190	Ramea Is.: Lt	47 31 N	57 25 W
7200	Burgeo Is., Boar I.: Lt	47 36 N	57 36 W
210	Ireland I.: Lt	47 38 N	58 22 W
220	Rose Blanche Head: Lt	47 36 N	58 42 W
230	*Port Aux Basques*	47 34 N	59 08 W
240	Cape Ray: Lt	47 37 N	59 18 W
250	Cape Anguille: Lt	47 54 N	59 25 W
260	*St. Georges*	48 26 N	58 30 W
270	Indian Head: Lt	48 30 N	58 31 W
280	Stephenville Pond	48 31 N	58 32 W
290	Red I.: Lt	48 34 N	59 14 W
7300	Long Point: Lt	48 47 N	58 47 W
310	Port Au Port (Aguathuna)	48 34 N	58 46 W
320	Little Port Head: Lt	49 07 N	58 25 W
330	Frenchman Head: Lt	49 03 N	58 09 W
340	*Corner Brook*	48 57 N	57 57 W
350	Lobster Cove Head: Lt	49 36 N	57 57 W
360	Cow Head Harbor: Lt	49 55 N	57 49 W
370	Keppel I.: Lt	50 38 N	57 20 W
380	Riche Point: Lt	50 42 N	57 25 W
390	Ferolle Point: Lt	51 01 N	57 05 W
7400	**Quebec**		
7410	Greenly I.: Lt	51 22 N	57 11 W
420	Flat I.: Lt	50 45 N	58 45 W
430	St. Mary Is.: Lt	50 18 N	59 39 W
440	Cape Whittle, Cormorant Rocks: Lt	50 10 N	60 04 W
450	Natashquan Point: Lt	50 05 N	61 44 W
460	Walrus I. (Île au Marteau): Lt.	50 12 N	63 34 W
470	*Havre-St.-Pierre*	50 14 N	63 36 W
480	Perroquet I.: Lt	50 13 N	64 12 W
7500	Anticosti Island		
510	—Cap de Rabast: Lt	49 57 N	64 09 W
520	—Carleton Point: Lt	49 44 N	62 57 W
530	—Table Head: Lt	49 21 N	61 54 W
540	—Heath Point: Lt	49 05 N	61 42 W
550	—Bagot Bluff: Lt	49 04 N	62 16 W
560	—Southwest Point: Lt	49 23 N	63 36 W
570	—*Port Menier*	49 49 N	64 21 W
580	—West Point: Lt	49 52 N	64 32 W
7600	St. Lawrence River		
610	—*Sept-Iles (Seven Is.)*	50 12 N	66 23 W
620	—Seven Is., Carrousel I.: Lt	50 05 N	66 23 W
630	—Pointe des Monts: Lt	49 20 N	67 22 W

APPENDIX S

MARITIME POSITIONS

EAST COAST OF NORTH AMERICA—Continued

Index No.	Place	Lat.	Long.
	Quebec—Continued	° ′	° ′
7640	—*Baie-Comeau*	49 14 N	68 08 W
650	—*Quebec*	46 49 N	71 13 W
660	—*Trois Rivières*	46 21 N	72 33 W
670	—*Montreal*	45 31 N	73 33 W
680	—*Rimouski*	48 27 N	68 31 W
690	—Father Point: Lt	48 31 N	68 28 W
7700	—Cap Chat: Lt	49 05 N	66 45 W
710	—*Sainte Anne-des-Monts*	49 08 N	66 29 W
720	—Cap de la Magdalen: Lt	49 15 N	65 20 W
730	—Fame Point: Lt	49 06 N	64 36 W
7800	Cap des Rosiers: Lt	48 51 N	64 12 W
810	*Gaspé*	48 50 N	64 29 W
820	Cap d'Espoir: Lt	48 26 N	64 19 W
830	*Chandler*	48 22 N	64 40 W
840	Pointe au Marquereau: Lt	48 12 N	64 46 W
850	*Paspebiac*	48 02 N	65 15 W
860	Carleton Centre, Tracadigash Point: Lt	48 05 N	66 07 W
7900	Magdalen Islands		
910	—*Havre Aubert* (*Amherst Harbor*)	47 14 N	61 50 W
920	—*Grand Entry Harbor*	47 34 N	61 34 W
8000	**New Brunswick**		
8010	*Campbellton*	48 01 N	66 40 W
020	*Dalhousie*	48 04 N	66 22 W
030	*Bathurst*	47 37 N	65 39 W
040	Miscou I., Birch Point: Lt	48 01 N	64 29 W
050	*Newcastle*	47 00 N	65 34 W
060	*Chatham*	47 02 N	65 28 W
070	Portage I.: Lt	47 10 N	65 02 W
080	Point Escuminac: Lt	47 05 N	64 48 W
090	*Richibucto*	46 41 N	64 52 W
8100	Caissie (Cassie) Point: Lt	46 19 N	64 31 W
110	Cape Jourimain: Lt	46 10 N	63 48 W
120	Cape Tormentine Harbor: Lt	46 08 N	63 47 W
8200	**Prince Edward Island**		
8210	Port Borden: Lt	46 15 N	63 42 W
220	Seacow Head: Lt	46 19 N	63 49 W
230	*Summerside*	46 24 N	63 47 W
240	Cape Egmont: Lt	46 24 N	64 08 W
250	West Point: Lt	46 38 N	64 23 W
260	Miminegash: Lt	46 53 N	64 14 W
270	North Point: Lt	47 04 N	63 59 W
280	Cape Tryon: Lt	46 32 N	63 30 W
290	Shipwreck Point: Lt	46 28 N	62 25 W
8300	East Point: Lt	46 27 N	61 58 W
310	Knight Point: Lt	46 21 N	62 14 W
320	*Souris*	46 21 N	62 15 W
330	*Georgetown*	46 11 N	62 32 W
340	Panmure I., Cardigan Bay: Lt	46 09 N	62 28 W
350	Cape Bear: Lt	46 01 N	62 27 W
360	Wood Is.: Lt	45 57 N	62 44 W
370	Prim Point: Lt	46 03 N	63 02 W
380	*Charlottetown*	46 14 N	63 08 W
390	St. Peters I.: Lt	46 07 N	63 11 W
8400	**Nova Scotia**		
8410	Coldspring Head: Lt	45 58 N	63 52 W
420	*Pugwash*	45 51 N	63 40 W
430	Amet I.: Lt	45 50 N	63 11 W
440	Caribou Point, Gull I.: Lt	45 46 N	62 41 W
450	Pictou I., Seal Point (East End): Lt	45 49 N	62 31 W
460	*Pictou*	45 40 N	62 43 W
470	Cape George: Lt	45 53 N	61 54 W
480	North Canso: Lt	45 42 N	61 29 W
8500	**Cape Breton Island**		
8510	Henry I.: Lt	45 59 N	61 36 W
520	*Port Hood*	46 01 N	61 32 W
530	*Mabou*	46 06 N	61 28 W
540	Sea Wolfe (Margaree) I.: Lt	46 21 N	61 16 W
550	*Margaree Harbor*	46 23 N	61 04 W
560	*Cheticamp I.*	46 38 N	61 00 W
570	Cape St. Lawrence: Lt	47 02 N	60 36 W
580	Cape North: Lt	47 02 N	60 24 W
590	St. Paul I., Atlantic Cove: Lt	47 12 N	60 09 W

Index No.	Place	Lat.	Long.
	Cape Breton Island—Cont.	° ′	° ′
8600	*Dingwall*	46 54 N	60 28 W
610	White Point: Lt	46 53 N	60 21 W
620	Neil Harbor: Lt	46 48 N	60 19 W
630	*Ingonish*	46 38 N	60 23 W
640	St. Anns Harbor, Beach Point: Lt	46 17 N	60 33 W
650	Ciboux I.: Lt	46 23 N	60 23 W
660	Great Bras d'Or, Blackrock Point: Lt	46 18 N	60 24 W
670	*Sydney*	46 09 N	60 12 W
680	Low (Flat) Point: Lt	46 16 N	60 08 W
690	*Glace Bay*	46 12 N	59 57 W
8700	Flint I.: Lt	46 11 N	59 46 W
710	Scatari I., Mainadieu: Lt	46 00 N	59 48 W
720	*Louisburg*	45 55 N	59 58 W
730	Guyon (Guion) I.: Lt	45 46 N	60 07 W
740	St. Esprit I.: Lt	45 37 N	60 29 W
750	Green I.: Lt	45 29 N	60 54 W
8800	Madame Island		
810	—Cap Rond: Lt	45 35 N	60 53 W
820	—*Arichat*	45 31 N	61 01 W
8900	Bear I.: Lt	45 33 N	61 18 W
910	*Port Hawkesbury*	45 37 N	61 22 W
920	Balache Point	45 39 N	61 25 W
9000	**Nova Scotia**		
9010	Cape Porcupine	45 38 N	61 25 W
020	Eddy (Sand) Point: Lt	45 31 N	61 15 W
030	*Guysborough*	45 24 N	61 30 W
040	Queensport, Rook I.: Lt	45 21 N	61 16 W
050	*Canso*	45 20 N	61 00 W
060	Cranberry Is.: Lt	45 19 N	60 56 W
070	Sable I., West Point: Lt	43 56 N	60 02 W
080	White Head I.: Lt	45 12 N	61 08 W
090	*Whitehead* (*Whitehaven*)	45 14 N	61 11 W
9100	Tor Bay, Berry Head: Lt	45 11 N	61 19 W
110	Country I.: Lt	45 06 N	61 33 W
120	Wedge I.: Lt	45 01 N	61 53 W
130	Liscomb I.: Lt	44 59 N	61 58 W
140	Beaver I.: Lt	44 50 N	62 20 W
150	*Sheet Harbor*	44 54 N	62 30 W
160	Tomlee Head, Spry Bay: Lt	44 49 N	62 36 W
170	*Ship Harbor*	44 47 N	62 49 W
180	Egg I.: Lt	44 40 N	62 52 W
190	Jeddore Rock: Lt	44 40 N	63 01 W
9200	Devils I.: Lt	44 35 N	63 28 W
210	*Halifax*	44 39 N	63 35 W
220	Chebucto Head: Lt	44 30 N	63 31 W
230	Sambro I.: Lt	44 26 N	63 34 W
240	Betty I., Brig Point: Lt	44 26 N	63 46 W
250	Peggy Point: Lt	44 30 N	63 55 W
260	*Ingramport*	44 41 N	63 58 W
270	East Ironbound I.: Lt	44 26 N	64 05 W
280	Pearl I.: Lt	44 23 N	64 03 W
290	*Chester*	44 32 N	64 15 W
9300	*Mahone Harbor*	44 27 N	64 23 W
310	Cross I.: Lt	44 19 N	64 10 W
320	*Lunenburg*	44 23 N	64 19 W
330	West Ironbound I.: Lt	44 14 N	64 16 W
340	*Bridgewater*	44 23 N	64 31 W
350	Medway Head: Lt	44 06 N	64 32 W
360	Coffin I.: Lt	44 02 N	64 38 W
370	*Liverpool*	44 02 N	64 43 W
380	Port Mouton, Spectacle I.: Lt	43 55 N	64 48 W
390	Little Hope I.: Lt	43 48 N	64 47 W
9400	Lockeport Harbor, Gull Rock: Lt	43 39 N	65 06 W
410	*Shelburne*	43 45 N	65 19 W
420	Cape Roseway: Lt	43 37 N	65 16 W
430	Cape Negro I.: Lt	43 30 N	65 21 W
440	Baccaro Point: Lt	43 27 N	65 28 W
450	Cape Sable: Lt	43 23 N	65 37 W
460	Bon Portage I.: Lt	43 27 N	65 45 W
470	Seal I.: Lt	43 24 N	66 01 W
480	Pubnico Harbor, Beach Point: Lt	43 36 N	65 47 W
490	Pease I.: Lt	43 38 N	66 02 W
9500	*Yarmouth*	43 50 N	66 07 W
510	Cape Fourchu: Lt	43 48 N	66 09 W
520	Cape St. Mary: Lt	44 05 N	66 13 W
530	Brier I.: Lt	44 15 N	66 24 W
540	Boars Head: Lt	44 24 N	66 13 W

APPENDIX S

MARITIME POSITIONS

EAST COAST OF NORTH AMERICA—Continued

Index No.	Place	Lat.	Long.
	Nova Scotia—Continued	° ′	° ′
9550	Prim Point: Lt	44 42 N	65 47 W
560	*Digby*	44 37 N	65 46 W
570	Port Lorne: Lt	44 57 N	65 16 W
580	Cape Split, Minas Channel	45 20 N	64 30 W
590	*Parrsboro*	45 24 N	64 20 W
9600	Cape d'Or: Lt	45 18 N	64 47 W
610	Île Haute: Lt	45 15 N	65 01 W
620	Apple River, Cape Capstan: Lt	45 28 N	64 52 W
9700	**New Brunswick**		
9710	*Moncton*	46 05 N	64 46 W
720	Grindstone I.: Lt	45 43 N	64 37 W
730	Cape Enragé: Lt	45 36 N	64 47 W
740	Martin Head: Lt	45 29 N	65 12 W
750	Quaco Head: Lt	45 20 N	65 32 W
760	Cape Spencer: Lt	45 12 N	65 55 W
770	*St. John*	45 16 N	66 03 W
780	Partridge I.: Lt	45 14 N	66 03 W
790	Musquash: Lt	45 09 N	66 14 W
9800	Point Lepreau: Lt	45 04 N	66 28 W
810	Bliss I.: Lt	45 01 N	66 51 W
820	South Wolf I.: Lt	44 56 N	66 44 W
9900	GRAND MANAN ISLAND		
910	—Swallowtail: Lt	44 46 N	66 44 W
920	—Gull Cove: Lt	44 38 N	66 42 W
930	—Southwest Head: Lt	44 36 N	66 54 W
10000	Gannet Rock: Lt	44 31 N	66 47 W
010	Machias Seal I.: Lt	44 30 N	67 06 W
020	Campobello I., Mulholland Point: Lt	44 52 N	66 59 W
030	Quoddy Head, Head Harbor I.: Lt	44 58 N	66 54 W
10100	PASSAMAQUODDY BAY		
110	—*St. Andrews*	45 04 N	67 03 W
120	—St. Stephen	45 12 N	67 17 W
10200	**Maine**		
10210	*Calais*	45 11 N	67 17 W
220	*Eastport*	44 54 N	66 59 W
230	*Lubec*	44 52 N	66 59 W
240	West Quoddy Head: Lt	44 49 N	66 57 W
250	Little River: Lt	44 39 N	67 12 W
260	Libby Is.: Lt	44 34 N	67 22 W
270	Petit Manan I.: Lt	44 22 N	67 52 W
280	Baker I.: Lt	44 14 N	68 12 W
290	Mount Desert Rock: Lt	43 58 N	68 08 W
10300	*Castine*	44 23 N	68 44 W
310	*Bangor*	44 48 N	68 46 W
320	*Searsport*	44 27 N	68 55 W
330	*Rockland*	44 06 N	69 06 W
340	Matinicus Rock: Lt	43 47 N	68 51 W
350	Monhegan I.: Lt	43 46 N	69 19 W
360	*Boothbay Harbor*	43 51 N	69 38 W
370	Seguin I.: Lt	43 42 N	69 46 W
380	*Bath*	43 55 N	69 49 W
390	Halfway Rock: Lt	43 39 N	70 02 W
10400	*Portland*	43 40 N	70 15 W
410	Cape Elizabeth: Lt	43 34 N	70 12 W
420	Wood I.: Lt	43 27 N	70 20 W
430	Boon I.: Lt	43 07 N	70 29 W
440	Whaleback Reef: Lt	43 04 N	70 42 W
10500	**New Hampshire**		
10510	*Portsmouth*	43 05 N	70 45 W
520	Isles of Shoals, White I.: Lt	42 58 N	70 37 W
10600	**Massachusetts**		
10610	*Newburyport*	42 49 N	70 52 W
620	Annisquam: Lt	42 40 N	70 41 W
630	Cape Ann, Thacher I.: Lt	42 38 N	70 34 W
640	Eastern Point: Lt	42 35 N	70 40 W
650	*Gloucester*	42 36 N	70 40 W
660	Bakers I.: Lt	42 32 N	70 47 W
670	*Salem*	42 31 N	70 53 W
680	*Marblehead*	42 30 N	70 51 W
690	The Graves: Lt	42 22 N	70 52 W
10700	Charlestown	42 23 N	71 03 W
710	*Boston*	42 21 N	71 03 W
	Massachusetts—Continued	° ′	° ′
10720	*Quincy*	42 15 N	70 58 W
730	Minots Ledge: Lt	42 16 N	70 46 W
740	Gurnet Point: Lt	42 00 N	70 36 W
750	*Plymouth*	41 57 N	70 40 W
760	Cape Cod Canal Eastern Entrance: Breakwater Lt	41 47 N	70 29 W
770	*Provincetown*	42 03 N	70 11 W
780	Race Point: Lt	42 04 N	70 15 W
790	Cape Cod: Lt	42 02 N	70 04 W
10800	Nauset Beach: Lt	41 52 N	69 57 W
810	*Chatham*	41 41 N	69 57 W
820	Monomoy Point: Tower	41 34 N	70 00 W
830	*Hyannis*	41 39 N	70 17 W
840	Sankaty Head: Lt	41 17 N	69 58 W
850	*Nantucket*	41 17 N	70 06 W
860	*Vineyard Haven*	41 27 N	70 36 W
870	West Chop: Lt	41 29 N	70 36 W
880	Cuttyhunk I.: Lt	41 25 N	70 57 W
890	*Woods Hole*	41 31 N	70 40 W
10900	Buzzards Bay	41 45 N	70 37 W
910	Cleveland Ledge: Lt	41 38 N	70 42 W
920	*New Bedford*	41 38 N	70 55 W
930	*Fall River*	41 42 N	71 10 W
11000	**Rhode Island**		
11010	*Providence*	41 48 N	71 24 W
020	*Davisville Depot*	41 37 N	71 24 W
030	*Quonset Point*	41 35 N	71 24 W
040	*Newport*	41 30 N	71 20 W
050	Beavertail Point: Lt	41 27 N	71 24 W
060	Point Judith: Lt	41 22 N	71 29 W
070	Watch Hill Point: Lt	41 18 N	71 52 W
080	Block I., Southeast Point: Lt	41 09 N	71 33 W
11100	**Connecticut and New York**		
11200	LONG ISLAND, NEW YORK		
210	—Montauk Point: Lt	41 04 N	71 51 W
220	—Shinnecock Inlet: Lt	40 50 N	72 29 W
230	—Fire I.: Lt	40 38 N	73 13 W
240	—Rockaway Inlet: Breakwater Lt	40 32 N	73 56 W
250	—*Port Jefferson*	40 58 N	73 05 W
260	—Eatons Point: Lt	40 57 N	73 24 W
270	—Kings Point: U.S. Merchant Marine Academy	40 49 N	73 46 W
11300	Race Rock: Lt	41 15 N	72 03 W
310	Little Gull I.: Lt	41 12 N	72 06 W
320	New London, Connecticut: U.S. Coast Guard Academy	41 23 N	72 06 W
330	Falkner I.: Lt	41 13 N	72 39 W
340	*New Haven, Connecticut*	41 14 N	72 55 W
350	Stratford Point: Lt	41 09 N	73 06 W
360	*Bridgeport, Connecticut*	41 10 N	73 11 W
370	Execution Rocks: Lt	40 53 N	73 44 W
380	City I.: Spire	40 51 N	73 47 W
390	*New York, New York*	40 42 N	74 01 W
11400	**New Jersey**		
11410	Sandy Hook: Lt	40 28 N	74 00 W
420	Sea Girt: Lt	40 08 N	74 02 W
430	Barnegat Inlet: Breakwater Lt	39 45 N	74 06 W
440	*Atlantic City*	39 22 N	74 25 W
450	Hereford Inlet: Lt	39 00 N	74 48 W
11500	**Delaware Bay**		
11510	Cape May Point: Lt	38 56 N	74 58 W
520	*Camden, New Jersey*	39 57 N	75 08 W
530	*Philadelphia, Pennsylvania*	39 57 N	75 08 W
540	*Chester, Pennsylvania*	39 51 N	75 21 W
550	*Wilmington, Delaware*	39 44 N	75 33 W
560	Reedy Point, C. & D. Canal East Entrance: Lt	39 34 N	75 34 W
570	Cape Henlopen: Harbor of Refuge Lt	38 49 N	75 06 W
11600	**Delaware and Virginia**		
11610	Fenwick I.: Lt	38 27 N	75 03 W
620	Assateague I.: Lt	37 55 N	75 21 W
630	Hog I.: Little Machipongo Lt	37 27 N	75 40 W

APPENDIX S

MARITIME POSITIONS

EAST COAST OF NORTH AMERICA—Continued

Index No.	Place	Lat.	Long.
11700	**Chesapeake Bay**	° ′	° ′
11710	Cape Charles: Lt	37 07 N	75 54 W
720	Chesapeake City, Maryland; C. & D. Canal: Spire	39 32 N	75 49 W
730	*Baltimore, Maryland*	39 16 N	76 35 W
740	Annapolis, Maryland: U. S. Naval Academy	38 59 N	76 29 W
750	Point Lookout: Lt	38 02 N	76 19 W
760	*Washington, District of Columbia*	38 52 N	77 02 W
770	Old Point Comfort: Lt	37 00 N	76 18 W
780	*Newport News, Virginia*	36 58 N	76 26 W
790	*Portsmouth, Virginia*	36 49 N	76 18 W
11800	*Norfolk, Virginia*	36 51 N	76 18 W
810	Cape Henry: Lt	36 56 N	76 00 W
11900	**North Carolina**		
11910	Currituck Beach: Lt	36 23 N	75 50 W
920	Bodie I.: Lt	35 49 N	75 34 W
930	Cape Hatteras: Lt	35 15 N	75 31 W
940	Ocracoke I.: Lt	35 06 N	75 59 W
950	Cape Lookout: Lt	34 37 N	76 32 W
960	*Beaufort*	34 43 N	76 40 W
970	Cape Fear: Lt	33 51 N	77 58 W
980	*Wilmington*	34 14 N	77 57 W
12000	**South Carolina**		
12010	North I.: Georgetown Lt	33 13 N	79 11 W
020	*Charleston*	32 47 N	79 55 W
12100	**Georgia**		
12110	*Savannah*	32 05 N	81 05 W
120	Tybee I.: Lt	32 01 N	80 51 W
130	St. Simons I.: Lt	31 08 N	81 24 W
140	*Brunswick*	31 09 N	81 30 W
12200	**Florida**		
12210	Amelia I.: Lt	30 40 N	81 27 W
220	*Jacksonville*	30 19 N	81 39 W
230	St. Johns Point: Lt	30 23 N	81 24 W
240	*St. Augustine*	29 54 N	81 19 W
250	Ponce De Leon Inlet: Lt	29 05 N	80 56 W
260	Cape Canaveral: Lt	28 28 N	80 33 W
270	Jupiter Inlet: Lt	26 57 N	80 05 W
280	*Palm Beach*	26 46 N	80 03 W
290	Hillsboro Inlet: Lt	26 16 N	80 05 W
12300	*Port Everglades*	26 06 N	80 07 W
310	*Miami*	25 47 N	80 11 W
320	Fowey Rocks: Lt	25 35 N	80 06 W
330	Carysfort Reef: Lt	25 13 N	80 13 W
340	Alligator Reef: Lt	24 51 N	80 37 W
350	Sombrero Key: Lt	24 38 N	81 07 W
360	American Shoal: Lt	24 32 N	81 31 W
370	*Key West*	24 33 N	81 49 W
380	Sand Key: Lt	24 27 N	81 53 W
390	Rebecca Shoal: Lt	24 35 N	82 35 W
12400	Dry Tortugas, Loggerhead Key: Lt	24 38 N	82 55 W
410	Sanibel I.: Lt	26 27 N	82 01 W
420	Gasparilla I.: Lt	26 43 N	82 16 W
430	Egmont Key: Lt	27 36 N	82 46 W
440	*Tampa*	27 55 N	82 27 W
450	*St. Petersburg*	27 46 N	82 37 W
460	Anclote Keys: Lt	28 10 N	82 51 W
470	Seahorse Reef: Lt	28 58 N	83 09 W
480	Cedar Keys: North Bank Lt. 1	29 08 N	83 06 W
490	*St. Marks*	30 09 N	84 13 W
12500	Crooked River: Lt	29 50 N	84 42 W
510	*Apalachicola*	29 43 N	84 59 W
520	Cape St. George: Lt	29 35 N	85 03 W
530	Cape San Blas: Lt	29 40 N	85 21 W
540	*Port St. Joe*	29 49 N	85 19 W
550	*Panama City*	30 08 N	85 39 W
560	*Pensacola*	30 24 N	87 13 W
12600	**Alabama**		
12610	Sand I.: Lt	30 11 N	88 03 W
620	*Mobile*	30 41 N	88 07 W

Index No.	Place	Lat.	Long.
12700	**Mississippi**	° ′	° ′
12710	Horn I.: Lt	30 13 N	88 29 W
720	*Pascagoula*	30 21 N	88 34 W
730	*Biloxi*	30 23 N	88 53 W
740	Ship I.: Lt	30 13 N	88 58 W
750	*Gulfport*	30 21 N	89 05 W
12800	**Louisiana**		
12810	Chandeleur Is.: Lt	30 03 N	88 52 W
12900	Mississippi River		
910	—South Pass: Lt	29 01 N	89 10 W
920	—Southwest Pass: Lt	28 54 N	89 26 W
13000	*New Orleans*	29 57 N	90 03 W
010	Barataria Bay: Lt	29 16 N	89 57 W
020	Ship Shoal: Lt	28 55 N	91 04 W
030	Point-au-Fer Reef: Lt	29 22 N	91 23 W
040	*Lake Charles*	30 13 N	93 15 W
13100	**Texas**		
13110	Sabine Pass: Coast Guard Station	29 42 N	93 51 W
120	*Port Arthur*	29 50 N	93 58 W
130	*Beaumont*	30 05 N	94 05 W
140	*Galveston*	29 19 N	94 47 W
150	*Texas City*	29 23 N	94 55 W
160	*Houston*	29 45 N	95 17 W
170	Matagorda I.: Lt	28 20 N	96 25 W
180	Aransas Pass: Lt	27 50 N	97 04 W
190	*Corpus Christi*	27 49 N	97 24 W
13200	Brazos Santiago: Lt	26 04 N	97 10 W
210	*Brownsville*	25 57 N	97 24 W
13300	**Mexico**		
13310	Punta Jerez: Lt	22 54 N	97 46 W
320	*Tampico*	22 13 N	97 51 W
330	Isla de Lobos: Lt	21 28 N	97 13 W
340	*Tuxpan*	20 59 N	97 20 W
350	Tecolutla: Lt	20 30 N	97 01 W
360	Rio Nautla: Lt	20 17 N	96 47 W
370	Punta del Morro: Lt	19 51 N	96 28 W
380	Arrecife Blanquilla: Lt	19 14 N	96 06 W
390	Anegada de Adentro: Lt	19 14 N	96 04 W
13400	*Veracruz*	19 12 N	96 08 W
410	Isla Verde: Lt	19 12 N	96 04 W
420	Isla Sacrificios: Lt	19 10 N	96 05 W
430	Isla Blanquia (Blanca Reef): Lt	19 05 N	96 00 W
440	Arrecife de Enmedio: Lt	19 06 N	95 56 W
450	Arrecife Santiaguillo: Lt	19 09 N	95 49 W
460	Punta Roca Partida: Lt	18 44 N	95 11 W
470	Punta Zapotitlán: Lt	18 33 N	94 48 W
480	*Coatzacoalcos (Puerto Mexico)*	18 09 N	94 25 W
490	Tonalá: Lt	18 12 N	94 08 W
13500	*Alvaro Obregon (Frontera)*	18 35 N	92 39 W
510	Punta Xicalango: Lt	18 38 N	91 53 W
520	Isla del Carmen: Lt	18 39 N	91 50 W
530	Aguada: Lt	18 47 N	91 29 W
540	Champotón: Lt	19 21 N	90 43 W
550	Punta Morro: Lt	19 41 N	90 42 W
560	*Campeche*	19 51 N	90 33 W
570	Cayos Arcas: Lt	20 13 N	91 58 W
580	Triangulo Oeste Arrecife: Lt	20 58 N	92 19 W
590	Celestun: Lt	20 51 N	90 24 W
13600	Punta Palmas: Lt	21 02 N	90 17 W
610	Sisal: Lt	21 10 N	90 03 W
620	Cayo Arenas: Lt	22 07 N	91 24 W
630	Arrecife Alacrán: Lt	22 24 N	89 42 W
640	*Progreso*	21 17 N	89 40 W
650	Yalkubul: Lt	21 32 N	88 37 W
660	El Cuyo (Monte de Cuyo): Lt	21 31 N	87 43 W
670	Cabo Catoche: Lt	21 37 N	87 04 W
680	Isla Mujeres: Lt	21 12 N	86 44 W
13700	Isla de Cozumel		
710	—Punta Molas: Lt	20 36 N	86 44 W
720	—*San Miguel de Cozumel*	20 30 N	86 58 W
730	—Punta Celerain: Lt	20 16 N	86 59 W
13800	Punta Herrero: Lt	19 18 N	87 27 W
810	Banco Chinchorro, Cayo Norte: Lt	18 46 N	87 19 W
820	Xcalak: Lt	18 16 N	87 50 W

APPENDIX S

MARITIME POSITIONS

EAST COAST OF NORTH AMERICA—Continued

Index No.	Place	Lat.	Long.
13900	**British Honduras**	° ′	° ′
13910	Rocky Point: Lt	18 21 N	88 05 W
14000	Turneffe Cays		
010	—Mauger Cay: Lt	17 36 N	87 46 W
020	—Cay Bokel: Lt	17 10 N	87 54 W
14100	Lighthouse Reef		
110	—Northern Two Cays, Sanbore Cay: Lt	17 28 N	87 29 W
120	—Half Moon Cay: Lt	17 12 N	87 32 W
14200	*Belize*	17 30 N	88 11 W
210	Bugle Cays: Lt	16 29 N	88 19 W
220	East Snake Cay: Lt	16 13 N	88 31 W
14300	**Guatemala**		
14310	*Puerto Barrios*	15 44 N	88 36 W
320	Cabo Tres Puntas (Cape Three Points): Lt	15 57 N	88 36 W
14400	**Honduras**		
14410	*Puerto Cortés*	15 50 N	87 57 W
420	Punta Caballos: Lt	15 52 N	87 58 W
430	*Tela*	15 46 N	87 27 W
440	Utila: Lt	16 08 N	86 53 W

Index No.	Place	Lat.	Long.
	Honduras—Continued	° ′	° ′
14450	Isla Roatan: Lt	16 18 N	86 38 W
460	Cabo Falso (False Cape): Lt	15 13 N	83 21 W
14500	**Nicaragua**		
14510	Cabo Gracias á Dios: Lt	15 00 N	83 09 W
520	Punta Gorda: Lt	14 21 N	83 12 W
530	*Puerto Cabezas*	14 01 N	83 23 W
540	Little Corn I.: Lt	12 18 N	82 59 W
550	*Bluefields*	12 01 N	83 45 W
560	*San Juan del Norte* (*Greytown*)	10 56 N	83 43 W
14600	**Costa Rica**		
14610	*Limon*	10 00 N	83 01 W
14700	**Panama**		
14710	*Almirante*	9 18 N	82 24 W
720	Punta Toro (Cape Toro): Lt	9 22 N	82 12 W
730	Toro Point: Lt	9 22 N	79 57 W
740	*Cristobal, Canal Zone*	9 21 N	79 55 W
750	Colón	9 22 N	79 54 W
760	Farallón Sucio: Lt	9 39 N	79 38 W
770	Isla Grande: Lt	9 38 N	79 34 W
780	*Nombre de Dios*	9 35 N	79 28 W

WEST COAST OF NORTH AMERICA

Index No.	Place	Lat.	Long.
15000	**Panama**	° ′	° ′
15010	Isla Pantiñito: Lt	8 16 N	78 19 W
020	Isla San José: Lt	8 13 N	79 08 W
030	Isla Pacheca: Lt	8 40 N	79 04 W
040	Isla Chepillo: Lt	8 57 N	79 08 W
050	*Panamá*	8 57 N	79 32 W
060	Flamenco I.: Lt	8 55 N	79 31 W
070	*Balboa, Canal Zone*	8 57 N	79 34 W
080	Isla Taboguilla: Lt	8 48 N	79 31 W
090	Isla Boná: Lt	8 34 N	79 35 W
15100	Punta Mala (Cape Mala): Lt	7 28 N	80 00 W
110	Frailes del Sur: Lt	7 20 N	80 08 W
120	Morro de Puercos: Lt	7 14 N	80 25 W
130	Isla Jicarita: Lt	7 12 N	81 48 W
140	*Puerto Armuelles*	8 16 N	82 52 W
150	Isla Burica: Lt	8 01 N	82 53 W
15200	**Costa Rica**		
15210	*Golfito*	8 39 N	83 10 W
220	Isla del Caño: Lt	8 43 N	83 54 W
230	*Punta Quepos*	9 24 N	84 10 W
240	Isla Herradura (Isla Caño): Lt	9 37 N	84 40 W
250	*Puntarenas*	9 59 N	84 50 W
260	Isla Cabo Blanco: Lt	9 32 N	85 07 W
15300	**Nicaragua**		
15310	*San Juan del Sur*	11 15 N	85 53 W
320	*Corinto*	12 28 N	87 11 W
15400	**Honduras**		
15410	*Amapala*	13 18 N	87 39 W
15500	**El Salvador**		
15510	*La Unión*	13 20 N	87 50 W
520	*La Libertad*	13 29 N	89 19 W
530	*Acajutla*	13 35 N	89 50 W
15600	**Guatemala**		
15610	*Puerto de San José*	13 55 N	90 50 W
620	*Champerico*	14 18 N	91 56 W
630	Ocós: Lt	14 30 N	92 11 W

Index No.	Place	Lat.	Long.
15700	**Mexico**	° ′	° ′
15710	*San Benito*	14 43 N	92 27 W
720	*Salina Cruz, Golfo de Tehuantepec*	16 10 N	95 12 W
730	*Puerto Angel* (*Port Angeles*)	15 39 N	96 31 W
740	Punta Galera: Lt	15 58 N	97 41 W
750	Punta Maldonado: Lt	16 20 N	98 35 W
760	*Acapulco*	16 51 N	99 56 W
770	La Roqueta (Grifo I.): Lt	16 49 N	99 56 W
780	Punta San Telmo: Lt	18 19 N	103 30 W
790	Punta Campos: Lt	19 01 N	104 21 W
15800	*Manzanillo*	19 03 N	104 20 W
810	Cabo Corrientes: Lt	20 24 N	105 43 W
820	San Blas: Lt	21 32 N	105 19 W
830	Isla Maria Madre: Lt	21 36 N	106 33 W
840	*Mazatlan*	23 12 N	106 26 W
850	*Yavaros*	26 42 N	109 31 W
860	Isla Lobos: Lt	27 20 N	110 38 W
870	*Guaymas*	27 55 N	110 55 W
880	Cabo Haro: Lt	27 50 N	110 54 W
890	*Santa Rosalia*	27 20 N	112 17 W
15900	Mulejé: Lt	26 54 N	111 58 W
910	Loreto: Lt	26 01 N	111 21 W
920	*La Paz*	24 10 N	110 19 W
930	Punta Prieta: Lt	24 13 N	110 18 W
940	Bahía San Jose del Cabo: Lt	23 04 N	109 40 W
950	Cabo San Lucas	22 52 N	109 53 W
960	Cabo Falso: Lt	22 52 N	109 58 W
970	Punta Tosca: Lt	24 19 N	111 43 W
980	Punta Redonda: Lt	24 31 N	112 01 W
990	Cabo San Lazaro: Lt	24 48 N	112 19 W
16000	Isla Natividad: Lt	27 52 N	115 10 W
010	Islas San Benito, Benito del Oeste: Lt	28 18 N	115 36 W
020	Isla Cedros (Cerros I.): Lt	28 22 N	115 12 W
030	Isla Todos Santos: Lt	31 49 N	116 49 W
040	*Ensenada*	31 52 N	116 38 W
050	Islas Los Coronados: Lt	32 24 N	117 14 W
16100	**California**		
16110	*National City*	32 40 N	117 08 W
120	*San Diego*	32 43 N	117 11 W
130	Point Loma: Lt	32 40 N	117 14 W
140	*Newport Beach*	33 37 N	117 54 W
150	*Long Beach*	33 46 N	118 11 W
160	*Los Angeles*	33 45 N	118 15 W
170	San Pedro	33 44 N	118 16 W
180	Wilmington	33 46 N	118 16 W
190	Point Fermin: Lt	33 42 N	118 18 W

APPENDIX S

MARITIME POSITIONS

WEST COAST OF NORTH AMERICA—Continued

Index No.	Place	Lat.	Long.
	California—Continued	° ′	° ′
16200	Point Vicente: Lt	33 44 N	118 25 W
16300	SANTA BARBARA ISLANDS (CHANNEL ISLANDS)		
310	—San Clemente I., Pyramid Head: Lt	32 49 N	118 21 W
320	—Santa Catalina I., West End: Lt	33 29 N	118 36 W
330	—Santa Barbara I., South End: Lt	33 28 N	119 02 W
340	—San Nicolas I., East End: Lt	33 14 N	119 26 W
350	—Santa Rosa I., South Point: Lt	33 54 N	120 07 W
360	—Santa Cruz I., Gull I.: Lt	33 57 N	119 50 W
370	—Anacapa I.: Lt	34 01 N	119 22 W
16400	*Port Hueneme*	34 09 N	119 12 W
410	*Santa Barbara*	34 25 N	119 41 W
420	Point Conception: Lt	34 27 N	120 28 W
430	Point Arguello: Lt	34 35 N	120 39 W
440	San Luis Obispo: Lt	35 10 N	120 46 W
450	Point Piedras Blancas: Lt	35 40 N	121 17 W
460	Point Sur: Lt	36 18 N	121 54 W
470	Point Pinos: Lt	36 38 N	121 56 W
480	*Monterey*	36 37 N	121 53 W
490	*Santa Cruz*	36 58 N	122 01 W
16500	Pigeon Point: Lt	37 11 N	122 24 W
510	Point Montara: Lt	37 32 N	122 31 W
520	Mile Rocks: Lt	37 48 N	122 31 W
16600	SAN FRANCISCO BAY		
610	—Alcatraz I.: Lt	37 50 N	122 25 W
620	—*San Francisco*	37 49 N	122 25 W
630	—*Redwood City*	37 32 N	122 12 W
640	—*Alameda*	37 47 N	122 16 W
650	—*Oakland*	37 49 N	122 20 W
660	—*Richmond*	37 55 N	122 22 W
670	—*Port Chicago*	38 03 N	122 01 W
680	—*Stockton*	37 57 N	121 18 W
690	—*Vallejo*	38 06 N	122 15 W
16700	—*Mare I.*	38 06 N	122 16 W
16800	Point Bonita: Lt	37 49 N	122 32 W
810	Southeast Farallon I.: Lt	37 42 N	123 00 W
820	Point Reyes: Lt	38 00 N	123 01 W
830	Point Arena: Lt	38 57 N	123 44 W
840	Point Cabrillo: Lt	39 21 N	123 50 W
850	Cape Mendocino: Lt	40 26 N	124 24 W
860	Table Bluff: Lt	40 42 N	124 16 W
870	*Eureka*	40 48 N	124 11 W
880	Trinidad Head: Lt	41 03 N	124 09 W
890	St. George Reef: Lt	41 50 N	124 22 W
16900	**Oregon**		
16910	Cape Blanco: Lt	42 50 N	124 34 W
920	Cape Arago: Lt	43 20 N	124 22 W
930	*Coos Bay (Marshfield)*	43 22 N	124 13 W
940	Heceta Head: Lt	44 08 N	124 08 W
950	Yaquina Head: Lt	44 41 N	124 05 W
960	Cape Meares: Lt	45 29 N	123 59 W
970	Tillamook Rock: Lt	45 56 N	124 01 W
17000	**Columbia River**		
17010	*Astoria, Oregon*	46 12 N	123 50 W
020	*Longview, Washington*	46 08 N	122 56 W
030	*Portland, Oregon*	45 31 N	122 40 W
040	*Vancouver, Washington*	45 38 N	122 41 W
17100	**Washington**		
17110	Cape Disappointment: Lt	46 17 N	124 03 W
120	North Head: Lt	46 18 N	124 05 W
130	Cape Shoalwater: Willapa Bay Lt	46 44 N	124 04 W
140	Point Chehalis: Grays Harbor Lt	46 53 N	124 07 W
150	*Aberdeen*	46 59 N	123 49 W
160	*Hoquiam*	46 58 N	123 54 W
170	Destruction I.: Lt	47 40 N	124 29 W
180	Cape Flattery, Tatoosh I.: Lt	48 24 N	124 44 W
190	Slip Point: Lt	48 16 N	124 15 W
17200	Ediz Hook: Lt	48 08 N	123 24 W
210	*Port Angeles*	48 07 N	123 26 W
220	New Dungeness: Lt	48 11 N	123 07 W
230	*Port Townsend*	48 07 N	122 45 W

Index No.	Place	Lat.	Long.
	Washington—Continued	° ′	° ′
17300	PUGET SOUND		
310	—*Seattle*	47 36 N	122 20 W
320	—*Bremerton*	47 34 N	122 39 W
330	—*Tacoma*	47 17 N	122 25 W
340	—*Olympia*	47 03 N	122 54 W
17400	*Everett*	48 00 N	122 13 W
410	Smith I.: Lt	48 19 N	122 51 W
420	*Anacortes*	48 31 N	122 37 W
430	*Bellingham*	48 45 N	122 30 W
440	Point Roberts: Lt	48 58 N	123 05 W
450	Patos I.: Lt	48 47 N	122 58 W
460	Skipjack I.: Lt	48 44 N	123 02 W
470	Turn Point, Stuart I.: Lt	48 41 N	123 14 W
480	Kellett Bluff: Lt	48 35 N	123 12 W
490	San Juan I.: Lime Kiln Lt	48 31 N	123 09 W
17500	**British Columbia**		
17600	VANCOUVER ISLAND		
610	—*Victoria*	48 25 N	123 24 W
620	—*Esquimalt*	48 26 N	123 26 W
630	—Race Rocks: Lt	48 18 N	123 32 W
640	—Sheringham Point: Lt	48 23 N	123 55 W
650	—Carmanah: Lt	48 37 N	124 45 W
660	—Pachena Point: Lt	48 43 N	125 06 W
670	—Cape Beale: Lt	48 47 N	125 13 W
680	—*Alberni*	49 14 N	124 48 W
690	—Amphitrite Point: Lt	48 55 N	125 32 W
17700	—Lennard I.: Lt	49 07 N	125 55 W
710	—Estevan Point: Lt	49 23 N	126 33 W
720	—Lookout I.: Lt	50 00 N	127 27 W
730	—Cape Cook, Solander I.: Lt	50 07 N	127 57 W
740	—*Port Alice*	50 23 N	127 28 W
750	—Kains I.: Lt	50 26 N	128 02 W
760	—Cape Scott: Lt	50 47 N	128 26 W
770	—*Nanaimo*	49 10 N	123 56 W
17800	*New Westminster*	49 12 N	122 55 W
810	*Vancouver*	49 17 N	123 07 W
820	Egg I.: Lt	51 15 N	127 50 W
17900	QUEEN CHARLOTTE ISLANDS		
910	—Cape St. James: Lt	51 56 N	131 01 W
920	—Langara I.: Lt	54 15 N	133 03 W
930	—Lawn Point: Lt	53 25 N	131 55 W
18000	*Ocean Falls*	52 21 N	127 42 W
010	*Prince Rupert*	54 19 N	130 20 W
18100	**Alaska**		
18110	Tree Point: Lt	54 48 N	130 56 W
120	Barren I.: Lt	54 45 N	131 21 W
130	*Ketchikan*	55 20 N	131 38 W
140	Cape Chacon: Lt	54 41 N	132 01 W
150	Cape Muzon: Lt	54 40 N	132 41 W
160	Cape Bartolome: Lt	55 14 N	133 37 W
170	*Wrangell*	56 28 N	132 23 W
180	*Petersburg*	56 49 N	132 57 W
190	Cape Decision: Lt	56 00 N	134 08 W
18200	Cape Ommaney: Lt	56 10 N	134 40 W
210	*Sitka*	57 03 N	135 20 W
220	Cape Edgecumbe: Lt	57 00 N	135 51 W
230	Klokachef I.: Lt	57 24 N	135 54 W
240	Lisianski Strait Entrance: Lt	57 51 N	136 26 W
250	*Juneau*	58 18 N	134 25 W
260	*Skagway*	59 27 N	135 20 W
270	Cape Spencer: Lt	58 12 N	136 38 W
280	Ocean Cape: Lt	59 32 N	139 51 W
290	*Yakutat*	59 33 N	139 44 W
18300	Cape St. Elias: Lt	59 48 N	144 36 W
310	Cape Hinchinbrook: Lt	60 14 N	146 39 W
320	*Cordova*	60 33 N	145 46 W
330	*Valdez*	61 07 N	146 15 W
340	*Whittier*	60 50 N	148 40 W
350	Point Elrington: Lt	59 56 N	148 15 W
360	Cape Resurrection, Barwell I.: Lt	59 52 N	149 17 W
370	*Seward*	60 06 N	149 26 W
380	Pilot Rock: Lt	59 44 N	149 28 W
390	Seal Rocks: Lt	59 31 N	149 38 W
18400	East Chugach I.: Lt	59 06 N	151 26 W
410	Perl I.: Lt	59 07 N	151 38 W
420	East Amatuli I.: Lt	58 55 N	151 57 W
430	Flat I.: Lt	59 20 N	152 00 W
440	*Seldovia*	59 26 N	151 44 W

APPENDIX S

MARITIME POSITIONS

WEST COAST OF NORTH AMERICA—Continued

Index No.	Place	Lat.	Long.
	Alaska—Continued	° ′	° ′
18450	Anchor Point: Lt	59 46 N	151 52 W
460	*Kenai*	60 33 N	151 16 W
470	Chisik I.: Lt	60 06 N	152 34 W
480	Kalgin I.: Lt	60 29 N	151 50 W
490	East Foreland: Lt	60 43 N	151 24 W
18500	*Anchorage*	61 13 N	149 54 W
510	Afognak I., Tonki Cape: Lt	58 21 N	151 59 W
520	Spruce Cape: Loran Station	57 49 N	152 20 W
530	*Kodiak*	57 47 N	152 24 W
540	*Womens Bay*	57 43 N	152 31 W
550	Cape Chiniak: Lt	57 38 N	152 09 W
560	Dangerous Cape: Lt	57 16 N	152 43 W
570	Sitkinak I., Whirlpool Point: Lt	56 37 N	154 06 W
580	Cape Alitak: Lt	56 51 N	154 18 W
590	Cape Uyak: Lt	57 38 N	154 21 W
18600	Raspberry Strait, Cape Nuniliak: Lt	58 10 N	153 13 W
610	Alligator I.: Lt	58 28 N	152 48 W
620	Cape Igvak: Lt	57 26 N	156 02 W
630	Foggy Cape: Lt	56 32 N	156 59 W
640	*Chignik*	56 18 N	158 24 W
650	Mitrofania I.: Lt	55 50 N	158 42 W
18700	SHUMAGIN ISLANDS		
710	—Cape Wedge: Lt	55 18 N	159 53 W
720	—*Sand Point*	55 20 N	160 32 W
730	—Unga Spit: Lt	55 25 N	160 44 W
18800	Arch Point: Lt	55 12 N	161 54 W
810	Iliasik Is.: Lt	55 02 N	161 56 W
820	*King Cove*	55 03 N	162 19 W
830	Hague Rock: Lt	54 33 N	162 24 W
840	*Sanak*	54 29 N	162 49 W
18900	UNIMAK ISLAND		
910	—Cape Pankof: Lt	54 40 N	163 04 W
920	—Scotch Cap: Lt	54 24 N	164 45 W
930	—Cape Sarichef: Lt	54 36 N	164 56 W
19000	**Aleutian Islands**		
19010	Akutan Harbor: Lt	54 09 N	165 44 W
020	*Unalaska*	53 52 N	166 32 W
	Aleutian Islands—Continued	° ′	° ′
19030	*Dutch Harbor*	53 54 N	166 32 W
040	*Makushin*	53 46 N	166 59 W
050	Seguam I.: Lt	52 23 N	172 26 W
060	Atka I., North Cape: Lt	52 26 N	174 11 W
070	Great Sitkin I., Swallow Head: Lt	52 07 N	176 09 W
080	*Adak I., Sweeper Cove*	51 52 N	176 38 W
090	*Amchitka I., Constantine Harbor*	51 24 N	179 18 E
19100	*Kiska Harbor*	51 58 N	177 33 E
110	*Shemya I., Alcan Harbor*	52 44 N	174 04 E
120	*Attu I., Massacre Bay*	52 50 N	173 14 E
19200	PRIBILOF ISLANDS		
210	—*St. Paul I., Village Cove*	57 08 N	170 16 W
19300	**Alaska**		
19310	Sealion Rocks: Lt	55 28 N	163 11 W
320	*Port Moller*	55 59 N	160 34 W
330	Cape Seniavin: Lt	56 23 N	160 08 W
340	*Port Heiden*	56 53 N	158 42 W
350	*Pilot Point*	57 33 N	157 36 W
360	*Ugashik*	57 32 N	157 25 W
370	*Egegik*	58 12 N	157 22 W
380	*Naknek*	58 43 N	157 01 W
390	*Clarks Point*	58 51 N	158 33 W
19400	*Dillingham*	59 02 N	158 29 W
19500	NUNIVAK ISLAND		
510	—Cape Mohican: Lt	60 13 N	167 27 W
520	—*Nash Harbor*	60 12 N	166 59 W
19600	*St. Lawrence I., Savoonga*	63 41 N	170 24 W
610	Point Romanof: Lt	63 12 N	162 50 W
620	*St. Michael*	63 29 N	162 02 W
630	*Unalakleet*	63 52 N	160 46 W
640	Cape Darby: Lt	64 20 N	162 47 W
650	*Solomon*	64 33 N	164 24 W
660	*Nome*	64 30 N	165 25 W
670	Sledge I.: Lt	64 30 N	166 11 W
680	Cape Rodney: Lt	64 40 N	166 24 W
690	Point Spencer: Lt	65 17 N	166 50 W
19700	*Grantley Harbor*	65 16 N	166 20 W

HAWAIIAN ISLANDS

Index No.	Place	Lat.	Long.
		° ′	° ′
20000	HAWAII		
010	—Kauhola Point: Lt	20 15 N	155 46 W
020	—*Kukuihaele Landing*	20 08 N	155 34 W
030	—Laupahoehoe Point: Lt	20 00 N	155 15 W
040	—Pepeekeo Point: Lt	19 51 N	155 05 W
050	—Paukaa Point: Lt	19 46 N	155 06 W
060	—*Hilo*	19 44 N	155 04 W
070	—Cape Kumukahi: Lt	19 31 N	154 49 W
080	—Ka Lae: Lt	18 55 N	155 41 W
090	—Kauna Point: Lt	19 02 N	155 53 W
20100	—*Napoopoo*	19 29 N	155 56 W
110	—*Kailua*	19 38 N	156 00 W
120	—Keahole Point: Lt	19 44 N	156 04 W
130	—*Kawaihae*	20 02 N	155 50 W
140	—*Mahukona*	20 11 N	155 54 W
20200	MAUI		
210	—Kauiki Head: Lt	20 46 N	155 59 W
220	—*Hana Bay*	20 45 N	155 59 W
230	—Pauwela Point: Lt	20 57 N	156 20 W
240	—*Kahului*	20 54 N	156 28 W
250	—Nakalele Point: Lt	21 02 N	156 36 W
260	—Hawea Point: Lt	21 00 N	156 40 W
270	—*Lahaina*	20 52 N	156 40 W
280	—Cape Hanamanioa: Lt	20 35 N	156 25 W
20300	Molokini I.: Lt	20 38 N	156 30 W
310	Kahoolawe I.: Southwest Point Lt	20 30 N	156 40 W
20400	LANAI		
410	—Cape Kaea: Lt	20 44 N	156 58 W
420	—*Kaumalapau Harbor*	20 47 N	157 00 W
20500	MOLOKAI		
510	—Laau Point: Lt	21 06 N	157 18 W
520	—*Kaunakakai*	21 05 N	157 02 W
	MOLOKAI—Continued		
20530	—Makanalua Peninsula: Molokai Lt	21 13 N	156 58 W
20600	OAHU		
610	—Makapuu Point: Lt	21 19 N	157 39 W
620	—Diamond Head: Lt	21 16 N	157 49 W
630	—*Honolulu*	21 18 N	157 52 W
640	—Pearl Harbor	21 22 N	157 58 W
650	—Barbers Point: Lt	21 18 N	158 06 W
660	—Kauna Point: Lt	21 35 N	158 17 W
670	—Kaneohe Bay, Pyramid Rock: Lt	21 28 N	157 46 W
20700	KAUAI		
710	—Kilauea Point: Lt	22 14 N	159 24 W
720	—Kahala Point: Lt	22 09 N	159 18 W
730	—*Nawiliwili Bay*	21 57 N	159 21 W
740	—Makahuena Point: Lt	21 52 N	159 27 W
750	—*Port Allen*	21 54 N	159 35 W
760	—Hanapepe Bay: Puolo Point Lt	21 54 N	159 36 W
770	—Kokole Point: Lt	21 59 N	159 46 W
20800	Lehua I.: Lt	22 01 N	160 06 W
810	*Niihau I., Nonopapa*	21 52 N	160 14 W
820	Nihoa I	23 04 N	161 55 W
830	Necker I	23 35 N	164 42 W
840	French Frigate Shoals, East I	23 47 N	166 13 W
850	Gardner Pinnacles	25 00 N	168 00 W
860	Laysan I	25 46 N	171 44 W
870	Lisianski I	26 04 N	173 58 W
20900	MIDWAY ISLANDS		
910	—Sand I.: Aero Lt	28 13 N	177 23 W
920	—*Welles Harbor, Midway I*	28 13 N	177 24 W

APPENDIX S

MARITIME POSITIONS

WEST INDIES

Index No.	Place	Lat.	Long.
		° ′	° ′
21000	Grand Bahama, Southwest Point: Lt	26 30 N	78 46 W
010	Great Isaac: Lt	26 02 N	79 05 W
020	Gun Cay: Lt	25 34 N	79 18 W
030	Great Stirrup Cay: Lt	25 50 N	77 53 W
040	Great Abaco I. (Hole in the Wall): Lt	25 51 N	77 11 W
050	Elbow Cay (Little Guana): Lt	26 32 N	76 57 W
060	Andros I., N. end: Lt	25 08 N	78 00 W
070	*Nassau*	25 05 N	77 20 W
080	Eleuthera Point: Lt	24 38 N	76 08 W
090	Cat I., Devils Point: Lt	24 07 N	75 28 W
21100	San Salvador (Watling I.), Dixon Hill: Lt	24 06 N	74 26 W
110	Long I., South Point: Lt	22 51 N	74 52 W
120	Bird Rock, Crooked I.: Lt	22 51 N	74 22 W
130	Castle I.: Lt	22 07 N	74 20 W
140	Mayaguana, Northwest Point: Lt	22 28 N	73 07 W
150	South Caicos: Lt	21 30 N	71 31 W
160	Grand Turk: Lt	21 31 N	71 08 W
170	Great Inagua: Lt	20 56 N	73 40 W
21200	**Cuba**		
21210	Punta Maisí (Cape Maysi): Lt	20 15 N	74 09 W
220	*Baracoa*	20 21 N	74 30 W
230	Cayo Grande de Moa: Lt	20 42 N	74 54 W
240	*Puerto Tanamo*	20 42 N	75 20 W
250	Punta Mayari, Bahia de Nipe: Lt	20 48 N	75 31 W
260	*Puerto Banes*	20 55 N	75 42 W
270	Cabo Lucrecia: Lt	21 05 N	75 37 W
280	*Puerto Vita*	21 05 N	75 57 W
290	*Gibara*	21 07 N	76 07 W
21300	*Puerto Padre*	21 17 N	76 32 W
310	Punta Practicos: Lt	21 37 N	77 06 W
320	*Santa Lucia*	22 40 N	83 58 W
330	Maternillos: Lt	21 40 N	77 08 W
340	Cayo Verde: Lt	22 07 N	77 39 W
350	Cayo Paredon Grande: Lt	22 29 N	78 10 W
360	Cayo Caiman Grande: Lt	22 41 N	78 53 W
370	Cayo Frances: Lt	22 39 N	79 14 W
380	*Caibarién*	22 32 N	79 28 W
390	Cayo Fragoso: Lt	22 49 N	79 35 W
21400	*La Isabela* (*Sagua la Grande*)	22 57 N	80 01 W
410	Cayo Hicacal: Lt	23 05 N	80 05 W
420	Cayo Bahia de Cadiz: Lt	23 13 N	80 29 W
430	Cayo Cruz del Padre: Lt	23 17 N	80 54 W
440	Cayo Diana: Lt	23 10 N	81 06 W
450	*Cárdenas*	23 03 N	81 12 W
460	Cayo Piedras del Norte: Lt	23 15 N	81 08 W
470	Punta de Maya: Lt	23 06 N	81 28 W
480	*Matanzas*	23 03 N	81 34 W
490	Punta Seboruco: Lt	23 09 N	81 36 W
21500	Castillo del Morro (Morro Castle): Lt	23 09 N	82 21 W
510	*Habana* (*Havana*)	23 09 N	82 21 W
520	*Puerto Vita*	21 05 N	75 57 W
530	*Cabañas*	22 59 N	82 56 W
540	Punta Gobermadora: Lt	23 00 N	83 13 W
550	Cayo Jutias: Lt	22 43 N	84 01 W
560	Cayo de Buenavista: Lt	22 24 N	84 27 W
570	Cabo San Antonio: Lt	21 52 N	84 57 W
580	Cabo Corrientes: Lt	21 46 N	84 31 W
590	Cabo Francés: Lt	21 55 N	84 02 W
21600	Isla de Pinos, Caleta Carapachibey: Lt	21 27 N	82 55 W
610	*Surgidero de Batabanó*	22 41 N	82 18 W
620	Cayo Guane del Este: Lt	21 40 N	81 02 W
630	*Cienfuegos*	22 09 N	80 27 W
640	Punta Colorados: Lt	22 02 N	80 27 W
650	Cayo Blanco de Casilda: Lt	21 38 N	79 53 W
660	Cayo Blanco de Tunas: Lt	21 36 N	79 36 W
670	Cayo Breton: Lt	21 07 N	79 27 W
680	Cayo Cachiboca: Lt	20 41 N	78 45 W
690	Cayo La Perla: Lt	20 22 N	77 15 W
21700	*Manzanillo*	20 21 N	77 07 W
710	Cabo Cruz: Lt	19 51 N	77 44 W
720	*Santiago de Cuba*	20 01 N	75 50 W
730	Morro: Lt	19 57 N	75 52 W
740	Windward Point, Bahia de Guantanamo: Lt	19 53 N	75 10 W
750	Punta Caleta: Lt	20 04 N	74 17 W
21800	**Jamaica**	° ′	° ′
21810	Morant Point: Lt	17 55 N	76 11 W
820	*Port Antonio*	18 12 N	76 27 W
830	Galina Point: Lt	18 25 N	76 55 W
840	*Montego Bay*	18 29 N	77 56 W
850	South Negril Point: Lt	18 15 N	78 23 W
860	Little Pedro Point (Port Kaiser)	17 52 N	77 34 W
870	Portland Point: Lt	17 44 N	77 10 W
880	*Port Royal*	17 56 N	76 51 W
890	*Kingston*	17 58 N	76 47 W
21900	**Caribbean Sea**		
21910	Navassa I.: Lt	18 24 N	75 01 W
22000	CAYMAN ISLANDS		
010	—Cayman Brac: Lt	19 45 N	79 44 W
020	—Gorling Bluff, Grand Cayman: Lt	19 18 N	81 06 W
22100	Swan Is.: Aviation Lt	17 24 N	83 56 W
110	Quita Sueño Bank: Lt	14 28 N	81 07 W
120	Isla de Providencia (Old Providence I.): Lt	13 19 N	81 23 W
130	Cayos del Ese (Courtown Cays): Lt	12 24 N	81 28 W
140	Roncador Bank: Lt	13 34 N	80 05 W
150	Serrana Bank: Lt	14 17 N	80 24 W
22200	**Haiti**		
22210	*Cap-Haïtien*	19 46 N	72 12 W
220	Pointe Picolet: Lt	19 48 N	72 12 W
230	Pointe Ouest, Tortuga: Lt	20 01 N	72 38 W
240	Cap du Môle (Cape St. Nicolas Mole): Lt	19 50 N	73 25 W
250	Pointe Saint-Marc: Lt	19 02 N	72 49 W
260	Les Arcadins: Lt	18 48 N	72 39 W
270	*Port Au Prince*	18 33 N	72 21 W
280	Pointe Lamentin: Lt	18 33 N	72 24 W
290	Île de la Gonâve, Pointe Fantasque: Lt	18 42 N	72 49 W
22300	Banc de Rochelois: Lt	18 39 N	73 12 W
310	Cap Dame-Marie: Lt	18 37 N	74 26 W
320	Île à Vache: Lt	18 04 N	73 34 W
330	*Jacmel*	18 13 N	72 31 W
22400	**Dominican Republic**		
22410	Isla Alto Velo: Lt	17 28 N	71 38 W
420	Punta Palenque: Lt	18 13 N	70 09 W
430	Jaina (Puerto Rio Haina)	18 25 N	70 00 W
440	*Santo Domingo* (*Ciudad Trujillo*)	18 28 N	69 53 W
450	*San Pedro de Macoris*	18 27 N	69 18 W
460	*La Romana*	18 25 N	68 57 W
470	Isla Saona: Lt	18 07 N	68 34 W
480	Cabo Engaño: Lt	18 36 N	68 19 W
490	Cabo Samaná: Lt	19 18 N	69 08 W
22500	Cabo Viejo Francés: Lt	19 41 N	69 55 W
510	*Puerto Plata*	19 49 N	70 41 W
520	*Puerto Libertador*	19 43 N	71 45 W
22600	**Puerto Rico**		
22610	Isla Mona: Lt	18 05 N	67 51 W
620	Punta Jiguero: Lt	18 22 N	67 16 W
630	Punta Borinquen: Lt	18 30 N	67 09 W
640	*San Juan*	18 28 N	66 07 W
650	Cabo San Juan: Lt	18 23 N	65 37 W
660	Isla Cabeza de Perro: Lt	18 15 N	65 35 W
670	*Ensenada Honda*	18 14 N	65 38 W
680	Punta Tuna: Lt	17 59 N	65 53 W
690	Punta Figuras, Puerto Arroyo: Lt	17 57 N	66 03 W
22700	Isla Cajas de Muertos: Lt	17 54 N	66 31 W
710	*Ponce*	17 59 N	66 37 W
720	*Guanico*	17 58 N	66 55 W
730	Cabo Rojo: Lt	17 56 N	67 11 W
740	*Mayagüez*	18 12 N	67 09 W
22800	**Lesser Antilles**		
22810	Isla de Vieques, Punta Este: Lt	18 08 N	65 16 W
820	Isla de Culebra, Punta del Soldado: Lt	18 17 N	65 17 W

APPENDIX S

MARITIME POSITIONS

WEST INDIES—Continued

Index No.	Place	Lat.	Long.
	Lesser Antilles—Continued	° ′	° ′
22900	VIRGIN ISLANDS		
910	—Buck I.: Lt	18 17 N	64 54 W
920	—*Charlotte Amalie, St. Thomas I.*	18 21 N	64 56 W
930	—St. John I	18 20 N	64 45 W
940	—*Frederiksted, St. Croix I*	17 43 N	64 53 W
950	—*Road Town, Tortola I*	18 25 N	64 37 W
960	—Anegada I	18 45 N	64 20 W
23000	Sombrero: Lt	18 36 N	63 26 W
010	Anguilla: Lt	18 12 N	63 06 W
23100	ÎLE SAINT-MARTIN		
110	—*Marigot (France)*	18 04 N	63 06 W
120	—*Filipsburg (Netherlands)*	18 01 N	63 03 W
23200	*Gustavia, Île Saint-Barthelémy*	17 54 N	62 51 W
210	Saba	17 38 N	63 14 W
220	*Oranjestad, Sint Eustatius*	17 29 N	62 59 W
230	*Basseterre, St. Christopher (St. Kitts)*	17 26 N	62 48 W
240	*Charlestown, Nevis I*	17 08 N	62 37 W
250	Barbuda	17 38 N	62 48 W
260	*Saint Johns, Antigua*	17 07 N	61 50 W
270	*Plymouth, Montserrat*	16 42 N	62 14 W
280	*Basse-Terre, Guadeloupe*	16 00 N	61 44 W
290	*Pointè-a-Pître, Grand Terre*	16 14 N	61 32 W
23300	La Desírade: Lt	16 20 N	61 01 W
310	*Grande Bourg, Marie Galante*	15 53 N	61 19 W
320	Isla Aves (Bird I.)	15 42 N	63 38 W
330	*Roseau, Dominica*	15 17 N	61 24 W
23400	MARTINIQUE		
410	—*Trinité*	14 45 N	60 58 W
420	—Caravelle: Lt	14 46 N	60 53 W
430	—*Fort-de-France*	14 36 N	61 05 W

Index No.	Place	Lat.	Long.
	Lesser Antilles—Continued	° ′	° ′
23500	ST. LUCIA		
510	—*Castries*	14 01 N	61 00 W
520	—Brandon Point (Cape Moule a Chique): Lt	13 43 N	60 57 W
23600	BARBADOS		
610	—Harrison Point: Lt	13 18 N	59 39 W
620	—Ragged Point: Lt	13 10 N	59 26 W
630	—*Bridgetown*	13 05 N	59 37 W
23700	*Kingstown, St. Vincent*	13 09 N	61 14 W
710	The Grenadines, Carriacou I	12 28 N	61 27 W
23800	GRENADA		
810	—*Saint Georges*	12 03 N	61 45 W
820	—Saline Point: Lt	12 00 N	61 48 W
23900	*Scarborough, Tobago*	11 11 N	60 44 W
910	Isla Testigo Grande	11 23 N	63 07 W
920	Los Roques: Lt	11 58 N	66 41 W
24000	BONAIRE		
010	—Boca Spelonk: Lt	12 14 N	68 12 W
020	—Lacre Punt: Lt	12 02 N	68 14 W
030	—*Kralendijk*	12 10 N	68 17 W
24100	Klein Curaçao: Lt	12 00 N	68 39 W
24200	CURAÇAO		
210	—*Caracas Baai*	12 05 N	68 52 W
220	—*Willemstad*	12 07 N	68 56 W
230	—*Bullen Baai*	12 11 N	69 01 W
24300	ARUBA		
310	—*Sint Nicolaas*	12 26 N	69 54 W
320	—*Oranjestad*	12 31 N	70 02 W
330	—*Druif*	12 32 N	70 04 W

EAST COAST OF SOUTH AMERICA

Index No.	Place	Lat.	Long.
25000	**Colombia**	° ′	° ′
25010	Isla Fuerte: Lt	9 24 N	76 11 W
020	Isla Tesoro: Lt	10 14 N	75 44 W
030	Isla Tierra Bomba: Lt	10 21 N	75 35 W
040	*Cartagena*	10 19 N	75 35 W
050	Punta Canoas: Lt	10 35 N	75 30 W
060	Punta Hermosa: Lt	10 58 N	75 01 W
070	*Barranquilla*	11 00 N	74 48 W
080	*Santa Marta*	11 15 N	74 13 W
090	Cabo de la Vela: Lt	12 13 N	72 10 W
25100	**Venezuela**		
25110	*Maracaibo*	10 38 N	71 37 W
120	*Las Piedras*	11 42 N	70 13 W
130	Punta Macolla: Lt	12 06 N	70 13 W
140	Cabo San Román: Lt	12 12 N	70 00 W
150	Cayo Borracho: Lt	10 59 N	68 15 W
160	*Puerto Cabello*	10 29 N	68 01 W
170	*La Guaira*	10 37 N	66 56 W
180	La Tortuga, Punta Oriental: Lt	10 55 N	65 13 W
190	*Guanta*	10 14 N	64 36 W
25200	*Puerto Sucre*	10 28 N	64 11 W
210	Isla de Margarita, Cabo de la Isla (Cabo Negro): Lt	11 10 N	63 53 W
220	*Carúpano*	10 40 N	63 15 W
230	*Güiria*	10 34 N	62 18 W
240	*Caripito*	10 10 N	63 03 W
25300	RIO ORINOCO		
310	—Punta Barima: Lt	8 36 N	60 25 W
320	—Puerto Ordaz	8 22 N	62 42 W
25400	**Trinidad**		
25410	Icacos Point: Lt	10 04 N	61 56 W
420	*Port-of-Spain*	10 39 N	61 31 W
430	Chacachacare, Dragons Mouth (Boca de Dragon): Lt	10 42 N	61 45 W
440	Galera Point: Lt	10 50 N	60 55 W
25500	**British Guiana**		
25510	*Georgetown*	6 50 N	58 10 W

Index No.	Place	Lat.	Long.
25600	**Surinam**	° ′	° ′
25610	*Paramaribo*	5 49 N	55 09 W
620	Galibi, Maroni River: Lt	5 45 N	53 59 W
25700	**French Guiana**		
25710	Île Royale, Îles du Salut: Lt	5 17 N	52 36 W
720	L'Enfant Perdu: Lt	5 03 N	52 22 W
730	*Cayenne*	4 56 N	52 20 W
25800	**Brazil**		
25810	Ilha de Maracá: Lt	2 12 N	50 17 W
820	Ilha Bailique, Amazon River (Rio Amazonas): Lt	1 00 N	49 55 W
830	Ilha Machadinho: Lt	0 09 S	48 44 W
840	Cabo Maguari, Simao Grande: Lt	0 17 S	48 25 W
850	*Belém (Pará)*	1 27 S	48 30 W
860	Ilhas das Gaivotas: Lt	0 35 S	48 03 W
870	Ponta Atalaia (Salinópolis): Lt	0 36 S	47 22 W
880	Ilha Boiuçucanga (Camara-Assu I.)	0 50 S	46 38 W
890	Ilha Maiau (Sao Joao I.): Lt	1 17 S	44 55 W
25900	Ponta Itacolomí: Lt	2 10 S	44 25 W
910	*São Luís (Maranhão)*	2 32 S	44 17 W
920	Ilha de Santana: Lt	2 16 S	43 36 W
930	*Tutóia*	2 46 S	42 16 W
940	Ponta Pedro do Sal: Lt	2 50 S	41 44 W
950	*Parnaíba*	3 00 S	41 46 W
960	Ponta de Itapagé: Lt	2 50 S	40 00 W
970	*Fortaleza (Ceara)*	3 43 S	38 32 W
980	Ponta de Mucuripe: Lt	3 42 S	38 28 W
990	Rochedos de São Pedro e São Paulo (St. Paul Rocks)	0 56 N	29 22 W
26000	ARQUIPÉLAGO DE FERNANDO DE NORONHA		
010	—Atol das Rocas: Lt	3 51 S	33 49 W
020	—Ilha Rata: Lt	3 49 S	32 23 W
26100	Cabo Calcanhar: Lt	5 10 S	35 29 W
110	Cabo de São Roque: Lt	5 29 S	35 15 W
120	*Natal*	5 47 S	35 12 W
130	Ponta Pinto: Lt	5 48 S	35 11 W

APPENDIX S

MARITIME POSITIONS

EAST COAST OF SOUTH AMERICA—Continued

Index No.	Place	Lat.	Long.
	Brazil—Continued	° ′	° ′
26140	*Cabedelo*	6 58 S	34 50 W
150	Ponta de Pedras: Lt	7 38 S	34 49 W
160	Olinda: Lt	8 01 S	34 51 W
170	*Recife (Pernambuco)*	8 04 S	34 53 W
180	Cabo Santo Agostinho: Lt	8 20 S	34 56 W
190	*Tamandaré*	8 44 S	35 05 W
26200	*Maceió*	9 40 S	35 44 W
210	Aracaju: Lt	10 58 S	37 03 W
220	Ponta Itapuã (Itapoan): Lt	12 57 S	38 21 W
230	Ponta de Santo Antônio: Lt	13 01 S	38 32 W
240	*Salvador (Baía) (Bahia)*	12 58 S	38 31 W
250	Morro de São Paulo: Lt	13 23 S	38 55 W
260	*Camamu*	13 54 S	39 02 W
270	*Ilhéus (São Jorge dos Ilheos)*	14 48 S	39 02 W
280	Morro Pernambuco: Lt	14 48 S	39 01 W
290	Belmonte: Lt	15 51 S	38 54 W
26300	Pôrto Seguro: Lt	16 26 S	39 04 W
310	Ponta Corumbau: Lt	16 52 S	39 07 W
320	Ponta de Baleia: Lt	17 41 S	39 08 W
330	Parcel dos Abrolhos: Lt	17 58 S	38 42 W
340	Rio Doce: Lt	19 37 S	39 49 W
350	*Vitória*	20 19 S	40 20 W
360	Ponta de Santa Luzia: Lt	20 19 S	40 16 W
370	Ilha Escalvada: Lt	20 44 S	40 26 W
380	Ilha do Francês: Lt	20 54 S	40 46 W
390	Cabo de São Tomé (São Thomé): Lt	22 03 S	41 03 W
26400	Ilha de Santana: Lt	22 26 S	41 42 W
410	Cabo Frio: Lt	23 01 S	42 00 W
420	Ponta Negra: Lt	22 58 S	42 40 W
430	Ilhas Maricás: Lt	23 01 S	42 55 W
440	Ilha Rasa (Raza): Lt	23 04 S	43 09 W
450	*Rio de Janeiro*	22 54 S	43 10 W
460	Ilha de Palmas: Lt	23 02 S	43 12 W
470	Ponta de Guaratiba: Lt	23 05 S	43 34 W
480	Laje da Marambaia: Lt	23 07 S	43 50 W
490	Ilha Grande, Ponta da Castelhanos: Lt	23 10 S	44 06 W
26500	Ilha Pau a Pino: Lt	23 06 S	44 07 W
510	*Angra dos Reis*	23 01 S	44 19 W
520	Laje do Coronel: Lt	23 06 S	44 24 W
530	Ponta Joatinga: Lt	23 18 S	44 30 W
540	Ilha da Vitória: Lt	23 45 S	45 01 W
550	Ilha de São Sebastião, Ponta do Boi: Lt	23 58 S	45 15 W
560	Ilha de Alcatrazes: Lt	24 06 S	45 42 W
570	Laje de Santos: Lt	24 19 S	46 10 W
580	Ilha Moela: Lt	24 03 S	46 16 W
590	*Santos*	23 56 S	46 19 W
26600	Laje da Conceição: Lt	24 14 S	46 40 W
610	Ilha Queimada Grande: Lt	24 29 S	46 41 W
620	Ilha de Bom Abrigo: Lt	25 07 S	47 52 W
630	Ponta das Conchas, Ilha do Mel: Lt	25 33 S	48 17 W
640	*Paranaguá*	25 30 S	48 30 W
650	Ilha Caiobá (Caiova I.): Lt	25 52 S	48 33 W
660	*São Francisco do Sul*	26 15 S	48 38 W
670	Ilha da Paz: Lt	26 11 S	48 29 W
680	Ponta das Cabeçudas: Lt	26 56 S	48 37 W
690	Ilha da Galé: Lt	27 11 S	48 25 W
26700	Ilha do Arvoredo: Lt	27 18 S	48 22 W
710	*Florianópolis*	27 36 S	48 34 W
720	Ponta dos Naufragados: Lt	27 50 S	48 35 W
730	Ponta de Imbituba (Ponta Grande): Lt	28 17 S	48 40 W
740	Ilhas das Araras: Lt	28 21 S	48 40 W
750	*Laguna*	28 29 S	48 47 W
760	Cabo de Santa Marta Grande: Lt	28 37 S	48 50 W
770	Capão da Canoa (Tramandaí): Lt	29 47 S	50 03 W
780	Cidreira: Lt	30 11 S	50 12 W
790	Ponta da Mostardas: Lt	31 15 S	50 54 W
26800	*Rio Grande*	32 03 S	52 06 W
810	*Pôrto Alegre*	30 00 S	51 13 W
820	Albardão: Lt	33 12 S	52 45 W
830	Chuí: Lt	33 44 S	53 22 W
26900	**Uruguay**		
26910	Cabo Polonio: Lt	34 24 S	53 48 W
920	Cabo Santa Maria: Lt	34 40 S	54 09 W
930	Isla de Lobos: Lt	35 02 S	54 53 W
27000	**Río de la Plata**	° ′	° ′
27010	Punta del Este: Lt	34 58 S	54 57 W
020	*Maldonado, Uruguay*	34 55 S	54 58 W
030	Punta Negra: Lt	34 54 S	55 15 W
040	Isla de Flores: Lt	34 57 S	55 56 W
050	Punta Brava: Lt	34 56 S	56 10 W
060	*Montevideo, Uruguay*	34 54 S	56 13 W
070	La Panela: Lt	34 55 S	56 27 W
080	*Colonia, Uruguay*	34 28 S	57 51 W
090	*Rosario, Argentina*	32 57 S	60 38 W
27100	Isla Martín García: Lt	34 11 S	58 15 W
110	*Buenos Aires, Argentina*	34 36 S	58 22 W
120	Puerto de La Plata, Argentina	34 52 S	57 54 W
130	Punta Piedras: Lt	35 27 S	57 09 W
140	Cabo San Antonio, Punta Rasa: Lt	36 18 S	56 47 W
27200	**Argentina**		
27210	Punta Médanos: Lt	36 53 S	56 41 W
220	Faro Querandí: Lt	37 28 S	57 07 W
230	Punta Mogotes: Lt	38 06 S	57 33 W
240	*Quequén*	38 35 S	58 42 W
250	Baineario Claromecó: Lt	38 51 S	60 03 W
260	Faro Recalada: Lt	39 00 S	61 16 W
27300	Bahía Blanca		
310	—*Punta Alta*	38 53 S	62 06 W
320	—*Ingeniero White*	38 48 S	62 16 W
27400	El Rincon: Lt	39 23 S	61 01 W
410	Faro Segunda Barranca: Lt	40 47 S	62 16 W
420	Río Negro: Lt	41 04 S	62 50 W
430	Faro San Matías: Lt	40 49 S	64 43 W
440	*San Antonio Oeste*	40 44 S	64 55 W
450	Punta Norte: Lt	42 05 S	63 46 W
460	Punta Delgada: Lt	42 46 S	63 38 W
470	Morro Nuevo: Lt	42 53 S	64 09 W
480	*Puerto Madryn*	42 46 S	65 02 W
490	Punta Ninfas: Lt	42 58 S	64 19 W
27500	Cabo San José: Lt	44 31 S	65 18 W
510	Isla Rasa: Lt	45 06 S	65 24 W
520	Isla Leones: Lt	45 03 S	65 37 W
530	Cabo Aristazábal: Lt	45 13 S	66 32 W
540	Cabo San Jorge: Lt	45 47 S	67 23 W
550	*Comodoro Rivadavia*	45 52 S	67 28 W
560	Cabo Blanco: Lt	47 12 S	65 45 W
570	*Deseado*	47 45 S	65 54 W
580	Isla Pingüino (Isla Penguino): Lt	47 55 S	65 43 W
590	Cabo Curioso: Lt	49 11 S	67 37 W
27600	Cabo San Francisco de Paula: Lt	49 44 S	67 43 W
610	*Santa Cruz*	50 01 S	68 31 W
620	Cabo Buen Tiempo: Lt	51 33 S	68 57 W
630	*Río Gallegos*	51 37 S	69 13 W
640	Cabo Vírgenes: Lt	52 20 S	68 21 W
650	Punta de Arenas: Lt	53 09 S	68 13 W
660	Cabo Peñas: Lt	53 51 S	67 35 W
670	Cabo San Diego: Lt	54 40 S	65 07 W
680	Islas Año Nuevo, Isla Observatorio: Lt	54 39 S	64 08 W
690	Estrecho de Le Maire, Isla de los Estados: Lt	54 47 S	64 44 W
27700	Cabo Buen Suceso: Lt	54 49 S	65 13 W
710	Cabo San Pío: Lt	55 04 S	66 32 W
27800	**Chile**		
27810	Cape Horn	55 59 S	67 16 W
27900	Magellan Strait		
910	—Punta Dungeness: Lt	52 24 S	68 26 W
920	—Cabo Posesion: Lt	52 18 S	68 58 W
930	—Cerro Direccíon: Lt	52 22 S	69 30 W
940	—*Punta Delgada*	52 28 S	69 33 W
950	—Punta Mendez: Lt	52 32 S	69 35 W
960	—Punta Satelite: Lt	52 33 S	69 40 W
970	—Cerro Cono (Cone Hill): Lt	52 40 S	70 23 W
980	—Cabo San Vincente: Lt	52 47 S	70 26 W
990	—Isla Santa Magdalena: Lt	52 55 S	70 34 W
28000	—Isla Contramaestre (Quartermaster I.): Lt	52 57 S	70 22 W
010	—*Punta Arenas (Magallanes)*	53 10 S	70 54 W
020	—Cabo San Isidro: Lt	53 47 S	70 58 W
030	—Cabo Froward: Lt	53 54 S	71 18 W

APPENDIX S

MARITIME POSITIONS

WEST COAST OF SOUTH AMERICA

Index No.	Place	Lat.	Long.
29000	**Chile**	° ′	° ′
29100	Magellan Strait		
110	—Isla Rupert, English Reach: Lt	53 40 S	72 13 W
120	—Isla Cohorn: Lt	53 33 S	72 20 W
130	—Paso Tortuoso (Crooked Reach), El Morrión: Lt	53 34 S	72 31 W
140	—Monte Radford (Radford Hill): Lt	53 26 S	72 57 W
150	—Cabo Cooper Key, Paso Largo (Long Reach): Lt	53 15 S	73 13 W
160	—Isla Centinela (Sentinel I.): Lt	53 05 S	73 35 W
170	—Bahía Félix: Lt	52 58 S	74 04 W
180	—Isla Fairway: Lt	52 44 S	73 47 W
190	—Cabo Pilar	52 43 S	74 41 W
29200	Groupo Evangelistas: Lt	52 24 S	75 06 W
210	Isla San Pedro: Lt	47 43 S	74 55 W
220	Cabo Raper: Lt	46 50 S	75 37 W
230	Isla Falsa: Lt	43 53 S	73 44 W
240	Melinca: Lt	43 55 S	73 44 W
250	Isla Guafo (Huafo): Lt	43 34 S	74 50 W
260	Cabo Corcovado	43 08 S	72 55 W
270	*Puerto Montt*	41 29 S	72 57 W
29300	Isla Chiloe		
310	—Isla Laitec: Lt	43 17 S	73 35 W
320	—Punta Corona: Lt	41 47 S	73 53 W
330	—Punta Ahuí: Lt	41 50 S	73 52 W
340	—*Ancud*	41 52 S	73 50 W
29400	Cabo Quedal: Lt	40 58 S	73 56 W
410	Punta Galera: Lt	40 00 S	73 45 W
420	Morro Gonzalo: Lt	39 50 S	73 28 W
430	*Corral*	39 52 S	73 26 W
440	*Valdivia*	39 48 S	73 15 W
450	Punta Rocura: Lt	39 47 S	73 24 W
29500	Isla Mocha		
510	—Morro de las Torrecillas: Lt	38 22 S	73 58 W
520	—Punta Anegadiza: Lt	38 23 S	73 54 W
29600	Punta Morguilla: Lt	37 47 S	73 42 W
610	*Lebu*	37 38 S	73 40 W
620	Punta Lavapié: Lt	37 09 S	73 35 W
630	Isla Santa Maria: Lt	36 59 S	73 32 W
640	*Lota*	37 06 S	73 09 W
650	*Coronel*	37 02 S	73 10 W
660	Punta Gaulpén: Lt	36 45 S	73 11 W
670	Punta Tumbes: Lt	36 37 S	73 07 W
680	*Talcahuano*	36 42 S	73 06 W
690	Isla Quiriquina: Lt	36 36 S	73 03 W
29700	Cabo Carranza: Lt	35 34 S	72 38 W
710	Punta Topocalma: Lt	34 08 S	72 01 W
720	Isla Juan Fernández, San Juan Bautista: Lt	33 38 S	78 50 W
730	*San Antonio*	33 35 S	71 38 W
740	Punta Panual: Lt	33 34 S	71 38 W
750	Punta Curaumilla: Lt	33 04 S	71 45 W
760	Punta Angeles: Lt	33 01 S	71 39 W
770	*Valparaiso*	33 02 S	71 37 W
780	Cabo Tablas: Lt	31 51 S	71 34 W
790	Punta Lengua de Vaca: Lt	30 15 S	71 38 W
29800	Punta Tortuga: Lt	29 56 S	71 22 W
810	*Coquimbo*	29 57 S	71 21 W
820	Islas Pajáros: Lt	29 35 S	71 33 W
830	*Cruz Grande*	29 27 S	71 20 W
840	Isla Chañaral: Lt	29 02 S	71 36 W
850	*Huasco*	28 28 S	71 14 W
860	*Caldera*	27 03 S	70 51 W
	Chile—Continued	° ′	° ′
29870	Punta Ballenita: Lt	25 45 S	70 47 W
880	*Taltal*	25 24 S	70 29 W
890	*Antofagasta*	23 38 S	70 25 W
29900	Punta Tetas: Lt	23 31 S	70 38 W
910	Punta Angamos: Lt	23 02 S	70 32 W
920	*Mejillones*	23 07 S	70 28 W
930	*Tocopilla*	22 05 S	70 14 W
940	Punta Gruesa: Lt	20 22 S	70 12 W
950	*Iquique*	20 12 S	70 10 W
960	Isla Alacran: Lt	18 29 S	70 21 W
970	*Arica*	18 29 S	70 20 W
30000	**Peru**		
30010	Punta Coles: Lt	17 42 S	71 22 W
020	*Mollendo*	17 01 S	72 02 W
030	Punta Islay: Lt	17 01 S	72 07 W
040	*Matarani*	17 00 S	72 07 W
050	*Atico*	16 14 S	73 37 W
060	Punta San Juan	15 20 S	75 10 W
070	Punta Doña Maria, Islotes Infiernillo: Lt	14 40 S	75 56 W
080	*Pisco*	13 43 S	76 15 W
090	Islas de Chincha: Lt	13 39 S	76 25 W
30100	Grupo de Palominos: Lt	12 08 S	77 15 W
110	Isla San Lorenzo: Lt	12 05 S	77 13 W
120	*Callao*	12 03 S	77 10 W
130	Isla Mazorca: Lt	11 24 S	77 44 W
140	Punta Cabeza Lagarto, Puerto Huarmey: Lt	10 07 S	78 11 W
150	Chimbote: Lt	9 05 S	78 36 W
160	Guañape (Huanape Is.): Lt	8 35 S	78 57 W
170	*Salaverry*	8 14 S	78 58 W
180	Isla de Macabi: Lt	7 49 S	79 29 W
190	*Puerto Eten*	6 56 S	79 52 W
30200	Islas Lobos de Afuera: Lt	6 57 S	80 43 W
210	*Isla Lobos de Tierra*	6 26 S	80 50 W
220	Punta Aguja: Lt	5 55 S	81 09 W
230	Isla Foca: Lt	5 13 S	81 13 W
240	*Paita*	5 05 S	81 07 W
250	Punta Pariñas: Lt	4 40 S	81 20 W
260	*Talara*	4 34 S	81 17 W
270	*Cabo Blanco*	4 16 S	81 15 W
30300	**Ecuador**		
30310	Isla Santa Clara: Lt	3 10 S	80 25 W
320	Punta Jambeli: Lt	3 13 S	80 02 W
330	Punta Arena: Lt	3 01 S	80 07 W
340	*Puná*	2 44 S	79 55 W
350	*Guayaquil*	2 12 S	79 53 W
360	Punta Santa Elena: Lt	2 11 S	81 00 W
370	Isla La Plata: Lt	1 15 S	81 06 W
380	Cabo San Lorenzo: Lt	1 03 S	80 55 W
390	Cabo Pasado: Lt	0 21 S	80 30 W
30400	Punta Galera: Lt	0 50 N	80 06 W
410	*Esmeraldas*	0 58 N	79 42 W
30500	**Colombia**		
30510	*Tumaco*	1 50 N	78 45 W
520	Isla Gorgona (Gorgonilla I.): Lt	2 56 N	78 14 W
530	*Buenaventura*	3 54 N	77 05 W
540	Punta Charambirá: Lt	4 15 N	77 32 W
550	Punta San Francisco Solano: Lt	6 18 N	77 29 W

ISLANDS OF THE ATLANTIC OCEAN

Index No.	Place	Lat.	Long.
31000	**Bermuda**	° ′	° ′
31010	North Rock: Lt	32 28 N	64 46 W
020	*St. George's*	32 23 N	64 41 W
030	St. David's I.: Lt	32 22 N	64 39 W
040	Gibbs Hill: Lt	32 15 N	64 50 W
050	*Hamilton*	32 18 N	64 47 W
31100	**Azores**		
31110	Ilha do Corvo, Ponta Negra: Lt	39 40 N	31 07 W
	Azores—Continued	° ′	° ′
31200	Ilha das Flores		
210	—Ponta do Albernas: Lt	39 31 N	31 15 W
220	—*Santa Cruz*	39 27 N	31 08 W
230	—Ponta Lagens (Lages) (Lajes): Lt	39 22 N	31 11 W
31300	Ilha do Faial (Fayal Island)		
310	—Ponta Comprida (Capellinhos): Lt	38 36 N	28 50 W
320	—*Horta*	38 32 N	28 38 W
330	—Ponta da Ribeirinha: Lt	38 36 N	28 36 W

APPENDIX S

MARITIME POSITIONS

ISLANDS OF THE ATLANTIC OCEAN—Continued

Index No.	Place	Lat.	Long.
	Azores—Continued	° ′	° ′
31400	Ilha do Pico		
410	—Ponta da Areia Larga: Lt	38 32 N	28 32 W
420	—Ponta da Ilha: Lt	38 25 N	28 03 W
31500	São Jorge, Ponta do Topo: Lt	38 33 N	27 47 W
510	Graciosa, Ponta da Barca: Lt	39 06 N	28 03 W
31600	Terceira		
610	—Ponta da Serreta: Lt	38 46 N	27 23 W
620	—*Angra do Heroísmo*	38 39 N	27 13 W
630	—*Praia da Vitoria (Bahia Praia)*	38 43 N	27 03 W
31700	San Miguel		
710	—Ponta da Ferraria: Lt	37 51 N	25 52 W
720	—*Ponta Delgada*	37 44 N	25 40 W
730	—Ponta do Arnel: Lt	37 49 N	25 08 W
31800	Ilha de Santa Maria		
810	—Ponta do Castelo (Gonçalo Velho): Lt	36 55 N	25 01 W
820	—*Vila do Porto*	36 56 N	25 09 W
31900	Ilhéus Formigas (Rocas Formigas): Lt	37 16 N	24 46 W
32000	**Madeira Islands**		
32100	Ilha da Madeira		
110	—Ponta do Pargo: Lt	32 48 N	17 16 W
120	—*Funchal*	32 38 N	16 55 W
32200	Ilhéu de Fora, Ponta de Barlavento: Lt	32 43 N	16 39 W
210	Ilha de Porto Santa, Ilhéu de Cima: Lt	33 03 N	16 16 W
32300	**Canary Islands**		
32400	La Palma		
410	—Punta Cumplida: Lt	28 50 N	17 47 W
420	—Punta de Fuencaliente: Lt	28 26 N	17 50 W
430	—*Santa Cruz de La Palma*	28 40 N	17 45 W
32500	Hierro, Punta Orchilla: Lt	27 42 N	18 10 W
510	Gomera, Punta de San Cristobal: Lt	28 06 N	17 06 W
32600	Tenerife		
610	—Punta de Teno: Lt	28 21 N	16 56 W
620	—Punta Rasca: Lt	28 00 N	16 41 W
630	—*Santa Cruz de Tenerife*	28 28 N	16 14 W
640	—Punta de Anaga, Roque Bermejo: Lt	28 35 N	16 08 W
32700	Gran Canaria		
710	—Punta Sardina: Lt	28 10 N	15 43 W
720	—Punta Mas Palomas, Morro Colchas: Lt	27 44 N	15 35 W
730	—*Puerto de la Luz*	28 09 N	15 25 W
740	—Isleta: Lt	28 11 N	15 25 W
32800	Isla Fuerteventura		
810	—Punta de Jandía: Lt	28 03 N	14 31 W
820	—*Puerto de Cabras*	28 29 N	13 51 W
830	—Punta de Tostón: Lt	28 42 N	14 01 W
32900	Isla de Lobos: Lt	28 45 N	13 49 W
910	Isla Lanzarote, Punta Pechiguera: Lt	28 51 N	13 52 W
920	Isla Alegranza, Punta Delgada: Lt	29 24 N	13 29 W
33000	**Cape Verde Islands**		
33100	Ilha de Santo Antão (St. Antonio Island)		
110	—Ponta do Chão de Mangrade (Ponta Oeste): Lt	17 04 N	25 22 W
120	—Ponta do Sol: Lt	17 12 N	25 07 W
	Cape Verde Islands—Continued	° ′	° ′
33130	—Ponta de Tumba (Fontes Pereira de Melo): Lt	17 07 N	24 59 W
33200	Ilha de São Vicente (St. Vincent Island)		
210	—Ilhéu dos Passaros (D. Luiz): Lt	16 55 N	25 01 W
220	—*Pôrto Grande (Mindelo)*	16 53 N	25 00 W
230	—Ponta Machado (D. Amelia): Lt	16 50 N	25 05 W
33300	Ilha de São Nicolau (St. Nicholas Island)		
310	—Ponta do Barril: Lt	16 36 N	24 25 W
320	—*Preguiça*	16 34 N	24 17 W
330	—Ponta Calheta (Ponta Leste): Lt	16 34 N	24 00 W
33400	Ilha do Sal		
410	—Ponta Norte: Lt	16 51 N	22 55 W
420	—*Santa Maria*	16 35 N	22 54 W
33500	Ilha da Boa Vista		
510	—Ponta do Sol: Lt	16 13 N	22 56 W
520	—Ilhéu do Sal-Rei: Lt	16 10 N	22 57 W
33600	Ilha de Maio, Forte de S. Jose: Lt	15 08 N	23 13 W
33700	Ilha de São Tiago		
710	—Ponta do Lobo: Lt	14 59 N	23 25 W
720	—*Praia*	14 55 N	23 31 W
730	—Ponta Temerosa (Maria Pia): Lt	14 54 N	23 31 W
740	—Ponta Moreira: Lt	15 20 N	23 45 W
33800	Ilha do Fogo, Ponta do Alcatraz: Lt	14 50 N	24 19 W
810	Ilha Brava, Ponta Nho Martinho: Lt	14 49 N	24 43 W
33900	**Islands of the South Atlantic**		
33910	*Ascension I., Georgetown*	7 56 S	14 25 W
920	*St. Helena, Jamestown*	15 55 S	5 43 W
930	Ilhas Martim Vaz	20 30 S	28 51 W
940	Ilha de Trinidade (Trinidad I.)	20 30 S	29 19 W
950	*Tristan I. (Tristan da Cunha), Tristan Settlement*	37 03 S	12 18 W
960	Gough I	40 20 S	10 00 W
970	Bouvetøya (Bouvet I.)	54 26 S	3 24 E
34000	Falkland Islands		
010	—Cape Meredith: Lt	52 14 S	60 39 W
020	—Cape Pembroke: Lt	51 41 S	57 42 W
030	—*Port Stanley*	51 42 S	57 50 W
040	—Shag Rocks	53 33 S	42 03 W
34100	South Georgia Island		
110	—Cape Saunders: Lt	54 07 S	36 38 W
120	—Jason Islet: Lt	54 10 S	36 30 W
130	—*Grytviken Harbor*	54 16 S	36 31 W
140	—Right Whale Rocks: Lt	54 13 S	36 24 W
34200	South Sandwich Islands		
210	—Zavodoski I	56 20 S	27 35 W
220	—Thule I	59 27 S	27 18 W
34300	South Orkney Islands		
310	—*Laurie I., Scotia Bay*	60 46 S	44 41 W
320	—Signy I., Borge Bay	60 43 S	45 36 W
34400	South Shetland Islands		
410	—*King George I., Admiralty Bay*	62 00 S	58 30 W
420	—Nelson I., Harmony Point: Lt	62 17 S	59 12 W
430	—Robert I.: Lt	62 28 S	59 30 W
440	—Cape Morris: Lt	62 22 S	59 48 W
450	—Deception I., Collins Point: Lt	63 00 S	60 33 W

BRITISH ISLES

Index No.	Place	Lat.	Long.
35000	**England**	° ′	° ′
35100	Scilly Isles		
110	—Bishop Rock: Lt	49 52 N	6 27 W
120	—St. Mary's, Penninis Head: Lt	49 54 N	6 18 W
130	—Round I.: Lt	49 59 N	6 19 W
35200	Lands End, Longships: Lt	50 04 N	5 45 W
210	Wolf Rock: Lt	49 57 N	5 48 W
	England—Continued	° ′	° ′
35220	*Penzance*	50 07 N	5 32 W
230	Lizard Point (Lizard Head): Lt	49 58 N	5 12 W
240	*Falmouth*	50 09 N	5 04 W
250	St. Anthony Head: Lt	50 08 N	5 01 W
260	St. Catherines Point: Lt	50 20 N	4 39 W
270	*Fowey*	50 20 N	4 38 W

APPENDIX S

MARITIME POSITIONS

BRITISH ISLES—Continued

Index No.	Place	Lat.	Long.
	England—Continued	° ′	° ′
35280	Eddystone Rocks: Lt	50 11 N	4 16 W
290	*Plymouth*	50 22 N	4 09 W
35300	Start Point: Lt	50 13 N	3 38 W
310	*Dartmouth*	50 21 N	3 35 W
320	Berry Head: Lt	50 24 N	3 29 W
330	Bill of Portland: Lt	50 31 N	2 27 W
340	*Portland Harbor*	50 34 N	2 26 W
350	Anvil Point: Lt	50 35 N	1 57 W
360	Hurst Point: Range Lts	50 42 N	1 33 W
35400	Isle of Wight		
410	—Needles: Lt	50 40 N	1 35 W
420	—St. Catherines Point: Lt	50 34 N	1 18 W
430	—Egypt Point: Lt	50 46 N	1 19 W
35500	*Southampton*	50 54 N	1 24 W
510	*Portsmouth*	50 48 N	1 06 W
520	Southsea Castle: Lt	50 47 N	1 05 W
530	Nab Tower: Lt	50 40 N	0 57 W
540	Beachy Head: Lt	50 44 N	0 15 E
550	Dungeness: Lt	50 55 N	0 58 E
560	*Dover*	51 07 N	1 19 E
570	South Foreland: Lt	51 08 N	1 22 E
580	North Foreland: Lt	51 22 N	1 27 E
590	*Chatham*	51 24 N	0 33 E
35600	River Thames		
610	—*Woolwich*	51 30 N	0 04 E
620	—*Greenwich*	51 29 N	0 00
630	—*London*	51 30 N	0 05 W
35700	Orfordness: Lt	52 05 N	1 35 E
710	*Lowestoft*	52 28 N	1 45 E
720	Happisburgh (Haisborough): Lt	52 49 N	1 32 E
730	Cromer: Lt	52 55 N	1 19 E
740	Spurn Head: Lt	53 35 N	0 07 E
35800	River Humber		
810	—*Grimsby*	53 34 N	0 04 W
820	—*Hull*	53 45 N	0 17 W
35900	Flamborough Head: Lt	54 07 N	0 05 W
910	Ling Hill: Lt	54 29 N	0 34 W
920	*Whitby*	54 30 N	0 37 W
930	*Hartlepool*	54 42 N	1 11 W
940	The Heugh: Lt	54 42 N	1 11 W
950	*Sunderland*	54 54 N	1 22 W
960	Lizard Point: Souter Lt	54 58 N	1 21 W
36000	River Tyne		
010	—*Shields*	55 01 N	1 26 W
020	—*Newcastle*	54 58 N	1 35 W
36100	St. Marys I.: Lt	55 04 N	1 27 W
110	*Blyth*	55 08 N	1 30 W
120	Coquet I.: Lt	55 20 N	1 32 W
130	Farne Is., Longstone: Lt	55 39 N	1 37 W
140	*Berwick upon Tweed*	55 46 N	2 00 W
36200	**Scotland**		
36210	*Eyemouth*	55 53 N	2 05 W
220	St. Abbs Head: Lt	55 55 N	2 08 W
230	Barns Ness: Lt	55 59 N	2 27 W
36300	Firth of Forth		
310	—Bass Rock: Lt	56 05 N	2 38 W
320	—Fidra: Lt	56 04 N	2 47 W
330	—*Leith (Edinburgh)*	55 59 N	3 11 W
340	—Inchkeith: Lt	56 02 N	3 08 W
350	—*Rosyth*	56 01 N	3 26 W
360	—Elie Ness: Lt	56 11 N	2 49 W
370	—Isle of May: Lt	56 11 N	2 33 W
36400	Bell Rock: Lt	56 26 N	2 23 W
410	*Dundee*	56 28 N	2 58 W
420	Scurdie Ness: Lt	56 42 N	2 26 W
430	*Montrose*	56 43 N	2 28 W
440	Todhead Point: Lt	56 53 N	2 13 W
450	Girdle Ness: Lt	57 08 N	2 03 W
460	*Aberdeen*	57 09 N	2 05 W
470	Buchan Ness: Lt	57 28 N	1 46 W
480	Rattray Head: Lt	57 37 N	1 49 W
490	Kinnairds Head: Lt	57 42 N	2 00 W
36500	Covesea Skerries: Lt	57 43 N	3 20 W
510	*Inverness*	57 29 N	4 14 W
520	*Cromarty*	57 41 N	4 02 W
530	Tarbat Ness: Lt	57 52 N	3 46 W
540	Clyth Ness: Lt	58 19 N	3 13 W
550	Noss Head: Lt	58 28 N	3 03 W
560	Duncansby Head: Lt	58 39 N	3 01 W
570	Dunnet Head: Lt	58 40 N	3 22 W

Index No.	Place	Lat.	Long.
	Scotland—Continued	° ′	° ′
36600	Pentland Firth		
610	—Pentland Skerries, Muckle Skerry: Lt	58 41 N	2 55 W
620	—Swilkie Point, Stroma: Lt	58 42 N	3 07 W
630	—Swona: Lt	58 44 N	3 04 W
36700	Orkney Islands		
710	—*Scapa Bay*	58 57 N	2 59 W
720	—Copinsay: Lt	58 54 N	2 40 W
730	—Auskerry: Lt	59 01 N	2 34 W
740	—Start Point: Lt	59 17 N	2 22 W
750	—North Ronaldsay, Dennis Ness: Lt	59 23 N	2 23 W
760	—Noup Head: Lt	59 20 N	3 04 W
770	—Brough of Birsay: Lt	59 08 N	3 20 W
780	—Tor Ness: Lt	58 47 N	3 18 W
36800	Shetland Islands		
810	—Skadan: Lt	59 31 N	1 39 W
820	—Fair Isle, Skroo: Lt	59 33 N	1 36 W
830	—Sumburgh Head: Lt	59 51 N	1 16 W
840	—Out Skerries: Lt	60 25 N	0 43 W
850	—North Unst, Muckle Flugga: Lt	60 51 N	0 53 W
860	—Esha Ness: Lt	60 29 N	1 38 W
36900	Sule Skerry: Lt	59 05 N	4 24 W
910	Cape Wrath: Lt	58 37 N	5 00 W
920	Point of Stoer: Lt	58 14 N	5 24 W
930	Rudh' Ré: Lt	57 51 N	5 48 W
37000	Hebrides		
010	—Tiumpan Head: Lt	58 16 N	6 08 W
020	—Butt of Lewis: Lt	58 31 N	6 16 W
030	—Loch Carloway: Lt	58 17 N	6 50 W
040	—Flannan Isles: Lt	58 17 N	7 35 W
050	—Barra Head: Lt	56 47 N	7 39 W
060	—Usinish: Lt	57 18 N	7 11 W
070	—Scalpay: Lt	57 51 N	6 38 W
37100	Eilean Trodday: Lt	57 43 N	6 18 W
110	Neist (Ness) (Eist) Point: Lt	57 25 N	6 47 W
120	Oigh Sgeir: Lt	56 58 N	6 41 W
130	Point of Ardnamurchan: Lt	56 44 N	6 13 W
140	Skerryvore: Lt	56 19 N	7 07 W
150	Dubh Artach: Lt	56 08 N	6 38 W
160	Rhinns of Islay, Orsay I.: Lt	55 40 N	6 31 W
170	Mull of Kintyre: Lt	55 19 N	5 48 W
180	*Campbeltown*	55 26 N	5 36 W
190	Pladda: Lt	55 26 N	5 07 W
37200	Firth of Clyde		
210	—Little Cumbrae I.: Lt	55 43 N	4 58 W
220	—*Glasgow*	55 52 N	4 17 W
230	—*Greenock*	55 57 N	4 45 W
37300	*Ayr*	55 28 N	4 38 W
310	Turnberry Point: Lt	55 19 N	4 50 W
320	Ailsa Craig: Lt	55 15 N	5 06 W
330	Corsewall Point: Lt	55 00 N	5 09 W
340	Black Head, Killantringan Bay: Lt	54 52 N	5 09 W
350	Mull of Galloway: Lt	54 38 N	4 51 W
360	Little Ross: Lt	54 46 N	4 05 W
37400	**England**		
37500	Isle of Man		
510	—Point of Ayre: Lt	54 25 N	4 22 W
520	—*Peel*	54 14 N	4 42 W
530	—Chicken Rock: Lt	54 02 N	4 50 W
540	—Langness: Lt	54 03 N	4 37 W
550	—*Douglas*	54 09 N	4 28 W
560	—Maughold Head: Lt	54 18 N	4 18 W
37600	St. Bees Head: Lt	54 31 N	3 38 W
610	*Barrow in Furness*	54 07 N	3 14 W
620	*Preston*	53 46 N	2 42 W
630	*Liverpool*	53 25 N	3 00 W
640	*Manchester*	53 29 N	2 14 W
37700	**Wales**		
37710	Great Ormes Head: Lt	53 21 N	3 52 W
720	Point Lynas: Lt	53 25 N	4 17 W
730	The Skerries: Lt	53 25 N	4 36 W
740	*Holyhead*	53 19 N	4 38 W
750	South Stack: Lt	53 18 N	4 42 W
760	*Caernarvon*	53 09 N	4 17 W
770	Bardsey I.: Lt	52 45 N	4 48 W
780	Saint Tudwals I. West: Lt	52 48 N	4 28 W

APPENDIX S

MARITIME POSITIONS

BRITISH ISLES—Continued

Index No.	Place	Lat.	Long.
	Wales—Continued	° ′	° ′
37790	*Fishguard*	52 01 N	4 59 W
37800	Strumble Head: Lt	52 02 N	5 04 W
810	South Bishop: Lt	51 51 N	5 25 W
820	The Smalls: Lt	51 43 N	5 40 W
830	Skokholm I.: Lt	51 42 N	5 17 W
840	St. Anns Head: Lt	51 41 N	5 10 W
850	*Milford Haven*	51 43 N	5 02 W
37900	**Bristol Channel**		
37910	Caldy I.: Lt	51 38 N	4 41 W
920	Mumbles Head: Lt	51 34 N	3 58 W
930	*Swansea, Wales*	51 37 N	3 57 W
940	*Barry Docks, Wales*	51 24 N	3 16 W
950	*Cardiff, Wales*	51 27 N	3 10 W
960	*Newport, Wales*	51 35 N	2 59 W
970	*Bristol, England*	51 28 N	2 37 W
980	Flat Holm: Lt	51 22 N	3 07 W
990	Foreland Point (Lynmouth Foreland): Lt	51 15 N	3 47 W
38000	Bull Point: Lt	51 12 N	4 12 W
38100	**England**		
38110	Lundy I.: North Lt	51 12 N	4 41 W
120	Hartland Point: Lt	51 01 N	4 32 W
130	Trevose Head: Lt	50 33 N	5 02 W
140	Godrevy I.: Lt	50 14 N	5 24 W
150	St. Ives: Lt	50 12 N	5 28 W
160	Pendeen: Lt	50 10 N	5 40 W
38200	**Ireland (Éire)**		
38210	Fastnet Rock: Lt	51 23 N	9 36 W
220	*Baltimore*	51 29 N	9 22 W
230	Galley Head: Lt	51 32 N	8 57 W
240	Kinsale, Old Head: Lt	51 36 N	8 32 W
250	*Cork*	51 54 N	8 27 W
260	*Cóbh (Queenstown)*	51 51 N	8 18 W
270	Roches Point: Lt	51 47 N	8 15 W
280	Ballycotton I.: Lt	51 49 N	7 59 W
290	*Youghal*	51 57 N	7 50 W
38300	Mine Head: Lt	51 59 N	7 35 W
310	*Waterford*	52 15 N	7 07 W
320	Hook Head: Lt	52 07 N	6 56 W
330	Tuskar Rock: Lt	52 12 N	6 12 W
340	Wicklow Head: Lt	52 58 N	6 00 W
350	Muglins: Lt	53 16 N	6 05 W
360	*Kingstown*	53 18 N	6 08 W
370	*Dublin (Baile Atha Cliath)*	53 21 N	6 13 W
380	The Bailey: Lt	53 22 N	6 03 W
390	Rockabill: Lt	53 36 N	6 00 W
38400	*Drogheda*	53 43 N	6 21 W
410	*Dundalk*	54 00 N	6 24 W
420	Haulbowline Rock: Lt	54 01 N	6 05 W
430	*Carlingford*	54 03 N	6 11 W

Index No.	Place	Lat.	Long.
38500	**Northern Ireland**	° ′	° ′
38510	*Dundrum*	54 15 N	5 50 W
520	St. Johns Point: Lt	54 13 N	5 39 W
530	*Ardglass*	54 16 N	5 37 W
540	Mew I.: Lt	54 42 N	5 31 W
550	*Bangor*	54 40 N	5 40 W
560	*Belfast*	54 36 N	5 56 W
570	Black Head: Lt	54 46 N	5 41 W
580	Maidens: Lt	54 56 N	5 44 W
590	Rathlin I., Rue Point: Lt	55 15 N	6 11 W
38600	*Londonderry*	55 00 N	7 20 W
38700	**Ireland (Éire)**		
38710	Inishowen Head, Dunagree Point: Lt	55 13 N	6 56 W
720	Inishtrahull: Lt	55 26 N	7 14 W
730	Malin Head	55 23 N	7 24 W
740	Fanad Head: Lt	55 16 N	7 38 W
750	Tory I.: Lt	55 16 N	8 15 W
760	Aran I., Rinrawros Point: Lt	55 01 N	8 34 W
770	Rathlin O Birne I.: Lt	54 40 N	8 50 W
780	*Killybegs*	54 38 N	8 27 W
790	*Donegal*	54 39 N	8 07 W
38800	*Sligo*	54 16 N	8 28 W
810	Eagle I.: Lt	54 17 N	10 05 W
820	Blackrock: Lt	54 04 N	10 19 W
830	Clare I.: Lt	53 49 N	9 59 W
840	*Westport*	53 48 N	9 32 W
850	Slyne Head: Lt	53 24 N	10 14 W
860	Rock Islet (Eeragh): Lt	53 09 N	9 52 W
870	*Galway*	53 16 N	9 03 W
880	Inisheer: Lt	53 03 N	9 31 W
890	Loop Head: Lt	52 34 N	9 56 W
38900	River Shannon		
910	—Kilcredaun Point: Lt	52 35 N	9 43 W
920	—*Limerick*	52 40 N	8 38 W
930	—*Foynes*	52 37 N	9 07 W
39000	*Tralee*	52 16 N	9 42 W
010	Tearaght I. (Inishtearaght): Lt	52 04 N	10 40 W
020	Valencia I., Fort Point: Lt	51 56 N	10 19 W
030	Skellig Rocks: Lt	51 46 N	10 32 W
040	The Bull: Lt	51 35 N	10 18 W
39100	**Channel Islands**		
39110	Casquets: Lt	49 43 N	2 23 W
120	Alderney, Quenard Point: Lt	49 44 N	2 10 W
39200	Jersey		
210	—Sorel Point: Lt	49 16 N	2 10 W
220	—*St. Helier*	49 11 N	2 06 W
230	—La Corbière: Lt	49 11 N	2 15 W
39300	Sark, Point Robert: Lt	49 26 N	2 21 W
39400	Guernsey		
410	—Platte Fougère: Lt	49 31 N	2 29 W
420	—*St. Peter Port*	49 27 N	2 32 W
430	—Les Hanois Rocks: Lt	49 26 N	2 42 W

WEST COAST OF EUROPE

Index No.	Place	Lat.	Long.
40000	**Norway**	° ′	° ′
40010	Bøkfjord: Lt	69 53 N	30 11 E
020	Bugøynes, Oterneset: Lt	69 58 N	29 40 E
030	*Vadsø*	70 04 N	29 44 E
040	*Vardø*	70 22 N	31 06 E
050	Hornøy: Lt	70 23 N	31 10 E
060	Kjølnes: Lt	70 51 N	29 15 E
070	Sletnes: Lt	71 05 N	28 13 E
080	Helnes: Lt	71 04 N	26 14 E
090	Knivskjelloden: Lt	71 11 N	25 41 E
40100	Fruholmen: Lt	71 06 N	23 59 E
110	*Hammerfest*	70 40 N	23 40 E
120	Tarhalsen: Lt	70 52 N	23 19 E
130	Hasvik: Lt	70 28 N	22 10 E
140	Fugløykalven: Lt	70 19 N	20 10 E
150	Torsvåg: Lt	70 15 N	19 30 E
160	Lille Lyngøy: Lt	69 55 N	18 28 E
170	*Tromsø*	69 39 N	18 58 E
180	Hekkingen: Lt	69 36 N	17 50 E

Index No.	Place	Lat.	Long.
	Norway—Continued	° ′	° ′
40200	Vesterålen		
210	—Andenes: Lt	69 19 N	16 07 E
220	—Anda: Lt	69 04 N	15 11 E
230	—Frugga: Lt	68 50 N	14 34 E
240	—Litløy: Lt	68 36 N	14 19 E
250	—Kleivheia: Lt	68 17 N	13 35 E
40300	Lofoten		
310	—Skomvær: Lt	67 25 N	11 53 E
320	—Værøy: Lt	67 39 N	12 44 E
330	—Glåpen: Lt	67 53 N	13 03 E
340	—Moholmen: Lt	68 09 N	14 25 E
350	—Skrova (Skraven), Saltværholmen: Lt	68 09 N	14 39 E
40400	*Narvik*	68 26 N	17 25 E
410	Tranøy: Lt	68 11 N	15 36 E
420	Måløy-Skarholmen: Lt	67 46 N	14 25 E
430	Landegode, Eggeløysa: Lt	67 27 N	14 23 E
440	Grytøy: Lt	67 23 N	13 51 E

APPENDIX S

MARITIME POSITIONS

WEST COAST OF EUROPE—Continued

Index No.	Place	Lat.	Long.
	Norway—Continued	° ′	° ′
40450	Tennholman: Lt	67 18 N	13 30 E
460	Kalsholmen: Lt	66 55 N	13 06 E
470	Myken: Lt	66 46 N	12 29 E
480	Træna, Sørholmen: Lt	66 26 N	11 58 E
490	Åsvær: Lt	66 16 N	12 18 E
40500	Ytterholmen: Lt	66 01 N	11 42 E
510	Bremsteinen, Heimøy: Lt	65 37 N	11 24 E
520	Sklinna: Lt	65 12 N	11 00 E
530	Nordøyan: Lt	64 48 N	10 33 E
540	Gjeslingene: Lt	64 44 N	10 52 E
550	Ellingråsa: Lt	64 34 N	10 49 E
560	Kya: Lt	64 28 N	10 13 E
570	Buholmråsa: Lt	64 24 N	10 28 E
580	Kaura, Kaurleden: Lt	64 14 N	10 09 E
590	Storskjær: Lt	64 03 N	9 51 E
40600	Asenvågøy: Lt	63 56 N	9 47 E
610	Tarven: Lt	63 49 N	9 23 E
620	Halten: Lt	64 10 N	9 25 E
630	Finnvær: Lt	64 04 N	9 07 E
640	Sulen: Lt	63 51 N	8 28 E
650	Sletringen: Lt	63 40 N	8 16 E
660	*Trondheim*	63 26 N	10 24 E
670	Haugjegla: Lt	63 32 N	7 58 E
680	Skalmen: Lt	63 28 N	7 47 E
690	Grip: Lt	63 14 N	7 37 E
40700	*Kristiansund*	63 07 N	7 45 E
710	Hestskær: Lt	63 05 N	7 30 E
720	Kvitholmen: Lt	63 01 N	7 15 E
730	Ona: Lt	62 52 N	6 33 E
740	Storholmen: Lt	62 38 N	5 56 E
750	*Ålesund*	62 28 N	6 09 E
760	Alnes, Godøy: Lt	62 29 N	5 58 E
770	Gressøyene: Lt	62 26 N	5 46 E
780	Rundøy: Lt	62 25 N	5 36 E
790	Svinøy: Lt	62 20 N	5 16 E
40800	Kråkenes: Lt	62 02 N	5 00 E
810	Kvanhovden: Lt	61 42 N	4 50 E
820	Ytterøyane: Lt	61 34 N	4 41 E
830	Geita: Lt	61 16 N	4 49 E
840	Utvær: Lt	61 02 N	4 31 E
850	Holmengrå: Lt	60 51 N	4 39 E
860	Hellesøy: Lt	60 45 N	4 43 E
870	Skarvøy: Lt	60 30 N	4 50 E
880	*Bergen*	60 24 N	5 19 E
890	Marsteinen: Lt	60 08 N	5 01 E
40900	Slåtterøy: Lt	59 54 N	5 04 E
910	Ryvarden, Mylstrevåg: Lt	59 32 N	5 14 E
920	*Haugesund*	59 25 N	5 16 E
930	Røværsholmen: Lt	59 27 N	5 04 E
940	Utsira: Lt	59 18 N	4 53 E
950	Geitungen: Lt	59 08 N	5 15 E
960	*Stavanger*	58 59 N	5 45 E
970	Feistein: Lt	58 49 N	5 31 E
980	Obrestad: Lt	58 39 N	5 34 E
990	Egerøy: Lt	58 26 N	5 52 E
41000	Lille Prestskjær: Lt	58 19 N	6 16 E
010	Egdeholm: Lt	58 16 N	6 23 E
020	Varnes: Lt	58 11 N	6 38 E
030	Lista: Lt	58 06 N	6 34 E
040	Rauna (Listerrauna): Lt	58 03 N	6 41 E
050	Lindesnes (The Naze): Lt	57 59 N	7 03 E
060	Ryvingen: Lt	57 58 N	7 30 E
070	Songvår: Lt	58 01 N	7 49 E
080	Oksøy: Lt	58 04 N	8 04 E
090	*Kristiansand*	58 09 N	8 00 E
41100	Homborsund: Lt	58 15 N	8 32 E
110	Torungen: Lt	58 24 N	8 48 E
120	Ytre Møkkalasset: Lt	58 32 N	9 01 E
130	Lyngør: Lt	58 38 N	9 09 E
140	Jomfruland: Lt	58 52 N	9 36 E
150	Tvesten: Lt	58 56 N	9 57 E
160	Svenner (Svenöer): Lt	58 58 N	10 09 E
170	Færder: Lt	59 02 N	10 32 E
180	Fulehuk: Lt	59 10 N	10 36 E
190	*Tønsberg*	59 16 N	10 24 E
41200	*Oslo*	59 54 N	10 44 E
210	Guldholmen: Lt	59 26 N	10 35 E
220	*Moss*	59 26 N	10 40 E
230	Torgauten (Strömtangen): Lt	59 09 N	10 50 E
240	*Fredrikstad*	59 12 N	10 57 E
250	Struten: Lt	59 07 N	10 45 E

Index No.	Place	Lat.	Long.
	Norway—Continued	° ′	° ′
41260	*Halden*	59 07 N	11 23 E
270	Torbjørnskjær: Lt	59 00 N	10 47 E
41300	**Sweden**		
41310	*Strömstad*	58 56 N	11 10 E
320	Ursholmarna: Lt	58 50 N	11 00 E
330	Ramskär: Lt	58 46 N	11 00 E
340	Väderöbod: Lt	58 33 N	11 02 E
350	Hållö: Lt	58 20 N	11 13 E
360	*Lysekil*	58 16 N	11 26 E
370	Måseskär: Lt	58 06 N	11 20 E
380	Pater Noster Skären: Lt	57 54 N	11 28 E
390	Vinga: Lt	57 38 N	11 36 E
41400	*Göteborg*	57 42 N	11 58 E
410	Yttre Tistlarna: Lt	57 31 N	11 44 E
420	Nidingen: Lt	57 18 N	11 54 E
430	*Varberg*	57 07 N	12 15 E
440	Morups Tånge: Lt	56 55 N	12 22 E
450	*Falkenberg*	56 54 N	12 30 E
460	Tylö: Lt	56 39 N	12 43 E
470	*Halmstad*	56 40 N	12 51 E
480	Hallands Väderö: Lt	56 27 N	12 33 E
490	*Torekov*	56 25 N	12 37 E
41500	Kullen: Lt	56 18 N	12 27 E
510	*Höganäs*	56 12 N	12 33 E
520	*Hälsingborg*	56 03 N	12 42 E
530	Ven, Haken: Lt	55 55 N	12 44 E
540	*Landskrona*	55 52 N	12 50 E
550	Barsebäck: Lt	55 45 N	12 54 E
560	*Malmö*	55 36 N	13 00 E
570	*Klagshamn*	55 31 N	12 54 E
580	Falsterbo: Lt	55 23 N	12 49 E
590	*Trelleborg*	55 22 N	13 09 E
41600	Smygehuk: Lt	55 20 N	13 21 E
610	*Ystad*	55 26 N	13 50 E
620	Sandhammaren: Lt	55 23 N	14 11 E
630	*Simrishamn*	55 33 N	14 21 E
640	Stenshuvud: Lt	55 40 N	14 17 E
650	*Åhus*	55 56 N	14 19 E
660	*Sölvesborg*	56 03 N	14 35 E
670	Hanö: Lt	56 01 N	14 51 E
680	*Karlshamn*	56 10 N	14 51 E
690	Tärnö: Lt	56 07 N	14 58 E
41700	*Ronneby*	56 11 N	15 17 E
710	*Karlskrona*	56 10 N	15 36 E
720	Utklipporna: Lt	55 57 N	15 42 E
730	*Kalmar*	56 40 N	16 22 E
41800	ÖLAND		
810	—Ölands Södra Udde: Lt	56 12 N	16 24 E
820	—Kapelludden: Lt	56 49 N	16 51 E
830	—Ölands Norra Udde: Lt	57 22 N	17 06 E
41900	GOTLAND		
910	—Stora Karlsö: Lt	57 17 N	17 58 E
920	—Hoborg: Lt	56 55 N	18 09 E
930	—Östergarn: Lt	57 27 N	18 59 E
940	—*Slite*	57 42 N	18 49 E
950	—*Fårösund*	57 52 N	19 04 E
960	—Fårö (Holmudde): Lt	57 57 N	19 21 E
970	—*Visby*	57 38 N	18 17 E
42000	*Oskarshamn*	57 16 N	16 27 E
010	*Västervik*	57 45 N	16 38 E
020	Häradskär: Lt	58 09 N	16 59 E
030	*Arkösund*	58 29 N	16 57 E
040	*Norrköping*	58 36 N	16 12 E
050	*Oxelösund*	58 40 N	17 07 E
060	Grankubben: Lt	58 48 N	17 45 E
070	Landsort: Lt	58 44 N	17 52 E
080	Huvudskär: Lt	58 58 N	18 34 E
090	*Sandhamn*	59 17 N	18 57 E
42100	*Stockholm*	59 20 N	18 05 E
110	Grönskär: Lt	59 17 N	19 02 E
120	Svenska Högarna: Lt	59 27 N	19 30 E
130	Söderarm: Lt	59 45 N	19 25 E
140	Tjärven: Lt	59 48 N	19 22 E
150	Svartklubben: Lt	60 11 N	18 50 E
160	Understen: Lt	60 17 N	18 55 E
170	*Öregrund*	60 21 N	18 27 E
180	Örskär: Lt	60 32 N	18 23 E
190	Björn: Lt	60 38 N	17 59 E

APPENDIX S

MARITIME POSITIONS

WEST COAST OF EUROPE—Continued

Index No.	Place	Lat.	Long.
	Sweden—Continued	° ′	° ′
42200	Eggegrund: Lt	60 44 N	17 34 E
210	*Gävle*	60 41 N	17 10 E
220	Storjungfrun: Lt	61 10 N	17 21 E
230	*Söderhamn*	61 18 N	17 05 E
240	Hallgrund: Lt	61 16 N	17 25 E
250	Agö: Lt	61 33 N	17 28 E
260	*Hudiksvall*	61 44 N	17 07 E
270	Bålsön: Lt	61 43 N	17 34 E
280	Gran: Lt	62 01 N	17 38 E
290	Brämön: Lt	62 13 N	17 45 E
42300	*Sundsvall*	62 23 N	17 19 E
310	Åstholmsudde: Lt	62 23 N	17 44 E
320	Härnöklubb: Lt	62 36 N	18 03 E
330	*Härnösand*	62 38 N	17 57 E
340	Lungö: Lt	62 39 N	18 06 E
350	Högbonden: Lt	62 52 N	18 28 E
360	Ulvöarna: Lt	63 01 N	18 41 E
370	*Örnsköldsvik*	63 17 N	18 43 E
380	Skag: Lt	63 12 N	19 03 E
390	Storbådan: Lt	63 25 N	19 36 E
42400	Bonden: Lt	63 26 N	20 03 E
410	*Umeå*	63 49 N	20 17 E
420	Gadden (Holmogadd): Lt	63 36 N	20 45 E
430	Jägarören: Lt	63 41 N	20 56 E
440	Stora Fjäderägg: Lt	63 48 N	21 00 E
450	*Ratan*	64 00 N	20 54 E
460	Yttre Vännskär: Lt	64 10 N	21 08 E
470	Blackkallen: Lt	64 20 N	21 31 E
480	Bjuröklubb: Lt	64 29 N	21 35 E
490	Rönnskär: Lt	65 02 N	21 34 E
42500	Rödkallen: Lt	65 19 N	22 22 E
510	*Luleå*	65 35 N	22 10 E
520	Malören: Lt	65 32 N	23 34 E
42600	**Finland**		
42610	*Tornio*	65 51 N	24 09 E
620	*Kemi*	65 44 N	24 34 E
630	Ajossaari (Ajosholm): Lt	65 41 N	24 31 E
640	*Oulu*	65 01 N	25 28 E
650	Hailuoto (Karlö), Marjaniemi: Lt	65 02 N	24 34 E
660	Ulkokalla: Lt	64 20 N	23 27 E
670	Tankar: Lt	63 57 N	22 51 E
680	Hällgrund (Khelgrund): Lt	63 39 N	22 25 E
690	Valsörarne (Valassaari): Lt	63 25 N	21 04 E
42700	Norrskär (North Quarken): Lt	63 14 N	20 36 E
710	*Vaasa* (*Vasa*)	63 05 N	21 34 E
720	Strömmingsbådan: Lt	62 59 N	20 45 E
730	Själgrund (Salgrund): Lt	62 20 N	21 11 E
740	*Kristiinankaupunki* (*Kristinestad*)	62 17 N	21 24 E
750	Yttergrund: Lt	61 59 N	21 18 E
760	Säppi (Sebbskär): Lt	61 29 N	21 21 E
770	Nurmisaari (Nurmes I.): Lt	61 12 N	21 20 E
780	*Rauma*	61 08 N	21 30 E
790	Kylmäpihlaja: Lt	61 09 N	21 18 E
42800	Enskär: Lt	60 43 N	21 01 E
810	Salskar, Södra Salskar: Lt	60 25 N	19 36 E
820	Märket: Lt	60 18 N	19 09 E
830	Heligman (Hellman): Lt	60 13 N	19 19 E
840	Gisslan: Lt	60 10 N	19 18 E
850	Korsö: Lt	60 02 N	19 54 E
860	Nyhamn: Lt	59 58 N	19 57 E
870	Lågskär: Lt	59 51 N	19 55 E
880	Bogskären: Lt	59 30 N	20 21 E
890	Kokarsören: Lt	59 46 N	21 01 E
42900	Utö. Lt	59 47 N	21 22 E
910	*Turku* (*Åbo*)	60 27 N	22 16 E
920	Bengtskär: Lt	59 43 N	22 31 E
930	Russarö: Lt	59 46 N	22 57 E
940	*Hangö* (*Hanko*)	59 49 N	22 57 E
950	Stor Jussarö, Sundharu: Lt	59 47 N	23 33 E
960	Porkala Kallbäda: Lt	59 52 N	24 20 E
970	Kytö (Kytö Karingen): Lt	60 04 N	24 45 E
980	*Helsinki*	60 10 N	24 58 E
990	Harmaja (Gråhara): Lt	60 06 N	25 00 E
43000	Söderskär: Lt	60 07 N	25 26 E
010	Orrengrund: Lt	60 16 N	26 27 E
020	Rödskär (Ruuskeri): Lt	59 58 N	26 42 E
030	Someri (Sommars): Lt	60 12 N	27 40 E
43100	**USSR**	° ′	° ′
43110	Ostrov Sur-Sari (Hogland), Mys Launat-Revi: Lt	60 01 N	27 02 E
120	Ostrov Narvi (Nerva): Lt	60 15 N	27 09 E
130	Ostrov Halli: Lt	60 24 N	28 09 E
140	*Vyborg* (*Viborg*)	60 43 N	28 46 E
150	Serkoluoto (Särkkäluoto): Lt	60 18 N	28 47 E
160	Mys Styursudd (Seivästö): Lt	60 11 N	29 02 E
170	*Leningrad*	59 56 N	30 18 E
180	*Kronshtadt*	59 59 N	29 47 E
190	Ostrov Kotlin, Ostrov Kronshlot: Lt	59 59 N	29 45 E
43200	Tolbukhin: Lt	60 03 N	29 33 E
210	Ostrov Karavalday, Shepelevski: Lt	59 59 N	29 08 E
220	Ostrov Bol'shoy Tyutyarsari (Iso Tytarsaari): Lt	59 51 N	27 11 E
43300	**Estonia**		
43310	*Narva-Jõesuu*	59 28 N	28 02 E
320	Põhja-Uhtju: Lt	59 41 N	26 31 E
330	Vaindlo (Stenskar): Lt	59 49 N	26 22 E
340	Mohnisaar (Ekholm): Lt	59 41 N	25 49 E
350	Keri (Kokskar): Lt	59 42 N	25 01 E
360	Aegna (Wulf I.): Lt	59 36 N	24 44 E
370	*Tallinn* (*Reval*)	59 27 N	24 46 E
380	Naissar (Nargen): Lt	59 36 N	24 31 E
390	Suurup (Sourop): Lt	59 28 N	24 23 E
43400	Pakrineem (Paker Ort): Lt	59 23 N	24 02 E
410	Osmussaar (Oldensholm): Lt	59 18 N	23 23 E
420	Takhuna Nina: Lt	59 05 N	22 36 E
430	Kõpu Poolsaar (Dagerort): Lt	58 55 N	22 12 E
440	Ristna: Lt	58 56 N	22 03 E
450	Vilsandi (Filsand): Lt	58 23 N	21 49 E
460	Sõrve Nina (Svalferort): Lt	57 54 N	22 04 E
470	Allirahu: Lt	58 10 N	22 47 E
480	Kübassaare: Lt	58 26 N	23 18 E
490	Kihnu (Kinö): Lt	58 06 N	23 58 E
43500	*Pärnu*	58 23 N	24 30 E
43600	**Latvia**		
43610	*Ainaži*	57 52 N	24 23 E
620	Kurmrags: Lt	57 33 N	24 22 E
630	*Rīga* (*Riga*)	56 57 N	24 06 E
640	Daugavgrīva: Lt	57 04 N	24 01 E
650	Mērsrags: Lt	57 22 N	23 07 E
660	Kolkasrags: Lt	57 48 N	22 38 E
670	Miķelbāka: Lt	57 36 N	21 59 E
680	Oviši (Lyser Ort): Lt	57 34 N	21 43 E
690	*Ventspils* (*Vindau*) (*Windau*)	57 24 N	21 32 E
43700	Užava (Backofen): Lt	57 13 N	21 25 E
710	Akmenrags (Stein Ort): Lt	56 50 N	21 04 E
720	*Liepāja* (*Libau*)	56 32 N	20 59 E
43800	**Lithuania**		
43810	*Klaipėda* (*Memel*)	55 42 N	21 09 E
43900	**USSR**		
43910	Mys Taran (Mys Bryusterort) (Brusterort): Lt	54 58 N	19 59 E
920	*Baltiysk* (*Pillau*)	54 38 N	19 54 E
930	*Kaliningrad* (*Konigsberg*)	54 42 N	20 32 E
44000	**Poland**		
44010	Łysica (Kahlberg): Lt	54 23 N	19 27 E
020	*Gdańsk* (*Danzig*)	54 21 N	18 40 E
030	*Nowy Port* (*Neufahrwasser*)	54 24 N	18 40 E
040	*Gdynia*	54 32 N	18 34 E
050	Hel: Lt	54 36 N	18 49 E
060	Rozewie (Rixhöft): Lt	54 50 N	18 20 E
070	Stilo (Stilo Kathen): Lt	54 47 N	17 44 E
080	Łeba: Radiobeacon	54 46 N	17 33 E
44100	**Germany**		
130	Czołpino (Scholpin): Lt	54 43 N	17 15 E
140	*Ustka* (*Stolpmünde*)	54 35 N	16 52 E
150	Jarosławiec (Jershöft): Lt	54 32 N	16 33 E
160	*Darłowko* (*Rügenwaldermünde*)	54 27 N	16 23 E

APPENDIX S

MARITIME POSITIONS

WEST COAST OF EUROPE—Continued

Index No.	Place	Lat.	Long.
	Germany—Continued	° ′	° ′
170	Gąski (Funkenhagen): Lt	54 15 N	15 52 E
180	*Kołobrzeg (Kolberg)*	54 11 N	15 35 E
190	Niechorze (Horst) (Gross Horst): Lt	54 06 N	15 04 E
44200	*Swinoujście (Swinemünde)*	53 55 N	14 16 E
210	*Szczecin (Stettin)*	53 26 N	14 34 E
220	Greifswalder Oie (Greifswald I.): Lt	54 15 N	13 56 E
44300	Rügen		
310	—*Sassnitz*	54 31 N	13 39 E
320	—Kollickerort: Lt	54 34 N	13 41 E
330	—Ranzow: Lt	54 35 N	13 38 E
340	—Arkona: Lt	54 41 N	13 26 E
350	—Der Dornbusch: Lt	54 36 N	13 07 E
44400	*Stralsund*	54 19 N	13 06 E
410	*Barhöft*	54 26 N	13 02 E
420	Darsser Ort: Lt	54 28 N	12 30 E
430	Wustrow: Lt	54 20 N	12 23 E
440	*Rostock*	54 06 N	12 08 E
450	*Warnemünde*	54 11 N	12 06 E
460	Buk Spitze: Lt	54 08 N	11 42 E
470	Timmendorf: Lt	54 00 N	11 23 E
480	*Wismar*	53 54 N	11 29 E
490	*Lübeck*	53 52 N	10 41 E
44500	*Travemünde*	53 58 N	10 53 E
510	Pelzerhaken: Lt	54 05 N	10 52 E
520	Dahmeshöved: Lt	54 12 N	11 06 E
44600	Fehmarn		
610	—Staberhuk: Lt	54 24 N	11 19 E
620	—Marienleuchte, Ohlenburgs Huk: Lt	54 30 N	11 14 E
630	—Westermarkelsdorf: Lt	54 32 N	11 04 E
640	—Flügge: Lt	54 27 N	11 01 E
44700	Neuland: Lt	54 22 N	10 36 E
710	*Kiel, Nord-Ostsee-Kanal*	54 20 N	10 08 E
720	*Wik (Vik)*	54 21 N	10 09 E
730	Friedrichsort: Lt	54 24 N	10 12 E
740	Bülk: Lt	54 27 N	10 12 E
750	*Eckernförde*	54 29 N	9 50 E
760	Schleimünde: Lt	54 40 N	10 02 E
770	*Schleswig*	54 31 N	9 34 E
780	Falshöft: Lt	54 46 N	9 58 E
790	*Flensburg*	54 47 N	9 26 E
44800	**Denmark**		
44810	Christiansø: Lt	55 19 N	15 11 E
44900	Bornholm		
910	—Hammer Odde: Lt	55 18 N	14 47 E
920	—Sandkaas Odde: Lt	55 08 N	15 09 E
930	—Dueodde: Lt	55 00 N	15 05 E
940	—*Rønne*	55 06 N	14 42 E
45000	Møn		
010	—Hellehavns Nakke: Lt	55 00 N	12 32 E
020	—Møn (Möen): Lt	54 57 N	12 33 E
45100	Falster		
110	—*Stubbekøbing*	54 53 N	12 03 E
120	—Hestehoved: Lt	54 50 N	12 10 E
130	—Gedser (Gjedser) Odde: Lt	54 34 N	11 58 E
140	—*Nykøbing*	54 46 N	11 52 E
45200	Lolland (Laaland)		
210	—*Nysted*	54 40 N	11 44 E
220	—Hyllekrog: Lt	54 36 N	11 30 E
230	—*Nakskov*	54 50 N	11 08 E
240	—*Kragenæs Havn*	54 55 N	11 22 E
250	—*Bandholm*	54 50 N	11 30 E
45300	Sjælland		
310	—*Præstø*	55 08 N	12 04 E
320	—*Faxe Havn*	55 13 N	12 10 E
330	—Stevns Klint: Lt	55 17 N	12 27 E
340	—*Køge (Kioge)*	55 27 N	12 12 E
350	—Drogden: Lt	55 32 N	12 43 E
360	—Nordre Røse: Lt	55 38 N	12 41 E
370	—*København (Copenhagen)*	55 42 N	12 36 E
380	—Trekroner: Lt	55 42 N	12 37 E
390	—Middelgrund: Lt	55 43 N	12 40 E
45400	—Flakfort: Lt	55 42 N	12 44 E
410	—*Helsingør*	56 02 N	12 37 E
420	—Kronborg: Lt	56 02 N	12 38 E
430	—Nakkehoved: Lt	56 07 N	12 21 E
440	—*Gilleleje*	56 08 N	12 19 E
450	—Lysegrund: Lt	56 18 N	11 48 E
460	—Hesselø: Lt	56 12 N	11 43 E
470	—Spodsbjerg: Lt	55 59 N	11 52 E

Index No.	Place	Lat.	Long.
	Denmark—Continued	° ′	° ′
45480	—*Nykøbing*	55 55 N	11 41 E
490	—Sjællands Rev: Lt	56 05 N	11 13 E
45500	—Sejerø (Sejrö): Lt	55 55 N	11 05 E
510	—Røsnæs (Revsnæs) Puller: Lt	55 45 N	10 51 E
520	—*Kalundborg*	55 41 N	11 06 E
530	—*Korsør*	55 20 N	11 08 E
540	—*Karrebæksminde*	55 11 N	11 38 E
550	—*Vordingborg*	55 00 N	11 55 E
45600	Vejrø (Veiro): Lt	55 02 N	11 22 E
610	Omø, Langelands Øre: Lt	55 10 N	11 08 E
620	Agersø, Helleholm: Lt	55 11 N	11 13 E
630	Sprogø: Lt	55 20 N	10 58 E
45700	Fyn		
710	—*Nyborg*	55 18 N	10 48 E
720	—Knudshoved: Lt	55 17 N	10 51 E
730	—Romsø: Lt	55 31 N	10 48 E
740	—Korshavn: Lt	55 36 N	10 37 E
750	—*Odense*	55 24 N	10 23 E
760	—Æbelø: Lt	55 39 N	10 10 E
770	—*Strib*	55 32 N	9 46 E
780	—*Middelfart*	55 30 N	9 44 E
790	—Tvingsbjerg: Lt	55 19 N	9 53 E
45800	—*Assens*	55 16 N	9 54 E
810	—Helnæs: Lt	55 08 N	9 59 E
820	—*Faaborg*	55 06 N	10 15 E
830	—*Svendborg*	55 03 N	10 37 E
840	—Elsehoved: Lt	55 06 N	10 47 E
45900	Langeland		
910	—*Rudkøbing*	54 56 N	10 43 E
920	—Frankeklint: Lt	55 10 N	10 56 E
930	—Hov: Lt	55 09 N	10 58 E
940	—Tranekær: Lt	54 59 N	10 53 E
950	—Keldsnor (Kjelsnor): Lt	54 44 N	10 44 E
46000	Ærø		
010	—Vejsnæs (Veisnæs) Nakke: Lt	54 49 N	10 26 E
020	—Skjoldnæs: Lt	54 58 N	10 13 E
46100	Als		
110	—*Sønderborg*	54 54 N	9 47 E
120	—Kegnæs (Kekenis): Lt	54 51 N	9 59 E
130	—Pøls Huk: Lt	54 53 N	10 04 E
140	—Traner Odde (Tranerort): Lt	55 03 N	9 51 E
150	—Nordborg: Lt	55 05 N	9 43 E
46200	*Aabenraa*	55 03 N	9 26 E
210	*Aarøsund Havn*	55 16 N	9 43 E
220	Baagø: Lt	55 18 N	9 48 E
230	Fænø: Lt	55 28 N	9 42 E
240	*Kolding*	55 29 N	9 30 E
250	Damgaard: Lt	55 32 N	9 40 E
260	*Fredericia*	55 34 N	9 46 E
270	Trelde (Trælle) Næs: Lt	55 38 N	9 52 E
280	*Vejle*	55 42 N	9 33 E
290	Hjarnø: Lt	55 50 N	10 04 E
46300	*Horsens*	55 52 N	9 51 E
310	*Hov Havn*	55 55 N	10 15 E
320	Samsø, Vesborg: Lt	55 46 N	10 33 E
330	Tunø: Lt	55 57 N	10 27 E
340	*Aarhus*	56 09 N	10 13 E
350	Sletterhage: Lt	56 06 N	10 31 E
360	*Æbeltoft (Ebeltoft)*	56 12 N	10 41 E
370	Hjelm: Lt	56 08 N	10 49 E
380	*Grenaa*	56 25 N	10 56 E
390	Fornæs: Lt	56 27 N	10 58 E
46400	Gerrild, Knudshoved: Lt	56 32 N	10 50 E
410	Udbyhøj (Elkjærbakke): Lt	56 35 N	10 19 E
420	Anholt: Lt	56 44 N	11 39 E
430	Hals Barre: Lt	56 57 N	10 26 E
440	*Aalborg*	57 03 N	9 56 E
450	Læsø, Syrodde: Lt	57 19 N	11 12 E
460	Nordre Rønner: Lt	57 22 N	10 56 E
470	*Frederikshavn*	57 26 N	10 33 E
480	Hirsholm: Lt	57 29 N	10 38 E
490	*Skagen, Jutland (Jylland)*	57 43 N	10 36 E
46500	Gamle Skagen (Hojen): Lt	57 45 N	10 34 E
510	Hirtshals: Lt	57 35 N	9 57 E
520	Rubjerg Knude: Lt	57 27 N	9 47 E
530	Hanstholm: Lt	57 07 N	8 36 E
540	Lodbjerg Kirke: Lt	56 49 N	8 16 E
550	*Thyborøn, Jutland (Jylland)*	56 42 N	8 13 E
560	Bovbjerg: Lt	56 31 N	8 07 E
570	Lyngvig, Holmsland Klit: Lt	56 03 N	8 06 E
580	*Ringkøbing, Jutland (Jylland)*	56 05 N	8 15 E

APPENDIX S

MARITIME POSITIONS

WEST COAST OF EUROPE—Continued

Index No.	Place	Lat.	Long.
	Denmark—Continued	° ′	° ′
46590	Blaavands Huk: Lt.	55 33 N	8 05 E
46600	Graadyb, Skallingen: Lt.	55 28 N	8 19 E
610	*Esbjerg, Jutland (Jylland)*	55 28 N	8 27 E
620	Fanø, Fanø Lo: Lt.	55 28 N	8 25 E
46700	**Germany**		
46800	SYLT		
810	—Ellenbogen: Lt.	55 03 N	8 24 E
820	—Rote Kliff: Lt.	54 57 N	8 21 E
830	—Hörnum Odde: Lt.	54 45 N	8 17 E
46900	Amrum, Norddorf: Lt.	54 40 N	8 19 E
910	*Husum*	54 29 N	9 03 E
920	Westerhever Sand: Lt.	54 22 N	8 39 E
930	*Tönning*	54 19 N	8 57 E
940	Büsum: Lt.	54 08 N	8 52 E
950	Helgoland: Lt.	54 11 N	7 53 E
960	Neuwerk: Lt.	53 55 N	8 30 E
47000	ELBE		
010	—*Brunsbüttelkoog, Nord-Ostsee-Kanal (Kiel Canal)*	53 53 N	9 09 E
020	—*Altona*	53 33 N	9 56 E
030	—*Hamburg*	53 33 N	9 58 E
040	—*Harburg-Wilhelmsburg*	53 28 N	9 59 E
050	—*Cuxhaven*	53 52 N	8 43 E
47100	WESER		
110	—Roter Sand: Lt.	53 51 N	8 05 E
120	—Hoher Weg: Lt.	53 43 N	8 15 E
130	—*Bremerhaven*	53 33 N	8 34 E
140	—*Wesermünde*	53 32 N	8 34 E
150	—*Nordenham*	53 30 N	8 30 E
160	—*Bremen*	53 07 N	8 43 E
47200	*Wilhelmshaven*	53 31 N	8 09 E
210	Wangerooge: Lt.	53 47 N	7 54 E
220	Norderney: Lt.	53 43 N	7 14 E
230	Borkum: Lt.	53 35 N	6 40 E
240	*Emden*	53 22 N	7 13 E
47300	**Netherlands**		
47310	*Delfzijl*	53 20 N	6 56 E
320	Schiermonnikoog: Lt.	53 29 N	6 09 E
330	Ameland, Amelander Gat: Lt.	53 27 N	5 38 E
340	Terschelling, Brandaris: Lt.	53 22 N	5 13 E
350	Vlieland, Vuurboetsduin: Lt.	53 18 N	5 04 E
360	Texel, Eierland: Lt.	53 11 N	4 51 E
370	Zeegat van Texel, Kijkduin: Lt.	52 57 N	4 44 E
380	Zanddijk (Grootekaap): Lt.	52 53 N	4 43 E
390	Egmond aan Zee: Lt.	52 37 N	4 37 E
47400	*Ijmuiden*	52 28 N	4 34 E
410	*Amsterdam*	52 22 N	4 54 E
420	Noordwijk aan Zee: Lt.	52 15 N	4 26 E
430	*Scheveningen*	52 06 N	4 15 E
440	*Hoek Van Holland*	51 59 N	4 07 E
450	*Rotterdam*	51 55 N	4 30 E
460	*Dordrecht*	51 48 N	4 39 E
470	Goeree, Westhoofd: Lt.	51 49 N	3 52 E
480	Schouwen: West Schouwen Lt.	51 43 N	3 41 E
490	Westkapelle: Lt.	51 32 N	3 27 E
47500	*Vlissingen (Flushing)*	51 27 N	3 36 E
510	*Terneuzen (Neuzen)*	51 20 N	3 49 E
520	Nieuwesluis: Lt.	51 24 N	3 30 E
47600	**Belgium**		
47610	*Antwerpen (Antwerp)*	51 14 N	4 24 E
620	*Gent (Ghent)*	51 03 N	3 44 E
630	*Zeebrugge*	51 20 N	3 12 E
640	*Brugge (Bruges)*	51 13 N	3 13 E
650	*Oostende (Ostend)*	51 14 N	2 55 E
660	*Nieuwpoort (Nieuport)*	51 08 N	2 44 E
47700	**France**		
47710	*Dunkerque*	51 03 N	2 21 E
720	*Gravelines*	51 00 N	2 07 E
730	*Calais*	50 58 N	1 51 E
740	Cap Gris-Nez: Lt.	50 52 N	1 35 E
750	*Boulogne*	50 44 N	1 35 E
760	Cap d'Alprech: Lt.	50 42 N	1 34 E
770	Le Touquet: Lt.	50 31 N	1 36 E
780	Pointe du Haut Banc: Lt.	50 24 N	1 34 E

Index No.	Place	Lat.	Long.
	France—Continued	° ′	° ′
47790	Somme, Le Hourdel: Lt.	50 13 N	1 34 E
47800	*Tréport*	50 04 N	1 22 E
810	*Dieppe*	49 56 N	1 05 E
820	Pointe d'Ailly: Lt.	49 55 N	0 58 E
830	*Saint-Valery-En-Caux*	49 52 N	0 43 E
840	*Fécamp*	49 46 N	0 23 E
850	Cap d'Antifer: Lt.	49 41 N	0 10 E
860	Cap de la Hève: Lt.	49 31 N	0 04 E
47900	SEINE		
910	—*Le Havre*	49 29 N	0 07 E
920	—*Rouen*	49 27 N	1 06 E
48000	*Trouville*	49 22 N	0 05 E
010	Ouistreham: Lt.	49 17 N	0 15 W
020	*Caen*	49 17 N	0 15 W
030	Pointe de Ver: Lt.	49 20 N	0 31 W
040	Îles Saint-Marcouf, Île du Large: Lt.	49 30 N	1 09 W
050	Pointe de Barfleur: Lt.	49 42 N	1 16 W
060	Cap Lévy: Lt.	49 42 N	1 28 W
070	*Cherbourg*	49 39 N	1 38 W
080	Cap de la Hague, Gros du Raz: Lt.	49 43 N	1 57 W
090	Cap de Carteret: Lt.	49 22 N	1 48 W
48100	Senequet: Lt.	49 06 N	1 40 W
110	Grande Île Chausey: Lt.	48 52 N	1 49 W
120	Pointe du Roc (Cap Lihou): Lt.	48 50 N	1 37 W
130	*Granville*	48 50 N	1 36 W
140	Pierre de Herpin: Lt.	48 44 N	1 49 W
150	Rochebonne: Lt.	48 40 N	1 59 W
160	*Saint-Malo*	48 39 N	2 01 W
170	Cap Fréhel: Lt.	48 41 N	2 19 W
180	Grand Léjon: Lt.	48 45 N	2 40 W
190	Roches Douvres: Lt.	49 06 N	2 49 W
48200	Les Heaux de Brehat: Lt.	48 55 N	3 05 W
210	Les Sept Îles: Lt.	48 53 N	3 29 W
220	Plateau des Triagoz: Lt.	48 52 N	3 39 W
230	Île de Batz (Bas): Lt.	48 45 N	4 02 W
240	Île Vierge: Lt.	48 38 N	4 34 W
48300	ÎLE D'OUESSANT (USHANT)		
310	—Le Stiff: Lt.	48 28 N	5 03 W
320	—Creach: Lt.	48 27 N	5 08 W
330	—La Jument: Lt.	48 25 N	5 08 W
48400	Binte de Corsen: Lt.	48 25 N	4 48 W
410	Presqu'île de Kermorvan: Lt.	48 22 N	4 47 W
420	Pointe de St. Mathieu: Lt.	48 20 N	4 46 W
430	Chaussée des Pierres Noires: Lt.	48 19 N	4 55 W
440	*Brest*	48 23 N	4 30 W
450	Pointe du Toulinguet: Lt.	48 17 N	4 38 W
460	*Douarnenez*	48 06 N	4 19 W
470	Chaussée de Sein, Ar Men: Lt.	48 03 N	5 00 W
480	Île de Sein: Lt.	48 03 N	4 52 W
490	La Vieille: Lt.	48 02 N	4 45 W
48500	*Audierne*	48 01 N	4 32 W
510	Pointe de Penmarc'h (Eckmühl): Lt.	47 48 N	4 22 W
520	*Concarneau*	47 52 N	3 55 W
530	Île de Penfret: Lt.	47 43 N	3 57 W
540	Île de Groix, Pen Men: Lt.	47 39 N	3 31 W
550	*Lorient*	47 45 N	3 21 W
48600	BELLE-ÎLE		
610	—Pointe des Poulains (Poulains Islet): Lt.	47 23 N	3 15 W
620	—Goulphar: Lt.	47 19 N	3 14 W
48700	*Le Palais*	47 21 N	3 09 W
710	Les Grands Cardinaux: Lt.	47 19 N	2 50 W
720	*Port Navalo*	47 33 N	2 55 W
730	*Le Croisic*	47 18 N	2 31 W
48800	LOIRE		
810	—*Saint-Nazaire*	47 17 N	2 12 W
820	—*Donges*	47 18 N	2 04 W
830	—*Paimboeuf*	47 17 N	2 02 W
840	—*Nantes*	47 12 N	1 34 W
48900	Pointe de Saint-Gildas: Lt.	47 08 N	2 15 W
910	Île du Pilier: Lt.	47 03 N	2 22 W
920	Île d'Yeu, Petite-Foule: Lt.	46 43 N	2 23 W
930	*St. Gilles sur Vie*	46 42 N	1 56 W
940	*Les Sables-d'Olonne*	46 30 N	1 48 W

APPENDIX S

MARITIME POSITIONS

WEST COAST OF EUROPE—Continued

Index No.	Place	Lat.	Long.
	France—Continued	° ′	° ′
48950	Île de Ré, Pointe des Baleines: Lt	46 15 N	1 34 W
960	*La Pallice*	46 10 N	1 13 W
970	*La Rochelle*	46 09 N	1 09 W
980	Île d'Aix: Lt	46 01 N	1 11 W
990	*Rochefort*	45 56 N	0 57 W
49000	Île d'Oleron, Pointe de Chassiron: Lt	46 03 N	1 25 W
49100	GIRONDE		
110	—Pointe de la Coubre: Lt	45 42 N	1 14 W
120	—Plateau de Cordouan: Lt	45 35 N	1 10 W
130	—Terre Negre: Lt	45 39 N	1 06 W
140	—*Royan*	45 38 N	1 02 W
150	—*Bordeaux*	44 51 N	0 34 W
49200	Hourtin: Lt	45 08 N	1 10 W
210	Cap Ferret: Lt	44 39 N	1 15 W
220	*Arcachon*	44 40 N	1 10 W
230	Contis-les-Bains: Lt	44 05 N	1 19 W
240	*Boucau*	43 31 N	1 29 W
250	*Bayonne*	43 30 N	1 28 W
260	Pointe St. Martin: Lt	43 30 N	1 33 W
270	*Biarritz*	43 29 N	1 34 W
280	Pointe Ste. Barbe: Lt	43 24 N	1 40 W
290	*St.-Jean-de-Luz*	43 24 N	1 40 W
49300	*Socoa*	43 24 N	1 41 W
49400	**Spain**		
49410	Cabo Higuer: Lt	43 24 N	1 48 W
420	*Pasajes de San Juan*	43 20 N	1 56 W
430	Cabo La Plata: Lt	43 20 N	1 56 W
440	Isla Santa Clara: Lt	43 19 N	2 00 W
450	*San Sebastian*	43 19 N	1 59 W
460	Punta Santa Catalina: Lt	43 23 N	2 31 W
470	Cabo Machichaco: Lt	43 27 N	2 45 W
480	Punta Galea: Lt	43 22 N	3 02 W
490	*Bilbao*	43 16 N	2 57 W
49500	*Castro-Urdiales*	43 23 N	3 13 W
510	*Santoña*	43 26 N	3 27 W
520	Cabo de Ajo: Lt	43 31 N	3 35 W
530	*Santander*	43 28 N	3 47 W
540	Cabo Mayor: Lt	43 29 N	3 47 W
550	Suances (San Martin de la Arena): Lt	43 27 N	4 03 W
560	*San Vicente de la Barquera*	43 24 N	4 24 W
570	*Ribadesella*	43 28 N	5 05 W
580	Monte Somos: Lt	43 28 N	5 05 W
590	*Gijón*	43 33 N	5 40 W
49600	*Puerto del Musel*	43 34 N	5 42 W
610	Cabo de Torres: Lt	43 34 N	5 42 W
620	Cabo de Peñas: Lt	43 39 N	5 51 W
630	*Avilés*	43 36 N	5 56 W
640	*San Esteban*	43 34 N	6 05 W
650	Cabo Busto: Lt	43 34 N	6 28 W
660	Isla Tapia: Lt	43 34 N	6 57 W
670	Punta de la Estaca de Bares: Lt	43 47 N	7 41 W
680	Punta de la Candelaria: Lt	43 43 N	8 03 W
690	Cabo Prior: Lt	43 34 N	8 19 W
49700	Cabo Prioriño Chico: Lt	43 28 N	8 20 W
710	*El Ferrol*	43 29 N	8 14 W
	Spain—Continued	° ′	° ′
49720	*La Coruña*	43 22 N	8 24 W
730	Torre (Tower) de Hercules: Lt	43 23 N	8 24 W
740	Isla Sisarga Grande: Lt	43 22 N	8 51 W
750	Cabo Villano: Lt	43 10 N	9 13 W
760	Cabo Toriñana: Lt	43 03 N	9 18 W
770	Cabo Finisterre: Lt	42 53 N	9 16 W
780	Cabo Corrubedo: Lt	42 35 N	9 05 W
790	Isla Salvora: Lt	42 28 N	9 01 W
49800	*Villagarcia*	42 36 N	8 46 W
810	Isla de Ons: Lt	42 23 N	8 56 W
820	*Marin*	42 24 N	8 42 W
830	Cabo del Home: Lt	42 15 N	8 52 W
840	*Vigo*	42 14 N	8 43 W
850	Islas Cies, Isla del Faro: Lt	42 13 N	8 55 W
860	Isla de San Martin, Cabo Vicos: Lt	42 11 N	8 53 W
870	Cabo Silleiro: Lt	42 06 N	8 54 W
49900	**Portugal**		
49910	Cabo Montedor: Lt	41 45 N	8 52 W
920	Leça: Lt	41 12 N	8 43 W
930	*Porto de Leixões*	41 11 N	8 42 W
940	*Pôrto (Oporto)*	41 09 N	8 36 W
950	*Aveiro*	40 39 N	8 39 W
960	Cabo Mondego: Lt	40 11 N	8 54 W
970	Penedo da Saüdade: Lt	39 46 N	9 02 W
980	Farilhão Grande: Lt	39 29 N	9 33 W
990	Ilhas Berlengas: Lt	39 25 N	9 30 W
50000	Cabo Carvoeiro: Lt	39 22 N	9 24 W
010	Cabo da Roca: Lt	38 47 N	9 30 W
020	Cabo Raso: Lt	38 42 N	9 29 W
030	*Lisboa (Lisbon)*	38 42 N	9 10 W
040	Forte Bugio, Tagus River: Lt	38 40 N	9 18 W
050	Cabo de Espichel: Lt	38 25 N	9 13 W
060	*Setúbal*	38 31 N	8 54 W
070	Cabo de Sines: Lt	37 57 N	8 53 W
080	Cabo Sardão: Lt	37 36 N	8 49 W
090	Cabo de São Vicente: Lt	37 01 N	9 00 W
50100	Ponta de Sagres: Lt	37 00 N	8 57 W
110	Ponta da Piedade: Lt	37 05 N	8 40 W
120	*Lagos*	37 06 N	8 40 W
130	Ponta de Alfanzina (Cabo Carvoeiro do Algarve): Lt	37 05 N	8 26 W
140	Cabo de Santa Maria: Lt	36 58 N	7 52 W
150	*Vila Real de Santo António*	37 11 N	7 24 W
50200	**Spain**		
50210	Punta del Rompido (Rompido de Cartaya): Lt	37 13 N	7 08 W
220	*Huelva*	37 15 N	6 57 W
230	Punta del Picacho: Lt	37 08 N	6 50 W
240	*Sevilla (Seville)*	37 23 N	6 00 W
250	Chipiona: Lt	36 44 N	6 26 W
260	*Rota*	36 37 N	6 22 W
270	*Cádiz*	36 32 N	6 18 W
280	Castillo de San Sebastian: Lt	36 31 N	6 19 W
290	Cabo Trafalgar: Lt	36 11 N	6 02 W
50300	*Tarifa*	36 00 N	5 36 W
310	Punta Carnero: Lt	36 04 N	5 26 W
320	*Algeciras*	36 08 N	5 26 W

MEDITERRANEAN AND BLACK SEAS

Index No.	Place	Lat.	Long.
51000	**Gibraltar**	° ′	° ′
51010	*Gibraltar*	36 08 N	5 21 W
020	Europa Point: Lt	36 06 N	5 21 W
51100	**Spain**		
51110	Punta de la Doncella: Lt	36 25 N	5 09 W
120	*Marbella*	36 30 N	4 53 W
130	Punta de Calaburras: Lt	36 30 N	4 38 W
140	*Málaga*	36 44 N	4 25 W
150	Punta de Torrox: Lt	36 43 N	3 57 W
160	Cabo Sacratif: Lt	36 41 N	3 28 W
170	*Adra*	36 45 N	3 01 W
180	Punta del Sabinal: Lt	36 41 N	2 42 W
	Spain—Continued	° ′	° ′
51190	*Almería*	36 50 N	2 28 W
51200	Cabo de Gata: Lt	36 43 N	2 11 W
210	Mesa de Roldan: Lt	36 56 N	1 54 W
220	Cabo Tiñoso: Lt	37 32 N	1 06 W
230	*Cartagena*	37 35 N	0 59 W
240	Isleta de Escombrera: Lt	37 33 N	0 58 W
250	Cabo de Palos: Lt	37 38 N	0 41 W
260	Isla Hormiga: Lt	37 39 N	0 39 W
270	Isla de Tabarca (Plana): Lt	38 10 N	0 28 W
280	Cabo de Santa Pola: Lt	38 12 N	0 31 W
290	*Alicante*	38 20 N	0 29 W
51300	Cabo de las Huertas: Lt	38 21 N	0 24 W
310	Punta del Albir: Lt	38 34 N	0 03 W

APPENDIX S

MARITIME POSITIONS

MEDITERRANEAN AND BLACK SEAS—Continued

Index No.	Place	Lat.	Long.
	Spain—Continued	° ′	° ′
51320	Cabo de la Nao: Lt	38 44 N	0 14 E
330	Cabo de San Antonio: Lt	38 48 N	0 12 E
340	*Denia*	38 50 N	0 07 E
350	*Gandia*	39 00 N	0 09 W
360	Cabo Cullera: Lt	39 11 N	0 13 W
370	*Valencia*	39 27 N	0 19 W
380	Cabo Canet: Lt	39 40 N	0 12 W
390	*Burriana*	39 53 N	0 03 W
51400	Castellón de la Plana: Lt	39 58 N	0 01 E
410	Islas Columbretes: Lt	39 54 N	0 41 E
420	Cabo de Oropesa: Lt	40 05 N	0 09 E
430	Peñíscola: Lt	40 21 N	0 24 E
440	*Vinaroz*	40 28 N	0 29 E
450	*Puerto de las Alfaques*	40 37 N	0 36 E
460	Punta de la Baña: Lt	40 34 N	0 40 E
470	Cabo Tortosa: Lt	40 43 N	0 54 E
480	Cabo Salou: Lt	41 03 N	1 10 E
490	*Tarragona*	41 06 N	1 15 E
51500	Villanueva y Geltrú: Lt	41 13 N	1 44 E
510	Río Llobregat: Lt	41 19 N	2 09 E
520	Castillo de Montjuich: Lt	41 22 N	2 10 E
530	*Barcelona*	41 21 N	2 10 E
540	Calella: Lt	41 36 N	2 39 E
550	Cabo de Tossa: Lt	41 43 N	2 56 E
560	*Palamós*	41 50 N	3 07 E
570	Punta del Molino: Lt	41 50 N	3 08 E
580	Cabo de San Sebastian: Lt	41 54 N	3 12 E
590	Islas Medas: Lt	42 03 N	3 13 E
51600	Punta de la Poncella: Lt	42 15 N	3 11 E
610	Punta de Cala Nans: Lt	42 16 N	3 17 E
620	Cabo Creus: Lt	42 19 N	3 19 E
51700	**Balearic Islands**		
51800	FORMENTARA		
810	—Punta Codolar: Lt	38 40 N	1 35 E
820	—Punta Cala Sabina: Lt	38 44 N	1 25 E
51900	IBIZA (IVIZA)		
910	—Isla Conejera: Lt	39 00 N	1 13 E
920	—Isla de Tagomago: Lt	39 02 N	1 39 E
930	—Isla Botafoch: Lt	38 54 N	1 27 E
52000	Isla Cabrera, Punta Anciola: Lt	39 08 N	2 55 E
010	Isla Dragonera, Cabo Llebeitx: Lt	39 34 N	2 18 E
52100	MALLORCA (MAJORCA)		
110	—Cabo de Salinas: Lt	39 16 N	3 03 E
120	—*Palma*	39 34 N	2 38 E
130	—Cabo Cala Figuera: Lt	39 27 N	2 31 E
140	—*Sóller*	39 48 N	2 41 E
150	—Cabo de Formentor: Lt	39 58 N	3 13 E
160	—Cabo de Pera: Lt	39 43 N	3 29 E
52200	MENORCA (MINORCA)		
210	—Cabo Dartuch: Lt	39 55 N	3 49 E
220	—Cabo Nati: Lt	40 03 N	3 49 E
230	—Cabo de Caballeria: Lt	40 05 N	4 05 E
240	—*Puerto de Mahón*	39 52 N	4 19 E
52300	Isla del Aire: Lt	39 48 N	4 18 E
52400	**France**		
52410	Cap Béar: Lt	42 31 N	3 08 E
420	*Port-Vendres*	42 31 N	3 07 E
430	Cap Leucate: Lt	42 55 N	3 03 E
440	*La Nouvelle*	43 01 N	3 04 E
450	Îlot de Brescou: Lt	43 16 N	3 30 E
460	Mont St. Clair: Lt	43 24 N	3 41 E
470	*Sète*	43 24 N	3 42 E
480	Pointe de l'Espiguette: Lt	43 29 N	4 09 E
490	Pointe du Sablon (Pointe de Beauduc): Lt	43 22 N	4 35 E
52500	Faraman (La Camargue): Lt	43 21 N	4 41 E
510	*Port-St.-Louis-du-Rhône*	43 23 N	4 49 E
520	*Port-de-Bouc*	43 24 N	4 59 E
530	Cap Couronne: Lt	43 20 N	5 03 E
540	*Marseille*	43 19 N	5 22 E
550	Pointe de Mourepiane: Lt	43 21 N	5 20 E
560	Île d'If: Lt	43 17 N	5 20 E
570	Île du Planier: Lt	43 12 N	5 14 E
580	*Cassis*	43 13 N	5 32 E
590	*La Ciotat*	43 10 N	5 37 E
52600	Île du Grand Rouveau: Lt	43 05 N	5 46 E
610	*Toulon*	43 06 N	5 55 E

Index No.	Place	Lat.	Long.
	France—Continued	° ′	° ′
52620	Cap Cépet: Lt	43 04 N	5 57 E
630	Île Grand Ribaud: Lt	43 01 N	6 08 E
640	Cap d'Armes: Lt	42 59 N	6 12 E
650	Cap Bénat: Lt	43 05 N	6 22 E
660	Île du Levant (Titan): Lt	43 03 N	6 31 E
670	Cap Camarat: Lt	43 12 N	6 40 E
680	*St. Tropez*	43 17 N	6 39 E
690	Agay: Lt	43 26 N	6 52 E
52700	*Cannes*	43 33 N	7 01 E
710	La Garoupe: Lt	43 34 N	7 08 E
720	*Antibes*	43 35 N	7 07 E
730	*Nice*	43 42 N	7 16 E
740	*Villefranche*	43 42 N	7 19 E
750	Cap Ferrat: Lt	43 40 N	7 26 E
52800	**Monaco**		
52810	*Monte-Carlo*	43 44 N	7 26 E
52900	**Corsica**		
52910	Capo Senetosa (Aquila Point): Lt	41 33 N	8 48 E
920	*Ajaccio*	41 55 N	8 45 E
930	Île Sanguinaire: Lt	41 53 N	8 36 E
940	Pointe de Revellata: Lt	42 35 N	8 44 E
950	Cap Corse: Lt	43 02 N	9 24 E
960	*Bastia*	42 42 N	9 27 E
970	Alistro: Lt	42 16 N	9 32 E
980	Pointe de Chiappa: Lt	41 36 N	9 22 E
990	Île de Lavezzi: Lt	41 20 N	9 16 E
53000	Cap Pertusato: Lt	41 22 N	9 11 E
53100	**Sardinia**		
53110	Capo Testa: Lt	41 15 N	9 09 E
120	Isola Razzoli: Lt	41 18 N	9 20 E
130	Capo Ferro: Lt	41 09 N	9 31 E
140	*Olbia*	40 55 N	9 30 E
150	Isola Tavolara: Lt	40 56 N	9 44 E
160	Capo Comino: Lt	40 32 N	9 49 E
170	Capo Bellavista: Lt	39 56 N	9 43 E
180	Isola dei Cavoli: Lt	39 05 N	9 32 E
190	Capo San Elia: Lt	39 11 N	9 09 E
53200	*Cagliari*	39 12 N	9 07 E
210	Capo di Pula: Lt	38 59 N	9 01 E
220	Capo Spartivento: Lt	38 53 N	8 51 E
230	Capo Sandalo: Lt	39 09 N	8 13 E
240	Capo San Marco: Lt	39 52 N	8 26 E
250	Capo Caccia: Lt	40 34 N	8 10 E
260	Punta dello Scorno (Punta Caprara): Lt	41 07 N	8 19 E
53300	**Italy**		
53310	Capo dell' Arma: Lt	43 49 N	7 50 E
320	*Porto Maurizio*	43 53 N	8 02 E
330	Capo Mele: Lt	43 57 N	8 10 E
340	Capo di Vado: Lt	44 15 N	8 27 E
350	*Savona*	44 19 N	8 30 E
360	Capo del Faro: Lt	44 24 N	8 54 E
370	*Genova (Genoa)*	44 24 N	8 56 E
380	Punta Vagno: Lt	44 23 N	8 57 E
390	Punta di Portofino: Lt	44 18 N	9 13 E
53400	Isola del Tino: Lt	44 02 N	9 51 E
410	*La Spezia*	44 06 N	9 49 E
420	Secche della Meloria: Lt	43 33 N	10 13 E
430	*Livorno (Leghorn)*	43 33 N	10 19 E
440	Secche di Vada: Lt	43 19 N	10 22 E
450	Isola di Gorgona, Punta Cala Scirocco: Lt	43 25 N	9 54 E
460	Isola Capraia, Capo Ferraione: Lt	43 03 N	9 51 E
470	Isola Palmaiola: Lt	42 52 N	10 29 E
53500	ELBA		
510	—Punta Polveraia: Lt	42 48 N	10 07 E
520	—*Portoferraio*	42 49 N	10 20 E
530	—*Porto Azzurro*	42 46 N	10 24 E
53600	Isola Pianosa: Lt	42 35 N	10 06 E
610	Scoglio d'Africa: Lt	42 21 N	10 04 E
53700	ISOLA DEL GIGLIO		
710	—Punta del Fienaio: Lt	42 23 N	10 53 E
720	—Punta del Capel Rosso: Lt	42 19 N	10 55 E

APPENDIX S

MARITIME POSITIONS

MEDITERRANEAN AND BLACK SEAS—Continued

Index No.	Place	Lat.	Long.
	Italy—Continued	° ′	° ′
53800	Punta Lividonia: Lt	42 27 N	11 06 E
810	*Port' Ercole*	42 23 N	11 13 E
820	Forte la Rocca: Lt	42 23 N	11 13 E
830	Isola di Giannutri: Lt	42 14 N	11 07 E
840	*Civitavecchia*	42 05 N	11 47 E
850	Fiumicino: Lt	41 46 N	12 13 E
860	*Anzio*	41 27 N	12 38 E
870	Monte Circeo: Lt	41 13 N	13 04 E
880	Isola Zannone: Lt	40 58 N	13 03 E
890	Isola di Ponza, Punta della Guardia: Lt	40 53 N	12 57 E
53900	Isola d'Ischia, Punta Imperatore: Lt	40 43 N	13 51 E
910	Isola di Procida, Punta Pioppeto: Lt	40 46 N	14 01 E
920	*Napoli* (*Naples*)	40 50 N	14 16 E
930	*Castellammare di Stabia*	40 42 N	14 29 E
940	Punta Campanella: Lt	40 34 N	14 20 E
54000	ISOLA DI CAPRI		
010	—Punta Carena: Lt	40 32 N	14 12 E
020	—Lo Capo: Lt	40 34 N	14 16 E
54100	Capo d'Orso: Lt	40 38 N	14 41 E
110	*Salerno*	40 40 N	14 46 E
120	Isola Licosa: Lt	40 15 N	14 54 E
130	Capo Palinuro: Lt	40 01 N	15 17 E
140	Capo Bonifati: Lt	39 33 N	15 53 E
150	Capo Suvero: Lt	38 57 N	16 10 E
160	Capo Vaticano: Lt	38 37 N	15 50 E
170	Scilla: Lt	38 15 N	15 43 E
180	Punta Pezzo: Lt	38 14 N	15 38 E
54200	ISOLE EOLIE (ISOLE LIPARI)		
210	—Isola Vulcano: Lt	38 22 N	15 00 E
220	—Isola Stromboli (Isolotto Strombolicchio): Lt	38 49 N	15 15 E
230	—Isola Salina, Capo Faro: Lt	38 35 N	14 52 E
54300	ISOLA D'USTICA		
310	—Punta Uomo Morto: Lt	38 43 N	13 12 E
320	—Punta Gavazzi: Lt	38 42 N	13 10 E
54400	ISOLE EGADI (AEGADEAN ISLANDS)		
410	—Isola di Levanzo, Capo Grosso: Lt	38 01 N	12 20 E
420	—Isola Marettimo, Punta Libeccio: Lt	37 57 N	12 04 E
430	—Isola Favignana, Punta Sottile: Lt	37 56 N	12 16 E
54500	ISOLA DI PANTELLERIA		
510	—Punta Spadillo: Lt	36 49 N	12 01 E
520	—Punta Limarsi: Lt	36 44 N	12 02 E
54600	ISOLE PELAGIE		
610	—Isola di Lampione: Lt	35 33 N	12 19 E
620	—Lampedusa, Capo Grecale: Lt	35 31 N	12 38 E
630	—Isola di Linosa, Punta Beppe Tuccio: Lt	35 52 N	12 53 E
54700	**Sicily**		
54710	Capo Peloro: Lt	38 16 N	15 39 E
720	Capo di Milazzo: Lt	38 16 N	15 14 E
730	Capo d'Orlando: Lt	38 10 N	14 45 E
740	Capo Zafferano: Lt	38 07 N	13 32 E
750	*Palermo*	38 08 N	13 22 E
760	Capo Gallo: Lt	38 13 N	13 19 E
770	Capo San Vito: Lt	38 11 N	12 44 E
780	*Trapani*	38 00 N	12 29 E
790	*Marsala*	37 47 N	12 26 E
54800	Capo Granitola: Lt	37 34 N	12 40 E
810	Capo Rossello: Lt	37 18 N	13 27 E
820	*Porto Empedocle*	37 17 N	13 32 E
830	*Licata*	37 06 N	13 57 E
840	*Gela*	37 04 N	14 15 E
850	Capo Scaramia (Capo Scalambri): Lt	36 47 N	14 30 E
860	Isola delle Correnti: Lt	36 38 N	15 06 E
870	Capo Passero: Lt	36 41 N	15 09 E
880	Capo Murro di Porco: Lt	37 00 N	15 20 E
890	*Siracusa* (*Syracuse*)	37 03 N	15 18 E
54900	*Augusta*	37 13 N	15 15 E
910	*Catania*	37 30 N	15 07 E
920	Capo Molini: Lt	37 35 N	15 11 E
930	*Messina*	38 12 N	15 34 E
940	Punta San Raineri: Lt	38 12 N	15 35 E

Index No.	Place	Lat.	Long.
55000	**Italy**	° ′	° ′
55010	Capo dell' Armi: Lt	37 57 N	15 41 E
020	Capo Spartivento: Lt	37 55 N	16 04 E
030	Capo Stilo: Lt	38 27 N	16 35 E
040	Capo Rizzuto: Lt	38 54 N	17 06 E
050	Capo Colonne: Lt	39 01 N	17 12 E
060	*Crotone*	39 05 N	17 08 E
070	Punta dell' Alice: Lt	39 24 N	17 09 E
080	Capo Trionto: Lt	39 37 N	16 46 E
090	*Taranto*	40 26 N	17 12 E
55100	Capo San Vito: Lt	40 25 N	17 12 E
110	*Gallipoli*	40 04 N	17 59 E
120	San Andrea: Lt	40 03 N	17 57 E
130	Capo Santa Maria di Leuca: Lt	39 48 N	18 22 E
140	Capo d'Otranto: Lt	40 06 N	18 31 E
150	Punta San Cataldo: Lt	40 23 N	18 19 E
160	*Brindisi*	40 39 N	17 59 E
170	Capo Gallo: Lt	40 41 N	17 56 E
180	*Bari*	41 08 N	16 52 E
190	*Molfetta*	41 13 N	16 36 E
55200	*Barletta*	41 19 N	16 17 E
210	*Manfredonia*	41 37 N	15 55 E
220	Vieste, Scoglio Santa Croce: Lt	41 53 N	16 11 E
230	Isola Pianosa: Lt	42 13 N	15 45 E
240	Isola Caprara: Lt	42 08 N	15 31 E
250	Isola San Domino, Punta del Diavolo: Lt	42 06 N	15 29 E
260	Punta della Penna: Lt	42 10 N	14 43 E
270	*Ancona*	43 37 N	13 31 E
280	*Rimini*	44 04 N	12 35 E
290	*Ravenna*	44 29 N	12 17 E
55300	Punta della Maestra: Lt	44 58 N	12 29 E
310	*Chioggia*	45 14 N	12 17 E
320	Porto di Lido: NE Breakwater Lt	45 25 N	12 26 E
330	*Venezia* (*Venice*)	45 25 N	12 26 E
340	Porto di Piave Vecchia: Lt	45 29 N	12 35 E
350	Punta del Tagliamento: Lt	45 38 N	13 06 E
360	*Monfalcone*	45 48 N	13 32 E
370	*Trieste*	45 39 N	13 46 E
380	*Muggia*	45 36 N	13 46 E
55400	**Yugoslavia**		
55410	Rt Savudrija (Capo Salvore): Lt	45 29 N	13 30 E
420	Rt Zub: Lt	45 18 N	13 34 E
430	*Poreč* (*Porenzo*)	45 14 N	13 36 E
440	*Rovinj* (*Rovigno*)	45 05 N	13 38 E
450	Hrid Sveti Ivan na Pučini: Lt	45 03 N	13 37 E
460	*Pula*	44 52 N	13 50 E
470	Hrid Porer: Lt	44 45 N	13 53 E
480	Hrid Galiola: Lt	44 44 N	14 11 E
490	Rt Mrlera (Pta. Merlera): Lt	44 48 N	14 00 E
55500	Rt Crna (Nera Pt.): Lt	44 57 N	14 09 E
510	*Rijeka*	45 20 N	14 25 E
520	*Sušak*	45 19 N	14 26 E
530	*Bakar*	45 18 N	14 32 E
540	Ostrvo Sušak (Isola Sansego): Lt	44 31 N	14 18 E
550	Ostrvo Grujica: Lt	44 25 N	14 34 E
560	Veli Rat: Lt	44 09 N	14 49 E
570	*Zadar*	44 08 N	15 12 E
580	Ostrvo Sestrice (Port Tajer): Lt	43 51 N	15 12 E
590	Ostrvo Blitvenica: Lt	43 38 N	15 35 E
55600	Hrid Mulo: Lt	43 31 N	15 55 E
610	*Split*	43 30 N	16 26 E
620	Rt Ražanj: Lt	43 19 N	16 24 E
630	Ostrvo Hvar, Rt Pelegrin: Lt	43 12 N	16 22 E
640	Ostrvo Vis, Rt Stončica: Lt	43 04 N	16 15 E
650	Ostrvo Sušac: Lt	42 45 N	16 29 E
660	Ostrvo Lastovo, Rt Struga: Lt	42 43 N	16 53 E
670	Lastovski Otočiči (Lagostini Is.): Lt	42 46 N	17 09 E
680	Ostrvo Lirica: Lt	42 52 N	17 26 E
690	Ostrvo Sveti Andrija: Lt	42 39 N	17 57 E
55700	*Gruž*	42 40 N	18 05 E
710	*Dubrovnik*	42 38 N	18 07 E
720	Oštri Rt: Lt	42 24 N	18 32 E
730	*Kotor*	42 25 N	18 46 E
740	Ostrvo Sveti Nikola: Lt	42 16 N	18 52 E
750	Rt Volovica: Lt	42 05 N	19 04 E
760	Mendre Rt: Lt	41 57 N	19 09 E

APPENDIX S

MARITIME POSITIONS

MEDITERRANEAN AND BLACK SEAS—Continued

Index No.	Place	Lat.	Long.
55800	**Albania**	° ′	° ′
55810	Kep i Rodonit (Cape Rodoni): Lt	41 35 N	19 27 E
820	*Durrës*	41 19 N	19 27 E
830	Sazan: Lt	40 30 N	19 16 E
840	*Vlonë*	40 28 N	19 30 E
850	Kavadoni: Lt	40 03 N	19 47 E
55900	**Greece**		
55910	Othonoí (Fano I.), Ákra Kastrí: Lt	39 51 N	19 26 E
920	Tignózon: Lt	39 47 N	19 58 E
930	Kérkira (Corfu), Ákra Sídhero: Lt	39 37 N	19 57 E
940	Sívota: Lt	39 24 N	20 13 E
950	Paxoí, Lákka: Lt	39 14 N	20 09 E
960	Andípaxoi, Ákra Ovoroú (Novara Pt.): Lt	39 08 N	20 16 E
970	Ákra Doukáton (Cape Dukato): Lt	38 34 N	20 34 E
980	Ákra Yerogómbos (Cape Gherogambo): Lt	38 11 N	20 21 E
990	Vardhiánoi: Lt	38 08 N	20 26 E
56000	*Argostólion*	38 11 N	20 29 E
010	Zákinthos, Ákra Skinári: Lt	37 56 N	20 42 E
020	Ákra Kerí: Lt	37 39 N	20 49 E
030	*Astakós*	38 32 N	21 06 E
040	Oxiá: Lt	38 17 N	21 07 E
050	*Mesolóngion*	38 22 N	21 25 E
060	Áyios Sóstis: Lt	38 19 N	21 22 E
070	Ákra Andírrion: Lt	38 20 N	21 46 E
080	Ákra Melangávi: Lt	38 02 N	22 50 E
090	*Kórinthos*	37 57 N	22 57 E
56100	*Pátrai*	38 15 N	21 43 E
110	Ákra Pápas (Áraxos): Lt	38 13 N	21 23 E
120	Ákra Killíni (Cape Glaréntza): Lt	37 57 N	21 07 E
130	Ákra Katákolon: Lt	37 38 N	21 19 E
140	Strofádhes (Stamphani I.): Lt	37 15 N	21 00 E
150	*Pílos* (*Navarino*)	36 55 N	21 41 E
160	Sapiéntza (Sapiénza): Lt	36 44 N	21 42 E
170	*Koróni*	36 48 N	21 58 E
180	*Kalámai*	37 01 N	22 07 E
190	Ákra Kitriés: Lt	36 55 N	22 08 E
56200	Ákra Tainaron (Cape Matapan): Lt	36 23 N	22 29 E
210	*Yíthion*	36 45 N	22 34 E
220	Ákra Maléa: Lt	36 27 N	23 12 E
230	Kíthira, Ákra Spathi: Lt	36 22 N	22 57 E
240	Andikíthira, Ákra Apolitáres: Lt	35 49 N	23 19 E
250	Parapóla (Belo Pulo): Lt	35 56 N	23 27 E
260	*Navplion*	37 34 N	22 48 E
270	Ákra Zourva: Lt	37 22 N	23 35 E
280	Áyios Yeóryios (Agios Georgio I.): Lt	37 28 N	23 57 E
290	Aiyina (Aegina I.), Vrakhos Tourlos (Cape Turlo): Lt	37 46 N	23 34 E
56300	Psittália (Lipso I.): Lt	37 56 N	23 36 E
310	*Piraiévs* (*Piraeus*)	37 56 N	23 39 E
320	Fléves (Phleva I.): Lt	37 46 N	23 46 E
330	Kéa (Zea), Ákra Tamélos: Lt	37 31 N	24 17 E
340	Sérifos (Serpho I.), Ákra Spáthi: Lt	37 06 N	24 31 E
350	Folégandros, Ákra Aspróponda: Lt	36 38 N	24 52 E
360	Thíra (Santorin), Akrotiri (Cape Akroterion): Lt	36 21 N	25 22 E
370	Páros (Pharos I.), Kórax (Korakis): Lt	37 09 N	25 14 E
380	Míkonos, Ákra Armenistí: Lt	37 29 N	25 19 E
390	Síros (Syros), Ákra Trímeson: Lt	37 31 N	24 53 E
56400	Ándros, Ákra Fássa (Cape Phassa): Lt	37 57 N	24 42 E
	Greece—Continued	° ′	° ′
56410	Ákra Kafirévs (Cape Doro): Lt	38 09 N	24 37 E
420	Vrakhonisís Kaloyéri: Lt	38 10 N	25 17 E
430	Prasoúdha (Prassudo Islet): Lt	38 40 N	24 15 E
440	Skíros (Skyros), Ákra Lithári: Lt	38 47 N	24 41 E
450	*Vólos*	39 21 N	22 57 E
460	Skópelos: Lt	39 12 N	23 36 E
470	Psathoúra: Lt	39 30 N	24 11 E
480	Ákra Posídhion (Kassándra Point): Lt	39 57 N	23 23 E
490	*Thessaloníki*	40 38 N	22 56 E
56500	Kómbi: Lt	39 47 N	25 14 E
56600	LÍMNOS		
610	—Kástron (Kastro): Lt	39 52 N	25 03 E
620	—Ákra Pláka: Lt	40 02 N	25 27 E
56700	*Kavála*	40 56 N	24 25 E
710	*Alexandroúpolis*	40 50 N	25 53 E
56800	**Turkey**		
56900	DARDANELLES		
910	—İlyasbala Burnu (Cape Helles): Lt	40 03 N	26 11 E
920	—Kumkale (Mendires Cape): Lt	40 01 N	26 12 E
930	—*Çanakkale*	40 09 N	26 24 E
940	—Gelibolu (Gallipoli): Lt	40 24 N	26 41 E
57000	*Tekirdağ*	40 58 N	27 31 E
010	*Marmaraereğlisi*	40 58 N	27 58 E
020	Yeşilköy Burnu (Stefano Pt.): Lt	40 57 N	28 51 E
030	*İstanbul*	41 01 N	28 58 E
040	Rumeli Burnu (Cape Rumili): Lt	41 14 N	29 07 E
050	Kara Burun: Lt	41 20 N	28 40 E
060	Iğneada Burnu (Cape Kuri): Lt	41 52 N	28 04 E
57100	**Bulgaria**		
57110	Ostrov Sveti Ivan (Megalo-Nisi I.): Lt	42 26 N	27 42 E
120	*Burgas*	42 29 N	27 29 E
130	Nos Emine (Cape Emineh): Lt	42 42 N	27 56 E
140	Nos Galata: Lt	43 10 N	27 59 E
150	*Varna* (*Stalin*)	43 11 N	27 58 E
160	Nos Kaliakra: Lt	43 21 N	28 30 E
57200	**Rumania**		
57210	Capul Tuzla: Lt	43 59 N	28 42 E
220	*Constanta*	44 10 N	28 40 E
230	Insula Serpilor (Fidonisi I.): Lt	45 16 N	30 13 E
57300	**USSR**		
57310	Mys Bol'shoy Fontan (Cape Fontana): Lt	46 23 N	30 45 E
320	*Odessa*	46 30 N	30 45 E
330	*Nikolayev*	46 57 N	31 59 E
340	*Kherson*	46 37 N	32 36 E
350	Tendrovskiy (Tendra Pt.): Lt	46 22 N	31 32 E
360	Mys Dzharylgach: Lt	46 01 N	33 04 E
370	Mys Tarkhankut (Cape Tarkhan): Lt	45 21 N	32 30 E
380	Mys Yevpatoriyskiy (Eupatoria Pt.): Lt	45 09 N	33 16 E
390	*Sevastopol'*	44 37 N	33 32 E
57400	Mys Khersonesskiy (Cape Khersonese): Lt	44 35 N	33 23 E
410	Mys Sarych (Sarich Pt.): Lt	44 23 N	33 45 E
420	Mys Aytodor (Cape Aitodor): Lt	44 26 N	34 07 E
430	*Yalta*	44 29 N	34 10 E
440	Mys Meganom: Lt	44 48 N	35 05 E
450	Mys Il'i (Cape St. Ili) (Elias): Lt	45 01 N	35 26 E
460	*Feodosiya*	45 01 N	35 26 E
470	Mys Chauda: Lt	45 00 N	35 50 E
480	*Kerch'*	45 21 N	36 29 E

APPENDIX S

MARITIME POSITIONS

MEDITERRANEAN AND BLACK SEAS—Continued

Index No.	Place	Lat.	Long.
		° ′	° ′
	USSR—Continued		
57500	Azovskoye More (Sea of Azov)		
510	—Mys Yenikale (Fonar): Lt	45 23 N	36 39 E
520	—*Osipenko* (*Osipyenko*)	46 45 N	36 47 E
530	—Belosarayskaya Kosa (Byelosarai): Lt	46 53 N	37 20 E
540	—*Zhdanov*	47 04 N	37 35 E
550	—*Rostov Na Don*	47 12 N	39 42 E
560	—Primorsko-Akhtarskaya (Akhtar): Lt	46 06 N	38 11 E
570	—Mys Akhilleon: Lt	45 26 N	36 47 E
57600	Mys Anapskiy (Anapa): Lt	44 53 N	37 18 E
610	*Novorossiysk*	44 43 N	37 47 E
620	Mys Kodosh: Lt	44 06 N	39 02 E
630	Mys Pitsunda: Lt	43 09 N	40 21 E
640	Mys Sukhumiyskiy (Sukhum Pt.): Lt	42 59 N	40 59 E
650	*Poti*	42 09 N	41 35 E
660	*Batumi* (*Batumskaya*)	41 39 N	41 39 E
57700	**Turkey**		
57710	Trabzon: Lt	41 01 N	39 46 E
720	Sinop Burnu (Cape Sinub): Lt	42 01 N	35 13 E
730	İnce (İnjeh) Burun: Lt	42 06 N	34 58 E
740	Kerempe Burnu: Lt	42 01 N	33 17 E
750	Ölüce Burun (Kisi Agsi): Lt	41 19 N	31 26 E
760	Şile (Kilia) Burnu: Lt	41 10 N	29 37 E
770	*Haydarpaşa*	40 59 N	29 01 E
780	Fener (Fanar) Burun: Lt	40 58 N	29 02 E
790	Yelken Kaya Burnu: Lt	40 45 N	29 21 E
57800	*İzmit*	40 46 N	29 55 E
810	Boz Burun: Lt	40 32 N	28 47 E
820	Fener Adasi: Lt	40 38 N	27 46 E
830	Hayirsiz Adasi (Khairsiz Ada): Lt	40 39 N	27 29 E
57900	**Aegean Sea**		
57910	Bozca Ada (Tenedos I.), Bati Burnu (Ponente Pt.): Lt	39 50 N	25 58 E
920	Baba Burnu: Lt	39 29 N	26 05 E
930	Sígri (Megalonisi): Lt	39 13 N	25 50 E
940	Kara Burun: Lt	38 40 N	26 23 E
950	Orak Adasi (Oghlak I.): Lt	38 41 N	26 43 E
960	*İzmir* (*Smyrna*), *Turkey*	38 26 N	27 08 E
970	Psará: Lt	38 32 N	25 37 E
980	Pashá: Lt	38 30 N	26 19 E
990	Venétiko: Lt	38 08 N	26 02 E
58000	Samós, Ákra Pankosi: Lt	37 48 N	26 40 E
010	Ikaría, Ákra Pápas: Lt	37 31 N	26 00 E
58100	Dodecanese		
110	—Levítha, Ákra Spanó: Lt	37 00 N	26 31 E
120	—Andileoúsa (Kandeliusa I.): Lt	36 30 N	26 59 E
130	—Ákra Prasonisi, Ródhos: Lt	35 52 N	27 47 E
140	—*Ródhos* (*Rhodes*)	36 26 N	28 14 E
150	—Strongilí (Hypsili I.): Lt	36 06 N	29 41 E
58200	**Crete (Kríti)**		
58210	Ágria Gramvoúsa (Grabusa): Lt	35 38 N	23 34 E
220	Elafónisos	35 15 N	23 31 E
230	Gávdhos, Ákra Tripití: Lt	34 48 N	24 07 E
240	Ákra Líthinon (Cape Littinos): Lt	34 55 N	24 45 E
250	Koufonísi: Lt	34 56 N	26 08 E
260	Ákra Sídheros (Cape Sidero): Lt	35 19 N	26 20 E
270	Día (Standia): Lt	35 28 N	25 14 E
280	*Iráklion*	35 20 N	25 09 E
290	Ákra Dhrápanon (Cape Drepano): Lt	35 28 N	24 15 E
58300	*Soúdha* (*Suda*)	35 29 N	24 04 E
310	Ákra Melékhas: Lt	35 35 N	24 10 E
320	*Khaniá*	35 31 N	24 01 E
58400	**Turkey**		
58410	*Antalya* (*Adalia*)	36 52 N	30 46 E
420	Kaleardi Burnu (Cape Kiloarda), Alanya: Lt	36 31 N	32 02 E
430	Anamur Burnu: Lt	36 01 N	32 51 E
440	*Mersin*	36 48 N	34 38 E
450	Fener Burnu (Karatas Burnu): Lt	36 33 N	35 20 E
460	*İskenderun* (*Alexandretta*)	36 36 N	36 10 E
470	Hinzir Burun (Domuz Burnu): Lt	36 19 N	35 46 E
58500	**Cyprus**		
58510	Klidhes Islet: Lt	35 41 N	34 37 E
520	Cape Kormakiti: Lt	35 24 N	32 56 E
530	Paphos Point: Lt	34 45 N	32 24 E
540	Cape Gata: Lt	34 34 N	33 01 E
550	*Limassol*	34 40 N	33 03 E
560	Cape Greco: Lt	34 56 N	34 06 E
570	*Famagusta*	35 08 N	33 56 E
58600	**Syria and Lebanon**		
58610	Ra's Ibn Hānī: Lt	35 35 N	35 43 E
620	*Al Lādhiqīyah* (*Latakia*), *Syria*	35 31 N	35 45 E
630	Jazīrat Ramkīn: Lt	34 30 N	35 45 E
640	*Ṭarābulus* (*Tripoli*), *Lebanon*	34 26 N	35 50 E
650	*Bayrūt* (*Beirut*), *Lebanon*	33 54 N	35 30 E
660	*Ṣaydā* (*Sidon*), *Lebanon*	33 35 N	35 22 E
670	*Ṣūr* (*Tyre*), *Lebanon*	33 17 N	35 12 E
58700	**Israel**		
58710	*Acre*	32 55 N	35 05 E
720	*Haifa*	32 49 N	35 00 E
730	Har Hakarmel (Mount Carmel): Lt	32 50 N	34 58 E
740	*Tel Aviv*	32 04 N	34 46 E
750	*Jaffa*	32 03 N	34 45 E
58800	**United Arab Republic (Egypt)**		
58810	*Port Said* (*Bor Sa'īd*)	31 16 N	32 18 E
820	Damietta Mouth: Lt	31 32 N	31 51 E
830	Cape Burullus (Brulos): Lt	31 35 N	31 05 E
840	Rosetta: Lt	31 30 N	30 19 E
850	Ras et Tin: Lt	31 12 N	29 52 E
860	*Alexandria*	31 11 N	29 54 E
870	*Matrūh*	31 22 N	27 14 E
58900	**Libya**		
58910	Ras Azzaz: Lt	31 58 N	24 59 E
920	*Tobruch*	32 05 N	23 59 E
930	*Derna*	32 45 N	22 39 E
940	Ras el Hilal: Lt	32 55 N	22 10 E
950	*Bengasi*	32 07 N	20 03 E
960	*Sirte*	31 13 N	16 35 E
970	Ras Zarrugh (Raz Zorug): Lt	32 22 N	15 13 E
980	Ras el Hallab: Lt	32 48 N	13 48 E
990	*Tripoli*	32 54 N	13 11 E
59000	*Zuara*	32 56 N	12 07 E
59100	**Gozo and Malta**		
59110	Gozo: Lt	36 04 N	14 13 E
59200	Malta		
210	—Ponta ta Delimara: Lt	35 49 N	14 34 E
220	—*Valletta*	35 54 N	14 31 E
59300	**Tunisia**		
59310	Île de Djerba (Jerba I.), Rass Tourgueness: Lt	33 49 N	11 03 E
320	*Gabès*	33 51 N	10 07 E
330	*Sfax*	34 44 N	10 46 E
340	*Mahdia* (*Mahedia*)	35 30 N	11 04 E
350	Île Kuriate: Lt	35 48 N	11 02 E
360	*Sousse*	35 50 N	10 38 E
370	Hammamet: Lt	36 24 N	10 37 E
380	Kélibia: Lt	36 50 N	11 08 E
390	Cap Bon: Lt	37 05 N	11 03 E

APPENDIX S

MARITIME POSITIONS

MEDITERRANEAN AND BLACK SEAS—Continued

Index No.	Place	Lat.	Long.
	Tunisia—Continued	° ′	° ′
59400	*Tunis*	36 48 N	10 11 E
410	Cap Carthage: Lt	36 52 N	10 21 E
420	Île Plane (El Kamela): Lt	37 11 N	10 20 E
430	Îles Cani (Cani Rocks): Lt	37 21 N	10 07 E
440	*Bizerte*	37 17 N	9 53 E
450	Rass Engela (Ras Enghela): Lt	37 21 N	9 45 E
460	Cap Serrat: Lt	37 14 N	9 13 E
470	Galitone, Galitons de l'Ouest: Lt	37 30 N	8 53 E
480	*Tabarka (Tabarca)*	36 58 N	8 46 E
59500	**Algeria**		
59510	*La Calle*	36 54 N	8 27 E
520	Cap Rosa: Lt	36 57 N	8 14 E
530	*Bône*	36 55 N	7 46 E
540	Cap de Garde: Lt	36 58 N	7 47 E
550	Cap de Fer: Lt	37 05 N	7 10 E
560	*Philippeville*	36 53 N	6 55 E
570	Île Srigina: Lt	36 56 N	6 53 E
580	Cap Bougaroun (Cap Bougaroni): Lt	37 05 N	6 28 E
590	Rass Atia: Lt	37 02 N	6 16 E
59600	*Djidjelli*	36 50 N	5 47 E
610	*Bougie*	36 45 N	5 05 E
620	Cap Carbon: Lt	36 47 N	5 06 E
630	Cap Sigli: Lt	36 54 N	4 46 E
640	Cap Corbelin: Lt	36 55 N	4 26 E
650	Cap Benngut (Cap Bengut): Lt	36 55 N	3 54 E

Index No.	Place	Lat.	Long.
	Algeria—Continued	° ′	° ′
59660	Cap Matifou: Lt	36 49 N	3 15 E
670	*Alger (Algiers)*	36 47 N	3 04 E
680	Cap Caxine: Lt	36 49 N	2 57 E
690	*Tipasa (Tipaza)*	36 36 N	2 27 E
59700	*Cherchel*	36 37 N	2 11 E
710	Cap Ténès (Ténèz): Lt	36 33 N	1 21 E
720	Cap Ivi: Lt	36 07 N	0 14 E
730	*Mostaganem*	35 56 N	0 05 E
740	*Arzew*	35 51 N	0 18 W
750	Pointe de l'Aiguille: Lt	35 53 N	0 29 W
760	*Oran*	35 43 N	0 38 W
770	*Mers el Kébir*	35 44 N	0 42 W
780	Cap Falcon: Lt	35 46 N	0 48 W
790	Îles Habibas: Lt	35 43 N	1 08 W
59800	Île Rachgoun (Rashgun): Lt	35 20 N	1 29 W
810	*Nemours*	35 06 N	1 51 W
59900	**Morocco**		
59910	Islas Chafarinas (Zafarin Is.), Isla Isabel Segunda: Lt	35 11 N	2 26 W
920	*Melilla*	35 18 N	2 56 W
930	Cabo de Tres Forcas: Lt	35 27 N	2 58 W
940	Isla de Alborán: Lt	35 56 N	3 02 W
950	Cabo Quilates: Lt	35 17 N	3 41 W
960	Peñón de Velez de la Gomera: Lt	35 10 N	4 18 W
970	Río Martin, Ensenada de Ta-merabel (Tetuan Bay): Lt	35 37 N	5 16 W
980	*Ceuta*	35 54 N	5 19 W
990	Punta Almina: Lt	35 54 N	5 17 W
60000	Punta Malabata: Lt	35 49 N	5 45 W

WEST COAST OF AFRICA

Index No.	Place	Lat.	Long.
61000	**Morocco**	° ′	° ′
61010	Le Charf: Lt	35 46 N	5 47 W
020	*Tangier*	35 47 N	5 48 W
030	Cap Spartel: Lt	35 47 N	5 55 W
61100	**Morocco**		
61110	*Larache*	35 12 N	6 09 W
120	Punta Nador: Lt	35 12 N	6 10 W
61200	**Morocco**		
61210	*Mehdia*	34 16 N	6 41 W
220	*Kénitra (Port-Lyautey)*	34 18 N	6 36 W
230	*Rabat*	34 02 N	6 50 W
240	*Casablanca*	33 36 N	7 37 W
250	El Hank (Pointe el Hank): Lt	33 37 N	7 39 W
260	*Mazagan*	33 15 N	8 30 W
270	Cap Cantin (Cap Kantin): Lt	32 32 N	9 14 W
280	*Safi*	32 18 N	9 15 W
290	*Mogador*	31 31 N	9 46 W
61300	Sidi Mogdoul: Lt	31 30 N	9 46 W
310	Cap Sim: Lt	31 24 N	9 50 W
320	Cap Rhir (Cap Ghir): Lt	30 38 N	9 53 W
330	*Agadir*	30 25 N	9 38 W
61400	**Spanish Sahara (Rio de Oro)**		
61410	Cabo Juby: Aviation Lt	27 57 N	12 55 W
420	Punta Durnford: Lt	23 38 N	16 00 W
61500	**Mauritania**		
61510	Cap Blanc: Lt	20 46 N	17 03 W
520	*Port-Étienne*	20 55 N	17 03 W
61600	**Senegal**		
61610	*Saint-Louis*	16 02 N	16 30 W
620	Cap Vert (Cap Verde): Lt	14 43 N	17 31 W
630	Cap Manuel: Lt	14 39 N	17 27 W
640	*Dakar*	14 41 N	17 26 W

Index No.	Place	Lat.	Long.
61700	**Gambia**	° ′	° ′
61710	*Bathurst*	13 27 N	16 34 W
720	Bijol Is.: Lt	13 23 N	16 48 W
61800	**Senegal**		
61810	Casamance (Kasamanze River), Pointe de Diogué (Jogue Point): Lt	12 35 N	16 48 W
820	*Carabane*	12 34 N	16 42 W
61900	**Portuguese Guinea**		
61910	Ilhéu de Caío (Cayo I.): Lt	11 50 N	16 23 W
920	Ilha Orangosinho, Cabo Cameleao (Cameleon): Lt	11 03 N	15 54 W
930	Ilha Poilão: Lt	10 52 N	15 45 W
62000	**Guinea**		
62010	Île Tamara: Lt	9 27 N	13 50 W
020	*Conakry*	9 31 N	13 43 W
030	Île Matakong: Lt	9 16 N	13 26 W
62100	**Sierra Leone**		
62110	*Pepel*	8 35 N	13 03 W
120	*Freetown*	8 30 N	13 14 W
130	Cape Sierra Leone: Lt	8 30 N	13 18 W
62200	**Liberia**		
62210	*Robertsport (Robert Port)*	6 45 N	11 22 W
220	*Monrovia*	6 19 N	10 49 W
230	Cape Mesurado: Lt	6 19 N	10 49 W
240	Grand Bassa Point: Lt	5 52 N	10 04 W
250	Sinoe (Sinu) Bay: Lt	4 59 N	9 02 W
260	Cape Palmas: Lt	4 22 N	7 44 W
62300	**Ivory Coast**		
62310	Pointe Tafou: Lt	4 25 N	7 22 W
320	Sassandra: Lt	4 57 N	6 04 W
330	*Grand-Lahou*	5 09 N	5 00 W

APPENDIX S

MARITIME POSITIONS

WEST COAST OF AFRICA—Continued

Index No.	Place	Lat. ° ′	Long. ° ′
	Ivory Coast—Continued		
62340	*Abidjan*	5 19 N	4 00 W
350	Grand-Bassam: Lt	5 12 N	3 43 W
62400	**Ghana**		
62410	*Axim*	4 52 N	2 15 W
420	Bobowasi I.: Lt	4 52 N	2 15 W
430	Cape Three Points: Lt	4 45 N	2 06 W
440	*Dixcove*	4 48 N	1 57 W
450	*Takoradi*	4 53 N	1 45 W
460	*Sekondi*	4 56 N	1 42 W
470	Cape Coast Castle, Fort William: Lt	5 06 N	1 14 W
480	*Accra (Akkra)*	5 33 N	0 12 W
485	*Tema*	5 38 N	0 00
490	Cape St. Paul: Lt	5 50 N	0 58 E
62500	**Togo and Dahomey**		
62510	*Lomé, Togo*	6 07 N	1 13 E
520	*Cotonou (Kotonu), Dahomey*	6 21 N	2 25 E
62600	**Nigeria**		
62610	Beecroft Point: Lt	6 24 N	3 23 E
620	*Lagos*	6 24 N	3 24 E
630	*Forcados*	5 22 N	5 26 E
640	Palm Point, Cape Formosa: Lt	4 16 N	6 05 E
62700	**Cameroon**		
62710	Debundscha (Debundga) Point: Lt	4 06 N	9 00 E
720	Cape Nachtigal: Lt	3 57 N	9 13 E
730	*Douala (Duala), French Cameroons*	4 03 N	9 41 E
62800	**Spanish Guinea (Rio Muni and Fernando Póo)**		
62810	*Bata*	1 51 N	9 45 E
820	Cabo San Juan: Lt	1 10 N	9 21 E
62900	FERNANDO PÓO		
910	—Punta Europa (Los Frailes): Lt	3 46 N	8 47 E
920	—*Santa Isabel*	3 45 N	8 46 E
930	—Islote Horacio: Lt	3 46 N	8 55 E
63000	**São Tomé e Príncipe**		
63100	ILHA DO PRÍNCIPE (PRINCE'S ISLAND)		
110	—Ponta da Garça: Lt	1 38 N	7 27 E
120	—*Santo António*	1 38 N	7 26 E
130	—Ilhéu Bombom: Lt	1 42 N	7 24 E
63200	ILHA DE SÃO TOMÉ (SÃO THOMÉ) (ST. THOMAS ISLAND)		
210	—Ilhéu das Cabras: Lt	0 24 N	6 43 E
220	—*São Tomé*	0 21 N	6 44 E
230	—Ilhéu Gago Coutinho (Ilhéu das Rôlas): Lt	0 00	6 31 E

Index No.	Place	Lat. ° ′	Long. ° ′
63300	**Gabon and Congo**		
63310	Libreville, Gabon	0 23 N	9 26 E
320	Pointe Gombé: Lt	0 18 N	9 18 E
330	Port-Gentil, Gabon	0 43 S	8 48 E
340	Cap Lopez: Lt	0 38 S	8 42 E
350	Loango, Congo	4 38 S	11 49 E
360	Baie de Pointe Noire (Black Point Bay), Congo	4 46 S	11 50 E
63400	**Cabinda**		
63410	Landana: Lt	5 14 S	12 09 E
420	*Cabinda (Kabinda)*	5 32 S	12 14 E
63500	**Republic of the Congo**		
63510	Moanda: Lt	5 57 S	12 20 E
520	*Boma*	5 51 S	13 03 E
63600	**Angola**		
63610	Ponta do Padrao (Padron Pt.), Congo River: Lt	6 05 S	12 20 E
620	Ponta de Moita Seca (Mouta Seca): Lt	6 07 S	12 16 E
630	Ambrizete (Foreland Bluff): Lt	7 16 S	12 52 E
640	*Ambriz*	7 50 S	13 06 E
650	Lagostas: Lt	8 45 S	13 18 E
660	*Luanda (Loanda)*	8 49 S	13 14 E
670	Ponta das Palmeirinhas, Cabo Lombo: Lt	9 04 S	13 00 E
680	*Pôrto Amboim*	10 44 S	13 45 E
690	*Lobito*	12 20 S	13 34 E
63700	*Benguela*	12 35 S	13 24 E
710	Ponta das Salinas: Lt	12 50 S	12 56 E
720	Giraúl (Ponta do Giraúl): Lt	15 08 S	12 07 E
730	*Moçâmedes (Mossâmedes)*	15 12 S	12 09 E
740	Ponta Albina (Albino Pt.): Lt	15 53 S	11 43 E
750	Baía dos Tigres (Great Fish Bay): Lt	16 31 S	11 44 E
63800	**South-West Africa**		
63810	Swakopmund: Lt	22 41 S	14 31 E
820	*Walvisbaai (Walvis Bay)*	22 57 S	14 30 E
830	Pelican Point: Lt	22 54 S	14 25 E
840	*Lüderitz*	26 39 S	15 09 E
850	Diaz Point: Lt	26 38 S	15 06 E
63900	**Republic of South Africa**		
63910	*Port Nolloth*	29 15 S	16 52 E
920	Cape Columbine: Lt	32 50 S	17 51 E
930	Dasseneiland (Dassen I.): Lt	33 26 S	18 05 E
940	Robbeneiland (Robben I.): Lt	33 49 S	18 23 E
950	*Cape Town (Capetown)*	33 54 S	18 26 E
960	Table Bay, Green Point: Lt	33 54 S	18 24 E
970	Slangkoppunt (Slang Kop Point): Lt	34 09 S	18 19 E
980	Cape of Good Hope: Lt	34 21 S	18 29 E

EAST COAST OF AFRICA

Index No.	Place	Lat. ° ′	Long. ° ′
64000	**Republic of South Africa**		
64010	*Simonstown*	34 11 S	18 26 E
020	Roman Rock: Lt	34 11 S	18 27 E
030	Danger Point: Lt	34 37 S	19 18 E
040	Cape Agulhas: Lt	34 50 S	20 01 E
050	Cape St. Blaize: Lt	34 11 S	22 09 E
060	*Mosselbaai (Mossel Bay)*	34 11 S	22 09 E
070	Cape St. Francis: Lt	34 12 S	24 50 E
080	Cape Recife: Lt	34 02 S	25 42 E
090	*Port Elizabeth*	33 58 S	25 37 E
64100	Bird Is.: Lt	33 50 S	26 17 E
110	Great Fish Point: Lt	33 31 S	27 06 E
120	Hood Point: Lt	33 02 S	27 54 E
130	Castle Point: Lt	33 02 S	27 55 E
140	*East London*	33 02 S	27 55 E
150	Bashee Entrance: Lt	33 14 S	28 55 E

Index No.	Place	Lat. ° ′	Long. ° ′
	Republic of South Africa—Cont.		
64160	Cape Hermes: Lt	31 28 S	29 33 E
170	*Port St. Johns*	31 38 S	29 33 E
180	Port Shepstone: Lt	30 45 S	30 28 E
190	Green Point: Lt	30 15 S	30 47 E
64200	Cape Natal (Natal Bluff): Lt	29 52 S	31 04 E
210	*Durban (Port Natal)*	29 52 S	31 04 E
220	Durnford Point: Lt	28 55 S	31 55 E
230	Cape St. Lucia: Lt	28 31 S	32 24 E
64300	**Mozambique**		
64310	Ponta do Ouro: Lt	26 50 S	32 54 E
320	Cabo da Inhaca: Lt	25 58 S	33 00 E
330	*Lourenço Marques*	25 58 S	32 35 E
340	Monte Belo: Lt	25 11 S	33 30 E

APPENDIX S

MARITIME POSITIONS

EAST COAST OF AFRICA—Continued

Index No.	Place	Lat.	Long.
	Mozambique—Continued	° ′	° ′
64350	Ponta Závora: Lt	24 31 S	35 12 E
360	Cabo das Currentes: Lt	24 06 S	35 30 E
370	Ponta da Barra: Lt	23 47 S	35 32 E
380	Ilha do Bazaruto: Lt	21 32 S	35 29 E
390	*Beira*	19 50 S	34 50 E
64400	Ponta Macúti: Lt	19 51 S	34 54 E
410	Zambezi River, Ilha Timbué: Lt	18 49 S	36 23 E
420	*Chinde*	18 32 S	36 30 E
430	Vilhena: Lt	18 06 S	36 55 E
440	Ponta Matirre: Lt	17 16 S	38 11 E
450	Ponta Caldeira: Lt	16 38 S	39 30 E
460	Ilha de Mafamede: Lt	16 21 S	40 02 E
470	Rio Sangage Entrance: Lt	15 59 S	40 09 E
480	Ponta Namalungo: Lt	15 38 S	40 25 E
490	Ilha de Gôa (St. George I.): Lt	15 03 S	40 47 E
64500	*Moçambique*	15 02 S	40 44 E
510	Baía de Memba, Ponta Cogune (Cape Loguno): Lt	14 12 S	40 43 E
520	Ponta Maunhane: Lt	12 58 S	40 36 E
530	*Porto Amélia*	12 58 S	40 30 E
540	Ilha Ibo: Lt	12 20 S	40 30 E
550	Cabo Delgado: Lt	10 41 S	40 39 E
64600	**Tanganyika**		
64610	*Lindi*	9 59 S	39 44 E
620	Fanjove I.: Lt	8 34 S	39 34 E
630	Ras Mkumbi (Moresby Pt.), Mafia I.: Lt	7 38 S	39 55 E
640	Ras Kanzi: Lt	7 01 S	39 33 E
650	*Dar es Salaam*	6 49 S	39 18 E
660	*Tanga*	5 05 S	39 07 E
670	Ulenga: Lt	5 01 S	39 10 E
64700	**Zanzibar**		
64710	Pungume I.: Lt	6 26 S	39 20 E
720	Chumbe: Lt	6 17 S	39 11 E

Index No.	Place	Lat.	Long.
	Zanzibar—Continued	° ′	° ′
64730	*Zanzibar*	6 10 S	39 11 E
740	Mwana Mwana: Lt	5 45 S	39 13 E
750	Ras Nungwe: Lt	5 43 S	39 18 E
760	Ras Kegomacha, Pemba: Lt	4 53 S	39 41 E
64800	**Kenya**		
64810	*Mombasa*	4 04 S	39 41 E
820	Kilifi Entrance: Lt	3 38 S	39 52 E
830	*Malindi*	3 13 S	40 08 E
840	*Lamu*	2 15 S	40 54 E
64900	**Somali Republic**		
64910	*Chisimaio (Kisimayu)*	0 22 S	42 33 E
920	Giumbo: Lt	0 15 S	42 38 E
930	*Brava*	1 06 N	44 03 E
940	*Mogadiscio*	2 02 N	45 21 E
950	Itala: Lt	2 45 N	46 19 E
960	Obbia (Obiat): Lt	5 21 N	48 31 E
970	Eil Marina: Lt	7 58 N	49 51 E
980	Ras Hafun: Lt	10 26 N	51 25 E
990	Capo Guardafui: Lt	11 50 N	51 17 E
65000	Ras Illaue (Alula): Lt	11 58 N	50 46 E
010	*Bender Cassim (Bandar Kassim)*	11 17 N	49 11 E
65100	**Gulf of Aden**		
65110	Suquṭrá (Socotra)	12 30 N	54 00 E
120	*Berbera, Somali Republic*	10 27 N	45 02 E
130	*Djibouti, French Somaliland*	11 36 N	43 09 E
140	Îles Moucha: Lt	11 44 N	43 13 E
150	*Obock, French Somaliland*	11 59 N	43 19 E
160	Ras Bir: Lt	11 59 N	43 22 E

RED SEA

Index No.	Place	Lat.	Long.
		° ′	° ′
66000	Barīm (Perim I.), Balfe Point: Lt	12 39 N	43 23 E
010	*Assab, Ethiopia*	13 00 N	42 45 E
020	*Al Mukhā (Mocha), Yemen*	13 19 N	43 15 E
030	Abu Ail Is., Quoin I.: Lt	14 05 N	42 49 E
040	Punta Shab Shakhs: Lt	14 39 N	41 07 E
050	Jazā'ir az Zubayr (Zubair Is.), Centre Peak	15 01 N	42 10 E
060	Jabal aṭ Ṭā'ir: Lt	15 32 N	41 49 E
070	Isola Sciumma: Lt	15 32 N	40 00 E
080	*Massaua, Ethiopia*	15 37 N	39 28 E
090	Isola Sceic el Abu (Sheikh al Abu I.): Lt	16 02 N	39 26 E
66100	Isola Difnein: Lt	16 37 N	39 19 E
110	Masamirit: Lt	18 50 N	38 45 E
120	*Sawākin, Sudan*	19 08 N	37 21 E
130	*Port Sudan, Sudan*	19 36 N	37 14 E
66140	Sanganeb: Lt	19 43 N	37 26 E
150	*Juddah (Jidda), Saudi Arabia*	21 29 N	39 11 E
160	Daydalās (Daedalus Reef): Lt	24 55 N	35 52 E
170	Al Ikhwān (El-Akhawein) (The Brothers): Lt	26 19 N	34 51 E
180	Jazīrat Shākir (Shadwan I.): Lt	27 27 N	34 02 E
190	Jazīrat Jūbāl aş Şaghīrah: Lt	27 41 N	33 48 E
66200	Juzur Ashrāfī (Ashrafi Is.): Lt	27 47 N	33 42 E
210	*Aṭ Ṭūr (Tor), U.A.R. (Egypt)*	28 13 N	33 37 E
220	*Ra's Ghārib, U.A.R. (Egypt)*	28 21 N	33 06 E
230	Ra's Za'fāranah: Lt	29 06 N	32 39 E
240	Ra's Abū Daraj: Lt	29 23 N	32 34 E
250	Newport Rock (Zenobia): Lt	29 53 N	32 33 E
260	*Suez (As Suways), U.A.R. (Egypt)*	29 58 N	32 33 E
270	*Ismailia (Al Ismā'īlīyah), U.A.R. (Egypt)*	30 35 N	32 17 E

ISLANDS OF THE INDIAN OCEAN

Index No.	Place	Lat.	Long.
		° ′	° ′
67000	Île de la Réunion		
010	—*Saint-Pierre*	21 20 S	55 29 E
020	—Saint-Paul: Lt	20 55 S	55 17 E
030	—Pointe des Galets: Lt	20 55 S	55 18 E
040	—*Saint-Dennis*	20 52 S	55 28 E
67100	Mauritius		
110	—Caves Point: Lt	20 11 S	57 25 E
120	—*Port Louis*	20 10 S	57 30 E
130	—Flat I.: Lt	19 53 S	57 39 E
140	—*Mahébourg*	20 25 S	57 42 E
67200	*Rodriguez I., Port Mathurin*	19 41 S	63 25 E
67300	Seychelles Group		
310	—*Victoria, Mahé I.*	4 37 S	55 27 E
320	—Mamelle Islet: Lt	4 29 S	55 32 E
330	—Dennis I.: Lt	3 48 S	55 40 E
67400	*Amirante Isles, Eagle I.*	5 07 S	53 19 E
67500	Chagos Archipelago		
510	—*Île Sudest*	6 40 S	71 24 E
520	—*Île Boddam*	5 21 S	72 13 E
530	—*Diego Garcia*	7 21 S	72 28 E
67600	*Maldive Is., Male I.*	4 10 N	73 30 E
67700	Cocos (Keeling) Islands		
710	—*Home I.*	12 07 S	96 54 E
720	—Direction I.: Lt	12 05 S	96 53 E
67800	*Christmas I., Flying Fish Cove*	10 25 S	105 43 E
810	Île Amsterdam	37 50 S	77 32 E
820	Île Saint Paul	38 43 S	77 31 E
830	Îles de Kergulen	49 35 S	69 30 E
840	Heard I.	53 12 S	73 34 E

APPENDIX S

MARITIME POSITIONS

ISLANDS OF THE INDIAN OCEAN—Continued

Index No.	Place	Lat.	Long.
		° ′	° ′
67900	ÎLES CROZET		
910	—Île de l'Est	46 25 S	52 10 E
920	—Île aux Cochons	46 06 S	50 10 E
68000	Prince Edward I.	46 36 S	37 57 E
68100	ARCHIPEL DES COMORES		
110	—*Moroni, Grande Comore*	11 42 S	43 15 E
120	—*Fumboni, Mohéli*	12 16 S	43 45 E
130	—*Mutsamudu, Anjouan*	12 09 S	44 24 E
140	—Mayotte, Îlot Dzaoudzi: Lt	12 47 S	45 16 E
68200	**Madagascar**		
68210	Cap d'Ambre (Cape Andre): Lt	11 57 S	49 17 E
68300	BAIE DE DIÉGO-SUAREZ		
310	—Îlot des Aigrettes (Nosy Langoro): Lt	12 13 S	49 20 E
320	—*Diégo-Suarez* (*Antsirana*)	12 16 S	49 18 E
68400	Miné (Cap Andran Omody): Lt	12 14 S	49 22 E
410	Nosy Akao: Lt	12 49 S	49 50 E
420	Cap Est: Lt	15 17 S	50 28 E
	Madagascar—Continued		
68500	ÎLE SAINTE MARIE		
510	—Pointe Albrand: Lt	16 44 S	50 00 E
520	—Île aux Nattes, Pointe Blevec: Lt	17 07 S	49 51 E
68600	Pointe Tanio: Lt	18 08 S	49 26 E
610	*Tamatave*	18 10 S	49 25 E
620	Pointe Hastie: Lt	18 09 S	49 25 E
630	*Mahanoro*	19 55 S	48 50 E
640	*Mananjary*	21 15 S	48 20 E
650	Pointe d'Itaperina: Lt	24 58 S	47 07 E
660	*Fort Dauphin*	25 01 S	47 01 E
670	*Tulear*	23 22 S	43 40 E
680	Massif Katsepe: Lt	15 43 S	46 13 E
690	*Majunga*	15 42 S	46 19 E
68700	Pointe Anorombato: Lt	15 43 S	46 18 E
710	*Analalava*	14 38 S	47 46 E
720	Nosi Lava: Lt	14 33 S	47 36 E
730	Nosi Iranja: Lt	13 35 S	47 50 E
740	Tany Kely: Lt	13 29 S	48 15 E
750	*Hellville*	13 24 S	48 17 E
760	Nosi Faly: Lt	13 19 S	48 29 E
770	Nosi Anambo (Woody I.): Lt	12 16 S	48 30 E

SOUTH COAST OF ASIA

Index No.	Place	Lat.	Long.
		° ′	° ′
69000	*Aden, Colony of Aden*	12 47 N	44 58 E
010	Elephants Back: Lt	12 46 N	44 59 E
020	Ras Marshaq: Lt	12 46 N	45 03 E
030	*Al Mukallā, Aden*	14 31 N	49 07 E
040	Kuria Muria Is.	17 30 N	56 00 E
050	Al Masīrah: Lt	20 41 N	58 55 E
060	*Al Ḥadd, Muscat and Oman*	22 32 N	59 48 E
070	*Masqaṭ, Muscat and Oman*	23 37 N	58 37 E
69100	**Persian Gulf**		
69110	Little Quoin: Lt	26 28 N	56 33 E
120	*Ash Shāriqah* (*Sharjah*), *Trucial Coast*	25 22 N	55 23 E
130	*Ad Dawḥah, Qatar*	25 17 N	51 32 E
69200	AL BAḤRAYN (BAHREIN ISLAND)		
210	—*Jazīrat Sitrah*	26 10 N	50 40 E
220	—*Bahrein Harbor*	26 14 N	50 35 E
69300	*Ad Dammām, Al Minṭaqah ash Sharqīyah* (*Hasa*)	26 26 N	50 06 E
310	*Ra's at Tannūrah, Al Minṭaqah ash Sharqīyah*	26 38 N	50 10 E
320	Jazīreh-ye Fārsī (Jezirat Tarsi): Lt	28 00 N	50 10 E
330	Mina Saud, Ra's az Zaur, Saudi Arabia	28 45 N	48 26 E
340	*Al Fuḥayḥīl* (*Faḥayḥīl*), *Kuwait*	29 04 N	48 09 E
350	Al Kubr: Lt	29 04 N	48 30 E
360	*Al Kuwayt* (*Kuwait*), *Kuwait*	29 25 N	47 58 E
370	*Al Baṣrah, Iraq*	30 30 N	47 50 E
380	*Khorramshahr, Iran*	30 26 N	48 10 E
390	*Abādān, Iran*	30 20 N	48 17 E
69400	*Bandar-e Shāhpūr, Iran*	30 26 N	49 05 E
410	*Būshehr, Iran*	28 59 N	50 50 E
420	Jazīreh-ye Qeys (Jezirat Qais) (Kais I.): Lt	26 31 N	53 59 E
430	Jazīreh-ye Tanb-e Bozorg: Lt	26 16 N	55 19 E
440	*Bandar 'Abbās, Iran*	27 10 N	56 17 E
450	Ra's-e Jāsk: Lt	25 38 N	57 46 E
460	*Gwādar, Muscat and Oman*	25 07 N	62 19 E
69500	**Pakistan**		
69510	Ras Muāri (Cape Monze): Lt	24 50 N	66 40 E
520	Manora Point: Lt	24 48 N	66 59 E
530	*Karāchi*	24 47 N	66 59 E
69600	**India**		
69610	*Māndvi*	22 50 N	69 21 E
620	Navinar Point: Lt	22 44 N	69 43 E
625	*Kandla*	23 02 N	70 13 E
630	Pirotan I.: Lt	22 36 N	69 57 E
640	*Okha* (*Beyt Harbor*)	22 28 N	69 05 E
	India—Continued		
650	Dwarka Point: Lt	22 14 N	68 57 E
69660	*Porbandar*	21 38 N	69 36 E
670	*Verāval*	20 54 N	70 22 E
680	Diu Head: Lt	20 41 N	70 50 E
690	*Diu*	20 43 N	70 59 E
69700	*Jāfarābād*	20 52 N	71 22 E
710	Gopnath Point: Lt	21 12 N	72 06 E
720	Pīram I.: Lt	21 35 N	72 20 E
730	*Damão*	20 25 N	72 50 E
740	Arnāla I.: Lt	19 27 N	72 44 E
750	Prongs Reef: Lt	18 53 N	72 48 E
760	*Bombay*	18 56 N	72 50 E
770	Khānderi I.: Lt	18 42 N	72 49 E
780	Rājpuri Point: Lt	18 17 N	72 56 E
790	*Dābhol*	17 35 N	73 11 E
69800	*Jaigarh*	17 18 N	73 13 E
810	*Ratnāgiri*	16 59 N	73 18 E
820	Rājāpur: Lt	16 36 N	73 19 E
830	*Vijayadurg*	16 33 N	73 20 E
840	*Devgarh*	16 22 N	73 22 E
850	*Mālvan*	16 03 N	73 28 E
860	Vengurla Rocks (Burnt Is.): Lt	15 53 N	73 27 E
870	*Vengurla*	15 51 N	73 38 E
880	Aguada: Lt	15 29 N	73 46 E
890	*Mormugão, Goa*	15 24 N	73 49 E
69900	Oyster Rocks: Lt	14 49 N	74 03 E
910	*Kārwār*	14 48 N	74 08 E
920	Bhatkal: Lt	13 58 N	74 32 E
930	Kāp (Kahp): Lt	13 13 N	74 44 E
940	*Mangalore*	12 51 N	74 50 E
950	*Cannanore*	11 52 N	75 22 E
960	*Tellicherry*	11 45 N	75 32 E
970	*Mahé*	11 42 N	75 32 E
980	Kadalur Point: Lt	11 28 N	75 39 E
990	*Calicut* (*Kozhikode*)	11 15 N	75 46 E
70000	*Cochin*	9 58 N	76 15 E
010	*Alleppey*	9 30 N	76 20 E
020	Tangasseri Point: Lt	8 53 N	76 34 E
030	*Quilon*	8 53 N	76 35 E
040	*Trivandrum*	8 28 N	76 56 E
050	Muttum Point: Lt	8 07 N	77 19 E
060	Cape Comorin	8 05 N	77 33 E
070	Laccadive Is., Kiltān I.: Lt	11 30 N	73 00 E
70100	**Ceylon**		
70110	Mannar I.: Lt	9 06 N	79 44 E
120	*Colombo*	6 57 N	79 51 E
130	Barberyn, Welmaduwa I.: Lt	6 28 N	79 58 E
140	*Galle*	6 02 N	80 13 E
150	Dondra Head: Lt	5 55 N	80 36 E
160	*Hambantota*	6 07 N	81 08 E

APPENDIX S

MARITIME POSITIONS

SOUTH COAST OF ASIA—Continued

Index No.	Place	Lat.	Long.
	Ceylon—Continued	° ′	° ′
70170	Great Basses Reef: Lt	6 11 N	81 29 E
180	Little Basses Rocks: Lt	6 25 N	81 44 E
190	Sangamankanda Point: Lt	7 01 N	81 52 E
70200	*Batticaloa*	7 45 N	81 41 E
210	Foul Point: Lt	8 32 N	81 19 E
220	*Trincomalee*	8 34 N	81 14 E
230	Mullaittivu: Lt	9 16 N	80 49 E
240	Point Pedro: Lt	9 51 N	80 15 E
250	*Kankesanturai*	9 49 N	80 03 E
260	Kovilan Point: Lt	9 46 N	79 50 E
270	*Jaffna*	9 40 N	80 00 E
70300	**India**		
70310	Manappādu Point: Lt	8 22 N	78 04 E
320	Pāndyan Tivu (Hare I.): Lt	8 47 N	78 11 E
330	*Tuticorin*	8 48 N	78 09 E
340	*Pāmban*	9 17 N	79 13 E
350	Tondi: Lt	9 45 N	79 01 E
360	Point Calimere: Lt	10 18 N	79 52 E
370	*Negapatam*	10 46 N	79 50 E
380	*Karikal*	10 55 N	79 51 E
390	*Tranquebar*	11 01 N	79 51 E
70400	*Cuddalore*	11 43 N	79 46 E
410	*Pondichéry*	11 56 N	79 50 E
420	Mahābalipur: Lt	12 37 N	80 11 E
430	*Madras*	13 05 N	80 17 E
440	Pulicat: Lt	13 25 N	80 20 E
450	*Masulipatam*	16 09 N	81 13 E
460	Sacramento: Lt	16 35 N	82 17 E
470	*Cocanada*	16 56 N	82 15 E
480	Vākalapūdi: Lt	17 01 N	82 17 E
490	*Visākhāpatnam (Vizagapatam)*	17 42 N	83 18 E
70500	*Bimlipatam*	17 53 N	83 28 E
510	Santapilli: Lt	18 04 N	83 38 E
520	*Kalingapatam*	18 20 N	84 09 E
530	*Bāruva*	18 53 N	84 36 E
540	*Gopālpur*	19 15 N	84 55 E
550	*Puri*	19 47 N	85 50 E
560	False Point: Lt	20 20 N	86 44 E
570	Shortts I.: Lt	20 47 N	87 05 E
580	Hooghly River, Sāgar I.: Lt	21 39 N	88 03 E
590	*Calcutta*	22 33 N	88 19 E
70600	ANDAMAN ISLANDS		
610	—Table I.: Lt	14 11 N	93 22 E
620	—*Port Blair*	11 40 N	92 46 E
70700	**Pakistan**		
70710	Chalna	22 36 N	89 31 E
720	*Chittagong*	22 20 N	91 50 E
730	Kutubdia I.: Lt	21 52 N	91 50 E
740	*Cox's Bāzār*	21 27 N	91 58 E
70800	**Burma**		
70810	Oyster I.: Lt	20 12 N	92 32 E
820	*Akyab*	20 08 N	92 53 E
830	Savage I.: Lt	20 05 N	92 54 E
840	*Kyaukpyu*	19 27 N	93 34 E
850	Beacon I.: Lt	18 56 N	93 27 E
860	*Bassein*	16 47 N	94 45 E
870	Diamond I.: Lt	15 52 N	94 17 E
880	Alguada Reef: Lt	15 42 N	94 12 E

Index No.	Place	Lat.	Long.
	Burma—Continued	° ′	° ′
70890	*Rangoon*	16 46 N	96 10 E
70900	Eastern Grove Flats: Lt	16 30 N	96 23 E
910	*Moulmein*	16 29 N	97 37 E
920	Green I.: Lt	16 04 N	97 33 E
930	Double I.: Lt	15 52 N	97 35 E
940	Mibya Kyun (Reef I.): Lt	13 36 N	98 13 E
950	*Tavoy*	14 04 N	98 11 E
960	*Mergui*	12 26 N	98 36 E
71000	**Thailand**		
71010	Ko Phi (Goh Pee), Pakchan River: Lt	9 58 N	98 35 E
020	Ko Ra (Takua Pa): Lt	9 15 N	98 19 E
030	Ko Kaeo Noi (Goh Keonoi): Lt	7 44 N	98 18 E
040	Ko Taphao Noi: Lt	7 50 N	98 26 E
050	*Phuket*	7 53 N	98 24 E
060	*Phangnga*	8 24 N	98 31 E
070	*Khlong Krabi Yai*	8 04 N	98 55 E
080	Ka Chom Fai Ko Liang (Goh Beng): Lt	7 05 N	99 24 E
71100	**Malaya**		
71110	Sungei Kedah Entrance: Lt	[illegible] 06 N	100 17 E
71200	PENANG ISLAND		
210	—Muka Head: Lt	5 28 N	100 11 E
220	—Pulau Tikus (Rat I.): Lt	5 29 N	100 18 E
230	—*Penang*	5 25 N	100 21 E
240	—Pulau Rimau: Lt	5 14 N	100 17 E
71300	*Prai*	5 23 N	100 22 E
310	Tanjong Hantu: Lt	4 19 N	100 33 E
320	*Pangkor*	4 14 N	100 32 E
330	Pulau Katak: Lt	4 09 N	100 37 E
340	White Rock, Sembilan Is.: Lt	4 00 N	100 30 E
350	*Bagan Datoh*	3 59 N	100 47 E
360	Sungei Selangor: Lt	3 20 N	101 15 E
370	Batu Penyu (Glamorganshire Rock): Lt	3 14 N	101 13 E
380	Pulau Angsa: Lt	3 11 N	101 13 E
390	*Port Swettenham*	3 00 N	101 24 E
71400	*Klang*	3 02 N	101 27 E
410	Pulau Pintu Gedong: Lt	2 54 N	101 15 E
420	One Fathom Bank: Lt	2 53 N	101 00 E
430	*Port Dickson*	2 31 N	101 47 E
440	Cape Rachado: Lt	2 24 N	101 51 E
450	*Malacca*	2 12 N	102 15 E
460	St. Pauls Hill: Lt	2 12 N	102 15 E
470	Pulau Undan: Lt	2 03 N	102 20 E
480	Pulau Pisang: Lt	1 28 N	103 15 E
490	Sultan Shoal: Lt	1 14 N	103 39 E
71500	Raffles, Pulau Satumu (Coney Islet): Lt	1 10 N	103 45 E
510	Pulau Sakijang Pelepah (E. St. John's I.): Lt	1 13 N	103 51 E
520	Keppel Harbor	1 16 N	103 50 E
530	Fort Canning: Lt	1 18 N	103 51 E
540	*Singapore*	1 17 N	103 51 E
550	*Serangoon*	1 23 N	103 58 E
560	Pulo Mungging (S. Lima Islet): Lt	1 22 N	104 18 E
570	Petra Branca (Pedra Branca) (Horsburgh): Lt	1 20 N	104 24 E

INDONESIA

Index No.	Place	Lat.	Long.
		° ′	° ′
72000	PULAU-PULAU NATUNA (NATUNA ISLANDS)		
010	—Pulau Subi-Ketjil: Lt	3 03 N	108 51 E
020	—Pulau Merundung: Lt	2 04 N	109 06 E
72100	PULAU-PULAU ANAMBAS (ANAMBAS ISLANDS)		
110	—Pulau Mangkai: Lt	3 05 N	105 36 E
120	—*Terampa*	3 13 N	106 13 E
72200	Nipa (Tree I.): Lt	1 09 N	103 39 E
210	Takong Ketjil: Lt	1 06 N	103 43 E
220	Batu Berhanti: Lt	1 11 N	103 53 E
230	*Pulau Sambu*	1 10 N	103 54 E
72240	Karang Galang (Pan Reef), Selat Riouw: Lt	1 10 N	104 11 E
250	Tandjung Berakit, Pulau Bintan: Lt	1 13 N	104 34 E
260	Pulau Mantang: Lt	0 45 N	104 31 E
270	Pulau Karas-ketjil: Lt	0 44 N	104 22 E
280	Pulau Kentar: Lt	0 02 N	104 47 E
290	Pulau Lingga, Tandjung Djang: Lt	0 18 S	105 00 E
72300	Pulau Berhala: Lt	0 52 S	104 24 E
310	Pulau Mutji: Lt	0 32 S	104 02 E

APPENDIX S

MARITIME POSITIONS

INDONESIA—Continued

Index No.	Place	Lat.	Long.
		° ′	° ′
72400	SELAT DURIAN (DURIAN STRAIT)		
410	—South Brother: Lt	0 33 N	103 46 E
420	—North Brother: Lt	0 37 N	103 46 E
430	—Melvill Reef: Lt	0 52 N	103 37 E
72500	Pulau Iju Ketjil (The Brothers): Lt	1 11 N	103 21 E
72600	**Sumatra**		
72610	*Bengkalis*	1 28 N	102 06 E
620	Pulau Djemur: Lt	2 53 N	100 34 E
630	Pulau Pandang: Lt	3 25 N	99 45 E
640	*Belawan*	3 47 N	98 41 E
650	*Teluk Aru (Aru Bay)*	4 14 N	98 14 E
660	*Langsa*	4 32 N	98 01 E
670	Diamond Point (Diamant Punt) (Jambu Ayer): Lt	5 16 N	97 29 E
680	Pulau Buru: Lt	5 41 N	95 23 E
72700	PULAU WE		
710	—Le Muele: Lt	5 54 N	95 20 E
720	—*Sabang*	5 53 N	95 19 E
72800	Pulau Breueh (Bras) (Willemstoren): Lt	5 45 N	95 03 E
810	Pulau Rusa: Lt	5 17 N	95 12 E
820	*Meulaboh*	4 08 N	96 08 E
830	*Tapaktuan*	3 15 N	97 11 E
840	Teluk Sinabang, Pulau Simeulue: Lt	2 30 N	96 24 E
850	*Singkil*	2 16 N	97 48 E
860	Tandjung Mbana, Pulau Nias: Lt	1 18 N	97 36 E
870	Hinako, Pulau-pulau Hinako: Lt	0 52 N	97 20 E
880	Pulau Labu: Lt	0 52 N	98 56 E
890	Pulau Temang: Lt	0 22 N	99 05 E
72900	Pulau Pangkal: Lt	0 08 N	99 17 E
910	Pulau Sigata: Lt	0 08 S	98 12 E
920	Pulau Bodjo: Lt	0 39 S	98 31 E
930	Pulau Karsik: Lt	0 36 S	100 04 E
940	*Padang*	1 00 S	100 22 E
950	Udjung Batumandi (Ujung Sungei Bramei): Lt	1 02 S	100 22 E
960	Pulau Najamuk: Lt	1 16 S	100 18 E
970	Pulau Katangkatang: Lt	1 53 S	100 34 E
980	*Bengkulu*	3 47 S	102 15 E
990	Pulau Tikus: Lt	3 51 S	102 11 E
73000	Manna: Lt	4 30 S	102 54 E
010	Tandjung Selasih (Tandjung Bandar): Lt	4 49 S	103 20 E
020	Pulau Pisang: Lt	5 08 S	103 51 E
030	*Kroe (Kru)*	5 11 S	103 56 E
040	Tjukuh Belimbing (Flat Cape) (Vlakke Hoek): Lt	5 56 S	104 33 E
050	*Telukbetung*	5 27 S	105 16 E
060	Pulau Sebuku, Tjukuh Banding: Lt	5 51 S	105 32 E
070	*Sungai Gerong*	2 59 S	104 50 E
080	*Pladju*	2 59 S	104 50 E
090	*Palembang*	2 59 S	104 46 E
73100	BANGKA		
110	—Tandjung Ular: Lt	1 58 S	105 07 E
120	—Tandjung Kelian: Lt	2 05 S	105 08 E
130	—*Muntok*	2 04 S	105 10 E
140	—Pulau Pelepasan (West Nangka): Lt	2 23 S	105 45 E
150	—Pulau Besar: Lt	2 53 S	106 08 E
160	—Pulau Dapur: Lt	3 08 S	106 31 E
170	—Tandjung Berikat: Lt	2 34 S	106 51 E
73200	SELAT-SELAT GASPAR (GASPAR STRAIT)		
210	—Pulau Lepar: Lt	2 57 S	106 55 E
220	—Pulau Tjelaka: Lt	2 52 S	107 01 E
230	—Shoalwater Is. (Shallow Wate Is.): Lt	3 19 S	107 13 E
73300	BILLITON (BELITUNG)		
310	—Pulau Langkuas: Lt	2 32 S	107 37 E
320	—Pulau Kanis: Lt	2 37 S	108 12 E
73400	Discovery East Bank: Lt	3 35 S	109 10 E
410	Pulau Menjawak (Boompjes I.): Lt	5 56 S	108 23 E
420	Etna Bank: Lt	5 18 S	106 54 E
430	Arnemuiden Bank: Lt	5 12 S	106 44 E
440	Pulau Tuguan (North Watcher): Lt	5 12 S	106 28 E
	Sumatra—Continued		
73500	SUNDA STRAIT (SELAT SUNDA)		
510	—Tandjung Lajar (First Pt.): Lt	6 45 S	105 13 E
520	—Tandjung Tjikoneng (Fourth Pt.): Lt	6 04 S	105 53 E
530	—Pulau Tampurung (Toppershoedje): Lt	5 54 S	105 56 E
73600	Pulau Tunda (Toenda Eilanden): Lt	5 49 S	106 17 E
610	Pulau Pajung: Lt	5 49 S	106 33 E
620	Pulau Damar-besar (Edam): Lt	5 57 S	106 50 E
73700	**Java**		
73710	*Djakarta (Batavia)*	6 07 S	106 48 E
720	Tandjungpriok: Lt	6 05 S	106 53 E
730	*Tjirebon (Cheribon)*	6 43 S	108 34 E
740	*Pekalongan*	6 51 S	109 42 E
750	*Semarang*	6 57 S	110 25 E
760	Pulau Mondoliko: Lt	6 23 S	110 55 E
770	Sangkapura, Pulau Bawean: Lt	5 51 S	112 39 E
780	*Surabaja*	7 12 S	112 44 E
73800	MADURA (MADOERA)		
810	—Sembilangan: Lt	7 04 S	112 40 E
820	—Tandjung: Lt	7 08 S	113 54 E
830	—Pulau Sapudi: Lt	7 05 S	114 16 E
73900	Zwaantjes Reef: Lt	7 28 S	113 07 E
910	*Probolinggo*	7 43 S	113 13 E
920	*Panarukan*	7 42 S	113 56 E
930	Pulau Karangmas (Pulau Meiderts Reef): Lt	7 40 S	114 26 E
940	Pulau Tabuan (Duiven I.): Lt	8 02 S	114 28 E
950	Tandjung Bansering: Lt	8 04 S	114 26 E
960	*Banjuwangi*	8 11 S	114 23 E
970	Tandjung Bantenan: Lt	8 46 S	114 31 E
980	*Tjilatjap*	7 44 S	109 00 E
990	*Palabuhan Ratu*	6 59 S	106 32 E
74000	**Lesser Sunda Islands**		
74100	BALI		
110	—Tandjung Pasir: Lt	8 06 S	114 26 E
120	—Tandjung Pengambengan: Lt	8 24 S	114 35 E
130	—*Buleleng*	8 06 S	115 06 E
74200	Nusa Lembongan, Selat Badung: Lt	8 40 S	115 27 E
74300	LOMBOK		
310	—*Ampenan*	8 34 S	116 04 E
320	—*Labuanhadji*	8 42 S	116 34 E
74400	Sakuntji (Maria Reigersbergen Bank): Lt	7 51 S	117 13 E
410	*Bima, Sumbawa*	8 27 S	118 43 E
420	Pulau Kelapa: Lt	8 40 S	119 14 E
74500	FLORES		
510	—*Pulau Badjo (Bajo)*	8 29 S	119 53 E
520	—*Ende*	8 50 S	121 39 E
530	—*Larantuka*	8 21 S	122 59 E
74600	*Kalabhi, Pulau Alor*	8 12 S	124 31 E
610	*Waingapu, Sumba (Soemba)*	9 39 S	120 16 E
620	*Seba, Pulau Sawu*	10 29 S	121 51 E
630	*Baa Roadstead, Pulau Roti*	10 43 S	123 03 E
640	Pulau Semau: Lt	10 08 S	123 27 E
74700	TIMOR		
710	—*Kupang*	10 10 S	123 35 E
720	—*Atapupu*	9 00 S	124 52 E
730	—*Dili (Dilly) (Portugal)*	8 32 S	125 35 E
74800	**Moluccas**		
74810	Pulau Liran: Lt	8 03 S	125 44 E
820	Meatij Miarang, Pulau-pulau Sermata: Lt	8 20 S	128 29 E
830	*Tepa, Pulau Babar*	7 52 S	129 36 E
74900	PULAU-PULAU TANIMBAR		
910	—*Saumlakki*	7 59 S	131 18 E
920	—*Ritabel*	7 09 S	131 43 E
75000	*Dobo, Pulau-pulau Aru (Aroe Is.)*	5 45 S	134 13 E
010	*Elat, Pulau-pulau Ewab (Kai Is.)*	5 39 S	132 59 E
020	*Pulau Naira, Pulau-pulau Banda*	4 32 S	129 54 E

APPENDIX S

MARITIME POSITIONS

INDONESIA—Continued

Index No.	Place	Lat.	Long.
	Moluccas—Continued	° ′	° ′
75100	CERAM		
110	—*Geser*	3 53 S	130 54 E
120	—*Amahai*	3 20 S	128 55 E
130	—*Piru*	3 04 S	128 11 E
75200	Pulau Saparua: Lt	3 34 S	128 39 E
210	Amboina (Ambon I.): Lt	3 47 S	128 06 E
220	Pulau Suanggi: Lt	3 18 S	127 28 E
230	*Leksula, Buru (Boeroe I.)*	3 47 S	126 31 E
240	*Pulau Sanana, Pulau-pulau Sula*	2 03 S	125 59 E
250	*Labuha, Pulau Batjan*	0 38 S	127 28 E
260	*Laiwui, Obi*	1 20 S	127 38 E
75300	HALMAHERA		
310	—*Weda*	0 20 N	127 53 E
320	—*Buli-serani*	0 52 N	128 18 E
330	—*Kau*	1 10 N	127 55 E
340	—*Galela*	1 50 N	127 51 E
350	—*Wajabula*	2 16 N	128 12 E
75400	*Lirung, Pulau-pulau Talaud*	3 56 N	126 42 E
410	*Tahuna, Pulau-pulau Sangihe*	3 37 N	125 29 E
75500	**Celebes**		
75510	Tandjung Arus, Pulau Talise: Lt	1 53 N	125 05 E
520	Pulau Pondang: Lt	0 26 N	124 29 E
530	*Gorontalo*	0 30 N	123 03 E
540	Tomini Road	0 31 N	120 33 E
550	Selat Walea: Lt	0 25 S	122 25 E
560	Pulau Banggai: Lt	1 35 S	123 29 E
570	Pulau Wangiwangi: Lt	5 15 S	123 32 E
580	Tandjung Djenemedja, Teluk Bone: Lt	3 15 S	120 26 E
590	Pulau Pasitanete: Lt	5 45 S	120 30 E
75600	Pulau Sabalana (Postiljon I.): Lt	6 49 S	119 12 E
610	Taka Rewataje (De Bril): Lt	6 05 S	118 54 E
620	Pulau Dewakang-lompo: Lt	5 24 S	118 26 E
630	Pulau Dajangdajangan: Lt	5 24 S	119 11 E
640	*Makasar (Makassar)*	5 08 S	119 24 E
650	Pulau Kapoposang: Lt	4 42 S	118 57 E
660	Tandjung Rangasa (Huk Mandar): Lt	3 34 S	118 56 E
670	Tandjung Rangas (William Cape): Lt	2 38 S	118 49 E
680	Teluk Palu, Tandjung Karang: Lt	0 38 S	119 44 E
	Celebes—Continued	° ′	° ′
75690	Pulau Tuguan (North Watcher I.): Lt	0 35 N	119 48 E
75700	Tandjung Benar (Stroomen Kaap): Lt	1 20 N	120 48 E
710	Pulau Hulawa: Lt	0 58 N	122 54 E
720	*Manado (Menado)*	1 30 N	124 50 E
75800	**Borneo**		
75810	Tandjung Mangkalihat: Lt	0 59 N	118 59 E
820	Muaras (Moearas) Reef: Lt	1 46 N	119 02 E
830	*Tarakan (Linkas), Indonesia*	3 17 N	117 36 E
840	*Tawau, North Borneo*	4 15 N	117 53 E
850	Batu Tinagat: Lt	4 13 N	117 59 E
860	Tanjong Labian: Lt	5 09 N	119 13 E
870	Tanjong Trang: Lt	5 25 N	119 13 E
880	*Sandakan, North Borneo*	5 50 N	118 07 E
890	*Kudat, North Borneo*	6 53 N	116 51 E
75900	Pulau Kalampunian: Lt	7 03 N	116 45 E
910	Mantanani Is.: Lt	6 43 N	116 18 E
920	*Jesselton, North Borneo*	5 59 N	116 04 E
930	Pulau Papan: Lt	5 15 N	115 16 E
940	*Victoria, Labuan, North Borneo*	5 17 N	115 15 E
950	Pulau Karaman: Lt	5 14 N	115 08 E
960	*Brunei, Brunei*	4 53 N	114 56 E
970	Tanjong Baram: Lt	4 36 N	113 58 E
980	*Lutong, Sarawak*	4 28 N	114 00 E
990	*Miri, Sarawak*	4 23 N	113 58 E
76000	Tanjong Lobang: Lt	4 22 N	113 59 E
010	Tanjong Sirik: Lt	2 47 N	111 19 E
020	*Kuching, Sarawak*	1 34 N	110 21 E
030	Tanjong Po: Lt	1 43 N	110 31 E
040	Tanjong Datu: Lt	2 05 N	109 39 E
050	Pulau Murih (Saint Peters I.): Lt	1 54 N	108 40 E
060	Pulau Karimata	1 36 S	108 54 E
070	Pulau Serutu: Lt	1 43 S	108 42 E
080	Tandjung Selatan: Lt	4 11 S	114 39 E
090	Pulau Kunjit: Lt	4 05 S	116 02 E
76100	Dwaalder: Lt	4 14 S	116 07 E
110	The Brothers (Sambargalong Is.): Lt	4 24 S	116 10 E
120	*Kotabaru, Indonesia*	3 14 S	116 13 E
130	Buton Butona I.: Lt	3 39 S	116 36 E
140	Little Paternoster Is., Pulau Balabalangan: Lt	2 32 S	117 57 E
150	Aru Bank: Lt	2 15 S	116 40 E
160	*Balikpapan, Indonesia*	1 16 S	116 49 E

AUSTRALIA

Index No.	Place	Lat.	Long.
		° ′	° ′
77000	Sandy Cape: Lt	24 44 S	153 13 E
010	Double I. Point: Lt	25 56 S	153 13 E
020	Caloundra Head: Lt	26 49 S	153 08 E
030	*Brisbane*	27 28 S	153 02 E
040	Cape Moreton: Lt	27 02 S	153 28 E
050	Point Lookout: Lt	27 26 S	153 33 E
060	Fingal Head: Lt	28 11 S	153 35 E
070	Cape Byron: Lt	28 38 S	153 39 E
080	Richmond River, North Head: Lt	28 52 S	153 37 E
090	Clarence River, South Head: Lt	29 26 S	153 23 E
77100	South Solitary I.: Lt	30 12 S	153 16 E
110	*Coffs Harbour*	30 18 S	153 08 E
120	Smoky Cape: Lt	30 56 S	153 05 E
130	*Port Macquarie*	31 26 S	152 55 E
140	Tacking Point: Lt	31 29 S	152 57 E
150	Crowdy Head: Lt	31 51 S	152 45 E
160	Sugarloaf Point: Lt	32 26 S	152 33 E
170	*Port Stephens*	32 43 S	152 12 E
180	Nobbys Head: Lt	32 55 S	151 48 E
190	*Newcastle*	32 56 S	151 46 E
77200	Norah Head: Lt	33 17 S	151 35 E
210	Barrenjoey Head: Lt	33 35 S	151 20 E
220	*Sydney (Port Jackson)*	33 53 S	151 12 E
230	Hornby (Inner South Head): Lt	33 50 S	151 18 E
240	Macquarie (Outer South Head): Lt	33 51 S	151 17 E
		° ′	° ′
77250	*Wollongong*	34 25 S	150 54 E
260	*Port Kembla*	34 29 S	150 55 E
270	*Kiama*	34 40 S	150 52 E
280	Point Perpendicular: Lt	35 05 S	150 50 E
290	Warden Head: Lt	35 22 S	150 31 E
77300	Montagu I.: Lt	36 15 S	150 14 E
310	*Eden*	37 04 S	149 55 E
320	Green Cape: Lt	37 16 S	150 04 E
330	Gabo I.: Lt	37 34 S	149 55 E
340	Cape Everard: Lt	37 48 S	149 16 E
350	Cliffy I.: Lt	38 57 S	146 42 E
360	Wilson Promontory, SE Point: Lt	39 08 S	146 25 E
370	Citadel I.: Lt	39 07 S	146 14 E
380	Cape Liptrap: Lt	38 55 S	145 56 E
390	Westernport, Grant Point: Lt	38 31 S	145 07 E
77400	Cape Schanck: Lt	38 30 S	144 53 E
77500	PORT PHILLIP		
510	—*Melbourne*	37 49 S	144 57 E
520	—*Williamstown*	37 52 S	144 54 E
530	—*Geelong*	38 09 S	144 22 E
77600	Point Lonsdale: Lt	38 18 S	144 37 E
610	Split Point (Eagle Nest Point): Lt	38 28 S	144 06 E
620	Cape Otway: Lt	38 52 S	143 31 E
630	*Warrnambool*	38 24 S	142 28 E
640	Griffith I.: Lt	38 24 S	142 15 E
650	*Portland*	38 21 S	141 37 E
660	Cape Nelson: Lt	38 25 S	141 33 E

APPENDIX S

MARITIME POSITIONS

AUSTRALIA—Continued

Index No.	Place	Lat.	Long.	Index No.	Place	Lat.	Long.
		° ′	° ′			° ′	° ′
77670	Cape Northumberland: Lt	38 04 S	140 40 E	78350	Airlie I.: Lt	21 19 S	115 10 E
680	Cape Banks: Lt	37 54 S	140 23 E	360	North Sandy I.: Lt	21 06 S	115 39 E
690	Penguin Islet (Rivoli Bay): Lt	37 30 S	140 01 E	370	Legendre I.: Lt	20 21 S	116 51 E
77700	Cape Jaffa (Bernoulli): Lt	36 58 S	139 36 E	380	*Port Walcott*	20 39 S	117 11 E
710	*Victor Harbour*	35 34 S	138 37 E	390	*Port Hedland*	20 18 S	118 35 E
720	Cape Jervis: Lt	35 37 S	138 06 E	78400	Bedout I.: Lt	19 35 S	119 06 E
77800	Kangaroo Island			410	Cape Bossut: Lt	18 43 S	121 39 E
810	—Cape St. Alban: Lt	35 49 S	138 08 E	420	*Broome*	17 58 S	122 14 E
820	—Cape Willoughby: Lt	35 51 S	138 08 E	430	Gantheaume Point: Lt	17 58 S	122 11 E
830	—Cape Couedie: Lt	36 04 S	136 42 E	440	Cape Leveque: Lt	16 24 S	122 56 E
840	—Cape Borda: Lt	35 46 S	136 35 E	450	Adele I.: Lt	15 31 S	123 09 E
850	—Marsden Point: Lt	35 34 S	137 37 E	460	Browse I.: Lt	14 07 S	123 34 E
77900	*Glenelg*	35 01 S	138 30 E	470	*Wyndham*	15 28 S	128 06 E
910	*Port Adelaide*	34 51 S	138 30 E	480	Cape Fourcroy: Lt	11 47 S	130 02 E
920	*Wakefield*	34 12 S	138 09 E	490	Point Charles: Lt	12 24 S	130 38 E
930	Troubridge Shoal: Lt	35 08 S	137 51 E	78500	*Darwin*	12 28 S	130 51 E
940	Althorpe I.: Lt	35 23 S	136 51 E	510	East Vernon I.: Lt	12 05 S	131 06 E
950	Wedge I.: Lt	35 10 S	136 29 E	520	Cape Hotham: Lt	12 03 S	131 18 E
960	Corny Point: Lt	34 54 S	137 01 E	530	Cape Don: Lt	11 18 S	131 46 E
970	Wauraltee (Wardang) I.: Lt	34 30 S	137 21 E	540	Booby I.: Lt	10 36 S	141 55 E
980	Tipara Reef: Lt	34 04 S	137 24 E	550	Good's I.: Lt	10 33 S	142 09 E
990	Winceby I.: Lt	34 29 S	136 17 E	560	Wednesday I.: Lt	10 31 S	142 19 E
78000	Boston Point: Lt	34 39 S	135 56 E	570	Eborac I.: Lt	10 41 S	142 32 E
010	*Port Lincoln*	34 43 S	135 51 E	580	Albany Rock: Lt	10 43 S	142 38 E
020	Cape Donington: Lt	34 44 S	136 00 E	590	Hannibal Is.: Lt	11 36 S	142 56 E
030	Dangerous Reef: Lt	34 49 S	136 12 E	78600	Clerke I.: Lt	11 58 S	143 17 E
040	Neptune Isles, S. Neptune I.:			610	Piper Is.: Lt	12 15 S	143 15 E
	Lt	35 20 S	136 07 E	620	Chapman Reef: Lt	12 53 S	143 36 E
050	Four Hummocks, Whidbey Is.:			630	Heath Reef: Lt	13 29 S	143 41 E
	Lt	34 47 S	135 01 E	640	Hannah I.: Lt	13 52 S	143 43 E
060	Flinders I.: Lt	33 40 S	134 30 E	650	Wharton Reef: Lt	14 08 S	144 00 E
070	*Streaky Bay*	32 48 S	134 13 E	660	Pipon I.: Lt	14 08 S	144 31 E
080	*Thevenard*	32 10 S	133 39 E	670	Coquet I.: Lt	14 32 S	144 59 E
090	St. Francis I.: Lt	32 31 S	133 19 E	680	Palfrey Islet: Lt	14 42 S	145 27 E
78100	*Esperance*	33 52 S	121 54 E	690	Archer Point: Lt	15 36 S	145 20 E
110	Breaksea I.: Lt	35 04 S	118 04 E	78700	Low Isles: Lt	16 23 S	145 34 E
120	*Albany*	35 02 S	117 53 E	710	*Port Douglas*	16 29 S	148 28 E
130	Eclipse I.: Lt	35 11 S	117 53 E	720	*Cairns*	16 55 S	145 47 E
140	Cape Leeuwin: Lt	34 22 S	115 08 E	730	Fitzroy I.: Lt	16 55 S	146 00 E
150	Hamelin I.: Lt	34 13 S	115 01 E	740	North Barnard Is.: Lt	17 41 S	146 11 E
160	Cape Naturaliste: Lt	33 32 S	115 02 E	750	Brook Is.: Lt	18 09 S	146 19 E
170	*Busselton*	33 39 S	115 20 E	760	White Rock, Palm Isles: Lt	18 46 S	146 43 E
180	Casuarina Point: Lt	33 19 S	115 39 E	770	*Townsville*	19 16 S	146 49 E
190	*Bunbury*	33 19 S	115 38 E	780	Cape Cleveland: Lt	19 11 S	147 01 E
78200	Woodman Point: Lt	32 08 S	115 47 E	790	Cape Bowling Green: Lt	19 20 S	147 26 E
210	*Fremantle*	32 03 S	115 45 E	78800	*Bowen*	20 01 S	148 15 E
220	*Perth*	31 57 S	115 52 E	810	Eshelby I.: Lt	20 01 S	148 38 E
230	Bathurst Point, Rottnest I.: Lt	31 59 S	115 33 E	820	Dent I.: Lt	20 22 S	148 57 E
240	Escape I.: Lt	30 19 S	115 00 E	830	*Mackay*	21 07 S	149 13 E
250	Moore Point: Lt	28 47 S	114 35 E	840	Flat-top I.: Lt	21 09 S	149 16 E
260	*Geraldton*	28 47 S	114 36 E	850	Pine Islet, Percy Isles: Lt	21 39 S	150 14 E
270	Cape Inscription: Lt	25 29 S	112 58 E	860	High Peak I.: Lt	21 57 S	150 43 E
280	Babbage I.: Lt	24 52 S	113 38 E	870	North Reef: Lt	23 11 S	151 55 E
290	*Carnarvon*	24 53 S	113 39 E	880	*Rockhampton*	23 23 S	150 32 E
78300	Quobba Point, Beagle Hill: Lt	24 30 S	113 25 E	890	Cape Capricorn: Lt	23 29 S	151 15 E
310	Frazer Islet: Lt	22 38 S	113 38 E	78900	Gatcombe Head: Lt	23 53 S	151 23 E
320	Vlaming Head: Lt	21 49 S	114 07 E	910	*Gladstone*	23 51 S	151 15 E
330	Anchor I.: Lt	21 32 S	114 46 E	920	Bustard Head: Lt	24 01 S	151 47 E
340	*Onslow*	21 39 S	115 06 E	930	Lady Elliot Islet: Lt	24 07 S	152 43 E

TASMANIA

		° ′	° ′			° ′	° ′
79000	Deal I.: Lt	39 30 S	147 20 E	79220	—Currie Harbor: Lt	39 56 S	143 51 E
010	Goose I.: Lt	40 19 S	147 48 E	230	—Stokes Point: Lt	40 09 S	143 55 E
020	Swan I.: Lt	40 44 S	148 08 E	79300	West Point: Lt	40 58 S	144 38 E
030	Low Head, Tamar River: Lt	41 03 S	146 49 E	310	Sandy Cape: Lt	41 26 S	144 45 E
040	*Devonport*	41 10 S	146 24 E	320	*Macquarie Harbor*	42 10 S	145 20 E
050	Mersey Bluff: Lt	41 10 S	146 21 E	330	Cape Sorrell: Lt	42 11 S	145 10 E
060	*Burnie*	41 03 S	145 57 E	340	Maatsuyker Isles: Lt	43 40 S	146 18 E
070	Table Cape: Lt	40 57 S	145 45 E	350	Cape Bruny: Lt	43 30 S	147 09 E
080	Hyfield (Highfield) Point: Lt	40 44 S	145 17 E	360	*Hobart*	42 53 S	147 20 E
090	Cape Rochon, Three Hummock I.: Lt	40 24 S	144 57 E	370	Iron Pot I.: Lt	43 04 S	147 26 E
79100	Hunter I.: Lt	40 29 S	144 43 E	380	Tasman I.: Lt	43 14 S	148 02 E
79200	King Island			390	Cape Forestier: Lt	42 11 S	148 23 E
210	—Cape Wickham: Lt	39 36 S	143 57 E	79400	Eddystone Point: Lt	41 00 S	148 21 E

APPENDIX S

MARITIME POSITIONS

NEW ZEALAND

Index No.	Place	Lat.	Long.
80000	**South Island**	° ′	° ′
80010	Cape Campbell: Lt	41 43 S	174 17 E
020	*Kaikoura*	42 24 S	173 41 E
030	Point Kean: Lt	42 25 S	173 43 E
040	Godley (Cachalot) Head: Lt	43 35 S	172 48 E
050	*Lyttelton*	43 36 S	172 43 E
060	Le Bon's Bay: Lt	43 45 S	173 09 E
070	*Akaroa*	43 48 S	172 58 E
080	*Timaru*	44 24 S	171 15 E
090	Jacks Point: Lt	44 27 S	171 17 E
80100	*Oamaru*	45 07 S	170 59 E
110	Moeraki: Lt	45 24 S	170 52 E
120	Heyward Point: Lt	45 45 S	170 41 E
130	*Port Chalmers*	45 49 S	170 38 E
140	*Dunedin*	45 53 S	170 31 E
150	Otago Harbor, Taiaroa Head: Lt	45 47 S	170 45 E
160	Cape Saunders: Lt	45 53 S	170 44 E
170	Nugget Point: Lt	46 27 S	169 50 E
180	Waipapa Point: Lt	46 40 S	168 52 E
190	Dog I.: Lt	46 40 S	168 26 E
80200	Stewart Island		
210	—Akers Point: Lt	46 54 S	168 11 E
220	—*Port Pegasus*	47 13 S	167 42 E
80300	*Bluff Harbor*	46 36 S	168 21 E
310	*Invercargill*	46 25 S	168 21 E
320	Centre I.: Lt	46 28 S	167 52 E
330	Puysegur Point: Lt	46 10 S	166 37 E
340	St. Anns Point: Lt	44 34 S	167 46 E
350	*Hokitika*	42 43 S	170 59 E
360	*Greymouth*	42 26 S	171 13 E
370	Cape Foulwind: Lt	41 45 S	171 28 E
380	*Westport*	41 45 S	171 36 E
390	Kahurangi Point: Lt	40 46 S	172 13 E
80400	Cape Farewell: Lt	40 30 S	172 43 E
410	Bush End Point: Lt	40 33 S	173 02 E
420	*Nelson*	41 16 S	173 16 E
430	Stephens I.: Lt	40 40 S	174 01 E
440	Cape Jackson: Lt	40 59 S	174 20 E
450	The Brothers: Lt	41 06 S	174 27 E

Index No.	Place	Lat.	Long.
80500	**North Island**	° ′	° ′
80510	Ohau Point: Lt	41 14 S	174 39 E
520	Karori Rock: Lt	41 21 S	174 41 E
530	*Wellington (Port Nicholson)*	41 17 S	174 47 E
540	Pencarrow Head: Lt	41 21 S	174 51 E
550	Baring Head: Lt	41 24 S	174 52 E
560	Cape Palliser: Lt	41 37 S	175 19 E
570	Castle Point (Rangiwha Raoma): Lt	40 55 S	176 14 E
580	Cape Kidnappers: Lt	39 38 S	177 06 E
590	*Napier Harbor*	39 29 S	176 55 E
80600	Portland I. (Te Houra): Lt	39 18 S	177 53 E
610	*Gisborne*	38 40 S	178 02 E
620	Tuahina Point: Lt	38 42 S	178 04 E
630	Gable Islet: Lt	38 32 S	178 18 E
640	East Cape (Otiki): Lt	37 41 S	178 33 E
650	Matakaoa Point: Lt	37 33 S	178 19 E
660	Ohena I.: Lt	36 44 S	175 53 E
670	Cuvier I.: Lt	36 26 S	175 47 E
680	*Thames*	37 09 S	175 33 E
690	*Auckland*	36 51 S	174 46 E
80700	Tiritiri I.: Lt	36 36 S	174 54 E
710	Flat Rock: Lt	36 27 S	174 56 E
720	Burgess Islet: Lt	35 54 S	175 07 E
730	Moro Tiri: Lt	35 53 S	174 47 E
740	*Whangarei*	35 45 S	174 20 E
750	Sugarloaf: Lt	35 52 S	174 31 E
760	Cape Brett: Lt	35 10 S	174 20 E
770	*Whangaroa*	35 03 S	173 46 E
780	North Cape: Lt	34 25 S	173 04 E
790	Cape Reinga: Lt	34 26 S	172 40 E
80800	Cape Maria van Diemen: Lt	34 29 S	172 38 E
810	Kaipara, North Head: Lt	36 23 S	174 08 E
820	Manukau, South Head: Lt	37 04 S	174 32 E
830	*New Plymouth*	39 04 S	174 02 E
840	Mikotahi I.: Lt	39 03 S	174 02 E
850	Cape Egmont: Lt	39 17 S	173 46 E
860	*Patea*	39 47 S	174 30 E
870	*Wanganui*	39 57 S	175 02 E
880	Kapiti I.: Lt	40 50 S	174 58 E

EAST COAST OF ASIA

Index No.	Place	Lat.	Long.
81000	**Malaya**	° ′	° ′
81010	Tanjong Tenggaroh: Lt	2 15 N	103 59 E
020	Mersing: Lt	2 27 N	103 49 E
030	Sungei Pahang: Lt	3 32 N	103 29 E
040	*Kuantan*	3 48 N	103 20 E
050	Pulau Tenggol: Lt	4 49 N	103 42 E
060	*Kuala Trengganu*	5 21 N	103 08 E
070	Tumpat: Lt	6 12 N	102 12 E
81100	**Thailand**		
81110	Khlong Sai Buri (Taluban River): Lt	6 42 N	101 40 E
120	Laem Pho (Lem Tachee): Lt	6 57 N	101 18 E
130	Songkhla: Lt	7 13 N	100 37 E
140	Laem Talumphuk: Lt	8 28 N	100 13 E
150	Ko Prap (Goh Prab): Lt	9 14 N	99 26 E
160	Ko Tawan Tok (Goh Wang Nai): Lt	9 17 N	99 54 E
170	Pak Nam Lang Suan: Lt	9 57 N	99 09 E
180	Ko Rang: Lt	10 49 N	99 30 E
190	Ko Ract (Go Rad): Lt	11 48 N	99 50 E
81200	*Krung Thep (Bangkok)*	13 45 N	100 30 E
210	Ko Sampayu: Lt	13 11 N	100 48 E
220	Ko Phai (Goh Pai): Lt	12 56 N	100 40 E
230	Ko Chuang: Lt	12 31 N	100 58 E
240	Ko Samet: Lt	12 35 N	101 27 E
250	Ko Man Nok: Lt	12 34 N	101 42 E
260	Laem Sing: Lt	12 28 N	102 04 E
270	Ko Chik Nok: Lt	12 18 N	102 14 E
280	Laem Sok (Lem Nam): Lt	12 03 N	102 35 E
81300	**Cambodia**		
81305	*Sihanoukville (Kompong Som)*	10 39 N	103 30 E
81310	*Ream*	10 30 N	103 36 E
320	*Duong Dong*	10 13 N	103 58 E

Index No.	Place	Lat.	Long.
81400	**Vietnam**	° ′	° ′
81410	*Ha Tien*	10 23 N	104 29 E
420	Île Poulo Obi: Lt	8 26 N	104 50 E
430	*Poulo Condore*	8 41 N	106 36 E
440	Hon Bai Can: Lt	8 40 N	106 42 E
450	Song Cua Tieu Entrance: Lt	10 15 N	106 47 E
460	*Saigon*	10 46 N	106 42 E
470	*Vung Tau (Cap Saint-Jacques)*	10 21 N	107 04 E
480	Pointe de Ke Ga: Lt	10 42 N	107 59 E
490	*Phan Thiet*	10 55 N	108 07 E
81500	Cap Padaran: Lt	11 22 N	109 01 E
510	Baie de Phan Rang: Lt	11 35 N	109 03 E
520	Hon Chut: Lt	11 47 N	109 13 E
530	Île Tre (Mui Rachtrang): Lt	12 12 N	109 20 E
540	*Nha Trang*	12 15 N	109 11 E
550	Cap Varella: Lt	12 54 N	109 27 E
560	Poulo Gambir: Lt	13 37 N	109 22 E
570	*Qui Nhon*	13 46 N	109 14 E
580	Vung Moi: Lt	14 15 N	109 11 E
590	Cu Lao Re: Lt	15 23 N	109 08 E
81600	Hsi-sha Ch'ün-tao (Paracel Islands) (China)		
610	—Shan-hu Tao (Pattle I.): Lt	16 32 N	111 36 E
620	—Shih Tao (Rocky I.): Lt	16 51 N	112 20 E
81700	Presqu'île de Tien Sha: Lt	16 08 N	108 19 E
710	*Da Nang (Tourane)*	16 04 N	108 13 E
720	*Ben Thuy*	18 39 N	105 42 E
730	Île de Bien Son: Lt	19 20 N	105 49 E
740	*Hanoi*	21 02 N	105 50 E
750	Hon Dau: Lt	20 40 N	106 49 E
760	*Haiphong*	20 52 N	106 41 E
770	Îles Norway: Lt	20 37 N	107 09 E
780	*Cam Pha*	21 02 N	107 22 E

APPENDIX S

MARITIME POSITIONS

EAST COAST OF ASIA—Continued

Index No.	Place	Lat.	Long.
81800	**China**	° ′	° ′
81810	*Pei-hai* (*Pakhoi*)	21 29 N	109 06 E
820	Wei-chou Tao: Lt	21 00 N	109 06 E
830	Teng-lou Chiao (Cape Cami): Lt	20 13 N	109 55 E
81900	HAI-NAN TAO (HAINAN)		
910	—Lin-kao Chiao (Lamko): Lt	20 00 N	109 42 E
920	—Yü-lin	18 11 N	109 31 E
930	—Hai-nan Tsui: Lt	20 10 N	110 41 E
940	—*Hai-k'ou*	20 03 N	110 20 E
82000	Nao Chou (Île Nao-chow): Lt	20 54 N	110 36 E
010	Aigrettes I., Colline Verte: Lt	21 06 N	110 33 E
020	*Chan-chiang*	21 12 N	110 24 E
030	Tien-pai (Tien Pak)	21 31 N	111 18 E
82100	**Macao**		
82110	Ponta de Ká-hó: Lt	22 08 N	113 35 E
120	*Macao*	22 12 N	113 33 E
82200	**China**		
82210	*Kuang-chou* (*Canton*)	23 06 N	113 14 E
220	Wan-shan Ch'ün-tao (Ladrone Is.), Chu Chou: Lt	22 00 N	113 49 E
230	Wen-wei Chou (Gap Rock): Lt	21 49 N	113 56 E
82300	**Hong Kong**		
82310	Tsing Chau (Green I.): Lt	22 17 N	114 07 E
320	*Hong Kong*	22 18 N	114 10 E
330	Tung Lung (Lamtong I.), Tathong Point: Lt	22 14 N	114 17 E
340	Wang Lan: Lt	22 11 N	114 18 E
82400	**China**		
82410	Ta-hsing-tsan Yen (Pedro Blanco)	22 19 N	115 07 E
420	Che-lang Chiao: Lt	22 39 N	115 34 E
430	Lien-hua-feng Chiao (Breaker Pt.): Lt	22 56 N	116 30 E
440	Piao Chiao (Cape Good Hope): Lt	23 14 N	116 48 E
450	Lu Hsü (Sugarloaf I.): Lt	23 20 N	116 46 E
460	*Shan-t'ou* (*Swatow*)	23 22 N	116 41 E
470	Nan-p'eng Ch'ün-tao (High Lamock I.): Lt	23 16 N	117 17 E
480	*Hsia-men Tao* (*Amoy*)	24 27 N	118 04 E
82500	**Taiwan (Formosa)**		
82510	Pai-sha-t'un (Hakushatou): Lt	25 03 N	121 04 E
520	Tan-shui Ho-k'ou (Tansui Harbor): Lt	25 11 N	121 25 E
530	Fu-kuei Chiao (Fukikaku): Lt	25 18 N	121 32 E
540	P'eng-chia Hsü (Hoka Sho) (Agincourt I.): Lt	25 38 N	122 04 E
550	Wan-jen-t'ui Pi (Banjintai Bi): Lt	25 09 N	121 44 E
560	*Chi-lung Chiang* (*Keelung*) (*Kii-run Ko*)	25 08 N	121 44 E
570	San-tao Chao (Sancho-Kaku): Lt	25 01 N	122 00 E
580	*Su-ao*	24 36 N	121 51 E
590	*Hua-lien Shih* (*Karen Ko*)	23 59 N	121 36 E
82600	San-hsien T'ai (Sansendai I.)	23 08 N	121 25 E
610	T'ai-tung (Taito Ko): Lt	22 45 N	121 09 E
620	Huo-shao Tao (Kasho-To): Lt	22 41 N	121 28 E
630	Hung-t'ou Hsü (Koto Sho)	22 03 N	121 32 E
640	O-luan-Pi (Garan Bi): Lt	21 54 N	120 51 E
650	Liu-chiu Hsü (Ryukyu Sho): Lt	22 20 N	120 22 E
660	*Kao-hsiung Shih* (*Takao Ko*)	22 38 N	120 16 E
670	An-p'ing: Lt	23 00 N	120 09 E
82700	P'ENG-HU LIEH-TAO (PESCADORES ISLANDS)		
710	—Tung-chi Hsü: Lt	23 16 N	119 40 E
720	—Hua Hsü: Lt	23 24 N	119 19 E
730	—*P'eng-hu Tao* (*Boko Ko*)	23 34 N	119 33 E
740	—Yü-weng Tao (Kissi): Lt	23 34 N	119 27 E
750	—Mu-tou Hsü (Mokuto Sho): Lt	23 47 N	119 36 E
760	—Ch'a-mu Hsü (Sabo Sho): Lt	23 32 N	119 43 E
82800	**China**		
82810	Chin-men Tao (Quemoy I.)	24 28 N	118 25 E
820	Pei-ting Tao (Dodd I.): Lt	24 26 N	118 30 E

Index No.	Place	Lat.	Long.
82800	**China—Continued**	° ′	° ′
830	Wu-ch'iu Hsü (Ockseu Is.): Lt	24 59 N	119 27 E
840	Niu-shan Tao (Turnabout Is.): Lt	25 26 N	119 56 E
850	Tung-ch'üan Tao (Tungkuen I.): Lt	25 58 N	119 59 E
860	*Fu-chou* (*Foochow*)	26 04 N	119 18 E
870	Ma-tsu Shan (Matsu I.)	26 09 N	119 55 E
880	Tung-yin Shan (Tung Yung): Lt	26 22 N	120 30 E
890	Ch'ih-chu Tao (Spider I.): Lt	26 31 N	120 04 E
82900	Yin-k'ou-kou Lieh-tao (Incog Is.): Lt	26 59 N	120 28 E
910	Tung-kua Hsu (Shroud I.): Lt	27 38 N	121 03 E
920	Pei-yü Shan: Lt	28 53 N	122 16 E
930	Tung-t'ing Shan: Lt	29 52 N	122 35 E
940	Lo-chia Shan (Loka I.): Lt	29 58 N	122 27 E
950	Hsiao-pan Tao (Steep I.): Lt	30 13 N	122 35 E
960	Hsia-san-hsing Shan (Elgar I.): Lt	30 26 N	122 31 E
970	Pai-chieh Shan (Bonham I.): Lt	30 37 N	122 25 E
980	Pan-yang Shan (Button Rock): Lt	30 38 N	122 22 E
990	Ta-ch'i Shan (Gutzlaff I.): Lt	30 49 N	122 10 E
83000	Hua-niao Shan (North Saddle I.): Lt	30 52 N	122 40 E
010	She Shan (Shaweishan I.): Lt	31 25 N	122 14 E
020	*Shang-hai*	31 15 N	121 30 E
030	*Han-k'ou*	30 35 N	114 17 E
040	Ch'in-shan Tao: Lt	35 00 N	119 49 E
050	*Ch'ing-tao* (*Tsing-Tao*)	36 05 N	120 19 E
060	Ch'ao-lien Tao (Ts'ang-chou): Lt	35 54 N	120 52 E
070	Mu-yeh Tao (Muitao I.) (Southeast Promontory): Lt	36 54 N	122 32 E
080	Ch'eng-shan Chiao (Shantung Promontory): Lt	37 24 N	122 42 E
090	*Wei-hai-wei*	37 30 N	122 07 E
83100	K'ung-t'ung Tao: Lt	37 34 N	121 32 E
110	*Yen-t'ai* (*Chefoo*)	37 32 N	121 24 E
120	Hou-chi Tao (Miaotao): Lt	38 04 N	120 39 E
130	Mu-chi-tao Chiao (Chimatao Promontory): Lt	37 41 N	120 13 E
140	*T'ang-ku* (*Taku Bar*)	39 00 N	117 42 E
150	*Ch'in-huang-tao* (*Chinwantao*)	39 56 N	119 36 E
160	*Hu-lu-tao*	40 43 N	121 00 E
170	*Ying-k'ou*	40 41 N	122 14 E
180	Wu-tao-kou Tsui-tzu: Lt	39 32 N	121 13 E
190	Hsiao-lung-shan Tao: Lt	38 57 N	120 58 E
83200	Lao-t'ieh Shan (Rotetsu San): Lt	38 44 N	121 08 E
210	Lao-hu-wei Shan (Rokobi): Lt	38 48 N	121 15 E
220	*Lü-shun* (*Port Arthur*)	38 48 N	121 15 E
230	Hsi-k'ou Chiao (Kohahu Shi): Lt	38 54 N	121 43 E
240	*Ta-lien* (*Dairen*)	38 55 N	121 38 E
250	Ta-san-shan Tao: Lt	38 52 N	121 49 E
260	Hai-yang Tao: Lt	39 05 N	123 08 E
270	Ta-wang-chia Tao: Lt	39 26 N	123 05 E
280	Ta-lu Tao: Lt	39 45 N	123 45 E
290	*An-tung*	40 07 N	124 23 E
83300	**Korea**		
83310	Yalu River Entrance, Suun Do (Suiun To): Lt	39 41 N	124 25 E
320	Taehwa Do (Daiwa To): Lt	39 26 N	124 35 E
330	*Chinnamp'o*	38 43 N	125 24 E
340	Taedong Gang, Chamae Do (Shimai To): Lt	38 41 N	124 59 E
350	Sŏ Do: Lt	38 33 N	124 46 E
360	Soch'ŏng Do (Shōsei To): Lt	37 46 N	124 44 E
370	Sŏnmi Do: Lt	37 17 N	126 05 E
380	*Inch'ŏn* (*Jinsen*) (*Chemulpo*)	37 28 N	126 37 E
390	An Do (An To): Lt	36 58 N	126 10 E
83400	Moktŏk To (Mokutoku To): Lt	36 56 N	125 47 E
410	Tonggyŏngnyŏlbi Do: Lt	36 37 N	125 34 E
420	Ong Do (Ō To): Lt	36 39 N	126 00 E
430	Ŏch'ŏng Do (Osei To): Lt	36 08 N	125 58 E
440	*Kunsan Hang*	35 59 N	126 43 E
450	Mal To (Matsu To): Lt	35 52 N	126 19 E
460	Taerorok To (Dairoruko To): Lt	35 06 N	125 59 E

APPENDIX S

MARITIME POSITIONS

EAST COAST OF ASIA—Continued

Index No.	Place	Lat.	Long.
	Korea—Continued	° ′	° ′
83470	*Mokp'o*	34 47 N	126 23 E
480	Ch'ilbal To (Shichihatsu To): Lt	34 47 N	125 47 E
490	Hong Do: Lt	34 43 N	125 12 E
83500	Sohŭksan Do (Kokuzan To): Lt	34 06 N	125 06 E
510	Chuk To: Lt	34 13 N	125 51 E
520	Hajo Do (Kacho Tō): Lt	34 19 N	126 05 E
530	Sangch'uja Do: Lt	33 57 N	126 18 E
540	Mara Do: Lt	33 07 N	126 16 E
550	*Cheju (Saishu)*	33 31 N	126 32 E
560	U Do (Gyu To): Lt	33 29 N	126 58 E
570	Chaji Do (Shashi To): Lt	34 06 N	126 36 E
580	Kŏmun Do (Kyobun To): Lt	34 00 N	127 19 E
590	Habaek To: Lt	34 03 N	127 35 E
83600	Sori Do (Shori To): Lt	34 25 N	127 48 E
610	*Yŏsu*	34 44 N	127 45 E
620	Somaemul To: Lt	34 37 N	128 33 E
630	Hong Do (Kō Tō): Lt	34 32 N	128 44 E
640	Kadŏk To: Lt	34 59 N	128 50 E
650	Mok To (Makino Shima): Lt	35 03 N	129 06 E
660	*Pusan*	35 06 N	129 03 E
670	Kanjŏl Gap (Konzetsu Ko): Lt	35 21 N	129 22 E
680	Ul Gi (Uru Saki): Lt	35 29 N	129 27 E
690	Changgi Gap (Choki Ko): Lt	36 05 N	129 34 E
83700	*P'ohang Dong*	36 02 N	129 22 E
710	Chukpyŏn Man (Yonshu Gap): Lt	37 03 N	129 26 E
720	Suwŏn Dan (Suigen Tan): Lt	38 41 N	128 22 E
730	*Wŏnsan Hang (Gensan Ko)*	39 10 N	127 26 E
740	*Hŭngnam*	39 50 N	127 37 E
750	*Sinp'o*	40 02 N	128 12 E
760	*Sŏngjin*	40 40 N	129 12 E
770	Musu Dan (Busui Tan): Lt	40 50 N	129 43 E
780	Ŏrang Dan (Gyoro Tan): Lt	41 23 N	129 48 E
790	*Ch'ŏngjin*	41 47 N	129 50 E
83800	*Najin*	42 14 N	130 18 E
810	*Unggi*	42 20 N	130 24 E
83900	**USSR**		
83910	Mys Gamova: Lt	42 33 N	131 13 E
920	Ostrov Rimskogo-Korsakova: Lt	42 40 N	131 28 E
930	Mys Bryusa: Lt	42 53 N	131 28 E
940	*Vladivostok*	43 07 N	131 54 E
950	Ostrov Skrypleva: Lt	43 02 N	131 57 E
960	Ostrov Askol'd: Lt	42 44 N	132 20 E
970	*Nakhodka*	42 49 N	132 54 E
980	Mys Povorotnyy: Lt	42 40 N	133 03 E
990	Mys Ostrovnoy: Lt	42 48 N	133 44 E
84000	Mys Nizmennyy: Lt	43 31 N	135 09 E
010	*Ol'ga*	43 44 N	135 17 E
020	Mys Yegorova: Lt	44 47 N	136 26 E
030	Mys Belkina (Cape Disappointment): Lt	45 50 N	137 41 E
040	Mys Zolotoy: Lt	47 19 N	138 59 E
050	Mys Peschanyy: Lt	48 27 N	140 11 E
060	Mys Krasnyy Partizan: Lt	48 58 N	140 23 E
070	Sovetskaya Gavan'	48 58 N	140 17 E
080	Mys Syurkum: Lt	50 06 N	140 42 E
090	Mys Kloster-Kamp: Lt	51 26 N	140 53 E
84100	*De-Kastri*	51 28 N	140 47 E
84200	**Sakhalin**		
84210	*Aleksandrovsk*	50 54 N	142 08 E
220	Mys Lamanon (Chirai Misaki): Lt	48 47 N	141 51 E

Index No.	Place	Lat.	Long.
84200	**Sakhalin**	° ′	° ′
230	Mys Slepikovskogo (Konotoro Misaki) (Naka-notoro Misaki): Lt	47 18 N	141 59 E
240	*Kholmsk (Maoka)*	47 03 N	142 03 E
250	Ostrov Moneron (Kaiba Tō): Lt	46 15 N	141 16 E
260	Mys Kril'on (Nishi-notoro Misaki) (Cape Notoro): Lt	45 54 N	142 05 E
270	*Korsakov (Ōtomari)*	46 38 N	142 46 E
280	Skala Sivuch'ya (Gojo Iwa): Lt	46 02 N	143 24 E
290	Mys Svobodnyy (Airō Misaki): Lt	46 51 N	143 26 E
84300	*Vostochnyy (Motodomari)*	48 16 N	142 38 E
310	Mys Terpeniya (Kita-shiretoko Misaki): Lt	48 39 N	144 45 E
320	Mys Yelizavety: Lt	54 25 N	142 43 E
330	Mys Marii: Lt	54 18 N	142 15 E
84400	**USSR**		
84410	*Nikolayevsk*	53 09 N	140 44 E
420	Mys Men'shikova: Lt	53 18 N	141 25 E
430	*Okhotsk*	59 22 N	143 12 E
440	Ostrov Spafar'yeva: Lt	59 10 N	149 06 E
450	*Nagayevo*	59 34 N	150 43 E
460	Ostrov Zav'yalova: Lt	58 58 N	150 27 E
470	*Nayakhan*	61 54 N	159 00 E
480	Ust'-Bol'sheretsk	52 50 N	156 17 E
490	Mys Lopatka: Lt	50 52 N	156 40 E
84500	Mys Povorotnyy: Lt	52 19 N	158 35 E
510	*Petropavlovsk*	53 01 N	158 39 E
520	Mys Mayachnyy (Dalni Pt.): Lt	52 53 N	158 43 E
530	Mys Shipunskiy: Lt	53 06 N	160 00 E
540	Mys Kronotskiy: Lt	54 45 N	162 10 E
550	*Ust'-Kamchatsk*	56 13 N	162 25 E
560	Komandorskiye Ostrova, Ostrov Beringa (Bering I.)	54 54 N	166 24 E
570	Mys Afrika: Lt	56 10 N	163 19 E
580	Mys Navarin: Lt	62 15 N	179 08 E
590	Mys Barykova: Lt	63 03 N	179 28 E
84600	Mys Geka: Lt	64 25 N	178 15 E
610	*Anadyr'*	64 44 N	177 30 E
620	Bukhta Provideniya, Mys Lesovskogo: Lt	64 20 N	173 30 W
630	Mys Chaplina: Lt	64 24 N	172 14 W
640	Mys Kygynin: Lt	64 45 N	172 04 W
650	Mys Nygligan: Lt	65 04 N	172 06 W
660	Mys Krigugon: Lt	65 29 N	171 03 W
84700	**Kuril Islands**		
84710	Mys Kurbatova (Kokutan Zaki): Lt	50 52 N	156 29 E
720	Imai Saki, Vtoroy Kuril'skiy Proliv (Paramushiru Kaikyo): Lt	50 46 N	156 12 E
730	Mys Vasil'yeva (Kurabu Saki): Lt	50 00 N	155 24 E
740	Ostrov Toporkova (Iwaki Jima) (Banjō Tō): Lt	48 05 N	153 18 E
750	Ostrov Simushir (Shimushiru Tō): Lt	46 52 N	151 49 E
760	Ostrov Urup (Uruppu Tō): Lt	46 14 N	150 21 E
770	Ostrov Hurup, Kasatka (Etorofu Tō, Toshimoe): Lt	45 00 N	147 44 E
780	Mys Lovtsova (Atoiya Misaki): Lt	44 27 N	146 34 E
790	*Yuzhno-Kuril'sk (Furukamappu)*	44 01 N	145 51 E
84800	Shikotan Tō: Lt	43 50 N	146 55 E
810	*Shakotan*	43 52 N	146 50 E

JAPAN

Index No.	Place	Lat.	Long.
85000	**Hokkaidō**	° ′	° ′
85010	*Nemuro*	43 20 N	145 35 E
020	Nosappu Saki: Lt	43 23 N	145 49 E
030	*Hanasaki*	43 17 N	145 35 E
040	Ochiishi Saki: Lt	43 10 N	145 31 E
050	*Akkeshi*	43 03 N	144 51 E
060	*Kushiro*	42 59 N	144 22 E
070	Erimo Saki: Lt	41 55 N	143 15 E
080	Urakawa Kō: Lt	42 11 N	142 47 E

Index No.	Place	Lat.	Long.
	Hokkaidō—Continued	° ′	° ′
090	Chikyū Misaki: Lt	42 18 N	141 00 E
85100	*Muroran*	42 20 N	140 59 E
110	Esan Saki: Lt	41 49 N	141 11 E
120	Shiokubi Misaki: Lt	41 43 N	140 58 E
85130	*Hakodate*	41 47 N	140 43 E
140	Kattoshi Misaki: Lt	41 44 N	140 36 E
150	Shirakami Saki: Lt	41 24 N	140 12 E

APPENDIX S

MARITIME POSITIONS

JAPAN—Continued

Index No.	Place	Lat.	Long.
85200	**Honshū**	° ′	° ′
85210	Tappi Saki: Lt	41 15 N	140 21 E
220	Tairadate: Lt	41 10 N	140 39 E
230	*Aomori*	40 50 N	140 45 E
240	Ōma Saki: Lt	41 33 N	140 55 E
250	Shiriya Saki: Lt	41 26 N	141 28 E
260	Todo Saki: Lt	39 33 N	142 05 E
85270	*Kamaishi*	39 16 N	141 53 E
280	Kinkazan Tō: Lt	38 16 N	141 35 E
290	Shioya Misaki: Lt	37 00 N	140 59 E
85300	*Chōshi*	35 44 N	140 51 E
310	Inubō Saki: Lt	35 42 N	140 52 E
320	Katsuura Wan: Lt	35 08 N	140 19 E
330	Nojima Zaki: Lt	34 54 N	139 53 E
340	Suno Saki: Lt	34 58 N	139 46 E
350	*Tateyama*	34 59 N	139 52 E
360	Daini Kaihō (Fort No. 2): Lt	35 19 N	139 45 E
370	*Tōkyō*	35 40 N	139 45 E
380	Haneda Su: Lt	35 32 N	139 48 E
390	*Yokohama*	35 27 N	139 35 E
85400	*Yokosuka*	35 17 N	139 40 E
410	Daisan Kaihō (Fort No. 3): Lt	35 17 N	139 44 E
420	Kannon Zaki: Lt	35 15 N	139 45 E
430	*Uraga*	35 14 N	139 43 E
440	Ajika Jima (Ashika Shima): Lt	35 13 N	139 44 E
450	Tsurugi Saki: Lt	35 08 N	139 41 E
85500	Ō Shima		
510	—Kazahaya Zaki: Lt	34 48 N	139 22 E
520	—Habu Kō: Lt	34 41 N	139 26 E
85600	Mikomoto Jima: Lt	34 34 N	138 57 E
610	Irō Saki: Lt	34 36 N	138 51 E
620	*Shimizu*	35 01 N	138 29 E
630	Fukiaino Misaki: Lt	35 00 N	138 32 E
640	Omae (Omai) Zaki: Lt	34 36 N	138 14 E
650	Kaketsuka: Lt	34 39 N	137 49 E
660	Irago Zaki: Lt	34 35 N	137 01 E
670	*Taketoyo*	34 51 N	136 56 E
680	Fugu Saki: Lt	34 45 N	136 51 E
690	*Nagoya*	35 05 N	136 54 E
85700	*Yokkaichi*	34 57 N	136 38 E
710	Kami Jima: Lt	34 33 N	136 59 E
720	*Toba*	34 29 N	136 51 E
730	Daiō Zaki: Lt	34 16 N	136 54 E
740	Ko Shima: Lt	34 14 N	136 49 E
750	Miki Zaki: Lt	33 58 N	136 16 E
760	Kandori (Kantori) Zaki: Lt	33 35 N	135 58 E
770	Kashino Zaki: Lt	33 28 N	135 52 E
780	Shiono Misaki: Lt	33 26 N	135 45 E
790	Ichie Zaki: Lt	33 35 N	135 24 E
85800	Hino Misaki: Lt	33 53 N	135 04 E
810	*Wakayama*	34 13 N	135 09 E
820	Tomoga Shima (Okino Shima): Lt	34 17 N	135 00 E
85900	**Naikai (Inland Sea)**		
85910	*Ōsaka, Honshū*	34 39 N	135 26 E
920	*Kōbe, Honshū*	34 39 N	135 11 E
930	Wada Misaki: Lt	34 39 N	135 11 E
940	Akashi Kō: Lt	34 38 N	134 59 E
950	*Itozakichō, Honshū*	34 23 N	133 07 E
960	*Kure, Honshū*	34 14 N	132 33 E
970	*Hiroshima, Honshū*	34 21 N	132 28 E
980	*Tokuyama, Honshū*	34 02 N	131 49 E
990	*Shimonoseki, Honshū*	33 57 N	130 56 E
86000	*Moji, Kyūshū*	33 57 N	130 58 E
010	He Zaki, Shimonoseki Kaikyō: Lt	33 57 N	131 02 E
020	*Beppu, Kyūshū*	33 16 N	131 31 E
030	Jizō Misaki (Seki Saki): Lt	33 16 N	131 54 E
040	Sada Misaki: Lt	33 20 N	132 01 E
050	*Imabari, Shikoku*	34 04 N	133 00 E
060	*Takamatsu, Shikoku*	34 20 N	134 03 E
070	E Saki: Lt	34 36 N	134 59 E
080	*Sumoto, Awaji Shima*	34 21 N	134 54 E
86100	**Shikoku**		
86110	*Komatsushima*	34 01 N	134 36 E
120	Gamōda Misaki (Kamata Saki): Lt	33 50 N	134 45 E
130	Kannoura Kō: Lt	33 32 N	134 18 E
140	Muroto Zaki: Lt	33 15 N	134 11 E
150	*Kōchi*	33 30 N	133 34 E
160	Ashizuri Zaki: Lt	32 43 N	133 01 E
170	Mizunoko Shima: Lt	33 02 N	132 11 E

Index No.	Place	Lat.	Long.
86200	**Kyūshū**	° ′	° ′
86210	Toi Misaki: Lt	31 22 N	131 21 E
220	Sata Misaki: Lt	30 59 N	130 40 E
86230	*Kagoshima*	31 35 N	130 34 E
240	Bōno Misaki: Lt	31 15 N	130 13 E
250	Tsurikake Zaki: Lt	31 37 N	129 42 E
260	Nagasaki Bana: Lt	32 07 N	130 07 E
270	Gotsū Iwa: Lt	32 34 N	130 07 E
280	*Misumi*	32 36 N	130 28 E
290	*Miike*	33 00 N	130 25 E
86300	*Shimabara*	32 47 N	130 23 E
310	Nomo Saki: Lt	32 33 N	129 47 E
320	Iō Shima: Lt	32 43 N	129 46 E
330	*Nagasaki*	32 45 N	129 53 E
340	Ō-date Shima: Lt	33 01 N	129 26 E
350	*Sakito*	33 01 N	129 34 E
360	Shira Se: Lt	33 05 N	129 38 E
370	*Sasebo*	33 10 N	129 43 E
380	Kōgo Zaki: Lt	33 06 N	129 40 E
390	Shimo-kareki Shima: Lt	33 12 N	129 30 E
86400	Ogami Shima: Lt	33 11 N	129 20 E
410	Danjo Guntō, Me Shima: Lt	31 59 N	128 21 E
86500	Gotō Rettō		
510	—Hirashima: Lt	33 00 N	129 13 E
520	—Ō Shima: Lt	32 34 N	128 54 E
530	—*Tomie, Fukue Shima*	32 37 N	128 46 E
540	—Ōse Zaki: Lt	32 37 N	128 36 E
550	—Shiro Se: Lt	33 11 N	128 48 E
560	—Koshiki Jima: Lt	33 18 N	129 10 E
86600	**Tsushima**		
86610	Kō Saki: Lt	34 05 N	129 13 E
620	*Izuhara*	34 12 N	129 18 E
630	Kuro Shima: Lt	34 19 N	129 25 E
640	Mitsu Shima: Lt	34 43 N	129 27 E
86700	**Kyūshū**		
86710	*Imari*	33 17 N	129 53 E
720	Futagami Shima: Lt	33 36 N	129 33 E
730	Nyaku Shima: Lt	33 52 N	129 41 E
740	Eboshi Jima: Lt	33 41 N	129 59 E
750	Genkai Jima: Lt	33 42 N	130 14 E
760	*Hakata Kō*	33 36 N	130 24 E
770	Okino Shima: Lt	34 15 N	130 06 E
780	*Wakamatsu*	33 53 N	130 48 E
790	*Kokura*	33 53 N	130 53 E
86800	**Honshū**		
86810	Mutsure Jima: Lt	33 59 N	130 52 E
820	Futaoi Jima: Lt	34 06 N	130 47 E
830	Tsuno Shima: Lt	34 21 N	130 51 E
840	*Hagi Kō*	34 25 N	131 25 E
850	*Hamada*	34 54 N	132 05 E
860	Hinomisaki: Lt	35 26 N	132 38 E
870	Jizō Zaki: Lt	35 34 N	133 20 E
880	*Sakai*	35 33 N	133 14 E
890	*Saigō, Oki Guntō*	36 12 N	133 20 E
86900	Kyōga Saki: Lt	35 46 N	135 14 E
910	*Miyazu*	35 32 N	135 12 E
920	*Maizuru*	35 27 N	135 20 E
930	Tateishi Saki: Lt	35 46 N	136 01 E
940	*Tsuruga*	35 39 N	136 04 E
950	Echizen Misaki: Lt	35 59 N	135 58 E
960	Saruyama Zaki: Lt	37 19 N	136 43 E
970	Rokugō Saki: Lt	37 32 N	137 20 E
980	*Nanao*	37 03 N	136 58 E
990	Kannon Zaki: Lt	37 06 N	137 03 E
87000	*Fushiki*	36 47 N	137 04 E
010	Torigakubi Saki: Lt	37 10 N	138 06 E
87100	Sado		
110	—Sawazaki Bana: Lt	37 49 N	138 12 E
120	—Hajiki Saki: Lt	38 20 N	138 31 E
130	—*Ryōtsu (Ryozu)*	38 05 N	138 26 E
140	—Hime Saki: Lt	38 05 N	138 34 E
87200	*Niigata*	37 55 N	139 03 E
210	*Sakata*	38 55 N	139 50 E
220	*Funagawa*	39 53 N	139 51 E
230	Nyūdō Saki: Lt	40 00 N	139 42 E
87300	**Hokkaidō**		
87310	Benten Jima: Lt	41 25 N	140 05 E
320	Ko Jima: Lt	41 22 N	139 49 E

APPENDIX S

MARITIME POSITIONS

JAPAN—Continued

Index No.	Place	Lat.	Long.
	Hokkaidō—Continued	° ′	° ′
87330	Kamome Shima: Lt	41 52 N	140 07 E
340	Inaho Misaki, Okushiro Shima: Lt	42 15 N	139 34 E
350	Motsutano Saki: Lt	42 37 N	139 50 E
360	Benkei Misaki: Lt	42 49 N	140 12 E
370	*Iwanai*	42 59 N	140 31 E
380	Kamui Misaki: Lt	43 20 N	140 21 E
390	Takashima Misaki: Lt	43 14 N	141 01 E
87400	*Otaru*	43 12 N	141 01 E
410	*Mashike*	43 51 N	141 32 E
420	Yakishiri Jima (Yangeshiri Shima): Lt	44 26 N	141 26 E
430	Oshidomari, Rishiri Tō: Lt	45 15 N	141 14 E
440	*Wakkanai*	45 25 N	141 41 E
450	Sōya Misaki: Lt	45 31 N	141 56 E
460	Notoro Misaki: Lt	44 07 N	144 15 E
87500	**Nampō Shotō (Nanpō Shotō)**		
87510	Hachijō Jima, Ishizumiga Hana: Lt	33 05 N	139 51 E
520	Aoga Shima	32 27 N	139 46 E
530	Urania I. (Existence doubtful)	31 54 N	140 00 E
540	Tori Shima	30 29 N	140 19 E

Index No.	Place	Lat.	Long.
	Nampō Shotō (Nanpō Shotō)—Continued	° ′	° ′
87550	Chichi Shima, Ogasawara Guntō (Bonin Is.): Lt	27 05 N	142 12 E
560	Haha Jima	26 39 N	142 09 E
570	Iwo Jima: Aviation Lt	24 47 N	141 19 E
87600	**Nansei Shotō (Ryukyu Islands)**		
87700	TANEGA SHIMA		
710	—Nishinoomote Kō: Lt	30 44 N	130 59 E
720	—Ōtake (Take) Zaki: Lt	30 23 N	130 58 E
87800	Yaku Shima, Nagata Misaki: Lt	30 24 N	130 23 E
810	Kusakaki Shima: Lt	30 51 N	129 28 E
820	Gaja Shima: Lt	29 54 N	129 32 E
87900	AMAMI-Ō SHIMA		
910	—*Nase*	28 23 N	129 30 E
920	—Sotsukō Zaki: Lt	28 15 N	129 08 E
88000	OKINAWA GUNTŌ		
010	—Tsuken Jima: Lt	26 15 N	127 56 E
020	—*Naha, Okinawa Jima*	26 13 N	127 41 E
88100	Miyako Jima: Loran Station	24 44 N	125 26 E
110	*Okino-daitō Jima*	24 28 N	131 11 E
120	*Kita-daitō Jima*	25 56 N	131 18 E

PHILIPPINES

Index No.	Place	Lat.	Long.
		° ′	° ′
89000	Batan I.: Peak	20 28 N	122 01 E
010	Babuyan I.: Peak	19 32 N	121 57 E
020	Didicas Rocks	19 05 N	122 12 E
030	Cape Engaño: Lt	18 35 N	122 08 E
040	*Aparri, Luzon*	18 22 N	121 38 E
050	Pata Point: Lt	18 37 N	121 09 E
060	Cape Bojeador: Lt	18 31 N	120 36 E
070	*Laoag, Luzon*	18 12 N	120 35 E
080	*Currimao, Luzon*	18 01 N	120 29 E
090	*Salomague, Luzon*	17 47 N	120 25 E
89100	*Pandan, Luzon*	17 32 N	120 22 E
110	Candon: Lt	17 12 N	120 25 E
120	Tagudin: Lt	16 57 N	120 26 E
130	*San Fernando, Luzon*	16 37 N	120 19 E
140	*Poro, Luzon*	16 37 N	120 18 E
150	Dagupan, Guecet Point: Lt	16 04 N	120 20 E
160	*Bolinao, Luzon*	16 23 N	119 54 E
170	Piedra Point, Cape Bolinao: Lt	16 18 N	119 47 E
180	Hermana Mayor I.: Lt	15 48 N	119 48 E
190	Palauig Point: Lt	15 26 N	119 54 E
89200	Capones I.: Lt	14 55 N	120 00 E
210	Subic Bay, Sueste Point: Lt	14 45 N	120 11 E
220	*Olongapo, Luzon*	14 49 N	120 16 E
230	La Monja I.: Lt	14 23 N	120 31 E
240	*Mariveles, Luzon*	14 25 N	120 30 E
250	Corregidor I.: Lt	14 23 N	120 35 E
260	*Manila, Luzon*	14 35 N	120 58 E
270	*Cavite, Luzon*	14 29 N	120 55 E
280	Sangley Point: Lt	14 30 N	120 55 E
290	San Nicolas Shoals: Lt	14 26 N	120 46 E
89300	Caballo I.: Lt	14 22 N	120 37 E
310	Fortune I.: Lt	14 03 N	120 29 E
320	Cabra I.: Lt	13 53 N	120 01 E
330	Golo I.: Lt	13 38 N	120 25 E
340	Cape Santiago: Lt	13 46 N	120 39 E
350	Balayan: Lt	13 56 N	120 44 E
360	*Batangas, Luzon*	13 45 N	121 03 E
370	Malabrigo Point: Lt	13 36 N	121 16 E
380	*Port Ragay, Luzon*	13 47 N	122 39 E
390	Donsol, Luzon	12 54 N	123 35 E
89400	*Sorsogon, Luzon*	12 58 N	124 00 E
410	Bagatao I.: Lt	12 50 N	123 47 E
420	*Bulan, Luzon*	12 40 N	123 52 E
430	*Legaspi, Luzon*	13 09 N	123 45 E
440	Ungay Point: Lt	13 11 N	124 13 E
450	*Tabaco, Luzon*	13 22 N	123 44 E
460	Malinao: Lt	13 24 N	123 43 E
470	Sabang: Lt	13 43 N	123 35 E
480	Sialat Point: Lt	13 40 N	124 02 E
490	*Virac, Catanduanes*	13 35 N	124 15 E
89500	Pandan: Lt	14 03 N	124 10 E
510	Ocata I.: Lt	13 59 N	123 50 E

Index No.	Place	Lat.	Long.
		° ′	° ′
89520	Canimo I.: Lt	14 08 N	123 03 E
530	Tailon I.: Lt	14 25 N	122 40 E
540	Baliscan I.: Lt	14 15 N	121 54 E
550	*Polillo, Polillo I.*	14 44 N	121 56 E
560	*Baler, Luzon*	15 46 N	121 34 E
570	Cape Calavite: Lt	13 27 N	120 18 E
580	Escarceo Point: Lt	13 31 N	120 59 E
590	Verde I.: Peak	13 34 N	121 05 E
89600	*Calapan, Mindoro*	13 25 N	121 10 E
610	Dumali Point: Lt	13 07 N	121 33 E
89700	**Sibuyan Sea**		
89710	Tres Reyes Is., Baltasar I.: Lt	13 14 N	121 49 E
720	*Gasan, Marinduque I.*	13 20 N	121 51 E
730	*Balanacan, Marinduque I.*	13 32 N	121 52 E
740	Santa Cruz Harbor: Lt	13 30 N	122 03 E
750	Simara I., Corcuera Point: Lt	12 48 N	122 01 E
760	Tablas I., Gorda Point: Lt	12 40 N	122 09 E
770	Apunan Point: Lt	12 29 N	122 17 E
780	*Romblon, Romblon I.*	12 35 N	122 16 E
790	Sabang Point: Lt	12 36 N	122 16 E
89800	Burias I.	13 00 N	123 05 E
810	San Miguel I.: Lt	12 43 N	123 35 E
820	Ticao I.	12 35 N	123 40 E
830	Bugui Point: Lt	12 36 N	123 14 E
840	Colorada Point: Lt	12 33 N	123 23 E
850	*Port Barrera, Masbate I.*	12 31 N	123 23 E
860	*Masbate, Masbate I.*	12 22 N	123 37 E
870	Jintotolo I.: Lt	11 50 N	123 07 E
880	Floripon Point: Lt	11 37 N	122 30 E
890	*Port Capiz, Panay I.*	11 36 N	122 43 E
89900	**Visayan Sea**		
89910	Manigonigo Islet: Lt	11 36 N	123 11 E
920	North Gigante I.: Lt	11 38 N	123 21 E
930	Baliguian I.: Lt	11 12 N	123 20 E
940	Calabazas I.: Lt	11 05 N	123 01 E
950	Tanguingui Islet: Lt	11 29 N	123 43 E
960	Malapascua I.: Lt	11 21 N	124 07 E
90000	**Samar Sea**		
90010	Matabao I.: Lt	12 19 N	123 48 E
020	Cabilison I.: Lt	11 53 N	124 17 E
030	*Catbalogan, Samar*	11 46 N	124 53 E
040	Calbayog: Lt	12 04 N	124 35 E
050	Capul I.: Lt	12 29 N	124 08 E
060	Calantas Rock: Lt	12 31 N	124 05 E
070	San Bernardino Islet: Lt	12 45 N	124 17 E
080	Batag I.: Lt	12 40 N	125 04 E
090	*Borongan, Samar*	11 36 N	125 26 E
90100	Divinubo I.: Lt	11 36 N	125 30 E
110	Suluan I.: Lt	10 45 N	125 58 E
120	Mariquitdaquit Islet: Lt	11 04 N	125 09 E

APPENDIX S

MARITIME POSITIONS

PHILIPPINES—Continued

Index No.	Place	Lat.	Long.
	Samar Sea—Continued	° ′	° ′
90130	*Tacloban, Leyte*	11 15 N	125 00 E
140	Hibuson I.: Lt.	10 28 N	125 28 E
90200	**Mindanao Sea**		
90210	*Liloan, Panaon I.*	10 10 N	125 07 E
220	*Malitbog, Leyte*	10 10 N	125 00 E
230	*Maasin, Leyte*	10 08 N	124 50 E
240	Balicasag I.: Lt.	9 31 N	123 41 E
250	Siquijor I., Port Canoan: Lt.	9 15 N	123 36 E
260	*Dumaguete, Negros*	9 19 N	123 18 E
270	Apo I.: Lt.	9 05 N	123 16 E
280	Tagolo Point: Lt.	8 44 N	123 23 E
290	Polo Point: Lt.	8 36 N	123 45 E
90300	*Port Misamis, Mindanao*	8 08 N	123 51 E
310	*Iligan, Mindanao*	8 14 N	124 14 E
320	*Cagayan Anchorage, Mindanao*	8 30 N	124 40 E
330	*Bugo, Mindanao*	8 31 N	124 45 E
340	*Mambajao, Camiguin I.*	9 15 N	124 43 E
350	*Butuan, Mindanao*	8 57 N	125 32 E
360	*Surigao, Mindanao*	9 47 N	125 30 E
370	Rasa I.: Lt.	9 48 N	125 35 E
380	Dahakit Point, Bucas Grande I.: Lt.	9 34 N	125 56 E
390	Cauit Point: Lt.	9 18 N	126 12 E
90400	*Arasasan, Mindanao*	8 53 N	126 19 E
410	*Mati, Mindanao*	6 57 N	126 13 E
420	Cape San Augustin: Lt.	6 16 N	126 12 E
430	Santa Ana: Lt.	7 05 N	125 37 E
440	*Davao, Mindanao*	7 04 N	125 37 E
450	*Daliao, Mindanao*	7 01 N	125 30 E
460	*Malita, Mindanao*	6 24 N	125 37 E
470	Tinaca Point: Lt.	5 33 N	125 20 E
480	*Cotabato, Mindanao*	7 14 N	124 15 E
490	*Parang, Mindanao*	7 22 N	124 15 E
90500	Sibago I.: Lt.	6 45 N	122 24 E
510	*Zamboanga, Mindanao*	6 54 N	122 04 E
520	Little Santa Cruz I.: Lt.	6 53 N	122 02 E
90600	**Camotes Sea**		
90610	Canigao I.: Lt.	10 15 N	124 45 E
620	*Baybay, Leyte*	10 41 N	124 48 E
630	*Pilar, Ponson I.*	10 48 N	124 34 E
640	*Ormoc, Leyte*	11 00 N	124 36 E
650	*Palompon, Leyte*	11 03 N	124 23 E
660	*Bogo, Cebu*	11 03 N	124 00 E
670	Capitancillo I.: Lt.	10 59 N	124 06 E
680	Bagacay Point: Lt.	10 23 N	124 01 E
	Camotes Sea—Continued	° ′	° ′
90690	Mactan I., Bantolinao Point: Lt.	10 20 N	123 59 E
90700	*Cebu City, Cebu*	10 18 N	123 54 E
710	Lauis Ledge: Lt.	10 14 N	123 53 E
90800	**Tañon Strait**		
90810	Amblan Point: Lt.	9 28 N	123 14 E
820	Pescador I.: Lt.	9 55 N	123 21 E
830	*Dumanjug, Cebu*	10 04 N	123 26 E
840	Refugio I.: Lt.	10 28 N	123 27 E
850	*San Carlos, Negros*	10 28 N	123 25 E
860	Balamban: Lt.	10 30 N	123 43 E
90900	**Panay Gulf**		
90910	*Bacolod, Negros*	10 40 N	122 57 E
920	Siete Pecados: Lt.	10 46 N	122 41 E
930	*Iloilo, Panay*	10 41 N	122 35 E
940	Guimaras I., Lusaran Point: Lt.	10 29 N	122 28 E
91000	**Sulu Sea**		
91010	Nogas I.: Lt.	10 25 N	121 55 E
020	Tubigan Point, San Jose de Buenavista: Lt.	10 44 N	121 56 E
030	Cuyo I.: Lt.	10 51 N	121 00 E
040	Maniguin I.: Lt.	11 36 N	121 42 E
050	Ambulong I.: Lt.	12 13 N	121 00 E
060	Apo Reef: Lt.	12 40 N	120 25 E
070	Culion I.: Lt.	11 54 N	120 01 E
080	Langoy I.: Lt.	10 30 N	120 00 E
090	Manucan Islet: Lt.	9 38 N	121 21 E
91100	*Puerto Princesa, Palawan*	9 44 N	118 44 E
110	Sir J. Brooke Point: Lt.	8 46 N	117 50 E
120	Tubbataha Reef: Beacon	8 44 N	119 49 E
130	*Balabac I., Calandorang Bay*	8 00 N	117 04 E
140	Cape Melville, Balabac I.: Lt.	7 49 N	117 00 E
150	Cagayan Sulu I.: Lt.	6 59 N	118 33 E
160	Taganak I.: Lt.	6 05 N	118 19 E
170	Saluag I.: Lt.	4 35 N	119 28 E
180	*Bongao, Tawitawi I.*	5 02 N	119 46 E
190	Pearl Bank: Lt.	5 50 N	119 44 E
91200	North Ubian I.: Lt.	6 10 N	120 27 E
210	*Jolo, Jolo I.*	6 03 N	121 00 E
220	Tatalan I.: Lt.	6 13 N	121 50 E
230	Mataja I.: Lt.	6 34 N	121 42 E
240	Malamaui I.: Lt.	6 45 N	121 59 E

LESSER ISLANDS OF THE PACIFIC

Index No.	Place	Lat.	Long.
		° ′	° ′
92000	New Guinea		
010	—Waigeo	0 10 S	130 50 E
020	—Tandjoeng Sorong: Lt.	0 49 S	131 13 E
030	—*Manokwari, Neth. New Guinea*	0 52 S	134 05 E
040	Poelau Noemfoor	1 00 S	134 53 E
050	—Biak	1 00 S	136 00 E
060	—Japen	1 45 S	136 15 E
070	—*Hollandia, Neth. New Guinea*	2 32 S	140 43 E
080	—*Madang, North East New Guinea*	5 13 S	145 50 E
090	—Cape Cretin, Nussing I.: Lt.	6 40 S	147 52 E
92100	—*Salamaua, North East New Guinea*	7 02 S	147 03 E
110	—Mitre Rock: Lt.	8 02 S	148 08 E
120	—Cape Vogel: Lt.	9 38 S	150 01 E
130	—*Samarai, Papua*	10 37 S	150 40 E
140	—Brumer Is.: Lt.	10 45 S	150 23 E
150	—*Port Moresby, Papua*	9 28 S	147 09 E
160	—Bramble Cay: Lt.	9 09 S	143 53 E
170	—*Merauke, Neth. New Guinea*	8 29 S	140 23 E
180	—*Fakfak, Neth. New Guinea*	2 56 S	132 17 E
190	—Misoöl	1 50 S	130 10 E
92200	Bismarck Archipelago		
210	—*Lorengau, Manus, Admiralty Is.*	2 01 S	147 17 E
220	—New Hanover	2 30 S	150 15 E
230	—*Karieng, New Ireland*	2 35 S	150 49 E
240	—*Rabaul, New Britain*	4 13 S	152 12 E
92300	Rossel I.	11 20 S	154 10 E
92400	Solomon Islands		
410	—*Kieta, Bougainville*	6 12 S	155 40 E
420	—Choiseul	7 00 S	157 00 E
430	—Vella Lavella	7 45 S	156 40 E
440	—*Munda, New Georgia*	8 20 S	157 15 E
450	—Rendova	8 30 S	157 20 E
460	—Santa Isabel	8 00 S	159 00 E
470	—Malaita	9 00 S	161 00 E
480	—Florida	9 05 S	160 15 E
490	—Savo	9 08 S	159 49 E
92500	—Guadalcanal	9 35 S	160 15 E
510	—San Cristobal	10 35 S	161 45 E
520	—Ndeni (Santa Cruz I.)	10 45 S	165 55 E
92600	New Hebrides		
610	—Espiritu Santo	15 25 S	167 00 E
620	—Efate (Sandwich I.)	17 40 S	168 25 E
92700	Loyalty Is.	21 00 S	167 15 E
710	*Nouméa, New Caledonia*	22 17 S	166 26 E
720	Norfolk I.	29 02 S	167 57 E
730	Lord Howe I.	31 33 S	159 05 E
740	Macquarie I.	54 37 S	158 54 E
750	Auckland I.	50 40 S	166 10 E
760	Antipodes I.	49 41 S	178 48 E
770	Chatham Is.	44 00 S	177 30 E
780	Île Tubuaï	23 23 S	149 27 W
790	Île Rapa	27 36 S	144 19 W
92800	Pitcairn I.	25 04 S	130 05 W
810	Henderson (Elizabeth) I.	24 22 S	128 19 W
820	Isla de Pascua (Easter I.)	27 09 S	109 27 W
830	Isla Sala y Gómez	26 27 S	105 21 W

APPENDIX S

MARITIME POSITIONS

LESSER ISLANDS OF THE PACIFIC—Continued

Index No.	Place	Lat.	Long.
		° ′	° ′
92900	Islas Juan Fernandez		
910	—Mas a Tierra	33 38 S	78 50 W
920	—Mas Afuera	33 45 S	80 45 W
93000	Isla San Ambrosia	26 21 S	79 52 W
010	Archipiélago de Colón (Galapagos Is.), Isla San Cristóbal: Lt	0 54 S	89 36 W
020	Isla de Malpelo	3 59 N	81 36 W
030	Isla del Coco (Cocos I.)	5 32 N	87 04 W
040	Île Clipperton	10 17 N	109 13 W
050	Johnston I.: Aviation Lt	16 45 N	169 31 W
060	Palmyra I.	5 53 N	162 05 W
070	Washington I.	4 43 N	160 25 W
080	Fanning I.	3 54 N	159 23 W
090	Christmas I.	1 59 N	157 27 W
93100	Jarvis I.	0 23 S	160 01 W
110	Malden I.	4 03 S	154 59 W
120	Starbuck I.	5 37 S	155 53 W
130	Tongareva (Penrhyn I.)	9 00 S	158 03 W
140	Rakahanga (Reirson I.)	10 03 S	161 06 W
150	Manihiki (Humphrey I.)	10 24 S	161 03 W
160	Vostok I.	10 06 S	152 23 W
170	Flint I.	11 25 S	151 48 W
180	Caroline I.	10 00 S	150 14 W
93200	Marquesas Islands		
210	—Île Nuku-hiva: Lt	8 56 S	140 05 W
220	—Île Hiva-oa: Lt	9 48 S	139 01 W
93300	Tuamotu Archipelago		
310	—Mangareva	23 07 S	134 57 W
320	—Nihiru	16 47 S	142 51 W
330	—Takapoto: Lt	14 41 S	145 11 W
340	—Fakarava, Rotoava	16 02 S	145 36 W
350	—Makatea: Lt	15 50 S	148 15 W
93400	Society Islands		
410	—Île Bora-Bora	16 30 S	151 45 W
420	—Pointe Venus, Île Tahiti: Lt	17 29 S	149 29 W
430	—*Papeete, Île Tahiti*	17 32 S	149 34 W
93500	Cook Islands		
510	—Mangaia	21 55 S	157 56 W
520	—Rarotonga	21 14 S	159 46 W
530	—Aitutaki	18 53 S	159 46 W
93600	Kermadec Is., Raoul I.	29 16 S	177 55 W
93700	Tonga		
710	—Tongatapu	21 08 S	175 12 W
720	—Malinoa: Lt	21 02 S	175 08 W
730	—Vava'u	18 39 S	173 59 W
93800	Niue	19 02 S	169 52 W
93900	Fiji		
910	—Kandavu, Cape Washington: Lt	19 07 S	177 58 E
920	—*Suva, Viti Levu*	18 08 S	178 25 E
930	—*Levuka, Ovalau*	17 41 S	178 50 E
940	—Koro: Lt	17 24 S	179 22 E
950	—Vanua Levu, Undu Point: Lt	16 08 S	179 56 W
960	—Wailangilala: Lt	16 45 S	179 06 W
94000	Samoa		
010	—*Apia, Upolu (New Zealand)*	13 49 S	171 46 W
020	—*Pago Pago, Tutuila I. (U. S. A.)*	14 17 S	170 40 W
94100	Tokelau (Union) Islands		
110	—Fakaofo	9 23 S	171 15 W
120	—Nukunono	9 12 S	171 54 W
130	—Atafu	8 32 S	172 31 W
94200	Phoenix I.	3 43 S	170 43 W
210	Hull I.	4 31 S	172 13 W
220	Gardner I.	4 40 S	174 32 W
230	McKean I.	3 36 S	174 08 W
240	Canton (Musick) I.	2 49 S	171 43 W
250	Enderbury I.	3 08 S	171 05 W
260	Baker I.	0 12 N	176 29 W
270	Howland I.	0 48 N	176 38 W
94300	Ellice Islands		
310	—Funafuti	8 31 S	179 13 E
320	—Nanomea	5 41 S	176 09 E
94400	Ocean I.	0 52 S	169 35 E
410	Nauru	0 31 S	166 56 E
94500	Gilbert Islands		
510	—Tabiteuea (Drummond I.)	1 30 S	175 00 E
520	—Tarawa	1 22 N	172 58 E
530	—Makin	3 01 N	172 46 E
94600	Marshall Islands		
610	—Jaluit	5 55 N	169 39 E
620	—Majuro	7 06 N	171 15 E
630	—Wotje	9 28 N	170 14 E
640	—Kwajalein	8 43 N	167 44 E
650	—Bikini	11 37 N	165 33 E
660	—Eniwetok	11 21 N	162 20 E
94700	Wake I.	19 17 N	166 39 E
710	Marcus I.	24 18 N	153 58 E
94800	Mariana Islands		
810	—Asuncion	19 40 N	145 24 E
820	—Agrihan	18 46 N	145 40 E
830	—Alamagan	17 36 N	145 50 E
840	—Anatahan	16 22 N	145 39 E
850	—Saipan	15 10 N	145 45 E
860	—Tinian	15 00 N	145 38 E
870	—*Rota*	14 08 N	145 09 E
880	—*Apra Harbor, Guam*	13 27 N	144 37 E
94900	Caroline Islands		
910	—Kusaie	5 20 N	162 58 E
920	—*Ponape, Senyavin Is*	6 59 N	158 12 E
930	—*Dublon I., Truk Is*	7 22 N	151 53 E
940	—Ulithi	10 00 N	139 40 E
950	—Tomil Harbor, Yap	9 30 N	138 08 E
95000	Palau (Pelew) Islands		
010	—Babelthuap	7 30 N	134 35 E
020	—Peleliu	7 00 N	134 15 E
030	—Angaur	6 54 N	134 09 E
95100	Sonsorol Is.	5 20 N	132 13 E
110	Merir	4 20 N	132 19 E
120	Tobi (Nevil I.)	3 00 N	131 11 E

ANTARCTICA

Index No.	Place	Lat.	Long.
		° ′	° ′
96000	Palmer Peninsula		
010	—Hope Bay	63 23 S	57 00 W
020	—Cape Legoupil (Joupil)	63 19 S	57 51 W
030	—Melchior Harbor	64 19 S	62 56 W
040	—Andersen Harbor	64 20 S	63 00 W
050	—Lambda I.: Lt	64 18 S	62 55 W
060	—Anvers I.: IGY Station	64 50 S	64 00 W
070	—The Waifs: Lt	64 32 S	62 40 W
080	—Wilhelmina Bay: Lt	64 33 S	62 26 W
090	—Port Lockroy	64 50 S	63 27 W
96100	—Useful Islet (Isla Lautaro): Lt	64 44 S	62 52 W
110	—Doumer I.: Lt	64 53 S	63 37 W
120	—Horseshoe I.: IGY Station	67 50 S	67 20 W
130	—Marguerite Bay (East Base)	68 12 S	67 03 W
96200	Charcot I., Cape Byrd	70 00 S	76 00 W
210	Peter First I.: IGY Station	68 50 S	90 35 W
220	Cape Dart, Mount Siple	73 15 S	122 50 W
230	Cape Colbeck	77 06 S	158 10 W
240	Little America: IGY Station	78 32 S	164 00 W
96300	Ross Island		
310	—Cape Bird	77 08 S	166 30 E
320	—Cape Armitage	77 49 S	166 45 E
96400	McMurdo Sound: IGY Station	77 50 S	166 45 E
410	Franklin I.	76 10 S	168 20 E
420	Cape Adare: IGY Station	71 17 S	170 15 E
430	Sturge I.	67 20 S	164 10 E
440	Cape Denison, George V Coast	67 00 S	142 40 E
450	Pont Géologie, Adélie Coast	66 37 S	140 12 E
460	Budd Coast, Cape Poinsett	66 00 S	113 00 E
470	Bowman I.	65 30 S	103 45 E
480	Mill I.	65 45 S	101 00 E
490	Drygalski I.	65 40 S	92 45 E
96500	Vestfold Hills: IGY Station	68 30 S	78 00 E
510	Cape Darnley	67 40 S	69 50 E
520	Cape Boothby	66 33 S	57 15 E
530	Proclamation I.	65 50 S	53 30 E
540	Riiser-Larsen Peninsula, Prince Harald Coast	68 40 S	34 00 E
550	Cape Norvegia	71 20 S	12 00 W
560	Duke Ernst Bay (Vahsel Bay): IGY Station	76 30 S	35 00 W

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S
MARITIME POSITIONS
INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S
MARITIME POSITIONS
INDEX

APPENDIX S
MARITIME POSITIONS
INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S
MARITIME POSITIONS
INDEX

Index No.

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S
MARITIME POSITIONS
INDEX

APPENDIX S
MARITIME POSITIONS
INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S
MARITIME POSITIONS
INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S
MARITIME POSITIONS
INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S
MARITIME POSITIONS
INDEX

APPENDIX S

MARITIME POSITIONS

INDEX

APPENDIX S
MARITIME POSITIONS
INDEX

APPENDIX T

EXTRACTS FROM TIDE TABLES

NEW YORK (The Battery), N.Y., 1958

Times and Heights of High and Low Waters

JANUARY

Day	Time	Ht.	Day	Time	Ht.
	h. m.	ft.		h. m.	ft.
1 W	0430	4.0	16 Th	0524	4.6
	1111	0.2		1151	-0.5
	1651	3.3		1752	3.8
	2313	0.0		2359	-0.6
2 Th	0522	4.2	17 F	0619	4.8
	1158	-0.1		1241	-0.7
	1745	3.4		1845	3.9
	2358	-0.2			
3 F	0610	4.5	18 Sa	0049	-0.6
	1245	-0.4		0707	4.9
	1834	3.6		1329	-0.8
				1932	4.0
4 Sa	0044	-0.4	19 Su	0138	-0.7
	0654	4.8		0751	4.9
	1331	-0.6		1415	-0.8
	1919	3.8		2016	4.1
5 Su	0131	-0.6	20 M	0223	-0.6
	0736	5.0		0832	4.8
	1415	-0.9		1456	-0.9
	2002	4.0		2100	4.0
6 M	0217	-0.7	21 Tu	0305	-0.6
	0819	5.1		0913	4.6
	1458	-1.0		1535	-0.8
	2049	4.2		2143	4.0
7 Tu	0302	-0.9	22 W	0343	-0.4
	0906	5.1		0953	4.4
	1541	-1.1		1611	-0.7
	2139	4.3		2225	3.9
8 W	0347	-0.9	23 Th	0421	-0.2
	0956	5.0		1032	4.2
	1623	-1.1		1646	-0.5
	2233	4.4		2307	3.8
9 Th	0434	-0.8	24 F	0455	0.0
	1051	4.8		1111	3.9
	1708	-1.0		1718	-0.2
	2329	4.4		2346	3.7
10 F	0525	-0.6	25 Sa	0530	0.2
	1146	4.6		1147	3.7
	1758	-0.8		1749	0.0
11 Sa	0024	4.5	26 Su	0024	3.7
	0627	-0.3		0610	0.5
	1242	4.3		1222	3.5
	1858	-0.6		1821	0.2
12 Su	0120	4.5	27 M	0101	3.7
	0740	-0.2		0714	0.7
	1340	4.1		1258	3.3
	2005	-0.4		1912	0.4
13 M	0219	4.4	28 Tu	0144	3.7
	0853	-0.1		0832	0.7
	1440	3.8		1346	3.2
	2112	-0.4		2032	0.4
14 Tu	0320	4.5	29 W	0234	3.7
	0958	-0.2		0938	0.6
	1546	3.7		1446	3.1
	2211	-0.4		2139	0.4
15 W	0424	4.5	30 Th	0334	3.9
	1056	-0.4		1034	0.3
	1651	3.7		1600	3.2
	2307	-0.5		2235	0.2
			31 F	0439	4.1
				1126	0.0
				1709	3.4
				2327	-0.1

FEBRUARY

Day	Time	Ht.	Day	Time	Ht.
	h. m.	ft.		h. m.	ft.
1 Sa	0537	4.4	16 Su	0032	-0.4
	1215	-0.3		0648	4.6
	1805	3.7		1306	-0.6
				1913	4.1
2 Su	0019	-0.4	17 M	0119	-0.5
	0628	4.7		0731	4.7
	1303	-0.7		1350	-0.7
	1855	4.1		1955	4.2
3 M	0110	-0.7	18 Tu	0203	-0.5
	0716	5.0		0810	4.6
	1350	-1.0		1430	-0.7
	1942	4.4		2034	4.3
4 Tu	0200	-1.0	19 W	0244	-0.5
	0802	5.2		0848	4.5
	1435	-1.2		1508	-0.7
	2029	4.6		2112	4.2
5 W	0247	-1.2	20 Th	0322	-0.5
	0850	5.2		0924	4.4
	1520	-1.3		1542	-0.6
	2119	4.8		2149	4.2
6 Th	0335	-1.2	21 F	0356	-0.3
	0941	5.1		1000	4.2
	1602	-1.3		1613	-0.4
	2212	4.8		2225	4.1
7 F	0422	-1.1	22 Sa	0428	-0.2
	1035	4.9		1033	3.9
	1647	-1.2		1639	-0.2
	2307	4.8		2258	4.0
8 Sa	0512	-0.9	23 Su	0457	0.0
	1131	4.6		1104	3.7
	1735	-0.9		1700	0.0
				2330	3.9
9 Su	0003	4.8	24 M	0526	0.3
	0610	-0.6		1137	3.5
	1226	4.3		1723	0.2
	1831	-0.6			
10 M	0059	4.6	25 Tu	0004	3.9
	0718	-0.3		0603	0.5
	1323	4.0		1215	3.4
	1939	-0.3		1757	0.4
11 Tu	0157	4.5	26 W	0047	3.9
	0831	-0.1		0709	0.6
	1423	3.8		1304	3.3
	2049	-0.1		1851	0.5
12 W	0258	4.4	27 Th	0138	3.9
	0938	-0.1		0850	0.6
	1528	3.6		1403	3.2
	2153	-0.1		2047	0.6
13 Th	0402	4.3	28 F	0241	3.9
	1036	-0.2		0956	0.4
	1634	3.6		1518	3.3
	2250	-0.2		2202	0 3
14 F	0505	4.4			
	1130	-0.3			
	1735	3.7			
	2342	-0.2			
15 Sa	0600	4.5			
	1220	-0.5			
	1828	3.9			

MARCH

Day	Time	Ht.	Day	Time	Ht.
	h. m.	ft.		h. m.	ft.
1 Sa	0356	4.1	16 Su	0535	4.3
	1052	0.1		1153	-0.2
	1634	3.6		1805	4.1
	2301	0.0			
2 Su	0504	4.4	17 M	0012	-0.1
	1144	-0.3		0623	4.4
	1737	4.0		1237	-0.4
	2356	-0.4		1849	4.3
3 M	0603	4.7	18 Tu	0057	-0.3
	1233	-0.6		0706	4.5
	1831	4.4		1320	-0.4
				1928	4.4
4 Tu	0049	-0.8	19 W	0140	-0.4
	0655	5.0		0744	4.5
	1322	-1.0		1400	-0.5
	1919	4.9		2005	4.5
5 W	0141	-1.1	20 Th	0221	-0.4
	0743	5.2		0821	4.4
	1409	-1.2		1437	-0.5
	2007	5.2		2040	4.6
6 Th	0231	-1.3	21 F	0258	-0.4
	0832	5.3		0855	4.3
	1455	-1.3		1511	-0.4
	2057	5.3		2112	4.5
7 F	0319	-1.4	22 Sa	0333	-0.3
	0923	5.2		0929	4.1
	1540	-1.3		1540	-0.2
	2149	5.3		2143	4.4
8 Sa	0407	-1.3	23 Su	0405	-0.2
	1017	4.9		1001	3.9
	1624	-1.1		1605	0.0
	2244	5.2		2211	4.3
9 Su	0456	-1.0	24 M	0434	0.0
	1113	4.7		1029	3.8
	1712	-0.8		1626	0.1
	2340	5.0		2240	4.3
10 M	0551	-0.6	25 Tu	0502	0.1
	1209	4.3		1104	3.6
	1806	-0.4		1651	0.3
				2319	4.2
11 Tu	0037	4.8	26 W	0535	0.3
	0655	-0.2		1149	3.5
	1307	4.0		1726	0.4
	1913	0.0			
12 W	0134	4.5	27 Th	0007	4.2
	0806	0.0		0627	0.5
	1406	3.8		1241	3.5
	2026	0.2		1817	0.6
13 Th	0234	4.3	28 F	0103	4.1
	0913	0.1		0801	0.5
	1508	3.6		1343	3.5
	2133	0.2		1954	0.7
14 F	0336	4.2	29 Sa	0208	4.1
	1013	0.0		0919	0.4
	1613	3.7		1453	3.6
	2231	0.1		2134	0.5
15 Sa	0439	4.2	30 Su	0320	4.2
	1105	-0.1		1019	0.1
	1713	3.8		1605	4.0
	2323	0.0		2239	0.1
			31 M	0433	4.4
				1112	-0.3
				1710	4.4
				2335	-0.4

Time meridian 75° W. 0000 is midnight. 1200 is noon.

Heights are reckoned from the datum of soundings on charts of the locality which is mean low water.

TABLE 2.—TIDAL DIFFERENCES AND OTHER CONSTANTS

No.	PLACE	POSITION		DIFFERENCES				RANGES		Mean Tide Level
				Time		Height				
		Lat.	Long.	High water	Low water	High water	Low water	Mean	Spring	
		° ′	° ′	h. m.	h. m.	feet	feet	feet	feet	feet
	NEW YORK—Continued Long Island, South Side—Continued	N.	W.	on **SANDY HOOK**, p. 70 Time meridian, 75° W.						
	Hempstead Bay									
1501	Deep Creek Meadow	40 36	73 32	+1 02	+1 09	*0.52	*0.52	2.4	2.9	1.2
1503	Green Island	40 37	73 30	+1 22	+1 29	*0.41	*0.41	1.9	2.3	0.9
1505	Cuba Island	40 37	73 31	+1 08	+1 20	*0.50	*0.50	2.3	2.8	1.1
1507	Bellmore, Bellmore Creek	40 40	73 31	+1 29	+1 56	*0.43	*0.43	2.0	2.4	1.0
1509	Neds Creek	40 37	73 33	+0 50	+0 52	−1.9	0.0	2.7	3.3	1.3
1511	Freeport Creek	40 38	73 34	+0 34	+0 27	−1.5	0.0	3.1	3.8	1.5
1513	Freeport, Baldwin Bay	40 38	73 35	+0 38	+0 53	−1.6	0.0	3.0	3.6	1.5
1515	Long Beach	40 36	73 39	+0 19	0 00	−0.7	0.0	3.9	4.7	1.9
1517	Long Beach, outer coast	40 35	73 39	−0 29	−0 35	−0.1	0.0	4.5	5.4	2.2
	Hempstead Bay—Continued									
1519	East Rockaway	40 38	73 40	+0 42	+0 45	−0.7	0.0	3.9	4.7	1.9
1521	Woodmere, Brosewere Bay	40 37	73 42	+0 35	+0 48	−0.7	0.0	3.9	4.7	1.9
1523	East Rockaway Inlet	40 36	73 44	−0 06	−0 16	−0.5	0.0	4.1	5.0	2.0
	Jamaica Bay									
1525	Plumb Beach Channel	40 35	73 55	+0 03	−0 05	+0.3	0.0	4.9	5.9	2.4
1527	Barren Island, Rockaway Inlet	40 35	73 53	0 00	−0 06	+0.4	0.0	5.0	6.0	2.5
1529	Beach Channel (bridge)	40 35	73 49	+0 38	+0 22	+0.5	0.0	5.1	6.2	2.5
1531	Motts Basin	40 37	73 46	+0 40	+0 46	+0.8	0.0	5.4	6.5	2.7
1533	Norton Point, Head of Bay	40 38	73 45	+0 39	+0 43	+0.8	0.0	5.4	6.5	2.7
1535	New York International Airport	40 37	73 47	+0 26	+0 43	+0.7	0.0	5.3	6.4	2.6
1537	Grassy Bay (bridge)	40 39	73 50	+0 44	+0 45	+0.6	0.0	5.2	6.3	2.6
1539	Canarsie	40 38	73 53	+0 28	+0 06	+0.6	0.0	5.2	6.3	2.6
1541	Mill Basin	40 37	73 55	+0 29	+0 02	+0.6	0.0	5.2	6.3	2.6
	NEW YORK AND NEW JERSEY New York Harbor									
1543	Coney Island	40 34	73 59	−0 03	−0 19	+0.1	0.0	4.7	5.7	2.3
1545	Norton Point, Gravesend Bay	40 35	74 00	−0 03	+0 01	+0.1	0.0	4.7	5.7	2.3
1547	Fort Wadsworth, The Narrows	40 36	74 03	+0 02	+0 12	−0.3	0.0	4.3	5.2	2.1
1549	Fort Hamilton, The Narrows	40 37	74 02	+0 03	+0 05	+0.1	0.0	4.7	5.7	2.3
				on **NEW YORK**, p. 62						
1551	Bay Ridge	40 38	74 02	−0 24	−0 24	+0.2	0.0	4.6	5.5	2.3
1553	St. George, Staten Island	40 39	74 04	−0 21	−0 18	+0.1	0.0	4.5	5.4	2.2
1555	Bayonne, New Jersey	40 41	74 06	−0 19	−0 08	+0.1	0.0	4.5	5.4	2.2
1557	Gowanus Bay	40 40	74 01	−0 19	−0 15	0.0	0.0	4.4	5.3	2.2
1559	Governors Island	40 42	74 01	−0 11	−0 06	0.0	0.0	4.4	5.3	2.2
1561	**New York** (The Battery)	40 42	74 01	Daily predictions				4.4	5.3	2.2
	Hudson River‡									
1563	Jersey City, Pa. RR. Ferry, N. J	40 43	74 02	+0 07	+0 07	0.0	0.0	4.4	5.3	2.2
1565	New York, Desbrosses Street	40 43	74 01	+0 10	+0 10	0.0	0.0	4.4	5.3	2.2
1567	New York, Chelsea Docks	40 45	74 01	+0 17	+0 16	−0.1	0.0	4.3	5.2	2.1
1569	Hoboken, Castle Point, N. J	40 45	74 01	+0 17	+0 16	−0.1	0.0	4.3	5.2	2.1
1571	Weehawken, Days Point, N. J	40 46	74 01	+0 24	+0 23	−0.2	0.0	4.2	5.0	2.1
1573	New York, Union Stock Yards	40 47	74 00	+0 27	+0 26	−0.2	0.0	4.2	5.0	2.1
1575	New York, 130th Street	40 49	73 58	+0 37	+0 35	−0.4	0.0	4.0	4.8	2.0
1577	George Washington Bridge	40 51	73 57	+0 46	+0 43	−0.5	0.0	3.9	4.6	1.9
1579	Spuyten Duyvil, West of RR. bridge	40 53	73 56	+0 58	+0 53	−0.6	0.0	3.8	4.5	1.9
1581	Yonkers	40 56	73 54	+1 09	+1 10	−0.7	0.0	3.7	4.4	1.8
1583	Dobbs Ferry	41 01	73 53	+1 29	+1 40	−1.0	0.0	3.4	4.0	1.7
1585	Tarrytown	41 05	73 52	+1 45	+1 54	−1.2	0.0	3.2	3.7	1.6

*Ratio.

‡Values for the Hudson River above George Washington Bridge are based upon averages for the six months May to October, when the fresh-water discharge is a minimum.

TABLE 3.—HEIGHT OF TIDE AT ANY TIME

Duration of rise or fall, see footnote	Time from the nearest high water or low water														
h. m.	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*
4 00	0 08	0 16	0 24	0 32	0 40	0 48	0 56	1 04	1 12	1 20	1 28	1 36	1 44	1 52	2 00
4 20	0 09	0 17	0 26	0 35	0 43	0 52	1 01	1 09	1 18	1 27	1 35	1 44	1 53	2 01	2 10
4 40	0 09	0 19	0 28	0 37	0 47	0 56	1 05	1 15	1 24	1 33	1 43	1 52	2 01	2 11	2 20
5 00	0 10	0 20	0 30	0 40	0 50	1 00	1 10	1 20	1 30	1 40	1 50	2 00	2 10	2 20	2 30
5 20	0 11	0 21	0 32	0 43	0 53	1 04	1 15	1 25	1 36	1 47	1 57	2 08	2 19	2 29	2 40
5 40	0 11	0 23	0 34	0 45	0 57	1 08	1 19	1 31	1 42	1 53	2 05	2 16	2 27	2 39	2 50
6 00	0 12	0 24	0 36	0 48	1 00	1 12	1 24	1 36	1 48	2 00	2 12	2 24	2 36	2 48	3 00
6 20	0 13	0 25	0 38	0 51	1 03	1 16	1 29	1 41	1 54	2 07	2 19	2 32	2 45	2 57	3 10
6 40	0 13	0 27	0 40	0 53	1 07	1 20	1 33	1 47	2 00	2 13	2 27	2 40	2 53	3 07	3 20
7 00	0 14	0 28	0 42	0 56	1 10	1 24	1 38	1 52	2 06	2 20	2 34	2 48	3 02	3 16	3 30
7 20	0 15	0 29	0 44	0 59	1 13	1 28	1 43	1 57	2 12	2 27	2 41	2 56	3 11	3 25	3 40
7 40	0 15	0 31	0 46	1 01	1 17	1 32	1 47	2 03	2 18	2 33	2 49	3 04	3 19	3 35	3 50
8 00	0 16	0 32	0 48	1 04	1 20	1 36	1 52	2 08	2 24	2 40	2 56	3 12	3 28	3 44	4 00
8 20	0 17	0 33	0 50	1 07	1 23	1 40	1 57	2 13	2 30	2 47	3 03	3 20	3 37	3 53	4 10
8 40	0 17	0 35	0 52	1 09	1 27	1 44	2 01	2 19	2 36	2 53	3 11	3 28	3 45	4 03	4 20
9 00	0 18	0 36	0 54	1 12	1 30	1 48	2 06	2 24	2 42	3 00	3 18	3 36	3 54	4 12	4 30
9 20	0 19	0 37	0 56	1 15	1 33	1 52	2 11	2 29	2 48	3 07	3 25	3 44	4 03	4 21	4 40
9 40	0 19	0 39	0 58	1 17	1 37	1 56	2 15	2 35	2 54	3 13	3 33	3 52	4 11	4 31	4 50
10 00	0 20	0 40	1 00	1 20	1 40	2 00	2 20	2 40	3 00	3 20	3 40	4 00	4 20	4 40	5 00
10 20	0 21	0 41	1 02	1 23	1 43	2 04	2 25	2 45	3 06	3 27	3 47	4 08	4 29	4 49	5 10
10 40	0 21	0 43	1 04	1 25	1 47	2 08	2 29	2 51	3 12	3 33	3 55	4 16	4 37	4 59	5 20

Range of tide, see footnote	Correction to height														
Ft.	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2
1.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.4	0.4	0.5
1.5	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	0.7	0.8
2.0	0.0	0.0	0.0	0.1	0.1	0.2	0.3	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
2.5	0.0	0.0	0.1	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.9	1.0	1.1	1.2
3.0	0.0	0.0	0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1.0	1.2	1.3	1.5
3.5	0.0	0.0	0.1	0.2	0.2	0.3	0.4	0.6	0.7	0.9	1.0	1.2	1.4	1.6	1.8
4.0	0.0	0.0	0.1	0.2	0.3	0.4	0.5	0.7	0.8	1.0	1.2	1.4	1.6	1.8	2.0
4.5	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.9	1.1	1.3	1.6	1.8	2.0	2.2
5.0	0.0	0.1	0.1	0.2	0.3	0.5	0.6	0.8	1.0	1.2	1.5	1.7	2.0	2.2	2.5
5.5	0.0	0.1	0.1	0.2	0.4	0.5	0.7	0.9	1.1	1.4	1.6	1.9	2.2	2.5	2.8
6.0	0.0	0.1	0.1	0.3	0.4	0.6	0.8	1.0	1.2	1.5	1.8	2.1	2.4	2.7	3.0
6.5	0.0	0.1	0.2	0.3	0.4	0.6	0.8	1.1	1.3	1.6	1.9	2.2	2.6	2.9	3.2
7.0	0.0	0.1	0.2	0.3	0.5	0.7	0.9	1.2	1.4	1.8	2.1	2.4	2.8	3.1	3.5
7.5	0.0	0.1	0.2	0.3	0.5	0.7	1.0	1.2	1.5	1.9	2.2	2.6	3.0	3.4	3.8
8.0	0.0	0.1	0.2	0.3	0.5	0.8	1.0	1.3	1.6	2.0	2.4	2.8	3.2	3.6	4.0
8.5	0.0	0.1	0.2	0.4	0.6	0.8	1.1	1.4	1.8	2.1	2.5	2.9	3.4	3.8	4.2
9.0	0.0	0.1	0.2	0.4	0.6	0.9	1.2	1.5	1.9	2.2	2.7	3.1	6.6	4.0	4.5
9.5	0.0	0.1	0.2	0.4	0.6	0.9	1.2	1.6	2.0	2.4	2.8	3.3	3.8	4.3	4.8
10.0	0.0	0.1	0.2	0.4	0.7	1.0	1.3	1.7	2.1	2.5	3.0	3.5	4.0	4.5	5.0
10.5	0.0	0.1	0.3	0.5	0.7	1.0	1.3	1.7	2.2	2.6	3.1	3.6	4.2	4.7	5.2
11.0	0.0	0.1	0.3	0.5	0.7	1.1	1.4	1.8	2.3	2.8	3.3	3.8	4.4	4.9	5.5
11.5	0.0	0.1	0.3	0.5	0.8	1.1	1.5	1.9	2.4	2.9	3.4	4.0	4.6	5.1	5.8
12.0	0.0	0.1	0.3	0.5	0.8	1.1	1.5	2.0	2.5	3.0	3.6	4.1	4.8	5.4	6.0
12.5	0.0	0.1	0.3	0.5	0.8	1.2	1.6	2.1	2.6	3.1	3.7	4.3	5.0	5.6	6.2
13.0	0.0	0.1	0.3	0.6	0.9	1.2	1.7	2.2	2.7	3.2	3.9	4.5	5.1	5.8	6.5
13.5	0.0	0.1	0.3	0.6	0.9	1.3	1.7	2.2	2.8	3.4	4.0	4.7	5.3	6.0	6.8
14.0	0.0	0.2	0.3	0.6	0.9	1.3	1.8	2.3	2.9	3.5	4.2	4.8	5.5	6.3	7.0
14.5	0.0	0.2	0.4	0.6	1.0	1.4	1.9	2.4	3.0	3.6	4.3	5.0	5.7	6.5	7.2
15.0	0.0	0.2	0.4	0.6	1.0	1.4	1.9	2.5	3.1	3.8	4.4	5.2	5.9	6.7	7.5
15.5	0.0	0.2	0.4	0.7	1.0	1.5	2.0	2.6	3.2	3.9	4.6	5.4	6.1	6.9	7.8
16.0	0.0	0.2	0.4	0.7	1.1	1.5	2.1	2.6	3.3	4.0	4.7	5.5	6.3	7.2	8.0
16.5	0.0	0.2	0.4	0.7	1.1	1.6	2.1	2.7	3.4	4.1	4.9	5.7	6.5	7.4	8.2
17.0	0.0	0.2	0.4	0.7	1.1	1.6	2.2	2.8	3.5	4.2	5.0	5.9	6.7	7.6	8.5
17.5	0.0	0.2	0.4	0.8	1.2	1.7	2.2	2.9	3.6	4.4	5.2	6.0	6.9	7.8	8.8
18.0	0.0	0.2	0.4	0.8	1.2	1.7	2.3	3.0	3.7	4.5	5.3	6.2	7.1	8.1	9.0
18.5	0.1	0.2	0.5	0.8	1.2	1.8	2.4	3.1	3.8	4.6	5.5	6.4	7.3	8.3	9.2
19.0	0.1	0.2	0.5	0.8	1.3	1.8	2.4	3.1	3.9	4.8	5.6	6.6	7.5	8.5	9.5
19.5	0.1	0.2	0.5	0.8	1.3	1.9	2.5	3.2	4.0	4.9	5.8	6.7	7.7	8.7	9.8
20.0	0.1	0.2	0.5	0.9	1.3	1.9	2.6	3.3	4.1	5.0	5.9	6.9	7.9	9.0	10.0

Obtain from the predictions the high water and low water, one of which is before and the other after the time for which the height is required. The difference between the times of occurrence of these tides is the duration of rise or fall, and the difference between their heights is the range of tide for the above table. Find the difference between the nearest high or low water and the time for which the height is required.

Enter the table with the duration of rise or fall, printed in heavy-faced type, which most nearly agrees with the actual value, and on that horizontal line find the time from the nearest high or low water which agrees most nearly with the corresponding actual difference. The correction sought is in the column directly below, on the line with the range of tide.

When the nearest tide is high water, subtract the correction.

When the nearest tide is low water, add the correction.

APPENDIX U

EXTRACTS FROM TIDAL CURRENT TABLES

THE NARROWS, NEW YORK HARBOR, N. Y., 1958

f — flood, direction 340° true. e — ebb, direction 160° true.

JANUARY

Day	Slack Water Time	Maximum Current Time	Maximum Current Vel.
	h. m.	h. m.	kn.
1 W	0023	0253	1.6f
	0611	0928	1.8e
	1316	1523	1.1f
	1816	2134	1.8e
2 Th	0110	0347	1.7f
	0659	1014	2.0e
	1407	1618	1.3f
	1905	2221	1.9e
3 F	0156	0435	1.9f
	0745	1101	2.1e
	1454	1705	1.4f
	1951	2308	2.0e
4 Sa	0241	0520	2.1f
	0829	1150	2.2e
	1537	1748	1.6f
	2038	2357	2.1e
5 Su	0326	0603	2.2f
	0915	1237	2.3e
	1619	1830	1.7f
	2125		
6 M		0046	2.2e
	0409	0647	2.3f
	1001	1324	2.4e
	1700	1913	1.8f
	2213		
7 Tu		0134	2.2e
	0455	0732	2.3f
	1047	1409	2.5e
	1743	2001	1.9f
	2304		
8 W		0222	2.3e
	0543	0821	2.2f
	1135	1454	2.5e
	1828	2051	1.9f
	2357		
9 Th		0311	2.3e
	0636	0914	2.1f
	1224	1539	2.4e
	1917	2145	1.9f
10 F	0050	0401	2.2e
	0735	1009	2.0f
	1314	1628	2.3e
	2011	2240	1.9f
11 Sa	0145	0458	2.1e
	0839	1104	1.8f
	1404	1724	2.2e
	2106	2336	2.0f
12 Su	0244	0602	2.0e
	0944	1159	1.7f
	1459	1826	2.1e
	2203		
13 M		0032	1.9f
	0346	0710	2.0e
	1046	1258	1.5f
	1557	1928	2.0e
	2300		
14 Tu		0132	1.9f
	0451	0812	2.0e
	1148	1405	1.4f
	1659	2027	2.0e
	2357		
15 W		0248	1.9f
	0554	0910	2.1e
	1249	1527	1.4f
	1800	2121	2.0e

Day	Slack Water Time	Maximum Current Time	Maximum Current Vel.
	h. m.	h. m.	kn.
16 Th	0053	0356	1.9f
	0651	1003	2.1e
	1347	1633	1.4f
	1856	2213	2.1e
17 F	0149	0454	2.0f
	0742	1055	2.2e
	1440	1725	1.5f
	1947	2305	2.0e
18 Sa	0240	0540	2.0f
	0828	1147	2.2e
	1530	1809	1.6f
	2037	2357	2.0e
19 Su	0329	0619	2.1f
	0912	1234	2.2e
	1614	1846	1.6f
	2124		
20 M		0046	2.0e
	0414	0654	2.0f
	0955	1320	2.2e
	1655	1918	1.6f
	2210		
21 Tu		0132	2.0e
	0457	0727	1.9f
	1036	1401	2.2e
	1737	1955	1.6f
	2256		
22 W		0216	2.0e
	0541	0806	1.8f
	1117	1440	2.2e
	1819	2035	1.5f
	2341		
23 Th		0256	1.9e
	0625	0850	1.7f
	1159	1520	2.1e
	1901	2119	1.5f
24 F	0026	0337	1.8e
	0712	0935	1.5f
	1239	1558	2.0e
	1946	2206	1.5f
25 Sa	0111	0420	1.7e
	0805	1022	1.4f
	1323	1640	1.8e
	2031	2253	1.5f
26 Su	0158	0509	1.6e
	0900	1110	1.3f
	1405	1728	1.7e
	2119	2339	1.5f
27 M	0248	0606	1.6e
	0957	1159	1.2f
	1454	1824	1.6e
	2208		
28 Tu		0026	1.5f
	0340	0706	1.6e
	1053	1249	1.1f
	1547	1921	1.6e
	2256		
29 W		0117	1.5f
	0437	0803	1.7e
	1148	1343	1.1f
	1645	2015	1.6e
	2345		
30 Th		0212	1.6f
	0534	0855	1.8e
	1241	1442	1.1f
	1742	2103	1.8e
31 F	0036	0309	1.7f
	0628	0944	1.9e
	1333	1542	1.2f
	1836	2152	1.9e

FEBRUARY

Day	Slack Water Time	Maximum Current Time	Maximum Current Vel.
	h. m.	h. m.	kn.
1 Sa	0127	0405	1.9f
	0717	1032	2.1e
	1421	1636	1.5f
	1927	2240	2.0e
2 Su	0216	0456	2.1f
	0804	1120	2.2e
	1507	1724	1.7f
	2015	2331	2.2e
3 M	0305	0542	2.2f
	0850	1210	2.4e
	1549	1808	1.9f
	2104		
4 Tu		0023	2.3e
	0351	0627	2.3f
	0937	1259	2.5e
	1631	1851	2.1f
	2153		
5 W		0115	2.5e
	0440	0713	2.4f
	1025	1346	2.6e
	1714	1938	2.1f
	2245		
6 Th		0205	2.5e
	0528	0802	2.3f
	1112	1431	2.6e
	1759	2028	2.2f
	2337		
7 F		0253	2.5e
	0621	0852	2.1f
	1202	1517	2.5e
	1847	2121	2.1f
8 Sa	0030	0343	2.4e
	0718	0948	2.0f
	1251	1605	2.4e
	1941	2218	2.1f
9 Su	0125	0437	2.3e
	0821	1044	1.8f
	1342	1658	2.2e
	2039	2314	2.0f
10 M	0223	0539	2.1e
	0924	1141	1.6f
	1436	1801	2.0e
	2138		
11 Tu		0010	1.9f
	0323	0645	2.0e
	1027	1240	1.4f
	1534	1905	1.9e
	2239		
12 W		0111	1.9f
	0427	0750	1.9e
	1128	1348	1.3f
	1637	2007	1.9e
	2338		
13 Th		0226	1.8f
	0532	0850	2.0e
	1228	1514	1.3f
	1742	2103	1.9e
14 F	0036	0344	1.8f
	0630	0944	2.0e
	1325	1618	1.4f
	1841	2156	1.9e
15 Sa	0132	0441	1.8f
	0722	1034	2.0e
	1417	1711	1.5f
	1933	2247	2.0e

Day	Slack Water Time	Maximum Current Time	Maximum Current Vel.
	h. m.	h. m.	kn.
16 Su	0224	0527	1.9f
	0807	1122	2.1e
	1504	1753	1.6f
	2019	2338	2.0e
17 M	0312	0606	1.9f
	0848	1208	2.1e
	1548	1827	1.7f
	2103		
18 Tu		0024	2.0e
	0356	0636	1.9f
	0929	1251	2.1e
	1628	1857	1.7f
	2146		
19 W		0109	2.0e
	0438	0704	1.8f
	1009	1332	2.1e
	1706	1927	1.7f
	2228		
20 Th		0151	2.0e
	0519	0740	1.7f
	1048	1411	2.1e
	1744	2003	1.7f
	2310		
21 F		0231	2.0e
	0601	0819	1.6f
	1128	1449	2.0e
	1822	2043	1.6f
	2354		
22 Sa		0308	1.9e
	0645	0902	1.5f
	1208	1525	1.9e
	1903	2127	1.6f
23 Su	0036	0349	1.8e
	0734	0951	1.4f
	1249	1602	1.8e
	1946	2215	1.6f
24 M	0121	0432	1.7e
	0827	1039	1.3f
	1332	1643	1.7e
	2033	2303	1.5f
25 Tu	0209	0523	1.6e
	0921	1128	1.2f
	1420	1736	1.6e
	2125	2352	1.5f
26 W	0300	0625	1.6e
	1019	1217	1.1f
	1512	1840	1.5e
	2216		
27 Th		0042	1.5f
	0357	0726	1.7e
	1113	1310	1.1f
	1609	1940	1.6e
	2309		
28 F		0136	1.6f
	0456	0821	1.8e
	1205	1408	1.2f
	1710	2036	1.8e

Time meridian 75° W. 0000 is midnight. 1200 is noon.

TABLE 2.—CURRENT DIFFERENCES AND OTHER CONSTANTS

No.	PLACE	POSITION		TIME DIFFERENCES		VELOCITY RATIOS		MAXIMUM CURRENTS			
								Flood		Ebb	
		Lat.	Long.	Slack water	Maximum current	Maximum flood	Maximum ebb	Direction (true)	Average velocity	Direction (true)	Average velocity
		° ′	° ′	h. m.	h. m.			deg.	knots	deg.	knots
	HUDSON RIVER, Midchannel [1]	N.	W.	on THE NARROWS, p. 52 Time meridian, 75° W.							
1001	The Battery, northwest of	40 43	74 02	+1 30	+1 35	0.9	1.2	15	1.5	195	2.3
1003	Desbrosses Street	40 43	74 01	+1 35	+1 40	0.9	1.2	10	1.5	-------	2.3
1005	Chelsea Docks	40 45	74 01	+1 30	+1 40	0.9	1.2	10	1.6	-------	2.3
1007	Forty-second Street	40 46	74 00	+1 35	+1 45	1.0	1.2	30	1.7	-------	2.3
1009	Ninety-sixth Street	40 48	73 59	+1 40	+1 50	1.0	1.2	30	1.7	-------	2.3
1011	Grants Tomb, 123d Street	40 49	73 58	+1 45	+1 55	0.9	1.2	25	1.6	-------	2.3
1013	George Washington Bridge	40 51	73 57	+1 45	+2 00	0.9	1.1	20	1.6	200	2.2
1015	Spuyten Duyvil	40 53	73 56	+2 00	+2 10	0.9	1.1	20	1.6	-------	2.1
1017	Riverdale	40 54	73 55	+2 05	+2 20	0.8	1.0	15	1.4	200	2.0
1019	Dobbs Ferry	41 01	73 53	+2 25	+2 40	0.8	0.9	10	1.3	-------	1.7
1021	Tarrytown	41 05	73 53	+2 40	+2 55	0.6	0.8	0	1.1	-------	1.5
1023	Ossining	41 10	73 54	+2 55	+3 10	0.5	0.7	320	0.9	-------	1.3
1025	Haverstraw	41 12	73 57	+3 05	+3 15	0.5	0.7	335	0.8	-------	1.3
1027	Peekskill	41 17	73 57	+3 20	+3 35	0.5	0.6	0	0.8	-------	1.2
1029	Bear Mountain Bridge	41 19	73 59	+3 25	+3 40	0.5	0.6	0	0.8	-------	1.1
1031	Highland Falls	41 22	73 58	+3 35	+3 50	0.6	0.6	5	1.0	185	1.2
1033	West Point, off Duck Island	41 24	73 57	+3 40	+3 55	0.5	0.6	10	1.0	-------	1.1
1035	Newburgh	41 30	74 00	+3 55	+4 15	0.5	0.6	5	0.9	-------	1.1
1037	New Hamburg	41 35	73 57	+4 10	+4 25	0.6	0.6	5	1.0	-------	1.1
1039	Poughkeepsie	41 42	73 57	+4 25	+4 45	0.6	0.6	5	1.1	-------	1.2
1041	Hyde Park	41 47	73 57	+4 35	+4 55	0.7	0.7	5	1.2	-------	1.3
1043	Kingston Point [2]	41 56	73 57	+5 00	+5 15	0.8	0.8	5	1.3	-------	1.6
1045	Barrytown	42 00	73 56	+5 20	+5 25	0.8	0.9	10	1.4	-------	1.7
1047	Saugerties	42 04	73 56	+5 35	+5 40	0.9	1.0	0	1.5	-------	1.9
1049	Silver Point	42 09	73 54	+5 55	+6 00	0.9	1.0	30	1.5	-------	2.0
1051	Catskill	42 13	73 51	+6 10	+6 20	0.9	1.0	355	1.6	-------	2.0
1053	Hudson	42 15	73 48	+6 20	+6 30	0.9	1.0	30	1.6	-------	2.0
1055	Coxsackie	42 21	73 47	+6 50	+6 50	0.9	0.9	350	1.6	-------	1.8
1057	New Baltimore	42 27	73 47	+7 10	+7 05	0.8	0.8	355	1.3	-------	1.5
1059	Castleton-on-Hudson	42 32	73 46	+7 25	+7 20	0.5	0.6	15	0.9	-------	1.2
1061	Albany	42 39	73 45	+7 35	+7 40	0.2	0.4	20	0.3	-------	0.8
1063	Troy (below the locks)	42 44	73 42	--------	--------	-------	-------	(3)	(3)	190	0.7
	NEW YORK HARBOR, Lower Bay										
1065	False Hook Channel	40 28	74 00	−1 45	−1 30	1.1	0.7	320	1.8	135	1.4
1067	Sandy Hook and South Channels (junction)	40 29	73 59	−1 20	−1 20	0.8	0.8	300	1.3	115	1.7
1069	Sandy Hook Channel, off Sandy Hook Point	40 29	74 01	−1 55	−1 55	1.1	0.9	255	1.8	55	1.8
1071	Sandy Hook Point, 2 miles W. of (channel)	40 29	74 04	−1 45	−1 50	0.4	0.3	265	0.6	85	0.6
1073	New Dorp Beach, 1¼ miles south of	40 32	74 06	−4 25	−3 55	0.2	0.2	225	0.4	30	0.5
1075	New Dorp Beach, 1¾ miles SE. of	40 33	74 04	(4)	(4)	0.3	0.3	-------	0.5	-------	0.5
1077	Hoffman Island, ¼ mile west of	40 35	74 04	(5)	(5)	0.5	0.4	20	0.9	210	0.8
1079	Rockaway Inlet Jetty, 1 mile SW. of	40 32	73 57	−1 50	−1 55	0.7	0.7	285	1.2	140	1.4
1081	Coney Island Channel, west end	40 34	74 00	−0 50	−0 45	0.6	0.6	295	1.1	100	1.2
	SANDY HOOK BAY [6]										
1083	Highlands Bridge, Shrewsbury River	40 24	73 59	+0 25	+0 25	1.5	1.3	170	2.6	-------	2.5
1085	Seabright Bridge, Shrewsbury River	40 22	73 58	+0 55	+1 00	0.8	0.9	185	1.4	-------	1.7

[1] The values for the Hudson River are for the summer months, when the fresh-water discharge is a minimum.

[2] In Roundout Creek entrance between lights, eddies on the flood make navigation difficult. Little difficulty will be experienced on the ebb.

[3] Current does not flood.

[4] Current is rotary, turning clockwise. It flows NW. at time of "Slack, flood begins" at The Narrows; NE. 1 hour after maximum flood; SE. 1½ hours after "Slack, ebb begins"; and SW. 2 hours after maximum ebb.

[5] Flood begins, $-1^h\,45^m$; maximum flood, $-1^h\,50^m$; ebb begins, $-0^h\,15^m$; maximum ebb, $-0^h\,50^m$.

[6] In Sandy Hook Bay (except in southern extremity) the current is weak.

TABLE 3.—VELOCITY OF CURRENT AT ANY TIME

TABLE A

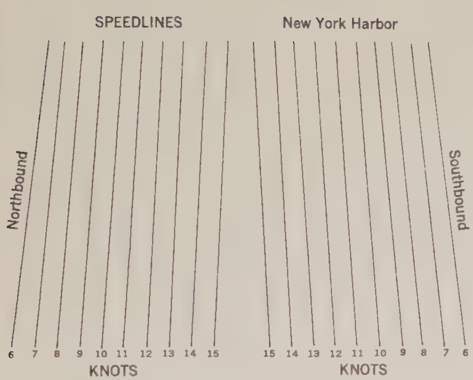
Interval between slack and desired time (h. m.)	Interval between slack and maximum current: 1 20	1 40	2 00	2 20	2 40	3 00	3 20	3 40	4 00	4 20	4 40	5 00	5 20	5 40
h. m.	f.	f.	f.	f.	f.	f.	f.	f.	f.	f.	f.	f.	f.	f.
0 20	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0 40	0.7	0.6	0.5	0.4	0.4	0.3	0.3	0.3	0.3	0.2	0.2	0.2	0.2	0.2
1 00	0.9	0.8	0.7	0.6	0.6	0.5	0.5	0.4	0.4	0.4	0.3	0.3	0.3	0.3
1 20	1.0	1.0	0.9	0.8	0.7	0.6	0.6	0.5	0.5	0.5	0.4	0.4	0.4	0.4
1 40	------	1.0	1.0	0.9	0.8	0.8	0.7	0.7	0.6	0.6	0.5	0.5	0.5	0.4
2 00	------	------	1.0	1.0	0.9	0.9	0.8	0.8	0.7	0.7	0.6	0.6	0.6	0.5
2 20	------	------	------	1.0	1.0	0.9	0.9	0.8	0.8	0.7	0.7	0.7	0.6	0.6
2 40	------	------	------	------	1.0	1.0	1.0	0.9	0.9	0.8	0.8	0.7	0.7	0.7
3 00	------	------	------	------	------	1.0	1.0	1.0	0.9	0.9	0.8	0.8	0.8	0.7
3 20	------	------	------	------	------	------	1.0	1.0	1.0	0.9	0.9	0.9	0.8	0.8
3 40	------	------	------	------	------	------	------	1.0	1.0	1.0	0.9	0.9	0.9	0.9
4 00	------	------	------	------	------	------	------	------	1.0	1.0	1.0	1.0	0.9	0.9
4 20	------	------	------	------	------	------	------	------	------	1.0	1.0	1.0	1.0	0.9
4 40	------	------	------	------	------	------	------	------	------	------	1.0	1.0	1.0	1.0
5 00	------	------	------	------	------	------	------	------	------	------	------	1.0	1.0	1.0
5 20	------	------	------	------	------	------	------	------	------	------	------	------	1.0	1.0
5 40	------	------	------	------	------	------	------	------	------	------	------	------	------	1.0

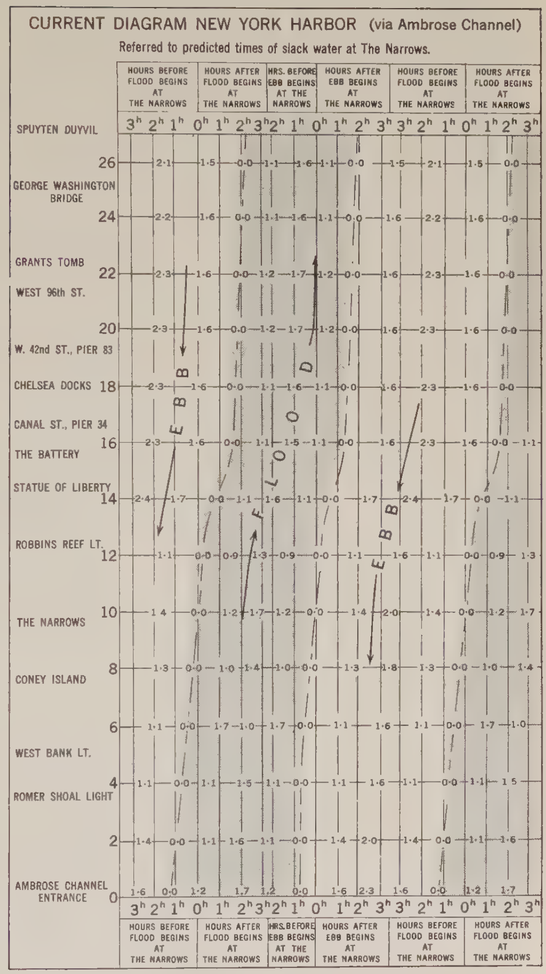
TABLE B

Interval between slack and desired time (h. m.)	Interval between slack and maximum current: 1 20	1 40	2 00	2 20	2 40	3 00	3 20	3 40	4 00	4 20	4 40	5 00	5 20	5 40
h. m.	f.	f.	f.	f.	f.	f.	f.	f.	f.	f.	f.	f.	f.	f.
0 20	0.5	0.4	0.4	0.3	0.3	0.3	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.2
0 40	0.8	0.7	0.6	0.5	0.5	0.5	0.4	0.4	0.4	0.4	0.3	0.3	0.3	0.3
1 00	0.9	0.8	0.8	0.7	0.7	0.6	0.6	0.5	0.5	0.5	0.4	0.4	0.4	0.4
1 20	1.0	1.0	0.9	0.8	0.8	0.7	0.7	0.6	0.6	0.6	0.5	0.5	0.5	0.5
1 40	------	1.0	1.0	0.9	0.9	0.8	0.8	0.7	0.7	0.7	0.6	0.6	0.6	0.6
2 00	------	------	1.0	1.0	0.9	0.9	0.9	0.8	0.8	0.7	0.7	0.7	0.7	0.6
2 20	------	------	------	1.0	1.0	1.0	0.9	0.9	0.8	0.8	0.8	0.7	0.7	0.7
2 40	------	------	------	------	1.0	1.0	1.0	0.9	0.9	0.9	0.8	0.8	0.8	0.7
3 00	------	------	------	------	------	1.0	1.0	1.0	0.9	0.9	0.9	0.9	0.8	0.8
3 20	------	------	------	------	------	------	1.0	1.0	1.0	1.0	0.9	0.9	0.9	0.8
3 40	------	------	------	------	------	------	------	1.0	1.0	1.0	1.0	0.9	0.9	0.9
4 00	------	------	------	------	------	------	------	------	1.0	1.0	1.0	1.0	0.9	0.9
4 20	------	------	------	------	------	------	------	------	------	1.0	1.0	1.0	1.0	0.9
4 40	------	------	------	------	------	------	------	------	------	------	1.0	1.0	1.0	1.0
5 00	------	------	------	------	------	------	------	------	------	------	------	1.0	1.0	1.0
5 20	------	------	------	------	------	------	------	------	------	------	------	------	1.0	1.0
5 40	------	------	------	------	------	------	------	------	------	------	------	------	------	1.0

Use Table A for all places except those listed below for Table B.
Use Table B for Cape Cod Canal, Hell Gate, Chesapeake and Delaware Canal and all stations in Table 2 which are referred to them.

1. From predictions find the time of slack water and the time and velocity of maximum current (flood or ebb), one of which is immediately before and the other after the time for which the velocity is desired.
2. Find the interval of time between the above slack and maximum current, and enter the top of Table A or B with the interval which most nearly agrees with this value.
3. Find the interval of time between the above slack and the time desired, and enter the side of Table A or B with the interval which most nearly agrees with this value.
4. Find, in the table, the factor corresponding to the above two intervals and multiply the maximum velocity by this factor. The result will be the approximate velocity at the time desired.


SPEEDLINES
New York Harbor
Northbound
Southbound
6 7 8 9 10 11 12 13 14 15
KNOTS
15 14 13 12 11 10 9 8 7 6
KNOTS


CURRENT DIAGRAM NEW YORK HARBOR (via Ambrose Channel)
Referred to predicted times of slack water at The Narrows.
HOURS BEFORE FLOOD BEGINS AT THE NARROWS
HOURS AFTER FLOOD BEGINS AT THE NARROWS
HRS. BEFORE EBB BEGINS AT THE NARROWS
HOURS AFTER EBB BEGINS AT THE NARROWS
SPUYTEN DUYVIL
GEORGE WASHINGTON BRIDGE
GRANTS TOMB
WEST 96th ST.
W. 42nd ST., PIER 83
CHELSEA DOCKS
CANAL ST., PIER 34
THE BATTERY
STATUE OF LIBERTY
ROBBINS REEF LT.
THE NARROWS
CONEY ISLAND
WEST BANK LT.
ROMER SHOAL LIGHT
AMBROSE CHANNEL ENTRANCE
EBB
FLOOD
EBB

APPENDIX V

EXTRACTS FROM *NAUTICAL ALMANAC*

ALTITUDE CORRECTION TABLES 10°–90°—SUN, STARS, PLANETS

SUN

OCT.–MAR. App. Alt.	Lower Limb	Upper Limb	APR.–SEPT. App. Alt.	Lower Limb	Upper Limb
° ′	′	′	° ′	′	′
9 34	+10·8	−22·7	9 39	+10·6	−22·4
9 45	+10·9	−22·6	9 51	+10·7	−22·3
9 56	+11·0	−22·5	10 03	+10·8	−22·2
10 08	+11·1	−22·4	10 15	+10·9	−22·1
10 21	+11·2	−22·3	10 27	+11·0	−22·0
10 34	+11·3	−22·2	10 40	+11·1	−21·9
10 47	+11·4	−22·1	10 54	+11·2	−21·8
11 01	+11·5	−22·0	11 08	+11·3	−21·7
11 15	+11·6	−21·9	11 23	+11·4	−21·6
11 30	+11·7	−21·8	11 38	+11·5	−21·5
11 46	+11·8	−21·7	11 54	+11·6	−21·4
12 02	+11·9	−21·6	12 10	+11·7	−21·3
12 19	+12·0	−21·5	12 28	+11·8	−21·2
12 37	+12·1	−21·4	12 46	+11·9	−21·1
12 55	+12·2	−21·3	13 05	+12·0	−21·0
13 14	+12·3	−21·2	13 24	+12·1	−20·9
13 35	+12·4	−21·1	13 45	+12·2	−20·8
13 56	+12·5	−21·0	14 07	+12·3	−20·7
14 18	+12·6	−20·9	14 30	+12·4	−20·6
14 42	+12·7	−20·8	14 54	+12·5	−20·5
15 06	+12·8	−20·7	15 19	+12·6	−20·4
15 32	+12·9	−20·6	15 46	+12·7	−20·3
15 59	+13·0	−20·5	16 14	+12·8	−20·2
16 28	+13·1	−20·4	16 44	+12·9	−20·1
16 59	+13·2	−20·3	17 15	+13·0	−20·0
17 32	+13·3	−20·2	17 48	+13·1	−19·9
18 06	+13·4	−20·1	18 24	+13·2	−19·8
18 42	+13·5	−20·0	19 01	+13·3	−19·7
19 21	+13·6	−19·9	19 42	+13·4	−19·6
20 03	+13·7	−19·8	20 25	+13·5	−19·5
20 48	+13·8	−19·7	21 11	+13·6	−19·4
21 35	+13·9	−19·6	22 00	+13·7	−19·3
22 26	+14·0	−19·5	22 54	+13·8	−19·2
23 22	+14·1	−19·4	23 51	+13·9	−19·1
24 21	+14·2	−19·3	24 53	+14·0	−19·0
25 26	+14·3	−19·2	26 00	+14·1	−18·9
26 36	+14·4	−19·1	27 13	+14·2	−18·8
27 52	+14·5	−19·0	28 33	+14·3	−18·7
29 15	+14·6	−18·9	30 00	+14·4	−18·6
30 46	+14·7	−18·8	31 35	+14·5	−18·5
32 26	+14·8	−18·7	33 20	+14·6	−18·4
34 17	+14·9	−18·6	35 17	+14·7	−18·3
36 20	+15·0	−18·5	37 26	+14·8	−18·2
38 36	+15·1	−18·4	39 50	+14·9	−18·1
41 08	+15·2	−18·3	42 31	+15·0	−18·0
43 59	+15·3	−18·2	45 31	+15·1	−17·9
47 10	+15·4	−18·1	48 55	+15·2	−17·8
50 46	+15·5	−18·0	52 44	+15·3	−17·7
54 49	+15·6	−17·9	57 02	+15·4	−17·6
59 23	+15·7	−17·8	61 51	+15·5	−17·5
64 30	+15·8	−17·7	67 17	+15·6	−17·4
70 12	+15·9	−17·6	73 16	+15·7	−17·3
76 26	+16·0	−17·5	79 43	+15·8	−17·2
83 05	+16·1	−17·4	86 32	+15·9	−17·1
90 00			90 00		

STARS AND PLANETS

App. Alt.	Corrⁿ
° ′	′
9 56	−5·3
10 08	−5·2
10 20	−5·1
10 33	−5·0
10 46	−4·9
11 00	−4·8
11 14	−4·7
11 29	−4·6
11 45	−4·5
12 01	−4·4
12 18	−4·3
12 35	−4·2
12 54	−4·1
13 13	−4·0
13 33	−3·9
13 54	−3·8
14 16	−3·7
14 40	−3·6
15 04	−3·5
15 30	−3·4
15 57	−3·3
16 26	−3·2
16 56	−3·1
17 28	−3·0
18 02	−2·9
18 38	−2·8
19 17	−2·7
19 58	−2·6
20 42	−2·5
21 28	−2·4
22 19	−2·3
23 13	−2·2
24 11	−2·1
25 14	−2·0
26 22	−1·9
27 36	−1·8
28 56	−1·7
30 24	−1·6
32 00	−1·5
33 45	−1·4
35 40	−1·3
37 48	−1·2
40 08	−1·1
42 44	−1·0
45 36	−0·9
48 47	−0·8
52 18	−0·7
56 11	−0·6
60 28	−0·5
65 08	−0·4
70 11	−0·3
75 34	−0·2
81 13	−0·1
87 03	0·0
90 00	

App. Alt. — Additional Corrⁿ

1958

VENUS

Period	App. Alt. (°)	Additional Corrⁿ (′)
Jan. 1–Jan. 10	0	+0·5
	6	+0·6
	20	+0·7
	31	
Jan. 11–Feb. 14	0	+0·6
	4	+0·7
	12	+0·8
	22	
Feb. 15–Feb. 21	0	+0·5
	6	+0·6
	20	+0·7
	31	
Feb. 22–Mar. 9	0	+0·4
	11	+0·5
	41	
Mar. 10–Apr. 4	0	+0·3
	46	
Apr. 5–May 19	0	+0·2
	47	
May 20–Dec. 31	0	+0·1
	42	

MARS

Period	App. Alt. (°)	Additional Corrⁿ (′)
Jan. 1–Sept. 3	0	+0·1
	60	
Sept. 4–Dec. 31	0	+0·3
	34	+0·2
	60	+0·1
	80	

DIP

Ht. of Eye	Corrⁿ	Ht. of Eye	Corrⁿ
ft.	′	ft.	′
1·1	−1·1	44	− 6·5
1·4	−1·2	45	− 6·6
1·6	−1·3	47	− 6·7
1·9	−1·4	48	− 6·8
2·2	−1·5	49	− 6·9
2·5	−1·6	51	− 7·0
2·8	−1·7	52	− 7·1
3·2	−1·8	54	− 7·2
3·6	−1·9	55	− 7·3
4·0	−2·0	57	− 7·4
4·4	−2·1	58	− 7·5
4·9	−2·2	60	− 7·6
5·3	−2·3	62	− 7·7
5·8	−2·4	63	− 7·8
6·3	−2·5	65	− 7·9
6·9	−2·6	67	− 8·0
7·4	−2·7	68	− 8·1
8·0	−2·8	70	− 8·2
8·6	−2·9	72	− 8·3
9·2	−3·0	74	− 8·4
9·8	−3·1	75	− 8·5
10·5	−3·2	77	− 8·6
11·2	−3·3	79	− 8·7
11·9	−3·4	81	− 8·8
12·6	−3·5	83	− 8·9
13·3	−3·6	85	− 9·0
14·1	−3·7	87	− 9·1
14·9	−3·8	88	− 9·2
15·7	−3·9	90	− 9·3
16·5	−4·0	92	− 9·4
17·4	−4·1	94	− 9·5
18·3	−4·2	96	− 9·6
19·1	−4·3	98	− 9·7
20·1	−4·4	101	− 9·8
21·0	−4·5	103	− 9·9
22·0	−4·6	105	−10·0
22·9	−4·7	107	−10·1
23·9	−4·8	109	−10·2
24·9	−4·9	111	−10·3
26·0	−5·0	113	−10·4
27·1	−5·1	116	−10·5
28·1	−5·2	118	−10·6
29·2	−5·3	120	−10·7
30·4	−5·4	122	−10·8
31·5	−5·5	125	−10·9
32·7	−5·6	127	−11·0
33·9	−5·7	129	−11·1
35·1	−5·8	132	−11·2
36·3	−5·9	134	−11·3
37·6	−6·0	136	−11·4
38·9	−6·1	139	−11·5
40·1	−6·2	141	−11·6
41·5	−6·3	144	−11·7
42·8	−6·4	146	−11·8
44·2		149	

App. Alt. = Apparent altitude = Sextant altitude corrected for index error and dip.

ALTITUDE CORRECTION TABLES 0°–10°—SUN, STARS, PLANETS

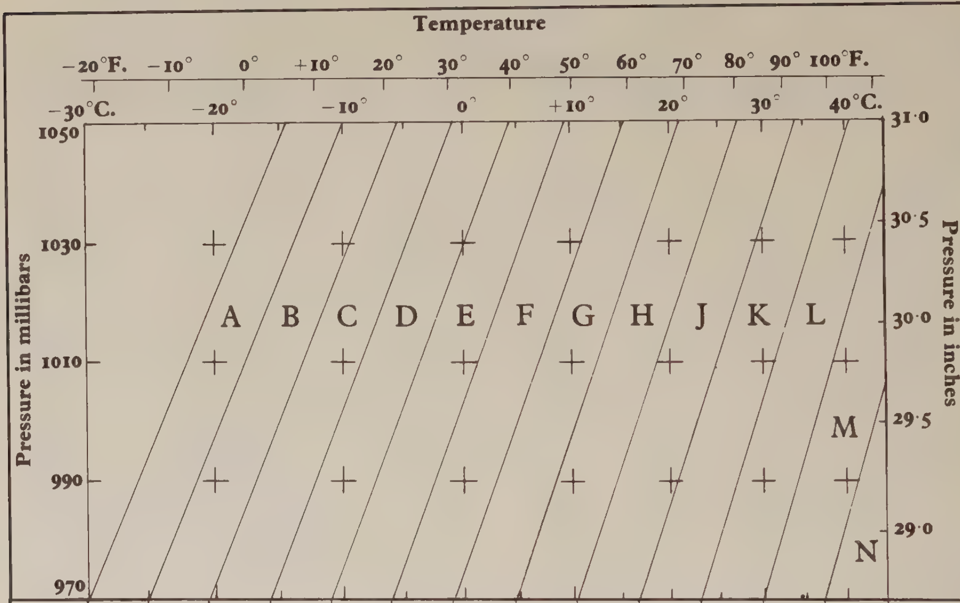
App. Alt.	OCT.–MAR. SUN Lower Limb	OCT.–MAR. SUN Upper Limb	APR.–SEPT. SUN Lower Limb	APR.–SEPT. SUN Upper Limb	STARS PLANETS
° ′	′	′	′	′	′
0 00	−18·2	−51·7	−18·4	−51·4	−34·5
03	17·5	51·0	17·8	50·8	33·8
06	16·9	50·4	17·1	50·1	33·2
09	16·3	49·8	16·5	49·5	32·6
12	15·7	49·2	15·9	48·9	32·0
15	15·1	48·6	15·3	48·3	31·4
0 18	−14·5	−48·0	−14·8	−47·8	−30·8
21	14·0	47·5	14·2	47·2	30·3
24	13·5	47·0	13·7	46·7	29·8
27	12·9	46·4	13·2	46·2	29·2
30	12·4	45·9	12·7	45·7	28·7
33	11·9	45·4	12·2	45·2	28·2
0 36	−11·5	−45·0	−11·7	−44·7	−27·8
39	11·0	44·5	11·2	44·2	27·3
42	10·5	44·0	10·8	43·8	26·8
45	10·1	43·6	10·3	43·3	26·4
48	9·6	43·1	9·9	42·9	25·9
51′	9·2	42·7	9·5	42·5	25·5
0 54	− 8·8	−42·3	− 9·1	−42·1	−25·1
0 57	8·4	41·9	8·7	41·7	24·7
1 00	8·0	41·5	8·3	41·3	24·3
03	7·7	41·2	7·9	40·9	24·0
06	7·3	40·8	7·5	40·5	23·6
09	6·9	40·4	7·2	40·2	23·2
1 12	− 6·6	−40·1	− 6·8	−39·8	−22·9
15	6·2	39·7	6·5	39·5	22·5
18	5·9	39·4	6·2	39·2	22·2
21	5·6	39·1	5·8	38·8	21·9
24	5·3	38·8	5·5	38·5	21·6
27	4·9	38·4	5·2	38·2	21·2
1 30	− 4·6	−38·1	− 4·9	−37·9	−20·9
35	4·2	37·7	4·4	37·4	20·5
40	3·7	37·2	4·0	37·0	20·0
45	3·2	36·7	3·5	36·5	19·5
50	2·8	36·3	3·1	36·1	19·1
1 55	2·4	35·9	2·6	35·6	18·7
2 00	− 2·0	−35·5	− 2·2	−35·2	−18·3
05	1·6	35·1	1·8	34·8	17·9
10	1·2	34·7	1·5	34·5	17·5
15	0·9	34·4	1·1	34·1	17·2
20	0·5	34·0	0·8	33·8	16·8
25	− 0·2	33·7	0·4	33·4	16·5
2 30	+ 0·2	−33·3	− 0·1	−33·1	−16·1
35	0·5	33·0	+ 0·2	32·8	15·8
40	0·8	32·7	0·5	32·5	15·5
45	1·1	32·4	0·8	32·2	15·2
50	1·4	32·1	1·1	31·9	14·9
2 55	1·6	31·9	1·4	31·6	14·7
3 00	+ 1·9	−31·6	+ 1·7	−31·3	−14·4
05	2·2	31·3	1·9	31·1	14·1
10	2·4	31·1	2·1	30·9	13·9
15	2·6	30·9	2·4	30·6	13·7
20	2·9	30·6	2·6	30·4	13·4
25	3·1	30·4	2·9	30·1	13·2
3 30	+ 3·3	−30·2	+ 3·1	−29·9	−13·0

App. Alt.	OCT.–MAR. SUN Lower Limb	OCT.–MAR. SUN Upper Limb	APR.–SEPT. SUN Lower Limb	APR.–SEPT. SUN Upper Limb	STARS PLANETS
° ′	′	′	′	′	′
3 30	+ 3·3	−30·2	+ 3·1	−29·9	−13·0
35	3·6	29·9	3·3	29·7	12·7
40	3·8	29·7	3·5	29·5	12·5
45	4·0	29·5	3·7	29·3	12·3
50	4·2	29·3	3·9	29·1	12·1
3 55	4·4	29·1	4·1	28·9	11·9
4 00	+ 4·5	−29·0	+ 4·3	−28·7	−11·8
05	4·7	28·8	4·5	28·5	11·6
10	4·9	28·6	4·6	28·4	11·4
15	5·1	28·4	4·8	28·2	11·2
20	5·2	28·3	5·0	28·0	11·1
25	5·4	28·1	5·1	27·9	10·9
4 30	+ 5·6	−27·9	+ 5·3	−27·7	−10·7
35	5·7	27·8	5·5	27·5	10·6
40	5·9	27·6	5·6	27·4	10·4
45	6·0	27·5	5·8	27·2	10·3
50	6·2	27·3	5·9	27·1	10·1
4 55	6·3	27·2	6·0	27·0	10·0
5 00	+ 6·4	−27·1	+ 6·2	−26·8	− 9·9
05	6·6	26·9	6·3	26·7	9·7
10	6·7	26·8	6·4	26·6	9·6
15	6·8	26·7	6·6	26·4	9·5
20	6·9	26·6	6·7	26·3	9·4
25	7·1	26·4	6·8	26·2	9·2
5 30	+ 7·2	−26·3	+ 6·9	−26·1	− 9·1
35	7·3	26·2	7·0	26·0	9·0
40	7·4	26·1	7·2	25·8	8·9
45	7·5	26·0	7·3	25·7	8·8
50	7·6	25·9	7·4	25·6	8·7
5 55	7·7	25·8	7·5	25·5	8·6
6 00	+ 7·8	−25·7	+ 7·6	−25·4	− 8·5
10	8·0	25·5	7·8	25·2	8·3
20	8·2	25·3	8·0	25·0	8·1
30	8·4	25·1	8·1	24·9	7·9
40	8·6	24·9	8·3	24·7	7·7
6 50	8·7	24·8	8·5	24·5	7·6
7 00	+ 8·9	−24·6	+ 8·6	−24·4	− 7·4
10	9·1	24·4	8·8	24·2	7·2
20	9·2	24·3	9·0	24·0	7·1
30	9·3	24·2	9·1	23·9	7·0
40	9·5	24·0	9·2	23·8	6·8
7 50	9·6	23·9	9·4	23·6	6·7
8 00	+ 9·7	−23·8	+ 9·5	−23·5	− 6·6
10	9·9	23·6	9·6	23·4	6·4
20	10·0	23·5	9·7	23·3	6·3
30	10·1	23·4	9·8	23·2	6·2
40	10·2	23·3	10·0	23·0	6·1
8 50	10·3	23·2	10·1	22·9	6·0
9 00	+10·4	−23·1	+10·2	−22·8	− 5·9
10	10·5	23·0	10·3	22·7	5·8
20	10·6	22·9	10·4	22·6	5·7
30	10·7	22·8	10·5	22·5	5·6
40	10·8	22·7	10·6	22·4	5·5
9 50	10·9	22·6	10·6	22·4	5·4
10 00	+11·0	−22·5	+10·7	−22·3	− 5·3

Additional corrections for temperature and pressure are given on the following page.
For bubble sextant observations ignore dip and use the star corrections for Sun, planets, and stars.

ALTITUDE CORRECTION TABLES—ADDITIONAL CORRECTIONS

ADDITIONAL REFRACTION CORRECTIONS FOR NON-STANDARD CONDITIONS



App. Alt.	A	B	C	D	E	F	G	H	J	K	L	M	N	App. Alt.
° ′	′	′	′	′	′	′	′	′	′	′	′	′	′	° ′
0 00	−6·9	−5·7	−4·6	−3·4	−2·3	−1·1	0·0	+1·1	+2·3	+3·4	+4·6	+5·7	+6·9	0 00
0 30	5·2	4·4	3·5	2·6	1·7	0·9	0·0	0·9	1·7	2·6	3·5	4·4	5·2	0 30
1 00	4·3	3·5	2·8	2·1	1·4	0·7	0·0	0·7	1·4	2·1	2·8	3·5	4·3	1 00
1 30	3·5	2·9	2·4	1·8	1·2	0·6	0·0	0·6	1·2	1·8	2·4	2·9	3·5	1 30
2 00	3·0	2·5	2·0	1·5	1·0	0·5	0·0	0·5	1·0	1·5	2·0	2·5	3·0	2 00
2 30	−2·5	−2·1	−1·6	−1·2	−0·8	−0·4	0·0	+0·4	+0·8	+1·2	+1·6	+2·1	+2·5	2 30
3 00	2·2	1·8	1·5	1·1	0·7	0·4	0·0	0·4	0·7	1·1	1·5	1·8	2·2	3 00
3 30	2·0	1·6	1·3	1·0	0·7	0·3	0·0	0·3	0·7	1·0	1·3	1·6	2·0	3 30
4 00	1·8	1·5	1·2	0·9	0·6	0·3	0·0	0·3	0·6	0·9	1·2	1·5	1·8	4 00
4 30	1·6	1·4	1·1	0·8	0·5	0·3	0·0	0·3	0·5	0·8	1·1	1·4	1·6	4 30
5 00	−1·5	−1·3	−1·0	−0·8	−0·5	−0·2	0·0	+0·2	+0·5	+0·8	+1·0	+1·3	+1·5	5 00
6	1·3	1·1	0·9	0·6	0·4	0·2	0·0	0·2	0·4	0·6	0·9	1·1	1·3	6
7	1·1	0·9	0·7	0·6	0·4	0·2	0·0	0·2	0·4	0·6	0·7	0·9	1·1	7
8	1·0	0·8	0·7	0·5	0·3	0·2	0·0	0·2	0·3	0·5	0·7	0·8	1·0	8
9	0·9	0·7	0·6	0·4	0·3	0·1	0·0	0·1	0·3	0·4	0·6	0·7	0·9	9
10 00	−0·8	−0·7	−0·5	−0·4	−0·3	−0·1	0·0	+0·1	+0·3	+0·4	+0·5	+0·7	+0·8	10 00
12	0·7	0·6	0·5	0·3	0·2	0·1	0·0	0·1	0·2	0·3	0·5	0·6	0·7	12
14	0·6	0·5	0·4	0·3	0·2	0·1	0·0	0·1	0·2	0·3	0·4	0·5	0·6	14
16	0·5	0·4	0·3	0·3	0·2	0·1	0·0	0·1	0·2	0·3	0·3	0·4	0·5	16
18	0·4	0·4	0·3	0·2	0·2	0·1	0·0	0·1	0·2	0·2	0·3	0·4	0·4	18
20 00	−0·4	−0·3	−0·3	−0·2	−0·1	−0·1	0·0	+0·1	+0·1	+0·2	+0·3	+0·3	+0·4	20 00
25	0·3	0·3	0·2	0·2	0·1	−0·1	0·0	+0·1	0·1	0·2	0·2	0·3	0·3	25
30	0·3	0·2	0·2	0·1	0·1	0·0	0·0	0·0	0·1	0·1	0·2	0·2	0·3	30
35	0·2	0·2	0·1	0·1	0·1	0·0	0·0	0·0	0·1	0·1	0·1	0·2	0·2	35
40	0·2	0·1	0·1	0·1	−0·1	0·0	0·0	0·0	+0·1	0·1	0·1	0·1	0·2	40
50 00	−0·1	−0·1	−0·1	−0·1	0·0	0·0	0·0	0·0	0·0	+0·1	+0·1	+0·1	+0·1	50 00

The graph is entered with arguments temperature and pressure to find a zone letter; using as arguments this zone letter and apparent altitude (sextant altitude corrected for dip), a correction is taken from the table. This correction is to be applied to the sextant altitude in addition to the corrections for standard conditions (for the Sun, planets and stars from the inside front cover and for the Moon from the inside back cover).

CALENDAR, 1958

DAYS OF THE WEEK AND DAYS OF THE YEAR

Day of Month	JAN. Week	JAN. Year	FEB. Week	FEB. Year	MAR. Week	MAR. Year	APR. Week	APR. Year	MAY Week	MAY Year	JUNE Week	JUNE Year
1	W.	1	S.	32	S.	60	Tu.	91	Th.	121	✻.	152
2	Th.	2	✻.	33	✻.	61	W.	92	F.	122	M.	153
3	F.	3	M.	34	M.	62	Th.	93	S.	123	Tu.	154
4	S.	4	Tu.	35	Tu.	63	F.	94	✻.	124	W.	155
5	✻.	5	W.	36	W.	64	S.	95	M.	125	Th.	156
6	M.	6	Th.	37	Th.	65	✻.	96	Tu.	126	F.	157
7	Tu.	7	F.	38	F.	66	M.	97	W.	127	S.	158
8	W.	8	S.	39	S.	67	Tu.	98	Th.	128	✻.	159
9	Th.	9	✻.	40	✻.	68	W.	99	F.	129	M.	160
10	F.	10	M.	41	M.	69	Th.	100	S.	130	Tu.	161
11	S.	11	Tu.	42	Tu.	70	F.	101	✻.	131	W.	162
12	✻.	12	W.	43	W.	71	S.	102	M.	132	Th.	163
13	M.	13	Th.	44	Th.	72	✻.	103	Tu.	133	F.	164
14	Tu.	14	F.	45	F.	73	M.	104	W.	134	S.	165
15	W.	15	S.	46	S.	74	Tu.	105	Th.	135	✻.	166
16	Th.	16	✻.	47	✻.	75	W.	106	F.	136	M.	167
17	F.	17	M.	48	M.	76	Th.	107	S.	137	Tu.	168
18	S.	18	Tu.	49	Tu.	77	F.	108	✻.	138	W.	169
19	✻.	19	W.	50	W.	78	S.	109	M.	139	Th.	170
20	M.	20	Th.	51	Th.	79	✻.	110	Tu.	140	F.	171
21	Tu.	21	F.	52	F.	80	M.	111	W.	141	S.	172
22	W.	22	S.	53	S.	81	Tu.	112	Th.	142	✻.	173
23	Th.	23	✻.	54	✻.	82	W.	113	F.	143	M.	174
24	F.	24	M.	55	M.	83	Th.	114	S.	144	Tu.	175
25	S.	25	Tu.	56	Tu.	84	F.	115	✻.	145	W.	176
26	✻.	26	W.	57	W.	85	S.	116	M.	146	Th.	177
27	M.	27	Th.	58	Th.	86	✻.	117	Tu.	147	F.	178
28	Tu.	28	F.	59	F.	87	M.	118	W.	148	S.	179
29	W.	29			S.	88	Tu.	119	Th.	149	✻.	180
30	Th.	30			✻.	89	W.	120	F.	150	M.	181
31	F.	31			M.	90			S.	151		

Day of Month	JULY Week	JULY Year	AUG. Week	AUG. Year	SEPT. Week	SEPT. Year	OCT. Week	OCT. Year	NOV. Week	NOV. Year	DEC. Week	DEC. Year
1	Tu.	182	F.	213	M.	244	W.	274	S.	305	M.	335
2	W.	183	S.	214	Tu.	245	Th.	275	✻.	306	Tu.	336
3	Th.	184	✻.	215	W.	246	F.	276	M.	307	W.	337
4	F.	185	M.	216	Th.	247	S.	277	Tu.	308	Th.	338
5	S.	186	Tu.	217	F.	248	✻.	278	W.	309	F.	339
6	✻.	187	W.	218	S.	249	M.	279	Th.	310	S.	340
7	M.	188	Th.	219	✻.	250	Tu.	280	F.	311	✻.	341
8	Tu.	189	F.	220	M.	251	W.	281	S.	312	M.	342
9	W.	190	S.	221	Tu.	252	Th.	282	✻.	313	Tu.	343
10	Th.	191	✻.	222	W.	253	F.	283	M.	314	W.	344
11	F.	192	M.	223	Th.	254	S.	284	Tu.	315	Th.	345
12	S.	193	Tu.	224	F.	255	✻.	285	W.	316	F.	346
13	✻.	194	W.	225	S.	256	M.	286	Th.	317	S.	347
14	M.	195	Th.	226	✻.	257	Tu.	287	F.	318	✻.	348
15	Tu.	196	F.	227	M.	258	W.	288	S.	319	M.	349
16	W.	197	S.	228	Tu.	259	Th.	289	✻.	320	Tu.	350
17	Th.	198	✻.	229	W.	260	F.	290	M.	321	W.	351
18	F.	199	M.	230	Th.	261	S.	291	Tu.	322	Th.	352
19	S.	200	Tu.	231	F.	262	✻.	292	W.	323	F.	353
20	✻.	201	W.	232	S.	263	M.	293	Th.	324	S.	354
21	M.	202	Th.	233	✻.	264	Tu.	294	F.	325	✻.	355
22	Tu.	203	F.	234	M.	265	W.	295	S.	326	M.	356
23	W.	204	S.	235	Tu.	266	Th.	296	✻.	327	Tu.	357
24	Th.	205	✻.	236	W.	267	F.	297	M.	328	W.	358
25	F.	206	M.	237	Th.	268	S.	298	Tu.	329	Th.	359
26	S	207	Tu.	238	F.	269	✻.	299	W.	330	F.	360
27	✻.	208	W.	239	S.	270	M.	300	Th.	331	S.	361
28	M.	209	Th.	240	✻.	271	Tu.	301	F.	332	✻.	362
29	Tu.	210	F.	241	M.	272	W.	302	S.	333	M.	363
30	W.	211	S.	242	Tu.	273	Th.	303	✻.	334	Tu.	364
31	Th.	212	✻.	243			F.	304			W.	365

ECLIPSES

There will be three eclipses, two of the Sun and one of the Moon.

I. *An Annular Eclipse of the Sun,* April 19. See map on page 6. The maximum duration of the annular phase is $7^{m}\ 07^{s}$.

II. *A Partial Eclipse of the Moon,* May 3. The eclipse begins at $12^{h}\ 00^{m}$ and ends at $12^{h}\ 26^{m}$; at the time of maximum eclipse 0·02 of the Moon's diameter is obscured. It is visible from the western part of North America, the Pacific Ocean, eastern Asia, the south-eastern part of the Indian Ocean, Australia, and Antarctica.

III. *A Total Eclipse of the Sun,* October 12. See map on page 7. The maximum duration of the total phase is $5^{m}\ 11^{s}$.

1958 MAY 31, JUNE 1, 2 (SAT., SUN., MON.)

G.M.T.		ARIES	VENUS −3·5		MARS +0·6		JUPITER −1·9		SATURN +0·3	
d	h	G.H.A. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′
31	00	247 59·2	220 35·9	N 9 01·6	251 58·8	S 3 55·0	46 50·0	S 7 21·7	345 03·6	S21 50·9
	01	263 01·6	235 35·7	02·6	266 59·6	54·3	61 52·6	21·7	0 06·2	50·9
	02	278 04·1	250 35·4	03·6	282 00·4	53·6	76 55·2	21·6	15 08·9	50·9
	03	293 06·6	265 35·2	· · 04·6	297 01·2	· · 52·9	91 57·8	· · 21·6	30 11·5	· · 50·9
	04	308 09·0	280 34·9	05·5	312 02·0	52·2	107 00·4	21·6	45 14·2	50·8
	05	323 11·5	295 34·6	06·5	327 02·8	51·5	122 03·0	21·5	60 16·8	50·8
	06	338 14·0	310 34·4	N 9 07·5	342 03·6	S 3 50·9	137 05·6	S 7 21·5	75 19·5	S21 50·8
SATURDAY	07	353 16·4	325 34·1	08·5	357 04·4	50·2	152 08·2	21·4	90 22·2	50·8
	08	8 18·9	340 33·9	09·5	12 05·2	49·5	167 10·8	21·4	105 24·8	50·8
	09	23 21·4	355 33·6	· · 10·5	27 06·0	· · 48·8	182 13·4	· · 21·3	120 27·5	· · 50·8
	10	38 23·8	10 33·4	11·5	42 06·8	48·1	197 16·0	21·3	135 30·1	50·8
	11	53 26·3	25 33·1	12·5	57 07·6	47·4	212 18·5	21·3	150 32·8	50·8
	12	68 28·7	40 32·8	N 9 13·4	72 08·4	S 3 46·7	227 21·1	S 7 21·2	165 35·4	S21 50·8
	13	83 31·2	55 32·6	14·4	87 09·2	46·0	242 23·7	21·2	180 38·1	50·8
	14	98 33·7	70 32·3	15·4	102 10·0	45·4	257 26·3	21·1	195 40·7	50·8
	15	113 36·1	85 32·1	· · 16·4	117 10·8	· · 44·7	272 28·9	· · 21·1	210 43·4	· · 50·8
	16	128 38·6	100 31·8	17·4	132 11·5	44·0	287 31·5	21·1	225 46·0	50·8
	17	143 41·1	115 31·5	18·4	147 12·3	43·3	302 34·1	21·0	240 48·7	50·7
	18	158 43·5	130 31·3	N 9 19·4	162 13·1	S 3 42·6	317 36·7	S 7 21·0	255 51·3	S21 50·7
	19	173 46·0	145 31·0	20·3	177 13·9	41·9	332 39·3	20·9	270 54·0	50·7
	20	188 48·5	160 30·7	21·3	192 14·7	41·2	347 41·9	20·9	285 56·6	50·7
	21	203 50·9	175 30·5	· · 22·3	207 15·5	· · 40·5	2 44·5	· · 20·9	300 59·3	· · 50·7
	22	218 53·4	190 30·2	23·3	222 16·3	39·9	17 47·1	20·8	316 02·0	50·7
	23	233 55·8	205 29·9	24·3	237 17·1	39·2	32 49·7	20·8	331 04·6	50·7
1	00	248 58·3	220 29·7	N 9 25·3	252 17·9	S 3 38·5	47 52·3	S 7 20·7	346 07·3	S21 50·7
	01	264 00·8	235 29·4	26·3	267 18·7	37·8	62 54·9	20·7	1 09·9	50·7
	02	279 03·2	250 29·2	27·2	282 19·5	37·1	77 57·5	20·7	16 12·6	50·7
	03	294 05·7	265 28·9	· · 28·2	297 20·3	· · 36·4	93 00·1	· · 20·6	31 15·2	· · 50·7
	04	309 08·2	280 28·6	29·2	312 21·1	35·7	108 02·7	20·6	46 17·9	50·7
	05	324 10·6	295 28·4	30·2	327 21·9	35·0	123 05·2	20·5	61 20·5	50·7
	06	339 13·1	310 28·1	N 9 31·2	342 22·7	S 3 34·3	138 07·8	S 7 20·5	76 23·2	S21 50·7
	07	354 15·6	325 27·8	32·2	357 23·5	33·7	153 10·4	20·5	91 25·8	50·6
	08	9 18·0	340 27·6	33·1	12 24·3	33·0	168 13·0	20·4	106 28·5	50·6
	09	24 20·5	355 27·3	· · 34·1	27 25·1	· · 32·3	183 15·6	· · 20·4	121 31·2	· · 50·6
SUNDAY	10	39 23·0	10 27·0	35·1	42 25·9	31·6	198 18·2	20·3	136 33·8	50·6
	11	54 25·4	25 26·7	36·1	57 26·7	30·9	213 20·8	20·3	151 36·5	50·6
	12	69 27·9	40 26·5	N 9 37·1	72 27·5	S 3 30·2	228 23·4	S 7 20·3	166 39·1	S21 50·6
	13	84 30·3	55 26·2	38·1	87 28·3	29·5	243 26·0	20·2	181 41·8	50·6
	14	99 32·8	70 25·9	39·0	102 29·1	28·8	258 28·6	20·2	196 44·4	50·6
	15	114 35·3	85 25·7	· · 40·0	117 29·9	· · 28·2	273 31·1	· · 20·1	211 47·1	· · 50·6
	16	129 37·7	100 25·4	41·0	132 30·7	27·5	288 33·7	20·1	226 49·7	50·6
	17	144 40·2	115 25·1	42·0	147 31·5	26·8	303 36·3	20·1	241 52·4	50·6
	18	159 42·7	130 24·9	N 9 43·0	162 32·3	S 3 26·1	318 38·9	S 7 20·0	256 55·0	S21 50·6
	19	174 45·1	145 24·6	43·9	177 33·1	25·4	333 41·5	20·0	271 57·7	50·6
	20	189 47·6	160 24·3	44·9	192 33·9	24·7	348 44·1	20·0	287 00·4	50·6
	21	204 50·1	175 24·0	· · 45·9	207 34·7	· · 24·0	3 46·7	· · 19·9	302 03·0	· · 50·5
	22	219 52·5	190 23·8	46·9	222 35·5	23·3	18 49·3	19·9	317 05·7	50·5
	23	234 55·0	205 23·5	47·9	237 36·3	22·6	33 51·8	19·8	332 08·3	50·5
2	00	249 57·5	220 23·2	N 9 48·8	252 37·1	S 3 22·0	48 54·4	S 7 19·8	347 11·0	S21 50·5
	01	264 59·9	235 22·9	49·8	267 37·9	21·3	63 57·0	19·8	2 13·6	50·5
	02	280 02·4	250 22·7	50·8	282 38·7	20·6	78 59·6	19·7	17 16·3	50·5
	03	295 04·8	265 22·4	· · 51·8	297 39·5	· · 19·9	94 02·2	· · 19·7	32 18·9	· · 50·5
	04	310 07·3	280 22·1	52·8	312 40·3	19·2	109 04·8	19·7	47 21·6	50·5
	05	325 09·8	295 21·8	53·7	327 41·1	18·5	124 07·4	19·6	62 24·3	50·5
	06	340 12·2	310 21·6	N 9 54·7	342 41·9	S 3 17·8	139 09·9	S 7 19·6	77 26·9	S21 50·5
	07	355 14·7	325 21·3	55·7	357 42·7	17·1	154 12·5	19·5	92 29·6	50·5
	08	10 17·2	340 21·0	56·7	12 43·5	16·5	169 15·1	19·5	107 32·2	50·5
	09	25 19·6	355 20·7	· · 57·6	27 44·3	· · 15·8	184 17·7	· · 19·5	122 34·9	· · 50·5
	10	40 22·1	10 20·5	58·6	42 45·1	15·1	199 20·3	19·4	137 37·5	50·5
	11	55 24·6	25 20·2	9 59·6	57 45·9	14·4	214 22·9	19·4	152 40·2	50·4
MONDAY	12	70 27·0	40 19·9	N10 00·6	72 46·7	S 3 13·7	229 25·5	S 7 19·4	167 42·8	S21 50·4
	13	85 29·5	55 19·6	01·5	87 47·5	13·0	244 28·0	19·3	182 45·5	50·4
	14	100 31·9	70 19·4	02·5	102 48·3	12·3	259 30·6	19·3	197 48·2	50·4
	15	115 34·4	85 19·1	· · 03·5	117 49·1	· · 11·6	274 33·2	· · 19·3	212 50·8	· · 50·4
	16	130 36·9	100 18·8	04·5	132 49·9	11·0	289 35·8	19·2	227 53·5	50·4
	17	145 39·3	115 18·5	05·4	147 50·7	10·3	304 38·4	19·2	242 56·1	50·4
	18	160 41·8	130 18·2	N10 06·4	162 51·5	S 3 09·6	319 40·9	S 7 19·1	257 58·8	S21 50·4
	19	175 44·3	145 18·0	07·4	177 52·3	08·9	334 43·5	19·1	273 01·4	50·4
	20	190 46·7	160 17·7	08·4	192 53·1	08·2	349 46·1	19·1	288 04·1	50·4
	21	205 49·2	175 17·4	· · 09·3	207 53·9	· · 07·5	4 48·7	· · 19·0	303 06·8	· · 50·4
	22	220 51·7	190 17·1	10·3	222 54·7	06·8	19 51·3	19·0	318 09·4	50·4
	23	235 54·1	205 16·8	11·3	237 55·5	06·1	34 53·9	19·0	333 12·1	50·4
Mer. Pass.		h m 7 22·9	*v* −0·3	*d* 1·0	*v* 0·8	*d* 0·7	*v* 2·6	*d* 0·0	*v* 2·7	*d* 0·0

STARS

Name	S.H.A. ° ′	Dec. ° ′
Acamar	315 50·1	S 40 28·2
Achernar	335 58·0	S 57 26·7
Acrux	173 55·2	S 62 52·5
Adhara	255 45·4	S 28 55·1
Aldebaran	291 37·2	N 16 25·5
Alioth	166 56·7	N 56 11·3
Alkaid	153 31·1	N 49 31·4
Al Na'ir	28 35·6	S 47 09·5
Alnilam	276 28·7	S 1 13·8
Alphard	218 36·8	S 8 28·9
Alphecca	126 45·7	N 26 51·3
Alpheratz	358 26·4	N 28 51·5
Altair	62 48·3	N 8 45·5
Ankaa	353 56·7	S 42 31·7
Antares	113 16·7	S 26 20·4
Arcturus	146 33·2	N 19 24·0
Atria	108 55·1	S 68 57·2
Avior	234 35·3	S 59 22·9
Bellatrix	279 16·7	N 6 18·6
Betelgeuse	271 46·4	N 7 23·9
Canopus	264 15·0	S 52 40·6
Capella	281 36·1	N 45 57·3
Deneb	49 59·4	N 45 07·8
Denebola	183 15·7	N 14 48·3
Diphda	349 37·6	S 18 12·8
Dubhe	194 42·4	N 61 58·7
Elnath	279 05·3	N 28 34·3
Eltanin	91 04·9	N 51 29·6
Enif	34 27·6	N 9 41·0
Fomalhaut	16 09·6	S 29 50·4
Gacrux	172 46·7	S 56 53·1
Gienah	176 34·8	S 17 18·9
Hadar	149 46·1	S 60 10·6
Hamal	328 47·8	N 23 15·8
Kaus Aust.	84 38·3	S 34 24·2
Kochab	137 17·2	N 74 19·7
Markab	14 19·6	N 14 58·8
Menkar	314 58·6	N 3 55·6
Menkent	148 56·1	S 36 10·1
Miaplacidus	221 48·8	S 69 33·2
Mirfak	309 40·1	N 49 42·7
Nunki	76 49·3	S 26 20·8
Peacock	54 24·0	S 56 51·9
Pollux	244 18·6	N 28 07·6
Procyon	245 43·3	N 5 19·8
Rasalhague	96 44·5	N 12 35·4
Regulus	203 27·6	N 12 10·2
Rigel	281 52·2	S 8 15·1
Rigil Kent.	140 47·7	S 60 40·0
Sabik	102 59·7	S 15 40·4
Schedar	350 28·0	N 56 18·3
Shaula	97 17·7	S 37 04·4
Sirius	259 10·5	S 16 39·7
Spica	159 14·7	S 10 56·8
Suhail	223 23·1	S 43 16·2
Vega	81 06·6	N 38 44·7
Zuben'ubi	137 50·9	S 15 52·2

	S.H.A. ° ′	Mer. Pass. h m
Venus	331 31·4	9 18
Mars	3 19·6	7 10
Jupiter	158 54·0	20 45
Saturn	97 09·0	0 55

1958 MAY 31, JUNE 1, 2 (SAT., SUN., MON.)

G.M.T.		SUN		MOON				
d	h	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	v ′	Dec. ° ′	d ′	H.P. ′
31	00	180 38·4	N21 48·6	26 24·6	6·4	S14 50·5	7·8	60·2
	01	195 38·3	48·9	40 50·0	6·3	14 58·3	7·6	60·2
	02	210 38·2	49·3	55 15·3	6·3	15 05·9	7·5	60·1
	03	225 38·1	· · 49·7	69 40·6	6·2	15 13·4	7·5	60·1
	04	240 38·0	50·0	84 05·8	6·3	15 20·9	7·3	60·1
	05	255 37·9	50·4	98 31·1	6·2	15 28·2	7·2	60·1
	06	270 37·9	N21 50·8	112 56·3	6·1	S15 35·4	7·1	60·1
	07	285 37·8	51·1	127 21·4	6·1	15 42·5	7·0	60·1
S	08	300 37·7	51·5	141 46·5	6·1	15 49·5	6·9	60·1
A	09	315 37·6	· · 51·8	156 11·6	6·1	15 56·4	6·8	60·1
T	10	330 37·5	52·2	170 36·7	6·0	16 03·2	6·7	60·1
U	11	345 37·4	52·6	185 01·7	6·0	16 09·9	6·6	60·1
R	12	0 37·3	N21 52·9	199 26·7	6·0	S16 16·5	6·4	60·1
D	13	15 37·2	53·3	213 51·7	5·9	16 22·9	6·4	60·0
A	14	30 37·2	53·6	228 16·6	5·9	16 29·3	6·2	60·0
Y	15	45 37·1	· · 54·0	242 41·5	5·9	16 35·5	6·1	60·0
	16	60 37·0	54·4	257 06·4	5·8	16 41·6	6·0	60·0
	17	75 36·9	54·7	271 31·2	5·9	16 47·6	5·9	60·0
	18	90 36·8	N21 55·1	285 56·1	5·8	S16 53·5	5·8	60·0
	19	105 36·7	55·4	300 20·9	5·8	16 59·3	5·6	60·0
	20	120 36·6	55·8	314 45·7	5·7	17 04·9	5·5	60·0
	21	135 36·5	· · 56·1	329 10·4	5·8	17 10·4	5·5	59·9
	22	150 36·4	56·5	343 35·2	5·7	17 15·9	5·3	59·9
	23	165 36·3	56·8	357 59·9	5·7	17 21·2	5·1	59·9
1	00	180 36·3	N21 57·2	12 24·6	5·7	S17 26·3	5·1	59·9
	01	195 36·2	57·5	26 49·3	5·7	17 31·4	4·9	59·9
	02	210 36·1	57·9	41 14·0	5·6	17 36·3	4·8	59·9
	03	225 36·0	· · 58·2	55 38·6	5·7	17 41·1	4·7	59·9
	04	240 35·9	58·6	70 03·3	5·6	17 45·8	4·6	59·8
	05	255 35·8	58·9	84 27·9	5·7	17 50·4	4·4	59·8
	06	270 35·7	N21 59·3	98 52·6	5·6	S17 54·8	4·3	59·8
	07	285 35·6	21 59·6	113 17·2	5·6	17 59·1	4·2	59·8
	08	300 35·5	22 00·0	127 41·8	5·6	18 03·3	4·1	59·8
S	09	315 35·4	· · 00·3	142 06·4	5·6	18 07·4	3·9	59·7
U	10	330 35·3	00·7	156 31·0	5·6	18 11·3	3·8	59·7
N	11	345 35·3	01·0	170 55·6	5·6	18 15·1	3·7	59·7
D	12	0 35·2	N22 01·4	185 20·2	5·6	S18 18·8	3·6	59·7
A	13	15 35·1	01·7	199 44·8	5·6	18 22·4	3·4	59·7
Y	14	30 35·0	02·0	214 09·4	5·6	18 25·8	3·3	59·6
	15	45 34·9	· · 02·4	228 34·0	5·5	18 29·1	3·2	59·6
	16	60 34·8	02·7	242 58·5	5·6	18 32·3	3·1	59·6
	17	75 34·7	03·1	257 23·1	5·6	18 35·4	2·9	59·6
	18	90 34·6	N22 03·4	271 47·7	5·6	S18 38·3	2·8	59·6
	19	105 34·5	03·7	286 12·3	5·7	18 41·1	2·6	59·5
	20	120 34·4	04·1	300 37·0	5·6	18 43·7	2·6	59·5
	21	135 34·3	· · 04·4	315 01·6	5·6	18 46·3	2·4	59·5
	22	150 34·2	04·8	329 26·2	5·7	18 48·7	2·3	59·5
	23	165 34·1	05·1	343 50·9	5·6	18 51·0	2·1	59·4
2	00	180 34·0	N22 05·4	358 15·5	5·7	S18 53·1	2·1	59·4
	01	195 33·9	05·8	12 40·2	5·7	18 55·2	1·9	59·4
	02	210 33·9	06·1	27 04·9	5·7	18 57·1	1·7	59·4
	03	225 33·8	· · 06·4	41 29·6	5·8	18 58·8	1·7	59·3
	04	240 33·7	06·8	55 54·4	5·7	19 00·5	1·5	59·3
	05	255 33·6	07·1	70 19·1	5·8	19 02·0	1·4	59·3
	06	270 33·5	N22 07·4	84 43·9	5·8	S19 03·4	1·2	59·3
	07	285 33·4	07·8	99 08·7	5·8	19 04·6	1·1	59·2
	08	300 33·3	08·1	113 33·5	5·8	19 05·7	1·0	59·2
M	09	315 33·2	· · 08·4	127 58·3	5·9	19 06·7	0·9	59·2
O	10	330 33·1	08·7	142 23·2	5·9	19 07·6	0·8	59·2
N	11	345 33·0	09·1	156 48·1	5·9	19 08·4	0·6	59·1
D	12	0 32·9	N22 09·4	171 13·0	6·0	S19 09·0	0·5	59·1
A	13	15 32·8	09·7	185 38·0	6·0	19 09·5	0·3	59·1
Y	14	30 32·7	10·1	200 03·0	6·0	19 09·8	0·3	59·1
	15	45 32·6	· · 10·4	214 28·0	6·1	19 10·1	0·1	59·0
	16	60 32·5	10·7	228 53·1	6·1	19 10·2	0·1	59·0
	17	75 32·4	11·0	243 18·2	6·1	19 10·1	0·1	59·0
	18	90 32·3	N22 11·4	257 43·3	6·2	S19 10·0	0·3	58·9
	19	105 32·2	11·7	272 08·5	6·2	19 09·7	0·4	58·9
	20	120 32·1	12·0	286 33·7	6·3	19 09·3	0·5	58·9
	21	135 32·0	· · 12·3	300 59·0	6·3	19 08·8	0·6	58·9
	22	150 31·9	12·6	315 24·3	6·3	19 08·2	0·8	58·8
	23	165 31·8	13·0	329 49·6	6·4	19 07·4	0·9	58·8
		S.D. 15·8	d 0·3	S.D.	16·4		16·3	16·1

Lat.	Twilight Naut.	Twilight Civil	Sunrise	Moonrise 31	Moonrise 1	Moonrise 2	Moonrise 3
°	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	22 07	■	■	■
N 70	□	□	□	20 59	22 49	23 58	24 19
68	□	□	□	20 23	21 55	23 00	23 37
66	////	////	01 14	19 58	21 23	22 27	23 08
64	////	////	01 59	19 38	20 59	22 02	22 47
62	////	00 10	02 28	19 22	20 40	21 42	22 29
60	////	01 27	02 50	19 08	20 24	21 26	22 14
N 58	////	02 01	03 08	18 57	20 11	21 13	22 02
56	00 09	02 25	03 23	18 47	20 00	21 01	21 51
54	01 20	02 44	03 35	18 38	19 50	20 51	21 42
52	01 51	03 00	03 46	18 30	19 41	20 42	21 33
50	02 13	03 13	03 56	18 23	19 33	20 34	21 26
45	02 53	03 41	04 17	18 08	19 16	20 17	21 09
N 40	03 21	04 01	04 33	17 56	19 02	20 02	20 56
35	03 42	04 18	04 47	17 46	18 50	19 50	20 45
30	04 00	04 32	04 59	17 36	18 40	19 40	20 35
20	04 27	04 56	05 20	17 21	18 23	19 22	20 18
N 10	04 48	05 15	05 38	17 07	18 07	19 06	20 03
0	05 06	05 32	05 54	16 54	17 53	18 52	19 49
S 10	05 22	05 48	06 10	16 42	17 39	18 37	19 35
20	05 37	06 04	06 28	16 28	17 24	18 22	19 20
30	05 52	06 21	06 47	16 13	17 07	18 04	19 03
35	06 00	06 31	06 59	16 04	16 57	17 53	18 53
40	06 08	06 42	07 12	15 54	16 45	17 41	18 41
45	06 18	06 54	07 27	15 42	16 32	17 27	18 28
S 50	06 28	07 09	07 46	15 28	16 16	17 10	18 11
52	06 33	07 16	07 55	15 22	16 08	17 02	18 04
54	06 38	07 23	08 06	15 14	15 59	16 53	17 55
56	06 43	07 31	08 17	15 06	15 50	16 43	17 46
58	06 49	07 41	08 30	14 57	15 39	16 32	17 35
S 60	06 56	07 51	08 46	14 46	15 27	16 19	17 22

Lat.	Sunset	Twilight Civil	Twilight Naut.	Moonset 31	Moonset 1	Moonset 2	Moonset 3
°	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	00 32	00 00	■	■
N 70	□	□	□	01 05	01 08	01 22	02 15
68	□	□	□	01 29	01 45	02 16	03 12
66	22 45	////	////	01 48	02 11	02 49	03 46
64	21 58	////	////	02 03	02 31	03 13	04 11
62	21 29	////	////	02 16	02 48	03 32	04 30
60	21 07	22 31	////	02 27	03 02	03 48	04 46
N 58	20 49	21 56	////	02 36	03 14	04 01	04 59
56	20 34	21 32	////	02 44	03 24	04 13	05 11
54	20 21	21 12	22 38	02 52	03 33	04 23	05 21
52	20 10	20 56	22 06	02 58	03 41	04 32	05 30
50	20 00	20 43	21 43	03 05	03 49	04 40	05 38
45	19 39	20 15	21 03	03 18	04 04	04 57	05 55
N 40	19 22	19 54	20 35	03 28	04 17	05 11	06 09
35	19 08	19 38	20 13	03 38	04 28	05 23	06 21
30	18 56	19 23	19 56	03 46	04 38	05 33	06 31
20	18 36	19 00	19 29	04 00	04 54	05 51	06 49
N 10	18 18	18 41	19 07	04 12	05 09	06 07	07 05
0	18 01	18 24	18 50	04 24	05 22	06 21	07 19
S 10	17 45	18 08	18 34	04 35	05 36	06 36	07 34
20	17 28	17 51	18 19	04 48	05 50	06 51	07 49
30	17 08	17 34	18 03	05 02	06 07	07 09	08 07
35	16 56	17 24	17 55	05 10	06 17	07 19	08 17
40	16 43	17 13	17 47	05 20	06 28	07 31	08 28
45	16 28	17 01	17 38	05 31	06 41	07 45	08 42
S 50	16 09	16 46	17 27	05 44	06 57	08 02	08 59
52	16 00	16 39	17 22	05 50	07 04	08 10	09 07
54	15 49	16 32	17 17	05 57	07 12	08 19	09 15
56	15 38	16 24	17 12	06 05	07 22	08 29	09 25
58	15 25	16 14	17 06	06 13	07 32	08 41	09 36
S 60	15 09	16 04	16 59	06 23	07 44	08 54	09 49

Day	SUN Eqn. of Time 00ʰ	SUN Eqn. of Time 12ʰ	SUN Mer. Pass.	MOON Mer. Pass. Upper	MOON Mer. Pass. Lower	Age	Phase
	m s	m s	h m	h m	h m	d	
31	02 34	02 30	11 58	23 08	10 39	13	
1	02 25	02 21	11 58	24 07	11 38	14	○
2	02 16	02 12	11 58	00 07	12 37	15	

1958 JUNE 12, 13, 14 (THURS., FRI., SAT.)

G.M.T.		ARIES	VENUS −3·5		MARS +0·5		JUPITER −1·8		SATURN +0·2	
d	h	G.H.A. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′
12 THURSDAY	00	259 48·8	219 05·8	N13 35·1	255 51·9	S 0 37·5	59 07·1	S 7 14·2	357 49·3	S21 48·8
	01	274 51·3	234 05·4	35·9	270 52·7	36·8	74 09·6	14·1	12 52·0	48·7
	02	289 53·8	249 05·0	36·8	285 53·5	36·1	89 12·1	14·1	27 54·6	48·7
	03	304 56·2	264 04·7	· · 37·7	300 54·4	· · 35·4	104 14·6	· · 14·1	42 57·3	· · 48·7
	04	319 58·7	279 04·3	38·6	315 55·2	34·8	119 17·1	14·1	58 00·0	48·7
	05	335 01·2	294 03·9	39·5	330 56·0	34·1	134 19·7	14·1	73 02·6	48·7
	06	350 03·6	309 03·5	N13 40·4	345 56·8	S 0 33·4	149 22·2	S 7 14·1	88 05·3	S21 48·7
	07	5 06·1	324 03·2	41·3	0 57·7	32·7	164 24·7	14·1	103 08·0	48·7
	08	20 08·6	339 02·8	42·2	15 58·5	32·0	179 27·2	14·1	118 10·6	48·7
	09	35 11·0	354 02·4	· · 43·1	30 59·3	· · 31·4	194 29·7	· · 14·1	133 13·3	· · 48·7
	10	50 13·5	9 02·0	44·0	46 00·1	30·7	209 32·2	14·1	148 15·9	48·7
	11	65 15·9	24 01·6	44·9	61 01·0	30·0	224 34·8	14·1	163 18·6	48·7
	12	80 18·4	39 01·3	N13 45·8	76 01·8	S 0 29·3	239 37·3	S 7 14·0	178 21·3	S21 48·7
	13	95 20·9	54 00·9	46·7	91 02·6	28·6	254 39·8	14·0	193 23·9	48·7
	14	110 23·3	69 00·5	47·6	106 03·4	27·9	269 42·3	14·0	208 26·6	48·7
	15	125 25·8	84 00·1	· · 48·5	121 04·3	· · 27·3	284 44·8	· · 14·0	223 29·3	· · 48·6
	16	140 28·3	98 59·7	49·4	136 05·1	26·6	299 47·3	14·0	238 31·9	48·6
	17	155 30·7	113 59·4	50·2	151 05·9	25·9	314 49·8	14·0	253 34·6	48·6
	18	170 33·2	128 59·0	N13 51·1	166 06·3	S 0 25·2	329 52·4	S 7 14·0	268 37·2	S21 48·6
	19	185 35·7	143 58·6	52·0	181 07·6	24·5	344 54·9	14·0	283 39·9	48·6
	20	200 38·1	158 58·2	52·9	196 08·4	23·9	359 57·4	14·0	298 42·6	48·6
	21	215 40·6	173 57·8	· · 53·8	211 09·2	· · 23·2	14 59·9	· · 14·0	313 45·2	· · 48·6
	22	230 43·1	188 57·4	54·7	226 10·1	22·5	30 02·4	14·0	328 47·9	48·6
	23	245 45·5	203 57·1	55·6	241 10·9	21·8	45 04·9	14·0	343 50·6	48·6
13 FRIDAY	00	260 48·0	218 56·7	N13 56·5	256 11·7	S 0 21·2	60 07·4	S 7 14·0	358 53·2	S21 48·6
	01	275 50·4	233 56·3	57·4	271 12·5	20·5	75 09·9	14·0	13 55·9	48·6
	02	290 52·9	248 55·9	58·2	286 13·4	19·8	90 12·4	13·9	28 58·5	48·6
	03	305 55·4	263 55·5	13 59·1	301 14·2	· · 19·1	105 15·0	· · 13·9	44 01·2	· · 48·6
	04	320 57·8	278 55·1	14 00·0	316 15·0	18·4	120 17·5	13·9	59 03·9	48·6
	05	336 00·3	293 54·7	00·9	331 15·9	17·8	135 20·0	13·9	74 06·5	48·5
	06	351 02·8	308 54·3	N14 01·8	346 16·7	S 0 17·1	150 22·5	S 7 13·9	89 09·2	S21 48·5
	07	6 05·2	323 54·0	02·7	1 17·5	16·4	165 25·0	13·9	104 11·9	48·5
	08	21 07·7	338 53·6	03·5	16 18·3	15·7	180 27·5	13·9	119 14·5	48·5
	09	36 10·2	353 53·2	· · 04·4	31 19·2	· · 15·0	195 30·0	· · 13·9	134 17·2	· · 48·5
	10	51 12·6	8 52·8	05·3	46 20·0	14·4	210 32·5	13·9	149 19·8	48·5
	11	66 15·1	23 52·4	06·2	61 20·8	13·7	225 35·0	13·9	164 22·5	48·5
	12	81 17·6	38 52·0	N14 07·1	76 21·7	S 0 13·0	240 37·5	S 7 13·9	179 25·2	S21 48·5
	13	96 20·0	53 51·6	08·0	91 22·5	12·3	255 40·0	13·9	194 27·8	48·5
	14	111 22·5	68 51·2	08·8	106 23·3	11·6	270 42·5	13·9	209 30·5	48·5
	15	126 24·9	83 50·8	· · 09·7	121 24·1	· · 11·0	285 45·1	· · 13·9	224 33·2	· · 48·5
	16	141 27·4	98 50·4	10·6	136 25·0	10·3	300 47·6	13·9	239 35·8	48·5
	17	156 29·9	113 50·0	11·5	151 25·8	09·6	315 50·1	13·9	254 38·5	48·5
	18	171 32·3	128 49·6	N14 12·4	166 26·6	S 0 08·9	330 52·6	S 7 13·9	269 41·1	S21 48·5
	19	186 34·8	143 49·3	13·2	181 27·5	08·3	345 55·1	13·9	284 43·8	48·4
	20	201 37·3	158 48·9	14·1	196 28·3	07·6	0 57·6	13·8	299 46·5	48·4
	21	216 39·7	173 48·5	· · 15·0	211 29·1	· · 06·9	16 00·1	· · 13·8	314 49·1	· · 48·4
	22	231 42·2	188 48·1	15·9	226 30·0	06·2	31 02·6	13·8	329 51·8	48·4
	23	246 44·7	203 47·7	16·8	241 30·8	05·6	46 05·1	13·8	344 54·5	48·4
14 SATURDAY	00	261 47·1	218 47·3	N14 17·6	256 31·6	S 0 04·9	61 07·6	S 7 13·8	359 57·1	S21 48·4
	01	276 49·6	233 46·9	18·5	271 32·4	04·2	76 10·1	13·8	14 59·8	48·4
	02	291 52·0	248 46·5	19·4	286 33·3	03·5	91 12·6	13·8	30 02·4	48·4
	03	306 54·5	263 46·1	· · 20·3	301 34·1	· · 02·8	106 15·1	· · 13·8	45 05·1	· · 48·4
	04	321 57·0	278 45·7	21·1	316 34·9	02·2	121 17·6	13·8	60 07·8	48·4
	05	336 59·4	293 45·3	22·0	331 35·8	01·5	136 20·1	13·8	75 10·4	48·4
	06	352 01·9	308 44·9	N14 22·9	346 36·6	S 0 00·8	151 22·6	S 7 13·8	90 13·1	S21 48·4
	07	7 04·4	323 44·5	23·7	1 37·4	S 0 00·1	166 25·1	13·8	105 15·8	48·4
	08	22 06·8	338 44·1	24·6	16 38·3	N 0 00·5	181 27·6	13·8	120 18·4	48·3
	09	37 09·3	353 43·7	· · 25·5	31 39·1	· · 01·2	196 30·1	· · 13·8	135 21·1	· · 48·3
	10	52 11·8	8 43·3	26·4	46 39·9	01·9	211 32·6	13·8	150 23·7	48·3
	11	67 14·2	23 42·9	27·2	61 40·8	02·6	226 35·1	13·8	165 26·4	48·3
	12	82 16·7	38 42·5	N14 28·1	76 41·6	N 0 03·3	241 37·6	S 7 13·8	180 29·1	S21 48·3
	13	97 19·2	53 42·1	29·0	91 42·4	03·9	256 40·1	13·8	195 31·7	48·3
	14	112 21·6	68 41·7	29·8	106 43·3	04·6	271 42·6	13·8	210 34·4	48·3
	15	127 24·1	83 41·3	· · 30·7	121 44·1	· · 05·3	286 45·1	· · 13·8	225 37·1	· · 48·3
	16	142 26·5	98 40·9	31·6	136 44·9	06·0	301 47·6	13·8	240 39·7	48·3
	17	157 29·0	113 40·5	32·4	151 45·8	06·6	316 50·1	13·8	255 42·4	48·3
	18	172 31·5	128 40·0	N14 33·3	166 46·6	N 0 07·3	331 52·6	S 7 13·8	270 45·1	S21 48·3
	19	187 33·9	143 39·6	34·2	181 47·4	08·0	346 55·1	13·8	285 47·7	48·3
	20	202 36·4	158 39·2	35·0	196 48·3	08·7	1 57·6	13·8	300 50·4	48·3
	21	217 38·9	173 38·8	· · 35·9	211 49·1	· · 09·3	17 00·1	· · 13·8	315 53·0	· · 48·2
	22	232 41·3	188 38·4	36·8	226 49·9	10·0	32 02·6	13·8	330 55·7	48·2
	23	247 43·8	203 38·0	37·6	241 50·7	10·7	47 05·1	13·8	345 58·4	48·2
Mer. Pass.		6 35·7 (h m)	v −0·4	d 0·9	v 0·8	d 0·7	v 2·5	d 0·0	v 2·7	d 0·0

STARS Name	S.H.A. ° ′	Dec. ° ′
Acamar	315 50·1	S 40 28·2
Achernar	335 57·9	S 57 26·6
Acrux	173 55·3	S 62 52·5
Adhara	255 45·4	S 28 55·1
Aldebaran	291 37·2	N 16 25·5
Alioth	166 56·8	N 56 11·3
Alkaid	153 31·2	N 49 31·4
Al Na'ir	28 35·5	S 47 09·5
Alnilam	276 28·7	S 1 13·8
Alphard	218 36·9	S 8 28·9
Alphecca	126 45·7	N 26 51·4
Alpheratz	358 26·3	N 28 51·5
Altair	62 48·3	N 8 45·5
Ankaa	353 56·6	S 42 31·6
Antares	113 16·6	S 26 20·4
Arcturus	146 33·2	N 19 24·0
Atria	108 55·0	S 68 57·2
Avior	234 35·4	S 59 22·9
Bellatrix	279 16·7	N 6 18·7
Betelgeuse	271 46·4	N 7 23·9
Canopus	264 15·0	S 52 40·6
Capella	281 36·0	N 45 57·3
Deneb	49 59·3	N 45 07·8
Denebola	183 15·8	N 14 48·3
Diphda	349 37·5	S 18 12·8
Dubhe	194 42·5	N 61 58·7
Elnath	279 05·3	N 28 34·3
Eltanin	91 04·8	N 51 29·7
Enif	34 27·5	N 9 41·1
Fomalhaut	16 09·5	S 29 50·3
Gacrux	172 46·8	S 56 53·2
Gienah	176 34·8	S 17 18·8
Hadar	149 46·1	S 60 10·6
Hamal	328 47·7	N 23 15·9
Kaus Aust.	84 38·2	S 34 24·2
Kochab	137 17·4	N 74 19·8
Markab	14 19·5	N 14 58·9
Menkar	314 58·6	N 3 55·6
Menkent	143 56·1	S 36 10·1
Miaplacidus	221 48·9	S 69 33·2
Mirfak	309 40·0	N 49 42·7
Nunki	76 49·2	S 26 20·8
Peacock	54 23·9	S 56 51·9
Pollux	244 18·6	N 28 07·6
Procyon	245 43·3	N 5 19·8
Rasalhague	96 44·5	N 12 35·4
Regulus	208 27·6	N 12 10·2
Rigel	281 52·1	S 8 15·0
Rigil Kent.	140 47·7	S 60 40·1
Sabik	102 59·6	S 15 40·4
Schedar	350 27·8	N 56 18·3
Shaula	97 17·6	S 37 04·4
Sirius	259 10·5	S 16 39·7
Spica	159 14·7	S 10 56·8
Suhail	223 23·1	S 43 16·2
Vega	81 06·5	N 38 44·7
Zuben'ubi	137 50·9	S 15 52·2

	S.H.A. ° ′	Mer. Pass. h m
Venus	318 08·7	9 25
Mars	355 23·7	6 55
Jupiter	159 19·4	19 56
Saturn	98 05·2	0 04

1958 JUNE 12, 13, 14 (THURS., FRI., SAT.)

G.M.T.	SUN G.H.A.	SUN Dec.	MOON G.H.A.	v	MOON Dec.	d	H.P.
d h	° ′	° ′	° ′	′	° ′	′	′
12 00	180 07·1	N23 06·2	241 41·3	14·8	N 8 40·2	8·8	54·3
01	195 06·9	06·4	256 15·1	14·7	8 49·0	8·7	54·3
02	210 06·8	06·6	270 48·8	14·7	8 57·7	8·6	54·3
03	225 06·7	· · 06·8	285 22·5	14·6	9 06·3	8·7	54·3
04	240 06·5	06·9	299 56·1	14·7	9 15·0	8·6	54·3
05	255 06·4	07·1	314 29·3	14·6	9 23·6	8·5	54·3
06	270 06·3	N23 07·3	329 03·4	14·5	N 9 32·1	8·6	54·3
07	285 06·2	07·4	343 36·9	14·6	9 40·7	8·4	54·3
T 08	300 06·0	07·6	358 10·5	14·4	9 49·1	8·5	54·3
H 09	315 05·9	· · 07·8	12 43·9	14·5	9 57·6	8·4	54·3
U 10	330 05·8	07·9	27 17·4	14·4	10 06·0	6·4	54·3
R 11	345 05·6	08·1	41 50·8	14·4	10 14·4	8·3	54·3
S 12	0 05·5	N23 08·2	56 24·2	14·4	N10 22·7	8·3	54·3
D 13	15 05·4	08·4	70 57·6	14·3	10 31·0	8·3	54·4
A 14	30 05·3	08·6	85 30·9	14·2	10 39·3	8·2	54·4
Y 15	45 05·1	· · 08·7	100 04·1	14·3	10 47·5	6·1	54·4
16	60 05·0	08·9	114 37·4	14·2	10 55·6	6·2	54·4
17	75 04·9	09·0	129 10·6	14·1	11 03·8	8·0	54·4
18	90 04·7	N23 09·2	143 43·7	14·1	N11 11·8	8·1	54·4
19	105 04·6	09·4	158 16·8	14·1	11 19·9	8·0	54·4
20	120 04·5	09·5	172 49·9	14·0	11 27·9	7·9	54·4
21	135 04·4	· · 09·7	187 22·9	14·0	11 35·8	7·9	54·4
22	150 04·2	09·8	201 55·9	14·0	11 43·7	7·9	54·4
23	165 04·1	10·0	216 28·9	13·9	11 51·6	7·8	54·5
13 00	180 04·0	N23 10·1	231 01·8	13·9	N11 59·4	7·7	54·5
01	195 03·8	10·3	245 34·7	13·8	12 07·1	7·8	54·5
02	210 03·7	10·4	260 07·5	13·8	12 14·9	7·6	54·5
03	225 03·6	· · 10·6	274 40·3	13·7	12 22·5	7·6	54·5
04	240 03·4	10·7	289 13·0	13·7	12 30·1	7·6	54·5
05	255 03·3	10·9	303 45·7	13·6	12 37·7	7·5	54·5
06	270 03·2	N23 11·0	318 18·3	13·6	N12 45·2	7·5	54·5
07	285 03·1	11·2	332 50·9	13·6	12 52·7	7·4	54·5
08	300 02·9	11·3	347 23·5	13·5	13 00·1	7·3	54·6
F 09	315 02·8	· · 11·5	1 56·0	13·4	13 07·4	7·3	54·6
R 10	330 02·7	11·6	16 28·4	13·4	13 14·7	7·3	54·6
I 11	345 02·5	11·8	31 00·8	13·4	13 22·0	7·2	54·6
D 12	0 02·4	N23 11·9	45 33·2	13·3	N13 29·2	7·1	54·6
A 13	15 02·3	12·1	60 05·5	13·3	13 36·3	7·1	54·6
Y 14	30 02·1	12·2	74 37·8	13·2	13 43·4	7·0	54·6
15	45 02·0	· · 12·3	89 10·0	13·2	13 50·4	7·0	54·7
16	60 01·9	12·5	103 42·2	13·1	13 57·4	6·9	54·7
17	75 01·7	12·6	118 14·3	13·1	14 04·3	6·9	54·7
18	90 01·6	N23 12·8	132 46·4	13·0	N14 11·2	6·8	54·7
19	105 01·5	12·9	147 18·4	13·0	14 18·0	6·7	54·7
20	120 01·4	13·0	161 50·4	12·9	14 24·7	6·7	54·7
21	135 01·2	· · 13·2	176 22·3	12·9	14 31·4	6·6	54·7
22	150 01·1	13·3	190 54·2	12·8	14 38·0	6·5	54·8
23	165 01·0	13·5	205 26·0	12·8	14 44·5	6·5	54·8
14 00	180 00·8	N23 13·6	219 57·8	12·7	N14 51·0	6·4	54·8
01	195 00·7	13·7	234 29·5	12·6	14 57·4	6·4	54·8
02	210 00·6	13·9	249 01·1	12·7	15 03·8	6·3	54·8
03	225 00·4	· · 14·0	263 32·8	12·5	15 10·1	6·2	54·8
04	240 00·3	14·1	278 04·3	12·5	15 16·3	6·2	54·9
05	255 00·2	14·3	292 35·8	12·5	15 22·5	6·1	54·9
06	270 00·0	N23 14·4	307 07·3	12·4	N15 28·6	6·0	54·9
07	284 59·9	14·5	321 38·7	12·4	15 34·6	6·0	54·9
S 08	299 59·8	14·7	336 10·1	12·3	15 40·6	5·8	54·9
A 09	314 59·6	· · 14·8	350 41·4	12·2	15 46·4	5·9	54·9
T 10	329 59·5	14·9	5 12·6	12·2	15 52·3	5·7	55·0
U 11	344 59·4	15·0	19 43·8	12·2	15 58·0	5·7	55·0
R 12	359 59·2	N23 15·2	34 15·0	12·1	N16 03·7	5·6	55·0
D 13	14 59·1	15·3	48 46·1	12·0	16 09·3	5·5	55·0
A 14	29 59·0	15·4	63 17·1	12·0	16 14·8	5·5	55·0
Y 15	44 58·9	· · 15·5	77 48·1	12·0	16 20·3	5·4	55·0
16	59 58·7	15·7	92 19·1	11·9	16 25·7	5·3	55·1
17	74 58·6	15·8	106 50·0	11·8	16 31·0	5·2	55·1
18	89 58·5	N23 15·9	121 20·8	11·8	N16 36·2	5·2	55·1
19	104 58·3	16·0	135 51·6	11·7	16 41·4	5·1	55·1
20	119 58·2	16·2	150 22·3	11·7	16 46·5	5·0	55·1
21	134 58·1	· · 16·3	164 53·0	11·6	16 51·5	4·9	55·2
22	149 57·9	16·4	179 23·6	11·6	16 56·4	4·9	55·2
23	164 57·8	16·5	193 54·2	11·5	17 01·3	4·8	55·2
	S.D. 15·8	d 0·1	S.D. 14·8		14·9		15·0

Lat.	Twilight Naut.	Twilight Civil	Sunrise	Moonrise 12	Moonrise 13	Moonrise 14	Moonrise 15
°	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	{00 03 / 23 54}	23 44	23 26	□
N 70	□	□	□	00 15	00 14	00 15	00 18
68	□	□	□	00 26	00 30	00 38	00 50
66	□	□	□	00 34	00 43	00 56	01 14
64	////	////	01 36	00 42	00 54	01 10	01 33
62	////	////	02 12	00 48	01 03	01 23	01 48
60	////	00 57	02 37	00 53	01 11	01 33	02 01
N 58	////	01 43	02 57	00 58	01 18	01 42	02 12
56	////	02 12	03 14	01 03	01 25	01 51	02 22
54	00 52	02 34	03 28	01 06	01 30	01 58	02 31
52	01 35	02 51	03 40	01 10	01 35	02 04	02 38
50	02 02	03 06	03 50	01 13	01 40	02 10	02 45
45	02 46	03 35	04 13	01 20	01 50	02 23	03 00
N 40	03 16	03 58	04 30	01 26	01 58	02 33	03 13
35	03 39	04 16	04 45	01 31	02 05	02 42	03 23
30	03 58	04 31	04 58	01 36	02 11	02 50	03 32
20	04 26	04 55	05 20	01 44	02 22	03 04	03 48
N 10	04 49	05 16	05 39	01 51	02 32	03 16	04 02
0	05 08	05 34	05 56	01 57	02 41	03 27	04 15
S 10	05 24	05 51	06 13	02 04	02 50	03 39	04 28
20	05 40	06 08	06 32	02 11	03 00	03 51	04 43
30	05 57	06 26	06 53	02 19	03 11	04 05	04 59
35	06 05	06 37	07 05	02 23	03 18	04 13	05 08
40	06 15	06 48	07 19	02 29	03 25	04 22	05 19
45	06 25	07 02	07 36	02 35	03 34	04 33	05 32
S 50	06 36	07 18	07 56	02 42	03 44	04 46	05 47
52	06 41	07 25	08 06	02 46	03 49	04 52	05 55
54	06 47	07 33	08 17	02 50	03 55	04 59	06 03
56	06 53	07 42	08 29	02 54	04 01	05 07	06 12
58	07 00	07 52	08 44	02 59	04 07	05 16	06 22
S 60	07 07	08 04	09 01	03 04	04 15	05 26	06 34

Lat.	Sunset	Twilight Civil	Twilight Naut.	Moonset 12	Moonset 13	Moonset 14	Moonset 15
°	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	16 49	18 34	20 29	□
N 70	□	□	□	16 30	18 04	19 39	21 12
68	□	□	□	16 15	17 41	19 07	20 28
66	□	□	□	16 03	17 24	18 44	19 58
64	22 25	////	////	15 54	17 10	18 25	19 36
62	21 49	////	////	15 45	16 59	18 10	19 18
60	21 23	23 04	////	15 38	16 49	17 58	19 04
N 58	21 03	22 17	////	15 31	16 40	17 47	18 51
56	20 46	21 48	////	15 26	16 32	17 38	18 40
54	20 32	21 26	23 09	15 21	16 26	17 29	18 31
52	20 20	21 09	22 26	15 16	16 20	17 22	18 22
50	20 10	20 54	21 59	15 12	16 14	17 15	18 15
45	19 47	20 25	21 14	15 03	16 02	17 01	17 59
N 40	19 29	20 02	20 44	14 56	15 53	16 49	17 46
35	19 15	19 44	20 21	14 50	15 44	16 39	17 34
30	19 02	19 29	20 02	14 44	15 37	16 31	17 25
20	18 40	19 04	19 33	14 34	15 24	16 15	17 08
N 10	18 21	18 44	19 11	14 26	15 13	16 02	16 53
0	18 03	18 26	18 52	14 18	15 03	15 50	16 39
S 10	17 46	18 09	18 35	14 10	14 53	15 38	16 25
20	17 28	17 52	18 19	14 02	14 42	15 24	16 11
30	17 07	17 33	18 03	13 52	14 29	15 10	15 54
35	16 55	17 23	17 54	13 47	14 22	15 01	15 44
40	16 41	17 11	17 45	13 41	14 14	14 51	15 33
45	16 24	16 58	17 35	13 33	14 04	14 39	15 20
S 50	16 04	16 42	17 23	13 25	13 53	14 25	15 04
52	15 54	16 35	17 18	13 21	13 48	14 19	14 56
54	15 43	16 26	17 12	13 16	13 42	14 12	14 48
56	15 30	16 17	17 06	13 12	13 35	14 04	14 39
58	15 16	16 07	17 00	13 06	13 28	13 55	14 28
S 60	14 59	15 56	16 52	13 00	13 20	13 44	14 16

Day	SUN Eqn. of Time 00h	SUN Eqn. of Time 12h	SUN Mer. Pass.	MOON Mer. Pass. Upper	MOON Mer. Pass. Lower	Age	Phase
	m s	m s	h m	h m	h m	d	
12	00 28	00 22	12 00	08 07	20 30	25	
13	00 16	00 10	12 00	08 52	21 15	26	
14	00 04	$\boxed{00\ 03}$	12 00	09 38	22 02	27	

STARS, 1958 JANUARY—JUNE

Mag.	Name and No.	S.H.A.	JAN.	FEB.	MAR.	APR.	MAY	JUNE	Declination	JAN.	FEB.	MAR.	APR.	MAY	JUNE
		°	′	′	′	′	′	′	°	′	′	′	′	′	′
3·1	γ Ursæ Minoris †	129	49·1	48·5	48·0	47·7	47·6	47·7	N. 71	58·8	58·7	58·7	58·9	59·0	59·2
2·7	β Libræ	131	18·8	18·5	18·3	18·2	18·1	18·1	S. 9	13·7	13·8	13·8	13·9	13·9	13·8
3·1	γ Trianguli Aust.	131	15·4	14·9	14·4	14·0	13·8	13·8	S. 68	31·3	31·3	31·4	31·5	31·7	31·8
2·8	β Lupi	136	03·3	02·9	02·7	02·5	02·4	02·4	S. 42	57·8	57·8	57·9	58·0	58·1	58·2
2·2	β Ursæ Minoris 40	137	18·6	18·0	17·5	17·2	17·1	17·4	N. 74	19·3	19·3	19·4	19·5	19·6	19·8
2·9	α Libræ 39	137	51·6	51·4	51·2	51·0	51·0	51·0	S. 15	52·1	52·1	52·2	52·2	52·2	52·2
2·6	ε Bootis	139	12·7	12·5	12·3	12·1	12·1	12·1	N. 27	14·8	14·8	14·8	14·9	15·0	15·1
2·9	α Lupi	140	12·9	12·6	12·3	12·1	12·0	12·1	S. 47	12·4	12·4	12·5	12·6	12·7	12·8
0·1	α Centauri 38	140	48·7	48·3	48·0	47·8	47·7	47·7	S. 60	39·6	39·6	39·7	39·9	40·0	40·1
2·6	η Centauri	141	47·3	47·0	46·8	46·6	46·5	46·6	S. 41	58·3	58·4	58·5	58·6	58·7	58·7
3·0	γ Bootis	142	24·2	24·0	23·8	23·6	23·6	23·7	N. 38	29·2	29·2	29·2	29·3	29·4	29·5
0·2	α Bootis 37	146	33·7	33·5	33·3	33·2	33·2	33·2	N. 19	23·9	23·8	23·8	23·8	23·9	24·0
2·3	θ Centauri 36	148	56·7	56·4	56·2	56·1	56·1	56·1	S. 36	09·7	09·8	09·9	10·0	10·1	10·1
0·9	β Centauri 35	149	47·0	46·6	46·3	46·1	46·1	46·1	S. 60	10·0	10·1	10·3	10·4	10·5	10·6
3·1	ζ Centauri	151	46·1	45·7	45·5	45·4	45·4	45·4	S. 47	04·8	04·9	05·0	05·1	05·2	05·3
2·8	η Bootis	151	49·6	49·4	49·2	49·1	49·1	49·2	N. 18	36·3	36·2	36·2	36·3	36·3	36·4
1·9	η Ursæ Majoris 34	153	31·7	31·4	31·2	31·1	31·1	31·2	N. 49	31·0	31·0	31·1	31·2	31·3	31·4
2·6	ε Centauri †	155	41·4	41·1	40·8	40·7	40·7	40·8	S. 53	15·0	15·1	15·2	15·4	15·5	15·6
1·2	α Virginis 33	159	15·1	14·9	14·7	14·7	14·7	14·7	S. 10	56·6	56·7	56·8	56·8	56·8	56·8
2·2	ζ Ursæ Majoris	159	26·4	26·1	25·9	25·8	25·9	26·0	N. 55	08·3	08·3	08·4	08·5	08·7	08·7
2·9	ι Centauri	160	26·2	26·0	25·8	25·7	25·7	25·8	S. 36	29·4	29·5	29·6	29·7	29·8	29·8
3·0	ε Virginis	164	58·5	58·3	58·1	58·1	58·1	58·2	N. 11	10·9	10·8	10·8	10·9	10·9	11·0
2·9	α Canum Venat.	166	28·9	28·6	28·5	28·4	28·5	28·6	N. 38	32·4	32·4	32·4	32·5	32·6	32·7
1·7	ε Ursæ Majoris 32	166	57·1	56·8	56·6	56·6	56·6	56·8	N. 56	10·9	10·9	11·0	11·1	11·3	11·3
1·5	β Crucis	168	40·6	40·3	40·1	40·0	40·1	40·3	S. 59	27·4	27·5	27·7	27·8	27·9	28·0
2·9	γ Virginis	170	06·8	06·6	06·5	06·4	06·4	06·5	S. 1	13·3	13·4	13·4	13·4	13·4	13·4
2·4	γ Centauri	170	11·7	11·4	11·2	11·2	11·2	11·4	S. 48	43·6	43·8	43·9	44·0	44·1	44·2
2·9	α Muscæ	171	19·4	18·9	18·7	18·6	18·8	19·0	S. 68	54·1	54·2	54·4	54·5	54·7	54·7
2·8	β Corvi	171	57·0	56·8	56·7	56·6	56·7	56·7	S. 23	09·9	10·0	10·1	10·2	10·2	10·2
1·6	γ Crucis 31	172	47·1	46·8	46·6	46·6	46·6	46·8	S. 56	52·5	52·7	52·8	53·0	53·1	53·2
1·1	α Crucis 30	173	55·5	55·2	55·0	55·0	55·1	55·3	S. 62	51·8	52·0	52·1	52·3	52·4	52·5
2·8	γ Corvi 29	176	35·0	34·8	34·7	34·7	34·7	34·8	S. 17	18·6	18·7	18·8	18·8	18·8	18·8
2·9	δ Centauri	178	26·9	26·6	26·5	26·5	26·6	26·7	S. 50	29·2	29·4	29·5	29·6	29·7	29·8
2·5	γ Ursæ Majoris	182	05·3	05·0	04·9	04·9	05·0	05·2	N. 53	55·3	55·4	55·5	55·6	55·7	55·7
2·2	β Leonis 28	183	15·9	15·7	15·6	15·6	15·7	15·8	N. 14	48·2	48·1	48·1	48·2	48·2	48·3
2·6	δ Leonis	192	01·5	01·3	01·3	01·3	01·4	01·5	N. 20	45·0	45·0	45·0	45·0	45·1	45·1
3·2	ψ Ursæ Majoris	193	10·0	09·8	09·7	09·8	09·9	10·1	N. 44	43·3	43·3	43·4	43·5	43·6	43·6
2·0	α Ursæ Majoris 27	194	42·3	42·0	41·9	42·0	42·3	42·5	N. 61	58·3	58·4	58·5	58·6	58·7	58·7
2·4	β Ursæ Majoris	195	09·8	09·6	09·5	09·6	09·8	10·0	N. 56	36·1	36·2	36·3	36·4	36·5	36·5
2·8	μ Velorum*	198	45·0	44·8	44·8	44·9	45·0	45·2	S. 49	11·9	12·0	12·2	12·3	12·4	12·4
3·0	θ Carinæ*†	199	37·3	37·1	37·1	37·2	37·5	37·8	S. 64	10·4	10·6	10·7	10·9	11·0	11·0
2·3	γ Leonis	205	34·7	34·5	34·5	34·6	34·7	34·8	N. 20	03·0	03·0	03·0	03·1	03·1	03·1
1·3	α Leonis 26	208	27·5	27·4	27·4	27·4	27·5	27·6	N. 12	10·1	10·1	10·1	10·1	10·2	10·2
3·1	ε Leonis	214	07·5	07·4	07·4	07·5	07·6	07·7	N. 23	57·8	57·9	57·9	57·9	58·0	58·0
3·0	N Velorum	217	30·0	30·0	30·0	30·2	30·5	30·7	S. 56	51·0	51·1	51·3	51·4	51·4	51·4
2·2	α Hydræ 25	218	36·7	36·6	36·6	36·7	36·8	36·9	S. 8	28·7	28·8	28·9	28·9	28·9	28·8
2·6	κ Velorum*	219	47·1	47·0	47·1	47·3	47·5	47·7	S. 54	49·9	50·1	50·2	50·3	50·4	50·3
2·2	ι Carinæ*	220	59·7	59·6	59·7	59·9	60·2	60·5	S. 59	06·0	06·2	06·3	06·4	06·4	06·4
1·8	β Carinæ* 24	221	47·6	47·6	47·8	48·1	48·6	48·9	S. 69	32·7	32·9	33·0	33·2	33·2	33·1
2·2	λ Velorum* 23	223	22·6	22·6	22·6	22·8	23·0	23·1	S. 43	15·8	16·0	16·1	16·2	16·2	16·2
3·1	ι Ursæ Majoris	225	54·3	54·2	54·2	54·4	54·6	54·7	N. 48	12·2	12·3	12·4	12·4	12·4	12·4
2·0	δ Velorum*	229	06·1	06·1	06·2	06·4	06·7	06·9	S. 54	33·4	33·5	33·7	33·7	33·8	33·7
1·7	ε Carinæ* 22	234	34·4	34·4	34·6	34·9	35·1	35·4	S. 59	22·6	22·7	22·9	22·9	22·9	22·9
1·9	γ Velorum*	237	55·8	55·8	55·9	56·1	56·3	56·5	S. 47	12·9	13·1	13·2	13·2	13·2	13·1
2·9	ρ Puppis*	238	33·1	33·1	33·2	33·3	33·5	33·6	S. 24	11·1	11·2	11·3	11·3	11·3	11·2
2·3	ζ Puppis*	239	27·8	27·8	27·9	28·1	28·3	28·4	S. 39	53·2	53·4	53·5	53·5	53·5	53·4
1·2	β Geminorum 21	244	18·2	18·2	18·3	18·4	18·5	18·6	N. 28	07·6	07·6	07·6	07·6	07·6	07·6
0·5	α Canis Minoris 20	245	42·9	42·9	43·0	43·1	43·2	43·3	N. 5	19·8	19·8	19·8	19·8	19·8	19·8

* Formerly Argus † Not suitable for use with H.O. 214 (H.D. 486)

POLARIS (POLE STAR) TABLES, 1958

FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

L.H.A. ARIES	240°–249°	250°–259°	260°–269°	270°–279°	280°–289°	290°–299°	300°–309°	310°–319°	320°–329°	330°–339°	340°–349°	350°–359°
	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	1 46·6	1 41·0	1 34·0	1 26·1	1 17·3	1 07·9	0 58·2	0 48·5	0 39·1	0 30·4	0 22·4	0 15·6
1	46·1	40·3	33·3	25·2	16·3	06·9	57·2	47·6	38·2	29·5	21·7	15·0
2	45·6	39·7	32·5	24·4	15·4	06·0	56·3	46·6	37·3	28·7	21·0	14·4
3	45·1	39·0	31·7	23·5	14·5	05·0	55·3	45·7	36·4	27·9	20·2	13·8
4	44·5	38·3	31·0	22·6	13·6	04·0	54·3	44·7	35·5	27·1	19·5	13·2
5	1 43·9	1 37·6	1 30·2	1 21·7	1 12·6	1 03·1	0 53·3	0 43·8	0 34·7	0 26·3	0 18·9	0 12·7
6	43·4	36·9	29·4	20·9	11·7	02·1	52·4	42·8	33·8	25·5	18·2	12·1
7	42·8	36·2	28·6	20·0	10·7	01·1	51·4	41·9	32·9	24·7	17·5	11·6
8	42·2	35·5	27·7	19·1	09·8	1 00·1	50·4	41·0	32·1	23·9	16·9	11·1
9	41·6	34·8	26·9	18·2	08·8	0 59·2	49·5	40·1	31·2	23·2	16·2	10·6
10	1 41·0	1 34·0	1 26·1	1 17·3	1 07·9	0 58·2	0 48·5	0 39·1	0 30·4	0 22·4	0 15·6	0 10·1
Lat.	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
°	′	′	′	′	′	′	′	′	′	′	′	′
0	0·4	0·3	0·2	0·2	0·1	0·1	0·1	0·1	0·2	0·3	0·3	0·4
10	·4	·4	·3	·2	·2	·1	·1	·2	·2	·3	·4	·5
20	·5	·4	·3	·3	·3	·2	·2	·3	·3	·4	·4	·5
30	·5	·5	·4	·4	·3	·3	·3	·3	·4	·4	·5	·5
40	0·5	0·5	0·5	0·5	0·5	0·4	0·4	0·5	0·5	0·5	0·5	0·5
45	·6	·6	·5	·5	·5	·5	·5	·5	·5	·5	·6	·6
50	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6
55	·6	·7	·7	·7	·7	·7	·7	·7	·7	·7	·7	·6
60	·7	·7	·8	·8	·8	·8	·8	·8	·8	·8	·7	·7
62	0·7	0·8	0·8	0·9	0·9	0·9	0·9	0·9	0·8	0·8	0·8	0·7
64	·7	·8	·9	0·9	1·0	1·0	1·0	1·0	0·9	·9	·8	·7
66	·8	·9	0·9	1·0	1·0	1·1	1·1	1·0	1·0	0·9	·8	·8
68	0·8	0·9	1·0	1·1	1·1	1·2	1·2	1·1	1·1	1·0	0·9	0·8
Month	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
	′	′	′	′	′	′	′	′	′	′	′	′
Jan.	0·5	0·5	0·5	0·5	0·5	0·5	0·6	0·6	0·6	0·6	0·6	0·7
Feb.	·4	·4	·4	·4	·4	·4	·4	·4	·5	·5	·5	·6
Mar.	·4	·4	·3	·3	·3	·3	·3	·3	·3	·4	·4	·4
Apr.	0·5	0·4	0·4	0·3	0·3	0·2	0·2	0·2	0·2	0·2	0·2	0·3
May	·6	·6	·5	·4	·4	·3	·3	·2	·2	·2	·2	·2
June	·8	·7	·7	·6	·5	·4	·4	·3	·3	·2	·2	·2
July	0·9	0·8	0·8	0·7	0·7	0·6	0·5	0·5	0·4	0·3	0·3	0·3
Aug.	·9	·9	·9	·8	·8	·8	·7	·6	·6	·5	·5	·4
Sept.	·9	·9	·9	·9	·9	·9	·8	·8	·7	·7	·6	·6
Oct.	0·8	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·8	0·8	0·8
Nov.	·7	·7	·8	·8	·9	·9	1·0	1·0	1·0	1·0	0·9	0·9
Dec.	0·5	0·6	0·6	0·7	0·8	0·8	0·9	0·9	1·0	1·0	1·0	1·0
Lat.	AZIMUTH											
°	°	°	°	°	°	°	°	°	°	°	°	°
0	0·5	0·7	0·8	0·8	0·9	0·9	0·9	0·9	0·8	0·7	0·6	0·5
20	0·6	0·7	0·8	0·9	0·9	1·0	1·0	0·9	0·9	0·8	0·7	0·5
40	0·7	0·9	1·0	1·1	1·2	1·2	1·2	1·2	1·1	1·0	0·8	0·7
50	0·8	1·0	1·2	1·3	1·4	1·4	1·4	1·4	1·3	1·2	1·0	0·8
55	0·9	1·1	1·3	1·5	1·6	1·6	1·6	1·6	1·5	1·3	1·1	0·9
60	1·1	1·3	1·5	1·7	1·8	1·8	1·8	1·8	1·7	1·5	1·3	1·0
65	1·3	1·5	1·8	1·9	2·1	2·2	2·2	2·1	2·0	1·8	1·6	1·3

$$\text{Latitude} = \text{corrected sextant altitude} - 1^\circ + a_0 + a_1 + a_2$$

The table is entered with L.H.A. Aries to determine the column to be used; each column refers to a range of 10°. a_0 is taken, with mental interpolation, from the upper table with the units of L.H.A. Aries in degrees as argument; a_1, a_2 are taken, without interpolation, from the second and third tables with arguments latitude and month respectively. a_0, a_1, a_2 are always positive. The final table gives the azimuth of *Polaris*.

CONVERSION OF ARC TO TIME

0°–59°		60°–119°		120°–179°		180°–239°		240°–299°		300°–359°			0′·00	0′·25	0′·50	0′·75
°	h m	°	h m	°	h m	°	h m	°	h m	°	h m	′	m s	m s	m s	m s
0	0 00	60	4 00	120	8 00	180	12 00	240	16 00	300	20 00	0	0 00	0 01	0 02	0 03
1	0 04	61	4 04	121	8 04	181	12 04	241	16 04	301	20 04	1	0 04	0 05	0 06	0 07
2	0 08	62	4 08	122	8 08	182	12 08	242	16 08	302	20 08	2	0 08	0 09	0 10	0 11
3	0 12	63	4 12	123	8 12	183	12 12	243	16 12	303	20 12	3	0 12	0 13	0 14	0 15
4	0 16	64	4 16	124	8 16	184	12 16	244	16 16	304	20 16	4	0 16	0 17	0 18	0 19
5	0 20	65	4 20	125	8 20	185	12 20	245	16 20	305	20 20	5	0 20	0 21	0 22	0 23
6	0 24	66	4 24	126	8 24	186	12 24	246	16 24	306	20 24	6	0 24	0 25	0 26	0 27
7	0 28	67	4 28	127	8 28	187	12 28	247	16 28	307	20 28	7	0 28	0 29	0 30	0 31
8	0 32	68	4 32	128	8 32	188	12 32	248	16 32	308	20 32	8	0 32	0 33	0 34	0 35
9	0 36	69	4 36	129	8 36	189	12 36	249	16 36	309	20 36	9	0 36	0 37	0 38	0 39
10	0 40	70	4 40	130	8 40	190	12 40	250	16 40	310	20 40	10	0 40	0 41	0 42	0 43
11	0 44	71	4 44	131	8 44	191	12 44	251	16 44	311	20 44	11	0 44	0 45	0 46	0 47
12	0 48	72	4 48	132	8 48	192	12 48	252	16 48	312	20 48	12	0 48	0 49	0 50	0 51
13	0 52	73	4 52	133	8 52	193	12 52	253	16 52	313	20 52	13	0 52	0 53	0 54	0 55
14	0 56	74	4 56	134	8 56	194	12 56	254	16 56	314	20 56	14	0 56	0 57	0 58	0 59
15	1 00	75	5 00	135	9 00	195	13 00	255	17 00	315	21 00	15	1 00	1 01	1 02	1 03
16	1 04	76	5 04	136	9 04	196	13 04	256	17 04	316	21 04	16	1 04	1 05	1 06	1 07
17	1 08	77	5 08	137	9 08	197	13 08	257	17 08	317	21 08	17	1 08	1 09	1 10	1 11
18	1 12	78	5 12	138	9 12	198	13 12	258	17 12	318	21 12	18	1 12	1 13	1 14	1 15
19	1 16	79	5 16	139	9 16	199	13 16	259	17 16	319	21 16	19	1 16	1 17	1 18	1 19
20	1 20	80	5 20	140	9 20	200	13 20	260	17 20	320	21 20	20	1 20	1 21	1 22	1 23
21	1 24	81	5 24	141	9 24	201	13 24	261	17 24	321	21 24	21	1 24	1 25	1 26	1 27
22	1 28	82	5 28	142	9 28	202	13 28	262	17 28	322	21 28	22	1 28	1 29	1 30	1 31
23	1 32	83	5 32	143	9 32	203	13 32	263	17 32	323	21 32	23	1 32	1 33	1 34	1 35
24	1 36	84	5 36	144	9 36	204	13 36	264	17 36	324	21 36	24	1 36	1 37	1 38	1 39
25	1 40	85	5 40	145	9 40	205	13 40	265	17 40	325	21 40	25	1 40	1 41	1 42	1 43
26	1 44	86	5 44	146	9 44	206	13 44	266	17 44	326	21 44	26	1 44	1 45	1 46	1 47
27	1 48	87	5 48	147	9 48	207	13 48	267	17 48	327	21 48	27	1 48	1 49	1 50	1 51
28	1 52	88	5 52	148	9 52	208	13 52	268	17 52	328	21 52	28	1 52	1 53	1 54	1 55
29	1 56	89	5 56	149	9 56	209	13 56	269	17 56	329	21 56	29	1 56	1 57	1 58	1 59
30	2 00	90	6 00	150	10 00	210	14 00	270	18 00	330	22 00	30	2 00	2 01	2 02	2 03
31	2 04	91	6 04	151	10 04	211	14 04	271	18 04	331	22 04	31	2 04	2 05	2 06	2 07
32	2 08	92	6 08	152	10 08	212	14 08	272	18 08	332	22 08	32	2 08	2 09	2 10	2 11
33	2 12	93	6 12	153	10 12	213	14 12	273	18 12	333	22 12	33	2 12	2 13	2 14	2 15
34	2 16	94	6 16	154	10 16	214	14 16	274	18 16	334	22 16	34	2 16	2 17	2 18	2 19
35	2 20	95	6 20	155	10 20	215	14 20	275	18 20	335	22 20	35	2 20	2 21	2 22	2 23
36	2 24	96	6 24	156	10 24	216	14 24	276	18 24	336	22 24	36	2 24	2 25	2 26	2 27
37	2 28	97	6 28	157	10 28	217	14 28	277	18 28	337	22 28	37	2 28	2 29	2 30	2 31
38	2 32	98	6 32	158	10 32	218	14 32	278	18 32	338	22 32	38	2 32	2 33	2 34	2 35
39	2 36	99	6 36	159	10 36	219	14 36	279	18 36	339	22 36	39	2 36	2 37	2 38	2 39
40	2 40	100	6 40	160	10 40	220	14 40	280	18 40	340	22 40	40	2 40	2 41	2 42	2 43
41	2 44	101	6 44	161	10 44	221	14 44	281	18 44	341	22 44	41	2 44	2 45	2 46	2 47
42	2 48	102	6 48	162	10 48	222	14 48	282	18 48	342	22 48	42	2 48	2 49	2 50	2 51
43	2 52	103	6 52	163	10 52	223	14 52	283	18 52	343	22 52	43	2 52	2 53	2 54	2 55
44	2 56	104	6 56	164	10 56	224	14 56	284	18 56	344	22 56	44	2 56	2 57	2 58	2 59
45	3 00	105	7 00	165	11 00	225	15 00	285	19 00	345	23 00	45	3 00	3 01	3 02	3 03
46	3 04	106	7 04	166	11 04	226	15 04	286	19 04	346	23 04	46	3 04	3 05	3 06	3 07
47	3 08	107	7 08	167	11 08	227	15 08	287	19 08	347	23 08	47	3 08	3 09	3 10	3 11
48	3 12	108	7 12	168	11 12	228	15 12	288	19 12	348	23 12	48	3 12	3 13	3 14	3 15
49	3 16	109	7 16	169	11 16	229	15 16	289	19 16	349	23 16	49	3 16	3 17	3 18	3 19
50	3 20	110	7 20	170	11 20	230	15 20	290	19 20	350	23 20	50	3 20	3 21	3 22	3 23
51	3 24	111	7 24	171	11 24	231	15 24	291	19 24	351	23 24	51	3 24	3 25	3 26	3 27
52	3 28	112	7 28	172	11 28	232	15 28	292	19 28	352	23 28	52	3 28	3 29	3 30	3 31
53	3 32	113	7 32	173	11 32	233	15 32	293	19 32	353	23 32	53	3 32	3 33	3 34	3 35
54	3 36	114	7 36	174	11 36	234	15 36	294	19 36	354	23 36	54	3 36	3 37	3 38	3 39
55	3 40	115	7 40	175	11 40	235	15 40	295	19 40	355	23 40	55	3 40	3 41	3 42	3 43
56	3 44	116	7 44	176	11 44	236	15 44	296	19 44	356	23 44	56	3 44	3 45	3 46	3 47
57	3 48	117	7 48	177	11 48	237	15 48	297	19 48	357	23 48	57	3 48	3 49	3 50	3 51
58	3 52	118	7 52	178	11 52	238	15 52	298	19 52	358	23 52	58	3 52	3 53	3 54	3 55
59	3 56	119	7 56	179	11 56	239	15 56	299	19 56	359	23 56	59	3 56	3 57	3 58	3 59

The above table is for converting expressions in arc to their equivalent in time; its main use in this Almanac is for the conversion of longitude for application to L.M.T. (*added* if *west*, *subtracted* if *east*) to give G.M.T. or vice versa, particularly in the case of sunrise, sunset, etc.

24ᵐ INCREMENTS AND CORRECTIONS 25ᵐ

24ᵐ	SUN PLANETS	ARIES	MOON	v or d	Corrⁿ	v or d	Corrⁿ	v or d	Corrⁿ
s	° ′	° ′	° ′	′	′	′	′	′	′
00	6 00.0	6 01.0	5 43.6	0.0	0.0	6.0	2.5	12.0	4.9
01	6 00.3	6 01.2	5 43.8	0.1	0.0	6.1	2.5	12.1	4.9
02	6 00.5	6 01.5	5 44.1	0.2	0.1	6.2	2.5	12.2	5.0
03	6 00.8	6 01.7	5 44.3	0.3	0.1	6.3	2.6	12.3	5.0
04	6 01.0	6 02.0	5 44.6	0.4	0.2	6.4	2.6	12.4	5.1
05	6 01.3	6 02.2	5 44.8	0.5	0.2	6.5	2.7	12.5	5.1
06	6 01.5	6 02.5	5 45.0	0.6	0.2	6.6	2.7	12.6	5.1
07	6 01.8	6 02.7	5 45.3	0.7	0.3	6.7	2.7	12.7	5.2
08	6 02.0	6 03.0	5 45.5	0.8	0.3	6.8	2.8	12.8	5.2
09	6 02.3	6 03.2	5 45.7	0.9	0.4	6.9	2.8	12.9	5.3
10	6 02.5	6 03.5	5 46.0	1.0	0.4	7.0	2.9	13.0	5.3
11	6 02.8	6 03.7	5 46.2	1.1	0.4	7.1	2.9	13.1	5.3
12	6 03.0	6 04.0	5 46.5	1.2	0.5	7.2	2.9	13.2	5.4
13	6 03.3	6 04.2	5 46.7	1.3	0.5	7.3	3.0	13.3	5.4
14	6 03.5	6 04.5	5 46.9	1.4	0.6	7.4	3.0	13.4	5.5
15	6 03.8	6 04.7	5 47.2	1.5	0.6	7.5	3.1	13.5	5.5
16	6 04.0	6 05.0	5 47.4	1.6	0.7	7.6	3.1	13.6	5.6
17	6 04.3	6 05.2	5 47.7	1.7	0.7	7.7	3.1	13.7	5.6
18	6 04.5	6 05.5	5 47.9	1.8	0.7	7.8	3.2	13.8	5.6
19	6 04.8	6 05.7	5 48.1	1.9	0.8	7.9	3.2	13.9	5.7
20	6 05.0	6 06.0	5 48.4	2.0	0.8	8.0	3.3	14.0	5.7
21	6 05.3	6 06.3	5 48.6	2.1	0.9	8.1	3.3	14.1	5.8
22	6 05.5	6 06.5	5 48.8	2.2	0.9	8.2	3.3	14.2	5.8
23	6 05.8	6 06.8	5 49.1	2.3	0.9	8.3	3.4	14.3	5.8
24	6 06.0	6 07.0	5 49.3	2.4	1.0	8.4	3.4	14.4	5.9
25	6 06.3	6 07.3	5 49.6	2.5	1.0	8.5	3.5	14.5	5.9
26	6 06.5	6 07.5	5 49.8	2.6	1.1	8.6	3.5	14.6	6.0
27	6 06.8	6 07.8	5 50.0	2.7	1.1	8.7	3.6	14.7	6.0
28	6 07.0	6 08.0	5 50.3	2.8	1.1	8.8	3.6	14.8	6.0
29	6 07.3	6 08.3	5 50.5	2.9	1.2	8.9	3.6	14.9	6.1
30	6 07.5	6 08.5	5 50.8	3.0	1.2	9.0	3.7	15.0	6.1
31	6 07.8	6 08.8	5 51.0	3.1	1.3	9.1	3.7	15.1	6.2
32	6 08.0	6 09.0	5 51.2	3.2	1.3	9.2	3.8	15.2	6.2
33	6 08.3	6 09.3	5 51.5	3.3	1.3	9.3	3.8	15.3	6.2
34	6 08.5	6 09.5	5 51.7	3.4	1.4	9.4	3.8	15.4	6.3
35	6 08.8	6 09.8	5 52.0	3.5	1.4	9.5	3.9	15.5	6.3
36	6 09.0	6 10.0	5 52.2	3.6	1.5	9.6	3.9	15.6	6.4
37	6 09.3	6 10.3	5 52.4	3.7	1.5	9.7	4.0	15.7	6.4
38	6 09.5	6 10.5	5 52.7	3.8	1.6	9.8	4.0	15.8	6.5
39	6 09.8	6 10.8	5 52.9	3.9	1.6	9.9	4.0	15.9	6.5
40	6 10.0	6 11.0	5 53.1	4.0	1.6	10.0	4.1	16.0	6.5
41	6 10.3	6 11.3	5 53.4	4.1	1.7	10.1	4.1	16.1	6.6
42	6 10.5	6 11.5	5 53.6	4.2	1.7	10.2	4.2	16.2	6.6
43	6 10.8	6 11.8	5 53.9	4.3	1.8	10.3	4.2	16.3	6.7
44	6 11.0	6 12.0	5 54.1	4.4	1.8	10.4	4.2	16.4	6.7
45	6 11.3	6 12.3	5 54.3	4.5	1.8	10.5	4.3	16.5	6.7
46	6 11.5	6 12.5	5 54.6	4.6	1.9	10.6	4.3	16.6	6.8
47	6 11.8	6 12.8	5 54.8	4.7	1.9	10.7	4.4	16.7	6.8
48	6 12.0	6 13.0	5 55.1	4.8	2.0	10.8	4.4	16.8	6.9
49	6 12.3	6 13.3	5 55.3	4.9	2.0	10.9	4.5	16.9	6.9
50	6 12.5	6 13.5	5 55.5	5.0	2.0	11.0	4.5	17.0	6.9
51	6 12.8	6 13.8	5 55.8	5.1	2.1	11.1	4.5	17.1	7.0
52	6 13.0	6 14.0	5 56.0	5.2	2.1	11.2	4.6	17.2	7.0
53	6 13.3	6 14.3	5 56.2	5.3	2.2	11.3	4.6	17.3	7.1
54	6 13.5	6 14.5	5 56.5	5.4	2.2	11.4	4.7	17.4	7.1
55	6 13.8	6 14.8	5 56.7	5.5	2.2	11.5	4.7	17.5	7.1
56	6 14.0	6 15.0	5 57.0	5.6	2.3	11.6	4.7	17.6	7.2
57	6 14.3	6 15.3	5 57.2	5.7	2.3	11.7	4.8	17.7	7.2
58	6 14.5	6 15.5	5 57.4	5.8	2.4	11.8	4.8	17.8	7.3
59	6 14.8	6 15.8	5 57.7	5.9	2.4	11.9	4.9	17.9	7.3
60	6 15.0	6 16.0	5 57.9	6.0	2.5	12.0	4.9	18.0	7.4

25ᵐ	SUN PLANETS	ARIES	MOON	v or d	Corrⁿ	v or d	Corrⁿ	v or d	Corrⁿ
s	° ′	° ′	° ′	′	′	′	′	′	′
00	6 15.0	6 16.0	5 57.9	0.0	0.0	6.0	2.6	12.0	5.1
01	6 15.3	6 16.3	5 58.2	0.1	0.0	6.1	2.6	12.1	5.1
02	6 15.5	6 16.5	5 58.4	0.2	0.1	6.2	2.6	12.2	5.2
03	6 15.8	6 16.8	5 58.6	0.3	0.1	6.3	2.7	12.3	5.2
04	6 16.0	6 17.0	5 58.9	0.4	0.2	6.4	2.7	12.4	5.3
05	6 16.3	6 17.3	5 59.1	0.5	0.2	6.5	2.8	12.5	5.3
06	6 16.5	6 17.5	5 59.3	0.6	0.3	6.6	2.8	12.6	5.4
07	6 16.8	6 17.8	5 59.6	0.7	0.3	6.7	2.8	12.7	5.4
08	6 17.0	6 18.0	5 59.8	0.8	0.3	6.8	2.9	12.8	5.4
09	6 17.3	6 18.3	6 00.1	0.9	0.4	6.9	2.9	12.9	5.5
10	6 17.5	6 18.5	6 00.3	1.0	0.4	7.0	3.0	13.0	5.5
11	6 17.8	6 18.8	6 00.5	1.1	0.5	7.1	3.0	13.1	5.6
12	6 18.0	6 19.0	6 00.8	1.2	0.5	7.2	3.1	13.2	5.6
13	6 18.3	6 19.3	6 01.0	1.3	0.6	7.3	3.1	13.3	5.7
14	6 18.5	6 19.5	6 01.3	1.4	0.6	7.4	3.1	13.4	5.7
15	6 18.8	6 19.8	6 01.5	1.5	0.6	7.5	3.2	13.5	5.7
16	6 19.0	6 20.0	6 01.7	1.6	0.7	7.6	3.2	13.6	5.8
17	6 19.3	6 20.3	6 02.0	1.7	0.7	7.7	3.3	13.7	5.8
18	6 19.5	6 20.5	6 02.2	1.8	0.8	7.8	3.3	13.8	5.9
19	6 19.8	6 20.8	6 02.5	1.9	0.8	7.9	3.4	13.9	5.9
20	6 20.0	6 21.0	6 02.7	2.0	0.9	8.0	3.4	14.0	6.0
21	6 20.3	6 21.3	6 02.9	2.1	0.9	8.1	3.4	14.1	6.0
22	6 20.5	6 21.5	6 03.2	2.2	0.9	8.2	3.5	14.2	6.0
23	6 20.8	6 21.8	6 03.4	2.3	1.0	8.3	3.5	14.3	6.1
24	6 21.0	6 22.0	6 03.6	2.4	1.0	8.4	3.6	14.4	6.1
25	6 21.3	6 22.3	6 03.9	2.5	1.1	8.5	3.6	14.5	6.2
26	6 21.5	6 22.5	6 04.1	2.6	1.1	8.6	3.7	14.6	6.2
27	6 21.8	6 22.8	6 04.4	2.7	1.1	8.7	3.7	14.7	6.2
28	6 22.0	6 23.0	6 04.6	2.8	1.2	8.8	3.7	14.8	6.3
29	6 22.3	6 23.3	6 04.8	2.9	1.2	8.9	3.8	14.9	6.3
30	6 22.5	6 23.5	6 05.1	3.0	1.3	9.0	3.8	15.0	6.4
31	6 22.8	6 23.8	6 05.3	3.1	1.3	9.1	3.9	15.1	6.4
32	6 23.0	6 24.0	6 05.6	3.2	1.4	9.2	3.9	15.2	6.5
33	6 23.3	6 24.3	6 05.8	3.3	1.4	9.3	4.0	15.3	6.5
34	6 23.5	6 24.5	6 06.0	3.4	1.4	9.4	4.0	15.4	6.5
35	6 23.8	6 24.8	6 06.3	3.5	1.5	9.5	4.0	15.5	6.6
36	6 24.0	6 25.1	6 06.5	3.6	1.5	9.6	4.1	15.6	6.6
37	6 24.3	6 25.3	6 06.7	3.7	1.6	9.7	4.1	15.7	6.7
38	6 24.5	6 25.6	6 07.0	3.8	1.6	9.8	4.2	15.8	6.7
39	6 24.8	6 25.8	6 07.2	3.9	1.7	9.9	4.2	15.9	6.8
40	6 25.0	6 26.1	6 07.5	4.0	1.7	10.0	4.3	16.0	6.8
41	6 25.3	6 26.3	6 07.7	4.1	1.7	10.1	4.3	16.1	6.8
42	6 25.5	6 26.6	6 07.9	4.2	1.8	10.2	4.3	16.2	6.9
43	6 25.8	6 26.8	6 08.2	4.3	1.8	10.3	4.4	16.3	6.9
44	6 26.0	6 27.1	6 08.4	4.4	1.9	10.4	4.4	16.4	7.0
45	6 26.3	6 27.3	6 08.7	4.5	1.9	10.5	4.5	16.5	7.0
46	6 26.5	6 27.6	6 08.9	4.6	2.0	10.6	4.5	16.6	7.1
47	6 26.8	6 27.8	6 09.1	4.7	2.0	10.7	4.5	16.7	7.1
48	6 27.0	6 28.1	6 09.4	4.8	2.0	10.8	4.6	16.8	7.1
49	6 27.3	6 28.3	6 09.6	4.9	2.1	10.9	4.6	16.9	7.2
50	6 27.5	6 28.6	6 09.8	5.0	2.1	11.0	4.7	17.0	7.2
51	6 27.8	6 28.8	6 10.1	5.1	2.2	11.1	4.7	17.1	7.3
52	6 28.0	6 29.1	6 10.3	5.2	2.2	11.2	4.8	17.2	7.3
53	6 28.3	6 29.3	6 10.6	5.3	2.3	11.3	4.8	17.3	7.4
54	6 28.5	6 29.6	6 10.8	5.4	2.3	11.4	4.8	17.4	7.4
55	6 28.8	6 29.8	6 11.0	5.5	2.3	11.5	4.9	17.5	7.4
56	6 29.0	6 30.1	6 11.3	5.6	2.4	11.6	4.9	17.6	7.5
57	6 29.3	6 30.3	6 11.5	5.7	2.4	11.7	5.0	17.7	7.5
58	6 29.5	6 30.6	6 11.8	5.8	2.5	11.8	5.0	17.8	7.6
59	6 29.8	6 30.8	6 12.0	5.9	2.5	11.9	5.1	17.9	7.6
60	6 30.0	6 31.1	6 12.2	6.0	2.6	12.0	5.1	18.0	7.7

TABLES FOR INTERPOLATING SUNRISE, MOONRISE, ETC.

TABLE I—FOR LATITUDE

Tabular Interval			Difference between the times for consecutive latitudes															
10°	5°	2°	5m	10m	15m	20m	25m	30m	35m	40m	45m	50m	55m	60m	1h 05m	1h 10m	1h 15m	1h 20m
° ′	° ′	° ′	m	m	m	m	m	m	m	m	m	m	m	m	h m	h m	h m	h m
0 30	0 15	0 06	0	0	1	1	1	1	1	2	2	2	2	2	0 02	0 02	0 02	0 02
1 00	0 30	0 12	0	1	1	2	2	3	3	3	4	4	4	5	05	05	05	05
1 30	0 45	0 18	1	1	2	3	3	4	4	5	5	6	7	7	07	07	07	07
2 00	1 00	0 24	1	2	3	4	5	5	6	7	7	8	9	10	10	10	10	10
2 30	1 15	0 30	1	2	4	5	6	7	8	9	9	10	11	12	12	13	13	13
3 00	1 30	0 36	1	3	4	6	7	8	9	10	11	12	13	14	0 15	0 15	0 16	0 16
3 30	1 45	0 42	2	3	5	7	8	10	11	12	13	14	16	17	18	18	19	19
4 00	2 00	0 48	2	4	6	8	9	11	13	14	15	16	18	19	20	21	22	22
4 30	2 15	0 54	2	4	7	9	11	13	15	16	18	19	21	22	23	24	25	26
5 00	2 30	1 00	2	5	7	10	12	14	16	18	20	22	23	25	26	27	28	29
5 30	2 45	1 06	3	5	8	11	13	16	18	20	22	24	26	28	0 29	0 30	0 31	0 32
6 00	3 00	1 12	3	6	9	12	14	17	20	22	24	26	29	31	32	33	34	36
6 30	3 15	1 18	3	6	10	13	16	19	22	24	26	29	31	34	36	37	38	40
7 00	3 30	1 24	3	7	10	14	17	20	23	26	29	31	34	37	39	41	42	44
7 30	3 45	1 30	4	7	11	15	18	22	25	28	31	34	37	40	43	44	46	48
8 00	4 00	1 36	4	8	12	16	20	23	27	30	34	37	41	44	0 47	0 48	0 51	0 53
8 30	4 15	1 42	4	8	13	17	21	25	29	33	36	40	44	48	0 51	0 53	0 56	0 58
9 00	4 30	1 48	4	9	13	18	22	27	31	35	39	43	47	52	0 55	0 58	1 01	1 04
9 30	4 45	1 54	5	9	14	19	24	28	33	38	42	47	51	56	1 00	1 04	1 08	1 12
10 00	5 00	2 00	5	10	15	20	25	30	35	40	45	50	55	60	1 05	1 10	1 15	1 20

Table I is for interpolating the L.M.T. of sunrise, twilight, moonrise, etc. for latitude. It is to be noted that the interpolation is not linear, so that when using this table it is essential to take out the required phenomenon for the latitude *less* than the true latitude. The table is entered with the nearest value of the difference between the times for the tabular latitude and the next higher one, and, in the appropriate column, with the difference between true latitude and tabular latitude; the correction so obtained is applied to the time for the tabular latitude; the sign of the correction can be seen by inspection.

TABLE II—FOR LONGITUDE

Long. East or West	Difference between the times for given date and preceding date (for east longitude) or for given date and following date (for west longitude)																	
	10m	20m	30m	40m	50m	60m	1h + 10m	1h + 20m	1h + 30m	1h + 40m	1h + 50m	1h + 60m	2h 10m	2h 20m	2h 30m	2h 40m	2h 50m	3h 00m
°	m	m	m	m	m	m	m	m	m	m	m	m	h m	h m	h m	h m	h m	h m
0	0	0	0	0	0	0	0	0	0	0	0	0	0 00	0 00	0 00	0 00	0 00	0 00
10	0	1	1	1	1	2	2	2	2	3	3	3	04	04	04	04	05	05
20	1	1	2	2	3	3	4	4	5	6	6	7	07	08	08	09	09	10
30	1	2	2	3	4	5	6	7	7	8	9	10	11	12	12	13	14	15
40	1	2	3	4	6	7	8	9	10	11	12	13	14	16	17	18	19	20
50	1	3	4	6	7	8	10	11	12	14	15	17	0 18	0 19	0 21	0 22	0 24	0 25
60	2	3	5	7	8	10	12	13	15	17	18	20	22	23	25	27	28	30
70	2	4	6	8	10	12	14	16	17	19	21	23	25	27	29	31	33	35
80	2	4	7	9	11	13	16	18	20	22	24	27	29	31	33	36	38	40
90	2	5	7	10	12	15	17	20	22	25	27	30	32	35	37	40	42	45
100	3	6	8	11	14	17	19	22	25	28	31	33	0 36	0 39	0 42	0 44	0 47	0 50
110	3	6	9	12	15	18	21	24	27	31	34	37	40	43	46	49	0 52	0 55
120	3	7	10	13	17	20	23	27	30	33	37	40	43	47	50	53	0 57	1 00
130	4	7	11	14	18	22	25	29	32	36	40	43	47	51	54	0 58	1 01	1 05
140	4	8	12	16	19	23	27	31	35	39	43	47	51	54	0 58	1 02	1 06	1 10
150	4	8	13	17	21	25	29	33	38	42	46	50	0 54	0 58	1 03	1 07	1 11	1 15
160	4	9	13	18	22	27	31	36	40	44	49	53	0 58	1 02	1 07	1 11	1 16	1 20
170	5	9	14	19	24	28	33	38	42	47	52	57	1 01	1 06	1 11	1 16	1 20	1 25
180	5	10	15	20	25	30	35	40	45	50	55	60	1 05	1 10	1 15	1 20	1 25	1 30

Table II is for interpolating the L.M.T. of moonrise, moonset and the Moon's meridian passage for longitude. It is entered with longitude and with the difference between the times for the given date and for the preceding date (in east longitudes) or following date (in west longitudes). The correction is normally *added* for west longitudes and *subtracted* for east longitudes, but if, as occasionally happens, the times become earlier each day instead of later, the signs of the corrections must be reversed.

INDEX TO SELECTED STARS

Name	No.	Mag.	S.H.A.	Dec.
Acamar	7	3·1	316°	S. 40°
Achernar	5	0·6	336	S. 57
Acrux	30	1·1	174	S. 63
Adhara	19	1·6	256	S. 29
Aldebaran	10	1·1	292	N. 16
Alioth	32	1·7	167	N. 56
Alkaid	34	1·9	154	N. 50
Al Na'ir	55	2·2	29	S. 47
Alnilam	15	1·8	276	S. 1
Alphard	25	2·2	219	S. 8
Alphecca	41	2·3	127	N. 27
Alpheratz	1	2·2	358	N. 29
Altair	51	0·9	63	N. 9
Ankaa	2	2·4	354	S. 43
Antares	42	1·2	113	S. 26
Arcturus	37	0·2	147	N. 19
Atria	43	1·9	109	S. 69
Avior	22	1·7	235	S. 59
Bellatrix	13	1·7	279	N. 6
Betelgeuse	16	Var.*	272	N. 7
Canopus	17	−0·9	264	S. 53
Capella	12	0·2	282	N. 46
Deneb	53	1·3	50	N. 45
Denebola	28	2·2	183	N. 15
Diphda	4	2·2	350	S. 18
Dubhe	27	2·0	195	N. 62
Elnath	14	1·8	279	N. 29
Eltanin	47	2·4	91	N. 51
Enif	54	2·5	34	N. 10
Fomalhaut	56	1·3	16	S. 30
Gacrux	31	1·6	173	S. 57
Gienah	29	2·8	177	S. 17
Hadar	35	0·9	150	S. 60
Hamal	6	2·2	329	N. 23
Kaus Australis	48	2·0	85	S. 34
Kochab	40	2·2	137	N. 74
Markab	57	2·6	14	N. 15
Menkar	8	2·8	315	N. 4
Menkent	36	2·3	149	S. 36
Miaplacidus	24	1·8	222	S. 70
Mirfak	9	1·9	310	N. 50
Nunki	50	2·1	77	S. 26
Peacock	52	2·1	54	S. 57
Pollux	21	1·2	244	N. 28
Procyon	20	0·5	246	N. 5
Rasalhague	46	2·1	97	N. 13
Regulus	26	1·3	208	N. 12
Rigel	11	0·3	282	S. 8
Rigil Kentaurus	38	0·1	141	S. 61
Sabik	44	2·6	103	S. 16
Schedar	3	2·5	350	N. 56
Shaula	45	1·7	97	S. 37
Sirius	18	−1·6	259	S. 17
Spica	33	1·2	159	S. 11
Suhail	23	2·2	223	S. 43
Vega	49	0·1	81	N. 39
Zubenelgenubi	39	2·9	138	S. 16

* 0·1—1·2

No.	Name	Mag.	S.H.A.	Dec.
1	*Alpheratz*	2·2	358°	N. 29°
2	*Ankaa*	2·4	354	S. 43
3	*Schedar*	2·5	350	N. 56
4	*Diphda*	2·2	350	S. 18
5	*Achernar*	0·6	336	S. 57
6	*Hamal*	2·2	329	N. 23
7	*Acamar*	3·1	316	S. 40
8	*Menkar*	2·8	315	N. 4
9	*Mirfak*	1·9	310	N. 50
10	*Aldebaran*	1·1	292	N. 16
11	*Rigel*	0·3	282	S. 8
12	*Capella*	0·2	282	N. 46
13	*Bellatrix*	1·7	279	N. 6
14	*Elnath*	1·8	279	N. 29
15	*Alnilam*	1·8	276	S. 1
16	*Betelgeuse*	Var.*	272	N. 7
17	*Canopus*	−0·9	264	S. 53
18	*Sirius*	−1·6	259	S. 17
19	*Adhara*	1·6	256	S. 29
20	*Procyon*	0·5	246	N. 5
21	*Pollux*	1·2	244	N. 28
22	*Avior*	1·7	235	S. 59
23	*Suhail*	2·2	223	S. 43
24	*Miaplacidus*	1·8	222	S. 70
25	*Alphard*	2·2	219	S. 8
26	*Regulus*	1·3	208	N. 12
27	*Dubhe*	2·0	195	N. 62
28	*Denebola*	2·2	183	N. 15
29	*Gienah*	2·8	177	S. 17
30	*Acrux*	1·1	174	S. 63
31	*Gacrux*	1·6	173	S. 57
32	*Alioth*	1·7	167	N. 56
33	*Spica*	1·2	159	S. 11
34	*Alkaid*	1·9	154	N. 50
35	*Hadar*	0·9	150	S. 60
36	*Menkent*	2·3	149	S. 36
37	*Arcturus*	0·2	147	N. 19
38	*Rigil Kentaurus*	0·1	141	S. 61
39	*Zubenelgenubi*	2·9	138	S. 16
40	*Kochab*	2·2	137	N. 74
41	*Alphecca*	2·3	127	N. 27
42	*Antares*	1·2	113	S. 26
43	*Atria*	1·9	109	S. 69
44	*Sabik*	2·6	103	S. 16
45	*Shaula*	1·7	97	S. 37
46	*Rasalhague*	2·1	97	N. 13
47	*Eltanin*	2·4	91	N. 51
48	*Kaus Australis*	2·0	85	S. 34
49	*Vega*	0·1	81	N. 39
50	*Nunki*	2·1	77	S. 26
51	*Altair*	0·9	63	N. 9
52	*Peacock*	2·1	54	S. 57
53	*Deneb*	1·3	50	N. 45
54	*Enif*	2·5	34	N. 10
55	*Al Na'ir*	2·2	29	S. 47
56	*Fomalhaut*	1·3	16	S. 30
57	*Markab*	2·6	14	N. 15

ALTITUDE CORRECTION TABLES 0°–35°—MOON

App. Alt.	0°–4° Corr^{n}	5°–9° Corr^{n}	10°–14° Corr^{n}	15°–19° Corr^{n}	20°–24° Corr^{n}	25°–29° Corr^{n}	30°–34° Corr^{n}	App. Alt.
′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	′
00	0 33·8	5 58·2	10 62·1	15 62·8	20 62·2	25 60·8	30 58·9	00
10	35·9	58·5	62·2	62·8	62·1	60·8	58·8	10
20	37·8	58·7	62·2	62·8	62·1	60·7	58·8	20
30	39·6	58·9	62·3	62·8	62·1	60·7	58·7	30
40	41·2	59·1	62·3	62·8	62·0	60·6	58·6	40
50	42·6	59·3	62·4	62·7	62·0	60·6	58·5	50
00	1 44·0	6 59·5	11 62·4	16 62·7	21 62·0	26 60·5	31 58·5	00
10	45·2	59·7	62·4	62·7	61·9	60·4	58·4	10
20	46·3	59·9	62·5	62·7	61·9	60·4	58·3	20
30	47·3	60·0	62·5	62·7	61·9	60·3	58·2	30
40	48·3	60·2	62·5	62·7	61·8	60·3	58·2	40
50	49·2	60·3	62·6	62·7	61·8	60·2	58·1	50
00	2 50·0	7 60·5	12 62·6	17 62·7	22 61·7	27 60·1	32 58·0	00
10	50·8	60·6	62·6	62·6	61·7	60·1	57·9	10
20	51·4	60·7	62·6	62·6	61·6	60·0	57·8	20
30	52·1	60·9	62·7	62·6	61·6	59·9	57·8	30
40	52·7	61·0	62·7	62·6	61·5	59·9	57·7	40
50	53·3	61·1	62·7	62·6	61·5	59·8	57·6	50
00	3 53·8	8 61·2	13 62·7	18 62·5	23 61·5	28 59·7	33 57·5	00
10	54·3	61·3	62·7	62·5	61·4	59·7	57·4	10
20	54·8	61·4	62·7	62·5	61·4	59·6	57·4	20
30	55·2	61·5	62·8	62·5	61·3	59·6	57·3	30
40	55·6	61·6	62·8	62·4	61·3	59·5	57·2	40
50	56·0	61·6	62·8	62·4	61·2	59·4	57·1	50
00	4 56·4	9 61·7	14 62·8	19 62·4	24 61·2	29 59·3	34 57·0	00
10	56·7	61·8	62·8	62·3	61·1	59·3	56·9	10
20	57·1	61·9	62·8	62·3	61·1	59·2	56·9	20
30	57·4	61·9	62·8	62·3	61·0	59·1	56·8	30
40	57·7	62·0	62·8	62·2	60·9	59·1	56·7	40
50	57·9	62·1	62·8	62·2	60·9	59·0	56·6	50

H.P.	L	U	L	U	L	U	L	U	L	U	L	U	L	U	H.P.
′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′
54·0	0·3	0·9	0·3	0·9	0·4	1·0	0·5	1·1	0·6	1·2	0·7	1·3	0·9	1·5	54·0
54·3	0·7	1·1	0·7	1·2	0·7	1·2	0·8	1·3	0·9	1·4	1·1	1·5	1·2	1·7	54·3
54·6	1·1	1·4	1·1	1·4	1·1	1·4	1·2	1·5	1·3	1·6	1·4	1·7	1·5	1·8	54·6
54·9	1·4	1·6	1·5	1·6	1·5	1·6	1·6	1·7	1·6	1·8	1·8	1·9	1·9	2·0	54·9
55·2	1·8	1·8	1·8	1·8	1·9	1·9	1·9	1·9	2·0	2·0	2·1	2·1	2·2	2·2	55·2
55·5	2·2	2·0	2·2	2·0	2·3	2·1	2·3	2·1	2·4	2·2	2·4	2·3	2·5	2·4	55·5
55·8	2·6	2·2	2·6	2·2	2·6	2·3	2·7	2·3	2·7	2·4	2·8	2·4	2·9	2·5	55·8
56·1	3·0	2·4	3·0	2·5	3·0	2·5	3·0	2·5	3·1	2·6	3·1	2·6	3·2	2·7	56·1
56·4	3·4	2·7	3·4	2·7	3·4	2·7	3·4	2·7	3·4	2·8	3·5	2·8	3·5	2·9	56·4
56·7	3·7	2·9	3·7	2·9	3·8	2·9	3·8	2·9	3·8	3·0	3·8	3·0	3·9	3·0	56·7
57·0	4·1	3·1	4·1	3·1	4·1	3·1	4·1	3·1	4·2	3·1	4·2	3·2	4·2	3·2	57·0
57·3	4·5	3·3	4·5	3·3	4·5	3·3	4·5	3·3	4·5	3·3	4·5	3·4	4·6	3·4	57·3
57·6	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·6	57·6
57·9	5·3	3·8	5·3	3·8	5·2	3·8	5·2	3·7	5·2	3·7	5·2	3·7	5·2	3·7	57·9
58·2	5·6	4·0	5·6	4·0	5·6	4·0	5·6	4·0	5·6	3·9	5·6	3·9	5·6	3·9	58·2
58·5	6·0	4·2	6·0	4·2	6·0	4·2	6·0	4·2	6·0	4·1	5·9	4·1	5·9	4·1	58·5
58·8	6·4	4·4	6·4	4·4	6·4	4·4	6·3	4·4	6·3	4·3	6·3	4·3	6·2	4·2	58·8
59·1	6·8	4·6	6·8	4·6	6·7	4·6	6·7	4·6	6·7	4·5	6·6	4·5	6·6	4·4	59·1
59·4	7·2	4·8	7·1	4·8	7·1	4·8	7·1	4·8	7·0	4·7	7·0	4·7	6·9	4·6	59·4
59·7	7·5	5·1	7·5	5·0	7·5	5·0	7·5	5·0	7·4	4·9	7·3	4·8	7·2	4·7	59·7
60·0	7·9	5·3	7·9	5·3	7·9	5·2	7·8	5·2	7·8	5·1	7·7	5·0	7·6	4·9	60·0
60·3	8·3	5·5	8·3	5·5	8·2	5·4	8·2	5·4	8·1	5·3	8·0	5·2	7·9	5·1	60·3
60·6	8·7	5·7	8·7	5·7	8·6	5·7	8·6	5·6	8·5	5·5	8·4	5·4	8·2	5·3	60·6
60·9	9·1	5·9	9·0	5·9	9·0	5·9	8·9	5·8	8·8	5·7	8·7	5·6	8·6	5·4	60·9
61·2	9·5	6·2	9·4	6·1	9·4	6·1	9·3	6·0	9·2	5·9	9·1	5·8	8·9	5·6	61·2
61·5	9·8	6·4	9·8	6·3	9·7	6·3	9·7	6·2	9·5	6·1	9·4	5·9	9·2	5·8	61·5

DIP

Ht. of Eye	Corr^{n}	Ht. of Eye	Corr^{n}	Ht. of Eye	Corr^{n}
ft.	′	ft.	′	ft.	′
4·0		24		63	
	−2·0		−4·9		−7·8
4·4		26		65	
	−2·1		−5·0		−7·9
4·9		27		67	
	−2·2		−5·1		−8·0
5·3		28		68	
	−2·3		−5·2		−8·1
5·8		29		70	
	−2·4		−5·3		−8·2
6·3		30		72	
	−2·5		−5·4		−8·3
6·9		31		74	
	−2·6		−5·5		−8·4
7·4		32		75	
	−2·7		−5·6		−8·5
8·0		33		77	
	−2·8		−5·7		−8·6
8·6		35		79	
	−2·9		−5·8		−8·7
9·2		36		81	
	−3·0		−5·9		−8·8
9·8		37		83	
	−3·1		−6·0		−8·9
10·5		38		85	
	−3·2		−6·1		−9·0
11·2		40		87	
	−3·3		−6·2		−9·1
11·9		41		88	
	−3·4		−6·3		−9·2
12·6		42		90	
	−3·5		−6·4		−9·3
13·3		44		92	
	−3·6		−6·5		−9·4
14·1		45		94	
	−3·7		−6·6		−9·5
14·9		47		96	
	−3·8		−6·7		−9·6
15·7		48		98	
	−3·9		−6·8		−9·7
16·5		49		101	
	−4·0		−6·9		−9·8
17·4		51		103	
	−4·1		−7·0		−9·9
18·3		52		105	
	−4·2		−7·1		−10·0
19·1		54		107	
	−4·3		−7·2		−10·1
20·1		55		109	
	−4·4		−7·3		−10·2
21·0		57		111	
	−4·5		−7·4		−10·3
22·0		58		113	
	−4·6		−7·5		−10·4
22·9		60		116	
	−4·7		−7·6		−10·5
23·9		62		118	
	−4·8		−7·7		−10·6
24·9		63		120	

MOON CORRECTION TABLE

The correction is in two parts; the first correction is taken from the upper part of the table with argument apparent altitude, and the second from the lower part, with argument H.P., in the same column as that from which the first correction was taken. Separate corrections are given in the lower part for lower (L) and upper (U) limbs. All corrections are to be **added** to apparent altitude, *but 30′ is to be subtracted from the altitude of the upper limb.*

For corrections for pressure and temperature see page A4.

For bubble sextant observations ignore dip, take the mean of upper and lower limb corrections and subtract 15′ from the altitude.

App. Alt. = Apparent altitude = Sextant altitude corrected for index error and dip.

ALTITUDE CORRECTION TABLES 35°–90°—MOON

App. Alt.	35°–39° Corr^n	40°–44° Corr^n	45°–49° Corr^n	50°–54° Corr^n	55°–59° Corr^n	60°–64° Corr^n	65°–69° Corr^n	70°–74° Corr^n	75°–79° Corr^n	80°–84° Corr^n	85°–89° Corr^n	App. Alt.
′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	′
00	35 56.5	40 53.7	45 50.5	50 46.9	55 43.1	60 38.9	65 34.6	70 30.1	75 25.3	80 20.5	85 15.6	00
10	56.4	53.6	50.4	46.8	42.9	38.8	34.4	29.9	25.2	20.4	15.5	10
20	56.3	53.5	50.2	46.7	42.8	38.7	34.3	29.7	25.0	20.2	15.3	20
30	56.2	53.4	50.1	46.5	42.7	38.5	34.1	29.6	24.9	20.0	15.1	30
40	56.2	53.3	50.0	46.4	42.5	38.4	34.0	29.4	24.7	19.9	15.0	40
50	56.1	53.2	49.9	46.3	42.4	38.2	33.8	29.3	24.5	19.7	14.8	50
00	36 56.0	41 53.1	46 49.8	51 46.2	56 42.3	61 38.1	66 33.7	71 29.1	76 24.4	81 19.6	86 14.6	00
10	55.9	53.0	49.7	46.0	42.1	37.9	33.5	29.0	24.2	19.4	14.5	10
20	55.8	52.8	49.5	45.9	42.0	37.8	33.4	28.8	24.1	19.2	14.3	20
30	55.7	52.7	49.4	45.8	41.8	37.7	33.2	28.7	23.9	19.1	14.1	30
40	55.6	52.6	49.3	45.7	41.7	37.5	33.1	28.5	23.8	18.9	14.0	40
50	55.5	52.5	49.2	45.5	41.6	37.4	32.9	28.3	23.6	18.7	13.8	50
00	37 55.4	42 52.4	47 49.1	52 45.4	57 41.4	62 37.2	67 32.8	72 28.2	77 23.4	82 18.6	87 13.7	00
10	55.3	52.3	49.0	45.3	41.3	37.1	32.6	28.0	23.3	18.4	13.5	10
20	55.2	52.2	48.8	45.2	41.2	36.9	32.5	27.9	23.1	18.2	13.3	20
30	55.1	52.1	48.7	45.0	41.0	36.8	32.3	27.7	22.9	18.1	13.2	30
40	55.0	52.0	48.6	44.9	40.9	36.6	32.2	27.6	22.8	17.9	13.0	40
50	55.0	51.9	48.5	44.8	40.8	36.5	32.0	27.4	22.6	17.8	12.8	50
00	38 54.9	43 51.8	48 48.4	53 44.6	58 40.6	63 36.4	68 31.9	73 27.2	78 22.5	83 17.6	88 12.7	00
10	54.8	51.7	48.2	44.5	40.5	36.2	31.7	27.1	22.3	17.4	12.5	10
20	54.7	51.6	48.1	44.4	40.3	36.1	31.6	26.9	22.1	17.3	12.3	20
30	54.6	51.5	48.0	44.2	40.2	35.9	31.4	26.8	22.0	17.1	12.2	30
40	54.5	51.4	47.9	44.1	40.1	35.8	31.3	26.6	21.8	16.9	12.0	40
50	54.4	51.2	47.8	44.0	39.9	35.6	31.1	26.5	21.7	16.8	11.8	50
00	39 54.3	44 51.1	49 47.6	54 43.9	59 39.8	64 35.5	69 31.0	74 26.3	79 21.5	84 16.6	89 11.7	00
10	54.2	51.0	47.5	43.7	39.6	35.3	30.8	26.1	21.3	16.5	11.5	10
20	54.1	50.9	47.4	43.6	39.5	35.2	30.7	26.0	21.2	16.3	11.4	20
30	54.0	50.8	47.3	43.5	39.4	35.0	30.5	25.8	21.0	16.1	11.2	30
40	53.9	50.7	47.2	43.3	39.2	34.9	30.4	25.7	20.9	16.0	11.0	40
50	53.8	50.6	47.0	43.2	39.1	34.7	30.2	25.5	20.7	15.8	10.9	50

H.P.	L U	L U	L U	L U	L U	L U	L U	L U	L U	L U	L U	H.P.
′	′ ′	′ ′	′ ′	′ ′	′ ′	′ ′	′ ′	′ ′	′ ′	′ ′	′ ′	′
54.0	1.1 1.7	1.3 1.9	1.5 2.1	1.7 2.4	2.0 2.6	2.3 2.9	2.6 3.2	2.9 3.5	3.2 3.8	3.5 4.1	3.8 4.5	54.0
54.3	1.4 1.8	1.6 2.0	1.8 2.2	2.0 2.5	2.3 2.7	2.5 3.0	2.8 3.2	3.0 3.5	3.3 3.8	3.6 4.1	3.9 4.4	54.3
54.6	1.7 2.0	1.9 2.2	2.1 2.4	2.3 2.6	2.5 2.8	2.7 3.0	3.0 3.3	3.2 3.5	3.5 3.8	3.7 4.1	4.0 4.3	54.6
54.9	2.0 2.2	2.2 2.3	2.3 2.5	2.5 2.7	2.7 2.9	2.9 3.1	3.2 3.3	3.4 3.5	3.6 3.8	3.9 4.0	4.1 4.3	54.9
55.2	2.3 2.3	2.5 2.4	2.6 2.6	2.8 2.8	3.0 2.9	3.2 3.1	3.4 3.3	3.6 3.5	3.8 3.7	4.0 4.0	4.2 4.2	55.2
55.5	2.7 2.5	2.8 2.6	2.9 2.7	3.1 2.9	3.2 3.0	3.4 3.2	3.6 3.4	3.7 3.5	3.9 3.7	4.1 3.9	4.3 4.1	55.5
55.8	3.0 2.6	3.1 2.7	3.2 2.8	3.3 3.0	3.5 3.1	3.6 3.3	3.8 3.4	3.9 3.6	4.1 3.7	4.2 3.9	4.4 4.0	55.8
56.1	3.3 2.8	3.4 2.9	3.5 3.0	3.6 3.1	3.7 3.2	3.8 3.3	4.0 3.4	4.1 3.6	4.2 3.7	4.4 3.8	4.5 4.0	56.1
56.4	3.6 2.9	3.7 3.0	3.8 3.1	3.9 3.2	3.9 3.3	4.0 3.4	4.1 3.5	4.3 3.6	4.4 3.7	4.5 3.8	4.6 3.9	56.4
56.7	3.9 3.1	4.0 3.1	4.1 3.2	4.1 3.3	4.2 3.3	4.3 3.4	4.3 3.5	4.4 3.6	4.5 3.7	4.6 3.8	4.7 3.8	56.7
57.0	4.3 3.2	4.3 3.3	4.3 3.3	4.4 3.4	4.4 3.4	4.5 3.5	4.5 3.5	4.6 3.6	4.7 3.6	4.7 3.7	4.8 3.8	57.0
57.3	4.6 3.4	4.6 3.4	4.6 3.4	4.6 3.5	4.7 3.5	4.7 3.5	4.7 3.6	4.8 3.6	4.8 3.6	4.8 3.7	4.9 3.7	57.3
57.6	4.9 3.6	4.9 3.6	4.9 3.6	4.9 3.6	4.9 3.6	4.9 3.6	4.9 3.6	4.9 3.6	5.0 3.6	5.0 3.6	5.0 3.6	57.6
57.9	5.2 3.7	5.2 3.7	5.2 3.7	5.2 3.7	5.2 3.7	5.1 3.6	5.1 3.6	5.1 3.6	5.1 3.6	5.1 3.6	5.1 3.6	57.9
58.2	5.5 3.9	5.5 3.8	5.5 3.8	5.4 3.8	5.4 3.7	5.4 3.7	5.3 3.7	5.3 3.6	5.2 3.6	5.2 3.5	5.2 3.5	58.2
58.5	5.9 4.0	5.8 4.0	5.8 3.9	5.7 3.9	5.6 3.8	5.6 3.8	5.5 3.7	5.5 3.6	5.4 3.6	5.3 3.5	5.3 3.4	58.5
58.8	6.2 4.2	6.1 4.1	6.0 4.1	6.0 4.0	5.9 3.9	5.8 3.8	5.7 3.7	5.6 3.6	5.5 3.5	5.4 3.5	5.3 3.4	58.8
59.1	6.5 4.3	6.4 4.3	6.3 4.2	6.2 4.1	6.1 4.0	6.0 3.9	5.9 3.8	5.8 3.6	5.7 3.5	5.6 3.4	5.4 3.3	59.1
59.4	6.8 4.5	6.7 4.4	6.6 4.3	6.5 4.2	6.4 4.1	6.2 3.9	6.1 3.8	6.0 3.7	5.8 3.5	5.7 3.4	5.5 3.2	59.4
59.7	7.1 4.6	7.0 4.5	6.9 4.4	6.8 4.3	6.6 4.1	6.5 4.0	6.3 3.8	6.2 3.7	6.0 3.5	5.8 3.3	5.6 3.2	59.7
60.0	7.5 4.8	7.3 4.7	7.2 4.5	7.0 4.4	6.9 4.2	6.7 4.0	6.5 3.9	6.3 3.7	6.1 3.5	5.9 3.3	5.7 3.1	60.0
60.3	7.8 5.0	7.6 4.8	7.5 4.7	7.3 4.5	7.1 4.3	6.9 4.1	6.7 3.9	6.5 3.7	6.3 3.5	6.0 3.2	5.8 3.0	60.3
60.6	8.1 5.1	7.9 5.0	7.7 4.8	7.6 4.6	7.3 4.4	7.1 4.2	6.9 3.9	6.7 3.7	6.4 3.4	6.2 3.2	5.9 2.9	60.6
60.9	8.4 5.3	8.2 5.1	8.0 4.9	7.8 4.7	7.6 4.5	7.3 4.2	7.1 4.0	6.8 3.7	6.6 3.4	6.3 3.2	6.0 2.9	60.9
61.2	8.7 5.4	8.5 5.2	8.3 5.0	8.1 4.8	7.8 4.5	7.6 4.3	7.3 4.0	7.0 3.7	6.7 3.4	6.4 3.1	6.1 2.8	61.2
61.5	9.1 5.6	8.8 5.4	8.6 5.1	8.3 4.9	8.1 4.6	7.8 4.3	7.5 4.0	7.2 3.7	6.9 3.4	6.5 3.1	6.2 2.7	61.5

APPENDIX W

EXTRACTS FROM *AIR ALMANAC*

STARS, MAY—AUG., 1958

No.	Name		Mag.	S.H.A.	Dec.
				° ′	° ′
7	*Acamar*		3.1	315 50	S. 40 28
5*	*Achernar*	*1*	0.6	335 58	S. 57 27
30*	*Acrux*	*2*	1.1	173 55	S. 62 52
19	*Adhara*	†	1.6	255 45	S. 28 55
10*	*Aldebaran*	*3*	1.1	291 37	N. 16 26
32*	*Alioth*		1.7	166 57	N. 56 11
34*	*Alkaid*		1.9	153 31	N. 49 31
55	*Al Na'ir*		2.2	28 35	S. 47 10
15	*Alnilam*	†	1.8	276 29	S. 1 14
25*	*Alphard*	†	2.2	218 37	S. 8 29
41	*Alphecca*	†	2.3	126 46	N. 26 51
1*	*Alpheratz*	*4*	2.2	358 26	N. 28 52
51*	*Altair*	*5*	0.9	62 48	N. 8 46
2	*Ankaa*		2.4	353 56	S. 42 32
42*	*Antares*	*6*	1.2	113 17	S. 26 20
37*	*Arcturus*	*7*	0.2	146 33	N. 19 24
43	*Atria*		1.9	108 55	S. 68 57
22	*Avior*		1.7	234 35	S. 59 23
13	*Bellatrix*	†	1.7	279 17	N. 6 19
16*	*Betelgeuse*	*8*	0.1–1.2	271 46	N. 7 24
17*	*Canopus*	*9*	−0.9	264 15	S. 52 40
12*	*Capella*	*10*	0.2	281 36	N. 45 57
53*	*Deneb*	*11*	1.3	49 59	N. 45 08
28*	*Denebola*	†	2.2	183 16	N. 14 48
4*	*Diphda*	†	2.2	349 37	S. 18 13
27*	*Dubhe*	*12*	2.0	194 43	N. 61 59
14	*Elnath*	†	1.8	279 05	N. 28 34
47	*Eltanin*		2.4	91 05	N. 51 30
54	*Enif*	†	2.5	34 28	N. 9 41
56*	*Fomalhaut*	*13*	1.3	16 09	S. 29 50
31	*Gacrux*		1.6	172 47	S. 56 53
29	*Gienah*	†	2.8	176 35	S. 17 19
35	*Hadar*		0.9	149 46	S. 60 11
6*	*Hamal*	†	2.2	328 48	N. 23 16
48	*Kaus Aust.*		2.0	84 38	S. 34 24
40*	*Kochab*		2.2	137 18	N. 74 20
57	*Markab*	†	2.6	14 19	N. 14 59
8	*Menkar*	†	2.8	314 58	N. 3 56
36	*Menkent*		2.3	148 56	S. 36 10
24	*Miaplacidus*		1.8	221 49	S. 69 33
9	*Mirfak*		1.9	309 40	N. 49 43
50*	*Nunki*	†	2.1	76 49	S. 26 21
52*	*Peacock*	*14*	2.1	54 24	S. 56 52
21*	*Pollux*	*15*	1.2	244 19	N. 28 08
20*	*Procyon*	*16*	0.5	245 43	N. 5 20
46*	*Rasalhague*	†	2.1	96 45	N. 12 35
26*	*Regulus*	*17*	1.3	208 28	N. 12 10
11*	*Rigel*	*18*	0.3	281 52	S. 8 15
38*	*Rigil Kent.*	*19*	0.1	140 48	S. 60 40
44	*Sabik*	†	2.6	103 00	S. 15 40
3*	*Schedar*		2.5	350 28	N. 56 18
45*	*Shaula*		1.7	97 18	S. 37 04
18*	*Sirius*	*20*	−1.6	259 10	S. 16 40
33*	*Spica*	*21*	1.2	159 15	S. 10 57
23*	*Suhail*		2.2	223 23	S 43 16
49*	*Vega*	*22*	0.1	81 07	N. 38 45
39	*Zuben'ubi*	†	2.9	137 51	S. 15 52

INTERPOLATION OF G.H.A.

Increment to be added for intervals of G.M.T. to G.H.A. of:
Sun, Aries (♈) and planets; Moon.

SUN, etc.		MOON	SUN, etc.		MOON	SUN, etc.		MOON
m s	° ′	m s	m s	° ′	m s	m s	° ′	m s
00 00	0 00	00 00	03 17	0 50	03 25	06 37	1 40	06 52
01	0 01	00 02	21	0 51	03 29	41	1 41	06 56
05	0 02	00 06	25	0 52	03 33	45	1 42	07 00
09	0 03	00 10	29	0 53	03 37	49	1 43	07 04
13	0 04	00 14	33	0 54	03 41	53	1 44	07 08
17	0 05	00 18	37	0 55	03 45	06 57	1 45	07 13
21	0 06	00 22	41	0 56	03 49	07 01	1 46	07 17
25	0 07	00 26	45	0 57	03 54	05	1 47	07 21
29	0 08	00 31	49	0 58	03 58	09	1 48	07 25
33	0 09	00 35	53	0 59	04 02	13	1 49	07 29
37	0 10	00 39	03 57	1 00	04 06	17	1 50	07 33
41	0 11	00 43	04 01	1 01	04 10	21	1 51	07 37
45	0 12	00 47	05	1 02	04 14	25	1 52	07 42
49	0 13	00 51	09	1 03	04 19	29	1 53	07 46
53	0 14	00 55	13	1 04	04 23	33	1 54	07 50
00 57	0 15	01 00	17	1 05	04 27	37	1 55	07 54
01 01	0 16	01 04	21	1 06	04 31	41	1 56	07 58
05	0 17	01 08	25	1 07	04 35	45	1 57	08 02
09	0 18	01 12	29	1 08	04 39	49	1 58	08 06
13	0 19	01 16	33	1 09	04 43	53	1 59	08 11
17	0 20	01 20	37	1 10	04 48	07 57	2 00	08 15
21	0 21	01 24	41	1 11	04 52	08 01	2 01	08 19
25	0 22	01 29	45	1 12	04 56	05	2 02	08 23
29	0 23	01 33	49	1 13	05 00	09	2 03	08 27
33	0 24	01 37	53	1 14	05 04	13	2 04	08 31
37	0 25	01 41	04 57	1 15	05 08	17	2 05	08 35
41	0 26	01 45	05 01	1 16	05 12	21	2 06	08 40
45	0 27	01 49	05	1 17	05 17	25	2 07	08 44
49	0 28	01 53	09	1 18	05 21	29	2 08	08 48
53	0 29	01 58	13	1 19	05 25	33	2 09	08 52
01 57	0 30	02 02	17	1 20	05 29	37	2 10	08 56
02 01	0 31	02 06	21	1 21	05 33	41	2 11	09 00
05	0 32	02 10	25	1 22	05 37	45	2 12	09 04
09	0 33	02 14	29	1 23	05 41	49	2 13	09 09
13	0 34	02 18	33	1 24	05 46	53	2 14	09 13
17	0 35	02 22	37	1 25	05 50	08 57	2 15	09 17
21	0 36	02 27	41	1 26	05 54	09 01	2 16	09 21
25	0 37	02 31	45	1 27	05 58	05	2 17	09 25
29	0 38	02 35	49	1 28	06 02	09	2 18	09 29
33	0 39	02 39	53	1 29	06 06	13	2 19	09 33
37	0 40	02 43	05 57	1 30	06 10	17	2 20	09 38
41	0 41	02 47	06 01	1 31	06 15	21	2 21	09 42
45	0 42	02 51	05	1 32	06 19	25	2 22	09 46
49	0 43	02 56	09	1 33	06 23	29	2 23	09 50
53	0 44	03 00	13	1 34	06 27	33	2 24	09 54
02 57	0 45	03 04	17	1 35	06 31	37	2 25	09 58
03 01	0 46	03 08	21	1 36	06 35	41	2 26	10 00
05	0 47	03 12	25	1 37	06 39	45	2 27	
09	0 48	03 16	29	1 38	06 44	49	2 28	
13	0 49	03 20	33	1 39	06 48	53	2 29	
17	0 50	03 25	37	1 40	06 52	09 57	2 30	
03 21		03 29	06 41		06 56	10 00		

* Stars used in H.O. 249 (A.P. 3270).
† Stars so indicated may be used with declination tables.
The numbers *following* the names are those used in A.P. 1618 (H.O. 218).

CORRECTIONS TO BE APPLIED TO MARINE SEXTANT ALTITUDES

MARINE SEXTANT ERROR

Sextant No.

Index Error

CORRECTIONS

In addition to sextant error and dip, corrections are to be applied for :

- Refraction
- Semi-diameter (for Sun and Moon)
- Parallax (for the Moon)
- Dome refraction if applicable.

CORRECTION FOR DIP OF THE HORIZON

To be subtracted from sextant altitude.

Ht.	Dip	Ht.	Dip	Ht.	Dip	Ht.	Dip	Ht.	Dip
Ft.		Ft.		Ft.		Ft.		Ft.	
0	′	114	′	437	′	968	′	1,707	′
	1		11		21		31		41
2		137		481		1,033		1,792	
	2		12		22		32		42
6		162		527		1,099		1,880	
	3		13		23		33		43
12		189		575		1,168		1,970	
	4		14		24		34		44
21		218		625		1,239		2,061	
	5		15		25		35		45
31		250		677		1,311		2,155	
	6		16		26		36		46
43		283		731		1,386		2,251	
	7		17		27		37		47
58		318		787		1,463		2,349	
	8		18		28		38		48
75		356		845		1,543		2,449	
	9		19		29		39		49
93		395		906		1,624		2,551	
	10		20		30		40		50
114		437		968		1,707		2,655	

LIST OF CONTENTS

GREENWICH P. M. 1958 JUNE 1 (SUNDAY)

GMT	☉ SUN GHA	SUN Dec.	ARIES GHA ♈	VENUS −3.6 GHA	VENUS Dec.
h m	° ′	° ′	° ′	° ′	° ′
12 00	0 35	N22 02	69 28	40 27	N 9 38
10	3 05		71 58	42 56	
20	5 35		74 29	45 26	
30	8 05	· ·	76 59	47 56	· ·
40	10 35		79 30	50 26	
50	13 05		82 00	52 56	
13 00	15 35	N22 02	84 30	55 26	N 9 39
10	18 05		87 01	57 56	
20	20 35		89 31	60 26	
30	23 05	· ·	92 02	62 56	· ·
40	25 35		94 32	65 26	
50	28 05		97 02	67 56	
14 00	30 35	N22 02	99 33	70 26	N 9 40
10	33 05		102 03	72 56	
20	35 35		104 34	75 26	
30	38 05	· ·	107 04	77 56	· ·
40	40 35		109 34	80 26	
50	43 05		112 05	82 56	
15 00	45 35	N22 03	114 35	85 26	N 9 41
10	48 05		117 06	87 56	
20	50 35		119 36	90 26	
30	53 05	· ·	122 07	92 56	· ·
40	55 35		124 37	95 26	
50	58 05		127 07	97 55	
16 00	60 35	N22 03	129 38	100 25	N 9 42
10	63 05		132 08	102 55	
20	65 35		134 39	105 25	
30	68 05	· ·	137 09	107 55	· ·
40	70 35		139 39	110 25	
50	73 05		142 10	112 55	
17 00	75 35	N22 03	144 40	115 25	N 9 43
10	78 05		147 11	117 55	
20	80 35		149 41	120 25	
30	83 05	· ·	152 11	122 55	· ·
40	85 35		154 42	125 25	
50	88 05		157 12	127 55	
18 00	90 35	N22 04	159 43	130 25	N 9 43
10	93 05		162 13	132 55	
20	95 35		164 44	135 25	
30	98 05	· ·	167 14	137 55	· ·
40	100 35		169 44	140 25	
50	103 05		172 15	142 55	
19 00	105 35	N22 04	174 45	145 25	N 9 44
10	108 04		177 16	147 55	
20	110 34		179 46	150 25	
30	113 04	· ·	182 16	152 54	· ·
40	115 34		184 47	155 24	
50	118 04		187 17	157 54	
20 00	120 34	N22 04	189 48	160 24	N 9 45
10	123 04		192 18	162 54	
20	125 34		194 48	165 24	
30	128 04	· ·	197 19	167 54	· ·
40	130 34		199 49	170 24	
50	133 04		202 20	172 54	
21 00	135 34	N22 05	204 50	175 24	N 9 46
10	138 04		207 21	177 54	
20	140 34		209 51	180 24	
30	143 04	· ·	212 21	182 54	· ·
40	145 34		214 52	185 24	
50	148 04		217 22	187 54	
22 00	150 34	N22 05	219 53	190 24	N 9 47
10	153 04		222 23	192 54	
20	155 34		224 53	195 24	
30	158 04	· ·	227 24	197 54	· ·
40	160 34		229 54	200 24	
50	163 04		232 25	202 54	
23 00	165 34	N22 05	234 55	205 24	N 9 48
10	168 04		237 25	207 53	
20	170 34		239 56	210 23	
30	173 04	· ·	242 26	212 53	· ·
40	175 34		244 57	215 23	
50	178 04		247 27	217 53	

GMT	MARS 0.6 GHA	MARS Dec.	JUPITER −1.9 GHA	JUPITER Dec.	○ MOON GHA	MOON Dec.
h m	° ′	° ′	° ′	° ′	° ′	° ′
12 00	72 28	S 3 30	228 23	S 7 20	185 20	S18 19
10	74 58		230 54		187 44	20
20	77 28		233 24		190 08	20
30	79 58	· ·	235 55	· ·	192 32	· 21
40	82 28		238 25		194 56	22
50	84 58		240 56		197 20	22
13 00	87 28	S 3 29	243 26	S 7 20	199 44	S18 23
10	89 58		245 56		202 09	23
20	92 29		248 27		204 33	24
30	94 59	· ·	250 57	· ·	206 57	· 24
40	97 29		253 28		209 21	25
50	99 59		255 58		211 45	26
14 00	102 29	S 3 29	258 29	S 7 20	214 09	S18 26
10	104 59		260 59		216 33	27
20	107 29		263 29		218 57	27
30	110 00	· ·	266 00	· ·	221 21	· 28
40	112 30		268 30		223 45	28
50	115 00		271 01		226 10	29
15 00	117 30	S 3 28	273 31	S 7 20	228 34	S18 29
10	120 00		276 02		230 58	30
20	122 30		278 32		233 22	30
30	125 00	· ·	281 02	· ·	235 46	· 31
40	127 30		283 33		238 10	32
50	130 01		286 03		240 34	32
16 00	132 31	S 3 27	288 34	S 7 20	242 58	S18 33
10	135 01		291 04		245 22	33
20	137 31		293 35		247 46	34
30	140 01	· ·	296 05	· ·	250 10	· 34
40	142 31		298 35		252 35	35
50	145 01		301 06		254 59	35
17 00	147 32	S 3 26	303 36	S 7 20	257 23	S18 36
10	150 02		306 07		259 47	36
20	152 32		308 37		262 11	37
30	155 02	· ·	311 08	· ·	264 35	· 37
40	157 32		313 38		266 59	38
50	160 02		316 08		269 23	38
18 00	162 32	S 3 26	318 39	S 7 20	271 47	S18 39
10	165 02		321 09		274 11	39
20	167 33		323 40		276 36	39
30	170 03	· ·	326 10	· ·	279 00	· 40
40	172 33		328 41		281 24	40
50	175 03		331 11		283 48	41
19 00	177 33	S 3 25	333 42	S 7 20	286 12	S18 41
10	180 03		336 12		288 36	42
20	182 33		338 42		291 00	42
30	185 04	· ·	341 13	· ·	293 24	· 43
40	187 34		343 43		295 48	43
50	190 04		346 14		298 13	43
20 00	192 34	S 3 24	348 44	S 7 20	300 37	S18 44
10	195 04		351 15		303 01	44
20	197 34		353 45		305 25	45
30	200 04	· ·	356 15	· ·	307 49	· 45
40	202 34		358 46		310 13	46
50	205 05		1 16		312 37	46
21 00	207 35	S 3 24	3 47	S 7 20	315 01	S18 47
10	210 05		6 17		317 25	47
20	212 35		8 48		319 49	47
30	215 05	· ·	11 18	· ·	322 14	· 48
40	217 35		13 48		324 38	48
50	220 05		16 19		327 02	49
22 00	222 36	S 3 23	18 49	S 7 20	329 26	S18 49
10	225 06		21 20		331 50	49
20	227 36		23 50		334 14	50
30	230 06	· ·	26 21	· ·	336 38	· 50
40	232 36		28 51		339 02	50
50	235 06		31 21		341 26	51
23 00	237 36	S 3 22	33 52	S 7 20	343 51	S18 51
10	240 06		36 22		346 15	52
20	242 37		38 53		348 39	52
30	245 07	· ·	41 23	· ·	351 03	· 52
40	247 37		43 54		353 27	53
50	250 07		46 24		355 51	53

Lat.	Sun-rise	Twi-light	Moon-rise	Diff.
N				
°	h m	m	h m	m
72	▭	▭	■	*
70	▭	▭	22 49	45
68	▭	▭	21 55	39
66	01 14	////	21 23	37
64	01 59	////	20 59	36
62	02 28	138	20 40	35
60	02 50	84	20 24	35
58	03 08	67	20 11	34
56	03 22	58	20 00	34
54	03 35	51	19 50	33
52	03 46	47	19 41	33
50	03 56	43	19 33	33
45	04 17	36	19 16	32
40	04 33	32	19 02	32
35	04 47	29	18 50	31
30	04 59	27	18 40	31
20	05 20	24	18 23	30
10	05 38	23	18 07	30
0	05 54	22	17 53	30
10	06 10	23	17 39	29
20	06 28	24	17 24	29
30	06 48	26	17 07	28
35	06 59	28	16 57	27
40	07 12	30	16 45	27
45	07 28	33	16 32	26
50	07 47	37	16 16	26
52	07 56	40	16 08	25
54	08 06	42	15 59	25
56	08 17	46	15 50	24
58	08 30	50	15 39	24
60	08 46	55	15 27	23
S				

Lat.	Sun-set	Twi-light	Moon-set	Diff.
N				
°	h m	m	h m	m
72	▭	▭	00 00	*
70	▭	▭	01 08	04
68	▭	▭	01 45	12
66	22 45	////	02 11	15
64	21 59	////	02 31	18
62	21 29	////	02 48	19
60	21 07	84	03 02	20
58	20 49	67	03 14	21
56	20 34	58	03 24	22
54	20 21	51	03 33	23
52	20 10	47	03 41	24
50	20 00	43	03 49	24
45	19 39	36	04 04	25
40	19 22	32	04 17	26
35	19 08	29	04 28	26
30	18 56	27	04 38	27
20	18 36	24	04 54	28
10	18 18	23	05 09	29
0	18 01	22	05 22	29
10	17 45	23	05 36	30
20	17 28	24	05 50	31
30	17 08	26	06 07	32
35	16 57	28	06 17	32
40	16 43	30	06 28	33
45	16 28	33	06 41	34
50	16 09	37	06 57	35
52	16 00	40	07 04	35
54	15 50	42	07 12	36
56	15 38	46	07 22	36
58	15 25	50	07 32	37
60	15 10	55	07 44	38
S				

GREENWICH A. M. 1958 JUNE 2 (MONDAY)

GMT	☉ SUN GHA	SUN Dec.	ARIES GHA ♈	VENUS −3.6 GHA	VENUS Dec.	MARS 0.6 GHA	MARS Dec.	JUPITER −1.9 GHA	JUPITER Dec.	☾ MOON GHA	MOON Dec.
h m	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
00 00	180 34	N22 06	249 58	220 23	N 9 49	252 37	S 3 22	48 54	S 7 20	358 15	S18 53
10	183 04		252 28	222 53		255 07		51 25		0 39	54
20	185 34		254 58	225 23		257 37		53 55		3 03	54
30	188 04	· ·	257 29	227 53	· ·	260 08	· ·	56 26	· ·	5 28	· 54
40	190 34		259 59	230 23		262 38		58 56		7 52	55
50	193 04		262 30	232 53		265 08		61 27		10 16	55
01 00	195 34	N22 06	265 00	235 23	N 9 50	267 38	S 3 21	63 57	S 7 20	12 40	S18 55
10	198 04		267 30	237 53		270 08		66 27		15 04	56
20	200 34		270 01	240 23		272 38		68 58		17 28	56
30	203 04	· ·	272 31	242 53	· ·	275 08	· ·	71 28	· ·	19 52	· 56
40	205 34		275 02	245 23		277 38		73 59		22 16	57
50	208 04		277 32	247 53		280 09		76 29		24 40	57
02 00	210 34	N22 06	280 02	250 23	N 9 51	282 39	S 3 20	79 00	S 7 20	27 05	S18 57
10	213 04		282 33	252 53		285 09		81 30		29 29	58
20	215 34		285 03	255 23		287 39		84 00		31 53	58
30	218 04	· ·	287 34	257 53	· ·	290 09	· ·	86 31	· ·	34 17	· 58
40	220 34		290 04	260 23		292 39		89 01		36 41	58
50	223 04		292 34	262 52		295 09		91 32		39 05	59
03 00	225 34	N22 07	295 05	265 22	N 9 52	297 40	S 3 20	94 02	S 7 20	41 29	S18 59
10	228 04		297 35	267 52		300 10		96 33		43 53	18 59
20	230 34		300 06	270 22		302 40		99 03		46 18	19 00
30	233 04	· ·	302 36	272 52	· ·	305 10	· ·	101 34	· ·	48 42	· 00
40	235 34		305 06	275 22		307 40		104 04		51 06	00
50	238 04		307 37	277 52		310 10		106 34		53 30	00
04 00	240 34	N22 07	310 07	280 22	N 9 53	312 40	S 3 19	109 05	S 7 20	55 54	S19 01
10	243 04		312 38	282 52		315 10		111 35		58 18	01
20	245 34		315 08	285 22		317 41		114 06		60 42	01
30	248 04	· ·	317 39	287 52	· ·	320 11	· ·	116 36	· ·	63 06	· 01
40	250 34		320 09	290 22		322 41		119 07		65 31	02
50	253 04		322 39	292 52		325 11		121 37		67 55	02
05 00	255 34	N22 07	325 10	295 22	N 9 54	327 41	S 3 18	124 07	S 7 20	70 19	S19 02
10	258 04		327 40	297 52		330 11		126 38		72 43	02
20	260 34		330 11	300 22		332 41		129 08		75 07	03
30	263 04	· ·	332 41	302 52	· ·	335 12	· ·	131 39	· ·	77 31	· 03
40	265 34		335 11	305 22		337 42		134 09		79 55	03
50	268 04		337 42	307 52		340 12		136 39		82 19	03
06 00	270 34	N22 08	340 12	310 22	N 9 55	342 42	S 3 17	139 10	S 7 20	84 44	S19 04
10	273 03		342 43	312 52		345 12		141 40		87 08	04
20	275 33		345 13	315 22		347 42		144 11		89 32	04
30	278 03	· ·	347 43	317 51	· ·	350 12	· ·	146 41	· ·	91 56	· 04
40	280 33		350 14	320 21		352 42		149 12		94 20	04
50	283 03		352 44	322 51		355 13		151 42		96 44	05
07 00	285 33	N22 08	355 15	325 21	N 9 56	357 43	S 3 17	154 13	S 7 20	99 08	S19 05
10	288 03		357 45	327 51		0 13		156 43		101 33	05
20	290 33		0 16	330 21		2 43		159 13		103 57	05
30	293 03	· ·	2 46	332 51	· ·	5 13	· ·	161 44	· ·	106 21	· 05
40	295 33		5 16	335 21		7 43		164 14		108 45	05
50	298 03		7 47	337 51		10 13		166 45		111 09	06
08 00	300 33	N22 08	10 17	340 21	N 9 57	12 44	S 3 16	169 15	S 7 20	113 33	S19 06
10	303 03		12 48	342 51		15 14		171 46		115 57	06
20	305 33		15 18	345 21		17 44		174 16		118 21	06
30	308 03	· ·	17 48	347 51	· ·	20 14	· ·	176 46	· ·	120 46	· 06
40	310 33		20 19	350 21		22 44		179 17		123 10	06
50	313 03		22 49	352 51		25 14		181 47		125 34	07
09 00	315 33	N22 09	25 20	355 21	N 9 58	27 44	S 3 15	184 18	S 7 19	127 58	S19 07
10	318 03		27 50	357 51		30 14		186 48		130 22	07
20	320 33		30 20	0 21		32 45		189 19		132 46	07
30	323 03	· ·	32 51	2 51	· ·	35 15	· ·	191 49	· ·	135 10	· 07
40	325 33		35 21	5 21		37 45		194 19		137 35	07
50	328 03		37 52	7 51		40 15		196 50		139 59	08
10 00	330 33	N22 09	40 22	10 21	N 9 59	42 45	S 3 15	199 20	S 7 19	142 23	S19 08
10	333 03		42 53	12 50		45 15		201 51		144 47	08
20	335 33		45 23	15 20		47 45		204 21		147 11	08
30	338 03	· ·	47 53	17 50	· ·	50 16	· ·	206 52	· ·	149 35	· 08
40	340 33		50 24	20 20		52 46		209 22		151 59	08
50	343 03		52 54	22 50		55 16		211 52		154 24	08
11 00	345 33	N22 09	55 25	25 20	N10 00	57 46	S 3 14	214 23	S 7 19	156 48	S19 08
10	348 03		57 55	27 50		60 16		216 53		159 12	09
20	350 33		60 25	30 20		62 46		219 24		161 36	09
30	353 03	· ·	62 56	32 50	· ·	65 16	· ·	221 54	· ·	164 00	· 09
40	355 33		65 26	35 20		67 46		224 25		166 24	09
50	358 03		67 57	37 50		70 17		226 55		168 49	09

Moon's P. in A.

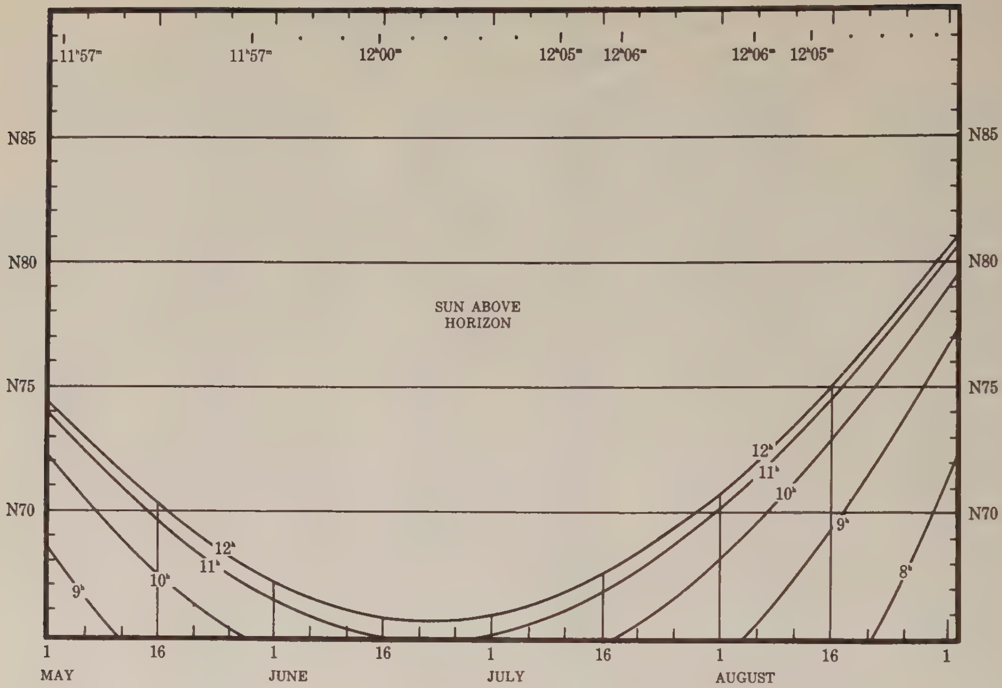
Alt. °	Corr. + ′
0	
	59
7	
	58
13	
	57
17	
	56
20	
	55
22	
	54
25	
	53
27	
	52
29	
	51
31	
	50
33	
	49
34	
	48
36	
	47
38	
	46
39	
	45
41	
	44
42	
	43
43	
	42
45	
	41
46	
	40
48	
	39
49	
	38
50	
	37
51	
	36
53	
	35
54	
	34
55	
	33
56	
	32
57	
	31
58	
	30
60	
	29
61	
	28
62	
	27
63	
	26
64	
	25
65	
	24
66	
	23
67	
	22
68	
	21
69	
	20
70	
	19
71	
	18
72	
	17
73	
	16
74	
	15
75	
	14
76	
	13
77	
	12
78	
	11
79	
	10
80	

☉Sun SD 16′

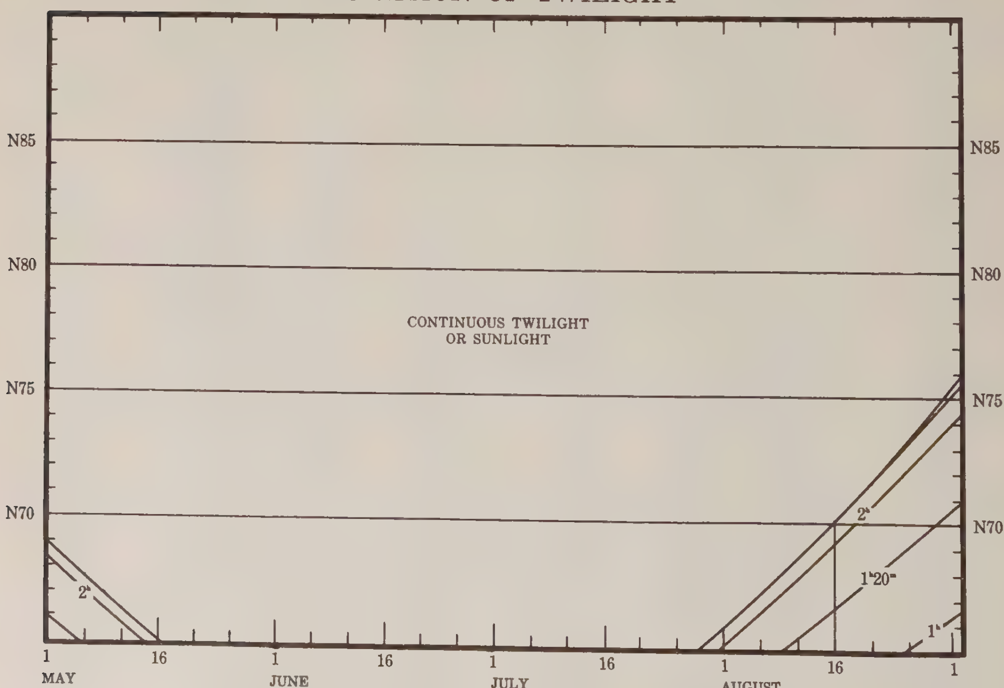
Moon SD 16′

Age 15

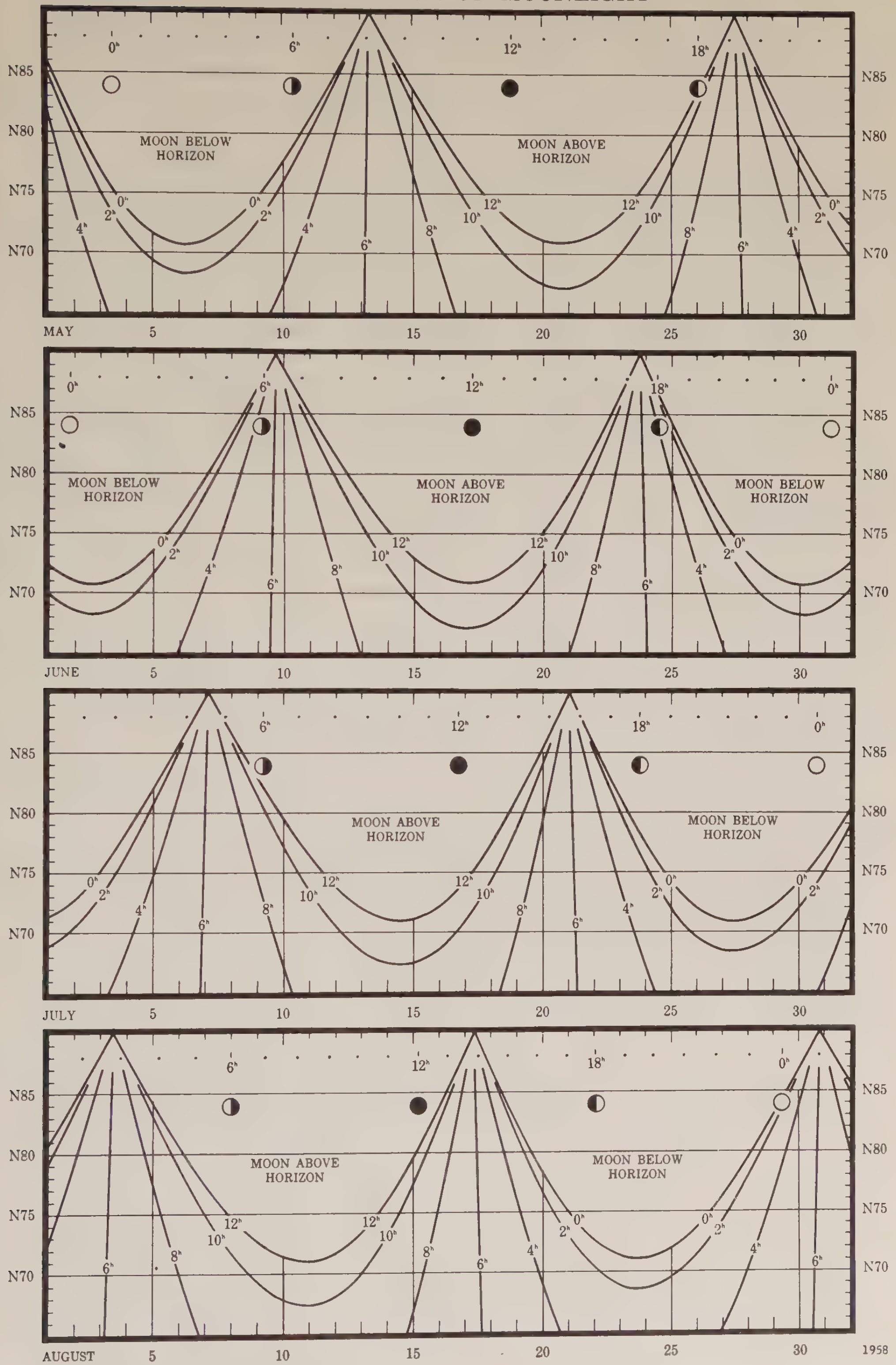
SEMIDURATION OF SUNLIGHT



DURATION OF TWILIGHT



SEMIDURATION OF MOONLIGHT



CONVERSION OF ARC TO TIME

°	h m	°	h m	°	h m	°	h m	′	m s
0	0 00	45	3 00	90	6 00	135	9 00	0	0 00
1	0 04	46	3 04	91	6 04	136	9 04	1	0 04
2	0 08	47	3 08	92	6 08	137	9 08	2	0 08
3	0 12	48	3 12	93	6 12	138	9 12	3	0 12
4	0 16	49	3 16	94	6 16	139	9 16	4	0 16
5	0 20	50	3 20	95	6 20	140	9 20	5	0 20
6	0 24	51	3 24	96	6 24	141	9 24	6	0 24
7	0 28	52	3 28	97	6 28	142	9 28	7	0 28
8	0 32	53	3 32	98	6 32	143	9 32	8	0 32
9	0 36	54	3 36	99	6 36	144	9 36	9	0 36
10	0 40	55	3 40	100	6 40	145	9 40	10	0 40
11	0 44	56	3 44	101	6 44	146	9 44	11	0 44
12	0 48	57	3 48	102	6 48	147	9 48	12	0 48
13	0 52	58	3 52	103	6 52	148	9 52	13	0 52
14	0 56	59	3 56	104	6 56	149	9 56	14	0 56
15	1 00	60	4 00	105	7 00	150	10 00	15	1 00
16	1 04	61	4 04	106	7 04	151	10 04	16	1 04
17	1 08	62	4 08	107	7 08	152	10 08	17	1 08
18	1 12	63	4 12	108	7 12	153	10 12	18	1 12
19	1 16	64	4 16	109	7 16	154	10 16	19	1 16
20	1 20	65	4 20	110	7 20	155	10 20	20	1 20
21	1 24	66	4 24	111	7 24	156	10 24	21	1 24
22	1 28	67	4 28	112	7 28	157	10 28	22	1 28
23	1 32	68	4 32	113	7 32	158	10 32	23	1 32
24	1 36	69	4 36	114	7 36	159	10 36	24	1 36
25	1 40	70	4 40	115	7 40	160	10 40	25	1 40
26	1 44	71	4 44	116	7 44	161	10 44	26	1 44
27	1 48	72	4 48	117	7 48	162	10 48	27	1 48
28	1 52	73	4 52	118	7 52	163	10 52	28	1 52
29	1 56	74	4 56	119	7 56	164	10 56	29	1 56
30	2 00	75	5 00	120	8 00	165	11 00	30	2 00
31	2 04	76	5 04	121	8 04	166	11 04	31	2 04
32	2 08	77	5 08	122	8 08	167	11 08	32	2 08
33	2 12	78	5 12	123	8 12	168	11 12	33	2 12
34	2 16	79	5 16	124	8 16	169	11 16	34	2 16
35	2 20	80	5 20	125	8 20	170	11 20	35	2 20
36	2 24	81	5 24	126	8 24	171	11 24	36	2 24
37	2 28	82	5 28	127	8 28	172	11 28	37	2 28
38	2 32	83	5 32	128	8 32	173	11 32	38	2 32
39	2 36	84	5 36	129	8 36	174	11 36	39	2 36
40	2 40	85	5 40	130	8 40	175	11 40	40	2 40
41	2 44	86	5 44	131	8 44	176	11 44	41	2 44
42	2 48	87	5 48	132	8 48	177	11 48	42	2 48
43	2 52	88	5 52	133	8 52	178	11 52	43	2 52
44	2 56	89	5 56	134	8 56	179	11 56	44	2 56
45	3 00	90	6 00	135	9 00	180	12 00	45	3 00
								46	3 04
								47	3 08
								48	3 12
								49	3 16
								50	3 20
								51	3 24
								52	3 28
								53	3 32
								54	3 36
								55	3 40
								56	3 44
								57	3 48
								58	3 52
								59	3 56
								60	4 00

For angles greater than 180°, subtract 180° and add 12^h to the result.

The above table is for converting expressions in arc to their equivalent in time; its main use in this Almanac is for the conversion of longitude for application to L.M.T. (*added* if *west*, *subtracted* if *east*) to give G.M.T. or vice versa, particularly in the case of sunrise, sunset, etc.

INTERPOLATION OF MOONRISE, MOONSET FOR LONGITUDE

Add if longitude *west*.
Subtract if longitude *east*.

Longitude	Diff.*					
	05	10	15	20	25	30
°	m	m	m	m	m	m
0	00	00	00	00	00	00
20	01	01	02	02	03	03
40	01	02	03	04	06	07
60	02	03	05	07	08	10
80	02	04	07	09	11	13
100	03	06	08	11	14	17
120	03	07	10	13	17	20
140	04	08	12	16	19	23
160	04	09	13	18	22	27
180	05	10	15	20	25	30

Longitude	Diff.*					
	35	40	45	50	55	60
°	m	m	m	m	m	m
0	00	00	00	00	00	00
15	03	03	04	04	05	05
30	06	07	08	08	09	10
45	09	10	11	12	14	15
60	12	13	15	17	18	20
75	15	17	19	21	23	25
90	18	20	22	25	28	30
105	20	23	26	29	32	35
120	23	27	30	33	37	40
135	26	30	34	38	41	45
150	29	33	38	42	46	50
165	32	37	41	46	50	55
180	35	40	45	50	55	60

Longitude	Diff.*					
	65	70	75	80	85	90
°	m	m	m	m	m	m
0	00	00	00	00	00	00
10	04	04	04	04	05	05
20	07	08	08	09	09	10
30	11	12	12	13	14	15
40	14	16	17	18	19	20
50	18	19	21	22	24	25
60	22	23	25	27	28	30
70	25	27	29	31	33	35
80	29	31	33	36	38	40
90	32	35	38	40	42	45
100	36	39	42	44	47	50
110	40	43	46	49	52	55
120	43	47	50	53	57	60
130	47	51	54	58	61	65
140	51	54	58	62	66	70
150	54	58	62	67	71	75
160	58	62	67	71	76	80
170	61	66	71	76	80	85
180	65	70	75	80	85	90

* When the Diff. is negative *subtract* correction if longitude *west* and *add* correction if longitude *east*.

CORRECTIONS TO BE APPLIED TO SEXTANT ALTITUDE

REFRACTION

To be subtracted from sextant altitude (referred to as observed altitude in A.P. 3270).

Height above sea level in units of 1,000 ft. — Sextant Altitude

R_0	0	5	10	15	20	25	30	35	40	45	50	55
′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
	90	90	90	90	90	90	90	90	90	90	90	90
0												
	63	59	55	51	46	41	36	31	26	20	17	13
1												
	33	29	26	22	19	16	14	11	9	7	6	4
2												
	21	19	16	14	12	10	8	7	5	4	2 40	1 40
3												
	16	14	12	10	8	7	6	5	3 10	2 20	1 30	0 40
4												
	12	11	9	8	7	5	4 00	3 10	2 10	1 30	0 39	+0 05
5												
	10	9	7	5 50	4 50	3 50	3 10	2 20	1 30	0 49	+0 11	−0 19
6												
	8 10	6 50	5 50	4 50	4 00	3 00	2 20	1 50	1 10	0 24	−0 11	−0 38
7												
	6 50	5 50	5 00	4 00	3 10	2 30	1 50	1 20	0 38	+0 04	−0 28	−0 54
8												
	6 00	5 10	4 10	3 20	2 40	2 00	1 30	1 00	0 19	−0 13	−0 42	−1 08
9												
	5 20	4 30	3 40	2 50	2 10	1 40	1 10	0 35	+0 03	−0 27	−0 53	−1 18
10												
	4 30	3 40	2 50	2 20	1 40	1 10	0 37	+0 11	−0 16	−0 43	−1 08	−1 31
12												
	3 30	2 50	2 10	1 40	1 10	0 34	+0 09	− 0 14	−0 37	−1 00	−1 23	−1 44
14												
	2 50	2 10	1 40	1 10	0 37	+0 10	−0 13	−0 34	−0 53	−1 14	−1 35	−1 56
16												
	2 20	1 40	1 20	0 43	+0 15	−0 08	−0 31	−0 52	−1 08	−1 27	−1 46	−2 05
18												
	1 50	1 20	0 49	+0 23	−0 02	−0 26	−0 46	−1 06	−1 22	−1 39	−1 57	−2 14
20												
	1 12	0 44	+0 19	−0 06	−0 28	−0 48	−1 09	−1 27	−1 42	−1 58	−2 14	−2 30
25												
	0 34	+0 10	−0 13	−0 36	−0 55	−1 14	−1 32	−1 51	−2 06	−2 21	−2 34	−2 49
30												
	+0 06	−0 16	−0 37	−0 59	−1 17	−1 33	−1 51	−2 07	−2 23	−2 37	−2 51	−3 04
35												
	−0 18	−0 37	−0 58	−1 16	−1 34	−1 49	−2 06	−2 22	−2 35	−2 49	−3 03	−3 16
40												
		−0 53	−1 14	−1 31	−1 47	−2 03	−2 18	−2 33	−2 47	−2 59	−3 13	−3 25
45												
		−1 10	−1 28	−1 44	−1 59	−2 15	−2 28	−2 43	−2 56	−3 08	−3 22	−3 33
50												
			−1 40	−1 53	−2 09	−2 24	−2 38	−2 52	−3 04	−3 17	−3 29	−3 41
55												
				−2 03	−2 18	−2 33	−2 46	−3 01	−3 12	−3 25	−3 37	−3 48
60												
							−2 53	−3 07	−3 19	−3 31	−3 42	−3 53

$R=R_0\times f$

R_0	0·9	1·0	1·1	1·2
′	′	′	′	′
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	4
4	4	4	4	5
5	5	5	5	6
6	5	6	7	7
7	6	7	8	8
8	7	8	9	10
9	8	9	10	11
10	9	10	11	12
12	11	12	13	14
14	13	14	15	17
16	14	16	18	19
18	16	18	20	22
20	18	20	22	24
25	22	25	28	30
30	27	30	33	36
35	31	35	38	42
40	36	40	44	48
45	40	45	50	54
50	45	50	55	60
55	49	55	60	66
60	54	60	66	72

Temperature in °C

f	0	5	10	15	20	25	30	35	40	45	50	55
	+47	+36	+27	+18	+10	+ 3	− 5	−13				
0·9												
	+26	+16	+ 6	− 4	−13	−22	−31	−40				
1·0												
	+ 5	− 5	−15	−25	−36	−46	−57	−68				
1·1												
	−16	−25	−36	−46	−58	−71	−83	−95				
1·2												
	−37	−45	−56	−67	−81	−95						

For these heights no temperature correction is necessary: take $f=1{\cdot}0$ and use $R=R_0$

When R_0 is less than 10′ or the height is greater than 35,000 ft. take $f=1{\cdot}0$ and use $R=R_0$

Choose the column appropriate to height, in units of 1,000 ft., and find the range of altitude in which the sextant altitude lies; the corresponding value of R_0 is the refraction, to be subtracted from sextant altitude, unless conditions are extreme. In that case find f from the lower table, with critical argument temperature. Use the table on the right to form the refraction, $R=R_0\times f$.

CORIOLIS (*Z*) CORRECTION

To be applied by moving the position line a distance *Z* to starboard (right) of the track in northern latitudes and to port (left) in southern latitudes. The argument is given as T.A.S. (True Air Speed) in A.P. 3270.

G/S KNOTS	Latitude 0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	G/S KNOTS	Latitude 0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
	′	′	′	′	′	′	′	′	′	′		′	′	′	′	′	′	′	′	′	′
150	0	1	1	2	3	3	3	4	4	4	450	0	2	4	6	8	9	10	11	12	12
200	0	1	2	3	3	4	5	5	5	5	500	0	2	4	7	8	10	11	12	13	13
250	0	1	2	3	4	5	6	6	6	7	550	0	3	5	7	9	11	12	14	14	14
300	0	1	3	4	5	6	7	7	8	8	600	0	3	5	8	10	12	14	15	16	16
350	0	2	3	5	6	7	8	9	9	9	650	0	3	6	9	11	13	15	16	17	17
400	0	2	4	5	7	8	9	10	10	10	700	0	3	6	9	12	14	16	17	18	18

APPENDIX X

LONG-TERM ALMANAC

This appendix is intended for use when a more complete almanac is not available. It is based principally upon the fact that approximately correct values for the Greenwich hour angle and declination of the sun, and the Greenwich hour angle of Aries, can be obtained from an almanac that is exactly four years out of date. The differences in these values at intervals of exactly four years can be largely removed by applying an average correction to the values obtained from the tables of this appendix. The maximum error in an altitude computed by means of this appendix should not exceed 2′.0 for the sun or 1′.3 for stars.

This four-year, or quadrennial, correction varies throughout the year for the GHA of the sun (between about plus and minus one-fourth of a minute) and for the declination of the sun (between about plus and minus three-fourths of a minute). For the GHA of Aries the quadrennial correction is a constant, (+)1′.84. The appropriate quadrennial correction is applied once for each full four years which has passed since the base year of the tabulation (1956 in this appendix).

The tabulated values for GHA and declination of the sun and GHA of Aries are given in four columns, labeled 0, 1, 2, and 3. The "0" column contains the data for the leap year in each four-year cycle and the 1, 2, and 3 columns contain data for, respectively, the first, second, and third years following each leap year.

The GHA and declination of the sun are given at intervals of three days throughout the four-year cycle, except for the final days of each month, when the interval varies between one and four days. Linear interpolation is made between entries to obtain data for a given day. Additional corrections to the GHA of the sun of 15° per hour, 15′ per minute, and 15″ per second are made to obtain the GHA at a given time. Declination of the sun is obtained to sufficient accuracy by linear interpolation alone.

The GHA of Aries is given for each month of the four-year cycle. Additional corrections of 0°59′.14 per day, 15°02′.5 per hour, 15′ per minute, and 15″ per second are made to obtain the GHA at a given time.

The SHA and declination of 38 navigational stars are given for the base year, 1956.0. Annual (not quadrennial) corrections are made to these data to obtain the values for a given year and tenth of a year.

A multiplication table is included as an aid in applying corrections to tabulated values.

Sun tables. **1.** Subtract 1956 from the year and divide the difference by four, obtaining (*a*) a whole number, and (*b*) a remainder. Enter column indicated by remainder (*b*) and take out values on either side of given time and date.

2. Multiply quadrennial correction for each value by whole number (*a*) obtained in step 1 and apply to tabulated values.

3. Divide difference between corrected values by number of days (usually three) between them to determine daily change.

4. Multiply daily change by number of days and tenths since 0^h GMT of earlier tabulated date, and mark correction plus (+) or minus (−) as appropriate.

5. (GHA only.) Enter multiplication table with hours, minutes, and seconds of GMT, and take out corrections *A*, *B*, and *C*, respectively. These are all positive.

6. Apply corrections of steps 4 and 5 to corrected *earlier* values of step 2.

Example.—Find GHA and declination of sun at GMT $17^h13^m49^s$ on July 18, 1986.

Solution.—*Steps* 1 *and* 2: (1986−1956)÷4=7, remainder 2. Use column 2, and multiply quadrennial corrections by 7. Corrected values: GHA, July 16, 178°32′.0−(7×0′.23)=178°30′.4; July 19, 178°28′.0−(7×0′.20)=178°26′.6. Dec., July 16, 21°29′.2 N−(7×0′.35)=21°26′.8 N; July 19, 20°58′.9 N−(7×0′.39)=20°56′.2 N.

	GHA	
July 16	178°30′.4	*Step 3*
July 19	178°26′.6	*Step 3*
3-day change	(−)3′.8	*Step 3*
daily change	(−)1′.3	*Step 3*, *Step 4*
days and tenths	2.7	*Step 4*
corr.	(−)3′.5	*Step 4*, *Step 6*
A	255°00′.0	*Step 5*, *Step 6*
B	3°15′.0	*Step 5*, *Step 6*
C	12′.2	*Step 5*, *Step 6*
0^h July 16	178°30′.4	*Step 6*
GHA	76°54′.1	*Step 6*

	Declination	
July 16	21°26′.8 N	*Step 3*
July 19	20°56′.2 N	*Step 3*
3-day change	(−)30′.6	*Step 3*
daily change	(−)10′.2	*Step 3*, *Step 4*
days and tenths	2.7	*Step 4*
corr.	(−)27′.5	*Step 4*, *Step 6*
0^h July 16	21°26′.8 N	*Step 6*
d	20°59′.3 N	*Step 6*

Aries table. **1.** Subtract 1956 from the year and divide the difference by four, obtaining (*a*) a whole number, and (*b*) a remainder. Enter column indicated by remainder (*b*) and take out value for given month.

2. Enter multiplication table with whole number (*a*) of step 1, day of month, hours of GMT, minutes of GMT, and seconds of GMT, and take out corrections *D*, *E*, *F*, *G*, and *C*, respectively.

3. Add values of steps 1 and 2.

Example.—Find GHA ♈ at GMT $11^h06^m33^s$ on November 28, 1979.

Solution.—*Step* 1: (1979−1956)÷4=5, remainder 3. Use column 3.

	GHA ♈	
Nov.	38°33′.0	*Step 1*
D	9′.2	*Step 2*
E	27°35′.9	*Step 2*
F	165°27′.1	*Step 2*
G	1°30′.2	*Step 2*
C	8′.2	*Step 2*
GHA ♈	233°23′.6	*Step 3*

Stars table. **1.** Enter table with star name, and take out tabulated values.

2. Subtract 1956.0 from given year and tenth, and multiply annual correction by difference. Apply as correction (+or−, as appropriate) to value of step 1.

Example.—Find SHA and declination of Spica on September 11, 1995.

Solution.—From decimal table, September 11, 1995=1995.7. 1995.7−1956.0=39.7.

	SHA	
1956.0	159°16′.9	*Step 1*
39.7×(−)0′.79	(−)31′.4	*Step 2*
SHA	158°45′.5	*Step 2*

	Declination	
1956.0	10°55′.9 S	*Step 1*
39.7×0′.31	(+)12′.3	*Step 2*
d	11°08′.2 S	*Step 2*

To determine GHA of star, add GHA ♈ and SHA☆ for given time and date.

SUN										
0		Quad. GHA Corr.	1		Date	2		Quad. Dec. Corr.	3	
GHA	Dec.		GHA	Dec.		GHA	Dec.		GHA	Dec.
JANUARY										
° ′	° ′	′	° ′	° ′		° ′	° ′	′	° ′	° ′
179 14.9	23 06.1 S	−0.03	179 09.2	23 02.5 S	1	179 10.9	23 03.6 S	−0.15	179 12.9	23 04.7 S
178 53.7	22 50.8 S	−0.03	178 48.1	22 46.2 S	4	178 49.9	22 47.7 S	−0.20	178 51.8	22 49.1 S
178 33.4	22 31.5 S	−0.02	178 28.0	22 25.9 S	7	178 29.8	22 27.7 S	−0.22	178 31.5	22 29.4 S
178 14.0	22 08.2 S	−0.01	178 09.0	22 01.6 S	10	178 10.8	22 03.7 S	−0.26	178 12.2	22 05.8 S
177 55.9	21 41.0 S	0.00	177 51.3	21 33.4 S	13	177 52.9	21 35.9 S	−0.29	177 54.2	21 38.2 S
177 39.1	21 10.0 S	+0.01	177 35.0	21 01.5 S	16	177 36.4	21 04.2 S	−0.34	177 37.6	21 06.9 S
177 23.8	20 35.3 S	+0.04	177 20.2	20 26.0 S	19	177 21.5	20 28.9 S	−0.36	177 22.5	20 31.9 S
177 10.2	19 57.2 S	+0.05	177 07.1	19 47.0 S	22	177 08.1	19 50.2 S	−0.40	177 09.1	19 53.4 S
176 58.4	19 15.7 S	+0.08	176 55.7	19 04.6 S	25	176 56.4	19 08.1 S	−0.42	176 57.5	19 11.7 S
176 48.3	18 31.0 S	+0.10	176 46.0	18 19.1 S	28	176 46.6	18 22.9 S	−0.45	176 47.6	18 26.7 S
FEBRUARY										
176 37.7	17 26.7 S	+0.12	176 35.9	17 13.9 S	1	176 36.4	17 18.0 S	−0.48	176 37.2	17 22.1 S
176 31.9	16 35.3 S	+0.13	176 30.4	16 21.8 S	4	176 31.0	16 26.1 S	−0.51	176 31.5	16 30.4 S
176 27.9	15 41.2 S	+0.15	176 26.9	15 27.1 S	7	176 27.4	15 31.7 S	−0.54	176 27.6	15 36.1 S
176 25.6	14 44.7 S	+0.15	176 25.1	14 30.0 S	10	176 25.5	14 34.8 S	−0.55	176 25.5	14 39.4 S
176 25.1	13 46.0 S	+0.17	176 25.2	13 30.8 S	13	176 25.4	13 35.8 S	−0.58	176 25.2	13 40.6 S
176 26.3	12 45.3 S	+0.18	176 27.0	12 29.6 S	16	176 26.9	12 34.7 S	−0.61	176 26.6	12 39.7 S
176 29.2	11 42.7 S	+0.20	176 30.3	11 26.6 S	19	176 30.0	11 31.8 S	−0.62	176 29.6	11 36.9 S
176 33.7	10 38.4 S	+0.22	176 35.2	10 22.0 S	22	176 34.7	10 27.3 S	−0.63	176 34.3	10 32.6 S
176 39.6	9 32.7 S	+0.23	176 41.4	9 15.9 S	25	176 40.8	9 21.3 S	−0.64	176 40.4	9 26.7 S
176 47.0	8 25.7 S	+0.24	176 49.0	8 08.6 S	28	176 48.3	8 14.1 S	−0.66	176 47.8	8 19.6 S
MARCH										
176 52.5	7 40.4 S	+0.24	176 51.8	7 45.9 S	1	176 51.1	7 51.4 S	−0.66	176 50.6	7 57.0 S
177 01.8	6 31.7 S	+0.24	177 00.9	6 37.2 S	4	177 00.2	6 42.8 S	−0.67	176 59.6	6 48.5 S
177 12.1	5 22.1 S	+0.24	177 11.0	5 27.6 S	7	177 10.4	5 33.4 S	−0.68	177 09.5	5 39.1 S
177 23.2	4 11.8 S	+0.24	177 22.1	4 17.5 S	10	177 21.5	4 23.3 S	−0.69	177 20.4	4 28.9 S
177 35.1	3 01.1 S	+0.24	177 34.0	3 06.8 S	13	177 33.3	3 12.6 S	−0.71	177 32.1	3 18.3 S
177 47.6	1 50.0 S	+0.24	177 46.6	1 55.8 S	16	177 45.8	2 01.7 S	−0.71	177 44.5	2 07.4 S
178 00.6	0 38.9 S	+0.23	177 59.7	0 44.7 S	19	177 58.7	0 50.5 S	−0.70	177 57.5	0 56.2 S
178 14.0	0 32.2 N	+0.23	178 13.1	0 26.4 N	22	178 12.0	0 20.6 N	+0.70	178 10.8	0 14.9 N
178 27.7	1 43.1 N	+0.23	178 26.8	1 37.3 N	25	178 25.5	1 31.6 N	+0.70	178 24.5	1 25.8 N
178 41.4	2 53.7 N	+0.22	178 40.4	2 47.9 N	28	178 39.2	2 42.2 N	+0.70	178 38.2	2 36.5 N
APRIL										
178 59.6	4 26.9 N	+0.22	178 58.5	4 21.3 N	1	178 57.4	4 15.6 N	+0.70	178 56.4	4 09.9 N
179 13.0	5 36.0 N	+0.19	179 11.9	5 30.5 N	4	179 10.9	5 24.8 N	+0.70	179 09.8	5 19.2 N
179 26.0	6 44.3 N	+0.18	179 24.9	6 38.8 N	7	179 24.0	6 33.2 N	+0.68	179 22.9	6 27.7 N
179 38.4	7 51.5 N	+0.17	179 37.4	7 46.1 N	10	179 36.6	7 40.6 N	+0.68	179 35.5	7 35.2 N
179 50.3	8 57.5 N	+0.14	179 49.5	8 52.2 N	13	179 48.6	8 46.8 N	+0.67	179 47.5	8 41.5 N
180 01.5	10 02.2 N	+0.12	180 00.8	9 56.9 N	16	179 59.9	9 51.7 N	+0.65	179 58.8	9 46.0 N
180 11.8	11 05.4 N	+0.10	180 11.3	11 00.2 N	19	180 10.4	10 55.2 N	+0.64	180 09.4	10 50.1 N
180 21.3	12 06.9 N	+0.09	180 20.9	12 01.9 N	22	180 19.9	11 57.0 N	+0.62	180 19.2	11 52.0 N
180 29.9	13 06.7 N	+0.08	180 29.4	13 01.8 N	25	180 28.5	12 57.1 N	+0.60	180 27.9	12 52.2 N
180 37.3	14 04.5 N	+0.06	180 36.8	13 59.8 N	28	180 36.0	13 55.2 N	+0.59	180 35.6	13 50.5 N
MAY										
180 43.6	15 00.2 N	+0.04	180 43.1	14 55.7 N	1	180 42.5	14 51.3 N	+0.56	180 42.1	14 46.7 N
180 48.6	15 53.7 N	0.00	180 48.2	15 49.4 N	4	180 47.8	15 45.1 N	+0.54	180 47.4	15 40.8 N
180 52.4	16 44.8 N	−0.02	180 52.1	16 40.7 N	7	180 51.8	16 36.6 N	+0.52	180 51.4	16 32.5 N
180 54.8	17 33.4 N	−0.04	180 54.7	17 29.5 N	10	180 54.6	17 25.6 N	+0.50	180 54.2	17 21.8 N
180 56.0	18 19.4 N	−0.08	180 56.1	18 15.7 N	13	180 56.0	18 12.0 N	+0.47	180 55.7	18 08.4 N
180 55.9	19 02.6 N	−0.10	180 56.2	18 59.1 N	16	180 56.1	18 55.7 N	+0.44	180 55.9	18 52.3 N
180 54.5	19 42.9 N	−0.12	180 55.0	19 39.7 N	19	180 54.8	19 36.5 N	+0.40	180 54.9	19 33.3 N
180 52.0	20 20.2 N	−0.14	180 52.6	20 17.2 N	22	180 52.4	20 14.3 N	+0.37	180 52.7	20 11.3 N
180 48.3	20 54.4 N	−0.14	180 48.9	20 51.6 N	25	180 48.8	20 49.0 N	+0.34	180 49.2	20 46.3 N
180 43.5	21 25.3 N	−0.16	180 44.1	21 22.8 N	28	180 44.1	21 20.4 N	+0.30	180 44.7	21 18.0 N
JUNE										
180 35.5	22 01.3 N	−0.18	180 36.0	21 59.2 N	1	180 36.3	21 57.2 N	+0.25	180 36.9	21 55.1 N
180 28.3	22 24.3 N	−0.22	180 28.9	22 22.5 N	4	180 29.3	22 20.8 N	+0.21	180 29.9	22 18.9 N
180 20.3	22 43.8 N	−0.24	180 21.0	22 42.3 N	7	180 21.5	22 40.8 N	+0.18	180 22.0	22 39.3 N
180 11.7	22 59.7 N	−0.25	180 12.5	22 58.5 N	10	180 13.0	22 57.3 N	+0.14	180 13.5	22 56.1 N
180 02.5	23 11.9 N	−0.26	180 03.5	23 11.0 N	13	180 04.0	23 10.1 N	+0.10	180 04.5	23 09.2 N
179 53.0	23 20.5 N	−0.27	179 54.1	23 19.9 N	16	179 54.5	23 19.3 N	+0.06	179 55.1	23 18.7 N
179 43.3	23 25.4 N	−0.27	179 44.4	23 25.1 N	19	179 44.7	23 24.8 N	+0.01	179 45.4	23 24.5 N
179 33.6	23 26.6 N	−0.27	179 34.7	23 26.6 N	22	179 34.9	23 26.5 N	−0.02	179 35.7	23 26.5 N
179 24.0	23 24.0 N	−0.26	179 24.9	23 24.3 N	25	179 25.1	23 24.6 N	−0.07	179 26.1	23 24.9 N
179 14.6	23 17.8 N	−0.26	179 15.4	23 18.3 N	28	179 15.7	23 18.9 N	−0.12	179 16.6	23 19.5 N

SUN

0 GHA	0 Dec.	Quad. GHA Corr.	1 GHA	1 Dec.	Date	2 GHA	2 Dec.	Quad. Dec. Corr.	3 GHA	3 Dec.
					JULY					
° ′	° ′	′	° ′	° ′		° ′	° ′	′	° ′	° ′
179 05.7	23 07.8 N	−0.26	179 06.3	23 08.7 N	1	179 06.7	23 09.6 N	−0.15	179 07.5	23 10.4 N
178 57.2	22 54.2 N	−0.26	178 57.8	22 55.4 N	4	178 58.2	22 56.6 N	−0.19	178 58.8	22 57.7 N
178 49.4	22 37.0 N	−0.26	178 50.0	22 38.5 N	7	178 50.4	22 40.0 N	−0.24	178 50.8	22 41.4 N
178 42.4	22 16.3 N	−0.25	178 43.1	22 18.1 N	10	178 43.3	22 19.8 N	−0.27	178 43.7	22 21.5 N
178 36.4	21 52.2 N	−0.23	178 37.1	21 54.3 N	13	178 37.2	21 56.2 N	−0.31	178 37.4	21 58.2 N
178 31.5	21 24.7 N	−0.23	178 32.2	21 27.0 N	16	178 32.0	21 29.2 N	−0.35	178 32.3	21 31.5 N
178 27.8	20 53.9 N	−0.20	178 28.4	20 56.5 N	19	178 28.0	20 58.9 N	−0.39	178 28.3	21 01.5 N
178 25.3	20 19.9 N	−0.17	178 25.7	20 22.8 N	22	178 25.3	20 25.5 N	−0.41	178 25.6	20 28.3 N
178 24.1	19 42.9 N	−0.15	178 24.3	19 46.0 N	25	178 23.8	19 48.9 N	−0.44	178 24.1	19 52.0 N
178 24.3	19 02.9 N	−0.13	178 24.2	19 06.2 N	28	178 23.7	19 09.4 N	−0.47	178 23.9	19 12.8 N
					AUGUST					
178 26.5	18 05.2 N	−0.11	178 26.2	18 08.8 N	1	178 25.7	18 12.4 N	−0.52	178 25.6	18 16.0 N
178 29.7	17 18.8 N	−0.11	178 29.3	17 22.7 N	4	178 28.8	17 26.4 N	−0.56	178 28.5	17 30.2 N
178 34.2	16 29.9 N	−0.09	178 33.9	16 34.0 N	7	178 33.2	16 37.9 N	−0.58	178 32.7	16 41.9 N
178 40.1	15 38.6 N	−0.07	178 39.7	15 42.9 N	10	178 39.0	15 47.0 N	−0.60	178 38.3	15 51.1 N
178 47.3	14 45.0 N	−0.05	178 46.9	14 49.5 N	13	178 45.9	14 53.8 N	−0.62	178 45.2	14 58.1 N
178 55.8	13 49.4 N	−0.03	178 55.3	13 54.0 N	16	178 54.1	13 58.4 N	−0.64	178 53.4	14 03.0 N
179 05.4	12 51.7 N	0.00	179 04.9	12 56.5 N	19	179 03.5	13 01.1 N	−0.66	179 02.9	13 05.8 N
179 16.2	11 52.3 N	+0.02	179 15.5	11 57.2 N	22	179 14.1	12 01.9 N	−0.68	179 13.4	12 06.8 N
179 28.1	10 51.1 N	+0.04	179 27.1	10 56.1 N	25	179 25.7	11 01.0 N	−0.69	179 25.0	11 06.0 N
179 40.8	9 48.4 N	+0.06	179 39.7	9 53.5 N	28	179 38.3	9 58.5 N	−0.70	179 37.5	10 03.7 N
					SEPTEMBER					
179 58.9	8 22.6 N	+0.07	179 57.7	8 27.8 N	1	179 56.4	8 33.0 N	−0.72	179 55.4	8 38.3 N
180 13.3	7 16.8 N	+0.07	180 12.1	7 22.1 N	4	180 10.8	7 27.4 N	−0.73	180 09.6	7 32.8 N
180 28.2	6 09.9 N	+0.08	180 27.1	6 15.4 N	7	180 25.7	6 20.8 N	−0.74	180 24.5	6 26.2 N
180 43.6	5 02.1 N	+0.10	180 42.6	5 07.7 N	10	180 41.1	5 13.1 N	−0.75	180 39.8	5 18.7 N
180 59.4	3 53.6 N	+0.11	180 58.3	3 59.3 N	13	180 56.8	4 04.7 N	−0.75	180 55.5	4 10.3 N
181 15.3	2 44.5 N	+0.12	181 14.3	2 50.2 N	16	181 12.6	2 55.6 N	−0.75	181 11.5	3 01.3 N
181 31.4	1 34.8 N	+0.13	181 30.3	1 40.6 N	19	181 28.6	1 46.1 N	−0.75	181 27.5	1 51.8 N
181 47.3	0 24.9 N	+0.13	181 46.1	0 30.6 N	22	181 44.5	0 36.2 N	−0.75	181 43.5	0 42.0 N
182 03.0	0 45.2 S	+0.14	182 01.7	0 39.5 S	25	182 00.3	0 33.9 S	+0.74	181 59.3	0 28.1 S
182 18.3	1 55.3 S	+0.14	182 17.0	1 49.7 S	28	182 15.7	1 44.0 S	+0.74	182 14.7	1 38.3 S
					OCTOBER					
182 33.1	3 05.4 S	+0.14	182 31.9	2 59.7 S	1	182 30.7	2 54.1 S	+0.74	182 29.6	2 48.3 S
182 47.2	4 15.1 S	+0.13	182 46.1	4 09.5 S	4	182 45.0	4 03.8 S	+0.73	182 43.9	3 58.1 S
183 00.6	5 24.4 S	+0.12	182 59.6	5 18.7 S	7	182 58.5	5 13.1 S	+0.72	182 57.4	5 07.5 S
183 13.0	6 33.0 S	+0.12	183 12.2	6 27.4 S	10	183 11.1	6 21.9 S	+0.71	183 10.2	6 16.3 S
183 24.4	7 40.9 S	+0.12	183 23.8	7 35.3 S	13	183 22.7	7 29.9 S	+0.70	183 21.9	7 24.4 S
183 34.7	8 47.8 S	+0.11	183 34.2	8 42.3 S	16	183 33.1	8 37.0 S	+0.68	183 32.5	8 31.5 S
183 43.8	9 53.5 S	+0.11	183 43.3	9 48.2 S	19	183 42.3	9 42.9 S	+0.66	183 41.9	9 37.5 S
183 51.4	10 58.0 S	+0.10	183 50.9	10 52.8 S	22	183 50.1	10 47.6 S	+0.64	183 49.9	10 42.3 S
183 57.6	12 01.0 S	+0.10	183 57.1	11 55.9 S	25	183 56.5	11 50.9 S	+0.63	183 56.4	11 45.7 S
184 02.0	13 02.3 S	+0.08	184 01.6	12 57.4 S	28	184 01.3	12 52.5 S	+0.62	184 01.2	12 47.4 S
					NOVEMBER					
184 05.3	14 21.3 S	+0.06	184 05.1	14 16.6 S	1	184 05.1	14 11.8 S	+0.57	184 05.0	14 07.0 S
184 05.6	15 18.1 S	+0.04	184 05.7	15 13.5 S	4	184 05.8	15 08.9 S	+0.55	184 05.8	15 04.3 S
184 04.1	16 12.6 S	+0.03	184 04.4	16 08.2 S	7	184 04.7	16 03.8 S	+0.53	184 04.8	15 59.4 S
184 00.8	17 04.7 S	+0.02	184 01.3	17 00.5 S	10	184 01.6	16 56.3 S	+0.50	184 01.9	16 52.1 S
183 55.5	17 54.2 S	0.00	183 56.3	17 50.2 S	13	183 56.6	17 46.2 S	+0.46	183 57.2	17 42.2 S
183 48.4	18 40.8 S	−0.01	183 49.3	18 37.1 S	16	183 49.7	18 33.4 S	+0.42	183 50.6	18 29.6 S
183 39.5	19 24.5 S	−0.01	183 40.4	19 21.0 S	19	183 41.0	19 17.6 S	+0.40	183 42.1	19 14.0 S
183 28.7	20 05.0 S	−0.02	183 29.6	20 01.8 S	22	183 30.5	19 58.6 S	+0.36	183 31.7	19 55.3 S
183 16.0	20 42.3 S	−0.03	183 17.0	20 39.3 S	25	183 18.1	20 36.4 S	+0.33	183 19.5	20 33.4 S
183 01.6	21 16.0 S	−0.06	183 02.7	21 13.4 S	28	183 04.1	21 10.7 S	+0.29	183 05.4	21 08.0 S
					DECEMBER					
182 45.6	21 46.2 S	−0.08	182 46.9	21 43.8 S	1	182 48.5	21 41.4 S	+0.29	182 49.8	21 39.0 S
182 28.1	22 12.6 S	−0.08	182 29.6	22 10.5 S	4	182 31.3	22 08.4 S	+0.22	182 32.6	22 06.4 S
182 09.3	22 35.1 S	−0.09	182 11.1	22 33.4 S	7	182 12.8	22 31.6 S	+0.18	182 14.2	22 29.9 S
181 49.5	22 53.7 S	−0.09	181 51.4	22 52.3 S	10	181 53.0	22 50.8 S	+0.15	181 54.6	22 49.4 S
181 28.7	23 08.1 S	−0.08	181 30.7	23 07.1 S	13	181 32.3	23 06.0 S	+0.11	181 34.0	23 04.9 S
181 07.2	23 18.5 S	−0.08	181 09.2	23 17.8 S	16	181 10.8	23 17.0 S	+0.06	181 12.7	23 16.3 S
180 45.2	23 24.7 S	−0.08	180 47.1	23 24.3 S	19	180 48.8	23 23.9 S	+0.02	180 50.8	23 23.4 S
180 22.9	23 26.6 S	−0.08	180 24.7	23 26.5 S	22	180 26.5	23 26.5 S	−0.01	180 28.5	23 26.4 S
180 00.5	23 24.3 S	−0.08	180 02.2	23 24.6 S	25	180 04.2	23 24.9 S	−0.05	180 06.1	23 25.2 S
179 38.3	23 17.8 S	−0.07	179 39.9	23 18.4 S	28	179 41.9	23 19.1 S	−0.09	179 43.7	23 19.6 S

ARIES (♈)

0	1	Month	2	3
° ′	° ′		° ′	° ′
98 38.9	99 23.6	Jan.	99 09.3	98 54.9
129 12.2	129 57.0	Feb.	129 42.6	129 28.2
157 47.2	157 32.8	Mar.	157 18.4	157 04.0
188 20.5	188 06.1	Apr.	187 51.7	187 37.3
217 54.6	217 40.3	May	217 25.9	217 11.5
248 27.9	248 13.6	June	247 59.2	247 44.8
278 02.1	277 47.7	July	277 33.3	277 18.9
308 35.4	308 21.1	Aug.	308 06.7	307 52.2
339 08.7	338 54.3	Sept.	338 39.9	338 25.5
8 42.9	8 28.5	Oct.	8 14.1	7 59.7
39 16.2	39 01.8	Nov.	38 47.4	38 33.0
68 50.3	68 35.9	Dec.	68 21.5	68 07.1

STARS

SHA (1956.0)	Annual Corr.	Star	Dec. (1956.0)	Annual Corr.
° ′	′		° ′	′
315 51.1	−0.57	Acamar	40 28.8 S	−0.24
335 58.8	−0.56	Achernar	57 27.6 S	−0.30
173 57.9	−0.84	Acrux	62 51.3 S	+0.33
291 39.1	−0.86	Aldebaran	16 25.3 N	+0.12
153 32.9	−0.59	Alkaid	49 31.9 N	−0.30
218 38.6	−0.74	Alphard	8 28.0 S	+0.26
126 47.7	−0.64	Alphecca	26 51.7 N	−0.20
358 28.4	−0.78	Alpheratz	28 50.9 N	+0.33
62 50.5	−0.73	Altair	8 45.0 N	+0.16
113 19.4	−0.92	Antares	26 20.2 S	+0.13
146 35.2	−0.68	Arcturus	19 24.6 N	−0.31
109 00.2	−1.59	Atria	68 57.0 S	+0.11
271 48.2	−0.81	Betelgeuse	7 24.0 N	+0.01
264 15.4	−0.33	Canopus	52 40.3 S	+0.03
281 38.5	−1.11	Capella	45 57.3 N	+0.06
50 01.0	−0.51	Deneb	45 07.3 N	+0.21
183 17.8	−0.76	Denebola	14 49.1 N	−0.34
349 39.3	−0.75	Diphda	18 13.7 S	−0.33
194 44.6	−0.92	Dubhe	61 59.3 N	−0.32
34 29.6	−0.74	Enif	9 40.3 N	+0.28
16 11.6	−0.83	Fomalhaut	29 51.4 S	−0.32
328 49.7	−0.85	Hamal	23 15.3 N	+0.28
137 17.8	+0.04	Kochab	74 20.1 N	−0.25
148 58.7	−0.88	Menkent	36 09.3 S	+0.29
309 42.5	−1.07	Mirfak	49 42.4 N	+0.21
76 51.9	−0.93	Nunki	26 21.2 S	−0.08
54 27.3	−1.19	Peacock	56 52.7 S	−0.19
244 20.6	−0.92	Pollux	28 08.0 N	−0.15
245 45.0	−0.78	Procyon	5 20.4 N	−0.15
96 46.6	−0.70	Rasalhague	12 35.4 N	−0.04
208 29.5	−0.80	Regulus	12 11.0 N	−0.29
281 53.7	−0.72	Rigel	8 15.1 S	−0.07
140 51.0	−1.02	Rigil Kent.	60 39.3 S	+0.25
350 30.1	−0.85	Schedar	56 17.8 N	+0.33
259 11.9	−0.66	Sirius	16 39.3 S	+0.08
159 16.9	−0.79	Spica	10 55.9 S	+0.31
223 24.4	−0.55	Suhail	43 15.2 S	+0.24
81 08.3	−0.51	Vega	38 44.5 N	+0.06

MULTIPLICATION TABLE

No.	A	B	C	D	E	F	G
	°	° ′	′	′	° ′	° ′	° ′
1	15	0 15	0.2	1.8	0 59.1	15 02.5	0 15.0
2	30	0 30	0.5	3.7	1 58.3	30 04.9	0 30.1
3	45	0 45	0.8	5.5	2 57.4	45 07.4	0 45.1
4	60	1 00	1.0	7.4	3 56.6	60 09.9	1 00.2
5	75	1 15	1.2	9.2	4 55.7	75 12.3	1 15.2
6	90	1 30	1.5	11.0	5 54.8	90 14.8	1 30.2
7	105	1 45	1.8	12.9	6 54.0	105 17.2	1 45.3
8	120	2 00	2.0	14.7	7 53.1	120 19.7	2 00.3
9	135	2 15	2.2	16.6	8 52.3	135 22.2	2 15.4
10	150	2 30	2.5	18.4	9 51.4	150 24.6	2 30.4
11	165	2 45	2.8	20.2	10 50.5	165 27.1	2 45.5
12	180	3 00	3.0	22.1	11 49.7	180 29.6	3 00.5
13	195	3 15	3.2	23.9	12 48.8	195 32.0	3 15.5
14	210	3 30	3.5	25.8	13 48.0	210 34.5	3 30.6
15	225	3 45	3.8	27.6	14 47.1	225 37.0	3 45.6
16	240	4 00	4.0	29.4	15 46.2	240 39.4	4 00.7
17	255	4 15	4.2	31.3	16 45.4	255 41.9	4 15.7
18	270	4.30	4.5	33.1	17 44.5	270 44.4	4 30.7
19	285	4 45	4.8	35.0	18 43.7	285 46.8	4 45.8
20	300	5 00	5.0	36.8	19 42.8	300 49.3	5 00.8
21	315	5 15	5.2	38.6	20 41.9	315 51.7	5 15.9
22	330	5 30	5.5	40.5	21 41.1	330 54.2	5 30.9
23	345	5 45	5.8	42.3	22 40.2	345 56.7	5 45.9
24	360	6 00	6.0	44.2	23 39.4	360 59.1	6 01.0
25	—	6 15	6.2	46.0	24 38.5	—	6 16.0
26	—	6 30	6.5	47.8	25 37.6	—	6 31.1
27	—	6 45	6.8	49.7	26 36.8	—	6 46.1
28	—	7.00	7.0	51.5	27 35.9	—	7 01.1
29	—	7 15	7.2	53.4	28 35.1	—	7 16.2
30	—	7 30	7.5	55.2	29 34.2	—	7 31.2
31	—	7 45	7.8	57.0	30 33.3	—	7 46.3
32	—	8 00	8.0	58.9	—	—	8 01.3
33	—	8 15	8.2	60.7	—	—	8 16.4
34	—	8 30	8.5	62.6	—	—	8 31.4
35	—	8 45	8.8	64.4	—	—	8 46.4
36	—	9 00	9.0	66.2	—	—	9 01.5
37	—	9 15	9.2	68.1	—	—	9 16.5
38	—	9 30	9.5	69.9	—	—	9 31.6
39	—	9 45	9.8	71.8	—	—	9 46.6
40	—	10 00	10.0	73.6	—	—	10 01.6
41	—	10 15	10.2	75.4	—	—	10 16.7
42	—	10 30	10.5	77.3	—	—	10 31.7
43	—	10 45	10.8	79.1	—	—	10 46.8
44	—	11 00	11.0	81.0	—	—	11 01.8
45	—	11 15	11.2	82.8	—	—	11 16.8
46	—	11 30	11.5	84.6	—	—	11 31.9
47	—	11.45	11.8	86.5	—	—	11 46.9
48	—	12 00	12.0	88.3	—	—	12 02.0
49	—	12 15	12.2	90.2	—	—	12 17.0
50	—	12 30	12.5	92.0	—	—	12 32.1
51	—	12 45	12.8	93.8	—	—	12 47.1
52	—	13 00	13.0	95.7	—	—	13 02.1
53	—	13 15	13.2	97.5	—	—	13 17.2
54	—	13 30	13.5	99.4	—	—	13 32.2
55	—	13 45	13.8	—	—	—	13 47.3
56	—	14 00	14.0	—	—	—	14 02.3
57	—	14 15	14.2	—	—	—	14 17.3
58	—	14 30	14.5	—	—	—	14 32.4
59	—	14 45	14.8	—	—	—	14 47.4
60	—	15 00	15.0	—	—	—	15 02.5

DECIMAL PARTS OF DAY AND YEAR

Decimal	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Hour of Day	0000 to 0112	0112 to 0336	0336 to 0600	0600 to 0824	0824 to 1048	1048 to 1312	1312 to 1536	1536 to 1800	1800 to 2024	2024 to 2248	2248 to 2400
Day of Year	Jan. 1 to Jan. 18	Jan. 19 to Feb. 23	Feb. 24 to Apr. 1	Apr 2 to May 7	May 8 to June 13	June 14 to July 19	July 20 to Aug. 25	Aug. 26 to Sept. 30	Oct. 1 to Nov. 6	Nov. 7 to Dec. 12	Dec. 13 to Dec. 31

APPENDIX Y

EXTRACTS FROM H.O. PUB. NO. 260

TRUE BEARING OR AZIMUTH.

LATITUDE 23°.

DECLINATION—SAME NAME AS—LATITUDE.

Dec.		12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	Dec.	
Apparent Time. A. M.		April.			May.							June.		Apparent Time. P. M.	
		22	25	28	1	5	8	12	16	21	26	1	10		
		August.							July.						
		22	19	16	12	9	5	2	28	24	19	12	3		
		October.			November.							December.			
		25	28	31	3	6	10	14	17	22	27	3	11		
		February.						January.							
		18	15	12	9	5	2	29	25	21	16	10	2		
h.	m.	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	h.	m.
V	20	......	......	......	......	......	......	......	......	......	......	......	65 02		40
	30	......	......	......	......	......	......	70 33	69 38	68 43	67 49	66 54	65 59		30
	40	77 01	76 06	75 11	74 16	73 21	72 26	71 31	70 36	69 40	68 45	67 50	66 54		20
	50	77 59	77 04	76 08	75 13	74 18	73 22	72 26	71 31	70 35	69 39	68 44	67 48		10
VI	00	78 56	78 00	77 04	76 09	75 13	74 17	73 21	72 25	71 29	70 32	69 36	68 39	VI	00
	10	79 52	78 56	78 00	77 03	76 07	75 11	74 14	73 17	72 21	71 24	70 27	69 30		50
	20	80 47	79 51	78 54	77 57	77 00	76 03	75 06	74 09	73 11	72 14	71 16	70 18		40
	30	81 42	80 44	79 47	78 50	77 52	76 55	75 57	74 59	74 01	73 02	72 03	71 05		30
	40	82 36	81 39	80 41	79 43	78 44	77 46	76 47	75 48	74 49	73 50	72 51	71 51		20
	50	83 30	82 32	81 33	80 34	79 35	78 36	77 36	76 37	75 37	74 36	73 36	72 35		10
VII	00	84 24	83 25	82 26	81 26	80 26	79 26	78 25	77 24	76 23	75 22	74 20	73 19	V	00
	10	85 19	84 19	83 18	82 17	81 16	80 15	79 13	78 11	77 09	76 07	75 04	74 01		50
	20	86 14	85 12	84 11	83 09	82 06	81 04	80 01	78 58	77 54	76 50	75 45	74 42		40
	30	87 09	86 06	85 03	84 00	82 56	81 52	80 48	79 44	78 39	77 34	76 28	75 22		30
	40	88 04	87 00	85 56	84 52	83 47	82 41	81 36	80 29	79 23	78 16	77 09	76 01		20
	50	89 01	87 56	86 50	85 44	84 37	83 30	82 23	81 15	80 07	78 58	77 49	76 40		10
VIII	00	89 58	88 52	87 44	86 37	85 28	84 19	83 10	82 00	80 50	79 40	78 29	77 17	IV	00
	10	90 58	89 49	88 40	87 31	86 20	85 09	83 58	82 46	81 34	80 21	79 08	77 54		50
	20	91 59	90 49	89 37	88 25	87 13	86 00	84 46	83 32	82 17	81 02	79 46	78 30		40
	30	93 03	91 50	90 36	89 22	88 07	86 51	85 35	84 18	83 01	81 43	80 25	79 06		30
	40	94 09	92 53	91 37	90 20	89 03	87 44	86 25	85 05	83 45	82 24	81 03	79 40		20
	50	95 18	94 00	92 41	91 21	90 01	88 39	87 16	85 53	84 30	83 05	81 40	80 15		10
IX	00	96 30	95 09	93 47	92 24	91 00	89 35	88 09	86 43	85 15	83 47	82 18	80 48	III	00
	10	97 47	96 23	94 57	93 30	92 03	90 34	89 04	87 33	86 01	84 29	82 56	81 22		50
	20	99 10	97 41	96 12	94 41	93 09	91 36	90 01	88 26	86 49	85 12	83 33	81 54		40
	30	100 38	99 06	97 32	95 56	94 19	92 41	91 01	89 21	87 39	85 56	84 12	82 27		30
	40	102 14	100 37	98 59	97 18	95 35	93 51	92 06	90 19	88 30	86 41	84 50	82 59		20
	50	103 59	102 17	100 33	98 47	56 58	95 07	93 15	91 21	89 25	87 28	85 29	83 30		10
X	00	105 56	104 08	102 17	100 23	98 27	96 30	94 29	92 27	90 22	88 17	86 09	84 01	II	00
	10	108 06	106 11	104 14	102 13	100 09	98 02	95 52	93 40	91 26	89 10	86 52	84 32		50
	20	110 33	108 32	106 26	104 16	102 02	99 45	97 25	95 02	92 36	90 06	87 35	85 03		40
	30	113 22	111 13	108 58	106 48	104 13	101 45	99 11	96 34	93 53	91 09	88 22	85 33		30
	40	116 38	114 20	111 55	109 24	106 48	104 04	101 15	98 21	95 22	92 19	89 12	86 04		20
	50	120 29	118 02	115 27	112 44	109 52	106 53	103 45	100 30	97 08	93 40	90 08	86 34		10
XI	00	125 05	122 30	119 45	116 48	113 40	110 21	106 50	103 09	99 18	95 19	91 13	87 03	I	00
	10	130 39	127 59	125 04	121 53	118 30	114 48	110 50	106 36	102 07	97 24	92 32	87 33		50
	20	137 27	134 46	131 48	128 30	124 50	120 46	116 16	111 20	105 58	100 16	94 14	88 03		40
	30	145 45	143 16	140 25	137 09	133 25	129 04	124 06	118 21	111 51	104 37	96 47	88 32		30
	40	155 46	153 45	151 23	148 34	145 10	141 03	135 58	129 40	121 53	112 23	101 15	89 01		20
XI	50	167 21	166 14	164 50	163 07	160 58	158 11	154 27	149 17	141 44	130 19	112 59	89 31	XII	10
		h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.		
Sun rises . . .		5 39	5 38	5 36	5 34	5 32	5 30	5 28	5 26	5 24	5 22	5 21	5 18	Sun rises.	
Sun sets . . .		6 21	6 22	6 24	6 26	6 28	6 30	6 32	6 34	6 36	6 38	6 39	6 42	Sun sets.	
		° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′		
Azimuth . . .		76 57	75 51	74 46	73 40	72 35	71 29	70 23	69 17	68 11	67 05	65 59	64 53	Azimuth.	

In **North latitude**, when the body is rising or East of the meridian, the tabulated azimuths are reckoned from North to East; and when the body is setting or West of the meridian, the tabulated azimuths are reckoned from North to West.

In **South latitude**, when the body is rising or East of the meridian, the tabulated azimuths are reckoned from South to East; and when the body is setting or West of the meridian, the tabulated azimuths are reckoned from South to West.

TRUE BEARING OR AZIMUTH.

LATITUDE 24°.

DECLINATION—SAME NAME AS—LATITUDE.

Dec.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	Dec.
Apparent Time. A. M.	March. 21	23	26	28	31	April. 3	5	8	11	13	16	19	Apparent Time. P. M.
	September. 23	21	18	16	13	10	8	5	2	August. 30	28	25	
	September. 23	26	28	October. 1	4	6	9	11	14	17	19	22	
	March. 21	18	16	13	11	8	6	3	1	February. 26	23	20	
h. m.	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	h. m.
V 50	......	......	......	......	......	......	83 30	82 35	81 41	80 46	79 51	78 56	10
VI 00	90 00	89 05	88 10	87 16	86 21	85 26	84 31	83 36	82 41	81 46	80 51	79 56	VI 00
10	91 01	90 06	89 11	88 16	87 21	86 26	85 31	84 36	83 41	82 46	81 50	80 55	50
20	92 02	91 07	90 12	89 17	88 22	87 27	86 31	85 36	84 40	83 44	82 49	81 53	40
30	93 04	92 09	91 13	90 18	89 23	88 27	87 31	86 35	85 39	84 43	83 47	82 50	30
40	94 06	93 11	92 15	91 19	90 24	89 28	88 31	87 35	86 38	85 42	84 45	83 48	20
50	95 09	94 13	93 17	92 21	91 25	90 29	89 32	88 35	87 38	86 41	85 43	84 46	10
VII 00	96 13	95 17	94 21	93 24	92 27	91 30	90 33	89 36	88 38	87 40	86 42	85 43	V 00
10	97 18	96 22	95 25	94 28	93 30	92 32	91 34	90 36	89 38	88 39	87 40	86 41	50
20	98 25	97 28	96 31	95 33	94 35	93 36	92 37	91 38	90 39	89 39	88 40	87 39	40
30	99 34	98 36	97 38	96 39	95 40	94 41	93 42	92 42	91 41	90 41	89 40	88 38	30
40	100 44	99 46	98 47	97 48	96 48	95 48	94 47	93 46	92 45	91 43	90 41	89 38	20
50	101 57	100 58	99 58	98 58	97 57	96 56	95 55	94 53	93 50	92 47	91 44	90 40	10
VIII 00	103 13	102 13	101 12	100 11	99 09	98 07	97 04	96 01	94 57	93 53	92 48	91 42	IV 00
10	104 32	103 30	102 28	101 26	100 24	99 20	98 16	97 11	96 06	95 00	93 54	92 47	50
20	105 54	104 51	103 48	102 45	101 41	100 36	99 31	98 25	97 18	96 10	95 02	93 53	40
30	107 20	106 17	105 13	104 08	103 02	101 56	100 49	99 42	98 33	97 23	96 13	95 03	30
40	108 51	107 46	106 41	105 35	104 28	103 20	102 11	101 02	99 51	98 40	97 28	96 15	20
50	110 26	109 21	108 14	107 06	105 58	104 48	103 38	102 26	101 14	100 01	98 46	97 31	10
IX 00	112 08	111 01	109 53	108 44	107 33	106 22	105 10	103 56	102 41	101 26	100 09	98 51	III 00
10	113 56	112 48	111 38	110 27	109 15	108 02	106 48	105 32	104 15	102 56	101 37	100 16	50
20	115 52	114 42	113 31	112 18	111 04	109 49	108 32	107 14	105 54	104 33	103 11	101 47	40
30	117 56	116 45	115 32	114 18	113 02	111 45	110 26	109 05	107 42	106 18	104 52	103 25	30
40	120 09	118 57	117 43	116 27	115 09	113 49	112 28	111 04	109 38	108 11	106 42	105 11	20
50	122 33	121 20	120 04	118 47	117 27	116 05	114 41	113 15	111 46	110 16	108 42	107 07	10
X 00	125 10	123 56	122 39	121 20	119 58	118 34	117 08	115 39	114 07	112 33	110 55	109 15	II 00
10	128 00	126 45	125 28	124 07	122 44	121 18	119 50	118 18	116 43	115 05	113 23	111 38	50
20	131 06	129 51	128 33	127 12	125 48	124 20	122 49	121 15	119 37	117 55	116 09	114 20	40
30	134 29	133 14	131 56	130 35	129 11	127 43	126 11	124 34	122 53	121 07	119 18	117 23	30
40	138 11	136 58	135 41	134 20	132 57	131 26	129 54	128 18	126 35	124 47	122 53	120 53	20
50	142 13	141 04	139 50	138 31	137 08	135 41	134 09	132 31	130 48	128 58	127 02	124 58	10
XI 00	146 38	145 32	144 22	143 07	141 48	140 25	138 55	137 20	135 37	133 48	131 51	129 45	I 00
10	151 24	150 25	149 21	148 13	147 00	145 42	144 18	142 47	141 09	139 23	137 29	135 25	50
20	156 34	155 42	154 47	153 47	152 43	151 34	150 20	148 59	147 29	145 50	144 03	142 05	40
30	162 04	161 23	160 38	159 50	158 58	158 01	156 59	155 51	154 36	153 13	151 40	149 57	30
40	167 52	167 23	166 52	166 18	165 41	165 00	164 15	163 26	162 31	161 30	160 21	159 03	20
XI 50	173 52	173 38	173 22	173 04	172 44	172 23	172 00	171 34	171 05	170 32	169 54	169 12	XII 10
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m	h. m.	
Sun rises . . .	6 00	5 58	5 56	5 55	5 53	5 51	5 49	5 47	5 46	5 44	5 42	5 40	Sun rises.
Sun sets . . .	6 00	6 02	6 04	6 05	6 07	6 09	6 11	6 13	6 14	6 16	6 18	6 20	Sun sets.
Azimuth . . .	90° 00′	88° 54′	87° 49′	86° 43′	85° 37′	84° 32′	83° 26′	82° 20′	81° 14′	80° 08′	79° 03′	77° 57′	Azimuth.

In **North latitude**, when the body is rising or East of the meridian, the tabulated azimuths are reckoned from North to East; and when the body is setting or West of the meridian, the tabulated azimuths are reckoned from North to West.

In **South latitude**, when the body is rising or East of the meridian, the tabulated azimuths are reckoned from South to East; and when the body is setting or West of the meridian, the tabulated azimuths are reckoned from South to West.

TRUE BEARING OR AZIMUTH.

LATITUDE 24°.

DECLINATION—SAME NAME AS—LATITUDE.

Dec.	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	Dec.
Apparent Time. A. M.	April.			May.							June.		Apparent Time. P. M.
	22	25	28	1	5	8	12	16	21	26	1	10	
	August.							July.					
	22	19	16	12	9	5	2	28	24	19	12	3	
	October.			November.							December.		
	25	28	31	3	6	10	14	17	22	27	3	11	
	February.						January.						
	18	15	12	9	5	2	29	25	21	16	10	2	
h. m.	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	h. m.
V 20	....	....	....	....	....	....	....	....	....	....	65 56	65 02	40
30	....	....	....	....	....	71 28	70 33	69 39	68 45	67 50	66 56	66 02	30
40	77 01	76 07	75 12	74 17	73 23	72 28	71 33	70 38	69 44	68 49	67 54	66 59	20
50	78 01	77 06	76 11	75 17	74 21	73 26	72 31	71 36	70 41	69 45	68 50	67 54	10
VI 00	79 01	78 06	77 10	76 15	75 19	74 24	73 28	72 32	71 36	70 41	69 44	68 48	VI 00
10	79 59	79 04	78 08	77 12	76 16	75 20	74 24	73 27	72 31	71 34	70 38	69 41	50
20	80 58	80 01	79 04	78 08	77 11	76 15	75 18	74 21	73 24	72 26	71 29	70 31	40
30	81 54	80 57	80 00	79 03	78 06	77 09	76 11	75 13	74 15	73 18	72 19	71 21	30
40	82 51	81 54	80 56	79 58	79 00	78 02	77 04	76 05	75 07	74 08	73 08	72 09	20
50	83 48	82 50	81 51	80 53	79 54	78 55	77 56	76 56	75 57	74 57	73 57	72 56	10
VII 00	84 44	83 45	82 46	81 47	80 47	79 47	78 47	77 47	76 46	75 45	74 43	73 42	V 00
10	85 41	84 41	83 41	82 41	81 40	80 39	79 38	78 36	77 34	76 32	75 30	74 27	50
20	86 39	85 38	84 36	83 35	82 33	81 31	80 28	79 25	78 22	77 19	76 15	75 11	40
30	87 37	86 35	85 32	84 29	83 26	82 23	81 19	80 14	79 10	78 05	76 59	75 54	30
40	88 35	87 32	86 28	85 23	84 20	83 14	82 09	81 03	79 57	78 51	77 44	76 36	20
50	89 35	88 30	87 25	86 19	85 13	84 06	82 59	81 52	80 44	79 36	78 27	77 18	10
VIII 00	90 36	89 30	88 23	87 16	86 08	84 59	83 50	82 41	81 31	80 21	79 10	77 59	IV 00
10	91 39	90 31	89 22	88 13	87 03	85 53	84 42	83 30	82 18	81 06	79 53	78 39	50
20	92 44	91 34	90 23	89 12	88 00	86 47	85 34	84 20	83 06	81 51	80 35	79 19	40
30	93 51	92 39	91 26	90 12	88 58	87 43	86 27	85 10	83 53	82 36	81 17	79 59	30
40	95 01	93 46	92 31	91 15	89 58	88 40	87 21	86 02	84 42	83 21	81 59	80 38	20
50	96 14	94 57	93 39	92 20	91 00	89 39	88 17	86 54	85 31	84 07	82 43	81 17	10
IX 00	97 32	96 11	94 50	93 28	92 05	90 40	89 15	87 49	86 22	84 54	83 26	81 56	III 00
10	98 54	97 30	96 06	94 40	93 13	91 45	90 16	88 45	87 14	85 42	84 09	82 35	50
20	100 21	98 54	97 26	95 56	94 25	92 52	91 19	89 44	88 08	86 31	84 53	83 14	40
30	101 55	100 24	98 51	97 17	95 42	94 05	92 26	90 46	89 04	87 22	85 39	83 54	30
40	103 37	102 02	100 25	98 46	97 04	95 22	93 38	91 52	90 04	88 15	86 25	84 34	20
50	105 29	103 49	102 06	100 22	98 35	96 46	94 55	93 02	91 07	89 11	87 13	85 15	10
X 00	107 33	105 47	103 59	102 08	100 14	98 18	96 19	94 18	92 16	90 11	88 04	85 56	II 00
10	109 50	107 59	106 04	104 06	102 05	100 01	97 53	95 43	93 30	91 15	88 58	86 39	50
20	112 26	110 28	108 26	106 20	104 10	101 56	99 39	97 17	94 53	92 26	89 56	87 25	40
30	115 23	113 18	111 09	108 54	106 34	104 09	101 39	99 04	96 27	93 45	91 00	88 11	30
40	118 47	116 36	114 17	111 53	109 22	106 44	104 00	101 11	98 15	95 16	92 12	89 04	20
50	122 47	120 29	118 03	115 27	112 44	109 52	106 51	103 43	100 27	97 04	93 35	90 02	10
XI 00	127 31	125 07	122 32	119 47	116 50	113 42	110 22	106 48	103 08	99 16	95 15	91 08	I 00
10	133 09	130 42	128 02	125 08	121 59	118 33	114 51	110 52	106 37	102 06	97 22	92 28	50
20	139 55	137 31	134 52	131 54	128 35	124 57	120 51	116 20	111 24	106 02	100 16	94 13	40
30	148 01	145 50	143 21	140 31	137 15	133 31	129 13	124 13	118 28	111 56	104 40	96 46	30
40	157 33	155 50	153 51	151 29	148 41	145 19	141 10	136 07	129 50	122 03	112 30	101 18	20
XI 50	168 23	167 25	166 16	164 54	163 12	161 03	158 17	154 35	149 26	141 55	130 34	113 56	XII 10
	h. m.	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	*h. m.*	
Sun rises . . .	5 38	5 36	5 35	5 33	5 31	5 29	5 27	5 25	5 23	5 21	5 19	5 16	Sun rises.
Sun sets . . .	6 22	6 24	6 25	6 27	6 29	6 31	6 33	6 35	6 37	6 39	6 41	6 44	Sun sets.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	
Azimuth . . .	76 51	75 45	74 39	73 33	72 26	71 20	70 14	69 07	68 01	66 54	65 47	64 41	Azimuth.

In **North latitude**, when the body is rising or East of the meridian, the tabulated azimuths are reckoned from North to East; and when the body is setting or West of the meridian, the tabulated azimuths are reckoned from North to West.

In **South latitude**, when the body is rising or East of the meridian, the tabulated azimuths are reckoned from South to East; and when the body is setting or West of the meridian, the tabulated azimuths are reckoned from South to West.

APPENDIX Z

EXTRACTS FROM H.O. PUB. NO. 261

AZIMUTH OR TRUE BEARING.

LATITUDE 51°. DECLINATION—SAME NAME AS—LATITUDE. **LATITUDE 51°.**

Hour Angle. h.	m.	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°	Hour Angle. h.	m.
		° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′		
0	00	180 00	180 00	180 00	....	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0	00
	10	150 37	140 15	121 11	89 02	56 18	36 36	25 54	19 37	15 34	12 45	10 43	9 08		10
	20	130 49	119 58	105 31	88 03	70 15	55 02	43 27	34 59	28 45	24 06	20 31	17 42		20
	30	118 45	109 37	98 55	87 05	75 00	63 39	53 45	45 30	38 46	33 19	28 52	25 14		30
	40	110 56	103 24	95 03	86 07	76 59	68 06	59 52	52 28	46 00	40 27	35 41	31 37		40
	50	105 26	99 07	92 18	85 08	77 48	70 33	63 36	57 06	51 10	45 49	41 04	36 52		50
1	00	101 18	95 53	90 09	84 10	78 02	71 56	65 57	60 14	54 50	49 50	45 15	41 06	1	00
	10	98 00	93 16	88 20	83 11	77 56	72 39	67 25	62 20	57 28	52 51	48 31	44 29		10
	20	95 15	91 03	86 42	82 12	77 36	72 57	68 20	63 47	59 21	55 04	51 01	47 10		20
	30	92 51	89 06	85 13	81 13	77 07	72 58	68 50	64 43	60 40	56 44	52 56	49 17		30
	40	90 44	87 20	83 50	80 13	76 32	72 48	69 02	65 18	61 35	57 56	54 23	50 56		40
	50	88 49	85 43	82 32	79 14	75 53	72 28	69 03	65 36	62 11	58 47	55 28	52 12		50
2	00	87 02	84 11	81 15	78 14	75 10	72 02	68 53	65 42	62 32	59 22	56 15	53 10	2	00
	10	85 22	82 44	80 01	77 14	74 24	71 31	68 35	65 38	62 41	59 44	56 48	53 53		10
	20	83 48	81 20	78 50	76 14	73 35	70 55	68 11	65 27	62 40	59 54	57 09	54 23		20
	30	82 18	79 59	77 38	75 13	72 45	70 15	67 42	65 07	62 32	59 55	57 19	54 42		30
	40	80 51	78 41	76 29	74 12	71 54	69 33	67 09	64 44	62 18	59 49	57 21	54 52		40
	50	79 27	77 24	75 19	73 11	71 00	68 48	66 32	64 15	61 57	59 37	57 16	54 55		50
3	00	78 05	76 09	74 11	72 09	70 07	68 00	65 52	63 43	61 31	59 19	57 06	54 51	3	00
	10	76 44	74 54	73 02	71 07	69 10	67 11	65 10	63 08	61 02	58 57	56 49	54 41		10
	20	75 24	73 38	71 54	70 05	68 14	66 20	64 25	62 29	60 30	58 29	56 28	54 26		20
	30	74 06	72 27	70 46	69 02	67 16	65 29	63 39	61 47	59 54	57 59	56 03	54 06		30
	40	72 50	71 14	69 38	67 58	66 17	64 35	62 50	61 04	59 15	57 26	55 35	53 42		40
	50	71 33	70 02	68 29	66 54	65 18	63 39	62 00	60 18	58 34	56 49	55 02	53 15		50
4	00	70 17	68 50	67 21	65 50	64 18	62 43	61 07	59 30	57 51	56 10	54 28	52 45	4	00
	10	69 01	67 38	66 12	64 45	63 17	61 45	60 14	58 41	57 05	55 28	53 50	52 10		10
	20	67 46	66 25	65 03	63 40	62 14	60 47	59 19	57 49	56 18	54 45	53 11	51 34		20
	30	66 31	65 13	63 55	62 33	61 12	59 48	58 23	56 56	55 28	53 59	52 29	50 56		30
	40	65 16	64 00	62 44	61 27	60 08	58 48	57 25	56 02	54 37	53 11	51 44	50 15		40
	50	64 00	62 47	61 34	60 20	59 03	57 46	56 27	55 07	53 45	52 21	50 57	49 32		50
5	00	62 45	61 35	60 23	59 11	57 58	56 43	55 28	54 10	52 51	51 31	50 09	48 47	5	00
	10	61 29	60 21	59 13	58 03	56 52	55 40	54 26	53 12	51 56	50 38	49 19	47 59		10
	20	60 12	59 07	r8 01	56 54	55 45	54 35	53 25	52 12	50 59	49 44	48 28	47 11		20
	30	58 56	57 53	56 49	55 43	54 37	53 30	52 22	51 12	50 00	48 48	47 35	46 21		30
	40	57 39	56 37	55 35	54 33	53 29	52 24	51 17	50 10	49 01	47 51	46 41	45 28		40
	50	56 2	55 21	54 22	53 21	52 20	51 16	50 13	49 07	48 01	46 53	45 45	44 36		50
6	00	55 04	54 05	53 08	52 09	51 09	50 08	49 06	48 03	46 59	45 54	44 48	43 40	6	00
	10	53 45	52 48	51 53	50 56	49 58	48 59	47 59	46 58	45 56	44 53	43 49	42 44		10
	20	52 26	51 32	50 37	49 42	48 46	47 49	46 52	45 52	44 52	43 51	42 50	41 46		20
	30	51 05	50 13	49 20	48 27	47 33	46 37	45 41	44 45	43 47	42 48	41 49	40 48		30
	40	49 45	48 54	48 03	47 12	46 19	45 26	44 32	43 37	42 41	41 44	40 47	39 47		40
	50	48 24	47 35	46 46	45 55	45 05	44 13	43 21	42 28	41 34	40 39	39 43	38 45		50
7	00	47 01	46 14	45 27	44 38	43 49	43 00	42 09	41 17	40 24	39 32	38 38	37 43	7	00
	10	45 39	44 52	44 07	43 20	42 33	41 45	40 56	40 06	39 15	38 25	37 33	36 39		10
	20	44 15	43 30	42 46	42 01	41 15	40 29	39 42	38 54	38 05	37 16	36 26	35 35		20
	30	42 51	42 07	41 25	40 41	39 58	39 12	38 27	37 41	36 54	36 07	35 18	34 29		30
	40	41 25	40 43	40 03	39 21	38 38	37 55	37 11	36 27	35 42	34 56	34 10	33 22		40
	50	39 59	39 20	38 40	37 59	37 19	36 37	35 54	35 12	34 28	33 44	33 00	32 15		50
8	00	38 32	37 54	37 15	36 36	35 57	35 17	34 37	33 56	33 14	32 31	31 49	31 05	8	00
	10	37 04	36 27	35 51	35 13	34 36	33 57	33 19	32 39	31 59	31 18	30 38	29 56		10
	20	35 36	35 00	34 25	33 49	33 13	32 36	31 59	31 21	30 43	30 04	29 25	28 45		20
	30	34 06	33 32	32 58	32 24	31 49	31 14	30 38	30 03	29 26	28 49	28 12	27 33		30
	40	32 36	32 03	31 31	30 58	30 26	29 51	29 17	28 43	28 08	27 33	26 57	26 21		40
	50	31 04	30 33	30 03	29 31	29 00	28 28	27 55	27 22	26 49	26 15	25 42	25 07		50
9	00	29 31	29 02	28 34	28 03	27 34	27 03	26 33	26 01	25 30	24 58	24 25	23 53	9	00
	10	27 59	27 30	27 03	26 35	26 06	25 38	25 09	24 40	24 10	23 39	23 09	22 38		10
	20	26 25	25 58	25 32	25 06	24 40	24 12	23 44	23 17	22 49	22 21	21 52	21 22		20
	30	24 50	24 26	24 01	23 36	23 11	22 45	22 20	21 53	21 27	21 00	20 33	20 05		30
	40	23 15	22 52	22 29	22 05	21 42	21 17	20 53	20 29	20 05	19 40	19 14	18 48		40
	50	21 38	21 17	20 56	20 34	20 12	19 50	19 27	19 04	18 42	18 18	17 55	17 30		50
10	00	20 02	19 41	19 22	19 02	18 41	18 21	18 00	17 39	17 17	16 56	16 34	16 12	10	00
	10	18 24	18 06	17 48	17 29	17 10	16 51	16 32	16 12	15 52	15 33	15 13	14 53		10
	20	16 46	16 29	16 12	15 55	15 38	15 21	15 04	14 46	14 28	14 11	13 52	13 33		20
	30	15 08	14 52	14 36	14 21	14 05	13 51	13 35	13 19	13 03	12 47	12 30	12 14		30
	40	13 27	13 13	13 00	12 47	12 33	12 20	12 06	11 52	11 38	11 23	11 08	10 53		40
	50	11 48	11 35	11 24	11 12	11 01	10 48	10 36	10 23	10 11	9 58	9 45	9 33		50
11	00	10 07	9 57	9 47	9 37	9 27	9 16	9 06	8 55	8 44	8 34	8 22	8 11	11	00
	10	8 27	8 18	8 10	8 01	7 53	7 44	7 35	7 26	7 17	7 08	6 59	6 50		10
	20	6 46	6 39	6 32	6 25	6 19	6 11	6 05	5 58	5 50	5 43	5 35	5 28		20
	30	5 05	4 59	4 54	4 49	4 44	4 39	4 33	4 28	4 22	4 17	4 12	4 07		30
	40	3 23	3 20	3 16	3 13	3 10	3 06	3 03	2 59	2 55	2 52	2 48	2 45		40
	50	1 42	1 40	1 38	1 37	1 35	1 33	1 32	1 30	1 28	1 26	1 24	1 22		50
12	00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	12	00

In **North latitude**, when the star is rising or East of the meridian, the tabulated azimuths are reckoned from North to East; and when the star is setting or West of the meridian, the tabulated azimuths are reckoned from North to West.

In **South latitude**, when the star is rising or East of the meridian, the tabulated azimuths are reckoned from South to East; and when the star is setting or West of the meridian, the tabulated azimuths are reckoned from South to West.

When the **latitude** and **declination** are of **different name**, the tables are to be entered with the supplement of the hour angle and the supplement of the tabulated azimuth is to be taken for the required true bearing.

AZIMUTH OR TRUE BEARING.

LATITUDE 52°. DECLINATION—SAME NAME AS—LATITUDE. **LATITUDE 52°.**

Hour Angle.		48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°	Hour Angle.	
h.	m.	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	h.	m.
0	00	180 00	180 00	180 00	180 00	.....	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0	00
	10	157 08	151 06	140 50	121 44	89 01	55 42	35 58	25 23	19 10	15 11	12 25	10 25		10
	20	139 16	131 22	120 28	105 51	88 02	69 50	54 24	42 46	34 19	28 08	23 31	19 59		20
	30	126 45	119 13	110 01	99 09	87 03	74 41	63 08	53 06	44 48	38 03	32 36	28 12		30
	40	117 59	111 19	103 42	95 11	86 03	76 43	67 40	59 17	51 47	45 16	39 42	34 56		40
	50	111 34	105 46	99 21	92 24	85 04	77 35	70 10	63 05	56 28	50 27	45 04	40 17		50
1	00	106 39	101 34	96 03	90 12	84 05	77 49	71 35	65 29	59 38	54 11	49 06	44 29	1	00
	10	102 42	98 12	93 24	88 20	83 05	77 43	72 19	67 00	61 48	56 50	52 08	47 45		10
	20	99 25	95 23	91 08	86 44	82 05	77 23	72 39	67 55	63 16	58 45	54 24	50 17		20
	30	96 36	92 57	89 09	85 11	81 06	76 55	72 40	68 27	64 14	60 07	56 06	52 13		30
	40	94 07	90 48	87 21	83 46	80 05	76 19	72 30	68 39	64 50	61 03	57 20	53 42		40
	50	91 54	88 51	85 42	82 27	79 05	75 40	72 11	68 40	65 10	61 39	58 12	54 48		50
2	00	89 50	87 03	84 08	81 09	78 05	74 56	71 44	68 30	65 15	62 02	58 48	55 36	2	00
	10	87 57	85 21	82 40	79 54	77 04	74 10	71 13	68 14	65 12	62 11	59 10	56 10		10
	20	86 11	83 45	81 16	78 41	76 03	73 21	70 37	67 49	65 01	62 12	59 22	56 31		20
	30	84 30	82 14	79 53	77 29	75 01	72 31	69 57	67 21	64 43	62 03	59 23	56 44		30
	40	82 53	80 45	78 33	76 18	74 00	71 38	69 14	66 47	64 19	61 49	59 18	56 46		40
	50	81 21	79 20	77 16	75 09	72 58	70 44	68 29	66 11	63 50	61 29	59 06	56 42		50
3	00	79 51	77 56	75 59	73 59	71 56	69 49	67 42	65 31	63 18	61 04	58 49	56 32	3	00
	10	78 23	76 35	74 43	72 49	70 53	68 53	66 52	64 48	62 42	60 35	58 26	56 16		10
	20	76 58	75 15	73 28	71 40	69 49	67 56	66 00	64 04	62 04	60 03	58 00	55 56		20
	30	75 34	73 56	72 15	70 32	68 46	66 58	65 08	63 16	61 22	59 26	57 29	55 31		30
	40	74 12	72 38	71 01	69 23	67 42	65 58	64 14	62 27	60 39	58 48	56 55	55 02		40
	50	72 50	71 20	69 48	68 14	66 37	64 59	63 18	61 37	59 52	58 07	56 19	54 31		50
4	00	71 30	70 04	68 35	67 04	65 32	63 58	62 22	60 44	59 05	57 23	55 40	53 56	4	00
	10	70 10	68 47	67 22	65 55	64 27	62 56	61 24	59 50	58 15	56 38	54 59	53 19		10
	20	68 50	67 31	66 09	64 46	63 21	61 54	60 26	58 56	57 24	55 51	54 15	52 39		20
	30	67 32	66 14	64 56	63 36	62 14	60 50	59 26	57 59	56 30	55 01	53 29	51 57		30
	40	66 12	64 59	63 43	62 26	61 07	59 47	58 25	57 01	55 36	54 10	52 43	51 13		40
	50	64 54	63 43	62 29	61 15	59 59	58 42	57 23	56 03	54 40	53 17	51 52	50 27		50
5	00	63 35	62 26	61 15	60 04	58 50	57 36	56 20	55 03	53 44	52 24	51 02	49 39	5	00
	10	62 16	61 09	60 01	58 52	57 41	56 30	55 16	54 02	52 45	51 28	50 09	48 49		10
	20	60 57	59 53	58 47	57 40	56 32	55 22	54 11	53 00	51 46	50 31	49 16	47 58		20
	30	59 38	58 35	57 32	56 27	55 21	54 14	53 05	51 56	50 45	49 33	48 20	47 06		30
	40	58 18	57 18	56 16	55 14	54 10	53 05	51 59	50 52	49 44	48 34	47 23	46 10		40
	50	56 59	56 01	55 01	54 00	52 58	51 55	50 52	49 47	48 40	47 34	46 25	45 15		50
6	00	55 39	54 41	53 43	52 45	51 46	50 45	49 43	48 41	47 36	46 31	45 25	44 18	6	00
	10	54 16	53 22	52 27	51 30	50 32	49 34	48 34	47 33	46 31	45 29	44 25	43 20		10
	20	52 58	52 04	51 09	50 14	49 19	48 22	47 24	46 25	45 25	44 25	43 23	42 20		20
	30	51 35	50 43	49 51	48 58	48 04	47 09	46 13	45 16	44 18	43 20	42 20	41 20		30
	40	50 11	49 23	48 31	47 41	46 48	45 55	45 01	44 06	43 10	42 13	41 16	40 18		40
	50	48 49	48 00	47 11	46 23	45 32	44 40	43 48	42 56	42 01	41 08	40 11	39 14		50
7	00	47 25	46 38	45 51	45 03	44 15	43 25	42 34	41 44	40 51	39 59	39 06	38 10	7	00
	10	46 00	45 16	44 30	43 43	42 56	42 08	41 20	40 31	39 40	38 50	37 57	37 05		10
	20	44 35	43 52	43 08	42 23	41 37	40 51	40 04	39 17	38 28	37 39	36 50	35 59		20
	30	43 10	42 28	41 45	41 02	40 18	39 33	38 48	38 03	37 16	36 29	35 41	34 52		30
	40	41 43	41 02	40 21	39 40	38 58	38 15	37 31	36 47	36 02	35 17	34 30	33 43		40
	50	40 15	39 37	38 57	38 17	37 36	36 55	36 13	35 31	34 47	34 04	33 20	32 34		50
8	00	38 47	38 10	37 31	36 53	36 14	35 34	34 54	34 14	33 31	32 50	32 07	31 24	8	00
	10	37 17	36 42	36 05	35 28	34 51	34 13	33 34	32 55	32 15	31 35	30 54	30 13		10
	20	35 48	35 14	34 39	34 03	33 27	32 51	32 14	31 36	30 58	30 20	29 41	29 01		20
	30	34 17	33 45	33 11	32 37	32 03	31 28	30 53	30 17	29 41	29 04	28 26	27 48		30
	40	32 46	32 14	31 43	31 11	30 37	30 04	29 30	28 56	28 21	27 47	27 11	26 34		40
	50	31 13	30 43	30 13	29 42	29 11	28 39	28 07	27 34	27 01	26 28	25 55	25 20		50
9	00	29 40	29 12	28 43	28 14	27 44	27 14	26 44	26 12	25 41	25 10	24 37	24 05	9	00
	10	28 07	27 39	27 12	26 44	26 16	25 48	25 19	24 50	24 20	23 51	23 20	22 49		10
	20	26 32	26 06	25 41	25 15	24 48	24 21	23 54	23 26	22 58	22 30	22 02	21 32		20
	30	24 57	24 33	24 08	23 43	23 18	22 53	22 28	22 02	21 36	21 09	20 43	20 15		30
	40	23 20	22 58	22 35	22 12	21 48	21 24	21 01	20 37	20 12	19 48	19 22	18 57		40
	50	21 43	21 22	21 01	20 40	20 18	19 56	19 34	19 12	18 49	18 25	18 03	17 39		50
10	00	20 06	19 47	19 27	19 07	18 47	18 27	18 06	17 45	17 24	17 03	16 42	16 19	10	00
	10	18 28	18 10	17 51	17 34	17 15	16 56	16 38	16 19	15 59	15 40	15 20	14 59		10
	20	16 50	16 33	16 16	16 00	15 43	15 26	15 09	14 51	14 34	14 16	13 57	13 40		20
	30	15 10	14 55	14 40	14 26	14 10	13 55	13 39	13 24	13 07	12 52	12 36	12 19		30
	40	13 30	13 17	13 04	12 51	12 37	12 23	12 09	11 55	11 41	11 27	11 13	10 58		40
	50	11 50	11 39	11 27	11 15	11 03	10 51	10 39	10 27	10 15	10 02	9 50	9 37		50
11	00	10 10	10 00	9 50	9 39	9 29	9 18	9 09	8 58	8 47	8 37	8 26	8 15	11	00
	10	8 28	8 21	8 11	8 03	7 55	7 47	7 38	7 28	7 20	7 11	7 02	6 52		10
	20	6 47	6 40	6 33	6 27	6 20	6 14	6 06	6 00	5 52	5 45	5 38	5 31		20
	30	5 05	5 00	4 55	4 50	4 45	4 40	4 34	4 30	4 24	4 19	4 13	4 08		30
	40	3 24	3 20	3 17	3 14	3 10	3 07	3 04	3 00	2 56	2 53	2 49	2 46		40
	50	1 42	1 40	1 38	1 37	1 35	1 34	1 32	1 30	1 28	1 26	1 24	1 23		50
12	00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	12	00

In **North latitude,** when the star is rising or East of the meridian, the tabulated azimuths are reckoned from North to East; and when the star is setting or West of the meridian, the tabulated azimuths are reckoned from North to West.

In **South latitude,** when the star is rising or East of the meridian, the tabulated azimuths are reckoned from South to East; and when the star is setting or West of the meridian, the tabulated azimuths are reckoned from South to West.

When the **latitude** and **declination** are of **different name,** the tables are to be entered with the supplement of the hour angle and the supplement of the tabulated azimuth is to be taken for the required true bearing.

APPENDIX AA

EXTRACTS FROM H.O. PUB. NO. 214

Lat. 41°

DECLINATION SAME NAME AS LATITUDE

H.A.	19° 00′ Alt.	Δd Δt	Az.	19° 30′ Alt.	Δd Δt	Az.	20° 00′ Alt.	Δd Δt	Az.	20° 30′ Alt.	Δd Δt	Az.
°	° ′		°	° ′		°	° ′		°	° ′		°
00	68 00.0	1.0 02	180.0	68 30.0	1.0 02	180.0	69 00.0	1.0 02	180.0	69 30.0	1.0 02	180.0
1	67 59.0	1.0 05	177.5	68 29.0	1.0 05	177.4	68 59.0	1.0 05	177.4	69 28.9	1.0 05	177.3
2	67 56.0	1.0 08	175.0	68 25.9	1.0 08	174.9	68 55.9	1.0 09	174.8	69 25.8	1.0 09	174.7
3	67 51.1	99 12	172.5	68 20.9	99 12	172.3	68 50.7	99 12	172.2	69 20.5	99 12	172.0
4	67 44.1	99 15	170.0	68 13.8	99 15	169.8	68 43.5	99 15	169.6	69 13.2	99 16	169.4
05	67 35.3	99 18	167.5	68 04.8	98 18	167.3	68 34.4	98 19	167.0	69 03.9	98 19	166.8
6	67 24.6	98 21	165.1	67 53.9	98 21	164.8	68 23.2	98 22	164.5	68 52.5	98 22	164.2
7	67 12.0	97 24	162.7	67 41.1	97 24	162.4	68 10.2	97 25	162.1	68 39.3	97 25	161.7
8	66 57.7	96 27	160.4	67 26.5	96 27	160.0	67 55.4	96 28	159.6	68 24.2	96 28	159.3
9	66 41.6	95 30	158.0	67 10.2	95 30	157.7	67 38.7	95 31	157.3	68 07.2	95 31	156.8
10	66 23.8	94 32	155.8	66 52.1	94 33	155.4	67 20.4	94 33	154.9	67 48.6	94 34	154.5
1	66 04.5	93 35	153.6	66 32.5	93 35	153.1	67 00.4	93 36	152.7	67 28.2	93 36	152.2
2	65 43.6	92 37	151.4	66 11.2	92 38	151.0	66 38.8	92 38	150.5	67 06.3	92 39	150.0
3	65 21.2	91 40	149.3	65 48.5	91 40	148.8	66 15.8	91 41	148.3	66 42.9	90 41	147.8
4	64 57.4	90 42	147.3	65 24.4	90 42	146.8	65 51.3	89 43	146.2	66 18.1	89 44	145.7
30	56 15.7	73 65	121.7	56 37.6	73 65	121.0	56 59.3	72 65	120.4	57 20.9	71 66	119.8
1	55 36.9	73 65	120.4	55 58.6	72 66	119.8	56 20.0	71 66	119.2	56 41.3	71 67	118.5
2	54 57.6	72 66	119.2	55 19.0	71 67	118.6	55 40.3	71 67	118.0	56 01.3	70 67	117.4
3	54 17.9	71 67	118.1	54 39.1	70 67	117.5	55 00.1	70 68	116.8	55 20.9	69 68	116.2
4	53 37.7	70 68	116.9	53 58.7	70 68	116.3	54 19.5	69 68	115.7	54 40.1	68 69	115.1
35	52 57.1	70 68	115.8	53 17.9	69 69	115.2	53 38.5	68 69	114.6	53 58.9	68 69	114.0
6	52 16.2	69 69	114.7	52 36.8	68 69	114.1	52 57.1	68 69	113.5	53 17.4	67 70	112.9
7	51 34.9	68 69	113.7	51 55.3	68 70	113.1	52 15.5	67 70	112.5	52 35.5	66 70	111.9
8	50 53.3	68 70	112.7	51 13.5	67 70	112.1	51 33.5	66 70	111.5	51 53.3	66 71	110.9
9	50 11.3	67 70	111.7	50 31.4	66 71	111.1	50 51.2	66 71	110.5	51 10.9	65 71	109.9
60	34 46.6	61 75	94.5	35 04.7	60 75	94.0	35 22.7	60 75	93.5	35 40.6	59 75	93.0
1	34 01.5	60 75	93.8	34 19.6	60 75	93.3	34 37.5	60 75	92.8	34 55.4	59 75	92.3
2	33 16.3	60 75	93.1	33 34.3	60 75	92.6	33 52.3	60 75	92.2	34 10.1	59 75	91.7
3	32 31.0	60 75	92.4	32 49.1	60 75	92.0	33 07.0	60 75	91.5	33 24.9	59 75	91.0
4	31 45.8	60 75	91.8	32 03.8	60 75	91.3	32 21.8	60 75	90.8	32 39.6	59 75	90.3
80	19 43.5	61 75	81.6	20 01.9	61 75	81.1	20 20.2	61 74	80.7	20 38.5	61 74	80.3
1	18 58.7	62 74	81.0	19 17.2	61 74	80.5	19 35.5	61 74	80.1	19 53.9	61 74	79.7
2	18 14.1	62 74	80.3	18 32.5	62 74	79.9	18 51.0	61 74	79.5	19 09.4	61 74	79.1
3	17 29.5	62 74	79.7	17 48.0	62 74	79.3	18 06.5	62 74	78.9	18 24.9	61 74	78.5
4	16 45.0	62 74	79.1	17 03.6	62 74	78.7	17 22.1	62 74	78.3	17 40.6	62 74	77.9
85	16 00.5	62 74	78.5	16 19.2	62 74	78.1	16 37.8	62 74	77.7	16 56.4	62 74	77.3
6	15 16.2	63 74	77.9	15 34.9	62 74	77.5	15 53.6	62 73	77.1	16 12.3	62 73	76.7
7	14 32.0	63 74	77.3	14 50.8	63 73	76.9	15 09.6	62 73	76.5	15 28.3	62 73	76.1
8	13 47.9	63 73	76.7	14 06.7	63 73	76.3	14 25.6	63 73	75.9	14 44.4	63 73	75.5
9	13 03.9	63 73	76.0	13 22.8	63 73	75.6	13 41.7	63 73	75.3	14 00.6	63 73	74.8
90	12 20.0	63 73	75.4	12 39.0	63 73	75.0	12 58.0	63 73	74.6	13 17.0	63 73	74.2

H.A.	21° 00′ Alt.	Δd Δt	Az.	21° 30′ Alt.	Δd Δt	Az.	22° 00′ Alt.	Δd Δt	Az.	22° 30′ Alt.	Δd Δt	Az.
°	° ′		°	° ′		°	° ′		°	° ′		°
00	70 00.0	1.0 02	180.0	70 30.0	1.0 02	180.0	71 00.0	1.0 02	180.0	71 30.0	1.0 02	180.0
1	69 58.9	1.0 05	177.3	70 28.9	1.0 06	177.2	70 58.9	1.0 06	177.2	71 28.8	1.0 06	177.1
2	69 55.7	1.0 09	174.6	70 25.6	1.0 09	174.4	70 55.5	1.0 09	174.3	71 25.4	1.0 10	174.2
3	69 50.3	99 12	171.9	70 20.1	99 13	171.7	70 49.9	99 13	171.5	71 19.7	99 13	171.3
4	69 42.9	99 16	169.2	70 12.5	99 16	168.9	70 42.1	99 17	168.7	71 11.7	99 17	168.5
05	69 33.3	98 19	166.5	70 02.8	98 20	166.3	70 32.2	98 20	166.0	71 01.6	98 20	165.7
6	69 21.8	97 22	163.9	69 51.0	97 23	163.6	70 20.2	97 23	163.3	70 49.3	97 24	162.9
7	69 08.3	97 26	161.4	69 37.2	96 26	161.0	70 06.1	96 27	160.6	70 35.0	96 27	160.2
8	68 52.9	96 29	158.9	69 21.5	95 29	158.4	69 50.2	95 30	158.0	70 18.7	95 30	157.6
9	68 35.6	95 32	156.4	69 04.0	94 32	156.0	69 32.3	94 33	155.5	70 00.5	94 33	155.0
10	68 16.7	94 34	154.0	68 44.7	93 35	153.5	69 12.6	93 36	153.0	69 40.4	93 36	152.5
1	67 56.0	92 37	151.7	68 23.7	92 38	151.2	68 51.2	92 38	150.6	69 18.7	91 39	150.1
2	67 33.7	91 40	149.4	68 01.0	91 40	148.9	68 28.2	90 41	148.3	68 55.3	90 42	147.7
3	67 10.0	90 42	147.2	67 36.9	90 43	146.7	68 03.7	89 43	146.1	68 30.3	89 44	145.4
4	66 44.8	89 44	145.1	67 11.3	88 45	144.5	67 37.7	88 46	143.9	68 03.9	87 46	143.2
30	57 42.2	71 66	119.1	58 03.4	70 67	118.4	58 24.3	69 67	117.8	58 45.0	69 68	117.1
1	57 02.4	70 67	117.9	57 23.3	69 67	117.2	57 44.0	69 68	116.6	58 04.4	68 68	115.9
2	56 22.2	69 68	116.7	56 42.8	68 68	116.1	57 03.3	68 68	115.4	57 23.5	67 69	114.7
3	55 41.5	68 68	115.6	56 02.0	68 69	114.9	56 22.2	67 69	114.2	56 42.2	66 69	113.6
4	55 00.5	68 69	114.4	55 20.7	67 69	113.8	55 40.7	66 70	113.1	56 00.5	66 70	112.5
35	54 19.1	67 70	113.4	54 39.1	66 70	112.7	54 58.9	66 70	112.1	55 18.5	65 71	111.4
6	53 37.4	66 70	112.3	53 57.2	66 70	111.7	54 16.8	65 71	111.0	54 36.2	64 71	110.4
7	52 55.3	66 71	111.3	53 14.9	65 71	110.6	53 34.4	64 71	110.0	53 53.6	64 71	109.3
8	52 13.0	65 71	110.3	52 32.4	65 71	109.6	52 51.7	64 72	109.0	53 10.7	63 72	108.4
9	51 30.4	65 71	109.3	51 49.7	64 72	108.7	52 08.8	63 72	108.0	52 27.6	63 72	107.4
60	35 58.4	59 75	92.5	36 16.0	59 75	92.0	36 33.5	58 75	91.5	36 50.9	58 75	91.0
1	35 13.1	59 75	91.9	35 30.7	59 75	91.4	35 48.2	58 75	90.9	36 05.6	58 75	90.4
2	34 27.9	59 75	91.2	34 45.5	58 75	90.7	35 02.9	58 75	90.2	35 20.3	58 75	89.7
3	33 42.6	59 75	90.5	34 00.2	58 75	90.0	34 17.7	58 75	89.5	34 35.0	58 75	89.1
4	32 57.3	59 75	89.9	33 14.9	58 75	89.4	33 32.4	58 75	88.9	33 49.8	58 75	88.4
80	20 56.7	61 74	79.9	21 14.8	60 74	79.5	21 32.9	60 74	79.0	21 50.9	60 74	78.6
1	20 12.1	61 74	79.3	20 30.3	61 74	78.9	20 48.5	60 74	78.4	21 06.5	60 74	78.0
2	19 27.7	61 74	78.7	19 45.9	61 74	78.2	20 04.1	61 74	77.8	20 22.3	60 74	77.4
3	18 43.3	61 74	78.1	19 01.7	61 74	77.6	19 19.9	61 74	77.2	19 38.1	61 73	76.8
4	17 59.1	61 74	77.5	18 17.5	61 73	77.0	18 35.8	61 73	76.6	18 54.1	61 73	76.2
85	17 14.9	62 73	76.9	17 33.4	61 73	76.4	17 51.8	61 73	76.0	18 10.2	61 73	75.6
6	16 30.9	62 73	76.3	16 49.4	62 73	75.8	17 07.9	62 73	75.4	17 26.4	61 73	75.0
7	15 46.9	62 73	75.7	16 05.6	62 73	75.2	16 24.2	62 73	74.8	16 42.7	62 73	74.4
8	15 03.1	62 73	75.1	15 21.8	62 73	74.6	15 40.5	62 73	74.2	15 59.1	62 72	73.8
9	14 19.4	63 73	74.4	14 38.2	63 72	74.0	14 57.0	62 72	73.6	15 15.7	62 72	73.2
90	13 35.9	63 72	73.8	13 54.8	63 72	73.4	14 13.6	63 72	73.0	14 32.4	63 72	72.6

Lat. 41°

DECLINATION CONTRARY NAME TO LATITUDE

H.A.	19° 00′ Alt.	Δd Δt	Az.	19° 30′ Alt.	Δd Δt	Az.	20° 00′ Alt.	Δd Δt	Az.	20° 30′ Alt.	Δd Δt	Az.
°	° ′		°	° ′		°	° ′		°	° ′		°
00	30 00.0	1.0 01	180.0	29 30.0	1.0 01	180.0	29 00.0	1.0 01	180.0	28 30.0	1.0 01	180.0
1	29 59.6	1.0 02	178.9	29 29.6	1.0 02	178.9	28 59.6	1.0 02	178.9	28 29.6	1.0 02	178.9
2	29 58.3	1.0 04	177.8	29 28.3	1.0 04	177.8	28 58.3	1.0 04	177.9	28 28.3	1.0 03	177.9
3	29 56.1	1.0 05	176.7	29 26.2	1.0 05	176.8	28 56.2	1.0 05	176.8	28 26.2	1.0 05	176.8
4	29 53.1	1.0 06	175.6	29 23.2	1.0 06	175.7	28 53.2	1.0 06	175.7	28 23.3	1.0 06	175.7
05	29 49.2	1.0 08	174.5	29 19.3	1.0 08	174.6	28 49.4	1.0 08	174.6	28 19.5	1.0 08	174.7
6	29 44.5	1.0 09	173.5	29 14.6	1.0 09	173.5	28 44.7	1.0 09	173.6	28 14.9	1.0 09	173.6
7	29 38.9	99 11	172.4	29 09.1	99 11	172.4	28 39.3	99 11	172.5	28 09.4	99 10	172.6
8	29 32.5	99 12	171.3	29 02.7	99 12	171.4	28 32.9	99 12	171.4	28 03.1	99 12	171.5
9	29 25.2	99 14	170.2	28 55.5	99 13	170.3	28 25.8	99 13	170.4	27 56.0	99 13	170.5
10	29 17.1	99 15	169.1	28 47.5	99 15	169.2	28 17.8	99 15	169.3	27 48.1	99 15	169.4
1	29 08.2	99 16	168.1	28 38.6	99 16	168.2	28 09.0	99 16	168.3	27 39.4	99 16	168.4
2	28 58.4	98 18	167.0	28 28.9	98 18	167.1	27 59.4	98 17	167.2	27 29.9	98 17	167.3
3	28 47.8	98 19	166.0	28 18.4	98 19	166.1	27 49.0	98 19	166.2	27 19.5	98 19	166.3
4	28 36.4	98 20	164.9	28 07.1	98 20	165.0	27 37.7	98 20	165.1	27 08.4	98 20	165.2
15	28 24.2	98 22	163.8	27 55.0	98 22	164.0	27 25.7	98 21	164.1	26 56.5	98 21	164.2
6	28 11.2	97 23	162.8	27 42.1	97 23	162.9	27 12.9	97 23	163.1	26 43.8	97 22	163.2
7	27 57.5	97 24	161.8	27 28.4	97 24	161.9	26 59.3	97 24	162.0	26 30.3	97 24	162.2
8	27 42.9	96 26	160.7	27 14.0	97 25	160.9	26 45.0	97 25	161.0	26 16.0	97 25	161.2
9	27 27.6	96 27	159.7	26 58.7	96 27	159.9	26 29.9	96 26	160.0	26 01.0	96 26	160.2
20	27 11.5	96 28	158.7	26 42.8	96 28	158.8	26 14.0	96 28	159.0	25 45.3	96 27	159.2
1	26 54.7	95 29	157.7	26 26.1	95 29	157.8	25 57.5	95 29	158.0	25 28.8	95 29	158.2
2	26 37.1	95 31	156.7	26 08.6	95 30	156.8	25 40.1	95 30	157.0	25 11.6	95 30	157.2
3	26 18.8	94 32	155.7	25 50.4	94 31	155.8	25 22.1	95 31	156.0	24 53.7	95 31	156.2
4	25 59.8	94 33	154.7	25 31.6	94 33	154.9	25 03.3	94 32	155.0	24 35.1	94 32	155.2
25	25 40.0	94 34	153.7	25 12.0	94 34	153.9	24 43.9	94 34	154.1	24 15.8	94 33	154.3
6	25 19.6	93 35	152.7	24 51.7	93 35	152.9	24 23.7	93 35	153.1	23 55.8	93 34	153.3
7	24 58.5	93 36	151.7	24 30.7	93 36	151.9	24 02.9	93 36	152.2	23 35.1	93 36	152.4
8	24 36.7	92 37	150.8	24 09.1	92 37	151.0	23 41.4	92 37	151.2	23 13.8	92 37	151.4
9	24 14.3	92 38	149.8	23 46.8	92 38	150.0	23 19.3	92 38	150.3	22 51.8	92 38	150.5
55	11 17.2	78 60	127.8	10 53.9	78 60	128.2	10 30.5	78 59	128.5	10 07.2	78 59	128.8
6	10 41.2	77 60	127.1	10 18.1	77 60	127.4	9 54.9	77 60	127.7	9 31.7	77 60	128.1
7	10 04.9	77 61	126.3	9 41.9	77 61	126.7	9 18.9	77 61	127.0	8 55.9	77 60	127.3
8	9 28.3	76 62	125.6	9 05.4	76 61	125.9	8 42.6	76 61	126.3	8 19.7	76 61	126.6
9	8 51.3	76 62	124.9	8 28.6	76 62	125.2	8 05.9	76 62	125.6	7 43.2	76 61	125.9
60	8 14.0	75 63	124.2	7 51.5	75 62	124.5	7 28.9	75 62	124.8	7 06.3	75 62	125.2
1	7 36.4	75 63	123.5	7 14.0	75 63	123.8	6 51.6	75 63	124.1	6 29.2	75 62	124.5
2	6 58.4	74 64	122.7	6 36.2	74 63	123.1	6 13.9	74 63	123.4	5 51.7	74 63	123.8
3	6 20.2	74 64	122.0	5 58.1	74 64	122.4	5 36.0	74 64	122.7	5 13.9	74 63	123.1
4	5 41.7	73 65	121.3	5 19.7	73 64	121.7						

H.A.	21° 00′ Alt.	Δd Δt	Az.	21° 30′ Alt.	Δd Δt	Az.	22° 00′ Alt.	Δd Δt	Az.	22° 30′ Alt.	Δd Δt	Az.	H.A.
°	° ′		°	° ′		°	° ′		°	° ′		°	°
00	28 00.0	1.0 01	180.0	27 30.0	1.0 01	180.0	27 00.0	1.0 01	180.0	26 30.0	1.0 01	180.0	00
1	27 59.6	1.0 02	178.9	27 29.6	1.0 02	179.0	26 59.6	1.0 02	179.0	26 29.6	1.0 02	179.0	1
2	27 58.3	1.0 04	177.9	27 28.3	1.0 03	177.9	26 58.4	1.0 03	177.9	26 28.4	1.0 03	177.9	2
3	27 56.2	1.0 05	176.8	27 26.3	1.0 05	176.9	26 56.3	1.0 05	176.9	26 26.3	1.0 05	176.9	3
4	27 53.3	1.0 06	175.8	27 23.4	1.0 06	175.8	26 53.4	1.0 06	175.8	26 23.5	1.0 06	175.9	4
05	27 49.6	1.0 08	174.7	27 19.7	1.0 08	174.8	26 49.7	1.0 08	174.8	26 19.8	1.0 07	174.8	05
6	27 45.0	1.0 09	173.7	27 15.1	1.0 09	173.7	26 45.2	1.0 09	173.8	26 15.3	1.0 09	173.8	6
7	27 39.6	99 10	172.6	27 09.7	99 10	172.7	26 39.9	99 10	172.7	26 10.1	99 10	172.8	7
8	27 33.4	99 12	171.6	27 03.6	99 12	171.6	26 33.8	99 12	171.7	26 04.0	99 11	171.8	8
9	27 26.3	99 13	170.5	26 56.6	99 13	170.6	26 26.8	99 13	170.7	25 57.1	99 13	170.8	9
10	27 18.5	99 14	169.5	26 48.8	99 14	169.6	26 19.1	99 14	169.7	25 49.4	99 14	169.7	10
1	27 09.8	99 16	168.5	26 40.2	99 16	168.5	26 10.6	99 16	168.6	25 41.0	99 15	168.7	1
2	27 00.3	98 17	167.4	26 30.8	98 17	167.5	26 01.3	98 17	167.6	25 31.7	98 17	167.7	2
3	26 50.1	98 18	166.4	26 20.6	98 18	166.5	25 51.2	98 18	166.6	25 21.7	98 18	166.7	3
4	26 39.0	98 20	165.4	26 09.6	98 20	165.5	25 40.3	98 19	165.6	25 10.9	98 19	165.7	4
15	26 27.2	98 21	164.3	25 57.9	98 21	164.5	25 28.6	98 21	164.6	24 59.3	98 21	164.7	15
6	26 14.6	97 22	163.3	25 45.4	97 22	163.5	25 16.2	97 22	163.6	24 47.0	97 22	163.7	6
7	26 01.2	97 24	162.3	25 32.1	97 23	162.5	25 03.0	97 23	162.6	24 33.9	97 23	162.7	7
8	25 47.1	97 25	161.3	25 18.1	97 25	161.5	24 49.1	97 24	161.6	24 20.1	97 24	161.7	8
9	25 32.2	96 26	160.3	25 03.3	96 26	160.5	24 34.4	96 26	160.6	24 05.5	96 25	160.8	9
20	25 16.6	96 27	159.3	24 47.8	96 27	159.5	24 19.0	96 27	159.6	23 50.3	96 27	159.8	20
1	25 00.2	95 28	158.3	24 31.6	95 28	158.5	24 02.9	96 28	158.7	23 34.3	96 28	158.8	1
2	24 43.1	95 30	157.4	24 14.6	95 29	157.5	23 46.1	95 29	157.7	23 17.6	95 29	157.9	2
3	24 25.3	95 31	156.4	23 57.0	95 31	156.6	23 28.6	95 30	156.7	23 00.1	95 30	156.9	3
4	24 06.9	94 32	155.4	23 38.6	94 32	155.6	23 10.3	94 32	155.8	22 42.0	94 31	156.0	4
25	23 47.7	94 33	154.5	23 19.5	94 33	154.6	22 51.4	94 33	154.8	22 23.3	94 32	155.0	25
6	23 27.8	93 34	153.5	22 59.8	93 34	153.7	22 31.8	93 34	153.9	22 03.8	93 34	154.1	6
7	23 07.3	93 35	152.6	22 39.4	93 35	152.8	22 11.6	93 35	153.0	21 43.7	93 35	153.2	7
8	22 46.1	92 36	151.6	22 18.4	92 36	151.8	21 50.6	92 36	152.0	21 22.9	92 36	152.2	8
9	22 24.2	92 38	150.7	21 56.7	92 37	150.9	21 29.1	92 37	151.1	21 01.5	92 37	151.3	9
55	9 43.9	78 59	129.1	9 20.5	78 59	129.4	8 57.1	78 58	129.7	8 33.7	78 58	130.1	55
6	9 08.5	77 59	128.4	8 45.3	77 59	128.7	8 22.1	77 59	129.0	7 58.8	78 59	129.3	6
7	8 32.9	77 60	127.7	8 09.8	77 60	128.0	7 46.7	77 60	128.3	7 23.6	77 59	128.6	7
8	7 56.8	76 61	126.9	7 33.9	76 60	127.3	7 11.0	76 60	127.6	6 48.1	76 60	127.9	8
9	7 20.5	76 61	126.2	6 57.7	76 61	126.5	6 35.0	76 61	126.9	6 12.2	76 60	127.2	9
60	6 43.8	75 62	125.5	6 21.2	75 61	125.8	5 58.6	75 61	126.2	5 35.9	75 61	126.5	60
1	6 06.7	75 62	124.8	5 44.3	75 62	125.1	5 21.8	75 62	125.5				1
2	5 29.4	74 63	124.1	5 07.1	74 62	124.4							2
3													3
4													4

STAR IDENTIFICATION TABLE

Lat. 41°

ALTITUDE

AZ.	4° Dec.	4° H.A.	8° Dec.	8° H.A.	12° Dec.	12° H.A.	16° Dec.	16° H.A.	20° Dec.	20° H.A.	24° Dec.	24° H.A.	28° Dec.	28° H.A.	32° Dec.	32° H.A.	36° Dec.	36° H.A.	40° Dec.	40° H.A.	44° Dec.	44° H.A.	AZ.
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
80	10	94	13	90	15	87	18	84	20	81	23	77	25	74	27	70	29	66	31	62	33	58	80
84	07	91	10	88	12	85	15	82	17	78	20	75	22	72	24	68	27	64	29	60	31	56	84
88	04	88	07	85	09	82	12	79	14	76	17	73	19	69	22	66	24	62	26	59	28	55	88
92	01	86	04	83	06	80	09	77	12	73	14	70	17	67	19	64	21	60	24	57	26	53	92
96	*02*	83	01	80	03	77	06	74	09	71	11	68	14	65	16	61	19	58	21	55	24	51	96
100	*05*	80	*02*	77	00	74	03	72	06	68	08	65	11	62	14	59	16	56	19	53	21	49	100
104	*08*	78	*05*	75	*02*	72	00	69	03	66	06	63	08	60	11	57	14	54	16	51	19	48	104
108	*11*	75	*08*	72	*05*	69	*02*	66	00	63	03	60	06	58	09	55	11	52	14	49	17	46	108
112	*14*	72	*11*	69	*08*	66	*05*	64	*02*	61	00	58	03	55	06	52	09	49	12	47	15	44	112
116	*17*	69	*14*	66	*11*	64	*08*	61	*05*	58	*02*	55	01	53	04	50	07	47	10	44	13	41	116
120	*19*	66	*16*	63	*13*	61	*10*	58	*07*	55	*04*	53	*01*	50	02	47	05	45	08	42	11	39	120
124	*22*	63	*19*	60	*16*	58	*13*	55	*10*	52	*07*	50	*04*	47	*01*	45	03	42	06	40	09	37	124
128	*25*	60	*22*	57	*19*	54	*15*	52	*12*	49	*09*	47	*06*	44	*03*	42	01	40	04	37	07	35	128
132	*27*	57	*24*	54	*21*	51	*18*	49	*14*	46	*11*	44	*08*	41	*05*	39	*01*	37	02	35	05	32	132
136	*30*	53	*27*	50	*23*	48	*20*	45	*17*	43	*13*	41	*10*	39	*06*	36	*03*	34	00	32	04	30	136
140	*32*	49	*29*	47	*25*	44	*22*	42	*19*	40	*15*	37	*12*	35	*08*	33	*05*	31	*01*	30	02	28	140
144	*34*	45	*31*	43	*27*	40	*24*	38	*20*	36	*17*	34	*13*	32	*10*	30	*06*	29	*03*	27	01	25	144
148	*36*	41	*33*	39	*29*	36	*26*	34	*22*	33	*19*	31	*15*	29	*11*	27	*08*	26	*04*	24	*00*	22	148
152	*38*	37	*35*	34	*31*	32	*27*	31	*24*	29	*20*	27	*16*	26	*13*	24	*09*	23	*05*	21	*01*	20	152
156	*40*	32	*36*	30	*33*	28	*29*	27	*25*	25	*21*	24	*18*	22	*14*	21	*10*	20	*06*	18	*02*	17	156
	4°		8°		12°		16°		20°		24°		28°		32°		36°		40°		44°		

FIGURES IN ITALICS INDICATE THAT DECLINATION IS OF CONTRARY NAME TO LATITUDE

Lat. 42°

DECLINATION CONTRARY NAME TO LATITUDE

H.A.	19° 00′ Alt.	Δd Δt	Az.	19° 30′ Alt.	Δd Δt	Az.	20° 00′ Alt.	Δd Δt	Az.	20° 30′ Alt.	Δd Δt	Az.
°	° ′		°	° ′		°	° ′		°	° ′		°
00	29 00.0	1.0 01	180.0	28 30.0	1.0 01	180.0	28 00.0	1.0 01	180.0	27 30.0	1.0 01	180.0
1	28 59.6	1.0 02	178.9	28 29.6	1.0 02	178.9	27 59.6	1.0 02	178.9	27 29.6	1.0 02	178.9
2	28 58.3	1.0 04	177.8	28 28.3	1.0 03	177.9	27 58.3	1.0 03	177.9	27 28.4	1.0 03	177.9
3	28 56.2	1.0 05	176.8	28 26.2	1.0 05	176.8	27 56.3	1.0 05	176.8	27 26.3	1.0 05	176.8
4	28 53.3	1.0 06	175.7	28 23.3	1.0 06	175.7	27 53.4	1.0 06	175.7	27 23.4	1.0 06	175.8
05	28 49.5	1.0 08	174.6	28 19.6	1.0 08	174.6	27 49.7	1.0 08	174.7	27 19.7	1.0 08	174.7
6	28 44.9	1.0 09	173.5	28 15.0	1.0 09	173.6	27 45.1	1.0 09	173.6	27 15.2	1.0 09	173.7
7	28 39.4	99 10	172.5	28 09.6	99 10	172.5	27 39.8	99 10	172.6	27 09.9	99 10	172.6
8	28 33.2	99 12	171.4	28 03.4	99 12	171.5	27 33.6	99 12	171.5	27 03.8	99 12	171.6
9	28 26.1	99 13	170.3	27 56.4	99 13	170.4	27 26.6	99 13	170.5	26 56.9	99 13	170.5
10	28 18.2	99 15	169.3	27 48.5	99 14	169.3	27 18.8	99 14	169.4	26 49.1	99 14	169.5
1	28 09.5	99 16	168.2	27 39.9	99 16	168.3	27 10.2	99 16	168.4	26 40.6	99 16	168.5
2	27 59.9	98 17	167.1	27 30.4	98 17	167.2	27 00.9	98 17	167.3	26 31.3	98 17	167.4
3	27 49.6	98 19	166.1	27 20.1	98 18	166.2	26 50.7	98 18	166.3	26 21.2	98 18	166.4
4	27 38.5	98 20	165.0	27 09.1	98 20	165.2	26 39.7	98 20	165.3	26 10.3	98 19	165.4
15	27 26.6	98 21	164.0	26 57.3	98 21	164.1	26 28.0	98 21	164.2	25 58.7	98 21	164.4
6	27 13.9	97 22	163.0	26 44.7	97 22	163.1	26 15.5	97 22	163.2	25 46.3	97 22	163.3
7	27 00.4	97 24	161.9	26 31.4	97 24	162.1	26 02.2	97 23	162.2	25 33.1	97 23	162.3
8	26 46.2	97 25	160.9	26 17.2	97 25	161.0	25 48.2	97 25	161.2	25 19.2	97 24	161.3
9	26 31.3	96 26	159 9	26 02.4	96 26	160.0	25 33.5	96 26	160.2	25 04.6	96 26	160.3
20	26 15.6	96 27	158.9	25 46.8	96 27	159.0	25 18.0	96 27	159.2	24 49.2	96 27	159.3
1	25 59.1	96 29	157.9	25 30.5	96 28	158.0	25 01.8	96 28	158.2	24 33.1	96 28	158.3
2	25 41.9	95 30	156.9	25 13.4	95 30	157.0	24 44.9	95 29	157.2	24 16.3	95 29	157.4
3	25 24.1	95 31	155.9	24 55.7	95 31	156.0	24 27.2	95 31	156.2	23 58.8	95 30	156.4
4	25 05.5	94 32	154.9	24 37.2	94 32	155.1	24 08.9	94 32	155.2	23 40.6	94 32	155.4
25	24 46.2	94 33	153.9	24 18.0	94 33	154.1	23 49.9	94 33	154.3	23 21.7	94 33	154.5
6	24 26.2	93 34	152.9	23 58.2	93 34	153.1	23 30.2	93 34	153.3	23 02.1	94 34	153.5
7	24 05.6	93 36	152.0	23 37.7	93 35	152.2	23 09.8	93 35	152.4	22 41.9	93 35	152.6
8	23 44.3	92 37	151.0	23 16.6	93 36	151.2	22 48.8	93 36	151.4	22 21.0	93 36	151.6
9	23 22.4	92 38	150.0	22 54.8	92 37	150.3	22 27.1	92 37	150.5	21 59.5	92 37	150.7
55	10 40.3	78 59	128.0	10 16.7	79 59	128.3	9 53.2	79 58	128.6	9 29.6	79 58	128.9
6	10 05.0	78 59	127.2	9 41.6	78 59	127.6	9 18.1	78 59	127.9	8 54.7	79 59	128.2
7	9 29.3	78 60	126.5	9 06.0	78 60	126.8	8 42.8	78 60	127.1	8 19.5	78 59	127.4
8	8 53.3	77 61	125.7	8 30.2	77 60	126.1	8 07.0	77 60	126.4	7 43.9	77 60	126.7
9	8 16.9	77 61	125.0	7 54.0	77 61	125.3	7 31.0	77 61	125.7	7 08.0	77 60	126.0
60	7 40.2	76 62	124.3	7 17.4	76 61	124.6	6 54.6	76 61	124.9	6 31.7	76 61	125.3
1	7 03.2	76 62	123.6	6 40.6	76 62	123.9	6 17.9	76 62	124.2	5 55.2	76 61	124.6
2	6 25.9	75 63	122.8	6 03.4	75 62	123.2	5 40.9	75 62	123.5	5 18.3	75 62	123.8
3	5 48.3	75 63	122.1	5 25.9	75 63	122.5	5 03.5	75 63	122.8			
4	5 10.4	74 64	121.4									

H.A.	21° 00′ Alt.	Δd Δt	Az.	21° 30′ Alt.	Δd Δt	Az.	22° 00′ Alt.	Δd Δt	Az.	22° 30′ Alt.	Δd Δt	Az.
°	° ′		°	° ′		°	° ′		°	° ′		°
00	27 00.0	1.0 01	180.0	26 30.0	1.0 01	180.0	26 00.0	1.0 01	180.0	25 30.0	1.0 01	180.0
1	26 59.6	1.0 02	179.0	26 29.6	1.0 02	179.0	25 59.6	1.0 02	179.0	25 29.6	1.0 02	179.0
2	26 58.4	1.0 03	177.9	26 28.4	1.0 03	177.9	25 58.4	1.0 03	177.9	25 28.4	1.0 03	178.0
3	26 56.3	1.0 05	176.9	26 26.4	1.0 05	176.9	25 56.4	1.0 05	176.9	25 26.4	1.0 05	176.9
4	26 53.5	1.0 06	175.8	26 23.5	1.0 06	175.8	25 53.6	1.0 06	175.9	25 23.6	1.0 06	175.9
05	26 49.8	1.0 07	174.8	26 19.9	1.0 07	174.8	25 50.0	1.0 07	174.8	25 20.1	1.0 07	174.9
6	26 45.4	1.0 09	173.7	26 15.5	1.0 09	173.8	25 45.6	1.0 09	173.8	25 15.7	1.0 09	173.9
7	26 40.1	99 10	172.7	26 10.2	99 10	172.7	25 40.4	99 10	172.8	25 10.5	99 10	172.9
8	26 34.0	99 11	171.6	26 04.2	99 11	171.7	25 34.4	99 11	171.8	25 04.6	99 11	171.8
9	26 27.1	99 13	170.6	25 57.4	99 13	170.7	25 27.6	99 13	170.8	24 57.9	99 12	170.8
10	26 19.5	99 14	169.6	25 49.8	99 14	169.7	25 20.1	99 14	169.7	24 50.4	99 14	169.8
1	26 11.0	99 15	168.6	25 41.4	99 15	168.6	25 11.7	99 15	168.7	24 42.1	99 15	168.8
2	26 01.8	99 17	167.5	25 32.2	99 17	167.6	25 02.6	99 16	167.7	24 33.1	99 16	167.8
3	25 51.7	98 18	166.5	25 22.3	98 18	166.6	24 52.8	98 18	166.7	24 23.3	98 18	166.8
4	25 40.9	98 19	165.5	25 11.5	98 19	165.6	24 42.1	98 19	165.7	24 12.7	98 19	165.8
15	25 29.4	98 21	164.5	25 00.1	98 20	164.6	24 30.8	98 20	164.7	24 01.4	98 20	164.8
6	25 17.1	97 22	163.5	24 47.9	97 22	163.6	24 18.6	97 21	163.7	23 49.4	97 21	163.8
7	25 04.0	97 23	162.5	24 34.9	97 23	162.6	24 05.7	97 23	162.7	23 36.6	97 22	162.9
8	24 50.2	97 24	161.5	24 21.2	97 24	161.6	23 52.1	97 24	161.7	23 23.1	97 24	161.9
9	24 35.7	96 25	160.5	24 06.7	96 25	160.6	23 37.8	96 25	160.8	23 08.9	96 25	160.9
20	24 20.4	96 27	159.5	23 51.6	96 26	159.6	23 22.8	96 26	159.8	22 53.9	96 26	159.9
1	24 04.4	96 28	158.5	23 35.7	96 28	158.7	23 07.0	96 27	158.8	22 38.3	96 27	159.0
2	23 47.7	95 29	157.5	23 19.1	95 29	157.7	22 50.5	95 29	157.9	22 21.9	95 28	158.0
3	23 30.3	95 30	156.6	23 01.9	95 30	156.7	22 33.4	95 30	156.9	22 04.9	95 30	157.1
4	23 12.3	94 31	155.6	22 43.9	94 31	155.8	22 15.6	95 31	156.0	21 47.2	95 31	156.1
25	22 53.5	94 32	154.6	22 25.3	94 32	154.8	21 57.1	94 32	155.0	21 28.8	94 32	155.2
6	22 34.1	94 34	153.7	22 06.0	94 33	153.9	21 37.9	94 33	154.1	21 09.8	94 33	154.3
7	22 14.0	93 35	152.8	21 46.0	93 34	152.9	21 18.1	93 34	153.1	20 50.1	93 34	153.3
8	21 53.2	93 36	151.8	21 25.4	93 35	152.0	20 57.6	93 35	152.2	20 29.8	93 35	152.4
9	21 31.9	92 37	150.9	21 04.2	92 36	151.1	20 36.5	92 36	151.3	20 08.8	92 36	151.5
55	9 06.0	79 58	129.2	8 42.3	79 58	129.6	8 18.7	79 57	129.9	7 55.0	79 57	130.2
6	8 31.2	78 58	128.5	8 07.8	78 58	128.8	7 44.3	78 58	129.1	7 20.8	78 58	129.4
7	7 56.2	78 59	127.8	7 32.8	78 59	128.1	7 09.5	78 59	128.4	6 46.2	78 58	128.7
8	7 20.7	77 60	127.0	6 57.6	77 59	127.4	6 34.4	77 59	127.7	6 11.2	77 59	128.0
9	6 45.0	77 60	126.3	6 22.0	77 60	126.6	5 58.9	77 60	127.0	5 35.9	77 59	127.3
60	6 08.9	76 61	125.6	5 46.0	76 60	125.9	5 23.1	76 60	126.2	5 00.2	76 60	126.6
1	5 32.5	76 61	124.9	5 09.7	76 61	125.2						
2												
3												
4												

ALTITUDE CORRECTION FOR D. R. LATITUDE

Az. °	Az. °	LATITUDE DIFFERENCE (minutes of arc) 1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	11′	12′	13′	14′	15′
0	180	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
1	179	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
2	178	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
3	177	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
4	176	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
5	175	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	13.9	14.9
6	174	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	9.9	10.9	11.9	12.9	13.9	14.9
7	173	1.0	2.0	3.0	4.0	5.0	6.0	6.9	7.9	8.9	9.9	10.9	11.9	12.9	13.9	14.9
8	172	1.0	2.0	3.0	4.0	5.0	5.9	6.9	7.9	8.9	9.9	10.9	11.9	12.9	13.9	14.9
9	171	1.0	2.0	3.0	4.0	4.9	5.9	6.9	7.9	8.9	9.9	10.9	11.9	12.8	13.8	14.8
10	170	1.0	2.0	3.0	3.9	4.9	5.9	6.9	7.9	8.9	9.8	10.8	11.8	12.8	13.8	14.8
11	169	1.0	2.0	2.9	3.9	4.9	5.9	6.9	7.9	8.8	9.8	10.8	11.8	12.8	13.7	14.7
12	168	1.0	2.0	2.9	3.9	4.9	5.9	6.8	7.8	8.8	9.8	10.8	11.7	12.7	13.7	14.7
13	167	1.0	1.9	2.9	3.9	4.9	5.8	6.8	7.8	8.8	9.7	10.7	11.7	12.6	13.6	14.6
14	166	1.0	1.9	2.9	3.9	4.9	5.8	6.8	7.8	8.7	9.7	10.7	11.6	12.6	13.6	14.6
15	165	1.0	1.9	2.9	3.9	4.8	5.8	6.8	7.7	8.7	9.7	10.6	11.6	12.6	13.5	14.5
16	164	1.0	1.9	2.9	3.8	4.8	5.8	6.7	7.7	8.7	9.6	10.6	11.5	12.5	13.5	14.4
17	163	1.0	1.9	2.9	3.8	4.8	5.7	6.7	7.7	8.6	9.6	10.5	11.5	12.4	13.4	14.3
18	162	1.0	1.9	2.9	3.8	4.8	5.7	6.7	7.6	8.6	9.5	10.5	11.4	12.4	13.3	14.3
19	161	0.9	1.9	2.8	3.8	4.7	5.7	6.6	7.6	8.5	9.5	10.4	11.3	12.3	13.2	14.2
20	160	0.9	1.9	2.8	3.8	4.7	5.6	6.6	7.5	8.5	9.4	10.3	11.3	12.2	13.2	14.1
21	159	0.9	1.9	2.8	3.7	4.7	5.6	6.5	7.5	8.4	9.3	10.3	11.2	12.1	13.1	14.0
22	158	0.9	1.9	2.8	3.7	4.6	5.6	6.5	7.4	8.3	9.3	10.2	11.1	12.1	13.0	13.9
23	157	0.9	1.8	2.8	3.7	4.6	5.5	6.4	7.4	8.3	9.2	10.1	11.0	12.0	12.9	13.8
24	156	0.9	1.8	2.7	3.7	4.6	5.5	6.4	7.3	8.2	9.1	10.0	11.0	11.9	12.8	13.7
25	155	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.3	8.2	9.1	10.0	10.9	11.8	12.7	13.6
26	154	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8	11.7	12.6	13.5
27	153	0.9	1.8	2.7	3.6	4.5	5.3	6.2	7.1	8.0	8.9	9.8	10.7	11.6	12.5	13.4
28	152	0.9	1.8	2.6	3.5	4.4	5.3	6.2	7.1	7.9	8.8	9.7	10.6	11.5	12.4	13.2
29	151	0.9	1.7	2.6	3.5	4.4	5.2	6.1	7.0	7.9	8.7	9.6	10.5	11.4	12.2	13.1
60	120	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5
61	119	0.5	1.0	1.5	1.9	2.4	2.9	3.4	3.9	4.4	4.8	5.3	5.8	6.3	6.8	7.3
62	118	0.5	0.9	1.4	1.9	2.3	2.8	3.3	3.8	4.2	4.7	5.2	5.6	6.1	6.6	7.0
63	117	0.5	0.9	1.4	1.8	2.3	2.7	3.2	3.6	4.1	4.5	5.0	5.4	5.9	6.4	6.8
64	116	0.4	0.9	1.3	1.8	2.2	2.6	3.1	3.5	3.9	4.4	4.8	5.3	5.7	6.1	6.6
75	105	0.3	0.5	0.8	1.0	1.3	1.6	1.8	2.1	2.3	2.6	2.8	3.1	3.4	3.6	3.9
76	104	0.2	0.5	0.7	1.0	1.2	1.5	1.7	1.9	2.2	2.4	2.7	2.9	3.1	3.4	3.6
77	103	0.2	0.4	0.7	0.9	1.1	1.3	1.6	1.8	2.0	2.2	2.5	2.7	2.9	3.1	3.4
78	102	0.2	0.4	0.6	0.8	1.0	1.2	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1
79	101	0.2	0.4	0.6	0.8	1.0	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9
80	100	0.2	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.7	1.9	2.1	2.3	2.4	2.6
81	99	0.2	0.3	0.5	0.6	0.8	0.9	1.1	1.3	1.4	1.6	1.7	1.9	2.0	2.2	2.3
82	98	0.1	0.3	0.4	0.6	0.7	0.8	1.0	1.1	1.3	1.4	1.5	1.7	1.8	1.9	2.1
83	97	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1	1.2	1.3	1.5	1.6	1.7	1.8
84	96	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.3	1.4	1.5	1.6
Az.		1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	11′	12′	13′	14′	15′

Az. °	Az. °	LAT. DIFF. (tenths of minutes of arc) 0.1′	0.2′	0.3′	0.4′	0.5′	0.6′	0.7′	0.8′	0.9′	Az. °	Az. °
0	180	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0	180
1	179										1	179
2	178										2	178
3	177										3	177
4	176										4	176
5	175	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	5	175
6	174										6	174
7	173										7	173
8	172										8	172
9	171										9	171
10	170	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	10	170
11	169										11	169
12	168										12	168
13	167										13	167
14	166										14	166
15	165	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	15	165
16	164										16	164
17	163										17	163
18	162										18	162
19	161									0.9	19	161
20	160	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.8	20	160
21	159							0.7	0.7		21	159
22	158							0.6			22	158
23	157						0.6				23	157
24	156						0.5				24	156
25	155	0.1	0.2	0.3	0.4	0.5	0.5	0.6	0.7	0.8	25	155
26	154					0.4					26	154
27	153										27	153
28	152				0.4						28	152
29	151				0.3						29	151
60	120	0.0	0.1	0.2	0.2	0.2	0.3	0.4	0.4	0.4	60	120
61	119			0.1				0.3			61	119
62	118										62	118
63	117										63	117
64	116								0.4		64	116
75	105	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	75	105
76	104		0.0				0.1				76	104
77	103							0.2			77	103
78	102							0.1			78	102
79	101								0.2		79	101
80	100	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.2	80	100
81	99			0.0						0.1	81	99
82	98				0.1						82	98
83	97				0.0						83	97
84	96					0.1					84	96
Az.		0.1′	0.2′	0.3′	0.4′	0.5′	0.6′	0.7′	0.8′	0.9′	Az.	

Azimuth angle greater than 90°:
If DR latitude is greater than selected tabulated latitude, ΔL correction is *minus;* but for DR latitude less than selected tabulated latitude, the correction is *plus.*

Azimuth angle less than 90°:
If DR latitude is greater than selected tabulated latitude, ΔL correction is *plus;* but for DR latitude less than selected tabulated latitude, the correction is *minus.*

ALTITUDE CORRECTION FOR D. R. LATITUDE

		LATITUDE DIFFERENCE (minutes of arc)																	LAT. DIFF. (tenths of minutes of arc)										
Az.		16′	17′	18′	19′	20′	21′	22′	23′	24′	25′	26′	27′	28′	29′	30′	Az.		0.1′	0.2′	0.3′	0.4′	0.5′	0.6′	0.7′	0.8′	0.9′	Az.	
°	°																°	°										°	°
0	180	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	25.0	26.0	27.0	28.0	29.0	30.0	0	180	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0	180
1	179	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	25.0	26.0	27.0	28.0	29.0	30.0	1	179										1	179
2	178	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	25.0	26.0	27.0	28.0	29.0	30.0	2	178										2	178
3	177	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	25.0	26.0	27.0	28.0	29.0	30.0	3	177										3	177
4	176	16.0	17.0	18.0	19.0	20.0	20.9	21.9	22.9	23.9	24.9	25.9	26.9	27.9	28.9	29.9	4	176										4	176
5	175	15.9	16.9	17.9	18.9	19.9	20.9	21.9	22.9	23.9	24.9	25.9	26.9	27.9	28.9	29.9	5	175	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	5	175
6	174	15.9	16.9	17.9	18.9	19.9	20.9	21.9	22.9	23.9	24.9	25.9	26.9	27.8	28.8	29.8	6	174										6	174
7	173	15.9	16.9	17.9	18.9	19.9	20.8	21.8	22.8	23.8	24.8	25.8	26.8	27.8	28.8	29.8	7	173										7	173
8	172	15.8	16.8	17.8	18.8	19.8	20.8	21.8	22.8	23.8	24.8	25.7	26.7	27.7	28.7	29.7	8	172										8	172
9	171	15.8	16.8	17.8	18.8	19.8	20.7	21.7	22.7	23.7	24.7	25.7	26.7	27.7	28.6	29.6	9	171										9	171
10	170	15.8	16.7	17.7	18.7	19.7	20.7	21.7	22.7	23.6	24.6	25.6	26.6	27.6	28.6	29.5	10	170	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	10	170
11	169	15.7	16.7	17.7	18.7	19.6	20.6	21.6	22.6	23.6	24.5	25.5	26.5	27.5	28.5	29.4	11	169										11	169
12	168	15.7	16.6	17.6	18.6	19.6	20.5	21.5	22.5	23.5	24.5	25.4	26.4	27.4	28.4	29.3	12	168										12	168
13	167	15.6	16.6	17.5	18.5	19.5	20.5	21.4	22.4	23.4	24.4	25.3	26.3	27.3	28.3	29.2	13	167										13	167
14	166	15.5	16.5	17.5	18.4	19.4	20.4	21.3	22.3	23.3	24.3	25.2	26.2	27.2	28.1	29.1	14	166										14	166
15	165	15.5	16.4	17.4	18.4	19.3	20.3	21.3	22.2	23.2	24.1	25.1	26.1	27.0	28.0	29.0	15	165	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	15	165
16	164	15.4	16.3	17.3	18.3	19.2	20.2	21.1	22.1	23.1	24.0	25.0	26.0	26.9	27.9	28.8	16	164										16	164
17	163	15.3	16.3	17.2	18.2	19.1	20.1	21.0	22.0	23.0	23.9	24.9	25.8	26.8	27.7	28.7	17	163										17	163
18	162	15.2	16.2	17.1	18.1	19.0	20.0	20.9	21.9	22.8	23.8	24.7	25.7	26.6	27.6	28.5	18	162										18	162
19	161	15.1	16.1	17.0	18.0	18.9	19.9	20.8	21.7	22.7	23.6	24.6	25.5	26.5	27.4	28.4	19	161									0.9	19	161
20	160	15.0	16.0	16.9	17.9	18.8	19.7	20.7	21.6	22.6	23.5	24.4	25.4	26.3	27.3	28.2	20	160	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.8	20	160
21	159	14.9	15.9	16.8	17.7	18.7	19.6	20.5	21.5	22.4	23.3	24.3	25.2	26.1	27.1	28.0	21	159							0.7	0.7		21	159
22	158	14.8	15.8	16.7	17.6	18.5	19.5	20.4	21.3	22.3	23.2	24.1	25.0	26.0	26.9	27.8	22	158							0.6			22	158
23	157	14.7	15.6	16.6	17.5	18.4	19.3	20.3	21.2	22.1	23.0	23.9	24.9	25.8	26.7	27.6	23	157						0.6				23	157
24	156	14.6	15.5	16.4	17.4	18.3	19.2	20.1	21.0	21.9	22.8	23.8	24.7	25.6	26.5	27.4	24	156						0.5				24	156
25	155	14.5	15.4	16.3	17.2	18.1	19.0	19.9	20.8	21.8	22.7	23.6	24.5	25.4	26.3	27.2	25	155	0.1	0.2	0.3	0.4	0.5	0.5	0.6	0.7	0.8	25	155
26	154	14.4	15.3	16.2	17.1	18.0	18.9	19.8	20.7	21.6	22.5	23.4	24.3	25.2	26.1	27.0	26	154					0.4					26	154
27	153	14.3	15.1	16.0	16.9	17.8	18.7	19.6	20.5	21.4	22.3	23.2	24.1	24.9	25.8	26.7	27	153										27	153
28	152	14.1	15.0	15.9	16.8	17.7	18.5	19.4	20.3	21.2	22.1	23.0	23.8	24.7	25.6	26.5	28	152				0.4						28	152
29	151	14.0	14.9	15.7	16.6	17.5	18.4	19.2	20.1	21.0	21.9	22.7	23.6	24.5	25.4	26.2	29	151				0.3						29	151
60	120	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	60	120	0.0	0.1	0.2	0.2	0.2	0.3	0.4	0.4	0.4	60	120
61	119	7.8	8.2	8.7	9.2	9.7	10.2	10.7	11.2	11.6	12.1	12.6	13.1	13.6	14.1	14.5	61	119			0.1				0.3			61	119
62	118	7.5	8.0	8.5	8.9	9.4	9.9	10.3	10.8	11.3	11.7	12.2	12.7	13.1	13.6	14.1	62	118										62	118
63	117	7.3	7.7	8.2	8.6	9.1	9.5	10.0	10.4	10.9	11.3	11.8	12.3	12.7	13.2	13.6	63	117										63	117
64	116	7.0	7.5	7.9	8.3	8.8	9.2	9.6	10.1	10.5	11.0	11.4	11.8	12.3	12.7	13.2	64	116								0.4		64	116
75	105	4.1	4.4	4.7	4.9	5.2	5.4	5.7	6.0	6.2	6.5	6.7	7.0	7.2	7.5	7.8	75	105	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	75	105
76	104	3.9	4.1	4.4	4.6	4.8	5.1	5.3	5.6	5.8	6.0	6.3	6.5	6.8	7.0	7.3	76	104		0.0				0.1				76	104
77	103	3.6	3.8	4.0	4.3	4.5	4.7	5.0	5.2	5.4	5.6	5.8	6.1	6.3	6.5	6.7	77	103							0.2			77	103
78	102	3.3	3.5	3.7	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	78	102							0.1			78	102
79	101	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.3	5.5	5.7	79	101								0.2		79	101
80	100	2.8	3.0	3.1	3.3	3.5	3.6	3.8	4.0	4.2	4.3	4.5	4.7	4.9	5.0	5.2	80	100	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.2	80	100
81	99	2.5	2.7	2.8	3.0	3.1	3.3	3.4	3.6	3.8	3.9	4.1	4.2	4.4	4.5	4.7	81	99			0.0						0.1	81	99
82	98	2.2	2.4	2.5	2.6	2.8	2.9	3.1	3.2	3.3	3.5	3.6	3.8	3.9	4.0	4.2	82	98				0.1						82	98
83	97	1.9	2.1	2.2	2.3	2.4	2.6	2.7	2.8	2.9	3.0	3.2	3.3	3.4	3.5	3.7	83	97				0.0						83	97
84	96	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	84	96					0.1					84	96
Az.		16′	17′	18′	19′	20′	21′	22′	23′	24′	25′	26′	27′	28′	29′	30′	Az.		0.1′	0.2′	0.3′	0.4′	0.5′	0.6′	0.7′	0.8′	0.9′	Az.	

Azimuth angle greater than 90°:
If DR latitude is greater than selected tabulated latitude, ΔL correction is *minus;* but for DR latitude less than selected tabulated latitude, the correction is *plus.*
Azimuth angle less than 90°:
If DR latitude is greater than selected tabulated latitude, ΔL correction is *plus;* but for DR latitude less than selected tabulated latitude, the correction is *minus.*

MULTIPLICATION TABLE

Δd or Δt

	DEC. DIFF. OR H. A. DIFF. (minutes of arc)																DEC. DIFF. OR H. A. DIFF. (tenths of minutes)									
Δ	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	11′	12′	13′	14′	15′	Δ	0.1′	0.2′	0.3′	0.4′	0.5′	0.6′	0.7′	0.8′	0.9′	Δ
01	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	01
2	.0	.0	.1	.1	.1	.1	.1	.2	.2	.2	.2	.2	.3	.3	.3	2										2
3	.0	.1	.1	.1	.2	.2	.2	.2	.3	.3	.3	.4	.4	.4	.5	3										3
4	.0	.1	.1	.2	.2	.2	.3	.3	.4	.4	.4	.5	.5	.6	.6	4										4
25	0.3	0.5	0.8	1.0	1.3	1.5	1.8	2.0	2.3	2.5	2.8	3.0	3.3	3.5	3.8	25	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	25
6	.3	.5	.8	1.0	1.3	1.6	1.8	2.1	2.3	2.6	2.9	3.1	3.4	3.6	3.9	6										6
7	.3	.5	.8	1.1	1.4	1.6	1.9	2.2	2.4	2.7	3.0	3.2	3.5	3.8	4.1	7									0.2	7
8	.3	.6	.8	1.1	1.4	1.7	2.0	2.2	2.5	2.8	3.1	3.4	3.6	3.9	4.2	8									0.3	8
9	.3	.6	.9	1.2	1.5	1.7	2.0	2.3	2.6	2.9	3.2	3.5	3.8	4.1	4.4	9					0.1					9
30	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	30	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.3	30
1	.3	.6	.9	1.2	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4.0	4.3	4.7	1								0.2		1
2	.3	.6	1.0	1.3	1.6	1.9	2.2	2.6	2.9	3.2	3.5	3.8	4.2	4.5	4.8	2								0.3		2
3	.3	.7	1.0	1.3	1.7	2.0	2.3	2.6	3.0	3.3	3.6	4.0	4.3	4.6	5.0	3										3
4	.3	.7	1.0	1.4	1.7	2.0	2.4	2.7	3.1	3.4	3.7	4.1	4.4	4.8	5.1	4										4
60	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4	6.0	6.6	7.2	7.8	8.4	9.0	60	0.1	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.5	60
1	.6	1.2	1.8	2.4	3.1	3.7	4.3	4.9	5.5	6.1	6.7	7.3	7.9	8.5	9.2	1									0.5	1
2	.6	1.2	1.9	2.5	3.1	3.7	4.3	5.0	5.6	6.2	6.8	7.4	8.1	8.7	9.3	2				0.2					0.6	2
3	.6	1.3	1.9	2.5	3.2	3.8	4.4	5.0	5.7	6.3	6.9	7.6	8.2	8.8	9.5	3				0.3						3
4	.6	1.3	1.9	2.6	3.2	3.8	4.5	5.1	5.8	6.4	7.0	7.7	8.3	9.0	9.6	4							0.4			4
65	0.7	1.3	2.0	2.6	3.3	3.9	4.6	5.2	5.9	6.5	7.2	7.8	8.5	9.1	9.8	65	0.1	0.1	0.2	0.3	0.3	0.4	0.5	0.5	0.6	65
6	.7	1.3	2.0	2.6	3.3	4.0	4.6	5.3	5.9	6.6	7.3	7.9	8.6	9.2	9.9	6										6
7	.7	1.3	2.0	2.7	3.4	4.0	4.7	5.4	6.0	6.7	7.4	8.0	8.7	9.4	10.1	7										7
8	.7	1.4	2.0	2.7	3.4	4.1	4.8	5.4	6.1	6.8	7.5	8.2	8.8	9.5	10.2	8								0.5		8
9	.7	1.4	2.1	2.8	3.5	4.1	4.8	5.5	6.2	6.9	7.6	8.3	9.0	9.7	10.4	9					0.3			0.6		9
70	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	7.0	7.7	8.4	9.1	9.8	10.5	70	0.1	0.1	0.2	0.3	0.4	0.4	0.5	0.6	0.6	70
1	.7	1.4	2.1	2.8	3.6	4.3	5.0	5.7	6.4	7.1	7.8	8.5	9.2	9.9	10.7	1										1
2	.7	1.4	2.2	2.9	3.6	4.3	5.0	5.8	6.5	7.2	7.9	8.6	9.4	10.1	10.8	2									0.6	2
3	.7	1.5	2.2	2.9	3.7	4.4	5.1	5.8	6.6	7.3	8.0	8.8	9.5	10.2	11.0	3									0.7	3
4	.7	1.5	2.2	3.0	3.7	4.4	5.2	5.9	6.7	7.4	8.1	8.9	9.6	10.4	11.1	4		0.1				0.4				4
75	0.8	1.5	2.3	3.0	3.8	4.5	5.3	6.0	6.8	7.5	8.3	9.0	9.8	10.5	11.3	75	0.1	0.2	0.2	0.3	0.4	0.5	0.5	0.6	0.7	75
6	.8	1.5	2.3	3.0	3.8	4.6	5.3	6.1	6.8	7.6	8.4	9.1	9.9	10.6	11.4	6										6
7	.8	1.5	2.3	3.1	3.9	4.6	5.4	6.2	6.9	7.7	8.5	9.2	10.0	10.8	11.6	7										7
8	.8	1.6	2.3	3.1	3.9	4.7	5.5	6.2	7.0	7.8	8.6	9.4	10.1	10.9	11.7	8							0.5			8
9	.8	1.6	2.4	3.2	4.0	4.7	5.5	6.3	7.1	7.9	8.7	9.5	10.3	11.1	11.9	9							0.6			9
90	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8	11.7	12.6	13.5	90	0.1	0.2	0.3	0.4	0.5	0.5	0.6	0.7	0.8	90
1	.9	1.8	2.7	3.6	4.6	5.5	6.4	7.3	8.2	9.1	10.0	10.9	11.8	12.7	13.7	1						0.5				1
2	.9	1.8	2.8	3.7	4.6	5.5	6.4	7.4	8.3	9.2	10.1	11.0	12.0	12.9	13.8	2						0.6	0.6			2
3	.9	1.9	2.8	3.7	4.7	5.6	6.5	7.4	8.4	9.3	10.2	11.2	12.1	13.0	14.0	3							0.7	0.7		3
4	.9	1.9	2.8	3.8	4.7	5.6	6.6	7.5	8.5	9.4	10.3	11.3	12.2	13.2	14.1	4								0.8	0.8	4
95	1.0	1.9	2.9	3.8	4.8	5.7	6.7	7.6	8.6	9.5	10.5	11.4	12.4	13.3	14.3	95	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	95
6	1.0	1.9	2.9	3.8	4.8	5.8	6.7	7.7	8.6	9.6	10.6	11.5	12.5	13.4	14.4	6										6
7	1.0	1.9	2.9	3.9	4.9	5.8	6.8	7.8	8.7	9.7	10.7	11.6	12.6	13.6	14.6	7										7
8	1.0	2.0	2.9	3.9	4.9	5.9	6.9	7.8	8.8	9.8	10.8	11.8	12.7	13.7	14.7	8										8
99	1.0	2.0	3.0	4.0	5.0	5.9	6.9	7.9	8.9	9.9	10.9	11.9	12.9	13.9	14.9	99										99
Δ	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	11′	12′	13′	14′	15′	Δ	0.1′	0.2′	0.3′	0.4′	0.5′	0.6′	0.7′	0.8′	0.9′	Δ

Δd or Δt

APPENDIX BB
EXTRACTS FROM H.O. PUB. NO. 221 (VOLUME ONE)

T	1H 2-2200		1H 2-2220		1H 2-2240		1H 2-2260		1H 2-2280		T
Lat ° ′	Lo ° ′	Δ	Lo ° ′	Δ	Lo ° ′	Δ	Lo ° ′	Δ	Lo ° ′	Δ	Long ° ′
51 N	70 55.2W	-82	70 39.2W	-79	70 23.7W	-76	70 08.6W	-74	69 53.9W	-73	
50	69 37.1	66	69 24.0	65	69 11.2	63	68 58.8	62	68 46.6	60	
49 N	68 23.5W	-54	68 12.9W	-52	68 02.6W	-51	67 52.6W	-50	67 42.7W	-49	
48	67 13.8	42	67 05.6	40	66 57.6	40	66 49.7	39	66 42.0	38	
47	66 07.9	31	66 01.8	30	65 55.9	30	65 50.0	29	65 44.3	28	
46 30	65 36.5	26	65 31.3	26	65 26.3	25	65 21.3	25	65 16.4	24	
46	65 06.1	22	65 01.8	21	64 57.6	21	64 53.4	21	64 49.3	20	
45 30N	64 36.9W	-18	64 33.3W	-18	64 29.8W	-17	64 26.4W	-17	64 23.0W	-17	
45	64 09.2	15	64 06.2	15	64 03.2	15	64 00.3	14	63 57.4	14	
44 30	63 43.3	13	63 40.7	13	63 38.0	13	63 35.4	13	63 32.8	13	
44	63 19.5	13	63 16.9	13	63 14.3	13	63 11.7	13	63 09.1	13	
43 30	62 57.6	14	62 54.8	14	62 51.9	14	62 49.1	14	62 46.2	14	
43 N	62 37.1W	-16	62 33.9W	-16	62 30.7W	-16	62 27.4W	-16	62 24.2W	-16	
42 30	62 17.6	18	62 13.9	18	62 10.2	18	62 06.5	19	62 02.7	19	
42	61 58.9	21	61 54.7	21	61 50.4	22	61 46.1	22	61 41.7	22	
41	61 23.0	27	61 17.7	27	61 12.2	28	61 06.7	28	61 01.1	28	
40	60 48.8	33	60 42.2	33	60 35.6	34	60 28.8	34	60 22.0	34	
	Lo	Δ	Lo	Δ	Lo	Δ	Lo	Δ	Lo	Δ	
T	1H 2-2200		1H 2-2220		1H 2-2240		1H 2-2260		1H 2-2280		T

SH

SKY WAVE CORRECTION

	LONGITUDE WEST															
	74	73	72	71	70	69	68	67	66	65	64	63	62	61	60	
46	+36	41	+46	+50	+52	+52	+49	+45	41	35	+31	27	23	20	16	46
45	+46	+56							+53	+45	+38	32	27	22	19	45
			DO NOT USE									ADD				
44	+59									+56	+46	+37	+30	+25	+21	44
43			SKY WAVES								+52	41	33	26	22	43
42											+56	43	34	27	22	42
41											+55	42	33	26	21	41
40	+46		IN THIS REGION								+49	38	30	24	20	40
39	+4	+35								+51	+40	+32	+26	+21	+18	39
38	-50	-20	+12	+38	+55		+59	+52	+44	37	+31	26	21	18	15	38
37	DO		-36	-7	+13	+24	29	30	27	25	+22	19	16	14	12	37
36				-36	-13	+1	+9	14	15	15	+15	13	12	11	10	36
35	NOT			-51	27	-12	-3	+3	6	8	+9	9	9	8	7	35
	USE															
34				-53	-33	-19	-10	-4	+1	+3	+4	+5	+6	+5	+5	34
33	HERE			-47	32	-21	13	7	-3	-1	+1	+2	3	3	3	33
32			-53	40	29	-20	14	9	6	3	-1	0	+1	2	2	32
31	-57	-49	41	32	25	-19	14	10	7	4	-2	-1	0	+1	+1	31
30	-41	36	31	26	21	-16	13	9	7	4	-3	2	-1	0	0	30
			SUBTRACT						SUBTRACT							
29	-30	-28	-24	-21	-17	-14	-11	-9	-6	-5	-3	-2	-1	-1	0	29
28	-23	21	19	17	14	-12	10	7	6	4	-3	2	1	-1	0	28
	74	73	72	71	70	69	68	67	66	65	64	63	62	61	60	
	LONGITUDE WEST															

SH

T	1H 4-6300		1H 4-6320		1H 4-6340		1H 4-6360		1H 4-6380		T
Lat ° ′	Lo ° ′	Δ	Lo ° ′	Δ	Lo ° ′	Δ	Lo ° ′	Δ	Lo ° ′	Δ	Long °
44 N	70 18.3W	-42	70 09.7W	-45	70 00.5W	-47	69 50.8W	-51	69 40.2W	-56	
43 30	70 25.6	37	70 18.1	39	70 10.1	41	70 01.6	44	69 52.4	48	
43	70 31.7	31	70 25.3	33	70 18.5	35	70 11.3	38	70 03.5	41	
42 30	70 36.0	26	70 30.7	27	70 25.2	29	70 19.2	31	70 12.9	33	
42	70 37.4	20	70 33.3	21	70 29.0	22	70 24.5	23	70 19.7	25	
	Lo	Δ	Lo	Δ	Lo	Δ	Lo	Δ	Lo	Δ	
	L ° ′	Δ	L ° ′	Δ	L ° ′	Δ	L ° ′	Δ	L ° ′	Δ	
	42 23.8N	+46	42 33.2N	+49	42 43.4N	+53	42 54.4N	+58	43 06.4N	+64	64 W
	42 43.1	51	42 53.6	55	43 04.9	59	43 17.2	64	43 30.6	71	63
	43 02.0	56	43 13.6	60	43 26.0	65	43 39.5	71	43 54.3	78	62
	43 20.5	61	43 33.1	65	43 46.6	71	44 01.3	77	44 17.3	85	61
	43 38.5	66	43 52.1	71	44 06.7	76	44 22.5	83	44 39.8	91	60
	43 55.9N	+71	44 10.5N	+76	44 26.1N	+81	44 43.0N	+89	45 01.5N	+98	59 W
	44 12.8	75	44 28.3	81	44 45.0	87	45 02.9	94	45 22.6	104	58
	44 29.1	80	44 45.6	85	45 03.2	92	45 22.2	100	45 42.9	109	57
	44 44.8	84	45 02.2	90	45 20.7	96	45 40.7	105	46 02.5	115	56
	45 00.0N	+89	45 18.2N	+94	45 37.6N	+101	45 58.6N	+110	46 21.5N	+121	55 W
	L	Δ	L	Δ	L	Δ	L	Δ	L	Δ	
T	1H 4-6300		1H 4-6320		1H 4-6340		1H 4-6360		1H 4-6380		T

EXTRACTS FROM H.O. PUB. NO. 249

LAT 41°S

LHA ♈	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn
	ALDEBARAN		SIRIUS		CANOPUS		ACHERNAR		FOMALHAUT		Hamal	
45	28 36	026	36 27	080	54 12	127	68 48	212	39 58	262	24 28	346
46	28 56	025	37 11	079	54 49	127	68 23	213	39 13	262	24 17	345
47	29 14	024	37 56	079	55 25	127	67 58	214	38 29	261	24 05	344
48	29 32	023	38 40	078	56 02	127	67 33	215	37 44	261	23 52	343
49	29 49	022	39 24	077	56 38	127	67 06	216	36 59	260	23 38	342
50	30 05	020	40 08	076	57 14	127	66 40	216	36 15	260	23 24	341
51	30 21	019	40 52	076	57 51	127	66 13	217	35 30	259	23 09	340
52	30 35	018	41 36	075	58 27	127	65 45	218	34 46	259	22 54	339
53	30 49	017	42 19	074	59 03	127	65 17	219	34 01	258	22 37	338
54	31 02	016	43 03	073	59 39	127	64 49	219	33 17	257	22 20	337
55	31 14	015	43 46	072	60 16	127	64 20	220	32 33	257	22 02	336
56	31 26	014	44 29	071	60 52	127	63 51	220	31 49	256	21 44	336
57	31 36	013	45 12	070	61 28	127	63 21	221	31 05	256	21 25	335
58	31 46	012	45 54	069	62 04	128	62 52	221	30 21	255	21 05	334
59	31 55	011	46 37	068	62 40	128	62 22	222	29 37	255	20 45	333
	ALDEBARAN		SIRIUS		CANOPUS		RIGIL KENT.		ACHERNAR		FOMALHAUT	
60	32 03	010	47 19	068	63 15	128	13 05	170	61 51	222	28 54	254
61	32 10	008	48 00	067	63 51	128	13 13	169	61 21	222	28 10	254
62	32 16	007	48 42	066	64 26	129	13 22	169	60 50	223	27 27	253
63	32 21	006	49 23	065	65 02	129	13 31	168	60 20	223	26 44	253
64	32 25	005	50 03	063	65 37	129	13 40	168	59 49	223	26 01	252
65	32 29	004	50 44	062	66 12	130	13 50	167	59 17	224	25 18	251
66	32 32	003	51 24	061	66 46	130	14 00	167	58 46	224	24 35	251
67	32 33	002	52 03	060	67 21	131	14 11	166	58 14	224	23 52	250
68	32 34	000	52 42	059	67 55	131	14 21	166	57 43	224	23 09	250
69	32 34	359	53 21	058	68 29	132	14 33	165	57 11	225	22 27	249
70	32 33	358	53 59	057	69 02	133	14 44	165	56 39	225	21 45	249
71	32 31	357	54 37	055	69 35	133	14 56	165	56 07	225	21 03	248
72	32 28	356	55 14	054	70 08	134	15 08	164	55 35	225	20 21	248
73	32 25	355	55 50	053	70 41	135	15 21	164	55 03	225	19 39	247
74	32 20	354	56 26	051	71 12	136	15 34	163	54 31	225	18 57	247
	SIRIUS		Suhail		RIGIL KENT.		ACHERNAR		Diphda		ALDEBARAN	
75	57 01	050	45 18	114	15 47	163	53 59	225	30 50	272	32 15	353
76	57 35	049	45 59	114	16 01	162	53 27	225	30 04	271	32 08	351
77	58 09	047	46 41	114	16 15	162	52 55	225	29 19	271	32 01	350
78	58 42	046	47 22	113	16 29	161	52 22	225	28 34	270	31 53	349
79	59 14	044	48 04	113	16 44	161	51 50	225	27 48	269	31 44	348
80	59 45	043	48 46	113	16 59	160	51 18	225	27 03	269	31 34	347
81	60 15	041	49 27	112	17 14	160	50 46	225	26 18	268	31 24	346
82	60 44	039	50 09	112	17 30	160	50 13	225	25 33	268	31 12	345
83	61 12	038	50 51	112	17 46	159	49 41	225	24 47	267	31 00	344
84	61 39	036	51 33	111	18 02	159	49 09	225	24 02	266	30 47	343
85	62 05	034	52 16	111	18 19	158	48 37	225	23 17	266	30 33	341
86	62 30	032	52 58	111	18 36	158	48 04	225	22 32	265	30 18	340
87	62 53	030	53 40	111	18 53	157	47 32	225	21 47	264	30 02	339
88	63 15	028	54 23	110	19 10	157	47 00	225	21 02	264	29 46	338
89	63 36	026	55 05	110	19 28	157	46 28	225	20 17	263	29 29	337

LAT 41°S

LHA ♈	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn
	REGULUS		SPICA		RIGIL KENT.		CANOPUS		SIRIUS		PROCYON	
135	34 41	020	25 23	082	37 32	142	61 11	233	51 51	299	39 57	333
136	34 55	019	26 08	082	38 00	142	60 35	233	51 11	298	39 35	331
137	35 10	017	26 53	081	38 28	142	59 59	233	50 31	297	39 13	330
138	35 23	016	27 38	080	38 56	142	59 23	233	49 51	296	38 50	329
139	35 35	015	28 22	079	39 24	141	58 46	233	49 10	295	38 27	328
140	35 46	014	29 07	079	39 53	141	58 10	233	48 29	294	38 02	327
141	35 57	013	29 51	078	40 21	141	57 34	233	47 48	293	37 37	326
142	36 06	012	30 35	077	40 50	141	56 57	233	47 06	292	37 11	324
143	36 15	010	31 19	076	41 18	141	56 21	233	46 24	291	36 45	323
144	36 23	009	32 03	076	41 47	141	55 45	233	45 41	290	36 17	322
145	36 29	008	32 47	075	42 16	140	55 08	233	44 59	289	35 49	321
146	36 35	007	33 31	074	42 45	140	54 32	233	44 16	288	35 20	320
147	36 40	006	34 14	073	43 14	140	53 55	233	43 33	288	34 51	319
148	36 44	004	34 57	073	43 43	140	53 19	233	42 50	287	34 21	318
149	36 47	003	35 40	072	44 12	140	52 43	233	42 06	286	33 50	317
	REGULUS		SPICA		RIGIL KENT.		CANOPUS		SIRIUS		PROCYON	
150	36 49	002	36 23	071	44 41	140	52 06	233	41 22	285	33 19	316
151	36 50	001	37 06	070	45 10	140	51 30	233	40 39	284	32 47	315
152	36 50	359	37 48	069	45 39	140	50 54	233	39 55	283	32 15	314
153	36 49	358	38 31	068	46 08	140	50 18	233	39 11	283	31 42	313
154	36 47	357	39 13	067	46 38	140	49 42	233	38 26	282	31 09	312
155	36 44	356	39 54	066	47 07	140	49 06	233	37 42	281	30 35	311
156	36 41	355	40 36	066	47 36	140	48 30	233	36 57	280	30 01	310
157	36 36	353	41 17	065	48 06	140	47 54	232	36 13	280	29 26	309
158	36 30	352	41 57	064	48 35	140	47 18	232	35 28	279	28 50	308
159	36 24	351	42 38	063	49 05	140	46 42	232	34 43	278	28 15	307
160	36 16	350	43 18	062	49 34	140	46 06	232	33 58	277	27 38	306
161	36 07	349	43 58	061	50 03	140	45 31	232	33 13	277	27 02	306
162	35 58	347	44 37	060	50 33	140	44 55	232	32 28	276	26 25	305
163	35 48	346	45 16	059	51 02	140	44 20	231	31 43	275	25 47	304
164	35 36	345	45 54	058	51 31	140	43 44	231	30 58	275	25 09	303
	SPICA		ANTARES		RIGIL KENT.		CANOPUS		SIRIUS		REGULUS	
165	46 32	057	22 51	106	52 01	140	43 09	231	30 13	274	35 24	344
166	47 10	055	23 35	105	52 30	140	42 34	231	29 28	273	35 11	343
167	47 47	054	24 18	105	52 59	140	41 59	231	28 43	272	34 57	341
168	48 23	053	25 02	104	53 28	140	41 24	230	27 57	272	34 42	340
169	48 59	052	25 46	103	53 57	140	40 49	230	27 12	271	34 27	339
170	49 35	051	26 30	103	54 26	140	40 15	230	26 27	270	34 10	338
171	50 10	050	27 15	102	54 55	141	39 40	230	25 42	270	33 53	337
172	50 44	048	27 59	102	55 24	141	39 06	229	24 56	269	33 35	336
173	51 17	047	28 43	101	55 52	141	38 31	229	24 11	269	33 16	335
174	51 50	046	29 28	101	56 21	141	37 57	229	23 26	268	32 56	334
175	52 22	044	30 12	100	56 49	141	37 23	229	22 41	267	32 36	333
176	52 53	043	30 57	099	57 17	142	36 49	228	21 55	267	32 14	331
177	53 24	042	31 42	099	57 45	142	36 16	228	21 10	266	31 52	330
178	53 53	040	32 26	098	58 13	142	35 42	228	20 25	265	31 29	329
179	54 22	039	33 11	098	58 41	143	35 09	227	19 40	265	31 06	328

DECLINATION (15°-29°) SAME NAME AS LATITUDE LAT 41°

LHA	23° Hc	d	Z	24° Hc	d	Z	25° Hc	d	Z	26° Hc	d	Z	27° Hc	d	Z	28° Hc	d	Z	29° Hc	d	Z	LHA
	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	
110	01 04	+43	60	01 47	+42	59	02 29	+42	58	03 11	+42	58	03 53	+43	57	04 36	+42	56	05 18	+42	56	250
111	00 25	43	59	01 08	43	59	01 51	42	58	02 33	43	57	03 16	42	56	03 58	43	56	04 41	42	55	249
112	−0 13	42	59	00 29	43	58	01 12	43	57	01 55	43	56	02 38	43	56	03 21	43	55	04 04	42	54	248
113	−0 52	43	58	−0 09	43	57	00 34	44	57	01 18	43	56	02 01	43	55	02 44	43	54	03 27	43	54	247
114	−1 30	43	57	−0 47	44	57	−0 03	43	56	00 40	44	55	01 24	43	55	02 07	44	54	02 51	43	53	246
115	−2 08	+44	57	−1 24	+43	56	−0 41	+44	55	00 03	+44	55	00 47	+44	54	01 31	+43	53	02 14	+44	53	245
116	−2 46	44	56	−2 02	44	55	−1 18	44	55	−0 34	45	54	00 11	44	53	00 55	44	53	01 39	44	52	244
117	−3 23	44	55	−2 39	45	55	−1 54	44	54	−1 10	44	53	−0 26	45	53	00 19	44	52	01 03	45	51	243
118	−4 00	44	55	−3 16	45	54	−2 31	45	53	−1 46	45	53	−1 01	44	52	−0 17	45	51	00 28	45	51	242
119	−4 37	45	54	−3 52	45	53	−3 07	45	53	−2 22	45	52	−1 37	45	51	−0 52	45	51	−0 07	45	50	241
120	−5 13	+45	53	−4 28	+45	53	−3 43	+46	52	−2 57	+45	51	−2 12	+45	51	−1 27	+46	50	−0 41	+45	49	240
121	−5 49	45	52	−5 04	46	52	−4 18	46	51	−3 32	45	51	−2 47	46	50	−2 01	46	49	−1 15	45	49	239
			122	−5 39	46	51	−4 53	46	50	−4 07	46	50	−3 21	46	49	−2 35	46	49	−1 49	46	48	238
						123	−5 28	46	50	−4 42	47	49	−3 55	46	48	−3 09	46	48	−2 23	47	47	237
						124	−6 02	46	49	−5 16	47	48	−4 29	47	48	−3 42	46	47	−2 56	47	47	236
									125	−5 49	+47	48	−5 02	+47	47	−4 15	+47	46	−3 28	+47	46	235
												126	−5 35	47	46	−4 48	47	46	−4 01	48	45	234
															127	−5 20	47	45	−4 33	48	44	233
															128	−5 52	48	44	−5 04	48	44	232
																		129	−5 35	48	43	231

LHA	23° Hc	d	Z	24° Hc	d	Z	25° Hc	d	Z	26° Hc	d	Z	27° Hc	d	Z	28° Hc	d	Z	29° Hc	d	Z	LHA
78	282																					
77	−5 45	40	116	283																		
76	−5 04	41	116	−5 45	40	117	284															
75	−4 23	−41	117	−5 04	−41	118	−5 45	−41	118	285												
74	−3 43	41	118	−4 24	41	118	−5 05	42	119	−5 47	41	120	286									
73	−3 03	42	118	−3 45	41	119	−4 26	41	120	−5 07	42	120	−5 49	41	121	287						
72	−2 23	42	119	−3 05	42	120	−3 47	41	120	−4 28	42	121	−5 10	42	122	−5 52	41	122	288			
71	−1 44	42	119	−2 26	42	120	−3 08	42	121	−3 50	42	122	−4 32	42	122	−5 14	41	123	−5 55	42	124	289
70	−1 04	−43	120	−1 47	−42	121	−2 29	−42	122	−3 11	−42	122	−3 53	−43	123	−4 36	−42	124	−5 18	−42	124	290
34	18 38	53	147	17 45	54	148	16 51	54	148	15 57	54	149	15 03	54	149	14 09	54	149	13 15	54	150	326
33	19 03	54	148	18 09	55	148	17 14	54	149	16 20	54	149	15 26	54	150	14 32	55	150	13 37	54	151	327
32	19 26	54	149	18 32	54	149	17 38	55	150	16 43	54	150	15 49	55	151	14 54	55	151	13 59	54	152	328
31	19 50	55	150	18 55	55	150	18 00	55	151	17 05	54	151	16 11	55	152	15 16	55	152	14 21	55	152	329
30	20 12	−55	151	19 17	−55	151	18 22	−55	152	17 27	−55	152	16 32	−55	152	15 37	−56	153	14 41	−55	153	330
29	20 34	55	152	19 39	56	152	18 43	55	152	17 48	55	153	16 53	56	153	15 57	55	154	15 02	56	154	331
28	20 55	55	152	20 00	56	153	19 04	56	153	18 08	55	154	17 13	56	154	16 17	56	154	15 21	56	155	332
27	21 16	56	153	20 20	56	154	19 24	56	154	18 28	56	155	17 32	56	155	16 36	56	155	15 40	56	156	333
26	21 36	56	154	20 40	56	155	19 44	57	155	18 47	56	155	17 51	56	156	16 55	56	156	15 59	57	157	334
25	21 55	−56	155	20 59	−57	156	20 02	−56	156	19 06	−57	156	18 09	−56	157	17 13	−57	157	16 16	−56	157	335
24	22 14	57	156	21 17	57	157	20 20	56	157	19 24	57	157	18 27	57	158	17 30	57	158	16 33	56	158	336
23	22 32	57	157	21 35	57	157	20 38	57	158	19 41	57	158	18 44	57	158	17 47	57	159	16 50	57	159	337
22	22 49	57	158	21 52	57	158	20 55	57	159	19 58	58	159	19 00	57	159	18 03	57	160	17 06	58	160	338
21	23 06	58	159	22 08	57	159	21 11	58	160	20 13	57	160	19 16	58	160	18 18	57	161	17 21	58	161	339
20	23 22	−58	160	22 24	−58	160	21 26	−57	161	20 29	−58	161	19 31	−58	161	18 33	−58	161	17 35	−57	162	340
19	23 37	58	161	22 39	58	161	21 41	58	162	20 43	58	162	19 45	58	162	18 47	58	162	17 49	58	163	341
18	23 51	58	162	22 53	58	162	21 55	58	162	20 57	58	163	19 59	58	163	19 01	58	163	18 03	59	164	342
17	24 05	58	163	23 07	59	163	22 08	58	163	21 10	58	164	20 12	59	164	19 13	58	164	18 15	58	164	343
16	24 18	59	164	23 19	58	164	22 21	59	164	21 22	58	165	20 24	59	165	19 25	58	165	18 27	59	165	344
15	24 30	−59	165	23 31	−58	165	22 33	−59	165	21 34	−58	166	20 36	−59	166	19 37	−59	166	18 38	−59	166	345
14	24 42	59	166	23 43	59	166	22 44	59	166	21 45	59	167	20 46	59	167	19 47	58	167	18 49	59	167	346
13	24 52	59	167	23 53	59	167	22 54	59	167	21 55	59	167	20 56	59	168	19 57	59	168	18 58	59	168	347
12	25 02	59	168	24 03	59	168	23 04	59	168	22 05	59	168	21 06	59	169	20 07	60	169	19 07	59	169	348
11	25 11	59	169	24 12	59	169	23 13	59	169	22 14	60	169	21 14	59	170	20 15	59	170	19 16	60	170	349
10	25 20	−60	170	24 20	−59	170	23 21	−59	170	22 22	−60	170	21 22	−59	170	20 23	−60	171	19 23	−59	171	350
9	25 27	59	171	24 28	60	171	23 28	59	171	22 29	60	171	21 29	59	171	20 30	60	172	19 30	59	172	351
8	25 34	59	172	24 35	60	172	23 35	60	172	22 35	59	172	21 36	60	172	20 36	59	173	19 37	60	173	352
7	25 40	59	173	24 41	60	173	23 41	60	173	22 41	60	173	21 41	59	173	20 42	60	173	19 42	60	174	353
6	25 46	60	174	24 46	60	174	23 46	60	174	22 46	60	174	21 46	59	174	20 47	60	174	19 47	60	174	354
5	25 50	−60	175	24 50	−60	175	23 50	−60	175	22 50	−59	175	21 51	−60	175	20 51	−60	175	19 51	−60	175	355
4	25 54	60	176	24 54	60	176	23 54	60	176	22 54	60	176	21 54	60	176	20 54	60	176	19 54	60	176	356
3	25 56	60	177	24 56	59	177	23 57	60	177	22 57	60	177	21 57	60	177	20 57	60	177	19 57	60	177	357
2	25 58	60	178	24 58	60	178	23 58	59	178	22 59	60	178	21 59	60	178	20 59	60	178	19 59	60	178	358
1	26 00	60	179	25 00	60	179	24 00	60	179	23 00	60	179	22 00	60	179	21 00	60	179	20 00	60	179	359
0	26 00	−60	180	25 00	−60	180	24 00	−60	180	23 00	−60	180	22 00	−60	180	21 00	−60	180	20 00	−60	180	360
	23°			24°			25°			26°			27°			28°			29°			

DECLINATION (15°-29°) CONTRARY NAME TO LATITUDE

TABLE III.—Correction to Tabulated Altitude for Minutes of Declination

d / ′	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	d / ′
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3
4	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4
5	3	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5
6	3	3	3	3	4	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6
7	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6	7	7	7	7	7	7
8	4	4	4	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8	8	8	8
9	5	5	5	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	8	8	9	9	9	9	9
10	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	8	8	9	9	9	9	9	10	10	10	10	10
11	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	10	10	10	10	10	10	11	11	11	11
12	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9	9	10	10	10	10	10	11	11	11	11	11	12	12	12	12
13	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	10	10	11	11	11	11	11	12	12	12	12	13	13	13	13
14	7	7	8	8	8	8	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	12	13	13	13	13	14	14	14	14
15	8	8	8	8	9	9	9	10	10	10	10	10	11	11	11	12	12	12	12	12	13	13	13	14	14	14	14	14	15	15	15
16	8	9	9	9	9	10	10	10	10	11	11	11	11	12	12	12	13	13	13	13	14	14	14	14	15	15	15	15	16	16	16
17	9	9	9	10	10	10	10	11	11	11	12	12	12	12	13	13	13	14	14	14	14	15	15	15	16	16	16	16	17	17	17
18	9	10	10	10	10	11	11	11	12	12	12	13	13	13	14	14	14	14	15	15	15	16	16	16	16	17	17	17	18	18	18
19	10	10	10	11	11	11	12	12	12	13	13	13	14	14	14	15	15	15	16	16	16	16	17	17	17	18	18	18	19	19	19
20	10	11	11	11	12	12	12	13	13	13	14	14	14	15	15	15	16	16	16	17	17	17	18	18	18	19	19	19	20	20	20
21	11	11	12	12	12	13	13	13	14	14	14	15	15	15	16	16	16	17	17	18	18	18	19	19	19	20	20	20	21	21	21
22	11	12	12	12	13	13	14	14	14	15	15	15	16	16	16	17	17	18	18	18	19	19	19	20	20	21	21	21	22	22	22
23	12	12	13	13	13	14	14	15	15	15	16	16	16	17	17	18	18	18	19	19	20	20	20	21	21	21	22	22	23	23	23
24	12	13	13	14	14	14	15	15	16	16	16	17	17	18	18	18	19	19	20	20	20	21	21	22	22	22	23	23	24	24	24
25	13	13	14	14	15	15	15	16	16	17	17	18	18	18	19	19	20	20	20	21	21	22	22	22	23	23	24	24	25	25	25
26	13	14	14	15	15	16	16	16	17	17	18	18	19	19	20	20	20	21	21	22	22	23	23	23	24	24	25	25	26	26	26
27	14	14	15	15	16	16	17	17	18	18	18	19	19	20	20	21	21	22	22	22	23	23	24	24	25	25	26	26	27	27	27
28	14	15	15	16	16	17	17	18	18	19	19	20	20	21	21	21	22	22	23	23	24	24	25	25	26	26	27	27	28	28	28
29	15	15	16	16	17	17	18	18	19	19	20	20	21	21	22	22	23	23	24	24	25	25	26	26	27	27	28	28	29	29	29
30	16	16	16	17	18	18	18	19	20	20	20	21	22	22	22	23	24	24	24	25	26	26	26	27	28	28	28	29	30	30	30
31	16	17	17	18	18	19	19	20	20	21	21	22	22	23	23	24	24	25	25	26	26	27	27	28	28	29	29	30	30	31	31
32	17	17	18	18	19	19	20	20	21	21	22	22	23	23	24	25	25	26	26	27	27	28	28	29	29	30	30	31	31	32	32
33	17	18	18	19	19	20	20	21	21	22	23	23	24	24	25	25	26	26	27	28	28	29	29	30	30	31	31	32	32	33	33
34	18	18	19	19	20	20	21	22	22	23	23	24	24	25	26	26	27	27	28	28	29	29	30	31	31	32	32	33	33	34	34
35	18	19	19	20	20	21	22	22	23	23	24	24	25	26	26	27	27	28	29	29	30	30	31	32	32	33	33	34	34	35	35
36	19	19	20	20	21	22	22	23	23	24	25	25	26	26	27	28	28	29	29	30	31	31	32	32	33	34	34	35	35	36	36
37	19	20	20	21	22	22	23	23	24	25	25	26	27	27	28	28	29	30	30	31	31	32	33	33	34	35	35	36	36	37	37
38	20	20	21	22	22	23	23	24	25	25	26	27	27	28	28	29	30	30	31	32	32	33	34	34	35	35	36	37	37	38	38
39	20	21	21	22	23	23	24	25	25	26	27	27	28	29	29	30	31	31	32	32	33	34	34	35	36	36	37	38	38	39	39
40	21	21	22	23	23	24	25	25	26	27	27	28	29	29	30	31	31	32	33	33	34	35	35	36	37	37	38	39	39	40	40
41	21	22	23	23	24	25	25	26	27	27	28	29	29	30	31	31	32	33	33	34	35	36	36	37	38	38	39	40	40	41	41
42	22	22	23	24	24	25	26	27	27	28	29	29	30	31	32	32	33	34	34	35	36	36	37	38	38	39	40	41	41	42	42
43	22	23	24	24	25	26	27	27	28	29	29	30	31	32	32	33	34	34	35	36	37	37	38	39	39	40	41	42	42	43	43
44	23	23	24	25	26	26	27	28	29	29	30	31	32	32	33	34	34	35	36	37	37	38	39	40	40	41	42	43	43	44	44
45	23	24	25	26	26	27	28	28	29	30	31	32	32	33	34	34	35	36	37	38	38	39	40	40	41	42	43	44	44	45	45
46	24	25	25	26	27	28	28	29	30	31	31	32	33	34	34	35	36	37	38	38	39	40	41	41	42	43	44	44	45	46	46
47	24	25	26	27	27	28	29	30	31	31	32	33	34	34	35	36	37	38	38	39	40	41	42	42	43	44	45	45	46	47	47
48	25	26	26	27	28	29	30	30	31	32	33	34	34	35	36	37	38	38	39	40	41	42	42	43	44	45	46	46	47	48	48
49	25	26	27	28	29	29	30	31	32	33	33	34	35	36	37	38	38	39	40	41	42	42	43	44	45	46	47	47	48	49	49
50	26	27	28	28	29	30	31	32	32	33	34	35	36	37	38	38	39	40	41	42	42	43	44	45	46	47	48	48	49	50	50
51	26	27	28	29	30	31	31	32	33	34	35	36	37	37	38	39	40	41	42	42	43	44	45	46	47	48	48	49	50	51	51
52	27	28	29	29	30	31	32	33	34	35	36	36	37	38	39	40	41	42	42	43	44	45	46	47	48	49	49	50	51	52	52
53	27	28	29	30	31	32	33	34	34	35	36	37	38	39	40	41	42	42	43	44	45	46	47	48	49	49	50	51	52	53	53
54	28	29	30	31	32	32	33	34	35	36	37	38	39	40	40	41	42	43	44	45	46	47	48	49	50	50	51	52	53	54	54
55	28	29	30	31	32	33	34	35	36	37	38	38	39	40	41	42	43	44	45	46	47	48	49	50	50	51	52	53	54	55	55
56	29	30	31	32	33	34	35	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	49	50	51	52	53	54	55	56	56
57	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	48	49	50	51	52	53	54	55	56	57	57
58	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	58
59	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	59

TABLES

TABLES

EXPLANATION OF TABLES

Table 1. Conversion Angle.—The angles listed in this table are the differences between the great-circle and rhumb line (Mercator) directions between various points. The table can be used either for converting rhumb line directions to equivalent great-circle directions, as in great-circle sailing; or the reverse, as in converting radio bearings to equivalent rhumb line bearings for plotting on a Mercator chart. The sign to be used for each process is indicated at the bottom of the table. As indicated, the sign given in the tabulation is reversed if the conversion angle is shown in italics.

The first one and one-half pages of the table, for differences of longitude of not more than 4°5, and middle latitudes between 0° and 85°, is intended primarily for use in converting radio bearings observed near a coast. In high latitudes it may be needed for converting visual bearings of objects a considerable distance away (art. 2522). This part of the table is entered with (1) the middle latitude between the craft and radio station or object, and (2) the difference of longitude between these two points. Do not use this part of the table if the latitudes are of contrary name (one north, the other south), if the difference of latitude is more than 10°, or if the difference of longitude is more than 4°5. Under any of these conditions, use the second part of the table.

For this part, select the page for the latitude nearest the latitude of departure in the case of great-circle sailing, or the latitude of the receiver in the case of radio bearings. On the selected page, the entering arguments are (1) difference of longitude (DLo) between the two points involved, and (2) the latitude of destination in the case of great-circle sailing, or the latitude of the transmitter in the case of radio bearings. For 0° latitude of departure or receiver, there is a single table. For all other latitudes, separate tables are provided for latitudes of same name and contrary name. For accurate results, use triple interpolation, as explained in article P4.

Use of the table is explained in articles 821 (great-circle sailing), 1204 (radio bearings), 1206 (consol), 2404 (azimuths by submarine periscope), and 2522 (visual bearings in high latitudes).

The conversion angles on the first one and one-half pages of the table were computed by means of the formula:

$$\tan \text{ conversion angle} = \sin Lm \tan \tfrac{1}{2} DLo,$$

in which Lm is the middle latitude, and DLo is the difference of longitude. This formula is based upon the assumption that the plot of a great-circle track on a Mercator chart is symmetrical, the axis of symmetry being the perpendicular bisector of the rhumb line connecting the two points. No error of practical significance in ordinary navigation is introduced by this erroneous assumption over the range covered by the first one and one-half pages of the table.

The remainder of the table was computed by means of the formula: conversion angle = initial great-circle direction $\sim$ rhumb line direction. The initial great-circle directions were computed by means of the formula:

$$\text{hav } C = \csc D \sec L_1 \, [\text{hav co } L_2 - \text{hav } (D \sim \text{co } L_1)],$$

in which C is the initial great-circle direction, D is the great-circle distance in arc units, L_1 is the latitude of departure (or receiver), and L_2 is the latitude of the destination (or transmitter). The distance D was computed by means of the formula:

$$\text{hav } D = \text{hav } DLo \cos L_1 \cos L_2 + \text{hav } (L_1 \sim L_2),$$

in which the notation is the same as above.

The rhumb line directions were computed by means of the formula:

$$\tan C = \frac{DLo}{m},$$

in which C is the rhumb line direction, and $m = M_1 \sim M_2$, the meridional parts of the two latitudes.

Table 2. Conversion of Compass Points to Degrees.—In this table the compass is boxed to 128 quarter points and the equivalent angle is given in degrees, minutes, and seconds. The naming of the quarter points, as given here, is one of several systems that have been used.

Table 3. Traverse Table.—This table provides the quantities needed for solution of plane right triangles, arranged in a form for convenient solution of the problems encountered in the various sailings (ch. VIII). Two sets of column headings are given to indicate the corresponding values in different problems. Thus, if DLo is used as entering argument in the first column, p is taken from the second column, but if D is used as entering argument in the first column, l is taken from the second column and p is taken from the third column. When the top line of column headings is used, the individual table is selected by means of the *latitude*. When the second line is used, the table is selected by means of the *course*.

The entering argument can be multiplied by any power of 10, including negative powers (art. O2), if the corresponding values taken from the table are multiplied by the same power. Thus, using the table for course 205°, if D=6 miles, l=5′.438, and p=2.536 miles; but if D=600 miles, l=543′.8, and p=253.6 miles; or if D=0.6 mile, l=0′.5438, and p=0.2536 mile.

In this table, DLo is difference of longitude, p is departure, D is distance, l is difference of latitude, and m is meridional difference (difference of meridional parts at two latitudes). In the solution of any right triangle, D can be considered the hypotenuse, and l and p the other two sides. The angle is that opposite side p. Also, if m is one of the short sides of a plane right triangle, DLo is the other side if the given angle is that opposite DLo. If the two short sides are known, the angle opposite one of them can be determined from the tabulation on the right if the table is entered with the quotient found by dividing this side by the other short side.

The use of this table in the solution of problems of the various sailings is given in chapter VIII.

The top decimal in each individual table is a natural trigonometric function, the p, l column being cosines, the p column being sines, the DLo, D column being secants, and the DLo column being tangents. The decimals below the top line are multiples of the top value. The decimals in the center column to the right of the double line are natural tangents at intervals of 0°.1. For additional decimal places, use table 31.

Table 4. Conversion Table for Meridional Parts.—The meridional parts given in table 5 are for the Clarke spheroid of 1866. The values given in table 4 can be applied as corrections to those of table 5 to obtain the meridional parts for the international spheroid, the Clarke spheroid of 1880, and the sphere. Data on these spheroids are given in appendix D.

An additional decimal place is given in this table to provide greater accuracy for interpolated values.

Table 5. Meridional Parts.—In this table the meridional parts used in the construction of Mercator charts and in Mercator sailing are tabulated to one decimal place for each minute of latitude from the equator to the poles. The use of the table is explained in articles 307 and 817.

The table was computed by means of the formula:

$$M = a \log_e 10 \log \tan\left(45° + \frac{L}{2}\right) - a\left(e^2 \sin L + \frac{e^4}{3} \sin^3 L + \frac{e^6}{5} \sin^5 L + \ldots\right),$$

in which

M is the number of meridional parts between the equator and the given latitude,

a is the equatorial radius of the earth, expressed in minutes of arc of the equator, or

$$a = \frac{21{,}600}{2\pi} = 3437.74677078 \ (\log = 3.5362738827),$$

$\log_e$ is the natural (Naperian) logarithm, using the base e=2.71828182846,
$\log_e 10=2.30258509299$ (log=0.3622156886),
L is the latitude,
e is the ellipticity of the earth, or $\sqrt{2f-f^2}=0.08227185422$ (log=8.9152512855−10),
and
f is the flattening of the earth, or $f=\frac{1}{294.98}=0.00339006034$ (log=7.5302074283−10).

Using these values,

$a \log_e 10=7915.704468$ (log=3.8984895715)

$ae^2=23.268932$ (log=1.3667764504)

$\frac{ae^4}{3}=0.052500$ (log=8.7201593034−10)

$\frac{ae^6}{5}=0.000213$ (log=6.3283796034−10).

Hence, the formula becomes

$$M=7915.704468 \log \tan\left(45°+\frac{L}{2}\right)-23.268932 \sin L-0.052500 \sin^3 L-0.000213 \sin^5 L \ldots$$

The constants used in this derivation and in the table are based upon the Clarke spheroid of 1866 (app. D), the standard reference spheroid used for charting North America.

Table 6. Length of a Degree of Latitude and Longitude.—This table gives the length of one degree of latitude and longitude at intervals of 1° from the equator to the poles. In the case of latitude, the values given are the lengths of the arcs extending half a degree on each side of the tabulated latitudes. Lengths are given in nautical miles, statute miles, feet, and meters.

The values were computed in meters, using the Clarke spheroid of 1866 (app. D), and converted to other units by the factors given in appendix D. The following formulas were used:

$$M=111{,}132.09-566.05 \cos 2L+1.20 \cos 4L-0.002 \cos 6L+ \ldots$$
$$P=111{,}415.13 \cos L-94.55 \cos 3L+0.12 \cos 5L- \ldots$$

in which M is the length of 1° of the meridian (latitude), L is the latitude, and P is the length of 1° of the parallel (longitude).

Table 7. Distance of an Object by Two Bearings.—To determine the distance of an object as a vessel passes it, observe two relative bearings (right or left) of the object, and note the time interval between bearings. Enter this table with the two bearings. Multiply the distance run between bearings by the number in the first column to find the distance of the object at the time of the second bearing, and by the number in the second column to find the distance when abeam. Use of the table is explained in article 910.

The table was computed by solving plane oblique and right triangles (art. O42).

Table 8. Distance of the Horizon.—This table gives the distance, in nautical and statute miles, of the visible horizon for various heights of eye from 1 to 200,000 feet. The actual distance varies somewhat as refraction changes. Also, the formulas used contain an approximation which introduces an error of a few tenths of a mile at the greatest heights tabulated. However, the error is generally less than that introduced by nonstandard atmospheric conditions. Since the earth's ellipticity is not considered, the table can be used at any place on the earth, without appreciable error.

Use of the table is explained in articles 916 (visibility of lights), 1208 (radar), 1606 (dip), and 1608 (wave height).

The table was computed by means of the formulas:

nautical miles: $D=1.144\sqrt{h}$,

statute miles: $D=1.317\sqrt{h}$,

in which D is the distance of the horizon in miles, and h is the height above the surface in feet. The constants 1.144 and 1.317 are based upon the mean radius of the earth according to the Clarke spheroid of 1866 (app. D).

Table 9. Distance by Vertical Angle.—This table provides means for determining the distance of an object of known height above sea level. The vertical angle between the top of the object and the visible (sea) horizon (the sextant altitude) is measured and corrected for index error and dip only. If the visible horizon is not available as a reference, the angle should be measured to the bottom of the object, and dip short of the horizon (tab. 22) used in place of the usual dip correction. This may require several approximations of distance by alternate entries of tables 9 and 22 until the same value is obtained twice. The table is entered with the difference in the height of the object and the height of eye of the observer, in feet, and the corrected vertical angle; and the distance in nautical miles is taken directly from the table. An error may be introduced if refraction differs from the standard value used in the computation of the table. Use of the table is explained in article 905. Other references to its use are given in articles 609, 4119, and 4127.

The table was computed by means of the formula:

$$D=\sqrt{\left(\frac{\tan \alpha}{0.000246}\right)^2+\frac{H-h}{0.74736}}-\frac{\tan \alpha}{0.000246},$$

in which D is the distance in nautical miles, α is the corrected vertical angle, H is the height of the top of the object in feet, and h is the height of eye of the observer, in feet. The constants 0.000246 and 0.74736 are based upon the mean refraction (0.0784).

Table 10. Direction and Speed of True Wind.—This table provides a means of converting apparent wind, observed aboard a moving craft, to true wind. To use the table, divide the apparent wind, in knots, by the vessel's speed, also in knots. This gives the apparent wind speed in units of ship's speed. Enter the table with this value and the difference between the heading and the apparent wind direction. The values taken from the table are (1) the difference between the heading and the true wind direction, and (2) the speed of the true wind in units of ship's speed. The true wind is on the same side as the apparent wind, and from a point farther aft. To convert wind speed in units of ship's speed to speed in knots, multiply by the vessel's speed in knots. The steadiness of the wind and the accuracy of its measurement are seldom sufficient to warrant interpolation in this table. If speed of the true wind and relative direction of the apparent wind are known, enter the column for direction of the apparent wind, and find the speed of the true wind, in units of ship's speed. The number to the left is the relative direction of the true wind. The number on the same line in the side columns is the speed of the apparent wind in units of ship's speed. Two solutions are possible if speed of the true wind is less than the ship's speed. Article 3709 explains the use of this table, and also a graphical solution of the problem.

The table was computed by solving the triangle involved in a graphical solution, using the formulas:

$$\tan \alpha=\frac{\sin B_A}{S_A-\cos B_A},$$

$$B_T=B_A+\alpha,$$

$$S_T=\frac{\sin B_A}{\sin \alpha},$$

in which α is an auxiliary angle, B_A is the difference between the heading and the apparent wind direction, S_A is the speed of the apparent wind in units of ship's speed, B_T is the difference between the heading and the true wind direction, and S_T is the speed of the true wind in units of ship's speed.

Table 11. Correction of Barometer Reading for Height Above Sea Level.—If simultaneous barometer readings at different heights are to be of maximum value in weather analysis, they should be converted to the corresponding readings at a standard height, usually sea level. To convert the observed barometer reading to this level, enter this table with the outside temperature and the height of the barometer above

sea level. The height of a barometer is the height of its sensitive element; in the case of a mercurial barometer, this is the height of the free surface of mercury in the cistern. The correction taken from this table applies to the readings of any type barometer, and is always *added* to the observed readings, unless the barometer is below sea level. Use of the table is explained in articles 3706 and 4119.

The correction was computed by means of the formula:

$$C=29.92126\left[1-\frac{1}{\text{antilog}\left(\frac{0.0081350H}{T+0.00178308H}\right)}\right],$$

in which

C is the correction in inches of mercury,

H is the height of the barometer above sea level in feet, and,

T is the mean temperature, in degrees Rankine (degrees Fahrenheit plus 459°.67), of the air between the barometer and sea level. At sea, the outside air temperature is sufficiently accurate for this purpose.

Table 12. Correction of Barometer Reading for Gravity.—The height of the mercury column of a mercurial barometer is affected by the force of gravity, which changes with latitude and is approximately equal along any parallel of latitude. The average gravitational force at latitude 45°32′40″ is used as the standard for calibration. This table provides a correction to convert the observed reading at any other latitude to the corresponding value at latitude 45°32′40″, so that it will have maximum value in weather analysis of an area. Enter the table with the latitude, take out the correction, and apply in accordance with the sign given. This correction does not apply to aneroid barometers. Use of the table is further explained in article 3706.

The correction was computed by means of the formula:

$$C=B\,(-0.002637\cos 2L+0.000006\cos^2 2L-0.000050),$$

in which

C is the correction in inches,

B is the observed reading of the barometer (corrected for temperature and instrumental errors) in inches of mercury. This table was computed for a standard height of 30 inches, and

L is the latitude.

Table 13. Correction of Barometer Reading for Temperature.—Because of the difference in expansion of the mercury column of a mercurial barometer and that of the brass scale by which the height is measured, a correction should be applied to the reading when the temperature differs from the standard used for calibration of the instrument. To find the correction, enter this table with the temperature in degrees Fahrenheit, and the barometer reading. Apply the correction in accordance with the sign given. This correction does not apply to aneroid barometers. Use of the table is further explained in article 3706.

The standard temperature used for calibration is 32° F for the mercury, and 62° F for the brass. The correction was computed by means of the formula:

$$C=-B\,\frac{m\,(T-32^\circ)-l\,(T-62^\circ)}{1+m\,(T-32^\circ)},$$

in which

C is the correction in inches,

B is the observed reading of the barometer in inches of mercury,

m is the coefficient of cubical expansion of mercury=0.0001010 cubic inches per degree F,

l is the coefficient of linear expansion of brass=0.0000102 inches per degree F, and

T is the temperature of the attached thermometer in degrees F.

Substituting the values for m and l and simplifying:

$$C=-B\,\frac{T-28^\circ.630}{1.1123T+10978^\circ}.$$

The minus sign before B indicates that the correction is negative if the temperature is more than 28°.630.

Table 14. Conversion Table for Millibars, Inches of Mercury, and Millimeters of Mercury.—The reading of a barometer in inches or millimeters of mercury corresponding to a given reading in millibars can be found directly from this table. Use of the various units is discussed in article 3702.

The formula for the pressure in millibars is:

$$P=\frac{B_mDg}{1000},$$

in which

P is the atmospheric pressure in millibars,

B_m is the height of the column of mercury in millimeters,

D is the density of mercury = 13.5951 grams per cubic centimeter, and

g is the standard value of gravity = 980.665 dynes. Substituting numerical values:

$$P=1.33322B_m,$$

and

$$B_m=\frac{P}{1.33322}=0.750064P.$$

Since one millimeter = 0.03937 inches,

$$B_i=\frac{0.03937P}{1.33322}=0.0295300P,$$

in which B_i is the height of the column of mercury, in inches.

Table 15. Conversion Table for Thermometer Scales.—Enter this table with temperature Fahrenheit, F; Celsius (centigrade), C; or Kelvin, K; and take out the corresponding readings on the other two temperature scales. Temperature measurement is discussed in article 3711.

On the Fahrenheit scale, the freezing temperature of pure water at standard sea-level pressure is 32°, and the boiling point under the same conditions is considered 212°. The corresponding temperatures are 0° and 100°, respectively, on the Celsius scale and 273°15 and 373°15, respectively, on the Kelvin scale. The value of (−) 273°15 C for absolute zero, the starting point of the Kelvin scale, is the value recognized officially by the National Bureau of Standards of the United States.

The formulas for converting the reading of one scale to the corresponding values of the others, derived from the figures given above, are:

$$C=\frac{5}{9}(F-32^\circ)=K-273^\circ 15,$$

$$F=\frac{9}{5}C+32^\circ=\frac{9}{5}K-459^\circ 67,$$

$$K=\frac{5}{9}(F+459^\circ 67)=C+273^\circ 15,$$

in which all temperatures are in degrees.

Table 16. Relative Humidity.—To determine the relative humidity of the atmosphere, enter this table with the dry-bulb (air) temperature (F), and the *difference* between the dry-bulb and wet-bulb temperatures (F). The value taken from the table is the approximate percentage of relative humidity. If the dry-bulb and wet-bulb temperatures are the same, relative humidity is 100 percent. Use of the table is explained in article 3713.

The table was computed by means of the formula:

$$R=\frac{100e}{e_w},$$

in which

R is the approximate relative humidity in percent,

e is the ambient vapor pressure, and

e_w is the saturation vapor pressure over water at dry-bulb temperature.

Professor Ferrel's psychrometric formula was used for computation of e:

$$e=e'-\left[0.000367P\ (t-t')\left(1+\frac{t'-32^\circ}{1571}\right)\right],$$

in which

e is the ambient vapor pressure in millibars,

e′ is the saturation vapor pressure in millibars at wet-bulb temperature with respect to water.

P is the atmospheric pressure (the millibar equivalent of 30 inches of mercury is used for this table),

t is the dry-bulb temperature in degrees Fahrenheit, and

t′ is the wet-bulb temperature in degrees Fahrenheit.

The values of e_w were taken from the International Meteorological Organization Publication Number 79, 1951, table 2, pages 82–83.

Table 17. Dew Point.—To determine the dew point, enter this table with the dry-bulb (air) temperature (F), and the *difference* between the dry-bulb and wet-bulb temperatures (F). The value taken from the table is the dew point in degrees Fahrenheit. If the dry-bulb and wet-bulb temperatures are the same, the air is at or below the dew point. Use of the table is explained in articles 3713 and 3715.

The values given in this table were obtained (1) by determining the saturation vapor pressure e′ for the given temperature T (in degrees Rankine) by means of the following formula:

$$\log_{10} e' = -7.90298\left(\frac{671.67}{T}-1\right)+5.02808\log_{10}\frac{671.67}{T}-1.3816\times10^{-7}\left(10^{11.344\left(1-\frac{T}{671.67}\right)}-1\right)$$
$$+8.1328\times10^{-3}\left(10^{-3.49149\left(\frac{671.67}{T}-1\right)}-1\right)+\log_{10}1013.246,$$

(2) by determining the ambient vapor pressure by means of Ferrel's formula (see explanation to table 16), (3) by substituting e for e′ in the formula of (1) to obtain the temperature T of the wet bulb when saturation occurs (to the precision of table 17), and (4) by converting the wet-bulb temperature (T) to the dry-bulb temperature T′ by means of the equation:

$$T'=T+(t-t'),$$

where $(t-t')$ is the depression of the wet-bulb temperature. Tables evaluating e′ in terms of T for use in steps (1) and (3) are given in International Meteorological Organization Publication Number 79, 1951, and the Smithsonian Meteorological Tables, Sixth Revised Edition, 1951.

Table 18. Speed Table for Measured Mile.—To find the speed of a vessel traversing a measured nautical mile in a given number of minutes and seconds of time, enter this table at the top or bottom with the number of minutes, and at either side with the number of seconds. The number taken from the table is speed in knots. Accurate results can be obtained by interpolating to the nearest 0.1 second. Use of the table is explained in articles 608 and 615.

This table was computed by means of the formula:

$$S=\frac{3600}{T},$$

in which S is speed in knots, and T is elapsed time in seconds.

Table 19. Speed, Time, and Distance.—To find the distance steamed at any given speed between 0.5 and 40 knots in any given number of minutes from 1 to 60, enter this table at the top with the speed, and at the left with the number of minutes. The number taken from the table is the distance in nautical miles. If hours are substituted for minutes, the tabulated distance should be multiplied by 60; if seconds are substituted for minutes, the tabulated distance should be divided by 60. Use of the table is explained in articles 608, 801, and P1.

The table was computed by means of the formula:

$$D=\frac{ST}{60},$$

in which D is distance in nautical miles, S is speed in knots, and T is elapsed time in minutes.

Table 20. Conversion Table for Nautical and Statute Miles.—This table gives the number of statute miles corresponding to any whole number of nautical miles from 1 to 100, and the number of nautical miles corresponding to any whole number of statute miles within the same range. The entering value can be multiplied by any

power of 10, including negative powers, if the corresponding value of the other unit is multiplied by the same power. Thus, 2,700 nautical miles are equivalent to 3,107.1 statute miles, and 0.3 statute mile is equivalent to 0.2607 nautical mile. Hence, to find the number of statute miles equal to 2463.2 nautical miles:

Nautical miles	*Statute miles*
2400. 0	2761. 9
63. 0	72. 5
0. 2	0. 2
2463. 2	2834. 6

Use of the table is explained in articles 205 and 607.

The table was computed by means of the conversion factors of appendix D:

1 nautical mile = 1.15077945 statute miles,
1 statute mile = 0.86897624 nautical mile.

Table 21. Conversion Table for Meters, Feet, and Fathoms.—The number of feet and fathoms corresponding to a given number of meters, and vice versa, can be taken directly from this table for any value of the entering argument from 1 to 120. The entering value can be multiplied by any power of 10, including negative powers, if the corresponding values of the other units are multiplied by the same power. Thus, 420 meters are equivalent to 1378.0 feet, and 11.2 fathoms are equivalent to 20.483 meters. Hence, to find the number of meters equal to 2163 feet:

Feet	*Meters*
2100	640
63	19
2163	659.

These units of measurement are discussed in article 607.

The table was computed by means of the relationships given in appendix D:

1 meter = 39.370079 inches,
1 foot = 12 inches,
1 fathom = 6 feet.

Approximately the same results would be obtained by using the direct conversion factors given in appendix D.

Table 22. Dip of the Sea Short of the Horizon.—If land, another vessel, or other obstruction is between the observer and the sea horizon, use the water line of the obstruction as the horizontal reference for altitude measurements, and substitute dip from this table for the dip of the horizon (height of eye correction) given in the *American Nautical Almanac* or other source. The values below the bold rules are for normal dip, the visible horizon being between the observer and the obstruction. Use of the table is explained in article 1606 and in the explanation of table 9.

The table was computed by means of the formula:

$$D_s = 0.4156d + 0.5658\frac{h}{d},$$

in which D_s is the dip short of the sea horizon, in minutes; d is the distance to the water line of the obstruction, in nautical miles; and h is the height of eye of the observer above sea level, in feet.

Table 23. Altitude Correction for Air Temperature.—This table provides a correction to be applied to the altitude of a celestial body when the air temperature varies from the 50° F used for determining mean refraction by means of the *Nautical Almanac*. For maximum accuracy, apply index correction and dip to sextant altitude first, obtaining rectified (apparent) altitude for use in entering this table. Enter the table

with altitude and air temperature in degrees Fahrenheit. Apply the correction, in accordance with its tabulated sign, to altitude. Use of the table is explained principally in chapter XVI, and especially in articles 1614 and 1632.

The table was computed by means of the formula:

$$\text{Correction} = R_m\left(1-\frac{510}{460+T}\right),$$

in which R_m is mean refraction and T is temperature in degrees Fahrenheit.

Table 24. Altitude Correction for Atmospheric Pressure.—This table provides a correction to be applied to the altitude of a celestial body when the atmospheric pressure varies from the 29.83 inches (1010 millibars) used for determining mean refraction by means of the *Nautical Almanac*. For most accurate results, apply index correction and dip to sextant altitude first, obtaining rectified (apparent) altitude for use in entering this table. Enter the table with altitude and atmospheric pressure. Apply the correction to altitude, *adding* if the pressure is less than 29.83 inches and *subtracting* if it is more than 29.83 inches. Use of the table is explained principally in chapter XVI, and especially in articles 1615 and 1632.

The table was computed by means of the formula:

$$\text{Correction} = R_m\left(1-\frac{P}{29.83}\right),$$

in which R_m is mean refraction and P is atmospheric pressure in inches of mercury.

Table 25. Meridian Angle and Altitude of a Body on the Prime Vertical Circle.—A celestial body having a declination of contrary name to the latitude does not cross the prime vertical above the celestial horizon, its nearest approach being at rising or setting.

If the declination and latitude are of the same name, and the declination is numerically greater, the body does not cross the prime vertical, but makes its nearest approach (in azimuth) when its meridian angle, east or west, and altitude are as shown in this table, these values being given in italics above the heavy line. At this time the body is stationary in azimuth.

If the declination and latitude are of the same name and numerically equal, the body passes through the zenith as it crosses both the celestial meridian and the prime vertical, as shown in the table.

If the declination and latitude are of the same name, and the declination is numerically less, the body crosses the prime vertical when its meridian angle, east or west, and altitude are as tabulated in vertical type below the heavy line.

The table is entered with declination of the celestial body and the latitude of the observer. Computed altitudes are given, no allowance having been made for refraction, dip, parallax, etc. The tabulated values apply to any celestial body, but values are not given for declination greater than 23° because the tabulated information is generally desired for the sun only. Use of the information given in this table is discussed in articles 2107, 2125, and 2306.

The table was computed by means of the following formulas, derived by Napier's rules (art. O42):

Nearest approach (in azimuth) to the prime vertical:

$$\csc h = \sin d \csc L,$$
$$\sec t = \tan d \cot L.$$

On the prime vertical:

$$\sin h = \sin d \csc L,$$
$$\cos t = \tan d \cot L.$$

In these formulas, h is the altitude, d is the declination, L is the latitude, t is the meridian angle.

Table 26. Latitude and Longitude Factors.—The latitude obtained by solution of an ex-meridian sight (art. 2103) is inaccurate if the longitude used in determining the meridian angle is incorrect. Similarly, the longitude obtained by solution of a time sight (art. 2106) is inaccurate if the latitude used in the solution is incorrect, unless the celestial body is on the prime vertical. This table gives the errors resulting from unit errors in the assumed values used in the computations. There are two columns for each tabulated value of latitude. The first gives the latitude factor, f, which is the error in minutes of latitude for a one-minute error of longitude. The second gives the longitude factor, F, which is the error in minutes of longitude for a one-minute error of latitude. In each case, the total error is the factor multiplied by the number of minutes error in the assumed value. Although the factors were originally intended for use in correcting ex-meridian altitudes and time-sight longitudes, they have other uses which may suggest themselves.

The azimuth angle used for entering the table can be measured from *either* the north or south, through 90°; or it may be measured from the elevated pole, through 180°. If the celestial body is in the southeast (090°–180°) or northwest (270°–360°) quadrant, the f correction is applied to the northward if the correct longitude is east of that used in the solution, and to the southward if the correct longitude is west of that used; while the F correction is applied to the eastward if the correct latitude is north of that used in the solution, and to the westward if the correct latitude is south of that used. If the body is in the northeast (000°–090°) or southwest (180°–270°) quadrant, the correction is applied in the opposite direction. These rules apply in both north and south latitude.

The table was computed by means of the formulas:

$$f=\cos L \tan Z=\frac{1}{\sec L \cot Z}=\frac{1}{F},$$

$$F=\sec L \cot Z=\frac{1}{\cos L \tan Z}=\frac{1}{f},$$

in which f is the tabulated latitude factor, L is the latitude, Z is the azimuth angle, and F is the tabulated longitude factor.

Table 27. Amplitudes.—This table lists amplitudes of celestial bodies at rising and setting. Enter with the declination of the body and the latitude of the observer. The value taken from the table is the amplitude when the *center* of the body is on the *celestial* horizon. For the sun, this occurs when the lower limb is a little more than half a diameter above the visible horizon. For the moon it occurs when the upper limb is about on the horizon. Use the prefix E if the body is rising, and W if it is setting; use the suffix N or S to agree with the declination of the body. Table 28 can be used with reversed sign to correct the tabulations to the values for the visible horizon. Use of table 27 is explained in article 2125.

The table was computed by means of the following formula, derived by Napier's rules (art. O42):

$$\sin A=\sec L \sin d,$$

in which A is the amplitude, L is the latitude of the observer, and d is the declination of the celestial body.

Table 28. Correction of Amplitude as Observed on the Visible Horizon.—This table contains a correction to be applied to the amplitude observed when the center of a celestial body is on the visible horizon, to obtain the corresponding amplitude when the center of the body is on the celestial horizon. For the sun, a planet, or a star, apply the correction in the direction *away* from the elevated pole, thus *increasing* the *azimuth angle.* For the moon apply *half* the correction *toward* the elevated pole. This correction can be applied in the opposite direction to a value taken from table 27, to

find the corresponding amplitude when the center of a celestial body is on the visible horizon. The table was computed for a height of eye of 41 feet. For other heights normally encountered, the error is too small to be of practical significance in ordinary navigation. Use of the table is explained in article 2125.

The values in the table were determined by computing the azimuth angle when the center of the celestial body is on the visible horizon, converting this to amplitude, and determining the difference between this value and the corresponding value from table 27. Computation of azimuth angle was made for an altitude of (−)0°42.′0, determined as follows:

Dip at 41 feet height of eye	(−) 6.′2
Refraction at (−)6.′2 alt.	(−)35.′3
Irradiation of horizon	(−) 0.′6
Parallax (value for sun)	(+) 0.′1
	(−)42.′0.

Azimuth angle was computed by means of the formula:

$$\cos Z = \frac{\sin d + \sin h \sin L}{\cos h \cos L},$$

in which Z is the azimuth angle, d is the declination of the celestial body, h is the altitude (−0°42.′0), and L is the latitude of the observer.

Table 29. Altitude Factor.—In one minute of time from meridian transit the altitude of a celestial body changes by the amount shown in this table if the altitude is between 6° and 86°, the latitude is not more than 60°, and the declination is not more than 63°. The values taken from this table are used to enter table 30 for solving reduction to the meridian (ex-meridian) problems, explained in article 2103.

For upper transit, use the left-hand pages if the declination and latitude are of the same name (both north or both south) and the right-hand pages if of contrary name. For lower transit, use the values below the heavy lines on the last three contrary-name pages. When a factor is taken from this part of the table, the correction from table 30 is *subtracted* from the observed altitude to obtain the corresponding meridian altitude. All other corrections are added.

The table was computed by means of the formula:

$$a = 1.''9635 \cos L \cos d \csc (L \sim d),$$

in which a is the change of altitude in one minute from meridian transit (the tabulated value), L is the latitude of the observer, and d is the declination of the celestial body.

This formula can be used to compute values outside the limits of the table, but is not accurate if the altitude is greater than 86°.

Table 30. Change of Altitude in Given Time from Meridian Transit.—Enter this table with the altitude factor from table 29 and the meridian angle, in either arc or time units, and take out the difference between the altitude at the given time and the altitude at meridian transit. Enter the table separately with whole numbers and tenths of a, interpolating for t if necessary, and add the two values to obtain the total difference. This total can be applied as a correction to observed altitude to obtain the corresponding meridian altitude, adding for upper transit and subtracting for lower transit. This problem is further discussed in article 2103.

The table was computed by means of the formula:

$$C = \frac{at^2}{60},$$

in which C is the tabulated difference to be used as a correction to observed altitude, in minutes of arc; a is the altitude factor from table 29, in seconds of arc; and t is the meridian angle, in minutes of time.

This formula should not be used for determining values beyond the limits of the table, unless reduced accuracy is acceptable.

Table 31. Natural Trigonometric Functions.—This table gives the values of natural sines, cosecants, tangents, cotangents, secants, and cosines of angles from 0° to 180°, at intervals of 1′. For angles between 0° and 45° use the column labels at the top and the minutes at the left; for angles between 45° and 90° use the column labels at the bottom and the minutes at the right; for angles between 90° and 135° use the column labels at the bottom and the minutes at the left; and for angles between 135° and 180° use the column labels at the top and the minutes at the right. These combinations are indicated by the arrows accompanying the figures representing the number of degrees. For angles between 180° and 360°, subtract 180° and proceed as indicated above to obtain the numerical values of the various functions.

Differences between consecutive entries are shown in the "Diff. 1′" column to the right of each column of values of a trigonometric function, as an aid to interpolation. These differences are one-half line out of step with the numbers to which they apply, as in a critical table. Each difference applies to the values half a line above and half a line below. To determine the correction to apply to the value for the smaller entering angle, multiply the difference by the number of tenths of a minute $\left(\text{or } \frac{\text{seconds}}{60}\right)$ of the entering angle. Note whether the function is increasing or decreasing, and add or subtract the correction as appropriate, so that the interpolated value lies between the two values between which interpolation is made.

The logarithms of values given in this table are given in table 33. The trigonometric functions are explained in article O39.

Table 32. Logarithms of Numbers.—The first page of this table gives the complete common logarithm (characteristic and mantissa) of numbers 1 through 250. The succeeding pages give the mantissa only of the common logarithm of any number. Values are given for four significant digits of entering values, the first three being in the left-hand column, and the fourth at the heading of one of the other columns. Thus, the mantissa of a three-digit number is given in the column headed 0, on the line with the given number; while the mantissa of a four-digit number is given in the column headed by the fourth digit, on the line with the first three digits. As an example, the mantissa of 328 is 51587, while that of 3.284 is 51640. For additional digits, interpolation should be used. The difference between each tabulated mantissa and the next larger tabulated mantissa is given in the "d" column to the right of the smaller mantissa. This difference can be used to enter the appropriate proportional parts ("Prop. parts") auxiliary table to interpolate for the fifth digit of the given number. If an accuracy of more than five significant digits is to be preserved in a computation, a table of logarithms to additional decimal places should be used. For a number of one or two digits, use the first page of the table or add zeros to make three digits. That is, the mantissa of 3, 30, and 300 is the same, 47712. Interpolation on the first page of the table is not recommended. The second part should be used for values not listed on the first page.

Additional information on the nature and use of logarithms is given in article O10.

Table 33. Logarithms of Trigonometric Functions.—This table gives the common logarithms (+10) of sines, cosecants, tangents, cotangents, secants, and cosines of angles from 0° to 180°, at intervals of 1′. For angles between 0° and 45° use the column labels at the top and the minutes at the left; for angles between 45° and 90° use the column labels at the bottom and the minutes at the right; for angles between 90° and 135° use the column labels at the bottom and the minutes at the left; and for angles between 135° and 180° use the column labels at the top and the minutes at the right. These combinations are indicated by the arrows accompanying the figures representing the number of degrees. For angles between 180° and 360°, subtract 180° and proceed as indicated above to obtain the numerical values of the various functions.

Differences between consecutive entries are shown in the "Diff. 1′" columns as in table 31, except that one difference column is used for both sines and cosecants, another for both tangents and cotangents, and a third for both secants and cosines. These differences, given as an aid to interpolation, are one-half line out of step with the numbers to which they apply, as in a critical table. Each difference applies to the values half a line above and half a line below. To determine the correction to apply to the value for the smaller entering angle, multiply the difference by the number of tenths of a minute $\left(\text{or } \frac{\text{seconds}}{60}\right)$ of the entering angle. Note whether the function is increasing or decreasing, and add or subtract the correction as appropriate, so that the interpolated value lies between the two values between which interpolation is made.

Natural trigonometric functions are given in table 31. The trigonometric functions, both natural and logarithmic, are explained in article O39.

Table 34. Haversines.—This table lists the common logarithms (+10) of haversines, and natural haversines, of angles from 0° to 360°, at intervals of 1′. For angles between 0° and 180° use the degrees as given at the tops of the columns and the minutes at the left; for angles between 180° and 360° use the degrees as given at the bottoms of the columns and the minutes at the right.

A haversine is half of a versed sine:

$$\text{hav } A = \tfrac{1}{2} \text{ ver } A = \tfrac{1}{2}\,(1-\cos A) = \sin^2 \tfrac{1}{2} A.$$

It is further discussed in article O39. Examples of the use of haversines are given in articles 822 and 2109.

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Mid lat.	Difference of longitude										Mid lat.
	0°	0°5	1°	1°5	2°	2°5	3°	3°5	4°	4°5	
°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	2
4	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.2	4
6	0.0	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	6
8	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	8
10	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.3	0.4	0.4	10
11	0.0	0.0	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	11
12	0.0	0.1	0.1	0.1	0.2	0.3	0.3	0.4	0.4	0.5	12
13	0.0	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	0.5	13
14	0.0	0.1	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	14
15	0.0	0.1	0.1	0.2	0.3	0.3	0.4	0.4	0.5	0.6	15
16	0.0	0.1	0.1	0.2	0.3	0.4	0.4	0.5	0.6	0.6	16
17	0.0	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	0.6	17
18	0.0	0.1	0.2	0.2	0.3	0.4	0.5	0.5	0.6	0.7	18
19	0.0	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.6	0.7	19
20	0.0	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	20
21	0.0	0.1	0.2	0.3	0.4	0.5	0.5	0.6	0.7	0.8	21
22	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.6	0.8	0.8	22
23	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	23
24	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	24
25	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	1.0	25
26	0.0	0.1	0.2	0.3	0.4	0.6	0.6	0.8	0.9	1.0	26
27	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.8	0.9	1.0	27
28	0.0	0.1	0.2	0.4	0.5	0.6	0.7	0.8	0.9	1.1	28
29	0.0	0.1	0.2	0.4	0.5	0.6	0.7	0.8	1.0	1.1	29
30	0.0	0.1	0.2	0.4	0.5	0.6	0.8	0.9	1.0	1.1	30
31	0.0	0.1	0.2	0.4	0.5	0.6	0.8	0.9	1.0	1.2	31
32	0.0	0.1	0.3	0.4	0.5	0.7	0.8	0.9	1.1	1.2	32
33	0.0	0.1	0.3	0.4	0.6	0.7	0.8	1.0	1.1	1.2	33
34	0.0	0.1	0.3	0.4	0.6	0.7	0.8	1.0	1.1	1.2	34
35	0.0	0.1	0.3	0.4	0.6	0.7	0.9	1.0	1.2	1.3	35
36	0.0	0.1	0.3	0.4	0.6	0.7	0.9	1.0	1.2	1.3	36
37	0.0	0.2	0.3	0.4	0.6	0.8	0.9	1.1	1.2	1.4	37
38	0.0	0.2	0.3	0.5	0.6	0.8	0.9	1.1	1.2	1.4	38
39	0.0	0.2	0.3	0.5	0.6	0.8	1.0	1.1	1.2	1.4	39
40	0.0	0.2	0.3	0.5	0.6	0.8	1.0	1.1	1.3	1.4	40
41	0.0	0.2	0.3	0.5	0.6	0.8	1.0	1.2	1.3	1.5	41
42	0.0	0.2	0.3	0.5	0.7	0.8	1.0	1.2	1.3	1.5	42
43	0.0	0.2	0.3	0.5	0.7	0.8	1.0	1.2	1.4	1.5	43
44	0.0	0.2	0.4	0.5	0.7	0.9	1.1	1.2	1.4	1.6	44
45	0.0	0.2	0.4	0.5	0.7	0.9	1.1	1.2	1.4	1.6	45
46	0.0	0.2	0.4	0.5	0.7	0.9	1.1	1.3	1.4	1.6	46
47	0.0	0.2	0.4	0.6	0.7	0.9	1.1	1.3	1.5	1.7	47
48	0.0	0.2	0.4	0.6	0.8	0.9	1.1	1.3	1.5	1.7	48
49	0.0	0.2	0.4	0.6	0.8	1.0	1.1	1.3	1.5	1.7	49
50	0.0	0.2	0.4	0.6	0.8	1.0	1.1	1.3	1.5	1.7	50
51	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	51
52	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	52
53	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	53
54	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	54
55	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	55
56	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.7	1.9	56
57	0.0	0.2	0.4	0.6	0.8	1.1	1.2	1.5	1.7	1.9	57
58	0.0	0.2	0.4	0.6	0.8	1.1	1.3	1.5	1.7	1.9	58
59	0.0	0.2	0.4	0.6	0.8	1.1	1.3	1.5	1.7	1.9	59
60	0.0	0.2	0.4	0.7	0.9	1.1	1.3	1.5	1.7	2.0	60

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Mid lat.	Difference of longitude										Mid lat.
	0°	0°5	1°	1°5	2°	2°5	3°	3°5	4°	4°5	
°	°	°	°	°	°	°	°	°	°	°	°
61	0.0	0.2	0.4	0.7	0.9	1.1	1.3	1.5	1.7	2.0	61
62	0.0	0.2	0.4	0.7	0.9	1.1	1.3	1.5	1.8	2.0	62
63	0.0	0.2	0.4	0.7	0.9	1.1	1.3	1.6	1.8	2.0	63
64	0.0	0.2	0.4	0.7	0.9	1.1	1.3	1.6	1.8	2.0	64
65	0.0	0.2	0.5	0.7	0.9	1.1	1.4	1.6	1.8	2.0	65
66	0.0	0.2	0.5	0.7	0.9	1.1	1.4	1.6	1.8	2.1	66
67	0.0	0.2	0.5	0.7	0.9	1.2	1.4	1.6	1.8	2.1	67
68	0.0	0.2	0.5	0.7	0.9	1.2	1.4	1.6	1.9	2.1	68
69	0.0	0.2	0.5	0.7	0.9	1.2	1.4	1.6	1.9	2.1	69
70	0.0	0.2	0.5	0.7	0.9	1.2	1.4	1.6	1.9	2.1	70
71	0.0	0.2	0.5	0.7	0.9	1.2	1.4	1.7	1.9	2.1	71
72	0.0	0.2	0.5	0.7	1.0	1.2	1.4	1.7	1.9	2.1	72
73	0.0	0.2	0.5	0.7	1.0	1.2	1.4	1.7	1.9	2.2	73
74	0.0	0.2	0.5	0.7	1.0	1.2	1.4	1.7	1.9	2.2	74
75	0.0	0.2	0.5	0.7	1.0	1.2	1.4	1.7	1.9	2.2	75
76	0.0	0.2	0.5	0.7	1.0	1.2	1.5	1.7	1.9	2.2	76
77	0.0	0.2	0.5	0.7	1.0	1.2	1.5	1.7	1.9	2.2	77
78	0.0	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2	78
79	0.0	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2	79
80	0.0	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2	80
81	0.0	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2	81
82	0.0	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2	82
83	0.0	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2	83
84	0.0	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2	84
85	0.0	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2	85

Latitude of Departure—0°—Latitude of Receiver

DLo	Latitude of destination—Latitude of transmitter																			DLo
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	0.0	0.3	0.3	0.3	0.4	0.4	0.5	0.6	0.6	0.7	0.8	0.9	0.9	1.0	1.1	1.1	1.2	1.2	0.0	5
10	0.0	0.3	0.5	0.6	0.7	0.9	1.0	1.1	1.3	1.4	1.6	1.7	1.9	2.0	2.2	2.3	2.4	2.3	0.0	10
15	0.0	0.3	0.6	0.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.6	2.8	3.0	3.2	3.4	3.5	3.5	0.0	15
20	0.0	0.4	0.8	1.1	1.4	1.7	2.0	2.2	2.5	2.8	3.1	3.4	3.7	4.0	4.3	4.6	4.7	4.7	0.0	20
25	0.0	0.5	0.9	1.3	1.7	2.1	2.4	2.8	3.2	3.6	3.9	4.3	4.7	5.1	5.4	5.7	5.9	5.8	0.0	25
30	0.0	0.5	1.0	1.5	2.0	2.5	2.9	3.4	3.8	4.3	4.7	5.2	5.7	6.1	6.5	6.9	7.1	7.0	0.0	30
35	0.0	0.6	1.2	1.7	2.3	2.9	3.4	3.9	4.5	5.0	5.6	6.1	6.7	7.2	7.7	8.1	8.3	8.2	0.0	35
40	0.0	0.7	1.3	2.0	2.6	3.3	3.9	4.5	5.2	5.8	6.4	7.1	7.7	8.3	8.8	9.3	9.6	9.4	0.0	40
45	0.0	0.8	1.5	2.2	3.0	3.7	4.4	5.2	5.9	6.6	7.3	8.0	8.7	9.4	10.0	10.5	10.8	10.6	0.0	45
50	0.0	0.8	1.7	2.5	3.3	4.2	5.0	5.8	6.6	7.4	8.2	9.0	9.8	10.5	11.2	11.8	12.1	11.8	0.0	50
55	0.0	0.9	1.9	2.8	3.7	4.6	5.6	6.5	7.4	8.3	9.2	10.1	10.9	11.7	12.4	13.0	13.3	13.0	0.0	55
60	0.0	1.0	2.1	3.1	4.1	5.1	6.2	7.2	8.2	9.2	10.2	11.1	12.1	12.9	13.7	14.3	14.6	14.2	0.0	60
65	0.0	1.1	2.3	3.4	4.6	5.7	6.8	7.9	9.0	10.1	11.2	12.3	13.2	14.2	15.0	15.7	16.0	15.4	0.0	65
70	0.0	1.3	2.5	3.8	5.0	6.3	7.5	8.7	9.9	11.1	12.3	13.4	14.5	15.5	16.4	17.0	17.3	16.7	0.0	70
75	0.0	1.4	2.8	4.1	5.5	6.9	8.2	9.6	10.9	12.2	13.5	14.7	15.8	16.9	17.8	18.4	18.7	17.9	0.0	75
80	0.0	1.5	3.0	4.6	6.1	7.6	9.0	10.5	11.9	13.3	14.7	16.0	17.2	18.3	19.2	19.9	20.0	19.2	0.0	80
85	0.0	1.7	3.3	5.0	6.6	8.3	9.9	11.5	13.0	14.5	16.0	17.4	18.6	19.8	20.7	21.3	21.4	20.4	0.0	85
90	0.0	1.8	3.7	5.5	7.3	9.1	10.8	12.6	14.2	15.8	17.4	18.8	20.2	21.3	22.3	22.9	22.9	21.7	0.0	90
95	0.0	2.0	4.0	6.0	8.0	10.0	11.9	13.7	15.5	17.2	18.9	20.4	21.8	23.0	23.9	24.4	24.4	23.0	0.0	95
100	0.0	2.2	4.5	6.7	8.8	10.9	13.0	15.0	16.9	18.8	20.5	22.1	23.5	24.7	25.6	26.0	25.8	24.3	0.0	100
105	0.0	2.5	4.9	7.3	9.7	12.0	14.3	16.4	18.5	20.4	22.2	23.9	25.3	26.5	27.3	27.7	27.4	25.6	0.0	105
110	0.0	2.7	5.4	8.1	10.7	13.3	15.7	18.0	20.2	22.2	24.1	25.8	27.2	28.3	29.1	29.4	28.9	26.9	0.0	110
115	0.0	3.0	6.0	9.0	11.9	14.6	17.3	19.8	22.1	24.2	26.1	27.8	29.2	30.3	31.0	31.2	30.5	28.2	0.0	115
120	0.0	3.4	6.8	10.0	13.2	16.2	19.1	21.7	24.2	26.4	28.4	30.0	31.4	32.4	33.0	33.0	32.1	29.5	0.0	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—5°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

Same Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	0.0	0.3	0.5	0.5	0.6	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.2	1.2	1.2	0.0	5
10	0.1	0.4	0.7	0.9	1.0	1.2	1.3	1.4	1.5	1.7	1.8	1.9	2.1	2.2	2.3	2.4	2.5	2.4	0.0	10
15	0.3	0.7	1.0	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.6	3.7	3.6	0.0	15
20	0.5	0.9	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.3	3.6	3.9	4.2	4.4	4.7	4.9	5.0	4.8	0.0	20
25	0.7	1.1	1.6	2.0	2.4	2.8	3.1	3.5	3.8	4.2	4.5	4.9	5.2	5.6	5.8	6.1	6.2	6.0	0.0	25
30	0.8	1.3	1.8	2.4	2.8	3.3	3.7	4.2	4.6	5.0	5.5	5.9	6.3	6.7	7.1	7.3	7.5	7.2	0.0	30
35	1.0	1.6	2.2	2.7	3.3	3.8	4.4	4.9	5.4	5.9	6.4	6.9	7.4	7.9	8.3	8.6	8.7	8.5	0.0	35
40	1.2	1.8	2.5	3.1	3.8	4.4	5.0	5.6	6.2	6.8	7.4	8.0	8.5	9.0	9.5	9.9	10.0	9.7	0.0	40
45	1.3	2.1	2.8	3.5	4.3	5.0	5.7	6.4	7.1	7.7	8.4	9.0	9.7	10.2	10.8	11.2	11.3	10.9	0.0	45
50	1.5	2.3	3.2	4.0	4.8	5.6	6.4	7.2	7.9	8.7	9.4	10.1	10.8	11.5	12.0	12.5	12.6	12.2	0.0	50
55	1.7	2.6	3.5	4.4	5.3	6.2	7.1	8.0	8.8	9.7	10.5	11.3	12.0	12.8	13.4	13.8	14.0	13.4	0.0	55
60	1.9	2.9	3.9	4.9	5.9	6.9	7.9	8.8	9.8	10.7	11.6	12.5	13.3	14.1	14.7	15.2	15.3	14.7	0.0	60
65	2.1	3.2	4.3	5.4	6.5	7.6	8.7	9.7	10.8	11.8	12.8	13.7	14.6	15.4	16.1	16.6	16.7	15.9	0.0	65
70	2.3	3.5	4.7	6.0	7.2	8.4	9.5	10.7	11.8	12.9	14.0	15.0	16.0	16.8	17.5	18.0	18.1	17.2	0.0	70
75	2.5	3.8	5.2	6.5	7.9	9.2	10.5	11.7	12.9	14.1	15.3	16.4	17.4	18.3	19.0	19.4	19.5	18.5	0.0	75
80	2.7	4.2	5.7	7.2	8.6	10.0	11.4	12.8	14.1	15.4	16.6	17.8	18.8	19.7	20.5	20.9	20.9	19.7	0.0	80
85	2.9	4.6	6.2	7.8	9.4	11.0	12.5	14.0	15.4	16.8	18.1	19.3	20.4	21.3	22.0	22.5	22.3	21.0	0.0	85
90	3.2	5.0	6.8	8.5	10.3	12.0	13.6	15.2	16.7	18.2	19.6	20.8	22.0	22.9	23.7	24.0	23.8	22.3	0.0	90
95	3.4	5.4	7.4	9.3	11.2	13.1	14.9	16.6	18.2	19.8	21.2	22.5	23.7	24.6	25.3	25.6	25.3	23.6	0.0	95
100	3.7	5.9	8.1	10.2	12.3	14.3	16.2	18.0	19.8	21.4	22.9	24.3	25.5	26.4	27.1	27.3	26.8	24.9	0.0	100
105	4.1	6.5	8.9	11.2	13.4	15.6	17.7	19.7	21.5	23.2	24.8	26.2	27.3	28.3	28.8	29.0	28.4	26.3	0.0	105
110	4.4	7.1	9.7	12.3	14.7	17.1	19.3	21.4	23.4	25.2	26.8	28.2	29.3	30.2	30.7	30.7	29.9	27.6	0.0	110
115	4.8	7.8	10.7	13.5	16.2	18.8	21.1	23.4	25.4	27.3	28.9	30.3	31.4	32.2	32.6	32.5	31.5	28.9	0.0	115
120	5.3	8.6	11.8	14.9	17.9	20.6	23.2	25.5	27.7	29.5	31.2	32.6	33.6	34.3	34.6	34.3	33.2	30.3	0.0	120

Contrary Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	0
5	0.0	*0.1*	*0.1*	*0.2*	*0.2*	*0.3*	*0.3*	*0.4*	*0.5*	*0.6*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*	*1.1*	*1.1*	*1.1*	*0.0*	5
10	0.1	0.0	*0.2*	*0.3*	*0.4*	*0.6*	*0.7*	*0.8*	*1.0*	*1.1*	*1.3*	*1.5*	*1.6*	*1.8*	*2.0*	*2.1*	*2.2*	*2.2*	*0.0*	10
15	0.3	0.0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*	*1.3*	*1.5*	*1.7*	*2.0*	*2.2*	*2.5*	*2.7*	*3.0*	*3.2*	*3.3*	*3.4*	*0.0*	15
20	0.5	0.1	*0.2*	*0.5*	*0.8*	*1.1*	*1.4*	*1.7*	*2.0*	*2.3*	*2.6*	*3.0*	*3.3*	*3.6*	*4.0*	*4.3*	*4.5*	*4.5*	*0.0*	20
25	0.7	0.2	*0.2*	*0.6*	*1.0*	*1.3*	*1.7*	*2.1*	*2.5*	*2.9*	*3.3*	*3.7*	*4.1*	*4.6*	*5.0*	*5.3*	*5.6*	*5.6*	*0.0*	25
30	0.8	0.3	*0.2*	*0.6*	*1.1*	*1.6*	*2.1*	*2.5*	*3.0*	*3.5*	*4.0*	*4.5*	*5.0*	*5.5*	*6.0*	*6.4*	*6.8*	*6.8*	*0.0*	30
35	1.0	0.4	*0.2*	*0.7*	*1.3*	*1.8*	*2.4*	*3.0*	*3.5*	*4.1*	*4.7*	*5.3*	*5.9*	*6.5*	*7.0*	*7.6*	*7.9*	*7.9*	*0.0*	35
40	1.2	0.5	*0.2*	*0.8*	*1.5*	*2.1*	*2.8*	*3.4*	*4.1*	*4.8*	*5.4*	*6.1*	*6.8*	*7.5*	*8.1*	*8.7*	*9.1*	*9.0*	*0.0*	40
45	1.3	0.6	*0.2*	*0.9*	*1.7*	*2.4*	*3.1*	*3.9*	*4.7*	*5.4*	*6.2*	*7.0*	*7.7*	*8.5*	*9.2*	*9.8*	*10.3*	*10.2*	*0.0*	45
50	1.5	0.7	*0.2*	*1.0*	*1.8*	*2.7*	*3.5*	*4.4*	*5.3*	*6.1*	*7.0*	*7.8*	*8.7*	*9.5*	*10.3*	*11.0*	*11.5*	*11.4*	*0.0*	50
55	1.7	0.7	*0.2*	*1.1*	*2.1*	*3.0*	*4.0*	*4.9*	*5.9*	*6.8*	*7.8*	*8.8*	*9.7*	*10.6*	*11.5*	*12.2*	*12.7*	*12.5*	*0.0*	55
60	1.9	0.8	*0.2*	*1.2*	*2.3*	*3.3*	*4.4*	*5.5*	*6.5*	*7.6*	*8.7*	*9.7*	*10.7*	*11.7*	*12.7*	*13.4*	*13.9*	*13.7*	*0.0*	60
65	2.1	0.9	*0.2*	*1.4*	*2.5*	*3.7*	*4.9*	*6.0*	*7.2*	*8.4*	*9.6*	*10.7*	*11.8*	*12.9*	*13.9*	*14.7*	*15.2*	*14.9*	*0.0*	65
70	2.3	1.0	*0.3*	*1.5*	*2.8*	*4.1*	*5.4*	*6.7*	*8.0*	*9.3*	*10.5*	*11.8*	*13.0*	*14.1*	*15.2*	*16.0*	*16.5*	*16.1*	*0.0*	70
75	2.5	1.1	*0.3*	*1.7*	*3.1*	*4.5*	*5.9*	*7.4*	*8.8*	*10.2*	*11.5*	*12.9*	*14.2*	*15.4*	*16.5*	*17.4*	*17.8*	*17.3*	*0.0*	75
80	2.7	1.2	*0.4*	*1.9*	*3.5*	*5.0*	*6.6*	*8.1*	*9.6*	*11.2*	*12.6*	*14.1*	*15.5*	*16.7*	*17.9*	*18.7*	*19.2*	*18.6*	*0.0*	80
85	2.9	1.2	*0.4*	*2.1*	*3.8*	*5.5*	*7.2*	*8.9*	*10.6*	*12.2*	*13.8*	*15.4*	*16.8*	*18.1*	*19.3*	*20.2*	*20.5*	*19.8*	*0.0*	85
90	3.2	1.3	*0.5*	*2.4*	*4.2*	*6.1*	*8.0*	*9.8*	*11.6*	*13.4*	*15.1*	*16.7*	*18.2*	*19.6*	*20.8*	*21.6*	*21.9*	*21.0*	*0.0*	90
95	3.4	1.4	*0.6*	*2.7*	*4.7*	*6.8*	*8.8*	*10.8*	*12.7*	*14.6*	*16.4*	*18.2*	*19.8*	*21.2*	*22.3*	*23.2*	*23.4*	*22.3*	*0.0*	95
100	3.7	1.5	*0.7*	*3.0*	*5.3*	*7.5*	*9.7*	*11.9*	*14.0*	*16.0*	*17.9*	*19.7*	*21.4*	*22.8*	*24.0*	*24.7*	*24.8*	*23.6*	*0.0*	100
105	4.1	1.6	*0.9*	*3.4*	*5.9*	*8.3*	*10.7*	*13.1*	*15.3*	*17.5*	*19.5*	*21.4*	*23.1*	*24.5*	*25.7*	*26.4*	*26.3*	*24.9*	*0.0*	105
110	4.4	1.7	*1.1*	*3.8*	*6.6*	*9.3*	*11.9*	*14.4*	*16.9*	*19.2*	*21.3*	*23.2*	*24.9*	*26.4*	*27.4*	*28.0*	*27.8*	*26.1*	*0.0*	110
115	4.8	1.8	*1.3*	*4.4*	*7.4*	*10.3*	*13.2*	*16.0*	*18.6*	*21.0*	*23.2*	*25.2*	*26.9*	*28.3*	*29.3*	*29.8*	*29.4*	*27.4*	*0.0*	115
120	5.3	1.9	*1.6*	*5.0*	*8.3*	*11.6*	*14.7*	*17.7*	*20.5*	*23.0*	*25.3*	*27.3*	*29.0*	*30.4*	*31.2*	*31.5*	*31.0*	*28.7*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—10°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

Same Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	0.1	0.2	0.5	0.7	0.7	0.8	0.8	0.8	0.9	1.0	1.0	1.1	1.1	1.2	1.2	1.3	1.3	1.2	0.0	5
10	0.4	0.6	0.9	1.2	1.3	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.3	2.4	2.5	2.6	2.6	2.5	0.0	10
15	0.7	1.0	1.3	1.6	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.2	3.4	3.6	3.7	3.8	3.9	3.7	0.0	15
20	1.0	1.4	1.8	2.1	2.5	2.8	3.1	3.3	3.6	3.8	4.1	4.3	4.6	4.8	5.0	5.1	5.2	5.0	0.0	20
25	1.3	1.8	2.2	2.6	3.1	3.4	3.8	4.1	4.5	4.8	5.1	5.4	5.7	6.0	6.3	6.4	6.5	6.2	0.0	25
30	1.7	2.2	2.7	3.2	3.6	4.1	4.5	5.0	5.4	5.8	6.2	6.6	6.9	7.2	7.5	7.8	7.8	7.5	0.0	30
35	2.0	2.6	3.1	3.7	4.3	4.8	5.3	5.8	6.3	6.8	7.2	7.7	8.1	8.5	8.8	9.1	9.1	8.7	0.0	35
40	2.3	3.0	3.6	4.3	4.9	5.5	6.1	6.7	7.2	7.8	8.3	8.8	9.3	9.8	10.2	10.4	10.5	10.0	0.0	40
45	2.7	3.4	4.1	4.8	5.5	6.2	6.9	7.6	8.2	8.8	9.4	10.0	10.6	11.1	11.5	11.8	11.8	11.3	0.0	45
50	3.0	3.8	4.6	5.4	6.2	7.0	7.8	8.5	9.2	9.9	10.6	11.2	11.9	12.4	12.9	13.2	13.2	12.5	0.0	50
55	3.4	4.3	5.2	6.1	6.9	7.8	8.6	9.5	10.3	11.0	11.8	12.5	13.2	13.8	14.3	14.6	14.6	13.8	0.0	55
60	3.7	4.7	5.7	6.7	7.7	8.6	9.5	10.5	11.3	12.2	13.0	13.8	14.5	15.1	15.7	16.0	16.0	15.1	0.0	60
65	4.1	5.2	6.3	7.4	8.5	9.5	10.5	11.5	12.5	13.4	14.3	15.1	15.9	16.6	17.1	17.4	17.4	16.4	0.0	65
70	4.5	5.7	6.9	8.1	9.3	10.4	11.5	12.6	13.7	14.7	15.6	16.5	17.4	18.1	18.6	18.9	18.8	17.7	0.0	70
75	4.9	6.3	7.6	8.9	10.2	11.4	12.6	13.8	14.9	16.0	17.0	18.0	18.8	19.6	20.1	20.4	20.2	19.0	0.0	75
80	5.4	6.9	8.3	9.7	11.1	12.5	13.8	15.0	16.3	17.4	18.5	19.5	20.4	21.2	21.7	22.0	21.7	20.3	0.0	80
85	5.8	7.5	9.1	10.6	12.1	13.6	15.0	16.4	17.7	18.9	20.0	21.1	22.0	22.8	23.3	23.6	23.2	21.6	0.0	85
90	6.3	8.1	9.9	11.6	13.2	14.8	16.3	17.8	19.2	20.5	21.7	22.8	23.7	24.5	25.0	25.2	24.7	23.0	0.0	90
95	6.9	8.8	10.7	12.6	14.4	16.1	17.7	19.3	20.8	22.2	23.4	24.5	25.5	26.3	26.7	26.8	26.2	24.3	0.0	95
100	7.5	9.6	11.7	13.7	15.7	17.5	19.3	21.0	22.5	24.0	25.3	26.4	27.4	28.1	28.5	28.5	27.8	25.6	0.0	100
105	8.1	10.5	12.8	15.0	17.1	19.1	21.0	22.7	24.4	25.9	27.2	28.4	29.3	30.0	30.3	30.2	29.4	27.0	0.0	105
110	8.8	11.4	13.9	16.3	18.6	20.8	22.8	24.7	26.4	27.9	29.3	30.4	31.3	32.0	32.2	32.0	31.0	28.3	0.0	110
115	9.6	12.5	15.3	17.9	20.4	22.7	24.8	26.8	28.6	30.1	31.5	32.6	33.5	34.0	34.2	33.8	32.6	29.7	0.0	115
120	10.5	13.7	16.7	19.6	22.3	24.8	27.1	29.1	30.9	32.5	33.9	34.9	35.7	36.2	36.2	35.6	34.2	31.0	0.0	120

Contrary Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	0
5	0.1	0.1	0.1	0.0	*0.1*	*0.1*	*0.2*	*0.3*	*0.4*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*	*1.0*	*1.1*	*0.0*	5
10	0.4	0.3	0.1	0.0	*0.1*	*0.3*	*0.4*	*0.6*	*0.7*	*0.9*	*1.1*	*1.2*	*1.4*	*1.6*	*1.8*	*2.0*	*2.1*	*2.1*	*0.0*	10
15	0.7	0.5	0.3	0.0	*0.2*	*0.4*	*0.6*	*0.8*	*1.1*	*1.3*	*1.6*	*1.9*	*2.1*	*2.4*	*2.7*	*2.9*	*3.2*	*3.2*	*0.0*	15
20	1.0	0.7	0.4	0.1	*0.2*	*0.5*	*0.8*	*1.1*	*1.4*	*1.8*	*2.1*	*2.5*	*2.8*	*3.2*	*3.6*	*3.9*	*4.2*	*4.3*	*0.0*	20
25	1.3	0.9	0.5	0.2	*0.2*	*0.6*	*1.0*	*1.4*	*1.8*	*2.2*	*2.7*	*3.1*	*3.6*	*4.1*	*4.5*	*4.9*	*5.3*	*5.4*	*0.0*	25
30	1.7	1.2	0.7	0.2	*0.2*	*0.7*	*1.2*	*1.7*	*2.2*	*2.7*	*3.2*	*3.8*	*4.3*	*4.9*	*5.5*	*6.0*	*6.4*	*6.5*	*0.0*	30
35	2.0	1.4	0.9	0.3	*0.3*	*0.8*	*1.4*	*2.0*	*2.6*	*3.2*	*3.8*	*4.5*	*5.1*	*5.8*	*6.4*	*7.0*	*7.5*	*7.6*	*0.0*	35
40	2.3	1.7	1.0	0.4	*0.3*	*0.9*	*1.6*	*2.3*	*3.0*	*3.7*	*4.4*	*5.2*	*5.9*	*6.7*	*7.4*	*8.1*	*8.6*	*8.7*	*0.0*	40
45	2.7	1.9	1.2	0.4	*0.3*	*1.1*	*1.8*	*2.6*	*3.4*	*4.2*	*5.0*	*5.9*	*6.7*	*7.6*	*8.4*	*9.1*	*9.7*	*9.8*	*0.0*	45
50	3.0	2.2	1.3	0.5	*0.3*	*1.2*	*2.1*	*3.0*	*3.9*	*4.8*	*5.7*	*6.6*	*7.6*	*8.5*	*9.4*	*10.2*	*10.9*	*11.0*	*0.0*	50
55	3.4	2.4	1.5	0.6	*0.4*	*1.3*	*2.3*	*3.3*	*4.3*	*5.3*	*6.4*	*7.4*	*8.5*	*9.5*	*10.5*	*11.4*	*12.0*	*12.1*	*0.0*	55
60	3.7	2.7	1.7	0.6	*0.4*	*1.5*	*2.6*	*3.7*	*4.8*	*6.0*	*7.1*	*8.2*	*9.4*	*10.5*	*11.6*	*12.5*	*13.2*	*13.2*	*0.0*	60
65	4.1	3.0	1.8	0.7	*0.5*	*1.7*	*2.9*	*4.1*	*5.4*	*6.6*	*7.9*	*9.1*	*10.4*	*11.6*	*12.7*	*13.7*	*14.4*	*14.4*	*0.0*	65
70	4.5	3.3	2.0	0.7	*0.6*	*1.9*	*3.2*	*4.6*	*5.9*	*7.3*	*8.7*	*10.1*	*11.4*	*12.7*	*13.9*	*15.0*	*15.7*	*15.6*	*0.0*	70
75	4.9	3.6	2.2	0.8	*0.7*	*2.1*	*3.6*	*5.1*	*6.6*	*8.1*	*9.6*	*11.0*	*12.5*	*13.9*	*15.2*	*16.2*	*17.0*	*16.8*	*0.0*	75
80	5.4	3.9	2.3	0.8	*0.8*	*2.4*	*4.0*	*5.6*	*7.3*	*8.9*	*10.5*	*12.1*	*13.7*	*15.1*	*16.5*	*17.6*	*18.3*	*18.0*	*0.0*	80
85	5.8	4.2	2.5	0.8	*0.9*	*2.7*	*4.5*	*6.2*	*8.0*	*9.8*	*11.5*	*13.2*	*14.9*	*16.4*	*17.8*	*18.9*	*19.6*	*19.2*	*0.0*	85
90	6.3	4.5	2.7	0.8	*1.1*	*3.0*	*5.0*	*6.9*	*8.9*	*10.8*	*12.7*	*14.5*	*16.2*	*17.8*	*19.2*	*20.4*	*20.9*	*20.4*	*0.0*	90
95	6.9	4.9	2.8	0.8	*1.3*	*3.4*	*5.6*	*7.7*	*9.8*	*11.9*	*13.9*	*15.8*	*17.6*	*19.3*	*20.7*	*21.8*	*22.3*	*21.6*	*0.0*	95
100	7.5	5.3	3.0	0.7	*1.6*	*3.9*	*6.2*	*8.5*	*10.8*	*13.0*	*15.2*	*17.2*	*19.2*	*20.9*	*22.3*	*23.4*	*23.8*	*22.9*	*0.0*	100
105	8.1	5.7	3.2	0.7	*1.9*	*4.5*	*7.0*	*9.5*	*12.0*	*14.4*	*16.7*	*18.8*	*20.8*	*22.5*	*24.0*	*25.0*	*25.2*	*24.1*	*0.0*	105
110	8.8	6.1	3.4	0.6	*2.3*	*5.1*	*7.9*	*10.6*	*13.3*	*15.9*	*18.3*	*20.5*	*22.6*	*24.3*	*25.7*	*26.6*	*26.7*	*25.4*	*0.0*	110
115	9.6	6.6	3.5	0.4	*2.7*	*5.8*	*8.9*	*11.9*	*14.8*	*17.5*	*20.1*	*22.4*	*24.5*	*26.2*	*27.5*	*28.3*	*28.3*	*26.7*	*0.0*	115
120	10.5	7.2	3.7	0.2	*3.3*	*6.7*	*10.1*	*13.4*	*16.5*	*19.4*	*22.1*	*24.4*	*26.5*	*28.2*	*29.4*	*30.1*	*29.9*	*28.0*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—15°—Latitude of Receiver

DLo	Latitude of destination—Latitude of transmitter																			DLo
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	
	Same Name																			
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	0.3	0.4	0.4	0.7	0.9	0.9	0.9	1.0	1.0	1.1	1.1	1.2	1.2	1.3	1.3	1.4	1.4	1.3	0.0	5
10	0.7	0.8	1.0	1.3	1.6	1.7	1.8	1.9	2.1	2.2	2.3	2.4	2.5	2.6	2.6	2.7	2.7	2.6	0.0	10
15	1.1	1.4	1.6	2.0	2.3	2.5	2.7	2.9	3.1	3.2	3.4	3.6	3.7	3.9	4.0	4.1	4.1	3.9	0.0	15
20	1.6	1.9	2.2	2.6	3.0	3.3	3.6	3.8	4.1	4.3	4.5	4.8	5.0	5.2	5.3	5.4	5.4	5.1	0.0	20
25	2.1	2.5	2.9	3.3	3.7	4.1	4.4	4.8	5.1	5.4	5.7	6.0	6.2	6.5	6.7	6.8	6.8	6.4	0.0	25
30	2.5	3.0	3.5	4.0	4.4	4.9	5.3	5.7	6.1	6.5	6.9	7.2	7.5	7.8	8.0	8.2	8.2	7.7	0.0	30
35	3.0	3.6	4.1	4.7	5.2	5.7	6.2	6.7	7.2	7.6	8.0	8.4	8.8	9.1	9.4	9.6	9.5	9.0	0.0	35
40	3.5	4.1	4.8	5.4	6.0	6.6	7.2	7.7	8.2	8.8	9.2	9.7	10.1	10.5	10.8	11.0	10.9	10.3	0.0	40
45	4.0	4.7	5.4	6.1	6.8	7.5	8.1	8.7	9.3	9.9	10.5	11.0	11.5	11.9	12.2	12.4	12.3	11.6	0.0	45
50	4.5	5.3	6.1	6.9	7.6	8.4	9.1	9.8	10.5	11.1	11.7	12.3	12.8	13.3	13.7	13.9	13.7	12.9	0.0	50
55	5.1	5.9	6.8	7.7	8.5	9.3	10.1	10.9	11.7	12.4	13.0	13.7	14.2	14.7	15.1	15.3	15.2	14.2	0.0	55
60	5.6	6.6	7.6	8.5	9.4	10.3	11.2	12.1	12.9	13.6	14.4	15.1	15.7	16.2	16.6	16.8	16.6	15.6	0.0	60
65	6.2	7.3	8.3	9.4	10.4	11.4	12.3	13.2	14.1	15.0	15.8	16.5	17.2	17.7	18.1	18.3	18.1	16.9	0.0	65
70	6.8	8.0	9.1	10.3	11.4	12.5	13.5	14.5	15.5	16.4	17.2	18.0	18.7	19.3	19.7	19.8	19.5	18.2	0.0	70
75	7.4	8.7	10.0	11.2	12.4	13.6	14.7	15.8	16.9	17.8	18.8	19.6	20.3	20.9	21.3	21.4	21.0	19.6	0.0	75
80	8.1	9.5	10.9	12.3	13.6	14.8	16.1	17.2	18.3	19.4	20.3	21.2	21.9	22.5	22.9	23.0	22.5	20.9	0.0	80
85	8.8	10.3	11.9	13.4	14.8	16.1	17.5	18.7	19.9	21.0	22.0	22.9	23.7	24.3	24.6	24.6	24.1	22.2	0.0	85
90	9.5	11.2	12.9	14.5	16.1	17.5	18.9	20.3	21.5	22.7	23.7	24.7	25.4	26.0	26.3	26.3	25.6	23.6	0.0	90
95	10.3	12.2	14.0	15.8	17.4	19.0	20.5	22.0	23.3	24.5	25.6	26.5	27.3	27.8	28.1	28.0	27.2	25.0	0.0	95
100	11.2	13.3	15.3	17.2	19.0	20.7	22.3	23.8	25.1	26.4	27.5	28.4	29.2	29.7	29.9	29.7	28.7	26.3	0.0	100
105	12.1	14.4	16.6	18.6	20.6	22.4	24.1	25.7	27.1	28.4	29.5	30.5	31.2	31.7	31.8	31.5	30.3	27.7	0.0	105
110	13.2	15.7	18.1	20.3	22.4	24.3	26.1	27.8	29.3	30.6	31.7	32.6	33.3	33.7	33.7	33.3	32.0	29.0	0.0	110
115	14.4	17.1	19.7	22.1	24.4	26.4	28.3	30.0	31.6	32.9	34.0	34.9	35.5	35.8	35.7	35.1	33.6	30.4	0.0	115
120	15.7	18.7	21.5	24.2	26.6	28.8	30.7	32.5	34.0	35.3	36.4	37.2	37.8	38.0	37.7	36.9	35.3	31.8	0.0	120
DLo	**Contrary Name**																			DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	0
5	0.3	0.3	0.2	0.2	0.1	0.0	0.0	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*	*1.0*	*0.0*	5
10	0.7	0.6	0.5	0.3	0.2	0.0	*0.1*	*0.3*	*0.4*	*0.6*	*0.8*	*1.0*	*1.2*	*1.4*	*1.6*	*1.8*	*2.0*	*2.1*	*0.0*	10
15	1.1	0.9	0.7	0.5	0.3	0.1	*0.2*	*0.4*	*0.7*	*0.9*	*1.2*	*1.5*	*1.8*	*2.1*	*2.4*	*2.7*	*3.0*	*3.1*	*0.0*	15
20	1.6	1.3	1.0	0.7	0.4	0.1	*0.2*	*0.5*	*0.9*	*1.2*	*1.6*	*2.0*	*2.4*	*2.8*	*3.2*	*3.6*	*4.0*	*4.1*	*0.0*	20
25	2.1	1.7	1.3	0.9	0.5	0.1	*0.3*	*0.7*	*1.1*	*1.6*	*2.0*	*2.5*	*3.0*	*3.5*	*4.0*	*4.5*	*5.0*	*5.2*	*0.0*	25
30	2.5	2.1	1.6	1.1	0.7	0.2	*0.3*	*0.8*	*1.3*	*1.9*	*2.4*	*3.0*	*3.6*	*4.2*	*4.9*	*5.5*	*6.0*	*6.2*	*0.0*	30
35	3.0	2.5	1.9	1.4	0.8	0.2	*0.4*	*1.0*	*1.6*	*2.2*	*2.9*	*3.6*	*4.3*	*5.0*	*5.7*	*6.4*	*7.0*	*7.3*	*0.0*	35
40	3.5	2.9	2.2	1.6	0.9	0.3	*0.4*	*1.1*	*1.8*	*2.6*	*3.3*	*4.1*	*5.0*	*5.8*	*6.6*	*7.4*	*8.1*	*8.4*	*0.0*	40
45	4.0	3.3	2.6	1.8	1.1	0.3	*0.5*	*1.3*	*2.1*	*3.0*	*3.8*	*4.7*	*5.7*	*6.6*	*7.5*	*8.4*	*9.1*	*9.4*	*0.0*	45
50	4.5	3.7	2.9	2.1	1.2	0.3	*0.5*	*1.5*	*2.4*	*3.4*	*4.3*	*5.4*	*6.4*	*7.4*	*8.5*	*9.4*	*10.2*	*10.5*	*0.0*	50
55	5.1	4.2	3.2	2.3	1.3	0.4	*0.6*	*1.7*	*2.7*	*3.8*	*4.9*	*6.0*	*7.2*	*8.3*	*9.4*	*10.5*	*11.3*	*11.6*	*0.0*	55
60	5.6	4.6	3.6	2.5	1.5	0.4	*0.7*	*1.9*	*3.0*	*4.2*	*5.5*	*6.7*	*8.0*	*9.2*	*10.4*	*11.6*	*12.5*	*12.7*	*0.0*	60
65	6.2	5.1	3.9	2.8	1.6	0.4	*0.9*	*2.1*	*3.4*	*4.7*	*6.1*	*7.5*	*8.8*	*10.2*	*11.5*	*12.7*	*13.6*	*13.9*	*0.0*	65
70	6.8	5.6	4.3	3.0	1.7	0.4	*1.0*	*2.4*	*3.8*	*5.3*	*6.8*	*8.2*	*9.7*	*11.2*	*12.6*	*13.9*	*14.8*	*15.0*	*0.0*	70
75	7.4	6.1	4.7	3.3	1.8	0.3	*1.2*	*2.7*	*4.3*	*5.9*	*7.5*	*9.1*	*10.7*	*12.3*	*13.8*	*15.1*	*16.0*	*16.2*	*0.0*	75
80	8.1	6.6	5.1	3.5	1.9	0.3	*1.4*	*3.1*	*4.8*	*6.5*	*8.3*	*10.0*	*11.7*	*13.4*	*15.0*	*16.3*	*17.3*	*17.3*	*0.0*	80
85	8.8	7.1	5.5	3.7	2.0	0.2	*1.6*	*3.5*	*5.3*	*7.2*	*9.1*	*11.0*	*12.9*	*14.6*	*16.3*	*17.6*	*18.6*	*18.5*	*0.0*	85
90	9.5	7.7	5.9	4.0	2.1	0.1	*1.9*	*3.9*	*6.0*	*8.0*	*10.1*	*12.1*	*14.1*	*15.9*	*17.6*	*19.0*	*19.9*	*19.7*	*0.0*	90
95	10.3	8.4	6.3	4.3	2.1	0.0	*2.2*	*4.5*	*6.7*	*8.9*	*11.1*	*13.3*	*15.4*	*17.3*	*19.0*	*20.4*	*21.3*	*20.9*	*0.0*	95
100	11.2	9.0	6.8	4.5	2.2	*0.2*	*2.6*	*5.1*	*7.5*	*9.9*	*12.3*	*14.6*	*16.8*	*18.8*	*20.5*	*21.9*	*22.7*	*22.1*	*0.0*	100
105	12.1	9.8	7.3	4.8	2.2	*0.4*	*3.1*	*5.8*	*8.4*	*11.1*	*13.6*	*16.0*	*18.3*	*20.4*	*22.1*	*23.5*	*24.1*	*23.4*	*0.0*	105
110	13.2	10.6	7.9	5.1	2.2	*0.7*	*3.7*	*6.6*	*9.5*	*12.3*	*15.1*	*17.6*	*20.0*	*22.1*	*23.8*	*25.1*	*25.6*	*24.6*	*0.0*	110
115	14.4	11.5	8.5	5.3	2.1	*1.1*	*4.4*	*7.6*	*10.7*	*13.8*	*16.7*	*19.4*	*21.8*	*23.9*	*25.6*	*26.8*	*27.1*	*25.9*	*0.0*	115
120	15.7	12.5	9.1	5.6	2.0	*1.6*	*5.2*	*8.7*	*12.2*	*15.4*	*18.5*	*21.3*	*23.8*	*25.9*	*27.5*	*28.5*	*28.7*	*27.2*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—20°—Latitude of Receiver

DLo	Latitude of destination—Latitude of transmitter 0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
	Same Name																			
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	0.5	0.6	0.6	0.6	0.9	1.1	1.1	1.1	1.2	1.2	1.3	1.3	1.3	1.4	1.4	1.4	1.4	1.3	0.0	5
10	1.0	1.2	1.3	1.4	1.7	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.8	2.8	2.8	2.7	0.0	10
15	1.6	1.8	2.0	2.3	2.6	2.9	3.1	3.3	3.5	3.6	3.7	3.9	4.0	4.1	4.2	4.3	4.2	4.0	0.0	15
20	2.2	2.5	2.8	3.1	3.5	3.8	4.1	4.4	4.6	4.8	5.0	5.2	5.4	5.5	5.6	5.7	5.6	5.3	0.0	20
25	2.8	3.2	3.6	3.9	4.3	4.7	5.1	5.4	5.7	6.0	6.3	6.5	6.7	6.9	7.1	7.1	7.1	6.6	0.0	25
30	3.4	3.9	4.3	4.8	5.2	5.7	6.1	6.5	6.9	7.2	7.5	7.8	8.1	8.3	8.5	8.6	8.5	8.0	0.0	30
35	4.1	4.6	5.1	5.6	6.2	6.7	7.2	7.6	8.0	8.4	8.8	9.2	9.5	9.8	10.0	10.1	9.9	9.3	0.0	35
40	4.7	5.3	5.9	6.5	7.1	7.7	8.2	8.7	9.2	9.7	10.1	10.6	10.9	11.2	11.4	11.5	11.4	10.6	0.0	40
45	5.4	6.1	6.8	7.4	8.1	8.7	9.3	9.9	10.5	11.0	11.5	11.9	12.3	12.7	12.9	13.0	12.8	12.0	0.0	45
50	6.1	6.8	7.6	8.4	9.1	9.8	10.5	11.1	11.7	12.3	12.9	13.4	13.8	14.2	14.4	14.5	14.3	13.3	0.0	50
55	6.8	7.6	8.5	9.3	10.1	10.9	11.6	12.3	13.0	13.7	14.3	14.8	15.3	15.7	16.0	16.0	15.8	14.7	0.0	55
60	7.5	8.5	9.5	10.3	11.2	12.0	12.8	13.6	14.4	15.1	15.7	16.3	16.8	17.3	17.5	17.6	17.2	16.0	0.0	60
65	8.3	9.3	10.4	11.3	12.3	13.2	14.1	15.0	15.8	16.5	17.2	17.9	18.4	18.9	19.1	19.2	18.8	17.4	0.0	65
70	9.1	10.2	11.4	12.4	13.5	14.5	15.4	16.4	17.2	18.1	18.8	19.5	20.0	20.5	20.8	20.7	20.3	18.7	0.0	70
75	9.9	11.2	12.4	13.6	14.7	15.8	16.8	17.8	18.8	19.6	20.4	21.1	21.7	22.2	22.4	22.4	21.8	20.1	0.0	75
80	10.8	12.2	13.5	14.8	16.0	17.2	18.3	19.4	20.4	21.3	22.1	22.8	23.5	23.9	24.1	24.0	23.4	21.5	0.0	80
85	11.7	13.2	14.7	16.1	17.4	18.7	19.9	21.0	22.0	23.0	23.9	24.6	25.2	25.7	25.9	25.7	24.9	22.9	0.0	85
90	12.7	14.4	16.0	17.5	18.9	20.2	21.5	22.7	23.8	24.8	25.7	26.5	27.1	27.5	27.6	27.4	26.5	24.2	0.0	90
95	13.8	15.6	17.3	18.9	20.5	21.9	23.3	24.5	25.7	26.7	27.7	28.4	29.0	29.4	29.5	29.1	28.1	25.6	0.0	95
100	14.9	16.9	18.8	20.5	22.2	23.7	25.2	26.5	27.7	28.8	29.7	30.4	31.0	31.3	31.3	30.9	29.7	27.0	0.0	100
105	16.2	18.3	20.4	22.3	24.0	25.7	27.2	28.6	29.8	30.9	31.8	32.5	33.1	33.3	33.2	32.7	31.3	28.4	0.0	105
110	17.6	19.9	22.1	24.2	26.0	27.8	29.4	30.8	32.0	33.1	34.0	34.7	35.2	35.4	35.2	34.5	33.0	29.8	0.0	110
115	19.1	21.6	24.0	26.2	28.2	30.1	31.7	33.2	34.4	35.5	36.4	37.0	37.4	37.5	37.2	36.4	34.6	31.2	0.0	115
120	20.8	23.6	26.2	28.5	30.7	32.6	34.3	35.7	37.0	38.1	38.9	39.4	39.7	39.7	39.3	38.2	36.3	32.6	0.0	120
DLo	**Contrary Name**																			DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	0
5	0.5	0.5	0.4	0.3	0.3	0.2	0.1	0.0	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*	*0.0*	5
10	1.0	0.9	0.8	0.6	0.5	0.4	0.2	0.0	*0.1*	*0.3*	*0.5*	*0.7*	*0.9*	*1.1*	*1.4*	*1.6*	*1.8*	*2.0*	*0.0*	10
15	1.6	1.4	1.2	1.0	0.8	0.5	0.3	0.1	*0.2*	*0.5*	*0.8*	*1.1*	*1.4*	*1.7*	*2.1*	*2.4*	*2.8*	*3.0*	*0.0*	15
20	2.2	1.9	1.6	1.3	1.0	0.7	0.4	0.1	*0.3*	*0.7*	*1.1*	*1.5*	*1.9*	*2.3*	*2.8*	*3.3*	*3.7*	*4.0*	*0.0*	20
25	2.8	2.4	2.1	1.7	1.3	0.9	0.5	0.1	*0.4*	*0.8*	*1.3*	*1.8*	*2.4*	*2.9*	*3.5*	*4.1*	*4.6*	*4.9*	*0.0*	25
30	3.4	3.0	2.5	2.1	1.6	1.1	0.6	0.1	*0.5*	*1.0*	*1.6*	*2.2*	*2.9*	*3.6*	*4.2*	*4.9*	*5.6*	*6.0*	*0.0*	30
35	4.1	3.5	3.0	2.5	1.9	1.3	0.7	0.1	*0.5*	*1.2*	*1.9*	*2.6*	*3.4*	*4.2*	*5.0*	*5.8*	*6.5*	*7.0*	*0.0*	35
40	4.7	4.1	3.5	2.8	2.2	1.5	0.8	0.1	*0.6*	*1.4*	*2.2*	*3.1*	*4.0*	*4.9*	*5.8*	*6.7*	*7.5*	*8.0*	*0.0*	40
45	5.4	4.7	4.0	3.2	2.5	1.7	0.9	0.1	*0.7*	*1.6*	*2.6*	*3.5*	*4.5*	*5.6*	*6.6*	*7.6*	*8.5*	*9.0*	*0.0*	45
50	6.1	5.3	4.5	3.7	2.8	1.9	1.0	0.1	*0.9*	*1.9*	*2.9*	*4.0*	*5.1*	*6.3*	*7.4*	*8.6*	*9.5*	*10.1*	*0.0*	50
55	6.8	5.9	5.0	4.1	3.1	2.1	1.1	0.1	*1.0*	*2.2*	*3.3*	*4.5*	*5.8*	*7.0*	*8.3*	*9.5*	*10.6*	*11.1*	*0.0*	55
60	7.5	6.5	5.5	4.5	3.4	2.3	1.2	0.0	*1.2*	*2.4*	*3.7*	*5.1*	*6.4*	*7.8*	*9.2*	*10.5*	*11.7*	*12.2*	*0.0*	60
65	8.3	7.2	6.1	4.9	3.8	2.5	1.3	0.0	*1.4*	*2.8*	*4.2*	*5.7*	*7.2*	*8.7*	*10.2*	*11.6*	*12.8*	*13.3*	*0.0*	65
70	9.1	7.9	6.6	5.4	4.1	2.7	1.3	*0.1*	*1.6*	*3.1*	*4.7*	*6.3*	*8.0*	*9.6*	*11.2*	*12.7*	*13.9*	*14.4*	*0.0*	70
75	9.9	8.6	7.2	5.8	4.4	2.9	1.4	*0.2*	*1.9*	*3.5*	*5.3*	*7.0*	*8.8*	*10.5*	*12.3*	*13.8*	*15.1*	*15.5*	*0.0*	75
80	10.8	9.3	7.9	6.3	4.7	3.1	1.4	*0.4*	*2.2*	*4.0*	*5.9*	*7.8*	*9.7*	*11.6*	*13.4*	*15.0*	*16.3*	*16.7*	*0.0*	80
85	11.7	10.1	8.5	6.8	5.1	3.2	1.4	*0.6*	*2.5*	*4.5*	*6.6*	*8.6*	*10.7*	*12.7*	*14.6*	*16.3*	*17.5*	*17.8*	*0.0*	85
90	12.7	11.0	9.2	7.3	5.4	3.4	1.3	*0.8*	*2.9*	*5.1*	*7.4*	*9.6*	*11.8*	*13.9*	*15.8*	*17.6*	*18.8*	*19.0*	*0.0*	90
95	13.8	11.9	9.9	7.9	5.7	3.5	1.3	*1.1*	*3.4*	*5.8*	*8.2*	*10.6*	*12.9*	*15.2*	*17.2*	*18.9*	*20.1*	*20.2*	*0.0*	95
100	14.9	12.9	10.7	8.4	6.1	3.7	1.1	*1.4*	*4.0*	*6.6*	*9.2*	*11.8*	*14.2*	*16.5*	*18.6*	*20.4*	*21.5*	*21.4*	*0.0*	100
105	16.2	13.9	11.5	9.0	6.4	3.7	1.0	*1.8*	*4.7*	*7.5*	*10.3*	*13.0*	*15.6*	*18.0*	*20.2*	*21.9*	*22.9*	*22.6*	*0.0*	105
110	17.6	15.1	12.4	9.7	6.8	3.8	0.8	*2.3*	*5.5*	*8.5*	*11.6*	*14.4*	*17.2*	*19.7*	*21.8*	*23.5*	*24.4*	*23.8*	*0.0*	110
115	19.1	16.3	13.4	10.4	7.1	3.8	0.5	*3.0*	*6.4*	*9.7*	*13.0*	*16.0*	*18.9*	*21.4*	*23.5*	*25.1*	*25.9*	*25.1*	*0.0*	115
120	20.8	17.8	14.5	11.1	7.5	3.8	0.0	*3.8*	*7.5*	*11.2*	*14.6*	*17.9*	*20.8*	*23.3*	*25.4*	*26.8*	*27.4*	*26.4*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—25°—Latitude of Receiver

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
	Latitude of destination—Latitude of transmitter																			
	Same Name																			
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	0.7	0.7	0.8	0.8	0.8	1.1	1.3	1.3	1.3	1.3	1.4	1.4	1.4	1.5	1.5	1.5	1.5	1.4	0.0	5
10	1.4	1.5	1.6	1.7	1.9	2.1	2.4	2.5	2.6	2.7	2.7	2.8	2.9	2.9	3.0	3.0	2.9	2.7	0.0	10
15	2.1	2.3	2.5	2.7	2.9	3.2	3.5	3.7	3.8	4.0	4.1	4.2	4.3	4.4	4.5	4.5	4.4	4.1	0.0	15
20	2.8	3.1	3.4	3.6	3.9	4.3	4.6	4.9	5.1	5.3	5.5	5.6	5.8	5.9	6.0	6.0	5.9	5.5	0.0	20
25	3.6	3.9	4.3	4.6	5.0	5.4	5.7	6.0	6.3	6.6	6.8	7.0	7.2	7.4	7.5	7.5	7.3	6.8	0.0	25
30	4.4	4.8	5.2	5.6	6.0	6.5	6.9	7.3	7.6	7.9	8.2	8.5	8.7	8.9	9.0	9.0	8.8	8.2	0.0	30
35	5.1	5.6	6.1	6.6	7.1	7.6	8.1	8.5	8.9	9.3	9.6	9.9	10.2	10.4	10.5	10.5	10.3	9.6	0.0	35
40	6.0	6.5	7.1	7.7	8.2	8.8	9.3	9.8	10.2	10.6	11.0	11.4	11.7	11.9	12.1	12.1	11.8	11.0	0.0	40
45	6.8	7.5	8.1	8.7	9.4	9.9	10.5	11.1	11.6	12.1	12.5	12.9	13.2	13.5	13.6	13.6	13.3	12.3	0.0	45
50	7.7	8.4	9.1	9.8	10.5	11.2	11.8	12.4	13.0	13.5	14.0	14.4	14.8	15.1	15.2	15.2	14.8	13.7	0.0	50
55	8.5	9.4	10.2	10.9	11.7	12.4	13.1	13.8	14.4	15.0	15.5	16.0	16.4	16.7	16.8	16.8	16.4	15.1	0.0	55
60	9.5	10.4	11.3	12.1	12.9	13.7	14.5	15.2	15.9	16.5	17.1	17.6	18.0	18.3	18.5	18.4	17.9	16.5	0.0	60
65	10.4	11.4	12.4	13.3	14.2	15.1	15.9	16.7	17.4	18.1	18.7	19.2	19.7	20.0	20.1	20.0	19.4	17.9	0.0	65
70	11.4	12.5	13.6	14.6	15.6	16.5	17.4	18.2	19.0	19.7	20.4	20.9	21.4	21.7	21.8	21.7	21.0	19.3	0.0	70
75	12.4	13.6	14.8	15.9	17.0	18.0	18.9	19.8	20.6	21.4	22.1	22.7	23.1	23.4	23.5	23.3	22.6	20.7	0.0	75
80	13.5	14.9	16.1	17.3	18.5	19.5	20.5	21.5	22.4	23.2	23.9	24.5	24.9	25.2	25.3	25.0	24.2	22.1	0.0	80
85	14.7	16.1	17.5	18.8	20.0	21.2	22.3	23.3	24.2	25.0	25.7	26.3	26.8	27.1	27.1	26.7	25.8	23.5	0.0	85
90	15.9	17.5	19.0	20.4	21.7	22.9	24.1	25.1	26.1	26.9	27.7	28.3	28.7	29.0	28.9	28.5	27.4	24.9	0.0	90
95	17.2	19.0	20.6	22.1	23.5	24.8	26.0	27.1	28.1	28.9	29.7	30.3	30.7	30.9	30.8	30.3	29.0	26.3	0.0	95
100	18.7	20.5	22.3	23.9	25.4	26.7	28.0	29.2	30.2	31.1	31.8	32.4	32.8	32.9	32.7	32.1	30.7	27.7	0.0	100
105	20.2	22.2	24.1	25.8	27.4	28.9	30.2	31.4	32.4	33.3	34.0	34.5	34.9	34.9	34.7	33.9	32.3	29.1	0.0	105
110	21.9	24.1	26.1	27.9	29.6	31.1	32.5	33.7	34.7	35.6	36.3	36.8	37.1	37.1	36.7	35.8	34.0	30.5	0.0	110
115	23.7	26.1	28.3	30.2	32.0	33.6	35.0	36.2	37.2	38.1	38.7	39.2	39.3	39.2	38.7	37.6	35.7	32.0	0.0	115
120	25.8	28.3	30.7	32.7	34.6	36.2	37.6	38.8	39.9	40.7	41.3	41.6	41.7	41.4	40.8	39.6	37.4	33.4	0.0	120
DLo	**Contrary Name**																			DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	0
5	0.7	0.6	0.6	0.5	0.4	0.3	0.3	0.2	0.1	0.0	*0.1*	*0.2*	*0.3*	*0.4*	*0.6*	*0.7*	*0.8*	*0.9*	*0.0*	5
10	1.4	1.2	1.1	1.0	0.8	0.7	0.5	0.3	0.2	0.0	*0.2*	*0.4*	*0.7*	*0.9*	*1.2*	*1.4*	*1.7*	*1.9*	*0.0*	10
15	2.1	1.9	1.7	1.5	1.3	1.0	0.8	0.5	0.2	*0.0*	*0.3*	*0.7*	*1.0*	*1.4*	*1.7*	*2.1*	*2.5*	*2.8*	*0.0*	15
20	2.8	2.5	2.3	2.0	1.7	1.4	1.0	0.7	0.3	*0.1*	*0.5*	*0.9*	*1.4*	*1.8*	*2.3*	*2.9*	*3.4*	*3.8*	*0.0*	20
25	3.6	3.2	2.9	2.5	2.1	1.7	1.3	0.9	0.4	*0.1*	*0.6*	*1.1*	*1.7*	*2.3*	*3.0*	*3.6*	*4.2*	*4.7*	*0.0*	25
30	4.4	3.9	3.5	3.0	2.6	2.1	1.6	1.0	0.5	*0.1*	*0.7*	*1.4*	*2.1*	*2.8*	*3.6*	*4.4*	*5.1*	*5.7*	*0.0*	30
35	5.1	4.6	4.1	3.6	3.0	2.4	1.8	1.2	0.5	*0.1*	*0.9*	*1.7*	*2.5*	*3.3*	*4.2*	*5.1*	*6.0*	*6.6*	*0.0*	35
40	6.0	5.4	4.8	4.1	3.5	2.8	2.1	1.4	0.6	*0.2*	*1.0*	*1.9*	*2.9*	*3.9*	*4.9*	*5.9*	*6.9*	*7.6*	*0.0*	40
45	6.8	6.1	5.4	4.7	4.0	3.2	2.4	1.5	0.7	*0.3*	*1.2*	*2.3*	*3.3*	*4.4*	*5.6*	*6.8*	*7.9*	*8.6*	*0.0*	45
50	7.7	6.9	6.1	5.3	4.4	3.6	2.7	1.7	0.7	*0.3*	*1.4*	*2.6*	*3.8*	*5.0*	*6.3*	*7.6*	*8.8*	*9.6*	*0.0*	50
55	8.5	7.7	6.8	5.9	4.9	4.0	2.9	1.9	0.7	*0.4*	*1.7*	*3.0*	*4.3*	*5.7*	*7.1*	*8.5*	*9.8*	*10.6*	*0.0*	55
60	9.5	8.5	7.5	6.5	5.4	4.4	3.2	2.0	0.8	*0.5*	*1.9*	*3.3*	*4.8*	*6.4*	*7.9*	*9.4*	*10.8*	*11.6*	*0.0*	60
65	10.4	9.4	8.3	7.1	6.0	4.8	3.5	2.1	0.8	*0.7*	*2.2*	*3.8*	*5.4*	*7.1*	*8.8*	*10.4*	*11.8*	*12.7*	*0.0*	65
70	11.4	10.3	9.1	7.8	6.5	5.1	3.7	2.3	0.7	*0.9*	*2.5*	*4.3*	*6.1*	*7.9*	*9.7*	*11.4*	*12.9*	*13.8*	*0.0*	70
75	12.4	11.2	9.9	8.5	7.0	5.6	4.0	2.4	0.7	*1.1*	*2.9*	*4.8*	*6.7*	*8.7*	*10.6*	*12.5*	*14.0*	*14.8*	*0.0*	75
80	13.5	12.1	10.7	9.2	7.6	6.0	4.2	2.4	0.6	*1.4*	*3.4*	*5.4*	*7.5*	*9.6*	*11.7*	*13.6*	*15.2*	*15.9*	*0.0*	80
85	14.7	13.2	11.6	9.9	8.2	6.4	4.5	2.5	0.5	*1.7*	*3.9*	*6.1*	*8.3*	*10.6*	*12.8*	*14.8*	*16.4*	*17.1*	*0.0*	85
90	15.9	14.3	12.5	10.7	8.8	6.8	4.7	2.5	0.3	*2.0*	*4.4*	*6.8*	*9.3*	*11.6*	*13.9*	*16.0*	*17.6*	*18.2*	*0.0*	90
95	17.2	15.4	13.5	11.5	9.4	7.2	4.9	2.5	0.1	*2.5*	*5.1*	*7.7*	*10.3*	*12.8*	*15.2*	*17.3*	*18.9*	*19.4*	*0.0*	95
100	18.7	16.7	14.6	12.4	10.1	7.7	5.1	2.5	*0.2*	*3.0*	*5.8*	*8.7*	*11.4*	*14.1*	*16.5*	*18.7*	*20.2*	*20.6*	*0.0*	100
105	20.2	18.1	15.8	13.3	10.8	8.1	5.3	2.4	*0.6*	*3.7*	*6.7*	*9.7*	*12.7*	*15.5*	*18.0*	*20.1*	*21.6*	*21.8*	*0.0*	105
110	21.9	19.6	17.0	14.4	11.5	8.5	5.4	2.2	*1.1*	*4.4*	*7.7*	*11.0*	*14.1*	*17.0*	*19.6*	*21.7*	*23.1*	*23.0*	*0.0*	110
115	23.7	21.2	18.4	15.5	12.3	9.0	5.5	2.0	*1.7*	*5.3*	*8.9*	*12.4*	*15.7*	*18.7*	*21.3*	*23.3*	*24.5*	*24.2*	*0.0*	115
120	25.8	23.0	20.0	16.7	13.2	9.5	5.6	1.6	*2.4*	*6.4*	*10.3*	*14.0*	*17.5*	*20.5*	*23.1*	*25.0*	*26.1*	*25.5*	*0.0*	120

Great-circle sailing

Latitude departure	Destination	Correction sign
N	Eastward	−
N	Westward	+
S	Eastward	+
S	Westward	−

Radio bearings

Latitude receiver	Transmitter	Correction sign
N	Eastward	+
N	Westward	−
S	Eastward	−
S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—30°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
	Same Name																			
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	0.8	0.9	1.0	1.0	1.0	1.0	1.3	1.4	1.4	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.5	1.4	0.0	5
10	1.7	1.8	1.9	2.0	2.1	2.3	2.5	2.7	2.8	2.9	3.0	3.0	3.1	3.1	3.1	3.1	3.0	2.8	0.0	10
15	2.6	2.8	2.9	3.1	3.3	3.5	3.8	4.0	4.2	4.3	4.4	4.5	4.6	4.7	4.7	4.7	4.6	4.2	0.0	15
20	3.5	3.7	4.0	4.2	4.5	4.7	5.0	5.3	5.6	5.8	5.9	6.0	6.2	6.3	6.3	6.3	6.1	5.6	0.0	20
25	4.4	4.7	5.0	5.3	5.7	6.0	6.3	6.7	6.9	7.2	7.4	7.6	7.7	7.8	7.9	7.8	7.6	7.0	0.0	25
30	5.3	5.7	6.1	6.5	6.9	7.3	7.6	8.0	8.3	8.6	8.9	9.1	9.3	9.4	9.5	9.4	9.2	8.5	0.0	30
35	6.3	6.7	7.2	7.7	8.1	8.5	9.0	9.4	9.7	10.1	10.4	10.7	10.9	11.0	11.1	11.0	10.7	9.9	0.0	35
40	7.2	7.8	8.3	8.8	9.3	9.9	10.3	10.8	11.2	11.6	11.9	12.2	12.5	12.6	12.7	12.6	12.3	11.3	0.0	40
45	8.2	8.9	9.5	10.1	10.6	11.2	11.7	12.2	12.7	13.1	13.5	13.8	14.1	14.3	14.3	14.2	13.8	12.7	0.0	45
50	9.3	10.0	10.7	11.3	11.9	12.6	13.1	13.7	14.2	14.7	15.1	15.4	15.7	15.9	16.0	15.9	15.4	14.1	0.0	50
55	10.3	11.1	11.9	12.6	13.3	14.0	14.6	15.2	15.8	16.3	16.7	17.1	17.4	17.6	17.7	17.5	17.0	15.5	0.0	55
60	11.4	12.3	13.1	13.9	14.7	15.4	16.1	16.8	17.4	17.9	18.4	18.8	19.1	19.3	19.4	19.2	18.6	17.0	0.0	60
65	12.6	13.5	14.4	15.3	16.1	16.9	17.7	18.4	19.0	19.6	20.1	20.6	20.9	21.1	21.1	20.9	20.1	18.4	0.0	65
70	13.8	14.8	15.8	16.8	17.7	18.5	19.3	20.0	20.7	21.3	21.9	22.3	22.7	22.9	22.9	22.6	21.8	19.8	0.0	70
75	15.0	16.2	17.3	18.3	19.2	20.2	21.0	21.8	22.5	23.2	23.7	24.2	24.5	24.7	24.7	24.3	23.4	21.3	0.0	75
80	16.3	17.6	18.7	19.9	20.9	21.9	22.8	23.6	24.4	25.0	25.6	26.1	26.4	26.6	26.5	26.0	25.0	22.7	0.0	80
85	17.7	19.1	20.3	21.5	22.6	23.7	24.6	25.5	26.3	27.0	27.6	28.0	28.3	28.5	28.3	27.8	26.7	24.1	0.0	85
90	19.2	20.6	22.0	23.3	24.5	25.6	26.6	27.5	28.3	29.0	29.6	30.1	30.3	30.4	30.2	29.6	28.3	25.6	0.0	90
95	20.7	22.3	23.8	25.2	26.4	27.6	28.6	29.6	30.4	31.1	31.7	32.1	32.4	32.4	32.2	31.4	30.0	27.0	0.0	95
100	22.4	24.1	25.7	27.2	28.5	29.7	30.8	31.8	32.6	33.3	33.9	34.3	34.5	34.5	34.1	33.3	31.7	28.5	0.0	100
105	24.2	26.1	27.8	29.3	30.7	32.0	33.1	34.1	34.9	35.6	36.2	36.5	36.7	36.6	36.1	35.1	33.4	29.9	0.0	105
110	26.2	28.2	30.0	31.6	33.1	34.4	35.5	36.5	37.4	38.0	38.5	38.9	38.9	38.7	38.1	37.0	35.1	31.3	0.0	110
115	28.4	30.5	32.4	34.1	35.6	37.0	38.1	39.1	39.9	40.6	41.0	41.2	41.2	40.9	40.2	38.9	36.8	32.8	0.0	115
120	30.7	33.0	35.0	36.8	38.4	39.7	40.9	41.9	42.6	43.2	43.6	43.7	43.6	43.2	42.3	40.9	38.5	34.2	0.0	120
DLo	**Contrary Name**																			DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	0
5	0.8	0.8	0.7	0.7	0.6	0.5	0.4	0.3	0.3	0.1	0.0	*0.1*	*0.2*	*0.3*	*0.5*	*0.6*	*0.7*	*0.9*	*0.0*	5
10	1.7	1.6	1.4	1.3	1.2	1.0	0.8	0.7	0.5	0.3	0.1	*0.1*	*0.4*	*0.6*	*0.9*	*1.2*	*1.5*	*1.8*	*0.0*	10
15	2.6	2.4	2.2	2.0	1.8	1.5	1.3	1.0	0.7	0.4	0.1	*0.2*	*0.6*	*1.0*	*1.4*	*1.8*	*2.3*	*2.6*	*0.0*	15
20	3.5	3.2	2.9	2.6	2.4	2.0	1.7	1.3	1.0	0.6	0.1	*0.3*	*0.8*	*1.3*	*1.9*	*2.5*	*3.1*	*3.5*	*0.0*	20
25	4.4	4.0	3.7	3.3	3.0	2.5	2.1	1.7	1.2	0.7	0.2	*0.4*	*1.0*	*1.7*	*2.4*	*3.1*	*3.8*	*4.4*	*0.0*	25
30	5.3	4.9	4.5	4.0	3.6	3.1	2.6	2.0	1.5	0.9	0.2	*0.5*	*1.2*	*2.0*	*2.9*	*3.8*	*4.6*	*5.3*	*0.0*	30
35	6.3	5.8	5.3	4.7	4.2	3.6	3.0	2.4	1.7	1.0	0.2	*0.6*	*1.5*	*2.4*	*3.4*	*4.4*	*5.4*	*6.3*	*0.0*	35
40	7.2	6.7	6.1	5.5	4.8	4.2	3.5	2.7	1.9	1.1	0.2	*0.7*	*1.8*	*2.8*	*4.0*	*5.1*	*6.3*	*7.2*	*0.0*	40
45	8.2	7.6	6.9	6.2	5.5	4.7	3.9	3.1	2.2	1.2	0.2	*0.9*	*2.0*	*3.3*	*4.5*	*5.9*	*7.1*	*8.1*	*0.0*	45
50	9.3	8.5	7.8	7.0	6.2	5.3	4.4	3.4	2.4	1.3	0.2	*1.1*	*2.4*	*3.7*	*5.1*	*6.6*	*8.0*	*9.1*	*0.0*	50
55	10.3	9.5	8.7	7.8	6.8	5.9	4.8	3.8	2.6	1.4	0.1	*1.3*	*2.7*	*4.2*	*5.8*	*7.4*	*8.9*	*10.1*	*0.0*	55
60	11.4	10.5	9.6	8.6	7.5	6.5	5.3	4.1	2.8	1.5	0.0	*1.5*	*3.1*	*4.8*	*6.5*	*8.2*	*9.9*	*11.0*	*0.0*	60
65	12.6	11.6	10.5	9.4	8.3	7.1	5.8	4.4	3.0	1.5	*0.1*	*1.8*	*3.5*	*5.3*	*7.2*	*9.1*	*10.8*	*12.0*	*0.0*	65
70	13.8	12.7	11.5	10.3	9.0	7.7	6.3	4.8	3.2	1.5	*0.2*	*2.1*	*4.0*	*6.0*	*8.0*	*10.0*	*11.8*	*13.1*	*0.0*	70
75	15.0	13.8	12.5	11.2	9.8	8.3	6.8	5.1	3.3	1.5	*0.4*	*2.4*	*4.5*	*6.7*	*8.9*	*11.0*	*12.9*	*14.1*	*0.0*	75
80	16.3	15.0	13.6	12.1	10.6	9.0	7.2	5.4	3.5	1.5	*0.6*	*2.8*	*5.1*	*7.4*	*9.8*	*12.0*	*14.0*	*15.2*	*0.0*	80
85	17.7	16.3	14.8	13.1	11.4	9.6	7.7	5.7	3.6	1.4	*0.9*	*3.3*	*5.8*	*8.3*	*10.8*	*13.1*	*15.1*	*16.3*	*0.0*	85
90	19.2	17.6	15.9	14.2	12.3	10.3	8.2	6.0	3.7	1.3	*1.3*	*3.9*	*6.5*	*9.2*	*11.8*	*14.3*	*16.3*	*17.4*	*0.0*	90
95	20.7	19.0	17.2	15.3	13.2	11.1	8.8	6.3	3.8	1.1	*1.7*	*4.5*	*7.4*	*10.2*	*13.0*	*15.5*	*17.6*	*18.5*	*0.0*	95
100	22.4	20.6	18.6	16.5	14.2	11 8	9.3	6.6	3.8	0.8	*2.2*	*5.3*	*8.3*	*11.4*	*14.2*	*16.8*	*18.9*	*19.7*	*0.0*	100
105	24.2	22.2	20.1	17.8	15.3	12.6	9.8	6.8	3.7	0.5	*2.8*	*6.1*	*9.4*	*12.6*	*15.6*	*18.2*	*20.2*	*20.9*	*0.0*	105
110	26.2	24.0	21.7	19.1	16.4	13.5	10.3	7.0	3.6	0.1	*3.6*	*7.2*	*10.7*	*14.0*	*17.1*	*19.7*	*21.6*	*22.1*	*0.0*	110
115	28.4	26.0	23.4	20.6	17.6	14.4	10.9	7.2	3.4	*0.5*	*4.5*	*8.4*	*12.1*	*15.6*	*18.7*	*21.3*	*23.1*	*23.3*	*0.0*	115
120	30.7	28.2	25.4	22.3	19.0	15.3	11.4	7.3	3.1	*1.2*	*5.6*	*9.7*	*13.7*	*17.3*	*20.5*	*23.0*	*24.6*	*24.6*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—35°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

Same Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	1.0	1.1	1.1	1.2	1.2	1.2	1.3	1.5	1.6	1.6	1.6	1.6	1.6	1.7	1.6	1.6	1.6	1.5	0.0	5
10	2.0	2.2	2.3	2.4	2.4	2.5	2.7	2.9	3.1	3.2	3.2	3.2	3.3	3.3	3.3	3.3	3.2	2.9	0.0	10
15	3.1	3.2	3.4	3.6	3.7	3.9	4.1	4.3	4.5	4.7	4.8	4.9	4.9	5.0	5.0	4.9	4.8	4.4	0.0	15
20	4.1	4.4	4.6	4.8	5.0	5.3	5.5	5.8	6.0	6.2	6.4	6.5	6.6	6.6	6.6	6.5	6.3	5.8	0.0	20
25	5.2	5.5	5.8	6.1	6.4	6.7	7.0	7.3	7.5	7.8	8.0	8.1	8.2	8.3	8.3	8.2	7.9	7.3	0.0	25
30	6.3	6.7	7.0	7.4	7.7	8.1	8.4	8.7	9.0	9.3	9.6	9.7	9.9	10.0	10.0	9.8	9.5	8.7	0.0	30
35	7.4	7.9	8.3	8.7	9.1	9.5	9.9	10.3	10.6	10.9	11.2	11.4	11.6	11.6	11.6	11.5	11.1	10.2	0.0	35
40	8.6	9.1	9.6	10.0	10.5	11.0	11.4	11.8	12.2	12.5	12.8	13.1	13.2	13.3	13.3	13.2	12.7	11.6	0.0	40
45	9.7	10.3	10.9	11.4	11.9	12.5	12.9	13.4	13.8	14.2	14.5	14.8	15.0	15.1	15.1	14.9	14.3	13.1	0.0	45
50	10.9	11.6	12.2	12.8	13.4	14.0	14.5	15.0	15.4	15.8	16.2	16.5	16.7	16.8	16.8	16.6	16.0	14.6	0.0	50
55	12.2	12.9	13.6	14.3	14.9	15.5	16.1	16.6	17.1	17.6	17.9	18.2	18.5	18.6	18.5	18.3	17.6	16.0	0.0	55
60	13.5	14.3	15.1	15.8	16.5	17.2	17.8	18.3	18.9	19.3	19.7	20.0	20.3	20.4	20.3	20.0	19.2	17.5	0.0	60
65	14.8	15.7	16.5	17.3	18.1	18.8	19.5	20.1	20.6	21.1	21.6	21.9	22.1	22.2	22.1	21.7	20.9	19.0	0.0	65
70	16.2	17.2	18.1	19.0	19.8	20.5	21.3	21.9	22.5	23.0	23.4	23.8	24.0	24.1	24.0	23.5	22.5	20.4	0.0	70
75	17 6	18.7	19.7	20.6	21.5	22.3	23.1	23.8	24.4	24.9	25.4	25.7	25.9	26.0	25.8	25.3	24.2	21.9	0.0	75
80	19.2	20.3	21.4	22.4	23.3	24.2	25.0	25.7	26.3	26.9	27.4	27.7	27.9	27.9	27.7	27.1	25.9	23.4	0.0	80
85	20.8	22.0	23.2	24.2	25.2	26.2	27.0	27.7	28.4	29.0	29.4	29.7	29.9	29.9	29.6	28.9	27.6	24.8	0.0	85
90	22.5	23.8	25.0	26.2	27.3	28.2	29.1	29.8	30.5	31.1	31.5	31.8	32.0	31.9	31.5	30.8	29.3	26.3	0.0	90
95	24.3	25.7	27.0	28.3	29.4	30.4	31.3	32.1	32.7	33.3	33.7	34.0	34.1	34.0	33.5	32.6	31.0	27.8	0.0	95
100	26.2	27.7	29.1	30.4	31.6	32.6	33.6	34.4	35.0	35.6	36.0	36.2	36.3	36.1	35.5	34.5	32.7	29.2	0.0	100
105	28.2	29.9	31.4	32.8	34.0	35.0	36.0	36.8	37.4	38.0	38.3	38.5	38.5	38.2	37.6	36.4	34.4	30.7	0.0	105
110	30.5	32.2	33.8	35.2	36.5	37.6	38.5	39.3	40.0	40.5	40.8	40.9	40.8	40.4	39.6	38.3	36.2	32.2	0.0	110
115	32.9	34.8	36.4	37.9	39.2	40.3	41.2	42.0	42.6	43.0	43.3	43.3	43.1	42.6	41.7	40.3	37.9	33.6	0.0	115
120	35.6	37.5	39.3	40.8	42.1	43.2	44.1	44.8	45.4	45.7	45.9	45.9	45.5	44.9	43.9	42.3	39.7	35.1	0.0	120

Contrary Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	0
5	1.0	1.0	0.9	0.8	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0	*0.2*	*0.3*	*0.5*	*0.7*	*0.8*	*0.0*	5
10	2.0	1.9	1.8	1.7	1.5	1.4	1.2	1.0	0.8	0.6	0.4	0.2	*0.1*	*0.4*	*0.7*	*1.0*	*1.3*	*1.6*	*0.0*	10
15	3.1	2.9	2.7	2.5	2.3	2.0	1.8	1.5	1.2	0.9	0.6	0.3	*0.1*	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*	*0.0*	15
20	4.1	3.9	3.6	3.3	3.0	2.7	2.4	2.0	1.7	1.3	0.8	0.3	*0.2*	*0.7*	*1.4*	*2.0*	*2.7*	*3.3*	*0.0*	20
25	5.2	4.9	4.5	4.2	3.8	3.4	3.0	2.6	2.1	1.6	1.0	0.4	*0.3*	*1.0*	*1.7*	*2.5*	*3.4*	*4.1*	*0.0*	25
30	6.3	5.9	5.5	5.1	4.6	4.1	3.6	3.1	2.5	1.9	1.2	0.5	*0.3*	*1.2*	*2.1*	*3.1*	*4.1*	*5.0*	*0.0*	30
35	7.4	6.9	6.5	6.0	5.4	4.8	4.2	3.6	2.9	2.2	1.4	0.5	*0.4*	*1.4*	*2.5*	*3.6*	*4.8*	*5.9*	*0.0*	35
40	8.6	8.0	7.5	6.9	6.2	5.6	4.9	4.1	3.3	2.5	1.6	0.6	*0.5*	*1.7*	*2.9*	*4.2*	*5.6*	*6.7*	*0.0*	40
45	9.7	9.1	8.5	7.8	7.1	6.3	5.5	4.7	3.7	2.8	1.7	0.6	*0.6*	*2.0*	*3.4*	*4.8*	*6.3*	*7.6*	*0.0*	45
50	10.9	10.2	9.5	8.8	7.9	7.1	6.2	5.2	4.2	3.0	1.9	0.6	*0.8*	*2.3*	*3.8*	*5.5*	*7.1*	*8.5*	*0.0*	50
55	12.2	11.4	10.6	9.7	8.8	7.9	6.8	5.7	4.6	3.3	2.0	0.6	*1.0*	*2.6*	*4.4*	*6.2*	*8.0*	*9.4*	*0.0*	55
60	13.5	12.6	11.7	10.7	9.7	8.7	7.5	6.3	5.0	3.6	2.1	0.5	*1.2*	*3.0*	*4.9*	*6.9*	*8.8*	*10.4*	*0.0*	60
65	14.8	13.9	12.9	11.8	10.7	9.5	8.2	6.9	5.4	3.9	2.2	0.4	*1.4*	*3.4*	*5.5*	*7.7*	*9.7*	*11.3*	*0.0*	65
70	16.2	15.2	14.1	12.9	11.6	10.3	8.9	7.4	5.8	4.1	2.3	0.3	*1.7*	*3.9*	*6.2*	*8.5*	*10.7*	*12.3*	*0.0*	70
75	17.6	16.5	15.3	14.0	12.7	11.2	9.6	8.0	6.2	4.3	2.3	0.2	*2.1*	*4.5*	*6.9*	*9.3*	*11.6*	*13.3*	*0.0*	75
80	19.2	17.9	16.6	15.2	13.7	12.1	10.4	8.6	6.6	4.5	2.3	0.0	*2.5*	*5.1*	*7.7*	*10.3*	*12.7*	*14.4*	*0.0*	80
85	20.8	19.4	18.0	16.5	14.8	13.0	11.2	9.1	7.0	4.7	2.3	*0.3*	*3.0*	*5.7*	*8.5*	*11.3*	*13.7*	*15.4*	*0.0*	85
90	22.5	21.0	19.4	17.8	16.0	14.0	11.9	9.7	7.4	4.8	2.2	*0.6*	*3.5*	*6.5*	*9.5*	*12.3*	*14.9*	*16.5*	*0.0*	90
95	24.3	22.7	21.0	19.2	17.2	15.1	12.8	10.3	7.7	4.9	2.0	*1.0*	*4.2*	*7.4*	*10.5*	*13.5*	*16.1*	*17.6*	*0.0*	95
100	26.2	24.5	22.7	20.6	18.5	16.2	13.6	10.9	8.1	5.0	1.8	*1.5*	*4.9*	*8.3*	*11.7*	*14.8*	*17.3*	*18.8*	*0.0*	100
105	28.2	26.4	24.4	22.3	19.9	17.3	14.5	11.6	8.4	5.0	1.5	*2.1*	*5.8*	*9.5*	*12.9*	*16.1*	*18.6*	*19.9*	*0.0*	105
110	30.5	28.5	26.4	24.0	21.4	18.6	15.5	12.2	8.7	4.9	1.1	*2.9*	*6.9*	*10.7*	*14.3*	*17.5*	*20.0*	*21.1*	*0.0*	110
115	32.9	30.8	28.5	25.9	23.0	19.9	16.5	12.8	8.9	4.8	0.5	*3.8*	*8.1*	*12.1*	*15.9*	*19.1*	*21.5*	*22.3*	*0.0*	115
120	35.6	33.3	30.8	28.0	24.8	21.4	17.6	13.5	9.1	4.5	*0.2*	*4.9*	*9.5*	*13.7*	*17.6*	*20.8*	*23.0*	*23.6*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—40°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

Same Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	1.2	1.2	1.3	1.3	1.4	1.4	1.4	1.5	1.6	1.7	1.7	1.7	1.7	1.8	1.7	1.7	1.6	1.5	0.0	5
10	2.4	2.5	2.6	2.7	2.8	2.9	2.9	3.0	3.2	3.4	3.4	3.5	3.5	3.5	3.5	3.4	3.3	3.0	0.0	10
15	3.6	3.8	3.9	4.1	4.2	4.3	4.5	4.6	4.8	5.0	5.1	5.2	5.2	5.2	5.2	5.1	4.9	4.5	0.0	15
20	4.8	5.0	5.3	5.5	5.7	5.8	6.0	6.3	6.5	6.7	6.8	6.9	7.0	7.0	7.0	6.8	6.6	6.0	0.0	20
25	6.1	6.3	6.6	6.9	7.1	7.4	7.6	7.9	8.1	8.3	8.5	8.6	8.7	8.7	8.7	8.6	8.2	7.5	0.0	25
30	7.3	7.7	8.0	8.3	8.6	8.9	9.2	9.5	9.8	10.0	10.2	10.4	10.5	10.5	10.4	10.3	9.9	9.0	0.0	30
35	8.6	9.0	9.4	9.8	10.2	10.5	10.8	11.2	11.5	11.7	12.0	12.1	12.3	12.3	12.2	12.0	11.5	10.5	0.0	35
40	9.9	10.4	10.9	11.3	11.7	12.1	12.5	12.9	13.2	13.5	13.7	13.9	14.0	14.1	14.0	13.8	13.2	12.0	0.0	40
45	11.3	11.8	12.3	12.8	13.3	13.7	14.2	14.6	14.9	15.2	15.5	15.7	15.9	15.9	15.8	15.5	14.9	13.5	0.0	45
50	12.7	13.3	13.9	14.4	14.9	15.4	15.9	16.3	16.7	17.0	17.3	17.6	17.7	17.7	17.6	17.3	16.6	15.0	0.0	50
55	14.1	14.8	15.4	16.0	16.6	17.1	17.6	18.1	18.5	18.9	19.2	19.4	19.6	19.6	19.4	19.0	18.2	16.5	0.0	55
60	15.6	16.3	17.0	17.7	18.3	18.9	19.4	19.9	20.4	20.7	21.1	21.3	21.5	21.4	21.3	20.8	19.9	18.0	0.0	60
65	17.1	17.9	18.7	19.4	20.1	20.7	21.3	21.8	22.3	22.7	23.0	23.3	23.4	23.4	23.2	22.6	21.6	19.5	0.0	65
70	18.7	19.6	20.4	21.2	21.9	22.6	23.2	23.8	24.2	24.7	25.0	25.2	25.3	25.3	25.0	24.5	23.3	21.0	0.0	70
75	20.3	21.3	22.2	23.1	23.8	24.5	25.2	25.8	26.3	26.7	27.0	27.3	27.3	27.3	27.0	26.3	25.1	22.5	0.0	75
80	22.1	23.1	24.1	25.0	25.8	26.6	27.2	27.8	28.4	28.8	29.1	29.3	29.4	29.3	28.9	28.2	26.8	24.1	0.0	80
85	23.9	25.0	26.0	27.0	27.9	28.7	29.4	30.0	30.5	30.9	31.3	31.4	31.5	31.3	30.9	30.0	28.5	25.6	0.0	85
90	25.8	27.0	28.1	29.1	30.0	30.9	31.6	32.2	32.7	33.2	33.5	33.6	33.6	33.4	32.9	31.9	30.3	27.1	0.0	90
95	27.8	29.1	30.3	31.3	32.3	33.1	33.9	34.5	35.1	35.5	35.7	35.9	35.8	35.5	34.9	33.9	32.0	28.6	0.0	95
100	30.0	31.4	32.6	33.7	34.7	35.6	36.3	37.0	37.5	37.8	38.1	38.2	38.0	37.7	37.0	35.8	33.8	30.1	0.0	100
105	32.3	33.7	35.0	36.2	37.2	38.1	38.8	39.5	40.0	40.3	40.5	40.5	40.3	39.9	39.1	37.7	35.6	31.6	0.0	105
110	34.7	36.3	37.6	38.8	39.9	40.8	41.5	42.1	42.6	42.9	43.0	42.9	42.7	42.1	41.2	39.7	37.3	33.1	0.0	110
115	37.4	39.0	40.4	41.6	42.7	43.6	44.3	44.9	45.3	45.5	45.6	45.4	45.1	44.4	43.3	41.7	39.1	34.6	0.0	115
120	40.3	42.0	43.4	44.6	45.7	46.5	47.2	47.7	48.1	48.2	48.2	48.0	47.5	46.7	45.5	43.7	40.9	36.1	0.0	120

Contrary Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	0
5	1.2	1.1	1.1	1.0	0.9	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.1	0.0	*0.2*	*0.4*	*0.6*	*0.7*	*0.0*	5
10	2.4	2.3	2.1	2.0	1.9	1.7	1.6	1.4	1.2	1.0	0.8	0.5	0.2	0.0	*0.4*	*0.7*	*1.1*	*1.5*	*0.0*	10
15	3.6	3.4	3.2	3.0	2.8	2.6	2.3	2.1	1.8	1.5	1.1	0.8	0.4	*0.1*	*0.6*	*1.1*	*1.7*	*2.3*	*0.0*	15
20	4.8	4.6	4.3	4.0	3.8	3.5	3.1	2.8	2.4	2.0	1.5	1.0	0.5	*0.1*	*0.8*	*1.5*	*2.3*	*3.0*	*0.0*	20
25	6.1	5.7	5.4	5.1	4.7	4.3	3.9	3.5	3.0	2.5	1.9	1.3	0.6	*0.2*	*1.0*	*1.9*	*2.9*	*3.8*	*0.0*	25
30	7.3	6.9	6.6	6.1	5.7	5.2	4.7	4.2	3.6	3.0	2.3	1.5	0.7	*0.2*	*1.2*	*2.3*	*3.5*	*4.6*	*0.0*	30
35	8.6	8.2	7.7	7.2	6.7	6.1	5.5	4.9	4.2	3.4	2.6	1.7	0.8	*0.3*	*1.5*	*2.8*	*4.1*	*5.4*	*0.0*	35
40	9.9	9.4	8.9	8.3	7.7	7.1	6.4	5.6	4.8	3.9	3.0	2.0	0.8	*0.4*	*1.8*	*3.2*	*4.8*	*6.2*	*0.0*	40
45	11.3	10.7	10.1	9.4	8.7	8.0	7.2	6.4	5.4	4.4	3.3	2.2	0.9	*0.5*	*2.1*	*3.7*	*5.4*	*7.0*	*0.0*	45
50	12.7	12.0	11.3	10.6	9.8	9.0	8.1	7.1	6.1	4.9	3.7	2.4	0.9	*0.7*	*2.4*	*4.3*	*6.2*	*7.9*	*0.0*	50
55	14.1	13.4	12.6	11.8	10.9	10.0	8.9	7.9	6.7	5.4	4.0	2.5	0.9	*0.9*	*2.8*	*4.8*	*6.9*	*8.8*	*0.0*	55
60	15.6	14.8	13.9	13.0	12.0	11.0	9.8	8.6	7.3	5.9	4.4	2.7	0.9	*1.1*	*3.2*	*5.4*	*7.7*	*9.6*	*0.0*	60
65	17.1	16.2	15.3	14.3	13.2	12.0	10.8	9.4	8.0	6.4	4.7	2.8	0.8	*1.3*	*3.6*	*6.1*	*8.5*	*10.5*	*0.0*	65
70	18.7	17.7	16.7	15.6	14.4	13.1	11.7	10.2	8.6	6.9	5.0	2.9	0.7	*1.6*	*4.1*	*6.7*	*9.3*	*11.5*	*0.0*	70
75	20.3	19.3	18.1	16.9	15.6	14.2	12.7	11.0	9.3	7.3	5.3	3.0	0.6	*2.0*	*4.7*	*7.5*	*10.2*	*12.4*	*0.0*	75
80	22.1	20.9	19.7	18.4	16.9	15.4	13.7	11.9	9.9	7.8	5.5	3.0	0.4	*2.4*	*5.3*	*8.3*	*11.2*	*13.4*	*0.0*	80
85	23.9	22.6	21.3	19.9	18.3	16.6	14.8	12.8	10.6	8.3	5.7	3.0	0.1	*2.9*	*6.1*	*9.2*	*12.2*	*14.5*	*0.0*	85
90	25.8	24.5	23.0	21.4	19.7	17.9	15.9	13.7	11.3	8.7	5.9	3.0	*0.2*	*3.5*	*6.9*	*10.2*	*13.3*	*15.5*	*0.0*	90
95	27.8	26.4	24.8	23.1	21.3	19.2	17.0	14.6	12.0	9.1	6.1	2.8	*0.6*	*4.2*	*7.7*	*11.2*	*14.4*	*16.6*	*0.0*	95
100	30.0	28.5	26.8	24.9	22.9	20.7	18.2	15.6	12.7	9.5	6.2	2.6	*1.1*	*4.9*	*8.8*	*12.4*	*15.6*	*17.7*	*0.0*	100
105	32.3	30.7	28.8	26.8	24.6	22.2	19.5	16.6	13.4	9.9	6.2	2.3	*1.7*	*5.9*	*9.9*	*13.7*	*16.9*	*18.8*	*0.0*	105
110	34.7	33.0	31.1	28.9	26.5	23.8	20.9	17.6	14.1	10.3	6.2	1.9	*2.5*	*6.9*	*11.2*	*15.0*	*18.2*	*20.0*	*0.0*	110
115	37.4	35.6	33.5	31.2	28.6	25.6	22.4	18.8	14.8	10.6	6.0	1.3	*3.5*	*8.2*	*12.6*	*16.5*	*19.6*	*21.2*	*0.0*	115
120	40.3	38.4	36.2	33.7	30.8	27.6	24.0	20.0	15.6	10.8	5.8	0.6	*4.6*	*9.6*	*14.2*	*18.1*	*21.2*	*22.5*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—45°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

Same Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	1.4	1.4	1.5	1.5	1.6	1.6	1.6	1.6	1.6	1.8	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.6	0.0	5
10	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.3	3.4	3.6	3.7	3.7	3.7	3.7	3.6	3.6	3.4	3.1	0.0	10
15	4.1	4.3	4.4	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.5	5.5	5.5	5.5	5.5	5.4	5.1	4.6	0.0	15
20	5.5	5.7	5.9	6.1	6.3	6.5	6.6	6.8	6.9	7.1	7.3	7.4	7.4	7.4	7.3	7.1	6.8	6.2	0.0	20
25	6.9	7.2	7.5	7.7	7.9	8.1	8.3	8.5	8.7	8.9	9.1	9.2	9.2	9.2	9.1	8.9	8.6	7.7	0.0	25
30	8.4	8.7	9.0	9.3	9.6	9.8	10.1	10.3	10.5	10.7	10.9	11.0	11.1	11.1	11.0	10.7	10.3	9.3	0.0	30
35	9.8	10.2	10.6	10.9	11.3	11.6	11.8	12.1	12.4	12.6	12.8	12.9	13.0	12.9	12.8	12.5	12.0	10.8	0.0	35
40	11.3	11.8	12.2	12.6	13.0	13.3	13.6	13.9	14.2	14.4	14.6	14.8	14.9	14.8	14.7	14.4	13.7	12.4	0.0	40
45	12.9	13.4	13.9	14.3	14.7	15.1	15.4	15.8	16.1	16.3	16.5	16.7	16.8	16.7	16.6	16.2	15.4	13.9	0.0	45
50	14.4	15.0	15.5	16.0	16.5	16.9	17.3	17.7	18.0	18.3	18.5	18.6	18.7	18.6	18.4	18.0	17.2	15.5	0.0	50
55	16.1	16.7	17.3	17.8	18.3	18.8	19.2	19.6	19.9	20.2	20.4	20.6	20.7	20.6	20.3	19.8	18.9	17.0	0.0	55
60	17.7	18.4	19.0	19.6	20.2	20.7	21.2	21.6	21.9	22.2	22.4	22.6	22.6	22.6	22.3	21.7	20.7	18.6	0.0	60
65	19.5	20.2	20.9	21.5	22.1	22.7	23.1	23.6	24.0	24.3	24.5	24.6	24.7	24.5	24.2	23.6	22.4	20.1	0.0	65
70	21.2	22.0	22.8	23.5	24.1	24.7	25.2	25.7	26.0	26.4	26.6	26.7	26.7	26.6	26.2	25.5	24.2	21.7	0.0	70
75	23.1	24.0	24.8	25.5	26.2	26.8	27.3	27.8	28.2	28.5	28.7	28.8	28.8	28.6	28.2	27.4	26.0	23.2	0.0	75
80	25.0	26.0	26.8	27.6	28.3	28.9	29.5	30.0	30.4	30.7	30.9	31.0	30.9	30.7	30.2	29.3	27.7	24.8	0.0	80
85	27.0	28.1	29.0	29.8	30.5	31.2	31.8	32.3	32.7	33.0	33.1	33.2	33.1	32.8	32.2	31.2	29.5	26.3	0.0	85
90	29.2	30.2	31.2	32.1	32.8	33.5	34.1	34.6	35.0	35.3	35.4	35.5	35.3	34.9	34.3	33.2	31.3	27.9	0.0	90
95	31.4	32.5	33.6	34.5	35.3	36.0	36.5	37.0	37.4	37.7	37.8	37.8	37.6	37.1	36.4	35.1	33.1	29.4	0.0	95
100	33.8	35.0	36.0	37.0	37.8	38.5	39.1	39.6	39.9	40.1	40.2	40.1	39.9	39.3	38.5	37.1	34.9	31.0	0.0	100
105	36.3	37.6	38.6	39.6	40.4	41.1	41.7	42.2	42.5	42.7	42.7	42.6	42.2	41.6	40.6	39.1	36.8	32.5	0.0	105
110	39.0	40.3	41.4	42.4	43.2	43.9	44.5	44.9	45.2	45.3	45.3	45.0	44.6	43.9	42.8	41.1	38.6	34.1	0.0	110
115	41.9	43.2	44.3	45.3	46.1	46.8	47.3	47.7	47.9	48.0	47.9	47.6	47.0	46.2	45.0	43.2	40.4	35.6	0.0	115
120	44.9	46.3	47.5	48.4	49.2	49.9	50.3	50.6	50.8	50.8	50.6	50.2	49.5	48.6	47.2	45.2	42.2	37.1	0.0	120

Contrary Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	*0.0*	*0.0*	0
5	1.4	1.3	1.3	1.2	1.1	1.1	1.0	0.9	0.8	0.7	0.6	0.5	0.3	0.1	0.0	*0.2*	*0.5*	*0.7*	*0.0*	5
10	2.7	2.6	2.5	2.4	2.2	2.1	1.9	1.8	1.6	1.4	1.1	0.9	0.6	0.3	*0.1*	*0.5*	*0.9*	*1.4*	*0.0*	10
15	4.1	4.0	3.8	3.6	3.4	3.2	2.9	2.7	2.4	2.1	1.7	1.3	0.9	0.4	*0.1*	*0.7*	*1.4*	*2.0*	*0.0*	15
20	5.5	5.3	5.1	4.8	4.5	4.2	3.9	3.6	3.2	2.7	2.3	1.8	1.2	0.6	*0.2*	*0.9*	*1.8*	*2.7*	*0.0*	20
25	6.9	6.7	6.4	6.0	5.7	5.3	4.9	4.4	4.0	3.4	2.9	2.2	1.5	0.7	*0.2*	*1.2*	*2.3*	*3.4*	*0.0*	25
30	8.4	8.0	7.7	7.3	6.9	6.4	5.9	5.4	4.8	4.1	3.4	2.6	1.8	0.8	*0.3*	*1.5*	*2.8*	*4.2*	*0.0*	30
35	9.8	9.4	9.0	8.5	8.0	7.5	6.9	6.3	5.6	4.8	4.0	3.1	2.0	0.9	*0.4*	*1.8*	*3.3*	*4.9*	*0.0*	35
40	11.3	10.9	10.4	9.8	9.3	8.6	7.9	7.2	6.4	5.5	4.6	3.5	2.3	1.0	*0.5*	*2.1*	*3.9*	*5.6*	*0.0*	40
45	12.9	12.3	11.8	11.2	10.5	9.8	9.0	8.2	7.2	6.2	5.1	3.9	2.6	1.0	*0.6*	*2.5*	*4.4*	*6.4*	*0.0*	45
50	14.4	13.8	13.2	12.5	11.8	11.0	10.1	9.1	8.1	6.9	5.7	4.3	2.8	1.1	*0.8*	*2.9*	*5.0*	*7.2*	*0.0*	50
55	16.1	15.4	14.7	13.9	13.1	12.2	11.2	10.1	8.9	7.7	6.3	4.7	3.0	1.1	*1.0*	*3.3*	*5.7*	*8.0*	*0.0*	55
60	17.7	17.0	16.2	15.3	14.4	13.4	12.3	11.1	9.8	8.4	6.8	5.1	3.2	1.1	*1.2*	*3.7*	*6.3*	*8.8*	*0.0*	60
65	19.5	18.6	17.8	16.8	15.8	14.7	13.5	12.2	10.7	9.1	7.4	5.5	3.3	1.0	*1.5*	*4.2*	*7.1*	*9.7*	*0.0*	65
70	21.2	20.3	19.4	18.4	17.2	16.0	14.7	13.2	11.6	9.8	7.9	5.8	3.5	0.9	*1.8*	*4.8*	*7.8*	*10.5*	*0.0*	70
75	23.1	22.1	21.1	20.0	18.7	17.4	15.9	14.3	12.5	10.6	8.5	6.1	3.6	0.8	*2.2*	*5.4*	*8.6*	*11.4*	*0.0*	75
80	25.0	24.0	22.9	21.6	20.3	18.8	17.2	15.4	13.5	11.3	9.0	6.4	3.6	0.6	*2.7*	*6.1*	*9.5*	*12.4*	*0.0*	80
85	27.0	25.9	24.7	23.4	21.9	20.3	18.5	16.6	14.5	12.1	9.5	6.7	3.6	0.3	*3.2*	*6.9*	*10.4*	*13.4*	*0.0*	85
90	29.2	28.0	26.7	25.2	23.6	21.9	20.0	17.8	15.5	12.9	10.0	6.9	3.6	0.0	*3.8*	*7.7*	*11.4*	*14.4*	*0.0*	90
95	31.4	30.1	28.7	27.2	25.5	23.6	21.4	19.1	16.5	13.7	10.5	7.1	3.4	*0.5*	*4.6*	*8.6*	*12.5*	*15.4*	*0.0*	95
100	33.8	32.5	30.9	29.3	27.4	25.3	23.0	20.4	17.6	14.4	11.0	7.2	3.2	*1.0*	*5.4*	*9.7*	*13.6*	*16.5*	*0.0*	100
105	36.3	34.9	33.3	31.5	29.5	27.2	24.7	21.9	18.7	15.3	11.4	7.3	2.9	*1.7*	*6.4*	*10.8*	*14.8*	*17.6*	*0.0*	105
110	39.0	37.5	35.8	33.9	31.7	29.3	26.5	23.4	19.9	16.1	11.8	7.3	2.4	*2.5*	*7.5*	*12.1*	*16.1*	*18.8*	*0.0*	110
115	41.9	40.3	38.5	36.5	34.1	31.5	28.4	25.0	21.2	16.9	12.2	7.1	1.9	*3.5*	*8.7*	*13.5*	*17.5*	*20.0*	*0.0*	115
120	44.9	43.3	41.5	39.3	36.8	33.9	30.6	26.8	22.5	17.7	12.5	6.9	1.1	*4.7*	*10.2*	*15.1*	*19.0*	*21.2*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—**50°**—Latitude of Receiver

DLo	Latitude of destination—Latitude of transmitter																			DLo
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	
	Same Name																			
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	1.6	1.6	1.6	1.7	1.7	1.8	1.8	1.8	1.8	1.8	1.9	2.0	2.0	1.9	1.9	1.9	1.8	1.6	0.0	5
10	3.1	3.2	3.3	3.4	3.5	3.5	3.6	3.6	3.7	3.7	3.8	3.9	3.9	3.9	3.8	3.7	3.6	3.2	0.0	10
15	4.7	4.8	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.6	5.8	5.9	5.9	5.8	5.7	5.6	5.3	4.8	0.0	15
20	6.3	6.5	6.6	6.8	7.0	7.1	7.2	7.3	7.5	7.6	7.7	7.8	7.8	7.8	7.7	7.5	7.1	6.4	0.0	20
25	7.9	8.1	8.3	8.6	8.7	8.9	9.1	9.2	9.4	9.5	9.6	9.7	9.8	9.7	9.6	9.3	8.9	8.0	0.0	25
30	9.5	9.8	10.1	10.3	10.6	10.8	11.0	11.2	11.3	11.5	11.6	11.7	11.7	11.7	11.5	11.2	10.7	9.6	0.0	30
35	11.1	11.5	11.8	12.1	12.4	12.6	12.9	13.1	13.3	13.5	13.6	13.7	13.7	13.6	13.4	13.1	12.5	11.2	0.0	35
40	12.8	13.2	13.6	13.9	14.3	14.5	14.8	15.1	15.3	15.5	15.6	15.7	15.7	15.6	15.4	15.0	14.3	12.8	0.0	40
45	14.6	15.0	15.4	15.8	16.2	16.5	16.8	17.0	17.3	17.5	17.6	17.7	17.7	17.6	17.3	16.9	16.0	14.4	0.0	45
50	16.3	16.8	17.3	17.7	18.1	18.5	18.8	19.1	19.3	19.5	19.7	19.7	19.7	19.6	19.3	18.8	17.8	16.0	0.0	50
55	18.1	18.7	19.2	19.7	20.1	20.5	20.8	21.1	21.4	21.6	21.8	21.8	21.8	21.6	21.3	20.7	19.7	17.6	0.0	55
60	20.0	20.6	21.2	21.7	22.1	22.5	22.9	23.2	23.5	23.7	23.9	23.9	23.9	23.7	23.3	22.6	21.5	19.2	0.0	60
65	21.9	22.6	23.2	23.7	24.2	24.7	25.1	25.4	25.7	25.9	26.0	26.1	26.0	25.8	25.3	24.6	23.3	20.8	0.0	65
70	23.9	24.6	25.2	25.8	26.4	26.8	27.3	27.6	27.9	28.1	28.2	28.2	28.1	27.9	27.4	26.5	25.1	22.4	0.0	70
75	25.9	26.7	27.4	28.0	28.6	29.1	29.5	29.9	30.1	30.4	30.5	30.5	30.3	30.0	29.4	28.5	26.9	24.0	0.0	75
80	28.1	28.9	29.6	30.3	30.9	31.4	31.8	32.2	32.5	32.7	32.7	32.7	32.5	32.2	31.5	30.5	28.8	25.6	0.0	80
85	30.3	31.2	32.0	32.6	33.2	33.8	34.2	34.6	34.8	35.0	35.1	35.0	34.8	34.3	33.6	32.5	30.6	27.2	0.0	85
90	32.6	33.6	34.4	35.1	35.7	36.2	36.7	37.0	37.3	37.4	37.5	37.4	37.1	36.6	35.7	34.5	32.5	28.8	0.0	90
95	35.1	36.0	36.9	37.6	38.3	38.8	39.2	39.6	39.8	39.9	39.9	39.7	39.4	38.8	37.9	36.5	34.3	30.4	0.0	95
100	37.7	38.7	39.5	40.3	40.9	41.5	41.9	42.2	42.4	42.5	42.4	42.2	41.8	41.1	40.1	38.6	36.2	32.0	0.0	100
105	40.4	41.4	42.3	43.1	43.7	44.2	44.6	44.9	45.1	45.1	45.0	44.7	44.2	43.4	42.3	40.6	38.1	33.6	0.0	105
110	43.2	44.3	45.2	46.0	46.6	47.1	47.5	47.7	47.8	47.8	47.6	47.2	46.6	45.7	44.5	42.7	39.9	35.1	0.0	110
115	46.3	47.4	48.3	49.0	49.6	50.1	50.4	50.6	50.6	50.5	50.3	49.8	49.1	48.1	46.7	44.8	41.8	36.7	0.0	115
120	49.5	50.6	51.5	52.2	52.8	53.2	53.4	53.6	53.5	53.4	53.0	52.4	51.6	50.5	49.0	46.8	43.7	38.3	0.0	120
DLo	**Contrary Name**																			DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	*0.0*	*0.0*	0
5	1.6	1.5	1.4	1.4	1.3	1.3	1.2	1.1	1.0	0.9	0.8	0.7	0.5	0.3	0.2	*0.1*	*0.3*	*0.6*	*0.0*	5
10	3.1	3.0	2.9	2.8	2.7	2.5	2.4	2.2	2.0	1.8	1.6	1.3	1.0	0.7	0.3	*0.1*	*0.6*	*1.2*	*0.0*	10
15	4.7	4.5	4.4	4.2	4.0	3.8	3.5	3.3	3.0	2.7	2.3	2.0	1.5	1.0	0.4	*0.2*	*1.0*	*1.8*	*0.0*	15
20	6.3	6.1	5.8	5.6	5.3	5.0	4.7	4.4	4.0	3.6	3.1	2.6	2.0	1.3	0.6	*0.3*	*1.3*	*2.4*	*0.0*	20
25	7.9	7.6	7.3	7.0	6.7	6.3	5.9	5.5	5.0	4.5	3.9	3.2	2.5	1.7	0.7	*0.4*	*1.7*	*3.0*	*0.0*	25
30	9.5	9.2	8.8	8.5	8.1	7.6	7.2	6.6	6.0	5.4	4.7	3.9	3.0	2.0	0.8	*0.5*	*2.0*	*3.6*	*0.0*	30
35	11.1	10.8	10.4	9.9	9.5	9.0	8.4	7.8	7.1	6.3	5.5	4.5	3.5	2.3	0.9	*0.7*	*2.4*	*4.3*	*0.0*	35
40	12.8	12.4	11.9	11.4	10.9	10.3	9.6	8.9	8.1	7.2	6.3	5.2	3.9	2.6	1.0	*0.8*	*2.8*	*5.0*	*0.0*	40
45	14.6	14.1	13.5	13.0	12.3	11.7	10.9	10.1	9.2	8.2	7.1	5.8	4.4	2.8	1.0	*1.0*	*3.3*	*5.6*	*0.0*	45
50	16.3	15.8	15.2	14.5	13.8	13.1	12.2	11.3	10.3	9.1	7.9	6.5	4.9	3.1	1.0	*1.2*	*3.8*	*6.3*	*0.0*	50
55	18.1	17.5	16.9	16.1	15.4	14.5	13.6	12.5	11.4	10.1	8.7	7.1	5.3	3.3	1.0	*1.5*	*4.3*	*7.1*	*0.0*	55
60	20.0	19.3	18.6	17.8	16.9	16.0	14.9	13.8	12.5	11.1	9.5	7.7	5.7	3.5	1.0	*1.8*	*4.8*	*7.8*	*0.0*	60
65	21.9	21.2	20.4	19.5	18.6	17.5	16.4	15.1	13.7	12.1	10.3	8.4	6.1	3.7	0.9	*2.1*	*5.4*	*8.6*	*0.0*	65
70	23.9	23.1	22.2	21.3	20.2	19.1	17.8	16.4	14.8	13.1	11.1	9.0	6.5	3.8	0.8	*2.5*	*6.0*	*9.4*	*0.0*	70
75	25.9	25.1	24.2	23.1	22.0	20.7	19.3	17.8	16.1	14.1	12.0	9.6	6.9	3.9	0.6	*3.0*	*6.8*	*10.3*	*0.0*	75
80	28.1	27.2	26.2	25.0	23.8	22.4	20.9	19.2	17.3	15.2	12.8	10.2	7.2	4.0	0.4	*3.5*	*7.5*	*11.2*	*0.0*	80
85	30.3	29.3	28.3	27.0	25.7	24.2	22.6	20.7	18.6	16.3	13.7	10.8	7.5	4.0	0.0	*4.1*	*8.3*	*12.1*	*0.0*	85
90	32.6	31.6	30.4	29.2	27.7	26.1	24.3	22.3	20.0	17.4	14.6	11.4	7.8	3.9	*0.3*	*4.8*	*9.2*	*13.1*	*0.0*	90
95	35.1	34.0	32.8	31.4	29.8	28.1	26.1	23.9	21.4	18.6	15.4	11.9	8.0	3.7	*0.8*	*5.6*	*10.2*	*14.1*	*0.0*	95
100	37.7	36.5	35.2	33.7	32.1	30.2	28.0	25.6	22.9	19.8	16.3	12.4	8.1	3.5	*1.4*	*6.5*	*11.3*	*15.1*	*0.0*	100
105	40.4	39.2	37.8	36.3	34.5	32.4	30.1	27.5	24.5	21.1	17.2	12.9	8.2	3.1	*2.2*	*7.5*	*12.4*	*16.2*	*0.0*	105
110	43.2	42.0	40.6	38.9	37.0	34.8	32.3	29.4	26.1	22.4	18.1	13.4	8.2	2.6	*3.1*	*8.6*	*13.7*	*17.4*	*0.0*	110
115	46.3	45.0	43.5	41.8	39.8	37.4	34.7	31.6	27.9	23.8	19.0	13.8	8.0	2.0	*4.1*	*10.0*	*15.0*	*18.6*	*0.0*	115
120	49.5	48.2	46.7	44.9	42.8	40.3	37.3	33.9	29.9	25.3	20.0	14.1	7.7	1.1	*5.4*	*11.4*	*16.5*	*19.8*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—**55°**—Latitude of Receiver

Latitude of destination—Latitude of transmitter

Same Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.0	2.0	2.0	2.1	2.1	2.1	2.0	2.0	1.9	1.7	0.0	5
10	3.5	3.6	3.7	3.8	3.8	3.9	3.9	4.0	4.0	4.0	4.0	4.1	4.2	4.1	4.0	3.9	3.7	3.3	0.0	10
15	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.0	6.0	6.1	6.2	6.2	6.1	6.0	5.9	5.6	5.0	0.0	15
20	7.1	7.2	7.4	7.5	7.7	7.8	7.9	8.0	8.0	8.1	8.2	8.2	8.2	8.2	8.0	7.8	7.4	6.6	0.0	20
25	8.9	9.1	9.3	9.4	9.6	9.8	9.9	10.0	10.1	10.2	10.2	10.3	10.3	10.2	10.1	9.8	9.3	8.3	0.0	25
30	10.7	10.9	11.2	11.4	11.6	11.8	11.9	12.1	12.2	12.3	12.3	12.4	12.4	12.3	12.1	11.7	11.1	10.0	0.0	30
35	12.5	12.8	13.1	13.4	13.6	13.8	14.0	14.1	14.3	14.4	14.5	14.5	14.5	14.3	14.1	13.7	13.0	11.6	0.0	35
40	14.4	14.8	15.1	15.4	15.6	15.9	16.1	16.2	16.4	16.5	16.6	16.6	16.6	16.4	16.1	15.7	14.8	13.3	0.0	40
45	16.3	16.7	17.1	17.4	17.7	17.9	18.2	18.4	18.5	18.7	18.7	18.8	18.7	18.5	18.2	17.6	16.7	14.9	0.0	45
50	18.3	18.7	19.1	19.5	19.8	20.1	20.3	20.5	20.7	20.8	20.9	20.9	20.8	20.6	20.2	19.6	18.6	16.6	0.0	50
55	20.3	20.8	21.2	21.6	21.9	22.3	22.5	22.7	22.9	23.1	23.1	23.1	23.0	22.8	22.3	21.6	20.4	18.3	0.0	55
60	22.3	22.9	23.3	23.8	24.1	24.5	24.8	25.0	25.2	25.3	25.4	25.3	25.2	24.9	24.4	23.6	22.3	19.9	0.0	60
65	24.5	25.0	25.5	26.0	26.4	26.7	27.0	27.3	27.5	27.6	27.6	27.6	27.4	27.1	26.5	25.7	24.2	21.6	0.0	65
70	26.6	27.2	27.8	28.3	28.7	29.1	29.4	29.6	29.8	29.9	29.9	29.8	29.6	29.3	28.6	27.7	26.1	23.2	0.0	70
75	28.9	29.5	30.1	30.6	31.1	31.5	31.8	32.0	32.2	32.3	32.3	32.2	31.9	31.5	30.8	29.7	28.0	24.9	0.0	75
80	31.2	31.9	32.5	33.1	33.5	33.9	34.2	34.5	34.6	34.7	34.7	34.5	34.2	33.8	32.9	31.8	29.9	26.5	0.0	80
85	33.7	34.4	35.0	35.6	36.0	36.4	36.7	37.0	37.1	37.2	37.1	36.9	36.5	36.0	35.1	33.8	31.8	28.2	0.0	85
90	36.2	36.9	37.6	38.2	38.6	39.0	39.3	39.5	39.7	39.7	39.6	39.3	38.9	38.3	37.3	35.9	33.7	29.8	0.0	90
95	38.8	39.6	40.3	40.9	41.3	41.7	42.0	42.2	42.3	42.3	42.1	41.8	41.3	40.6	39.5	38.0	35.6	31.5	0.0	95
100	41.6	42.4	43.1	43.6	44.1	44.5	44.7	44.9	44.9	44.9	44.7	44.3	43.8	42.9	41.8	40.1	37.6	33.1	0.0	100
105	44.5	45.3	46.0	46.5	47.0	47.3	47.6	47.7	47.7	47.6	47.3	46.9	46.2	45.3	44.0	42.2	39.5	34.7	0.0	105
110	47.5	48.3	49.0	49.6	50.0	50.3	50.5	50.6	50.5	50.3	50.0	49.5	48.7	47.7	46.3	44.4	41.4	36.4	0.0	110
115	50.7	51.5	52.2	52.7	53.1	53.4	53.5	53.5	53.4	53.2	52.7	52.1	51.3	50.2	48.6	46.5	43.3	38.0	0.0	115
120	54.1	54.9	55.5	56.0	56.3	56.5	56.6	56.6	56.4	56.0	55.5	54.8	53.9	52.6	51.0	48.6	45.3	39.6	0.0	120

Contrary Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	*0.0*	0
5	1.8	1.7	1.7	1.6	1.5	1.5	1.4	1.3	1.2	1.1	1.0	0.9	0.7	0.6	0.4	0.1	*0.2*	*0.5*	*0.0*	5
10	3.5	3.4	3.3	3.2	3.1	3.0	2.8	2.6	2.5	2.3	2.0	1.8	1.5	1.1	0.7	0.2	*0.3*	*1.0*	*0.0*	10
15	5.3	5.1	5.0	4.8	4.6	4.4	4.2	4.0	3.7	3.4	3.0	2.6	2.2	1.7	1.0	0.3	*0.5*	*1.5*	*0.0*	15
20	7.1	6.9	6.7	6.4	6.2	5.9	5.6	5.3	4.9	4.5	4.0	3.5	2.9	2.2	1.4	0.4	*0.7*	*2.0*	*0.0*	20
25	8.9	8.6	8.4	8.1	7.8	7.4	7.1	6.6	6.2	5.7	5.1	4.4	3.6	2.7	1.7	0.5	*0.9*	*2.5*	*0.0*	25
30	10.7	10.4	10.1	9.7	9.4	9.0	8.5	8.0	7.4	6.8	6.1	5.3	4.4	3.3	2.0	0.6	*1.1*	*3.0*	*0.0*	30
35	12.5	12.2	11.8	11.4	11.0	10.5	10.0	9.4	8.7	8.0	7.1	6.2	5.1	3.8	2.3	0.6	*1.4*	*3.6*	*0.0*	35
40	14.4	14.0	13.6	13.1	12.6	12.1	11.5	10.8	10.0	9.1	8.2	7.1	5.8	4.3	2.6	0.7	*1.6*	*4.2*	*0.0*	40
45	16.3	15.9	15.4	14.9	14.3	13.7	13.0	12.2	11.3	10.3	9.2	8.0	6.5	4.8	2.9	0.7	*1.9*	*4.8*	*0.0*	45
50	18.3	17.8	17.3	16.7	16.0	15.3	14.5	13.6	12.7	11.5	10.3	8.9	7.2	5.3	3.1	0.6	*2.2*	*5.4*	*0.0*	50
55	20.3	19.8	19.2	18.5	17.8	17.0	16.1	15.1	14.0	12.8	11.4	9.8	7.9	5.8	3.4	0.6	*2.6*	*6.0*	*0.0*	55
60	22.3	21.8	21.1	20.4	19.6	18.7	17.7	16.7	15.4	14.0	12.5	10.7	8.6	6.3	3.6	0.5	*3.0*	*6.7*	*0.0*	60
65	24.5	23.8	23.1	22.3	21.5	20.5	19.4	18.2	16.9	15.3	13.6	11.6	9.3	6.7	3.7	0.3	*3.5*	*7.4*	*0.0*	65
70	26.6	26.0	25.2	24.3	23.4	22.3	21.2	19.8	18.3	16.6	14.7	12.5	10.0	7.1	3.9	0.1	*4.0*	*8.1*	*0.0*	70
75	28.9	28.2	27.3	26.4	25.4	24.3	23.0	21.5	19.9	18.0	15.9	13.5	10.7	7.5	3.9	*0.1*	*4.5*	*8.9*	*0.0*	75
80	31.2	30.5	29.6	28.6	27.5	26.3	24.8	23.3	21.5	19.4	17.1	14.4	11.3	7.9	3.9	*0.4*	*5.1*	*9.7*	*0.0*	80
85	33.7	32.8	31.9	30.8	29.7	28.3	26.8	25.1	23.1	20.9	18.3	15.4	12.0	8.2	3.9	*0.8*	*5.8*	*10.6*	*0.0*	85
90	36.2	35.3	34.3	33.2	31.9	30.5	28.9	27.0	24.8	22.4	19.6	16.3	12.6	8.5	3.8	*1.3*	*6.6*	*11.5*	*0.0*	90
95	38.8	37.9	36.9	35.7	34.3	32.8	31.0	29.0	26.6	23.9	20.8	17.3	13.2	8.6	3.6	*1.9*	*7.5*	*12.5*	*0.0*	95
100	41.6	40.7	39.6	38.3	36.9	35.2	33.3	31.1	28.5	25.6	22.2	18.3	13.8	8.8	3.3	*2.6*	*8.4*	*13.5*	*0.0*	100
105	44.5	43.5	42.4	41.1	39.6	37.8	35.7	33.3	30.6	27.3	23.6	19.3	14.3	8.8	2.8	*3.4*	*9.5*	*14.6*	*0.0*	105
110	47.5	46.5	45.4	44.0	42.4	40.5	38.3	35.8	32.7	29.2	25.1	20.3	14.8	8.8	2.3	*4.4*	*10.7*	*15.7*	*0.0*	110
115	50.7	49.7	48.5	47.1	45.5	43.5	41.1	38.4	35.1	31.2	26.6	21.3	15.3	8.6	1.5	*5.5*	*12.0*	*16.8*	*0.0*	115
120	54.1	53.1	51.9	50.5	48.7	46.6	44.2	41.2	37.6	33.3	28.3	22.3	15.6	8.2	0.6	*6.9*	*13.4*	*18.1*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—60°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

Same Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	2.0	2.0	2.0	2.1	2.1	2.1	2.1	2.1	2.2	2.2	2.2	2.2	2.2	2.2	2.1	2.0	1.9	1.7	0.0	5
10	3.9	4.0	4.1	4.1	4.2	4.2	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.2	4.1	3.9	3.5	0.0	10
15	5.9	6.0	6.1	6.2	6.3	6.3	6.4	6.4	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.3	5.8	5.2	0.0	15
20	7.9	8.0	8.2	8.3	8.4	8.5	8.6	8.7	8.7	8.7	8.7	8.7	8.7	8.6	8.5	8.2	7.7	6.9	0.0	20
25	9.9	10.1	10.3	10.4	10.5	10.6	10.7	10.8	10.9	10.9	10.9	10.9	10.9	10.8	10.6	10.2	9.7	8.6	0.0	25
30	11.9	12.1	12.3	12.5	12.7	12.8	12.9	13.0	13.1	13.1	13.1	13.1	13.1	12.9	12.7	12.3	11.6	10.4	0.0	30
35	14.0	14.2	14.5	14.7	14.9	15.0	15.1	15.2	15.3	15.4	15.4	15.4	15.3	15.1	14.8	14.3	13.6	12.1	0.0	35
40	16.1	16.4	16.6	16.9	17.1	17.2	17.4	17.5	17.6	17.6	17.7	17.6	17.5	17.3	17.0	16.4	15.5	13.8	0.0	40
45	18.2	18.5	18.8	19.1	19.3	19.5	19.7	19.8	19.9	19.9	19.9	19.9	19.7	19.5	19.1	18.5	17.4	15.5	0.0	45
50	20.4	20.7	21.0	21.3	21.6	21.8	22.0	22.1	22.2	22.2	22.2	22.2	22.0	21.7	21.3	20.6	19.4	17.3	0.0	50
55	22.6	23.0	23.3	23.6	23.9	24.1	24.3	24.5	24.5	24.6	24.6	24.5	24.3	23.9	23.4	22.6	21.3	19.0	0.0	55
60	24.8	25.3	25.6	26.0	26.3	26.5	26.7	26.8	26.9	27.0	26.9	26.8	26.6	26.2	25.6	24.7	23.3	20.7	0.0	60
65	27.1	27.6	28.0	28.4	28.7	28.9	29.1	29.3	29.4	29.4	29.3	29.2	28.9	28.5	27.8	26.8	25.3	22.4	0.0	65
70	29.5	30.0	30.5	30.8	31.2	31.4	31.6	31.8	31.8	31.8	31.7	31.6	31.2	30.8	30.0	28.9	27.2	24.1	0.0	70
75	32.0	32.5	33.0	33.4	33.7	33.9	34.1	34.3	34.3	34.3	34.2	34.0	33.6	33.1	32.3	31.1	29.2	25.9	0.0	75
80	34.5	35.1	35.5	35.9	36.3	36.5	36.7	36.9	36.9	36.8	36.7	36.4	36.0	35.4	34.5	33.2	31.2	27.6	0.0	80
85	37.1	37.7	38.2	38.6	38.9	39.2	39.4	39.5	39.5	39.4	39.2	38.9	38.4	37.8	36.8	35.3	33.1	29.3	0.0	85
90	39.9	40.5	40.9	41.4	41.7	41.9	42.1	42.2	42.2	42.0	41.8	41.4	40.9	40.1	39.1	37.5	35.1	31.0	0.0	90
95	42.7	43.3	43.8	44.2	44.5	44.7	44.9	44.9	44.9	44.7	44.4	44.0	43.4	42.5	41.3	39.7	37.1	32.7	0.0	95
100	45.6	46.2	46.7	47.1	47.4	47.6	47.7	47.7	47.6	47.4	47.1	46.6	45.9	45.0	43.7	41.8	39.1	34.4	0.0	100
105	48.7	49.3	49.7	50.1	50.4	50.6	50.6	50.6	50.5	50.2	49.8	49.2	48.5	47.4	46.0	44.0	41.1	36.1	0.0	105
110	51.8	52.4	52.9	53.2	53.5	53.6	53.6	53.6	53.4	53.0	52.6	51.9	51.1	49.9	48.4	46.2	43.1	37.8	0.0	110
115	55.2	55.7	56.2	56.5	56.7	56.7	56.7	56.6	56.3	55.9	55.4	54.6	53.7	52.4	50.7	48.4	45.1	39.4	0.0	115
120	58.6	59.2	59.5	59.8	59.9	60.0	59.9	59.7	59.3	58.9	58.2	57.4	56.3	54.9	53.1	50.6	47.1	41.1	0.0	120

Contrary Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	0
5	2.0	1.9	1.9	1.8	1.8	1.7	1.6	1.6	1.5	1.4	1.3	1.1	1.0	0.8	0.6	0.3	0.0	*0.4*	*0.0*	5
10	3.9	3.9	3.8	3.7	3.5	3.4	3.3	3.1	3.0	2.8	2.5	2.3	2.0	1.6	1.2	0.7	0.0	*0.7*	*0.0*	10
15	5.9	5.8	5.6	5.5	5.3	5.1	4.9	4.7	4.4	4.1	3.8	3.4	2.9	2.4	1.8	1.0	0.0	*1.1*	*0.0*	15
20	7.9	7.7	7.5	7.3	7.1	6.9	6.6	6.3	5.9	5.5	5.1	4.5	3.9	3.2	2.3	1.3	0.0	*1.5*	*0.0*	20
25	9.9	9.7	9.5	9.2	8.9	8.6	8.3	7.9	7.4	6.9	6.4	5.7	4.9	4.0	2.9	1.6	0.0	*1.9*	*0.0*	25
30	11.9	11.7	11.4	11.1	10.8	10.4	10.0	9.5	9.0	8.3	7.7	6.8	5.9	4.8	3.5	1.9	0.0	*2.3*	*0.0*	30
35	14.0	13.7	13.4	13.0	12.6	12.2	11.7	11.1	10.5	9.8	9.0	8.0	6.9	5.6	4.0	2.2	*0.1*	*2.7*	*0.0*	35
40	16.1	15.7	15.4	15.0	14.5	14.0	13.4	12.8	12.1	11.2	10.3	9.2	7.9	6.4	4.6	2.4	*0.2*	*3.2*	*0.0*	40
45	18.2	17.8	17.4	16.9	16.4	15.8	15.2	14.5	13.6	12.7	11.6	10.4	8.9	7.2	5.1	2.7	*0.3*	*3.7*	*0.0*	45
50	20.4	19.9	19.5	19.0	18.4	17.7	17.0	16.2	15.3	14.2	13.0	11.6	9.9	7.9	5.6	2.9	*0.4*	*4.2*	*0.0*	50
55	22.6	22.1	21.6	21.0	20.4	19.7	18.9	18.0	16.9	15.7	14.4	12.8	10.9	8.7	6.1	3.1	*0.6*	*4.7*	*0.0*	55
60	24.8	24.3	23.8	23.2	22.5	21.7	20.8	19.8	18.6	17.3	15.8	14.0	11.9	9.5	6.6	3.2	*0.8*	*5.3*	*0.0*	60
65	27.1	26.6	26.0	25.3	24.6	23.7	22.7	21.6	20.4	18.9	17.2	15.2	12.9	10.2	7.1	3.3	*1.1*	*5.9*	*0.0*	65
70	29.5	29.0	28.3	27.6	26.8	25.8	24.8	23.6	22.2	20.5	18.7	16.5	14.0	11.0	7.5	3.4	*1.4*	*6.5*	*0.0*	70
75	32.0	31.4	30.7	29.9	29.0	28.0	26.9	25.5	24.0	22.3	20.2	17.8	15.0	11.7	7.9	3.4	*1.8*	*7.2*	*0.0*	75
80	34.5	33.9	33.2	32.3	31.4	30.3	29.0	27.6	25.9	24.0	21.8	19.1	16.1	12.4	8.2	3.3	*2.2*	*7.9*	*0.0*	80
85	37.1	36.5	35.7	34.8	33.8	32.6	31.3	29.7	27.9	25.8	23.4	20.5	17.1	13.1	8.5	3.2	*2.7*	*8.7*	*0.0*	85
90	39.9	39.2	38.4	37.4	36.4	35.1	33.7	32.0	30.0	27.8	25.1	21.9	18.2	13.8	8.7	2.9	*3.3*	*9.6*	*0.0*	90
95	42.7	42.0	41.1	40.2	39.0	37.7	36.2	34.4	32.2	29.8	26.8	23.4	19.3	14.5	8.9	2.6	*4.1*	*10.5*	*0.0*	95
100	45.6	44.9	44.0	43.0	41.8	40.4	38.8	36.9	34.6	31.9	28.7	24.9	20.4	15.1	9.0	2.2	*4.9*	*11.4*	*0.0*	100
105	48.7	47.9	47.1	46.0	44.8	43.3	41.6	39.5	37.0	34.1	30.6	26.4	21.5	15.6	9.0	1.7	*5.8*	*12.5*	*0.0*	105
110	51.8	51.1	50.2	49.2	47.9	46.3	44.5	42.3	39.7	36.5	32.7	28.1	22.6	16.1	8.8	1.0	*6.9*	*13.6*	*0.0*	110
115	55.2	54.4	53.6	52.5	51.2	49.6	47.7	45.3	42.5	39.1	34.9	29.8	23.7	16.6	8.5	0.1	*8.1*	*14.7*	*0.0*	115
120	58.6	57.9	57.1	56.0	54.7	53.0	51.0	48.6	45.6	41.8	37.2	31.6	24.8	16.9	8.1	*1.0*	*9.4*	*16.0*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—65°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

Same Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	2.2	2.2	2.2	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.2	2.2	2.0	1.8	0.0	5
10	4.4	4.4	4.5	4.5	4.6	4.6	4.6	4.6	4.7	4.7	4.6	4.6	4.6	4.5	4.5	4.3	4.1	3.6	0.0	10
15	6.6	6.7	6.7	6.8	6.9	6.9	7.0	7.0	7.0	7.0	7.0	6.9	6.9	6.8	6.7	6.5	6.1	5.4	0.0	15
20	8.8	8.9	9.0	9.1	9.2	9.2	9.3	9.3	9.3	9.3	9.3	9.3	9.2	9.1	8.9	8.6	8.1	7.2	0.0	20
25	11.0	11.2	11.3	11.4	11.5	11.6	11.6	11.7	11.7	11.7	11.7	11.6	11.5	11.4	11.1	10.8	10.1	9.0	0.0	25
30	13.3	13.4	13.6	13.7	13.8	13.9	14.0	14.0	14.1	14.1	14.0	14.0	13.8	13.7	13.4	12.9	12.2	10.8	0.0	30
35	15.5	15.7	15.9	16.1	16.2	16.3	16.4	16.4	16.5	16.5	16.4	16.3	16.2	16.0	15.6	15.1	14.2	12.6	0.0	35
40	17.8	18.1	18.3	18.5	18.6	18.7	18.8	18.9	18.9	18.9	18.8	18.7	18.5	18.3	17.9	17.2	16.2	14.4	0.0	40
45	20.2	20.5	20.7	20.9	21.0	21.2	21.2	21.3	21.3	21.3	21.2	21.1	20.9	20.6	20.1	19.4	18.3	16.2	0.0	45
50	22.6	22.9	23.1	23.3	23.5	23.6	23.7	23.8	23.8	23.8	23.7	23.5	23.3	22.9	22.4	21.6	20.3	18.0	0.0	50
55	25.0	25.3	25.6	25.8	26.0	26.1	26.2	26.3	26.3	26.3	26.1	26.0	25.7	25.3	24.7	23.8	22.4	19.8	0.0	55
60	27.5	27.8	28.1	28.3	28.5	28.7	28.8	28.8	28.8	28.8	28.6	28.4	28.1	27.6	26.9	26.0	24.4	21.6	0.0	60
65	30.0	30.4	30.7	30.9	31.1	31.3	31.4	31.4	31.4	31.3	31.2	30.9	30.5	30.0	29.3	28.2	26.4	23.4	0.0	65
70	32.6	33.0	33.3	33.5	33.7	33.9	34.0	34.0	34.0	33.9	33.7	33.4	33.0	32.4	31.6	30.4	28.5	25.2	0.0	70
75	35.3	35.6	36.0	36.2	36.4	36.6	36.7	36.7	36.6	36.5	36.3	36.0	35.5	34.8	33.9	32.6	30.6	27.0	0.0	75
80	38.0	38.4	38.7	39.0	39.2	39.3	39.4	39.4	39.3	39.2	38.9	38.5	38.0	37.3	36.2	34.8	32.6	28.8	0.0	80
85	40.8	41.2	41.5	41.8	42.0	42.1	42.2	42.2	42.0	41.9	41.5	41.1	40.5	39.7	38.6	37.0	34.7	30.6	0.0	85
90	43.7	44.1	44.5	44.7	44.9	45.0	45.0	45.0	44.8	44.6	44.2	43.7	43.1	42.2	41.0	39.3	36.8	32.4	0.0	90
95	46.7	47.1	47.4	47.7	47.8	47.9	47.9	47.8	47.6	47.4	47.0	46.4	45.7	44.7	43.5	41.5	38.8	34.1	0.0	95
100	49.8	50.2	50.5	50.7	50.8	50.9	50.9	50.7	50.5	50.2	49.7	49.1	48.3	47.2	45.8	43.8	40.9	35.9	0.0	100
105	53.0	53.4	53.6	53.8	53.9	54.0	53.9	53.7	53.4	53.0	52.5	51.8	50.9	49.8	48.2	46.1	43.0	37.7	0.0	105
110	56.3	56.6	56.9	57.1	57.1	57.1	57.0	56.7	56.4	56.0	55.4	54.6	53.6	52.3	50.6	48.4	45.0	39.4	0.0	110
115	59.7	60.0	60.3	60.4	60.4	60.3	60.1	59.8	59.4	58.9	58.2	57.4	56.3	54.9	53.1	50.6	47.1	41.2	0.0	115
120	63.3	63.5	63.7	63.8	63.7	63.6	63.3	63.0	62.5	61.9	61.2	60.2	59.0	57.5	55.6	52.9	49.2	42.9	0.0	120

Contrary Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	0
5	2.2	2.2	2.1	2.1	2.0	2.0	1.9	1.8	1.8	1.7	1.6	1.4	1.3	1.1	0.9	0.6	0.2	*0.2*	*0.0*	5
10	4.4	4.3	4.2	4.1	4.1	3.9	3.8	3.7	3.5	3.3	3.1	2.9	2.6	2.2	1.7	1.2	0.5	*0.4*	*0.0*	10
15	6.6	6.5	6.4	6.2	6.1	5.9	5.7	5.5	5.3	5.0	4.7	4.3	3.8	3.3	2.6	1.8	0.7	*0.6*	*0.0*	15
20	8.8	8.7	8.5	8.3	8.1	7.9	7.7	7.4	7.0	6.7	6.2	5.7	5.1	4.4	3.5	2.4	0.9	*0.9*	*0.0*	20
25	11.0	10.8	10.7	10.4	10.2	9.9	9.6	9.2	8.8	8.4	7.8	7.2	6.4	5.5	4.3	2.9	1.2	*1.1*	*0.0*	25
30	13.3	13.1	12.8	12.6	12.3	11.9	11.6	11.1	10.6	10.1	9.4	8.6	7.7	6.6	5.2	3.5	1.4	*1.4*	*0.0*	30
35	15.5	15.3	15.0	14.7	14.4	14.0	13.6	13.1	12.5	11.8	11.0	10.1	9.0	7.7	6.1	4.0	1.5	*1.6*	*0.0*	35
40	17.8	17.6	17.3	16.9	16.5	16.1	15.6	15.0	14.3	13.6	12.7	11.6	10.3	8.8	6.9	4.6	1.7	*1.9*	*0.0*	40
45	20.2	19.9	19.5	19.2	18.7	18.2	17.6	17.0	16.2	15.3	14.3	13.1	11.7	9.9	7.8	5.1	1.8	*2.3*	*0.0*	45
50	22.6	22.2	21.9	21.4	20.9	20.4	19.7	19.0	18.1	17.2	16.0	14.6	13.0	11.0	8.6	5.6	1.9	*2.6*	*0.0*	50
55	25.0	24.6	24.2	23.7	23.2	22.6	21.9	21.1	20.1	19.0	17.7	16.2	14.3	12.1	9.4	6.1	2.0	*3.0*	*0.0*	55
60	27.5	27.1	26.6	26.1	25.5	24.9	24.1	23.2	22.1	20.9	19.5	17.8	15.7	13.3	10.3	6.5	2.0	*3.4*	*0.0*	60
65	30.0	29.6	29.1	28.5	27.9	27.2	26.3	25.3	24.2	22.9	21.3	19.4	17.1	14.4	11.1	7.0	2.0	*3.9*	*0.0*	65
70	32.6	32.2	31.6	31.0	30.4	29.6	28.6	27.6	26.3	24.9	23.1	21.1	18.6	15.5	11.8	7.3	1.9	*4.4*	*0.0*	70
75	35.3	34.8	34.2	33.6	32.9	32.0	31.0	29.9	28.5	26.9	25.0	22.8	20.0	16.7	12.6	7.7	1.7	*5.0*	*0.0*	75
80	38.0	37.5	36.9	36.3	35.5	34.6	33.5	32.3	30.8	29.1	27.0	24.5	21.5	17.8	13.4	8.0	1.5	*5.6*	*0.0*	80
85	40.8	40.3	39.7	39.0	38.2	37.2	36.1	34.8	33.2	31.3	29.1	26.3	23.0	19.0	14.1	8.2	1.2	*6.3*	*0.0*	85
90	43.7	43.2	42.6	41.8	41.0	40.0	38.8	37.4	35.7	33.6	31.2	28.2	24.6	20.2	14.8	8.3	0.9	*7.1*	*0.0*	90
95	46.7	46.2	45.6	44.8	43.9	42.9	41.6	40.1	38.3	36.1	33.4	30.2	26.2	21.4	15.5	8.4	0.4	*7.9*	*0.0*	95
100	49.8	49.3	48.6	47.9	47.0	45.8	44.5	42.9	41.0	38.6	35.8	32.2	27.9	22.6	16.1	8.4	*0.2*	8.8	*0.0*	100
105	53.0	52.5	51.8	51.1	50.1	49.0	47.6	45.9	43.9	41.4	38.3	34.4	29.7	23.8	16.6	8.2	*0.9*	9.7	*0.0*	105
110	56.3	55.8	55.2	54.4	53.5	52.3	50.9	49.1	46.9	44.2	40.9	36.7	31.5	25.0	17.1	8.0	*1.8*	*10.8*	*0.0*	110
115	59.7	59.2	58.6	57.9	56.9	55.8	54.3	52.5	50.2	47.3	43.7	39.2	33.4	26.2	17.5	7.5	*2.8*	*11.9*	*0.0*	115
120	63.3	62.8	62.3	61.5	60.6	59.4	57.9	56.1	53.7	50.6	46.8	41.8	35.5	27.5	17.8	6.9	*4.0*	*13.2*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—70°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
	Same Name																			
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	2.4	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.4	2.4	2.4	2.3	2.1	1.9	0.0	5
10	4.9	4.9	4.9	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	4.9	4.9	4.8	4.7	4.5	4.3	3.8	0.0	10
15	7.3	7.4	7.4	7.5	7.5	7.5	7.6	7.6	7.5	7.5	7.5	7.4	7.3	7.2	7.1	6.8	6.4	5.7	0.0	15
20	9.8	9.8	9.9	10.0	10.0	10.1	10.1	10.1	10.1	10.0	10.0	9.9	9.8	9.6	9.4	9.1	8.6	7.6	0.0	20
25	12.2	12.3	12.4	12.5	12.6	12.6	12.6	12.6	12.6	12.6	12.5	12.4	12.2	12.0	11.8	11.4	10.7	9.5	0.0	25
30	14.7	14.9	15.0	15.0	15.1	15.2	15.2	15.2	15.2	15.1	15.0	14.9	14.7	14.5	14.1	13.6	12.8	11.4	0.0	30
35	17.2	17.4	17.5	17.6	17.7	17.7	17.8	17.8	17.7	17.7	17.6	17.4	17.2	16.9	16.5	15.9	15.0	13.3	0.0	35
40	19.8	19.9	20.1	20.2	20.3	20.3	20.4	20.4	20.3	20.2	20.1	19.9	19.7	19.4	18.9	18.2	17.1	15.2	0.0	40
45	22.4	22.5	22.7	22.8	22.9	23.0	23.0	23.0	22.9	22.8	22.7	22.5	22.2	21.8	21.3	20.5	19.3	17.1	0.0	45
50	25.0	25.2	25.3	25.5	25.6	25.6	25.6	25.6	25.6	25.4	25.3	25.1	24.7	24.3	23.7	22.8	21.4	19.0	0.0	50
55	27.6	27.8	28.0	28.2	28.2	28.3	28.3	28.3	28.2	28.1	27.9	27.6	27.3	26.8	26.1	25.1	23.6	20.9	0.0	55
60	30.3	30.6	30.7	30.9	31.0	31.0	31.0	31.0	30.9	30.8	30.5	30.2	29.8	29.3	28.5	27.4	25.7	22.8	0.0	60
65	33.1	33.3	33.5	33.6	33.7	33.8	33.8	33.7	33.6	33.5	33.2	32.9	32.4	31.8	30.9	29.7	27.9	24.6	0.0	65
70	35.9	36.1	36.3	36.5	36.5	36.6	36.6	36.5	36.4	36.2	35.9	35.5	35.0	34.3	33.4	32.0	30.0	26.5	0.0	70
75	38.7	39.0	39.2	39.3	39.4	39.4	39.4	39.3	39.2	38.9	38.6	38.2	37.6	36.8	35.8	34.4	32.2	28.4	0.0	75
80	41.7	41.9	42.1	42.2	42.3	42.3	42.3	42.2	42.0	41.7	41.3	40.9	40.2	39.4	38.3	36.7	34.4	30.3	0.0	80
85	44.7	44.9	45.1	45.2	45.3	45.3	45.2	45.1	44.8	44.5	44.1	43.6	42.9	42.0	40.7	39.0	36.5	32.2	0.0	85
90	47.8	48.0	48.2	48.3	48.3	48.3	48.2	48.0	47.7	47.4	46.9	46.3	45.6	44.6	43.2	41.4	38.7	34.0	0.0	90
95	50.9	51.1	51.3	51.4	51.4	51.3	51.2	51.0	50.7	50.3	49.8	49.1	48.3	47.2	45.7	43.8	40.9	35.9	0.0	95
100	54.2	54.4	54.5	54.5	54.5	54.4	54.3	54.0	53.7	53.2	52.6	51.9	51.0	49.8	48.2	46.1	43.0	37.8	0.0	100
105	57.5	57.7	57.8	57.8	57.7	57.6	57.4	57.1	56.7	56.2	55.5	54.7	53.7	52.4	50.8	48.5	45.2	39.6	0.0	105
110	60.9	61.1	61.1	61.1	61.0	60.8	60.6	60.2	59.8	59.2	58.5	57.6	56.5	55.1	53.3	50.9	47.4	41.5	0.0	110
115	64.4	64.5	64.6	64.5	64.4	64.1	63.8	63.4	62.9	62.2	61.4	60.5	59.3	57.8	55.9	53.3	49.6	43.3	0.0	115
120	68.0	68.1	68.1	68.0	67.8	67.5	67.1	66.6	66.0	65.3	64.4	63.4	62.1	60.5	58.4	55.7	51.7	45.1	0.0	120
DLo	**Contrary Name**																			DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	*0.0*	0
5	2.4	2.4	2.4	2.3	2.3	2.3	2.2	2.1	2.1	2.0	1.9	1.8	1.6	1.4	1.2	0.9	0.5	*0.0*	*0.0*	5
10	4.9	4.8	4.8	4.7	4.6	4.5	4.4	4.3	4.1	4.0	3.8	3.5	3.2	2.9	2.4	1.8	1.1	*0.0*	*0.0*	10
15	7.3	7.2	7.1	7.0	6.9	6.8	6.6	6.4	6.2	6.0	5.7	5.3	4.9	4.3	3.7	2.8	1.6	*0.0*	*0.0*	15
20	9.8	9.7	9.5	9.4	9.2	9.1	8.8	8.6	8.3	8.0	7.6	7.1	6.5	5.8	4.9	3.7	2.1	*0.0*	*0.0*	20
25	12.2	12.1	12.0	11.8	11.6	11.4	11.1	10.8	10.4	10.0	9.5	8.9	8.2	7.2	6.1	4.6	2.6	*0.1*	*0.0*	25
30	14.7	14.6	14.4	14.2	14.0	13.7	13.3	13.0	12.5	12.0	11.4	10.7	9.8	8.7	7.3	5.5	3.1	*0.1*	*0.0*	30
35	17.2	17.1	16.9	16.6	16.3	16.0	15.6	15.2	14.7	14.1	13.4	12.5	11.5	10.2	8.5	6.4	3.6	*0.2*	*0.0*	35
40	19.8	19.6	19.3	19.1	18.8	18.4	18.0	17.5	16.9	16.2	15.4	14.4	13.2	11.7	9.8	7.3	4.1	*0.3*	*0.0*	40
45	22.4	22.1	21.9	21.6	21.2	20.8	20.3	19.8	19.1	18.3	17.4	16.3	14.9	13.2	11.0	8.2	4.5	*0.4*	*0.0*	45
50	25.0	24.7	24.4	24.1	23.7	23.3	22.7	22.1	21.4	20.5	19.4	18.2	16.6	14.7	12.2	9.1	4.9	*0.6*	*0.0*	50
55	27.6	27.4	27.1	26.7	26.3	25.8	25.2	24.5	23.7	22.7	21.5	20.1	18.4	16.2	13.5	9.9	5.3	*0.7*	*0.0*	55
60	30.3	30.0	29.7	29.3	28.9	28.3	27.7	26.9	26.0	25.0	23.7	22.1	20.2	17.8	14.7	10.8	5.6	*1.0*	*0.0*	60
65	33.1	32.8	32.4	32.0	31.5	30.9	30.2	29.4	28.4	27.3	25.9	24.2	22.0	19.4	16.0	11.6	5.9	*1.3*	*0.0*	65
70	35.9	35.6	35.2	34.8	34.2	33.6	32.9	32.0	30.9	29.7	28.1	26.2	23.9	21.0	17.2	12.4	6.2	*1.6*	*0.0*	70
75	38.7	38.4	38.0	37.6	37.0	36.4	35.6	34.6	33.5	32.1	30.4	28.4	25.8	22.6	18.5	13.2	6.4	*2.0*	*0.0*	75
80	41.7	41.4	41.0	40.5	39.9	39.2	38.3	37.3	36.1	34.7	32.9	30.6	27.8	24.3	19.8	14.0	6.5	*2.4*	*0.0*	80
85	44.7	44.4	43.9	43.4	42.8	42.1	41.2	40.2	38.9	37.3	35.4	32.9	29.9	26.0	21.1	14.7	6.6	*3.0*	*0.0*	85
90	47.8	47.4	47.0	46.5	45.9	45.1	44.2	43.1	41.7	40.0	38.0	35.3	32.0	27.8	22.4	15.4	6.6	*3.6*	*0.0*	90
95	50.9	50.6	50.2	49.7	49.0	48.3	47.3	46.1	44.7	42.9	40.7	37.9	34.3	29.6	23.7	16.0	6.5	*4.3*	*0.0*	95
100	54.2	53.9	53.5	52.9	52.3	51.5	50.5	49.3	47.8	45.9	43.5	40.5	36.6	31.5	25.0	16.5	6.2	*5.0*	*0.0*	100
105	57.5	57.2	56.8	56.3	55.7	54.9	53.9	52.6	51.0	49.0	46.5	43.3	39.0	33.5	26.3	17.0	5.9	*5.9*	*0.0*	105
110	60.9	60.7	60.3	59.8	59.2	58.4	57.4	56.1	54.4	52.4	49.7	46.2	41.6	35.6	27.6	17.4	5.4	*6.9*	*0.0*	110
115	64.4	64.2	63.9	63.4	62.8	62.0	61.0	59.7	58.0	55.9	53.1	49.4	44.4	37.8	29.0	17.7	4.7	*8.0*	*0.0*	115
120	68.0	67.9	67.6	67.2	66.6	65.8	64.8	63.5	61.8	59.6	56.7	52.7	47.4	40.1	30.3	17.9	3.8	*9.2*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—75°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

Same Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.6	2.6	2.5	2.4	2.3	2.0	0.0	5
10	5.4	5.4	5.5	5.5	5.5	5.5	5.5	5.5	5.4	5.4	5.4	5.3	5.2	5.1	5.0	4.8	4.6	4.0	0.0	10
15	8.1	8.2	8.2	8.2	8.2	8.2	8.2	8.2	8.2	8.1	8.1	8.0	7.9	7.7	7.5	7.3	6.8	6.1	0.0	15
20	10.9	10.9	10.9	11.0	11.0	11.0	11.0	10.9	10.9	10.8	10.8	10.6	10.5	10.3	10.0	9.7	9.1	8.1	0.0	20
25	13.6	13.7	13.7	13.7	13.8	13.7	13.7	13.7	13.6	13.6	13.5	13.3	13.1	12.9	12.6	12.1	11.4	10.1	0.0	25
30	16.3	16.4	16.5	16.5	16.5	16.5	16.5	16.5	16.4	16.3	16.2	16.0	15.8	15.5	15.1	14.5	13.7	12.1	0.0	30
35	19.1	19.2	19.3	19.3	19.3	19.3	19.3	19.2	19.2	19.0	18.9	18.7	18.4	18.1	17.6	16.9	15.9	14.1	0.0	35
40	21.9	22.0	22.1	22.1	22.1	22.1	22.1	22.0	21.9	21.8	21.6	21.4	21.1	20.7	20.1	19.4	18.2	16.2	0.0	40
45	24.8	24.9	24.9	25.0	25.0	25.0	24.9	24.9	24.7	24.6	24.4	24.1	23.8	23.3	22.7	21.8	20.5	18.2	0.0	45
50	27.6	27.7	27.8	27.8	27.9	27.8	27.8	27.7	27.6	27.4	27.1	26.8	26.4	25.9	25.2	24.3	22.8	20.2	0.0	50
55	30.5	30.6	30.7	30.8	30.8	30.7	30.7	30.6	30.4	30.2	29.9	29.6	29.1	28.6	27.8	26.7	25.1	22.2	0.0	55
60	33.5	33.6	33.7	33.7	33.7	33.6	33.6	33.4	33.3	33.0	32.7	32.3	31.8	31.2	30.4	29.2	27.4	24.2	0.0	60
65	36.5	36.6	36.6	36.7	36.7	36.6	36.5	36.4	36.2	35.9	35.6	35.1	34.6	33.9	32.9	31.6	29.7	26.2	0.0	65
70	39.5	39.6	39.7	39.7	39.7	39.6	39.5	39.3	39.1	38.8	38.4	37.9	37.3	36.5	35.5	34.1	32.0	28.2	0.0	70
75	42.6	42.7	42.7	42.7	42.7	42.6	42.5	42.3	42.0	41.7	41.3	40.8	40.1	39.2	38.1	36.6	34.3	30.2	0.0	75
80	45.7	45.8	45.8	45.8	45.8	45.7	45.5	45.3	45.0	44.6	44.2	43.6	42.9	41.9	40.7	39.0	36.6	32.2	0.0	80
85	48.9	49.0	49.0	49.0	48.9	48.8	48.6	48.3	48.0	47.6	47.1	46.5	45.7	44.7	43.3	41.5	38.9	34.2	0.0	85
90	52.2	52.2	52.2	52.2	52.1	51.9	51.7	51.4	51.1	50.6	50.0	49.3	48.5	47.4	46.0	44.0	41.2	36.2	0.0	90
95	55.5	55.5	55.5	55.5	55.3	55.1	54.9	54.5	54.1	53.6	53.0	52.3	51.3	50.1	48.6	46.5	43.5	38.2	0.0	95
100	58.9	58.9	58.9	58.8	58.6	58.4	58.1	57.7	57.2	56.7	56.0	55.2	54.2	52.9	51.3	49.0	45.8	40.2	0.0	100
105	62.3	62.3	62.3	62.1	61.9	61.7	61.3	60.9	60.4	59.8	59.0	58.1	57.1	55.7	53.9	51.5	48.1	42.2	0.0	105
110	65.9	65.8	65.7	65.5	65.3	65.0	64.6	64.1	63.6	62.9	62.1	61.1	60.0	58.5	56.6	54.1	50.4	44.2	0.0	110
115	69.5	69.4	69.2	69.0	68.7	68.4	67.9	67.4	66.8	66.1	65.2	64.1	62.9	61.3	59.3	56.6	52.7	46.1	0.0	115
120	73.1	73.0	72.8	72.5	72.2	71.8	71.3	70.7	70.1	69.3	68.3	67.2	65.8	64.1	62.0	59.1	55.0	48.1	0.0	120

Contrary Name

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	*0.0*	0
5	2.7	2.7	2.7	2.6	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.2	2.0	1.9	1.7	1.4	0.9	0.3	*0.0*	5
10	5.4	5.4	5.3	5.3	5.2	5.2	5.1	5.0	4.9	4.7	4.5	4.3	4.1	3.7	3.3	2.7	1.9	0.6	*0.0*	10
15	8.1	8.1	8.0	7.9	7.9	7.8	7.6	7.5	7.3	7.1	6.8	6.5	6.1	5.6	5.0	4.1	2.8	0.9	*0.0*	15
20	10.9	10.8	10.7	10.6	10.5	10.4	10.2	10.0	9.8	9.5	9.1	8.7	8.2	7.5	6.6	5.4	3.7	1.2	*0.0*	20
25	13.6	13.5	13.4	13.3	13.2	13.0	12.8	12.5	12.2	11.9	11.4	10.9	10.2	9.4	8.3	6.8	4.6	1.5	*0.0*	25
30	16.3	16.2	16.1	16.0	15.8	15.6	15.4	15.1	14.7	14.3	13.8	13.1	12.3	11.3	10.0	8.1	5.5	1.7	*0.0*	30
35	19.1	19.0	18.9	18.7	18.5	18.3	18.0	17.7	17.2	16.7	16.1	15.4	14.5	13.2	11.6	9.5	6.4	1.9	*0.0*	35
40	21.9	21.8	21.7	21.5	21.2	21.0	20.6	20.3	19.8	19.2	18.5	17.7	16.6	15.2	13.4	10.9	7.3	2.1	*0.0*	40
45	24.8	24.6	24.5	24.3	24.0	23.7	23.3	22.9	22.4	21.7	21.0	20.0	18.7	17.2	15.1	12.2	8.2	2.3	*0.0*	45
50	27.6	27.5	27.3	27.1	26.8	26.5	26.1	25.6	25.0	24.3	23.4	22.3	21.0	19.2	16.8	13.6	9.1	2.5	*0.0*	50
55	30.5	30.4	30.2	29.9	29.6	29.3	28.8	28.3	27.7	26.9	25.9	24.7	23.2	21.2	18.6	15.0	9.9	2.6	*0.0*	55
60	33.5	33.3	33.1	32.8	32.5	32.1	31.7	31.1	30.4	29.5	28.5	27.2	25.5	23.3	20.4	16.3	10.7	2.6	*0.0*	60
65	36.5	36.3	36.1	35.8	35.5	35.1	34.6	33.9	33.2	32.3	31.1	29.7	27.8	25.4	22.2	17.7	11.5	2.6	*0.0*	65
70	39.5	39.3	39.1	38.8	38.5	38.0	37.5	36.9	36.1	35.1	33.8	32.2	30.2	27.6	24.0	19.1	12.2	2.6	*0.0*	70
75	42.6	42.4	42.2	41.9	41.5	41.1	40.5	39.8	39.0	37.9	36.6	34.9	32.7	29.8	25.9	20.5	13.0	2.5	*0.0*	75
80	45.7	45.5	45.3	45.0	44.7	44.2	43.6	42.9	42.0	40.9	39.4	37.6	35.2	32.1	27.8	21.9	13.6	2.3	*0.0*	80
85	48.9	48.8	48.5	48.2	47.9	47.4	46.8	46.0	45.1	43.9	42.4	40.4	37.9	34.5	29.8	23.3	14.3	2.0	*0.0*	85
90	52.2	52.0	51.8	51.5	51.1	50.7	50.0	49.3	48.3	47.0	45.4	43.4	40.6	36.9	31.9	24.8	14.8	1.7	*0.0*	90
95	55.5	55.4	55.2	54.9	54.5	54.0	53.4	52.6	51.6	50.3	48.6	46.4	43.5	39.5	34.0	26.2	15.3	1.3	*0.0*	95
100	58.9	58.8	58.6	58.3	58.0	57.5	56.9	56.0	55.0	53.7	51.9	49.6	46.5	42.2	36.2	27.6	15.7	0.7	*0.0*	100
105	62.3	62.3	62.1	61.9	61.5	61.1	60.4	59.6	58.6	57.2	55.3	52.9	49.6	45.0	38.4	29.1	16.0	0.0	*0.0*	105
110	65.9	65.8	65.7	65.5	65.2	64.7	64.1	63.3	62.2	60.8	59.0	56.4	52.9	48.0	40.8	30.5	16.2	*0.8*	*0.0*	110
115	69.5	69.5	69.4	69.2	68.9	68.5	67.9	67.1	66.1	64.7	62.7	60.1	56.4	51.1	43.4	32.0	16.3	*1.7*	*0.0*	115
120	73.1	73.2	73.2	73.0	72.8	72.4	71.9	71.1	70.1	68.7	66.7	64.0	60.2	54.5	46.1	33.5	16.1	*2.9*	*0.0*	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

Reverse sign of correction for *italic* figures

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—80°—Latitude of Receiver

Latitude of destination—Latitude of transmitter

DLo	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	DLo
	Same Name																			
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	2.9	2.9	2.8	2.8	2.7	2.6	2.6	2.2	0.0	5
10	6.1	6.1	6.1	6.1	6.0	6.0	6.0	6.0	5.9	5.9	5.8	5.8	5.7	5.6	5.4	5.2	4.9	4.4	0.0	10
15	9.1	9.1	9.1	9.1	9.1	9.0	9.0	9.0	8.9	8.8	8.8	8.7	8.5	8.4	8.1	7.8	7.4	6.6	0.0	15
20	12.1	12.1	12.1	12.1	12.1	12.1	12.0	12.0	11.9	11.8	11.7	11.5	11.4	11.2	10.9	10.5	9.9	8.8	0.0	20
25	15.2	15.2	15.2	15.2	15.1	15.1	15.0	15.0	14.9	14.8	14.6	14.4	14.2	13.9	13.6	13.1	12.3	11.0	0.0	25
30	18.2	18.2	18.2	18.2	18.2	18.1	18.1	18.0	17.9	17.7	17.6	17.3	17.1	16.8	16.3	15.7	14.8	13.2	0.0	30
35	21.3	21.3	21.3	21.3	21.2	21.2	21.1	21.0	20.9	20.7	20.5	20.3	19.9	19.6	19.0	18.3	17.3	15.4	0.0	35
40	24.4	24.4	24.4	24.4	24.3	24.3	24.2	24.0	23.9	23.7	23.5	23.2	22.8	22.4	21.8	20.9	19.7	17.5	0.0	40
45	27.5	27.5	27.5	27.5	27.4	27.3	27.2	27.1	26.9	26.7	26.4	26.1	25.7	25.2	24.5	23.6	22.2	19.7	0.0	45
50	30.7	30.7	30.7	30.6	30.6	30.5	30.3	30.2	30.0	29.7	29.4	29.1	28.6	28.0	27.3	26.2	24.7	21.9	0.0	50
55	33.9	33.9	33.8	33.8	33.7	33.6	33.4	33.3	33.0	32.8	32.4	32.0	31.5	30.8	30.0	28.9	27.2	24.1	0.0	55
60	37.1	37.1	37.0	37.0	36.9	36.7	36.6	36.4	36.1	35.8	35.4	35.0	34.4	33.7	32.8	31.5	29.6	26.3	0.0	60
65	40.3	40.3	40.3	40.2	40.1	39.9	39.7	39.5	39.2	38.9	38.5	38.0	37.3	36.6	35.6	34.2	32.1	28.5	0.0	65
70	43.6	43.6	43.5	43.4	43.3	43.1	42.9	42.6	42.3	42.0	41.5	41.0	40.3	39.4	38.3	36.8	34.6	30.7	0.0	70
75	46.9	46.9	46.8	46.7	46.5	46.4	46.1	45.8	45.5	45.1	44.6	44.0	43.2	42.3	41.1	39.5	37.1	32.9	0.0	75
80	50.3	50.2	50.1	50.0	49.8	49.6	49.4	49.0	48.6	48.2	47.7	47.0	46.2	45.2	43.9	42.2	39.6	35.0	0.0	80
85	53.7	53.6	53.5	53.3	53.1	52.9	52.6	52.3	51.9	51.4	50.8	50.1	49.2	48.1	46.7	44.8	42.1	37.2	0.0	85
90	57.1	57.0	56.9	56.7	56.5	56.2	55.9	55.5	55.1	54.5	53.9	53.1	52.2	51.0	49.6	47.5	44.6	39.4	0.0	90
95	60.6	60.5	60.4	60.2	59.9	59.6	59.2	58.8	58.3	57.7	57.0	56.2	55.2	54.0	52.4	50.2	47.1	41.6	0.0	95
100	64.2	64.0	63.9	63.6	63.3	63.0	62.6	62.1	61.6	61.0	60.2	59.3	58.3	56.9	55.2	52.9	49.6	43.7	0.0	100
105	67.8	67.6	67.4	67.1	66.8	66.4	66.0	65.5	64.9	64.2	63.4	62.5	61.3	59.9	58.1	55.7	52.1	45.9	0.0	105
110	71.4	71.2	71.0	70.7	70.3	69.9	69.4	68.9	68.2	67.5	66.6	65.6	64.4	62.9	61.0	58.4	54.6	48.1	0.0	110
115	75.1	74.9	74.6	74.2	73.8	73.4	72.9	72.3	71.6	70.8	69.9	68.8	67.5	65.9	63.8	61.1	57.1	50.2	0.0	115
120	78.9	78.6	78.3	77.9	77.4	76.9	76.4	75.7	75.0	74.1	73.2	72.0	70.6	68.9	66.7	63.8	59.6	52.4	0.0	120
DLo	**Contrary Name**																			DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	3.0	3.0	3.0	3.0	3.0	3.0	2.9	2.9	2.9	2.8	2.7	2.7	2.6	2.4	2.2	1.9	1.5	0.8	0.0	5
10	6.1	6.0	6.0	6.0	6.0	5.9	5.9	5.8	5.7	5.6	5.5	5.3	5.1	4.8	4.4	3.9	3.0	1.6	0.0	10
15	9.1	9.1	9.0	9.0	8.9	8.9	8.8	8.7	8.6	8.4	8.2	8.0	7.7	7.3	6.7	5.8	4.5	2.3	0.0	15
20	12.1	12.1	12.1	12.0	11.9	11.9	11.8	11.6	11.5	11.3	11.0	10.7	10.3	9.7	8.9	7.8	6.1	3.1	0.0	20
25	15.2	15.1	15.1	15.0	15.0	14.8	14.7	14.6	14.4	14.1	13.8	13.4	12.9	12.1	11.2	9.8	7.6	3.9	0.0	25
30	18.2	18.2	18.1	18.1	18.0	17.9	17.7	17.5	17.3	17.0	16.6	16.1	15.5	14.6	13.4	11.7	9.1	4.6	0.0	30
35	21.3	21.3	21.2	21.1	21.0	20.9	20.7	20.5	20.2	19.9	19.4	18.9	18.1	17.1	15.7	13.7	10.6	5.3	0.0	35
40	24.4	24.4	24.3	24.2	24.1	23.9	23.7	23.5	23.2	22.8	22.3	21.6	20.8	19.6	18.0	15.7	12.1	6.0	0.0	40
45	27.5	27.5	27.4	27.3	27.2	27.0	26.8	26.5	26.2	25.7	25.2	24.5	23.5	22.2	20.4	17.7	13.6	6.7	0.0	45
50	30.7	30.6	30.6	30.5	30.3	30.1	29.9	29.6	29.2	28.7	28.1	27.3	26.2	24.8	22.8	19.8	15.2	7.4	0.0	50
55	33.9	33.8	33.8	33.6	33.5	33.3	33.0	32.7	32.3	31.8	31.1	30.2	29.0	27.4	25.2	21.9	16.7	8.0	0.0	55
60	37.1	37.0	37.0	36.9	36.7	36.5	36.2	35.9	35.4	34.9	34.1	33.2	31.9	30.1	27.6	24.0	18.2	8.6	0.0	60
65	40.3	40.3	40.2	40.1	40.0	39.7	39.5	39.1	38.6	38.0	37.2	36.2	34.8	32.9	30.2	26.1	19.8	9.1	0.0	65
70	43.6	43.6	43.5	43.4	43.3	43.0	42.7	42.4	41.9	41.2	40.4	39.3	37.8	35.7	32.7	28.3	21.3	9.6	0.0	70
75	46.9	46.9	46.8	46.7	46.6	46.4	46.1	45.7	45.2	44.5	43.6	42.4	40.8	38.6	35.4	30.5	22.9	10.1	0.0	75
80	50.3	50.3	50.2	50.1	50.0	49.8	49.5	49.1	48.5	47.8	46.9	45.6	43.9	41.6	38.1	32.8	24.4	10.4	0.0	80
85	53.7	53.7	53.7	53.6	53.4	53.2	52.9	52.5	52.0	51.2	50.3	49.0	47.2	44.6	40.9	35.2	26.0	10.7	0.0	85
90	57.1	57.2	57.1	57.1	57.0	56.8	56.4	56.0	55.5	54.7	53.7	52.4	50.5	47.8	43.8	37.6	27.5	11.0	0.0	90
95	60.6	60.7	60.7	60.6	60.5	60.3	60.1	59.6	59.1	58.3	57.3	55.9	53.9	51.1	46.8	40.1	29.1	11.1	0.0	95
100	64.2	64.3	64.3	64.3	64.2	64.0	63.7	63.3	62.8	62.0	61.0	59.5	57.5	54.5	50.0	42.7	30.7	11.1	0.0	100
105	67.8	67.9	68.0	68.0	67.9	67.7	67.5	67.1	66.6	65.8	64.8	63.3	61.2	58.1	53.3	45.5	32.3	10.9	0.0	105
110	71.4	71.6	71.7	71.7	71.7	71.5	71.3	71.0	70.5	69.7	68.7	67.2	65.0	61.8	56.7	48.3	33.9	10.6	0.0	110
115	75.1	75.3	75.4	75.5	75.5	75.4	75.2	74.9	74.4	73.7	72.7	71.2	69.1	65.7	60.4	51.4	35.5	10.2	0.0	115
120	78.9	79.1	79.3	79.4	79.4	79.4	79.2	79.0	78.5	77.9	76.9	75.4	73.2	69.8	64.3	54.6	37.1	9.5	0.0	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

TABLE 1

Conversion Angle

Great-circle Sailing and Radio Bearings

Latitude of Departure—85°—Latitude of Receiver

DLo	Latitude of destination—Latitude of transmitter																			DLo
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	
	Same Name																			
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.3	3.3	3.3	3.3	3.2	3.2	3.1	3.0	2.9	2.8	2.6	0.0	5
10	6.8	6.8	6.8	6.8	6.8	6.7	6.7	6.7	6.6	6.6	6.5	6.4	6.3	6.2	6.1	5.9	5.5	5.0	0.0	10
15	10.3	10.2	10.2	10.2	10.2	10.1	10.1	10.0	9.9	9.8	9.7	9.6	9.5	9.3	9.1	8.8	8.3	7.5	0.0	15
20	13.7	13.7	13.6	13.6	13.5	13.5	13.4	13.3	13.2	13.1	13.0	12.8	12.6	12.4	12.1	11.7	11.1	10.0	0.0	20
25	17.1	17.1	17.1	17.0	16.9	16.9	16.8	16.7	16.6	16.4	16.2	16.0	15.8	15.5	15.1	14.6	13.8	12.5	0.0	25
30	20.6	20.5	20.5	20.4	20.3	20.2	20.1	20.0	19.9	19.7	19.5	19.3	19.0	18.6	18.2	17.6	16.6	14.9	0.0	30
35	24.0	24.0	23.9	23.8	23.7	23.6	23.5	23.4	23.2	23.0	22.8	22.5	22.2	21.7	21.2	20.5	19.4	17.4	0.0	35
40	27.5	27.5	27.4	27.3	27.2	27.0	26.9	26.7	26.5	26.3	26.0	25.7	25.3	24.9	24.3	23.4	22.2	19.9	0.0	40
45	31.0	30.9	30.8	30.7	30.6	30.5	30.3	30.1	29.9	29.6	29.3	29.0	28.5	28.0	27.3	26.4	25.0	22.4	0.0	45
50	34.5	34.4	34.3	34.2	34.1	33.9	33.7	33.5	33.2	33.0	32.6	32.2	31.7	31.1	30.4	29.3	27.7	24.9	0.0	50
55	38.0	37.9	37.8	37.7	37.5	37.3	37.1	36.9	36.6	36.3	35.9	35.5	34.9	34.3	33.4	32.3	30.5	27.4	0.0	55
60	41.6	41.5	41.3	41.2	41.0	40.8	40.6	40.3	40.0	39.6	39.2	38.7	38.2	37.4	36.5	35.2	33.3	29.9	0.0	60
65	45.1	45.0	44.9	44.7	44.5	44.3	44.0	43.7	43.4	43.0	42.6	42.0	41.4	40.6	39.6	38.2	36.1	32.4	0.0	65
70	48.7	48.6	48.4	48.2	48.0	47.8	47.5	47.2	46.8	46.4	45.9	45.3	44.6	43.8	42.6	41.1	38.9	34.9	0.0	70
75	52.3	52.2	52.0	51.8	51.5	51.3	51.0	50.6	50.2	49.8	49.2	48.6	47.9	46.9	45.8	44.1	41.7	37.4	0.0	75
80	56.0	55.8	55.6	55.4	55.1	54.8	54.5	54.1	53.7	53.2	52.6	51.9	51.1	50.1	48.9	47.1	44.5	39.9	0.0	80
85	59.6	59.4	59.2	59.0	58.7	58.4	58.0	57.6	57.1	56.6	56.0	55.3	54.4	53.3	52.0	50.1	47.4	42.4	0.0	85
90	63.3	63.1	62.9	62.6	62.3	61.9	61.6	61.1	60.6	60.1	59.4	58.6	57.7	56.6	55.1	53.1	50.2	44.9	0.0	90
95	67.0	66.8	66.5	66.2	65.9	65.5	65.1	64.6	64.1	63.5	62.8	62.0	61.0	59.8	58.2	56.1	53.0	47.4	0.0	95
100	70.8	70.5	70.2	69.9	69.5	69.1	68.7	68.2	67.6	67.0	66.2	65.4	64.3	63.0	61.4	59.2	55.9	49.9	0.0	100
105	74.6	74.3	74.0	73.6	73.2	72.8	72.3	71.8	71.2	70.5	69.7	68.8	67.6	66.3	64.5	62.2	58.7	52.4	0.0	105
110	78.4	78.1	77.7	77.3	76.9	76.4	75.9	75.4	74.7	74.0	73.2	72.2	71.0	69.6	67.7	65.2	61.5	54.9	0.0	110
115	82.2	81.9	81.5	81.1	80.6	80.1	79.6	79.0	78.3	77.5	76.6	75.6	74.4	72.9	70.9	68.3	64.4	57.4	0.0	115
120	86.1	85.7	85.3	84.9	84.4	83.9	83.3	82.6	81.9	81.1	80.2	79.1	77.8	76.2	74.1	71.4	67.3	59.9	0.0	120
DLo	**Contrary Name**																			DLo
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
5	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.3	3.3	3.2	3.1	3.0	2.8	2.5	1.7	0.0	5
10	6.8	6.9	6.9	6.9	6.9	6.9	6.8	6.8	6.8	6.7	6.7	6.6	6.5	6.3	6.0	5.6	4.9	3.4	0.0	10
15	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.2	10.2	10.1	10.0	9.9	9.7	9.5	9.1	8.5	7.4	5.1	0.0	15
20	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.6	13.5	13.4	13.2	13.0	12.6	12.1	11.3	9.9	6.8	0.0	20
25	17.1	17.2	17.2	17.2	17.2	17.2	17.1	17.1	17.0	16.9	16.7	16.5	16.2	15.8	15.2	14.1	12.3	8.6	0.0	25
30	20.6	20.6	20.6	20.7	20.7	20.6	20.6	20.5	20.4	20.3	20.1	19.9	19.5	19.0	18.3	17.0	14.8	10.3	0.0	30
35	24.0	24.1	24.1	24.1	24.1	24.1	24.1	24.0	23.9	23.7	23.5	23.2	22.8	22.2	21.4	19.9	17.4	12.0	0.0	35
40	27.5	27.6	27.6	27.6	27.6	27.6	27.6	27.5	27.4	27.2	27.0	26.6	26.2	25.5	24.5	22.8	19.9	13.7	0.0	40
45	31.0	31.1	31.1	31.1	31.1	31.1	31.1	31.0	30.9	30.7	30.4	30.1	29.5	28.8	27.6	25.8	22.5	15.4	0.0	45
50	34.5	34.6	34.6	34.7	34.7	34.6	34.6	34.5	34.4	34.2	33.9	33.5	32.9	32.1	30.9	28.8	25.1	17.1	0.0	50
55	38.0	38.1	38.2	38.2	38.2	38.2	38.2	38.1	37.9	37.7	37.4	37.0	36.4	35.5	34.1	31.9	27.7	18.9	0.0	55
60	41.6	41.7	41.7	41.8	41.8	41.8	41.7	41.6	41.5	41.3	41.0	40.5	39.9	38.9	37.4	34.9	30.4	20.6	0.0	60
65	45.1	45.2	45.3	45.4	45.4	45.4	45.4	45.3	45.1	44.9	44.6	44.1	43.4	42.4	40.8	38.1	33.2	22.3	0.0	65
70	48.7	48.8	48.9	49.0	49.0	49.0	49.0	48.9	48.8	48.5	48.2	47.7	47.0	45.9	44.2	41.3	36.0	24.0	0.0	70
75	52.3	52.5	52.6	52.6	52.7	52.7	52.7	52.6	52.5	52.2	51.9	51.4	50.6	49.5	47.7	44.6	38.8	25.8	0.0	75
80	56.0	56.1	56.2	56.3	56.4	56.4	56.4	56.3	56.2	56.0	55.6	55.1	54.3	53.1	51.2	48.0	41.8	27.5	0.0	80
85	59.6	59.8	59.9	60.0	60.1	60.2	60.2	60.1	60.0	59.8	59.4	58.9	58.1	56.9	54.9	51.4	44.8	29.3	0.0	85
90	63.3	63.5	63.7	63.8	63.9	63.9	64.0	63.9	63.8	63.6	63.3	62.7	61.9	60.7	58.6	55.0	47.9	31.0	0.0	90
95	67.0	67.3	67.4	67.6	67.7	67.8	67.8	67.8	67.7	67.5	67.2	66.6	65.8	64.5	62.4	58.7	51.1	32.8	0.0	95
100	70.8	71.0	71.2	71.4	71.5	71.6	71.7	71.7	71.6	71.4	71.1	70.6	69.8	68.5	66.4	62.5	54.5	34.5	0.0	100
105	74.6	74.8	75.1	75.2	75.4	75.5	75.6	75.6	75.6	75.4	75.1	74.7	73.9	72.6	70.4	66.4	58.0	36.3	0.0	105
110	78.4	78.7	78.9	79.1	79.3	79.5	79.6	79.6	79.6	79.5	79.2	78.8	78.0	76.8	74.6	70.5	61.7	38.0	0.0	110
115	82.2	82.5	82.8	83.1	83.3	83.5	83.6	83.7	83.7	83.6	83.4	83.0	82.3	81.0	78.9	74.7	65.5	39.8	0.0	115
120	86.1	86.4	86.7	87.0	87.3	87.5	87.7	87.8	87.8	87.8	87.6	87.2	86.6	85.4	83.3	79.2	69.6	41.6	0.0	120

Great-circle sailing			Radio bearings		
Latitude departure	Destination	Correction sign	Latitude receiver	Transmitter	Correction sign
N	Eastward	−	N	Eastward	+
N	Westward	+	N	Westward	−
S	Eastward	+	S	Eastward	−
S	Westward	−	S	Westward	+

TABLE 2

Conversion of Compass Points to Degrees

	Points	Angular measure ° ′ ″		Points	Angular measure ° ′ ″
NORTH TO EAST			SOUTH TO WEST		
North	0	0 00 00	South	16	180 00 00
N¼E	¼	2 48 45	S¼W	16¼	182 48 45
N½E	½	5 37 30	S½W	16½	185 37 30
N¾E	¾	8 26 15	S¾W	16¾	188 26 15
N by E	1	11 15 00	S by W	17	191 15 00
N by E¼E	1¼	14 03 45	S by W¼W	17¼	194 03 45
N by E½E	1½	16 52 30	S by W½W	17½	196 52 30
N by E¾E	1¾	19 41 15	S by W¾W	17¾	199 41 15
NNE	2	22 30 00	SSW	18	202 30 00
NNE¼E	2¼	25 18 45	SSW¼W	18¼	205 18 45
NNE½E	2½	28 07 30	SSW½W	18½	208 07 30
NNE¾E	2¾	30 56 15	SSW¾W	18¾	210 56 15
NE by N	3	33 45 00	SW by S	19	213 45 00
NE¾N	3¼	36 33 45	SW¾S	19¼	216 33 45
NE½N	3½	39 22 30	SW½S	19½	219 22 30
NE¼N	3¾	42 11 15	SW¼S	19¾	222 11 15
NE	4	45 00 00	SW	20	225 00 00
NE¼E	4¼	47 48 45	SW¼W	20¼	227 48 45
NE½E	4½	50 37 30	SW½W	20½	230 37 30
NE¾E	4¾	53 26 15	SW¾W	20¾	233 26 15
NE by E	5	56 15 00	SW by W	21	236 15 00
NE by E¼E	5¼	59 03 45	SW by W¼W	21¼	239 03 45
NE by E½E	5½	61 52 30	SW by W½W	21½	241 52 30
NE by E¾E	5¾	64 41 15	SW by W¾W	21¾	244 41 15
ENE	6	67 30 00	WSW	22	247 30 00
ENE¼E	6¼	70 18 45	WSW¼W	22¼	250 18 45
ENE½E	6½	73 07 30	WSW½W	22½	253 07 30
ENE¾E	6¾	75 56 15	WSW¾W	22¾	255 56 15
E by N	7	78 45 00	W by S	23	258 45 00
E¾N	7¼	81 33 45	W¾S	23¼	261 33 45
E½N	7½	84 22 30	W½S	23½	264 22 30
E¼N	7¾	87 11 15	W¼S	23¾	267 11 15
EAST TO SOUTH			WEST TO NORTH		
East	8	90 00 00	West	24	270 00 00
E¼S	8¼	92 48 45	W¼N	24¼	272 48 45
E½S	8½	95 37 30	W½N	24½	275 37 30
E¾S	8¾	98 26 15	W¾N	24¾	278 26 15
E by S	9	101 15 00	W by N	25	281 15 00
ESE¾E	9¼	104 03 45	WNW¾W	25¼	284 03 45
ESE½E	9½	106 52 30	WNW½W	25½	286 52 30
ESE¼E	9¾	109 41 15	WNW¼W	25¾	289 41 15
ESE	10	112 30 00	WNW	26	292 30 00
SE by E¾E	10¼	115 18 45	NW by W¾W	26¼	295 18 45
SE by E½E	10½	118 07 30	NW by W½W	26½	298 07 30
SE by E¼E	10¾	120 56 15	NW by W¼W	26¾	300 56 15
SE by E	11	123 45 00	NW by W	27	303 45 00
SE¾E	11¼	126 33 45	NW¾W	27¼	306 33 45
SE½E	11½	129 22 30	NW½W	27½	309 22 30
SE¼E	11¾	132 11 15	NW¼W	27¾	312 11 15
SE	12	135 00 00	NW	28	315 00 00
SE¼S	12¼	137 48 45	NW¼N	28¼	317 48 45
SE½S	12½	140 37 30	NW½N	28½	320 37 30
SE¾S	12¾	143 26 15	NW¾N	28¾	323 26 15
SE by S	13	146 15 00	NW by N	29	326 15 00
SSE¾E	13¼	149 03 45	NNW¾W	29¼	329 03 45
SSE½E	13½	151 52 30	NNW½W	29½	331 52 30
SSE¼E	13¾	154 41 15	NNW¼W	29¾	334 41 15
SSE	14	157 30 00	NNW	30	337 30 00
S by E¾E	14¼	160 18 45	N by W¾W	30¼	340 18 45
S by E½E	14½	163 07 30	N by W½W	30½	343 07 30
S by E¼E	14¾	165 56 15	N by W¼W	30¾	345 56 15
S by E	15	168 45 00	N by W	31	348 45 00
S¾E	15¼	171 33 45	N¾W	31¼	351 33 45
S½E	15½	174 22 30	N½W	31½	354 22 30
S¼E	15¾	177 11 15	N¼W	31¾	357 11 15
South	16	180 00 00	North	32	360 00 00

TABLE 3

Traverse Table

0°—180°—180°—360°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	1. 000	0. 000	1	1. 000	1	0. 000
2	2. 000	0. 000	2	2. 000	2	0. 000
3	3. 000	0. 000	3	3. 000	3	0. 000
4	4. 000	0. 000	4	4. 000	4	0. 000
5	5. 000	0. 000	5	5. 000	5	0. 000
6	6. 000	0. 000	6	6. 000	6	0. 000
7	7. 000	0. 000	7	7. 000	7	0. 000
8	8. 000	0. 000	8	8. 000	8	0. 000
9	9. 000	0. 000	9	9. 000	9	0. 000

Course		
0° 180°	p÷l DLo÷m	179° 359°
0. 0	0. 000	1. 0
0. 1	0. 002	0. 9
0. 2	0. 003	0. 8
0. 3	0. 005	0. 7
0. 4	0. 007	0. 6
0. 5	0. 009	0. 5
0. 6	0. 010	0. 4
0. 7	0. 012	0. 3
0. 8	0. 014	0. 2
0. 9	0. 016	0. 1

1°—179°—181°—359°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	1. 000	0. 017	1	1. 000	1	0. 017
2	2. 000	0. 035	2	2. 000	2	0. 035
3	3. 000	0. 052	3	3. 000	3	0. 052
4	3. 999	0. 070	4	4. 001	4	0. 070
5	4. 999	0. 087	5	5. 001	5	0. 087
6	5. 999	0. 105	6	6. 001	6	0. 105
7	6. 999	0. 122	7	7. 001	7	0. 122
8	7. 999	0. 140	8	8. 001	8	0. 140
9	8. 999	0. 157	9	9. 001	9	0. 157

Course		
1° 181°	p÷l DLo÷m	178° 358°
0. 0	0. 017	1. 0
0. 1	0. 019	0. 9
0. 2	0. 021	0. 8
0. 3	0. 023	0. 7
0. 4	0. 024	0. 6
0. 5	0. 026	0. 5
0. 6	0. 028	0. 4
0. 7	0. 030	0. 3
0. 8	0. 031	0. 2
0. 9	0. 033	0. 1

2°—178°—182°—358°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 999	0. 035	1	1. 001	1	0. 035
2	1. 999	0. 070	2	2. 001	2	0. 070
3	2. 998	0. 105	3	3. 002	3	0. 105
4	3. 998	0. 140	4	4. 002	4	0. 140
5	4. 997	0. 174	5	5. 003	5	0. 175
6	5. 996	0. 209	6	6. 004	6	0. 210
7	6. 996	0. 244	7	7. 004	7	0. 244
8	7. 995	0. 279	8	8. 005	8	0. 279
9	8. 995	0. 314	9	9. 005	9	0. 314

Course		
2° 182°	p÷l DLo÷m	177° 357°
0. 0	0. 035	1. 0
0. 1	0. 037	0. 9
0. 2	0. 038	0. 8
0. 3	0. 040	0. 7
0. 4	0. 042	0. 6
0. 5	0. 044	0. 5
0. 6	0. 045	0. 4
0. 7	0. 047	0. 3
0. 8	0. 049	0. 2
0. 9	0. 051	0. 1

3°—177°—183°—357°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 999	0. 052	1	1. 001	1	0. 052
2	1. 997	0. 105	2	2. 003	2	0. 105
3	2. 996	0. 157	3	3. 004	3	0. 157
4	3. 995	0. 209	4	4. 005	4	0. 210
5	4. 993	0. 262	5	5. 007	5	0. 262
6	5. 992	0. 314	6	6. 008	6	0. 314
7	6. 990	0. 366	7	7. 010	7	0. 367
8	7. 989	0. 419	8	8. 011	8	0. 419
9	8. 988	0. 471	9	9. 012	9	0. 472

Course		
3° 183°	p÷l DLo÷m	176° 356°
0. 0	0. 052	1. 0
0. 1	0. 054	0. 9
0. 2	0. 056	0. 8
0. 3	0. 058	0. 7
0. 4	0. 059	0. 6
0. 5	0. 061	0. 5
0. 6	0. 063	0. 4
0. 7	0. 065	0. 3
0. 8	0. 066	0. 2
0. 9	0. 068	0. 1

4°—176°—184°—356°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 998	0. 070	1	1. 002	1	0. 070
2	1. 995	0. 140	2	2. 005	2	0. 140
3	2. 993	0. 209	3	3. 007	3	0. 210
4	3. 990	0. 279	4	4. 010	4	0. 280
5	4. 988	0. 349	5	5. 012	5	0. 350
6	5. 985	0. 419	6	6. 015	6	0. 420
7	6. 983	0. 488	7	7. 017	7	0. 489
8	7. 981	0. 558	8	8. 020	8	0. 559
9	8. 978	0. 628	9	9. 022	9	0. 629

Course		
4° 184°	p÷l DLo÷m	175° 355°
0. 0	0. 070	1. 0
0. 1	0. 072	0. 9
0. 2	0. 073	0. 8
0. 3	0. 075	0. 7
0. 4	0. 077	0. 6
0. 5	0. 079	0. 5
0. 6	0. 080	0. 4
0. 7	0. 082	0. 3
0. 8	0. 084	0. 2
0. 9	0. 086	0. 1

TABLE 3

Traverse Table

5°—175°—185°—355°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 996	0. 087	1	1. 004	1	0. 087
2	1. 992	0. 174	2	2. 008	2	0. 175
3	2. 989	0. 261	3	3. 011	3	0. 262
4	3. 985	0. 349	4	4. 015	4	0. 350
5	4. 981	0. 436	5	5. 019	5	0. 437
6	5. 977	0. 523	6	6. 023	6	0. 525
7	6. 973	0. 610	7	7. 027	7	0. 612
8	7. 970	0. 697	8	8. 031	8	0. 700
9	8. 966	0. 784	9	9. 034	9	0. 787

Course		
5° 185°	p÷l DLo÷m	174° 354°
0. 0	0. 087	1. 0
0. 1	0. 089	0. 9
0. 2	0. 091	0. 8
0. 3	0. 093	0. 7
0. 4	0. 095	0. 6
0. 5	0. 096	0. 5
0. 6	0. 098	0. 4
0. 7	0. 100	0. 3
0. 8	0. 102	0. 2
0. 9	0. 103	0. 1

6°—174°—186°—354°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 995	0. 105	1	1. 006	1	0. 105
2	1. 989	0. 209	2	2. 011	2	0. 210
3	2. 984	0. 314	3	3. 017	3	0. 315
4	3. 978	0. 418	4	4. 022	4	0. 420
5	4. 973	0. 523	5	5. 028	5	0. 526
6	5. 967	0. 627	6	6. 033	6	0. 631
7	6. 962	0. 732	7	7. 039	7	0. 736
8	7. 956	0. 836	8	8. 044	8	0. 841
9	8. 951	0. 941	9	9. 050	9	0. 946

Course		
6° 186°	p÷l DLo÷m	173° 353°
0. 0	0. 105	1. 0
0. 1	0. 107	0. 9
0. 2	0. 109	0. 8
0. 3	0. 110	0. 7
0. 4	0. 112	0. 6
0. 5	0. 114	0. 5
0. 6	0. 116	0. 4
0. 7	0. 117	0. 3
0. 8	0. 119	0. 2
0. 9	0. 121	0. 1

7°—173°—187°—353°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 993	0. 122	1	1. 008	1	0. 123
2	1. 985	0. 244	2	2. 015	2	0. 246
3	2. 978	0. 366	3	3. 023	3	0. 368
4	3. 970	0. 487	4	4. 030	4	0. 491
5	4. 963	0. 609	5	5. 038	5	0. 614
6	5. 955	0. 731	6	6. 045	6	0. 737
7	6. 948	0. 853	7	7. 053	7	0. 859
8	7. 940	0. 975	8	8. 060	8	0. 982
9	8. 933	1. 097	9	9. 068	9	1. 105

Course		
7° 187°	p÷l DLo÷m	172° 352°
0. 0	0. 123	1. 0
0. 1	0. 125	0. 9
0. 2	0. 126	0. 8
0. 3	0. 128	0. 7
0. 4	0. 130	0. 6
0. 5	0. 132	0. 5
0. 6	0. 133	0. 4
0. 7	0. 135	0. 3
0. 8	0. 137	0. 2
0. 9	0. 139	0. 1

8°—172°—188°—352°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 990	0. 139	1	1. 010	1	0. 141
2	1. 981	0. 278	2	2. 020	2	0. 281
3	2. 971	0. 418	3	3. 029	3	0. 422
4	3. 961	0. 557	4	4. 039	4	0. 562
5	4. 951	0. 696	5	5. 049	5	0. 703
6	5. 942	0. 835	6	6. 059	6	0. 843
7	6. 932	0. 974	7	7. 069	7	0. 984
8	7. 922	1. 113	8	8. 079	8	1. 124
9	8. 912	1. 253	9	9. 088	9	1. 265

Course		
8° 188°	p÷l DLo÷m	171° 351°
0. 0	0. 141	1. 0
0. 1	0. 142	0. 9
0. 2	0. 144	0. 8
0. 3	0. 146	0. 7
0. 4	0. 148	0. 6
0. 5	0. 149	0. 5
0. 6	0. 151	0. 4
0. 7	0. 153	0. 3
0. 8	0. 155	0. 2
0. 9	0. 157	0. 1

9°—171°—189°—351°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 988	0. 156	1	1. 012	1	0. 158
2	1. 975	0. 313	2	2. 025	2	0. 317
3	2. 963	0. 469	3	3. 037	3	0. 475
4	3. 951	0. 626	4	4. 050	4	0. 634
5	4. 938	0. 782	5	5. 062	5	0. 792
6	5. 926	0. 939	6	6. 075	6	0. 950
7	6. 914	1. 095	7	7. 087	7	1. 109
8	7. 902	1. 251	8	8. 100	8	1. 267
9	8. 889	1. 408	9	9. 112	9	1. 425

Course		
9° 189°	p÷l DLo÷m	170° 350°
0. 0	0. 158	1. 0
0. 1	0. 160	0. 9
0. 2	0. 162	0. 8
0. 3	0. 164	0. 7
0. 4	0. 166	0. 6
0. 5	0. 167	0. 5
0. 6	0. 169	0. 4
0. 7	0. 171	0. 3
0. 8	0. 173	0. 2
0. 9	0. 175	0. 1

TABLE 3

Traverse Table

10°—170°—190°—350°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 985	0. 174	1	1. 015	1	0. 176
2	1. 970	0. 347	2	2. 031	2	0. 353
3	2. 954	0. 521	3	3. 046	3	0. 529
4	3. 939	0. 695	4	4. 062	4	0. 705
5	4. 924	0. 868	5	5. 077	5	0. 882
6	5. 909	1. 042	6	6. 093	6	1. 058
7	6. 894	1. 216	7	7. 108	7	1. 234
8	7. 878	1. 389	8	8. 123	8	1. 411
9	8. 863	1. 563	9	9. 139	9	1. 587

Course		
10° 190°	p÷l DLo÷m	169° 349°
0. 0	0. 176	1. 0
0. 1	0. 178	0. 9
0. 2	0. 180	0. 8
0. 3	0. 182	0. 7
0. 4	0. 184	0. 6
0. 5	0. 185	0. 5
0. 6	0. 187	0. 4
0. 7	0. 189	0. 3
0. 8	0. 191	0. 2
0. 9	0. 193	0. 1

11°—169°—191°—349°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 982	0. 191	1	1. 019	1	0. 194
2	1. 963	0. 382	2	2. 037	2	0. 389
3	2. 945	0. 572	3	3. 056	3	0. 583
4	3. 927	0. 763	4	4. 075	4	0. 778
5	4. 908	0. 954	5	5. 094	5	0. 972
6	5. 890	1. 145	6	6. 112	6	1. 166
7	6. 871	1. 336	7	7. 131	7	1. 361
8	7. 853	1. 526	8	8. 150	8	1. 555
9	8. 835	1. 717	9	9. 168	9	1. 749

Course		
11° 191°	p÷l DLo÷m	168° 348°
0. 0	0. 194	1. 0
0. 1	0. 196	0. 9
0. 2	0. 198	0. 8
0. 3	0. 200	0. 7
0. 4	0. 202	0. 6
0. 5	0. 203	0. 5
0. 6	0. 205	0. 4
0. 7	0. 207	0. 3
0. 8	0. 209	0. 2
0. 9	0. 211	0. 1

12°—168°—192°—348°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 978	0. 208	1	1. 022	1	0. 213
2	1. 956	0. 416	2	2. 045	2	0. 425
3	2. 934	0. 624	3	3. 067	3	0. 638
4	3. 913	0. 832	4	4. 089	4	0. 850
5	4. 891	1. 040	5	5. 112	5	1. 063
6	5. 869	1. 247	6	6. 134	6	1. 275
7	6. 847	1. 455	7	7. 156	7	1. 488
8	7. 825	1. 663	8	8. 179	8	1. 700
9	8. 803	1. 871	9	9. 201	9	1. 913

Course		
12° 192°	p÷l DLo÷m	167° 347°
0. 0	0. 213	1. 0
0. 1	0. 214	0. 9
0. 2	0. 216	0. 8
0. 3	0. 218	0. 7
0. 4	0. 220	0. 6
0. 5	0. 222	0. 5
0. 6	0. 224	0. 4
0. 7	0. 225	0. 3
0. 8	0. 227	0. 2
0. 9	0. 229	0. 1

13°—167°—193°—347°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 974	0. 225	1	1. 026	1	0. 231
2	1. 949	0. 450	2	2. 053	2	0. 462
3	2. 923	0. 675	3	3. 079	3	0. 693
4	3. 897	0. 900	4	4. 105	4	0. 923
5	4. 872	1. 125	5	5. 132	5	1. 154
6	5. 846	1. 350	6	6. 158	6	1. 385
7	6. 821	1. 575	7	7. 184	7	1. 616
8	7. 795	1. 800	8	8. 210	8	1. 847
9	8. 769	2. 025	9	9. 237	9	2. 078

Course		
13° 193°	p÷l DLo÷m	166° 346°
0. 0	0. 231	1. 0
0. 1	0. 233	0. 9
0. 2	0. 235	0. 8
0. 3	0. 236	0. 7
0. 4	0. 238	0. 6
0. 5	0. 240	0. 5
0. 6	0. 242	0. 4
0. 7	0. 244	0. 3
0. 8	0. 246	0. 2
0. 9	0. 247	0. 1

14°—166°—194°—346°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 970	0. 242	1	1. 031	1	0. 249
2	1. 941	0. 484	2	2. 061	2	0. 499
3	2. 911	0. 726	3	3. 092	3	0. 748
4	3. 881	0. 968	4	4. 122	4	0. 997
5	4. 851	1. 210	5	5. 153	5	1. 247
6	5. 822	1. 452	6	6. 184	6	1. 496
7	6. 792	1. 693	7	7. 214	7	1. 745
8	7. 762	1. 935	8	8. 245	8	1. 995
9	8. 733	2. 177	9	9. 276	9	2. 244

Course		
14° 194°	p÷l DLo÷m	165° 345°
0. 0	0. 249	1. 0
0. 1	0. 251	0. 9
0. 2	0. 253	0. 8
0. 3	0. 255	0. 7
0. 4	0. 257	0. 6
0. 5	0. 259	0. 5
0. 6	0. 260	0. 4
0. 7	0. 262	0. 3
0. 8	0. 264	0. 2
0. 9	0. 266	0. 1

TABLE 3

Traverse Table

15°—165°—195°—345°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 966	0. 259	1	1. 035	1	0. 268
2	1. 932	0. 518	2	2. 071	2	0. 536
3	2. 898	0. 776	3	3. 106	3	0. 804
4	3. 864	1. 035	4	4. 141	4	1. 072
5	4. 830	1. 294	5	5. 176	5	1. 340
6	5. 796	1. 553	6	6. 212	6	1. 608
7	6. 761	1. 812	7	7. 247	7	1. 876
8	7. 727	2. 071	8	8. 282	8	2. 144
9	8. 693	2. 329	9	9. 317	9	2. 412

Course		
15° 195°	p÷l / DLo÷m	164° 344°
0. 0	0. 268	1. 0
0. 1	0. 270	0. 9
0. 2	0. 272	0. 8
0. 3	0. 274	0. 7
0. 4	0. 275	0. 6
0. 5	0. 277	0. 5
0. 6	0. 279	0. 4
0. 7	0. 281	0. 3
0. 8	0. 283	0. 2
0. 9	0. 285	0. 1

16°—164°—196°—344°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 961	0. 276	1	1. 040	1	0. 287
2	1. 923	0. 551	2	2. 081	2	0. 573
3	2. 884	0. 827	3	3. 121	3	0. 860
4	3. 845	1. 103	4	4. 161	4	1. 147
5	4. 806	1. 378	5	5. 201	5	1. 434
6	5. 768	1. 654	6	6. 242	6	1. 720
7	6. 729	1. 929	7	7. 282	7	2. 007
8	7. 690	2. 205	8	8. 322	8	2. 294
9	8. 651	2. 481	9	9. 363	9	2. 581

Course		
16° 196°	p÷l / DLo÷m	163° 343°
0. 0	0. 287	1. 0
0. 1	0. 289	0. 9
0. 2	0. 291	0. 8
0. 3	0. 292	0. 7
0. 4	0. 294	0. 6
0. 5	0. 296	0. 5
0. 6	0. 298	0. 4
0. 7	0. 300	0. 3
0. 8	0. 302	0. 2
0. 9	0. 304	0. 1

17°—163°—197°—343°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 956	0. 292	1	1. 046	1	0. 306
2	1. 913	0. 585	2	2. 091	2	0. 611
3	2. 869	0. 877	3	3. 137	3	0. 917
4	3. 825	1. 169	4	4. 183	4	1. 223
5	4. 782	1. 462	5	5. 228	5	1. 529
6	5. 738	1. 754	6	6. 274	6	1. 834
7	6. 694	2. 047	7	7. 320	7	2. 140
8	7. 650	2. 339	8	8. 366	8	2. 446
9	8. 607	2. 631	9	9. 411	9	2. 752

Course		
17° 197°	p÷l / DLo÷m	162° 342°
0. 0	0. 306	1. 0
0. 1	0. 308	0. 9
0. 2	0. 310	0. 8
0. 3	0. 311	0. 7
0. 4	0. 313	0. 6
0. 5	0. 315	0. 5
0. 6	0. 317	0. 4
0. 7	0. 319	0. 3
0. 8	0. 321	0. 2
0. 9	0. 323	0. 1

18°—162°—198°—342°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 951	0. 309	1	1. 051	1	0. 325
2	1. 902	0. 618	2	2. 103	2	0. 650
3	2. 853	0. 927	3	3. 154	3	0. 975
4	3. 804	1. 236	4	4. 206	4	1. 300
5	4. 755	1. 545	5	5. 257	5	1. 625
6	5. 706	1. 854	6	6. 309	6	1. 950
7	6. 657	2. 163	7	7. 360	7	2. 274
8	7. 608	2. 472	8	8. 412	8	2. 599
9	8. 560	2. 781	9	9. 463	9	2. 924

Course		
18° 198°	p÷l / DLo÷m	161° 341°
0. 0	0. 325	1. 0
0. 1	0. 327	0. 9
0. 2	0. 329	0. 8
0. 3	0. 331	0. 7
0. 4	0. 333	0. 6
0. 5	0. 335	0. 5
0. 6	0. 337	0. 4
0. 7	0. 338	0. 3
0. 8	0. 340	0. 2
0. 9	0. 342	0. 1

19°—161°—199°—341°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 946	0. 326	1	1. 058	1	0. 344
2	1. 891	0. 651	2	2. 115	2	0. 689
3	2. 837	0. 977	3	3. 173	3	1. 033
4	3. 782	1. 302	4	4. 230	4	1. 377
5	4. 728	1. 628	5	5. 288	5	1. 722
6	5. 673	1. 953	6	6. 346	6	2. 066
7	6. 619	2. 279	7	7. 403	7	2. 410
8	7. 564	2. 605	8	8. 461	8	2. 755
9	8. 510	2. 930	9	9. 519	9	3. 099

Course		
19° 199°	p÷l / DLo÷m	160° 340°
0. 0	0. 344	1. 0
0. 1	0. 346	0. 9
0. 2	0. 348	0. 8
0. 3	0. 350	0. 7
0. 4	0. 352	0. 6
0. 5	0. 354	0. 5
0. 6	0. 356	0. 4
0. 7	0. 358	0. 3
0. 8	0. 360	0. 2
0. 9	0. 362	0. 1

TABLE 3

Traverse Table

20°—160°—200°—340°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 940	0. 342	1	1. 064	1	0. 364
2	1. 879	0. 684	2	2. 128	2	0. 728
3	2. 819	1. 026	3	3. 193	3	1. 092
4	3. 759	1. 368	4	4. 257	4	1. 456
5	4. 698	1. 710	5	5. 321	5	1. 820
6	5. 638	2. 052	6	6. 385	6	2. 184
7	6. 578	2. 394	7	7. 449	7	2. 548
8	7. 518	2. 736	8	8. 513	8	2. 912
9	8. 457	3. 078	9	9. 578	9	3. 276

Course		
20° 200°	p÷l DLo÷m	159° 339°
0. 0	0. 364	1. 0
0. 1	0. 366	0. 9
0. 2	0. 368	0. 8
0. 3	0. 370	0. 7
0. 4	0. 372	0. 6
0. 5	0. 374	0. 5
0. 6	0. 376	0. 4
0. 7	0. 378	0. 3
0. 8	0. 380	0. 2
0. 9	0. 382	0. 1

21°—159°—201°—339°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 934	0. 358	1	1. 071	1	0. 384
2	1. 867	0. 717	2	2. 142	2	0. 768
3	2. 801	1. 075	3	3. 213	3	1. 152
4	3. 734	1. 433	4	4. 285	4	1. 535
5	4. 668	1. 792	5	5. 356	5	1. 919
6	5. 601	2. 150	6	6. 427	6	2. 303
7	6. 535	2. 509	7	7. 498	7	2. 687
8	7. 469	2. 867	8	8. 569	8	3. 071
9	8. 402	3. 225	9	9. 640	9	3. 455

Course		
21° 201°	p÷l DLo÷m	158° 338°
0. 0	0. 384	1. 0
0. 1	0. 386	0. 9
0. 2	0. 388	0. 8
0. 3	0. 390	0. 7
0. 4	0. 392	0. 6
0. 5	0. 394	0. 5
0. 6	0. 396	0. 4
0. 7	0. 398	0. 3
0. 8	0. 400	0. 2
0. 9	0. 402	0. 1

22°—158°—202°—338°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 927	0. 375	1	1. 079	1	0. 404
2	1. 854	0. 749	2	2. 157	2	0. 808
3	2. 782	1. 124	3	3. 236	3	1. 212
4	3. 709	1. 498	4	4. 314	4	1. 616
5	4. 636	1. 873	5	5. 393	5	2. 020
6	5. 563	2. 248	6	6. 471	6	2. 424
7	6. 490	2. 622	7	7. 550	7	2. 828
8	7. 417	2. 997	8	8. 628	8	3. 232
9	8. 345	3. 371	9	9. 707	9	3. 636

Course		
22° 202°	p÷l DLo÷m	157° 337°
0. 0	0. 404	1. 0
0. 1	0. 406	0. 9
0. 2	0. 408	0. 8
0. 3	0. 410	0. 7
0. 4	0. 412	0. 6
0. 5	0. 414	0. 5
0. 6	0. 416	0. 4
0. 7	0. 418	0. 3
0. 8	0. 420	0. 2
0. 9	0. 422	0. 1

23°—157°—203°—337°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 921	0. 391	1	1. 086	1	0. 424
2	1. 841	0. 781	2	2. 173	2	0. 849
3	2. 762	1. 172	3	3. 259	3	1. 273
4	3. 682	1. 563	4	4. 345	4	1. 698
5	4. 603	1. 954	5	5. 432	5	2. 122
6	5. 523	2. 344	6	6. 518	6	2. 547
7	6. 444	2. 735	7	7. 605	7	2. 971
8	7. 364	3. 126	8	8. 691	8	3. 396
9	8. 285	3. 517	9	9. 777	9	3. 820

Course		
23° 203°	p÷l DLo÷m	156° 336°
0. 0	0. 424	1. 0
0. 1	0. 427	0. 9
0. 2	0. 429	0. 8
0. 3	0. 431	0. 7
0. 4	0. 433	0. 6
0. 5	0. 435	0. 5
0. 6	0. 437	0. 4
0. 7	0. 439	0. 3
0. 8	0. 441	0. 2
0. 9	0. 443	0. 1

24°—156°—204°—336°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 914	0. 407	1	1. 095	1	0. 445
2	1. 827	0. 813	2	2. 189	2	0. 890
3	2. 741	1. 220	3	3. 284	3	1. 336
4	3. 654	1. 627	4	4. 379	4	1. 781
5	4. 568	2. 034	5	5. 473	5	2. 226
6	5. 481	2. 440	6	6. 568	6	2. 671
7	6. 395	2. 847	7	7. 662	7	3. 117
8	7. 308	3. 254	8	8. 757	8	3. 562
9	8. 222	3. 661	9	9. 852	9	4. 007

Course		
24° 204°	p÷l DLo÷m	155° 335°
0. 0	0. 445	1. 0
0. 1	0. 447	0. 9
0. 2	0. 449	0. 8
0. 3	0. 452	0. 7
0. 4	0. 454	0. 6
0. 5	0. 456	0. 5
0. 6	0. 458	0. 4
0. 7	0. 460	0. 3
0. 8	0. 462	0. 2
0. 9	0. 464	0. 1

TABLE 3

Traverse Table

25°—155°—205°—335°

D Lo	p	/////////	p	D Lo	/////////	/////////
D	l	p	l	D	m	D Lo
1	0. 906	0. 423	1	1. 103	1	0. 466
2	1. 813	0. 845	2	2. 207	2	0. 933
3	2. 719	1. 268	3	3. 310	3	1. 399
4	3. 625	1. 690	4	4. 414	4	1. 865
5	4. 532	2. 113	5	5. 517	5	2. 332
6	5. 438	2. 536	6	6. 620	6	2. 798
7	6. 344	2. 958	7	7. 724	7	3. 264
8	7. 250	3. 381	8	8. 827	8	3. 730
9	8. 157	3. 804	9	9. 930	9	4. 197

Course		
25° 205°	p÷l DLo÷m	154° 334°
0. 0	0. 466	1. 0
0. 1	0. 468	0. 9
0. 2	0. 471	0. 8
0. 3	0. 473	0. 7
0. 4	0. 475	0. 6
0. 5	0. 477	0. 5
0. 6	0. 479	0. 4
0. 7	0. 481	0. 3
0. 8	0. 483	0. 2
0. 9	0. 486	0. 1

26°—154°—206°—334°

D Lo	p	/////////	p	D Lo	/////////	/////////
D	l	p	l	D	m	D Lo
1	0. 899	0. 438	1	1. 113	1	0. 488
2	1. 798	0. 877	2	2. 225	2	0. 975
3	2. 696	1. 315	3	3. 338	3	1. 463
4	3. 595	1. 753	4	4. 450	4	1. 951
5	4. 494	2. 192	5	5. 563	5	2. 439
6	5. 393	2. 630	6	6. 676	6	2. 926
7	6. 292	3. 069	7	7. 788	7	3. 414
8	7. 190	3. 507	8	8. 901	8	3. 902
9	8. 089	3. 945	9	10. 013	9	4. 390

Course		
26° 206°	p÷l DLo÷m	153° 333°
0. 0	0. 488	1. 0
0. 1	0. 490	0. 9
0. 2	0. 492	0. 8
0. 3	0. 494	0. 7
0. 4	0. 496	0. 6
0. 5	0. 499	0. 5
0. 6	0. 501	0. 4
0. 7	0. 503	0. 3
0. 8	0. 505	0. 2
0. 9	0. 507	0. 1

27°—153°—207°—333°

D Lo	p	/////////	p	D Lo	/////////	/////////
D	l	p	l	D	m	D Lo
1	0. 891	0. 454	1	1. 122	1	0. 510
2	1. 782	0. 908	2	2. 245	2	1. 019
3	2. 673	1. 362	3	3. 367	3	1. 529
4	3. 564	1. 816	4	4. 489	4	2. 038
5	4. 455	2. 270	5	5. 612	5	2. 548
6	5. 346	2. 724	6	6. 734	6	3. 057
7	6. 237	3. 178	7	7. 856	7	3. 567
8	7. 128	3. 632	8	8. 979	8	4. 076
9	8. 019	4. 086	9	10. 101	9	4. 586

Course		
27° 207°	p÷l DLo÷m	152° 332°
0. 0	0. 510	1. 0
0. 1	0. 512	0. 9
0. 2	0. 514	0. 8
0. 3	0. 516	0. 7
0. 4	0. 518	0. 6
0. 5	0. 521	0. 5
0. 6	0. 523	0. 4
0. 7	0. 525	0. 3
0. 8	0. 527	0. 2
0. 9	0. 529	0. 1

28°—152°—208°—332°

D Lo	p	/////////	p	D Lo	/////////	/////////
D	l	p	l	D	m	D Lo
1	0. 883	0. 469	1	1. 133	1	0. 532
2	1. 766	0. 939	2	2. 265	2	1. 063
3	2. 649	1. 408	3	3. 398	3	1. 595
4	3. 532	1. 878	4	4. 530	4	2. 127
5	4. 415	2. 347	5	5. 663	5	2. 659
6	5. 298	2. 817	6	6. 795	6	3. 190
7	6. 181	3. 286	7	7. 928	7	3. 722
8	7. 064	3. 756	8	9. 061	8	4. 254
9	7. 947	4. 225	9	10. 193	9	4. 785

Course		
28° 208°	p÷l DLo÷m	151° 331°
0. 0	0. 532	1. 0
0. 1	0. 534	0. 9
0. 2	0. 536	0. 8
0. 3	0. 538	0. 7
0. 4	0. 541	0. 6
0. 5	0. 543	0. 5
0. 6	0. 545	0. 4
0. 7	0. 547	0. 3
0. 8	0. 550	0. 2
0. 9	0. 552	0. 1

29°—151°—209°—331°

D Lo	p	/////////	p	D Lo	/////////	/////////
D	l	p	l	D	m	D Lo
1	0. 875	0. 485	1	1. 143	1	0. 554
2	1. 749	0. 970	2	2. 287	2	1. 109
3	2. 624	1. 454	3	3. 430	3	1. 663
4	3. 498	1. 939	4	4. 573	4	2. 217
5	4. 373	2. 424	5	5. 717	5	2. 772
6	5. 248	2. 909	6	6. 860	6	3. 326
7	6. 122	3. 394	7	8. 003	7	3. 880
8	6. 997	3. 878	8	9. 147	8	4. 434
9	7. 872	4. 363	9	10. 290	9	4. 989

Course		
29° 209°	p÷l DLo÷m	150° 330°
0. 0	0. 554	1. 0
0. 1	0. 557	0. 9
0. 2	0. 559	0. 8
0. 3	0. 561	0. 7
0. 4	0. 563	0. 6
0. 5	0. 566	0. 5
0. 6	0. 568	0. 4
0. 7	0. 570	0. 3
0. 8	0. 573	0. 2
0. 9	0. 575	0. 1

TABLE 3

Traverse Table

30°—150°—210°—330°

DLo	p	//////	p	DLo	//////	//////
D	l	p	l	D	m	DLo
1	0. 866	0. 500	1	1. 155	1	0. 577
2	1. 732	1. 000	2	2. 309	2	1. 155
3	2. 598	1. 500	3	3. 464	3	1. 732
4	3. 464	2. 000	4	4. 619	4	2. 309
5	4. 330	2. 500	5	5. 774	5	2. 887
6	5. 196	3. 000	6	6. 928	6	3. 464
7	6. 062	3. 500	7	8. 083	7	4. 041
8	6. 928	4. 000	8	9. 238	8	4. 619
9	7. 794	4. 500	9	10. 392	9	5. 196

Course		
30° 210°	p÷l DLo÷m	149° 329°
0. 0	0. 577	1. 0
0. 1	0. 580	0. 9
0. 2	0. 582	0. 8
0. 3	0. 584	0. 7
0. 4	0. 587	0. 6
0. 5	0. 589	0. 5
0. 6	0. 591	0. 4
0. 7	0. 594	0. 3
0. 8	0. 596	0. 2
0. 9	0. 598	0. 1

31°—149°—211°—329°

DLo	p	//////	p	DLo	//////	//////
D	l	p	l	D	m	DLo
1	0. 857	0. 515	1	1. 167	1	0. 601
2	1. 714	1. 030	2	2. 333	2	1. 202
3	2. 572	1. 545	3	3. 500	3	1. 803
4	3. 429	2. 060	4	4. 667	4	2. 403
5	4. 286	2. 575	5	5. 833	5	3. 004
6	5. 143	3. 090	6	7. 000	6	3. 605
7	6. 000	3. 605	7	8. 166	7	4. 206
8	6. 857	4. 120	8	9. 333	8	4. 807
9	7. 715	4. 635	9	10. 500	9	5. 408

Course		
31° 211°	p÷l DLo÷m	148° 328°
0. 0	0. 601	1. 0
0. 1	0. 603	0. 9
0. 2	0. 606	0. 8
0. 3	0. 608	0. 7
0. 4	0. 610	0. 6
0. 5	0. 613	0. 5
0. 6	0. 615	0. 4
0. 7	0. 618	0. 3
0. 8	0. 620	0. 2
0. 9	0. 622	0. 1

32°—148°—212°—328°

DLo	p	//////	p	DLo	//////	//////
D	l	p	l	D	m	DLo
1	0. 848	0. 530	1	1. 179	1	0. 625
2	1. 696	1. 060	2	2. 358	2	1. 250
3	2. 544	1. 590	3	3. 538	3	1. 875
4	3. 392	2. 120	4	4. 717	4	2. 499
5	4. 240	2. 650	5	5. 896	5	3. 124
6	5. 088	3. 180	6	7. 075	6	3. 749
7	5. 936	3. 709	7	8. 254	7	4. 374
8	6. 784	4. 239	8	9. 433	8	4. 999
9	7. 632	4. 769	9	10. 613	9	5. 624

Course		
32° 212°	p÷l DLo÷m	147° 327°
0. 0	0. 625	1. 0
0. 1	0. 627	0. 9
0. 2	0. 630	0. 8
0. 3	0. 632	0. 7
0. 4	0. 635	0. 6
0. 5	0. 637	0. 5
0. 6	0. 640	0. 4
0. 7	0. 642	0. 3
0. 8	0. 644	0. 2
0. 9	0. 647	0. 1

33°—147°—213°—327°

DLo	p	//////	p	DLo	//////	//////
D	l	p	l	D	m	DLo
1	0. 839	0. 545	1	1. 192	1	0. 649
2	1. 677	1. 089	2	2. 385	2	1. 299
3	2. 516	1. 634	3	3. 577	3	1. 948
4	3. 355	2. 179	4	4. 769	4	2. 598
5	4. 193	2. 723	5	5. 962	5	3. 247
6	5. 032	3. 268	6	7. 154	6	3. 896
7	5. 871	3. 812	7	8. 347	7	4. 546
8	6. 709	4. 357	8	9. 539	8	5. 195
9	7. 548	4. 902	9	10. 731	9	5. 845

Course		
33° 213°	p÷l DLo÷m	146° 326°
0. 0	0. 649	1. 0
0. 1	0. 652	0. 9
0. 2	0. 654	0. 8
0. 3	0. 657	0. 7
0. 4	0. 659	0. 6
0. 5	0. 662	0. 5
0. 6	0. 664	0. 4
0. 7	0. 667	0. 3
0. 8	0. 669	0. 2
0. 9	0. 672	0. 1

34°—146°—214°—326°

DLo	p	//////	p	DLo	//////	//////
D	l	p	l	D	m	DLo
1	0. 829	0. 559	1	1. 206	1	0. 675
2	1. 658	1. 118	2	2. 412	2	1. 349
3	2. 487	1. 678	3	3. 619	3	2. 024
4	3. 316	2. 237	4	4. 825	4	2. 698
5	4. 145	2. 796	5	6. 031	5	3. 373
6	4. 974	3. 355	6	7. 237	6	4. 047
7	5. 803	3. 914	7	8. 444	7	4. 722
8	6. 632	4. 474	8	9. 650	8	5. 396
9	7. 461	5. 033	9	10. 856	9	6. 071

Course		
34° 214°	p÷l DLo÷m	145° 325°
0. 0	0. 675	1. 0
0. 1	0. 677	0. 9
0. 2	0. 680	0. 8
0. 3	0. 682	0. 7
0. 4	0. 685	0. 6
0. 5	0. 687	0. 5
0. 6	0. 690	0. 4
0. 7	0. 692	0. 3
0. 8	0. 695	0. 2
0. 9	0. 698	0. 1

TABLE 3

Traverse Table

35°—145°—215°—325°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 819	0. 574	1	1. 221	1	0. 700
2	1. 638	1. 147	2	2. 442	2	1. 400
3	2. 457	1. 721	3	3. 662	3	2. 101
4	3. 277	2. 294	4	4. 883	4	2. 801
5	4. 096	2. 868	5	6. 104	5	3. 501
6	4. 915	3. 441	6	7. 325	6	4. 201
7	5. 734	4. 015	7	8. 545	7	4. 901
8	6. 553	4. 589	8	9. 766	8	5. 602
9	7. 372	5. 162	9	10. 987	9	6. 302

Course		
35° 215°	p÷l DLo÷m	144° 324°
0. 0	0. 700	1. 0
0. 1	0. 703	0. 9
0. 2	0. 705	0. 8
0. 3	0. 708	0. 7
0. 4	0. 711	0. 6
0. 5	0. 713	0. 5
0. 6	0. 716	0. 4
0. 7	0. 719	0. 3
0. 8	0. 721	0. 2
0. 9	0. 724	0. 1

36°—144°—216°—324°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 809	0. 588	1	1. 236	1	0. 727
2	1. 618	1. 176	2	2. 472	2	1. 453
3	2. 427	1. 763	3	3. 708	3	2. 180
4	3. 236	2. 351	4	4. 944	4	2. 906
5	4. 045	2. 939	5	6. 180	5	3. 633
6	4. 854	3. 527	6	7. 416	6	4. 359
7	5. 663	4. 114	7	8. 652	7	5. 086
8	6. 472	4. 702	8	9. 889	8	5. 812
9	7. 281	5. 290	9	11. 125	9	6. 539

Course		
36° 216°	p÷l DLo÷m	143° 323°
0. 0	0. 727	1. 0
0. 1	0. 729	0. 9
0. 2	0. 732	0. 8
0. 3	0. 735	0. 7
0. 4	0. 737	0. 6
0. 5	0. 740	0. 5
0. 6	0. 743	0. 4
0. 7	0. 745	0. 3
0. 8	0. 748	0. 2
0. 9	0. 751	0. 1

37°—143°—217°—323°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 799	0. 602	1	1. 252	1	0. 754
2	1. 597	1. 204	2	2. 504	2	1. 507
3	2. 396	1. 805	3	3. 756	3	2. 261
4	3. 195	2. 407	4	5. 009	4	3. 014
5	3. 993	3. 009	5	6. 261	5	3. 768
6	4. 792	3. 611	6	7. 513	6	4. 521
7	5. 590	4. 213	7	8. 765	7	5. 275
8	6. 389	4. 815	8	10. 017	8	6. 028
9	7. 188	5. 416	9	11. 269	9	6. 782

Course		
37° 217°	p÷l DLo÷m	142° 322°
0. 0	0. 754	1. 0
0. 1	0. 756	0. 9
0. 2	0. 759	0. 8
0. 3	0. 762	0. 7
0. 4	0. 765	0. 6
0. 5	0. 767	0. 5
0. 6	0. 770	0. 4
0. 7	0. 773	0. 3
0. 8	0. 776	0. 2
0. 9	0. 778	0. 1

38°—142°—218°—322°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 788	0. 616	1	1. 269	1	0. 781
2	1. 576	1. 231	2	2. 538	2	1. 563
3	2. 364	1. 847	3	3. 807	3	2. 344
4	3. 152	2. 463	4	5. 076	4	3. 125
5	3. 940	3. 078	5	6. 345	5	3. 906
6	4. 728	3. 694	6	7. 614	6	4. 688
7	5. 516	4. 310	7	8. 883	7	5. 469
8	6. 304	4. 925	8	10. 152	8	6. 250
9	7. 092	5. 541	9	11. 421	9	7. 032

Course		
38° 218°	p÷l DLo÷m	141° 321°
0. 0	0. 781	1. 0
0. 1	0. 784	0. 9
0. 2	0. 787	0. 8
0. 3	0. 790	0. 7
0. 4	0. 793	0. 6
0. 5	0. 795	0. 5
0. 6	0. 798	0. 4
0. 7	0. 801	0. 3
0. 8	0. 804	0. 2
0. 9	0. 807	0. 1

39°—141°—219°—321°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 777	0. 629	1	1. 287	1	0. 810
2	1. 554	1. 259	2	2. 574	2	1. 620
3	2. 331	1. 888	3	3. 860	3	2. 429
4	3. 109	2. 517	4	5. 147	4	3. 239
5	3. 886	3. 147	5	6. 434	5	4. 049
6	4. 663	3. 776	6	7. 721	6	4. 859
7	5. 440	4. 405	7	9. 007	7	5. 668
8	6. 217	5. 035	8	10. 294	8	6. 478
9	6. 994	5. 664	9	11. 581	9	7. 288

Course		
39° 219°	p÷l DLo÷m	140° 320°
0. 0	0. 810	1. 0
0. 1	0. 813	0. 9
0. 2	0. 816	0. 8
0. 3	0. 818	0. 7
0. 4	0. 821	0. 6
0. 5	0. 824	0. 5
0. 6	0. 827	0. 4
0. 7	0. 830	0. 3
0. 8	0. 833	0. 2
0. 9	0. 836	0. 1

TABLE 3

Traverse Table

40°—140°—220°—320°

DLo D	p l	 p	p l	DLo D	 m	 DLo
1	0. 766	0. 643	1	1. 305	1	0. 839
2	1. 532	1. 286	2	2. 611	2	1. 678
3	2. 298	1. 928	3	3. 916	3	2. 517
4	3. 064	2. 571	4	5. 222	4	3. 356
5	3. 830	3. 214	5	6. 527	5	4. 196
6	4. 596	3. 857	6	7. 832	6	5. 035
7	5. 362	4. 500	7	9. 138	7	5. 874
8	6. 128	5. 142	8	10. 443	8	6. 713
9	6. 894	5. 785	9	11. 749	9	7. 552

Course

40° 220°	p÷l DLo÷m	139° 319°
0. 0	0. 839	1. 0
0. 1	0. 842	0. 9
0. 2	0. 845	0. 8
0. 3	0. 848	0. 7
0. 4	0. 851	0. 6
0. 5	0. 854	0. 5
0. 6	0. 857	0. 4
0. 7	0. 860	0. 3
0. 8	0. 863	0. 2
0. 9	0. 866	0. 1

41°—139°—221°—319°

DLo D	p l	 p	p l	DLo D	 m	 DLo
1	0. 755	0. 656	1	1. 325	1	0. 869
2	1. 509	1. 312	2	2. 650	2	1. 739
3	2. 264	1. 968	3	3. 975	3	2. 608
4	3. 019	2. 624	4	5. 300	4	3. 477
5	3. 774	3. 280	5	6. 625	5	4. 346
6	4. 528	3. 936	6	7. 950	6	5. 216
7	5. 283	4. 592	7	9. 275	7	6. 085
8	6. 038	5. 248	8	10. 600	8	6. 954
9	6. 792	5. 905	9	11. 925	9	7. 824

Course

41° 221°	p÷l DLo÷m	138° 318°
0. 0	0. 869	1. 0
0. 1	0. 872	0. 9
0. 2	0. 875	0. 8
0. 3	0. 879	0. 7
0. 4	0. 882	0. 6
0. 5	0. 885	0. 5
0. 6	0. 888	0. 4
0. 7	0. 891	0. 3
0. 8	0. 894	0. 2
0. 9	0. 897	0. 1

42°—138°—222°—318°

DLo D	p l	 p	p l	DLo D	 m	 DLo
1	0. 743	0. 669	1	1. 346	1	0. 900
2	1. 486	1. 338	2	2. 691	2	1. 801
3	2. 229	2. 007	3	4. 037	3	2. 701
4	2. 973	2. 677	4	5. 383	4	3. 602
5	3. 716	3. 346	5	6. 728	5	4. 502
6	4. 459	4. 015	6	8. 074	6	5. 402
7	5. 202	4. 684	7	9. 419	7	6. 303
8	5. 945	5. 353	8	10. 765	8	7. 203
9	6. 688	6. 022	9	12. 111	9	8. 104

Course

42° 222°	p÷l DLo÷m	137° 317°
0. 0	0. 900	1. 0
0. 1	0. 904	0. 9
0. 2	0. 907	0. 8
0. 3	0. 910	0. 7
0. 4	0. 913	0. 6
0. 5	0. 916	0. 5
0. 6	0. 920	0. 4
0. 7	0. 923	0. 3
0. 8	0. 926	0. 2
0. 9	0. 929	0. 1

43°—137°—223°—317°

DLo D	p l	 p	p l	DLo D	 m	 DLo
1	0. 731	0. 682	1	1. 367	1	0. 933
2	1. 463	1. 364	2	2. 735	2	1. 865
3	2. 194	2. 046	3	4. 102	3	2. 798
4	2. 925	2. 728	4	5. 469	4	3. 730
5	3. 657	3. 410	5	6. 837	5	4. 663
6	4. 388	4. 092	6	8. 204	6	5. 595
7	5. 119	4. 774	7	9. 571	7	6. 528
8	5. 851	5. 456	8	10. 939	8	7. 460
9	6. 582	6. 138	9	12. 306	9	8. 393

Course

43° 223°	p÷l DLo÷m	136° 316°
0. 0	0. 933	1. 0
0. 1	0. 936	0. 9
0. 2	0. 939	0. 8
0. 3	0. 942	0. 7
0. 4	0. 946	0. 6
0. 5	0. 949	0. 5
0. 6	0. 952	0. 4
0. 7	0. 956	0. 3
0. 8	0. 959	0. 2
0. 9	0. 962	0. 1

44°—136°—224°—316°

DLo D	p l	 p	p l	DLo D	 m	 DLo
1	0. 719	0. 695	1	1. 390	1	0. 966
2	1. 439	1. 389	2	2. 780	2	1. 931
3	2. 158	2. 084	3	4. 170	3	2. 897
4	2. 877	2. 779	4	5. 561	4	3. 863
5	3. 597	3. 473	5	6. 951	5	4. 828
6	4. 316	4. 168	6	8. 341	6	5. 794
7	5. 035	4. 863	7	9. 731	7	6. 760
8	5. 755	5. 557	8	11. 121	8	7. 726
9	6. 474	6. 252	9	12. 511	9	8. 691

Course

44° 224°	p÷l DLo÷m	135° 315°
0. 0	0. 966	1. 0
0. 1	0. 969	0. 9
0. 2	0. 972	0. 8
0. 3	0. 976	0. 7
0. 4	0. 979	0. 6
0. 5	0. 983	0. 5
0. 6	0. 986	0. 4
0. 7	0. 990	0. 3
0. 8	0. 993	0. 2
0. 9	0. 997	0. 1

TABLE 3

Traverse Table

45°—135°—225°—315°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 707	0. 707	1	1. 414	1	1. 000
2	1. 414	1. 414	2	2. 828	2	2. 000
3	2. 121	2. 121	3	4. 243	3	3. 000
4	2. 828	2. 828	4	5. 657	4	4. 000
5	3. 536	3. 536	5	7. 071	5	5. 000
6	4. 243	4. 243	6	8. 485	6	6. 000
7	4. 950	4. 950	7	9. 899	7	7. 000
8	5. 657	5. 657	8	11. 314	8	8. 000
9	6. 364	6. 364	9	12. 728	9	9. 000

Course		
45° 225°	p÷l DLo÷m	134° 314°
0. 0	1. 000	1. 0
0. 1	1. 003	0. 9
0. 2	1. 007	0. 8
0. 3	1. 011	0. 7
0. 4	1. 014	0. 6
0. 5	1. 018	0. 5
0. 6	1. 021	0. 4
0. 7	1. 025	0. 3
0. 8	1. 028	0. 2
0. 9	1. 032	0. 1

46°—134°—226°—314°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 695	0. 719	1	1. 440	1	1. 036
2	1. 389	1. 439	2	2. 879	2	2. 071
3	2. 084	2. 158	3	4. 319	3	3. 107
4	2. 779	2. 877	4	5. 758	4	4. 142
5	3. 473	3. 597	5	7. 198	5	5. 178
6	4. 168	4. 316	6	8. 637	6	6. 213
7	4. 863	5. 035	7	10. 077	7	7. 249
8	5. 557	5. 755	8	11. 516	8	8. 284
9	6. 252	6. 474	9	12. 956	9	9. 320

Course		
46° 226°	p÷l DLo÷m	133° 313°
0. 0	1. 036	1. 0
0. 1	1. 039	0. 9
0. 2	1. 043	0. 8
0. 3	1. 046	0. 7
0. 4	1. 050	0. 6
0. 5	1. 054	0. 5
0. 6	1. 057	0. 4
0. 7	1. 061	0. 3
0. 8	1. 065	0. 2
0. 9	1. 069	0. 1

47°—133°—227°—313°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 682	0. 731	1	1. 466	1	1. 072
2	1. 364	1. 463	2	2. 933	2	2. 145
3	2. 046	2. 194	3	4. 399	3	3. 217
4	2. 728	2. 925	4	5. 865	4	4. 289
5	3. 410	3. 657	5	7. 331	5	5. 362
6	4. 092	4. 388	6	8. 798	6	6. 434
7	4. 774	5. 119	7	10. 264	7	7. 507
8	5. 456	5. 851	8	11. 730	8	8. 579
9	6. 138	6. 582	9	13. 197	9	9. 651

Course		
47° 227°	p÷l DLo÷m	132° 312°
0. 0	1. 072	1. 0
0. 1	1. 076	0. 9
0. 2	1. 080	0. 8
0. 3	1. 084	0. 7
0. 4	1. 087	0. 6
0. 5	1. 091	0. 5
0. 6	1. 095	0. 4
0. 7	1. 099	0. 3
0. 8	1. 103	0. 2
0. 9	1. 107	0. 1

48°—132°—228°—312°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 669	0. 743	1	1. 494	1	1. 111
2	1. 338	1. 486	2	2. 989	2	2. 221
3	2. 007	2. 229	3	4. 483	3	3. 332
4	2. 677	2. 973	4	5. 978	4	4. 442
5	3. 346	3. 716	5	7. 472	5	5. 553
6	4. 015	4. 459	6	8. 967	6	6. 664
7	4. 684	5. 202	7	10. 461	7	7. 774
8	5. 353	5. 945	8	11. 956	8	8. 885
9	6. 022	6. 688	9	13. 450	9	9. 996

Course		
48° 228°	p÷l DLo÷m	131° 311°
0. 0	1. 111	1. 0
0. 1	1. 115	0. 9
0. 2	1. 118	0. 8
0. 3	1. 122	0. 7
0. 4	1. 126	0. 6
0. 5	1. 130	0. 5
0. 6	1. 134	0. 4
0. 7	1. 138	0. 3
0. 8	1. 142	0. 2
0. 9	1. 146	0. 1

49°—131°—229°—311°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 656	0. 755	1	1. 524	1	1. 150
2	1. 312	1. 509	2	3. 049	2	2. 301
3	1. 968	2. 264	3	4. 573	3	3. 451
4	2. 624	3. 019	4	6. 097	4	4. 601
5	3. 280	3. 774	5	7. 621	5	5. 752
6	3. 936	4. 528	6	9. 146	6	6. 902
7	4. 592	5. 283	7	10. 670	7	8. 053
8	5. 248	6. 038	8	12. 194	8	9. 203
9	5. 905	6. 792	9	13. 718	9	10. 353

Course		
49° 229°	p÷l DLo÷m	130° 310°
0. 0	1. 150	1. 0
0. 1	1. 154	0. 9
0. 2	1. 159	0. 8
0. 3	1. 163	0. 7
0. 4	1. 167	0. 6
0. 5	1. 171	0. 5
0. 6	1. 175	0. 4
0. 7	1. 179	0. 3
0. 8	1. 183	0. 2
0. 9	1. 188	0. 1

TABLE 3

Traverse Table

50°—130°—230°—310°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 643	0. 766	1	1. 556	1	1. 192
2	1. 286	1. 532	2	3. 111	2	2. 384
3	1. 928	2. 298	3	4. 667	3	3. 575
4	2. 571	3. 064	4	6. 223	4	4. 767
5	3. 214	3. 830	5	7. 779	5	5. 959
6	3. 857	4. 596	6	9. 334	6	7. 151
7	4. 500	5. 362	7	10. 890	7	8. 342
8	5. 142	6. 128	8	12. 446	8	9. 534
9	5. 785	6. 894	9	14. 002	9	10. 726

Course		
50° 230°	p÷l DLo÷m	129° 309°
0. 0	1. 192	1. 0
0. 1	1. 196	0. 9
0. 2	1. 200	0. 8
0. 3	1. 205	0. 7
0. 4	1. 209	0. 6
0. 5	1. 213	0. 5
0. 6	1. 217	0. 4
0. 7	1. 222	0. 3
0. 8	1. 226	0. 2
0. 9	1. 230	0. 1

51°—129°—231°—309°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 629	0. 777	1	1. 589	1	1. 235
2	1. 259	1. 554	2	3. 178	2	2. 470
3	1. 888	2. 331	3	4. 767	3	3. 705
4	2. 517	3. 109	4	6. 356	4	4. 940
5	3. 147	3. 886	5	7. 945	5	6. 174
6	3. 776	4. 663	6	9. 534	6	7. 409
7	4. 405	5. 440	7	11. 123	7	8. 644
8	5. 035	6. 217	8	12. 712	8	9. 879
9	5. 664	6. 994	9	14. 301	9	11. 114

Course		
51° 231°	p÷l DLo÷m	128° 308°
0. 0	1. 235	1. 0
0. 1	1. 239	0. 9
0. 2	1. 244	0. 8
0. 3	1. 248	0. 7
0. 4	1. 253	0. 6
0. 5	1. 257	0. 5
0. 6	1. 262	0. 4
0. 7	1. 266	0. 3
0. 8	1. 271	0. 2
0. 9	1. 275	0. 1

52°—128°—232°—308°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 616	0. 788	1	1. 624	1	1. 280
2	1. 231	1. 576	2	3. 249	2	2. 560
3	1. 847	2. 364	3	4. 873	3	3. 840
4	2. 463	3. 152	4	6. 497	4	5. 120
5	3. 078	3. 940	5	8. 121	5	6. 400
6	3. 694	4. 728	6	9. 746	6	7. 680
7	4. 310	5. 516	7	11. 370	7	8. 960
8	4. 925	6. 304	8	12. 994	8	10. 240
9	5. 541	7. 092	9	14. 618	9	11. 519

Course		
52° 232°	p÷l DLo÷m	127° 307°
0. 0	1. 280	1. 0
0. 1	1. 285	0. 9
0. 2	1. 289	0. 8
0. 3	1. 294	0. 7
0. 4	1. 299	0. 6
0. 5	1. 303	0. 5
0. 6	1. 308	0. 4
0. 7	1. 313	0. 3
0. 8	1. 317	0. 2
0. 9	1. 322	0. 1

53°—127°—233°—307°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 602	0. 799	1	1. 662	1	1. 327
2	1. 204	1. 597	2	3. 323	2	2. 654
3	1. 805	2. 396	3	4. 985	3	3. 981
4	2. 407	3. 195	4	6. 647	4	5. 308
5	3. 009	3. 993	5	8. 308	5	6. 635
6	3. 611	4. 792	6	9. 970	6	7. 962
7	4. 213	5. 590	7	11. 631	7	9. 289
8	4. 815	6. 389	8	13. 293	8	10. 616
9	5. 416	7. 188	9	14. 955	9	11. 943

Course		
53° 233°	p÷l DLo÷m	126° 306°
0. 0	1. 327	1. 0
0. 1	1. 332	0. 9
0. 2	1. 337	0. 8
0. 3	1. 342	0. 7
0. 4	1. 347	0. 6
0. 5	1. 351	0. 5
0. 6	1. 356	0. 4
0. 7	1. 361	0. 3
0. 8	1. 366	0. 2
0. 9	1. 371	0. 1

54°—126°—234°—306°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 588	0. 809	1	1. 701	1	1. 376
2	1. 176	1. 618	2	3. 403	2	2. 753
3	1. 763	2. 427	3	5. 104	3	4. 129
4	2. 351	3. 236	4	6. 805	4	5. 506
5	2. 939	4. 045	5	8. 507	5	6. 882
6	3. 527	4. 854	6	10. 208	6	8. 258
7	4. 114	5. 663	7	11. 909	7	9. 635
8	4. 702	6. 472	8	13. 610	8	11. 011
9	5. 290	7. 281	9	15. 312	9	12. 387

Course		
54° 234°	p÷l DLo÷m	125° 305°
0. 0	1. 376	1. 0
0. 1	1. 381	0. 9
0. 2	1. 387	0. 8
0. 3	1. 392	0. 7
0. 4	1. 397	0. 6
0. 5	1. 402	0. 5
0. 6	1. 407	0. 4
0. 7	1. 412	0. 3
0. 8	1. 418	0. 2
0. 9	1. 423	0. 1

TABLE 3

Traverse Table

55°—125°—235°—305°

DLo	p	//////////////////	p	DLo	//////////	//////////////////////
D	l	p	l	D	m	DLo
1	0. 574	0. 819	1	1. 743	1	1. 428
2	1. 147	1. 638	2	3. 487	2	2. 856
3	1. 721	2. 457	3	5. 230	3	4. 284
4	2. 294	3. 277	4	6. 974	4	5. 713
5	2. 868	4. 096	5	8. 717	5	7. 141
6	3. 441	4. 915	6	10. 461	6	8. 569
7	4. 015	5. 734	7	12. 204	7	9. 997
8	4. 589	6. 553	8	13. 948	8	11. 425
9	5. 162	7. 372	9	15. 691	9	12. 853

Course		
55° 235°	p÷l DLo÷m	124° 304°
0. 0	1. 428	1. 0
0. 1	1. 433	0. 9
0. 2	1. 439	0. 8
0. 3	1. 444	0. 7
0. 4	1. 450	0. 6
0. 5	1. 455	0. 5
0. 6	1. 460	0. 4
0. 7	1. 466	0. 3
0. 8	1. 471	0. 2
0. 9	1. 477	0. 1

56°—124°—236°—304°

DLo	p	//////////////////	p	DLo	//////////	//////////////////////
D	l	p	l	D	m	DLo
1	0. 559	0. 829	1	1. 788	1	1. 483
2	1. 118	1. 658	2	3. 577	2	2. 965
3	1. 678	2. 487	3	5. 365	3	4. 448
4	2. 237	3. 316	4	7. 153	4	5. 930
5	2. 796	4. 145	5	8. 941	5	7. 413
6	3. 355	4. 974	6	10. 730	6	8. 895
7	3. 914	5. 803	7	12. 518	7	10. 378
8	4. 474	6. 632	8	14. 306	8	11. 860
9	5. 033	7. 461	9	16. 095	9	13. 343

Course		
56° 236°	p÷l DLo÷m	123° 303°
0. 0	1. 483	1. 0
0. 1	1. 488	0. 9
0. 2	1. 494	0. 8
0. 3	1. 499	0. 7
0. 4	1. 505	0. 6
0. 5	1. 511	0. 5
0. 6	1. 517	0. 4
0. 7	1. 522	0. 3
0. 8	1. 528	0. 2
0. 9	1. 534	0. 1

57°—123°—237°—303°

DLo	p	//////////////////	p	DLo	//////////	//////////////////////
D	l	p	l	D	m	DLo
1	0. 545	0. 839	1	1. 836	1	1. 540
2	1. 089	1. 677	2	3. 672	2	3. 080
3	1. 634	2. 516	3	5. 508	3	4. 620
4	2. 179	3. 355	4	7. 344	4	6. 159
5	2. 723	4. 193	5	9. 180	5	7. 699
6	3. 268	5. 032	6	11. 016	6	9. 239
7	3. 812	5. 871	7	12. 853	7	10. 779
8	4. 357	6. 709	8	14. 689	8	12. 319
9	4. 902	7. 548	9	16. 525	9	13. 859

Course		
57° 237°	p÷l DLo÷m	122° 302°
0. 0	1. 540	1. 0
0. 1	1. 546	0. 9
0. 2	1. 552	0. 8
0. 3	1. 558	0. 7
0. 4	1. 564	0. 6
0. 5	1. 570	0. 5
0. 6	1. 576	0. 4
0. 7	1. 582	0. 3
0. 8	1. 588	0. 2
0. 9	1. 594	0. 1

58°—122°—238°—302°

DLo	p	//////////////////	p	DLo	//////////	//////////////////////
D	l	p	l	D	m	DLo
1	0. 530	0. 848	1	1. 887	1	1. 600
2	1. 060	1. 696	2	3. 774	2	3. 201
3	1. 590	2. 544	3	5. 661	3	4. 801
4	2. 120	3. 392	4	7. 548	4	6. 401
5	2. 650	4. 240	5	9. 435	5	8. 002
6	3. 180	5. 088	6	11. 322	6	9. 602
7	3. 709	5. 936	7	13. 210	7	11. 202
8	4. 239	6. 784	8	15. 097	8	12. 803
9	4. 769	7. 632	9	16. 984	9	14. 403

Course		
58° 238°	p÷l DLo÷m	121° 301°
0. 0	1. 600	1. 0
0. 1	1. 607	0. 9
0. 2	1. 613	0. 8
0. 3	1. 619	0. 7
0. 4	1. 625	0. 6
0. 5	1. 632	0. 5
0. 6	1. 638	0. 4
0. 7	1. 645	0. 3
0. 8	1. 651	0. 2
0. 9	1. 658	0. 1

59°—121°—239°—301°

DLo	p	//////////////////	p	DLo	//////////	//////////////////////
D	l	p	l	D	m	DLo
1	0. 515	0. 857	1	1. 942	1	1. 664
2	1. 030	1. 714	2	3. 883	2	3. 329
3	1. 545	2. 572	3	5. 825	3	4. 993
4	2. 060	3. 429	4	7. 766	4	6. 657
5	2. 575	4. 286	5	9. 708	5	8. 321
6	3. 090	5. 143	6	11. 650	6	9. 986
7	3. 605	6. 000	7	13. 591	7	11. 650
8	4. 120	6. 857	8	15. 533	8	13. 314
9	4. 635	7. 715	9	17. 474	9	14. 979

Course		
59° 239°	p÷l DLo÷m	120° 300°
0. 0	1. 664	1. 0
0. 1	1. 671	0. 9
0. 2	1. 678	0. 8
0. 3	1. 684	0. 7
0. 4	1. 691	0. 6
0. 5	1. 698	0. 5
0. 6	1. 704	0. 4
0. 7	1. 711	0. 3
0. 8	1. 718	0. 2
0. 9	1. 725	0. 1

TABLE 3

Traverse Table

60°—120°—240°—300°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 500	0. 866	1	2. 000	1	1. 732
2	1. 000	1. 732	2	4. 000	2	3. 464
3	1. 500	2. 598	3	6. 000	3	5. 196
4	2. 000	3. 464	4	8. 000	4	6. 928
5	2. 500	4. 330	5	10. 000	5	8. 660
6	3. 000	5. 196	6	12. 000	6	10. 392
7	3. 500	6. 062	7	14. 000	7	12. 124
8	4. 000	6. 928	8	16. 000	8	13. 856
9	4. 500	7. 794	9	18. 000	9	15. 588

Course		
60° 240°	p÷l DLo÷m	119° 299°
0. 0	1. 732	1. 0
0. 1	1. 739	0. 9
0. 2	1. 746	0. 8
0. 3	1. 753	0. 7
0. 4	1. 760	0. 6
0. 5	1. 767	0. 5
0. 6	1. 775	0. 4
0. 7	1. 782	0. 3
0. 8	1. 789	0. 2
0. 9	1. 797	0. 1

61°—119°—241°—299°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 485	0. 875	1	2. 063	1	1. 804
2	0. 970	1. 749	2	4. 125	2	3. 608
3	1. 454	2. 624	3	6. 188	3	5. 412
4	1. 939	3. 498	4	8. 251	4	7. 216
5	2. 424	4. 373	5	10. 313	5	9. 020
6	2. 909	5. 248	6	12. 376	6	10. 824
7	3. 394	6. 122	7	14. 439	7	12. 628
8	3. 878	6. 997	8	16. 501	8	14. 432
9	4. 363	7. 872	9	18. 564	9	16. 236

Course		
61° 241°	p÷l DLo÷m	118° 298°
0. 0	1. 804	1. 0
0. 1	1. 811	0. 9
0. 2	1. 819	0. 8
0. 3	1. 827	0. 7
0. 4	1. 834	0. 6
0. 5	1. 842	0. 5
0. 6	1. 849	0. 4
0. 7	1. 857	0. 3
0. 8	1. 865	0. 2
0. 9	1. 873	0. 1

62°—118°—242°—298°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 469	0. 883	1	2. 130	1	1. 881
2	0. 939	1. 766	2	4. 260	2	3. 761
3	1. 408	2. 649	3	6. 390	3	5. 642
4	1. 878	3. 532	4	8. 520	4	7. 523
5	2. 347	4. 415	5	10. 650	5	9. 404
6	2. 817	5. 298	6	12. 780	6	11. 284
7	3. 286	6. 181	7	14. 910	7	13. 165
8	3. 756	7. 064	8	17. 040	8	15. 046
9	4. 225	7. 947	9	19. 170	9	16. 927

Course		
62° 242°	p÷l DLo÷m	117° 297°
0. 0	1. 881	1. 0
0. 1	1. 889	0. 9
0. 2	1. 897	0. 8
0. 3	1. 905	0. 7
0. 4	1. 913	0. 6
0. 5	1. 921	0. 5
0. 6	1. 929	0. 4
0. 7	1. 937	0. 3
0. 8	1. 946	0. 2
0. 9	1. 954	0. 1

63°—117°—243°—297°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 454	0. 891	1	2. 203	1	1. 963
2	0. 908	1. 782	2	4. 405	2	3. 925
3	1. 362	2. 673	3	6. 608	3	5. 888
4	1. 816	3. 564	4	8. 811	4	7. 850
5	2. 270	4. 455	5	11. 013	5	9. 813
6	2. 724	5. 346	6	13. 216	6	11. 776
7	3. 178	6. 237	7	15. 419	7	13. 738
8	3. 632	7. 128	8	17. 622	8	15. 701
9	4. 086	8. 019	9	19. 824	9	17. 663

Course		
63° 243°	p÷l DLo÷m	116° 296°
0. 0	1. 963	1. 0
0. 1	1. 971	0. 9
0. 2	1. 980	0. 8
0. 3	1. 988	0. 7
0. 4	1. 997	0. 6
0. 5	2. 006	0. 5
0. 6	2. 014	0. 4
0. 7	2. 023	0. 3
0. 8	2. 032	0. 2
0. 9	2. 041	0. 1

64°—116°—244°—296°

DLo	p	//////////	p	DLo	//////////	//////////
D	l	p	l	D	m	DLo
1	0. 438	0. 899	1	2. 281	1	2. 050
2	0. 877	1. 798	2	4. 562	2	4. 101
3	1. 315	2. 696	3	6. 844	3	6. 151
4	1. 753	3. 595	4	9. 125	4	8. 201
5	2. 192	4. 494	5	11. 406	5	10. 252
6	2. 630	5. 393	6	13. 687	6	12. 302
7	3. 069	6. 292	7	15. 968	7	14. 352
8	3. 507	7. 190	8	18. 249	8	16. 402
9	3. 945	8. 089	9	20. 531	9	18. 453

Course		
64° 244°	p÷l DLo÷m	115° 295°
0. 0	2. 050	1. 0
0. 1	2. 059	0. 9
0. 2	2. 069	0. 8
0. 3	2. 078	0. 7
0. 4	2. 087	0. 6
0. 5	2. 097	0. 5
0. 6	2. 106	0. 4
0. 7	2. 116	0. 3
0. 8	2. 125	0. 2
0. 9	2. 135	0. 1

TABLE 3

Traverse Table

65°—115°—245°—295°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 423	0. 906	1	2. 366	1	2. 145
2	0. 845	1. 813	2	4. 732	2	4. 289
3	1. 268	2. 719	3	7. 099	3	6. 434
4	1. 690	3. 625	4	9. 465	4	8. 578
5	2. 113	4. 532	5	11. 831	5	10. 723
6	2. 536	5. 438	6	14. 197	6	12. 867
7	2. 958	6. 344	7	16. 563	7	15. 012
8	3. 381	7. 250	8	18. 930	8	17. 156
9	3. 804	8. 157	9	21. 296	9	19. 301

Course		
65°	p÷l	114°
245°	DLo÷m	294°
0. 0	2. 145	1. 0
0. 1	2. 154	0. 9
0. 2	2. 164	0. 8
0. 3	2. 174	0. 7
0. 4	2. 184	0. 6
0. 5	2. 194	0. 5
0. 6	2. 204	0. 4
0. 7	2. 215	0. 3
0. 8	2. 225	0. 2
0. 9	2. 236	0. 1

66°—114°—246°—294°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 407	0. 914	1	2. 459	1	2. 246
2	0. 813	1. 827	2	4. 917	2	4. 492
3	1. 220	2. 741	3	7. 376	3	6. 738
4	1. 627	3. 654	4	9. 834	4	8. 984
5	2. 034	4. 568	5	12. 293	5	11. 230
6	2. 440	5. 481	6	14. 752	6	13. 476
7	2. 847	6. 395	7	17. 210	7	15. 722
8	3. 254	7. 308	8	19. 669	8	17. 968
9	3. 661	8. 222	9	22. 127	9	20. 214

Course		
66°	p÷l	113°
246°	DLo÷m	293°
0. 0	2. 246	1. 0
0. 1	2. 257	0. 9
0. 2	2. 267	0. 8
0. 3	2. 278	0. 7
0. 4	2. 289	0. 6
0. 5	2. 300	0. 5
0. 6	2. 311	0. 4
0. 7	2. 322	0. 3
0. 8	2. 333	0. 2
0. 9	2. 344	0. 1

67°—113°—247°—293°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 391	0. 921	1	2. 559	1	2. 356
2	0. 781	1. 841	2	5. 119	2	4. 712
3	1. 172	2. 762	3	7. 678	3	7. 068
4	1. 563	3. 682	4	10. 237	4	9. 423
5	1. 954	4. 603	5	12. 797	5	11. 779
6	2. 344	5. 523	6	15. 356	6	14. 135
7	2. 735	6. 444	7	17. 915	7	16. 491
8	3. 126	7. 364	8	20. 474	8	18. 847
9	3. 517	8. 285	9	23. 034	9	21. 203

Course		
67°	p÷l	112°
247°	DLo÷m	292°
0. 0	2. 356	1. 0
0. 1	2. 367	0. 9
0. 2	2. 379	0. 8
0. 3	2. 391	0. 7
0. 4	2. 402	0. 6
0. 5	2. 414	0. 5
0. 6	2. 426	0. 4
0. 7	2. 438	0. 3
0. 8	2. 450	0. 2
0. 9	2. 463	0. 1

68°—112°—248°—292°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 375	0. 927	1	2. 669	1	2. 475
2	0. 749	1. 854	2	5. 339	2	4. 950
3	1. 124	2. 782	3	8. 008	3	7. 425
4	1. 498	3. 709	4	10. 678	4	9. 900
5	1. 873	4. 636	5	13. 347	5	12. 375
6	2. 248	5. 563	6	16. 017	6	14. 851
7	2. 622	6. 490	7	18. 686	7	17. 326
8	2. 997	7. 417	8	21. 356	8	19. 801
9	3. 371	8. 345	9	24. 025	9	22. 276

Course		
68°	p÷l	111°
248°	DLo÷m	291°
0. 0	2. 475	1. 0
0. 1	2. 488	0. 9
0. 2	2. 500	0. 8
0. 3	2. 513	0. 7
0. 4	2. 526	0. 6
0. 5	2. 539	0. 5
0. 6	2. 552	0. 4
0. 7	2. 565	0. 3
0. 8	2. 578	0. 2
0. 9	2. 592	0. 1

69°—111°—249°—291°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 358	0. 934	1	2. 790	1	2. 605
2	0. 717	1. 867	2	5. 581	2	5. 210
3	1. 075	2. 801	3	8. 371	3	7. 815
4	1. 433	3. 734	4	11. 162	4	10. 420
5	1. 792	4. 668	5	13. 952	5	13. 025
6	2. 150	5. 601	6	16. 743	6	15. 631
7	2. 509	6. 535	7	19. 533	7	18. 236
8	2. 867	7. 469	8	22. 323	8	20. 841
9	3. 225	8. 402	9	25. 114	9	23. 446

Course		
69°	p÷l	110°
249°	DLo÷m	290°
0. 0	2. 605	1. 0
0. 1	2. 619	0. 9
0. 2	2. 633	0. 8
0. 3	2. 646	0. 7
0. 4	2. 660	0. 6
0. 5	2. 675	0. 5
0. 6	2. 689	0. 4
0. 7	2. 703	0. 3
0. 8	2. 718	0. 2
0. 9	2. 733	0. 1

TABLE 3

Traverse Table

70°—110°—250°—290°

DLo D	p l	 p	p l	DLo D	 m	 DLo
1	0. 342	0. 940	1	2. 924	1	2. 747
2	0. 684	1. 879	2	5. 848	2	5. 495
3	1. 026	2. 819	3	8. 771	3	8. 242
4	1. 368	3. 759	4	11. 695	4	10. 990
5	1. 710	4. 698	5	14. 619	5	13. 737
6	2. 052	5. 638	6	17. 543	6	16. 485
7	2. 394	6. 578	7	20. 467	7	19. 232
8	2. 736	7. 518	8	23. 390	8	21. 980
9	3. 078	8. 457	9	26. 314	9	24. 727

Course

70° 250°	p÷l DLo÷m	109° 289°
0. 0	2. 747	1. 0
0. 1	2. 762	0. 9
0. 2	2. 778	0. 8
0. 3	2. 793	0. 7
0. 4	2. 808	0. 6
0. 5	2. 824	0. 5
0. 6	2. 840	0. 4
0. 7	2. 856	0. 3
0. 8	2. 872	0. 2
0. 9	2. 888	0. 1

71°—109°—251°—289°

DLo D	p l	 p	p l	DLo D	 m	 DLo
1	0. 326	0. 946	1	3. 072	1	2. 904
2	0. 651	1. 891	2	6. 143	2	5. 808
3	0. 977	2. 837	3	9. 215	3	8. 713
4	1. 302	3. 782	4	12. 286	4	11. 617
5	1. 628	4. 728	5	15. 358	5	14. 521
6	1. 953	5. 673	6	18. 429	6	17. 425
7	2. 279	6. 619	7	21. 501	7	20. 329
8	2. 605	7. 564	8	24. 572	8	23. 234
9	2. 930	8. 510	9	27. 644	9	26. 138

Course

71° 251°	p÷l DLo÷m	108° 288°
0. 0	2. 904	1. 0
0. 1	2. 921	0. 9
0. 2	2. 937	0. 8
0. 3	2. 954	0. 7
0. 4	2. 971	0. 6
0. 5	2. 989	0. 5
0. 6	3. 006	0. 4
0. 7	3. 024	0. 3
0. 8	3. 042	0. 2
0. 9	3. 060	0. 1

72°—108°—252°—288°

DLo D	p l	 p	p l	DLo D	 m	 DLo
1	0. 309	0. 951	1	3. 236	1	3. 078
2	0. 618	1. 902	2	6. 472	2	6. 155
3	0. 927	2. 853	3	9. 708	3	9. 233
4	1. 236	3. 804	4	12. 944	4	12. 311
5	1. 545	4. 755	5	16. 180	5	15. 388
6	1. 854	5. 706	6	19. 416	6	18. 466
7	2. 163	6. 657	7	22. 652	7	21. 544
8	2. 472	7. 608	8	25. 889	8	24. 621
9	2. 781	8. 560	9	29. 125	9	27. 699

Course

72° 252°	p÷l DLo÷m	107° 287°
0. 0	3. 078	1. 0
0. 1	3. 096	0. 9
0. 2	3. 115	0. 8
0. 3	3. 133	0. 7
0. 4	3. 152	0. 6
0. 5	3. 172	0. 5
0. 6	3. 191	0. 4
0. 7	3. 211	0. 3
0. 8	3. 230	0. 2
0. 9	3. 251	0. 1

73°—107°—253°—287°

DLo D	p l	 p	p l	DLo D	 m	 DLo
1	0. 292	0. 956	1	3. 420	1	3. 271
2	0. 585	1. 913	2	6. 841	2	6. 542
3	0. 877	2. 869	3	10. 261	3	9. 813
4	1. 169	3. 825	4	13. 681	4	13. 083
5	1. 462	4. 782	5	17. 102	5	16. 354
6	1. 754	5. 738	6	20. 522	6	19. 625
7	2. 047	6. 694	7	23. 942	7	22. 896
8	2. 339	7. 650	8	27. 362	8	26. 167
9	2. 631	8. 607	9	30. 783	9	29. 438

Course

73° 253°	p÷l DLo÷m	106° 286°
0. 0	3. 271	1. 0
0. 1	3. 291	0. 9
0. 2	3. 312	0. 8
0. 3	3. 333	0. 7
0. 4	3. 354	0. 6
0. 5	3. 376	0. 5
0. 6	3. 398	0. 4
0. 7	3. 420	0. 3
0. 8	3. 442	0. 2
0. 9	3. 465	0. 1

74°—106°—254°—286°

DLo D	p l	 p	p l	DLo D	 m	 DLo
1	0. 276	0. 961	1	3. 628	1	3. 487
2	0. 551	1. 923	2	7. 256	2	6. 975
3	0. 827	2. 884	3	10. 884	3	10. 462
4	1. 103	3. 845	4	14. 512	4	13. 950
5	1. 378	4. 806	5	18. 140	5	17. 437
6	1. 654	5. 768	6	21. 768	6	20. 924
7	1. 929	6. 729	7	25. 396	7	24. 412
8	2. 205	7. 690	8	29. 024	8	27. 899
9	2. 481	8. 651	9	32. 652	9	31. 387

Course

74° 254°	p÷l DLo÷m	105° 285°
0. 0	3. 487	1. 0
0. 1	3. 511	0. 9
0. 2	3. 534	0. 8
0. 3	3. 558	0. 7
0. 4	3. 582	0. 6
0. 5	3. 606	0. 5
0. 6	3. 630	0. 4
0. 7	3. 655	0. 3
0. 8	3. 681	0. 2
0. 9	3. 706	0. 1

TABLE 3

Traverse Table

75°—105°—255°—285°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 259	0. 966	1	3. 864	1	3. 732
2	0. 518	1. 932	2	7. 727	2	7. 464
3	0. 776	2. 898	3	11. 591	3	11. 196
4	1. 035	3. 864	4	15. 455	4	14. 928
5	1. 294	4. 830	5	19. 319	5	18. 660
6	1. 553	5. 796	6	23. 182	6	22. 392
7	1. 812	6. 761	7	27. 046	7	26. 124
8	2. 071	7. 727	8	30. 910	8	29. 856
9	2. 329	8. 693	9	34. 773	9	33. 588

Course		
75° 255°	p÷l DLo÷m	104° 284°
0. 0	3. 732	1. 0
0. 1	3. 758	0. 9
0. 2	3. 785	0. 8
0. 3	3. 812	0. 7
0. 4	3. 839	0. 6
0. 5	3. 867	0. 5
0. 6	3. 895	0. 4
0. 7	3. 923	0. 3
0. 8	3. 952	0. 2
0. 9	3. 981	0. 1

76°—104°—256°—284°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 242	0. 970	1	4. 134	1	4. 011
2	0. 484	1. 941	2	8. 267	2	8. 022
3	0. 726	2. 911	3	12. 401	3	12. 032
4	0. 968	3. 881	4	16. 534	4	16. 043
5	1. 210	4. 851	5	20. 668	5	20. 054
6	1. 452	5. 822	6	24. 801	6	24. 065
7	1. 693	6. 792	7	28. 935	7	28. 075
8	1. 935	7. 762	8	33. 069	8	32. 086
9	2. 177	8. 733	9	37. 202	9	36. 097

Course		
76° 256°	p÷l DLo÷m	103° 283°
0. 0	4. 011	1. 0
0. 1	4. 041	0. 9
0. 2	4. 071	0. 8
0. 3	4. 102	0. 7
0. 4	4. 134	0. 6
0. 5	4. 165	0. 5
0. 6	4. 198	0. 4
0. 7	4. 230	0. 3
0. 8	4. 264	0. 2
0. 9	4. 297	0. 1

77°—103°—257°—283°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 225	0. 974	1	4. 445	1	4. 331
2	0. 450	1. 949	2	8. 891	2	8. 663
3	0. 675	2. 923	3	13. 336	3	12. 994
4	0. 900	3. 897	4	17. 782	4	17. 326
5	1. 125	4. 872	5	22. 227	5	21. 657
6	1. 350	5. 846	6	26. 672	6	25. 989
7	1. 575	6. 821	7	31. 118	7	30. 320
8	1. 800	7. 795	8	35. 563	8	34. 652
9	2. 025	8. 769	9	40. 009	9	38. 983

Course		
77° 257°	p÷l DLo÷m	102° 282°
0. 0	4. 331	1. 0
0. 1	4. 366	0. 9
0. 2	4. 402	0. 8
0. 3	4. 437	0. 7
0. 4	4. 474	0. 6
0. 5	4. 511	0. 5
0. 6	4. 548	0. 4
0. 7	4. 586	0. 3
0. 8	4. 625	0. 2
0. 9	4. 665	0. 1

78°—102°—258°—282°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 208	0. 978	1	4. 810	1	4. 705
2	0. 416	1. 956	2	9. 619	2	9. 409
3	0. 624	2. 934	3	14. 429	3	14. 114
4	0. 832	3. 913	4	19. 239	4	18. 819
5	1. 040	4. 891	5	24. 049	5	23. 523
6	1. 247	5. 869	6	28. 858	6	28. 228
7	1. 455	6. 847	7	33. 668	7	32. 932
8	1. 663	7. 825	8	38. 478	8	37. 637
9	1. 871	8. 803	9	43. 288	9	42. 342

Course		
78° 258°	p÷l DLo÷m	101° 281°
0. 0	4. 705	1. 0
0. 1	4. 745	0. 9
0. 2	4. 787	0. 8
0. 3	4. 829	0. 7
0. 4	4. 872	0. 6
0. 5	4. 915	0. 5
0. 6	4. 959	0. 4
0. 7	5. 005	0. 3
0. 8	5. 050	0. 2
0. 9	5. 097	0. 1

79°—101°—259°—281°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 191	0. 982	1	5. 241	1	5. 145
2	0. 382	1. 963	2	10. 482	2	10. 289
3	0. 572	2. 945	3	15. 723	3	15. 434
4	0. 763	3. 927	4	20. 963	4	20. 578
5	0. 954	4. 908	5	26. 204	5	25. 723
6	1. 145	5. 890	6	31. 445	6	30. 867
7	1. 336	6. 871	7	36. 686	7	36. 012
8	1. 526	7. 853	8	41. 927	8	41. 156
9	1. 717	8. 835	9	47. 168	9	46. 301

Course		
79° 259°	p÷l DLo÷m	100° 280°
0. 0	5. 145	1. 0
0. 1	5. 193	0. 9
0. 2	5. 242	0. 8
0. 3	5. 292	0. 7
0. 4	5. 343	0. 6
0. 5	5. 396	0. 5
0. 6	5. 449	0. 4
0. 7	5. 503	0. 3
0. 8	5. 558	0. 2
0. 9	5. 614	0. 1

TABLE 3

Traverse Table

80°—100°—260°—280°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 174	0. 985	1	5. 759	1	5. 671
2	0. 347	1. 970	2	11. 518	2	11. 343
3	0. 521	2. 954	3	17. 276	3	17. 014
4	0. 695	3. 939	4	23. 035	4	22. 685
5	0. 868	4. 924	5	28. 794	5	28. 356
6	1. 042	5. 909	6	34. 553	6	34. 028
7	1. 216	6. 894	7	40. 311	7	39. 699
8	1. 389	7. 878	8	46. 070	8	45. 370
9	1. 563	8. 863	9	51. 829	9	51. 042

Course		
80° 260°	p÷l DLo÷m	99° 279°
0. 0	5. 671	1. 0
0. 1	5. 730	0. 9
0. 2	5. 789	0. 8
0. 3	5. 850	0. 7
0. 4	5. 912	0. 6
0. 5	5. 976	0. 5
0. 6	6. 041	0. 4
0. 7	6. 107	0. 3
0. 8	6. 174	0. 2
0. 9	6. 243	0. 1

81°—99°—261°—279°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 156	0. 988	1	6. 392	1	6. 314
2	0. 313	1. 975	2	12. 785	2	12. 628
3	0. 469	2. 963	3	19. 177	3	18. 941
4	0. 626	3. 951	4	25. 570	4	25. 255
5	0. 782	4. 938	5	31. 962	5	31. 569
6	0. 939	5. 926	6	38. 355	6	37. 883
7	1. 095	6. 914	7	44. 747	7	44. 196
8	1. 251	7. 902	8	51. 140	8	50. 510
9	1. 408	8. 889	9	57. 532	9	56. 824

Course		
81° 261°	p÷l DLo÷m	98° 278°
0. 0	6. 314	1. 0
0. 1	6. 386	0. 9
0. 2	6. 460	0. 8
0. 3	6. 535	0. 7
0. 4	6. 612	0. 6
0. 5	6. 691	0. 5
0. 6	6. 772	0. 4
0. 7	6. 855	0. 3
0. 8	6. 940	0. 2
0. 9	7. 026	0. 1

82°—98°—262°—278°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 139	0. 990	1	7. 185	1	7. 115
2	0. 278	1. 981	2	14. 371	2	14. 231
3	0. 418	2. 971	3	21. 556	3	21. 346
4	0. 557	3. 961	4	28. 741	4	28. 461
5	0. 696	4. 951	5	35. 926	5	35. 577
6	0. 835	5. 942	6	43. 112	6	42. 692
7	0. 974	6. 932	7	50. 297	7	49. 808
8	1. 113	7. 922	8	57. 482	8	56. 923
9	1. 253	8. 912	9	64. 668	9	64. 038

Course		
82° 262°	p÷l DLo÷m	97° 277°
0. 0	7. 115	1. 0
0. 1	7. 207	0. 9
0. 2	7. 300	0. 8
0. 3	7. 396	0. 7
0. 4	7. 495	0. 6
0. 5	7. 596	0. 5
0. 6	7. 700	0. 4
0. 7	7. 806	0. 3
0. 8	7. 916	0. 2
0. 9	8. 028	0. 1

83°—97°—263°—277°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 122	0. 993	1	8. 206	1	8. 144
2	0. 244	1. 985	2	16. 411	2	16. 289
3	0. 366	2. 978	3	24. 617	3	24. 433
4	0. 487	3. 970	4	32. 822	4	32. 577
5	0. 609	4. 963	5	41. 028	5	40. 722
6	0. 731	5. 955	6	49. 233	6	48. 866
7	0. 853	6. 948	7	57. 439	7	57. 010
8	0. 975	7. 940	8	65. 644	8	65. 155
9	1. 097	8. 933	9	73. 850	9	73. 299

Course		
83° 263°	p÷l DLo÷m	96° 276°
0. 0	8. 144	1. 0
0. 1	8. 264	0. 9
0. 2	8. 386	0. 8
0. 3	8. 513	0. 7
0. 4	8. 643	0. 6
0. 5	8. 777	0. 5
0. 6	8. 915	0. 4
0. 7	9. 058	0. 3
0. 8	9. 205	0. 2
0. 9	9. 357	0. 1

84°—96°—264°—276°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 105	0. 995	1	9. 567	1	9. 514
2	0. 209	1. 989	2	19. 134	2	19. 029
3	0. 314	2. 984	3	28. 700	3	28. 543
4	0. 418	3. 978	4	38. 267	4	38. 057
5	0. 523	4. 973	5	47. 834	5	47. 572
6	0. 627	5. 967	6	57. 401	6	57. 086
7	0. 732	6. 962	7	66. 967	7	66. 601
8	0. 836	7. 956	8	76. 534	8	76. 115
9	0. 941	8. 951	9	86. 101	9	85. 629

Course		
84° 264°	p÷l DLo÷m	95° 275°
0. 0	9. 514	1. 0
0. 1	9. 677	0. 9
0. 2	9. 845	0. 8
0. 3	10. 019	0. 7
0. 4	10. 199	0. 6
0. 5	10. 385	0. 5
0. 6	10. 579	0. 4
0. 7	10. 780	0. 3
0. 8	10. 988	0. 2
0. 9	11. 205	0. 1

TABLE 3

Traverse Table

85°—95°—265°—275°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 087	0. 996	1	11. 474	1	11. 430
2	0. 174	1. 992	2	22. 947	2	22. 860
3	0. 261	2. 989	3	34. 421	3	34. 290
4	0. 349	3. 985	4	45. 895	4	45. 720
5	0. 436	4. 981	5	57. 369	5	57. 150
6	0. 523	5. 977	6	68. 842	6	68. 580
7	0. 610	6. 973	7	80. 316	7	80. 010
8	0. 697	7. 970	8	91. 790	8	91. 440
9	0. 784	8. 966	9	103. 263	9	102. 870

Course		
85° 265°	p÷l DLo÷m	94° 274°
0. 0	11. 430	1. 0
0. 1	11. 664	0. 9
0. 2	11. 909	0. 8
0. 3	12. 163	0. 7
0. 4	12. 429	0. 6
0. 5	12. 706	0. 5
0. 6	12. 996	0. 4
0. 7	13. 300	0. 3
0. 8	13. 617	0. 2
0. 9	13. 951	0. 1

86°—94°—266°—274°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 070	0. 998	1	14. 336	1	14. 301
2	0. 140	1. 995	2	28. 671	2	28. 601
3	0. 209	2. 993	3	43. 007	3	42. 902
4	0. 279	3. 990	4	57. 342	4	57. 203
5	0. 349	4. 988	5	71. 678	5	71. 503
6	0. 419	5. 985	6	86. 014	6	85. 804
7	0. 488	6. 983	7	100. 349	7	100. 105
8	0. 558	7. 981	8	114. 685	8	114. 405
9	0. 628	8. 978	9	129. 020	9	128. 706

Course		
86° 266°	p÷l DLo÷m	93° 273°
0. 0	14. 301	1. 0
0. 1	14. 669	0. 9
0. 2	15. 056	0. 8
0. 3	15. 464	0. 7
0. 4	15. 895	0. 6
0. 5	16. 350	0. 5
0. 6	16. 832	0. 4
0. 7	17. 343	0. 3
0. 8	17. 886	0. 2
0. 9	18. 464	0. 1

87°—93°—267°—273°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 052	0. 999	1	19. 107	1	19. 081
2	0. 105	1. 997	2	38. 215	2	38. 162
3	0. 157	2. 996	3	57. 322	3	57. 243
4	0. 209	3. 995	4	76. 429	4	76. 325
5	0. 262	4. 993	5	95. 537	5	95. 406
6	0. 314	5. 992	6	114. 644	6	114. 487
7	0. 366	6. 990	7	133. 751	7	133. 568
8	0. 419	7. 989	8	152. 859	8	152. 649
9	0. 471	8. 988	9	171. 966	9	171. 730

Course		
87° 267°	p÷l DLo÷m	92° 272°
0. 0	19. 081	1. 0
0. 1	19. 740	0. 9
0. 2	20. 446	0. 8
0. 3	21. 205	0. 7
0. 4	22. 022	0. 6
0. 5	22. 904	0. 5
0. 6	23. 859	0. 4
0. 7	24. 898	0. 3
0. 8	26. 031	0. 2
0. 9	27. 271	0. 1

88°—92°—268°—272°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 035	0. 999	1	28. 654	1	28. 636
2	0. 070	1. 999	2	57. 307	2	57. 273
3	0. 105	2. 998	3	85. 961	3	85. 909
4	0. 140	3. 998	4	114. 615	4	114. 545
5	0. 174	4. 997	5	143. 269	5	143. 181
6	0. 209	5. 996	6	171. 922	6	171. 818
7	0. 244	6. 996	7	200. 576	7	200. 454
8	0. 279	7. 995	8	229. 230	8	229. 090
9	0. 314	8. 995	9	257. 883	9	257. 726

Course		
88° 268°	p÷l DLo÷m	91° 271°
0. 0	28. 636	1. 0
0. 1	30. 145	0. 9
0. 2	31. 821	0. 8
0. 3	33. 694	0. 7
0. 4	35. 801	0. 6
0. 5	38. 188	0. 5
0. 6	40. 917	0. 4
0. 7	44. 066	0. 3
0. 8	47. 740	0. 2
0. 9	52. 081	0. 1

89°—91°—269°—271°

DLo	p		p	DLo		
D	l	p	l	D	m	DLo
1	0. 017	1. 000	1	57. 299	1	57. 290
2	0. 035	2. 000	2	114. 597	2	114. 580
3	0. 052	3. 000	3	171. 896	3	171. 870
4	0. 070	3. 999	4	229. 195	4	229. 160
5	0. 087	4. 999	5	286. 493	5	286. 450
6	0. 105	5. 999	6	343. 792	6	343. 740
7	0. 122	6. 999	7	401. 091	7	401. 030
8	0. 140	7. 999	8	458. 390	8	458. 320
9	0. 157	8. 999	9	515. 688	9	515. 610

Course		
89° 269°	p÷l DLo÷m	90° 270°
0. 0	57. 290	1. 0
0. 1	63. 657	0. 9
0. 2	71. 615	0. 8
0. 3	81. 847	0. 7
0. 4	95. 489	0. 6
0. 5	114. 589	0. 5
0. 6	143. 237	0. 4
0. 7	190. 984	0. 3
0. 8	286. 478	0. 2
0. 9	572. 957	0. 1

TABLE 4

Conversion Table for Meridional Parts

Latitude	International	Clarke (1880)	Sphere	Latitude	International	Clarke (1880)	Sphere	Latitude	International	Clarke (1880)	Sphere
° ′				° ′				° ′			
0 00	0.00	0.00	0.00	30 00	+0.08	−0.06	+11.64	60 00	+0.14	−0.10	+20.19
0 30	0.00	0.00	+0.20	30 30	+0.08	−0.06	+11.82	60 30	+0.14	−0.10	+20.29
1 00	0.00	0.00	+0.41	31 00	+0.08	−0.06	+11.99	61 00	+0.14	−0.11	+20.39
1 30	0.00	0.00	+0.61	31 30	+0.08	−0.06	+12.17	61 30	+0.14	−0.11	+20.48
2 00	+0.01	0.00	+0.81	32 00	+0.08	−0.06	+12.34	62 00	+0.14	−0.11	+20.58
2 30	+0.01	−0.01	+1.02	32 30	+0.09	−0.06	+12.51	62 30	+0.14	−0.11	+20.68
3 00	+0.01	−0.01	+1.22	33 00	+0.09	−0.07	+12.68	63 00	+0.14	−0.11	+20.77
3 30	+0.01	−0.01	+1.42	33 30	+0.09	−0.07	+12.85	63 30	+0.14	−0.11	+20.86
4 00	+0.01	−0.01	+1.62	34 00	+0.09	−0.07	+13.02	64 00	+0.14	−0.11	+20.95
4 30	+0.01	−0.01	+1.83	34 30	+0.09	−0.07	+13.19	64 30	+0.14	−0.11	+21.04
5 00	+0.01	−0.01	+2.03	35 00	+0.09	−0.07	+13.36	65 00	+0.14	−0.11	+21.13
5 30	+0.02	−0.01	+2.23	35 30	+0.09	−0.07	+13.52	65 30	+0.14	−0.11	+21.21
6 00	+0.02	−0.01	+2.43	36 00	+0.09	−0.07	+13.69	66 00	+0.14	−0.11	+21.30
6 30	+0.02	−0.01	+2.63	36 30	+0.09	−0.07	+13.85	66 30	+0.15	−0.11	+21.38
7 00	+0.02	−0.01	+2.84	37 00	+0.10	−0.07	+14.01	67 00	+0.15	−0.11	+21.46
7 30	+0.02	−0.02	+3.04	37 30	+0.10	−0.07	+14.18	67 30	+0.15	−0.11	+21.54
8 00	+0.02	−0.02	+3.24	38 00	+0.10	−0.07	+14.34	68 00	+0.15	−0.11	+21.62
8 30	+0.02	−0.02	+3.44	38 30	+0.10	−0.07	+14.50	68 30	+0.15	−0.11	+21.69
9 00	+0.02	−0.02	+3.64	39 00	+0.10	−0.08	+14.66	69 00	+0.15	−0.11	+21.77
9 30	+0.03	−0.02	+3.84	39 30	+0.10	−0.08	+14.81	69 30	+0.15	−0.11	+21.84
10 00	+0.03	−0.02	+4.04	40 00	+0.10	−0.08	+14.97	70 00	+0.15	−0.11	+21.91
10 30	+0.03	−0.02	+4.24	40 30	+0.10	−0.08	+15.13	70 30	+0.15	−0.11	+21.98
11 00	+0.03	−0.02	+4.44	41 00	+0.10	−0.08	+15.28	71 00	+0.15	−0.11	+22.05
11 30	+0.03	−0.02	+4.64	41 30	+0.10	−0.08	+15.43	71 30	+0.15	−0.11	+22.11
12 00	+0.03	−0.02	+4.84	42 00	+0.11	−0.08	+15.59	72 00	+0.15	−0.11	+22.18
12 30	+0.03	−0.03	+5.04	42 30	+0.11	−0.08	+15.74	72 30	+0.15	−0.11	+22.24
13 00	+0.04	−0.03	+5.23	43 00	+0.11	−0.08	+15.89	73 00	+0.15	−0.12	+22.30
13 30	+0.04	−0.03	+5.43	43 30	+0.11	−0.08	+16.03	73 30	+0.15	−0.12	+22.36
14 00	+0.04	−0.03	+5.63	44 00	+0.11	−0.08	+16.18	74 00	+0.15	−0.12	+22.41
14 30	+0.04	−0.03	+5.83	44 30	+0.11	−0.08	+16.33	74 30	+0.15	−0.12	+22.47
15 00	+0.04	−0.03	+6.02	45 00	+0.11	−0.08	+16.47	75 00	+0.15	−0.12	+22.52
15 30	+0.04	−0.03	+6.22	45 30	+0.11	−0.09	+16.62	75 30	+0.15	−0.12	+22.58
16 00	+0.04	−0.03	+6.41	46 00	+0.11	−0.09	+16.76	76 00	+0.15	−0.12	+22.63
16 30	+0.04	−0.03	+6.61	46 30	+0.11	−0.09	+16.90	76 30	+0.15	−0.12	+22.67
17 00	+0.05	−0.04	+6.80	47 00	+0.12	−0.09	+17.04	77 00	+0.15	−0.12	+22.72
17 30	+0.05	−0.04	+7.00	47 30	+0.12	−0.09	+17.18	77 30	+0.16	−0.12	+22.77
18 00	+0.05	−0.04	+7.19	48 00	+0.12	−0.09	+17.31	78 00	+0.16	−0.12	+22.81
18 30	+0.05	−0.04	+7.38	48 30	+0.12	−0.09	+17.45	78 30	+0.16	−0.12	+22.85
19 00	+0.05	−0.04	+7.58	49 00	+0.12	−0.09	+17.58	79 00	+0.16	−0.12	+22.89
19 30	+0.05	−0.04	+7.77	49 30	+0.12	−0.09	+17.72	79 30	+0.16	−0.12	+22.93
20 00	+0.05	−0.04	+7.96	50 00	+0.12	−0.09	+17.85	80 00	+0.16	−0.12	+22.97
20 30	+0.06	−0.04	+8.15	50 30	+0.12	−0.09	+17.98	80 30	+0.16	−0.12	+23.00
21 00	+0.06	−0.04	+8.34	51 00	+0.12	−0.09	+18.11	81 00	+0.16	−0.12	+23.03
21 30	+0.06	−0.04	+8.53	51 30	+0.12	−0.09	+18.24	81 30	+0.16	−0.12	+23.06
22 00	+0.06	−0.04	+8.72	52 00	+0.12	−0.09	+18.36	82 00	+0.16	−0.12	+23.09
22 30	+0.06	−0.05	+8.91	52 30	+0.13	−0.10	+18.49	82 30	+0.16	−0.12	+23.12
23 00	+0.06	−0.05	+9.10	53 00	+0.13	−0.10	+18.61	83 00	+0.16	−0.12	+23.15
23 30	+0.06	−0.05	+9.28	53 30	+0.13	−0.10	+18.73	83 30	+0.16	−0.12	+23.17
24 00	+0.06	−0.05	+9.47	54 00	+0.13	−0.10	+18.85	84 00	+0.16	−0.12	+23.19
24 30	+0.07	−0.05	+9.65	54 30	+0.13	−0.10	+18.97	84 30	+0.16	−0.12	+23.21
25 00	+0.07	−0.05	+9.84	55 00	+0.13	−0.10	+19.09	85 00	+0.16	−0.12	+23.23
25 30	+0.07	−0.05	+10.02	55 30	+0.13	−0.10	+19.21	85 30	+0.16	−0.12	+23.25
26 00	+0.07	−0.05	+10.21	56 00	+0.13	−0.10	+19.32	86 00	+0.16	−0.12	+23.26
26 30	+0.07	−0.05	+10.39	56 30	+0.13	−0.10	+19.43	86 30	+0.16	−0.12	+23.28
27 00	+0.07	−0.05	+10.57	57 00	+0.13	−0.10	+19.55	87 00	+0.16	−0.12	+23.29
27 30	+0.07	−0.06	+10.75	57 30	+0.13	−0.10	+19.66	87 30	+0.16	−0.12	+23.30
28 00	+0.07	−0.06	+10.93	58 00	+0.13	−0.10	+19.77	88 00	+0.16	−0.12	+23.31
28 30	+0.08	−0.06	+11.11	58 30	+0.14	−0.10	+19.87	88 30	+0.16	−0.12	+23.31
29 00	+0.08	−0.06	+11.29	59 00	+0.14	−0.10	+19.98	89 00	+0.16	−0.12	+23.32
29 30	+0.08	−0.06	+11.46	59 30	+0.14	−0.10	+20.08	89 30	+0.16	−0.12	+23.32
30 00	+0.08	−0.06	+11.64	60 00	+0.14	−0.10	+20.19	90 00	—	—	—

TABLE 5

Meridional Parts

Lat.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	Lat.
′											′
0	0.0	59.6	119.2	178.9	238.6	298.4	358.2	418.2	478.3	538.6	0
1	1.0	60.6	20.2	79.9	39.6	299.4	59.2	19.2	79.3	39.6	1
2	2.0	61.6	21.2	80.9	40.6	300.3	60.2	20.2	80.3	40.6	2
3	3.0	62.6	22.2	81.8	41.6	01.3	61.2	21.2	81.3	41.6	3
4	4.0	63.6	23.2	82.8	42.6	02.3	62.2	22.2	82.3	42.6	4
5	5.0	64.6	124.2	183.8	243.6	303.3	363.2	423.2	483.3	543.6	5
6	6.0	65.6	25.2	84.8	44.5	04.3	64.2	24.2	84.3	44.6	6
7	7.0	66.6	26.2	85.8	45.5	05.3	65.2	25.2	85.4	45.6	7
8	7.9	67.5	27.2	86.8	46.5	06.3	66.2	26.2	86.4	46.6	8
9	8.9	68.5	28.2	87.8	47.5	07.3	67.2	27.2	87.4	47.6	9
10	9.9	69.5	129.2	188.8	248.5	308.3	368.2	428.2	488.4	548.7	10
11	10.9	70.5	30.1	89.8	49.5	09.3	69.2	29.2	89.4	49.7	11
12	11.9	71.5	31.1	90.8	50.5	10.3	70.2	30.2	90.4	50.7	12
13	12.9	72.5	32.1	91.8	51.5	11.3	71.2	31.2	91.4	51.7	13
14	13.9	73.5	33.1	92.8	52.5	12.3	72.2	32.2	92.4	52.7	14
15	14.9	74.5	134.1	193.8	253.5	313.3	373.2	433.2	493.4	553.7	15
16	15.9	75.5	35.1	94.8	54.5	14.3	74.2	34.2	94.4	54.7	16
17	16.9	76.5	36.1	95.8	55.5	15.3	75.2	35.2	95.4	55.7	17
18	17.9	77.5	37.1	96.8	56.5	16.3	76.2	36.2	96.4	56.7	18
19	18.9	78.5	38.1	97.8	57.5	17.3	77.2	37.2	97.4	57.7	19
20	19.9	79.5	139.1	198.8	258.5	318.3	378.2	438.2	498.4	558.7	20
21	20.9	80.5	40.1	199.8	59.5	19.3	79.2	39.2	499.4	59.7	21
22	21.9	81.5	41.1	200.7	60.5	20.3	80.2	40.2	500.4	60.7	22
23	22.8	82.4	42.1	01.7	61.5	21.3	81.2	41.2	01.4	61.7	23
24	23.8	83.4	43.1	02.7	62.5	22.3	82.2	42.2	02.4	62.7	24
25	24.8	84.4	144.1	203.7	263.5	323.3	383.2	443.2	503.4	563.8	25
26	25.8	85.4	45.1	04.7	64.5	24.3	84.2	44.2	04.4	64.8	26
27	26.8	86.4	46.1	05.7	65.5	25.3	85.2	45.2	05.4	65.8	27
28	27.8	87.4	47.0	06.7	66.5	26.3	86.2	46.2	06.4	66.8	28
29	28.8	88.4	48.0	07.7	67.5	27.3	87.2	47.3	07.4	67.8	29
30	29.8	89.4	149.0	208.7	268.5	328.3	388.2	448.3	508.4	568.8	30
31	30.8	90.4	50.0	09.7	69.4	29.3	89.2	49.3	09.4	69.8	31
32	31.8	91.4	51.0	10.7	70.4	30.3	90.2	50.3	10.5	70.8	32
33	32.8	92.4	52.0	11.7	71.4	31.3	91.2	51.3	11.5	71.8	33
34	33.8	93.4	53.0	12.7	72.4	32.3	92.2	52.3	12.5	72.8	34
35	34.8	94.4	154.0	213.7	273.4	333.3	393.2	453.3	513.5	573.8	35
36	35.8	95.4	55.0	14.7	74.4	34.3	94.2	54.3	14.5	74.8	36
37	36.8	96.4	56.0	15.7	75.4	35.3	95.2	55.3	15.5	75.8	37
38	37.7	97.4	57.0	16.7	76.4	36.3	96.2	56.3	16.5	76.8	38
39	38.7	98.3	58.0	17.7	77.4	37.3	97.2	57.3	17.5	77.9	39
40	39.7	99.3	159.0	218.7	278.4	338.3	398.2	458.3	518.5	578.9	40
41	40.7	100.3	60.0	19.7	79.4	39.3	399.2	59.3	19.5	79.9	41
42	41.7	01.3	61.0	20.7	80.4	40.3	400.2	60.3	20.5	80.9	42
43	42.7	02.3	62.0	21.6	81.4	41.3	01.2	61.3	21.5	81.9	43
44	43.7	03.3	63.0	22.6	82.4	42.3	02.2	62.3	22.5	82.9	44
45	44.7	104.3	163.9	223.6	283.4	343.2	403.2	463.3	523.5	583.9	45
46	45.7	05.3	64.9	24.6	84.4	44.2	04.2	64.3	24.5	84.9	46
47	46.7	06.3	65.9	25.6	85.4	45.2	05.2	65.3	25.5	85.9	47
48	47.7	07.3	66.9	26.6	86.4	46.2	06.2	66.3	26.5	86.9	48
49	48.7	08.3	67.9	27.6	87.4	47.2	07.2	67.3	27.5	87.9	49
50	49.7	109.3	168.9	228.6	288.4	348.2	408.2	468.3	528.5	588.9	50
51	50.7	10.3	69.9	29.6	89.4	49.2	09.2	69.3	29.5	90.0	51
52	51.7	11.3	70.9	30.6	90.4	50.2	10.2	70.3	30.5	91.0	52
53	52.6	12.3	71.9	31.6	91.4	51.2	11.2	71.3	31.6	92.0	53
54	53.6	13.2	72.9	32.6	92.4	52.2	12.2	72.3	32.6	93.0	54
55	54.6	114.2	173.9	233.6	293.4	353.2	413.2	473.3	533.6	594.0	55
56	55.6	15.2	74.9	34.6	94.4	54.2	14.2	74.3	34.6	95.0	56
57	56.6	16.2	75.9	35.6	95.4	55.2	15.2	75.3	35.6	96.0	57
58	57.6	17.2	76.9	36.6	96.4	56.2	16.2	76.3	36.6	97.0	58
59	58.6	18.2	77.9	37.6	97.4	57.2	17.2	77.3	37.6	98.0	59
60	59.6	119.2	178.9	238.6	298.4	358.2	418.2	478.3	538.6	599.0	60
Lat.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	Lat.

TABLE 5

Meridional Parts

Lat.	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	Lat.
′											′
0	599.0	659.7	720.5	781.5	842.9	904.4	966.3	1028.5	1091.0	1153.9	0
1	600.0	60.7	21.5	82.6	43.9	05.5	67.3	29.5	92.1	55.0	1
2	01.0	61.7	22.5	83.6	44.9	06.5	68.4	30.6	93.1	56.0	2
3	02.1	62.7	23.5	84.6	45.9	07.5	69.4	31.6	94.2	57.1	3
4	03.1	63.7	24.5	85.6	47.0	08.6	70.4	32.7	95.2	58.1	4
5	604.1	664.7	725.6	786.6	848.0	909.6	971.5	1033.7	1096.3	1159.2	5
6	05.1	65.7	26.6	87.7	49.0	10.6	72.5	34.7	97.3	60.2	6
7	06.1	66.7	27.6	88.7	50.0	11.6	73.5	35.8	98.3	61.3	7
8	07.1	67.8	28.6	89.7	51.1	12.7	74.6	36.8	1099.4	62.3	8
9	08.1	68.8	29.6	90.7	52.1	13.7	75.6	37.9	1100.4	63.4	9
10	609.1	669.8	730.6	791.7	853.1	914.7	976.7	1038.9	1101.5	1164.4	10
11	10.1	70.8	31.7	92.8	54.1	15.8	77.7	39.9	02.5	65.5	11
12	11.1	71.8	32.7	93.8	55.1	16.8	78.7	41.0	03.6	66.5	12
13	12.1	72.8	33.7	94.8	56.2	17.8	79.8	42.0	04.6	67.6	13
14	13.2	73.8	34.7	95.8	57.2	18.8	80.8	43.1	05.7	68.6	14
15	614.2	674.8	735.7	796.8	858.2	919.9	981.8	1044.1	1106.7	1169.7	15
16	15.2	75.9	36.7	97.9	59.3	20.9	82.9	45.1	07.8	70.7	16
17	16.2	76.9	37.8	98.9	60.3	21.9	83.9	46.2	08.8	71.8	17
18	17.2	77.9	38.8	799.9	61.3	23.0	84.9	47.2	09.9	72.8	18
19	18.2	78.9	39.8	800.9	62.3	24.0	86.0	48.3	10.9	73.9	19
20	619.2	679.9	740.8	802.0	863.4	925.0	987.0	1049.3	1111.9	1175.0	20
21	20.2	80.9	41.8	03.0	64.4	26.1	88.0	50.3	13.0	76.0	21
22	21.2	81.9	42.8	04.0	65.4	27.1	89.1	51.4	14.0	77.1	22
23	22.2	82.9	43.9	05.0	66.4	28.1	90.1	52.4	15.1	78.1	23
24	23.3	84.0	44.9	06.0	67.5	29.2	91.1	53.5	16.1	79.2	24
25	624.3	685.0	745.9	807.1	868.5	930.2	992.2	1054.5	1117.2	1180.2	25
26	25.3	86.0	46.9	08.1	69.5	31.2	93.2	55.6	18.2	81.3	26
27	26.3	87.0	47.9	09.1	70.5	32.2	94.3	56.6	19.3	82.3	27
28	27.3	88.0	48.9	10.1	71.6	33.3	95.3	57.6	20.3	83.4	28
29	28.3	89.0	50.0	11.1	72.6	34.3	96.3	58.7	21.4	84.4	29
30	629.3	690.0	751.0	812.2	873.6	935.3	997.4	1059.7	1122.4	1185.5	30
31	30.3	91.1	52.0	13.2	74.6	36.4	98.4	60.8	23.5	86.5	31
32	31.3	92.1	53.0	14.2	75.7	37.4	999.4	61.8	24.5	87.6	32
33	32.3	93.1	54.0	15.2	76.7	38.4	1000.5	62.8	25.6	88.7	33
34	33.4	94.1	55.1	16.3	77.7	39.5	01.5	63.9	26.6	89.7	34
35	634.4	695.1	756.1	817.3	878.7	940.5	1002.5	1064.9	1127.7	1190.8	35
36	35.4	96.1	57.1	18.3	79.8	41.5	03.6	66.0	28.7	91.8	36
37	36.4	97.1	58.1	19.3	80.8	42.6	04.6	67.0	29.8	92.9	37
38	37.4	98.2	59.1	20.3	81.8	43.6	05.7	68.1	30.8	93.9	38
39	38.4	699.2	60.1	21.4	82.9	44.6	06.7	69.1	31.9	95.0	39
40	639.4	700.2	761.2	822.4	883.9	945.7	1007.7	1070.1	1132.9	1196.0	40
41	40.4	01.2	62.2	23.4	84.9	46.7	08.8	71.2	34.0	97.1	41
42	41.4	02.2	63.2	24.4	85.9	47.7	09.8	72.2	35.0	98.2	42
43	42.5	03.2	64.2	25.5	87.0	48.7	10.8	73.3	36.1	1199.2	43
44	43.5	04.2	65.2	26.5	88.0	49.8	11.9	74.3	37.1	1200.3	44
45	644.5	705.3	766.3	827.5	889.0	950.8	1012.9	1075.4	1138.2	1201.3	45
46	45.5	06.3	67.3	28.5	90.0	51.8	14.0	76.4	39.2	02.4	46
47	46.5	07.3	68.3	29.5	91.1	52.9	15.0	77.4	40.3	03.4	47
48	47.5	08.3	69.3	30.6	92.1	53.9	16.0	78.5	41.3	04.5	48
49	48.5	09.3	70.3	31.6	93.1	54.9	17.1	79.5	42.4	05.5	49
50	649.5	710.3	771.4	832.6	894.2	956.0	1018.1	1080.6	1143.4	1206.6	50
51	50.5	11.3	72.4	33.6	95.2	57.0	19.2	81.6	44.5	07.7	51
52	51.6	12.4	73.4	34.7	96.2	58.0	20.2	82.7	45.5	08.7	52
53	52.6	13.4	74.4	35.7	97.2	59.1	21.2	83.7	46.6	09.8	53
54	53.6	14.4	75.4	36.7	98.3	60.1	22.3	84.8	47.6	10.8	54
55	654.6	715.4	776.4	837.7	899.3	961.1	1023.3	1085.8	1148.7	1211.9	55
56	55.6	16.4	77.5	38.8	900.3	62.2	24.3	86.8	49.7	12.9	56
57	56.6	17.4	78.5	39.8	01.4	63.2	25.4	87.9	50.8	14.0	57
58	57.6	18.5	79.5	40.8	02.4	64.2	26.4	88.9	51.8	15.1	58
59	58.6	19.5	80.5	41.8	03.4	65.3	27.5	90.0	52.9	16.1	59
60	659.7	720.5	781.5	842.9	904.4	966.3	1028.5	1091.0	1153.9	1217.2	60
Lat.	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	Lat.

TABLE 5

Meridional Parts

Lat.	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	Lat.
′											′
0	1217. 2	1280. 9	1345. 0	1409. 5	1474. 6	1540. 2	1606. 3	1672. 9	1740. 2	1808. 1	0
1	18. 2	81. 9	46. 0	10. 6	75. 7	41. 3	07. 4	74. 1	41. 4	09. 3	1
2	19. 3	83. 0	47. 1	11. 7	76. 8	42. 4	08. 5	75. 2	42. 5	10. 4	2
3	20. 4	84. 1	48. 2	12. 8	77. 9	43. 4	09. 6	76. 3	43. 6	11. 6	3
4	21. 4	85. 1	49. 3	13. 9	78. 9	44. 5	10. 7	77. 4	44. 7	12. 7	4
5	1222. 5	1286. 2	1350. 3	1414. 9	1480. 0	1545. 6	1611. 8	1678. 5	1745. 9	1813. 8	5
6	23. 5	87. 2	51. 4	16. 0	81. 1	46. 7	12. 9	79. 6	47. 0	15. 0	6
7	24. 6	88. 3	52. 5	17. 1	82. 2	47. 8	14. 0	80. 8	48. 1	16. 1	7
8	25. 6	89. 4	53. 5	18. 2	83. 3	48. 9	15. 1	81. 9	49. 3	17. 3	8
9	26. 7	90. 4	54. 6	19. 3	84. 4	50. 0	16. 2	83. 0	50. 4	18. 4	9
10	1227. 8	1291. 5	1355. 7	1420. 3	1485. 5	1551. 1	1617. 3	1684. 1	1751. 5	1819. 5	10
11	28. 8	92. 6	56. 8	21. 4	86. 6	52. 2	18. 4	85. 2	52. 6	20. 7	11
12	29. 9	93. 6	57. 8	22. 5	87. 7	53. 3	19. 6	86. 4	53. 8	21. 8	12
13	30. 9	94. 7	58. 9	23. 6	88. 8	54. 4	20. 7	87. 5	54. 9	23. 0	13
14	32. 0	95. 8	60. 0	24. 7	89. 8	55. 5	21. 8	88. 6	56. 0	24. 1	14
15	1233. 1	1296. 8	1361. 1	1425. 8	1490. 9	1556. 6	1622. 9	1689. 7	1757. 2	1825. 2	15
16	34. 1	97. 9	62. 1	26. 8	92. 0	57. 7	24. 0	90. 8	58. 3	26. 4	16
17	35. 2	1299. 0	63. 2	27. 9	93. 1	58. 8	25. 1	91. 9	59. 4	27. 5	17
18	36. 2	1300. 0	64. 3	29. 0	94. 2	59. 9	26. 2	93. 1	60. 5	28. 7	18
19	37. 3	01. 1	65. 4	30. 1	95. 3	61. 0	27. 3	94. 2	61. 7	29. 8	19
20	1238. 4	1302. 2	1366. 4	1431. 2	1496. 4	1562. 1	1628. 4	1695. 3	1762. 8	1830. 9	20
21	39. 4	03. 2	67. 5	32. 2	97. 5	63. 2	29. 5	96. 4	63. 9	32. 1	21
22	40. 5	04. 3	68. 6	33. 3	98. 6	64. 3	30. 6	97. 5	65. 1	33. 2	22
23	41. 5	05. 4	69. 7	34. 4	1499. 7	65. 4	31. 8	98. 7	66. 2	34. 4	23
24	42. 6	06. 4	70. 7	35. 5	1500. 8	66. 5	32. 9	1699. 8	67. 3	35. 5	24
25	1243. 7	1307. 5	1371. 8	1436. 6	1501. 8	1567. 6	1634. 0	1700. 9	1768. 5	1836. 6	25
26	44. 7	08. 6	72. 9	37. 7	02. 9	68. 7	35. 1	02. 0	69. 6	37. 8	26
27	45. 8	09. 6	74. 0	38. 7	04. 0	69. 8	36. 2	03. 1	70. 7	38. 9	27
28	46. 8	10. 7	75. 0	39. 8	05. 1	70. 9	37. 3	04. 3	71. 8	40. 1	28
29	47. 9	11. 8	76. 1	40. 9	06. 2	72. 0	38. 4	05. 4	73. 0	41. 2	29
30	1249. 0	1312. 9	1377. 2	1442. 0	1507. 3	1573. 1	1639. 5	1706. 5	1774. 1	1842. 4	30
31	50. 0	13. 9	78. 3	43. 1	08. 4	74. 2	40. 6	07. 6	75. 2	43. 5	31
32	51. 1	15. 0	79. 3	44. 2	09. 5	75. 3	41. 8	08. 8	76. 4	44. 6	32
33	52. 1	16. 1	80. 4	45. 3	10. 6	76. 4	42. 9	09. 9	77. 5	45. 8	33
34	53. 2	17. 1	81. 5	46. 3	11. 7	77. 6	44. 0	11. 0	78. 6	46. 9	34
35	1254. 3	1318. 2	1382. 6	1447. 4	1512. 8	1578. 7	1645. 1	1712. 1	1779. 8	1848. 1	35
36	55. 3	19. 3	83. 7	48. 5	13. 9	79. 8	46. 2	13. 2	80. 9	49. 2	36
37	56. 4	20. 3	84. 7	49. 6	15. 0	80. 9	47. 3	14. 4	82. 0	50. 4	37
38	57. 5	21. 4	85. 8	50. 7	16. 1	82. 0	48. 4	15. 5	83. 2	51. 5	38
39	58. 5	22. 5	86. 9	51. 8	17. 1	83. 1	49. 5	16. 6	84. 3	52. 7	39
40	1259. 6	1323. 5	1388. 0	1452. 8	1518. 2	1584. 2	1650. 7	1717. 7	1785. 4	1853. 8	40
41	60. 6	24. 6	89. 0	53. 9	19. 3	85. 3	51. 8	18. 9	86. 6	54. 9	41
42	61. 7	25. 7	90. 1	55. 0	20. 4	86. 4	52. 9	20. 0	87. 7	56. 1	42
43	62. 8	26. 8	91. 2	56. 1	21. 5	87. 5	54. 0	21. 1	88. 8	57. 2	43
44	63. 8	27. 8	92. 3	57. 2	22. 6	88. 6	55. 1	22. 2	90. 0	58. 4	44
45	1264. 9	1328. 9	1393. 3	1458. 3	1523. 7	1589. 7	1656. 2	1723. 4	1791. 1	1859. 5	45
46	66. 0	30. 0	94. 4	59. 4	24. 8	90. 8	57. 3	24. 5	92. 2	60. 7	46
47	67. 0	31. 0	95. 5	60. 5	25. 9	91. 9	58. 5	25. 6	93. 4	61. 8	47
48	68. 1	32. 1	96. 6	61. 5	27. 0	93. 0	59. 6	26. 7	94. 5	63. 0	48
49	69. 1	33. 2	97. 7	62. 6	28. 1	94. 1	60. 7	27. 9	95. 6	64. 1	49
50	1270. 2	1334. 2	1398. 7	1463. 7	1529. 2	1595. 2	1661. 8	1729. 0	1796. 8	1865. 3	50
51	71. 3	35. 3	1399. 8	64. 8	30. 3	96. 3	62. 9	30. 1	97. 9	66. 4	51
52	72. 3	36. 4	1400. 9	65. 9	31. 4	97. 4	64. 0	31. 2	1799. 1	67. 5	52
53	73. 4	37. 5	02. 0	67. 0	32. 5	98. 5	65. 1	32. 4	1800. 2	68. 7	53
54	74. 5	38. 5	03. 1	68. 1	33. 6	1599. 6	66. 3	33. 5	01. 3	69. 8	54
55	1275. 5	1339. 6	1404. 1	1469. 1	1534. 7	1600. 7	1667. 4	1734. 6	1802. 5	1871. 0	55
56	76. 6	40. 7	05. 2	70. 2	35. 8	01. 8	68. 5	35. 7	03. 6	72. 1	56
57	77. 7	41. 7	06. 3	71. 3	36. 9	02. 9	69. 6	36. 9	04. 7	73. 3	57
58	78. 7	42. 8	07. 4	72. 4	38. 0	04. 1	70. 7	38. 0	05. 9	74. 4	58
59	79. 8	43. 9	08. 5	73. 5	39. 1	05. 2	71. 8	39. 1	07. 0	75. 6	59
60	1280. 9	1345. 0	1409. 5	1474. 6	1540. 2	1606. 3	1672. 9	1740. 2	1808. 1	1876. 7	60
Lat.	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	Lat.

TABLE 5

Meridional Parts

Lat.	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	Lat.
′											′
0	1876. 7	1946. 0	2016. 0	2086. 8	2158. 5	2230. 9	2304. 3	2378. 6	2453. 9	2530. 3	0
1	77. 9	47. 2	17. 2	88. 0	59. 7	32. 1	05. 5	79. 9	55. 2	31. 6	1
2	79. 0	48. 3	18. 4	89. 2	60. 9	33. 4	06. 8	81. 1	56. 5	32. 8	2
3	80. 2	49. 5	19. 6	90. 4	62. 1	34. 6	08. 0	82. 4	57. 7	34. 1	3
4	81. 3	50. 7	20. 7	91. 6	63. 3	35. 8	09. 2	83. 6	59. 0	35. 4	4
5	1882. 5	1951. 8	2021. 9	2092. 8	2164. 5	2237. 0	2310. 5	2384. 9	2460. 2	2536. 7	5
6	83. 6	53. 0	23. 1	94. 0	65. 7	38. 2	11. 7	86. 1	61. 5	38. 0	6
7	84. 8	54. 2	24. 3	95. 2	66. 9	39. 4	12. 9	87. 3	62. 8	39. 3	7
8	85. 9	55. 3	25. 4	96. 3	68. 1	40. 7	14. 2	88. 6	64. 0	40. 5	8
9	87. 1	56. 5	26. 6	97. 5	69. 3	41. 9	15. 4	89. 8	65. 3	41. 8	9
10	1888. 2	1957. 6	2027. 8	2098. 7	2170. 5	2243. 1	2316. 6	2391. 1	2466. 6	2543. 1	10
11	89. 4	58. 8	29. 0	2099. 9	71. 7	44. 3	17. 9	92. 3	67. 8	44. 4	11
12	90. 5	60. 0	30. 1	2101. 1	72. 9	45. 5	19. 1	93. 6	69. 1	45. 7	12
13	91. 7	61. 1	31. 3	02. 3	74. 1	46. 7	20. 3	94. 8	70. 4	47. 0	13
14	92. 8	62. 3	32. 5	03. 5	75. 3	48. 0	21. 6	96. 1	71. 6	48. 2	14
15	1894. 0	1963. 5	2033. 7	2104. 7	2176. 5	2249. 2	2322. 8	2397. 3	2472. 9	2549. 5	15
16	95. 1	64. 6	34. 8	05. 9	77. 7	50. 4	24. 0	98. 6	74. 2	50. 8	16
17	96. 3	65. 8	36. 0	07. 1	78. 9	51. 6	25. 3	2399. 8	75. 4	52. 1	17
18	97. 4	66. 9	37. 2	08. 2	80. 1	52. 8	26. 5	2401. 1	76. 7	53. 4	18
19	98. 6	68. 1	38. 4	09. 4	81. 3	54. 1	27. 7	02. 4	78. 0	54. 7	19
20	1899. 8	1969. 3	2039. 6	2110. 6	2182. 5	2255. 3	2329. 0	2403. 6	2479. 3	2556. 0	20
21	1900. 9	70. 4	40. 7	11. 8	83. 7	56. 5	30. 2	04. 9	80. 5	57. 3	21
22	02. 1	71. 6	41. 9	13. 0	84. 9	57. 7	31. 4	06. 1	81. 8	58. 5	22
23	03. 2	72. 8	43. 1	14. 2	86. 1	58. 9	32. 7	07. 4	83. 1	59. 8	23
24	04. 4	73. 9	44. 3	15. 4	87. 3	60. 2	33. 9	08. 6	84. 3	61. 1	24
25	1905. 5	1975. 1	2045. 4	2116. 6	2188. 5	2261. 4	2335. 1	2409. 9	2485. 6	2562. 4	25
26	06. 7	76. 3	46. 6	17. 8	89. 8	62. 6	36. 4	11. 1	86. 9	63. 7	26
27	07. 8	77. 4	47. 8	19. 0	91. 0	63. 8	37. 6	12. 4	88. 1	65. 0	27
28	09. 0	78. 6	49. 0	20. 2	92. 2	65. 1	38. 9	13. 6	89. 4	66. 3	28
29	10. 1	79. 8	50. 2	21. 4	93. 4	66. 3	40. 1	14. 9	90. 7	67. 6	29
30	1911. 3	1980. 9	2051. 3	2122. 5	2194. 6	2267. 5	2341. 3	2416. 1	2492. 0	2568. 9	30
31	12. 4	82. 1	52. 5	23. 7	95. 8	68. 7	42. 6	17. 4	93. 2	70. 1	31
32	13. 6	83. 3	53. 7	24. 9	97. 0	69. 9	43. 8	18. 7	94. 5	71. 4	32
33	14. 8	84. 4	54. 9	26. 1	98. 2	71. 2	45. 1	19. 9	95. 8	72. 7	33
34	15. 9	85. 6	56. 1	27. 3	2199. 4	72. 4	46. 3	21. 2	97. 1	74. 0	34
35	1917. 1	1986. 8	2057. 2	2128. 5	2200. 6	2273. 6	2347. 5	2422. 4	2498. 3	2575. 3	35
36	18. 2	87. 9	58. 4	29. 7	01. 8	74. 8	48. 8	23. 7	2499. 6	76. 6	36
37	19. 4	89. 1	59. 6	30. 9	03. 0	76. 1	50. 0	24. 9	2500. 9	77. 9	37
38	20. 5	90. 3	60. 8	32. 1	04. 3	77. 3	51. 3	26. 2	02. 2	79. 2	38
39	21. 7	91. 5	62. 0	33. 3	05. 5	78. 5	52. 5	27. 4	03. 4	80. 5	39
40	1922. 8	1992. 6	2063. 2	2134. 5	2206. 7	2279. 7	2353. 7	2428. 7	2504. 7	2581. 8	40
41	24. 0	93. 8	64. 3	35. 7	07. 9	81. 0	55. 0	30. 0	06. 0	83. 1	41
42	25. 2	95. 0	65. 5	36. 9	09. 1	82. 2	56. 2	31. 2	07. 3	84. 4	42
43	26. 3	96. 1	66. 7	38. 1	10. 3	83. 4	57. 5	32. 5	08. 5	85. 7	43
44	27. 5	97. 3	67. 9	39. 3	11. 5	84. 6	58. 7	33. 7	09. 8	87. 0	44
45	1928. 6	1998. 5	2069. 1	2140. 5	2212. 7	2285. 9	2359. 9	2435. 0	2511. 1	2588. 3	45
46	29. 8	1999. 6	70. 3	41. 7	13. 9	87. 1	61. 2	36. 3	12. 4	89. 5	46
47	30. 9	2000. 8	71. 4	42. 9	15. 2	88. 3	62. 4	37. 5	13. 6	90. 8	47
48	32. 1	02. 0	72. 6	44. 1	16. 4	89. 6	63. 7	38. 8	14. 9	92. 1	48
49	33. 3	03. 2	73. 8	45. 3	17. 6	90. 8	64. 9	40. 0	16. 2	93. 4	49
50	1934. 4	2004. 3	2075. 0	2146. 5	2218. 8	2292. 0	2366. 2	2441. 3	2517. 5	2594. 7	50
51	35. 6	05. 5	76. 2	47. 7	20. 0	93. 2	67. 4	42. 6	18. 8	96. 0	51
52	36. 7	06. 7	77. 4	48. 9	21. 2	94. 5	68. 7	43. 8	20. 0	97. 3	52
53	37. 9	07. 8	78. 5	50. 1	22. 4	95. 7	69. 9	45. 1	21. 3	98. 6	53
54	39. 1	09. 0	79. 7	51. 3	23. 6	96. 9	71. 1	46. 3	22. 6	2599. 9	54
55	1940. 2	2010. 2	2080. 9	2152. 5	2224. 9	2298. 2	2372. 4	2447. 6	2523. 9	2601. 2	55
56	41. 4	11. 4	82. 1	53. 7	26. 1	2299. 4	73. 6	48. 9	25. 1	02. 5	56
57	42. 5	12. 5	83. 3	54. 9	27. 3	2300. 6	74. 9	50. 1	26. 4	03. 8	57
58	43. 7	13. 7	84. 5	56. 1	28. 5	01. 8	76. 1	51. 4	27. 7	05. 1	58
59	44. 9	14. 9	85. 7	57. 3	29. 7	03. 1	77. 4	52. 7	29. 0	06. 4	59
60	1946. 0	2016. 0	2086. 8	2158. 5	2230. 9	2304. 3	2378. 6	2453. 9	2530. 3	2607. 7	60
Lat.	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	Lat.

TABLE 5

Meridional Parts

Lat.	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	Lat.
′											′
0	2607.7	2686.3	2766.1	2847.2	2929.6	3013.5	3098.8	3185.7	3274.2	3364.5	0
1	09.0	87.6	67.5	48.6	31.0	14.9	3100.2	87.1	75.7	66.0	1
2	10.3	89.0	68.8	49.9	32.4	16.3	01.7	88.6	77.2	67.5	2
3	11.6	90.3	70.1	51.3	33.8	17.7	03.1	90.1	78.7	69.1	3
4	12.9	91.6	71.5	52.7	35.2	19.1	04.5	91.5	80.2	70.6	4
5	2614.2	2692.9	2772.8	2854.0	2936.6	3020.5	3106.0	3193.0	3281.7	3372.1	5
6	15.5	94.2	74.2	55.4	38.0	21.9	07.4	94.5	83.2	73.6	6
7	16.8	95.6	75.5	56.8	39.3	23.3	08.8	95.9	84.7	75.1	7
8	18.1	96.9	76.9	58.1	40.7	24.8	10.3	97.4	86.1	76.7	8
9	19.4	98.2	78.2	59.5	42.1	26.2	11.7	3198.8	87.6	78.2	9
10	2620.7	2699.5	2779.5	2860.9	2943.5	3027.6	3113.2	3200.3	3289.1	3379.7	10
11	22.0	2700.9	80.9	62.2	44.9	29.0	14.6	01.8	90.6	81.2	11
12	23.3	02.2	82.2	63.6	46.3	30.4	16.0	03.2	92.1	82.8	12
13	24.6	03.5	83.6	65.0	47.7	31.8	17.5	04.7	93.6	84.3	13
14	26.0	04.8	84.9	66.3	49.1	33.2	18.9	06.2	95.1	85.8	14
15	2627.3	2706.2	2786.3	2867.7	2950.5	3034.7	3120.4	3207.7	3296.6	3387.4	15
16	28.6	07.5	87.6	69.1	51.8	36.1	21.8	09.1	98.1	88.9	16
17	29.9	08.8	89.0	70.4	53.2	37.5	23.2	10.6	3299.6	90.4	17
18	31.2	10.1	90.3	71.8	54.6	38.9	24.7	12.1	3301.1	91.9	18
19	32.5	11.5	91.7	73.2	56.0	40.3	26.1	13.5	02.6	93.5	19
20	2633.8	2712.8	2793.0	2874.5	2957.4	3041.7	3127.6	3215.0	3304.1	3395.0	20
21	35.1	14.1	94.4	75.9	58.8	43.2	29.0	16.5	05.6	96.5	21
22	36.4	15.4	95.7	77.3	60.2	44.6	30.5	17.9	07.1	98.1	22
23	37.7	16.8	97.1	78.6	61.6	46.0	31.9	19.4	08.6	3399.6	23
24	39.0	18.1	98.4	80.0	63.0	47.4	33.4	20.9	10.1	3401.1	24
25	2640.3	2719.4	2799.8	2881.4	2964.4	3048.8	3134.8	3222.4	3311.6	3402.7	25
26	41.6	20.7	2801.1	82.8	65.8	50.3	36.2	23.8	13.1	04.2	26
27	42.9	22.1	02.5	84.1	67.2	51.7	37.7	25.3	14.6	05.7	27
28	44.3	23.4	03.8	85.5	68.6	53.1	39.1	26.8	16.1	07.3	28
29	45.6	24.7	05.2	86.9	70.0	54.5	40.6	28.3	17.6	08.8	29
30	2646.9	2726.1	2806.5	2888.2	2971.4	3055.9	3142.0	3229.7	3319.1	3410.3	30
31	48.2	27.4	07.9	89.6	72.8	57.4	43.5	31.2	20.6	11.9	31
32	49.5	28.7	09.2	91.0	74.2	58.8	44.9	32.7	22.1	13.4	32
33	50.8	30.1	10.6	92.4	75.6	60.2	46.4	34.2	23.6	14.9	33
34	52.1	31.4	11.9	93.7	77.0	61.6	47.8	35.6	25.2	16.5	34
35	2653.4	2732.7	2813.3	2895.1	2978.4	3063.1	3149.3	3237.1	3326.7	3418.0	35
36	54.7	34.1	14.6	96.5	79.8	64.5	50.7	38.6	28.2	19.5	36
37	56.0	35.4	16.0	97.9	81.2	65.9	52.2	40.1	29.7	21.1	37
38	57.4	36.7	17.3	2899.3	82.6	67.3	53.6	41.6	31.2	22.6	38
39	58.7	38.1	18.7	2900.6	84.0	68.8	55.1	43.0	32.7	24.2	39
40	2660.0	2739.4	2820.0	2902.0	2985.4	3070.2	3156.5	3244.5	3334.2	3425.7	40
41	61.3	40.7	21.4	03.4	86.8	71.6	58.0	46.0	35.7	27.2	41
42	62.6	42.1	22.7	04.8	88.2	73.0	59.4	47.5	37.2	28.8	42
43	63.9	43.4	24.1	06.1	89.6	74.5	60.9	49.0	38.7	30.3	43
44	65.2	44.7	25.5	07.5	91.0	75.9	62.3	50.4	40.2	31.9	44
45	2666.6	2746.1	2826.8	2908.9	2992.4	3077.3	3163.8	3251.9	3341.8	3433.4	45
46	67.9	47.4	28.2	10.3	93.8	78.7	65.3	53.4	43.3	35.0	46
47	69.2	48.7	29.5	11.7	95.2	80.2	66.7	54.9	44.8	36.5	47
48	70.5	50.1	30.9	13.0	96.6	81.6	68.2	56.4	46.3	38.0	48
49	71.8	51.4	32.2	14.4	98.0	83.0	69.6	57.9	47.8	39.6	49
50	2673.1	2752.7	2833.6	2915.8	2999.4	3084.5	3171.1	3259.3	3349.3	3441.1	50
51	74.5	54.1	35.0	17.2	3000.8	85.9	72.5	60.8	50.8	42.7	51
52	75.8	55.4	36.3	18.6	02.2	87.3	74.0	62.3	52.4	44.2	52
53	77.1	56.8	37.7	19.9	03.6	88.8	75.5	63.8	53.9	45.8	53
54	78.4	58.1	39.0	21.3	05.0	90.2	76.9	65.3	55.4	47.3	54
55	2679.7	2759.4	2840.4	2922.7	3006.4	3091.6	3178.4	3266.8	3356.9	3448.9	55
56	81.0	60.8	41.8	24.1	07.8	93.1	79.8	68.3	58.4	50.4	56
57	82.4	62.1	43.1	25.5	09.2	94.5	81.3	69.7	59.9	52.0	57
58	83.7	63.4	44.5	26.9	10.6	95.9	82.8	71.2	61.5	53.5	58
59	85.0	64.8	45.8	28.2	12.1	97.4	84.2	72.7	63.0	55.1	59
60	2686.3	2766.1	2847.2	2929.6	3013.5	3098.8	3185.7	3274.2	3364.5	3456.6	60
Lat.	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	Lat.

TABLE 5

Meridional Parts

Lat.	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°	Lat.
′											′
0	3456. 6	3550. 7	3646. 8	3745. 2	3845. 8	3948. 9	4054. 6	4163. 1	4274. 5	4389. 2	0
1	58. 2	52. 3	48. 5	46. 8	47. 5	50. 6	56. 4	64. 9	76. 4	91. 1	1
2	59. 7	53. 9	50. 1	48. 5	49. 2	52. 4	58. 2	66. 7	78. 3	93. 0	2
3	61. 3	55. 5	51. 7	50. 1	50. 9	54. 1	59. 9	68. 6	80. 2	95. 0	3
4	62. 8	57. 0	53. 3	51. 8	52. 6	55. 8	61. 7	70. 4	82. 1	96. 9	4
5	3464. 4	3558. 6	3654. 9	3753. 4	3854. 3	3957. 6	4063. 5	4172. 2	4284. 0	4398. 9	5
6	65. 9	60. 2	56. 6	55. 1	56. 0	59. 3	65. 3	74. 1	85. 8	4400. 8	6
7	67. 5	61. 8	58. 2	56. 8	57. 7	61. 1	67. 1	75. 9	87. 7	02. 8	7
8	69. 1	63. 4	59. 8	58. 4	59. 4	62. 8	68. 9	77. 8	89. 6	04. 7	8
9	70. 6	65. 0	61. 4	60. 1	61. 1	64. 6	70. 7	79. 6	91. 5	06. 6	9
10	3472. 2	3566. 6	3663. 1	3761. 8	3862. 8	3966. 3	4072. 5	4181. 4	4293. 4	4408. 6	10
11	73. 7	68. 2	64. 7	63. 4	64. 5	68. 1	74. 3	83. 3	95. 3	10. 5	11
12	75. 3	69. 8	66. 3	65. 1	66. 2	69. 8	76. 1	85. 1	97. 2	12. 5	12
13	76. 8	71. 3	67. 9	66. 8	67. 9	71. 6	77. 8	87. 0	4299. 1	14. 4	13
14	78. 4	72. 9	69. 6	68. 4	69. 6	73. 3	79. 6	88. 8	4301. 0	16. 4	14
15	3480. 0	3574. 5	3671. 2	3770. 1	3871. 3	3975. 1	4081. 4	4190. 7	4302. 9	4418. 3	15
16	81. 5	76. 1	72. 8	71. 8	73. 0	76. 8	83. 2	92. 5	04. 8	20. 3	16
17	83. 1	77. 7	74. 5	73. 4	74. 7	78. 6	85. 0	94. 3	06. 7	22. 2	17
18	84. 6	79. 3	76. 1	75. 1	76. 4	80. 3	86. 8	96. 2	08. 6	24. 2	18
19	86. 2	80. 9	77. 7	76. 8	78. 2	82. 1	88. 6	98. 0	10. 5	26. 2	19
20	3487. 8	3582. 5	3679. 4	3778. 4	3879. 9	3983. 8	4090. 4	4199. 9	4312. 4	4428. 1	20
21	89. 3	84. 1	81. 0	80. 1	81. 6	85. 6	92. 2	4201. 7	14. 3	30. 1	21
22	90. 9	85. 7	82. 6	81. 8	83. 3	87. 3	94. 0	03. 6	16. 2	32. 0	22
23	92. 4	87. 3	84. 3	83. 4	85. 0	89. 1	95. 8	05. 4	18. 1	34. 0	23
24	94. 0	88. 9	85. 9	85. 1	86. 7	90. 8	97. 6	07. 3	20. 0	35. 9	24
25	3495. 6	3590. 5	3687. 5	3786. 8	3888. 4	3992. 6	4099. 4	4209. 1	4321. 9	4437. 9	25
26	97. 1	92. 1	89. 2	88. 5	90. 1	94. 4	4101. 2	11. 0	23. 8	39. 9	26
27	3498. 7	93. 7	90. 8	90. 1	91. 9	96. 1	03. 0	12. 9	25. 7	41. 8	27
28	3500. 3	95. 3	92. 4	91. 8	93. 6	97. 9	04. 9	14. 7	27. 6	43. 8	28
29	01. 8	96. 9	94. 1	93. 5	95. 3	3999. 6	06. 7	16. 6	29. 5	45. 8	29
30	3503. 4	3598. 5	3695. 7	3795. 2	3897. 0	4001. 4	4108. 5	4218. 4	4331. 4	4447. 7	30
31	05. 0	3600. 1	97. 3	96. 8	3898. 7	03. 2	10. 3	20. 3	33. 3	49. 7	31
32	06. 5	01. 7	3699. 0	3798. 5	3900. 5	04. 9	12. 1	22. 1	35. 3	51. 7	32
33	08. 1	03. 3	3700. 6	3800. 2	02. 2	06. 7	13. 9	24. 0	37. 2	53. 6	33
34	09. 7	04. 9	02. 3	01. 9	03. 9	08. 4	15. 7	25. 9	39. 1	55. 6	34
35	3511. 3	3606. 5	3703. 9	3803. 6	3905. 6	4010. 2	4117. 5	4227. 7	4341. 0	4457. 6	35
36	12. 8	08. 1	05. 6	05. 2	07. 3	12. 0	19. 3	29. 6	42. 9	59. 6	36
37	14. 4	09. 7	07. 2	06. 9	09. 1	13. 7	21. 1	31. 4	44. 8	61. 5	37
38	16. 0	11. 3	08. 8	08. 6	10. 8	15. 5	23. 0	33. 3	46. 7	63. 5	38
39	17. 5	12. 9	10. 5	10. 3	12. 5	17. 3	24. 8	35. 2	48. 7	65. 5	39
40	3519. 1	3614. 6	3712. 1	3812. 0	3914. 2	4019. 0	4126. 6	4237. 0	4350. 6	4467. 5	40
41	20. 7	16. 2	13. 8	13. 7	16. 0	20. 8	28. 4	38. 9	52. 5	69. 4	41
42	22. 3	17. 8	15. 4	15. 3	17. 7	22. 6	30. 2	40. 8	54. 4	71. 4	42
43	23. 8	19. 4	17. 1	17. 0	19. 4	24. 4	32. 0	42. 6	56. 3	73. 4	43
44	25. 4	21. 0	18. 7	18. 7	21. 1	26. 1	33. 9	44. 5	58. 3	75. 4	44
45	3527. 0	3622. 6	3720. 4	3820. 4	3922. 9	4027. 9	4135. 7	4246. 4	4360. 2	4477. 3	45
46	28. 6	24. 2	22. 0	22. 1	24. 6	29. 7	37. 5	48. 2	62. 1	79. 3	46
47	30. 1	25. 8	23. 7	23. 8	26. 3	31. 5	39. 3	50. 1	64. 0	81. 3	47
48	31. 7	27. 4	25. 3	25. 5	28. 1	33. 2	41. 1	52. 0	66. 0	83. 3	48
49	33. 3	29. 0	27. 0	27. 2	29. 8	35. 0	43. 0	53. 9	67. 9	85. 3	49
50	3534. 9	3630. 7	3728. 6	3828. 8	3931. 5	4036. 8	4144. 8	4255. 7	4369. 8	4487. 3	50
51	36. 5	32. 3	30. 3	30. 5	33. 2	38. 6	46. 6	57. 6	71. 8	89. 3	51
52	38. 0	33. 9	31. 9	32. 2	35. 0	40. 3	48. 4	59. 5	73. 7	91. 2	52
53	39. 6	35. 5	33. 6	33. 9	36. 7	42. 1	50. 3	61. 4	75. 6	93. 2	53
54	41. 2	37. 1	35. 2	35. 6	38. 5	43. 9	52. 1	63. 2	77. 5	95. 2	54
55	3542. 8	3638. 7	3736. 9	3837. 3	3940. 2	4045. 7	4153. 9	4265. 1	4379. 5	4497. 2	55
56	44. 4	40. 4	38. 5	39. 0	41. 9	47. 5	55. 8	67. 0	81. 4	4499. 2	56
57	45. 9	42. 0	40. 2	40. 7	43. 7	49. 2	57. 6	68. 9	83. 4	4501. 2	57
58	47. 5	43. 6	41. 8	42. 4	45. 4	51. 0	59. 4	70. 8	85. 3	03. 2	58
59	49. 1	45. 2	43. 5	44. 1	47. 1	52. 8	61. 2	72. 6	87. 2	05. 2	59
60	3550. 7	3646. 8	3745. 2	3845. 8	3948. 9	4054. 6	4163. 1	4274. 5	4389. 2	4507. 2	60
Lat.	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°	Lat.

TABLE 5

Meridional Parts

Lat.	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	Lat.
′											′
0	4507. 2	4628. 8	4754. 4	4884. 2	5018. 5	5157. 7	5302. 2	5452. 5	5609. 2	5772. 8	0
1	09. 2	30. 9	56. 5	86. 4	20. 7	60. 0	04. 7	55. 1	11. 9	75. 6	1
2	11. 2	33. 0	58. 7	88. 6	23. 0	62. 4	07. 1	57. 7	14. 5	78. 4	2
3	13. 2	35. 0	60. 8	90. 8	25. 3	64. 8	09. 6	60. 2	17. 2	81. 2	3
4	15. 2	37. 1	62. 9	93. 0	27. 6	67. 1	12. 1	62. 8	19. 9	84. 0	4
5	4517. 2	4639. 1	4765. 0	4895. 2	5029. 9	5169. 5	5314. 5	5465. 4	5622. 6	5786. 8	5
6	19. 2	41. 2	67. 2	97. 4	32. 2	71. 9	17. 0	67. 9	25. 2	89. 6	6
7	21. 2	43. 3	69. 3	4899. 6	34. 5	74. 3	19. 4	70. 5	27. 9	92. 4	7
8	23. 2	45. 3	71. 5	4901. 8	36. 7	76. 6	21. 9	73. 1	30. 6	95. 2	8
9	25. 2	47. 4	73. 6	04. 0	39. 0	79. 0	24. 4	75. 6	33. 3	5798. 0	9
10	4527. 2	4649. 5	4775. 7	4906. 2	5041. 3	5181. 4	5326. 9	5478. 2	5636. 0	5800. 8	10
11	29. 2	51. 6	77. 9	08. 4	43. 6	83. 8	29. 3	80. 8	38. 7	03. 6	11
12	31. 2	53. 6	80. 0	10. 7	45. 9	86. 1	31. 8	83. 4	41. 3	06. 4	12
13	33. 2	55. 7	82. 1	12. 9	48. 2	88. 5	34. 3	85. 9	44. 0	09. 2	13
14	35. 2	57. 8	84. 3	15. 1	50. 5	90. 9	36. 8	88. 5	46. 7	12. 0	14
15	4537. 2	4659. 9	4786. 4	4917. 3	5052. 8	5193. 3	5339. 2	5491. 1	5649. 4	5814. 9	15
16	39. 3	61. 9	88. 6	19. 5	55. 1	95. 7	41. 7	93. 7	52. 1	17. 7	16
17	41. 3	64. 0	90. 7	21. 7	57. 4	5198. 1	44. 2	96. 3	54. 8	20. 5	17
18	43. 3	66. 1	92. 9	24. 0	59. 7	5200. 5	46. 7	5498. 9	57. 5	23. 3	18
19	45. 3	68. 2	95. 0	26. 2	62. 0	02. 9	49. 2	5501. 4	60. 2	26. 1	19
20	4547. 3	4670. 2	4797. 2	4928. 4	5064. 3	5205. 2	5351. 7	5504. 0	5662. 9	5829. 0	20
21	49. 3	72. 3	4799. 3	30. 6	66. 6	07. 6	54. 1	06. 6	65. 6	31. 8	21
22	51. 4	74. 4	4801. 5	32. 9	68. 9	10. 0	56. 6	09. 2	68. 3	34. 6	22
23	53. 4	76. 5	03. 6	35. 1	71. 2	12. 4	59. 1	11. 8	71. 1	37. 5	23
24	55. 4	78. 6	05. 8	37. 3	73. 5	14. 8	61. 6	14. 4	73. 8	40. 3	24
25	4557. 4	4680. 7	4807. 9	4939. 6	5075. 9	5217. 2	5364. 1	5517. 0	5676. 5	5843. 2	25
26	59. 4	82. 8	10. 1	41. 8	78. 2	19. 6	66. 6	19. 6	79. 2	46. 0	26
27	61. 5	84. 8	12. 3	44. 0	80. 5	22. 0	69. 1	22. 2	81. 9	48. 8	27
28	63. 5	86. 9	14. 4	46. 3	82. 8	24. 4	71. 6	24. 8	84. 6	51. 7	28
29	65. 5	89. 0	16. 6	48. 5	85. 1	26. 8	74. 1	27. 4	87. 4	54. 5	29
30	4567. 5	4691. 1	4818. 7	4950. 7	5087. 4	5229. 3	5376. 6	5530. 0	5690. 1	5857. 4	30
31	69. 6	93. 2	20. 9	53. 0	89. 8	31. 7	79. 1	32. 7	92. 8	60. 3	31
32	71. 6	95. 3	23. 1	55. 2	92. 1	34. 1	81. 6	35. 3	95. 5	63. 1	32
33	73. 6	97. 4	25. 2	57. 4	94. 4	36. 5	84. 1	37. 9	5698. 3	66. 0	33
34	75. 7	4699. 5	27. 4	59. 7	96. 7	38. 9	86. 7	40. 5	5701. 0	68. 8	34
35	4577. 7	4701. 6	4829. 6	4961. 9	5099. 1	5241. 3	5389. 2	5543. 1	5703. 7	5871. 7	35
36	79. 7	03. 7	31. 7	64. 2	5101. 4	43. 7	91. 7	45. 7	06. 5	74. 6	36
37	81. 8	05. 8	33. 9	66. 4	03. 7	46. 2	94. 2	48. 4	09. 2	77. 4	37
38	83. 8	07. 9	36. 1	68. 7	06. 0	48. 6	96. 7	51. 0	12. 0	80. 3	38
39	85. 8	10. 0	38. 3	70. 9	08. 4	51. 0	5399. 2	53. 6	14. 7	83. 2	39
40	4587. 9	4712. 1	4840. 4	4973. 2	5110. 7	5253. 4	5401. 8	5556. 2	5717. 5	5886. 0	40
41	89. 9	14. 2	42. 6	75. 4	13. 0	55. 8	04. 3	58. 9	20. 2	88. 9	41
42	91. 9	16. 3	44. 8	77. 7	15. 4	58. 3	06. 8	61. 5	22. 9	91. 8	42
43	94. 0	18. 4	47. 0	79. 9	17. 7	60. 7	09. 3	64. 1	25. 7	94. 7	43
44	96. 0	20. 5	49. 1	82. 2	20. 1	63. 1	11. 9	66. 8	28. 5	5897. 6	44
45	4598. 1	4722. 6	4851. 3	4984. 4	5122. 4	5265. 6	5414. 4	5569. 4	5731. 2	5900. 4	45
46	4600. 1	24. 7	53. 5	86. 7	24. 7	68. 0	16. 9	72. 1	34. 0	03. 3	46
47	02. 2	26. 8	55. 7	89. 0	27. 1	70. 4	19. 5	74. 7	36. 7	06. 2	47
48	04. 2	29. 0	57. 9	91. 2	29. 4	72. 9	22. 0	77. 3	39. 5	09. 1	48
49	06. 3	31. 1	60. 0	93. 5	31. 8	75. 3	24. 5	80. 0	42. 3	12. 0	49
50	4608. 3	4733. 2	4862. 2	4995. 8	5134. 1	5277. 7	5427. 1	5582. 6	5745. 0	5914. 9	50
51	10. 3	35. 3	64. 4	4998. 0	36. 5	80. 2	29. 6	85. 3	47. 8	17. 8	51
52	12. 4	37. 4	66. 6	5000. 3	38. 8	82. 6	32. 1	87. 9	50. 6	20. 7	52
53	14. 4	39. 5	68. 8	02. 6	41. 2	85. 1	34. 7	90. 6	53. 3	23. 6	53
54	16. 5	41. 7	71. 0	04. 8	43. 5	87. 5	37. 2	93. 2	56. 1	26. 5	54
55	4618. 6	4743. 8	4873. 2	5007. 1	5145. 9	5290. 0	5439. 8	5595. 9	5758. 9	5929. 4	55
56	20. 6	45. 9	75. 4	09. 4	48. 2	92. 4	42. 3	5598. 5	61. 7	32. 3	56
57	22. 7	48. 0	77. 6	11. 6	50. 6	94. 9	44. 9	5601. 2	64. 4	35. 3	57
58	24. 7	50. 1	79. 8	13. 9	53. 0	97. 3	47. 4	03. 9	67. 2	38. 2	58
59	26. 8	52. 3	82. 0	16. 2	55. 3	5299. 8	50. 0	06. 5	70. 0	41. 1	59
60	4628. 8	4754. 4	4884. 2	5018. 5	5157. 7	5302. 2	5452. 5	5609. 2	5772. 8	5944. 0	60
Lat.	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	Lat.

TABLE 5

Meridional Parts

Lat.	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	Lat.
′											′
0	5944.0	6123.7	6312.7	6512.1	6723.3	6947.8	7187.4	7444.5	7721.8	8022.8	0
1	46.9	26.7	15.9	15.5	27.0	51.7	91.6	48.9	26.6	28.1	1
2	49.9	29.8	19.1	19.0	30.6	55.5	95.7	53.4	31.4	33.3	2
3	52.8	32.9	22.4	22.4	34.2	59.4	7199.9	57.8	36.2	38.6	3
4	55.7	36.0	25.6	25.8	37.9	63.3	7204.0	62.3	41.0	43.8	4
5	5958.6	6139.0	6328.9	6529.3	6741.5	6967.2	7208.2	7466.8	7745.9	8049.1	5
6	61.6	42.1	32.1	32.7	45.2	71.1	12.3	71.2	50.7	54.4	6
7	64.5	45.2	35.4	36.1	48.8	75.0	16.5	75.7	55.6	59.7	7
8	67.5	48.3	38.6	39.6	52.5	78.8	20.7	80.2	60.4	65.0	8
9	70.4	51.4	41.9	43.0	56.1	82.7	24.8	84.7	65.3	70.3	9
10	5973.3	6154.5	6345.1	6546.5	6759.8	6986.6	7229.0	7489.2	7770.2	8075.6	10
11	76.3	57.6	48.4	49.9	63.4	90.6	33.2	93.7	75.0	80.9	11
12	79.2	60.7	51.7	53.4	67.1	94.5	37.4	7498.2	79.9	86.3	12
13	82.2	63.8	55.0	56.8	70.8	6998.4	41.6	7502.7	84.8	91.6	13
14	85.1	66.9	58.2	60.3	74.5	7002.3	45.8	07.3	89.7	8096.9	14
15	5988.1	6170.0	6361.5	6563.8	6778.1	7006.2	7250.0	7511.8	7794.6	8102.3	15
16	91.1	73.1	64.8	67.2	81.8	10.2	54.2	16.3	7799.5	07.7	16
17	94.0	76.2	68.1	70.7	85.5	14.1	58.4	20.9	7804.5	13.0	17
18	97.0	79.3	71.4	74.2	89.2	18.0	62.6	25.4	09.4	18.4	18
19	5999.9	82.4	74.6	77.7	92.9	22.0	66.8	29.9	14.3	23.8	19
20	6002.9	6185.6	6377.9	6581.2	6796.6	7025.9	7271.1	7534.5	7819.3	8129.2	20
21	05.9	88.7	81.2	84.6	6800.3	29.9	75.3	39.1	24.2	34.6	21
22	08.9	91.8	84.5	88.1	04.0	33.8	79.5	43.6	29.2	40.0	22
23	11.8	94.9	87.8	91.6	07.7	37.8	83.8	48.2	34.1	45.4	23
24	14.8	6198.1	91.1	95.1	11.4	41.7	88.0	52.8	39.1	50.9	24
25	6017.8	6201.2	6394.4	6598.6	6815.2	7045.7	7292.3	7557.4	7844.1	8156.3	25
26	20.8	04.3	6397.7	6602.1	18.9	49.7	7296.5	62.0	49.1	61.8	26
27	23.8	07.5	6401.1	05.6	22.6	53.6	7300.8	66.6	54.0	67.2	27
28	26.7	10.6	04.4	09.1	26.3	57.6	05.1	71.2	59.0	72.7	28
29	29.7	13.8	07.7	12.7	30.1	61.6	09.3	75.8	64.0	78.2	29
30	6032.7	6216.9	6411.0	6616.2	6833.8	7065.6	7313.6	7580.4	7869.1	8183.6	30
31	35.7	20.1	14.3	19.7	37.5	69.6	17.9	85.0	74.1	89.1	31
32	38.7	23.2	17.7	23.2	41.3	73.6	22.2	89.6	79.1	8194.6	32
33	41.7	26.4	21.0	26.7	45.0	77.6	26.5	94.3	84.1	8200.1	33
34	44.7	29.5	24.3	30.3	48.8	81.6	30.8	7598.9	89.2	05.7	34
35	6047.7	6232.7	6427.7	6633.8	6852.6	7085.6	7335.1	7603.6	7894.2	8211.2	35
36	50.7	35.9	31.0	37.3	56.3	89.6	39.4	08.2	7899.3	16.7	36
37	53.7	39.0	34.4	40.9	60.1	93.7	43.7	12.9	7904.3	22.3	37
38	56.8	42.2	37.7	44.4	63.9	7097.7	48.0	17.5	09.4	27.8	38
39	59.8	45.4	41.0	48.0	67.6	7101.7	52.4	22.2	14.5	33.4	39
40	6062.8	6248.6	6444.4	6651.5	6871.4	7105.8	7356.7	7626.9	7919.6	8238.9	40
41	65.8	51.7	47.8	55.1	75.2	09.8	61.0	31.6	24.7	44.5	41
42	68.8	54.9	51.1	58.6	79.0	13.8	65.4	36.3	29.7	50.1	42
43	71.8	58.1	54.5	62.2	82.8	17.9	69.7	41.0	34.9	55.7	43
44	74.9	61.3	57.8	65.8	86.6	21.9	74.1	45.7	40.0	61.3	44
45	6077.9	6264.5	6461.2	6669.3	6890.4	7126.0	7378.4	7650.4	7945.1	8266.9	45
46	80.9	67.7	64.6	72.9	94.2	30.1	82.8	55.1	50.2	72.5	46
47	84.0	70.9	68.0	76.5	6898.0	34.1	87.2	59.8	55.4	78.2	47
48	87.0	74.1	71.3	80.1	6901.8	38.2	91.6	64.5	60.5	83.8	48
49	90.0	77.3	74.7	83.7	05.6	42.3	7395.9	69.3	65.6	89.5	49
50	6093.1	6280.5	6478.1	6687.3	6909.4	7146.4	7400.3	7674.0	7970.8	8295.1	50
51	96.1	83.7	81.5	90.8	13.2	50.4	04.7	78.7	76.0	8300.8	51
52	6099.2	86.9	84.9	94.4	17.1	54.5	09.1	83.5	81.1	06.5	52
53	6102.2	90.1	88.3	6698.0	20.9	58.6	13.5	88.3	86.3	12.2	53
54	05.3	93.3	91.7	6701.6	24.7	62.7	17.9	93.0	91.5	17.9	54
55	6108.3	6296.5	6495.1	6705.2	6928.6	7166.8	7422.3	7697.8	7996.7	8323.6	55
56	11.4	6299.8	6498.5	08.9	32.4	71.0	26.8	7702.6	8001.9	29.3	56
57	14.5	6303.0	6501.9	12.5	36.2	75.1	31.2	07.4	07.1	35.0	57
58	17.5	06.2	05.3	16.1	40.1	79.2	35.6	12.2	12.4	40.7	58
59	20.6	09.4	08.7	19.7	44.0	83.3	40.0	17.0	17.6	46.5	59
60	6123.7	6312.7	6512.1	6723.3	6947.8	7187.4	7444.5	7721.8	8022.8	8352.2	60
Lat.	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	Lat.

TABLE 5

Meridional Parts

Lat.	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	Lat.
′											′
0	8352.2	8716.0	9122.4	9582.7	10113.7	10741.4	11509.3	12498.8	13893.1	16276.2	0
1	58.0	22.4	29.6	90.9	123.3	752.9	523.6	518.0	921.9	334.0	1
2	63.8	28.8	36.8	9599.1	132.9	764.4	538.0	537.2	950.9	392.8	2
3	69.5	35.3	44.0	9607.4	142.5	776.0	552.5	556.6	13980.2	452.6	3
4	75.3	41.7	51.2	15.6	152.2	787.6	567.1	576.1	14009.7	513.4	4
5	8381.1	8748.1	9158.5	9623.9	10161.9	10799.2	11581.7	12595.7	14039.5	16575.4	5
6	86.9	54.6	65.7	32.3	171.6	810.9	596.4	615.4	069.5	638.5	6
7	92.8	61.1	73.0	40.6	181.3	822.6	611.1	635.2	099.8	702.7	7
8	8398.6	67.5	80.3	48.9	191.1	834.4	625.9	655.2	130.3	768.2	8
9	8404.4	74.0	87.6	57.3	200.9	846.2	640.7	675.2	161.2	835.0	9
10	8410.3	8780.5	9195.0	9665.7	10210.7	10858.1	11655.7	12695.4	14192.3	16903.0	10
11	16.1	87.1	9202.3	74.1	220.6	870.0	670.7	715.7	223.7	16972.5	11
12	22.0	8793.6	09.7	82.6	230.4	881.9	685.7	736.1	255.4	17043.4	12
13	27.9	8800.1	17.0	91.0	240.3	893.9	700.8	756.6	287.4	115.8	13
14	33.8	06.7	24.4	9699.5	250.3	905.9	716.0	777.3	319.7	189.7	14
15	8439.7	8813.2	9231.8	9708.0	10260.2	10917.9	11731.3	12798.1	14352.2	17265.3	15
16	45.6	19.8	39.3	16.5	270.2	930.0	746.6	819.0	385.2	342.5	16
17	51.5	26.4	46.7	25.0	280.3	942.2	762.0	840.0	418.4	421.5	17
18	57.4	33.0	54.2	33.6	290.3	954.3	777.5	861.2	451.9	502.4	18
19	63.4	39.6	61.6	42.2	300.4	966.6	793.0	882.5	485.8	585.3	19
20	8469.3	8846.3	9269.1	9750.8	10310.5	10978.8	11808.6	12903.9	14520.0	17670.2	20
21	75.3	52.9	76.6	59.4	320.7	10991.2	824.3	925.4	554.6	757.2	21
22	81.3	59.6	84.1	68.0	330.8	11003.5	840.0	947.1	589.5	846.5	22
23	87.2	66.2	91.7	76.7	341.0	015.9	855.8	969.0	624.7	17938.2	23
24	93.2	72.9	9299.2	85.4	351.3	028.4	871.7	12990.9	660.4	18032.4	24
25	8499.2	8879.6	9306.8	9794.1	10361.5	11040.8	11887.7	13013.1	14696.4	18129.2	25
26	8505.2	86.3	14.4	9802.8	371.8	053.4	903.7	035.3	732.7	228.9	26
27	11.3	93.0	22.0	11.6	382.1	066.0	919.8	057.7	769.5	331.5	27
28	17.3	8899.8	29.6	20.4	392.5	078.6	936.0	080.3	806.7	437.3	28
29	23.3	8906.5	37.2	29.2	402.9	091.3	952.3	103.0	844.3	546.4	29
30	8529.4	8913.3	9344.9	9838.0	10413.3	11104.0	11968.6	13125.8	14882.2	18659.2	30
31	35.4	20.0	52.6	46.8	423.7	116.8	11985.0	148.8	920.7	775.7	31
32	41.5	26.8	60.2	55.7	434.2	129.6	12001.5	172.0	959.5	18896.3	32
33	47.6	33.6	67.9	64.6	444.7	142.4	018.1	195.3	14998.8	19021.4	33
34	53.7	40.4	75.7	73.5	455.3	155.3	034.8	218.8	15038.6	151.1	34
35	8559.8	8947.2	9383.4	9882.4	10465.9	11168.3	12051.5	13242.4	15078.8	19285.9	35
36	65.9	54.1	91.2	9891.4	476.5	181.3	068.3	266.2	119.5	426.3	36
37	72.0	60.9	9398.9	9900.4	487.1	194.4	085.2	290.2	160.6	572.6	37
38	78.2	67.8	9406.7	09.4	497.8	207.5	102.2	314.3	202.3	725.4	38
39	84.3	74.7	14.5	18.4	508.5	220.6	119.3	338.6	244.5	19885.3	39
40	8590.5	8981.6	9422.3	9927.5	10519.2	11233.9	12136.4	13363.1	15287.2	20053.1	40
41	8596.7	88.5	30.2	36.6	530.0	247.1	153.7	387.7	330.4	229.4	41
42	8602.8	8995.4	38.0	45.7	540.8	260.4	171.0	412.5	374.2	415.3	42
43	09.0	9002.3	45.9	54.8	551.7	273.8	188.4	437.6	418.6	611.8	43
44	15.2	09.3	53.8	63.9	562.5	287.2	205.9	462.7	463.5	20820.2	44
45	8621.5	9016.2	9461.7	9973.1	10573.4	11300.7	12223.5	13488.1	15509.1	21042.0	45
46	27.7	23.2	69.7	82.3	584.4	314.2	241.2	513.7	555.2	279.2	46
47	33.9	30.2	77.6	9991.5	595.4	327.8	259.0	539.4	602.0	534.0	47
48	40.2	37.2	85.6	10000.8	606.4	341.4	276.8	565.4	649.4	21809.2	48
49	46.4	44.2	9493.6	010.0	617.4	355.1	294.8	591.5	697.5	22108.3	49
50	8652.7	9051.3	9501.6	10019.3	10628.5	11368.8	12312.9	13617.9	15746.3	22435.9	50
51	59.0	58.3	09.6	028.7	639.6	382.6	331.0	644.5	795.7	22798.1	51
52	65.3	65.4	17.6	038.0	650.8	396.4	349.3	671.2	845.9	23203.1	52
53	71.6	72.4	25.7	047.4	662.0	410.3	367.6	698.2	896.9	23662.1	53
54	77.9	79.5	33.8	056.8	673.2	424.3	386.0	725.4	15948.6	24192.0	54
55	8684.2	9086.6	9541.9	10066.2	10684.5	11438.3	12404.6	13752.8	16001.1	24818.8	55
56	90.6	9093.7	50.0	075.6	695.8	452.4	423.2	780.4	054.4	25585.9	56
57	8696.9	9100.9	58.1	085.1	707.1	466.5	442.0	808.2	108.5	26574.9	57
58	8703.3	08.0	66.3	094.6	718.5	480.7	460.8	836.3	163.5	27968.9	58
59	09.6	15.2	74.5	104.1	729.9	494.9	479.8	864.6	219.4	30351.9	59
60	8716.0	9122.4	9582.7	10113.7	10741.4	11509.3	12498.8	13893.1	16276.2	—	60
Lat.	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	Lat.

TABLE 6

Length of a Degree of Latitude and Longitude

Lat.	Degree of latitude				Degree of longitude				Lat.
	Nautical miles	Statute miles	Feet	Meters	Nautical miles	Statute miles	Feet	Meters	
°									°
0	59. 701	68. 703	362 753	110 567	60. 109	69. 172	365 226	111 321	0
1	. 702	. 704	756	568	60. 099	69. 161	365 171	111 304	1
2	. 702	. 704	759	569	60. 072	69. 129	365 003	111 253	2
3	. 703	. 705	762	570	60. 026	69. 077	364 728	111 169	3
4	. 705	. 707	772	573	59. 963	69. 004	364 341	111 051	4
5	59. 706	68. 709	362 782	110 576	59. 881	68. 910	363 845	110 900	5
6	. 708	. 711	795	580	59. 781	68. 795	363 238	110 715	6
7	. 711	. 714	808	584	59. 664	68. 660	362 523	110 497	7
8	. 713	. 717	825	589	59. 528	68. 503	361 696	110 245	8
9	. 717	. 721	844	595	59. 373	68. 325	360 758	109 959	9
10	59. 720	68. 724	362 864	110 601	59. 201	68. 128	359 715	109 641	10
11	. 724	. 729	887	608	59. 011	67. 909	358 560	109 289	11
12	. 728	. 734	913	616	58. 803	67. 670	357 297	108 904	12
13	. 732	. 739	940	624	58. 578	67. 410	355 925	108 486	13
14	. 737	. 744	969	633	58. 335	67. 130	354 449	108 036	14
15	59. 742	68. 750	363 002	110 643	58. 074	66. 830	352 864	107 553	15
16	. 748	. 757	035	653	57. 795	66. 509	351 168	107 036	16
17	. 753	. 763	068	663	57. 498	66. 168	349 367	106 487	17
18	. 760	. 770	107	675	57. 185	65. 807	347 461	105 906	18
19	. 766	. 777	143	686	56. 854	65. 427	345 453	105 294	19
20	59. 773	68. 785	363 186	110 699	56. 506	65. 026	343 337	104 649	20
21	. 780	. 793	228	712	56. 140	64. 605	341 115	103 972	21
22	. 787	. 801	271	725	55. 758	64. 165	338 793	103 264	22
23	. 794	. 810	317	739	55. 359	63. 705	336 365	102 524	23
24	. 802	. 819	363	753	54. 943	63. 227	333 839	101 754	24
25	59. 810	68. 828	363 412	110 768	54. 510	62. 729	331 207	100 952	25
26	. 818	. 837	461	783	54. 060	62. 211	328 474	100 119	26
27	. 827	. 847	514	799	53. 594	61. 675	325 646	99 257	27
28	. 835	. 857	566	815	53. 112	61. 121	322 717	98 364	28
29	. 844	. 868	622	832	52. 614	60. 547	319 688	97 441	29
30	59. 853	68. 878	363 675	110 848	52. 099	59. 955	316 562	96 488	30
31	. 863	. 889	734	866	51. 569	59. 345	313 340	95 506	31
32	. 872	. 900	789	883	51. 023	58. 716	310 023	94 495	32
33	. 882	. 911	848	901	50. 462	58. 070	306 611	93 455	33
34	. 891	. 922	907	919	49. 885	57. 407	303 107	92 387	34
35	59. 902	68. 934	363 970	110 938	49. 293	56. 725	299 508	91 290	35
36	. 911	. 945	364 029	956	48. 686	56. 027	295 820	90 166	36
37	. 922	. 957	091	975	48. 064	55. 311	292 041	89 014	37
38	. 932	. 968	154	994	47. 427	54. 578	288 173	87 835	38
39	. 942	. 980	216	111 013	46. 776	53. 829	284 216	86 629	39
40	59. 953	68. 993	364 281	111 033	46. 110	53. 063	280 171	85 396	40
41	. 963	69. 005	344	052	45. 430	52. 280	276 040	84 137	41
42	. 974	. 017	409	072	44. 737	51. 482	271 827	82 853	42
43	. 984	. 029	472	091	44. 030	50. 668	267 530	81 543	43
44	. 995	. 041	537	111	43. 309	49. 839	263 150	80 208	44
45	60. 006	69. 054	364 603	111 131	42. 575	48. 994	258 691	78 849	45

TABLE 6

Length of a Degree of Latitude and Longitude

Lat.	Degree of latitude				Degree of longitude				Lat.
	Nautical miles	Statute miles	Feet	Meters	Nautical miles	Statute miles	Feet	Meters	
°									°
45	60. 006	69. 054	364 603	111 131	42. 575	48. 994	258 691	78 849	45
46	. 017	. 066	669	151	41. 828	48. 135	254 154	77 466	46
47	. 027	. 078	731	170	41. 068	47. 260	249 534	76 058	47
48	. 038	. 090	797	190	40. 296	46. 372	244 843	74 628	48
49	. 049	. 103	862	210	39. 511	45. 468	240 072	73 174	49
50	60. 059	69. 114	364 925	111 229	38. 714	44. 551	235 230	71 698	50
51	. 070	. 127	990	249	37. 905	43. 620	230 315	70 200	51
52	. 080	. 139	365 052	268	37. 084	42. 676	225 328	68 680	52
53	. 090	. 151	115	287	36. 253	41. 719	220 276	67 140	53
54	. 100	. 162	177	306	35. 409	40. 748	215 151	65 578	54
55	60. 111	69. 174	365 240	111 325	34. 555	39. 765	209 961	63 996	55
56	. 120	. 185	299	343	33. 691	38. 770	204 708	62 395	56
57	. 130	. 197	358	361	32. 815	37. 763	199 390	60 774	57
58	. 140	. 208	417	379	31. 930	36. 745	194 012	59 135	58
59	. 150	. 219	476	397	31. 036	35. 715	188 576	57 478	59
60	60. 159	69. 229	365 531	111 414	30. 131	34. 674	183 077	55 802	60
61	. 168	. 241	591	432	29. 217	33. 622	177 526	54 110	61
62	. 177	. 251	643	448	28. 294	32. 560	171 916	52 400	62
63	. 186	. 261	696	464	27. 362	31. 488	166 257	50 675	63
64	. 194	. 270	748	480	26. 422	30. 406	160 545	48 934	64
65	60. 203	69. 280	365 801	111 496	25. 474	29. 314	154 780	47 177	65
66	. 211	. 290	850	511	24. 518	28. 215	148 973	45 407	66
67	. 219	. 298	896	525	23. 554	27. 105	143 117	43 622	67
68	. 226	. 307	942	539	22. 583	25. 988	137 215	41 823	68
69	. 234	. 316	988	553	21. 605	24. 862	131 273	40 012	69
70	60. 241	69. 324	366 030	111 566	20. 620	23. 729	125 289	38 188	70
71	. 247	. 331	070	578	19. 629	22. 589	119 268	36 353	71
72	. 254	. 339	109	590	18. 632	21. 441	113 209	34 506	72
73	. 260	. 346	148	602	17. 629	20. 287	107 113	32 648	73
74	. 266	. 353	184	613	16. 620	19. 126	100 988	30 781	74
75	60. 272	69. 359	366 217	111 623	15. 606	17. 959	94 826	28 903	75
76	. 276	. 365	247	632	14. 588	16. 788	88 638	27 017	76
77	. 282	. 371	280	642	13. 565	15. 611	82 425	25 123	77
78	. 286	. 376	306	650	12. 538	14. 428	76 181	23 220	78
79	. 290	. 381	332	658	11. 507	13. 242	69 918	21 311	79
80	60. 294	69. 385	366 355	111 665	10. 472	12. 051	63 629	19 394	80
81	. 298	. 389	375	671	9. 434	10. 857	57 323	17 472	81
82	. 301	. 393	394	677	8. 394	9. 659	51 001	15 545	82
83	. 303	. 396	411	682	7. 350	8. 458	44 659	13 612	83
84	. 306	. 399	427	687	6. 304	7. 255	38 304	11 675	84
85	60. 308	69. 402	366 440	111 691	5. 256	6. 049	31 939	9 735	85
86	. 310	. 403	450	694	4. 207	4. 842	25 564	7 792	86
87	. 311	. 405	457	696	3. 157	3. 633	19 180	5 846	87
88	. 312	. 406	463	698	2. 105	2. 422	12 789	3 898	88
89	. 313	. 407	467	699	1. 052	1. 211	6 394	1 949	89
90	60. 313	69. 407	366 467	111 699	0. 000	0. 000	0	0	90

TABLE 7

Distance of an Object by Two Bearings

Difference between the course and second bearing	Difference between the course and first bearing													
	20°		22°		24°		26°		28°		30°		32°	
°														
30	1.97	0.98												
32	1.64	0.87	2.16	1.14										
34	1.41	0.79	1.80	1.01	2.34	1.31								
36	1.24	0.73	1.55	0.91	1.96	1.15	2.52	1.48						
38	1.11	0.68	1.36	0.84	1.68	1.04	2.11	1.30	2.70	1.66				
40	1.00	0.64	1.21	0.78	1.48	0.95	1.81	1.16	2.26	1.45	2.88	1.85		
42	0.91	0.61	1.10	0.73	1.32	0.88	1.59	1.06	1.94	1.30	2.40	1.61	3.05	2.04
44	0.84	0.58	1.00	0.69	1.19	0.83	1.42	0.98	1.70	1.18	2.07	1.44	2.55	1.77
46	0.78	0.56	0.92	0.66	1.09	0.78	1.28	0.92	1.52	1.09	1.81	1.30	2.19	1.58
48	0.73	0.54	0.85	0.64	1.00	0.74	1.17	0.87	1.37	1.02	1.62	1.20	1.92	1.43
50	0.68	0.52	0.80	0.61	0.93	0.71	1.08	0.83	1.25	0.96	1.46	1.12	1.71	1.31
52	0.65	0.51	0.75	0.59	0.87	0.68	1.00	0.79	1.15	0.91	1.33	1.05	1.55	1.22
54	0.61	0.49	0.71	0.57	0.81	0.66	0.93	0.76	1.07	0.87	1.23	0.99	1.41	1.14
56	0.58	0.48	0.67	0.56	0.77	0.64	0.88	0.73	1.00	0.83	1.14	0.95	1.30	1.08
58	0.56	0.47	0.64	0.54	0.73	0.62	0.83	0.70	0.94	0.80	1.07	0.90	1.21	1.03
60	0.53	0.46	0.61	0.53	0.69	0.60	0.78	0.68	0.89	0.77	1.00	0.87	1.13	0.98
62	0.51	0.45	0.58	0.51	0.66	0.58	0.75	0.66	0.84	0.74	0.94	0.83	1.06	0.94
64	0.49	0.44	0.56	0.50	0.63	0.57	0.71	0.64	0.80	0.72	0.89	0.80	1.00	0.90
66	0.48	0.43	0.54	0.49	0.61	0.56	0.68	0.62	0.76	0.70	0.85	0.78	0.95	0.87
68	0.46	0.43	0.52	0.48	0.59	0.54	0.66	0.61	0.73	0.68	0.81	0.75	0.90	0.84
70	0.45	0.42	0.50	0.47	0.57	0.53	0.63	0.59	0.70	0.66	0.78	0.73	0.86	0.81
72	0.43	0.41	0.49	0.47	0.55	0.52	0.61	0.58	0.68	0.64	0.75	0.71	0.82	0.78
74	0.42	0.41	0.48	0.46	0.53	0.51	0.59	0.57	0.65	0.63	0.72	0.69	0.79	0.76
76	0.41	0.40	0.46	0.45	0.52	0.50	0.57	0.56	0.63	0.61	0.70	0.67	0.76	0.74
78	0.40	0.39	0.45	0.44	0.50	0.49	0.56	0.54	0.61	0.60	0.67	0.66	0.74	0.72
80	0.39	0.39	0.44	0.44	0.49	0.48	0.54	0.53	0.60	0.59	0.65	0.64	0.71	0.70
82	0.39	0.38	0.43	0.43	0.48	0.47	0.53	0.52	0.58	0.57	0.63	0.63	0.69	0.69
84	0.38	0.38	0.42	0.42	0.47	0.47	0.52	0.51	0.57	0.56	0.62	0.61	0.67	0.67
86	0.37	0.37	0.42	0.42	0.46	0.46	0.51	0.51	0.55	0.55	0.60	0.60	0.66	0.65
88	0.37	0.37	0.41	0.41	0.45	0.45	0.50	0.50	0.54	0.54	0.59	0.59	0.64	0.64
90	0.36	0.36	0.40	0.40	0.45	0.45	0.49	0.49	0.53	0.53	0.58	0.58	0.62	0.62
92	0.36	0.36	0.40	0.40	0.44	0.44	0.48	0.48	0.52	0.52	0.57	0.57	0.61	0.61
94	0.36	0.35	0.39	0.39	0.43	0.43	0.47	0.47	0.51	0.51	0.56	0.55	0.60	0.60
96	0.35	0.35	0.39	0.39	0.43	0.43	0.47	0.46	0.51	0.50	0.55	0.54	0.59	0.59
98	0.35	0.35	0.39	0.38	0.42	0.42	0.46	0.46	0.50	0.50	0.54	0.53	0.58	0.57
100	0.35	0.34	0.38	0.38	0.42	0.41	0.46	0.45	0.49	0.49	0.53	0.52	0.57	0.56
102	0.35	0.34	0.38	0.37	0.42	0.41	0.45	0.44	0.49	0.48	0.53	0.51	0.56	0.55
104	0.34	0.33	0.38	0.37	0.41	0.40	0.45	0.43	0.48	0.47	0.52	0.50	0.56	0.54
106	0.34	0.33	0.38	0.36	0.41	0.39	0.45	0.43	0.48	0.46	0.52	0.50	0.55	0.53
108	0.34	0.32	0.38	0.36	0.41	0.39	0.44	0.42	0.48	0.45	0.51	0.49	0.55	0.52
110	0.34	0.32	0.37	0.35	0.41	0.38	0.44	0.41	0.47	0.44	0.51	0.48	0.54	0.51
112	0.34	0.32	0.37	0.35	0.41	0.38	0.44	0.41	0.47	0.44	0.50	0.47	0.54	0.50
114	0.34	0.31	0.37	0.34	0.41	0.37	0.44	0.40	0.47	0.43	0.50	0.46	0.54	0.49
116	0.34	0.31	0.38	0.34	0.41	0.37	0.44	0.39	0.47	0.42	0.50	0.45	0.53	0.48
118	0.35	0.31	0.38	0.33	0.41	0.36	0.44	0.39	0.47	0.41	0.50	0.44	0.53	0.47
120	0.35	0.30	0.38	0.33	0.41	0.36	0.44	0.38	0.47	0.41	0.50	0.43	0.53	0.46
122	0.35	0.30	0.38	0.32	0.41	0.35	0.44	0.37	0.47	0.40	0.50	0.42	0.53	0.45
124	0.35	0.29	0.38	0.32	0.41	0.34	0.44	0.37	0.47	0.39	0.50	0.42	0.53	0.44
126	0.36	0.29	0.39	0.31	0.42	0.34	0.45	0.36	0.47	0.38	0.50	0.41	0.53	0.43
128	0.36	0.28	0.39	0.31	0.42	0.33	0.45	0.35	0.48	0.38	0.50	0.40	0.53	0.42
130	0.36	0.28	0.39	0.30	0.42	0.32	0.45	0.35	0.48	0.37	0.51	0.39	0.54	0.41
132	0.37	0.27	0.40	0.30	0.43	0.32	0.46	0.34	0.48	0.36	0.51	0.38	0.54	0.40
134	0.37	0.27	0.40	0.29	0.43	0.31	0.46	0.33	0.49	0.35	0.52	0.37	0.54	0.39
136	0.38	0.26	0.41	0.28	0.44	0.30	0.47	0.32	0.49	0.34	0.52	0.36	0.55	0.38
138	0.39	0.26	0.42	0.28	0.45	0.30	0.47	0.32	0.50	0.33	0.53	0.35	0.55	0.37
140	0.39	0.25	0.42	0.27	0.45	0.29	0.48	0.31	0.51	0.33	0.53	0.34	0.56	0.36
142	0.40	0.25	0.43	0.27	0.46	0.28	0.49	0.30	0.51	0.32	0.54	0.33	0.56	0.35
144	0.41	0.24	0.44	0.26	0.47	0.28	0.50	0.29	0.52	0.31	0.55	0.32	0.57	0.34
146	0.42	0.24	0.45	0.25	0.48	0.27	0.51	0.28	0.53	0.30	0.56	0.31	0.58	0.32
148	0.43	0.23	0.46	0.25	0.49	0.26	0.52	0.27	0.54	0.29	0.57	0.30	0.59	0.31
150	0.45	0.22	0.48	0.24	0.50	0.25	0.53	0.26	0.55	0.28	0.58	0.29	0.60	0.30
152	0.46	0.22	0.49	0.23	0.52	0.24	0.54	0.25	0.57	0.27	0.59	0.28	0.61	0.29
154	0.48	0.21	0.50	0.22	0.53	0.23	0.56	0.24	0.58	0.25	0.60	0.26	0.62	0.27
156	0.49	0.20	0.52	0.21	0.55	0.22	0.57	0.23	0.60	0.24	0.62	0.25	0.64	0.26
158	0.51	0.19	0.54	0.20	0.57	0.21	0.59	0.22	0.61	0.23	0.63	0.24	0.66	0.25
160	0.53	0.18	0.56	0.19	0.59	0.20	0.61	0.21	0.63	0.22	0.65	0.22	0.67	0.23

TABLE 7
Distance of an Object by Two Bearings

Difference between the course and second bearing	Difference between the course and first bearing													
	34°		36°		38°		40°		42°		44°		46°	
°														
44	3.22	2.24												
46	2.69	1.93	3.39	2.43										
48	2.31	1.72	2.83	2.10	3.55	2.63								
50	2.03	1.55	2.43	1.86	2.96	2.27	3.70	2.84						
52	1.81	1.43	2.13	1.68	2.54	2.01	3.09	2.44	3.85	3.04				
54	1.63	1.32	1.90	1.54	2.23	1.81	2.66	2.15	3.22	2.60	4.00	3.24		
56	1.49	1.24	1.72	1.42	1.99	1.65	2.33	1.93	2.77	2.29	3.34	2.77	4.14	3.43
58	1.37	1.17	1.57	1.33	1.80	1.53	2.08	1.76	2.43	2.06	2.87	2.44	3.46	2.93
60	1.28	1.10	1.45	1.25	1.64	1.42	1.88	1.63	2.17	1.88	2.52	2.18	2.97	2.57
62	1.19	1.05	1.34	1.18	1.51	1.34	1.72	1.52	1.96	1.73	2.25	1.98	2.61	2.30
64	1.12	1.01	1.25	1.13	1.40	1.26	1.58	1.42	1.79	1.61	2.03	1.83	2.33	2.09
66	1.06	0.96	1.18	1.07	1.31	1.20	1.47	1.34	1.65	1.51	1.85	1.69	2.10	1.92
68	1.00	0.93	1.11	1.03	1.23	1.14	1.37	1.27	1.53	1.42	1.71	1.58	1.92	1.78
70	0.95	0.89	1.05	0.99	1.16	1.09	1.29	1.21	1.43	1.34	1.58	1.49	1.77	1.66
72	0.91	0.86	1.00	0.95	1.10	1.05	1.21	1.15	1.34	1.27	1.48	1.41	1.64	1.56
74	0.87	0.84	0.95	0.92	1.05	1.01	1.15	1.10	1.26	1.21	1.39	1.34	1.53	1.47
76	0.84	0.81	0.91	0.89	1.00	0.97	1.09	1.06	1.20	1.16	1.31	1.27	1.44	1.40
78	0.80	0.79	0.88	0.86	0.96	0.94	1.04	1.02	1.14	1.11	1.24	1.22	1.36	1.33
80	0.78	0.77	0.85	0.83	0.92	0.91	1.00	0.98	1.09	1.07	1.18	1.16	1.28	1.27
82	0.75	0.75	0.82	0.81	0.89	0.88	0.96	0.95	1.04	1.03	1.13	1.12	1.22	1.21
84	0.73	0.73	0.79	0.79	0.86	0.85	0.93	0.92	1.00	0.99	1.08	1.07	1.17	1.16
86	0.71	0.71	0.77	0.77	0.83	0.83	0.89	0.89	0.96	0.96	1.04	1.04	1.12	1.12
88	0.69	0.69	0.75	0.75	0.80	0.80	0.86	0.86	0.93	0.93	1.00	1.00	1.08	1.07
90	0.67	0.67	0.73	0.73	0.78	0.78	0.84	0.84	0.90	0.90	0.97	0.97	1.04	1.04
92	0.66	0.66	0.71	0.71	0.76	0.76	0.82	0.82	0.87	0.87	0.93	0.93	1.00	1.00
94	0.65	0.64	0.69	0.69	0.74	0.74	0.79	0.79	0.85	0.85	0.91	0.90	0.97	0.97
96	0.63	0.63	0.68	0 67	0.73	0.72	0.78	0.77	0.83	0.82	0.88	0.88	0.94	0.93
98	0.62	0.62	0.67	0.66	0.71	0.70	0.76	0.75	0.81	0.80	0.86	0.85	0.91	0.90
100	0.61	0.60	0.65	0.64	0.70	0.69	0.74	0.73	0.79	0.78	0.84	0.83	0.89	0.88
102	0.60	0.59	0.64	0.63	0.68	0.67	0.73	0.71	0.77	0.76	0.82	0.80	0.87	0.85
104	0.60	0.58	0.63	0.61	0.67	0.65	0.72	0.69	0.76	0.74	0.80	0.78	0.85	0.82
106	0.59	0.57	0.63	0.60	0.66	0.64	0.70	0.68	0.74	0.72	0.79	0.76	0.83	0.80
108	0.58	0.55	0.62	0.59	0.66	0.62	0.69	0.66	0.73	0.70	0.77	0.74	0.81	0.77
110	0.58	0.54	0.61	0.57	0.65	0.61	0.68	0.64	0.72	0.68	0.76	0.71	0.80	0.75
112	0.57	0.53	0.61	0.56	0.64	0.59	0.68	0.63	0.71	0.66	0.75	0.69	0.79	0.73
114	0.57	0.52	0.60	0.55	0.63	0.58	0.67	0.61	0.70	0.64	0.74	0.68	0.78	0.71
116	0.56	0.51	0.60	0.54	0.63	0.57	0.66	0.60	0.70	0.63	0.73	0.66	0.77	0.69
118	0.56	0.50	0.59	0.52	0.63	0.55	0.66	0.58	0.69	0.61	0.72	0.64	0.76	0.67
120	0.56	0.49	0.59	0.51	0.62	0.54	0.65	0.57	0.68	0.59	0.72	0.62	0.75	0.65
122	0.56	0.47	0.59	0.50	0.62	0.53	0.65	0.55	0.68	0.58	0.71	0.60	0.74	0.63
124	0.56	0.46	0.59	0.49	0.62	0.51	0.65	0.54	0.68	0.56	0.71	0.58	0.74	0.61
126	0.56	0.45	0.59	0.48	0.62	0.50	0.64	0.52	0.67	0.54	0.70	0.57	0.73	0.59
128	0.56	0.44	0.59	0.46	0.62	0.49	0.64	0.51	0.67	0.53	0.70	0.55	0.73	0.57
130	0.56	0.43	0.59	0.45	0.62	0.47	0.64	0.49	0.67	0.51	0.70	0.53	0.72	0.55
132	0.56	0.42	0.59	0.44	0.62	0.46	0.64	0.48	0.67	0.50	0.70	0.52	0.72	0.54
134	0.57	0.41	0.59	0.43	0.62	0.45	0.64	0.46	0.67	0.48	0.69	0.50	0.72	0.52
136	0.57	0.40	0.60	0.41	0.62	0.43	0.65	0.45	0.67	0.47	0.70	0.48	0.72	0.50
138	0.58	0.39	0.60	0.40	0.63	0.42	0.65	0.43	0.67	0.45	0.70	0.47	0.72	0.48
140	0.58	0.37	0.61	0.39	0.63	0.40	0.65	0.42	0.68	0.43	0.70	0.45	0.72	0.46
142	0.59	0.36	0.61	0.38	0.63	0.39	0.66	0.41	0.68	0.42	0.70	0.43	0.72	0.45
144	0.60	0.35	0.62	0.36	0.64	0.38	0.66	0.39	0.68	0.40	0.71	0.41	0.73	0.43
146	0.60	0.34	0.63	0.35	0.65	0.36	0.67	0.37	0.69	0.39	0.71	0.40	0.73	0.41
148	0.61	0.32	0.63	0.34	0.66	0.35	0.68	0.36	0.70	0.37	0.72	0.38	0.74	0.39
150	0.62	0.31	0.64	0.32	0.66	0.33	0.68	0.34	0.70	0.35	0.72	0.36	0.74	0.37
152	0.63	0.30	0.65	0.31	0.67	0.32	0.69	0.33	0.71	0.33	0.73	0.34	0.75	0.35
154	0.65	0.28	0.67	0.29	0.68	0.30	0.70	0.31	0.72	0.32	0.74	0.32	0.76	0.33
156	0.66	0.27	0.68	0.28	0.70	0.28	0.72	0.29	0.73	0.30	0.75	0.30	0.77	0.31
158	0.67	0.25	0.69	0.26	0.71	0.27	0.73	0.27	0.74	0.28	0.76	0.28	0.78	0.29
160	0.69	0.24	0.71	0.24	0.73	0.25	0.74	0.25	0.76	0.26	0.77	0.26	0.79	0.27

TABLE 7

Distance of an Object by Two Bearings

Difference between the course and second bearing	Difference between the course and first bearing													
	48°		50°		52°		54°		56°		58°		60°	
°														
58	4.28	3.63												
60	3.57	3.10	4.41	3.82										
62	3.07	2.71	3.68	3.25	4.54	4.01								
64	2.70	2.42	3.17	2.85	3.79	3.41	4.66	4.19						
66	2.40	2.20	2.78	2.54	3.26	2.98	3.89	3.55	4.77	4.36				
68	2.17	2.01	2.48	2.30	2.86	2.65	3.34	3.10	3.99	3.71	4.88	4.53		
70	1.98	1.86	2.24	2.10	2.55	2.39	2.94	2.76	3.43	3.22	4.08	3.83	4.99	4.69
72	1.83	1.74	2.04	1.94	2.30	2.19	2.62	2.49	3.01	2.86	3.51	3.33	4.17	3.96
74	1.70	1.63	1.88	1.81	2.10	2.02	2.37	2.27	2.68	2.58	3.08	2.96	3.58	3.44
76	1.58	1.54	1.75	1.70	1.94	1.88	2.16	2.10	2.42	2.35	2.74	2.66	3.14	3.05
78	1.49	1.45	1.63	1.60	1.80	1.76	1.99	1.95	2.21	2.16	2.48	2.43	2.80	2.74
80	1.40	1.38	1.53	1.51	1.68	1.65	1.85	1.82	2.04	2.01	2.26	2.23	2.53	2.49
82	1.33	1.32	1.45	1.43	1.58	1.56	1.72	1.71	1.89	1.87	2.08	2.06	2.31	2.29
84	1.26	1.26	1.37	1.36	1.49	1.48	1.62	1.61	1.77	1.76	1.93	1.92	2.13	2.12
86	1.21	1.20	1.30	1.30	1.41	1.41	1.53	1.52	1.66	1.65	1.81	1.80	1.98	1.97
88	1.16	1.16	1.24	1.24	1.34	1.34	1.45	1.45	1.56	1.56	1.70	1.70	1.84	1.84
90	1.11	1.11	1.19	1.19	1.28	1.28	1.38	1.38	1.48	1.48	1.60	1.60	1.73	1.73
92	1.07	1.07	1.14	1.14	1.23	1.23	1.31	1.31	1.41	1.41	1.52	1.52	1.63	1.63
94	1.03	1.03	1.10	1.10	1.18	1.17	1.26	1.26	1.35	1.34	1.44	1.44	1.55	1.54
96	1.00	0.99	1.06	1.06	1.13	1.13	1.21	1.20	1.29	1.28	1.38	1.37	1.47	1.47
98	0.97	0.96	1.03	1.02	1.10	1.08	1.16	1.15	1.24	1.23	1.32	1.31	1.41	1.39
100	0.94	0.93	1.00	0.98	1.06	1.04	1.12	1.11	1.19	1.18	1.27	1.25	1.35	1.33
102	0.92	0.90	0.97	0.95	1.03	1.01	1.09	1.06	1.15	1.13	1.22	1.19	1.29	1.27
104	0.90	0.87	0.95	0.92	1.00	0.97	1.06	1.02	1.12	1.08	1.18	1.14	1.25	1.21
106	0.88	0.84	0.92	0.89	0.97	0.94	1.03	0.99	1.09	1.04	1.14	1.10	1.20	1.16
108	0.86	0.82	0.90	0.86	0.95	0.90	1.00	0.95	1.05	1.00	1.11	1.05	1.17	1.11
110	0.84	0.79	0.88	0.83	0.93	0.87	0.98	0.92	1.02	0.96	1.08	1.01	1.13	1.06
112	0.83	0.77	0.87	0.80	0.91	0.84	0.95	0.88	1.00	0.93	1.05	0.97	1.10	1.02
114	0.81	0.74	0.85	0.78	0.89	0.82	0.93	0.85	0.98	0.89	1.02	0.93	1.07	0.98
116	0.80	0.72	0.84	0.75	0.88	0.79	0.92	0.82	0.96	0.85	1.00	0.90	1.04	0.94
118	0.79	0.70	0.83	0.73	0.86	0.76	0.90	0.79	0.94	0.83	0.98	0.86	1.02	0.90
120	0.78	0.68	0.82	0.71	0.85	0.74	0.89	0.77	0.91	0.80	0.96	0.83	1.00	0.87
122	0.77	0.66	0.81	0.68	0.84	0.71	0.87	0.74	0.90	0.77	0.95	0.80	0.98	0.83
124	0.77	0.63	0.80	0.66	0.83	0.69	0.86	0.71	0.90	0.74	0.93	0.77	0.96	0.80
126	0.76	0.61	0.79	0.64	0.82	0.66	0.85	0.69	0.88	0.71	0.91	0.74	0.95	0.77
128	0.75	0.59	0.78	0.62	0.81	0.64	0.84	0.66	0.87	0.69	0.90	0.71	0.93	0.74
130	0.75	0.57	0.78	0.60	0.81	0.62	0.83	0.64	0.86	0.66	0.89	0.68	0.92	0.71
132	0.75	0.56	0.77	0.57	0.80	0.59	0.83	0.61	0.85	0.64	0.88	0.66	0.91	0.68
134	0.74	0.54	0.77	0.55	0.80	0.57	0.82	0.59	0.85	0.61	0.87	0.63	0.90	0.65
136	0.74	0.52	0.77	0.53	0.80	0.55	0.82	0.57	0.84	0.58	0.87	0.60	0.89	0.62
138	0.74	0.50	0.77	0.51	0.79	0.53	0.81	0.54	0.84	0.56	0.86	0.58	0.89	0.59
140	0.74	0.48	0.77	0.49	0.79	0.51	0.81	0.52	0.83	0.54	0.86	0.55	0.88	0.57
142	0.74	0.46	0.77	0.47	0.79	0.49	0.81	0.50	0.83	0.51	0.85	0.52	0.87	0.54
144	0.75	0.44	0.77	0.45	0.79	0.46	0.81	0.48	0.83	0.49	0.85	0.50	0.87	0.51
146	0.75	0.42	0.77	0.43	0.79	0.44	0.81	0.45	0.83	0.46	0.85	0.47	0.87	0.49
148	0.76	0.40	0.77	0.41	0.79	0.42	0.81	0.43	0.83	0.44	0.85	0.45	0.87	0.46
150	0.76	0.38	0.78	0.39	0.80	0.40	0.81	0.41	0.83	0.42	0.85	0.42	0.87	0.43
152	0.77	0.36	0.78	0.37	0.80	0.38	0.82	0.38	0.83	0.39	0.85	0.40	0.87	0.41
154	0.77	0.34	0.79	0.35	0.81	0.35	0.82	0.36	0.84	0.37	0.85	0.37	0.87	0.38
156	0.78	0.32	0.80	0.32	0.81	0.33	0.83	0.34	0.84	0.34	0.86	0.35	0.87	0.35
158	0.79	0.30	0.81	0.30	0.82	0.31	0.83	0.31	0.85	0.32	0.86	0.32	0.87	0.33
160	0.80	0.27	0.82	0.28	0.83	0.28	0.84	0.29	0.85	0.29	0.86	0.30	0.88	0.30

TABLE 7

Distance of an Object by Two Bearings

Difference between the course and second bearing	Difference between the course and first bearing															
	62°		64°		66°		68°		70°		72°		74°		76°	
°																
72	5.08	4.84														
74	4.25	4.08	5.18	4.98												
76	3.65	3.54	4.32	4.19	5.26	5.10										
78	3.20	3.13	3.72	3.63	4.39	4.30	5.34	5.22								
80	2.86	2.81	3.26	3.21	3.78	3.72	4.46	4.39	5.41	5.33						
82	2.58	2.56	2.91	2.88	3.31	3.28	3.83	3.80	4.52	4.48	5.48	5.42				
84	2.36	2.34	2.63	2.61	2.96	2.94	3.36	3.35	3.88	3.86	4.57	4.55	5.54	5.51		
86	2.17	2.17	2.40	2.39	2.67	2.66	3.00	2.99	3.41	3.40	3.93	3.92	4.62	4.61	5.59	5.57
88	2.01	2.01	2.21	2.21	2.44	2.44	2.71	2.71	3.04	3.04	3.45	3.45	3.97	3.97	4.67	4.66
90	1.88	1.88	2.05	2.05	2.25	2.25	2.48	2.48	2.75	2.75	3.08	3.08	3.49	3.49	4.01	4.01
92	1.77	1.76	1.91	1.91	2.08	2.08	2.28	2.28	2.51	2.51	2.78	2.78	3.11	3.11	3.52	3.52
94	1.67	1.66	1.80	1.79	1.95	1.94	2.12	2.11	2.31	2.30	2.54	2.53	2.81	2.80	3.14	3.13
96	1.58	1.57	1.70	1.69	1.83	1.82	1.97	1.96	2.14	2.13	2.34	2.33	2.57	2.55	2.84	2.82
98	1.50	1.49	1.61	1.59	1.72	1.71	1.85	1.84	2.00	1.98	2.17	2.15	2.36	2.34	2.59	2.56
100	1.43	1.41	1.53	1.51	1.63	1.61	1.75	1.72	1.88	1.85	2.03	2.00	2.19	2.16	2.39	2.35
102	1.37	1.34	1.46	1.43	1.55	1.52	1.66	1.62	1.77	1.73	1.90	1.86	2.05	2.00	2.21	2.16
104	1.32	1.28	1.40	1.36	1.48	1.44	1.58	1.53	1.68	1.63	1.79	1.74	1.92	1.87	2.07	2.01
106	1.27	1.22	1.34	1.29	1.42	1.37	1.51	1.45	1.60	1.54	1.70	1.63	1.81	1.74	1.94	1.87
108	1.23	1.17	1.29	1.23	1.37	1.30	1.44	1.37	1.53	1.45	1.62	1.54	1.72	1.63	1.83	1.74
110	1.19	1.12	1.25	1.17	1.32	1.24	1.39	1.30	1.46	1.37	1.54	1.45	1.64	1.54	1.74	1.63
112	1.15	1.07	1.21	1.12	1.27	1.18	1.33	1.24	1.40	1.30	1.48	1.37	1.56	1.45	1.65	1.53
114	1.12	1.02	1.17	1.07	1.23	1.12	1.29	1.18	1.35	1.24	1.42	1.30	1.50	1.37	1.58	1.44
116	1.09	0.98	1.14	1.03	1.19	1.07	1.25	1.12	1.31	1.17	1.37	1.23	1.44	1.29	1.51	1.36
118	1.07	0.94	1.11	0.98	1.16	1.02	1.21	1.07	1.26	1.12	1.32	1.17	1.38	1.22	1.45	1.28
120	1.04	0.90	1.08	0.94	1.13	0.98	1.18	1.02	1.23	1.06	1.28	1.11	1.34	1.16	1.40	1.21
122	1.02	0.86	1.06	0.90	1.10	0.93	1.15	0.97	1.19	1.01	1.24	1.05	1.29	1.10	1.35	1.14
124	1.00	0.83	1.04	0.86	1.08	0.89	1.12	0.93	1.16	0.96	1.21	1.00	1.25	1.04	1.31	1.08
126	0.98	0.79	1.02	0.82	1.05	0.85	1.09	0.88	1.13	0.92	1.18	0.95	1.22	0.99	1.27	1.02
128	0.97	0.76	1.00	0.79	1.03	0.82	1.07	0.84	1.11	0.87	1.15	0.90	1.19	0.94	1.23	0.97
130	0.95	0.73	0.98	0.75	1.02	0.78	1.05	0.80	1.09	0.83	1.12	0.86	1.16	0.89	1.20	0.92
132	0.94	0.70	0.97	0.72	1.00	0.74	1.03	0.77	1.06	0.79	1.10	0.82	1.13	0.84	1.17	0.87
134	0.93	0.67	0.96	0.69	0.99	0.71	1.01	0.73	1.04	0.75	1.08	0.77	1.11	0.80	1.14	0.82
136	0.92	0.64	0.95	0.66	0.97	0.68	1.00	0.69	1.03	0.71	1.06	0.74	1.09	0.76	1.12	0.78
138	0.91	0.61	0.94	0.63	0.96	0.64	0.99	0.66	1.01	0.68	1.04	0.70	1.07	0.72	1.10	0.74
140	0.90	0.58	0.93	0.60	0.95	0.61	0.97	0.63	1.00	0.64	1.03	0.66	1.05	0.68	1.08	0.70
142	0.90	0.55	0.92	0.57	0.94	0.58	0.96	0.59	0.99	0.61	1.01	0.62	1.04	0.64	1.06	0.65
144	0.89	0.52	0.91	0.54	0.93	0.55	0.96	0.56	0.98	0.57	1.00	0.59	1.02	0.60	1.05	0.62
146	0.89	0.50	0.91	0.51	0.93	0.52	0.95	0.53	0.97	0.54	0.99	0.55	1.01	0.57	1.03	0.58
148	0.89	0.47	0.90	0.48	0.92	0.49	0.94	0.50	0.96	0.51	0.98	0.52	1.00	0.53	1.02	0.54
150	0.88	0.44	0.90	0.45	0.92	0.46	0.94	0.47	0.95	0.48	0.97	0.49	0.99	0.50	1.01	0.50
152	0.88	0.41	0.90	0.42	0.92	0.43	0.93	0.44	0.95	0.45	0.97	0.45	0.98	0.46	1.00	0.47
154	0.88	0.39	0.90	0.39	0.91	0.40	0.93	0.41	0.94	0.41	0.96	0.42	0.98	0.43	0.99	0.43
156	0.89	0.36	0.90	0.37	0.91	0.37	0.93	0.38	0.94	0.38	0.96	0.39	0.97	0.39	0.99	0.40
158	0.89	0.33	0.90	0.34	0.91	0.34	0.93	0.35	0.94	0.35	0.95	0.36	0.97	0.36	0.98	0.37
160	0.89	0.30	0.90	0.31	0.91	0.31	0.93	0.32	0.94	0.32	0.95	0.33	0.96	0.33	0.98	0.33

TABLE 7

Distance of an Object by Two Bearings

Difference between the course and second bearing	Difference between the course and first bearing															
	78°		80°		82°		84°		86°		88°		90°		92°	
°																
88	5.63	5.63														
90	4.70	4.70	5.67	5.67												
92	4.04	4.04	4.74	4.73	5.70	5.70										
94	3.55	3.54	4.07	4.06	4.76	4.75	5.73	5.71								
96	3.17	3.15	3.57	3.55	4.09	4.07	4.78	4.76	5.74	5.71						
98	2.86	2.83	3.19	3.16	3.59	3.56	4.11	4.07	4.80	4.75	5.76	5.70				
100	2.61	2.57	2.88	2.84	3.20	3.16	3.61	3.55	4.12	4.06	4.81	4.73	5.76	5.67		
102	2.40	2.35	2.63	2.57	2.90	2.83	3.22	3.15	3.62	3.54	4.13	4.04	4.81	4.70	5.76	5.63
104	2.23	2.16	2.42	2.35	2.64	2.56	2.91	2.82	3.23	3.13	3.63	3.52	4.13	4.01	4.81	4.66
106	2.08	2.00	2.25	2.16	2.43	2.34	2.65	2.55	2.92	2.80	3.23	3.11	3.63	3.49	4.13	3.97
108	1.96	1.86	2.10	2.00	2.26	2.15	2.45	2.33	2.66	2.53	2.92	2.78	3.24	3.08	3.63	3.45
110	1.85	1.73	1.97	1.85	2.11	1.98	2.27	2.13	2.45	2.31	2.67	2.51	2.92	2.75	3.23	3.04
112	1.75	1.62	1.86	1.72	1.98	1.83	2.12	1.96	2.28	2.11	2.46	2.28	2.67	2.48	2.92	2.71
114	1.66	1.52	1.76	1.61	1.87	1.71	1.99	1.82	2.12	1.94	2.28	2.08	2.46	2.25	2.67	2.44
116	1.59	1.43	1.68	1.51	1.77	1.59	1.88	1.69	2.00	1.79	2.13	1.91	2.28	2.05	2.46	2.21
118	1.52	1.34	1.60	1.41	1.68	1.49	1.78	1.57	1.88	1.66	2.00	1.76	2.13	1.88	2.28	2.01
120	1.46	1.27	1.53	1.33	1.61	1.39	1.69	1.47	1.78	1.54	1.89	1.63	2.00	1.73	2.13	1.84
122	1.41	1.19	1.47	1.25	1.54	1.31	1.62	1.37	1.70	1.44	1.79	1.52	1.89	1.60	2.00	1.70
124	1.36	1.13	1.42	1.18	1.48	1.23	1.55	1.28	1.62	1.34	1.70	1.41	1.79	1.48	1.89	1.56
126	1.32	1.06	1.37	1.11	1.43	1.15	1.48	1.20	1.55	1.26	1.62	1.31	1.70	1.38	1.79	1.45
128	1.28	1.01	1.33	1.04	1.38	1.08	1.43	1.13	1.49	1.17	1.55	1.23	1.62	1.28	1.70	1.34
130	1.24	0.95	1.29	0.98	1.33	1.02	1.38	1.06	1.44	1.10	1.49	1.14	1.56	1.19	1.62	1.24
132	1.21	0.90	1.25	0.93	1.29	0.96	1.34	0.99	1.39	1.03	1.44	1.07	1.49	1.11	1.55	1.16
134	1.18	0.85	1.22	0.88	1.26	0.90	1.30	0.93	1.34	0.97	1.39	1.00	1.44	1.04	1.49	1.07
136	1.15	0.80	1.19	0.83	1.22	0.85	1.26	0.88	1.30	0.90	1.34	0.93	1.39	0.97	1.44	1.00
138	1.13	0.76	1.16	0.78	1.19	0.80	1.23	0.82	1.27	0.85	1.30	0.87	1.35	0.90	1.39	0.93
140	1.11	0.71	1.14	0.73	1.17	0.75	1.20	0.77	1.23	0.79	1.27	0.82	1.31	0.84	1.34	0.86
142	1.09	0.67	1.12	0.69	1.14	0.70	1.17	0.72	1.20	0.74	1.24	0.76	1.27	0.78	1.30	0.80
144	1.07	0.63	1.10	0.64	1.12	0.66	1.15	0.67	1.18	0.69	1.21	0.71	1.24	0.73	1.27	0.75
146	1.05	0.59	1.08	0.60	1.10	0.62	1.13	0.63	1.15	0.64	1.18	0.66	1.21	0.67	1.24	0.69
148	1.04	0.55	1.06	0.56	1.08	0.57	1.11	0.59	1.13	0.60	1.15	0.61	1.18	0.62	1.21	0.64
150	1.03	0.51	1.05	0.52	1.07	0.53	1.09	0.54	1.11	0.55	1.13	0.57	1.15	0.58	1.18	0.59
152	1.02	0.48	1.04	0.49	1.05	0.49	1.07	0.50	1.09	0.51	1.11	0.52	1.13	0.53	1.15	0.54
154	1.01	0.44	1.02	0.45	1.04	0.46	1.06	0.46	1.08	0.47	1.09	0.48	1.11	0.49	1.13	0.50
156	1.00	0.41	1.01	0.41	1.03	0.42	1.05	0.43	1.06	0.43	1.08	0.44	1.09	0.45	1.11	0.45
158	0.99	0.37	1.01	0.38	1.02	0.38	1.03	0.39	1.05	0.39	1.06	0.40	1.08	0.40	1.09	0.41
160	0.99	0.34	1.00	0.34	1.01	0.35	1.02	0.35	1.04	0.35	1.05	0.36	1.06	0.36	1.08	0.37
	94°		96°		98°		100°		102°		104°		106°		108°	
°																
104	5.74	5.57														
106	4.80	4.61	5.73	5.51												
108	4.12	3.92	4.78	4.55	5.70	5.42										
110	3.62	3.40	4.11	3.86	4.76	4.48	5.67	5.33								
112	3.23	2.99	3.61	3.35	4.09	3.80	4.74	4.40	5.63	5.22						
114	2.92	2.66	3.22	2.94	3.59	3.28	4.07	3.72	4.70	4.30	5.59	5.10				
116	2.66	2.39	2.91	2.61	3.20	2.88	3.57	3.21	4.04	3.63	4.67	4.19	5.54	4.98		
118	2.45	2.17	2.65	2.34	2.90	2.56	3.19	2.81	3.55	3.13	4.01	3.54	4.62	4.08	5.48	4.84
120	2.28	1.97	2.45	2.12	2.64	2.29	2.88	2.49	3.17	2.74	3.52	3.05	3.97	3.44	4.57	3.96
122	2.12	1.80	2.27	1.92	2.43	2.06	2.63	2.23	2.86	2.43	3.14	2.66	3.49	2.96	3.93	3.33
124	2.00	1.65	2.12	1.76	2.26	1.87	2.42	2.01	2.61	2.16	2.84	2.35	3.11	2.58	3.45	2.86
126	1.88	1.52	1.99	1.61	2.11	1.71	2.25	1.82	2.40	1.95	2.59	2.10	2.81	2.27	3.08	2.49
128	1.78	1.41	1.88	1.48	1.98	1.56	2.10	1.65	2.23	1.76	2.39	1.88	2.57	2.02	2.78	2.19
130	1.70	1.30	1.78	1.36	1.87	1.43	1.97	1.51	2.08	1.60	2.21	1.70	2.36	1.81	2.54	1.94
132	1.62	1.20	1.69	1.26	1.77	1.32	1.86	1.38	1.96	1.45	2.07	1.54	2.19	1.63	2.34	1.74
134	1.55	1.12	1.62	1.16	1.68	1.21	1.76	1.27	1.85	1.33	1.94	1.40	2.05	1.47	2.17	1.56
136	1.49	1.04	1.55	1.07	1.61	1.12	1.68	1.16	1.75	1.22	1.83	1.27	1.92	1.34	2.03	1.41
138	1.44	0.96	1.49	0.99	1.54	1.03	1.60	1.07	1.66	1.11	1.74	1.16	1.81	1.21	1.90	1.27
140	1.39	0.89	1.43	0.92	1.48	0.95	1.53	0.98	1.59	1.02	1.65	1.06	1.72	1.10	1.79	1.15
142	1.34	0.83	1.38	0.85	1.43	0.88	1.47	0.91	1.52	0.94	1.58	0.97	1.64	1.01	1.70	1.05
144	1.30	0.77	1.34	0.79	1.38	0.81	1.42	0.83	1.46	0.86	1.51	0.89	1.56	0.92	1.62	0.95
146	1.27	0.71	1.30	0.73	1.33	0.75	1.37	0.77	1.41	0.79	1.45	0.81	1.50	0.84	1.54	0.86
148	1.23	0.65	1.26	0.67	1.29	0.69	1.33	0.70	1.36	0.72	1.40	0.74	1.44	0.76	1.48	0.78
150	1.20	0.60	1.23	0.61	1.26	0.63	1.29	0.64	1.32	0.66	1.35	0.67	1.38	0.69	1.42	0.71
152	1.18	0.55	1.20	0.56	1.22	0.57	1.25	0.59	1.28	0.60	1.31	0.61	1.34	0.63	1.37	0.64
154	1.15	0.50	1.17	0.51	1.19	0.52	1.22	0.53	1.24	0.54	1.27	0.56	1.29	0.57	1.32	0.58
156	1.13	0.46	1.15	0.47	1.17	0.47	1.19	0.48	1.21	0.49	1.23	0.50	1.25	0.51	1.28	0.52
158	1.11	0.42	1.13	0.42	1.14	0.43	1.16	0.44	1.18	0.44	1.20	0.45	1.22	0.46	1.24	0.47
160	1.09	0.37	1.11	0.38	1.12	0.38	1.14	0.39	1.15	0.39	1.17	0.40	1.19	0.41	1.21	0.41

TABLE 7

Distance of an Object by Two Bearings

Difference between the course and second bearing.	Difference between the course and first bearing.													
	110°		112°		114°		116°		118°		120°		122°	
°														
120	5.41	4.69												
122	4.52	3.83	5.34	4.53										
124	3.88	3.22	4.46	3.70	5.26	4.36								
126	3.41	2.76	3.83	3.10	4.39	3.55	5.18	4.19						
128	3.04	2.40	3.36	2.65	3.78	2.98	4.32	3.41	5.08	4.01				
130	2.75	2.10	3.00	2.30	3.31	2.54	3.72	2.85	4.25	3.25	4.99	3.82		
132	2.51	1.86	2.71	2.01	2.96	2.20	3.26	2.42	3.65	2.71	4.17	3.10	4.88	3.63
134	2.31	1.66	2.48	1.78	2.67	1.92	2.91	2.09	3.20	2.30	3.58	2.57	4.08	2.93
136	2.14	1.49	2.28	1.58	2.44	1.69	2.63	1.83	2.86	1.98	3.14	2.18	3.51	2.44
138	2.00	1.34	2.12	1.42	2.25	1.50	2.40	1.61	2.58	1.73	2.80	1.88	3.08	2.06
140	1.88	1.21	1.97	1.27	2.08	1.34	2.21	1.42	2.36	1.52	2.53	1.63	2.74	1.76
142	1.77	1.09	1.85	1.14	1.95	1.20	2.05	1.26	2.17	1.34	2.31	1.42	2.48	1.53
144	1.68	0.99	1.75	1.03	1.83	1.07	1.91	1.13	2.01	1.18	2.13	1.25	2.26	1.33
146	1.60	0.89	1.66	0.93	1.72	0.96	1.80	1.01	1.88	1.05	1.98	1.10	2.08	1.17
148	1.53	0.81	1.58	0.84	1.63	0.87	1.70	0.90	1.77	0.94	1.84	0.98	1.93	1.03
150	1.46	0.73	1.51	0.75	1.55	0.78	1.61	0.80	1.67	0.83	1.73	0.87	1.81	0.90
152	1.40	0.66	1.44	0.68	1.48	0.70	1.53	0.72	1.58	0.74	1.63	0.77	1.70	0.80
154	1.35	0.59	1.39	0.61	1.42	0.62	1.46	0.64	1.50	0.66	1.55	0.68	1.60	0.70
156	1.31	0.53	1.33	0.54	1.37	0.56	1.40	0.57	1.43	0.58	1.47	0.60	1.52	0.62
158	1.26	0.47	1.29	0.48	1.32	0.49	1.34	0.50	1.37	0.51	1.41	0.53	1.44	0.54
160	1.23	0.42	1.25	0.43	1.27	0.43	1.29	0.44	1.32	0.45	1.35	0.46	1.38	0.47

	124°		126°		128°		130°		132°		134°		136°	
134	4.77	3.43												
136	3.99	2.77	4.66	3.23										
138	3.43	2.29	3.89	2.60	4.54	3.04								
140	3.01	1.93	3.34	2.15	3.79	2.44	4.41	2.84						
142	2.68	1.65	2.94	1.81	3.26	2.01	3.68	2.27	4.28	2.63				
144	2.42	1.42	2.62	1.54	2.86	1.68	3.17	1.86	3.57	2.10	4.14	2.43		
146	2.21	1.24	2.37	1.32	2.55	1.43	2.78	1.55	3.07	1.72	3.46	1.93	4.00	2.24
148	2.04	1.08	2.16	1.14	2.30	1.22	2.48	1.31	2.70	1.43	2.97	1.58	3.34	1.77
150	1.89	0.95	1.99	0.99	2.10	1.05	2.24	1.12	2.40	1.20	2.61	1.30	2.87	1.44
152	1.77	0.83	1.85	0.87	1.94	0.91	2.04	0.96	2.17	1.02	2.33	1.09	2.52	1.18
154	1.66	0.73	1.72	0.76	1.80	0.79	1.88	0.83	1.98	0.87	2.10	0.92	2.25	0.99
156	1.56	0.64	1.62	0.66	1.68	0.68	1.75	0.71	1.83	0.74	1.92	0.78	2.03	0.83
158	1.48	0.56	1.53	0.57	1.58	0.59	1.63	0.61	1.70	0.64	1.77	0.66	1.85	0.69
160	1.41	0.48	1.45	0.49	1.49	0.51	1.53	0.52	1.58	0.54	1.64	0.56	1.71	0.58

	138°		140°		142°		144°		146°		148°		150°	
148	3.85	2.04												
150	3.22	1.61	3.70	1.85										
152	2.77	1.30	3.09	1.45	3.55	1.66								
154	2.43	1.06	2.66	1.16	2.96	1.30	3.38	1.48						
156	2.17	0.88	2.33	0.95	2.54	1.04	2.83	1.15	3.22	1.31				
158	1.96	0.73	2.08	0.78	2.23	0.84	2.43	0.91	2.69	1.01	3.05	1.14		
160	1.79	0.61	1.88	0.64	1.99	0.68	2.13	0.73	2.31	0.79	2.55	0.87	2.88	0.98

TABLE 8

Distance of the Horizon

Height feet	Nautical miles	Statute miles	Height feet	Nautical miles	Statute miles	Height feet	Nautical miles	Statute miles
1	1. 1	1. 3	120	12. 5	14. 4	940	35. 1	40. 4
2	1. 6	1. 9	125	12. 8	14. 7	960	35. 4	40. 8
3	2. 0	2. 3	130	13. 0	15. 0	980	35. 8	41. 2
4	2. 3	2. 6	135	13. 3	15. 3	1, 000	36. 2	41. 6
5	2. 6	2. 9	140	13. 5	15. 6	1, 100	37. 9	43. 7
6	2. 8	3. 2	145	13. 8	15. 9	1, 200	39. 6	45. 6
7	3. 0	3. 5	150	14. 0	16. 1	1, 300	41. 2	47. 5
8	3. 2	3. 7	160	14. 5	16. 7	1, 400	42. 8	49. 3
9	3. 4	4. 0	170	14. 9	17. 2	1, 500	44. 3	51. 0
10	3. 6	4. 2	180	15. 3	17. 7	1, 600	45. 8	52. 7
11	3. 8	4. 4	190	15. 8	18. 2	1, 700	47. 2	54. 3
12	4. 0	4. 6	200	16. 2	18. 6	1, 800	48. 5	55. 9
13	4. 1	4. 7	210	16. 6	19. 1	1, 900	49. 9	57. 4
14	4. 3	4. 9	220	17. 0	19. 5	2, 000	51. 2	58. 9
15	4. 4	5. 1	230	17. 3	20. 0	2, 100	52. 4	60. 4
16	4. 6	5. 3	240	17. 7	20. 4	2, 200	53. 7	61. 8
17	4. 7	5. 4	250	18. 1	20. 8	2, 300	54. 9	63. 2
18	4. 9	5. 6	260	18. 4	21. 2	2, 400	56. 0	64. 5
19	5. 0	5. 7	270	18. 8	21. 6	2, 500	57. 2	65. 8
20	5. 1	5. 9	280	19. 1	22. 0	2, 600	58. 3	67. 2
21	5. 2	6. 0	290	19. 5	22. 4	2, 700	59. 4	68. 4
22	5. 4	6. 2	300	19. 8	22. 8	2, 800	60. 5	69. 7
23	5. 5	6. 3	310	20. 1	23. 2	2, 900	61. 6	70. 9
24	5. 6	6. 5	320	20. 5	23. 6	3, 000	62. 7	72. 1
25	5. 7	6. 6	330	20. 8	23. 9	3, 100	63. 7	73. 3
26	5. 8	6. 7	340	21. 1	24. 3	3, 200	64. 7	74. 5
27	5. 9	6. 8	350	21. 4	24. 6	3, 300	65. 7	75. 7
28	6. 1	7. 0	360	21. 7	25. 0	3, 400	66. 7	76. 8
29	6. 2	7. 1	370	22. 0	25. 3	3, 500	67. 7	77. 9
30	6. 3	7. 2	380	22. 3	25. 7	3, 600	68. 6	79. 0
31	6. 4	7. 3	390	22. 6	26. 0	3, 700	69. 6	80. 1
32	6. 5	7. 5	400	22. 9	26. 3	3, 800	70. 5	81. 2
33	6. 6	7. 6	410	23. 2	26. 7	3, 900	71. 4	82. 2
34	6. 7	7. 7	420	23. 4	27. 0	4, 000	72. 4	83. 3
35	6. 8	7. 8	430	23. 7	27. 3	4, 100	73. 3	84. 3
36	6. 9	7. 9	440	24. 0	27. 6	4, 200	74. 1	85. 4
37	7. 0	8. 0	450	24. 3	27. 9	4, 300	75. 0	86. 4
38	7. 1	8. 1	460	24. 5	28. 2	4, 400	75. 9	87. 4
39	7. 1	8. 2	470	24. 8	28. 6	4, 500	76. 7	88. 3
40	7. 2	8. 3	480	25. 1	28. 9	4, 600	77. 6	89. 3
41	7. 3	8. 4	490	25. 3	29. 2	4, 700	78. 4	90. 3
42	7. 4	8. 5	500	25. 6	29. 4	4, 800	79. 3	91. 2
43	7. 5	8. 6	520	26. 1	30. 0	4, 900	80. 1	92. 2
44	7. 6	8. 7	540	26. 6	30. 6	5, 000	80. 9	93. 1
45	7. 7	8. 8	560	27. 1	31. 2	6, 000	88. 6	102. 0
46	7. 8	8. 9	580	27. 6	31. 7	7, 000	95. 7	110. 2
47	7. 8	9. 0	600	28. 0	32. 3	8, 000	102. 3	117. 8
48	7. 9	9. 1	620	28. 5	32. 8	9, 000	108. 5	124. 9
49	8. 0	9. 2	640	28. 9	33. 3	10, 000	114. 4	131. 7
50	8. 1	9. 3	660	29. 4	33. 8	15, 000	140. 1	161. 3
55	8. 5	9. 8	680	29. 8	34. 3	20, 000	161. 8	186. 3
60	8. 9	10. 2	700	30. 3	34. 8	25, 000	180. 9	208. 2
65	9. 2	10. 6	720	30. 7	35. 3	30, 000	198. 1	228. 1
70	9. 6	11. 0	740	31. 1	35. 8	35, 000	214. 0	246. 4
75	9. 9	11. 4	760	31. 5	36. 3	40, 000	228. 8	263. 4
80	10. 2	11. 8	780	31. 9	36. 8	45, 000	242. 7	279. 4
85	10. 5	12. 1	800	32. 4	37. 3	50, 000	255. 8	294. 5
90	10. 9	12. 5	820	32. 8	37. 7	60, 000	280. 2	322. 6
95	11. 2	12. 8	840	33. 2	38. 2	70, 000	302. 7	348. 4
100	11. 4	13. 2	860	33. 5	38. 6	80, 000	323. 6	372. 5
105	11. 7	13. 5	880	33. 9	39. 1	90, 000	343. 2	395. 1
110	12. 0	13. 8	900	34. 3	39. 5	100, 000	361. 8	416. 5
115	12. 3	14. 1	920	34. 7	39. 9	200, 000	511. 6	589. 0

TABLE 9

Distance by Vertical Angle

Angle	Difference in feet between height of object and height of eye of observer												Angle
	50	60	70	80	90	100	110	120	130	140	160	180	
° ′	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	° ′
0 10	2.55	3.01	3.45	3.89	4.31	4.72	5.11	5.51	5.89	6.26	6.99	7.68	0 10
0 11	2.36	2.79	3.20	3.61	4.01	4.40	4.78	5.15	5.51	5.87	6.57	7.24	0 11
0 12	2.19	2.59	2.99	3.37	3.75	4.12	4.48	4.83	5.18	5.53	6.19	6.84	0 12
0 13	2.04	2.42	2.80	3.16	3.52	3.87	4.21	4.55	4.89	5.21	5.85	6.48	0 13
0 14	1.91	2.27	2.62	2.97	3.31	3.64	3.97	4.30	4.61	4.93	5.54	6.14	0 14
0 15	1.79	2.13	2.46	2.79	3.12	3.43	3.75	4.06	4.36	4.66	5.25	5.83	0 15
0 20	1.37	1.64	1.90	2.16	2.42	2.68	2.93	3.18	3.43	3.68	4.16	4.64	0 20
0 25	1.11	1.33	1.54	1.76	1.97	2.18	2.39	2.60	2.81	3.02	3.42	3.83	0 25
0 30	0.93	1.11	1.29	1.47	1.65	1.83	2.01	2.19	2.37	2.54	2.89	3.24	0 30
0 35	0.80	0.96	1.11	1.27	1.43	1.58	1.74	1.89	2.05	2.20	2.51	2.81	0 35
0 40	0.70	0.84	0.98	1.12	1.26	1.39	1.53	1.67	1.81	1.94	2.21	2.48	0 40
0 45	0.62	0.74	0.87	0.99	1.12	1.24	1.36	1.48	1.61	1.73	1.97	2.21	0 45
0 50	0.56	0.67	0.78	0.89	1.01	1.12	1.23	1.34	1.45	1.56	1.78	2.00	0 50
0 55	0.51	0.61	0.72	0.82	0.92	1.02	1.12	1.22	1.32	1.43	1.63	1.83	0 55
1 00	0.47	0.56	0.65	0.75	0.84	0.93	1.02	1.12	1.21	1.30	1.49	1.67	1 00
1 10	0.40	0.48	0.56	0.64	0.72	0.80	0.88	0.96	1.04	1.12	1.28	1.44	1 10
1 20	0.35	0.42	0.49	0.56	0.63	0.70	0.77	0.84	0.91	0.98	1.12	1.26	1 20
1 30	0.31	0.38	0.44	0.50	0.56	0.63	0.69	0.75	0.81	0.87	1.00	1.12	1 30
1 40	0.28	0.34	0.39	0.45	0.51	0.57	0.62	0.67	0.73	0.79	0.90	1.02	1 40
1 50	0.26	0.31	0.36	0.42	0.47	0.52	0.56	0.61	0.66	0.72	0.82	0.92	1 50
2 00	0.24	0.28	0.33	0.38	0.43	0.47	0.53	0.57	0.62	0.65	0.77	0.85	2 00
2 15	0.21	0.26	0.30	0.34	0.38	0.42	0.46	0.50	0.54	0.57	0.65	0.73	2 15
2 30	0.19	0.23	0.26	0.30	0.34	0.38	0.41	0.45	0.49	0.53	0.60	0.68	2 30
2 45	0.17	0.21	0.24	0.27	0.31	0.34	0.37	0.41	0.44	0.48	0.55	0.62	2 45
3 00	0.16	0.19	0.22	0.25	0.28	0.31	0.35	0.38	0.41	0.44	0.50	0.56	3 00
3 20	0.14	0.17	0.20	0.22	0.25	0.28	0.31	0.34	0.37	0.39	0.45	0.51	3 20
3 40	0.13	0.15	0.18	0.21	0.23	0.26	0.28	0.31	0.33	0.36	0.41	0.46	3 40
4 00	0.12	0.14	0.16	0.19	0.21	0.24	0.26	0.28	0.31	0.33	0.38	0.42	4 00
4 20	0.11	0.13	0.15	0.17	0.19	0.22	0.24	0.26	0.28	0.30	0.35	0.39	4 20
4 40	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.32	0.36	4 40
5 00		0.11	0.13	0.15	0.17	0.19	0.21	0.22	0.25	0.26	0.30	0.34	5 00
5 20		0.11	0.12	0.14	0.16	0.17	0.19	0.21	0.23	0.25	0.28	0.32	5 20
5 40		0.10	0.12	0.13	0.15	0.17	0.18	0.20	0.21	0.23	0.27	0.30	5 40
6 00			0.11	0.12	0.14	0.15	0.17	0.19	0.20	0.22	0.25	0.28	6 00
6 20			0.10	0.12	0.13	0.15	0.16	0.18	0.19	0.21	0.23	0.27	6 20
6 40			0.10	0.11	0.12	0.14	0.15	0.17	0.18	0.19	0.22	0.25	6 40
7 00				0.10	0.12	0.13	0.14	0.16	0.17	0.18	0.21	0.24	7 00
7 20				0.10	0.12	0.13	0.14	0.15	0.17	0.18	0.20	0.23	7 20
7 40				0.10	0.11	0.12	0.13	0.15	0.16	0.17	0.19	0.22	7 40
8 00					0.10	0.11	0.13	0.14	0.15	0.16	0.19	0.21	8 00
8 20					0.10	0.11	0.13	0.13	0.14	0.15	0.18	0.20	8 20
8 40						0.11	0.12	0.13	0.14	0.15	0.17	0.19	8 40
9 00						0.10	0.12	0.12	0.14	0.14	0.17	0.19	9 00
9 30						0.10	0.11	0.12	0.13	0.14	0.16	0.18	9 30
10 00							0.10	0.11	0.12	0.13	0.15	0.17	10 00
10 30							0.10	0.11	0.11	0.12	0.14	0.16	10 30
11 00								0.10	0.11	0.12	0.13	0.15	11 00
11 30								0.10	0.11	0.12	0.13	0.14	11 30
12 00								0.10	0.10	0.11	0.12	0.14	12 00
12 30									0.10	0.11	0.12	0.14	12 30
13 00										0.10	0.11	0.12	13 00
13 30										0.10	0.11	0.12	13 30
14 00											0.10	0.12	14 00
14 30											0.10	0.12	14 30
15 00												0.11	15 00
16 00												0.10	16 00
17 00													17 00
18 00													18 00
19 00													19 00
20 00													20 00

TABLE 9

Distance by Vertical Angle

Angle	Difference in feet between height of object and height of eye of observer												Angle
	200	250	300	350	400	600	800	1,000	1,200	1,400	1,600	1,800	
° ′	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	° ′
0 10	8. 36	9. 95	11. 44	12. 83	14. 15	18. 87	22. 96	26. 61	29. 95	33. 04	35. 93	38. 65	0 10
0 11	7. 89	9. 44	10. 88	12. 24	13. 51	18. 17	22. 20	25. 81	29. 12	32. 18	35. 05	37. 76	0 11
0 12	7. 47	8. 96	10. 36	11. 69	12. 95	17. 50	21. 47	25. 05	28. 32	31. 36	34. 21	36. 90	0 12
0 13	7. 08	8. 52	9. 88	11. 17	12. 41	16. 87	20. 78	24. 31	27. 55	30. 56	33. 39	36. 06	0 13
0 14	6. 72	8. 11	9. 43	10. 69	11. 90	16. 26	20. 12	23. 60	26. 80	29. 79	32. 61	35. 24	0 14
0 15	6. 39	7. 74	9. 02	10. 25	11. 41	15. 69	19. 48	22. 91	26. 08	29. 03	31. 81	34. 44	0 15
0 20	5. 11	6. 25	7. 35	8. 41	9. 43	13. 26	16. 72	19. 91	22. 88	25. 67	28. 31	30. 83	0 20
0 25	4. 23	5. 20	6. 15	7. 07	7. 98	11. 39	14. 53	17. 47	20. 23	22. 85	25. 35	27. 73	0 25
0 30	3. 59	4. 44	5. 27	6. 08	6. 87	9. 92	12. 78	15. 48	18. 04	20. 48	22. 82	25. 08	0 30
0 35	3. 11	3. 86	4. 60	5. 32	6. 02	8. 77	11. 37	13. 84	16. 22	18. 49	20. 69	22. 81	0 35
0 40	2. 75	3. 42	4. 07	4. 72	5. 36	7. 88	10. 21	12. 49	14. 69	16. 81	18. 87	20. 86	0 40
0 45	2. 45	3. 06	3. 65	4. 23	4. 81	7. 06	9. 25	11. 35	13. 39	15. 37	17. 30	19. 17	0 45
0 50	2. 22	2. 76	3. 30	3. 84	4. 36	6. 43	8. 44	10. 40	12. 30	14. 14	15. 95	17. 71	0 50
0 55	2. 03	2. 52	3. 02	3. 51	3. 99	5. 90	7. 77	9. 58	11. 35	13. 09	14. 78	16. 44	0 55
1 00	1. 86	2. 32	2. 77	3. 23	3. 67	5. 44	7. 18	8. 87	10. 53	12. 15	13. 75	15. 31	1 00
1 10	1. 60	2. 00	2. 39	2. 78	3. 17	4. 71	6. 23	7. 72	9. 19	10. 63	12. 05	13. 45	1 10
1 20	1. 40	1. 75	2. 10	2. 44	2. 79	4. 15	5. 50	6. 82	8. 14	9. 43	10. 71	11. 97	1 20
1 30	1. 25	1. 56	1. 87	2. 18	2. 48	3. 71	4. 91	6. 11	7. 29	8. 46	9. 62	10. 77	1 30
1 40	1. 12	1. 41	1. 68	1. 96	2. 25	3. 35	4. 45	5. 52	6. 60	7. 66	8. 73	9. 78	1 40
1 50	1. 03	1. 28	1. 53	1. 79	2. 04	3. 04	4. 05	5. 05	6. 02	7. 01	7. 98	8. 95	1 50
2 00	0. 95	1. 17	1. 41	1. 64	1. 89	2. 80	3. 72	4. 64	5. 55	6. 44	7. 35	8. 25	2 00
2 15	0. 81	1. 04	1. 25	1. 46	1. 66	2. 49	3. 32	4. 14	4. 95	5. 77	6. 56	7. 36	2 15
2 30	0. 75	0. 94	1. 13	1. 32	1. 50	2. 25	2. 99	3. 73	4. 47	5. 20	5. 93	6. 66	2 30
2 45	0. 68	0. 86	1. 03	1. 19	1. 37	2. 05	2. 72	3. 40	4. 07	4. 74	5. 41	6. 07	2 45
3 00	0. 63	0. 78	0. 94	1. 10	1. 25	1. 88	2. 50	3. 12	3. 74	4. 35	4. 97	5. 58	3 00
3 20	0. 56	0. 71	0. 85	0. 99	1. 12	1. 69	2. 35	2. 81	3. 37	3. 92	4. 48	5. 03	3 20
3 40	0. 51	0. 64	0. 77	0. 90	1. 02	1. 54	2. 04	2. 56	3. 06	3. 57	4. 08	4. 58	3 40
4 00	0. 47	0. 59	0. 70	0. 82	0. 94	1. 41	1. 88	2. 35	2. 81	3. 27	3. 74	4. 21	4 00
4 20	0. 43	0. 54	0. 65	0. 76	0. 87	1. 30	1. 73	2. 17	2. 59	3. 02	3. 46	3. 88	4 20
4 40	0. 40	0. 50	0. 60	0. 70	0. 81	1. 21	1. 61	2. 01	2. 41	2. 81	3. 21	3. 61	4 40
5 00	0. 38	0. 47	0. 57	0. 66	0. 75	1. 13	1. 50	1. 88	2. 25	2. 62	3. 00	3. 37	5 00
5 20	0. 35	0. 44	0. 53	0. 61	0. 71	1. 05	1. 41	1. 76	2. 11	2. 46	2. 81	3. 16	5 20
5 40	0. 33	0. 42	0. 50	0. 58	0. 66	0. 99	1. 32	1. 65	1. 98	2. 32	2. 65	2. 97	5 40
6 00	0. 31	0. 39	0. 47	0. 55	0. 62	0. 94	1. 25	1. 56	1. 88	2. 19	2. 50	2. 81	6 00
6 20	0. 29	0. 37	0. 45	0. 52	0. 59	0. 89	1. 19	1. 48	1. 78	2. 07	2. 37	2. 66	6 20
6 40	0. 28	0. 35	0. 42	0. 49	0. 56	0. 85	1. 13	1. 41	1. 69	1. 97	2. 25	2. 53	6 40
7 00	0. 26	0. 33	0. 40	0. 47	0. 53	0. 80	1. 07	1. 34	1. 61	1. 87	2. 14	2. 41	7 00
7 20	0. 26	0. 32	0. 38	0. 44	0. 51	0. 76	1. 02	1. 28	1. 53	1. 78	2. 04	2. 30	7 20
7 40	0. 24	0. 31	0. 37	0. 43	0. 49	0. 73	0. 97	1. 22	1. 47	1. 71	1. 95	2. 19	7 40
8 00	0. 23	0. 29	0. 35	0. 41	0. 46	0. 70	0. 94	1. 17	1. 41	1. 64	1. 87	2. 10	8 00
8 20	0. 22	0. 28	0. 34	0. 39	0. 45	0. 67	0. 90	1. 12	1. 35	1. 57	1. 80	2. 02	8 20
8 40	0. 22	0. 27	0. 32	0. 38	0. 43	0. 64	0. 86	1. 08	1. 30	1. 51	1. 73	1. 94	8 40
9 00	0. 21	0. 26	0. 31	0. 36	0. 41	0. 62	0. 83	1. 04	1. 24	1. 46	1. 66	1. 87	9 00
9 30	0. 20	0. 24	0. 29	0. 35	0. 39	0. 59	0. 78	0. 98	1. 18	1. 37	1. 57	1. 77	9 30
10 00	0. 19	0. 24	0. 28	0. 33	0. 37	0. 56	0. 75	0. 93	1. 12	1. 30	1. 49	1. 68	10 00
10 30	0. 17	0. 22	0. 26	0. 31	0. 35	0. 53	0. 71	0. 89	1. 06	1. 24	1. 42	1. 60	10 30
11 00	0. 17	0. 21	0. 25	0. 30	0. 34	0. 51	0. 67	0. 85	1. 01	1. 19	1. 35	1. 53	11 00
11 30	0. 16	0. 20	0. 24	0. 28	0. 32	0. 48	0. 65	0. 81	0. 97	1. 13	1. 29	1. 46	11 30
12 00	0. 16	0. 19	0. 23	0. 27	0. 31	0. 47	0. 61	0. 77	0. 92	1. 08	1. 24	1. 39	12 00
12 30	0. 14	0. 19	0. 23	0. 26	0. 30	0. 44	0. 59	0. 74	0. 89	1. 04	1. 19	1. 33	12 30
13 00	0. 14	0. 18	0. 22	0. 25	0. 28	0. 42	0. 57	0. 71	0. 85	0. 99	1. 14	1. 28	13 00
13 30	0. 14	0. 18	0. 20	0. 24	0. 27	0. 41	0. 55	0. 68	0. 82	0. 96	1. 09	1. 23	13 30
14 00	0. 13	0. 16	0. 20	0. 23	0. 26	0. 40	0. 53	0. 66	0. 79	0. 92	1. 05	1. 19	14 00
14 30	0. 13	0. 16	0. 19	0. 22	0. 25	0. 38	0. 50	0. 63	0. 76	0. 88	1. 02	1. 15	14 30
15 00	0. 12	0. 15	0. 19	0. 22	0. 24	0. 37	0. 49	0. 61	0. 73	0. 85	0. 98	1. 10	15 00
16 00	0. 11	0. 14	0. 17	0. 20	0. 22	0. 34	0. 45	0. 57	0. 69	0. 80	0. 92	1. 03	16 00
17 00	0. 10	0. 14	0. 16	0. 19	0. 21	0. 32	0. 42	0. 53	0. 65	0. 75	0. 86	0. 97	17 00
18 00		0. 13	0. 16	0. 17	0. 20	0. 30	0. 40	0. 50	0. 61	0. 70	0. 81	0. 91	18 00
19 00		0. 13	0. 14	0. 17	0. 18	0. 28	0. 38	0. 48	0. 57	0. 67	0. 76	0. 85	19 00
20 00		0. 12	0. 13	0. 16	0. 18	0. 27	0. 36	0. 45	0. 54	0. 63	0. 72	0. 81	20 00

TABLE 9

Distance by Vertical Angle

Angle	Difference in feet between height of object and height of eye of observer												Angle
	2,000	2,200	2,400	2,600	2,800	3,000	3,200	3,400	3,600	3,800	4,000	4,200	
° ′	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	° ′
0 10	41.2	43.7	46.1	48.3	50.5	52.6	54.7	56.6	58.6	60.5	62.3	64.1	0 10
0 11	40.3	42.8	45.1	47.4	49.6	51.7	53.7	55.7	57.6	59.5	61.3	63.1	0 11
0 12	39.5	41.9	44.2	46.5	48.6	50.7	52.8	54.7	56.7	58.5	60.3	62.1	0 12
0 13	38.6	41.0	43.3	45.6	47.7	49.8	51.8	53.8	55.7	57.6	59.4	61.2	0 13
0 14	37.8	40.2	42.5	44.7	46.9	48.9	50.9	52.9	54.8	56.7	58.5	60.2	0 14
0 15	37.0	39.3	41.6	43.9	46.0	48.1	50.1	52.0	53.9	55.7	57.5	59.3	0 15
0 20	33.2	35.5	37.8	39.9	42.0	44.0	45.9	47.8	49.7	51.5	53.2	55.0	0 20
0 25	30.0	32.2	34.4	36.4	38.4	40.4	42.2	44.1	45.9	47.6	49.3	51.0	0 25
0 30	27.2	29.3	31.4	33.3	35.3	37.1	39.0	40.7	42.5	44.2	45.8	47.5	0 30
0 35	24.9	26.8	28.8	30.7	32.5	34.3	36.0	37.7	39.4	41.1	42.7	44.2	0 35
0 40	22.8	24.7	26.5	28.3	30.1	31.8	33.4	35.1	36.7	38.3	39.8	41.3	0 40
0 45	21.0	22.8	24.5	26.2	27.9	29.5	31.1	32.7	34.2	35.8	37.2	38.7	0 45
0 50	19.4	21.1	22.8	24.4	26.0	27.5	29.1	30.6	32.0	33.5	34.9	36.3	0 50
0 55	18.1	19.7	21.2	22.8	24.3	25.8	27.2	28.7	30.1	31.5	32.8	34.2	0 55
1 00	16.9	18.4	19.8	21.3	22.7	24.2	25.6	26.9	28.3	29.6	31.0	32.3	1 00
1 10	14.8	16.2	17.5	18.9	20.2	21.5	22.7	24.0	25.2	26.5	27.7	28.9	1 10
1 20	13.2	14.5	15.7	16.9	18.1	19.3	20.4	21.6	22.7	23.9	25.0	26.1	1 20
1 30	11.9	13.0	14.1	15.2	16.3	17.4	18.5	19.6	20.6	21.7	22.7	23.7	1 30
1 40	10.8	11.9	12.9	13.9	14.9	15.9	16.9	17.9	18.9	19.8	20.8	21.8	1 40
1 50	9.9	10.9	11.8	12.7	13.7	14.6	15.5	16.4	17.4	18.3	19.2	20.1	1 50
2 00	9.1	10.0	10.9	11.8	12.6	13.5	14.4	15.2	16.1	16.9	17.7	18.6	2 00
2 15	8.2	9.0	9.8	10.5	11.3	12.1	12.9	13.7	14.4	15.2	16.0	16.7	2 15
2 30	7.4	8.1	8.8	9.5	10.3	11.0	11.7	12.4	13.1	13.8	14.5	15.2	2 30
2 45	6.7	7.4	8.1	8.7	9.4	10.0	10.7	11.3	12.0	12.6	13.3	13.9	2 45
3 00	6.2	6.8	7.4	8.0	8.6	9.2	9.8	10.4	11.0	11.6	12.2	12.8	3 00
3 20	5.6	6.1	6.7	7.2	7.8	8.3	8.9	9.4	10.0	10.5	11.0	11.6	3 20
3 40	5.1	5.6	6.1	6.6	7.1	7.6	8.1	8.6	9.1	9.6	10.1	10.6	3 40
4 00	4.7	5.1	5.6	6.1	6.5	7.0	7.4	7.9	8.3	8.8	9.3	9.7	4 00
4 20	4.3	4.7	5.2	5.6	6.0	6.5	6.9	7.3	7.7	8.1	8.6	9.0	4 20
4 40	4.0	4.4	4.8	5.2	5.6	6.0	6.4	6.8	7.2	7.6	8.0	8.4	4 40
5 00	3.7	4.1	4.5	4.9	5.2	5.6	6.0	6.3	6.7	7.1	7.4	7.8	5 00
5 20	3.5	3.9	4.2	4.6	4.9	5.3	5.6	5.9	6.3	6.6	7.0	7.3	5 20
5 40	3.3	3.6	4.0	4.3	4.6	4.9	5.3	5.6	5.9	6.3	6.6	6.9	5 40
6 00	3.1	3.4	3.7	4.1	4.4	4.7	5.0	5.3	5.6	5.9	6.2	6.5	6 00
6 20	3.0	3.2	3.5	3.8	4.1	4.4	4.7	5.0	5.3	5.6	5.9	6.2	6 20
6 40	2.8	3.1	3.4	3.6	3.9	4.2	4.5	4.8	5.0	5.3	5.6	5.9	6 40
7 00	2.7	2.9	3.2	3.5	3.7	4.0	4.3	4.5	4.8	5.1	5.3	5.6	7 00
7 20	2.6	2.8	3.1	3.3	3.6	3.8	4.1	4.3	4.6	4.8	5.1	5.3	7 20
7 40	2.4	2.7	2.9	3.2	3.4	3.7	3.9	4.1	4.4	4.6	4.9	5.1	7 40
8 00	2.3	2.6	2.8	3.0	3.3	3.5	3.7	4.0	4.2	4.4	4.7	4.9	8 00
8 20	2.2	2.5	2.7	2.9	3.1	3.4	3.6	3.8	4.0	4.3	4.5	4.7	8 20
8 40	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.7	3.9	4.1	4.3	4.5	8 40
9 00	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.4	9 00
9 30	2.0	2.2	2.4	2.6	2.8	2.9	3.1	3.3	3.5	3.7	3.9	4.1	9 30
10 00	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.5	3.7	3.9	10 00
10 30	1.8	2.0	2.1	2.3	2.5	2.7	2.8	3.0	3.2	3.4	3.5	3.7	10 30
11 00	1.7	1.9	2.0	2.2	2.4	2.5	2.7	2.9	3.0	3.2	3.4	3.5	11 00
11 30	1.6	1.8	1.9	2.1	2.3	2.4	2.6	2.7	2.9	3.1	3.2	3.4	11 30
12 00	1.5	1.7	1.9	2.0	2.2	2.3	2.5	2.6	2.8	2.9	3.1	3.2	12 00
12 30	1.5	1.6	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.8	3.0	3.1	12 30
13 00	1.4	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.7	2.9	3.0	13 00
13 30	1.4	1.5	1.6	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.7	2.9	13 30
14 00	1.3	1.4	1.6	1.7	1.8	2.0	2.1	2.2	2.4	2.5	2.6	2.8	14 00
14 30	1.3	1.4	1.5	1.7	1.8	1.9	2.0	2.2	2.3	2.4	2.5	2.7	14 30
15 00	1.2	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.2	2.3	2.5	2.6	15 00
16 00	1.1	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.1	2.2	2.3	2.4	16 00
17 00	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.3	17 00
18 00	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	18 00
19 00	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	19 00
20 00	0.9	1.0	1.1	1.2	1.3	1.4	1.4	1.5	1.6	1.7	1.8	1.9	20 00

TABLE 9

Distance by Vertical Angle

Angle	Difference in feet between height of object and height of eye of observer												Angle
	4,400	4,600	4,800	5,000	5,200	5,400	5,600	5,800	6,000	6,200	6,400	6,600	
° ′	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	° ′
0 10	65. 8	67. 5	69. 2	70. 8	72. 4	74. 0	75. 5	77. 1	78. 5	80. 0	81. 5	82. 9	0 10
0 11	64. 8	66. 5	68. 2	69. 8	71. 4	73. 0	74. 5	76. 0	77. 5	79. 0	80. 4	81. 9	0 11
0 12	63. 8	65. 5	67. 2	68. 8	70. 4	72. 0	73. 5	75. 0	76. 5	78. 0	79. 4	80. 8	0 12
0 13	62. 9	64. 6	66. 2	67. 9	69. 4	71. 0	72. 5	74. 1	75. 5	77. 0	78. 4	79. 9	0 13
0 14	61. 9	63. 6	65. 3	66. 9	68. 5	70. 0	71. 6	73. 1	74. 6	76. 0	77. 5	78. 9	0 14
0 15	61. 0	62. 7	64. 3	66. 0	67. 5	69. 1	70. 6	72. 1	73. 6	75. 1	76. 5	77. 9	0 15
0 20	56. 6	58. 3	59. 9	61. 5	63. 1	64. 6	66. 1	67. 6	69. 0	70. 5	71. 9	73. 3	0 20
0 25	52. 7	54. 3	55. 9	57. 4	58. 9	60. 4	61. 9	63. 4	64. 8	66. 2	67. 6	69. 0	0 25
0 30	49. 1	50. 6	52. 2	53. 7	55. 2	56. 6	58. 1	59. 5	60. 9	62. 3	63. 6	65. 0	0 30
0 35	45. 8	47. 3	48. 8	50. 3	51. 7	53. 2	54. 6	55. 9	57. 3	58. 7	60. 0	61. 3	0 35
0 40	42. 8	44. 3	45. 8	47. 2	48. 6	50. 0	51. 3	52. 7	54. 0	55. 3	56. 6	57. 9	0 40
0 45	40. 2	41. 6	43. 0	44. 4	45. 7	47. 1	48. 4	49. 7	51. 0	52. 3	53. 5	54. 8	0 45
0 50	37. 7	39. 1	40. 5	41. 8	43. 1	44. 4	45. 7	47. 0	48. 2	49. 5	50. 7	51. 9	0 50
0 55	35. 5	36. 9	38. 2	39. 5	40. 7	42. 0	43. 2	44. 5	45. 7	46. 9	48. 1	49. 2	0 55
1 00	33. 5	34. 8	36. 1	37. 3	38. 6	39. 8	41. 0	42. 2	43. 3	44. 5	45. 7	46. 8	1 00
1 10	30. 1	31. 3	32. 4	33. 6	34. 7	35. 9	37. 0	38. 1	39. 2	40. 3	41. 4	42. 5	1 10
1 20	27. 2	28. 3	29. 4	30. 5	31. 5	32. 6	33. 6	34. 7	35. 7	36. 7	37. 7	38. 7	1 20
1 30	24. 8	25. 8	26. 8	27. 8	28. 8	29. 8	30. 8	31. 7	32. 7	33. 6	34. 6	35. 5	1 30
1 40	22. 7	23. 7	24. 6	25. 5	26. 5	27. 4	28. 3	29. 2	30. 1	31. 0	31. 9	32. 8	1 40
1 50	20. 9	21. 8	22. 7	23. 6	24. 4	25. 3	26. 2	27. 0	27. 9	28. 7	29. 5	30. 4	1 50
2 00	19. 4	20. 2	21. 1	21. 9	22. 7	23. 5	24. 3	25. 1	25. 9	26. 7	27. 5	28. 3	2 00
2 15	17. 5	18. 2	19. 0	19. 7	20. 5	21. 2	21. 9	22. 7	23. 4	24. 1	24. 9	25. 6	2 15
2 30	15. 9	16. 6	17. 3	17. 9	18. 6	19. 3	20. 0	20. 7	21. 3	22. 0	22. 7	23. 3	2 30
2 45	14. 5	15. 2	15. 8	16. 4	17. 1	17. 7	18. 3	19. 0	19. 6	20. 2	20. 8	21. 4	2 45
3 00	13. 4	14. 0	14. 6	15. 2	15. 7	16. 3	16. 9	17. 5	18. 1	18. 7	19. 2	19. 8	3 00
3 20	12. 1	12. 7	13. 2	13. 7	14. 3	14. 8	15. 3	15. 9	16. 4	16. 9	17. 4	18. 0	3 20
3 40	11. 1	11. 6	12. 0	12. 5	13. 0	13. 5	14. 0	14. 5	15. 0	15. 5	16. 0	16. 4	3 40
4 00	10. 2	10. 6	11. 1	11. 5	12. 0	12. 4	12. 9	13. 3	13. 8	14. 2	14. 7	15. 1	4 00
4 20	9. 4	9. 8	10. 3	10. 7	11. 1	11. 5	11. 9	12. 3	12. 8	13. 2	13. 6	14. 0	4 20
4 40	8. 8	9. 1	9. 5	9. 9	10. 3	10. 7	11. 1	11. 5	11. 9	12. 3	12. 7	13. 1	4 40
5 00	8. 2	8. 5	8. 9	9. 3	9. 7	10. 0	10. 4	10. 7	11. 1	11. 5	11. 8	12. 2	5 00
5 20	7. 7	8. 0	8. 4	8. 7	9. 1	9. 4	9. 7	10. 1	10. 4	10. 8	11. 1	11. 5	5 20
5 40	7. 2	7. 6	7. 9	8. 2	8. 5	8. 9	9. 2	9. 5	9. 8	10. 2	10. 5	10. 8	5 40
6 00	6. 8	7. 1	7. 5	7. 8	8. 1	8. 4	8. 7	9. 0	9. 3	9. 6	9. 9	10. 2	6 00
6 20	6. 5	6. 8	7. 1	7. 4	7. 6	7. 9	8. 2	8. 5	8. 8	9. 1	9. 4	9. 7	6 20
6 40	6. 2	6. 4	6. 7	7. 0	7. 3	7. 5	7. 8	8. 1	8. 4	8. 7	8. 9	9. 2	6 40
7 00	5. 9	6. 1	6. 4	6. 7	6. 9	7. 2	7. 5	7. 7	8. 0	8. 2	8. 5	8. 8	7 00
7 20	5. 6	5. 8	6. 1	6. 4	6. 6	6. 9	7. 1	7. 4	7. 6	7. 9	8. 1	8. 4	7 20
7 40	5. 4	5. 6	5. 8	6. 1	6. 3	6. 6	6. 8	7. 0	7. 3	7. 5	7. 8	8. 0	7 40
8 00	5. 1	5. 4	5. 6	5. 8	6. 1	6. 3	6. 5	6. 8	7. 0	7. 2	7. 4	7. 7	8 00
8 20	4. 9	5. 1	5. 4	5. 6	5. 8	6. 0	6. 3	6. 5	6. 7	6. 9	7. 1	7. 4	8 20
8 40	4. 7	4. 9	5. 2	5. 4	5. 6	5. 8	6. 0	6. 2	6. 4	6. 7	6. 9	7. 1	8 40
9 00	4. 6	4. 8	5. 0	5. 2	5. 4	5. 6	5. 8	6. 0	6. 2	6. 4	6. 6	6. 8	9 00
9 30	4. 3	4. 5	4. 7	4. 9	5. 1	5. 3	5. 5	5. 7	5. 9	6. 1	6. 3	6. 5	9 30
10 00	4. 1	4. 3	4. 5	4. 7	4. 8	5. 0	5. 2	5. 4	5. 6	5. 8	5. 9	6. 1	10 00
10 30	3. 9	4. 1	4. 2	4. 4	4. 6	4. 8	5. 0	5. 1	5. 3	5. 5	5. 7	5. 8	10 30
11 00	3. 7	3. 9	4. 1	4. 2	4. 4	4. 6	4. 7	4. 9	5. 1	5. 2	5. 4	5. 6	11 00
11 30	3. 6	3. 7	3. 9	4. 0	4. 2	4. 4	4. 5	4. 7	4. 8	5. 0	5. 2	5. 3	11 30
12 00	3. 4	3. 6	3. 7	3. 9	4. 0	4. 2	4. 2	4. 5	4. 6	4. 8	4. 9	5. 1	12 00
12 30	3. 3	3. 4	3. 6	3. 7	3. 9	4. 0	4. 1	4. 3	4. 4	4. 6	4. 7	4. 9	12 30
13 00	3. 1	3. 3	3. 4	3. 6	3. 7	3. 8	4. 0	4. 1	4. 3	4. 4	4. 6	4. 7	13 00
13 30	3. 0	3. 1	3. 3	3. 4	3. 6	3. 7	3. 8	4. 0	4. 1	4. 2	4. 4	4. 5	13 30
14 00	2. 9	3. 0	3. 2	3. 3	3. 4	3. 6	3. 7	3. 8	4. 0	4. 1	4. 2	4. 3	14 00
14 30	2. 8	2. 9	3. 0	3. 2	3. 3	3. 4	3. 6	3. 7	3. 8	3. 9	4. 1	4. 2	14 30
15 00	2. 7	2. 8	2. 9	3. 1	3. 2	3. 3	3. 4	3. 6	3. 7	3. 8	3. 9	4. 0	15 00
16 00	2. 5	2. 6	2. 8	2. 9	3. 0	3. 1	3. 2	3. 3	3. 4	3. 6	3. 7	3. 8	16 00
17 00	2. 4	2. 5	2. 6	2. 7	2. 8	2. 9	3. 0	3. 1	3. 2	3. 3	3. 4	3. 6	17 00
18 00	2. 2	2. 3	2. 4	2. 5	2. 6	2. 7	2. 8	2. 9	3. 0	3. 1	3. 2	3. 3	18 00
19 00	2. 1	2. 2	2. 3	2. 4	2. 5	2. 6	2. 7	2. 8	2. 9	3. 0	3. 1	3. 1	19 00
20 00	2. 0	2. 1	2. 2	2. 3	2. 3	2. 4	2. 5	2. 6	2. 7	2. 8	2. 9	3. 0	20 00

TABLE 9

Distance by Vertical Angle

Angle	Difference in feet between height of object and height of eye of observer												Angle
	6,800	7,000	7,500	8,000	8,500	9,000	9,500	10,000	10,500	11,000	11,500	12,000	
° ′	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	° ′
0 10	84.3	85.7	89.0	92.3	95.5	98.5	101.5	104.4	107.3	110.1	112.8	115.4	0 10
0 11	83.3	84.6	88.0	91.3	94.4	97.5	100.5	103.4	106.2	109.0	111.7	114.4	0 11
0 12	82.2	83.6	87.0	90.2	93.4	96.5	99.4	102.4	105.2	108.0	110.7	113.3	0 12
0 13	81.2	82.6	86.0	89.2	92.4	95.4	98.4	101.3	104.2	106.9	109.6	112.3	0 13
0 14	80.3	81.6	85.0	88.2	91.4	94.4	97.4	100.3	103.1	105.9	108.6	111.2	0 14
0 15	79.3	80.7	84.0	87.2	90.4	93.4	96.4	99.3	102.1	104.9	107.6	110.2	0 15
0 20	74.6	76.0	79.3	82.5	85.6	88.6	91.5	94.4	97.2	100.0	102.6	105.3	0 20
0 25	70.3	71.6	74.9	78.0	81.1	84.1	87.0	89.8	92.6	95.3	98.0	100.6	0 25
0 30	66.3	67.6	70.8	73.9	76.9	79.9	82.7	85.5	88.2	90.9	93.5	96.1	0 30
0 35	62.6	63.9	67.0	70.0	73.0	75.9	78.7	81.5	84.2	86.8	89.4	91.9	0 35
0 40	59.2	60.4	63.5	66.5	69.4	72.2	75.0	77.7	80.3	82.9	85.5	88.0	0 40
0 45	56.0	57.2	60.2	63.1	66.0	68.7	71.5	74.1	76.7	79.3	81.8	84.2	0 45
0 50	53.1	54.3	57.2	60.0	62.8	65.5	68.2	70.8	73.3	75.8	78.3	80.7	0 50
0 55	50.4	51.6	54.4	57.2	59.9	62.5	65.1	67.7	70.2	72.6	75.0	77.4	0 55
1 00	47.9	49.0	51.8	54.5	57.1	59.7	62.3	64.7	67.2	69.6	72.0	74.3	1 00
1 10	43.5	44.6	47.2	49.7	52.2	54.7	57.1	59.5	61.8	64.1	66.3	68.6	1 10
1 20	39.7	40.7	43.2	45.6	48.0	50.3	52.6	54.8	57.0	59.2	61.4	63.5	1 20
1 30	36.5	37.4	39.7	42.0	44.2	46.4	48.6	50.8	52.9	55.0	57.0	59.0	1 30
1 40	33.7	34.6	36.7	38.9	41.0	43.1	45.1	47.2	49.2	51.2	53.1	55.1	1 40
1 50	31.2	32.0	34.1	36.1	38.1	40.1	42.1	44.0	45.9	47.8	49.7	51.5	1 50
2 00	29.1	29.9	31.8	33.7	35.6	37.5	39.3	41.2	43.0	44.8	46.6	48.3	2 00
2 15	26.3	27.0	28.8	30.6	32.3	34.1	35.8	37.5	39.2	40.9	42.5	44.2	2 15
2 30	24.0	24.7	26.3	28.0	29.6	31.2	32.8	34.4	35.9	37.5	39.1	40.6	2 30
2 45	22.1	22.7	24.2	25.7	27.2	28.7	30.2	31.7	33.2	34.6	36.1	37.5	2 45
3 00	20.4	21.0	22.4	23.8	25.2	26.6	28.0	29.4	30.8	32.1	33.5	34.8	3 00
3 20	18.5	19.0	20.3	21.6	22.9	24.2	25.5	26.7	28.0	29.3	30.5	31.8	3 20
3 40	16.9	17.4	18.7	19.8	21.0	22.2	23.4	24.5	25.7	26.9	28.0	29.2	3 40
4 00	15.6	16.0	17.1	18.2	19.3	20.4	21.5	22.6	23.7	24.8	25.9	27.0	4 00
4 20	14.4	14.8	15.9	16.9	17.9	19.0	20.0	21.0	22.0	23.0	24.0	25.0	4 20
4 40	13.4	13.8	14.8	15.8	16.7	17.7	18.6	19.6	20.5	21.5	22.4	23.4	4 40
5 00	12.6	12.9	13.8	14.7	15.6	16.5	17.4	18.3	19.2	20.1	21.0	21.9	5 00
5 20	11.8	12.1	13.0	13.9	14.7	15.5	16.4	17.2	18.1	18.9	19.8	20.6	5 20
5 40	11.1	11.4	12.3	13.1	13.9	14.7	15.5	16.3	17.1	17.9	18.6	19.4	5 40
6 00	10.5	10.8	11.6	12.3	13.1	13.9	14.6	15.4	16.1	16.9	17.6	18.4	6 00
6 20	10.0	10.3	11.0	11.7	12.4	13.2	13.9	14.6	15.3	16.0	16.7	17.5	6 20
6 40	9.5	9.8	10.4	11.1	11.8	12.5	13.2	13.9	14.6	15.2	15.9	16.6	6 40
7 00	9.0	9.3	10.0	10.6	11.3	11.9	12.6	13.2	13.9	14.5	15.2	15.8	7 00
7 20	8.6	8.9	9.5	10.1	10.8	11.4	12.0	12.6	13.3	13.9	14.5	15.1	7 20
7 40	8.3	8.5	9.1	9.7	10.3	10.9	11.5	12.1	12.7	13.3	13.9	14.5	7 40
8 00	7.9	8.1	8.7	9.3	9.9	10.4	11.0	11.6	12.2	12.7	13.3	13.9	8 00
8 20	7.6	7.8	8.4	8.9	9.5	10.0	10.6	11.1	11.7	12.2	12.8	13.3	8 20
8 40	7.3	7.5	8.0	8.6	9.1	9.6	10.2	10.7	11.2	11.8	12.3	12.8	8 40
9 00	7.0	7.2	7.7	8.3	8.8	9.3	9.8	10.3	10.8	11.3	11.8	12.3	9 00
9 30	6.7	6.9	7.3	7.8	8.3	8.8	9.3	9.8	10.3	10.7	11.2	11.7	9 30
10 00	6.3	6.5	7.0	7.4	7.9	8.4	8.8	9.3	9.7	10.2	10.7	11.1	10 00
10 30	6.0	6.2	6.6	7.1	7.5	7.9	8.4	8.8	9.3	9.7	10.1	10.6	10 30
11 00	5.7	5.9	6.3	6.7	7.2	7.6	8.0	8.4	8.8	9.3	9.7	10.1	11 00
11 30	5.5	5.6	6.0	6.4	6.8	7.3	7.6	8.0	8.5	8.8	9.3	9.7	11 30
12 00	5.3	5.4	5.8	6.2	6.6	6.9	7.3	7.7	8.1	8.5	8.9	9.2	12 00
12 30	5.0	5.2	5.6	5.9	6.3	6.7	7.0	7.4	7.8	8.1	8.5	8.9	12 30
13 00	4.8	5.0	5.3	5.7	6.0	6.4	6.7	7.1	7.5	7.8	8.2	8.5	13 00
13 30	4.6	4.8	5.1	5.5	5.8	6.1	6.5	6.8	7.2	7.5	7.9	8.2	13 30
14 00	4.5	4.6	4.9	5.3	5.6	5.9	6.2	6.6	6.9	7.2	7.6	7.9	14 00
14 30	4.3	4.4	4.8	5.1	5.4	5.7	6.0	6.3	6.7	7.0	7.3	7.6	14 30
15 00	4.2	4.3	4.6	4.9	5.2	5.5	5.8	6.1	6.4	6.7	7.0	7.3	15 00
16 00	3.9	4.0	4.3	4.6	4.9	5.2	5.4	5.7	6.0	6.3	6.6	6.9	16 00
17 00	3.7	3.8	4.0	4.3	4.6	4.8	5.1	5.4	5.6	5.9	6.2	6.5	17 00
18 00	3.4	3.5	3.8	4.1	4.2	4.5	4.8	5.1	5.3	5.6	5.8	6.1	18 00
19 00	3.2	3.3	3.6	3.8	4.1	4.3	4.5	4.8	5.0	5.2	5.5	5.7	19 00
20 00	3.1	3.2	3.4	3.6	3.8	4.1	4.3	4.5	4.7	5.0	5.2	5.4	20 00

TABLE 10
Direction and Speed of True Wind in Units of Ship's Speed

Apparent wind speed	Difference between the heading and apparent wind direction 0°		10°		20°		30°		40°		Apparent wind speed
	°		°		°		°		°		
0.0	180	1.00	180	1.00	180	1.00	180	1.00	180	1.00	0.0
0.1	180	0.90	179	0.90	178	0.91	177	0.91	176	0.93	0.1
0.2	180	0.80	178	0.80	175	0.81	173	0.83	171	0.86	0.2
0.3	180	0.70	176	0.71	172	0.73	169	0.76	166	0.79	0.3
0.4	180	0.60	173	0.61	168	0.64	163	0.68	160	0.74	0.4
0.5	180	0.50	170	0.51	162	0.56	156	0.62	152	0.70	0.5
0.6	180	0.40	166	0.42	155	0.48	148	0.57	144	0.66	0.6
0.7	180	0.30	159	0.33	145	0.42	138	0.53	136	0.65	0.7
0.8	180	0.20	147	0.25	132	0.37	127	0.50	127	0.64	0.8
0.9	180	0.10	126	0.19	117	0.34	116	0.50	118	0.66	0.9
1.0	calm	0.00	95	0.17	100	0.35	105	0.52	110	0.68	1.0
1.1	0	0.10	66	0.21	85	0.38	95	0.55	103	0.72	1.1
1.2	0	0.20	49	0.28	73	0.43	86	0.60	96	0.78	1.2
1.3	0	0.30	39	0.36	64	0.50	79	0.66	90	0.84	1.3
1.4	0	0.40	33	0.45	57	0.57	73	0.73	85	0.90	1.4
1.5	0	0.50	29	0.54	51	0.66	68	0.81	81	0.98	1.5
1.6	0	0.60	26	0.64	47	0.74	64	0.89	78	1.05	1.6
1.7	0	0.70	24	0.74	44	0.83	61	0.97	75	1.13	1.7
1.8	0	0.80	22	0.83	42	0.93	58	1.06	72	1.22	1.8
1.9	0	0.90	21	0.93	40	1.02	56	1.15	70	1.30	1.9
2.0	0	1.00	20	1.03	38	1.11	54	1.24	68	1.39	2.0
2.5	0	1.50	17	1.52	32	1.60	47	1.71	60	1.85	2.5
3.0	0	2.00	15	2.02	29	2.09	43	2.19	56	2.32	3.0
3.5	0	2.50	14	2.52	28	2.58	41	2.68	53	2.81	3.5
4.0	0	3.00	13	3.02	26	3.08	39	3.17	51	3.30	4.0
4.5	0	3.50	13	3.52	25	3.58	38	3.67	50	3.79	4.5
5.0	0	4.00	12	4.02	25	4.08	37	4.16	49	4.28	5.0
6.0	0	5.00	12	5.02	24	5.07	36	5.16	47	5.27	6.0
7.0	0	6.00	12	6.02	23	6.07	35	6.15	46	6.27	7.0
8.0	0	7.00	11	7.02	23	7.07	34	7.15	45	7.26	8.0
9.0	0	8.00	11	8.02	22	8.07	34	8.15	44	8.26	9.0
10.0	0	9.00	11	9.02	22	9.06	33	9.15	44	9.26	10.0

Apparent wind speed	50°		60°		70°		80°		90°		Apparent wind speed
	°		°		°		°		°		
0.0	180	1.00	180	1.00	180	1.00	180	1.00	180	1.00	0.0
0.1	175	0.94	175	0.95	174	0.97	174	0.99	174	1.00	0.1
0.2	170	0.88	169	0.92	169	0.95	168	0.99	169	1.02	0.2
0.3	164	0.84	163	0.89	163	0.94	163	0.99	163	1.04	0.3
0.4	158	0.80	157	0.87	156	0.94	157	1.01	158	1.08	0.4
0.5	151	0.78	150	0.87	150	0.95	152	1.04	153	1.12	0.5
0.6	143	0.77	143	0.87	145	0.97	147	1.07	149	1.17	0.6
0.7	136	0.77	137	0.89	139	1.01	142	1.12	145	1.22	0.7
0.8	128	0.78	131	0.92	134	1.05	138	1.17	141	1.28	0.8
0.9	121	0.81	125	0.95	129	1.09	134	1.22	138	1.35	0.9
1.0	115	0.85	120	1.00	125	1.15	130	1.29	135	1.41	1.0
1.1	109	0.89	115	1.05	121	1.21	127	1.35	132	1.49	1.1
1.2	104	0.95	111	1.11	118	1.27	124	1.42	130	1.56	1.2
1.3	99	1.01	107	1.18	114	1.34	121	1.50	128	1.64	1.3
1.4	95	1.08	104	1.25	112	1.42	119	1.57	126	1.72	1.4
1.5	92	1.15	101	1.32	109	1.49	117	1.65	124	1.80	1.5
1.6	89	1.23	98	1.40	107	1.57	115	1.73	122	1.89	1.6
1.7	86	1.31	96	1.48	105	1.65	113	1.82	120	1.97	1.7
1.8	84	1.39	94	1.56	103	1.73	111	1.90	119	2.06	1.8
1.9	81	1.47	92	1.65	101	1.82	110	1.99	118	2.15	1.9
2.0	79	1.56	90	1.73	100	1.91	108	2.07	117	2.24	2.0
2.5	72	2.01	83	2.18	94	2.35	103	2.53	112	2.69	2.5
3.0	68	2.48	79	2.65	89	2.82	99	2.99	108	3.16	3.0
3.5	65	2.96	76	3.12	87	3.29	96	3.47	106	3.64	3.5
4.0	63	3.44	74	3.61	84	3.78	94	3.95	104	4.12	4.0
4.5	61	3.93	72	4.09	83	4.26	93	4.44	103	4.61	4.5
5.0	60	4.42	71	4.58	81	4.75	92	4.93	101	5.10	5.0
6.0	58	5.41	69	5.57	79	5.74	90	5.91	99	6.08	6.0
7.0	57	6.40	68	6.56	78	6.72	88	6.90	98	7.07	7.0
8.0	56	7.40	67	7.55	77	7.72	87	7.89	97	8.06	8.0
9.0	55	8.39	66	8.54	76	8.71	86	8.88	96	9.06	9.0
10.0	55	9.39	65	9.54	76	9.70	86	9.88	96	10.01	10.0

TABLE 10

Direction and Speed of True Wind in Units of Ship's Speed

Apparent wind speed	Difference between the heading and apparent wind direction										Apparent wind speed
	90°		100°		110°		120°		130°		
	°		°		°		°		°		
0. 0	180	1. 00	180	1. 00	180	1. 00	180	1. 00	180	1. 00	0. 0
0. 1	174	1. 00	174	1. 02	175	1. 04	175	1. 05	176	1. 07	0. 1
0. 2	169	1. 02	169	1. 05	170	1. 08	171	1. 11	172	1. 14	0. 2
0. 3	163	1. 04	164	1. 09	166	1. 14	167	1. 18	169	1. 21	0. 3
0. 4	158	1. 08	160	1. 14	162	1. 20	164	1. 25	166	1. 29	0. 4
0. 5	153	1. 12	156	1. 19	158	1. 26	161	1. 32	164	1. 38	0. 5
0. 6	149	1. 17	152	1. 25	155	1. 33	158	1. 40	162	1. 46	0. 6
0. 7	145	1. 22	148	1. 32	152	1. 40	156	1. 48	160	1. 55	0. 7
0. 8	141	1. 28	145	1. 38	149	1. 48	154	1. 56	158	1. 63	0. 8
0. 9	138	1. 35	143	1. 46	147	1. 56	152	1. 65	156	1. 72	0. 9
1. 0	135	1. 41	140	1. 53	145	1. 64	150	1. 73	155	1. 81	1. 0
1. 1	132	1. 49	138	1. 61	143	1. 72	148	1. 82	154	1. 90	1. 1
1. 2	130	1. 56	136	1. 69	141	1. 81	147	1. 91	153	2. 00	1. 2
1. 3	128	1. 64	134	1. 77	140	1. 89	146	2. 00	152	2. 09	1. 3
1. 4	126	1. 72	132	1. 86	138	1. 98	145	2. 09	151	2. 18	1. 4
1. 5	124	1. 80	130	1. 94	137	2. 07	143	2. 18	150	2. 28	1. 5
1. 6	122	1. 89	129	2. 03	136	2. 16	142	2. 27	149	2. 37	1. 6
1. 7	120	1. 97	128	2. 12	135	2. 25	141	2. 36	148	2. 46	1. 7
1. 8	119	2. 06	127	2. 21	134	2. 34	141	2. 46	147	2. 56	1. 8
1. 9	118	2. 15	125	2. 30	133	2. 43	140	2. 55	147	2. 66	1. 9
2. 0	117	2. 24	124	2. 39	132	2. 52	139	2. 65	146	2. 75	2. 0
2. 5	112	2. 69	120	2. 85	128	2. 99	136	3. 12	144	3. 23	2. 5
3. 0	108	3. 16	117	3. 32	126	3. 47	134	3. 61	142	3. 72	3. 0
3. 5	106	3. 64	115	3. 80	124	3. 96	132	4. 09	140	4. 21	3. 5
4. 0	104	4. 12	113	4. 29	122	4. 44	131	4. 58	139	4. 71	4. 0
4. 5	103	4. 61	112	4. 78	121	4. 93	130	5. 07	138	5. 20	4. 5
5. 0	101	5. 10	111	5. 27	120	5. 42	129	5. 57	138	5. 69	5. 0
6. 0	99	6. 08	109	6. 25	118	6. 41	128	6. 56	137	6. 69	6. 0
7. 0	98	7. 07	108	7. 24	117	7. 40	127	7. 55	136	7. 68	7. 0
8. 0	97	8. 06	107	8. 23	116	8. 39	126	8. 54	135	8. 68	8. 0
9. 0	96	9. 06	106	9. 23	116	9. 39	125	9. 54	135	9. 67	9. 0
10. 0	96	10. 01	106	10. 22	115	10. 39	125	10. 54	134	10. 67	10. 0

	140°		150°		160°		170°		180°		
	°		°		°		°		°		
0. 0	180	1. 00	180	1. 00	180	1. 00	180	1. 00	180	1. 00	0. 0
0. 1	177	1. 08	177	1. 09	178	1. 09	179	1. 10	180	1. 10	0. 1
0. 2	174	1. 16	175	1. 18	177	1. 19	178	1. 20	180	1. 20	0. 2
0. 3	171	1. 24	173	1. 27	175	1. 29	178	1. 30	180	1. 30	0. 3
0. 4	169	1. 33	172	1. 36	174	1. 38	177	1. 40	180	1. 40	0. 4
0. 5	167	1. 42	170	1. 45	173	1. 48	177	1. 50	180	1. 50	0. 5
0. 6	165	1. 51	169	1. 55	173	1. 58	176	1. 60	180	1. 60	0. 6
0. 7	164	1. 60	168	1. 64	172	1. 68	176	1. 69	180	1. 70	0. 7
0. 8	162	1. 69	167	1. 74	171	1. 77	176	1. 79	180	1. 80	0. 8
0. 9	161	1. 79	166	1. 84	171	1. 87	175	1. 89	180	1. 90	0. 9
1. 0	160	1. 88	165	1. 93	170	1. 97	175	1. 99	180	2. 00	1. 0
1. 1	159	1. 97	164	2. 03	170	2. 07	175	2. 09	180	2. 10	1. 1
1. 2	158	2. 07	164	2. 13	169	2. 17	175	2. 19	180	2. 20	1. 2
1. 3	157	2. 16	163	2. 22	169	2. 27	174	2. 29	180	2. 30	1. 3
1. 4	157	2. 26	162	2. 32	168	2. 36	174	2. 39	180	2. 40	1. 4
1. 5	156	2. 36	162	2. 42	168	2. 46	174	2. 49	180	2. 50	1. 5
1. 6	155	2. 45	161	2. 52	168	2. 56	174	2. 59	180	2. 60	1. 6
1. 7	155	2. 55	161	2. 61	167	2. 66	174	2. 69	180	2. 70	1. 7
1. 8	154	2. 65	161	2. 71	167	2. 76	174	2. 79	180	2. 80	1. 8
1. 9	154	2. 74	160	2. 81	167	2. 86	173	2. 89	180	2. 90	1. 9
2. 0	153	2. 84	160	2. 91	167	2. 96	173	2. 99	180	3. 00	2. 0
2. 5	151	3. 33	158	3. 40	166	3. 46	173	3. 49	180	3. 50	2. 5
3. 0	150	3. 82	157	3. 90	165	3. 95	172	3. 99	180	4. 00	3. 0
3. 5	149	4. 31	157	4. 39	164	4. 45	172	4. 49	180	4. 50	3. 5
4. 0	148	4. 81	156	4. 89	164	4. 95	172	4. 99	180	5. 00	4. 0
4. 5	147	5. 31	155	5. 39	164	5. 45	172	5. 49	180	5. 50	4. 5
5. 0	146	5. 80	155	5. 89	163	5. 95	172	5. 99	180	6. 00	5. 0
6. 0	145	6. 80	154	6. 88	163	6. 95	171	6. 99	180	7. 00	6. 0
7. 0	145	7. 79	154	7. 88	162	7. 95	171	7. 99	180	8. 00	7. 0
8. 0	144	8. 79	153	8. 88	162	8. 95	171	8. 99	180	9. 00	8. 0
9. 0	144	9. 79	153	9. 88	162	9. 95	171	9. 99	180	10. 00	9. 0
10. 0	143	10. 78	153	10. 88	162	10. 95	171	10. 98	180	11. 00	10. 0

TABLE 11

Correction of Barometer Reading for Height Above Sea Level

All barometers. All values positive.

Height in feet	Outside temperature in degrees Fahrenheit													Height in feet
	−20°	−10°	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	
	Inches	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	
5	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	5
10	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	0. 01	10
15	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	15
20	0. 03	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	0. 02	20
25	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	25
30	0. 04	0. 04	0. 04	0. 04	0. 04	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	0. 03	30
35	0. 04	0. 04	0. 04	0. 04	0. 04	0. 04	0. 04	0. 04	0. 04	0. 04	0. 04	0. 04	0. 04	35
40	0. 05	0. 05	0. 05	0. 05	0. 05	0. 05	0. 04	0. 04	0. 04	0. 04	0. 04	0. 04	0. 04	40
45	0. 06	0. 06	0. 05	0. 05	0. 05	0. 05	0. 05	0. 05	0. 05	0. 05	0. 05	0. 05	0. 05	45
50	0. 06	0. 06	0. 06	0. 06	0. 06	0. 06	0. 06	0. 06	0. 05	0. 05	0. 05	0. 05	0. 05	50
55	0. 07	0. 07	0. 07	0. 07	0. 06	0. 06	0. 06	0. 06	0. 06	0. 06	0. 06	0. 06	0. 06	55
60	0. 08	0. 07	0. 07	0. 07	0. 07	0. 07	0. 07	0. 07	0. 06	0. 06	0. 06	0. 06	0. 06	60
65	0. 08	0. 08	0. 08	0. 08	0. 08	0. 07	0. 07	0. 07	0. 07	0. 07	0. 07	0. 07	0. 07	65
70	0. 09	0. 09	0. 09	0. 08	0. 08	0. 08	0. 08	0. 08	0. 08	0. 07	0. 07	0. 07	0. 07	70
75	0. 10	0. 09	0. 09	0. 09	0. 09	0. 09	0. 08	0. 08	0. 08	0. 08	0. 08	0. 08	0. 08	75
80	0. 10	0. 10	0. 10	0. 10	0. 09	0. 09	0. 09	0. 09	0. 09	0. 08	0. 08	0. 08	0. 08	80
85	0. 11	0. 11	0. 10	0. 10	0. 10	0. 10	0. 10	0. 09	0. 09	0. 09	0. 09	0. 09	0. 09	85
90	0. 11	0. 11	0. 11	0. 11	0. 11	0. 10	0. 10	0. 10	0. 10	0. 10	0. 09	0. 09	0. 09	90
95	0. 12	0. 12	0. 12	0. 11	0. 11	0. 11	0. 11	0. 10	0. 10	0. 10	0. 10	0. 10	0. 10	95
100	0. 13	0. 12	0. 12	0. 12	0. 12	0. 11	0. 11	0. 11	0. 11	0. 11	0. 10	0. 10	0. 10	100
105	0. 13	0. 13	0. 13	0. 13	0. 12	0. 12	0. 12	0. 12	0. 11	0. 11	0. 11	0. 11	0. 11	105
110	0. 14	0. 14	0. 13	0. 13	0. 13	0. 13	0. 12	0. 12	0. 12	0. 12	0. 11	0. 11	0. 11	110
115	0. 15	0. 14	0. 14	0. 14	0. 13	0. 13	0. 13	0. 13	0. 12	0. 12	0. 12	0. 12	0. 12	115
120	0. 15	0. 15	0. 15	0. 14	0. 14	0. 14	0. 13	0. 13	0. 13	0. 13	0. 12	0. 12	0. 12	120
125	0. 16	0. 16	0. 15	0. 15	0. 15	0. 14	0. 14	0. 14	0. 13	0. 13	0. 13	0. 13	0. 12	125

TABLE 12

Correction of Barometer Reading for Gravity

Mercurial barometers only.

Latitude	Correction	Latitude	Correction	Latitude	Correction	Latitude	Correction
°	*Inches*	°	*Inches*	°	*Inches*	°	*Inches*
0	−0. 08	25	−0. 05	50	+0. 01	75	+0. 07
5	−0. 08	30	−0. 04	55	+0. 03	80	+0. 07
10	−0. 08	35	−0. 03	60	+0. 04	85	+0. 08
15	−0. 07	40	−0. 02	65	+0. 05	90	+0. 08
20	−0. 06	45	0. 00	70	+0. 06		

TABLE 13

Correction of Barometer Reading for Temperature

Mercurial barometers only.

Temp. F	Height of barometer in inches								Temp. F
	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0	
°	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	°
−20	+0.12	+0.12	+0.13	+0.13	+0.13	+0.13	+0.14	+0.14	−20
18	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.13	18
16	0.11	0.11	0.12	0.12	0.12	0.12	0.12	0.13	16
14	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.12	14
12	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.11	12
−10	+0.10	+0.10	+0.10	+0.10	+0.10	+0.11	+0.11	+0.11	−10
8	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10	8
6	0.09	0.09	0.09	0.09	0.09	0.09	0.10	0.10	6
4	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.09	4
−2	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	−2
0	+0.07	+0.07	+0.07	+0.08	+0.08	+0.08	+0.08	+0.08	0
+2	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.08	+2
4	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.07	4
6	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	6
8	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	8
+10	+0.05	+0.05	+0.05	+0.05	+0.05	+0.05	+0.05	+0.05	+10
12	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	12
14	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	14
16	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04	16
18	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	18
+20	+0.02	+0.02	+0.02	+0.02	+0.02	+0.02	+0.02	+0.02	+20
22	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	22
24	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	24
26	+0.01	+0.01	+0.01	+0.01	+0.01	+0.01	+0.01	+0.01	26
28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	28
+30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	+30
32	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	32
34	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	34
36	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	36
38	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	38
+40	−0.03	−0.03	−0.03	−0.03	−0.03	−0.03	−0.03	−0.03	+40
42	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	42
44	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	44
46	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	46
48	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	48
+50	−0.05	−0.05	−0.06	−0.06	−0.06	−0.06	−0.06	−0.06	+50
52	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.07	52
54	0.06	0.06	0.07	0.07	0.07	0.07	0.07	0.07	54
56	0.07	0.07	0.07	0.07	0.07	0.07	0.08	0.08	56
58	0.07	0.07	0.08	0.08	0.08	0.08	0.08	0.08	58
+60	−0.08	−0.08	−0.08	−0.08	−0.08	−0.09	−0.09	−0.09	+60
62	0.08	0.08	0.09	0.09	0.09	0.09	0.09	0.09	62
64	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10	64
66	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10	66
68	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11	68
+70	−0.10	−0.10	−0.11	−0.11	−0.11	−0.11	−0.11	−0.12	+70
72	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	72
74	0.11	0.11	0.12	0.12	0.12	0.12	0.13	0.13	74
76	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.13	76
78	0.12	0.12	0.13	0.13	0.13	0.13	0.14	0.14	78
+80	−0.13	−0.13	−0.13	−0.13	−0.14	−0.14	−0.14	−0.14	+80
82	0.13	0.14	0.14	0.14	0.14	0.14	0.15	0.15	82
84	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.16	84
86	0.14	0.15	0.15	0.15	0.15	0.16	0.16	0.16	86
88	0.15	0.15	0.15	0.16	0.16	0.16	0.16	0.17	88
+90	−0.15	−0.16	−0.16	−0.16	−0.16	−0.17	−0.17	−0.17	+90
92	0.16	0.16	0.16	0.17	0.17	0.17	0.17	0.18	92
94	0.16	0.17	0.17	0.17	0.17	0.18	0.18	0.18	94
96	0.17	0.17	0.17	0.18	0.18	0.18	0.19	0.19	96
98	0.17	0.18	0.18	0.18	0.18	0.19	0.19	0.19	98
100	0.18	0.18	0.18	0.19	0.19	0.19	0.20	0.20	100

TABLE 14

Conversion Table for Millibars, Inches of Mercury, and Millimeters of Mercury

Millibars	Inches	Millimeters	Millibars	Inches	Millimeters	Millibars	Inches	Millimeters
900	26. 58	675. 1	960	28. 35	720. 1	1020	30. 12	765. 1
901	26. 61	675. 8	961	28. 38	720. 8	1021	30. 15	765. 8
902	26. 64	676. 6	962	28. 41	721. 6	1022	30. 18	766. 6
903	26. 67	677. 3	963	28. 44	722. 3	1023	30. 21	767. 3
904	26. 70	678. 1	964	28. 47	723. 1	1024	30. 24	768. 1
905	26. 72	678. 8	965	28. 50	723. 8	1025	30. 27	768. 8
906	26. 75	679. 6	966	28. 53	724. 6	1026	30. 30	769. 6
907	26. 78	680. 3	967	28. 56	725. 3	1027	30. 33	770. 3
908	26. 81	681. 1	968	28. 58	726. 1	1028	30. 36	771. 1
909	26. 84	681. 8	969	28. 61	726. 8	1029	30. 39	771. 8
910	26. 87	682. 6	970	28. 64	727. 6	1030	30. 42	772. 6
911	26. 90	683. 3	971	28. 67	728. 3	1031	30. 45	773. 3
912	26. 93	684. 1	972	28. 70	729. 1	1032	30. 47	774. 1
913	26. 96	684. 8	973	28. 73	729. 8	1033	30. 50	774. 8
914	26. 99	685. 6	974	28. 76	730. 6	1034	30. 53	775. 6
915	27. 02	686. 3	975	28. 79	731. 3	1035	30. 56	776. 3
916	27. 05	687. 1	976	28. 82	732. 1	1036	30. 59	777. 1
917	27. 08	687. 8	977	28. 85	732. 8	1037	30. 62	777. 8
918	27. 11	688. 6	978	28. 88	733. 6	1038	30. 65	778. 6
919	27. 14	689. 3	979	28. 91	734. 3	1039	30. 68	779. 3
920	27. 17	690. 1	980	28. 94	735. 1	1040	30. 71	780. 1
921	27. 20	690. 8	981	28. 97	735. 8	1041	30. 74	780. 8
922	27. 23	691. 6	982	29. 00	736. 6	1042	30. 77	781. 6
923	27. 26	692. 3	983	29. 03	737. 3	1043	30. 80	782. 3
924	27. 29	693. 1	984	29. 06	738. 1	1044	30. 83	783. 1
925	27. 32	693. 8	985	29. 09	738. 8	1045	30. 86	783. 8
926	27. 34	694. 6	986	29. 12	739. 6	1046	30. 89	784. 6
927	27. 37	695. 3	987	29. 15	740. 3	1047	30. 92	785. 3
928	27. 40	696. 1	988	29. 18	741. 1	1048	30. 95	786. 1
929	27. 43	696. 8	989	29. 21	741. 8	1049	30. 98	786. 8
930	27. 46	697. 6	990	29. 23	742. 6	1050	31. 01	787. 6
931	27. 49	698. 3	991	29. 26	743. 3	1051	31. 04	788. 3
932	27. 52	699. 1	992	29. 29	744. 1	1052	31. 07	789. 1
933	27. 55	699. 8	993	29. 32	744. 8	1053	31. 10	789. 8
934	27. 58	700. 6	994	29. 35	745. 6	1054	31. 12	790. 6
935	27. 61	701. 3	995	29. 38	746. 3	1055	31. 15	791. 3
936	27. 64	702. 1	996	29. 41	747. 1	1056	31. 18	792. 1
937	27. 67	702. 8	997	29. 44	747. 8	1057	31. 21	792. 8
938	27. 70	703. 6	998	29. 47	748. 6	1058	31. 24	793. 6
939	27. 73	704. 3	999	29. 50	749. 3	1059	31. 27	794. 3
940	27. 76	705. 1	1000	29. 53	750. 1	1060	31. 30	795. 1
941	27. 79	705. 8	1001	29. 56	750. 8	1061	31. 33	795. 8
942	27. 82	706. 6	1002	29. 59	751. 6	1062	31. 36	796. 6
943	27. 85	707. 3	1003	29. 62	752. 3	1063	31. 39	797. 3
944	27. 88	708. 1	1004	29. 65	753. 1	1064	31. 42	798. 1
945	27. 91	708. 8	1005	29. 68	753. 8	1065	31. 45	798. 8
946	27. 94	709. 6	1006	29. 71	754. 6	1066	31. 48	799. 6
947	27. 96	710. 3	1007	29. 74	755. 3	1067	31. 51	800. 3
948	27. 99	711. 1	1008	29. 77	756. 1	1068	31. 54	801. 1
949	28. 02	711. 8	1009	29. 80	756. 8	1069	31. 57	801. 8
950	28. 05	712. 6	1010	29. 83	757. 6	1070	31. 60	802. 6
951	28. 08	713. 3	1011	29. 85	758. 3	1071	31. 63	803. 3
952	28. 11	714. 1	1012	29. 88	759. 1	1072	31. 66	804. 1
953	28. 14	714. 8	1013	29. 91	759. 8	1073	31. 69	804. 8
954	28. 17	715. 6	1014	29. 94	760. 6	1074	31. 72	805. 6
955	28. 20	716. 3	1015	29. 97	761. 3	1075	31. 74	806. 3
956	28. 23	717. 1	1016	30. 00	762. 1	1076	31. 77	807. 1
957	28. 26	717. 8	1017	30. 03	762. 8	1077	31. 80	807. 8
958	28. 29	718. 6	1018	30. 06	763. 6	1078	31. 83	808. 6
959	28. 32	719. 3	1019	30. 09	764. 3	1079	31. 86	809. 3
960	28. 35	720. 1	1020	30. 12	765. 1	1080	31. 89	810. 1

TABLE 15

Conversion Tables for Thermometer Scales

F=Fahrenheit, C=Celsius (centigrade), K=Kelvin

F	C	K	F	C	K	C	F	K	K	F	C
°	°	°	°	°	°	°	°	°	°	°	°
−20	−28.9	244.3	+40	+4.4	277.6	−25	−13.0	248.2	250	−9.7	−23.2
19	28.3	244.8	41	5.0	278.2	24	11.2	249.2	251	7.9	22.2
18	27.8	245.4	42	5.6	278.7	23	9.4	250.2	252	6.1	21.2
17	27.2	245.9	43	6.1	279.3	22	7.6	251.2	253	4.3	20.2
16	26.7	246.5	44	6.7	279.8	21	5.8	252.2	254	2.5	19.2
−15	−26.1	247.0	+45	+7.2	280.4	−20	−4.0	253.2	255	−0.7	−18.2
14	25.6	247.6	46	7.8	280.9	19	2.2	254.2	256	+1.1	17.2
13	25.0	248.2	47	8.3	281.5	18	−0.4	255.2	257	2.9	16.2
12	24.4	248.7	48	8.9	282.0	17	+1.4	256.2	258	4.7	15.2
11	23.9	249.3	49	9.4	282.6	16	3.2	257.2	259	6.5	14.2
−10	−23.3	249.8	+50	+10.0	283.2	−15	+5.0	258.2	260	+8.3	−13.2
9	22.8	250.4	51	10.6	283.7	14	6.8	259.2	261	10.1	12.2
8	22.2	250.9	52	11.1	284.3	13	8.6	260.2	262	11.9	11.2
7	21.7	251.5	53	11.7	284.8	12	10.4	261.2	263	13.7	10.2
6	21.1	252.0	54	12.2	285.4	11	12.2	262.2	264	15.5	9.2
−5	−20.6	252.6	+55	+12.8	285.9	−10	+14.0	263.2	265	+17.3	−8.2
4	20.0	253.2	56	13.3	286.5	9	15.8	264.2	266	19.1	7.2
3	19.4	253.7	57	13.9	287.0	8	17.6	265.2	267	20.9	6.2
2	18.9	254.3	58	14.4	287.6	7	19.4	266.2	268	22.7	5.2
−1	18.3	254.8	59	15.0	288.2	6	21.2	267.2	269	24.5	4.2
0	−17.8	255.4	+60	+15.6	288.7	−5	+23.0	268.2	270	+26.3	−3.2
+1	17.2	255.9	61	16.1	289.3	4	24.8	269.2	271	28.1	2.2
2	16.7	256.5	62	16.7	289.8	3	26.6	270.2	272	29.9	1.2
3	16.1	257.0	63	17.2	290.4	2	28.4	271.2	273	31.7	−0.2
4	15.6	257.6	64	17.8	290.9	−1	30.2	272.2	274	33.5	+0.8
+5	−15.0	258.2	+65	+18.3	291.5	0	+32.0	273.2	275	+35.3	+1.8
6	14.4	258.7	66	18.9	292.0	+1	33.8	274.2	276	37.1	2.8
7	13.9	259.3	67	19.4	292.6	2	35.6	275.2	277	38.9	3.8
8	13.3	259.8	68	20.0	293.2	3	37.4	276.2	278	40.7	4.8
9	12.8	260.4	69	20.6	293.7	4	39.2	277.2	279	42.5	5.8
+10	−12.2	260.9	+70	+21.1	294.3	+5	+41.0	278.2	280	+44.3	+6.8
11	11.7	261.5	71	21.7	294.8	6	42.8	279.2	281	46.1	7.8
12	11.1	262.0	72	22.2	295.4	7	44.6	280.2	282	47.9	8.8
13	10.6	262.6	73	22.8	295.9	8	46.4	281.2	283	49.7	9.8
14	10.0	263.2	74	23.3	296.5	9	48.2	282.2	284	51.5	10.8
+15	−9.4	263.7	+75	+23.9	297.0	+10	+50.0	283.2	285	+53.3	+11.8
16	8.9	264.3	76	24.4	297.6	11	51.8	284.2	286	55.1	12.8
17	8.3	264.8	77	25.0	298.2	12	53.6	285.2	287	56.9	13.8
18	7.8	265.4	78	25.6	298.7	13	55.4	286.2	288	58.7	14.8
19	7.2	265.9	79	26.1	299.3	14	57.2	287.2	289	60.5	15.8
+20	−6.7	266.5	+80	+26.7	299.8	+15	+59.0	288.2	290	+62.3	+16.8
21	6.1	267.0	81	27.2	300.4	16	60.8	289.2	291	64.1	17.8
22	5.6	267.6	82	27.8	300.9	17	62.6	290.2	292	65.9	18.8
23	5.0	268.2	83	28.3	301.5	18	64.4	291.2	293	67.7	19.8
24	4.4	268.7	84	28.9	302.0	19	66.2	292.2	294	69.5	20.8
+25	−3.9	269.3	+85	+29.4	302.6	+20	+68.0	293.2	295	+71.3	+21.8
26	3.3	269.8	86	30.0	303.2	21	69.8	294.2	296	73.1	22.8
27	2.8	270.4	87	30.6	303.7	22	71.6	295.2	297	74.9	23.8
28	2.2	270.9	88	31.1	304.3	23	73.4	296.2	298	76.7	24.8
29	1.7	271.5	89	31.7	304.8	24	75.2	297.2	299	78.5	25.8
+30	−1.1	272.0	+90	+32.2	305.4	+25	+77.0	298.2	300	+80.3	+26.8
31	0.6	272.6	91	32.8	305.9	26	78.8	299.2	301	82.1	27.8
32	0.0	273.2	92	33.3	306.5	27	80.6	300.2	302	83.9	28.8
33	+0.6	273.7	93	33.9	307.0	28	82.4	301.2	303	85.7	29.8
34	1.1	274.3	94	34.4	307.6	29	84.2	302.2	304	87.5	30.8
+35	+1.7	274.8	+95	+35.0	308.2	+30	+86.0	303.2	305	+89.3	+31.8
36	2.2	275.4	96	35.6	308.7	31	87.8	304.2	306	91.1	32.8
37	2.8	275.9	97	36.1	309.3	32	89.6	305.2	307	92.9	33.8
38	3.3	276.5	98	36.7	309.8	33	91.4	306.2	308	94.7	34.8
39	3.9	277.0	99	37.2	310.4	34	93.2	307.2	309	96.5	35.8
+40	+4.4	277.6	+100	+37.8	310.9	+35	+95.0	308.2	310	+98.3	+36.8

TABLE 16

Relative Humidity

Dry-bulb temp. F	Difference between dry-bulb and wet-bulb temperatures														Dry-bulb temp. F
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	
°	%	%	%	%	%	%	%	%	%	%	%	%	%	%	°
−20	7														−20
18	14														18
16	21														16
14	27														14
12	32														12
−10	37														−10
8	41	2													8
6	45	9													6
4	49	16													4
−2	52	22													−2
0	56	28													0
+2	59	33	7												+2
4	62	37	14												4
6	64	42	20												6
8	67	46	25	5											8
+10	69	50	30	11											+10
12	71	53	35	17											12
14	73	56	40	23	7										14
16	76	60	44	28	13										16
18	77	62	48	33	19	4									18
+20	79	65	51	37	24	10									+20
22	81	68	55	42	29	16	4								22
24	83	70	58	45	33	21	10								24
26	85	73	61	49	38	26	15	4							26
28	86	75	64	53	42	31	20	10							28
+30	88	77	66	56	45	35	25	15	6						+30
32	89	79	69	59	49	39	30	20	11	2					32
34	90	81	71	62	52	43	34	25	16	8					34
36	91	82	73	64	55	47	38	29	21	13	5				36
38	91	83	74	66	58	50	42	33	25	18	10	2			38
+40	92	84	76	68	60	52	45	37	30	22	15	7			+40
42	92	84	77	69	62	54	47	40	33	26	19	12	5		42
44	92	85	78	70	63	56	49	43	36	29	23	17	10	4	44
46	93	86	79	72	65	58	52	45	39	32	26	20	14	8	46
48	93	86	79	73	66	60	54	47	41	35	29	24	18	12	48
+50	93	87	80	74	68	61	55	49	44	38	32	27	21	16	+50
52	94	87	81	75	69	63	57	51	46	40	35	29	24	19	52
54	94	88	82	76	70	64	59	53	48	42	37	32	27	22	54
56	94	88	82	77	71	65	60	55	50	44	39	35	30	25	56
58	94	88	83	77	72	67	61	56	51	46	42	37	32	28	58
+60	94	89	83	78	73	68	63	58	53	48	43	39	34	30	+60
62	95	89	84	79	74	69	64	59	54	50	45	41	37	32	62
64	95	89	84	79	74	70	65	60	56	51	47	43	38	34	64
66	95	90	85	80	75	71	66	61	57	53	49	44	40	36	66
68	95	90	85	81	76	71	67	63	58	54	50	46	42	38	68
+70	95	90	86	81	77	72	68	64	59	55	51	48	44	40	+70
72	95	91	86	82	77	73	69	65	61	57	53	49	45	42	72
74	95	91	86	82	78	74	69	65	62	58	54	50	47	43	74
76	95	91	87	82	78	74	70	66	63	59	55	51	48	45	76
78	96	91	87	83	79	75	71	67	63	60	56	53	49	46	78
+80	96	91	87	83	79	75	72	68	64	61	57	54	50	47	+80
82	96	92	88	84	80	76	72	69	65	62	58	55	52	48	82
84	96	92	88	84	80	76	73	69	66	62	59	56	53	49	84
86	96	92	88	84	81	77	73	70	67	63	60	57	54	51	86
88	96	92	88	85	81	77	74	71	67	64	61	58	55	52	88
+90	96	92	89	85	81	78	74	71	68	65	61	58	55	52	+90
92	96	92	89	85	82	78	75	72	68	65	62	59	56	53	92
94	96	93	89	85	82	79	75	72	69	66	63	60	57	54	94
96	96	93	89	86	82	79	76	73	70	67	64	61	58	55	96
98	96	93	89	86	83	79	76	73	70	67	64	61	59	56	98
+100	96	93	90	86	83	80	77	74	71	68	65	62	59	57	+100

TABLE 16
Relative Humidity

Dry-bulb temp. F	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	Dry-bulb temp. F
°	%	%	%	%	%	%	%	%	%	%	%	%	%	%	°
+46	2														+46
48	7	1													48
+50	10	5													+50
52	14	9	4												52
54	17	12	7	3											54
56	20	16	11	7	2										56
58	23	19	14	10	6	2									58
+60	26	21	17	13	9	5	1								+60
62	28	24	20	16	12	8	4	1							62
64	30	26	22	19	15	11	8	4							64
66	32	29	25	21	17	14	10	7	4						66
68	34	31	27	23	20	16	13	10	7	3					68
+70	36	33	29	26	22	19	16	12	9	6	3				+70
72	38	34	31	28	24	21	18	15	12	9	6	3			72
74	40	36	33	30	26	23	20	17	14	11	8	6	3		74
76	41	38	35	31	28	25	22	19	16	14	11	8	5	3	76
78	43	39	36	33	30	27	24	21	18	16	13	10	8	5	78
+80	44	41	38	35	32	29	26	23	20	18	15	13	10	8	+80
82	45	42	39	36	33	30	28	25	22	20	17	15	12	10	82
84	46	43	40	38	35	32	29	27	24	21	19	17	14	12	84
86	48	45	42	39	36	33	31	28	26	23	21	18	16	14	86
88	49	46	43	40	37	35	32	30	27	25	22	20	18	16	88
+90	50	47	44	41	39	36	34	31	29	26	24	22	19	17	+90
92	51	48	45	42	40	37	35	32	30	28	25	23	21	19	92
94	51	49	46	44	41	39	36	34	31	29	27	25	23	20	94
96	52	50	47	45	42	40	37	35	33	30	28	26	24	22	96
98	53	51	48	45	43	41	38	36	34	32	29	27	25	23	98
+100	54	51	49	46	44	42	39	37	35	33	31	29	27	25	+100

Difference between dry-bulb and wet-bulb temperatures

Dry-bulb temp. F	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	Dry-bulb temp. F
°	%	%	%	%	%	%	%	%	%	%	%	%	%	%	°
+78	3														+78
+80	5	3													+80
82	7	5	3	1											82
84	10	7	5	3	1										84
86	11	9	7	5	3	1									86
88	13	11	9	7	5	3	1								88
+90	15	13	11	9	7	5	3	1							+90
92	17	15	13	11	9	7	5	3	1						92
94	18	16	14	12	11	9	7	5	3	2					94
96	20	18	16	14	12	10	9	7	5	4	2				96
98	21	19	17	16	14	12	10	9	7	5	4	2	1		98
+100	23	21	19	17	15	14	12	10	9	7	5	4	2	1	+100

TABLE 17

Dew Point

Dry-bulb temp. F	Difference between dry-bulb and wet-bulb temperatures 1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	Dry-bulb temp. F
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
-20															-20
18	-52														18
16	45														16
14	39														14
12	34														12
-10	-29														-10
8	25	-75													8
6	22	50													6
4	18	39													4
-2	15	32													-2
0	-12	-26													0
+2	9	21	-49												+2
4	6	16	35												4
6	3	12	27												6
8	-1	9	20	-50											8
+10	+2	-5	-15	-34											+10
12	5	-2	10	24											12
14	7	+1	6	17											14
16	10	4	-2	11											16
18	12	7	+1	6											18
+20	+15	+10	+5	-2											+20
22	17	13	8	+2											22
24	20	16	11	6											24
26	22	18	14	10	+4	-4									26
28	24	21	17	13	8	+1	-8	-22							28
+30	+27	+24	+20	+16	+11	+6	-1	-12	-31						+30
32	29	26	23	19	15	10	+4	-4	16	-47					32
34	32	29	26	22	18	14	9	+2	-7	22					34
36	34	31	28	25	22	18	13	7	0	11	-30				36
38	36	33	31	28	25	21	17	12	+6	-2	14	-42			38
+40	+38	+35	+33	+30	+27	+24	+20	+16	+11	+4	-4	-18	-79		+40
42	40	38	35	33	30	27	23	19	15	10	+3	-7	23		42
44	42	40	37	35	32	29	26	23	19	14	9	+2	-9	-29	44
46	44	42	40	37	35	32	29	26	22	18	13	7	0	11	46
48	46	44	42	40	37	35	32	29	26	22	18	13	+6	-2	48
+50	+48	+46	+44	+42	+40	+37	+35	+32	+29	+25	+21	+17	+12	+5	+50
52	50	48	46	44	42	40	37	35	32	29	25	21	17	11	52
54	52	50	49	47	44	42	40	37	35	32	28	25	21	16	54
56	54	53	51	49	47	45	42	40	37	35	32	28	25	21	56
58	56	55	53	51	49	47	45	43	40	38	35	32	28	25	58
+60	+58	+57	+55	+53	+51	+49	+47	+45	+43	+40	+38	+35	+32	+28	+60
62	60	59	57	55	54	52	50	48	45	43	41	38	35	32	62
64	62	61	59	57	56	54	52	50	48	46	43	41	38	35	64
66	64	63	61	60	58	56	54	52	50	48	46	44	41	39	66
68	67	65	63	62	60	58	57	55	53	51	49	46	44	42	68
+70	+69	+67	+66	+64	+62	+61	+59	+57	+55	+53	+51	+49	+47	+45	+70
72	71	69	68	66	64	63	61	59	58	56	54	52	50	47	72
74	73	71	70	68	67	65	63	62	60	58	56	54	52	50	74
76	75	73	72	70	69	67	66	64	62	61	59	57	55	53	76
78	77	75	74	72	71	69	68	66	65	63	61	59	57	55	78
+80	+79	+77	+76	+74	+73	+72	+70	+68	+67	+65	+64	+62	+60	+58	+80
82	81	79	78	77	75	74	72	71	69	67	66	64	62	61	82
84	83	81	80	79	77	76	74	73	71	70	68	67	65	63	84
86	85	83	82	81	79	78	76	75	74	72	70	69	67	66	86
88	87	85	84	83	81	80	79	77	76	74	73	71	70	68	88
+90	+89	+87	+86	+85	+84	+82	+81	+79	+78	+76	+75	+73	+72	+70	+90
92	91	89	88	87	86	84	83	82	80	79	77	76	74	73	92
94	93	92	90	89	88	86	85	84	82	81	79	78	76	75	94
96	95	94	92	91	90	88	87	86	84	83	82	80	79	77	96
98	97	96	94	93	92	91	89	88	87	85	84	82	81	80	98
+100	+99	+98	+96	+95	+94	+93	+91	+90	+89	+87	+86	+85	+83	+82	+100

TABLE 17
Dew Point

Dry-bulb temp. F	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	Dry-bulb temp. F
	Difference between dry-bulb and wet-bulb temperatures														
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
+46	−36														+46
48	14	−45													48
+50	−3	−17	−78												+50
52	+4	−5	21												52
54	10	+3	−7	−25											54
56	16	10	+2	−8	−29										56
58	20	16	10	+2	−10	−34									58
+60	+25	+20	+15	+9	+1	−11	−39								+60
62	29	25	20	15	9	+1	−12	−45							62
64	32	29	25	20	15	9	0	−13	−52						64
66	36	33	29	25	21	15	+9	0	−14	−59					66
68	39	36	33	29	25	21	16	+9	0	−14	−68				68
+70	+42	+39	+36	+33	+30	+26	+21	+16	+9	0	−14	−76			+70
72	45	43	40	37	34	30	26	22	16	+10	+1	−14	−77		72
74	48	46	43	40	37	34	31	27	22	17	10	+1	−13	−70	74
76	51	48	46	44	41	38	35	31	27	23	17	11	+2	−12	76
78	53	51	49	47	44	41	38	35	32	28	23	18	11	+3	78
+80	+56	+54	+52	+50	+47	+45	+42	+39	+36	+32	+28	+24	+19	+12	+80
82	59	57	55	53	50	48	45	43	40	37	33	29	25	20	82
84	61	59	57	55	53	51	49	46	43	41	37	34	30	26	84
86	64	62	60	58	56	54	52	49	47	44	41	38	35	31	86
88	66	64	63	61	59	57	55	52	50	48	45	42	39	36	88
+90	+69	+67	+65	+63	+62	+60	+58	+55	+53	+51	+48	+46	+43	+40	+90
92	71	69	68	66	64	62	60	58	56	54	52	49	47	44	92
94	73	72	70	68	67	65	63	61	59	57	55	52	50	47	94
96	76	74	73	71	69	67	66	64	62	60	58	56	53	51	96
98	78	77	75	73	72	70	68	67	65	63	61	59	57	54	98
+100	+80	+79	+77	+76	+74	+73	+71	+69	+67	+66	+64	+62	+60	+57	+100

Dry-bulb temp. F	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	Dry-bulb temp. F
	Difference between dry-bulb and wet-bulb temperatures														
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
+76	−61														+76
78	−11	−53													78
+80	+4	−10	−45												+80
82	13	+5	−8	−39											82
84	20	14	+6	−6	−33										84
86	27	21	15	+7	−4	−28									86
88	32	27	22	16	+9	−2	−23								88
+90	+36	+33	+28	+24	+18	+10	0	−18							+90
92	41	37	34	30	25	19	+12	+2	−14						92
94	45	42	38	35	31	26	20	13	+4	−10					94
96	48	46	43	39	36	32	27	22	15	+6	−7	−43			96
98	52	49	47	44	40	37	33	28	23	17	+9	−4	−30		98
+100	+55	+53	+50	+47	+45	+41	+38	+34	+30	+25	+19	+11	0	−21	+100

TABLE 18
Speed Table for Measured Mile

Sec.	Minutes												Sec.
	1	2	3	4	5	6	7	8	9	10	11	12	
	Knots	*Knots*	*Knots*	*Knots*	*Knots*	*Knots*	*Knots*	*Knots*	*Knots*	*Knots*	*Knots*	*Knots*	
0	60.000	30.000	20.000	15.000	12.000	10.000	8.571	7.500	6.667	6.000	5.455	5.000	0
1	59.016	29.752	19.890	14.938	11.960	9.972	8.551	7.484	6.654	5.990	5.446	4.993	1
2	58.065	29.508	19.780	14.876	11.921	9.945	8.531	7.469	6.642	5.980	5.438	4.986	2
3	57.143	29.268	19.672	14.815	11.881	9.917	8.511	7.453	6.630	5.970	5.430	4.979	3
4	56.250	29.032	19.565	14.754	11.842	9.890	8.491	7.438	6.618	5.960	5.422	4.972	4
5	55.385	28.800	19.459	14.694	11.803	9.863	8.471	7.423	6.606	5.950	5.414	4.966	5
6	54.545	28.571	19.355	14.634	11.765	9.836	8.451	7.407	6.593	5.941	5.405	4.959	6
7	53.731	28.346	19.251	14.575	11.726	9.809	8.431	7.392	6.581	5.931	5.397	4.952	7
8	52.941	28.125	19.149	14.516	11.688	9.783	8.411	7.377	6.569	5.921	5.389	4.945	8
9	52.174	27.907	19.048	14.458	11.650	9.756	8.392	7.362	6.557	5.911	5.381	4.938	9
10	51.429	27.692	18.947	14.400	11.613	9.730	8.372	7.347	6.545	5.902	5.373	4.932	10
11	50.704	27.481	18.848	14.343	11.576	9.704	8.353	7.332	6.534	5.892	5.365	4.925	11
12	50.000	27.273	18.750	14.286	11.538	9.677	8.333	7.317	6.522	5.882	5.357	4.918	12
13	49.315	27.068	18.653	14.229	11.502	9.651	8.314	7.302	6.510	5.873	5.349	4.911	13
14	48.649	26.866	18.557	14.173	11.465	9.626	8.295	7.287	6.498	5.863	5.341	4.905	14
15	48.000	26.667	18.462	14.118	11.429	9.600	8.276	7.273	6.486	5.854	5.333	4.898	15
16	47.368	26.471	18.367	14.062	11.392	9.574	8.257	7.258	6.475	5.844	5.325	4.891	16
17	46.753	26.277	18.274	14.008	11.356	9.549	8.238	7.243	6.463	5.835	5.318	4.885	17
18	46.154	26.087	18.182	13.953	11.321	9.524	8.219	7.229	6.452	5.825	5.310	4.878	18
19	45.570	25.899	18.090	13.900	11.285	9.499	8.200	7.214	6.440	5.816	5.302	4.871	19
20	45.000	25.714	18.000	13.846	11.250	9.474	8.182	7.200	6.429	5.806	5.294	4.865	20
21	44.444	25.532	17.910	13.793	11.215	9.449	8.163	7.186	6.417	5.797	5.286	4.858	21
22	43.902	25.352	17.822	13.740	11.180	9.424	8.145	7.171	6.406	5.788	5.279	4.852	22
23	43.373	25.175	17.734	13.688	11.146	9.399	8.126	7.157	6.394	5.778	5.271	4.845	23
24	42.857	25.000	17.647	13.636	11.111	9.375	8.108	7.143	6.383	5.769	5.263	4.839	24
25	42.353	24.828	17.561	13.585	11.077	9.351	8.090	7.129	6.372	5.760	5.255	4.832	25
26	41.860	24.658	17.476	13.534	11.043	9.326	8.072	7.115	6.360	5.751	5.248	4.826	26
27	41.379	24.490	17.391	13.483	11.009	9.302	8.054	7.101	6.349	5.742	5.240	4.819	27
28	40.909	24.324	17.308	13.433	10.976	9.278	8.036	7.087	6.338	5.732	5.233	4.813	28
29	40.449	24.161	17.225	13.383	10.942	9.254	8.018	7.073	6.327	5.723	5.225	4.806	29
30	40.000	24.000	17.143	13.333	10.909	9.231	8.000	7.059	6.316	5.714	5.217	4.800	30
31	39.560	23.841	17.062	13.284	10.876	9.207	7.982	7.045	6.305	5.705	5.210	4.794	31
32	39.130	23.684	16.981	13.235	10.843	9.184	7.965	7.031	6.294	5.696	5.202	4.787	32
33	38.710	23.529	16.901	13.187	10.811	9.160	7.947	7.018	6.283	5.687	5.195	4.781	33
34	38.298	23.377	16.822	13.139	10.778	9.137	7.930	7.004	6.272	5.678	5.187	4.775	34
35	37.895	23.226	16.744	13.091	10.746	9.114	7.912	6.990	6.261	5.669	5.180	4.768	35
36	37.500	23.077	16.667	13.043	10.714	9.091	7.895	6.977	6.250	5.660	5.172	4.762	36
37	37.113	22.930	16.590	12.996	10.682	9.068	7.877	6.963	6.239	5.651	5.165	4.756	37
38	36.735	22.785	16.514	12.950	10.651	9.045	7.860	6.950	6.228	5.643	5.158	4.749	38
39	36.364	22.642	16.438	12.903	10.619	9.023	7.843	6.936	6.218	5.634	5.150	4.743	39
40	36.000	22.500	16.364	12.857	10.588	9.000	7.826	6.923	6.207	5.625	5.143	4.737	40
41	35.644	22.360	16.290	12.811	10.557	8.978	7.809	6.910	6.196	5.616	5.136	4.731	41
42	35.294	22.222	16.216	12.766	10.526	8.955	7.792	6.897	6.186	5.607	5.128	4.724	42
43	34.951	22.086	16.143	12.721	10.496	8.933	7.775	6.883	6.175	5.599	5.121	4.718	43
44	34.615	21.951	16.071	12.676	10.465	8.911	7.759	6.870	6.164	5.590	5.114	4.712	44
45	34.286	21.818	16.000	12.632	10.435	8.889	7.742	6.857	6.154	5.581	5.106	4.706	45
46	33.962	21.687	15.929	12.587	10.405	8.867	7.725	6.844	6.143	5.573	5.099	4.700	46
47	33.645	21.557	15.859	12.544	10.375	8.845	7.709	6.831	6.133	5.564	5.092	4.694	47
48	33.333	21.429	15.789	12.500	10.345	8.824	7.692	6.818	6.122	5.556	5.085	4.688	48
49	33.028	21.302	15.721	12.457	10.315	8.802	7.676	6.805	6.112	5.547	5.078	4.681	49
50	32.727	21.176	15.652	12.414	10.286	8.780	7.660	6.792	6.102	5.538	5.070	4.675	50
51	32.432	21.053	15.584	12.371	10.256	8.759	7.643	6.780	6.091	5.530	5.063	4.669	51
52	32.143	20.930	15.517	12.329	10.227	8.738	7.627	6.767	6.081	5.521	5.056	4.663	52
53	31.858	20.809	15.451	12.287	10.198	8.717	7.611	6.754	6.071	5.513	5.049	4.657	53
54	31.579	20.690	15.385	12.245	10.169	8.696	7.595	6.742	6.061	5.505	5.042	4.651	54
55	31.304	20.571	15.319	12.203	10.141	8.675	7.579	6.729	6.050	5.496	5.035	4.645	55
56	31.034	20.455	15.254	12.162	10.112	8.654	7.563	6.716	6.040	5.488	5.028	4.639	56
57	30.769	20.339	15.190	12.121	10.084	8.633	7.547	6.704	6.030	5.479	5.021	4.633	57
58	30.508	20.225	15.126	12.081	10.056	8.612	7.531	6.691	6.020	5.471	5.014	4.627	58
59	30.252	20.112	15.063	12.040	10.028	8.592	7.516	6.679	6.010	5.463	5.007	4.621	59
60	30.000	20.000	15.000	12.000	10.000	8.571	7.500	6.667	6.000	5.455	5.000	4.615	60
Sec.	1	2	3	4	5	6	7	8	9	10	11	12	Sec.

TABLE 19

Speed, Time, and Distance

Minutes	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	Minutes
	Speed in knots																
	Miles	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	
1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	1
2	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	2
3	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.4	0.4	0.4	3
4	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.3	0.4	0.4	0.4	0.5	0.5	0.5	4
5	0.0	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.7	5
6	0.0	0.1	0.2	0.2	0.2	0.3	0.4	0.4	0.4	0.5	0.6	0.6	0.6	0.7	0.8	0.8	6
7	0.1	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.5	0.6	0.6	0.7	0.8	0.8	0.9	0.9	7
8	0.1	0.1	0.2	0.3	0.3	0.4	0.5	0.5	0.6	0.7	0.7	0.8	0.9	0.9	1.0	1.1	8
9	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	9
10	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.8	0.9	1.0	1.1	1.2	1.2	1.3	10
11	0.1	0.2	0.3	0.4	0.5	0.6	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	11
12	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	12
13	0.1	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	13
14	0.1	0.2	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.3	1.4	1.5	1.6	1.8	1.9	14
15	0.1	0.2	0.4	0.5	0.6	0.8	0.9	1.0	1.1	1.2	1.4	1.5	1.6	1.8	1.9	2.0	15
16	0.1	0.3	0.4	0.5	0.7	0.8	0.9	1.1	1.2	1.3	1.5	1.6	1.7	1.9	2.0	2.1	16
17	0.1	0.3	0.4	0.6	0.7	0.8	1.0	1.1	1.3	1.4	1.6	1.7	1.8	2.0	2.1	2.3	17
18	0.2	0.3	0.4	0.6	0.8	0.9	1.0	1.2	1.4	1.5	1.6	1.8	2.0	2.1	2.2	2.4	18
19	0.2	0.3	0.5	0.6	0.8	1.0	1.1	1.3	1.4	1.6	1.7	1.9	2.1	2.2	2.4	2.5	19
20	0.2	0.3	0.5	0.7	0.8	1.0	1.2	1.3	1.5	1.7	1.8	2.0	2.2	2.3	2.5	2.7	20
21	0.2	0.4	0.5	0.7	0.9	1.0	1.2	1.4	1.6	1.8	1.9	2.1	2.3	2.4	2.6	2.8	21
22	0.2	0.4	0.6	0.7	0.9	1.1	1.3	1.5	1.6	1.8	2.0	2.2	2.4	2.6	2.8	2.9	22
23	0.2	0.4	0.6	0.8	1.0	1.2	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	23
24	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	24
25	0.2	0.4	0.6	0.8	1.0	1.2	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	25
26	0.2	0.4	0.6	0.9	1.1	1.3	1.5	1.7	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.5	26
27	0.2	0.4	0.7	0.9	1.1	1.4	1.6	1.8	2.0	2.2	2.5	2.7	2.9	3.2	3.4	3.6	27
28	0.2	0.5	0.7	0.9	1.2	1.4	1.6	1.9	2.1	2.3	2.6	2.8	3.0	3.3	3.5	3.7	28
29	0.2	0.5	0.7	1.0	1.2	1.4	1.7	1.9	2.2	2.4	2.7	2.9	3.1	3.4	3.6	3.9	29
30	0.2	0.5	0.8	1.0	1.2	1.5	1.8	2.0	2.2	2.5	2.8	3.0	3.2	3.5	3.8	4.0	30
31	0.3	0.5	0.8	1.0	1.3	1.6	1.8	2.1	2.3	2.6	2.8	3.1	3.4	3.6	3.9	4.1	31
32	0.3	0.5	0.8	1.1	1.3	1.6	1.9	2.1	2.4	2.7	2.9	3.2	3.5	3.7	4.0	4.3	32
33	0.3	0.6	0.8	1.1	1.4	1.6	1.9	2.2	2.5	2.8	3.0	3.3	3.6	3.8	4.1	4.4	33
34	0.3	0.6	0.8	1.1	1.4	1.7	2.0	2.3	2.6	2.8	3.1	3.4	3.7	4.0	4.2	4.5	34
35	0.3	0.6	0.9	1.2	1.5	1.8	2.0	2.3	2.6	2.9	3.2	3.5	3.8	4.1	4.4	4.7	35
36	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	36
37	0.3	0.6	0.9	1.2	1.5	1.8	2.2	2.5	2.8	3.1	3.4	3.7	4.0	4.3	4.6	4.9	37
38	0.3	0.6	1.0	1.3	1.6	1.9	2.2	2.5	2.8	3.2	3.5	3.8	4.1	4.4	4.8	5.1	38
39	0.3	0.6	1.0	1.3	1.6	2.0	2.3	2.6	2.9	3.2	3.6	3.9	4.2	4.6	4.9	5.2	39
40	0.3	0.7	1.0	1.3	1.7	2.0	2.3	2.7	3.0	3.3	3.7	4.0	4.3	4.7	5.0	5.3	40
41	0.3	0.7	1.0	1.4	1.7	2.0	2.4	2.7	3.1	3.4	3.8	4.1	4.4	4.8	5.1	5.5	41
42	0.4	0.7	1.0	1.4	1.8	2.1	2.4	2.8	3.2	3.5	3.8	4.2	4.6	4.9	5.2	5.6	42
43	0.4	0.7	1.1	1.4	1.8	2.2	2.5	2.9	3.2	3.6	3.9	4.3	4.7	5.0	5.4	5.7	43
44	0.4	0.7	1.1	1.5	1.8	2.2	2.6	2.9	3.3	3.7	4.0	4.4	4.8	5.1	5.5	5.9	44
45	0.4	0.8	1.1	1.5	1.9	2.2	2.6	3.0	3.4	3.8	4.1	4.5	4.9	5.2	5.6	6.0	45
46	0.4	0.8	1.2	1.5	1.9	2.3	2.7	3.1	3.4	3.8	4.2	4.6	5.0	5.4	5.8	6.1	46
47	0.4	0.8	1.2	1.6	2.0	2.4	2.7	3.1	3.5	3.9	4.3	4.7	5.1	5.5	5.9	6.3	47
48	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0	6.4	48
49	0.4	0.8	1.2	1.6	2.0	2.4	2.9	3.3	3.7	4.1	4.5	4.9	5.3	5.7	6.1	6.5	49
50	0.4	0.8	1.2	1.7	2.1	2.5	2.9	3.3	3.8	4.2	4.6	5.0	5.4	5.8	6.2	6.7	50
51	0.4	0.8	1.3	1.7	2.1	2.6	3.0	3.4	3.8	4.2	4.7	5.1	5.5	6.0	6.4	6.8	51
52	0.4	0.9	1.3	1.7	2.2	2.6	3.0	3.5	3.9	4.3	4.8	5.2	5.6	6.1	6.5	6.9	52
53	0.4	0.9	1.3	1.8	2.2	2.6	3.1	3.5	4.0	4.4	4.9	5.3	5.7	6.2	6.6	7.1	53
54	0.4	0.9	1.4	1.8	2.2	2.7	3.2	3.6	4.0	4.5	5.0	5.4	5.8	6.3	6.8	7.2	54
55	0.5	0.9	1.4	1.8	2.3	2.8	3.2	3.7	4.1	4.6	5.0	5.5	6.0	6.4	6.9	7.3	55
56	0.5	0.9	1.4	1.9	2.3	2.8	3.3	3.7	4.2	4.7	5.1	5.6	6.1	6.5	7.0	7.5	56
57	0.5	1.0	1.4	1.9	2.4	2.8	3.3	3.8	4.3	4.8	5.2	5.7	6.2	6.6	7.1	7.6	57
58	0.5	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.8	5.3	5.8	6.3	6.8	7.2	7.7	58
59	0.5	1.0	1.5	2.0	2.5	3.0	3.4	3.9	4.4	4.9	5.4	5.9	6.4	6.9	7.4	7.9	59
60	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	60

TABLE 19

Speed, Time, and Distance

Minutes	Speed in knots																Minutes
	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	
	Miles	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	
1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	1
2	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.5	2
3	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.8	3
4	0.6	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.8	0.9	0.9	0.9	1.0	1.0	1.0	1.1	4
5	0.7	0.8	0.8	0.8	0.9	0.9	1.0	1.0	1.0	1.1	1.1	1.2	1.2	1.2	1.3	1.3	5
6	0.8	0.9	1.0	1.0	1.0	1.1	1.2	1.2	1.2	1.3	1.4	1.4	1.4	1.5	1.6	1.6	6
7	1.0	1.0	1.1	1.2	1.2	1.3	1.3	1.4	1.5	1.5	1.6	1.6	1.7	1.8	1.8	1.9	7
8	1.1	1.2	1.3	1.3	1.4	1.5	1.5	1.6	1.7	1.7	1.8	1.9	1.9	2.0	2.1	2.1	8
9	1.3	1.4	1.4	1.5	1.6	1.6	1.7	1.8	1.9	2.0	2.0	2.1	2.2	2.2	2.3	2.4	9
10	1.4	1.5	1.6	1.7	1.8	1.8	1.9	2.0	2.1	2.2	2.2	2.3	2.4	2.5	2.6	2.7	10
11	1.6	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.8	2.9	11
12	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	12
13	1.8	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.4	3.5	13
14	2.0	2.1	2.2	2.3	2.4	2.6	2.7	2.8	2.9	3.0	3.2	3.3	3.4	3.5	3.6	3.7	14
15	2.1	2.2	2.4	2.5	2.6	2.8	2.9	3.0	3.1	3.2	3.4	3.5	3.6	3.8	3.9	4.0	15
16	2.3	2.4	2.5	2.7	2.8	2.9	3.1	3.2	3.3	3.5	3.6	3.7	3.9	4.0	4.1	4.3	16
17	2.4	2.6	2.7	2.8	3.0	3.1	3.3	3.4	3.5	3.7	3.8	4.0	4.1	4.2	4.4	4.5	17
18	2.6	2.7	2.8	3.0	3.2	3.3	3.4	3.6	3.8	3.9	4.0	4.2	4.4	4.5	4.6	4.8	18
19	2.7	2.8	3.0	3.2	3.3	3.5	3.6	3.8	4.0	4.1	4.3	4.4	4.6	4.8	4.9	5.1	19
20	2.8	3.0	3.2	3.3	3.5	3.7	3.8	4.0	4.2	4.3	4.5	4.7	4.8	5.0	5.2	5.3	20
21	3.0	3.2	3.3	3.5	3.7	3.8	4.0	4.2	4.4	4.6	4.7	4.9	5.1	5.2	5.4	5.6	21
22	3.1	3.3	3.5	3.7	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.1	5.3	5.5	5.7	5.9	22
23	3.3	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	5.9	6.1	23
24	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	24
25	3.5	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.5	6.7	25
26	3.7	3.9	4.1	4.3	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.1	6.3	6.5	6.7	6.9	26
27	3.8	4.0	4.3	4.5	4.7	5.0	5.2	5.4	5.6	5.8	6.1	6.3	6.5	6.8	7.0	7.2	27
28	4.0	4.2	4.4	4.7	4.9	5.1	5.4	5.6	5.8	6.1	6.3	6.5	6.8	7.0	7.2	7.5	28
29	4.1	4.4	4.6	4.8	5.1	5.3	5.6	5.8	6.0	6.3	6.5	6.8	7.0	7.2	7.5	7.7	29
30	4.2	4.5	4.8	5.0	5.2	5.5	5.8	6.0	6.2	6.5	6.8	7.0	7.2	7.5	7.8	8.0	30
31	4.4	4.6	4.9	5.2	5.4	5.7	5.9	6.2	6.5	6.7	7.0	7.2	7.5	7.8	8.0	8.3	31
32	4.5	4.8	5.1	5.3	5.6	5.9	6.1	6.4	6.7	6.9	7.2	7.5	7.7	8.0	8.3	8.5	32
33	4.7	5.0	5.2	5.5	5.8	6.0	6.3	6.6	6.9	7.2	7.4	7.7	8.0	8.2	8.5	8.8	33
34	4.8	5.1	5.4	5.7	6.0	6.2	6.5	6.8	7.1	7.4	7.6	7.9	8.2	8.5	8.8	9.1	34
35	5.0	5.2	5.5	5.8	6.1	6.4	6.7	7.0	7.3	7.6	7.9	8.2	8.5	8.8	9.0	9.3	35
36	5.1	5.4	5.7	6.0	6.3	6.6	6.9	7.2	7.5	7.8	8.1	8.4	8.7	9.0	9.3	9.6	36
37	5.2	5.6	5.9	6.2	6.5	6.8	7.1	7.4	7.7	8.0	8.3	8.6	8.9	9.2	9.6	9.9	37
38	5.4	5.7	6.0	6.3	6.6	7.0	7.3	7.6	7.9	8.2	8.6	8.9	9.2	9.5	9.8	10.1	38
39	5.5	5.8	6.2	6.5	6.8	7.2	7.5	7.8	8.1	8.4	8.8	9.1	9.4	9.8	10.1	10.4	39
40	5.7	6.0	6.3	6.7	7.0	7.3	7.7	8.0	8.3	8.7	9.0	9.3	9.7	10.0	10.3	10.7	40
41	5.8	6.2	6.5	6.8	7.2	7.5	7.9	8.2	8.5	8.9	9.2	9.6	9.9	10.2	10.6	10.9	41
42	6.0	6.3	6.6	7.0	7.4	7.7	8.0	8.4	8.8	9.1	9.4	9.8	10.2	10.5	10.8	11.2	42
43	6.1	6.4	6.8	7.2	7.5	7.9	8.2	8.6	9.0	9.3	9.7	10.0	10.4	10.8	11.1	11.5	43
44	6.2	6.6	7.0	7.3	7.7	8.1	8.4	8.8	9.2	9.5	9.9	10.3	10.6	11.0	11.4	11.7	44
45	6.4	6.8	7.1	7.5	7.9	8.2	8.6	9.0	9.4	9.8	10.1	10.5	10.9	11.2	11.6	12.0	45
46	6.5	6.9	7.3	7.7	8.0	8.4	8.8	9.2	9.6	10.0	10.4	10.7	11.1	11.5	11.9	12.3	46
47	6.7	7.0	7.4	7.8	8.2	8.6	9.0	9.4	9.8	10.2	10.6	11.0	11.4	11.8	12.1	12.5	47
48	6.8	7.2	7.6	8.0	8.4	8.8	9.2	9.6	10.0	10.4	10.8	11.2	11.6	12.0	12.4	12.8	48
49	6.9	7.4	7.8	8.2	8.6	9.0	9.4	9.8	10.2	10.6	11.0	11.4	11.8	12.2	12.7	13.1	49
50	7.1	7.5	7.9	8.3	8.8	9.2	9.6	10.0	10.4	10.8	11.2	11.7	12.1	12.5	12.9	13.3	50
51	7.2	7.6	8.1	8.5	8.9	9.4	9.8	10.2	10.6	11.0	11.5	11.9	12.3	12.8	13.2	13.6	51
52	7.4	7.8	8.2	8.7	9.1	9.5	10.0	10.4	10.8	11.3	11.7	12.1	12.6	13.0	13.4	13.9	52
53	7.5	8.0	8.4	8.8	9.3	9.7	10.2	10.6	11.0	11.5	11.9	12.4	12.8	13.2	13.7	14.1	53
54	7.6	8.1	8.6	9.0	9.4	9.9	10.4	10.8	11.2	11.7	12.2	12.6	13.0	13.5	14.0	14.4	54
55	7.8	8.2	8.7	9.2	9.6	10.1	10.5	11.0	11.5	11.9	12.4	12.8	13.3	13.8	14.2	14.7	55
56	7.9	8.4	8.9	9.3	9.8	10.3	10.7	11.2	11.7	12.1	12.6	13.1	13.5	14.0	14.5	14.9	56
57	8.1	8.6	9.0	9.5	10.0	10.4	10.9	11.4	11.9	12.4	12.8	13.3	13.8	14.2	14.7	15.2	57
58	8.2	8.7	9.2	9.7	10.2	10.6	11.1	11.6	12.1	12.6	13.0	13.5	14.0	14.5	15.0	15.5	58
59	8.4	8.8	9.3	9.8	10.3	10.8	11.3	11.8	12.3	12.8	13.3	13.8	14.3	14.8	15.2	15.7	59
60	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	60

TABLE 19

Speed, Time, and Distance

Minutes	Speed in knots																Minutes
	16.5	17.0	17.5	18.0	18.5	19.0	19.5	20.0	20.5	21.0	21.5	22.0	22.5	23.0	23.5	24.0	
	Miles	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	
1	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4	1
2	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.7	0.7	0.7	0.7	0.7	0.8	0.8	0.8	0.8	2
3	0.8	0.8	0.9	0.9	0.9	1.0	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.2	1.2	1.2	3
4	1.1	1.1	1.2	1.2	1.2	1.3	1.3	1.3	1.4	1.4	1.4	1.5	1.5	1.5	1.6	1.6	4
5	1.4	1.4	1.5	1.5	1.5	1.6	1.6	1.7	1.7	1.8	1.8	1.8	1.9	1.9	2.0	2.0	5
6	1.6	1.7	1.8	1.8	1.8	1.9	2.0	2.0	2.0	2.1	2.2	2.2	2.2	2.3	2.4	2.4	6
7	1.9	2.0	2.0	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.7	2.8	7
8	2.2	2.3	2.3	2.4	2.5	2.5	2.6	2.7	2.7	2.8	2.9	2.9	3.0	3.1	3.1	3.2	8
9	2.5	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.2	3.3	3.4	3.4	3.5	3.6	9
10	2.8	2.8	2.9	3.0	3.1	3.2	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.8	3.9	4.0	10
11	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.1	4.2	4.3	4.4	11
12	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	12
13	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.6	4.7	4.8	4.9	5.0	5.1	5.2	13
14	3.8	4.0	4.1	4.2	4.3	4.4	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.4	5.5	5.6	14
15	4.1	4.2	4.4	4.5	4.6	4.8	4.9	5.0	5.1	5.2	5.4	5.5	5.6	5.8	5.9	6.0	15
16	4.4	4.5	4.7	4.8	4.9	5.1	5.2	5.3	5.5	5.6	5.7	5.9	6.0	6.1	6.3	6.4	16
17	4.7	4.8	5.0	5.1	5.2	5.4	5.5	5.7	5.8	6.0	6.1	6.2	6.4	6.5	6.7	6.8	17
18	5.0	5.1	5.2	5.4	5.6	5.7	5.8	6.0	6.2	6.3	6.4	6.6	6.8	6.9	7.0	7.2	18
19	5.2	5.4	5.5	5.7	5.9	6.0	6.2	6.3	6.5	6.6	6.8	7.0	7.1	7.3	7.4	7.6	19
20	5.5	5.7	5.8	6.0	6.2	6.3	6.5	6.7	6.8	7.0	7.2	7.3	7.5	7.7	7.8	8.0	20
21	5.8	6.0	6.1	6.3	6.5	6.6	6.8	7.0	7.2	7.4	7.5	7.7	7.9	8.0	8.2	8.4	21
22	6.0	6.2	6.4	6.6	6.8	7.0	7.2	7.3	7.5	7.7	7.9	8.1	8.2	8.4	8.6	8.8	22
23	6.3	6.5	6.7	6.9	7.1	7.3	7.5	7.7	7.9	8.0	8.2	8.4	8.6	8.8	9.0	9.2	23
24	6.6	6.8	7.0	7.2	7.4	7.6	7.8	8.0	8.2	8.4	8.6	8.8	9.0	9.2	9.4	9.6	24
25	6.9	7.1	7.3	7.5	7.7	7.9	8.1	8.3	8.5	8.8	9.0	9.2	9.4	9.6	9.8	10.0	25
26	7.2	7.4	7.6	7.8	8.0	8.2	8.4	8.7	8.9	9.1	9.3	9.5	9.8	10.0	10.2	10.4	26
27	7.4	7.6	7.9	8.1	8.3	8.6	8.8	9.0	9.2	9.4	9.7	9.9	10.1	10.4	10.6	10.8	27
28	7.7	7.9	8.2	8.4	8.6	8.9	9.1	9.3	9.6	9.8	10.0	10.3	10.5	10.7	11.0	11.2	28
29	8.0	8.2	8.5	8.7	8.9	9.2	9.4	9.7	9.9	10.2	10.4	10.6	10.9	11.1	11.4	11.6	29
30	8.2	8.5	8.8	9.0	9.2	9.5	9.8	10.0	10.2	10.5	10.8	11.0	11.2	11.5	11.8	12.0	30
31	8.5	8.8	9.0	9.3	9.6	9.8	10.1	10.3	10.6	10.8	11.1	11.4	11.6	11.9	12.1	12.4	31
32	8.8	9.1	9.3	9.6	9.9	10.1	10.4	10.7	10.9	11.2	11.5	11.7	12.0	12.3	12.5	12.8	32
33	9.1	9.4	9.6	9.9	10.2	10.4	10.7	11.0	11.3	11.6	11.8	12.1	12.4	12.6	12.9	13.2	33
34	9.4	9.6	9.9	10.2	10.5	10.8	11.0	11.3	11.6	11.9	12.2	12.5	12.8	13.0	13.3	13.6	34
35	9.6	9.9	10.2	10.5	10.8	11.1	11.4	11.7	12.0	12.2	12.5	12.8	13.1	13.4	13.7	14.0	35
36	9.9	10.2	10.5	10.8	11.1	11.4	11.7	12.0	12.3	12.6	12.9	13.2	13.5	13.8	14.1	14.4	36
37	10.2	10.5	10.8	11.1	11.4	11.7	12.0	12.3	12.6	13.0	13.3	13.6	13.9	14.2	14.5	14.8	37
38	10.4	10.8	11.1	11.4	11.7	12.0	12.4	12.7	13.0	13.3	13.6	13.9	14.2	14.6	14.9	15.2	38
39	10.7	11.0	11.4	11.7	12.0	12.4	12.7	13.0	13.3	13.6	14.0	14.3	14.6	15.0	15.3	15.6	39
40	11.0	11.3	11.7	12.0	12.3	12.7	13.0	13.3	13.7	14.0	14.3	14.7	15.0	15.3	15.7	16.0	40
41	11.3	11.6	12.0	12.3	12.6	13.0	13.3	13.7	14.0	14.4	14.7	15.0	15.4	15.7	16.1	16.4	41
42	11.6	11.9	12.2	12.6	13.0	13.3	13.6	14.0	14.4	14.7	15.0	15.4	15.8	16.1	16.4	16.8	42
43	11.8	12.2	12.5	12.9	13.3	13.6	14.0	14.3	14.7	15.0	15.4	15.8	16.1	16.5	16.8	17.2	43
44	12.1	12.5	12.8	13.2	13.6	13.9	14.3	14.7	15.0	15.4	15.8	16.1	16.5	16.9	17.2	17.6	44
45	12.4	12.8	13.1	13.5	13.9	14.2	14.6	15.0	15.4	15.8	16.1	16.5	16.9	17.2	17.6	18.0	45
46	12.6	13.0	13.4	13.8	14.2	14.6	15.0	15.3	15.7	16.1	16.5	16.9	17.2	17.6	18.0	18.4	46
47	12.9	13.3	13.7	14.1	14.5	14.9	15.3	15.7	16.1	16.4	16.8	17.2	17.6	18.0	18.4	18.8	47
48	13.2	13.6	14.0	14.4	14.8	15.2	15.6	16.0	16.4	16.8	17.2	17.6	18.0	18.4	18.8	19.2	48
49	13.5	13.9	14.3	14.7	15.1	15.5	15.9	16.3	16.7	17.2	17.6	18.0	18.4	18.8	19.2	19.6	49
50	13.8	14.2	14.6	15.0	15.4	15.8	16.2	16.7	17.1	17.5	17.9	18.3	18.8	19.2	19.6	20.0	50
51	14.0	14.4	14.9	15.3	15.7	16.2	16.6	17.0	17.4	17.8	18.3	18.7	19.1	19.6	20.0	20.4	51
52	14.3	14.7	15.2	15.6	16.0	16.5	16.9	17.3	17.8	18.2	18.6	19.1	19.5	19.9	20.4	20.8	52
53	14.6	15.0	15.5	15.9	16.3	16.8	17.2	17.7	18.1	18.6	19.0	19.4	19.9	20.3	20.8	21.2	53
54	14.8	15.3	15.8	16.2	16.6	17.1	17.6	18.0	18.4	18.9	19.4	19.8	20.2	20.7	21.2	21.6	54
55	15.1	15.6	16.0	16.5	17.0	17.4	17.9	18.3	18.8	19.2	19.7	20.2	20.6	21.1	21.5	22.0	55
56	15.4	15.9	16.3	16.8	17.3	17.7	18.2	18.7	19.1	19.6	20.1	20.5	21.0	21.5	21.9	22.4	56
57	15.7	16.2	16.6	17.1	17.6	18.0	18.5	19.0	19.5	20.0	20.4	20.9	21.4	21.8	22.3	22.8	57
58	16.0	16.4	16.9	17.4	17.9	18.4	18.8	19.3	19.8	20.3	20.8	21.3	21.8	22.2	22.7	23.2	58
59	16.2	16.7	17.2	17.7	18.2	18.7	19.2	19.7	20.2	20.6	21.1	21.6	22.1	22.6	23.1	23.6	59
60	16.5	17.0	17.5	18.0	18.5	19.0	19.5	20.0	20.5	21.0	21.5	22.0	22.5	23.0	23.5	24.0	60

TABLE 19

Speed, Time, and Distance

Minutes	Speed in knots																Minutes
	24.5	25.0	25.5	26.0	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0	31.5	32.0	
	Miles	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	
1	0.4	0.4	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1
2	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.1	2
3	1.2	1.2	1.3	1.3	1.3	1.4	1.4	1.4	1.4	1.4	1.5	1.5	1.5	1.6	1.6	1.6	3
4	1.6	1.7	1.7	1.7	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.1	4
5	2.0	2.1	2.1	2.2	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.5	2.5	2.6	2.6	2.7	5
6	2.4	2.5	2.6	2.6	2.6	2.7	2.8	2.8	2.8	2.9	3.0	3.0	3.0	3.1	3.2	3.2	6
7	2.9	2.9	3.0	3.0	3.1	3.2	3.2	3.3	3.3	3.4	3.4	3.5	3.6	3.6	3.7	3.7	7
8	3.3	3.3	3.4	3.5	3.5	3.6	3.7	3.7	3.8	3.9	3.9	4.0	4.1	4.1	4.2	4.3	8
9	3.7	3.8	3.8	3.9	4.0	4.0	4.1	4.2	4.3	4.4	4.4	4.5	4.6	4.6	4.7	4.8	9
10	4.1	4.2	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.1	5.2	5.2	5.3	10
11	4.5	4.6	4.7	4.8	4.9	5.0	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	11
12	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.1	6.2	6.3	6.4	12
13	5.3	5.4	5.5	5.6	5.7	5.8	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	13
14	5.7	5.8	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.8	6.9	7.0	7.1	7.2	7.4	7.5	14
15	6.1	6.2	6.4	6.5	6.6	6.8	6.9	7.0	7.1	7.2	7.4	7.5	7.6	7.8	7.9	8.0	15
16	6.5	6.7	6.8	6.9	7.1	7.2	7.3	7.5	7.6	7.7	7.9	8.0	8.1	8.3	8.4	8.5	16
17	6.9	7.1	7.2	7.4	7.5	7.6	7.8	7.9	8.1	8.2	8.4	8.5	8.6	8.8	8.9	9.1	17
18	7.4	7.5	7.6	7.8	8.0	8.1	8.2	8.4	8.6	8.7	8.8	9.0	9.2	9.3	9.4	9.6	18
19	7.8	7.9	8.1	8.2	8.4	8.6	8.7	8.9	9.0	9.2	9.3	9.5	9.7	9.8	10.0	10.1	19
20	8.2	8.3	8.5	8.7	8.8	9.0	9.2	9.3	9.5	9.7	9.8	10.0	10.2	10.3	10.5	10.7	20
21	8.6	8.8	8.9	9.1	9.3	9.4	9.6	9.8	10.0	10.2	10.3	10.5	10.7	10.8	11.0	11.2	21
22	9.0	9.2	9.4	9.5	9.7	9.9	10.1	10.3	10.4	10.6	10.8	11.0	11.2	11.4	11.6	11.7	22
23	9.4	9.6	9.8	10.0	10.2	10.4	10.5	10.7	10.9	11.1	11.3	11.5	11.7	11.9	12.1	12.3	23
24	9.8	10.0	10.2	10.4	10.6	10.8	11.0	11.2	11.4	11.6	11.8	12.0	12.2	12.4	12.6	12.8	24
25	10.2	10.4	10.6	10.8	11.0	11.2	11.5	11.7	11.9	12.1	12.3	12.5	12.7	12.9	13.1	13.3	25
26	10.6	10.8	11.0	11.3	11.5	11.7	11.9	12.1	12.4	12.6	12.8	13.0	13.2	13.4	13.6	13.9	26
27	11.0	11.2	11.5	11.7	11.9	12.2	12.4	12.6	12.8	13.0	13.3	13.5	13.7	14.0	14.2	14.4	27
28	11.4	11.7	11.9	12.1	12.4	12.6	12.8	13.1	13.3	13.5	13.8	14.0	14.2	14.5	14.7	14.9	28
29	11.8	12.1	12.3	12.6	12.8	13.0	13.3	13.5	13.8	14.0	14.3	14.5	14.7	15.0	15.2	15.5	29
30	12.2	12.5	12.8	13.0	13.2	13.5	13.8	14.0	14.2	14.5	14.8	15.0	15.2	15.5	15.8	16.0	30
31	12.7	12.9	13.2	13.4	13.7	14.0	14.2	14.5	14.7	15.0	15.2	15.5	15.8	16.0	16.3	16.5	31
32	13.1	13.3	13.6	13.9	14.1	14.4	14.7	14.9	15.2	15.5	15.7	16.0	16.3	16.5	16.8	17.1	32
33	13.5	13.8	14.0	14.3	14.6	14.8	15.1	15.4	15.7	16.0	16.2	16.5	16.8	17.0	17.3	17.6	33
34	13.9	14.2	14.4	14.7	15.0	15.3	15.6	15.9	16.2	16.4	16.7	17.0	17.3	17.6	17.8	18.1	34
35	14.3	14.6	14.9	15.2	15.5	15.8	16.0	16.3	16.6	16.9	17.2	17.5	17.8	18.1	18.4	18.7	35
36	14.7	15.0	15.3	15.6	15.9	16.2	16.5	16.8	17.1	17.4	17.7	18.0	18.3	18.6	18.9	19.2	36
37	15.1	15.4	15.7	16.0	16.3	16.6	17.0	17.3	17.6	17.9	18.2	18.5	18.8	19.1	19.4	19.7	37
38	15.5	15.8	16.2	16.5	16.8	17.1	17.4	17.7	18.0	18.4	18.7	19.0	19.3	19.6	20.0	20.3	38
39	15.9	16.2	16.6	16.9	17.2	17.6	17.9	18.2	18.5	18.8	19.2	19.5	19.8	20.2	20.5	20.8	39
40	16.3	16.7	17.0	17.3	17.7	18.0	18.3	18.7	19.0	19.3	19.7	20.0	20.3	20.7	21.0	21.3	40
41	16.7	17.1	17.4	17.8	18.1	18.4	18.8	19.1	19.5	19.8	20.2	20.5	20.8	21.2	21.5	21.9	41
42	17.2	17.5	17.8	18.2	18.6	18.9	19.2	19.6	20.0	20.3	20.6	21.0	21.4	21.7	22.0	22.4	42
43	17.6	17.9	18.3	18.6	19.0	19.4	19.7	20.1	20.4	20.8	21.1	21.5	21.9	22.2	22.6	22.9	43
44	18.0	18.3	18.7	19.1	19.4	19.8	20.2	20.5	20.9	21.3	21.6	22.0	22.4	22.7	23.1	23.5	44
45	18.4	18.8	19.1	19.5	19.9	20.2	20.6	21.0	21.4	21.8	22.1	22.5	22.9	23.2	23.6	24.0	45
46	18.8	19.2	19.6	19.9	20.3	20.7	21.1	21.5	21.8	22.2	22.6	23.0	23.4	23.8	24.2	24.5	46
47	19.2	19.6	20.0	20.4	20.8	21.2	21.5	21.9	22.3	22.7	23.1	23.5	23.9	24.3	24.7	25.1	47
48	19.6	20.0	20.4	20.8	21.2	21.6	22.0	22.4	22.8	23.2	23.6	24.0	24.4	24.8	25.2	25.6	48
49	20.0	20.4	20.8	21.2	21.6	22.0	22.5	22.9	23.3	23.7	24.1	24.5	24.9	25.3	25.7	26.1	49
50	20.4	20.8	21.2	21.7	22.1	22.5	22.9	23.3	23.8	24.2	24.6	25.0	25.4	25.8	26.2	26.7	50
51	20.8	21.2	21.7	22.1	22.5	23.0	23.4	23.8	24.2	24.6	25.1	25.5	25.9	26.4	26.8	27.2	51
52	21.2	21.7	22.1	22.5	23.0	23.4	23.8	24.3	24.7	25.1	25.6	26.0	26.4	26.9	27.3	27.7	52
53	21.6	22.1	22.5	23.0	23.4	23.8	24.3	24.7	25.2	25.6	26.1	26.5	26.9	27.4	27.8	28.3	53
54	22.0	22.5	23.0	23.4	23.8	24.3	24.8	25.2	25.6	26.1	26.6	27.0	27.4	27.9	28.4	28.8	54
55	22.5	22.9	23.4	23.8	24.3	24.8	25.2	25.7	26.1	26.6	27.0	27.5	28.0	28.4	28.9	29.3	55
56	22.9	23.3	23.8	24.3	24.7	25.2	25.7	26.1	26.6	27.1	27.5	28.0	28.5	28.9	29.4	29.9	56
57	23.3	23.8	24.2	24.7	25.2	25.6	26.1	26.6	27.1	27.6	28.0	28.5	29.0	29.4	29.9	30.4	57
58	23.7	24.2	24.6	25.1	25.6	26.1	26.6	27.1	27.6	28.0	28.5	29.0	29.5	30.0	30.4	30.9	58
59	24.1	24.6	25.1	25.6	26.1	26.6	27.0	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0	31.5	59
60	24.5	25.0	25.5	26.0	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0	31.5	32.0	60

TABLE 19

Speed, Time, and Distance

Minutes	Speed in knots																Minutes
	32.5	33.0	33.5	34.0	34.5	35.0	35.5	36.0	36.5	37.0	37.5	38.0	38.5	39.0	39.5	40.0	
	Miles	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	*Miles*	
1	0.5	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.7	0.7	1
2	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.3	1.3	1.3	1.3	1.3	2
3	1.6	1.6	1.7	1.7	1.7	1.8	1.8	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.0	3
4	2.2	2.2	2.2	2.3	2.3	2.3	2.4	2.4	2.4	2.5	2.5	2.5	2.6	2.6	2.6	2.7	4
5	2.7	2.8	2.8	2.8	2.9	2.9	3.0	3.0	3.0	3.1	3.1	3.2	3.2	3.2	3.3	3.3	5
6	3.2	3.3	3.4	3.4	3.4	3.5	3.6	3.6	3.6	3.7	3.8	3.8	3.8	3.9	4.0	4.0	6
7	3.8	3.8	3.9	4.0	4.0	4.1	4.1	4.2	4.3	4.3	4.4	4.4	4.5	4.6	4.6	4.7	7
8	4.3	4.4	4.5	4.5	4.6	4.7	4.7	4.8	4.9	4.9	5.0	5.1	5.1	5.2	5.3	5.3	8
9	4.9	5.0	5.0	5.1	5.2	5.2	5.3	5.4	5.5	5.6	5.6	5.7	5.8	5.8	5.9	6.0	9
10	5.4	5.5	5.6	5.7	5.8	5.8	5.9	6.0	6.1	6.2	6.2	6.3	6.4	6.5	6.6	6.7	10
11	6.0	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0	7.1	7.2	7.2	7.3	11
12	6.5	6.6	6.7	6.8	6.9	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0	12
13	7.0	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0	8.1	8.2	8.3	8.4	8.6	8.7	13
14	7.6	7.7	7.8	7.9	8.0	8.2	8.3	8.4	8.5	8.6	8.8	8.9	9.0	9.1	9.2	9.3	14
15	8.1	8.2	8.4	8.5	8.6	8.8	8.9	9.0	9.1	9.2	9.4	9.5	9.6	9.8	9.9	10.0	15
16	8.7	8.8	8.9	9.1	9.2	9.3	9.5	9.6	9.7	9.9	10.0	10.1	10.3	10.4	10.5	10.7	16
17	9.2	9.4	9.5	9.6	9.8	9.9	10.1	10.2	10.3	10.5	10.6	10.8	10.9	11.0	11.2	11.3	17
18	9.8	9.9	10.0	10.2	10.4	10.5	10.6	10.8	11.0	11.1	11.2	11.4	11.6	11.7	11.8	12.0	18
19	10.3	10.4	10.6	10.8	10.9	11.1	11.2	11.4	11.6	11.7	11.9	12.0	12.2	12.4	12.5	12.7	19
20	10.8	11.0	11.2	11.3	11.5	11.7	11.8	12.0	12.2	12.3	12.5	12.7	12.8	13.0	13.2	13.3	20
21	11.4	11.6	11.7	11.9	12.1	12.2	12.4	12.6	12.8	13.0	13.1	13.3	13.5	13.6	13.8	14.0	21
22	11.9	12.1	12.3	12.5	12.6	12.8	13.0	13.2	13.4	13.6	13.8	13.9	14.1	14.3	14.5	14.7	22
23	12.5	12.6	12.8	13.0	13.2	13.4	13.6	13.8	14.0	14.2	14.4	14.6	14.8	15.0	15.1	15.3	23
24	13.0	13.2	13.4	13.6	13.8	14.0	14.2	14.4	14.6	14.8	15.0	15.2	15.4	15.6	15.8	16.0	24
25	13.5	13.8	14.0	14.2	14.4	14.6	14.8	15.0	15.2	15.4	15.6	15.8	16.0	16.2	16.5	16.7	25
26	14.1	14.3	14.5	14.7	15.0	15.2	15.4	15.6	15.8	16.0	16.2	16.5	16.7	16.9	17.1	17.3	26
27	14.6	14.8	15.1	15.3	15.5	15.8	16.0	16.2	16.4	16.6	16.9	17.1	17.3	17.6	17.8	18.0	27
28	15.2	15.4	15.6	15.9	16.1	16.3	16.6	16.8	17.0	17.3	17.5	17.7	18.0	18.2	18.4	18.7	28
29	15.7	16.0	16.2	16.4	16.7	16.9	17.2	17.4	17.6	17.9	18.1	18.4	18.6	18.8	19.1	19.3	29
30	16.2	16.5	16.8	17.0	17.2	17.5	17.8	18.0	18.2	18.5	18.8	19.0	19.2	19.5	19.8	20.0	30
31	16.8	17.0	17.3	17.6	17.8	18.1	18.3	18.6	18.9	19.1	19.4	19.6	19.9	20.2	20.4	20.7	31
32	17.3	17.6	17.9	18.1	18.4	18.7	18.9	19.2	19.5	19.7	20.0	20.3	20.5	20.8	21.1	21.3	32
33	17.9	18.2	18.4	18.7	19.0	19.2	19.5	19.8	20.1	20.4	20.6	20.9	21.2	21.4	21.7	22.0	33
34	18.4	18.7	19.0	19.3	19.6	19.8	20.1	20.4	20.7	21.0	21.2	21.5	21.8	22.1	22.4	22.7	34
35	19.0	19.2	19.5	19.8	20.1	20.4	20.7	21.0	21.3	21.6	21.9	22.2	22.5	22.8	23.0	23.3	35
36	19.5	19.8	20.1	20.4	20.7	21.0	21.3	21.6	21.9	22.2	22.5	22.8	23.1	23.4	23.7	24.0	36
37	20.0	20.4	20.7	21.0	21.3	21.6	21.9	22.2	22.5	22.8	23.1	23.4	23.7	24.0	24.4	24.7	37
38	20.6	20.9	21.2	21.5	21.8	22.2	22.5	22.8	23.1	23.4	23.8	24.1	24.4	24.7	25.0	25.3	38
39	21.1	21.4	21.8	22.1	22.4	22.8	23.1	23.4	23.7	24.0	24.4	24.7	25.0	25.4	25.7	26.0	39
40	21.7	22.0	22.3	22.7	23.0	23.3	23.7	24.0	24.3	24.7	25.0	25.3	25.7	26.0	26.3	26.7	40
41	22.2	22.6	22.9	23.2	23.6	23.9	24.3	24.6	24.9	25.3	25.6	26.0	26.3	26.6	27.0	27.3	41
42	22.8	23.1	23.4	23.8	24.2	24.5	24.8	25.2	25.6	25.9	26.2	26.6	27.0	27.3	27.6	28.0	42
43	23.3	23.6	24.0	24.4	24.7	25.1	25.4	25.8	26.2	26.5	26.9	27.2	27.6	28.0	28.3	28.7	43
44	23.8	24.2	24.6	24.9	25.3	25.7	26.0	26.4	26.8	27.1	27.5	27.9	28.2	28.6	29.0	29.3	44
45	24.4	24.8	25.1	25.5	25.9	26.2	26.6	27.0	27.4	27.8	28.1	28.5	28.9	29.2	29.6	30.0	45
46	24.9	25.3	25.7	26.1	26.4	26.8	27.2	27.6	28.0	28.4	28.8	29.1	29.5	29.9	30.3	30.7	46
47	25.5	25.8	26.2	26.6	27.0	27.4	27.8	28.2	28.6	29.0	29.4	29.8	30.2	30.6	30.9	31.3	47
48	26.0	26.4	26.8	27.2	27.6	28.0	28.4	28.8	29.2	29.6	30.0	30.4	30.8	31.2	31.6	32.0	48
49	26.5	27.0	27.4	27.8	28.2	28.6	29.0	29.4	29.8	30.2	30.6	31.0	31.4	31.8	32.3	32.7	49
50	27.1	27.5	27.9	28.3	28.8	29.2	29.6	30.0	30.4	30.8	31.2	31.7	32.1	32.5	32.9	33.3	50
51	27.6	28.0	28.5	28.9	29.3	29.8	30.2	30.6	31.0	31.4	31.9	32.3	32.7	33.2	33.6	34.0	51
52	28.2	28.6	29.0	29.5	29.9	30.3	30.8	31.2	31.6	32.1	32.5	32.9	33.4	33.8	34.2	34.7	52
53	28.7	29.2	29.6	30.0	30.5	30.9	31.4	31.8	32.2	32.7	33.1	33.6	34.0	34.4	34.9	35.3	53
54	29.2	29.7	30.2	30.6	31.0	31.5	32.0	32.4	32.8	33.3	33.8	34.2	34.6	35.1	35.6	36.0	54
55	29.8	30.2	30.7	31.2	31.6	32.1	32.5	33.0	33.5	33.9	34.4	34.8	35.3	35.8	36.2	36.7	55
56	30.3	30.8	31.3	31.7	32.2	32.7	33.1	33.6	34.1	34.5	35.0	35.5	35.9	36.4	36.9	37.3	56
57	30.9	31.4	31.8	32.3	32.8	33.2	33.7	34.2	34.7	35.2	35.6	36.1	36.6	37.0	37.5	38.0	57
58	31.4	31.9	32.4	32.9	33.4	33.8	34.3	34.8	35.3	35.8	36.2	36.7	37.2	37.7	38.2	38.7	58
59	32.0	32.4	32.9	33.4	33.9	34.4	34.9	35.4	35.9	36.4	36.9	37.4	37.9	38.4	38.8	39.3	59
60	32.5	33.0	33.5	34.0	34.5	35.0	35.5	36.0	36.5	37.0	37.5	38.0	38.5	39.0	39.5	40.0	60

TABLE 20

Conversion Table for Nautical and Statute Miles

1 nautical mile=6,076.11548 . . . feet   1 statute mile=5,280 feet

Nautical miles to statute miles				Statute miles to nautical miles			
Nautical miles	Statute miles	Nautical miles	Statute miles	Statute miles	Nautical miles	Statute miles	Nautical miles
1	1. 151	51	58. 690	1	0. 869	51	44. 318
2	2. 302	52	59. 841	2	1. 738	52	45. 187
3	3. 452	53	60. 991	3	2. 607	53	46. 056
4	4. 603	54	62. 142	4	3. 476	54	46. 925
5	5. 754	55	63. 293	5	4. 345	55	47. 794
6	6. 905	56	64. 444	6	5. 214	56	48. 663
7	8. 055	57	65. 594	7	6. 083	57	49. 532
8	9. 206	58	66. 745	8	6. 952	58	50. 401
9	10. 357	59	67. 896	9	7. 821	59	51. 270
10	11. 508	60	69. 047	10	8. 690	60	52. 139
11	12. 659	61	70. 198	11	9. 559	61	53. 008
12	13. 809	62	71. 348	12	10. 428	62	53. 877
13	14. 960	63	72. 499	13	11. 297	63	54. 746
14	16. 111	64	73. 650	14	12. 166	64	55. 614
15	17. 262	65	74. 801	15	13. 035	65	56. 483
16	18. 412	66	75. 951	16	13. 904	66	57. 352
17	19. 563	67	77. 102	17	14. 773	67	58. 221
18	20. 714	68	78. 253	18	15. 642	68	59. 090
19	21. 865	69	79. 404	19	16. 511	69	59. 959
20	23. 016	70	80. 555	20	17. 380	70	60. 828
21	24. 166	71	81. 705	21	18. 249	71	61. 697
22	25. 317	72	82. 856	22	19. 117	72	62. 566
23	26. 468	73	84. 007	23	19. 986	73	63. 435
24	27. 619	74	85. 158	24	20. 855	74	64. 304
25	28. 769	75	86. 308	25	21. 724	75	65. 173
26	29. 920	76	87. 459	26	22. 593	76	66. 042
27	31. 071	77	88. 610	27	23. 462	77	66. 911
28	32. 222	78	89. 761	28	24. 331	78	67. 780
29	33. 373	79	90. 912	29	25. 200	79	68. 649
30	34. 523	80	92. 062	30	26. 069	80	69. 518
31	35. 674	81	93. 213	31	26. 938	81	70. 387
32	36. 825	82	94. 364	32	27. 807	82	71. 256
33	37. 976	83	95. 515	33	28. 676	83	72. 125
34	39. 127	84	96. 665	34	29. 545	84	72. 994
35	40. 277	85	97. 816	35	30. 414	85	73. 863
36	41. 428	86	98. 967	36	31. 283	86	74. 732
37	42. 579	87	100. 118	37	32. 152	87	75. 601
38	43. 730	88	101. 269	38	33. 021	88	76. 470
39	44. 880	89	102. 419	39	33. 890	89	77. 339
40	46. 031	90	103. 570	40	34. 759	90	78. 208
41	47. 182	91	104. 721	41	35. 628	91	79. 077
42	48. 333	92	105. 872	42	36. 497	92	79. 946
43	49. 484	93	107. 022	43	37. 366	93	80. 815
44	50. 634	94	108. 173	44	38. 235	94	81. 684
45	51. 785	95	109. 324	45	39. 104	95	82. 553
46	52. 936	96	110. 475	46	39. 973	96	83. 422
47	54. 087	97	111. 626	47	40. 842	97	84. 291
48	55. 237	98	112. 776	48	41. 711	98	85. 160
49	56. 388	99	113. 927	49	42. 580	99	86. 029
50	57. 539	100	115. 078	50	43. 449	100	86. 898

TABLE 21

Conversion Table for Meters, Feet, and Fathoms

Meters	Feet	Fathoms	Meters	Feet	Fathoms	Feet	Meters	Feet	Meters	Fathoms	Meters	Fathoms	Meters
1	3. 28	0. 55	61	200. 13	33. 36	1	0. 30	61	18. 59	1	1. 83	61	111. 56
2	6. 56	1. 09	62	203. 41	33. 90	2	0. 61	62	18. 90	2	3. 66	62	113. 39
3	9. 84	1. 64	63	206. 69	34. 45	3	0. 91	63	19. 20	3	5. 49	63	115. 21
4	13. 12	2. 19	64	209. 97	35. 00	4	1. 22	64	19. 51	4	7. 32	64	117. 04
5	16. 40	2. 73	65	213. 25	35. 54	5	1. 52	65	19. 81	5	9. 14	65	118. 87
6	19. 69	3. 28	66	216. 54	36. 09	6	1. 83	66	20. 12	6	10. 97	66	120. 70
7	22. 97	3. 83	67	219. 82	36. 64	7	2. 13	67	20. 42	7	12. 80	67	122. 53
8	26. 25	4. 37	68	223. 10	37. 18	8	2. 44	68	20. 73	8	14. 63	68	124. 36
9	29. 53	4. 92	69	226. 38	37. 73	9	2. 74	69	21. 03	9	16. 46	69	126. 19
10	32. 81	5. 47	70	229. 66	38. 28	10	3. 05	70	21. 34	10	18. 29	70	128. 02
11	36. 09	6. 01	71	232. 94	38. 82	11	3. 35	71	21. 64	11	20. 12	71	129. 84
12	39. 37	6. 56	72	236. 22	39. 37	12	3. 66	72	21. 95	12	21. 95	72	131. 67
13	42. 65	7. 11	73	239. 50	39. 92	13	3. 96	73	22. 25	13	23. 77	73	133. 50
14	45. 93	7. 66	74	242. 78	40. 46	14	4. 27	74	22. 56	14	25. 60	74	135. 33
15	49. 21	8. 20	75	246. 06	41. 01	15	4. 57	75	22. 86	15	27. 43	75	137. 16
16	52. 49	8. 75	76	249. 34	41. 56	16	4. 88	76	23. 16	16	29. 26	76	138. 99
17	55. 77	9. 30	77	252. 62	42. 10	17	5. 18	77	23. 47	17	31. 09	77	140. 82
18	59. 06	9. 84	78	255. 91	42. 65	18	5. 49	78	23. 77	18	32. 92	78	142. 65
19	62. 34	10. 39	79	259. 19	43. 20	19	5. 79	79	24. 08	19	34. 75	79	144. 48
20	65. 62	10. 94	80	262. 47	43. 74	20	6. 10	80	24. 38	20	36. 58	80	146. 30
21	68. 90	11. 48	81	265. 75	44. 29	21	6. 40	81	24. 69	21	38. 40	81	148. 13
22	72. 18	12. 03	82	269. 03	44. 84	22	6. 71	82	24. 99	22	40. 23	82	149. 96
23	75. 46	12. 58	83	272. 31	45. 38	23	7. 01	83	25. 30	23	42. 06	83	151. 79
24	78. 74	13. 12	84	275. 59	45. 93	24	7. 32	84	25. 60	24	43. 89	84	153. 62
25	82. 02	13. 67	85	278. 87	46. 48	25	7. 62	85	25. 91	25	45. 72	85	155. 45
26	85. 30	14. 22	86	282. 15	47. 03	26	7. 92	86	26. 21	26	47. 55	86	157. 28
27	88. 58	14. 76	87	285. 43	47. 57	27	8. 23	87	26. 52	27	49. 38	87	159. 11
28	91. 86	15. 31	88	288. 71	48. 12	28	8. 53	88	26. 82	28	51. 21	88	160. 93
29	95. 14	15. 86	89	291. 99	48. 67	29	8. 84	89	27. 13	29	53. 04	89	162. 76
30	98. 43	16. 40	90	295. 28	49. 21	30	9. 14	90	27. 43	30	54. 86	90	164. 59
31	101. 71	16. 95	91	298. 56	49. 76	31	9. 45	91	27. 74	31	56. 69	91	166. 42
32	104. 99	17. 50	92	301. 84	50. 31	32	9. 75	92	28. 04	32	58. 52	92	168. 25
33	108. 27	18. 04	93	305. 12	50. 85	33	10. 06	93	28. 35	33	60. 35	93	170. 08
34	111. 55	18. 59	94	308. 40	51. 40	34	10. 36	94	28. 65	34	62. 18	94	171. 91
35	114. 83	19. 14	95	311. 68	51. 95	35	10. 67	95	28. 96	35	64. 01	95	173. 74
36	118. 11	19. 69	96	314. 96	52. 49	36	10. 97	96	29. 26	36	65. 84	96	175. 56
37	121. 39	20. 23	97	318. 24	53. 04	37	11. 28	97	29. 57	37	67. 67	97	177. 39
38	124. 67	20. 78	98	321. 52	53. 59	38	11. 58	98	29. 87	38	69. 49	98	179. 22
39	127. 95	21. 33	99	324. 80	54. 13	39	11. 89	99	30. 18	39	71. 32	99	181. 05
40	131. 23	21. 87	100	328. 08	54. 68	40	12. 19	100	30. 48	40	73. 15	100	182. 88
41	134. 51	22. 42	101	331. 36	55. 23	41	12. 50	101	30. 78	41	74. 98	101	184. 71
42	137. 80	22. 97	102	334. 65	55. 77	42	12. 80	102	31. 09	42	76. 81	102	186. 54
43	141. 08	23. 51	103	337. 93	56. 32	43	13. 11	103	31. 39	43	78. 64	103	188. 37
44	144. 36	24. 06	104	341. 21	56. 87	44	13. 41	104	31. 70	44	80. 47	104	190. 20
45	147. 64	24. 61	105	344. 49	57. 41	45	13. 72	105	32. 00	45	82. 30	105	192. 02
46	150. 92	25. 15	106	347. 77	57. 96	46	14. 02	106	32. 31	46	84. 12	106	193. 85
47	154. 20	25. 70	107	351. 05	58. 51	47	14. 33	107	32. 61	47	85. 95	107	195. 68
48	157. 48	26. 25	108	354. 33	59. 06	48	14. 63	108	32. 92	48	87. 78	108	197. 51
49	160. 76	26. 79	109	357. 61	59. 60	49	14. 94	109	33. 22	49	89. 61	109	199. 34
50	164. 04	27. 34	110	360. 89	60. 15	50	15. 24	110	33. 53	50	91. 44	110	201. 17
51	167. 32	27. 89	111	364. 17	60. 70	51	15. 54	111	33. 83	51	93. 27	111	203. 00
52	170. 60	28. 43	112	367. 45	61. 24	52	15. 85	112	34. 14	52	95. 10	112	204. 83
53	173. 88	28. 98	113	370. 73	61. 79	53	16. 15	113	34. 44	53	96. 93	113	206. 65
54	177. 17	29. 53	114	374. 02	62. 34	54	16. 46	114	34. 75	54	98. 76	114	208. 48
55	180. 45	30. 07	115	377. 30	62. 88	55	16. 76	115	35. 05	55	100. 58	115	210. 31
56	183. 73	30. 62	116	380. 58	63. 43	56	17. 07	116	35. 36	56	102. 41	116	212. 14
57	187. 01	31. 17	117	383. 86	63. 98	57	17. 37	117	35. 66	57	104. 24	117	213. 97
58	190. 29	31. 71	118	387. 14	64. 52	58	17. 68	118	35. 97	58	106. 07	118	215. 80
59	193. 57	32. 26	119	390. 42	65. 07	59	17. 98	119	36. 27	59	107. 90	119	217. 63
60	196. 85	32. 81	120	393. 70	65. 62	60	18. 29	120	36. 58	60	109. 73	120	219. 46

TABLE 22

Dip of the Sea Short of the Horizon

Distance	Height of eye above the sea, in feet										Distance
	5	10	15	20	25	30	35	40	45	50	
Miles	′	′	′	′	′	′	′	′	′	′	*Miles*
0. 1	28. 3	56. 6	84. 9	113. 2	141. 5	169. 8	198. 0	226. 3	254. 6	282. 9	0. 1
0. 2	14. 2	28. 4	42. 5	56. 7	70. 8	84. 9	99. 1	113. 2	127. 4	141. 5	0. 2
0. 3	9. 6	19. 0	28. 4	37. 8	47. 3	56. 7	66. 1	75. 6	85. 0	94. 4	0. 3
0. 4	7. 2	14. 3	21. 4	28. 5	35. 5	42. 6	49. 7	56. 7	63. 8	70. 9	0. 4
0. 5	5. 9	11. 5	17. 2	22. 8	28. 5	34. 2	39. 8	45. 5	51. 1	56. 8	0. 5
0. 6	5. 0	9. 7	14. 4	19. 1	23. 8	28. 5	33. 3	38. 0	42. 7	47. 4	0. 6
0. 7	4. 3	8. 4	12. 4	16. 5	20. 5	24. 5	28. 6	32. 6	36. 7	40. 7	0. 7
0. 8	3. 9	7. 4	10. 9	14. 5	18. 0	21. 5	25. 1	28. 6	32. 2	35. 7	0. 8
0. 9	3. 5	6. 7	9. 8	12. 9	16. 1	19. 2	22. 4	25. 5	28. 7	31. 8	0. 9
1. 0	3. 2	6. 1	8. 9	11. 7	14. 6	17. 4	20. 2	23. 0	25. 9	28. 7	1. 0
1. 1	3. 0	5. 6	8. 2	10. 7	13. 3	15. 9	18. 5	21. 0	23. 6	26. 2	1. 1
1. 2	2. 9	5. 2	7. 6	9. 9	12. 3	14. 6	17. 0	19. 4	21. 7	24. 1	1. 2
1. 3	2. 7	4. 9	7. 1	9. 2	11. 4	13. 6	15. 8	17. 9	20. 1	22. 3	1. 3
1. 4	2. 6	4. 6	6. 6	8. 7	10. 7	12. 7	14. 7	16. 7	18. 8	20. 8	1. 4
1. 5	2. 5	4. 4	6. 3	8. 2	10. 0	11. 9	13. 8	15. 7	17. 6	19. 5	1. 5
1. 6	2. 4	4. 2	6. 0	7. 7	9. 5	11. 3	13. 0	14. 8	16. 6	18. 3	1. 6
1. 7	2. 4	4. 0	5. 7	7. 4	9. 0	10. 7	12. 4	14. 0	15. 7	17. 3	1. 7
1. 8	2. 3	3. 9	5. 5	7. 0	8. 6	10. 2	11. 7	13. 3	14. 9	16. 5	1. 8
1. 9	2. 3	3. 8	5. 3	6. 7	8. 2	9. 7	11. 2	12. 7	14. 2	15. 7	1. 9
2. 0	2. 2	3. 7	5. 1	6. 5	7. 9	9. 3	10. 7	12. 1	13. 6	15. 0	2. 0
2. 1	2. 2	3. 6	4. 9	6. 3	7. 6	9. 0	10. 3	11. 6	13. 0	14. 3	2. 1
2. 2	2. 2	3. 5	4. 8	6. 1	7. 3	8. 6	9. 9	11. 2	12. 5	13. 8	2. 2
2. 3	2. 2	3. 4	4. 6	5. 9	7. 1	8. 3	9. 6	10. 8	12. 0	13. 3	2. 3
2. 4	2. 2	3. 4	4. 5	5. 7	6. 9	8. 1	9. 2	10. 4	11. 6	12. 8	2. 4
2. 5	2. 2	3. 3	4. 4	5. 6	6. 7	7. 8	9. 0	10. 1	11. 2	12. 4	2. 5
2. 6	2. 2	3. 3	4. 3	5. 4	6. 5	7. 6	8. 7	9. 8	10. 9	12. 0	2. 6
2. 7	2. 2	3. 2	4. 3	5. 3	6. 4	7. 4	8. 4	9. 5	10. 6	11. 6	2. 7
2. 8	2. 2	3. 2	4. 2	5. 2	6. 2	7. 2	8. 2	9. 2	10. 3	11. 3	2. 8
2. 9	2. 2	3. 2	4. 1	5. 1	6. 1	7. 1	8. 0	9. 0	10. 0	11. 0	2. 9
3. 0	2. 2	3. 1	4. 1	5. 0	6. 0	6. 9	7. 8	8. 8	9. 7	10. 7	3. 0
3. 1	2. 2	3. 1	4. 0	4. 9	5. 9	6. 8	7. 7	8. 6	9. 5	10. 4	3. 1
3. 2	2. 2	3. 1	4. 0	4. 9	5. 7	6. 6	7. 5	8. 4	9. 3	10. 2	3. 2
3. 3	2. 2	3. 1	3. 9	4. 8	5. 7	6. 5	7. 4	8. 2	9. 1	9. 9	3. 3
3. 4	2. 2	3. 1	3. 9	4. 7	5. 6	6. 4	7. 2	8. 1	8. 9	9. 7	3. 4
3. 5	2. 2	3. 1	3. 9	4. 7	5. 5	6. 3	7. 1	7. 9	8. 7	9. 5	3. 5
3. 6	2. 2	3. 1	3. 8	4. 6	5. 4	6. 2	7. 0	7. 8	8. 6	9. 4	3. 6
3. 7	2. 2	3. 1	3. 8	4. 6	5. 4	6. 1	6. 9	7. 7	8. 4	9. 2	3. 7
3. 8	2. 2	3. 1	3. 8	4. 6	5. 3	6. 0	6. 8	7. 5	8. 3	9. 0	3. 8
3. 9	2. 2	3. 1	3. 8	4. 5	5. 2	6. 0	6. 7	7. 4	8. 1	8. 9	3. 9
4. 0	2. 2	3. 1	3. 8	4. 5	5. 2	5. 9	6. 6	7. 3	8. 0	8. 7	4. 0
4. 1	2. 2	3. 1	3. 8	4. 5	5. 1	5. 8	6. 5	7. 2	7. 9	8. 6	4. 1
4. 2	2. 2	3. 1	3. 8	4. 4	5. 1	5. 8	6. 5	7. 1	7. 8	8. 5	4. 2
4. 3	2. 2	3. 1	3. 8	4. 4	5. 1	5. 7	6. 4	7. 0	7. 7	8. 4	4. 3
4. 4	2. 2	3. 1	3. 8	4. 4	5. 0	5. 7	6. 3	7. 0	7. 6	8. 3	4. 4
4. 5	2. 2	3. 1	3. 8	4. 4	5. 0	5. 6	6. 3	6. 9	7. 5	8. 2	4. 5
4. 6	2. 2	3. 1	3. 8	4. 4	5. 0	5. 6	6. 2	6. 8	7. 4	8. 1	4. 6
4. 7	2. 2	3. 1	3. 8	4. 4	5. 0	5. 6	6. 2	6. 8	7. 4	8. 0	4. 7
4. 8	2. 2	3. 1	3. 8	4. 4	4. 9	5. 5	6. 1	6. 7	7. 3	7. 9	4. 8
4. 9	2. 2	3. 1	3. 8	4. 3	4. 9	5. 5	6. 1	6. 7	7. 2	7. 8	4. 9
5. 0	2. 2	3. 1	3. 8	4. 3	4. 9	5. 5	6. 0	6. 6	7. 2	7. 7	5. 0
5. 5	2. 2	3. 1	3. 8	4. 3	4. 9	5. 4	5. 9	6. 4	6. 9	7. 4	5. 5
6. 0	2. 2	3. 1	3. 8	4. 3	4. 9	5. 3	5. 8	6. 3	6. 7	7. 2	6. 0
6. 5	2. 2	3. 1	3. 8	4. 3	4. 9	5. 3	5. 7	6. 2	6. 6	7. 1	6. 5
7. 0	2. 2	3. 1	3. 8	4. 3	4. 9	5. 3	5. 7	6. 1	6. 5	6. 9	7. 0
7. 5	2. 2	3. 1	3. 8	4. 3	4. 9	5. 3	5. 7	6. 1	6. 5	6. 9	7. 5
8. 0	2. 2	3. 1	3. 8	4. 3	4. 9	5. 3	5. 7	6. 1	6. 5	6. 9	8. 0
8. 5	2. 2	3. 1	3. 8	4. 3	4. 9	5. 3	5. 7	6. 1	6. 5	6. 9	8. 5
9. 0	2. 2	3. 1	3. 8	4. 3	4. 9	5. 3	5. 7	6. 1	6. 5	6. 9	9. 0
9. 5	2. 2	3. 1	3. 8	4. 3	4. 9	5. 3	5. 7	6. 1	6. 5	6. 9	9. 5
10. 0	2. 2	3. 1	3. 8	4. 3	4. 9	5. 3	5. 7	6. 1	6. 5	6. 9	10. 0

TABLE 22

Dip of the Sea Short of the Horizon

Distance	Height of eye above the sea, in feet										Distance
	55	60	65	70	75	80	85	90	95	100	
Miles	′	′	′	′	′	′	′	′	′	′	*Miles*
0. 1	311. 2	339. 5	367. 8	396. 1	424. 4	452. 6	480. 9	509. 2	537. 5	565. 8	0. 1
0. 2	155. 6	169. 8	184. 0	198. 1	212. 2	226. 4	240. 5	254. 7	268. 8	283. 0	0. 2
0. 3	103. 8	113. 3	122. 7	132. 1	141. 6	151. 0	160. 4	169. 9	179. 3	188. 7	0. 3
0. 4	78. 0	85. 0	92. 1	99. 2	106. 2	113. 3	120. 4	127. 5	134. 5	141. 6	0. 4
0. 5	62. 4	68. 1	73. 8	79. 4	85. 1	90. 7	96. 4	102. 0	107. 7	113. 4	0. 5
0. 6	52. 1	56. 8	61. 5	66. 3	71. 0	75. 7	80. 4	85. 1	89. 8	94. 5	0. 6
0. 7	44. 7	48. 8	52. 8	56. 9	60. 9	64. 9	69. 0	73. 0	77. 1	81. 1	0. 7
0. 8	39. 2	42. 8	46. 3	49. 8	53. 4	56. 9	60. 4	64. 0	67. 5	71. 1	0. 8
0. 9	34. 9	38. 1	41. 2	44. 4	47. 5	50. 7	53. 8	56. 9	60. 1	63. 2	0. 9
1. 0	31. 5	34. 4	37. 2	40. 0	42. 8	45. 7	48. 5	51. 3	54. 2	57. 0	1. 0
1. 1	28. 7	31. 3	33. 9	36. 5	39. 0	41. 6	44. 2	46. 7	49. 3	51. 9	1. 1
1. 2	26. 4	28. 8	31. 1	33. 5	35. 9	38. 2	40. 6	42. 9	45. 3	47. 6	1. 2
1. 3	24. 5	26. 7	28. 8	31. 0	33. 2	35. 4	37. 5	39. 7	41. 9	44. 1	1. 3
1. 4	22. 8	24. 8	26. 8	28. 9	30. 9	32. 9	34. 9	37. 0	39. 0	41. 0	1. 4
1. 5	21. 4	23. 3	25. 1	27. 0	28. 9	30. 8	32. 7	34. 6	36. 5	38. 3	1. 5
1. 6	20. 1	21. 9	23. 6	25. 4	27. 2	29. 0	30. 7	32. 5	34. 3	36. 0	1. 6
1. 7	19. 0	20. 7	22. 3	24. 0	25. 7	27. 3	29. 0	30. 7	32. 3	34. 0	1. 7
1. 8	18. 0	19. 6	21. 2	22. 8	24. 3	25. 9	27. 5	29. 0	30. 6	32. 2	1. 8
1. 9	17. 2	18. 7	20. 1	21. 6	23. 1	24. 6	26. 1	27. 6	29. 1	30. 6	1. 9
2. 0	16. 4	17. 8	19. 2	20. 6	22. 0	23. 5	24. 9	26. 3	27. 7	29. 1	2. 0
2. 1	15. 7	17. 0	18. 4	19. 7	21. 1	22. 4	23. 8	25. 1	26. 5	27. 8	2. 1
2. 2	15. 1	16. 3	17. 6	18. 9	20. 2	21. 5	22. 7	24. 1	25. 3	26. 6	2. 2
2. 3	14. 5	15. 7	16. 9	18. 2	19. 4	20. 6	21. 9	23. 1	24. 3	25. 6	2. 3
2. 4	14. 0	15. 1	16. 3	17. 5	18. 7	19. 9	21. 0	22. 2	23. 4	24. 6	2. 4
2. 5	13. 5	14. 6	15. 7	16. 9	18. 0	19. 1	20. 3	21. 4	22. 5	23. 7	2. 5
2. 6	13. 0	14. 1	15. 2	16. 3	17. 4	18. 5	19. 6	20. 7	21. 8	22. 8	2. 6
2. 7	12. 6	13. 7	14. 7	15. 8	16. 8	17. 9	18. 9	20. 0	21. 0	22. 1	2. 7
2. 8	12. 3	13. 3	14. 3	15. 3	16. 3	17. 3	18. 3	19. 3	20. 4	21. 4	2. 8
2. 9	11. 9	12. 9	13. 9	14. 9	15. 8	16. 8	17. 8	18. 8	19. 7	20. 7	2. 9
3. 0	11. 6	12. 6	13. 5	14. 4	15. 4	16. 3	17. 3	18. 2	19. 2	20. 1	3. 0
3. 1	11. 3	12. 2	13. 2	14. 1	15. 0	15. 9	16. 8	17. 7	18. 6	19. 5	3. 1
3. 2	11. 1	11. 9	12. 8	13. 7	14. 6	15. 5	16. 4	17. 2	18. 1	19. 0	3. 2
3. 3	10. 8	11. 7	12. 5	13. 4	14. 2	15. 1	15. 9	16. 8	17. 7	18. 5	3. 3
3. 4	10. 6	11. 4	12. 2	13. 1	13. 9	14. 7	15. 6	16. 4	17. 2	18. 1	3. 4
3. 5	10. 3	11. 2	12. 0	12. 8	13. 6	14. 4	15. 2	16. 0	16. 8	17. 6	3. 5
3. 6	10. 1	10. 9	11. 7	12. 4	13. 3	14. 1	14. 9	15. 6	16. 4	17. 2	3. 6
3. 7	9. 9	10. 7	11. 5	12. 2	13. 0	13. 8	14. 5	15. 3	16. 1	16. 8	3. 7
3. 8	9. 8	10. 5	11. 3	12. 0	12. 7	13. 5	14. 2	15. 0	15. 7	16. 5	3. 8
3. 9	9. 6	10. 3	11. 1	11. 8	12. 5	13. 2	14. 0	14. 7	15. 4	16. 1	3. 9
4. 0	9. 4	10. 1	10. 9	11. 6	12. 3	13. 0	13. 7	14. 4	15. 1	15. 8	4. 0
4. 1	9. 3	10. 0	10. 7	11. 4	12. 1	12. 7	13. 4	14. 1	14. 8	15. 5	4. 1
4. 2	9. 2	9. 8	10. 5	11. 2	11. 8	12. 5	13. 2	13. 9	14. 5	15. 2	4. 2
4. 3	9. 0	9. 7	10. 3	11. 0	11. 7	12. 3	13. 0	13. 6	14. 3	14. 9	4. 3
4. 4	8. 9	9. 5	10. 2	10. 8	11. 5	12. 1	12. 8	13. 4	14. 0	14. 7	4. 4
4. 5	8. 8	9. 4	10. 0	10. 7	11. 3	11. 9	12. 6	13. 2	13. 8	14. 4	4. 5
4. 6	8. 7	9. 3	9. 9	10. 5	11. 1	11. 8	12. 4	13. 0	13. 6	14. 2	4. 6
4. 7	8. 6	9. 2	9. 8	10. 4	11. 0	11. 6	12. 2	12. 8	13. 4	14. 0	4. 7
4. 8	8. 5	9. 1	9. 7	10. 2	10. 8	11. 4	12. 0	12. 6	13. 2	13. 8	4. 8
4. 9	8. 4	9. 0	9. 5	10. 1	10. 7	11. 3	11. 9	12. 4	13. 0	13. 6	4. 9
5. 0	8. 3	8. 9	9. 4	10. 0	10. 6	11. 1	11. 7	12. 3	12. 8	13. 4	5. 0
5. 5	7. 9	8. 5	9. 0	9. 5	10. 0	10. 5	11. 0	11. 5	12. 1	12. 6	5. 5
6. 0	7. 7	8. 2	8. 6	9. 1	9. 6	10. 0	10. 5	11. 0	11. 5	11. 9	6. 0
6. 5	7. 5	7. 9	8. 4	8. 8	9. 2	9. 7	10. 1	10. 5	11. 0	11. 4	6. 5
7. 0	7. 4	7. 8	8. 2	8. 6	9. 0	9. 4	9. 8	10. 2	10. 6	11. 0	7. 0
7. 5	7. 3	7. 6	8. 0	8. 4	8. 8	9. 2	9. 5	9. 9	10. 3	10. 7	7. 5
8. 0	7. 2	7. 6	7. 9	8. 3	8. 6	9. 0	9. 3	9. 7	10. 0	10. 4	8. 0
8. 5	7. 2	7. 5	7. 9	8. 2	8. 5	8. 9	9. 2	9. 5	9. 9	10. 2	8. 5
9. 0	7. 2	7. 5	7. 8	8. 1	8. 5	8. 8	9. 1	9. 4	9. 7	10. 0	9. 0
9. 5	7. 2	7. 5	7. 8	8. 1	8. 4	8. 7	9. 0	9. 3	9. 6	9. 9	9. 5
10. 0	7. 2	7. 5	7. 8	8. 1	8. 4	8. 7	9. 0	9. 2	9. 5	9. 8	10. 0

TABLE 23

Altitude Correction for Air Temperature

Altitude	Temperature—degrees Fahrenheit								Altitude
	−40	−30	−20	−10	0	+10	+20	+30	
° ′	′	′	′	′	′	′	′	′	° ′
−0 10	−7.9	−6.8	−5.8	−4.9	−4.0	−3.1	−2.3	−1.5	−0 10
0 00	7.4	6.4	5.5	4.6	3.8	2.9	2.2	1.4	0 00
+0 10	6.9	6.0	5.2	4.3	3.5	2.8	2.0	1.3	+0 10
0 20	6.6	5.7	4.9	4.1	3.3	2.6	1.9	1.2	0 20
0 30	6.1	5.3	4.6	3.8	3.1	2.4	1.8	1.2	0 30
+0 45	−5.7	−4.9	−4.2	−3.5	−2.9	−2.2	−1.6	−1.1	+0 45
1 00	5.2	4.5	3.9	3.2	2.6	2.1	1.5	1.0	1 00
1 20	4.7	4.1	3.5	2.9	2.4	1.9	1.4	0.9	1 20
1 40	4.3	3.7	3.2	2.7	2.2	1.7	1.2	0.8	1 40
2 00	3.9	3.4	2.9	2.4	2.0	1.6	1.1	0.7	2 00
+2 30	−3.4	−3.0	−2.6	−2.1	−1.8	−1.4	−1.0	−0.7	+2 30
3 00	3.1	2.7	2.3	1.9	1.6	1.2	0.9	0.6	3 00
4	2.5	2.2	1.9	1.6	1.3	1.0	0.7	0.5	4
5	2.1	1.8	1.6	1.3	1.1	0.8	0.6	0.4	5
6	1.8	1.6	1.4	1.1	0.9	0.7	0.5	0.3	6
+7	−1.6	−1.4	−1.2	−1.0	−0.8	−0.6	−0.5	−0.3	+7
8	1.4	1.2	1.0	0.9	0.7	0.6	0.4	0.3	8
9	1.3	1.1	0.9	0.8	0.6	0.5	0.4	0.2	9
10	1.1	1.0	0.8	0.7	0.6	0.5	0.3	0.2	10
15	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	15
+20	−0.6	−0.5	−0.4	−0.3	−0.3	−0.2	−0.2	−0.1	+20
30	0.4	0.3	0.3	0.2	0.2	0.1	0.1	0.1	30
50	0.2	0.1	0.1	0.1	0.1	0.1	0.0	0.0	50
70	0.1	0.1	0.1	0.1	0.0	0.0	0.0	0.0	70
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	90

Altitude	Temperature—degrees Fahrenheit								Altitude
	+40	+50	+60	+70	+80	+90	+100	+110	
° ′	′		′	′	′	′	′	′	° ′
−0 10	−0.7	0.0	+0.7	+1.4	+2.0	+2.7	+3.3	+3.9	−0 10
0 00	0.7	0.0	0.7	1.3	1.9	2.5	3.1	3.6	0 00
+0 10	0.6	0.0	0.6	1.2	1.8	2.4	2.9	3.4	+0 10
0 20	0.6	0.0	0.6	1.2	1.7	2.2	2.7	3.2	0 20
0 30	0.6	0.0	0.6	1.1	1.6	2.1	2.6	3.0	0 30
+0 45	−0.5	0.0	+0.5	+1.0	+1.5	+1.9	+2.4	+2.8	+0 45
1 00	0.5	0.0	0.5	0.9	1.4	1.8	2.2	2.6	1 00
1 20	0.4	0.0	0.4	0.8	1.2	1.6	2.0	2.3	1 20
1 40	0.4	0.0	0.4	0.8	1.1	1.5	1.8	2.1	1 40
2 00	0.4	0.0	0.4	0.7	1.0	1.3	1.6	1.9	2 00
+2 30	−0.3	0.0	+0.3	+0.6	+0.9	+1.2	+1.4	+1.7	+2 30
3 00	0.3	0.0	0.3	0.5	0.8	1.0	1.3	1.5	3 00
4	0.2	0.0	0.2	0.4	0.7	0.9	1.1	1.2	4
5	0.2	0.0	0.2	0.4	0.6	0.7	0.9	1.0	5
6	0.2	0.0	0.2	0.3	0.5	0.6	0.8	0.9	6
+7	−0.1	0.0	+0.1	+0.3	+0.4	+0.5	+0.7	+0.8	+7
8	0.1	0.0	0.1	0.2	0.4	0.5	0.6	0.7	8
9	0.1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	9
10	0.1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	10
15	0.1	0.0	0.1	0.1	0.2	0.3	0.3	0.4	15
+20	−0.1	0.0	+0.1	+0.1	+0.1	+0.2	+0.2	+0.3	+20
30	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2	30
50	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	50
70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	70
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	90

TABLE 24
Altitude Correction for Atmospheric Pressure

Altitude	Pressure in inches or millibars—**Subtract** correction from sextant or rectified altitude								Altitude
	31. 2	31. 0	30. 8	30. 6	30. 4	30. 2	30. 0	29. 8	
	1056.56	1049.78	1043.01	1036.24	1029.46	1022.69	1015.92	1009.15	
° ′	′	′	′	′	′	′	′	′	° ′
−0 10	−1. 7	−1. 4	−1. 2	−1. 0	−0. 7	−0. 5	−0. 2	0. 0	−0 10
0 00	1. 6	1. 4	1. 1	0. 9	0. 7	0. 4	0. 2	0. 0	0 00
+0 10	1. 5	1. 3	1. 1	0. 8	0. 6	0. 4	0. 2	0. 0	+0 10
0 20	1. 4	1. 2	1. 0	0. 8	0. 6	0. 4	0. 2	0. 0	0 20
0 30	1. 3	1. 1	0. 9	0. 7	0. 6	0. 4	0. 2	0. 0	0 30
+0 45	−1. 2	−1. 0	−0. 9	−0. 7	−0. 5	−0. 3	−0. 2	0. 0	+0 45
1 00	1. 1	1. 0	0. 8	0. 6	0. 5	0. 3	0. 1	0. 0	1 00
1 20	1. 0	0. 9	0. 7	0. 6	0. 4	0. 3	0. 1	0. 0	1 20
1 40	0. 9	0. 8	0. 7	0. 5	0. 4	0. 3	0. 1	0. 0	1 40
2 00	0. 8	0. 7	0. 6	0. 5	0. 4	0. 2	0. 1	0. 0	2 00
+2 30	−0. 7	−0. 6	−0. 5	−0. 4	−0. 3	−0. 2	−0. 1	0. 0	+2 30
3 00	0. 7	0. 6	0. 5	0. 4	0. 3	0. 2	0. 1	0. 0	3 00
4	0. 5	0. 5	0. 4	0. 3	0. 2	0. 1	0. 1	0. 0	4
5	0. 5	0. 4	0. 3	0. 3	0. 2	0. 1	0. 0	0. 0	5
6	0. 4	0. 3	0. 3	0. 2	0. 2	0. 1	0. 0	0. 0	6
+7	−0. 3	−0. 3	−0. 2	−0. 2	−0. 1	−0. 1	0. 0	0. 0	+7
8	0. 3	0. 3	0. 2	0. 2	0. 1	0. 1	0. 0	0. 0	8
9	0. 3	0. 2	0. 2	0. 2	0. 1	0. 1	0. 0	0. 0	9
10	0. 2	0. 2	0. 2	0. 1	0. 1	0. 1	0. 0	0. 0	10
15	0. 2	0. 1	0. 1	0. 1	0. 1	0. 0	0. 0	0. 0	15
+20	−0. 1	−0. 1	−0. 1	−0. 1	−0. 1	0. 0	0. 0	0. 0	+20
30	0. 1	0. 1	0. 1	0. 0	0. 0	0. 0	0. 0	0. 0	30
50	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	50
70	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	70
+90	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	+90

Altitude	Pressure in inches or millibars—**Add** correction to sextant or rectified altitude								Altitude
	29. 6	29. 4	29. 2	29. 0	28. 8	28. 6	28. 4	28. 2	
	1002.37	995.60	988.83	982.05	975.28	968.51	961.74	954.96	
° ′	′	′	′	′	′	′	′	′	° ′
−0 10	+0. 3	+0. 5	+0. 8	+1. 0	+1. 3	+1. 5	+1. 8	+2. 0	−0 10
0 00	0. 3	0. 5	0. 7	1. 0	1. 2	1. 4	1. 6	1. 9	0 00
+0 10	0. 2	0. 5	0. 7	0. 9	1. 1	1. 3	1. 5	1. 8	+0 10
0 20	0. 2	0. 4	0. 6	0. 8	1. 1	1. 3	1. 5	1. 7	0 20
0 30	0. 2	0. 4	0. 6	0. 8	1. 0	1. 2	1. 4	1. 6	0 30
+0 45	+0. 2	+0. 4	+0. 6	+0. 7	+0. 9	+1. 1	+1. 3	+1. 4	+0 45
1 00	0. 2	0. 3	0. 5	0. 7	0. 8	1. 0	1. 2	1. 3	1 00
1 20	0. 2	0. 3	0. 5	0. 6	0. 8	0. 9	1. 1	1. 2	1 20
1 40	0. 2	0. 3	0. 4	0. 6	0. 7	0. 8	1. 0	1. 1	1 40
2 00	0. 1	0. 3	0. 4	0. 5	0. 6	0. 8	0. 9	1. 0	2 00
+2 30	+0. 1	+0. 2	+0. 3	+0. 4	+0. 6	+0. 7	+0. 8	+0. 9	+2 30
3 00	0. 1	0. 2	0. 3	0. 4	0. 5	0. 6	0. 7	0. 8	3 00
4	0. 1	0. 2	0. 2	0. 3	0. 4	0. 5	0. 6	0. 6	4
5	0. 1	0. 1	0. 2	0. 3	0. 3	0. 4	0. 5	0. 5	5
6	0. 1	0. 1	0. 2	0. 2	0. 3	0. 3	0. 4	0. 5	6
+7	+0. 1	+0. 1	+0. 2	+0. 2	+0. 3	+0. 3	+0. 4	+0. 4	+7
8	0. 0	0. 1	0. 1	0. 2	0. 2	0. 3	0. 3	0. 4	8
9	0. 0	0. 1	0. 1	0. 2	0. 2	0. 2	0. 3	0. 3	9
10	0. 0	0. 1	0. 1	0. 1	0. 2	0. 2	0. 3	0. 3	10
15	0. 0	0. 1	0. 1	0. 1	0. 1	0. 1	0. 2	0. 2	15
+20	0. 0	0. 0	+0. 1	+0. 1	+0. 1	+0. 1	+0. 1	+0. 1	+20
30	0. 0	0. 0	0. 0	0. 0	0. 1	0. 1	0. 1	0. 1	30
50	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	50
70	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	70
+90	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	+90

TABLE 25

Meridian Angle and Altitude of a Body on the Prime Vertical Circle

Latitude	Declination (same name as latitude)												Latitude
	0°		1°		2°		3°		4°		5°		
	t	Alt.	t	Alt.	t	Alt.	t	Alt.	t	Alt.	t	Alt.	
°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	—	—	*90.0*	*0.0*	*90.0*	*0.0*	*90.0*	*0.0*	*90.0*	*0.0*	*90.0*	*0.0*	0
1	90.0	0.0	0.0	90.0	*60.0*	*30.0*	*70.5*	*19.5*	*75.5*	*14.5*	*78.5*	*11.6*	1
2	90.0	0.0	60.0	30.0	0.0	90.0	*48.2*	*41.8*	*60.0*	*30.0*	*66.5*	*23.6*	2
3	90.0	0.0	70.5	19.5	48.2	41.8	0.0	90.0	*41.5*	*48.6*	*53.2*	*36.9*	3
4	90.0	0.0	75.5	14.5	60.0	30.0	41.5	48.6	0.0	90.0	*36.9*	*53.2*	4
5	90.0	0.0	78.5	11.6	66.5	23.6	53.2	36.9	36.9	53.2	0.0	90.0	5
6	90.0	0.0	80.4	9.6	70.6	19.5	60.1	30.0	48.3	41.9	33.7	56.5	6
7	90.0	0.0	81.8	8.2	73.5	16.6	64.7	25.4	55.3	34.9	44.6	45.7	7
8	90.0	0.0	82.9	7.2	75.6	14.5	68.1	22.1	60.2	30.1	51.5	38.8	8
9	90.0	0.0	83.7	6.4	77.3	12.9	70.7	19.5	63.8	26.5	56.5	33.9	9
10	90.0	0.0	84.3	5.8	78.6	11.6	72.7	17.5	66.6	23.7	60.3	30.1	10
11	90.0	0.0	84.8	5.2	79.7	10.5	74.4	15.9	68.9	21.4	63.3	27.2	11
12	90.0	0.0	85.3	4.8	80.5	9.7	75.7	14.6	70.8	19.6	65.7	24.8	12
13	90.0	0.0	85.7	4.4	81.3	8.9	76.9	13.5	72.4	18.1	67.7	22.8	13
14	90.0	0.0	86.0	4.1	81.9	8.3	77.9	12.5	73.7	16.8	69.5	21.1	14
15	90.0	0.0	86.3	3.9	82.5	7.7	78.7	11.7	74.9	15.6	70.9	19.7	15
16	90.0	0.0	86.5	3.6	83.0	7.3	79.5	10.9	75.9	14.7	72.2	18.4	16
17	90.0	0.0	86.7	3.4	83.4	6.9	80.1	10.3	76.8	13.8	73.4	17.3	17
18	90.0	0.0	86.9	3.2	83.8	6.5	80.7	9.8	77.6	13.0	74.4	16.4	18
19	90.0	0.0	87.1	3.1	84.2	6.2	81.2	9.3	78.3	12.4	75.3	15.5	19
20	90.0	0.0	87.3	2.9	84.5	5.9	81.7	8.8	78.9	11.8	76.1	14.8	20
21	90.0	0.0	87.4	2.8	84.8	5.6	82.2	8.4	79.5	11.2	76.8	14.1	21
22	90.0	0.0	87.5	2.7	85.0	5.3	82.5	8.0	80.0	10.7	77.5	13.5	22
23	90.0	0.0	87.6	2.6	85.3	5.1	82.9	7.7	80.5	10.3	78.1	12.9	23
24	90.0	0.0	87.8	2.5	85.5	4.9	83.2	7.4	81.0	9.9	78.7	12.4	24
25	90.0	0.0	87.9	2.4	85.7	4.7	83.5	7.1	81.4	9.5	79.2	11.9	25
26	90.0	0.0	87.9	2.3	85.9	4.6	83.8	6.9	81.8	9.2	79.7	11.5	26
27	90.0	0.0	88.0	2.2	86.1	4.4	84.1	6.6	82.1	8.8	80.1	11.1	27
28	90.0	0.0	88.1	2.1	86.2	4.3	84.3	6.4	82.4	8.5	80.5	10.7	28
29	90.0	0.0	88.2	2.1	86.4	4.1	84.6	6.2	82.8	8.3	80.9	10.4	29
30	90.0	0.0	88.3	2.0	86.5	4.0	84.8	6.0	83.0	8.0	81.3	10.0	30
31	90.0	0.0	88.3	1.9	86.7	3.9	85.0	5.8	83.3	7.8	81.6	9.7	31
32	90.0	0.0	88.4	1.9	86.8	3.8	85.2	5.7	83.6	7.6	82.0	9.5	32
33	90.0	0.0	88.5	1.8	86.9	3.7	85.4	5.5	83.8	7.4	82.3	9.2	33
34	90.0	0.0	88.5	1.8	87.0	3.6	85.5	5.4	84.0	7.2	82.5	9.0	34
35	90.0	0.0	88.6	1.7	87.1	3.5	85.7	5.2	84.3	7.0	82.8	8.7	35
36	90.0	0.0	88.6	1.7	87.2	3.4	85.9	5.1	84.5	6.8	83.1	8.5	36
37	90.0	0.0	88.7	1.7	87.3	3.3	86.0	5.0	84.7	6.7	83.3	8.3	37
38	90.0	0.0	88.7	1.6	87.4	3.2	86.2	4.9	84.9	6.5	83.6	8.1	38
39	90.0	0.0	88.8	1.6	87.5	3.2	86.3	4.8	85.0	6.4	83.8	8.0	39
40	90.0	0.0	88.8	1.6	87.6	3.1	86.4	4.7	85.2	6.2	84.0	7.8	40
41	90.0	0.0	88.8	1.5	87.7	3.0	86.5	4.6	85.4	6.1	84.2	7.6	41
42	90.0	0.0	88.9	1.5	87.8	3.0	86.7	4.5	85.5	6.0	84.4	7.5	42
43	90.0	0.0	88.9	1.5	87.9	2.9	86.8	4.4	85.7	5.9	84.6	7.3	43
44	90.0	0.0	89.0	1.4	87.9	2.9	86.9	4.3	85.8	5.8	84.8	7.2	44
45	90.0	0.0	89.0	1.4	88.0	2.8	87.0	4.2	86.0	5.7	85.0	7.1	45
46	90.0	0.0	89.0	1.4	88.1	2.8	87.1	4.2	86.1	5.6	85.2	7.0	46
47	90.0	0.0	89.1	1.4	88.1	2.7	87.2	4.1	86.3	5.5	85.3	6.8	47
48	90.0	0.0	89.1	1.3	88.2	2.7	87.3	4.0	86.4	5.4	85.5	6.7	48
49	90.0	0.0	89.1	1.3	88.3	2.7	87.4	4.0	86.5	5.3	85.6	6.6	49
50	90.0	0.0	89.2	1.3	88.3	2.6	87.5	3.9	86.6	5.2	85.8	6.5	50
52	90.0	0.0	89.2	1.3	88.4	2.5	87.7	3.8	86.9	5.1	86.1	6.4	52
54	90.0	0.0	89.3	1.2	88.5	2.5	87.8	3.7	87.1	4.9	86.4	6.2	54
56	90.0	0.0	89.3	1.2	88.7	2.4	88.0	3.6	87.3	4.8	86.6	6.0	56
58	90.0	0.0	89.4	1.2	88.7	2.4	88.1	3.5	87.5	4.7	86.9	5.9	58
60	90.0	0.0	89.4	1.2	88.8	2.3	88.3	3.5	87.7	4.6	87.1	5.8	60
65	90.0	0.0	89.5	1.1	89.1	2.2	88.6	3.3	88.1	4.4	87.7	5.5	65
70	90.0	0.0	89.6	1.1	89.3	2.1	88.9	3.2	88.5	4.3	88.2	5.3	70
75	90.0	0.0	89.7	1.0	89.5	2.1	89.2	3.1	88.9	4.1	88.7	5.2	75
80	90.0	0.0	89.8	1.0	89.6	2.0	89.5	3.0	89.3	4.1	89.1	5.1	80
85	90.0	0.0	89.9	1.0	89.8	2.0	89.7	3.0	89.6	4.0	89.6	5.0	85

Numbers in *italics* indicate nearest approach to prime vertical

TABLE 25

Meridian Angle and Altitude of a Body on the Prime Vertical Circle

Latitude	Declination (same name as latitude)												Latitude
	6°		7°		8°		9°		10°		11°		
	t	Alt.	t	Alt.	t	Alt.	t	Alt.	t	Alt.	t	Alt.	
°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	*90.0*	*0.0*	*90.0*	*0.0*	*90.0*	*0.0*	*90.0*	*0.0*	*90.0*	*0.0*	*90.0*	*0.0*	0
1	*80.4*	*9.6*	*81.8*	*8.2*	*82.9*	*7.2*	*83.7*	*6.4*	*84.3*	*5.8*	*84.8*	*5.2*	1
2	*70.6*	*19.5*	*73.5*	*16.6*	*75.6*	*14.5*	*77.3*	*12.9*	*78.6*	*11.6*	*79.7*	*10.5*	2
3	*60.1*	*30.0*	*64.7*	*25.4*	*68.1*	*22.1*	*70.7*	*19.5*	*72.7*	*17.5*	*74.4*	*15.9*	3
4	*48.3*	*41.9*	*55.3*	*34.9*	*60.2*	*30.1*	*63.8*	*26.5*	*66.6*	*23.7*	*68.9*	*21.4*	4
5	*33.7*	*56.5*	*44.6*	*45.7*	*51.5*	*38.8*	*56.5*	*33.9*	*60.3*	*30.1*	*63.3*	*27.2*	5
6	0.0	90.0	*31.1*	*59.1*	*41.6*	*48.7*	*48.4*	*41.9*	*53.4*	*37.0*	*57.3*	*33.2*	6
7	31.1	59.1	0.0	90.0	*29.1*	*61.1*	*39.2*	*51.2*	*45.9*	*44.6*	*50.8*	*39.7*	7
8	41.6	48.7	29.1	61.1	0.0	90.0	*27.5*	*62.8*	*37.2*	*53.3*	*43.7*	*46.8*	8
9	48.4	41.9	39.2	51.2	27.5	62.8	0.0	90.0	*26.1*	*64.3*	*35.4*	*55.1*	9
10	53.4	37.0	45.9	44.6	37.2	53.3	26.1	64.3	0.0	90.0	*24.9*	*65.5*	10
11	57.3	33.2	50.8	39.7	43.7	46.8	35.4	55.1	24.9	65.5	0.0	90.0	11
12	60.4	30.2	54.7	35.9	48.6	42.0	41.8	48.8	33.9	56.6	23.9	66.6	12
13	62.9	27.7	57.9	32.8	52.5	38.2	46.7	44.1	40.2	50.5	32.7	58.0	13
14	65.1	25.6	60.5	30.2	55.7	35.1	50.6	40.3	45.0	45.9	38.8	52.1	14
15	66.9	23.8	62.7	28.1	58.4	32.5	53.8	37.2	48.8	42.1	43.5	47.5	15
16	68.5	22.3	64.6	26.2	60.7	30.3	56.5	34.6	52.1	39.0	47.3	43.8	16
17	69.9	20.9	66.3	24.6	62.6	28.4	58.8	32.3	54.8	36.4	50.5	40.7	17
18	71.1	19.8	67.8	23.2	64.4	26.8	60.8	30.4	57.1	34.2	53.3	38.1	18
19	72.2	18.7	69.1	22.0	65.9	25.3	62.6	28.7	59.2	32.2	55.6	35.9	19
20	73.2	17.8	70.3	20.9	67.3	24.0	64.2	27.2	61.0	30.5	57.7	33.9	20
21	74.1	17.0	71.3	19.9	68.5	22.9	65.6	25.9	62.7	29.0	59.6	32.2	21
22	74.9	16.2	72.3	19.0	69.6	21.8	66.9	24.7	64.1	27.6	61.2	30.6	22
23	75.7	15.5	73.2	18.2	70.7	20.9	68.1	23.6	65.5	26.4	62.7	29.2	23
24	76.3	14.9	74.0	17.4	71.6	20.0	69.2	22.6	66.7	25.3	64.1	28.0	24
25	77.0	14.3	74.7	16.8	72.5	19.2	70.1	21.7	67.8	24.3	65.4	26.8	25
26	77.6	13.8	75.4	16.1	73.3	18.5	71.1	20.9	68.8	23.3	66.5	25.8	26
27	78.1	13.3	76.1	15.6	74.0	17.9	71.9	20.2	69.8	22.5	67.6	24.9	27
28	78.6	12.9	76.6	15.0	74.7	17.2	72.7	19.5	70.6	21.7	68.6	24.0	28
29	79.1	12.5	77.2	14.6	75.3	16.7	73.4	18.8	71.5	21.0	69.5	23.2	29
30	79.5	12.1	77.7	14.1	75.9	16.2	74.1	18.2	72.2	20.3	70.3	22.4	30
31	79.9	11.7	78.2	13.7	76.5	15.7	74.7	17.7	72.9	19.7	71.1	21.7	31
32	80.3	11.4	78.7	13.3	77.0	15.2	75.3	17.2	73.6	19.1	71.9	21.1	32
33	80.7	11.1	79.1	12.9	77.5	14.8	75.9	16.7	74.2	18.6	72.6	20.5	33
34	81.0	10.8	79.5	12.6	78.0	14.4	76.4	16.2	74.8	18.1	73.3	20.0	34
35	81.4	10.5	79.9	12.3	78.4	14.0	76.9	15.8	75.4	17.6	73.9	19.4	35
36	81.7	10.2	80.3	12.0	78.8	13.7	77.4	15.4	76.0	17.2	74.5	18.9	36
37	82.0	10.0	80.6	11.7	79.3	13.4	77.9	15.1	76.5	16.8	75.1	18.5	37
38	82.3	9.8	81.0	11.4	79.6	13.1	78.3	14.7	77.0	16.4	75.6	18.1	38
39	82.5	9.6	81.3	11.2	80.0	12.8	78.7	14.4	77.4	16.0	76.1	17.6	39
40	82.8	9.4	81.6	10.9	80.4	12.5	79.1	14.1	77.9	15.7	76.6	17.3	40
41	83.1	9.2	81.9	10.7	80.7	12.2	79.5	13.8	78.3	15.3	77.1	16.9	41
42	83.3	9.0	82.2	10.5	81.0	12.0	79.9	13.5	78.7	15.0	77.5	16.6	42
43	83.5	8.8	82.4	10.3	81.3	11.8	80.2	13.3	79.1	14.8	78.0	16.2	43
44	83.8	8.7	82.7	10.1	81.6	11.6	80.6	13.0	79.5	14.5	78.4	15.9	44
45	84.0	8.5	82.9	9.9	81.9	11.4	80.9	12.8	79.8	14.2	78.8	15.7	45
46	84.2	8.4	83.2	9.8	82.2	11.2	81.2	12.6	80.2	14.0	79.2	15.4	46
47	84.4	8.2	83.4	9.6	82.5	11.0	81.5	12.4	80.5	13.7	79.6	15.1	47
48	84.6	8.1	83.7	9.4	82.7	10.8	81.8	12.2	80.9	13.5	79.9	14.9	48
49	84.8	8.0	83.9	9.3	83.0	10.6	82.1	12.0	81.2	13.3	80.3	14.6	49
50	84.9	7.8	84.1	9.2	83.2	10.5	82.4	11.8	81.5	13.1	80.6	14.4	50
52	85.3	7.6	84.5	8.9	83.7	10.2	82.9	11.5	82.1	12.7	81.3	14.0	52
54	85.6	7.4	84.9	8.7	84.1	9.9	83.4	11.1	82.6	12.4	81.9	13.6	54
56	85.9	7.2	85.2	8.5	84.6	9.7	83.9	10.9	83.2	12.1	82.5	13.3	56
58	86.2	7.1	85.6	8.3	85.0	9.4	84.3	10.6	83.7	11.8	83.0	13.0	58
60	86.5	6.9	85.9	8.1	85.3	9.2	84.8	10.4	84.2	11.6	83.6	12.7	60
65	87.2	6.6	86.7	7.7	86.2	8.8	85.8	9.9	85.3	11.0	84.8	12.2	65
70	87.8	6.4	87.4	7.5	87.1	8.5	86.7	9.6	86.3	10.6	85.9	11.7	70
75	88.4	6.2	88.1	7.2	87.8	8.3	87.6	9.3	87.3	10.4	87.0	11.4	75
80	88.9	6.1	88.8	7.1	88.6	8.1	88.4	9.1	88.2	10.2	88.0	11.2	80
85	89.5	6.0	89.4	7.0	89.3	8.0	89.2	9.0	89.1	10.0	89.0	11.0	85

Numbers in *italics* indicate nearest approach to prime vertical

TABLE 25

Meridian Angle and Altitude of a Body on the Prime Vertical Circle

Latitude	Declination (same name as latitude)												Latitude
	12°		13°		14°		15°		16°		17°		
	t	Alt.	t	Alt.	t	Alt.	t	Alt.	t	Alt.	t	Alt.	
°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	*90. 0*	*0. 0*	*90. 0*	*0. 0*	*90. 0*	*0. 0*	*90. 0*	*0. 0*	*90. 0*	*0. 0*	*90. 0*	*0. 0*	0
1	*85. 3*	*4. 8*	*85. 7*	*4. 4*	*86. 0*	*4. 1*	*86. 3*	*3. 9*	*86. 5*	*3. 6*	*86. 7*	*3. 4*	1
2	*80. 5*	*9. 7*	*81. 3*	*8. 9*	*81. 9*	*8. 3*	*82. 5*	*7. 7*	*83. 0*	*7. 3*	*83. 4*	*6. 9*	2
3	*75. 7*	*14. 6*	*76. 9*	*13. 5*	*77. 9*	*12. 5*	*78. 7*	*11. 7*	*79. 5*	*10. 9*	*80. 1*	*10. 3*	3
4	*70. 8*	*19. 6*	*72. 4*	*18. 1*	*73. 7*	*16. 8*	*74. 9*	*15. 6*	*75. 9*	*14. 7*	*76. 8*	*13. 8*	4
5	*65. 7*	*24. 8*	*67. 7*	*22. 8*	*69. 5*	*21. 1*	*70. 9*	*19. 7*	*72. 2*	*18. 4*	*73. 4*	*17. 3*	5
6	*60. 4*	*30. 2*	*62. 9*	*27. 7*	*65. 1*	*25. 6*	*66. 9*	*23. 8*	*68. 5*	*22. 3*	*69. 9*	*20. 9*	6
7	*54. 7*	*35. 9*	*57. 9*	*32. 8*	*60. 5*	*30. 2*	*62. 7*	*28. 1*	*64. 6*	*26. 2*	*66. 3*	*24. 6*	7
8	*48. 6*	*42. 0*	*52. 5*	*38. 2*	*55. 7*	*35. 1*	*58. 4*	*32. 5*	*60. 7*	*30. 3*	*62. 6*	*28. 4*	8
9	*41. 8*	*48. 8*	*46. 7*	*44. 1*	*50. 6*	*40. 3*	*53. 8*	*37. 2*	*56. 5*	*34. 6*	*58. 8*	*32. 3*	9
10	*33. 9*	*56. 6*	*40. 2*	*50. 5*	*45. 0*	*45. 9*	*48. 8*	*42. 1*	*52. 1*	*39. 0*	*54. 8*	*36. 4*	10
11	*23. 9*	*66. 6*	*32. 7*	*58. 0*	*38. 8*	*52. 1*	*43. 5*	*47. 5*	*47. 3*	*43. 8*	*50. 5*	*40. 7*	11
12	0. 0	90. 0	*23. 0*	*67. 6*	*31. 5*	*59. 3*	*37. 5*	*53. 4*	*42. 2*	*49. 0*	*46. 0*	*45. 3*	12
13	23. 0	67. 6	0. 0	90. 0	*22. 2*	*68. 4*	*30. 5*	*60. 4*	*36. 4*	*54. 7*	*41. 0*	*50. 3*	13
14	31. 5	59. 3	22. 2	68. 4	0. 0	90. 0	*21. 5*	*69. 2*	*29. 6*	*61. 4*	*35. 4*	*55. 8*	14
15	37. 5	53. 4	30. 5	60. 4	21. 5	69. 2	0. 0	90. 0	*20. 9*	*69. 9*	*28. 8*	*62. 3*	15
16	42. 2	49. 0	36. 4	54. 7	29. 6	61. 4	20. 9	69. 9	0. 0	90. 0	*20. 3*	*70. 5*	16
17	46. 0	45. 3	41. 0	50. 3	35. 4	55. 8	28. 8	62. 3	20. 3	70. 5	0. 0	90. 0	17
18	49. 1	42. 3	44. 7	46. 7	39. 9	51. 5	34. 4	56. 9	28. 1	63. 1	19. 8	71. 1	18
19	51. 9	39. 7	47. 9	43. 7	43. 6	48. 0	38. 9	52. 7	33. 6	57. 8	27. 4	63. 9	19
20	54. 3	37. 4	50. 6	41. 1	46. 8	45. 0	42. 6	49. 2	38. 0	53. 7	32. 9	58. 7	20
21	56. 4	35. 5	53. 0	38. 9	49. 5	42. 4	45. 7	46. 2	41. 7	50. 3	37. 2	54. 7	21
22	58. 3	33. 7	55. 2	36. 9	51. 9	40. 2	48. 5	43. 7	44. 8	47. 4	40. 8	51. 3	22
23	60. 0	32. 1	57. 1	35. 1	54. 0	38. 3	50. 9	41. 5	47. 5	44. 9	43. 9	48. 4	23
24	61. 5	30. 7	58. 8	33. 6	55. 9	36. 5	53. 0	39. 5	49. 9	42. 7	46. 6	46. 0	24
25	62. 9	29. 5	60. 3	32. 2	57. 7	34. 9	54. 9	37. 8	52. 1	40. 7	49. 0	43. 8	25
26	64. 2	28. 3	61. 7	30. 9	59. 3	33. 5	56. 7	36. 2	54. 0	39. 0	51. 2	41. 8	26
27	65. 3	27. 3	63. 1	29. 7	60. 7	32. 2	58. 3	34. 8	55. 8	37. 4	53. 1	40. 1	27
28	66. 4	26. 3	64. 3	28. 6	62. 0	31. 0	59. 7	33. 5	57. 4	36. 0	54. 9	38. 5	28
29	67. 5	25. 4	65. 4	27. 6	63. 3	29. 9	61. 1	32. 3	58. 8	34. 6	56. 5	37. 1	29
30	68. 4	24. 6	66. 4	26. 7	64. 4	28. 9	62. 3	31. 2	60. 2	33. 5	58. 0	35. 8	30
31	69. 3	23. 8	67. 4	25. 9	65. 5	28. 0	63. 5	30. 2	61. 5	32. 4	59. 4	34. 6	31
32	70. 1	23. 1	68. 3	25. 1	66. 5	27. 2	64. 6	29. 2	62. 7	31. 3	60. 7	33. 5	32
33	70. 9	22. 4	69. 2	24. 4	67. 4	26. 4	65. 6	28. 4	63. 8	30. 4	61. 9	32. 5	33
34	71. 6	21. 8	70. 0	23. 7	68. 3	25. 6	66. 6	27. 6	64. 8	29. 5	63. 0	31. 5	34
35	72. 3	21. 3	70. 7	23. 1	69. 1	24. 9	67. 5	26. 8	65. 8	28. 7	64. 1	30. 6	35
36	73. 0	20. 7	71. 5	22. 5	69. 9	24. 3	68. 4	26. 1	66. 8	28. 0	65. 1	29. 8	36
37	73. 6	20. 2	72. 2	21. 9	70. 7	23. 7	69. 2	25. 5	67. 6	27. 3	66. 1	29. 1	37
38	74. 2	19. 7	72. 8	21. 4	71. 4	23. 1	69. 9	24. 9	68. 5	26. 6	67. 0	28. 4	38
39	74. 8	19. 3	73. 4	20. 9	72. 1	22. 6	70. 7	24. 3	69. 3	26. 0	67. 8	27. 7	39
40	75. 3	18. 9	74. 0	20. 5	72. 7	22. 1	71. 4	23. 7	70. 0	25. 4	68. 6	27. 1	40
41	75. 8	18. 5	74. 6	20. 1	73. 3	21. 6	72. 0	23. 2	70. 7	24. 8	69. 4	26. 5	41
42	76. 3	18. 1	75. 1	19. 6	73. 9	21. 2	72. 7	22. 8	71. 4	24. 3	70. 2	25. 9	42
43	76. 8	17. 7	75. 7	19. 3	74. 5	20. 8	73. 3	22. 3	72. 1	23. 8	70. 9	25. 4	43
44	77. 3	17. 4	76. 2	18. 9	75. 0	20. 4	73. 9	21. 9	72. 7	23. 4	71. 5	24. 9	44
45	77. 7	17. 1	76. 7	18. 5	75. 6	20. 0	74. 5	21. 5	73. 3	22. 9	72. 2	24. 4	45
46	78. 2	16. 8	77. 1	18. 2	76. 1	19. 7	75. 0	21. 1	73. 9	22. 5	72. 8	24. 0	46
47	78. 6	16. 5	77. 6	17. 9	76. 6	19. 3	75. 5	20. 7	74. 5	22. 1	73. 4	23. 6	47
48	79. 0	16. 2	78. 0	17. 6	77. 0	19. 0	76. 0	20. 4	75. 0	21. 8	74. 0	23. 2	48
49	79. 4	16. 0	78. 4	17. 3	77. 5	18. 7	76. 5	20. 1	75. 6	21. 4	74. 6	22. 8	49
50	79. 7	15. 7	78. 8	17. 1	77. 9	18. 4	77. 0	19. 7	76. 1	21. 1	75. 1	22. 4	50
52	80. 4	15. 3	79. 6	16. 6	78. 8	17. 9	77. 9	19. 2	77. 1	20. 5	76. 2	21. 8	52
54	81. 1	14. 9	80. 3	16. 1	79. 6	17. 4	78. 8	18. 7	78. 0	19. 9	77. 2	21. 2	54
56	81. 7	14. 5	81. 0	15. 7	80. 3	17. 0	79. 6	18. 2	78. 8	19. 4	78. 1	20. 7	56
58	82. 4	14. 2	81. 7	15. 4	81. 0	16. 6	80. 4	17. 8	79. 7	19. 0	79. 0	20. 2	58
60	83. 0	13. 9	82. 3	15. 1	81. 7	16. 2	81. 1	17. 4	80. 5	18. 6	79. 8	19. 7	60
65	84. 3	13. 3	83. 8	14. 4	83. 3	15. 5	82. 8	16. 6	82. 3	17. 7	81. 8	18. 8	65
70	85. 6	12. 8	85. 2	13. 8	84. 8	14. 9	84. 4	16. 0	84. 0	17. 1	83. 6	18. 1	70
75	86. 7	12. 4	86. 5	13. 5	86. 2	14. 5	85. 9	15. 5	85. 6	16. 6	85. 3	17. 6	75
80	87. 9	12. 2	87. 7	13. 2	87. 5	14. 2	87. 3	15. 2	87. 1	16. 3	86. 9	17. 3	80
85	88. 9	12. 0	88. 8	13. 1	88. 8	14. 1	88. 7	15. 1	88. 6	16. 1	88. 5	17. 1	85

Numbers in *italics* indicate nearest approach to prime vertical

TABLE 25

Meridian Angle and Altitude of a Body on the Prime Vertical Circle

Latitude	Declination (same name as latitude) 18° t	18° Alt.	19° t	19° Alt.	20° t	20° Alt.	21° t	21° Alt.	22° t	22° Alt.	23° t	23° Alt.	Latitude
°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	*90. 0*	*0. 0*	*90. 0*	*0. 0*	*90. 0*	*0. 0*	*90. 0*	*0. 0*	*90. 0*	*0. 0*	*90. 0*	*0. 0*	0
1	*86. 9*	*3. 2*	*87. 1*	*3. 1*	*87. 3*	*2. 9*	*87. 4*	*2. 8*	*87. 5*	*2. 7*	*87. 6*	*2. 6*	1
2	*83. 8*	*6. 5*	*84. 2*	*6. 2*	*84. 5*	*5. 9*	*84. 8*	*5. 6*	*85. 0*	*5. 3*	*85. 3*	*5. 1*	2
3	*80. 7*	*9. 8*	*81. 2*	*9. 3*	*81. 7*	*8. 8*	*82. 2*	*8. 4*	*82. 5*	*8. 0*	*82. 9*	*7. 7*	3
4	*77. 6*	*13. 0*	*78. 3*	*12. 4*	*78. 9*	*11. 8*	*79. 5*	*11. 2*	*80. 0*	*10. 7*	*80. 5*	*10. 3*	4
5	*74. 4*	*16. 4*	*75. 3*	*15. 5*	*76. 1*	*14. 8*	*76. 8*	*14. 1*	*77. 5*	*13. 5*	*78. 1*	*12. 9*	5
6	*71. 1*	*19. 8*	*72. 2*	*18. 7*	*73. 2*	*17. 8*	*74. 1*	*17. 0*	*74. 9*	*16. 2*	*75. 7*	*15. 5*	6
7	*67. 8*	*23. 2*	*69. 1*	*22. 0*	*70. 3*	*20. 9*	*71. 3*	*19. 9*	*72. 3*	*19. 0*	*73. 2*	*18. 2*	7
8	*64. 4*	*26. 8*	*65. 9*	*25. 3*	*67. 3*	*24. 0*	*68. 5*	*22. 9*	*69. 6*	*21. 8*	*70. 7*	*20. 9*	8
9	*60. 8*	*30. 4*	*62. 6*	*28. 7*	*64. 2*	*27. 2*	*65. 6*	*25. 9*	*66. 9*	*24. 7*	*68. 1*	*23. 6*	9
10	*57. 1*	*34. 2*	*59. 2*	*32. 2*	*61. 0*	*30. 5*	*62. 7*	*29. 0*	*64. 1*	*27. 6*	*65. 5*	*26. 4*	10
11	*53. 3*	*38. 1*	*55. 6*	*35. 9*	*57. 7*	*33. 9*	*59. 6*	*32. 2*	*61. 2*	*30. 6*	*62. 7*	*29. 2*	11
12	*49. 1*	*42. 3*	*51. 9*	*39. 7*	*54. 3*	*37. 4*	*56. 4*	*35. 5*	*58. 3*	*33. 7*	*60. 0*	*32. 1*	12
13	*44. 7*	*46. 7*	*47. 9*	*43. 7*	*50. 6*	*41. 1*	*53. 0*	*38. 9*	*55. 2*	*36. 9*	*57. 0*	*35. 2*	13
14	*39. 9*	*51. 5*	*43. 6*	*48. 0*	*46. 8*	*45. 0*	*49. 5*	*42. 5*	*51. 9*	*40. 2*	*54. 0*	*38. 3*	14
15	*34. 4*	*56. 9*	*38. 9*	*52. 7*	*42. 6*	*49. 2*	*45. 7*	*46. 2*	*48. 5*	*43. 7*	*50. 9*	*41. 5*	15
16	*28. 1*	*63. 1*	*33. 6*	*57. 8*	*38. 0*	*53. 7*	*41. 7*	*50. 3*	*44. 8*	*47. 4*	*47. 5*	*44. 9*	16
17	*19. 8*	*71. 1*	*27. 4*	*63. 9*	*32. 9*	*58. 7*	*37. 2*	*54. 7*	*40. 8*	*51. 3*	*43. 9*	*48. 4*	17
18	0. 0	90. 0	*19. 3*	*71. 7*	*26. 8*	*64. 6*	*32. 2*	*59. 6*	*36. 5*	*55. 6*	*40. 1*	*52. 3*	18
19	19. 3	71. 7	0. 0	90. 0	*18. 9*	*72. 2*	*26. 2*	*65. 3*	*31. 5*	*60. 4*	*35. 8*	*56. 4*	19
20	26. 8	64. 6	18. 9	72. 2	0. 0	90. 0	*18. 5*	*72. 6*	*25. 7*	*65. 9*	*31. 0*	*61. 1*	20
21	32. 2	59. 6	26. 2	65. 3	18. 5	72. 6	0. 0	90. 0	*18. 2*	*73. 1*	*25. 3*	*66. 5*	21
22	36. 5	55. 6	31. 5	60. 4	25. 7	65. 9	18. 2	73. 1	0. 0	90. 0	*17. 9*	*73. 5*	22
23	40. 1	52. 3	35. 8	56. 4	31. 0	61. 1	25. 3	66. 5	17. 9	73. 5	0. 0	90. 0	23
24	43. 1	49. 4	39. 3	53. 2	35. 2	57. 2	30. 4	61. 8	24. 8	67. 1	17. 6	73. 9	24
25	45. 8	47. 0	42. 4	50. 4	38. 7	54. 0	34. 6	58. 0	30. 0	62. 4	24. 5	67. 6	25
26	48. 2	44. 8	45. 1	48. 0	41. 7	51. 3	38. 1	54. 8	34. 1	58. 7	29. 5	63. 0	26
27	50. 4	42. 9	47. 5	45. 8	44. 4	48. 9	41. 1	52. 1	37. 5	55. 6	33. 6	59. 4	27
28	52. 3	41. 2	49. 6	43. 9	46. 8	46. 8	43. 8	49. 8	40. 5	52. 9	37. 0	56. 3	28
29	54. 1	39. 6	51. 6	42. 2	49. 0	44. 9	46. 2	47. 7	43. 2	50. 6	40. 0	53. 7	29
30	55. 8	38. 2	53. 4	40. 6	50. 9	43. 2	48. 3	45. 8	45. 6	48. 5	42. 7	51. 4	30
31	57. 3	36. 9	55. 0	39. 2	52. 7	41. 6	50. 3	44. 1	47. 7	46. 7	45. 1	49. 3	31
32	58. 7	35. 7	56. 6	37. 9	54. 4	40. 2	52. 1	42. 6	49. 7	45. 0	47. 2	47. 5	32
33	60. 0	34. 6	58. 0	36. 7	55. 9	38. 9	53. 8	41. 1	51. 5	43. 5	49. 2	45. 8	33
34	61. 2	33. 5	59. 3	35. 6	57. 3	37. 7	55. 3	39. 9	53. 2	42. 1	51. 0	44. 3	34
35	62. 4	32. 6	60. 5	34. 6	58. 7	36. 6	56. 8	38. 7	54. 8	40. 8	52. 7	42. 9	35
36	63. 4	31. 7	61. 7	33. 6	59. 9	35. 6	58. 1	37. 6	56. 2	39. 6	54. 3	41. 7	36
37	64. 5	30. 9	62. 8	32. 8	61. 1	34. 6	59. 4	36. 5	57. 6	38. 5	55. 7	40. 5	37
38	65. 4	30. 1	63. 9	31. 9	62. 2	33. 7	60. 6	35. 6	58. 9	37. 5	57. 1	39. 4	38
39	66. 3	29. 4	64. 8	31. 2	63. 3	32. 9	61. 7	34. 7	60. 1	36. 5	58. 4	38. 4	39
40	67. 2	28. 7	65. 8	30. 4	64. 3	32. 1	62. 8	33. 9	61. 2	35. 6	59. 6	37. 4	40
41	68. 1	28. 1	66. 7	29. 8	65. 2	31. 4	63. 8	33. 1	62. 3	34. 8	60. 8	36. 6	41
42	68. 8	27. 5	67. 5	29. 1	66. 2	30. 7	64. 8	32. 4	63. 3	34. 0	61. 9	35. 7	42
43	69. 6	26. 9	68. 3	28. 5	67. 0	30. 1	65. 7	31. 7	64. 3	33. 3	62. 9	35. 0	43
44	70. 3	26. 4	69. 1	27. 9	67. 9	29. 5	66. 6	31. 1	65. 3	32. 6	63. 9	34. 2	44
45	71. 0	25. 9	69. 9	27. 4	68. 7	28. 9	67. 4	30. 5	66. 2	32. 0	64. 9	33. 5	45
46	71. 7	25. 4	70. 6	26. 9	69. 4	28. 4	68. 2	29. 9	67. 0	31. 4	65. 8	32. 9	46
47	72. 4	25. 0	71. 3	26. 4	70. 2	27. 9	69. 0	29. 3	67. 9	30. 8	66. 7	32. 3	47
48	73. 0	24. 6	71. 9	26. 0	70. 9	27. 4	69. 8	28. 8	68. 7	30. 3	67. 5	31. 7	48
49	73. 6	24. 2	72. 6	25. 6	71. 6	26. 9	70. 5	28. 3	69. 4	29. 8	68. 3	31. 2	49
50	74. 2	23. 8	73. 2	25. 2	72. 2	26. 5	71. 2	27. 9	70. 2	29. 3	69. 1	30. 7	50
52	75. 3	23. 1	74. 4	24. 4	73. 5	25. 7	72. 5	27. 1	71. 6	28. 4	70. 6	29. 7	52
54	76. 3	22. 5	75. 5	23. 7	74. 7	25. 0	73. 8	26. 3	72. 9	27. 6	72. 0	28. 9	54
56	77. 3	21. 9	76. 6	23. 1	75. 8	24. 4	75. 0	25. 6	74. 2	26. 9	73. 4	28. 1	56
58	78. 3	21. 4	77. 6	22. 6	76. 9	23. 8	76. 1	25. 0	75. 4	26. 2	74. 6	27. 4	58
60	79. 2	20. 9	78. 5	22. 1	77. 9	23. 3	77. 2	24. 4	76. 5	25. 6	75. 8	26. 8	60
65	81. 3	19. 9	80. 8	21. 1	80. 2	22. 2	79. 7	23. 3	79. 1	24. 4	78. 6	25. 5	65
70	83. 2	19. 2	82. 8	20. 3	82. 4	21. 3	82. 0	22. 4	81. 5	23. 5	81. 1	24. 6	70
75	85. 0	18. 7	84. 7	19. 7	84. 4	20. 7	84. 1	21. 8	83. 8	22. 8	83. 5	23. 9	75
80	86. 7	18. 3	86. 5	19. 3	86. 3	20. 3	86. 1	21. 3	85. 9	22. 4	85. 7	23. 4	80
85	88. 4	18. 1	88. 3	19. 1	88. 2	20. 1	88. 1	21. 1	88. 0	22. 1	87. 9	23. 1	85

Numbers in *italics* indicate nearest approach to prime vertical

TABLE 26

Latitude and Longitude Factors
f, the change of latitude for a unit change in longitude
F, the change of longitude for a unit change in latitude

Azimuth angle	Latitude 0°		2°		4°		6°		8°		Azimuth angle
	f	F	f	F	f	F	f	F	f	F	
°											°
0	0.00	—	0.00	—	0.00	—	0.00	—	0.00	—	180
1	0.02	57.29	0.02	57.32	0.02	57.43	0.02	57.61	0.02	57.85	179
2	0.03	28.64	0.03	28.65	0.03	28.71	0.03	28.79	0.03	28.92	178
3	0.05	19.08	0.05	19.09	0.05	19.13	0.05	19.19	0.05	19.27	177
4	0.07	14.30	0.07	14.31	0.07	14.34	0.07	14.38	0.07	14.44	176
5	0.09	11.43	0.09	11.44	0.09	11.46	0.09	11.49	0.09	11.54	175
6	0.11	9.51	0.11	9.52	0.10	9.54	0.10	9.57	0.10	9.61	174
7	0.12	8.14	0.12	8.15	0.12	8.16	0.12	8.19	0.12	8.22	173
8	0.14	7.12	0.14	7.12	0.14	7.13	0.14	7.15	0.14	7.18	172
9	0.16	6.31	0.16	6.32	0.16	6.33	0.16	6.35	0.16	6.38	171
10	0.18	5.67	0.18	5.68	0.18	5.69	0.18	5.70	0.17	5.73	170
12	0.21	4.70	0.21	4.71	0.21	4.72	0.21	4.73	0.21	4.75	168
14	0.25	4.01	0.25	4.01	0.25	4.02	0.25	4.03	0.25	4.05	166
16	0.29	3.49	0.29	3.49	0.29	3.50	0.28	3.51	0.28	3.52	164
18	0.32	3.08	0.32	3.08	0.32	3.08	0.32	3.10	0.32	3.11	162
20	0.36	2.75	0.36	2.75	0.36	2.75	0.36	2.76	0.36	2.77	160
22	0.40	2.48	0.40	2.48	0.40	2.48	0.40	2.49	0.40	2.50	158
24	0.45	2.25	0.44	2.25	0.44	2.25	0.44	2.26	0.44	2.27	156
26	0.49	2.05	0.49	2.05	0.49	2.05	0.49	2.06	0.48	2.07	154
28	0.53	1.88	0.53	1.88	0.53	1.88	0.53	1.89	0.53	1.90	152
30	0.58	1.73	0.58	1.73	0.57	1.74	0.57	1.74	0.57	1.75	150
32	0.62	1.60	0.62	1.60	0.62	1.60	0.62	1.61	0.62	1.62	148
34	0.67	1.48	0.67	1.48	0.67	1.49	0.67	1.49	0.67	1.50	146
36	0.73	1.38	0.73	1.38	0.72	1.38	0.72	1.38	0.72	1.39	144
38	0.78	1.28	0.78	1.28	0.78	1.28	0.78	1.29	0.78	1.29	142
40	0.84	1.19	0.84	1.19	0.84	1.19	0.83	1.20	0.83	1.20	140
42	0.90	1.11	0.90	1.11	0.90	1.11	0.90	1.12	0.89	1.12	138
44	0.97	1.04	0.97	1.04	0.96	1.04	0.96	1.04	0.96	1.05	136
46	1.04	0.97	1.04	0.97	1.03	0.97	1.03	0.97	1.03	0.98	134
48	1.11	0.90	1.11	0.90	1.11	0.90	1.11	0.90	1.10	0.91	132
50	1.19	0.84	1.19	0.84	1.19	0.84	1.19	0.84	1.18	0.85	130
52	1.28	0.78	1.28	0.78	1.28	0.78	1.27	0.79	1.27	0.79	128
54	1.38	0.73	1.38	0.73	1.37	0.73	1.37	0.73	1.36	0.73	126
56	1.48	0.67	1.48	0.67	1.48	0.68	1.47	0.68	1.47	0.68	124
58	1.60	0.62	1.60	0.63	1.60	0.63	1.59	0.63	1.58	0.63	122
60	1.73	0.58	1.73	0.58	1.73	0.58	1.72	0.58	1.72	0.58	120
62	1.88	0.53	1.88	0.53	1.88	0.53	1.87	0.53	1.86	0.54	118
64	2.05	0.49	2.05	0.49	2.05	0.49	2.04	0.49	2.03	0.49	116
66	2.25	0.45	2.24	0.45	2.24	0.45	2.23	0.45	2.22	0.45	114
68	2.48	0.40	2.47	0.40	2.47	0.40	2.46	0.40	2.45	0.41	112
70	2.75	0.36	2.75	0.36	2.74	0.36	2.73	0.37	2.72	0.37	110
72	3.08	0.32	3.08	0.33	3.07	0.33	3.06	0.33	3.05	0.33	108
74	3.49	0.29	3.49	0.29	3.48	0.29	3.47	0.29	3.45	0.29	106
76	4.01	0.25	4.01	0.25	4.00	0.25	3.99	0.25	3.97	0.25	104
78	4.70	0.21	4.70	0.21	4.69	0.21	4.68	0.21	4.66	0.21	102
80	5.67	0.18	5.67	0.18	5.66	0.18	5.64	0.18	5.62	0.18	100
81	6.31	0.16	6.31	0.16	6.30	0.16	6.28	0.16	6.25	0.16	99
82	7.12	0.14	7.11	0.14	7.10	0.14	7.07	0.14	7.05	0.14	98
83	8.14	0.12	8.14	0.12	8.12	0.12	8.10	0.12	8.07	0.12	97
84	9.51	0.11	9.51	0.11	9.49	0.11	9.46	0.11	9.42	0.11	96
85	11.43	0.09	11.42	0.09	11.40	0.09	11.37	0.09	11.32	0.09	95
86	14.30	0.07	14.29	0.07	14.27	0.07	14.22	0.07	14.16	0.07	94
87	19.08	0.05	19.07	0.05	19.03	0.05	18.98	0.05	18.91	0.05	93
88	28.64	0.03	28.62	0.03	28.57	0.03	28.48	0.03	28.36	0.03	92
89	57.29	0.02	57.26	0.02	57.15	0.02	56.98	0.02	56.73	0.02	91
90	—	0.00	—	0.00	—	0.00	—	0.00	—	0.00	90
	0°		2°		4°		6°		8°		

Correction to latitude = f × error in longitude

Correction to longitude = F × error in latitude

TABLE 26

Latitude and Longitude Factors

f, the change of latitude for a unit change in longitude

F, the change of longitude for a unit change in latitude

Azimuth angle	Latitude 10° f	10° F	12° f	12° F	14° f	14° F	16° f	16° F	18° f	18° F	Azimuth angle
°											°
0	0.00	—	0.00	—	0.00	—	0.00	—	0.00	—	180
1	0.02	58.17	0.02	58.57	0.02	59.04	0.02	59.60	0.02	60.24	179
2	0.03	29.08	0.03	29.28	0.03	29.51	0.03	29.79	0.03	30.11	178
3	0.05	19.38	0.05	19.51	0.05	19.67	0.05	19.85	0.05	20.06	177
4	0.07	14.52	0.07	14.62	0.07	14.74	0.07	14.88	0.07	15.04	176
5	0.09	11.61	0.09	11.69	0.08	11.78	0.08	11.89	0.08	12.02	175
6	0.10	9.66	0.10	9.73	0.10	9.81	0.10	9.90	0.10	10.00	174
7	0.12	8.27	0.12	8.33	0.12	8.39	0.12	8.47	0.12	8.56	173
8	0.14	7.22	0.14	7.27	0.14	7.33	0.14	7.40	0.13	7.48	172
9	0.16	6.41	0.15	6.45	0.15	6.51	0.15	6.57	0.15	6.64	171
10	0.17	5.76	0.17	5.80	0.17	5.85	0.17	5.90	0.17	5.96	170
12	0.21	4.78	0.21	4.81	0.21	4.85	0.20	4.89	0.20	4.95	168
14	0.25	4.07	0.24	4.10	0.24	4.13	0.24	4.17	0.24	4.22	166
16	0.28	3.54	0.28	3.56	0.28	3.59	0.28	3.63	0.27	3.67	164
18	0.32	3.13	0.32	3.15	0.32	3.17	0.31	3.20	0.31	3.24	162
20	0.36	2.79	0.36	2.81	0.35	2.83	0.35	2.86	0.35	2.89	160
22	0.40	2.51	0.40	2.53	0.39	2.55	0.39	2.57	0.38	2.60	158
24	0.44	2.28	0.44	2.30	0.43	2.32	0.43	2.34	0.42	2.36	156
26	0.48	2.08	0.48	2.10	0.47	2.11	0.47	2.13	0.46	2.16	154
28	0.52	1.91	0.52	1.92	0.52	1.94	0.51	1.96	0.51	1.98	152
30	0.57	1.76	0.56	1.77	0.56	1.78	0.56	1.80	0.55	1.82	150
32	0.62	1.63	0.61	1.64	0.61	1.65	0.60	1.66	0.59	1.68	148
34	0.66	1.50	0.66	1.52	0.65	1.53	0.65	1.54	0.64	1.56	146
36	0.72	1.40	0.71	1.41	0.70	1.42	0.70	1.43	0.69	1.45	144
38	0.77	1.30	0.76	1.31	0.76	1.32	0.75	1.33	0.74	1.35	142
40	0.83	1.21	0.82	1.22	0.81	1.23	0.81	1.24	0.80	1.25	140
42	0.88	1.13	0.88	1.14	0.88	1.14	0.87	1.15	0.85	1.17	138
44	0.95	1.05	0.94	1.06	0.94	1.07	0.93	1.08	0.92	1.09	136
46	1.02	0.98	1.01	0.99	1.01	1.00	1.00	1.01	0.99	1.02	134
48	1.10	0.91	1.09	0.92	1.08	0.93	1.07	0.94	1.06	0.95	132
50	1.17	0.85	1.17	0.86	1.16	0.87	1.15	0.87	1.13	0.88	130
52	1.26	0.79	1.25	0.80	1.24	0.80	1.23	0.81	1.22	0.82	128
54	1.36	0.74	1.35	0.74	1.34	0.75	1.32	0.76	1.31	0.76	126
56	1.46	0.68	1.45	0.69	1.44	0.69	1.43	0.70	1.41	0.71	124
58	1.58	0.63	1.57	0.64	1.55	0.64	1.54	0.65	1.52	0.66	122
60	1.71	0.59	1.69	0.59	1.68	0.60	1.67	0.60	1.65	0.61	120
62	1.85	0.54	1.84	0.54	1.83	0.55	1.81	0.55	1.79	0.56	118
64	2.02	0.50	2.01	0.50	1.99	0.50	1.97	0.51	1.95	0.51	116
66	2.21	0.45	2.20	0.46	2.18	0.46	2.16	0.46	2.14	0.47	114
68	2.44	0.41	2.42	0.41	2.40	0.42	2.38	0.42	2.35	0.42	112
70	2.71	0.37	2.69	0.37	2.67	0.37	2.64	0.38	2.61	0.38	110
72	3.03	0.33	3.01	0.33	2.99	0.33	2.96	0.34	2.93	0.34	108
74	3.43	0.29	3.41	0.29	3.38	0.30	3.35	0.30	3.32	0.30	106
76	3.95	0.25	3.92	0.25	3.89	0.26	3.86	0.26	3.81	0.26	104
78	4.63	0.22	4.60	0.22	4.56	0.22	4.52	0.22	4.47	0.22	102
80	5.59	0.18	5.55	0.18	5.50	0.18	5.45	0.18	5.39	0.18	100
81	6.22	0.16	6.18	0.16	6.13	0.16	6.07	0.16	6.01	0.17	99
82	7.01	0.14	6.96	0.14	6.90	0.14	6.84	0.15	6.77	0.15	98
83	8.02	0.12	7.97	0.13	7.90	0.13	7.83	0.13	7.75	0.13	97
84	9.37	0.11	9.31	0.11	9.23	0.11	9.15	0.11	9.05	0.11	96
85	11.25	0.09	11.18	0.09	11.09	0.09	10.99	0.09	10.87	0.09	95
86	14.08	0.07	13.99	0.07	13.88	0.07	13.75	0.07	13.60	0.07	94
87	18.79	0.05	18.66	0.05	18.51	0.05	18.34	0.05	18.15	0.05	93
88	28.20	0.03	28.01	0.04	27.79	0.04	27.53	0.04	27.23	0.04	92
89	56.42	0.02	56.04	0.02	55.59	0.02	55.07	0.02	54.49	0.02	91
90	—	0.00	—	0.00	—	0.00	—	0.00	—	0.00	90
	10°		12°		14°		16°		18°		

Correction to latitude = f × error in longitude | Correction to longitude = F × error in latitude

TABLE 26

Latitude and Longitude Factors

f, the change of latitude for a unit change in longitude
F, the change of longitude for a unit change in latitude

Azimuth angle	Latitude 20° f	20° F	22° f	22° F	24° f	24° F	26° f	26° F	28° f	28° F	Azimuth angle
°											°
0	0. 00	—	0. 00	—	0. 00	—	0. 00	—	0. 00	—	180
1	0. 02	60. 97	0. 02	61. 79	0. 02	62. 71	0. 02	63. 74	0. 02	64. 88	179
2	0. 03	30. 47	0. 03	30. 89	0. 03	31. 35	0. 03	31. 86	0. 03	32. 43	178
3	0. 05	20. 31	0. 05	20. 58	0. 05	20. 89	0. 05	21. 23	0. 05	21. 61	177
4	0. 07	15. 22	0. 06	15. 42	0. 06	15. 65	0. 06	15. 91	0. 06	16. 20	176
5	0. 08	12. 16	0. 08	12. 33	0. 08	12. 51	0. 08	12. 72	0. 08	12. 95	175
6	0. 10	10. 12	0. 10	10. 26	0. 10	10. 41	0. 09	10. 59	0. 09	10. 78	174
7	0. 12	8. 67	0. 11	8. 78	0. 11	8. 91	0. 11	9. 06	0. 11	9. 22	173
8	0. 13	7. 57	0. 13	7. 67	0. 13	7. 79	0. 13	7. 92	0. 12	8. 06	172
9	0. 15	6. 72	0. 15	6. 81	0. 14	6. 91	0. 14	7. 02	0. 14	7. 15	171
10	0. 17	6. 03	0. 16	6. 12	0. 16	6. 21	0. 16	6. 31	0. 16	6. 42	170
12	0. 20	5. 01	0. 20	5. 07	0. 19	5. 15	0. 19	5. 23	0. 19	5. 33	168
14	0. 23	4. 27	0. 23	4. 33	0. 23	4. 39	0. 22	4. 46	0. 22	4. 54	166
16	0. 27	3. 71	0. 27	3. 76	0. 26	3. 82	0. 26	3. 88	0. 25	3. 95	164
18	0. 30	3. 28	0. 30	3. 32	0. 30	3. 37	0. 29	3. 42	0. 29	3. 49	162
20	0. 34	2. 92	0. 34	2. 96	0. 33	3. 01	0. 33	3. 06	0. 32	3. 11	160
22	0. 38	2. 63	0. 38	2. 67	0. 37	2. 71	0. 36	2. 75	0. 36	2. 80	158
24	0. 42	2. 39	0. 41	2. 42	0. 41	2. 46	0. 40	2. 50	0. 39	2. 54	156
26	0. 46	2. 18	0. 45	2. 21	0. 45	2. 24	0. 44	2. 28	0. 43	2. 32	154
28	0. 50	2. 00	0. 49	2. 03	0. 49	2. 06	0. 48	2. 09	0. 47	2. 13	152
30	0. 54	1. 84	0. 53	1. 87	0. 53	1. 90	0. 52	1. 93	0. 51	1. 96	150
32	0. 59	1. 70	0. 58	1. 73	0. 57	1. 75	0. 56	1. 78	0. 55	1. 81	148
34	0. 63	1. 58	0. 63	1. 60	0. 62	1. 62	0. 61	1. 65	0. 60	1. 68	146
36	0. 68	1. 47	0. 67	1. 48	0. 66	1. 51	0. 65	1. 53	0. 64	1. 56	144
38	0. 74	1. 36	0. 72	1. 38	0. 71	1. 40	0. 70	1. 42	0. 69	1. 45	142
40	0. 79	1. 27	0. 78	1. 28	0. 77	1. 30	0. 75	1. 33	0. 74	1. 35	140
42	0. 85	1. 18	0. 83	1. 20	0. 82	1. 22	0. 81	1. 24	0. 79	1. 26	138
44	0. 91	1. 10	0. 90	1. 12	0. 88	1. 13	0. 87	1. 15	0. 85	1. 17	136
46	0. 97	1. 03	0. 96	1. 04	0. 95	1. 06	0. 93	1. 07	0. 91	1. 09	134
48	1. 04	0. 96	1. 03	0. 97	1. 02	0. 99	1. 00	1. 00	0. 98	1. 02	132
50	1. 12	0. 89	1. 10	0. 91	1. 09	0. 92	1. 07	0. 93	1. 05	0. 95	130
52	1. 20	0. 83	1. 19	0. 84	1. 17	0. 85	1. 15	0. 87	1. 13	0. 88	128
54	1. 29	0. 77	1. 28	0. 78	1. 26	0. 79	1. 24	0. 81	1. 22	0. 82	126
56	1. 39	0. 72	1. 38	0. 73	1. 35	0. 74	1. 33	0. 75	1. 31	0. 76	124
58	1. 50	0. 66	1. 48	0. 67	1. 46	0. 68	1. 44	0. 70	1. 41	0. 71	122
60	1. 63	0. 61	1. 61	0. 62	1. 58	0. 63	1. 56	0. 64	1. 53	0. 65	120
62	1. 77	0. 57	1. 74	0. 57	1. 72	0. 58	1. 69	0. 59	1. 66	0. 60	118
64	1. 93	0. 52	1. 90	0. 53	1. 87	0. 53	1. 84	0. 54	1. 81	0. 55	116
66	2. 11	0. 47	2. 08	0. 48	2. 05	0. 49	2. 02	0. 50	1. 98	0. 50	114
68	2. 33	0. 43	2. 30	0. 44	2. 26	0. 44	2. 23	0. 45	2. 18	0. 46	112
70	2. 58	0. 39	2. 55	0. 39	2. 51	0. 40	2. 47	0. 40	2. 43	0. 41	110
72	2. 89	0. 35	2. 85	0. 35	2. 81	0. 36	2. 77	0. 36	2. 72	0. 37	108
74	3. 28	0. 31	3. 23	0. 31	3. 19	0. 31	3. 14	0. 32	3. 08	0. 33	106
76	3. 77	0. 27	3. 72	0. 27	3. 66	0. 27	3. 61	0. 28	3. 54	0. 28	104
78	4. 42	0. 23	4. 36	0. 23	4. 30	0. 23	4. 23	0. 24	4. 15	0. 24	102
80	5. 33	0. 19	5. 26	0. 19	5. 18	0. 19	5. 10	0. 20	5. 01	0. 20	100
81	5. 93	0. 17	5. 86	0. 17	5. 77	0. 17	5. 68	0. 18	5. 58	0. 18	99
82	6. 69	0. 15	6. 60	0. 15	6. 50	0. 15	6. 40	0. 16	6. 28	0. 16	98
83	7. 65	0. 13	7. 55	0. 13	7. 44	0. 13	7. 32	0. 14	7. 19	0. 14	97
84	8. 94	0. 11	8. 82	0. 11	8. 69	0. 12	8. 55	0. 12	8. 40	0. 12	96
85	10. 74	0. 09	10. 60	0. 09	10. 44	0. 10	10. 26	0. 10	10. 09	0. 10	95
86	13. 44	0. 07	13. 26	0. 08	13. 07	0. 08	12. 86	0. 08	12. 63	0. 08	94
87	17. 93	0. 06	17. 69	0. 06	17. 43	0. 06	17. 15	0. 06	16. 85	0. 06	93
88	26. 91	0. 04	26. 55	0. 04	26. 16	0. 04	25. 74	0. 04	25. 28	0. 04	92
89	53. 84	0. 02	53. 12	0. 02	52. 33	0. 02	51. 50	0. 02	50. 58	0. 02	91
90	—	0. 00	—	0. 00	—	0. 00	—	0. 00	—	0. 00	90
	20°		22°		24°		26°		28°		

Correction to latitude = f × error in longitude | Correction to longitude = F × error in latitude

TABLE 26

Latitude and Longitude Factors

f, the change of latitude for a unit change in longitude

F, the change of longitude for a unit change in latitude

Azimuth angle	Latitude 30° f	30° F	32° f	32° F	34° f	34° F	36° f	36° F	38° f	38° F	Azimuth angle
°											°
0	0.00	—	0.00	—	0.00	—	0.00	—	0.00	—	180
1	0.02	66.15	0.01	67.56	0.01	69.10	0.01	70.81	0.01	72.70	179
2	0.03	33.07	0.03	33.77	0.03	34.54	0.03	35.40	0.03	36.34	178
3	0.05	22.03	0.05	22.50	0.04	23.02	0.04	23.59	0.04	24.21	177
4	0.06	16.51	0.06	16.86	0.06	17.25	0.06	17.68	0.06	18.15	176
5	0.08	13.20	0.07	13.48	0.07	13.79	0.07	14.13	0.07	14.50	175
6	0.09	10.99	0.09	11.22	0.09	11.48	0.09	11.76	0.08	12.07	174
7	0.11	9.40	0.10	9.60	0.10	9.82	0.10	10.07	0.10	10.34	173
8	0.12	8.22	0.12	8.39	0.12	8.58	0.11	8.79	0.11	9.03	172
9	0.14	7.29	0.13	7.45	0.13	7.62	0.13	7.80	0.12	8.01	171
10	0.15	6.55	0.15	6.69	0.15	6.84	0.14	7.01	0.14	7.20	170
12	0.18	5.43	0.18	5.55	0.18	5.67	0.17	5.82	0.17	5.97	168
14	0.22	4.63	0.21	4.73	0.21	4.84	0.20	4.96	0.20	5.09	166
16	0.25	4.03	0.24	4.11	0.24	4.21	0.23	4.31	0.23	4.43	164
18	0.28	3.55	0.28	3.63	0.27	3.71	0.26	3.80	0.26	3.91	162
20	0.32	3.17	0.31	3.24	0.30	3.31	0.29	3.40	0.29	3.49	160
22	0.35	2.86	0.34	2.92	0.34	2.99	0.33	3.06	0.32	3.14	158
24	0.39	2.59	0.38	2.65	0.37	2.71	0.36	2.78	0.35	2.85	156
26	0.42	2.37	0.41	2.42	0.40	2.47	0.40	2.53	0.38	2.60	154
28	0.46	2.17	0.45	2.22	0.44	2.27	0.43	2.32	0.42	2.39	152
30	0.50	2.00	0.49	2.04	0.48	2.09	0.47	2.14	0.45	2.20	150
32	0.54	1.85	0.53	1.89	0.52	1.93	0.51	1.98	0.49	2.03	148
34	0.58	1.71	0.57	1.75	0.56	1.79	0.55	1.83	0.53	1.88	146
36	0.63	1.59	0.62	1.62	0.60	1.66	0.59	1.70	0.57	1.75	144
38	0.68	1.48	0.66	1.51	0.65	1.54	0.63	1.58	0.62	1.62	142
40	0.72	1.38	0.71	1.41	0.69	1.44	0.68	1.47	0.66	1.51	140
42	0.78	1.28	0.76	1.31	0.75	1.34	0.73	1.37	0.71	1.41	138
44	0.84	1.20	0.82	1.22	0.80	1.25	0.78	1.28	0.76	1.31	136
46	0.90	1.11	0.88	1.14	0.86	1.16	0.84	1.19	0.82	1.23	134
48	0.96	1.04	0.94	1.06	0.92	1.09	0.90	1.11	0.88	1.14	132
50	1.03	0.97	1.01	0.99	0.99	1.01	0.96	1.04	0.94	1.06	130
52	1.11	0.90	1.09	0.92	1.06	0.94	1.04	0.97	1.01	0.99	128
54	1.19	0.84	1.16	0.86	1.14	0.88	1.11	0.90	1.08	0.92	126
56	1.28	0.78	1.26	0.79	1.23	0.81	1.20	0.83	1.17	0.86	124
58	1.39	0.72	1.36	0.74	1.33	0.75	1.30	0.77	1.26	0.79	122
60	1.49	0.67	1.47	0.68	1.44	0.70	1.40	0.71	1.37	0.73	120
62	1.63	0.61	1.59	0.63	1.56	0.64	1.52	0.66	1.48	0.67	118
64	1.78	0.56	1.74	0.57	1.70	0.59	1.66	0.60	1.62	0.62	116
66	1.95	0.51	1.91	0.52	1.85	0.54	1.82	0.55	1.77	0.56	114
68	2.14	0.47	2.10	0.48	2.05	0.49	2.00	0.50	1.95	0.51	112
70	2.38	0.42	2.33	0.43	2.28	0.44	2.22	0.45	2.17	0.46	110
72	2.67	0.38	2.61	0.38	2.55	0.39	2.50	0.40	2.43	0.41	108
74	3.02	0.33	2.96	0.34	2.89	0.35	2.82	0.35	2.75	0.36	106
76	3.47	0.29	3.40	0.29	3.33	0.30	3.25	0.31	3.16	0.32	104
78	4.07	0.24	3.99	0.25	3.90	0.26	3.81	0.26	3.71	0.27	102
80	4.91	0.20	4.81	0.21	4.70	0.21	4.59	0.22	4.47	0.22	100
81	5.47	0.18	5.35	0.19	5.24	0.19	5.11	0.20	4.98	0.20	99
82	6.16	0.16	6.03	0.17	5.90	0.17	5.76	0.17	5.61	0.18	98
83	7.05	0.14	6.91	0.14	6.75	0.15	6.59	0.15	6.42	0.16	97
84	8.24	0.12	8.07	0.12	7.89	0.13	7.70	0.13	7.50	0.13	96
85	9.90	0.10	9.69	0.10	9.48	0.11	9.25	0.11	9.01	0.11	95
86	12.39	0.08	12.13	0.08	11.86	0.08	11.57	0.09	11.27	0.09	94
87	16.52	0.06	16.18	0.06	15.82	0.06	15.44	0.06	15.04	0.07	93
88	24.80	0.04	24.28	0.04	23.74	0.04	23.17	0.04	22.57	0.04	92
89	49.61	0.02	48.58	0.02	47.50	0.02	46.36	0.02	45.14	0.02	91
90	—	0.00	—	0.00	—	0.00	—	0.00	—	0.00	90
	30°		32°		34°		36°		38°		

Correction to latitude = f × error in longitude | Correction to longitude = F × error in latitude

TABLE 26

Latitude and Longitude Factors

f, the change of latitude for a unit change in longitude

F, the change of longitude for a unit change in latitude

Azimuth angle	Latitude 40° f	40° F	42° f	42° F	44° f	44° F	46° f	46° F	48° f	48° F	Azimuth angle
°											°
0	0.00	—	0.00	—	0.00	—	0.00	—	0.00	—	180
1	0.01	74.79	0.01	77.09	0.01	79.64	0.01	82.47	0.01	85.62	179
2	0.03	37.38	0.03	38.53	0.03	39.81	0.02	41.22	0.02	42.80	178
3	0.04	24.91	0.04	25.68	0.04	26.53	0.04	27.47	0.03	28.52	177
4	0.05	18.67	0.05	19.24	0.05	19.88	0.05	20.59	0.05	21.37	176
5	0.07	14.92	0.07	15.38	0.06	15.89	0.06	16.45	0.06	17.08	175
6	0.08	12.42	0.08	12.80	0.08	13.23	0.07	13.70	0.07	14.22	174
7	0.09	10.63	0.09	10.96	0.09	11.32	0.08	11.72	0.08	12.17	173
8	0.11	9.29	0.10	9.57	0.10	9.89	0.10	10.24	0.09	10.63	172
9	0.12	8.24	0.12	8.50	0.11	8.78	0.11	9.09	0.11	9.44	171
10	0.14	7.40	0.13	7.63	0.13	7.88	0.12	8.16	0.12	8.48	170
12	0.16	6.14	0.16	6.33	0.15	6.54	0.15	6.77	0.14	7.03	168
14	0.19	5.24	0.19	5.40	0.18	5.58	0.17	5.77	0.17	5.99	166
16	0.22	4.55	0.21	4.69	0.21	4.85	0.20	5.02	0.19	5.21	164
18	0.25	4.02	0.24	4.14	0.23	4.28	0.23	4.43	0.22	4.60	162
20	0.28	3.59	0.27	3.70	0.26	3.82	0.25	3.95	0.24	4.11	160
22	0.31	3.23	0.30	3.33	0.29	3.44	0.28	3.56	0.27	3.70	158
24	0.34	2.93	0.33	3.02	0.32	3.12	0.31	3.23	0.30	3.36	156
26	0.37	2.68	0.36	2.76	0.35	2.85	0.34	2.95	0.33	3.06	154
28	0.41	2.45	0.40	2.53	0.38	2.61	0.37	2.71	0.36	2.81	152
30	0.44	2.26	0.43	2.33	0.41	2.41	0.40	2.49	0.39	2.59	150
32	0.48	2.09	0.46	2.15	0.45	2.22	0.43	2.30	0.42	2.39	148
34	0.52	1.93	0.50	1.99	0.49	2.06	0.47	2.13	0.45	2.22	146
36	0.56	1.80	0.54	1.85	0.52	1.91	0.50	1.98	0.49	2.06	144
38	0.60	1.67	0.58	1.72	0.56	1.78	0.54	1.84	0.52	1.91	142
40	0.64	1.56	0.63	1.60	0.60	1.66	0.58	1.71	0.56	1.78	140
42	0.69	1.45	0.67	1.49	0.65	1.54	0.63	1.60	0.60	1.66	138
44	0.74	1.35	0.72	1.39	0.69	1.44	0.67	1.49	0.65	1.55	136
46	0.79	1.26	0.77	1.30	0.74	1.34	0.72	1.39	0.69	1.44	134
48	0.85	1.17	0.83	1.21	0.80	1.25	0.77	1.30	0.74	1.35	132
50	0.91	1.09	0.88	1.13	0.86	1.17	0.83	1.21	0.80	1.25	130
52	0.98	1.02	0.95	1.05	0.92	1.09	0.89	1.12	0.86	1.17	128
54	1.05	0.95	1.02	0.98	0.99	1.01	0.96	1.05	0.92	1.09	126
56	1.14	0.88	1.10	0.91	1.07	0.94	1.03	0.97	0.99	1.01	124
58	1.23	0.82	1.19	0.84	1.15	0.87	1.11	0.90	1.07	0.93	122
60	1.33	0.75	1.29	0.78	1.25	0.80	1.20	0.83	1.16	0.86	120
62	1.44	0.69	1.40	0.72	1.35	0.74	1.31	0.77	1.26	0.79	118
64	1.57	0.64	1.52	0.66	1.48	0.68	1.42	0.70	1.37	0.73	116
66	1.72	0.58	1.67	0.60	1.62	0.62	1.56	0.64	1.50	0.66	114
68	1.90	0.53	1.84	0.54	1.78	0.56	1.72	0.58	1.66	0.60	112
70	2.10	0.47	2.04	0.49	1.98	0.51	1.91	0.52	1.84	0.54	110
72	2.36	0.42	2.29	0.44	2.21	0.45	2.14	0.47	2.06	0.49	108
74	2.67	0.37	2.59	0.39	2.51	0.40	2.42	0.41	2.33	0.43	106
76	3.07	0.32	2.98	0.34	2.89	0.35	2.79	0.36	2.68	0.37	104
78	3.60	0.28	3.50	0.29	3.38	0.29	3.27	0.31	3.15	0.32	102
80	4.34	0.23	4.22	0.24	4.08	0.24	3.94	0.25	3.80	0.26	100
81	4.84	0.21	4.69	0.21	4.54	0.22	4.39	0.23	4.23	0.24	99
82	5.45	0.18	5.29	0.19	5.12	0.19	4.94	0.20	4.76	0.21	98
83	6.24	0.16	6.05	0.16	5.86	0.17	5.66	0.18	5.45	0.18	97
84	7.29	0.14	7.07	0.14	6.84	0.15	6.61	0.15	6.37	0.16	96
85	8.75	0.11	8.49	0.12	8.22	0.12	7.94	0.13	7.65	0.13	95
86	10.95	0.09	10.63	0.09	10.29	0.10	9.94	0.10	9.57	0.10	94
87	14.62	0.07	14.18	0.07	13.73	0.07	13.26	0.08	12.77	0.08	93
88	21.94	0.05	21.28	0.05	20.60	0.05	19.89	0.05	19.16	0.05	92
89	43.98	0.02	42.58	0.02	41.21	0.02	39.80	0.02	38.34	0.03	91
90	—	0.00	—	0.00	—	0.00	—	0.00	—	0.00	90
	40°		42°		44°		46°		48°		

Correction to latitude = f × error in longitude

Correction to longitude = F × error in latitude

TABLE 26

Latitude and Longitude Factors

f, the change of latitude for a unit change in longitude

F, the change of longitude for a unit change in latitude

Azimuth angle	Latitude 50° f	50° F	52° f	52° F	54° f	54° F	56° f	56° F	58° f	58° F	Azimuth angle
°											°
0	0.00	—	0.00	—	0.00	—	0.00	—	0.00	—	180
1	0.01	89.13	0.01	93.05	0.01	97.47	0.01	102.45	0.01	108.11	179
2	0.02	44.55	0.02	46.51	0.02	48.72	0.02	51.21	0.02	54.04	178
3	0.03	29.68	0.03	30.99	0.03	32.46	0.03	34.12	0.03	36.01	177
4	0.04	22.25	0.04	23.23	0.04	24.33	0.04	25.57	0.04	26.99	176
5	0.06	17.78	0.05	18.57	0.05	19.45	0.05	20.44	0.05	21.57	175
6	0.07	14.80	0.06	15.45	0.06	16.19	0.06	17.01	0.06	17.95	174
7	0.08	12.67	0.08	13.23	0.07	13.86	0.07	14.56	0.06	15.37	173
8	0.09	11.07	0.08	11.56	0.08	12.11	0.08	12.72	0.07	13.43	172
9	0.10	9.82	0.10	10.26	0.09	10.74	0.09	11.29	0.08	11.91	171
10	0.11	8.82	0.11	9.21	0.10	9.65	0.10	10.14	0.09	10.70	170
12	0.14	7.32	0.13	7.64	0.13	8.00	0.12	8.41	0.11	8.88	168
14	0.16	6.24	0.15	6.51	0.15	6.82	0.14	7.17	0.13	7.57	166
16	0.18	5.42	0.18	5.66	0.17	5.93	0.16	6.24	0.15	6.58	164
18	0.21	4.79	0.20	5.00	0.19	5.24	0.18	5.50	0.17	5.81	162
20	0.23	4.27	0.22	4.46	0.21	4.67	0.20	4.91	0.19	5.19	160
22	0.26	3.85	0.25	4.02	0.24	4.21	0.23	4.43	0.21	4.67	158
24	0.29	3.49	0.27	3.65	0.26	3.82	0.25	4.02	0.24	4.24	156
26	0.31	3.19	0.30	3.33	0.29	3.49	0.27	3.66	0.26	3.87	154
28	0.34	2.93	0.33	3.05	0.31	3.20	0.30	3.36	0.28	3.55	152
30	0.37	2.69	0.36	2.81	0.34	2.95	0.32	3.10	0.31	3.27	150
32	0.40	2.49	0.38	2.60	0.37	2.72	0.35	2.86	0.33	3.02	148
34	0.43	2.31	0.42	2.41	0.40	2.52	0.38	2.65	0.36	2.80	146
36	0.47	2.14	0.45	2.24	0.43	2.34	0.41	2.46	0.39	2.60	144
38	0.50	1.99	0.48	2.08	0.46	2.18	0.44	2.29	0.41	2.41	142
40	0.54	1.85	0.52	1.94	0.49	2.03	0.47	2.13	0.44	2.25	140
42	0.58	1.73	0.56	1.80	0.53	1.89	0.50	1.99	0.48	2.09	138
44	0.62	1.61	0.59	1.68	0.57	1.76	0.54	1.85	0.51	1.95	136
46	0.67	1.50	0.64	1.57	0.61	1.64	0.58	1.73	0.55	1.82	134
48	0.71	1.40	0.68	1.46	0.65	1.53	0.62	1.61	0.59	1.70	132
50	0.77	1.31	0.73	1.36	0.70	1.43	0.67	1.50	0.63	1.58	130
52	0.82	1.22	0.79	1.27	0.75	1.33	0.72	1.40	0.68	1.47	128
54	0.88	1.13	0.85	1.18	0.81	1.23	0.77	1.30	0.73	1.37	126
56	0.95	1.05	0.91	1.10	0.87	1.15	0.83	1.21	0.79	1.27	124
58	1.03	0.97	0.99	1.01	0.94	1.06	0.89	1.12	0.85	1.18	122
60	1.11	0.90	1.07	0.94	1.02	0.98	0.97	1.03	0.92	1.09	120
62	1.21	0.83	1.16	0.86	1.11	0.90	1.05	0.95	1.00	1.00	118
64	1.32	0.76	1.26	0.79	1.20	0.83	1.15	0.87	1.09	0.92	116
66	1.44	0.69	1.38	0.72	1.32	0.76	1.26	0.79	1.19	0.84	114
68	1.59	0.63	1.52	0.65	1.45	0.69	1.38	0.72	1.31	0.76	112
70	1.77	0.57	1.69	0.59	1.61	0.62	1.54	0.65	1.45	0.68	110
72	1.98	0.51	1.89	0.53	1.81	0.55	1.72	0.58	1.63	0.61	108
74	2.24	0.45	2.15	0.46	2.05	0.49	1.95	0.51	1.85	0.54	106
76	2.58	0.39	2.47	0.40	2.36	0.42	2.24	0.45	2.13	0.47	104
78	3.02	0.33	2.90	0.34	2.77	0.36	2.63	0.38	2.49	0.40	102
80	3.65	0.27	3.49	0.29	3.33	0.30	3.17	0.31	3.01	0.33	100
81	4.06	0.25	3.89	0.26	3.71	0.27	3.53	0.28	3.35	0.30	99
82	4.57	0.22	4.38	0.23	4.18	0.24	3.98	0.25	3.77	0.26	98
83	5.24	0.19	5.01	0.20	4.79	0.21	4.55	0.22	4.32	0.23	97
84	6.12	0.16	5.86	0.17	5.59	0.18	5.32	0.19	5.04	0.20	96
85	7.35	0.14	7.04	0.14	6.72	0.15	6.39	0.16	6.06	0.16	95
86	9.19	0.11	8.81	0.11	8.41	0.12	8.00	0.12	7.58	0.13	94
87	12.27	0.08	11.75	0.08	11.22	0.09	10.67	0.09	10.11	0.10	93
88	18.41	0.05	17.63	0.06	16.83	0.06	16.01	0.06	15.17	0.07	92
89	36.83	0.03	35.27	0.03	33.68	0.03	32.04	0.03	30.36	0.03	91
90	—	0.00	—	0.00	—	0.00	—	0.00	—	0.00	90
	50°		52°		54°		56°		58°		

Correction to latitude = f × error in longitude | Correction to longitude = F × error in latitude

TABLE 26

Latitude and Longitude Factors

f, the change of latitude for a unit change in longitude

F, the change of longitude for a unit change in latitude

Azimuth angle	Latitude 60° f	60° F	62° f	62° F	64° f	64° F	66° f	66° F	68° f	68° F	Azimuth angle
°											°
0	0. 00	—	0. 00	—	0. 00	—	0. 00	—	0. 00	—	180
1	0. 01	114. 58	0. 01	122. 03	0. 01	130. 69	0. 01	140. 85	0. 01	152. 93	179
2	0. 02	57. 27	0. 02	61. 00	0. 02	65. 32	0. 01	70. 40	0. 01	76. 44	178
3	0. 03	38. 16	0. 02	40. 64	0. 02	43. 53	0. 02	46. 91	0. 02	50. 94	177
4	0. 03	28. 60	0. 03	30. 46	0. 03	32. 62	0. 03	35. 16	0. 03	38. 18	176
5	0. 04	22. 86	0. 04	24. 35	0. 04	26. 07	0. 04	28. 10	0. 03	30. 51	175
6	0. 05	19. 03	0. 05	20. 27	0. 05	21. 70	0. 04	23. 39	0. 04	25. 40	174
7	0. 06	16. 29	0. 06	17. 35	0. 05	18. 58	0. 05	20. 02	0. 05	21. 74	173
8	0. 07	14. 23	0. 07	15. 16	0. 06	16. 23	0. 06	17. 49	0. 05	18. 99	172
9	0. 08	12. 63	0. 07	13. 45	0. 07	14. 40	0. 06	15. 52	0. 06	16. 85	171
10	0. 09	11. 34	0. 08	12. 08	0. 08	12. 94	0. 07	13. 94	0. 07	15. 14	170
12	0. 11	9. 41	0. 10	10. 02	0. 09	10. 73	0. 09	11. 57	0. 08	12. 56	168
14	0. 12	8. 02	0. 12	8. 54	0. 11	9. 15	0. 10	9. 86	0. 09	10. 71	166
16	0. 14	6. 97	0. 13	7. 43	0. 13	7. 96	0. 12	8. 57	0. 11	9. 31	164
18	0. 16	6. 15	0. 15	6. 56	0. 14	7. 02	0. 13	7. 57	0. 12	8. 22	162
20	0. 18	5. 49	0. 17	5. 85	0. 16	6. 27	0. 15	6. 75	0. 14	7. 33	160
22	0. 20	4. 95	0. 19	5. 27	0. 18	5. 65	0. 16	6. 09	0. 15	6. 61	158
24	0. 22	4. 49	0. 21	4. 78	0. 20	5. 12	0. 18	5. 52	0. 17	6. 00	156
26	0. 24	4. 10	0. 23	4. 37	0. 21	4. 68	0. 20	5. 04	0. 18	5. 47	154
28	0. 27	3. 76	0. 25	4. 01	0. 23	4. 29	0. 22	4. 62	0. 20	5. 02	152
30	0. 29	3. 46	0. 27	3. 69	0. 25	3. 95	0. 23	4. 26	0. 22	4. 62	150
32	0. 31	3. 20	0. 29	3. 41	0. 27	3. 65	0. 25	3. 93	0. 23	4. 27	148
34	0. 34	2. 96	0. 32	3. 16	0. 30	3. 38	0. 27	3. 65	0. 25	3. 96	146
36	0. 36	2. 75	0. 34	2. 93	0. 32	3. 14	0. 30	3. 38	0. 27	3. 67	144
38	0. 39	2. 56	0. 37	2. 73	0. 34	2. 92	0. 32	3. 15	0. 29	3. 42	142
40	0. 42	2. 38	0. 39	2. 54	0. 37	2. 72	0. 34	2. 93	0. 31	3. 18	140
42	0. 45	2. 22	0. 42	2. 37	0. 39	2. 53	0. 37	2. 73	0. 34	2. 96	138
44	0. 48	2. 07	0. 45	2. 21	0. 42	2. 36	0. 39	2. 55	0. 36	2. 76	136
46	0. 52	1. 93	0. 49	2. 06	0. 45	2. 20	0. 42	2. 37	0. 39	2. 58	134
48	0. 56	1. 80	0. 52	1. 92	0. 49	2. 05	0. 45	2. 21	0. 42	2. 40	132
50	0. 60	1. 68	0. 56	1. 79	0. 52	1. 91	0. 48	2. 06	0. 45	2. 24	130
52	0. 64	1. 56	0. 60	1. 66	0. 56	1. 78	0. 52	1. 92	0. 48	2. 09	128
54	0. 69	1. 45	0. 65	1. 55	0. 60	1. 66	0. 56	1. 79	0. 52	1. 94	126
56	0. 74	1. 35	0. 70	1. 44	0. 65	1. 54	0. 60	1. 66	0. 56	1. 80	124
58	0. 80	1. 25	0. 75	1. 33	0. 70	1. 43	0. 65	1. 54	0. 60	1. 67	122
60	0. 87	1. 15	0. 81	1. 23	0. 76	1. 32	0. 70	1. 42	0. 65	1. 54	120
62	0. 94	1. 06	0. 88	1. 13	0. 82	1. 21	0. 76	1. 31	0. 70	1. 42	118
64	1. 03	0. 97	0. 96	1. 04	0. 90	1. 11	0. 83	1. 20	0. 77	1. 30	116
66	1. 12	0. 89	1. 05	0. 95	0. 98	1. 02	0. 91	1. 09	0. 84	1. 19	114
68	1. 24	0. 81	1. 16	0. 86	1. 09	0. 92	1. 01	0. 99	0. 93	1. 08	112
70	1. 37	0. 73	1. 29	0. 78	1. 20	0. 83	1. 12	0. 89	1. 03	0. 97	110
72	1. 54	0. 65	1. 44	0. 69	1. 35	0. 74	1. 25	0. 80	1. 15	0. 87	108
74	1. 74	0. 57	1. 64	0. 61	1. 53	0. 65	1. 42	0. 70	1. 31	0. 77	106
76	2. 01	0. 50	1. 88	0. 53	1. 76	0. 57	1. 63	0. 61	1. 50	0. 67	104
78	2. 35	0. 42	2. 21	0. 45	2. 06	0. 48	1. 91	0. 52	1. 76	0. 57	102
80	2. 84	0. 35	2. 66	0. 38	2. 49	0. 40	2. 31	0. 43	2. 12	0. 47	100
81	3. 16	0. 32	2. 96	0. 34	2. 77	0. 36	2. 57	0. 39	2. 37	0. 42	99
82	3. 56	0. 28	3. 34	0. 30	3. 12	0. 32	2. 89	0. 35	2. 67	0. 38	98
83	4. 07	0. 25	3. 82	0. 26	3. 57	0. 28	3. 31	0. 30	3. 05	0. 33	97
84	4. 76	0. 21	4. 47	0. 22	4. 17	0. 24	3. 87	0. 26	3. 56	0. 28	96
85	5. 72	0. 17	5. 37	0. 19	5. 01	0. 20	4. 65	0. 22	4. 28	0. 23	95
86	7. 15	0. 14	6. 71	0. 15	6. 27	0. 16	5. 82	0. 17	5. 36	0. 19	94
87	9. 54	0. 10	8. 96	0. 11	8. 36	0. 12	7. 76	0. 13	7. 15	0. 14	93
88	14. 32	0. 07	13. 44	0. 07	12. 55	0. 08	11. 65	0. 09	10. 73	0. 09	92
89	28. 65	0. 03	26. 90	0. 04	25. 11	0. 04	23. 30	0. 04	21. 46	0. 05	91
90	—	0. 00	—	0. 00	—	0. 00	—	0. 00	—	0. 00	90
	60°		62°		64°		66°		68°		

Correction to latitude = f × error in longitude

Correction to longitude = F × error in latitude

TABLE 27
Amplitudes

Latitude	Declination 0°0	0°5	1°0	1°5	2°0	2°5	3°0	3°5	4°0	4°5	5°0	5°5	6°0	Latitude
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	0
10	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.6	4.1	4.6	5.1	5.6	6.1	10
15	0.0	0.5	1.0	1.6	2.1	2.6	3.1	3.6	4.1	4.7	5.2	5.7	6.2	15
20	0.0	0.5	1.1	1.6	2.1	2.7	3.2	3.7	4.3	4.8	5.3	5.9	6.4	20
25	0.0	0.6	1.1	1.7	2.2	2.8	3.3	3.9	4.4	5.0	5.5	6.1	6.6	25
30	0.0	0.6	1.2	1.7	2.3	2.9	3.5	4.0	4.6	5.2	5.8	6.4	6.9	30
32	0.0	0.6	1.2	1.8	2.4	2.9	3.5	4.1	4.7	5.3	5.9	6.5	7.1	32
34	0.0	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4	6.0	6.6	7.2	34
36	0.0	0.6	1.2	1.9	2.5	3.1	3.7	4.3	4.9	5.6	6.2	6.8	7.4	36
38	0.0	0.6	1.3	1.9	2.5	3.2	3.8	4.4	5.1	5.7	6.4	7.0	7.6	38
40	0.0	0.7	1.3	2.0	2.6	3.3	3.9	4.6	5.2	5.9	6.5	7.2	7.8	40
42	0.0	0.7	1.3	2.0	2.7	3.4	4.0	4.7	5.4	6.1	6.7	7.4	8.1	42
44	0.0	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	7.0	7.7	8.4	44
46	0.0	0.7	1.4	2.2	2.9	3.6	4.3	5.0	5.8	6.5	7.2	7.9	8.7	46
48	0.0	0.7	1.5	2.2	3.0	3.7	4.5	5.2	6.0	6.7	7.5	8.2	9.0	48
50	0.0	0.8	1.6	2.3	3.1	3.9	4.7	5.4	6.2	7.0	7.8	8.6	9.4	50
51	0.0	0.8	1.6	2.4	3.2	4.0	4.8	5.6	6.4	7.2	8.0	8.8	9.6	51
52	0.0	0.8	1.6	2.4	3.2	4.1	4.9	5.7	6.5	7.3	8.1	9.0	9.8	52
53	0.0	0.8	1.7	2.5	3.3	4.2	5.0	5.8	6.7	7.5	8.3	9.2	10.0	53
54	0.0	0.9	1.7	2.6	3.4	4.3	5.1	6.0	6.8	7.7	8.5	9.4	10.2	54
55	0.0	0.9	1.7	2.6	3.5	4.4	5.2	6.1	7.0	7.9	8.7	9.6	10.5	55
56	0.0	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8	56
57	0.0	0.9	1.8	2.8	3.7	4.6	5.5	6.4	7.4	8.3	9.2	10.1	11.1	57
58	0.0	0.9	1.9	2.8	3.8	4.7	5.7	6.6	7.6	8.5	9.5	10.4	11.4	58
59	0.0	1.0	1.9	2.9	3.9	4.9	5.8	6.8	7.8	8.8	9.7	10.7	11.7	59
60	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.1	12.1	60
61	0.0	1.0	2.1	3.1	4.1	5.2	6.2	7.2	8.3	9.3	10.3	11.4	12.5	61
62	0.0	1.1	2.1	3.2	4.3	5.3	6.4	7.5	8.5	9.6	10.7	11.8	12.9	62
63	0.0	1.1	2.2	3.3	4.4	5.5	6.6	7.7	8.8	10.0	11.1	12.2	13.3	63
64	0.0	1.1	2.3	3.4	4.6	5.7	6.9	8.0	9.2	10.3	11.5	12.6	13.8	64
65.0	0.0	1.2	2.4	3.6	4.7	5.9	7.1	8.3	9.5	10.7	11.9	13.1	14.3	65.0
65.5	0.0	1.2	2.4	3.6	4.8	6.0	7.3	8.5	9.7	10.9	12.1	13.4	14.6	65.5
66.0	0.0	1.2	2.5	3.7	4.9	6.2	7.4	8.6	9.9	11.1	12.4	13.6	14.9	66.0
66.5	0.0	1.3	2.5	3.8	5.0	6.3	7.5	8.8	10.1	11.3	12.6	13.9	15.2	66.5
67.0	0.0	1.3	2.6	3.8	5.1	6.4	7.7	9.0	10.3	11.6	12.9	14.2	15.5	67.0
67.5	0.0	1.3	2.6	3.9	5.2	6.5	7.9	9.2	10.5	11.8	13.2	14.5	15.9	67.5
68.0	0.0	1.3	2.7	4.0	5.3	6.7	8.0	9.4	10.7	12.1	13.5	14.8	16.2	68.0
68.5	0.0	1.4	2.7	4.1	5.5	6.8	8.2	9.6	11.0	12.4	13.8	15.2	16.6	68.5
69.0	0.0	1.4	2.8	4.2	5.6	7.0	8.4	9.8	11.2	12.6	14.1	15.5	17.0	69.0
69.5	0.0	1.4	2.9	4.3	5.7	7.2	8.6	10.0	11.5	12.9	14.4	15.9	17.4	69.5
70.0	0.0	1.5	2.9	4.4	5.9	7.3	8.8	10.3	11.8	13.3	14.8	16.3	17.8	70.0
70.5	0.0	1.5	3.0	4.5	6.0	7.5	9.0	10.5	12.1	13.6	15.1	16.7	18.2	70.5
71.0	0.0	1.5	3.1	4.6	6.2	7.7	9.3	10.8	12.4	13.9	15.5	17.1	18.7	71.0
71.5	0.0	1.6	3.2	4.7	6.3	7.9	9.5	11.1	12.7	14.3	15.9	17.6	19.2	71.5
72.0	0.0	1.6	3.2	4.9	6.5	8.1	9.8	11.4	13.0	14.7	16.4	18.1	19.8	72.0
72.5	0.0	1.7	3.3	5.0	6.7	8.3	10.0	11.7	13.4	15.1	16.8	18.6	20.3	72.5
73.0	0.0	1.7	3.4	5.1	6.9	8.6	10.3	12.1	13.8	15.6	17.3	19.1	20.9	73.0
73.5	0.0	1.8	3.5	5.3	7.1	8.8	10.6	12.4	14.2	16.0	17.9	19.7	21.6	73.5
74.0	0.0	1.8	3.6	5.4	7.3	9.1	10.9	12.8	14.7	16.5	18.4	20.3	22.3	74.0
74.5	0.0	1.9	3.7	5.6	7.5	9.4	11.3	13.2	15.1	17.1	19.0	21.0	23.0	74.5
75.0	0.0	1.9	3.9	5.8	7.7	9.7	11.7	13.6	15.6	17.6	19.7	21.7	23.8	75.0
75.5	0.0	2.0	4.0	6.0	8.0	10.0	12.1	14.1	16.2	18.3	20.4	22.5	24.7	75.5
76.0	0.0	2.1	4.1	6.2	8.3	10.4	12.5	14.6	16.8	18.9	21.1	23.3	25.6	76.0
76.5	0.0	2.1	4.3	6.4	8.6	10.8	13.0	15.2	17.4	19.6	21.9	24.2	26.6	76.5
77.0	0.0	2.2	4.4	6.7	8.9	11.2	13.5	15.7	18.1	20.4	22.8	25.2	27.7	77.0

TABLE 27

Amplitudes

Latitude	Declination 6°0	6°5	7°0	7°5	8°0	8°5	9°0	9°5	10°0	10°5	11°0	11°5	12°0	Latitude
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	0
10	6.1	6.6	7.1	7.6	8.1	8.6	9.1	9.6	10.2	10.7	11.2	11.7	12.2	10
15	6.2	6.7	7.2	7.8	8.3	8.8	9.3	9.8	10.4	10.9	11.4	11.9	12.4	15
20	6.4	6.9	7.5	8.0	8.5	9.0	9.6	10.1	10.6	11.2	11.7	12.2	12.8	20
25	6.6	7.2	7.7	8.3	8.8	9.4	9.9	10.5	11.0	11.6	12.2	12.7	13.3	25
30	6.9	7.5	8.1	8.7	9.2	9.8	10.4	11.0	11.6	12.1	12.7	13.3	13.9	30
32	7.1	7.7	8.3	8.9	9.4	10.0	10.6	11.2	11.8	12.4	13.0	13.6	14.2	32
34	7.2	7.8	8.5	9.1	9.7	10.3	10.9	11.5	12.1	12.7	13.3	13.9	14.5	34
36	7.4	8.0	8.7	9.3	9.9	10.5	11.1	11.8	12.4	13.0	13.6	14.3	14.9	36
38	7.6	8.3	8.9	9.5	10.2	10.8	11.5	12.1	12.7	13.4	14.0	14.7	15.3	38
40	7.8	8.5	9.2	9.8	10.5	11.1	11.8	12.4	13.1	13.8	14.4	15.1	15.7	40
42	8.1	8.8	9.4	10.1	10.8	11.5	12.1	12.8	13.5	14.2	14.9	15.6	16.2	42
44	8.4	9.1	9.8	10.5	11.2	11.9	12.6	13.3	14.0	14.7	15.4	16.1	16.8	44
46	8.7	9.4	10.1	10.8	11.6	12.3	13.0	13.7	14.5	15.2	15.9	16.7	17.4	46
48	9.0	9.7	10.5	11.2	12.0	12.8	13.5	14.3	15.0	15.8	16.6	17.3	18.1	48
50	9.4	10.1	10.9	11.7	12.5	13.3	14.1	14.9	15.7	16.5	17.3	18.1	18.9	50
51	9.6	10.4	11.2	12.0	12.8	13.6	14.4	15.2	16.0	16.8	17.7	18.5	19.3	51
52	9.8	10.6	11.4	12.2	13.1	13.9	14.7	15.6	16.4	17.2	18.1	18.9	19.7	52
53	10.0	10.8	11.7	12.5	13.4	14.2	15.1	15.9	16.8	17.6	18.5	19.3	20.2	53
54	10.2	11.1	12.0	12.8	13.7	14.6	15.4	16.3	17.2	18.1	18.9	19.8	20.7	54
55	10.5	11.4	12.3	13.2	14.0	14.9	15.8	16.7	17.6	18.5	19.4	20.3	21.3	55
56	10.8	11.7	12.6	13.5	14.4	15.3	16.2	17.2	18.1	19.0	20.0	20.9	21.8	56
57	11.1	12.0	12.9	13.9	14.8	15.7	16.7	17.6	18.6	19.6	20.5	21.5	22.4	57
58	11.4	12.3	13.3	14.3	15.2	16.2	17.2	18.1	19.1	20.1	21.1	22.1	23.1	58
59	11.7	12.7	13.7	14.7	15.7	16.7	17.7	18.7	19.7	20.7	21.7	22.8	23.8	59
60	12.1	13.1	14.1	15.1	16.2	17.2	18.2	19.3	20.3	21.4	22.4	23.5	24.6	60
61	12.5	13.5	14.6	15.6	16.7	17.8	18.8	19.9	21.0	22.1	23.2	24.3	25.4	61
62	12.9	14.0	15.0	16.1	17.2	18.4	19.5	20.6	21.7	22.8	24.0	25.1	26.3	62
63	13.3	14.4	15.6	16.7	17.9	19.0	20.2	21.3	22.5	23.7	24.9	26.0	27.3	63
64	13.8	15.0	16.2	17.3	18.5	19.7	20.9	22.1	23.3	24.6	25.8	27.1	28.3	64
65.0	14.3	15.5	16.8	18.0	19.2	20.5	21.7	23.0	24.3	25.5	26.8	28.1	29.5	65.0
65.5	14.6	15.8	17.1	18.3	19.6	20.9	22.2	23.5	24.8	26.1	27.4	28.7	30.1	65.5
66.0	14.9	16.2	17.4	18.7	20.0	21.3	22.6	23.9	25.3	26.6	28.0	29.4	30.7	66.0
66.5	15.2	16.5	17.8	19.1	20.4	21.8	23.1	24.5	25.8	27.2	28.6	30.0	31.4	66.5
67.0	15.5	16.8	18.2	19.5	20.9	22.2	23.6	25.0	26.4	27.8	29.2	30.7	32.1	67.0
67.5	15.9	17.2	18.6	19.9	21.3	22.7	24.1	25.5	27.0	28.4	29.9	31.4	32.9	67.5
68.0	16.2	17.6	19.0	20.4	21.8	23.2	24.7	26.1	27.6	29.1	30.6	32.2	33.7	68.0
68.5	16.6	18.0	19.4	20.9	22.3	23.8	25.3	26.8	28.3	29.8	31.4	33.0	34.6	68.5
69.0	17.0	18.4	19.9	21.4	22.9	24.4	25.9	27.4	29.0	30.6	32.2	33.8	35.5	69.0
69.5	17.4	18.9	20.4	21.9	23.4	25.0	26.5	28.1	29.7	31.4	33.0	34.7	36.4	69.5
70.0	17.8	19.3	20.9	22.4	24.0	25.6	27.2	28.9	30.5	32.2	33.9	35.7	37.4	70.0
70.5	18.2	19.8	21.4	23.0	24.6	26.3	27.9	29.6	31.3	33.1	34.9	36.7	38.5	70.5
71.0	18.7	20.3	22.0	23.6	25.3	27.0	28.7	30.5	32.2	34.0	35.9	37.8	39.7	71.0
71.5	19.2	20.9	22.6	24.3	26.0	27.8	29.5	31.3	33.2	35.1	37.0	38.9	40.9	71.5
72.0	19.8	21.5	23.2	25.0	26.8	28.6	30.4	32.3	34.2	36.1	38.1	40.2	42.3	72.0
72.5	20.3	22.1	23.9	25.7	27.6	29.4	31.3	33.3	35.3	37.3	39.4	41.5	43.7	72.5
73.0	20.9	22.8	24.6	26.5	28.4	30.4	32.3	34.4	36.4	38.6	40.7	43.0	45.3	73.0
73.5	21.6	23.5	25.4	27.4	29.3	31.4	33.4	35.5	37.7	39.9	42.2	44.6	47.1	73.5
74.0	22.3	24.2	26.2	28.3	30.3	32.4	34.6	36.8	39.0	41.4	43.8	46.3	49.0	74.0
74.5	23.0	25.1	27.1	29.3	31.4	33.6	35.8	38.1	40.5	43.0	45.6	48.2	51.1	74.5
75.0	23.8	25.9	28.1	30.3	32.5	34.8	37.2	39.6	42.1	44.8	47.5	50.4	53.4	75.0
75.5	24.7	26.9	29.1	31.4	33.8	36.2	38.7	41.2	43.9	46.7	49.6	52.8	56.1	75.5
76.0	25.6	27.9	30.2	32.7	35.1	37.7	40.3	43.0	45.9	48.9	52.1	55.5	59.3	76.0
76.5	26.6	29.0	31.5	34.0	36.6	39.3	42.1	45.0	48.1	51.3	54.8	58.7	63.0	76.5
77.0	27.7	30.2	32.8	35.5	38.2	41.1	44.1	47.2	50.5	54.1	58.0	62.4	67.6	77.0

TABLE 27

Amplitudes

Latitude	Declination 12°0	12°5	13°0	13°5	14°0	14°5	15°0	15°5	16°0	16°5	17°0	17°5	18°0	Latitude
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0	17.5	18.0	0
10	12.2	12.7	13.2	13.7	14.2	14.7	15.2	15.7	16.3	16.8	17.3	17.8	18.3	10
15	12.4	12.9	13.5	14.0	14.5	15.0	15.5	16.1	16.6	17.1	17.6	18.1	18.7	15
20	12.8	13.3	13.9	14.4	14.9	15.5	16.0	16.5	17.1	17.6	18.1	18.7	19.2	20
25	13.3	13.8	14.4	14.9	15.5	16.0	16.6	17.1	17.7	18.3	18.8	19.4	19.9	25
30	13.9	14.5	15.1	15.6	16.2	16.8	17.4	18.0	18.6	19.1	19.7	20.3	20.9	30
32	14.2	14.8	15.4	16.0	16.6	17.2	17.8	18.4	19.0	19.6	20.2	20.8	21.4	32
34	14.5	15.1	15.7	16.4	17.0	17.6	18.2	18.8	19.4	20.0	20.7	21.3	21.9	34
36	14.9	15.5	16.1	16.8	17.4	18.0	18.7	19.3	19.9	20.6	21.2	21.8	22.5	36
38	15.3	15.9	16.6	17.2	17.9	18.5	19.2	19.8	20.5	21.1	21.8	22.4	23.1	38
40	15.7	16.4	17.1	17.7	18.4	19.1	19.7	20.4	21.1	21.8	22.4	23.1	23.8	40
41	16.0	16.7	17.3	18.0	18.7	19.4	20.1	20.8	21.4	22.1	22.8	23.5	24.2	41
42	16.2	16.9	17.6	18.3	19.0	19.7	20.4	21.1	21.8	22.5	23.2	23.9	24.6	42
43	16.5	17.2	17.9	18.6	19.3	20.0	20.7	21.4	22.1	22.9	23.6	24.3	25.0	43
44	16.8	17.5	18.2	18.9	19.7	20.4	21.1	21.8	22.5	23.3	24.0	24.7	25.4	44
45	17.1	17.8	18.5	19.3	20.0	20.7	21.5	22.2	22.9	23.7	24.4	25.2	25.9	45
46	17.4	18.2	18.9	19.6	20.4	21.1	21.9	22.6	23.4	24.1	24.9	25.7	26.4	46
47	17.7	18.5	19.3	20.0	20.8	21.5	22.3	23.1	23.8	24.6	25.4	26.2	26.9	47
48	18.1	18.9	19.6	20.4	21.2	22.0	22.8	23.5	24.3	25.1	25.9	26.7	27.5	48
49	18.5	19.3	20.1	20.8	21.6	22.4	23.2	24.0	24.8	25.7	26.5	27.3	28.1	49
50	18.9	19.7	20.5	21.3	22.1	22.9	23.7	24.6	25.4	26.2	27.1	27.9	28.7	50
51	19.3	20.1	20.9	21.8	22.6	23.4	24.3	25.1	26.0	26.8	27.7	28.5	29.4	51
52	19.7	20.6	21.4	22.3	23.1	24.0	24.9	25.7	26.6	27.5	28.3	29.2	30.1	52
53	20.2	21.1	21.9	22.8	23.7	24.6	25.5	26.4	27.3	28.2	29.1	30.0	30.9	53
54	20.7	21.6	22.5	23.4	24.3	25.2	26.1	27.0	28.0	28.9	29.8	30.8	31.7	54
55	21.3	22.2	23.1	24.0	24.9	25.9	26.8	27.8	28.7	29.7	30.6	31.6	32.6	55
56	21.8	22.8	23.7	24.7	25.6	26.6	27.6	28.5	29.5	30.5	31.5	32.5	33.5	56
57	22.4	23.4	24.4	25.4	26.4	27.4	28.4	29.4	30.4	31.4	32.5	33.5	34.6	57
58	23.1	24.1	25.1	26.1	27.2	28.2	29.2	30.3	31.3	32.4	33.5	34.6	35.7	58
59	23.8	24.8	25.9	27.0	28.0	29.1	30.2	31.3	32.4	33.5	34.6	35.7	36.9	59
60	24.6	25.7	26.7	27.8	28.9	30.1	31.2	32.3	33.5	34.6	35.8	37.0	38.2	60
61	25.4	26.5	27.6	28.8	29.9	31.1	32.3	33.5	34.6	35.9	37.1	38.3	39.6	61
62	26.3	27.5	28.6	29.8	31.0	32.2	33.5	34.7	36.0	37.2	38.5	39.8	41.2	62
63	27.3	28.5	29.7	30.9	32.2	33.5	34.8	36.1	37.4	38.7	40.1	41.5	42.9	63
64	28.3	29.6	30.9	32.2	33.5	34.8	36.2	37.6	39.0	40.4	41.8	43.3	44.8	64
65.0	29.5	30.8	32.2	33.5	34.9	36.3	37.8	39.2	40.7	42.2	43.8	45.4	47.0	65.0
65.5	30.1	31.5	32.9	34.3	35.7	37.1	38.6	40.1	41.7	43.2	44.8	46.5	48.2	65.5
66.0	30.7	32.1	33.6	35.0	36.5	38.0	39.5	41.1	42.7	44.3	46.0	47.7	49.4	66.0
66.5	31.4	32.9	34.3	35.8	37.3	38.9	40.5	42.1	43.7	45.4	47.2	48.9	50.8	66.5
67.0	32.1	33.6	35.1	36.7	38.3	39.9	41.5	43.2	44.9	46.6	48.4	50.3	52.3	67.0
67.5	32.9	34.4	36.0	37.6	39.2	40.9	42.6	44.3	46.1	47.9	49.8	51.8	53.9	67.5
68.0	33.7	35.3	36.9	38.6	40.2	41.9	43.7	45.5	47.4	49.3	51.3	53.4	55.6	68.0
68.5	34.6	36.2	37.9	39.6	41.3	43.1	44.9	46.8	48.8	50.8	52.9	55.1	57.5	68.5
69.0	35.5	37.2	38.9	40.6	42.5	44.3	46.2	48.2	50.3	52.4	54.7	57.0	59.6	69.0
69.5	36.4	38.2	40.0	41.8	43.7	45.6	47.7	49.7	51.9	54.2	56.6	59.2	61.9	69.5
70.0	37.4	39.3	41.1	43.0	45.0	47.1	49.2	51.4	53.7	56.1	58.7	61.5	64.6	70.0
70.5	38.5	40.4	42.4	44.4	46.4	48.6	50.8	53.2	55.7	58.3	61.1	64.3	67.8	70.5
71.0	39.7	41.7	43.7	45.8	48.0	50.3	52.7	55.2	57.8	60.7	63.9	67.5	71.7	71.0
71.5	40.9	43.0	45.1	47.4	49.7	52.1	54.7	57.4	60.3	63.5	67.1	71.4	76.9	71.5
72.0	42.3	44.5	46.7	49.1	51.5	54.1	56.9	59.9	63.1	66.8	71.1	76.7	90.0	72.0
72.5	43.7	46.0	48.4	50.9	53.6	56.4	59.4	62.7	66.4	70.8	76.5	90.0		72.5
73.0	45.3	47.8	50.3	53.0	55.8	58.9	62.3	66.1	70.5	76.3	90.0			73.0
73.5	47.1	49.6	52.4	55.3	58.4	61.8	65.7	70.2	76.0	90.0				73.5
74.0	49.0	51.7	54.7	57.9	61.4	65.3	69.9	75.8	90.0					74.0
74.5	51.1	54.1	57.3	60.9	64.9	69.5	75.6	90.0						74.5

TABLE 27
Amplitudes

Latitude	Declination 18°.0	18°.5	19°.0	19°.5	20°.0	20°.5	21°.0	21°.5	22°.0	22°.5	23°.0	23°.5	24°.0	Latitude
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	18. 0	18. 5	19. 0	19. 5	20. 0	20. 5	21. 0	21. 5	22. 0	22. 5	23. 0	23. 5	24. 0	0
10	18. 3	18. 8	19. 3	19. 8	20. 3	20. 8	21. 3	21. 8	22. 4	22. 9	23. 4	23. 9	24. 4	10
15	18. 7	19. 2	19. 7	20. 2	20. 7	21. 3	21. 8	22. 3	22. 8	23. 3	23. 9	24. 4	24. 9	15
20	19. 2	19. 7	20. 3	20. 8	21. 3	21. 9	22. 4	23. 0	23. 5	24. 0	24. 6	25. 1	25. 6	20
25	19. 9	20. 5	21. 1	21. 6	22. 2	22. 7	23. 3	23. 9	24. 4	25. 0	25. 5	26. 1	26. 7	25
30	20. 9	21. 5	22. 1	22. 7	23. 3	23. 9	24. 4	25. 0	25. 6	26. 2	26. 8	27. 4	28. 0	30
32	21. 4	22. 0	22. 6	23. 2	23. 8	24. 4	25. 0	25. 6	26. 2	26. 8	27. 4	28. 0	28. 7	32
34	21. 9	22. 5	23. 1	23. 7	24. 4	25. 0	25. 6	26. 2	26. 9	27. 5	28. 1	28. 7	29. 4	34
36	22. 5	23. 1	23. 7	24. 4	25. 0	25. 7	26. 3	26. 9	27. 6	28. 2	28. 9	29. 5	30. 2	36
38	23. 1	23. 7	24. 4	25. 1	25. 7	26. 4	27. 1	27. 7	28. 4	29. 1	29. 7	30. 4	31. 1	38
40	23. 8	24. 5	25. 2	25. 8	26. 5	27. 2	27. 9	28. 6	29. 3	30. 0	30. 7	31. 4	32. 1	40
41	24. 2	24. 9	25. 6	26. 3	26. 9	27. 6	28. 3	29. 1	29. 8	30. 5	31. 2	31. 9	32. 6	41
42	24. 6	25. 3	26. 0	26. 7	27. 4	28. 1	28. 8	29. 5	30. 3	31. 0	31. 7	32. 5	33. 2	42
43	25. 0	25. 7	26. 4	27. 2	27. 9	28. 6	29. 3	30. 1	30. 8	31. 6	32. 3	33. 0	33. 8	43
44	25. 4	26. 2	26. 9	27. 6	28. 4	29. 1	29. 9	30. 6	31. 4	32. 1	32. 9	33. 7	34. 4	44
45	25. 9	26. 7	27. 4	28. 2	28. 9	29. 7	30. 5	31. 2	32. 0	32. 8	33. 5	34. 3	35. 1	45
46	26. 4	27. 2	27. 9	28. 7	29. 5	30. 3	31. 1	31. 8	32. 6	33. 4	34. 2	35. 0	35. 8	46
47	26. 9	27. 7	28. 5	29. 3	30. 1	30. 9	31. 7	32. 5	33. 3	34. 1	35. 0	35. 8	36. 6	47
48	27. 5	28. 3	29. 1	29. 9	30. 7	31. 6	32. 4	33. 2	34. 0	34. 9	35. 7	36. 6	37. 4	48
49	28. 1	28. 9	29. 8	30. 6	31. 4	32. 3	33. 1	34. 0	34. 8	35. 7	36. 6	37. 4	38. 3	49
50	28. 7	29. 6	30. 4	31. 3	32. 1	33. 0	33. 9	34. 8	35. 6	36. 5	37. 4	38. 3	39. 3	50
51	29. 4	30. 3	31. 2	32. 0	32. 9	33. 8	34. 7	35. 6	36. 5	37. 5	38. 4	39. 3	40. 3	51
52	30. 1	31. 0	31. 9	32. 8	33. 7	34. 7	35. 6	36. 5	37. 5	38. 4	39. 4	40. 4	41. 3	52
53	30. 9	31. 8	32. 8	33. 7	34. 6	35. 6	36. 5	37. 5	38. 5	39. 5	40. 5	41. 5	42. 5	53
54	31. 7	32. 7	33. 6	34. 6	35. 6	36. 6	37. 6	38. 6	39. 6	40. 6	41. 7	42. 7	43. 8	54
55	32. 6	33. 6	34. 6	35. 6	36. 6	37. 6	38. 7	39. 7	40. 8	41. 9	42. 9	44. 0	45. 2	55
56	33. 5	34. 6	35. 6	36. 7	37. 7	38. 8	39. 9	41. 0	42. 1	43. 2	44. 3	45. 5	46. 7	56
57	34. 6	35. 6	36. 7	37. 8	38. 9	40. 0	41. 1	42. 3	43. 5	44. 6	45. 8	47. 1	48. 3	57
58	35. 7	36. 8	37. 9	39. 1	40. 2	41. 4	42. 6	43. 8	45. 0	46. 2	47. 5	48. 8	50. 1	58
59	36. 9	38. 0	39. 2	40. 4	41. 6	42. 8	44. 1	45. 4	46. 7	48. 0	49. 3	50. 7	52. 2	59
60. 0	38. 2	39. 4	40. 6	41. 9	43. 2	44. 5	45. 8	47. 1	48. 5	49. 9	51. 4	52. 9	54. 4	60. 0
60. 5	38. 9	40. 1	41. 4	42. 7	44. 0	45. 3	46. 7	48. 1	49. 5	51. 0	52. 5	54. 1	55. 7	60. 5
61. 0	39. 6	40. 9	42. 2	43. 5	44. 9	46. 3	47. 7	49. 1	50. 6	52. 1	53. 7	55. 3	57. 0	61. 0
61. 5	40. 4	41. 7	43. 0	44. 4	45. 8	47. 2	48. 7	50. 2	51. 7	53. 3	55. 0	56. 7	58. 5	61. 5
62. 0	41. 2	42. 5	43. 9	45. 3	46. 8	48. 2	49. 8	51. 3	52. 9	54. 6	56. 3	58. 1	60. 0	62. 0
62. 5	42. 0	43. 4	44. 8	46. 3	47. 8	49. 3	50. 9	52. 5	54. 2	56. 0	57. 8	59. 7	61. 7	62. 5
63. 0	42. 9	44. 3	45. 8	47. 3	48. 9	50. 5	52. 1	53. 8	55. 6	57. 5	59. 4	61. 4	63. 6	63. 0
63. 5	43. 8	45. 3	46. 9	48. 4	50. 0	51. 7	53. 4	55. 2	57. 1	59. 1	61. 1	63. 4	65. 7	63. 5
64. 0	44. 8	46. 4	48. 0	49. 6	51. 3	53. 0	54. 8	56. 7	58. 7	60. 8	63. 0	65. 5	68. 1	64. 0
64. 5	45. 9	47. 5	49. 1	50. 8	52. 6	54. 4	56. 3	58. 4	60. 5	62. 7	65. 2	67. 9	70. 9	64. 5
65. 0	47. 0	48. 7	50. 4	52. 2	54. 0	56. 0	58. 0	60. 1	62. 4	64. 9	67. 6	70. 7	74. 2	65. 0
65. 5	48. 2	49. 9	51. 7	53. 6	55. 6	57. 6	59. 8	62. 1	64. 6	67. 3	70. 4	74. 1	78. 8	65. 5
66. 0	49. 4	51. 3	53. 2	55. 2	57. 2	59. 4	61. 8	64. 3	67. 1	70. 2	73. 9	78. 6	90. 0	66. 0
66. 5	50. 8	52. 7	54. 7	56. 8	59. 1	61. 4	64. 0	66. 8	70. 0	73. 7	78. 5	90. 0		66. 5
67. 0	52. 3	54. 3	56. 4	58. 7	61. 1	63. 7	66. 5	69. 7	73. 5	78. 4	90. 0			67. 0
67. 5	53. 9	56. 0	58. 3	60. 7	63. 3	66. 2	69. 5	73. 3	78. 2	90. 0				67. 5
68. 0	55. 6	57. 9	60. 4	63. 0	65. 9	69. 2	73. 1	78. 1	90. 0					68. 0
68. 5	57. 5	60. 0	62. 7	65. 6	68. 9	72. 9	77. 9	90. 0						68. 5
69. 0	59. 6	62. 3	65. 3	68. 7	72. 6	77. 7	90. 0							69. 0
69. 5	61. 9	65. 0	68. 4	72. 4	77. 6	90. 0								69. 5
70. 0	64. 6	68. 1	72. 2	77. 4	90. 0									70. 0
70. 5	67. 8	71. 9	77. 2	90. 0										70. 5
71. 0	71. 7	77. 1	90. 0											71. 0
71. 5	76. 9	90. 0												71. 5
72. 0	90. 0													72. 0

TABLE 28

Correction of Amplitude as Observed on the Visible Horizon

Latitude	Declination 0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°	Latitude
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
10	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	10
15	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	15
20	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	20
25	0.3	0.3	0.3	0.3	0.3	0.4	0.3	0.3	0.3	0.3	0.3	0.3	0.3	25
30	0.4	0.4	0.4	0.4	0.5	0.4	0.4	0.4	0.4	0.4	0.4	0.5	0.5	30
32	0.4	0.4	0.4	0.4	0.5	0.4	0.4	0.4	0.4	0.4	0.5	0.5	0.5	32
34	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	34
36	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.6	0.5	0.6	0.6	0.6	36
38	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	38
40	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.7	0.7	0.7	40
42	0.6	0.6	0.6	0.6	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	42
44	0.7	0.7	0.7	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.8	0.8	0.9	44
46	0.7	0.7	0.7	0.7	0.7	0.8	0.8	0.8	0.8	0.8	0.8	0.9	0.9	46
48	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.9	0.9	1.0	1.0	1.0	48
50	0.8	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	1.0	1.0	1.1	1.0	50
51	0.8	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	1.0	1.1	1.1	1.1	51
52	0.9	0.9	0.9	0.9	0.9	0.9	1.0	1.0	1.0	1.1	1.1	1.1	1.3	52
53	0.9	0.9	0.9	0.9	0.9	0.9	1.0	1.0	1.0	1.1	1.2	1.2	1.3	53
54	1.0	1.0	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.2	1.2	1.3	1.3	54
55	1.0	1.0	1.0	1.0	1.1	1.1	1.0	1.2	1.2	1.2	1.3	1.3	1.4	55
56	1.0	1.0	1.0	1.0	1.1	1.1	1.2	1.2	1.2	1.3	1.3	1.4	1.5	56
57	1.1	1.1	1.1	1.1	1.1	1.1	1.2	1.2	1.3	1.3	1.4	1.5	1.7	57
58	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.4	1.4	1.5	1.6	1.8	58
59	1.2	1.2	1.2	1.2	1.2	1.2	1.3	1.3	1.3	1.4	1.6	1.7	1.9	59
60	1.2	1.2	1.2	1.2	1.2	1.3	1.3	1.4	1.4	1.5	1.7	1.9	2.2	60
61	1.3	1.3	1.3	1.3	1.3	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.4	61
62	1.3	1.3	1.3	1.3	1.4	1.4	1.5	1.6	1.6	1.7	1.9	2.3	2.6	62
63	1.3	1.4	1.4	1.4	1.4	1.5	1.5	1.6	1.7	1.9	2.1	2.5	3.3	63
64	1.4	1.4	1.4	1.5	1.5	1.6	1.7	1.7	1.8	2.1	2.3	2.9	4.3	64
65.0	1.5	1.5	1.5	1.6	1.6	1.6	1.7	1.9	2.0	2.2	2.7	3.5	7.2	65.0
65.5	1.5	1.5	1.5	1.6	1.6	1.7	1.8	1.9	2.1	2.3	2.8	3.9		65.5
66.0	1.6	1.6	1.6	1.6	1.7	1.7	1.9	2.0	2.1	2.5	3.1	4.4		66.0
66.5	1.6	1.6	1.6	1.7	1.7	1.8	1.9	2.1	2.3	2.6	3.3	5.4		66.5
67.0	1.7	1.7	1.7	1.7	1.7	1.8	2.0	2.1	2.3	2.8	3.6	7.5		67.0
67.5	1.7	1.7	1.7	1.7	1.8	1.9	2.0	2.2	2.5	2.9	4.1			67.5
68.0	1.7	1.8	1.8	1.8	1.9	2.0	2.1	2.3	2.6	3.2	4.7			68.0
68.5	1.8	1.8	1.8	1.8	2.0	2.0	2.2	2.4	2.8	3.5	5.7			68.5
69.0	1.8	1.8	1.9	1.9	1.9	2.1	2.2	2.5	2.9	3.8	7.9			69.0
69.5	1.9	1.9	1.9	1.9	2.1	2.2	2.4	2.6	3.2	4.3				69.5
70.0	1.9	1.9	1.9	2.0	2.1	2.3	2.5	2.8	3.4	5.0				70.0
70.5	2.0	2.0	2.0	2.2	2.2	2.4	2.6	3.0	3.6	6.0				70.5
71.0	2.0	2.0	2.1	2.2	2.3	2.5	2.7	3.1	4.1	8.3				71.0
71.5	2.1	2.1	2.2	2.3	2.4	2.5	2.9	3.3	4.6					71.5
72.0	2.2	2.2	2.3	2.3	2.4	2.6	3.0	3.6	5.3					72.0
72.5	2.2	2.2	2.3	2.4	2.5	2.7	3.2	3.9	6.4					72.5
73.0	2.3	2.3	2.4	2.5	2.7	2.9	3.4	4.4	8.9					73.0
73.5	2.4	2.4	2.5	2.6	2.8	3.0	3.6	4.9						73.5
74.0	2.4	2.4	2.5	2.7	2.9	3.3	3.8	5.6						74.0
74.5	2.5	2.6	2.7	2.8	3.0	3.4	4.2	6.8						74.5
75.0	2.6	2.7	2.8	2.9	3.2	3.7	4.7	9.3						75.0
75.5	2.7	2.8	2.8	3.0	3.3	3.9	5.3							75.5
76.0	2.8	2.8	2.9	3.2	3.5	4.2	5.6							76.0
76.5	2.9	3.0	3.1	3.3	3.7	4.5	7.3							76.5
77.0	3.0	3.1	3.2	3.5	4.0	5.1	10.2							77.0

For the **sun**, a **planet**, or a **star**, apply the correction to the observed amplitude in the direction **away** from the elevated pole. For the **moon** apply **half** the correction **toward** the elevated pole.

TABLE 29

Altitude Factor

a, the change of altitude in one minute from meridian transit; used for entering table 30

Latitude	Declination same name as latitude, upper transit: add correction to observed altitude												Latitude
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	
°	″	″	″	″	″	″	″	″	″	″	″	″	°
0					28.1	22.4	18.7	16.0	14.0	12.4	11.1	10.1	0
1						28.0	22.4	18.6	16.0	13.9	12.4	11.1	1
2							28.0	22.3	18.6	15.9	13.9	12.3	2
3								27.9	22.3	18.5	15.8	13.8	3
4	28.1								27.8	22.2	18.5	15.8	4
5	22.4	28.0								27.7	22.1	18.4	5
6	18.7	22.4	28.0								27.6	22.0	6
7	16.0	18.6	22.3	27.9								27.4	7
8	14.0	16.0	18.6	22.3	27.8								8
9	12.4	13.9	15.9	18.5	22.2	27.7							9
10	11.1	12.4	13.9	15.8	18.5	22.1	27.6						10
11	10.1	11.1	12.3	13.8	15.8	18.4	22.0	27.4					11
12	9.2	10.1	11.1	12.3	13.8	15.7	18.3	21.9	27.3				12
13	8.5	9.2	10.0	11.0	12.2	13.7	15.6	18.2	21.7	27.1			13
14	7.9	8.5	9.2	10.0	10.9	12.1	13.6	15.5	18.0	21.6	26.9		14
15	7.3	7.8	8.4	9.1	9.9	10.9	12.1	13.5	15.4	17.9	21.4	26.7	15
16	6.8	7.3	7.8	8.4	9.1	9.8	10.8	12.0	13.4	15.3	17.8	21.3	16
17	6.4	6.8	7.2	7.8	8.3	9.0	9.8	10.7	11.9	13.3	15.2	17.6	17
18	6.0	6.4	6.8	7.2	7.7	8.3	8.9	9.7	10.6	11.8	13.2	15.0	18
19	5.7	6.0	6.3	6.7	7.2	7.6	8.2	8.9	9.6	10.6	11.7	13.1	19
20	5.4	5.7	6.0	6.3	6.7	7.1	7.6	8.1	8.8	9.5	10.5	11.6	20
21	5.1	5.4	5.6	5.9	6.3	6.6	7.0	7.5	8.1	8.7	9.5	10.4	21
22	4.9	5.1	5.3	5.6	5.9	6.2	6.6	7.0	7.5	8.0	8.6	9.4	22
23	4.6	4.8	5.0	5.3	5.5	5.8	6.1	6.5	6.9	7.4	7.9	8.5	23
24	4.4	4.6	4.8	5.0	5.2	5.5	5.8	6.1	6.4	6.8	7.3	7.8	24
25	4.2	4.4	4.6	4.7	5.0	5.2	5.4	5.7	6.0	6.4	6.8	7.2	25
26	4.0	4.2	4.3	4.5	4.7	4.9	5.1	5.4	5.7	6.0	6.3	6.7	26
27	3.9	4.0	4.1	4.3	4.5	4.7	4.9	5.1	5.3	5.6	5.9	6.2	27
28	3.7	3.8	4.0	4.1	4.3	4.4	4.6	4.8	5.0	5.3	5.5	5.8	28
29	3.5	3.7	3.8	3.9	4.1	4.2	4.4	4.6	4.7	5.0	5.2	5.5	29
30	3.4	3.5	3.6	3.7	3.9	4.0	4.2	4.3	4.5	4.7	4.9	5.1	30
31	3.3	3.4	3.5	3.6	3.7	3.8	4.0	4.1	4.3	4.4	4.6	4.8	31
32	3.1	3.2	3.3	3.4	3.5	3.7	3.8	3.9	4.1	4.2	4.4	4.6	32
33	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.9	4.0	4.2	4.3	33
34	2.9	3.0	3.1	3.2	3.2	3.3	3.4	3.6	3.7	3.8	3.9	4.1	34
35	2.8	2.9	3.0	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.9	35
36	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	36
37	2.6	2.7	2.7	2.8	2.9	2.9	3.0	3.1	3.2	3.3	3.4	3.5	37
38	2.5	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.0	3.2	3.2	3.3	38
39	2.4	2.5	2.5	2.6	2.7	2.7	2.8	2.9	2.9	3.0	3.1	3.2	39
40	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.7	2.8	2.9	3.0	3.0	40
41	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.6	2.7	2.8	2.8	2.9	41
42	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.6	2.7	2.8	42
43	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.7	43
44	2.0	2.1	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.5	44
45	2.0	2.0	2.0	2.1	2.1	2.2	2.2	2.2	2.3	2.3	2.4	2.4	45
46	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.2	2.2	2.2	2.3	2.3	46
47	1.8	1.9	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.1	2.2	2.2	47
48	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.1	48
49	1.7	1.7	1.8	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.1	49
50	1.6	1.7	1.7	1.7	1.8	1.8	1.8	1.8	1.9	1.9	1.9	2.0	50
51	1.6	1.6	1.6	1.7	1.7	1.7	1.7	1.8	1.8	1.8	1.9	1.9	51
52	1.5	1.6	1.6	1.6	1.6	1.6	1.7	1.7	1.7	1.8	1.8	1.8	52
53	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6	1.7	1.7	1.7	1.7	53
54	1.4	1.4	1.5	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6	1.7	54
55	1.4	1.4	1.4	1.4	1.5	1.5	1.5	1.5	1.5	1.6	1.6	1.6	55
56	1.3	1.3	1.4	1.4	1.4	1.4	1.4	1.4	1.5	1.5	1.5	1.5	56
57	1.3	1.3	1.3	1.3	1.3	1.4	1.4	1.4	1.4	1.4	1.4	1.5	57
58	1.2	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.4	1.4	1.4	58
59	1.2	1.2	1.2	1.2	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.3	59
60	1.1	1.1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.3	1.3	60
Latitude	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	Latitude
	Declination same name as latitude, upper transit: add correction to observed altitude												

TABLE 29

Altitude Factor
a, the change of altitude in one minute from meridian transit;
used for entering table 30

Latitude	Declination **contrary** name to latitude, **upper** transit: **add** correction to observed altitude											Latitude
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	
°	"	"	"	"	"	"	"	"	"	"	"	"	°
0					28.1	22.4	18.7	16.0	14.0	12.4	11.1	10.1	0
1				28.1	22.4	18.7	16.0	14.0	12.4	11.2	10.1	9.3	1
2			28.1	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	2
3		28.1	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0	3
4	28.1	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0	7.4	4
5	22.4	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0	7.4	7.0	5
6	18.7	16.0	14.0	12.5	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6	6
7	16.0	14.0	12.4	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6	6.2	7
8	14.0	12.4	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6	6.2	5.9	8
9	12.4	11.2	10.2	9.3	8.6	8.0	7.5	7.0	6.6	6.2	5.9	5.6	9
10	11.1	10.1	9.3	8.6	8.0	7.4	7.0	6.6	6.2	5.9	5.6	5.3	10
11	10.1	9.3	8.6	8.0	7.4	7.0	6.6	6.2	5.9	5.6	5.3	5.1	11
12	9.2	8.5	7.9	7.4	7.0	6.5	6.2	5.9	5.6	5.3	5.0	4.8	12
13	8.5	7.9	7.4	6.9	6.5	6.2	5.8	5.6	5.3	5.0	4.8	4.6	13
14	7.9	7.4	6.9	6.5	6.2	5.8	5.5	5.3	5.0	4.8	4.6	4.4	14
15	7.3	6.9	6.5	6.1	5.8	5.5	5.3	5.0	4.8	4.6	4.4	4.2	15
16	6.8	6.5	6.1	5.8	5.5	5.2	5.0	4.8	4.6	4.4	4.2	4.1	16
17	6.4	6.1	5.8	5.5	5.2	5.0	4.8	4.6	4.4	4.2	4.1	3.9	17
18	6.0	5.7	5.5	5.2	5.0	4.8	4.6	4.4	4.2	4.1	3.9	3.8	18
19	5.7	5.4	5.2	4.9	4.7	4.5	4.4	4.2	4.0	3.9	3.8	3.6	19
20	5.4	5.1	4.9	4.7	4.5	4.3	4.2	4.0	3.9	3.8	3.6	3.5	20
21	5.1	4.9	4.7	4.5	4.3	4.2	4.0	3.9	3.7	3.6	3.5	3.4	21
22	4.9	4.7	4.5	4.3	4.1	4.0	3.9	3.7	3.6	3.5	3.4	3.3	22
23	4.6	4.4	4.3	4.1	4.0	3.8	3.7	3.6	3.5	3.4	3.3	3.2	23
24	4.4	4.2	4.1	3.9	3.8	3.7	3.6	3.5	3.4	3.3	3.2	3.1	24
25	4.2	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.1	3.0	25
26	4.0	3.9	3.8	3.6	3.5	3.4	3.3	3.2	3.1	3.0	3.0	2.9	26
27	3.9	3.7	3.6	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8	27
28	3.7	3.6	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7	28
29	3.5	3.4	3.3	3.2	3.1	3.1	3.0	2.9	2.8	2.8	2.7	2.6	29
30	3.4	3.3	3.2	3.1	3.0	3.0	2.9	2.8	2.7	2.7	2.6	2.5	30
31	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.6	2.5	2.5	31
32	3.2	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.5	2.4	32
33	3.0	2.9	2.9	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	33
34	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.3	34
35	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	35
36	2.7	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	36
37	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.2	2.1	2.1	37
38	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.1	2.0	38
39	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.1	2.0	2.0	2.0	39
40	2.3	2.3	2.2	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9	40
41	2.3	2.2	2.2	2.1	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8	41
42	2.2	2.1	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	42
43	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	43
44	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	44
45	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6	45
46	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	46
47	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.6	47
48	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.6	1.5	1.5	48
49	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	49
50	1.6	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.4	50
51	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	51
52	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.3	52
53	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	53
54	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	54
55	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	55
56	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.2	56
57	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	57
58	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	58
59	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	59
60	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	60
Latitude	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	Latitude
	Declination **contrary** name to latitude, **upper** transit: **add** correction to observed altitude												

TABLE 29

Altitude Factor
a, the change of altitude in one minute from meridian transit; used for entering table 30

Latitude	Declination same name as latitude, upper transit: add correction to observed altitude													Latitude
	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	
°	"	"	"	"	"	"	"	"	"	"	"	"	"	°
0	9.2	8.5	7.9	7.3	6.8	6.4	6.0	5.7	5.4	5.1	4.9	4.6	4.4	0
1	10.1	9.2	8.5	7.8	7.3	6.8	6.4	6.0	5.7	5.4	5.1	4.8	4.6	1
2	11.1	10.0	9.2	8.4	7.8	7.2	6.8	6.3	6.0	5.6	5.3	5.0	4.8	2
3	12.3	11.0	10.0	9.1	8.4	7.8	7.2	6.7	6.3	5.9	5.6	5.3	5.0	3
4	13.8	12.2	10.9	9.9	9.1	8.3	7.7	7.2	6.7	6.3	5.9	5.5	5.2	4
5	15.7	13.7	12.1	10.9	9.8	9.0	8.3	7.6	7.1	6.6	6.2	5.8	5.5	5
6	18.3	15.6	13.6	12.1	10.8	9.8	8.9	8.2	7.6	7.0	6.6	6.1	5.8	6
7	21.9	18.2	15.5	13.5	12.0	10.7	9.7	8.9	8.1	7.5	7.0	6.5	6.1	7
8	27.3	21.7	18.0	15.4	13.4	11.9	10.6	9.6	8.8	8.1	7.5	6.9	6.4	8
9		27.1	21.6	17.9	15.3	13.3	11.8	10.6	9.5	8.7	8.0	7.4	6.8	9
10			26.9	21.4	17.8	15.2	13.2	11.7	10.5	9.5	8.6	7.9	7.3	10
11				26.7	21.3	17.6	15.0	13.1	11.6	10.4	9.4	8.5	7.8	11
12					26.5	21.1	17.5	14.9	13.0	11.5	10.3	9.3	8.4	12
13						26.2	20.9	17.3	14.8	12.8	11.3	10.1	9.2	13
14							26.0	20.7	17.1	14.6	12.7	11.2	10.0	14
15								25.7	20.4	16.9	14.4	12.5	11.1	15
16	26.5								25.4	20.2	16.7	14.3	12.4	16
17	21.1	26.2								25.1	20.0	16.5	14.1	17
18	17.5	20.9	26.0								24.8	19.7	16.3	18
19	14.9	17.3	20.7	25.7								24.5	19.5	19
20	13.0	14.8	17.1	20.4	25.4								24.2	20
21	11.5	12.8	14.6	16.9	20.2	25.1								21
22	10.3	11.3	12.7	14.4	16.7	20.0	24.8							22
23	9.3	10.1	11.2	12.5	14.3	16.5	19.7	24.5						23
24	8.4	9.2	10.0	11.1	12.4	14.1	16.3	19.5	24.2					24
25	7.7	8.3	9.0	9.9	10.9	12.2	13.9	16.1	19.2	23.8				25
26	7.1	7.6	8.2	8.9	9.8	10.8	12.1	13.7	15.9	18.9	23.5			26
27	6.6	7.0	7.5	8.1	8.8	9.6	10.6	11.9	13.5	15.6	18.6	23.1		27
28	6.2	6.5	7.0	7.4	8.0	8.7	9.5	10.5	11.7	13.3	15.4	18.3	22.7	28
29	5.7	6.1	6.4	6.9	7.3	7.9	8.6	9.4	10.3	11.5	13.1	15.1	18.0	29
30	5.4	5.7	6.0	6.4	6.8	7.2	7.8	8.4	9.2	10.1	11.3	12.8	14.9	30
31	5.1	5.3	5.6	5.9	6.3	6.7	7.1	7.7	8.3	9.0	10.0	11.1	12.6	31
32	4.8	5.0	5.2	5.5	5.8	6.2	6.5	7.0	7.5	8.1	8.9	9.8	10.9	32
33	4.5	4.7	4.9	5.1	5.4	5.7	6.1	6.4	6.9	7.4	8.0	8.7	9.6	33
34	4.3	4.4	4.6	4.8	5.1	5.3	5.6	5.9	6.3	6.8	7.3	7.8	8.6	34
35	4.0	4.2	4.4	4.5	4.7	5.0	5.2	5.5	5.8	6.2	6.6	7.1	7.7	35
36	3.8	4.0	4.1	4.3	4.5	4.7	4.9	5.1	5.4	5.7	6.1	6.5	7.0	36
37	3.6	3.8	3.9	4.0	4.2	4.4	4.6	4.8	5.0	5.3	5.6	6.0	6.4	37
38	3.4	3.6	3.7	3.8	4.0	4.1	4.3	4.5	4.7	4.9	5.2	5.5	5.8	38
39	3.3	3.4	3.5	3.6	3.8	3.9	4.0	4.2	4.4	4.6	4.8	5.1	5.4	39
40	3.1	3.2	3.3	3.4	3.6	3.7	3.8	4.0	4.1	4.3	4.5	4.7	5.0	40
41	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.9	4.0	4.2	4.4	4.6	41
42	2.9	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.7	3.8	4.0	4.1	4.3	42
43	2.7	2.8	2.9	3.0	3.0	3.1	3.2	3.3	3.5	3.6	3.7	3.9	4.0	43
44	2.6	2.7	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.8	44
45	2.5	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	45
46	2.4	2.4	2.5	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3	46
47	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.8	2.9	2.9	3.0	3.1	47
48	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.8	2.9	3.0	48
49	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6	2.7	2.8	49
50	2.0	2.0	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6	50
51	1.9	2.0	2.0	2.0	2.1	2.1	2.2	2.2	2.3	2.3	2.4	2.4	2.5	51
52	1.8	1.9	1.9	1.9	2.0	2.0	2.1	2.1	2.1	2.2	2.2	2.3	2.4	52
53	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.0	2.1	2.1	2.2	2.2	53
54	1.7	1.7	1.7	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.1	2.1	54
55	1.6	1.6	1.7	1.7	1.7	1.8	1.8	1.8	1.9	1.9	1.9	2.0	2.0	55
56	1.5	1.6	1.6	1.6	1.6	1.7	1.7	1.7	1.8	1.8	1.8	1.9	1.9	56
57	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6	1.7	1.7	1.7	1.8	1.8	57
58	1.4	1.4	1.5	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6	1.7	1.7	58
59	1.4	1.4	1.4	1.4	1.4	1.5	1.5	1.5	1.5	1.5	1.6	1.6	1.6	59
60	1.3	1.3	1.3	1.3	1.4	1.4	1.4	1.4	1.4	1.5	1.5	1.5	1.5	60
Latitude	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	Latitude
	Declination same name as latitude, upper transit: add correction to observed altitude													

TABLE 29

Altitude Factor

a, the change of altitude in one minute from meridian transit; used for entering table 30

Declination **contrary** name to latitude, **upper** transit: **add** correction to observed altitude

Latitude	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	Latitude
°	"	"	"	"	"	"	"	"	"	"	"	"	"	°
0	9.2	8.5	7.9	7.3	6.8	6.4	6.0	5.7	5.4	5.1	4.9	4.6	4.4	0
1	8.5	7.9	7.4	6.9	6.5	6.1	5.7	5.4	5.1	4.9	4.7	4.4	4.2	1
2	7.9	7.4	6.9	6.5	6.1	5.8	5.5	5.2	4.9	4.7	4.5	4.3	4.1	2
3	7.4	6.9	6.5	6.1	5.8	5.5	5.2	4.9	4.7	4.5	4.3	4.1	3.9	3
4	7.0	6.5	6.2	5.8	5.5	5.2	5.0	4.7	4.5	4.3	4.1	4.0	3.8	4
5	6.5	6.2	5.8	5.5	5.2	5.0	4.8	4.5	4.3	4.2	4.0	3.8	3.7	5
6	6.2	5.8	5.5	5.3	5.0	4.8	4.6	4.4	4.2	4.0	3.9	3.7	3.6	6
7	5.9	5.6	5.3	5.0	4.8	4.6	4.4	4.2	4.0	3.9	3.7	3.6	3.5	7
8	5.6	5.3	5.0	4.8	4.6	4.4	4.2	4.0	3.9	3.7	3.6	3.5	3.4	8
9	5.3	5.0	4.8	4.6	4.4	4.2	4.1	3.9	3.8	3.6	3.5	3.4	3.3	9
10	5.0	4.8	4.6	4.4	4.2	4.1	3.9	3.8	3.6	3.5	3.4	3.3	3.2	10
11	4.8	4.6	4.4	4.2	4.1	3.9	3.8	3.6	3.5	3.4	3.3	3.2	3.1	11
12	4.6	4.4	4.3	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	12
13	4.4	4.3	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	13
14	4.2	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	14
15	4.1	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8	15
16	3.9	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7	16
17	3.8	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7	2.6	17
18	3.7	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.5	18
19	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.6	2.5	19
20	3.4	3.3	3.2	3.1	3.0	2.9	2.9	2.8	2.7	2.6	2.6	2.5	2.4	20
21	3.3	3.2	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.4	2.4	21
22	3.2	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.4	2.4	2.3	22
23	3.1	3.0	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.4	2.4	2.3	2.3	23
24	3.0	2.9	2.8	2.8	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	24
25	2.9	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.2	25
26	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.3	2.2	2.1	2.1	26
27	2.7	2.7	2.6	2.5	2.5	2.4	2.4	2.3	2.2	2.2	2.1	2.1	2.1	27
28	2.6	2.6	2.5	2.5	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.1	2.0	28
29	2.6	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.0	2.0	2.0	29
30	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	30
31	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9	31
32	2.3	2.3	2.2	2.2	2.2	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8	32
33	2.3	2.2	2.2	2.1	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8	1.8	33
34	2.2	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	34
35	2.2	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.8	1.8	1.8	1.7	1.7	35
36	2.1	2.1	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	36
37	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.6	37
38	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6	38
39	1.9	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.6	1.6	39
40	1.9	1.8	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	40
41	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	41
42	1.8	1.7	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	42
43	1.7	1.7	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.4	1.4	43
44	1.7	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	44
45	1.6	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	45
46	1.6	1.6	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.3	1.3	46
47	1.5	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	47
48	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	48
49	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	49
50	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	50
51	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.2	51
52	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.1	1.1	52
53	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1	53
54	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1	54
55	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	55
56	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	56
57	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	57
58	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	58
59	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9	0.9	59
60	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9	0.9	0.9	0.9	0.9	0.9	60
Latitude	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	Latitude

Declination **contrary** name to latitude, **upper** transit: **add** correction to observed altitude

TABLE 29

Altitude Factor

a, the change of altitude in one minute from meridian transit; used for entering table 30

Latitude	Declination same name as latitude, upper transit: add correction to observed altitude													Latitude
	25°	26°	27°	28°	29°	30°	31°	32°	33°	34°	35°	36°	37°	
°	″	″	″	″	″	″	″	″	″	″	″	″	″	°
0	4.2	4.0	3.9	3.7	3.5	3.4	3.3	3.1	3.0	2.9	2.8	2.7	2.6	0
1	4.4	4.2	4.0	3.8	3.7	3.5	3.4	3.2	3.1	3.0	2.9	2.8	2.7	1
2	4.6	4.3	4.1	4.0	3.8	3.6	3.5	3.3	3.2	3.1	3.0	2.8	2.7	2
3	4.7	4.5	4.3	4.1	3.9	3.7	3.6	3.4	3.3	3.2	3.0	2.9	2.8	3
4	5.0	4.7	4.5	4.3	4.1	3.9	3.7	3.5	3.4	3.3	3.1	3.0	2.9	4
5	5.2	4.9	4.7	4.4	4.2	4.0	3.8	3.7	3.5	3.3	3.2	3.1	3.0	5
6	5.4	5.1	4.9	4.6	4.4	4.2	4.0	3.8	3.6	3.5	3.3	3.2	3.0	6
7	5.7	5.4	5.1	4.8	4.6	4.3	4.1	3.9	3.7	3.6	3.4	3.3	3.1	7
8	6.0	5.7	5.3	5.0	4.8	4.5	4.3	4.1	3.9	3.7	3.5	3.4	3.2	8
9	6.4	6.0	5.6	5.3	5.0	4.7	4.4	4.2	4.0	3.8	3.6	3.5	3.3	9
10	6.8	6.3	5.9	5.5	5.2	4.9	4.6	4.4	4.2	3.9	3.8	3.6	3.4	10
11	7.2	6.7	6.2	5.8	5.5	5.1	4.8	4.6	4.3	4.1	3.9	3.7	3.5	11
12	7.7	7.1	6.6	6.2	5.8	5.4	5.1	4.8	4.5	4.3	4.0	3.8	3.6	12
13	8.3	7.6	7.1	6.5	6.1	5.7	5.3	5.0	4.7	4.4	4.2	4.0	3.8	13
14	9.1	8.2	7.6	7.0	6.4	6.0	5.6	5.2	4.9	4.6	4.4	4.1	3.9	14
15	9.9	8.9	8.1	7.4	6.9	6.4	5.9	5.5	5.2	4.8	4.5	4.3	4.0	15
16	10.9	9.8	8.8	8.0	7.3	6.8	6.3	5.8	5.4	5.1	4.8	4.5	4.2	16
17	12.2	10.8	9.6	8.7	7.9	7.2	6.7	6.2	5.7	5.3	5.0	4.7	4.4	17
18	13.9	12.1	10.6	9.5	8.6	7.8	7.1	6.6	6.1	5.6	5.2	4.9	4.6	18
19	16.1	13.7	11.9	10.5	9.4	8.4	7.7	7.0	6.4	6.0	5.5	5.1	4.8	19
20	19.2	15.9	13.5	11.7	10.3	9.2	8.3	7.5	6.9	6.3	5.8	5.4	5.0	20
21	23.8	18.9	15.6	13.3	11.5	10.2	9.1	8.2	7.4	6.8	6.2	5.7	5.3	21
22		23.5	18.6	15.4	13.1	11.3	10.0	8.9	8.0	7.3	6.6	6.1	5.6	22
23			23.1	18.3	15.1	12.8	11.1	9.8	8.7	7.9	7.1	6.5	6.0	23
24				22.7	18.0	14.9	12.6	10.9	9.6	8.6	7.7	7.0	6.4	24
25					22.3	17.7	14.6	12.4	10.7	9.4	8.4	7.5	6.8	25
26						21.9	17.4	14.3	12.1	10.5	9.2	8.2	7.4	26
27							21.5	17.0	14.0	11.9	10.3	9.1	8.1	27
28								21.1	16.7	13.8	11.7	10.1	8.9	28
29	22.3								20.6	16.3	13.5	11.4	9.9	29
30	17.7	21.9								20.2	16.0	13.2	11.1	30
31	14.6	17.4	21.5								19.8	15.6	12.9	31
32	12.4	14.3	17.0	21.1								19.3	15.3	32
33	10.7	12.1	14.0	16.7	20.6								18.9	33
34	9.4	10.5	11.9	13.8	16.3	20.2								34
35	8.4	9.2	10.3	11.7	13.5	16.0	19.8							35
36	7.5	8.2	9.1	10.1	11.4	13.2	15.6	19.3						36
37	6.8	7.4	8.1	8.9	9.9	11.1	12.9	15.3	18.9					37
38	6.2	6.7	7.2	7.9	8.7	9.6	10.9	12.6	14.9	18.4				38
39	5.7	6.1	6.5	7.1	7.7	8.5	9.4	10.6	12.2	14.5	17.9			39
40	5.3	5.6	6.0	6.4	6.9	7.5	8.2	9.2	10.4	11.9	14.1	17.4		40
41	4.9	5.2	5.5	5.8	6.2	6.7	7.3	8.0	8.9	10.1	11.6	13.8	17.0	41
42	4.5	4.8	5.0	5.3	5.7	6.1	6.6	7.1	7.8	8.7	9.8	11.3	13.4	42
43	4.2	4.4	4.6	4.9	5.2	5.5	5.9	6.4	6.9	7.6	8.5	9.5	11.0	43
44	3.9	4.1	4.3	4.5	4.8	5.1	5.4	5.8	6.2	6.7	7.4	8.2	9.3	44
45	3.7	3.8	4.0	4.2	4.4	4.7	4.9	5.2	5.6	6.0	6.6	7.2	8.0	45
46	3.5	3.6	3.7	3.9	4.1	4.3	4.5	4.8	5.1	5.4	5.9	6.4	7.0	46
47	3.3	3.4	3.5	3.6	3.8	4.0	4.2	4.4	4.6	4.9	5.3	5.7	6.2	47
48	3.1	3.2	3.3	3.4	3.5	3.7	3.9	4.0	4.3	4.5	4.8	5.1	5.5	48
49	2.9	3.0	3.1	3.2	3.3	3.4	3.6	3.7	3.9	4.1	4.4	4.6	5.0	49
50	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.5	3.6	3.8	4.0	4.2	4.5	50
51	2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.4	3.5	3.7	3.9	4.1	51
52	2.4	2.5	2.6	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.4	3.6	3.7	52
53	2.3	2.3	2.4	2.5	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.3	3.4	53
54	2.2	2.2	2.3	2.3	2.4	2.5	2.5	2.6	2.7	2.8	2.9	3.0	3.2	54
55	2.0	2.1	2.1	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.7	2.8	2.9	55
56	1.9	2.0	2.0	2.1	2.1	2.2	2.2	2.3	2.4	2.4	2.5	2.6	2.7	56
57	1.8	1.9	1.9	2.0	2.0	2.0	2.1	2.2	2.2	2.3	2.3	2.4	2.5	57
58	1.7	1.8	1.8	1.8	1.9	1.9	2.0	2.0	2.1	2.1	2.2	2.3	2.3	58
59	1.6	1.7	1.7	1.7	1.8	1.8	1.9	1.9	1.9	2.0	2.0	2.1	2.2	59
60	1.6	1.6	1.6	1.6	1.7	1.7	1.7	1.8	1.8	1.9	1.9	2.0	2.0	60
Latitude	25°	26°	27°	28°	29°	30°	31°	32°	33°	34°	35°	36°	37°	Latitude
	Declination same name as latitude, upper transit: add correction to observed altitude													

TABLE 29

Altitude Factor

a, the change of altitude in one minute from meridian transit; used for entering table 30

Latitude	25°	26°	27°	28°	29°	30°	31°	32°	33°	34°	35°	36°	37°	Latitude
	Declination **contrary** name to latitude, **upper** transit: **add** correction to observed altitude													
°	″	″	″	″	″	″	″	″	″	″	″	″	″	°
0	4. 2	4. 0	3. 9	3. 7	3. 5	3. 4	3. 3	3. 1	3. 0	2. 9	2. 8	2. 7	2. 6	0
1	4. 1	3. 9	3. 7	3. 6	3. 4	3. 3	3. 2	3. 1	2. 9	2. 8	2. 7	2. 6	2. 6	1
2	3. 9	3. 8	3. 6	3. 5	3. 3	3. 2	3. 1	3. 0	2. 9	2. 8	2. 7	2. 6	2. 5	2
3	3. 8	3. 6	3. 5	3. 4	3. 2	3. 1	3. 0	2. 9	2. 8	2. 7	2. 6	2. 5	2. 4	3
4	3. 7	3. 5	3. 4	3. 3	3. 2	3. 0	2. 9	2. 8	2. 7	2. 6	2. 6	2. 5	2. 4	4
5	3. 6	3. 4	3. 3	3. 2	3. 1	3. 0	2. 9	2. 8	2. 7	2. 6	2. 5	2. 4	2. 3	5
6	3. 4	3. 3	3. 2	3. 1	3. 0	2. 9	2. 8	2. 7	2. 6	2. 5	2. 4	2. 4	2. 3	6
7	3. 3	3. 2	3. 1	3. 0	2. 9	2. 8	2. 7	2. 6	2. 5	2. 5	2. 4	2. 3	2. 2	7
8	3. 2	3. 1	3. 0	2. 9	2. 8	2. 7	2. 7	2. 6	2. 5	2. 4	2. 3	2. 3	2. 2	8
9	3. 1	3. 0	2. 9	2. 9	2. 8	2. 7	2. 6	2. 5	2. 4	2. 4	2. 3	2. 2	2. 2	9
10	3. 1	3. 0	2. 9	2. 8	2. 7	2. 6	2. 5	2. 5	2. 4	2. 3	2. 2	2. 2	2. 1	10
11	3. 0	2. 9	2. 8	2. 7	2. 6	2. 5	2. 5	2. 4	2. 3	2. 3	2. 2	2. 1	2. 1	11
12	2. 9	2. 8	2. 7	2. 6	2. 6	2. 5	2. 4	2. 3	2. 3	2. 2	2. 2	2. 1	2. 0	12
13	2. 8	2. 7	2. 7	2. 6	2. 5	2. 4	2. 4	2. 3	2. 2	2. 2	2. 1	2. 1	2. 0	13
14	2. 7	2. 7	2. 6	2. 5	2. 4	2. 4	2. 3	2. 3	2. 2	2. 1	2. 1	2. 0	2. 0	14
15	2. 7	2. 6	2. 5	2. 5	2. 4	2. 3	2. 3	2. 2	2. 1	2. 1	2. 0	2. 0	1. 9	15
16	2. 6	2. 5	2. 5	2. 4	2. 3	2. 3	2. 2	2. 2	2. 1	2. 0	2. 0	1. 9	1. 9	16
17	2. 5	2. 5	2. 4	2. 3	2. 3	2. 2	2. 2	2. 1	2. 1	2. 0	2. 0	1. 9	1. 9	17
18	2. 5	2. 4	2. 4	2. 3	2. 2	2. 2	2. 1	2. 1	2. 0	2. 0	1. 9	1. 9	1. 8	18
19	2. 4	2. 4	2. 3	2. 2	2. 2	2. 1	2. 1	2. 0	2. 0	1. 9	1. 9	1. 8	1. 8	19
20	2. 4	2. 3	2. 3	2. 2	2. 1	2. 1	2. 0	2. 0	1. 9	1. 9	1. 9	1. 8	1. 8	20
21	2. 3	2. 3	2. 2	2. 1	2. 1	2. 0	2. 0	2. 0	1. 9	1. 9	1. 8	1. 8	1. 7	21
22	2. 3	2. 2	2. 2	2. 1	2. 1	2. 0	2. 0	1. 9	1. 9	1. 8	1. 8	1. 7	1. 7	22
23	2. 2	2. 2	2. 1	2. 1	2. 0	2. 0	1. 9	1. 9	1. 8	1. 8	1. 8	1. 7	1. 7	23
24	2. 2	2. 1	2. 1	2. 0	2. 0	1. 9	1. 9	1. 8	1. 8	1. 8	1. 7	1. 7	1. 6	24
25	2. 1	2. 1	2. 0	2. 0	1. 9	1. 9	1. 8	1. 8	1. 8	1. 7	1. 7	1. 6	1. 6	25
26	2. 1	2. 0	2. 0	1. 9	1. 9	1. 9	1. 8	1. 8	1. 7	1. 7	1. 7	1. 6	1. 6	26
27	2. 0	2. 0	1. 9	1. 9	1. 9	1. 8	1. 8	1. 7	1. 7	1. 7	1. 6	1. 6	1. 6	27
28	2. 0	1. 9	1. 9	1. 9	1. 8	1. 8	1. 7	1. 7	1. 7	1. 6	1. 6	1. 6	1. 5	28
29	1. 9	1. 9	1. 9	1. 8	1. 8	1. 7	1. 7	1. 7	1. 6	1. 6	1. 6	1. 5	1. 5	29
30	1. 9	1. 8	1. 8	1. 8	1. 7	1. 7	1. 7	1. 6	1. 6	1. 6	1. 5	1. 5	1. 5	30
31	1. 8	1. 8	1. 8	1. 7	1. 7	1. 7	1. 6	1. 6	1. 6	1. 5	1. 5	1. 5	1. 5	31
32	1. 8	1. 8	1. 7	1. 7	1. 7	1. 6	1. 6	1. 6	1. 5	1. 5	1. 5	1. 5	1. 4	32
33	1. 8	1. 7	1. 7	1. 7	1. 6	1. 6	1. 6	1. 5	1. 5	1. 5	1. 5	1. 4	1. 4	33
34	1. 7	1. 7	1. 7	1. 6	1. 6	1. 6	1. 5	1. 5	1. 5	1. 5	1. 4	1. 4	1. 4	34
35	1. 7	1. 7	1. 6	1. 6	1. 6	1. 5	1. 5	1. 5	1. 5	1. 4	1. 4	1. 4	1. 4	35
36	1. 6	1. 6	1. 6	1. 6	1. 5	1. 5	1. 5	1. 5	1. 4	1. 4	1. 4	1. 4	1. 3	36
37	1. 6	1. 6	1. 6	1. 5	1. 5	1. 5	1. 5	1. 4	1. 4	1. 4	1. 4	1. 3	1. 3	37
38	1. 6	1. 5	1. 5	1. 5	1. 5	1. 5	1. 4	1. 4	1. 4	1. 4	1. 3	1. 3	1. 3	38
39	1. 5	1. 5	1. 5	1. 5	1. 4	1. 4	1. 4	1. 4	1. 4	1. 3	1. 3	1. 3	1. 3	39
40	1. 5	1. 5	1. 5	1. 4	1. 4	1. 4	1. 4	1. 3	1. 3	1. 3	1. 3	1. 3	1. 2	40
41	1. 5	1. 4	1. 4	1. 4	1. 4	1. 4	1. 3	1. 3	1. 3	1. 3	1. 3	1. 2	1. 2	41
42	1. 4	1. 4	1. 4	1. 4	1. 4	1. 3	1. 3	1. 3	1. 3	1. 2	1. 2	1. 2	1. 2	42
43	1. 4	1. 4	1. 4	1. 3	1. 3	1. 3	1. 3	1. 3	1. 2	1. 2	1. 2	1. 2	1. 2	43
44	1. 4	1. 4	1. 3	1. 3	1. 3	1. 3	1. 3	1. 2	1. 2	1. 2	1. 2	1. 2	1. 2	44
45	1. 3	1. 3	1. 3	1. 3	1. 3	1. 2	1. 2	1. 2	1. 2	1. 2	1. 2	1. 1	1. 1	45
46	1. 3	1. 3	1. 3	1. 3	1. 2	1. 2	1. 2	1. 2	1. 2	1. 2	1. 1	1. 1	1. 1	46
47	1. 3	1. 3	1. 2	1. 2	1. 2	1. 2	1. 2	1. 2	1. 1	1. 1	1. 1	1. 1	1. 1	47
48	1. 2	1. 2	1. 2	1. 2	1. 2	1. 2	1. 1	1. 1	1. 1	1. 1	1. 1	1. 1		48
49	1. 2	1. 2	1. 2	1. 2	1. 2	1. 1	1. 1	1. 1	1. 1	1. 1	1. 1			49
50	1. 2	1. 2	1. 2	1. 1	1. 1	1. 1	1. 1	1. 1	1. 1	1. 1				50
51	1. 2	1. 1	1. 1	1. 1	1. 1	1. 1	1. 1	1. 1	1. 0					51
52	1. 1	1. 1	1. 1	1. 1	1. 1	1. 1	1. 0	1. 0						52
53	1. 1	1. 1	1. 1	1. 1	1. 0	1. 0	1. 0							53
54	1. 1	1. 0	1. 0	1. 0	1. 0	1. 0								54
55	1. 0	1. 0	1. 0	1. 0	1. 0									55
56	1. 0	1. 0	1. 0	1. 0										56
57	1. 0	1. 0	1. 0											57
58	1. 0	0. 9												58
59	0. 9												0. 8	59
60												0. 8	0. 8	60
Latitude	25°	26°	27°	28°	29°	30°	31°	32°	33°	34°	35°	36°	37°	Latitude
	Declination **same** name as latitude, **lower** transit: **subtract** correction from observed altitude													

TABLE 29

Altitude Factor

a, the change of altitude in one minute from meridian transit; used for entering table 30

Latitude	Declination same name as latitude, upper transit: add correction to observed altitude													Latitude
	38°	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	
°	"	"	"	"	"	"	"	"	"	"	"	"	"	°
0	2.5	2.4	2.3	2.3	2.2	2.1	2.0	2.0	1.9	1.8	1.8	1.7	1.7	0
1	2.6	2.5	2.4	2.3	2.2	2.2	2.1	2.0	1.9	1.9	1.8	1.7	1.7	1
2	2.6	2.5	2.4	2.4	2.3	2.2	2.1	2.0	2.0	1.9	1.8	1.8	1.7	2
3	2.7	2.6	2.5	2.4	2.3	2.2	2.2	2.1	2.0	1.9	1.9	1.8	1.7	3
4	2.8	2.7	2.6	2.5	2.4	2.3	2.2	2.1	2.0	2.0	1.9	1.8	1.8	4
5	2.8	2.7	2.6	2.5	2.4	2.3	2.2	2.2	2.1	2.0	1.9	1.9	1.8	5
6	2.9	2.8	2.7	2.6	2.5	2.4	2.3	2.2	2.1	2.0	2.0	1.9	1.8	6
7	3.0	2.9	2.7	2.6	2.5	2.4	2.3	2.2	2.2	2.1	2.0	1.9	1.8	7
8	3.1	2.9	2.8	2.7	2.6	2.5	2.4	2.3	2.2	2.1	2.0	1.9	1.9	8
9	3.2	3.0	2.9	2.8	2.7	2.5	2.4	2.3	2.2	2.2	2.1	2.0	1.9	9
10	3.3	3.1	3.0	2.8	2.7	2.6	2.5	2.4	2.3	2.2	2.1	2.0	1.9	10
11	3.4	3.2	3.1	2.9	2.8	2.7	2.6	2.4	2.3	2.2	2.1	2.1	2.0	11
12	3.5	3.3	3.1	3.0	2.9	2.7	2.6	2.5	2.4	2.3	2.2	2.1	2.0	12
13	3.6	3.4	3.2	3.1	2.9	2.8	2.7	2.6	2.4	2.3	2.2	2.1	2.0	13
14	3.7	3.5	3.3	3.2	3.0	2.9	2.7	2.6	2.5	2.4	2.3	2.2	2.1	14
15	3.8	3.6	3.4	3.3	3.1	3.0	2.8	2.7	2.6	2.4	2.3	2.2	2.1	15
16	4.0	3.8	3.6	3.4	3.2	3.0	2.9	2.8	2.6	2.5	2.4	2.3	2.2	16
17	4.1	3.9	3.7	3.5	3.3	3.1	3.0	2.8	2.7	2.6	2.4	2.3	2.2	17
18	4.3	4.1	3.8	3.6	3.4	3.2	3.1	2.9	2.8	2.6	2.5	2.4	2.3	18
19	4.5	4.2	4.0	3.7	3.5	3.3	3.2	3.0	2.8	2.7	2.6	2.4	2.3	19
20	4.7	4.4	4.1	3.9	3.7	3.5	3.3	3.1	2.9	2.8	2.6	2.5	2.4	20
21	4.9	4.6	4.3	4.0	3.8	3.6	3.4	3.2	3.0	2.9	2.7	2.6	2.4	21
22	5.2	4.8	4.5	4.2	4.0	3.7	3.5	3.3	3.1	2.9	2.8	2.6	2.5	22
23	5.5	5.1	4.7	4.4	4.1	3.9	3.6	3.4	3.2	3.0	2.9	2.7	2.6	23
24	5.8	5.4	5.0	4.6	4.3	4.0	3.8	3.5	3.3	3.1	3.0	2.8	2.6	24
25	6.2	5.7	5.3	4.9	4.5	4.2	3.9	3.7	3.5	3.3	3.1	2.9	2.7	25
26	6.7	6.1	5.6	5.2	4.8	4.4	4.1	3.8	3.6	3.4	3.2	3.0	2.8	26
27	7.2	6.5	6.0	5.5	5.0	4.6	4.3	4.0	3.7	3.5	3.3	3.1	2.9	27
28	7.9	7.1	6.4	5.8	5.3	4.9	4.5	4.2	3.9	3.6	3.4	3.2	3.0	28
29	8.7	7.7	6.9	6.2	5.7	5.2	4.8	4.4	4.1	3.8	3.5	3.3	3.1	29
30	9.6	8.5	7.5	6.7	6.1	5.5	5.1	4.7	4.3	4.0	3.7	3.4	3.2	30
31	10.9	9.4	8.2	7.3	6.6	5.9	5.4	4.9	4.5	4.2	3.9	3.6	3.3	31
32	12.6	10.6	9.2	8.0	7.1	6.4	5.8	5.2	4.8	4.4	4.0	3.7	3.5	32
33	14.9	12.2	10.4	8.9	7.8	6.9	6.2	5.6	5.1	4.6	4.3	3.9	3.6	33
34	18.4	14.5	11.9	10.1	8.7	7.6	6.7	6.0	5.4	4.9	4.5	4.1	3.8	34
35		17.9	14.1	11.6	9.8	8.5	7.4	6.6	5.9	5.3	4.8	4.4	4.0	35
36			17.4	13.8	11.3	9.5	8.2	7.2	6.4	5.7	5.1	4.6	4.2	36
37				17.0	13.4	11.0	9.3	8.0	7.0	6.2	5.5	5.0	4.5	37
38					16.5	13.0	10.7	9.0	7.7	6.8	6.0	5.3	4.8	38
39						16.0	12.6	10.3	8.7	7.5	6.5	5.8	5.1	39
40							15.5	12.2	10.0	8.4	7.2	6.3	5.6	40
41								15.0	11.8	9.7	8.1	7.0	6.1	41
42	16.5								14.5	11.4	9.3	7.9	6.7	42
43	13.0	16.0								14.0	11.0	9.0	7.6	43
44	10.7	12.6	15.5								13.6	10.6	8.7	44
45	9.0	10.3	12.2	15.0								13.1	10.2	45
46	7.7	8.7	10.0	11.8	14.5								12.6	46
47	6.8	7.5	8.4	9.7	11.4	14.0								47
48	6.0	6.5	7.2	8.1	9.3	11.0	13.6							48
49	5.3	5.8	6.3	7.0	7.9	9.0	10.6	13.1						49
50	4.8	5.1	5.6	6.1	6.7	7.6	8.7	10.2	12.6					50
51	4.3	4.6	5.0	5.4	5.9	6.5	7.3	8.4	9.9	12.1				51
52	3.9	4.2	4.5	4.8	5.2	5.7	6.3	7.0	8.0	9.5	11.6			52
53	3.6	3.8	4.0	4.3	4.6	5.0	5.4	6.0	6.7	7.7	9.1	11.1		53
54	3.3	3.5	3.7	3.9	4.1	4.4	4.8	5.2	5.8	6.5	7.4	8.7	10.6	54
55	3.0	3.2	3.3	3.5	3.7	4.0	4.3	4.6	5.0	5.5	6.2	7.1	8.3	55
56	2.8	2.9	3.1	3.2	3.4	3.6	3.8	4.1	4.4	4.8	5.3	5.9	6.8	56
57	2.6	2.7	2.8	2.9	3.1	3.2	3.4	3.6	3.9	4.2	4.6	5.0	5.6	57
58	2.4	2.5	2.6	2.7	2.8	2.9	3.1	3.3	3.5	3.7	4.0	4.4	4.8	58
59	2.2	2.3	2.4	2.5	2.6	2.7	2.8	3.0	3.1	3.3	3.6	3.8	4.2	59
60	2.1	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	3.0	3.2	3.4	3.6	60
Latitude	38°	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	Latitude
	Declination same name as latitude, upper transit: add correction to observed altitude													

TABLE 29

Altitude Factor

a, the change of altitude in one minute from meridian transit; used for entering table 30

Lati-tude	Declination **contrary** name to latitude, **upper** transit: **add** correction to observed altitude													Lati-tude
	38°	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	
°	″	″	″	″	″	″	″	″	″	″	″	″	″	°
0	2.5	2.4	2.3	2.3	2.2	2.1	2.0	2.0	1.9	1.8	1.8	1.7	1.7	0
1	2.5	2.4	2.3	2.2	2.1	2.1	2.0	1.9	1.9	1.8	1.7	1.7	1.6	1
2	2.4	2.3	2.3	2.2	2.1	2.0	2.0	1.9	1.8	1.8	1.7	1.7	1.6	2
3	2.4	2.3	2.2	2.1	2.1	2.0	1.9	1.9	1.8	1.8	1.7	1.6	1.6	3
4	2.3	2.2	2.2	2.1	2.0	2.0	1.9	1.8	1.8	1.7	1.7	1.6	1.6	4
5	2.3	2.2	2.1	2.1	2.0	1.9	1.9	1.8	1.8	1.7	1.6	1.6	1.5	5
6	2.2	2.2	2.1	2.0	2.0	1.9	1.8	1.8	1.7	1.7	1.6	1.6	1.5	6
7	2.2	2.1	2.0	2.0	1.9	1.9	1.8	1.8	1.7	1.6	1.6	1.5	1.5	7
8	2.1	2.1	2.0	1.9	1.9	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	8
9	2.1	2.0	2.0	1.9	1.9	1.8	1.8	1.7	1.6	1.6	1.6	1.5	1.5	9
10	2.1	2.0	1.9	1.9	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4	10
11	2.0	2.0	1.9	1.8	1.8	1.7	1.7	1.6	1.6	1.6	1.5	1.5	1.4	11
12	2.0	1.9	1.9	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4	1.4	12
13	1.9	1.9	1.8	1.8	1.7	1.7	1.6	1.6	1.6	1.5	1.5	1.4	1.4	13
14	1.9	1.9	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4	1.4	1.4	14
15	1.9	1.8	1.8	1.7	1.7	1.6	1.6	1.6	1.5	1.5	1.4	1.4	1.4	15
16	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.5	1.5	1.4	1.4	1.4	1.3	16
17	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.5	1.4	1.4	1.4	1.3	17
18	1.8	1.7	1.7	1.6	1.6	1.6	1.5	1.5	1.4	1.4	1.4	1.3	1.3	18
19	1.7	1.7	1.7	1.6	1.6	1.5	1.5	1.5	1.4	1.4	1.4	1.3	1.3	19
20	1.7	1.7	1.6	1.6	1.6	1.5	1.5	1.4	1.4	1.4	1.3	1.3	1.3	20
21	1.7	1.6	1.6	1.6	1.5	1.5	1.5	1.4	1.4	1.4	1.3	1.3	1.3	21
22	1.7	1.6	1.6	1.5	1.5	1.5	1.4	1.4	1.4	1.3	1.3	1.3	1.2	22
23	1.6	1.6	1.6	1.5	1.5	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.2	23
24	1.6	1.6	1.5	1.5	1.5	1.4	1.4	1.4	1.3	1.3	1.3	1.2	1.2	24
25	1.6	1.5	1.5	1.5	1.4	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	25
26	1.6	1.5	1.5	1.5	1.4	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	26
27	1.5	1.5	1.5	1.4	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.2	27
28	1.5	1.5	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.1	28
29	1.5	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.1	29
30	1.5	1.4	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.1	1.1	30
31	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.1	1.1	31
32	1.4	1.4	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.1	1.1	1.1	32
33	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	33
34	1.4	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1	34
35	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1		35
36	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1			36
37	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1				37
38	1.3	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1					38
39	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1						39
40	1.2	1.2	1.2	1.2	1.1	1.1	1.1							40
41	1.2	1.2	1.2	1.1	1.1	1.1								41
42	1.2	1.2	1.1	1.1	1.1									42
43	1.2	1.1	1.1	1.1										43
44	1.1	1.1	1.1											44
45	1.1	1.1												45
46	1.1												0.9	46
47												0.9	0.9	47
48											0.9	0.9	0.9	48
49										0.9	0.9	0.9	0.8	49
50									0.9	0.9	0.9	0.8	0.8	50
51								0.9	0.9	0.9	0.8	0.8	0.8	51
52							0.9	0.9	0.9	0.8	0.8	0.8	0.8	52
53						0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.8	53
54					0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.8	54
55				0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	55
56			0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	56
57		0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	57
58	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	58
59	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.7	59
60	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	60
Lati-tude	38°	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	Lati-tude
	Declination **same** name as latitude, **lower** transit: **subtract** correction from observed altitude													

TABLE 29

Altitude Factor

a, the change of altitude in one minute from meridian transit; used for entering table 30

Latitude	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	61°	62°	63°	Latitude
	Declination **same** name as latitude, **upper** transit: **add** correction to observed altitude													
°	″	″	″	″	″	″	″	″	″	″	″	″	″	°
0	1.6	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.0	1.0	0
1	1.6	1.6	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.2	1.1	1.1	1.0	1
2	1.6	1.6	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.0	2
3	1.7	1.6	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.0	3
4	1.7	1.6	1.6	1.5	1.5	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.0	4
5	1.7	1.7	1.6	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.1	1.1	1.1	5
6	1.7	1.7	1.6	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	6
7	1.8	1.7	1.6	1.6	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	7
8	1.8	1.7	1.7	1.6	1.5	1.5	1.4	1.4	1.3	1.2	1.2	1.1	1.1	8
9	1.8	1.8	1.7	1.6	1.6	1.5	1.4	1.4	1.3	1.3	1.2	1.1	1.1	9
10	1.9	1.8	1.7	1.6	1.6	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	10
11	1.9	1.8	1.7	1.7	1.6	1.5	1.5	1.4	1.3	1.3	1.2	1.2	1.1	11
12	1.9	1.8	1.8	1.7	1.6	1.6	1.5	1.4	1.4	1.3	1.2	1.2	1.1	12
13	2.0	1.9	1.8	1.7	1.6	1.6	1.5	1.4	1.4	1.3	1.3	1.2	1.1	13
14	2.0	1.9	1.8	1.7	1.7	1.6	1.5	1.5	1.4	1.3	1.3	1.2	1.2	14
15	2.0	1.9	1.9	1.8	1.7	1.6	1.5	1.5	1.4	1.3	1.3	1.2	1.2	15
16	2.1	2.0	1.9	1.8	1.7	1.6	1.6	1.5	1.4	1.4	1.3	1.2	1.2	16
17	2.1	2.0	1.9	1.8	1.8	1.7	1.6	1.5	1.5	1.4	1.3	1.3	1.2	17
18	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.5	1.5	1.4	1.3	1.3	1.2	18
19	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.6	1.5	1.4	1.4	1.3	1.2	19
20	2.3	2.1	2.0	1.9	1.9	1.8	1.7	1.6	1.5	1.4	1.4	1.3	1.2	20
21	2.3	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.5	1.5	1.4	1.3	1.2	21
22	2.4	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.6	1.5	1.4	1.3	1.3	22
23	2.4	2.3	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.5	1.4	1.4	1.3	23
24	2.5	2.4	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.5	1.5	1.4	1.3	24
25	2.6	2.4	2.3	2.2	2.0	1.9	1.8	1.7	1.6	1.6	1.5	1.4	1.3	25
26	2.6	2.5	2.3	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.5	1.4	1.3	26
27	2.7	2.6	2.4	2.3	2.1	2.0	1.9	1.8	1.7	1.6	1.5	1.4	1.4	27
28	2.8	2.6	2.5	2.3	2.2	2.1	2.0	1.8	1.7	1.6	1.5	1.5	1.4	28
29	2.9	2.7	2.5	2.4	2.3	2.1	2.0	1.9	1.8	1.7	1.6	1.5	1.4	29
30	3.0	2.8	2.6	2.5	2.3	2.2	2.0	1.9	1.8	1.7	1.6	1.5	1.4	30
31	3.1	2.9	2.7	2.5	2.4	2.2	2.1	2.0	1.9	1.7	1.6	1.5	1.4	31
32	3.2	3.0	2.8	2.6	2.4	2.3	2.2	2.0	1.9	1.8	1.7	1.6	1.5	32
33	3.4	3.1	2.9	2.7	2.5	2.4	2.2	2.1	1.9	1.8	1.7	1.6	1.5	33
34	3.5	3.2	3.0	2.8	2.6	2.4	2.3	2.1	2.0	1.9	1.7	1.6	1.5	34
35	3.7	3.4	3.1	2.9	2.7	2.5	2.3	2.2	2.0	1.9	1.8	1.7	1.6	35
36	3.9	3.6	3.3	3.0	2.8	2.6	2.4	2.3	2.1	2.0	1.8	1.7	1.6	36
37	4.1	3.7	3.4	3.2	2.9	2.7	2.5	2.3	2.2	2.0	1.9	1.7	1.6	37
38	4.3	3.9	3.6	3.3	3.0	2.8	2.6	2.4	2.2	2.1	1.9	1.8	1.7	38
39	4.6	4.2	3.8	3.5	3.2	2.9	2.7	2.5	2.3	2.1	2.0	1.8	1.7	39
40	5.0	4.5	4.0	3.7	3.3	3.1	2.8	2.6	2.4	2.2	2.0	1.9	1.8	40
41	5.4	4.8	4.3	3.9	3.5	3.2	2.9	2.7	2.5	2.3	2.1	1.9	1.8	41
42	5.9	5.2	4.6	4.1	3.7	3.4	3.1	2.8	2.6	2.4	2.2	2.0	1.9	42
43	6.5	5.7	5.0	4.4	4.0	3.6	3.2	2.9	2.7	2.5	2.3	2.1	1.9	43
44	7.3	6.3	5.4	4.8	4.3	3.8	3.4	3.1	2.8	2.6	2.3	2.2	2.0	44
45	8.4	7.0	6.0	5.2	4.6	4.1	3.6	3.3	3.0	2.7	2.4	2.2	2.0	45
46	9.9	8.0	6.7	5.8	5.0	4.4	3.9	3.5	3.1	2.8	2.6	2.3	2.1	46
47	12.1	9.5	7.7	6.5	5.5	4.8	4.2	3.7	3.3	3.0	2.7	2.4	2.2	47
48		11.6	9.1	7.4	6.2	5.3	4.6	4.0	3.6	3.2	2.8	2.6	2.3	48
49			11.1	8.7	7.1	5.9	5.0	4.4	3.8	3.4	3.0	2.7	2.4	49
50				10.6	8.3	6.8	5.6	4.8	4.2	3.6	3.2	2.9	2.6	50
51					10.2	7.9	6.4	5.4	4.6	4.0	3.5	3.0	2.7	51
52						9.7	7.6	6.1	5.1	4.3	3.8	3.3	2.9	52
53							9.2	7.2	5.9	4.9	4.1	3.6	3.1	53
54								8.8	6.8	5.5	4.6	3.9	3.4	54
55	10.2								8.3	6.5	5.3	4.3	3.7	55
56	7.9	9.7								7.9	6.1	5.0	4.1	56
57	6.4	7.6	9.2								7.4	5.8	4.7	57
58	5.4	6.1	7.2	8.8								7.0	5.4	58
59	4.6	5.1	5.9	6.8	8.3								6.6	59
60	4.0	4.3	4.9	5.5	6.5	7.9								60
Latitude	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	61°	62°	63°	Latitude
	Declination **same** name as latitude, **upper** transit: **add** correction to observed altitude													

TABLE 29

Altitude Factor

a, the change of altitude in one minute from meridian transit; used for entering table 30

Latitude	Declination contrary name to latitude, upper transit: **add** correction to observed altitude													Latitude
	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	61°	62°	63°	
°	″	″	″	″	″	″	″	″	″	″	″	″	″	°
0	1.6	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.0	1.0	0
1	1.6	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.0	1.0	1
2	1.5	1.5	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.1	1.1	1.0	1.0	2
3	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.1	1.0	1.0	3
4	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.1	1.0	1.0	4
5	1.5	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.1	1.1	1.0	1.0	1.0	5
6	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.2	1.1	1.1	1.0	1.0	1.0	6
7	1.4	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.1	1.0	1.0	0.9	7
8	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.1	1.1	1.1	1.0	1.0	0.9	8
9	1.4	1.4	1.3	1.3	1.2	1.2	1.2	1.1	1.1	1.0	1.0	1.0	0.9	9
10	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	0.9	10
11	1.4	1.3	1.3	1.3	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	0.9	11
12	1.4	1.3	1.3	1.2	1.2	1.2	1.1	1.1	1.1	1.0	1.0	0.9	0.9	12
13	1.3	1.3	1.3	1.2	1.2	1.2	1.1	1.1	1.0	1.0	1.0	0.9	0.9	13
14	1.3	1.3	1.3	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	0.9	0.9	14
15	1.3	1.3	1.2	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	0.9	0.9	15
16	1.3	1.3	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	0.9	0.9	0.9	16
17	1.3	1.2	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	0.9	0.9	0.9	17
18	1.3	1.2	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	0.9	0.9	0.9	18
19	1.2	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	1.0	0.9	0.9	0.9	19
20	1.2	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	0.9	0.9	0.9	0.8	20
21	1.2	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	0.9	0.9	0.9	0.8	21
22	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	1.0	0.9	0.9	0.9		22
23	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0	0.9	0.9	0.9			23
24	1.2	1.1	1.1	1.1	1.1	1.0	1.0	1.0	0.9	0.9				24
25	1.2	1.1	1.1	1.1	1.0	1.0	1.0	1.0	0.9					25
26	1.1	1.1	1.1	1.1	1.0	1.0	1.0	0.9						26
27	1.1	1.1	1.1	1.0	1.0	1.0	1.0							27
28	1.1	1.1	1.1	1.0	1.0	1.0								28
29	1.1	1.1	1.0	1.0	1.0									29
30	1.1	1.1	1.0	1.0										30
31	1.1	1.0	1.0											31
32	1.1	1.0												32
33	1.1												0.8	33
34												0.8	0.7	34
35											0.8	0.8	0.7	35
36										0.8	0.8	0.8	0.7	36
37									0.8	0.8	0.8	0.7	0.7	37
38								0.8	0.8	0.8	0.8	0.7	0.7	38
39							0.8	0.8	0.8	0.8	0.8	0.7	0.7	39
40						0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	40
41					0.9	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	41
42				0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	42
43			0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	43
44		0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	44
45	0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	45
46	0.9	0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	46
47	0.9	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.6	47
48	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.6	48
49	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	49
50	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	50
51	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	51
52	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.6	52
53	0.8	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.6	53
54	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6	54
55	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6	55
56	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6	0.6	56
57	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6	0.6	0.6	57
58	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	58
59	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.5	59
60	0.7	0.7	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.5	60
Latitude	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	61°	62°	63°	Latitude
	Declination same name as latitude, lower transit: **subtract** correction from observed altitude													

TABLE 30

Change of Altitude in Given Time from Meridian Transit

a (table 29)	t, meridian angle 5′ 0m 20s	10′ 0m 40s	15′ 1m 00s	20′ 1m 20s	25′ 1m 40s	30′ 2m 00s	35′ 2m 20s	40′ 2m 40s	45′ 3m 00s	50′ 3m 20s	55′ 3m 40s	1°00′ 4m 00s	1°05′ 4m 20s	1°10′ 4m 40s	a (table 29)
″	′	′	′	′	′	′	′	′	′	′	′	′	′	′	″
0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2
0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.3
0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.4
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.5
0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.6
0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.7
0.8	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.3	0.3	0.8
0.9	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.9
1.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.4	1.0
2.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	0.7	2.0
3.0	0.0	0.0	0.0	0.1	0.1	0.2	0.3	0.4	0.4	0.6	0.7	0.8	0.9	1.1	3.0
4.0	0.0	0.0	0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.9	1.1	1.3	1.5	4.0
5.0	0.0	0.0	0.1	0.1	0.2	0.3	0.5	0.6	0.8	0.9	1.1	1.3	1.6	1.8	5.0
6.0	0.0	0.0	0.1	0.2	0.3	0.4	0.5	0.7	0.9	1.1	1.3	1.6	1.9	2.2	6.0
7.0	0.0	0.1	0.1	0.2	0.3	0.5	0.6	0.8	1.0	1.3	1.6	1.9	2.2	2.5	7.0
8.0	0.0	0.1	0.1	0.2	0.4	0.5	0.7	0.9	1.2	1.5	1.8	2.1	2.5	2.9	8.0
9.0	0.0	0.1	0.2	0.3	0.4	0.6	0.8	1.1	1.4	1.7	2.0	2.4	2.8	3.3	9.0
10.0	0.0	0.1	0.2	0.3	0.5	0.7	0.9	1.2	1.5	1.9	2.2	2.7	3.1	3.6	10.0
11.0	0.0	0.1	0.2	0.3	0.5	0.7	1.0	1.3	1.6	2.0	2.5	2.9	3.4	4.0	11.0
12.0	0.0	0.1	0.2	0.4	0.6	0.8	1.1	1.4	1.8	2.2	2.7	3.2	3.8	4.4	12.0
13.0	0.0	0.1	0.2	0.4	0.6	0.9	1.2	1.5	2.0	2.4	2.9	3.5	4.1	4.7	13.0
14.0	0.0	0.1	0.2	0.4	0.6	0.9	1.3	1.7	2.1	2.6	3.1	3.7	4.4	5.1	14.0
15.0	0.0	0.1	0.2	0.4	0.7	1.0	1.4	1.8	2.2	2.8	3.4	4.0	4.7	5.4	15.0
16.0	0.0	0.1	0.3	0.5	0.7	1.1	1.5	1.9	2.4	3.0	3.6	4.3	5.0	5.8	16.0
17.0	0.0	0.1	0.3	0.5	0.8	1.1	1.5	2.0	2.6	3.1	3.8	4.5	5.3	6.2	17.0
18.0	0.0	0.1	0.3	0.5	0.8	1.2	1.6	2.1	2.7	3.3	4.0	4.8	5.6	6.5	18.0
19.0	0.0	0.1	0.3	0.6	0.9	1.3	1.7	2.3	2.8	3.5	4.3	5.1	5.9	6.9	19.0
20.0	0.0	0.1	0.3	0.6	0.9	1.3	1.8	2.4	3.0	3.7	4.5	5.3	6.3	7.3	20.0
21.0	0.0	0.2	0.4	0.6	1.0	1.4	1.9	2.5	3.2	3.9	4.7	5.6	6.6	7.6	21.0
22.0	0.0	0.2	0.4	0.7	1.0	1.5	2.0	2.6	3.3	4.1	4.9	5.9	6.9	8.0	22.0
23.0	0.0	0.2	0.4	0.7	1.1	1.5	2.1	2.7	3.4	4.3	5.2	6.1	7.2	8.3	23.0
24.0	0.0	0.2	0.4	0.7	1.1	1.6	2.2	2.8	3.6	4.4	5.4	6.4	7.5	8.7	24.0
25.0	0.0	0.2	0.4	0.7	1.2	1.7	2.3	3.0	3.8	4.6	5.6	6.7	7.8	9.1	25.0
26.0	0.0	0.2	0.4	0.8	1.2	1.7	2.4	3.1	3.9	4.8	5.8	6.9	8.1	9.4	26.0
27.0	0.0	0.2	0.4	0.8	1.2	1.8	2.4	3.2	4.0	5.0	6.0	7.2	8.4	9.8	27.0
28.0	0.1	0.2	0.5	0.8	1.3	1.9	2.5	3.3	4.2	5.2	6.3	7.5	8.8	10.2	28.0

TABLE 30

Change of Altitude in Given Time from Meridian Transit

a (table 29)	t, meridian angle														a (table 29)
	1°15′	1°20′	1°25′	1°30′	1°35′	1°40′	1°45′	1°50′	1°55′	2°00′	2°05′	2°10′	2°15′	2°20′	
	$5^m 00^s$	$5^m 20^s$	$5^m 40^s$	$6^m 00^s$	$6^m 20^s$	$6^m 40^s$	$7^m 00^s$	$7^m 20^s$	$7^m 40^s$	$8^m 00^s$	$8^m 20^s$	$8^m 40^s$	$9^m 00^s$	$9^m 20^s$	
″	′	′	′	′	′	′	′	′	′	′	′	′	′	′	″
0. 1	0. 0	0. 0	0. 1	0. 1	0. 1	0. 1	0. 1	0. 1	0. 1	0. 1	0. 1	0. 1	0. 1	0. 1	0. 1
0. 2	0. 1	0. 1	0. 1	0. 1	0. 1	0. 1	0. 2	0. 2	0. 2	0. 2	0. 2	0. 3	0. 3	0. 3	0. 2
0. 3	0. 1	0. 1	0. 2	0. 2	0. 2	0. 2	0. 2	0. 3	0. 3	0. 3	0. 3	0. 4	0. 4	0. 4	0. 3
0. 4	0. 2	0. 2	0. 2	0. 2	0. 3	0. 3	0. 3	0. 4	0. 4	0. 4	0. 5	0. 5	0. 5	0. 6	0. 4
0. 5	0. 2	0. 2	0. 3	0. 3	0. 3	0. 4	0. 4	0. 5	0. 5	0. 5	0. 6	0. 6	0. 7	0. 7	0. 5
0. 6	0. 2	0. 3	0. 3	0. 4	0. 4	0. 4	0. 5	0. 5	0. 6	0. 6	0. 7	0. 8	0. 8	0. 9	0. 6
0. 7	0. 3	0. 3	0. 4	0. 4	0. 5	0. 5	0. 6	0. 6	0. 7	0. 7	0. 8	0. 9	0. 9	1. 0	0. 7
0. 8	0. 3	0. 4	0. 4	0. 5	0. 5	0. 6	0. 7	0. 7	0. 8	0. 9	0. 9	1. 0	1. 1	1. 2	0. 8
0. 9	0. 4	0. 4	0. 5	0. 5	0. 6	0. 7	0. 7	0. 8	0. 9	1. 0	1. 0	1. 1	1. 2	1. 3	0. 9
1. 0	0. 4	0. 5	0. 5	0. 6	0. 7	0. 7	0. 8	0. 9	1. 0	1. 1	1. 2	1. 3	1. 4	1. 5	1. 0
2. 0	0. 8	0. 9	1. 1	1. 2	1. 3	1. 5	1. 6	1. 8	2. 0	2. 1	2. 3	2. 5	2. 7	2. 9	2. 0
3. 0	1. 2	1. 4	1. 6	1. 8	2. 0	2. 2	2. 4	2. 7	2. 9	3. 2	3. 5	3. 8	4. 0	4. 4	3. 0
4. 0	1. 7	1. 9	2. 1	2. 4	2. 7	3. 0	3. 3	3. 6	3. 9	4. 3	4. 6	5. 0	5. 4	5. 8	4. 0
5. 0	2. 1	2. 4	2. 7	3. 0	3. 3	3. 7	4. 1	4. 5	4. 9	5. 3	5. 8	6. 3	6. 8	7. 3	5. 0
6. 0	2. 5	2. 8	3. 2	3. 6	4. 0	4. 4	4. 9	5. 4	5. 9	6. 4	6. 9	7. 5	8. 1	8. 7	6. 0
7. 0	2. 9	3. 3	3. 7	4. 2	4. 7	5. 2	5. 7	6. 3	6. 9	7. 5	8. 1	8. 8	9. 4	10. 2	7. 0
8. 0	3. 3	3. 8	4. 3	4. 8	5. 3	5. 9	6. 5	7. 2	7. 8	8. 5	9. 3	10. 0	10. 8	11. 6	8. 0
9. 0	3. 8	4. 3	4. 8	5. 4	6. 0	6. 7	7. 4	8. 1	8. 8	9. 6	10. 4	11. 3	12. 2	13. 1	9. 0
10. 0	4. 2	4. 7	5. 4	6. 0	6. 7	7. 4	8. 2	9. 0	9. 8	10. 7	11. 6	12. 5	13. 5	14. 5	10. 0
11. 0	4. 6	5. 2	5. 9	6. 6	7. 4	8. 1	9. 0	9. 9	10. 8	11. 7	12. 7	13. 8	14. 8	16. 0	11. 0
12. 0	5. 0	5. 7	6. 4	7. 2	8. 0	8. 9	9. 8	10. 8	11. 8	12. 8	13. 9	15. 0	16. 2	17. 4	12. 0
13. 0	5. 4	6. 2	7. 0	7. 8	8. 7	9. 6	10. 6	11. 7	12. 7	13. 9	15. 0	16. 3	17. 6	18. 9	13. 0
14. 0	5. 8	6. 6	7. 5	8. 4	9. 4	10. 4	11. 4	12. 5	13. 7	14. 9	16. 2	17. 5	18. 9	20. 3	14. 0
15. 0	6. 2	7. 1	8. 0	9. 0	10. 0	11. 1	12. 2	13. 4	14. 7	16. 0	17. 4	18. 8	20. 2	21. 8	15. 0
16. 0	6. 7	7. 6	8. 6	9. 6	10. 7	11. 9	13. 1	14. 3	15. 7	17. 1	18. 5	20. 0	21. 6	23. 2	16. 0
17. 0	7. 1	8. 1	9. 1	10. 2	11. 4	12. 6	13. 9	15. 2	16. 7	18. 1	19. 7	21. 3	23. 0	24. 7	17. 0
18. 0	7. 5	8. 5	9. 6	10. 8	12. 0	13. 3	14. 7	16. 1	17. 6	19. 2	20. 8	22. 5	24. 3	26. 1	18. 0
19. 0	7. 9	9. 0	10. 2	11. 4	12. 7	14. 1	15. 5	17. 0	18. 6	20. 3	22. 0	23. 8			19. 0
20. 0	8. 3	9. 5	10. 7	12. 0	13. 4	14. 8	16. 3	17. 9	19. 6	21. 3	23. 1				20. 0
21. 0	8. 8	10. 0	11. 2	12. 6	14. 0	15. 6	17. 2	18. 8	20. 6						21. 0
22. 0	9. 2	10. 4	11. 8	13. 2	14. 7	16. 3	18. 0	19. 7	21. 6						22. 0
23. 0	9. 6	10. 9	12. 3	13. 8	15. 4	17. 0	18. 8	20. 6							23. 0
24. 0	10. 0	11. 4	12. 8	14. 4	16. 0	17. 8	19. 6	21. 5							24. 0
25. 0	10. 4	11. 9	13. 4	15. 0	16. 7	18. 5	20. 4								25. 0
26. 0	10. 8	12. 3	13. 9	15. 6	17. 4	19. 3									26. 0
27. 0	11. 2	12. 8	14. 4	16. 2	18. 0	20. 0									27. 0

TABLE 30

Change of Altitude in Given Time from Meridian Transit

a (table 29)	t, meridian angle 2°25′	2°30′	2°35′	2°40′	2°45′	2°50′	2°55′	3°00′	3°05′	3°10′	3°15′	3°20′	3°25′	3°30′	a (table 29)
	$9^m 40^s$	$10^m 00^s$	$10^m 20^s$	$10^m 40^s$	$11^m 00^s$	$11^m 20^s$	$11^m 40^s$	$12^m 00^s$	$12^m 20^s$	$12^m 40^s$	$13^m 00^s$	$13^m 20^s$	$13^m 40^s$	$14^m 00^s$	
″	′	′	′	′	′	′	′	′	′	′	′	′	′	′	″
0. 1	0. 2	0. 2	0. 2	0. 2	0. 2	0. 2	0. 2	0. 2	0. 3	0. 3	0. 3	0. 3	0. 3	0. 3	0. 1
0. 2	0. 3	0. 3	0. 4	0. 4	0. 4	0. 4	0. 5	0. 5	0. 5	0. 5	0. 6	0. 6	0. 6	0. 7	0. 2
0. 3	0. 5	0. 5	0. 5	0. 6	0. 6	0. 6	0. 7	0. 7	0. 8	0. 8	0. 8	0. 9	0. 9	1. 0	0. 3
0. 4	0. 6	0. 7	0. 7	0. 8	0. 8	0. 9	0. 9	1. 0	1. 0	1. 1	1. 1	1. 2	1. 2	1. 3	0. 4
0. 5	0. 8	0. 8	0. 9	0. 9	1. 0	1. 1	1. 1	1. 2	1. 3	1. 3	1. 4	1. 5	1. 6	1. 6	0. 5
0. 6	0. 9	1. 0	1. 1	1. 1	1. 2	1. 3	1. 4	1. 4	1. 5	1. 6	1. 7	1. 8	1. 9	2. 0	0. 6
0. 7	1. 1	1. 2	1. 2	1. 3	1. 4	1. 5	1. 6	1. 7	1. 8	1. 9	2. 0	2. 1	2. 2	2. 3	0. 7
0. 8	1. 2	1. 3	1. 4	1. 5	1. 6	1. 7	1. 8	1. 9	2. 0	2. 1	2. 3	2. 4	2. 5	2. 6	0. 8
0. 9	1. 4	1. 5	1. 6	1. 7	1. 8	1. 9	2. 0	2. 2	2. 3	2. 4	2. 5	2. 7	2. 8	2. 9	0. 9
1. 0	1. 6	1. 7	1. 8	1. 9	2. 0	2. 1	2. 3	2. 4	2. 5	2. 7	2. 8	3. 0	3. 1	3. 3	1. 0
2. 0	3. 1	3. 3	3. 6	3. 8	4. 0	4. 3	4. 5	4. 8	5. 1	5. 3	5. 6	5. 9	6. 2	6. 5	2. 0
3. 0	4. 7	5. 0	5. 3	5. 7	6. 0	6. 4	6. 8	7. 2	7. 6	8. 0	8. 4	8. 9	9. 3	9. 8	3. 0
4. 0	6. 2	6. 7	7. 1	7. 6	8. 1	8. 6	9. 1	9. 6	10. 1	10. 7	11. 3	11. 9	12. 5	13. 1	4. 0
5. 0	7. 8	8. 3	8. 9	9. 5	10. 1	10. 7	11. 3	12. 0	12. 7	13. 4	14. 1	14. 8	15. 6	16. 3	5. 0
6. 0	9. 3	10. 0	10. 7	11. 4	12. 1	12. 8	13. 6	14. 4	15. 2	16. 0	16. 9	17. 8	18. 7	19. 6	6. 0
7. 0	10. 9	11. 7	12. 5	13. 3	14. 1	15. 0	15. 9	16. 8	17. 7	18. 7	19. 7	20. 7	21. 8	22. 9	7. 0
8. 0	12. 5	13. 3	14. 2	15. 2	16. 1	17. 1	18. 1	19. 2	20. 3	21. 4	22. 5	23. 7	24. 9	26. 1	8. 0
9. 0	14. 0	15. 0	16. 0	17. 1	18. 2	19. 3	20. 4	21. 6	22. 8	24. 1	25. 4	26. 7	28. 0	29. 4	9. 0
10. 0	15. 6	16. 7	17. 8	19. 0	20. 2	21. 4	22. 7	24. 0	25. 4	26. 7	28. 2	29. 6			10. 0
11. 0	17. 1	18. 3	19. 6	20. 9	22. 2	23. 5	25. 0	26. 4	27. 9	29. 4					11. 0
12. 0	18. 7	20. 0	21. 4	22. 8	24. 2	25. 7	27. 2	28. 8							12. 0
13. 0	20. 2	21. 7	23. 1	24. 7	26. 2	27. 8	29. 5								13. 0
14. 0	21. 8	23. 3	24. 9	26. 5	28. 2	30. 0									14. 0
15. 0	23. 4	25. 0	26. 7	28. 4	30. 2										15. 0
16. 0	24. 9	26. 7	28. 5	30. 3											16. 0
17. 0	26. 5	28. 3	30. 3												17. 0

a (table 29)	t, meridian angle 3°35′	3°40′	3°45′	3°50′	3°55′	4°00′	4°05′	4°10′	4°15′	4°20′	4°25′	4°30′	4°35′	4°40′	a (table 29)
	$14^m 20^s$	$14^m 40^s$	$15^m 00^s$	$15^m 20^s$	$15^m 40^s$	$16^m 00^s$	$16^m 20^s$	$16^m 40^s$	$17^m 00^s$	$17^m 20^s$	$17^m 40^s$	$18^m 00^s$	$18^m 20^s$	$18^m 40^s$	
″	′	′	′	′	′	′	′	′	′	′	′	′	′	′	″
0. 1	0. 3	0. 4	0. 4	0. 4	0. 4	0. 4	0. 4	0. 5	0. 5	0. 5	0. 5	0. 5	0. 6	0. 6	0. 1
0. 2	0. 7	0. 7	0. 8	0. 8	0. 8	0. 9	0. 9	0. 9	1. 0	1. 0	1. 0	1. 1	1. 1	1. 2	0. 2
0. 3	1. 0	1. 1	1. 1	1. 2	1. 2	1. 3	1. 3	1. 4	1. 4	1. 5	1. 6	1. 6	1. 7	1. 7	0. 3
0. 4	1. 4	1. 4	1. 5	1. 6	1. 6	1. 7	1. 8	1. 9	1. 9	2. 0	2. 1	2. 2	2. 2	2. 3	0. 4
0. 5	1. 7	1. 8	1. 9	2. 0	2. 0	2. 1	2. 2	2. 3	2. 4	2. 5	2. 6	2. 7	2. 8	2. 9	0. 5
0. 6	2. 1	2. 2	2. 2	2. 4	2. 5	2. 6	2. 7	2. 8	2. 9	3. 0	3. 1	3. 2	3. 4	3. 5	0. 6
0. 7	2. 4	2. 5	2. 6	2. 7	2. 9	3. 0	3. 1	3. 2	3. 4	3. 5	3. 6	3. 8	3. 9	4. 1	0. 7
0. 8	2. 7	2. 9	3. 0	3. 1	3. 3	3. 4	3. 6	3. 7	3. 9	4. 0	4. 2	4. 3	4. 5	4. 6	0. 8
0. 9	3. 1	3. 2	3. 4	3. 5	3. 7	3. 8	4. 0	4. 2	4. 3	4. 5	4. 7	4. 9	5. 0	5. 2	0. 9
1. 0	3. 4	3. 6	3. 8	3. 9	4. 1	4. 3	4. 4	4. 6	4. 8	5. 0	5. 2	5. 4	5. 6	5. 8	1. 0
2. 0	6. 8	7. 2	7. 5	7. 8	8. 2	8. 5	8. 9	9. 3	9. 6	10. 0	10. 4	10. 8	11. 2	11. 6	2. 0
3. 0	10. 3	10. 8	11. 2	11. 8	12. 3	12. 8	13. 3	13. 9	14. 4	15. 0	15. 6	16. 2	16. 8	17. 4	3. 0
4. 0	13. 7	14. 3	15. 0	15. 7	16. 4	17. 1	17. 8	18. 5	19. 3	20. 0	20. 8	21. 6	22. 4	23. 2	4. 0
5. 0	17. 1	17. 9	18. 8	19. 6	20. 5	21. 3	22. 2	23. 1	24. 1	25. 0	26. 0	27. 0	28. 0	29. 0	5. 0
6. 0	20. 5	21. 5	22. 5	23. 5	24. 5	25. 6	26. 7	27. 8							6. 0
7. 0	24. 0	25. 1	26. 2	27. 4											7. 0
8. 0	27. 4	28. 7	30. 0												8. 0

TABLE 30

Change of Altitude in Given Time from Meridian Transit

a (table 29)	t, meridian angle														a (table 29)
	4°45′	4°50′	4°55′	5°00′	5°05′	5°10′	5°15′	5°20′	5°25′	5°30′	5°35′	5°40′	5°45′	5°50′	
	19^m00^s	19^m20^s	19^m40^s	20^m00^s	20^m20^s	20^m40^s	21^m00^s	21^m20^s	21^m40^s	22^m00^s	22^m20^s	22^m40^s	23^m00^s	23^m20^s	
″	′	′	′	′	′	′	′	′	′	′	′	′	′	′	″
0.1	0.6	0.6	0.6	0.7	0.7	0.7	0.7	0.8	0.8	0.8	0.8	0.9	0.9	0.9	0.1
0.2	1.2	1.2	1.3	1.3	1.4	1.4	1.5	1.5	1.6	1.6	1.7	1.7	1.8	1.8	0.2
0.3	1.8	1.9	1.9	2.0	2.1	2.1	2.2	2.3	2.3	2.4	2.5	2.6	2.6	2.7	0.3
0.4	2.4	2.5	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	0.4
0.5	3.0	3.1	3.2	3.3	3.4	3.6	3.7	3.8	3.9	4.0	4.2	4.3	4.4	4.5	0.5
0.6	3.6	3.7	3.9	4.0	4.1	4.3	4.4	4.6	4.7	4.8	5.0	5.1	5.3	5.4	0.6
0.7	4.2	4.4	4.5	4.7	4.8	5.0	5.1	5.3	5.5	5.6	5.8	6.0	6.2	6.4	0.7
0.8	4.8	5.0	5.2	5.3	5.5	5.7	5.9	6.1	6.3	6.5	6.7	6.9	7.1	7.3	0.8
0.9	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0	7.3	7.5	7.7	7.9	8.2	0.9
1.0	6.0	6.2	6.4	6.7	6.9	7.1	7.4	7.6	7.8	8.1	8.3	8.6	8.8	9.1	1.0
2.0	12.0	12.5	12.9	13.3	13.8	14.2	14.7	15.2	15.6	16.1	16.6	17.1	17.6	18.1	2.0
3.0	18.0	18.7	19.3	20.0	20.7	21.4	22.0	22.8	23.5	24.2	24.9	25.7	26.4	27.2	3.0
4.0	24.1	24.9	25.8	26.7	27.6	28.5	29.4	30.3	31.3						4.0

a (table 29)	t, meridian angle														a (table 29)
	5°55′	6°00′	6°05′	6°10′	6°15′	6°20′	6°25′	6°30′	6°35′	6°40′	6°45′	6°50′	6°55′	7°00′	
	23^m40^s	24^m00^s	24^m20^s	24^m40^s	25^m00^s	25^m20^s	25^m40^s	26^m00^s	26^m20^s	26^m40^s	27^m00^s	27^m20^s	27^m40^s	28^m00^s	
″	′	′	′	′	′	′	′	′	′	′	′	′	′	′	″
0.1	0.9	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.3	1.3	0.1
0.2	1.9	1.9	2.0	2.0	2.1	2.1	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.6	0.2
0.3	2.8	2.9	3.0	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.6	3.7	3.8	3.9	0.3
0.4	3.7	3.8	3.9	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.9	5.0	5.1	5.2	0.4
0.5	4.7	4.8	4.9	5.1	5.2	5.3	5.5	5.6	5.8	5.9	6.1	6.2	6.4	6.5	0.5
0.6	5.6	5.8	5.9	6.1	6.2	6.4	6.6	6.8	6.9	7.1	7.3	7.5	7.7	7.8	0.6
0.7	6.5	6.7	6.9	7.1	7.3	7.5	7.7	7.9	8.1	8.3	8.5	8.7	8.9	9.1	0.7
0.8	7.5	7.7	7.9	8.1	8.3	8.6	8.8	9.0	9.2	9.5	9.7	10.0	10.2	10.5	0.8
0.9	8.4	8.6	8.9	9.1	9.4	9.6	9.9	10.1	10.4	10.7	10.9	11.2	11.5	11.8	0.9
1.0	9.3	9.6	9.9	10.1	10.4	10.7	11.0	11.3	11.6	11.9	12.2	12.5	12.8	13.1	1.0
2.0	18.7	19.2	19.7	20.3	20.8	21.4	22.0	22.5	23.1	23.7	24.3	24.9	25.5	26.1	2.0
3.0	28.0	28.8	29.6	30.4											3.0

TABLE 31

Natural Trigonometric Functions

0°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←179° ↓ ′
0	0. 00000	29	∞	—	0. 00000	29	∞	—	1. 00000	0	1. 00000	0	60
1	. 00029	29	3437. 75	1718. 88	. 00029	29	3437. 75	1718. 88	. 00000	0	. 00000	0	59
2	. 00058	29	1718. 87	572. 95	. 00058	29	1718. 87	572. 95	. 00000	0	. 00000	0	58
3	. 00087	29	1145. 92	286. 483	. 00087	29	1145. 92	286. 484	. 00000	0	. 00000	0	57
4	. 00116	29	859. 437	171. 887	. 00116	29	859. 436	171. 887	. 00000	0	. 00000	0	56
5	0. 00145	30	687. 550	114. 592	0. 00145	30	687. 549	114. 592	1. 00000	0	1. 00000	0	55
6	. 00175	29	572. 958	81. 851	. 00175	29	572. 957	81. 851	. 00000	0	. 00000	0	54
7	. 00204	29	491. 107	61. 388	. 00204	29	491. 106	61. 388	. 00000	0	. 00000	0	53
8	. 00233	29	429. 719	47. 747	. 00233	29	429. 718	47. 747	. 00000	0	. 00000	0	52
9	. 00262	29	381. 972	38. 197	. 00262	29	381. 971	38. 197	. 00000	0	. 00000	0	51
10	0. 00291	29	343. 775	31. 252	0. 00291	29	343. 774	31. 253	1. 00000	1	1. 00000	1	50
11	. 00320	29	312. 523	26. 044	. 00320	29	312. 521	26. 043	. 00001	0	0. 99999	0	49
12	. 00349	29	286. 479	22. 036	. 00349	29	286. 478	22. 037	. 00001	0	. 99999	0	48
13	. 00378	29	264. 443	18. 889	. 00378	29	264. 441	18. 889	. 00001	0	. 99999	0	47
14	. 00407	29	245. 554	16. 370	. 00407	29	245. 552	16. 370	. 00001	0	. 99999	0	46
15	0. 00436	29	229. 184	14. 324	0. 00436	29	229. 182	14. 324	1. 00001	0	0. 99999	0	45
16	. 00465	30	214. 860	12. 639	. 00465	30	214. 858	12. 639	. 00001	0	. 99999	0	44
17	. 00495	29	202. 221	11. 234	. 00495	29	202. 219	11. 235	. 00001	0	. 99999	0	43
18	. 00524	29	190. 987	10. 052	. 00524	29	190. 984	10. 052	. 00001	1	. 99999	1	42
19	. 00553	29	180. 935	9. 047	. 00553	29	180. 932	9. 047	. 00002	0	. 99998	0	41
20	0. 00582	29	171. 888	8. 185	0. 00582	29	171. 885	8. 185	1. 00002	0	0. 99998	0	40
21	. 00611	29	163. 703	7. 441	. 00611	29	163. 700	7. 441	. 00002	0	. 99998	0	39
22	. 00640	29	156. 262	6. 794	. 00640	29	156. 259	6. 794	. 00002	0	. 99998	0	38
23	. 00669	29	149. 468	6. 227	. 00669	29	149. 465	6. 228	. 00002	0	. 99998	0	37
24	. 00698	29	143. 241	5. 730	. 00698	29	143. 237	5. 730	. 00002	1	. 99998	1	36
25	0. 00727	29	137. 511	5. 289	0. 00727	29	137. 507	5. 288	1. 00003	0	0. 99997	0	35
26	. 00756	29	132. 222	4. 897	. 00756	29	132. 219	4. 898	. 00003	0	. 99997	0	34
27	. 00785	29	127. 325	4. 547	. 00785	30	127. 321	4. 547	. 00003	0	. 99997	0	33
28	. 00814	30	122. 778	4. 234	. 00815	29	122. 774	4. 234	. 00003	1	. 99997	1	32
29	. 00844	29	118. 544	3. 951	. 00844	29	118. 540	3. 951	. 00004	0	. 99996	0	31
30	0. 00873	29	114. 593	3. 696	0. 00873	29	114. 589	3. 697	1. 00004	0	0. 99996	0	30
31	. 00902	29	110. 897	3. 466	. 00902	29	110. 892	3. 466	. 00004	0	. 99996	0	29
32	. 00931	29	107. 431	3. 255	. 00931	29	107. 426	3. 255	. 00004	1	. 99996	1	28
33	. 00960	29	104. 176	3. 064	. 00960	29	104. 171	3. 064	. 00005	0	. 99995	0	27
34	. 00989	29	101. 112	2. 8890	. 00989	29	101. 107	2. 8891	. 00005	0	. 99995	0	26
35	0. 01018	29	98. 2230	2. 7283	0. 01018	29	98. 2179	2. 7284	1. 00005	0	0. 99995	0	25
36	. 01047	29	95. 4947	2. 5808	. 01047	29	95. 4895	2. 5810	. 00005	1	. 99995	1	24
37	. 01076	29	92. 9139	2. 4450	. 01076	29	92. 9085	2. 4452	. 00006	0	. 99994	0	23
38	. 01105	29	90. 4689	2. 3197	. 01105	30	90. 4633	2. 3197	. 00006	0	. 99994	0	22
39	. 01134	30	88. 1492	2. 2036	. 01135	29	88. 1436	2. 2038	. 00006	1	. 99994	1	21
40	0. 01164	29	85. 9456	2. 0961	0. 01164	29	85. 9398	2. 0963	1. 00007	0	0. 99993	0	20
41	. 01193	29	83. 8495	1. 9963	. 01193	29	83. 8435	1. 9965	. 00007	0	. 99993	0	19
42	. 01222	29	81. 8532	1. 9035	. 01222	29	81. 8470	1. 9036	. 00007	1	. 99993	1	18
43	. 01251	29	79. 9497	1. 8170	. 01251	29	79. 9434	1. 8171	. 00008	0	. 99992	0	17
44	. 01280	29	78. 1327	1. 7361	. 01280	29	78. 1263	1. 7363	. 00008	1	. 99992	1	16
45	0. 01309	29	76. 3966	1. 6607	0. 01309	29	76. 3900	1. 6608	1. 00009	0	0. 99991	0	15
46	. 01338	29	74. 7359	1. 5901	. 01338	29	74. 7292	1. 5902	. 00009	0	. 99991	0	14
47	. 01367	29	73. 1458	1. 5237	. 01367	29	73. 1390	1. 5239	. 00009	1	. 99991	1	13
48	. 01396	29	71. 6221	1. 4616	. 01396	29	71. 6151	1. 4618	. 00010	0	. 99990	0	12
49	. 01425	29	70. 1605	1. 4031	. 01425	30	70. 1533	1. 4032	. 00010	1	. 99990	1	11
50	0. 01454	29	68. 7574	1. 3481	0. 01455	29	68. 7501	1. 3482	1. 00011	0	0. 99989	0	10
51	. 01483	30	67. 4093	1. 2963	. 01484	29	67. 4019	1. 2964	. 00011	0	. 99989	0	9
52	. 01513	29	66. 1130	1. 2473	. 01513	29	66. 1055	1. 2475	. 00011	1	. 99989	1	8
53	. 01542	29	64. 8657	1. 2011	. 01542	29	64. 8580	1. 2013	. 00012	0	. 99988	0	7
54	. 01571	29	63. 6646	1. 1574	. 01571	29	63. 6567	1. 1575	. 00012	1	. 99988	1	6
55	0. 01600	29	62. 5072	1. 1161	0. 01600	29	62. 4992	1. 1163	1. 00013	0	0. 99987	0	5
56	. 01629	29	61. 3911	1. 0770	. 01629	29	61. 3829	1. 0771	. 00013	1	. 99987	1	4
57	. 01658	29	60. 3141	1. 0398	. 01658	29	60. 3058	1. 0399	. 00014	0	. 99986	0	3
58	. 01687	29	59. 2743	1. 0045	. 01687	29	59. 2659	1. 0047	. 00014	1	. 99986	1	2
59	. 01716	29	58. 2698	0. 9711	. 01716	30	58. 2612	0. 9712	. 00015	0	. 99985	0	1
60	0. 01745		57. 2987		0. 01746		57. 2900		1. 00015		0. 99985		0
↑ 90°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←89° ↑

TABLE 31

Natural Trigonometric Functions

1°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←178° ↓ ′
0	0. 01745	29	57. 2987	9392	0. 01746	29	57. 2900	9394	1. 00015	1	0. 99985	1	60
1	. 01774	29	56. 3595	9090	. 01775	29	56. 3506	9091	. 00016	0	. 99984	0	59
2	. 01803	29	55. 4505	8800	. 01804	29	55. 4415	8802	. 00016	1	. 99984	1	58
3	. 01832	30	54. 5705	8526	. 01833	29	54. 5613	8527	. 00017	0	. 99983	0	57
4	. 01862	29	53. 7179	8263	. 01862	29	53. 7086	8265	. 00017	1	. 99983	1	56
5	0. 01891	29	52. 8916	8013	0. 01891	29	52. 8821	8014	1. 00018	0	0. 99982	0	55
6	. 01920	29	52. 0903	7774	. 01920	29	52. 0807	7775	. 00018	1	. 99982	1	54
7	. 01949	29	51. 3129	7545	. 01949	29	51. 3032	7547	. 00019	1	. 99981	1	53
8	. 01978	29	50. 5584	7326	. 01978	29	50. 5485	7328	. 00020	0	. 99980	0	52
9	. 02007	29	49. 8258	7117	. 02007	29	49. 8157	7118	. 00020	1	. 99980	1	51
10	0. 02036	29	49. 1141	6917	0. 02036	30	49. 1039	6918	1. 00021	0	0. 99979	0	50
11	. 02065	29	48. 4224	6724	. 02066	29	48. 4121	6726	. 00021	1	. 99979	1	49
12	. 02094	29	47. 7500	6540	. 02095	29	47. 7395	6542	. 00022	1	. 99978	1	48
13	. 02123	29	47. 0960	6364	. 02124	29	47. 0853	6364	. 00023	0	. 99977	0	47
14	. 02152	29	46. 4596	6193	. 02153	29	46. 4489	6195	. 00023	1	. 99977	1	46
15	0. 02181	30	45. 8403	6031	0. 02182	29	45. 8294	6033	1. 00024	0	0. 99976	0	45
16	. 02211	29	45. 2372	5874	. 02211	29	45. 2261	5875	. 00024	1	. 99976	1	44
17	. 02240	29	44. 6498	5723	. 02240	29	44. 6386	5725	. 00025	1	. 99975	1	43
18	. 02269	29	44. 0775	5579	. 02269	29	44. 0661	5580	. 00026	0	. 99974	0	42
19	. 02298	29	43. 5196	5439	. 02298	30	43. 5081	5440	. 00026	1	. 99974	1	41
20	0. 02327	29	42. 9757	5305	0. 02328	29	42. 9641	5306	1. 00027	1	0. 99973	1	40
21	. 02356	29	42. 4452	5175	. 02357	29	42. 4335	5177	. 00028	0	. 99972	0	39
22	. 02385	29	41. 9277	5050	. 02386	29	41. 9158	5052	. 00028	1	. 99972	1	38
23	. 02414	29	41. 4227	4931	. 02415	29	41. 4106	4932	. 00029	1	. 99971	1	37
24	. 02443	29	40. 9296	4814	. 02444	29	40. 9174	4816	. 00030	1	. 99970	1	36
25	0. 02472	29	40. 4482	4702	0. 02473	29	40. 4358	4703	1. 00031	0	0. 99969	0	35
26	. 02501	29	39. 9780	4595	. 02502	29	39. 9655	4596	. 00031	1	. 99969	1	34
27	. 02530	30	39. 5185	4489	. 02531	29	39. 5059	4491	. 00032	1	. 99968	1	33
28	. 02560	29	39. 0696	4389	. 02560	29	39. 0568	4391	. 00033	1	. 99967	1	32
29	. 02589	29	38. 6307	4291	. 02589	30	38. 6177	4292	. 00034	0	. 99966	0	31
30	0. 02618	29	38. 2016	4198	0. 02619	29	38. 1885	4199	1. 00034	1	0. 99966	1	30
31	. 02647	29	37. 7818	4105	. 02648	29	37. 7686	4107	. 00035	1	. 99965	1	29
32	. 02676	29	37. 3713	4018	. 02677	29	37. 3579	4019	. 00036	1	. 99964	1	28
33	. 02705	29	36. 9695	3932	. 02706	29	36. 9560	3933	. 00037	0	. 99963	0	27
34	. 02734	29	36. 5763	3849	. 02735	29	36. 5627	3851	. 00037	1	. 99963	1	26
35	0. 02763	29	36. 1914	3769	0. 02764	29	36. 1776	3770	1. 00038	1	0. 99962	1	25
36	. 02792	29	35. 8145	3691	. 02793	29	35. 8006	3693	. 00039	1	. 99961	1	24
37	. 02821	29	35. 4454	3616	. 02822	29	35. 4313	3618	. 00040	1	. 99960	1	23
38	. 02850	29	35. 0838	3543	. 02851	30	35. 0695	3544	. 00041	0	. 99959	0	22
39	. 02879	29	34. 7295	3472	. 02881	29	34. 7151	3473	. 00041	1	. 99959	1	21
40	0. 02908	30	34. 3823	3403	0. 02910	29	34. 3678	3405	1. 00042	1	0. 99958	1	20
41	. 02938	29	34. 0420	3337	. 02939	29	34. 0273	3338	. 00043	1	. 99957	1	19
42	. 02967	29	33. 7083	3271	. 02968	29	33. 6935	3273	. 00044	1	. 99956	1	18
43	. 02996	29	33. 3812	3209	. 02997	29	33. 3662	3210	. 00045	1	. 99955	1	17
44	. 03025	29	33. 0603	3148	. 03026	29	33. 0452	3149	. 00046	1	. 99954	1	16
45	0. 03054	29	32. 7455	3088	0. 03055	29	32. 7303	3090	1. 00047	1	0. 99953	1	15
46	. 03083	29	32. 4367	3030	. 03084	30	32. 4213	3032	. 00048	0	. 99952	0	14
47	. 03112	29	32. 1337	2975	. 03114	29	32. 1181	2976	. 00048	1	. 99952	1	13
48	. 03141	29	31. 8362	2920	. 03143	29	31. 8205	2921	. 00049	1	. 99951	1	12
49	. 03170	29	31. 5442	2866	. 03172	29	31. 5284	2868	. 00050	1	. 99950	1	11
50	0. 03199	29	31. 2576	2815	0. 03201	29	31. 2416	2817	1. 00051	1	0. 99949	1	10
51	. 03228	29	30. 9761	2765	. 03230	29	30. 9599	2766	. 00052	1	. 99948	1	9
52	. 03257	29	30. 6996	2716	. 03259	29	30. 6833	2717	. 00053	1	. 99947	1	8
53	. 03286	30	30. 4280	2668	. 03288	29	30. 4116	2670	. 00054	1	. 99946	1	7
54	. 03316	29	30. 1612	2622	. 03317	29	30. 1446	2623	. 00055	1	. 99945	1	6
55	0. 03345	29	29. 8990	2576	0. 03346	30	29. 8823	2578	1. 00056	1	0. 99944	1	5
56	. 03374	29	29. 6414	2533	. 03376	29	29. 6245	2534	. 00057	1	. 99943	1	4
57	. 03403	29	29. 3881	2489	. 03405	29	29. 3711	2491	. 00058	1	. 99942	1	3
58	. 03432	29	29. 1392	2448	. 03434	29	29. 1220	2449	. 00059	1	. 99941	1	2
59	. 03461	29	28. 8944	2407	. 03463	29	28. 8771	2408	. 00060	1	. 99940	1	1
60	0. 03490		28. 6537		0. 03492		28. 6363		1. 00061		0. 99939		0
↑ 91°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←88° ↑

TABLE 31

Natural Trigonometric Functions

2° → ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	← 177° ↓ ′
0	0. 03490	29	28. 6537	2367	0. 03492	29	28. 6363	2369	1. 00061	1	0. 99939	1	60
1	. 03519	29	. 4170	2328	. 03521	29	. 3994	2330	. 00062	1	. 99938	1	59
2	. 03548	29	28. 1842	2291	. 03550	29	28. 1664	2292	. 00063	1	. 99937	1	58
3	. 03577	29	27. 9551	2253	. 03579	30	27. 9372	2255	. 00064	1	. 99936	1	57
4	. 03606	29	. 7298	2218	. 03609	29	. 7117	2218	. 00065	1	. 99935	1	56
5	0. 03635	29	27. 5080	2182	0. 03638	29	27. 4899	2184	1. 00066	1	0. 99934	1	55
6	. 03664	29	. 2898	2148	. 03667	29	. 2715	2149	. 00067	1	. 99933	1	54
7	. 03693	30	27. 0750	2114	. 03696	29	27. 0566	2116	. 00068	1	. 99932	1	53
8	. 03723	29	26. 8636	2081	. 03725	29	26. 8450	2083	. 00069	1	. 99931	1	52
9	. 03752	29	. 6555	2050	. 03754	29	. 6367	2051	. 00070	2	. 99930	1	51
10	0. 03781	29	26. 4505	2018	0. 03783	29	26. 4316	2020	1. 00072	1	0. 99929	2	50
11	. 03810	29	. 2487	1988	. 03812	30	. 2296	1989	. 00073	1	. 99927	1	49
12	. 03839	29	26. 0499	1957	. 03842	29	26. 0307	1959	. 00074	1	. 99926	1	48
13	. 03868	29	25. 8542	1929	. 03871	29	25. 8348	1930	. 00075	1	. 99925	1	47
14	. 03897	29	. 6613	1900	. 03900	29	. 6418	1901	. 00076	1	. 99924	1	46
15	0. 03926	29	25. 4713	1872	0. 03929	29	25. 4517	1873	1. 00077	1	0. 99923	1	45
16	. 03955	29	. 2841	1844	. 03958	29	. 2644	1846	. 00078	1	. 99922	1	44
17	. 03984	29	25. 0997	1818	. 03987	29	25. 0798	1820	. 00079	2	. 99921	2	43
18	. 04013	29	24. 9179	1792	. 04016	30	24. 8978	1793	. 00081	1	. 99919	1	42
19	. 04042	29	. 7387	1766	. 04046	29	. 7185	1767	. 00082	1	. 99918	1	41
20	0. 04071	29	24. 5621	1741	0. 04075	29	24. 5418	1743	1. 00083	1	0. 99917	1	40
21	. 04100	29	. 3880	1716	. 04104	29	. 3675	1718	. 00084	1	. 99916	1	39
22	. 04129	30	. 2164	1693	. 04133	29	. 1957	1694	. 00085	2	. 99915	2	38
23	. 04159	29	24. 0471	1669	. 04162	29	24. 0263	1670	. 00087	1	. 99913	1	37
24	. 04188	29	23. 8802	1646	. 04191	29	23. 8593	1648	. 00088	1	. 99912	1	36
25	0. 04217	29	23. 7156	1623	0. 04220	30	23. 6945	1624	1. 00089	1	0. 99911	1	35
26	. 04246	29	. 5533	1601	. 04250	29	. 5321	1603	. 00090	1	. 99910	1	34
27	. 04275	29	. 3932	1580	. 04279	29	. 3718	1581	. 00091	2	. 99909	2	33
28	. 04304	29	. 2352	1558	. 04308	29	. 2137	1560	. 00093	1	. 99907	1	32
29	. 04333	29	23. 0794	1538	. 04337	29	23. 0577	1539	. 00094	1	. 99906	1	31
30	0. 04362	29	22. 9256	1517	0. 04366	29	22. 9038	1519	1. 00095	2	0. 99905	1	30
31	. 04391	29	. 7739	1498	. 04395	29	. 7519	1499	. 00097	1	. 99904	2	29
32	. 04420	29	. 6241	1477	. 04424	30	. 6020	1479	. 00098	1	. 99902	1	28
33	. 04449	29	. 4764	1459	. 04454	29	. 4541	1460	. 00099	1	. 99901	1	27
34	. 04478	29	. 3305	1440	. 04483	29	. 3081	1441	. 00100	2	. 99900	2	26
35	0. 04507	29	22. 1865	1421	0. 04512	29	22. 1640	1423	1. 00102	1	0. 99898	1	25
36	. 04536	29	22. 0444	1403	. 04541	29	22. 0217	1404	. 00103	1	. 99897	1	24
37	. 04565	29	21. 9041	1385	. 04570	29	21. 8813	1387	. 00104	2	. 99896	2	23
38	. 04594	29	. 7656	1368	. 04599	29	. 7426	1370	. 00106	1	. 99894	1	22
39	. 04623	30	. 6288	1351	. 04628	30	. 6056	1352	. 00107	1	. 99893	1	21
40	0. 04653	29	21. 4937	1334	0. 04658	29	21. 4704	1335	1. 00108	2	0. 99892	2	20
41	. 04682	29	. 3603	1318	. 04687	29	. 3369	1320	. 00110	1	. 99890	1	19
42	. 04711	29	. 2285	1301	. 04716	29	. 2049	1302	. 00111	2	. 99889	1	18
43	. 04740	29	21. 0984	1286	. 04745	29	21. 0747	1287	. 00113	1	. 99888	2	17
44	. 04769	29	20. 9698	1270	. 04774	29	20. 9460	1272	. 00114	1	. 99886	1	16
45	0. 04798	29	20. 8428	1254	0. 04803	30	20. 8188	1256	1. 00115	2	0. 99885	2	15
46	. 04827	29	. 7174	1240	. 04833	29	. 6932	1241	. 00117	1	. 99883	1	14
47	. 04856	29	. 5934	1225	. 04862	29	. 5691	1226	. 00118	2	. 99882	1	13
48	. 04885	29	. 4709	1210	. 04891	29	. 4465	1212	. 00120	1	. 99881	2	12
49	. 04914	29	. 3499	1196	. 04920	29	. 3253	1197	. 00121	1	. 99879	1	11
50	0. 04943	29	20. 2303	1182	0. 04949	29	20. 2056	1184	1. 00122	2	0. 99878	2	10
51	. 04972	29	20. 1121	1169	. 04978	29	20. 0872	1170	. 00124	1	. 99876	1	9
52	. 05001	29	19. 9952	1154	. 05007	30	19. 9702	1156	. 00125	2	. 99875	2	8
53	. 05030	29	. 8798	1142	. 05037	29	. 8546	1143	. 00127	1	. 99873	1	7
54	. 05059	29	. 7656	1128	. 05066	29	. 7403	1130	. 00128	2	. 99872	2	6
55	0. 05088	29	19. 6528	1116	0. 05095	29	19. 6273	1117	1. 00130	1	0. 99870	1	5
56	. 05117	29	. 5412	1103	. 05124	29	. 5156	1105	. 00131	2	. 99869	2	4
57	. 05146	29	. 4309	1091	. 05153	29	. 4051	1092	. 00133	1	. 99867	1	3
58	. 05175	30	. 3218	1078	. 05182	30	. 2959	1080	. 00134	2	. 99866	2	2
59	. 05205	29	. 2140	1067	. 05212	29	. 1879	1068	. 00136	1	. 99864	1	1
60	0. 05234		19. 1073		0. 05241		19. 0811		1. 00137		0. 99863		0
↑ 92° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′ ←	↑ 87°

TABLE 31
Natural Trigonometric Functions

3°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←176° ↓ ′
0	0. 05234	29	19. 1073	1054	0. 05241	29	19. 0811	1056	1. 00137	2	0. 99863	2	60
1	. 05263	29	19. 0019	1044	. 05270	29	18. 9755	1044	. 00139	1	. 99861	1	59
2	. 05292	29	18. 8975	1031	. 05299	29	. 8711	1033	. 00140	2	. 99860	2	58
3	. 05321	29	. 7944	1021	. 05328	29	. 7678	1022	. 00142	1	. 99858	1	57
4	. 05350	29	. 6923	1009	. 05357	30	. 6656	1011	. 00143	2	. 99857	2	56
5	0. 05379	29	18. 5914	999	0. 05387	29	18. 5645	1000	1. 00145	2	0. 99855	1	55
6	. 05408	29	. 4915	988	. 05416	29	. 4645	990	. 00147	1	. 99854	2	54
7	. 05437	29	. 3927	977	. 05445	29	. 3655	978	. 00148	2	. 99852	1	53
8	. 05466	29	. 2950	967	. 05474	29	. 2677	969	. 00150	1	. 99851	2	52
9	. 05495	29	. 1983	957	. 05503	30	. 1708	958	. 00151	2	. 99849	2	51
10	0. 05524	29	18. 1026	947	0. 05533	29	18. 0750	948	1. 00153	2	0. 99847	1	50
11	. 05553	29	18. 0079	937	. 05562	29	17. 9802	939	. 00155	1	. 99846	2	49
12	. 05582	29	17. 9142	927	. 05591	29	. 8863	929	. 00156	2	. 99844	2	48
13	. 05611	29	. 8215	917	. 05620	29	. 7934	919	. 00158	1	. 99842	1	47
14	. 05640	29	. 7298	909	. 05649	29	. 7015	909	. 00159	2	. 99841	2	46
15	0. 05669	29	17. 6389	899	0. 05678	30	17. 6106	901	1. 00161	2	0. 99839	1	45
16	. 05698	29	. 5490	890	. 05708	29	. 5205	891	. 00163	1	. 99838	2	44
17	. 05727	29	. 4600	880	. 05737	29	. 4314	882	. 00164	2	. 99836	2	43
18	. 05756	29	. 3720	872	. 05766	29	. 3432	874	. 00166	2	. 99834	1	42
19	. 05785	29	. 2848	864	. 05795	29	. 2558	865	. 00168	1	. 99833	2	41
20	0. 05814	30	17. 1984	854	0. 05824	30	17. 1693	856	1. 00169	2	0. 99831	2	40
21	. 05844	29	. 1130	847	. 05854	29	17. 0837	847	. 00171	2	. 99829	2	39
22	. 05873	29	17. 0283	837	. 05883	29	16. 9990	840	. 00173	2	. 99827	1	38
23	. 05902	29	16. 9446	830	. 05912	29	. 9150	831	. 00175	1	. 99826	2	37
24	. 05931	29	. 8616	822	. 05941	29	. 8319	823	. 00176	2	. 99824	2	36
25	0. 05960	29	16. 7794	813	0. 05970	29	16. 7496	815	1. 00178	2	0. 99822	1	35
26	. 05989	29	. 6981	806	. 05999	30	. 6681	807	. 00180	2	. 99821	2	34
27	. 06018	29	. 6175	798	. 06029	29	. 5874	799	. 00182	1	. 99819	2	33
28	. 06047	29	. 5377	790	. 06058	29	. 5075	792	. 00183	2	. 99817	2	32
29	. 06076	29	. 4587	783	. 06087	29	. 4283	784	. 00185	2	. 99815	2	31
30	0. 06105	29	16. 3804	775	0. 06116	29	16. 3499	777	1. 00187	2	0. 99813	1	30
31	. 06134	29	. 3029	768	. 06145	30	. 2722	770	. 00189	1	. 99812	2	29
32	. 06163	29	. 2261	761	. 06175	29	. 1952	762	. 00190	2	. 99810	2	28
33	. 06192	29	. 1500	754	. 06204	29	. 1190	755	. 00192	2	. 99808	2	27
34	. 06221	29	16. 0746	747	. 06233	29	16. 0435	748	. 00194	2	. 99806	2	26
35	0. 06250	29	15. 9999	739	0. 06262	29	15. 9687	742	1. 00196	2	0. 99804	1	25
36	. 06279	29	. 9260	733	. 06291	30	. 8945	734	. 00198	2	. 99803	2	24
37	. 06308	29	. 8527	726	. 06321	29	. 8211	728	. 00200	1	. 99801	2	23
38	. 06337	29	. 7801	720	. 06350	29	. 7483	721	. 00201	2	. 99799	2	22
39	. 06366	29	. 7081	713	. 06379	29	. 6762	714	. 00203	2	. 99797	2	21
40	0. 06395	29	15. 6368	707	0. 06408	30	15. 6048	708	1. 00205	2	0. 99795	2	20
41	. 06424	29	. 5661	700	. 06438	29	. 5340	702	. 00207	2	. 99793	1	19
42	. 06453	29	. 4961	694	. 06467	29	. 4638	695	. 00209	2	. 99792	2	18
43	. 06482	29	. 4267	688	. 06496	29	. 3943	689	. 00211	2	. 99790	2	17
44	. 06511	29	. 3579	681	. 06525	29	. 3254	683	. 00213	2	. 99788	2	16
45	0. 06540	29	15. 2898	676	0. 06554	30	15. 2571	678	1. 00215	1	0. 99786	2	15
46	. 06569	29	. 2222	669	. 06584	29	. 1893	671	. 00216	2	. 99784	2	14
47	. 06598	29	. 1553	664	. 06613	29	. 1222	665	. 00218	2	. 99782	2	13
48	. 06627	29	. 0889	658	. 06642	29	15. 0557	659	. 00220	2	. 99780	2	12
49	. 06656	29	15. 0231	652	. 06671	29	14. 9898	654	. 00222	2	. 99778	2	11
50	0. 06685	29	14. 9579	647	0. 06700	30	14. 9244	648	1. 00224	2	0. 99776	2	10
51	. 06714	29	. 8932	641	. 06730	29	. 8596	642	. 00226	2	. 99774	2	9
52	. 06743	30	. 8291	635	. 06759	29	. 7954	637	. 00228	2	. 99772	2	8
53	. 06773	29	. 7656	630	. 06788	29	. 7317	632	. 00230	2	. 99770	2	7
54	. 06802	29	. 7026	625	. 06817	30	. 6685	626	. 00232	2	. 99768	2	6
55	0. 06831	29	14. 6401	619	0. 06847	29	14. 6059	621	1. 00234	2	0. 99766	2	5
56	. 06860	29	. 5782	614	. 06876	29	. 5438	615	. 00236	2	. 99764	2	4
57	. 06889	29	. 5168	609	. 06905	29	. 4823	611	. 00238	2	. 99762	2	3
58	. 06918	29	. 4559	604	. 06934	29	. 4212	605	. 00240	2	. 99760	2	2
59	. 06947	29	. 3955	599	. 06963	30	. 3607	600	. 00242	2	. 99758	2	1
60	0. 06976		14. 3356		0. 06993		14. 3007		1. 00244		0. 99756		0
↑ 93°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←86° ↑

TABLE 31

Natural Trigonometric Functions

4°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←175° ↓ ′
0	0. 06976	29	14. 3356	594	0. 06993	29	14. 3007	596	1. 00244	2	0. 99756	2	60
1	. 07005	29	. 2762	589	. 07022	29	. 2411	590	. 00246	2	. 99754	2	59
2	. 07034	29	. 2173	584	. 07051	29	. 1821	586	. 00248	2	. 99752	2	58
3	. 07063	29	. 1589	579	. 07080	30	. 1235	580	. 00250	2	. 99750	2	57
4	. 07092	29	. 1010	575	. 07110	29	. 0655	576	. 00252	2	. 99748	2	56
5	0. 07121	29	14. 0435	570	0. 07139	29	14. 0079	572	1. 00254	3	0. 99746	2	55
6	. 07150	29	13. 9865	565	. 07168	29	13. 9507	567	. 00257	2	. 99744	2	54
7	. 07179	29	. 9300	561	. 07197	30	. 8940	562	. 00259	2	. 99742	2	53
8	. 07208	29	. 8739	556	. 07227	29	. 8378	557	. 00261	2	. 99740	2	52
9	. 07237	29	. 8183	552	. 07256	29	. 7821	554	. 00263	2	. 99738	2	51
10	0. 07266	29	13. 7631	547	0. 07285	29	13. 7267	548	1. 00265	2	0. 99736	2	50
11	. 07295	29	. 7084	543	. 07314	30	. 6719	545	. 00267	2	. 99734	3	49
12	. 07324	29	. 6541	539	. 07344	29	. 6174	540	. 00269	2	. 99731	2	48
13	. 07353	29	. 6002	534	. 07373	29	. 5634	536	. 00271	3	. 99729	2	47
14	. 07382	29	. 5468	531	. 07402	29	. 5098	532	. 00274	2	. 99727	2	46
15	0. 07411	29	13. 4937	526	0. 07431	30	13. 4566	527	1. 00276	2	0. 99725	2	45
16	. 07440	29	. 4411	522	. 07461	29	. 4039	524	. 00278	2	. 99723	2	44
17	. 07469	29	. 3889	518	. 07490	29	. 3515	519	. 00280	2	. 99721	2	43
18	. 07498	29	. 3371	514	. 07519	29	. 2996	516	. 00282	2	. 99719	3	42
19	. 07527	29	. 2857	510	. 07548	30	. 2480	511	. 00284	3	. 99716	2	41
20	0. 07556	29	13. 2347	506	0. 07578	29	13. 1969	508	1. 00287	2	0. 99714	2	40
21	. 07585	29	. 1841	502	. 07607	29	. 1461	503	. 00289	2	. 99712	2	39
22	. 07614	29	. 1339	499	. 07636	29	. 0958	500	. 00291	2	. 99710	2	38
23	. 07643	29	. 0840	494	. 07665	30	13. 0458	496	. 00293	3	. 99708	3	37
24	. 07672	29	13. 0346	491	. 07695	29	12. 9962	493	. 00296	2	. 99705	2	36
25	0. 07701	29	12. 9855	487	0. 07724	29	12. 9469	488	1. 00298	2	0. 99703	2	35
26	. 07730	29	. 9368	484	. 07753	29	. 8981	485	. 00300	2	. 99701	2	34
27	. 07759	29	. 8884	480	. 07782	30	. 8496	482	. 00302	3	. 99699	3	33
28	. 07788	29	. 8404	476	. 07812	29	. 8014	478	. 00305	2	. 99696	2	32
29	. 07817	29	. 7928	473	. 07841	29	. 7536	474	. 00307	2	. 99694	2	31
30	0. 07846	29	12. 7455	469	0. 07870	29	12. 7062	471	1. 00309	3	0. 99692	3	30
31	. 07875	29	. 6986	466	. 07899	30	. 6591	467	. 00312	2	. 99689	2	29
32	. 07904	29	. 6520	463	. 07929	29	. 6124	464	. 00314	2	. 99687	2	28
33	. 07933	29	. 6057	459	. 07958	29	. 5660	461	. 00316	2	. 99685	2	27
34	. 07962	29	. 5598	456	. 07987	30	. 5199	457	. 00318	3	. 99683	3	26
35	0. 07991	29	12. 5142	452	0. 08017	29	12. 4742	454	1. 00321	2	0. 99680	2	25
36	. 08020	29	. 4690	449	. 08046	29	. 4288	450	. 00323	3	. 99678	2	24
37	. 08049	29	. 4241	446	. 08075	29	. 3838	448	. 00326	2	. 99676	3	23
38	. 08078	29	. 3795	443	. 08104	30	. 3390	444	. 00328	2	. 99673	2	22
39	. 08107	29	. 3352	439	. 08134	29	. 2946	441	. 00330	3	. 99671	3	21
40	0. 08136	29	12. 2913	437	0. 08163	29	12. 2505	438	1. 00333	2	0. 99668	2	20
41	. 08165	29	. 2476	433	. 08192	29	. 2067	435	. 00335	2	. 99666	2	19
42	. 08194	29	. 2043	431	. 08221	30	. 1632	431	. 00337	3	. 99664	3	18
43	. 08223	29	. 1612	427	. 08251	29	. 1201	429	. 00340	2	. 99661	2	17
44	. 08252	29	. 1185	424	. 08280	29	. 0772	426	. 00342	3	. 99659	2	16
45	0. 08281	29	12. 0761	421	0. 08309	30	12. 0346	423	1. 00345	2	0. 99657	3	15
46	. 08310	29	12. 0340	419	. 08339	29	11. 9923	419	. 00347	3	. 99654	2	14
47	. 08339	29	11. 9921	415	. 08368	29	. 9504	417	. 00350	2	. 99652	3	13
48	. 08368	29	. 9506	413	. 08397	30	. 9087	414	. 00352	2	. 99649	2	12
49	. 08397	29	. 9093	409	. 08427	29	. 8673	411	. 00354	3	. 99647	3	11
50	0. 08426	29	11. 8684	407	0. 08456	29	11. 8262	409	1. 00357	2	0. 99644	2	10
51	. 08455	29	. 8277	404	. 08485	29	. 7853	405	. 00359	3	. 99642	3	9
52	. 08484	29	. 7873	402	. 08514	30	. 7448	403	. 00362	2	. 99639	2	8
53	. 08513	29	. 7471	398	. 08544	29	. 7045	400	. 00364	3	. 99637	2	7
54	. 08542	29	. 7073	396	. 08573	29	. 6645	397	. 00367	2	. 99635	3	6
55	0. 08571	29	11. 6677	393	0. 08602	30	11. 6248	395	1. 00369	3	0. 99632	2	5
56	. 08600	29	. 6284	391	. 08632	29	. 5853	392	. 00372	2	. 99630	3	4
57	. 08629	29	. 5893	388	. 08661	29	. 5461	389	. 00374	3	. 99627	2	3
58	. 08658	29	. 5505	385	. 08690	30	. 5072	387	. 00377	2	. 99625	3	2
59	. 08687	29	. 5120	383	. 08720	29	. 4685	384	. 00379	3	. 99622	3	1
60	0. 08716		11. 4737		0. 08749		11. 4301		1. 00382		0. 99619		0
↑ 94°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←85° ↑

TABLE 31
Natural Trigonometric Functions

5°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←174° ↓ ′
0	0. 08716	29	11. 4737	380	0. 08749	29	11. 4301	382	1. 00382	3	0. 99619	2	60
1	. 08745	29	. 4357	378	. 08778	29	. 3919	379	. 00385	2	. 99617	3	59
2	. 08774	29	. 3979	375	. 08807	30	. 3540	377	. 00387	3	. 99614	2	58
3	. 08803	28	. 3604	373	. 08837	29	. 3163	374	. 00390	2	. 99612	3	57
4	. 08831	29	. 3231	370	. 08866	29	. 2789	372	. 00392	3	. 99609	2	56
5	0. 08860	29	11. 2861	368	0. 08895	30	11. 2417	369	1. 00395	2	0. 99607	3	55
6	. 08889	29	. 2493	365	. 08925	29	. 2048	367	. 00397	3	. 99604	2	54
7	. 08918	29	. 2128	363	. 08954	29	. 1681	365	. 00400	3	. 99602	3	53
8	. 08947	29	. 1765	361	. 08983	30	. 1316	362	. 00403	2	. 99599	3	52
9	. 08976	29	. 1404	359	. 09013	29	. 0954	360	. 00405	3	. 99596	2	51
10	0. 09005	29	11. 1045	356	0. 09042	29	11. 0594	357	1. 00408	3	0. 99594	3	50
11	. 09034	29	. 0689	353	. 09071	30	11. 0237	355	. 00411	2	. 99591	3	49
12	. 09063	29	11. 0336	352	. 09101	29	10. 9882	353	. 00413	3	. 99588	2	48
13	. 09092	29	10. 9984	349	. 09130	29	. 9529	351	. 00416	3	. 99586	3	47
14	. 09121	29	. 9635	347	. 09159	30	. 9178	349	. 00419	2	. 99583	3	46
15	0. 09150	29	10. 9288	345	0. 09189	29	10. 8829	346	1. 00421	3	0. 99580	2	45
16	. 09179	29	. 8943	343	. 09218	29	. 8483	344	. 00424	3	. 99578	3	44
17	. 09208	29	. 8600	340	. 09247	30	. 8139	342	. 00427	2	. 99575	3	43
18	. 09237	29	. 8260	339	. 09277	29	. 7797	340	. 00429	3	. 99572	2	42
19	. 09266	29	. 7921	336	. 09306	29	. 7457	338	. 00432	3	. 99570	3	41
20	0. 09295	29	10. 7585	334	0. 09335	30	10. 7119	336	1. 00435	3	0. 99567	3	40
21	. 09324	29	. 7251	332	. 09365	29	. 6783	333	. 00438	2	. 99564	2	39
22	. 09353	29	. 6919	330	. 09394	29	. 6450	332	. 00440	3	. 99562	3	38
23	. 09382	29	. 6589	328	. 09423	30	. 6118	329	. 00443	3	. 99559	3	37
24	. 09411	29	. 6261	326	. 09453	29	. 5789	327	. 00446	3	. 99556	3	36
25	0. 09440	29	10. 5935	324	0. 09482	29	10. 5462	326	1. 00449	2	0. 99553	2	35
26	. 09469	29	. 5611	322	. 09511	30	. 5136	323	. 00451	3	. 99551	3	34
27	. 09498	29	. 5289	320	. 09541	29	. 4813	322	. 00454	3	. 99548	3	33
28	. 09527	29	. 4969	319	. 09570	30	. 4491	319	. 00457	3	. 99545	3	32
29	. 09556	29	. 4650	316	. 09600	29	. 4172	318	. 00460	3	. 99542	2	31
30	0. 09585	29	10. 4334	314	0. 09629	29	10. 3854	316	1. 00463	2	0. 99540	3	30
31	. 09614	28	. 4020	312	. 09658	30	. 3538	314	. 00465	3	. 99537	3	29
32	. 09642	29	. 3708	311	. 09688	29	. 3224	311	. 00468	3	. 99534	3	28
33	. 09671	29	. 3397	308	. 09717	29	. 2913	311	. 00471	3	. 99531	3	27
34	. 09700	29	. 3089	307	. 09746	30	. 2602	308	. 00474	3	. 99528	2	26
35	0. 09729	29	10. 2782	305	0. 09776	29	10. 2294	306	1. 00477	3	0. 99526	3	25
36	. 09758	29	. 2477	303	. 09805	29	. 1988	305	. 00480	2	. 99523	3	24
37	. 09787	29	. 2174	301	. 09834	30	. 1683	302	. 00482	3	. 99520	3	23
38	. 09816	29	. 1873	300	. 09864	29	. 1381	301	. 00485	3	. 99517	3	22
39	. 09845	29	. 1573	298	. 09893	30	. 1080	300	. 00488	3	. 99514	3	21
40	0. 09874	29	10. 1275	296	0. 09923	29	10. 0780	297	1. 00491	3	0. 99511	3	20
41	. 09903	29	. 0979	294	. 09952	29	. 0483	296	. 00494	3	. 99508	2	19
42	. 09932	29	. 0685	293	. 09981	30	10. 0187	2939	. 00497	3	. 99506	3	18
43	. 09961	29	. 0392	291	. 10011	29	9. 98931	2924	. 00500	3	. 99503	3	17
44	. 09990	29	. 0101	2887	. 10040	29	. 96007	2906	. 00503	3	. 99500	3	16
45	0. 10019	29	9. 98123	2875	0. 10069	30	9. 93101	2890	1. 00506	3	0. 99497	3	15
46	. 10048	29	. 95248	2859	. 10099	29	. 90211	2873	. 00509	3	. 99494	3	14
47	. 10077	29	. 92389	2842	. 10128	30	. 87338	2856	. 00512	3	. 99491	3	13
48	. 10106	29	. 89547	2825	. 10158	29	. 84482	2841	. 00515	3	. 99488	3	12
49	. 10135	29	. 86722	2810	. 10187	29	. 81641	2824	. 00518	3	. 99485	3	11
50	0. 10164	28	9. 83912	2793	0. 10216	30	9. 78817	2808	1. 00521	3	0. 99482	3	10
51	. 10192	29	. 81119	2778	. 10246	29	. 76009	2792	. 00524	3	. 99479	3	9
52	. 10221	29	. 78341	2762	. 10275	30	. 73217	2776	. 00527	3	. 99476	3	8
53	. 10250	29	. 75579	2746	. 10305	29	. 70441	2761	. 00530	3	. 99473	3	7
54	. 10279	29	72833	2730	. 10334	29	. 67680	2745	. 00533	3	. 99470	3	6
55	0. 10308	29	9. 70103	2716	0. 10363	30	9. 64935	2730	1. 00536	3	0. 99467	3	5
56	. 10337	29	. 67387	2700	. 10393	29	. 62205	2715	. 00539	3	. 99464	3	4
57	. 10366	29	. 64687	2685	. 10422	30	. 59490	2699	. 00542	3	. 99461	3	3
58	. 10395	29	. 62002	2670	. 10452	29	. 56791	2685	. 00545	3	. 99458	3	2
59	. 10424	29	. 59332	2655	. 10481	29	. 54106	2670	. 00548	3	. 99455	3	1
60	0. 10453		9. 56677		0. 10510		9. 51436		1. 00551		0. 99452		0
↑ 95°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←84° ↑

TABLE 31

Natural Trigonometric Functions

6°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	←173° Diff. 1′	↓ ′
0	0.10453	29	9.56677	2640	0.10510	30	9.51436	2655	1.00551	3	0.99452	3	60
1	.10482	29	.54037	2626	.10540	29	.48781	2640	.00554	3	.99449	3	59
2	.10511	29	.51411	2611	.10569	30	.46141	2626	.00557	3	.99446	3	58
3	.10540	29	.48800	2597	.10599	29	.43515	2611	.00560	3	.99443	3	57
4	.10569	28	.46203	2583	.10628	29	.40904	2597	.00563	3	.99440	3	56
5	0.10597	29	9.43620	2568	0.10657	30	9.38307	2583	1.00566	3	0.99437	3	55
6	.10626	29	.41052	2555	.10687	29	.35724	2569	.00569	4	.99434	3	54
7	.10655	29	.38497	2540	.10716	30	.33155	2556	.00573	3	.99431	3	53
8	.10684	29	.35957	2527	.10746	29	.30599	2541	.00576	3	.99428	4	52
9	.10713	29	.33430	2513	.10775	30	.28058	2528	.00579	3	.99424	3	51
10	0.10742	29	9.30917	2500	0.10805	29	9.25530	2514	1.00582	3	0.99421	3	50
11	.10771	29	.28417	2486	.10834	29	.23016	2500	.00585	3	.99418	3	49
12	.10800	29	.25931	2472	.10863	30	.20516	2488	.00588	4	.99415	3	48
13	.10829	29	.23459	2460	.10893	29	.18028	2474	.00592	3	.99412	3	47
14	.10858	29	.20999	2446	.10922	30	.15554	2461	.00595	3	.99409	3	46
15	0.10887	29	9.18553	2433	0.10952	29	9.13093	2447	1.00598	3	0.99406	4	45
16	.10916	29	.16120	2421	.10981	30	.10646	2435	.00601	3	.99402	3	44
17	.10945	28	.13699	2407	.11011	29	.08211	2422	.00604	4	.99399	3	43
18	.10973	29	.11292	2395	.11040	30	.05789	2410	.00608	3	.99396	3	42
19	.11002	29	.08897	2382	.11070	29	.03379	2396	.00611	3	.99393	3	41
20	0.11031	29	9.06515	2369	0.11099	29	9.00983	2385	1.00614	3	0.99390	4	40
21	.11060	29	.04146	2358	.11128	30	8.98598	2371	.00617	4	.99386	3	39
22	.11089	29	9.01788	2344	.11158	29	.96227	2360	.00621	3	.99383	3	38
23	.11118	29	8.99444	2333	.11187	30	.93867	2347	.00624	3	.99380	3	37
24	.11147	29	.97111	2320	.11217	29	.91520	2335	.00627	3	.99377	3	36
25	0.11176	29	8.94791	2309	0.11246	30	8.89185	2323	1.00630	4	0.99374	4	35
26	.11205	29	.92482	2296	.11276	29	.86862	2311	.00634	3	.99370	3	34
27	.11234	29	.90186	2285	.11305	30	.84551	2299	.00637	3	.99367	3	33
28	.11263	28	.87901	2273	.11335	29	.82252	2288	.00640	4	.99364	4	32
29	.11291	29	.85628	2261	.11364	30	.79964	2275	.00644	3	.99360	3	31
30	0.11320	29	8.83367	2249	0.11394	29	8.77689	2264	1.00647	3	0.99357	3	30
31	.11349	29	.81118	2238	.11423	29	.75425	2253	.00650	4	.99354	3	29
32	.11378	29	.78880	2227	.11452	30	.73172	2241	.00654	3	.99351	4	28
33	.11407	29	.76653	2215	.11482	29	.70931	2230	.00657	3	.99347	3	27
34	.11436	29	.74438	2204	.11511	30	.68701	2219	.00660	4	.99344	3	26
35	0.11465	29	8.72234	2193	0.11541	29	8.66482	2207	1.00664	3	0.99341	4	25
36	.11494	29	.70041	2182	.11570	30	.64275	2197	.00667	4	.99337	3	24
37	.11523	29	.67859	2171	.11600	29	.62078	2185	.00671	3	.99334	3	23
38	.11552	28	.65688	2160	.11629	30	.59893	2175	.00674	3	.99331	4	22
39	.11580	29	.63528	2149	.11659	29	.57718	2163	.00677	4	.99327	3	21
40	0.11609	29	8.61379	2138	0.11688	30	8.55555	2153	1.00681	3	0.99324	4	20
41	.11638	29	.59241	2128	.11718	29	.53402	2143	.00684	4	.99320	3	19
42	.11667	29	.57113	2117	.11747	30	.51259	2131	.00688	3	.99317	3	18
43	.11696	29	.54996	2107	.11777	29	.49128	2121	.00691	4	.99314	4	17
44	.11725	29	.52889	2096	.11806	30	.47007	2111	.00695	3	.99310	3	16
45	0.11754	29	8.50793	2086	0.11836	29	8.44896	2101	1.00698	3	0.99307	4	15
46	.11783	29	.48707	2075	.11865	30	.42795	2090	.00701	4	.99303	3	14
47	.11812	28	.46632	2066	.11895	29	.40705	2080	.00705	3	.99300	3	13
48	.11840	29	.44566	2055	.11924	30	.38625	2070	.00708	4	.99297	4	12
49	.11869	29	.42511	2045	.11954	29	.36555	2059	.00712	3	.99293	3	11
50	0.11898	29	8.40466	2035	0.11983	30	8.34496	2050	1.00715	4	0.99290	4	10
51	.11927	29	.38431	2026	.12013	29	.32446	2040	.00719	3	.99286	3	9
52	.11956	29	.36405	2015	.12042	30	.30406	2030	.00722	4	.99283	4	8
53	.11985	29	.34390	2006	.12072	29	.28376	2021	.00726	4	.99279	3	7
54	.12014	29	.32384	1996	.12101	30	.26355	2010	.00730	3	.99276	4	6
55	0.12043	28	8.30388	1986	0.12131	29	8.24345	2001	1.00733	4	0.99272	3	5
56	.12071	29	.28402	1977	.12160	30	.22344	1992	.00737	3	.99269	4	4
57	.12100	29	.26425	1968	.12190	29	.20352	1982	.00740	4	.99265	3	3
58	.12129	29	.24457	1957	.12219	30	.18370	1972	.00744	3	.99262	4	2
59	.12158	29	.22500	1949	.12249	29	.16398	1963	.00747	4	.99258	3	1
60	0.12187		8.20551		0.12278		8.14435		1.00751		0.99255		0
↑ 96°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′←83°	↑

TABLE 31

Natural Trigonometric Functions

7°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←172° ↓ ′
0	0. 12187	29	8. 20551	1939	0. 12278	30	8. 14435	1954	1. 00751	4	0. 99255	4	60
1	. 12216	29	. 18612	1931	. 12308	30	. 12481	1945	. 00755	3	. 99251	3	59
2	. 12245	29	. 16681	1921	. 12338	29	. 10536	1936	. 00758	4	. 99248	4	58
3	. 12274	28	. 14760	1911	. 12367	30	. 08600	1926	. 00762	3	. 99244	4	57
4	. 12302	29	. 12849	1903	. 12397	29	. 06674	1918	. 00765	4	. 99240	3	56
5	0. 12331	29	8. 10946	1894	0. 12426	30	8. 04756	1908	1. 00769	4	0. 99237	4	55
6	. 12360	29	. 09052	1885	. 12456	29	. 02848	1900	. 00773	3	. 99233	3	54
7	. 12389	29	. 07167	1876	. 12485	30	8. 00948	1890	. 00776	4	. 99230	4	53
8	. 12418	29	. 05291	1868	. 12515	29	7. 99058	1882	. 00780	4	. 99226	4	52
9	. 12447	29	. 03423	1858	. 12544	30	. 97176	1874	. 00784	3	. 99222	3	51
10	0. 12476	28	8. 01565	1851	0. 12574	29	7. 95302	1864	1. 00787	4	0. 99219	4	50
11	. 12504	29	7. 99714	1841	. 12603	30	. 93438	1856	. 00791	4	. 99215	4	49
12	. 12533	29	. 97873	1833	. 12633	29	. 91582	1848	. 00795	4	. 99211	3	48
13	. 12562	29	. 96040	1824	. 12662	30	. 89734	1839	. 00799	3	. 99208	4	47
14	. 12591	29	. 94216	1817	. 12692	30	. 87895	1831	. 00802	4	. 99204	4	46
15	0. 12620	29	7. 92399	1807	0. 12722	29	7. 86064	1822	1. 00806	4	0. 99200	3	45
16	. 12649	29	. 90592	1800	. 12751	30	. 84242	1814	. 00810	3	. 99197	4	44
17	. 12678	28	. 88792	1791	. 12781	29	. 82428	1806	. 00813	4	. 99193	4	43
18	. 12706	29	. 87001	1783	. 12810	30	. 80622	1797	. 00817	4	. 99189	3	42
19	. 12735	29	. 85218	1775	. 12840	29	. 78825	1790	. 00821	4	. 99186	4	41
20	0. 12764	29	7. 83443	1766	0. 12869	30	7. 77035	1781	1. 00825	3	0. 99182	4	40
21	. 12793	29	. 81677	1759	. 12899	30	. 75254	1774	. 00828	4	. 99178	3	39
22	. 12822	29	. 79918	1751	. 12929	29	. 73480	1765	. 00832	4	. 99175	4	38
23	. 12851	29	. 78167	1743	. 12958	30	. 71715	1758	. 00836	4	. 99171	4	37
24	. 12880	28	. 76424	1735	. 12988	29	. 69957	1749	. 00840	4	. 99167	4	36
25	0. 12908	29	7. 74689	1727	0. 13017	30	7. 68208	1742	1. 00844	4	0. 99163	3	35
26	. 12937	29	. 72962	1720	. 13047	29	. 66466	1734	. 00848	3	. 99160	4	34
27	. 12966	29	. 71242	1712	. 13076	30	. 64732	1727	. 00851	4	. 99156	4	33
28	. 12995	29	. 69530	1704	. 13106	30	. 63005	1718	. 00855	4	. 99152	4	32
29	. 13024	29	. 67826	1696	. 13136	29	. 61287	1712	. 00859	4	. 99148	4	31
30	0. 13053	28	7. 66130	1689	0. 13165	30	7. 59575	1703	1. 00863	4	0. 99144	3	30
31	. 13081	29	. 64441	1682	. 13195	29	. 57872	1696	. 00867	4	. 99141	4	29
32	. 13110	29	. 62759	1674	. 13224	30	. 56176	1689	. 00871	4	. 99137	4	28
33	. 13139	29	. 61085	1667	. 13254	30	. 54487	1681	. 00875	3	. 99133	4	27
34	. 13168	29	. 59418	1659	. 13284	29	. 52806	1674	. 00878	4	. 99129	4	26
35	0. 13197	29	7. 57759	1652	0. 13313	30	7. 51132	1667	1. 00882	4	0. 99125	3	25
36	. 13226	28	. 56107	1645	. 13343	29	. 49465	1659	. 00886	4	. 99122	4	24
37	. 13254	29	. 54462	1637	. 13372	30	. 47806	1652	. 00890	4	. 99118	4	23
38	. 13283	29	. 52825	1631	. 13402	30	. 46154	1645	. 00894	4	. 99114	4	22
39	. 13312	29	. 51194	1623	. 13432	29	. 44509	1638	. 00898	4	. 99110	4	21
40	0. 13341	29	7. 49571	1616	0. 13461	30	7. 42871	1631	1. 00902	4	0. 99106	4	20
41	. 13370	29	. 47955	1609	. 13491	30	. 41240	1624	. 00906	4	. 99102	4	19
42	. 13399	28	. 46346	1603	. 13521	29	. 39616	1617	. 00910	4	. 99098	4	18
43	. 13427	29	. 44743	1595	. 13550	30	. 37999	1610	. 00914	4	. 99094	3	17
44	. 13456	29	. 43148	1588	. 13580	29	. 36389	1603	. 00918	4	. 99091	4	16
45	0. 13485	29	7. 41560	1582	0. 13609	30	7. 34786	1596	1. 00922	4	0. 99087	4	15
46	. 13514	29	. 39978	1575	. 13639	30	. 33190	1590	. 00926	4	. 99083	4	14
47	. 13543	29	. 38403	1568	. 13669	29	. 31600	1582	. 00930	4	. 99079	4	13
48	. 13572	28	. 36835	1561	. 13698	30	. 30018	1576	. 00934	4	. 99075	4	12
49	. 13600	29	. 35274	1555	. 13728	30	. 28442	1569	. 00938	4	. 99071	4	11
50	0. 13629	29	7. 33719	1548	0. 13758	29	7. 26873	1563	1. 00942	4	0. 99067	4	10
51	. 13658	29	. 32171	1541	. 13787	30	. 25310	1556	. 00946	4	. 99063	4	9
52	. 13687	29	. 30630	1535	. 13817	29	. 23754	1550	. 00950	4	. 99059	4	8
53	. 13716	28	. 29095	1529	. 13846	30	. 22204	1543	. 00954	4	. 99055	4	7
54	. 13744	29	. 27566	1522	. 13876	30	. 20661	1536	. 00958	4	. 99051	4	6
55	0. 13773	29	7. 26044	1515	0. 13906	29	7. 19125	1531	1. 00962	4	0. 99047	4	5
56	. 13802	29	. 24529	1510	. 13935	30	. 17594	1523	. 00966	4	. 99043	4	4
57	. 13831	29	. 23019	1502	. 13965	30	. 16071	1518	. 00970	5	. 99039	4	3
58	. 13860	29	. 21517	1497	. 13995	29	. 14553	1511	. 00975	4	. 99035	4	2
59	. 13889	28	. 20020	1490	. 14024	30	. 13042	1505	. 00979	4	. 99031	4	1
60	0. 13917		7. 18530		0. 14054		7. 11537		1. 00983		0. 99027		0
↑ 97°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ←82°

TABLE 31

Natural Trigonometric Functions

8°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←171° ↓ ′
0	0. 13917	29	7. 18530	1484	0. 14054	30	7. 11537	1499	1. 00983	4	0. 99027	4	60
1	. 13946	29	. 17046	1478	. 14084	29	. 10038	1492	. 00987	4	. 99023	4	59
2	. 13975	29	. 15568	1472	. 14113	30	. 08546	1487	. 00991	4	. 99019	4	58
3	. 14004	29	. 14096	1466	. 14143	30	. 07059	1480	. 00995	4	. 99015	4	57
4	. 14033	28	. 12630	1459	. 14173	29	. 05579	1474	. 00999	5	. 99011	5	56
5	0. 14061	29	7. 11171	1454	0. 14202	30	7. 04105	1468	1. 01004	4	0. 99006	4	55
6	. 14090	29	. 09717	1448	. 14232	30	. 02637	1463	. 01008	4	. 99002	4	54
7	. 14119	29	. 08269	1441	. 14262	29	7. 01174	1456	. 01012	4	. 98998	4	53
8	. 14148	29	. 06828	1436	. 14291	30	6. 99718	1450	. 01016	4	. 98994	4	52
9	. 14177	28	. 05392	1430	. 14321	30	. 98268	1445	. 01020	4	. 98990	4	51
10	0. 14205	29	7. 03962	1424	0. 14351	30	6. 96823	1438	1. 01024	5	0. 98986	4	50
11	. 14234	29	. 02538	1418	. 14381	29	. 95385	1433	. 01029	4	. 98982	4	49
12	. 14263	29	7. 01120	1412	. 14410	30	. 93952	1427	. 01033	4	. 98978	5	48
13	. 14292	28	6. 99708	1407	. 14440	30	. 92525	1421	. 01037	4	. 98973	4	47
14	. 14320	29	. 98301	1401	. 14470	29	. 91104	1416	. 01041	5	. 98969	4	46
15	0. 14349	29	6. 96900	1395	0. 14499	30	6. 89688	1410	1. 01046	4	0. 98965	4	45
16	. 14378	29	. 95505	1390	. 14529	30	. 88278	1404	. 01050	4	. 98961	4	44
17	. 14407	29	. 94115	1384	. 14559	29	. 86874	1399	. 01054	5	. 98957	4	43
18	. 14436	28	. 92731	1379	. 14588	30	. 85475	1393	. 01059	4	. 98953	5	42
19	. 14464	29	. 91352	1373	. 14618	30	. 84082	1388	. 01063	4	. 98948	4	41
20	0. 14493	29	6. 89979	1367	0. 14648	30	6. 82694	1382	1. 01067	4	0. 98944	4	40
21	. 14522	29	. 88612	1362	. 14678	29	. 81312	1376	. 01071	5	. 98940	4	39
22	. 14551	29	. 87250	1357	. 14707	30	. 79936	1372	. 01076	4	. 98936	5	38
23	. 14580	28	. 85893	1351	. 14737	30	. 78564	1365	. 01080	4	. 98931	4	37
24	. 14608	29	. 84542	1346	. 14767	29	. 77199	1361	. 01084	5	. 98927	4	36
25	0. 14637	29	6. 83196	1340	0. 14796	30	6. 75838	1355	1. 01089	4	0. 98923	4	35
26	. 14666	29	. 81856	1335	. 14826	30	. 74483	1350	. 01093	4	. 98919	5	34
27	. 14695	28	. 80521	1330	. 14856	30	. 73133	1344	. 01097	5	. 98914	4	33
28	. 14723	29	. 79191	1325	. 14886	29	. 71789	1339	. 01102	4	. 98910	4	32
29	. 14752	29	. 77866	1319	. 14915	30	. 70450	1334	. 01106	5	. 98906	4	31
30	0. 14781	29	6. 76547	1314	0. 14945	30	6. 69116	1329	1. 01111	4	0. 98902	5	30
31	. 14810	28	. 75233	1309	. 14975	30	. 67787	1324	. 01115	4	. 98897	4	29
32	. 14838	29	. 73924	1304	. 15005	29	. 66463	1319	. 01119	5	. 98893	4	28
33	. 14867	29	. 72620	1299	. 15034	30	. 65144	1313	. 01124	4	. 98889	5	27
34	. 14896	29	. 71321	1294	. 15064	30	. 63831	1308	. 01128	5	. 98884	4	26
35	0. 14925	29	6. 70027	1289	0. 15094	30	6. 62523	1304	1. 01133	4	0. 98880	4	25
36	. 14954	28	. 68738	1284	. 15124	29	. 61219	1298	. 01137	5	. 98876	5	24
37	. 14982	29	. 67454	1278	. 15153	30	. 59921	1294	. 01142	4	. 98871	4	23
38	. 15011	29	. 66176	1274	. 15183	30	. 58627	1288	. 01146	5	. 98867	4	22
39	. 15040	29	. 64902	1269	. 15213	30	. 57339	1284	. 01151	4	. 98863	5	21
40	0. 15069	28	6. 63633	1264	0. 15243	29	6. 56055	1278	1. 01155	5	0. 98858	4	20
41	. 15097	29	. 62369	1259	. 15272	30	. 54777	1274	. 01160	4	. 98854	5	19
42	. 15126	29	. 61110	1255	. 15302	30	. 53503	1269	. 01164	5	. 98849	4	18
43	. 15155	29	. 59855	1249	. 15332	30	. 52234	1264	. 01169	4	. 98845	4	17
44	. 15184	28	. 58606	1245	. 15362	29	. 50970	1260	. 01173	5	. 98841	5	16
45	0. 15212	29	6. 57361	1240	0. 15391	30	6. 49710	1254	1. 01178	4	0. 98836	4	15
46	. 15241	29	. 56121	1235	. 15421	30	. 48456	1250	. 01182	5	. 98832	5	14
47	. 15270	29	. 54886	1231	. 15451	30	. 47206	1245	. 01187	4	. 98827	4	13
48	. 15299	28	. 53655	1226	. 15481	30	. 45961	1241	. 01191	5	. 98823	5	12
49	. 15327	29	. 52429	1221	. 15511	29	. 44720	1236	. 01196	4	. 98818	4	11
50	0. 15356	29	6. 51208	1217	0. 15540	30	6. 43484	1231	1. 01200	5	0. 98814	5	10
51	. 15385	29	. 49991	1212	. 15570	30	. 42253	1227	. 01205	4	. 98809	4	9
52	. 15414	28	. 48779	1207	. 15600	30	. 41026	1222	. 01209	5	. 98805	5	8
53	. 15442	29	. 47572	1203	. 15630	30	. 39804	1217	. 01214	5	. 98800	4	7
54	. 15471	29	. 46369	1198	. 15660	29	. 38587	1213	. 01219	4	. 98796	5	6
55	0. 15500	29	6. 45171	1194	0. 15689	30	6. 37374	1209	1. 01223	5	0. 98791	4	5
56	. 15529	28	. 43977	1190	. 15719	30	. 36165	1204	. 01228	5	. 98787	5	4
57	. 15557	29	. 42787	1185	. 15749	30	. 34961	1200	. 01233	4	. 98782	4	3
58	. 15586	29	. 41602	1180	. 15779	30	. 33761	1195	. 01237	5	. 98778	5	2
59	. 15615	28	. 40422	1177	. 15809	29	. 32566	1191	. 01242	5	. 98773	4	1
60	0. 15643		6. 39245		0. 15838		6. 31375		1. 01247		0. 98769		0
↑ 98°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ←81°

TABLE 31

Natural Trigonometric Functions

9°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←170° ↓ ′
0	0.15643	29	6.39245	1172	0.15838	30	6.31375	1186	1.01247	4	0.98769	5	60
1	.15672	29	.38073	1167	.15868	30	.30189	1182	.01251	5	.98764	4	59
2	.15701	29	.36906	1163	.15898	30	.29007	1178	.01256	5	.98760	5	58
3	.15730	28	.35743	1159	.15928	30	.27829	1174	.01261	4	.98755	4	57
4	.15758	29	.34584	1155	.15958	30	.26655	1169	.01265	5	.98751	5	56
5	0.15787	29	6.33429	1150	0.15988	29	6.25486	1165	1.01270	5	0.98746	5	55
6	.15816	29	.32279	1146	.16017	30	.24321	1161	.01275	4	.98741	4	54
7	.15845	28	.31133	1142	.16047	30	.23160	1157	.01279	5	.98737	5	53
8	.15873	29	.29991	1138	.16077	30	.22003	1152	.01284	5	.98732	4	52
9	.15902	29	.28853	1134	.16107	30	.20851	1148	.01289	5	.98728	5	51
10	0.15931	28	6.27719	1129	0.16137	30	6.19703	1144	1.01294	4	0.98723	5	50
11	.15959	29	.26590	1126	.16167	29	.18559	1140	.01298	5	.98718	4	49
12	.15988	29	.25464	1121	.16196	30	.17419	1136	.01303	5	.98714	5	48
13	.16017	29	.24343	1117	.16226	30	.16283	1132	.01308	5	.98709	5	47
14	.16046	28	.23226	1113	.16256	30	.15151	1128	.01313	4	.98704	4	46
15	0.16074	29	6.22113	1109	0.16286	30	6.14023	1124	1.01317	5	0.98700	5	45
16	.16103	29	.21004	1106	.16316	30	.12899	1120	.01322	5	.98695	5	44
17	.16132	28	.19898	1101	.16346	30	.11779	1115	.01327	5	.98690	4	43
18	.16160	29	.18797	1097	.16376	29	.10664	1112	.01332	5	.98686	5	42
19	.16189	29	.17700	1093	.16405	30	.09552	1108	.01337	5	.98681	5	41
20	0.16218	28	6.16607	1090	0.16435	30	6.08444	1104	1.01342	4	0.98676	5	40
21	.16246	29	.15517	1085	.16465	30	.07340	1100	.01346	5	.98671	4	39
22	.16275	29	.14432	1082	.16495	30	.06240	1097	.01351	5	.98667	5	38
23	.16304	29	.13350	1077	.16525	30	.05143	1092	.01356	5	.98662	5	37
24	.16333	28	.12273	1074	.16555	30	.04051	1089	.01361	5	.98657	5	36
25	0.16361	29	6.11199	1070	0.16585	30	6.02962	1084	1.01366	5	0.98652	4	35
26	.16390	29	.10129	1067	.16615	30	.01878	1081	.01371	5	.98648	5	34
27	.16419	28	.09062	1062	.16645	29	6.00797	1077	.01376	5	.98643	5	33
28	.16447	29	.08000	1059	.16674	30	5.99720	1074	.01381	5	.98638	5	32
29	.16476	29	.06941	1055	.16704	30	.98646	1070	.01386	5	.98633	4	31
30	0.16505	28	6.05886	1052	0.16734	30	5.97576	1066	1.01391	4	0.98629	5	30
31	.16533	29	.04834	1047	.16764	30	.96510	1062	.01395	5	.98624	5	29
32	.16562	29	.03787	1044	.16794	30	.95448	1058	.01400	5	.98619	5	28
33	.16591	29	.02743	1041	.16824	30	.94390	1055	.01405	5	.98614	5	27
34	.16620	28	.01702	1036	.16854	30	.93335	1052	.01410	5	.98609	5	26
35	0.16648	29	6.00666	1033	0.16884	30	5.92283	1047	1.01415	5	0.98604	4	25
36	.16677	29	5.99633	1030	.16914	30	.91236	1045	.01420	5	.98600	5	24
37	.16706	28	.98603	1026	.16944	30	.90191	1040	.01425	5	.98595	5	23
38	.16734	29	.97577	1022	.16974	30	.89151	1037	.01430	5	.98590	5	22
39	.16763	29	.96555	1019	.17004	29	.88114	1034	.01435	5	.98585	5	21
40	0.16792	28	5.95536	1015	0.17033	30	5.87080	1029	1.01440	5	0.98580	5	20
41	.16820	29	.94521	1012	.17063	30	.86051	1027	.01445	5	.98575	5	19
42	.16849	29	.93509	1008	.17093	30	.85024	1023	.01450	5	.98570	5	18
43	.16878	28	.92501	1005	.17123	30	.84001	1019	.01455	5	.98565	4	17
44	.16906	29	.91496	1001	.17153	30	.82982	1016	.01460	6	.98561	5	16
45	0.16935	29	5.90495	998	0.17183	30	5.81966	1013	1.01466	5	0.98556	5	15
46	.16964	28	.89497	995	.17213	30	.80953	1009	.01471	5	.98551	5	14
47	.16992	29	.88502	991	.17243	30	.79944	1006	.01476	5	.98546	5	13
48	.17021	29	.87511	987	.17273	30	.78938	1002	.01481	5	.98541	5	12
49	.17050	28	.86524	985	.17303	30	.77936	999	.01486	5	.98536	5	11
50	0.17078	29	5.85539	981	0.17333	30	5.76937	996	1.01491	5	0.98531	5	10
51	.17107	29	.84558	977	.17363	30	.75941	992	.01496	5	.98526	5	9
52	.17136	28	.83581	975	.17393	30	.74949	989	.01501	5	.98521	5	8
53	.17164	29	.82606	971	.17423	30	.73960	986	.01506	6	.98516	5	7
54	.17193	29	.81635	968	.17453	30	.72974	982	.01512	5	.98511	5	6
55	0.17222	28	5.80667	964	0.17483	30	5.71992	979	1.01517	5	0.98506	5	5
56	.17250	29	.79703	961	.17513	30	.71013	976	.01522	5	.98501	5	4
57	.17279	29	.78742	959	.17543	30	.70037	973	.01527	5	.98496	5	3
58	.17308	28	.77783	954	.17573	30	.69064	970	.01532	5	.98491	5	2
59	.17336	29	.76829	952	.17603	30	.68094	966	.01537	6	.98486	5	1
60	0.17365		5.75877		0.17633		5.67128		1.01543		0.98481		0
↑ 99°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←80° ↑

TABLE 31

Natural Trigonometric Functions

10°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←169° ↓ ′
0	0. 17365	28	5. 75877	948	0. 17633	30	5. 67128	963	1. 01543	5	0. 98481	5	60
1	. 17393	29	. 74929	946	. 17663	30	. 66165	960	. 01548	5	. 98476	5	59
2	. 17422	29	. 73983	942	. 17693	30	. 65205	957	. 01553	5	. 98471	5	58
3	. 17451	28	. 73041	939	. 17723	30	. 64248	953	. 01558	6	. 98466	5	57
4	. 17479	29	. 72102	936	. 17753	30	. 63295	951	. 01564	5	. 98461	6	56
5	0. 17508	29	5. 71166	932	0. 17783	30	5. 62344	947	1. 01569	5	0. 98455	5	55
6	. 17537	28	. 70234	930	. 17813	30	. 61397	945	. 01574	5	. 98450	5	54
7	. 17565	29	. 69304	927	. 17843	30	. 60452	941	. 01579	6	. 98445	5	53
8	. 17594	29	. 68377	923	. 17873	30	. 59511	938	. 01585	5	. 98440	5	52
9	. 17623	28	. 67454	921	. 17903	30	. 58573	935	. 01590	5	. 98435	5	51
10	0. 17651	29	5. 66533	917	0. 17933	30	5. 57638	932	1. 01595	6	0. 98430	5	50
11	. 17680	28	. 65616	915	. 17963	30	. 56706	929	. 01601	5	. 98425	5	49
12	. 17708	29	. 64701	911	. 17993	30	. 55777	926	. 01606	5	. 98420	6	48
13	. 17737	29	. 63790	909	. 18023	30	. 54851	924	. 01611	5	. 98414	5	47
14	. 17766	28	. 62881	905	. 18053	30	. 53927	920	. 01616	6	. 98409	5	46
15	0. 17794	29	5. 61976	903	0. 18083	30	5. 53007	917	1. 01622	5	0. 98404	5	45
16	. 17823	29	. 61073	899	. 18113	30	. 52090	914	. 01627	6	. 98399	5	44
17	. 17852	28	. 60174	897	. 18143	30	. 51176	912	. 01633	5	. 98394	5	43
18	. 17880	29	. 59277	894	. 18173	30	. 50264	908	. 01638	5	. 98389	6	42
19	. 17909	28	. 58383	890	. 18203	30	. 49356	905	. 01643	6	. 98383	5	41
20	0. 17937	29	5. 57493	888	0. 18233	30	5. 48451	903	1. 01649	5	0. 98378	5	40
21	. 17966	29	. 56605	885	. 18263	30	. 47548	900	. 01654	5	. 98373	5	39
22	. 17995	28	. 55720	883	. 18293	30	. 46648	897	. 01659	6	. 98368	6	38
23	. 18023	29	. 54837	879	. 18323	30	. 45751	894	. 01665	5	. 98362	5	37
24	. 18052	29	. 53958	877	. 18353	31	. 44857	891	. 01670	6	. 98357	5	36
25	0. 18081	28	5. 53081	873	0. 18384	30	5. 43966	888	1. 01676	5	0. 98352	5	35
26	. 18109	29	. 52208	871	. 18414	30	. 43078	886	. 01681	6	. 98347	6	34
27	. 18138	28	. 51337	869	. 18444	30	. 42192	883	. 01687	5	. 98341	5	33
28	. 18166	29	. 50468	865	. 18474	30	. 41309	880	. 01692	6	. 98336	5	32
29	. 18195	29	. 49603	863	. 18504	30	. 40429	877	. 01698	5	. 98331	6	31
30	0. 18224	28	5. 48740	859	0. 18534	30	5. 39552	875	1. 01703	6	0. 98325	5	30
31	. 18252	29	. 47881	858	. 18564	30	. 38677	872	. 01709	5	. 98320	5	29
32	. 18281	28	. 47023	854	. 18594	30	. 37805	869	. 01714	6	. 98315	5	28
33	. 18309	29	. 46169	852	. 18624	30	. 36936	866	. 01720	5	. 98310	6	27
34	. 18338	29	. 45317	849	. 18654	30	. 36070	864	. 01725	6	. 98304	5	26
35	0. 18367	28	5. 44468	846	0. 18684	30	5. 35206	861	1. 01731	5	0. 98299	5	25
36	. 18395	29	. 43622	844	. 18714	31	. 34345	858	. 01736	6	. 98294	6	24
37	. 18424	28	. 42778	841	. 18745	30	. 33487	856	. 01742	5	. 98288	5	23
38	. 18452	29	. 41937	838	. 18775	30	. 32631	853	. 01747	6	. 98283	6	22
39	. 18481	28	. 41099	836	. 18805	30	. 31778	850	. 01753	5	. 98277	5	21
40	0. 18509	29	5. 40263	833	0. 18835	30	5. 30928	848	1. 01758	6	0. 98272	5	20
41	. 18538	29	. 39430	830	. 18865	30	. 30080	845	. 01764	5	. 98267	6	19
42	. 18567	28	. 38600	828	. 18895	30	. 29235	842	. 01769	6	. 98261	5	18
43	. 18595	29	. 37772	825	. 18925	30	. 28393	840	. 01775	6	. 98256	6	17
44	. 18624	28	. 36947	823	. 18955	31	. 27553	838	. 01781	5	. 98250	5	16
45	0. 18652	29	5. 36124	820	0. 18986	30	5. 26715	835	1. 01786	6	0. 98245	5	15
46	. 18681	29	. 35304	818	. 19016	30	. 25880	832	. 01792	6	. 98240	6	14
47	. 18710	28	. 34486	815	. 19046	30	. 25048	830	. 01798	5	. 98234	5	13
48	. 18738	29	. 33671	812	. 19076	30	. 24218	827	. 01803	6	. 98229	6	12
49	. 18767	28	. 32859	810	. 19106	30	. 23391	825	. 01809	6	. 98223	5	11
50	0. 18795	29	5. 32049	808	0. 19136	30	5. 22566	822	1. 01815	5	0. 98218	6	10
51	. 18824	28	. 31241	805	. 19166	31	. 21744	819	. 01820	6	. 98212	5	9
52	. 18852	29	. 30436	802	. 19197	30	. 20925	818	. 01826	6	. 98207	6	8
53	. 18881	29	. 29634	801	. 19227	30	. 20107	814	. 01832	5	. 98201	5	7
54	. 18910	28	. 28833	797	. 19257	30	. 19293	813	. 01837	6	. 98196	6	6
55	0. 18938	29	5. 28036	795	0. 19287	30	5. 18480	809	1. 01843	6	0. 98190	5	5
56	. 18967	28	. 27241	793	. 19317	30	. 17671	808	. 01849	5	. 98185	6	4
57	. 18995	29	. 26448	790	. 19347	31	. 16863	805	. 01854	6	. 98179	5	3
58	. 19024	28	. 25658	788	. 19378	30	. 16058	802	. 01860	6	. 98174	6	2
59	. 19052	29	. 24870	786	. 19408	30	. 15256	801	. 01866	6	. 98168	5	1
60	0. 19081		5. 24084		0. 19438		5. 14455		1. 01872		0. 98163		0
↑ 100°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ←79°

TABLE 31
Natural Trigonometric Functions

11°→ ′ ↓	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←168° ↓ ′
0	0.19081	28	5.24084	783	0.19438	30	5.14455	797	1.01872	5	0.98163	6	60
1	.19109	29	.23301	780	.19468	30	.13658	796	.01877	6	.98157	5	59
2	.19138	29	.22521	779	.19498	31	.12862	793	.01883	6	.98152	6	58
3	.19167	28	.21742	776	.19529	30	.12069	790	.01889	6	.98146	6	57
4	.19195	29	.20966	773	.19559	30	.11279	789	.01895	6	.98140	5	56
5	0.19224	28	5.20193	772	0.19589	30	5.10490	786	1.01901	5	0.98135	6	55
6	.19252	29	.19421	769	.19619	30	.09704	783	.01906	6	.98129	5	54
7	.19281	28	.18652	766	.19649	31	.08921	782	.01912	6	.98124	6	53
8	.19309	29	.17886	765	.19680	30	.08139	779	.01918	6	.98118	6	52
9	.19338	28	.17121	762	.19710	30	.07360	776	.01924	6	.98112	5	51
10	0.19366	29	5.16359	760	0.19740	30	5.06584	775	1.01930	6	0.98107	6	50
11	.19395	28	.15599	757	.19770	31	.05809	772	.01936	5	.98101	5	49
12	.19423	29	.14842	755	.19801	30	.05037	770	.01941	6	.98096	6	48
13	.19452	29	.14087	753	.19831	30	.04267	768	.01947	6	.98090	6	47
14	.19481	28	.13334	751	.19861	30	.03499	765	.01953	6	.98084	5	46
15	0.19509	29	5.12583	748	0.19891	30	5.02734	763	1.01959	6	0.98079	6	45
16	.19538	28	.11835	747	.19921	31	.01971	761	.01965	6	.98073	6	44
17	.19566	29	.11088	744	.19952	30	.01210	759	.01971	6	.98067	6	43
18	.19595	28	.10344	742	.19982	30	5.00451	756	.01977	6	.98061	5	42
19	.19623	29	.09602	739	.20012	30	4.99695	755	.01983	6	.98056	6	41
20	0.19652	28	5.08863	738	0.20042	31	4.98940	752	1.01989	6	0.98050	6	40
21	.19680	29	.08125	735	.20073	30	.98188	750	.01995	6	.98044	5	39
22	.19709	28	.07390	733	.20103	30	.97438	748	.02001	6	.98039	6	38
23	.19737	29	.06657	731	.20133	31	.96690	745	.02007	6	.98033	6	37
24	.19766	28	.05926	729	.20164	30	.95945	744	.02013	6	.98027	6	36
25	0.19794	29	5.05197	726	0.20194	30	4.95201	741	1.02019	6	0.98021	5	35
26	.19823	28	.04471	725	.20224	30	.94460	739	.02025	6	.98016	6	34
27	.19851	29	.03746	722	.20254	31	.93721	737	.02031	6	.98010	6	33
28	.19880	28	.03024	721	.20285	30	.92984	735	.02037	6	.98004	6	32
29	.19908	29	.02303	718	.20315	30	.92249	733	.02043	6	.97998	6	31
30	0.19937	28	5.01585	716	0.20345	31	4.91516	731	1.02049	6	0.97992	5	30
31	.19965	29	.00869	714	.20376	30	.90785	729	.02055	6	.97987	6	29
32	.19994	28	5.00155	712	.20406	30	.90056	726	.02061	6	.97981	6	28
33	.20022	29	4.99443	710	.20436	30	.89330	725	.02067	6	.97975	6	27
34	.20051	28	.98733	708	.20466	31	.88605	723	.02073	6	.97969	6	26
35	0.20079	29	4.98025	705	0.20497	30	4.87882	720	1.02079	6	0.97963	5	25
36	.20108	28	.97320	704	.20527	30	.87162	718	.02085	6	.97958	6	24
37	.20136	29	.96616	702	.20557	31	.86444	717	.02091	6	.97952	6	23
38	.20165	28	.95914	699	.20588	30	.85727	714	.02097	6	.97946	6	22
39	.20193	29	.95215	698	.20618	30	.85013	713	.02103	7	.97940	6	21
40	0.20222	28	4.94517	696	0.20648	31	4.84300	710	1.02110	6	0.97934	6	20
41	.20250	29	.93821	693	.20679	30	.83590	708	.02116	6	.97928	6	19
42	.20279	28	.93128	692	.20709	30	.82882	707	.02122	6	.97922	6	18
43	.20307	29	.92436	690	.20739	31	.82175	704	.02128	6	.97916	6	17
44	.20336	28	.91746	688	.20770	30	.81471	702	.02134	6	.97910	5	16
45	0.20364	29	4.91058	685	0.20800	30	4.80769	701	1.02140	6	0.97905	6	15
46	.20393	28	.90373	684	.20830	31	.80068	698	.02146	7	.97899	6	14
47	.20421	29	.89689	682	.20861	30	.79370	697	.02153	6	.97893	6	13
48	.20450	28	.89007	680	.20891	30	.78673	695	.02159	6	.97887	6	12
49	.20478	29	.88327	678	.20921	31	.77978	692	.02165	6	.97881	6	11
50	0.20507	28	4.87649	676	0.20952	30	4.77286	691	1.02171	7	0.97875	6	10
51	.20535	28	.86973	674	.20982	31	.76595	689	.02178	6	.97869	6	9
52	.20563	29	.86299	672	.21013	30	.75906	687	.02184	6	.97863	6	8
53	.20592	28	.85627	671	.21043	30	.75219	685	.02190	6	.97857	6	7
54	.20620	29	.84956	668	.21073	31	.74534	683	.02196	7	.97851	6	6
55	0.20649	28	4.84288	667	0.21104	30	4.73851	681	1.02203	6	0.97845	6	5
56	.20677	29	.83621	665	.21134	30	.73170	680	.02209	6	.97839	6	4
57	.20706	28	.82956	662	.21164	31	.72490	677	.02215	6	.97833	6	3
58	.20734	29	.82294	661	.21195	30	.71813	676	.02221	7	.97827	6	2
59	.20763	28	.81633	660	.21225	31	.71137	674	.02228	6	.97821	6	1
60	0.20791		4.80973		0.21256		4.70463		1.02234		0.97815		0
↑ 101°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←78° ↑

TABLE 31

Natural Trigonometric Functions

12° → ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←167° ↓ ′
0	0. 20791	29	4. 80973	657	0. 21256	30	4. 70463	672	1. 02234	6	0. 97815	6	60
1	. 20820	28	. 80316	655	. 21286	30	. 69791	670	. 02240	7	. 97809	6	59
2	. 20848	29	. 79661	654	. 21316	31	. 69121	669	. 02247	6	. 97803	6	58
3	. 20877	28	. 79007	652	. 21347	30	. 68452	666	. 02253	6	. 97797	6	57
4	. 20905	28	. 78355	650	. 21377	31	. 67786	665	. 02259	7	. 97791	7	56
5	0. 20933	29	4. 77705	648	0. 21408	30	4. 67121	663	1. 02266	6	0. 97784	6	55
6	. 20962	28	. 77057	646	. 21438	31	. 66458	661	. 02272	7	. 97778	6	54
7	. 20990	29	. 76411	645	. 21469	30	. 65797	659	. 02279	6	. 97772	6	53
8	. 21019	28	. 75766	643	. 21499	30	. 65138	658	. 02285	6	. 97766	6	52
9	. 21047	29	. 75123	641	. 21529	31	. 64480	655	. 02291	7	. 97760	6	51
10	0. 21076	28	4. 74482	639	0. 21560	30	4. 63825	654	1. 02298	6	0. 97754	6	50
11	. 21104	28	. 73843	638	. 21590	31	. 63171	653	. 02304	7	. 97748	6	49
12	. 21132	29	. 73205	636	. 21621	30	. 62518	650	. 02311	6	. 97742	7	48
13	. 21161	28	. 72569	634	. 21651	31	. 61868	649	. 02317	6	. 97735	6	47
14	. 21189	29	. 71935	632	. 21682	30	. 61219	647	. 02323	7	. 97729	6	46
15	0. 21218	28	4. 71303	630	0. 21712	31	4. 60572	645	1. 02330	6	0. 97723	6	45
16	. 21246	29	. 70673	629	. 21743	30	. 59927	644	. 02336	7	. 97717	6	44
17	. 21275	28	. 70044	627	. 21773	31	. 59283	642	. 02343	6	. 97711	6	43
18	. 21303	28	. 69417	626	. 21804	30	. 58641	640	. 02349	7	. 97705	7	42
19	. 21331	29	. 68791	624	. 21834	30	. 58001	638	. 02356	6	. 97698	6	41
20	0. 21360	28	4. 68167	622	0. 21864	31	4. 57363	637	1. 02362	7	0. 97692	6	40
21	. 21388	29	. 67545	620	. 21895	30	. 56726	635	. 02369	6	. 97686	6	39
22	. 21417	28	. 66925	618	. 21925	31	. 56091	633	. 02375	7	. 97680	7	38
23	. 21445	29	. 66307	617	. 21956	30	. 55458	632	. 02382	6	. 97673	6	37
24	. 21474	28	. 65690	616	. 21986	31	. 54826	630	. 02388	7	. 97667	6	36
25	0. 21502	28	4. 65074	613	0. 22017	30	4. 54196	628	1. 02395	7	0. 97661	6	35
26	. 21530	29	. 64461	612	. 22047	31	. 53568	627	. 02402	6	. 97655	7	34
27	. 21559	28	. 63849	611	. 22078	30	. 52941	625	. 02408	7	. 97648	6	33
28	. 21587	29	. 63238	608	. 22108	31	. 52316	623	. 02415	6	. 97642	6	32
29	. 21616	28	. 62630	607	. 22139	30	. 51693	622	. 02421	7	. 97636	6	31
30	0. 21644	28	4. 62023	606	0. 22169	31	4. 51071	620	1. 02428	7	0. 97630	7	30
31	. 21672	29	. 61417	604	. 22200	31	. 50451	619	. 02435	6	. 97623	6	29
32	. 21701	28	. 60813	602	. 22231	30	. 49832	617	. 02441	7	. 97617	6	28
33	. 21729	29	. 60211	600	. 22261	31	. 49215	615	. 02448	6	. 97611	7	27
34	. 21758	28	. 59611	599	. 22292	30	. 48600	614	. 02454	7	. 97604	6	26
35	0. 21786	28	4. 59012	598	0. 22322	31	4. 47986	612	1. 02461	7	0. 97598	6	25
36	. 21814	29	. 58414	595	. 22353	30	. 47374	610	. 02468	6	. 97592	7	24
37	. 21843	28	. 57819	595	. 22383	31	. 46764	609	. 02474	7	. 97585	6	23
38	. 21871	28	. 57224	592	. 22414	30	. 46155	607	. 02481	7	. 97579	6	22
39	. 21899	29	. 56632	591	. 22444	31	. 45548	606	. 02488	6	. 97573	7	21
40	0. 21928	28	4. 56041	590	0. 22475	30	4. 44942	604	1. 02494	7	0. 97566	6	20
41	. 21956	29	. 55451	588	. 22505	31	. 44338	603	. 02501	7	. 97560	7	19
42	. 21985	28	. 54863	586	. 22536	31	. 43735	601	. 02508	7	. 97553	6	18
43	. 22013	28	. 54277	585	. 22567	30	. 43134	600	. 02515	6	. 97547	6	17
44	. 22041	29	. 53692	583	. 22597	31	. 42534	598	. 02521	7	. 97541	7	16
45	0. 22070	28	4. 53109	582	0. 22628	30	4. 41936	596	1. 02528	7	0. 97534	6	15
46	. 22098	28	. 52527	580	. 22658	31	. 41340	595	. 02535	7	. 97528	7	14
47	. 22126	29	. 51947	579	. 22689	30	. 40745	593	. 02542	6	. 97521	6	13
48	. 22155	28	. 51368	577	. 22719	31	. 40152	592	. 02548	7	. 97515	7	12
49	. 22183	29	. 50791	575	. 22750	31	. 39560	591	. 02555	7	. 97508	6	11
50	0. 22212	28	4. 50216	574	0. 22781	30	4. 38969	588	1. 02562	7	0. 97502	6	10
51	. 22240	28	. 49642	573	. 22811	31	. 38381	588	. 02569	7	. 97496	7	9
52	. 22268	29	. 49069	571	. 22842	30	. 37793	586	. 02576	6	. 97489	6	8
53	. 22297	28	. 48498	570	. 22872	31	. 37207	584	. 02582	7	. 97483	7	7
54	. 22325	28	. 47928	568	. 22903	31	. 36623	583	. 02589	7	. 97476	6	6
55	0. 22353	29	4. 47360	567	0. 22934	30	4. 36040	581	1. 02596	7	0. 97470	7	5
56	. 22382	28	. 46793	565	. 22964	31	. 35459	580	. 02603	7	. 97463	6	4
57	. 22410	28	. 46228	564	. 22995	31	. 34879	579	. 02610	7	. 97457	7	3
58	. 22438	29	. 45664	562	. 23026	30	. 34300	577	. 02617	7	. 97450	6	2
59	. 22467	28	. 45102	561	. 23056	31	. 33723	575	. 02624	6	. 97444	7	1
60	0. 22495		4. 44541		0. 23087		4. 33148		1. 02630		0. 97437		0
↑ 102° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←77° ↑

TABLE 31

Natural Trigonometric Functions

13° → ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	← 166° ↓ ′
0	0. 22495	28	4. 44541	559	0. 23087	30	4. 33148	575	1. 02630	7	0. 97437	7	60
1	. 22523	29	. 43982	558	. 23117	31	. 32573	572	. 02637	7	. 97430	6	59
2	. 22552	28	. 43424	557	. 23148	31	. 32001	571	. 02644	7	. 97424	7	58
3	. 22580	28	. 42867	555	. 23179	30	. 31430	570	. 02651	7	. 97417	6	57
4	. 22608	29	. 42312	553	. 23209	31	. 30860	569	. 02658	7	. 97411	7	56
5	0. 22637	28	4. 41759	553	0. 23240	31	4. 30291	567	1. 02665	7	0. 97404	6	55
6	. 22665	28	. 41206	550	. 23271	30	. 29724	565	. 02672	7	. 97398	7	54
7	. 22693	29	. 40656	550	. 23301	31	. 29159	564	. 02679	7	. 97391	7	53
8	. 22722	28	. 40106	548	. 23332	31	. 28595	563	. 02686	7	. 97384	6	52
9	. 22750	28	. 39558	546	. 23363	30	. 28032	561	. 02693	7	. 97378	7	51
10	0. 22778	29	4. 39012	546	0. 23393	31	4. 27471	560	1. 02700	7	0. 97371	6	50
11	. 22807	28	. 38466	543	. 23424	31	. 26911	559	. 02707	7	. 97365	7	49
12	. 22835	28	. 37923	543	. 23455	30	. 26352	557	. 02714	7	. 97358	7	48
13	. 22863	29	. 37380	541	. 23485	31	. 25795	556	. 02721	7	. 97351	6	47
14	. 22892	28	. 36839	540	. 23516	31	. 25239	554	. 02728	7	. 97345	7	46
15	0. 22920	28	4. 36299	538	0. 23547	31	4. 24685	553	1. 02735	7	0. 97338	7	45
16	. 22948	29	. 35761	537	. 23578	30	. 24132	552	. 02742	7	. 97331	6	44
17	. 22977	28	. 35224	535	. 23608	31	. 23580	550	. 02749	7	. 97325	7	43
18	. 23005	28	. 34689	535	. 23639	31	. 23030	549	. 02756	7	. 97318	7	42
19	. 23033	29	. 34154	532	. 23670	30	. 22481	548	. 02763	7	. 97311	7	41
20	0. 23062	28	4. 33622	532	0. 23700	31	4. 21933	546	1. 02770	7	0. 97304	6	40
21	. 23090	28	. 33090	530	. 23731	31	. 21387	545	. 02777	7	. 97298	7	39
22	. 23118	28	. 32560	529	. 23762	31	. 20842	544	. 02784	7	. 97291	7	38
23	. 23146	29	. 32031	528	. 23793	30	. 20298	542	. 02791	8	. 97284	6	37
24	. 23175	28	. 31503	526	. 23823	31	. 19756	541	. 02799	7	. 97278	7	36
25	0. 23203	28	4. 30977	525	0. 23854	31	4. 19215	540	1. 02806	7	0. 97271	7	35
26	. 23231	29	. 30452	523	. 23885	31	. 18675	538	. 02813	7	. 97264	7	34
27	. 23260	28	. 29929	523	. 23916	30	. 18137	537	. 02820	7	. 97257	6	33
28	. 23288	28	. 29406	521	. 23946	31	. 17600	536	. 02827	7	. 97251	7	32
29	. 23316	29	. 28885	519	. 23977	31	. 17064	534	. 02834	8	. 97244	7	31
30	0. 23345	28	4. 28366	519	0. 24008	31	4. 16530	533	1. 02842	7	0. 97237	7	30
31	. 23373	28	. 27847	517	. 24039	30	. 15997	532	. 02849	7	. 97230	7	29
32	. 23401	28	. 27330	516	. 24069	31	. 15465	531	. 02856	7	. 97223	6	28
33	. 23429	29	. 26814	514	. 24100	31	. 14934	529	. 02863	7	. 97217	7	27
34	. 23458	28	. 26300	513	. 24131	31	. 14405	528	. 02870	8	. 97210	7	26
35	0. 23486	28	4. 25787	512	0. 24162	31	4. 13877	527	1. 02878	7	0. 97203	7	25
36	. 23514	28	. 25275	511	. 24193	30	. 13350	525	. 02885	7	. 97196	7	24
37	. 23542	29	. 24764	509	. 24223	31	. 12825	524	. 02892	7	. 97189	7	23
38	. 23571	28	. 24255	509	. 24254	31	. 12301	523	. 02899	8	. 97182	6	22
39	. 23599	28	. 23746	507	. 24285	31	. 11778	522	. 02907	7	. 97176	7	21
40	0. 23627	29	4. 23239	505	0. 24316	31	4. 11256	520	1. 02914	7	0. 97169	7	20
41	. 23656	28	. 22734	505	. 24347	30	. 10736	520	. 02921	7	. 97162	7	19
42	. 23684	28	. 22229	503	. 24377	31	. 10216	517	. 02928	8	. 97155	7	18
43	. 23712	28	. 21726	502	. 24408	31	. 09699	517	. 02936	7	. 97148	7	17
44	. 23740	29	. 21224	501	. 24439	31	. 09182	516	. 02943	7	. 97141	7	16
45	0. 23769	28	4. 20723	499	0. 24470	31	4. 08666	514	1. 02950	8	0. 97134	7	15
46	. 23797	28	. 20224	499	. 24501	31	. 08152	513	. 02958	7	. 97127	7	14
47	. 23825	28	. 19725	497	. 24532	30	. 07639	512	. 02965	7	. 97120	7	13
48	. 23853	29	. 19228	495	. 24562	31	. 07127	511	. 02972	8	. 97113	7	12
49	. 23882	28	. 18733	495	. 24593	31	. 06616	509	. 02980	7	. 97106	6	11
50	0. 23910	28	4. 18238	494	0. 24624	31	4. 06107	508	1. 02987	7	0. 97100	7	10
51	. 23938	28	. 17744	492	. 24655	31	. 05599	507	. 02994	8	. 97093	7	9
52	. 23966	29	. 17252	491	. 24686	31	. 05092	506	. 03002	7	. 97086	7	8
53	. 23995	28	. 16761	490	. 24717	30	. 04586	505	. 03009	8	. 97079	7	7
54	. 24023	28	. 16271	489	. 24747	31	. 04081	503	. 03017	7	. 97072	7	6
55	0. 24051	28	4. 15782	487	0. 24778	31	4. 03578	502	1. 03024	8	0. 97065	7	5
56	. 24079	29	. 15295	486	. 24809	31	. 03076	502	. 03032	7	. 97058	7	4
57	. 24108	28	. 14809	486	. 24840	31	. 02574	500	. 03039	7	. 97051	7	3
58	. 24136	28	. 14323	484	. 24871	31	. 02074	498	. 03046	8	. 97044	7	2
59	. 24164	28	. 13839	482	. 24902	31	. 01576	498	. 03054	7	. 97037	7	1
60	0. 24192		4. 13357		0. 24933		4. 01078		1. 03061		0. 97030		0
↑ 103° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	← 76° ↑

TABLE 31

Natural Trigonometric Functions

14°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←165° ↓ ′
0	0.24192	28	4.13357	482	0.24933	31	4.01078	496	1.03061	8	0.97030	7	60
1	.24220	29	.12875	481	.24964	31	.00582	496	.03069	7	.97023	8	59
2	.24249	28	.12394	479	.24995	31	4.00086	494	.03076	8	.97015	7	58
3	.24277	28	.11915	478	.25026	30	3.99592	493	.03084	7	.97008	7	57
4	.24305	28	.11437	477	.25056	31	.99099	492	.03091	8	.97001	7	56
5	0.24333	29	4.10960	476	0.25087	31	3.98607	490	1.03099	7	0.96994	7	55
6	.24362	28	.10484	475	.25118	31	.98117	490	.03106	8	.96987	7	54
7	.24390	28	.10009	474	.25149	31	.97627	488	.03114	7	.96980	7	53
8	.24418	28	.09535	472	.25180	31	.97139	488	.03121	8	.96973	7	52
9	.24446	28	.09063	472	.25211	31	.96651	486	.03129	8	.96966	7	51
10	0.24474	29	4.08591	470	0.25242	31	3.96165	485	1.03137	7	0.96959	7	50
11	.24503	28	.08121	469	.25273	31	.95680	484	.03144	8	.96952	7	49
12	.24531	28	.07652	468	.25304	31	.95196	483	.03152	7	.96945	8	48
13	.24559	28	.07184	467	.25335	31	.94713	481	.03159	8	.96937	7	47
14	.24587	28	.06717	466	.25366	31	.94232	481	.03167	8	.96930	7	46
15	0.24615	29	4.06251	465	0.25397	31	3.93751	480	1.03175	7	0.96923	7	45
16	.24644	28	.05786	464	.25428	31	.93271	478	.03182	8	.96916	7	44
17	.24672	28	.05322	462	.25459	31	.92793	477	.03190	7	.96909	7	43
18	.24700	28	.04860	462	.25490	31	.92316	477	.03197	8	.96902	8	42
19	.24728	28	.04398	460	.25521	31	.91839	475	.03205	8	.96894	7	41
20	0.24756	28	4.03938	459	0.25552	31	3.91364	474	1.03213	7	0.96887	7	40
21	.24784	29	.03479	459	.25583	31	.90890	473	.03220	8	.96880	7	39
22	.24813	28	.03020	457	.25614	31	.90417	472	.03228	8	.96873	7	38
23	.24841	28	.02563	456	.25645	31	.89945	471	.03236	8	.96866	8	37
24	.24869	28	.02107	455	.25676	31	.89474	470	.03244	7	.96858	7	36
25	0.24897	28	4.01652	454	0.25707	31	3.89004	468	1.03251	8	0.96851	7	35
26	.24925	29	.01198	453	.25738	31	.88536	468	.03259	8	.96844	7	34
27	.24954	28	.00745	452	.25769	31	.88068	467	.03267	8	.96837	8	33
28	.24982	28	4.00293	450	.25800	31	.87601	465	.03275	7	.96829	7	32
29	.25010	28	3.99843	450	.25831	31	.87136	465	.03282	8	.96822	7	31
30	0.25038	28	3.99393	449	0.25862	31	3.86671	463	1.03290	8	0.96815	8	30
31	.25066	28	.98944	447	.25893	31	.86208	463	.03298	8	.96807	7	29
32	.25094	28	.98497	447	.25924	31	.85745	461	.03306	7	.96800	7	28
33	.25122	29	.98050	446	.25955	31	.85284	460	.03313	8	.96793	7	27
34	.25151	28	.97604	444	.25986	31	.84824	460	.03321	8	.96786	8	26
35	0.25179	28	3.97160	444	0.26017	31	3.84364	458	1.03329	8	0.96778	7	25
36	.25207	28	.96716	442	.26048	31	.83906	457	.03337	8	.96771	7	24
37	.25235	28	.96274	442	.26079	31	.83449	457	.03345	8	.96764	8	23
38	.25263	28	.95832	440	.26110	31	.82992	455	.03353	7	.96756	7	22
39	.25291	29	.95392	440	.26141	31	.82537	454	.03360	8	.96749	7	21
40	0.25320	28	3.94952	438	0.26172	31	3.82083	453	1.03368	8	0.96742	8	20
41	.25348	28	.94514	438	.26203	32	.81630	453	.03376	8	.96734	7	19
42	.25376	28	.94076	436	.26235	31	.81177	451	.03384	8	.96727	8	18
43	.25404	28	.93640	436	.26266	31	.80726	450	.03392	8	.96719	7	17
44	.25432	28	.93204	434	.26297	31	.80276	449	.03400	8	.96712	7	16
45	0.25460	28	3.92770	433	0.26328	31	3.79827	449	1.03408	8	0.96705	8	15
46	.25488	28	.92337	433	.26359	31	.79378	447	.03416	8	.96697	7	14
47	.25516	29	.91904	431	.26390	31	.78931	446	.03424	8	.96690	8	13
48	.25545	28	.91473	431	.26421	31	.78485	445	.03432	7	.96682	7	12
49	.25573	28	.91042	429	.26452	31	.78040	445	.03439	8	.96675	8	11
50	0.25601	28	3.90613	429	0.26483	32	3.77595	443	1.03447	8	0.96667	7	10
51	.25629	28	.90184	428	.26515	31	.77152	443	.03455	8	.96660	7	9
52	.25657	28	.89756	426	.26546	31	.76709	441	.03463	8	.96653	8	8
53	.25685	28	.89330	426	.26577	31	.76268	440	.03471	8	.96645	7	7
54	.25713	28	.88904	425	.26608	31	.75828	440	.03479	8	.96638	8	6
55	0.25741	28	3.88479	423	0.26639	31	3.75388	438	1.03487	8	0.96630	7	5
56	.25769	29	.88056	423	.26670	31	.74950	438	.03495	8	.96623	8	4
57	.25798	28	.87633	422	.26701	32	.74512	437	.03503	8	.96615	7	3
58	.25826	28	.87211	421	.26733	31	.74075	435	.03511	9	.96608	8	2
59	.25854	28	.86790	420	.26764	31	.73640	435	.03520	8	.96600	7	1
60	0.25882		3.86370		0.26795		3.73205		1.03528		0.96593		0
↑ 104°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←75° ↑

TABLE 31

Natural Trigonometric Functions

15°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←164° ↓ ′
0	0. 25882	28	3. 86370	419	0. 26795	31	3. 73205	434	1. 03528	8	0. 96593	8	60
1	. 25910	28	. 85951	418	. 26826	31	. 72771	433	. 03536	8	. 96585	7	59
2	. 25938	28	. 85533	417	. 26857	31	. 72338	431	. 03544	8	. 96578	8	58
3	. 25966	28	. 85116	416	. 26888	32	. 71907	431	. 03552	8	. 96570	8	57
4	. 25994	28	. 84700	415	. 26920	31	. 71476	430	. 03560	8	. 96562	7	56
5	0. 26022	28	3. 84285	414	0. 26951	31	3. 71046	430	1. 03568	8	0. 96555	8	55
6	. 26050	29	. 83871	414	. 26982	31	. 70616	428	. 03576	8	. 96547	7	54
7	. 26079	28	. 83457	412	. 27013	31	. 70188	427	. 03584	8	. 96540	8	53
8	. 26107	28	. 83045	412	. 27044	32	. 69761	426	. 03592	9	. 96532	8	52
9	. 26135	28	. 82633	410	. 27076	31	. 69335	426	. 03601	8	. 96524	7	51
10	0. 26163	28	3. 82223	410	0. 27107	31	3. 68909	424	1. 03609	8	0. 96517	8	50
11	. 26191	28	. 81813	409	. 27138	31	. 68485	424	. 03617	8	. 96509	7	49
12	. 26219	28	. 81404	408	. 27169	32	. 68061	423	. 03625	8	. 96502	8	48
13	. 26247	28	. 80996	407	. 27201	31	. 67638	421	. 03633	9	. 96494	8	47
14	. 26275	28	. 80589	406	. 27232	31	. 67217	421	. 03642	8	. 96486	7	46
15	0. 26303	28	3. 80183	405	0. 27263	31	3. 66796	420	1. 03650	8	0. 96479	8	45
16	. 26331	28	. 79778	404	. 27294	32	. 66376	419	. 03658	8	. 96471	8	44
17	. 26359	28	. 79374	404	. 27326	31	. 65957	419	. 03666	8	. 96463	7	43
18	. 26387	28	. 78970	402	. 27357	31	. 65538	417	. 03674	9	. 96456	8	42
19	. 26415	28	. 78568	402	. 27388	31	. 65121	416	. 03683	8	. 96448	8	41
20	0. 26443	28	3. 78166	401	0. 27419	32	3. 64705	416	1. 03691	8	0. 96440	7	40
21	. 26471	29	. 77765	400	. 27451	31	. 64289	415	. 03699	9	. 96433	8	39
22	. 26500	28	. 77365	399	. 27482	31	. 63874	413	. 03708	8	. 96425	8	38
23	. 26528	28	. 76966	398	. 27513	32	. 63461	413	. 03716	8	. 96417	7	37
24	. 26556	28	. 76568	397	. 27545	31	. 63048	412	. 03724	8	. 96410	8	36
25	0. 26584	28	3. 76171	396	0. 27576	31	3. 62636	412	1. 03732	9	0. 96402	8	35
26	. 26612	28	. 75775	396	. 27607	31	. 62224	410	. 03741	8	. 96394	8	34
27	. 26640	28	. 75379	395	. 27638	32	. 61814	409	. 03749	8	. 96386	7	33
28	. 26668	28	. 74984	393	. 27670	31	. 61405	409	. 03757	9	. 96379	8	32
29	. 26696	28	. 74591	393	. 27701	31	. 60996	408	. 03766	8	. 96371	8	31
30	0. 26724	28	3. 74198	392	0. 27732	32	3. 60588	407	1. 03774	9	0. 96363	8	30
31	. 26752	28	. 73806	392	. 27764	31	. 60181	406	. 03783	8	. 96355	8	29
32	. 26780	28	. 73414	390	. 27795	31	. 59775	405	. 03791	8	. 96347	7	28
33	. 26808	28	. 73024	389	. 27826	32	. 59370	404	. 03799	9	. 96340	8	27
34	. 26836	28	. 72635	389	. 27858	31	. 58966	404	. 03808	8	. 96332	8	26
35	0. 26864	28	3. 72246	388	0. 27889	32	3. 58562	402	1. 03816	9	0. 96324	8	25
36	. 26892	28	. 71858	387	. 27921	31	. 58160	402	. 03825	8	. 96316	8	24
37	. 26920	28	. 71471	386	. 27952	31	. 57758	401	. 03833	9	. 96308	7	23
38	. 26948	28	. 71085	385	. 27983	32	. 57357	400	. 03842	8	. 96301	8	22
39	. 26976	28	. 70700	385	. 28015	31	. 56957	400	. 03850	8	. 96293	8	21
40	0. 27004	28	3. 70315	384	0. 28046	31	3. 56557	398	1. 03858	9	0. 96285	8	20
41	. 27032	28	. 69931	382	. 28077	32	. 56159	398	. 03867	8	. 96277	8	19
42	. 27060	28	. 69549	382	. 28109	31	. 55761	397	. 03875	9	. 96269	8	18
43	. 27088	28	. 69167	382	. 28140	32	. 55364	396	. 03884	8	. 96261	8	17
44	. 27116	28	. 68785	380	. 28172	31	. 54968	395	. 03892	9	. 96253	7	16
45	0. 27144	28	3. 68405	380	0. 28203	31	3. 54573	394	1. 03901	8	0. 96246	8	15
46	. 27172	28	. 68025	378	. 28234	32	. 54179	394	. 03909	9	. 96238	8	14
47	. 27200	28	. 67647	378	. 28266	31	. 53785	392	. 03918	9	. 96230	8	13
48	. 27228	28	. 67269	377	. 28297	32	. 53393	392	. 03927	8	. 96222	8	12
49	. 27256	28	. 66892	377	. 28329	31	. 53001	392	. 03935	9	. 96214	8	11
50	0. 27284	28	3. 66515	375	0. 28360	31	3. 52609	390	1. 03944	8	0. 96206	8	10
51	. 27312	28	. 66140	375	. 28391	32	. 52219	390	. 03952	9	. 96198	8	9
52	. 27340	28	. 65765	374	. 28423	31	. 51829	388	. 03961	8	. 96190	8	8
53	. 27368	28	. 65391	373	. 28454	32	. 51441	388	. 03969	9	. 96182	8	7
54	. 27396	28	. 65018	373	. 28486	31	. 51053	387	. 03978	9	. 96174	8	6
55	0. 27424	28	3. 64645	371	0. 28517	32	3. 50666	387	1. 03987	8	0. 96166	8	5
56	. 27452	28	. 64274	371	. 28549	31	. 50279	385	. 03995	9	. 96158	8	4
57	. 27480	28	. 63903	370	. 28580	32	. 49894	385	. 04004	9	. 96150	8	3
58	. 27508	28	. 63533	369	. 28612	31	. 49509	384	. 04013	8	. 96142	8	2
59	. 27536	28	. 63164	368	. 28643	32	. 49125	384	. 04021	9	. 96134	8	1
60	0. 27564		3. 62796		0. 28675		3. 48741		1. 04030		0. 96126		0
↑ 105°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←74° ↑

TABLE 31

Natural Trigonometric Functions

16°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←163° ↓ ′
0	0. 27564	28	3. 62796	368	0. 28675	31	3. 48741	382	1. 04030	9	0. 96126	8	60
1	. 27592	28	. 62428	367	. 28706	32	. 48359	382	. 04039	8	. 96118	8	59
2	. 27620	28	. 62061	366	. 28738	31	. 47977	381	. 04047	9	. 96110	8	58
3	. 27648	28	. 61695	365	. 28769	32	. 47596	380	. 04056	9	. 96102	8	57
4	. 27676	28	. 61330	365	. 28801	31	. 47216	379	. 04065	8	. 96094	8	56
5	0. 27704	27	3. 60965	364	0. 28832	32	3. 46837	379	1. 04073	9	0. 96086	8	55
6	. 27731	28	. 60601	363	. 28864	31	. 46458	378	. 04082	9	. 96078	8	54
7	. 27759	28	. 60238	362	. 28895	32	. 46080	377	. 04091	9	. 96070	8	53
8	. 27787	28	. 59876	362	. 28927	31	. 45703	376	. 04100	8	. 96062	8	52
9	. 27815	28	. 59514	360	. 28958	32	. 45327	376	. 04108	9	. 96054	8	51
10	0. 27843	28	3. 59154	360	0. 28990	31	3. 44951	375	1. 04117	9	0. 96046	9	50
11	. 27871	28	. 58794	360	. 29021	32	. 44576	374	. 04126	9	. 96037	8	49
12	. 27899	28	. 58434	358	. 29053	31	. 44202	373	. 04135	9	. 96029	8	48
13	. 27927	28	. 58076	358	. 29084	32	. 43829	373	. 04144	8	. 96021	8	47
14	. 27955	28	. 57718	357	. 29116	31	. 43456	372	. 04152	9	. 96013	8	46
15	0. 27983	28	3. 57361	356	0. 29147	32	3. 43084	371	1. 04161	9	0. 96005	8	45
16	. 28011	28	. 57005	356	. 29179	31	. 42713	370	. 04170	9	. 95997	8	44
17	. 28039	28	. 56649	355	. 29210	32	. 42343	370	. 04179	9	. 95989	8	43
18	. 28067	28	. 56294	354	. 29242	32	. 41973	369	. 04188	9	. 95981	9	42
19	. 28095	28	. 55940	353	. 29274	31	. 41604	368	. 04197	9	. 95972	8	41
20	0. 28123	27	3. 55587	353	0. 29305	32	3. 41236	367	1. 04206	8	0. 95964	8	40
21	. 28150	28	. 55234	351	. 29337	31	. 40869	367	. 04214	9	. 95956	8	39
22	. 28178	28	. 54883	352	. 29368	32	. 40502	366	. 04223	9	. 95948	8	38
23	. 28206	28	. 54531	350	. 29400	32	. 40136	365	. 04232	9	. 95940	9	37
24	. 28234	28	. 54181	350	. 29432	31	. 39771	365	. 04241	9	. 95931	8	36
25	0. 28262	28	3. 53831	349	0. 29463	32	3. 39406	364	1. 04250	9	0. 95923	8	35
26	. 28290	28	. 53482	348	. 29495	31	. 39042	363	. 04259	9	. 95915	8	34
27	. 28318	28	. 53134	347	. 29526	32	. 38679	362	. 04268	9	. 95907	9	33
28	. 28346	28	. 52787	347	. 29558	32	. 38317	362	. 04277	9	. 95898	8	32
29	. 28374	28	. 52440	346	. 29590	31	. 37955	361	. 04286	9	. 95890	8	31
30	0. 28402	27	3. 52094	346	0. 29621	32	3. 37594	360	1. 04295	9	0. 95882	8	30
31	. 28429	28	. 51748	344	. 29653	32	. 37234	359	. 04304	9	. 95874	9	29
32	. 28457	28	. 51404	344	. 29685	31	. 36875	359	. 04313	9	. 95865	8	28
33	. 28485	28	. 51060	344	. 29716	32	. 36516	358	. 04322	9	. 95857	8	27
34	. 28513	28	. 50716	342	. 29748	32	. 36158	358	. 04331	9	. 95849	8	26
35	0. 28541	28	3. 50374	342	0. 29780	31	3. 35800	357	1. 04340	9	0. 95841	9	25
36	. 28569	28	. 50032	341	. 29811	32	. 35443	356	. 04349	9	. 95832	8	24
37	. 28597	28	. 49691	341	. 29843	32	. 35087	355	. 04358	9	. 95824	8	23
38	. 28625	27	. 49350	340	. 29875	31	. 34732	355	. 04367	9	. 95816	9	22
39	. 28652	28	. 49010	339	. 29906	32	. 34377	354	. 04376	9	. 95807	8	21
40	0. 28680	28	3. 48671	338	0. 29938	32	3. 34023	353	1. 04385	9	0. 95799	8	20
41	. 28708	28	. 48333	338	. 29970	31	. 33670	353	. 04394	9	. 95791	9	19
42	. 28736	28	. 47995	337	. 30001	32	. 33317	352	. 04403	10	. 95782	8	18
43	. 28764	28	. 47658	337	. 30033	32	. 32965	351	. 04413	9	. 95774	8	17
44	. 28792	28	. 47321	335	. 30065	32	. 32614	350	. 04422	9	. 95766	9	16
45	0. 28820	27	3. 46986	335	0. 30097	31	3. 32264	350	1. 04431	9	0. 95757	8	15
46	. 28847	28	. 46651	335	. 30128	32	. 31914	349	. 04440	9	. 95749	9	14
47	. 28875	28	. 46316	333	. 30160	32	. 31565	349	. 04449	9	. 95740	8	13
48	. 28903	28	. 45983	333	. 30192	32	. 31216	348	. 04458	10	. 95732	8	12
49	. 28931	28	. 45650	333	. 30224	31	. 30868	347	. 04468	9	. 95724	9	11
50	0. 28959	28	3. 45317	331	0. 30255	32	3. 30521	347	1. 04477	9	0. 95715	8	10
51	. 28987	28	. 44986	331	. 30287	32	. 30174	345	. 04486	9	. 95707	9	9
52	. 29015	27	. 44655	331	. 30319	32	. 29829	346	. 04495	9	. 95698	8	8
53	. 29042	28	. 44324	329	. 30351	31	. 29483	344	. 04504	10	. 95690	9	7
54	. 29070	28	. 43995	329	. 30382	32	. 29139	344	. 04514	9	. 95681	8	6
55	0. 29098	28	3. 43666	329	0. 30414	32	3. 28795	343	1. 04523	9	0. 95673	9	5
56	. 29126	28	. 43337	327	. 30446	32	. 28452	343	. 04532	9	. 95664	8	4
57	. 29154	28	. 43010	327	. 30478	31	. 28109	342	. 04541	10	. 95656	9	3
58	. 29182	27	. 42683	327	. 30509	32	. 27767	341	. 04551	9	. 95647	8	2
59	. 29209	28	. 42356	326	. 30541	32	. 27426	341	. 04560	9	. 95639	9	1
60	0. 29237		3. 42030		0. 30573		3. 27085		1. 04569		0. 95630		0
↑ 106°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ←73°

TABLE 31

Natural Trigonometric Functions

17°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←162° ↓ ′
0	0. 29237	28	3. 42030	325	0. 30573	32	3. 27085	340	1. 04569	9	0. 95630	8	60
1	. 29265	28	. 41705	324	. 30605	32	. 26745	339	. 04578	10	. 95622	9	59
2	. 29293	28	. 41381	324	. 30637	32	. 26406	339	. 04588	9	. 95613	8	58
3	. 29321	27	. 41057	323	. 30669	31	. 26067	338	. 04597	9	. 95605	9	57
4	. 29348	28	. 40734	323	. 30700	32	. 25729	337	. 04606	10	. 95596	8	56
5	0. 29376	28	3. 40411	322	0. 30732	32	3. 25392	337	1. 04616	9	0. 95588	9	55
6	. 29404	28	. 40089	321	. 30764	32	. 25055	336	. 04625	10	. 95579	8	54
7	. 29432	28	. 39768	320	. 30796	32	. 24719	336	. 04635	9	. 95571	9	53
8	. 29460	27	. 39448	320	. 30828	32	. 24383	334	. 04644	9	. 95562	8	52
9	. 29487	28	. 39128	320	. 30860	31	. 24049	335	. 04653	10	. 95554	9	51
10	0. 29515	28	3. 38808	319	0. 30891	32	3. 23714	333	1. 04663	9	0. 95545	9	50
11	. 29543	28	. 38489	318	. 30923	32	. 23381	333	. 04672	10	. 95536	8	49
12	. 29571	28	. 38171	317	. 30955	32	. 23048	333	. 04682	9	. 95528	9	48
13	. 29599	27	. 37854	317	. 30987	32	. 22715	331	. 04691	9	. 95519	8	47
14	. 29626	28	. 37537	316	. 31019	32	. 22384	331	. 04700	10	. 95511	9	46
15	0. 29654	28	3. 37221	316	0. 31051	32	3. 22053	331	1. 04710	9	0. 95502	9	45
16	. 29682	28	. 36905	315	. 31083	32	. 21722	330	. 04719	10	. 95493	8	44
17	. 29710	27	. 36590	314	. 31115	32	. 21392	329	. 04729	9	. 95485	9	43
18	. 29737	28	. 36276	314	. 31147	31	. 21063	329	. 04738	10	. 95476	9	42
19	. 29765	28	. 35962	313	. 31178	32	. 20734	328	. 04748	9	. 95467	8	41
20	0. 29793	28	3. 35649	313	0. 31210	32	3. 20406	327	1. 04757	10	0. 95459	9	40
21	. 29821	28	. 35336	311	. 31242	32	. 20079	327	. 04767	9	. 95450	9	39
22	. 29849	27	. 35025	312	. 31274	32	. 19752	326	. 04776	10	. 95441	8	38
23	. 29876	28	. 34713	310	. 31306	32	. 19426	326	. 04786	9	. 95433	9	37
24	. 29904	28	. 34403	311	. 31338	32	. 19100	325	. 04795	10	. 95424	9	36
25	0. 29932	28	3. 34092	309	0. 31370	32	3. 18775	324	1. 04805	10	0. 95415	8	35
26	. 29960	27	. 33783	309	. 31402	32	. 18451	324	. 04815	9	. 95407	9	34
27	. 29987	28	. 33474	308	. 31434	32	. 18127	323	. 04824	10	. 95398	9	33
28	. 30015	28	. 33166	308	. 31466	32	. 17804	323	. 04834	9	. 95389	9	32
29	. 30043	28	. 32858	307	. 31498	32	. 17481	322	. 04843	10	. 95380	8	31
30	0. 30071	27	3. 32551	307	0. 31530	32	3. 17159	321	1. 04853	10	0. 95372	9	30
31	. 30098	28	. 32244	305	. 31562	32	. 16838	321	. 04863	9	. 95363	9	29
32	. 30126	28	. 31939	306	. 31594	32	. 16517	320	. 04872	10	. 95354	9	28
33	. 30154	28	. 31633	305	. 31626	32	. 16197	320	. 04882	9	. 95345	8	27
34	. 30182	27	. 31328	304	. 31658	32	. 15877	319	. 04891	10	. 95337	9	26
35	0. 30209	28	3. 31024	303	0. 31690	32	3. 15558	318	1. 04901	10	0. 95328	9	25
36	. 30237	28	. 30721	303	. 31722	32	. 15240	318	. 04911	9	. 95319	9	24
37	. 30265	27	. 30418	303	. 31754	32	. 14922	317	. 04920	10	. 95310	9	23
38	. 30292	28	. 30115	301	. 31786	32	. 14605	317	. 04930	10	. 95301	8	22
39	. 30320	28	. 29814	302	. 31818	32	. 14288	316	. 04940	10	. 95293	9	21
40	0. 30348	28	3. 29512	300	0. 31850	32	3. 13972	316	1. 04950	9	0. 95284	9	20
41	. 30376	27	. 29212	300	. 31882	32	. 13656	315	. 04959	10	. 95275	9	19
42	. 30403	28	. 28912	300	. 31914	32	. 13341	314	. 04969	10	. 95266	9	18
43	. 30431	28	. 28612	299	. 31946	32	. 13027	314	. 04979	10	. 95257	9	17
44	. 30459	27	. 28313	298	. 31978	32	. 12713	313	. 04989	9	. 95248	8	16
45	0. 30486	28	3. 28015	298	0. 32010	32	3. 12400	313	1. 04998	10	0. 95240	9	15
46	. 30514	28	. 27717	297	. 32042	32	. 12087	312	. 05008	10	. 95231	9	14
47	. 30542	28	. 27420	297	. 32074	32	. 11775	311	. 05018	10	. 95222	9	13
48	. 30570	27	. 27123	296	. 32106	33	. 11464	311	. 05028	10	. 95213	9	12
49	. 30597	28	. 26827	296	. 32139	32	. 11153	311	. 05038	9	. 95204	9	11
50	0. 30625	28	3. 26531	294	0. 32171	32	3. 10842	310	1. 05047	10	0. 95195	9	10
51	. 30653	27	. 26237	295	. 32203	32	. 10532	309	. 05057	10	. 95186	9	9
52	. 30680	28	. 25942	294	. 32235	32	. 10223	309	. 05067	10	. 95177	9	8
53	. 30708	28	. 25648	293	. 32267	32	. 09914	308	. 05077	10	. 95168	9	7
54	. 30736	27	. 25355	293	. 32299	32	. 09606	308	. 05087	10	. 95159	9	6
55	0. 30763	28	3. 25062	292	0. 32331	32	3. 09298	307	1. 05097	10	0. 95150	8	5
56	. 30791	28	. 24770	292	. 32363	33	. 08991	306	. 05107	9	. 95142	9	4
57	. 30819	27	. 24478	291	. 32396	32	. 08685	306	. 05116	10	. 95133	9	3
58	. 30846	28	. 24187	290	. 32428	32	. 08379	306	. 05126	10	. 95124	9	2
59	. 30874	28	. 23897	290	. 32460	32	. 08073	305	. 05136	10	. 95115	9	1
60	0. 30902		3. 23607		0. 32492		3. 07768		1. 05146		0. 95106		0
↑ 107°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←72° ↑

TABLE 31

Natural Trigonometric Functions

18°→ ↓ '	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←161° ↓ '
0	0.30902	27	3.23607	290	0.32492	32	3.07768	304	1.05146	10	0.95106	9	60
1	.30929	28	.23317	289	.32524	32	.07464	304	.05156	10	.95097	9	59
2	.30957	28	.23028	288	.32556	32	.07160	303	.05166	10	.95088	9	58
3	.30985	27	.22740	288	.32588	33	.06857	303	.05176	10	.95079	9	57
4	.31012	28	.22452	287	.32621	32	.06554	302	.05186	10	.95070	9	56
5	0.31040	28	3.22165	287	0.32653	32	3.06252	302	1.05196	10	0.95061	9	55
6	.31068	27	.21878	286	.32685	32	.05950	301	.05206	10	.95052	9	54
7	.31095	28	.21592	286	.32717	32	.05649	300	.05216	10	.95043	10	53
8	.31123	28	.21306	285	.32749	33	.05349	300	.05226	10	.95033	9	52
9	.31151	27	.21021	284	.32782	32	.05049	300	.05236	10	.95024	9	51
10	0.31178	28	3.20737	284	0.32814	32	3.04749	299	1.05246	10	0.95015	9	50
11	.31206	27	.20453	284	.32846	32	.04450	298	.05256	10	.95006	9	49
12	.31233	28	.20169	283	.32878	33	.04152	298	.05266	10	.94997	9	48
13	.31261	28	.19886	282	.32911	32	.03854	298	.05276	10	.94988	9	47
14	.31289	27	.19604	282	.32943	32	.03556	296	.05286	11	.94979	9	46
15	0.31316	28	3.19322	282	0.32975	32	3.03260	297	1.05297	10	0.94970	9	45
16	.31344	28	.19040	281	.33007	33	.02963	296	.05307	10	.94961	9	44
17	.31372	27	.18759	280	.33040	32	.02667	295	.05317	10	.94952	9	43
18	.31399	28	.18479	280	.33072	32	.02372	295	.05327	10	.94943	10	42
19	.31427	27	.18199	279	.33104	32	.02077	294	.05337	10	.94933	9	41
20	0.31454	28	3.17920	279	0.33136	33	3.01783	294	1.05347	10	0.94924	9	40
21	.31482	28	.17641	278	.33169	32	.01489	293	.05357	10	.94915	9	39
22	.31510	27	.17363	278	.33201	32	.01196	293	.05367	11	.94906	9	38
23	.31537	28	.17085	277	.33233	33	.00903	292	.05378	10	.94897	9	37
24	.31565	28	.16808	277	.33266	32	.00611	292	.05388	10	.94888	10	36
25	0.31593	27	3.16531	276	0.33298	32	3.00319	291	1.05398	10	0.94878	9	35
26	.31620	28	.16255	276	.33330	33	3.00028	290	.05408	10	.94869	9	34
27	.31648	27	.15979	275	.33363	32	2.99738	291	.05418	11	.94860	9	33
28	.31675	28	.15704	275	.33395	32	.99447	289	.05429	10	.94851	9	32
29	.31703	27	.15429	274	.33427	33	.99158	290	.05439	10	.94842	10	31
30	0.31730	28	3.15155	274	0.33460	32	2.98868	288	1.05449	10	0.94832	9	30
31	.31758	28	.14881	273	.33492	32	.98580	288	.05459	11	.94823	9	29
32	.31786	27	.14608	273	.33524	33	.98292	288	.05470	10	.94814	9	28
33	.31813	28	.14335	272	.33557	32	.98004	287	.05480	10	.94805	10	27
34	.31841	27	.14063	272	.33589	32	.97717	287	.05490	11	.94795	9	26
35	0.31868	28	3.13791	271	0.33621	33	2.97430	286	1.05501	10	0.94786	9	25
36	.31896	27	.13520	271	.33654	32	.97144	286	.05511	10	.94777	9	24
37	.31923	28	.13249	270	.33686	32	.96858	285	.05521	11	.94768	10	23
38	.31951	28	.12979	270	.33718	33	.96573	285	.05532	10	.94758	9	22
39	.31979	27	.12709	269	.33751	32	.96288	284	.05542	10	.94749	9	21
40	0.32006	28	3.12440	269	0.33783	33	2.96004	283	1.05552	11	0.94740	10	20
41	.32034	27	.12171	268	.33816	32	.95721	284	.05563	10	.94730	9	19
42	.32061	28	.11903	268	.33848	33	.95437	282	.05573	11	.94721	9	18
43	.32089	27	.11635	268	.33881	32	.95155	283	.05584	10	.94712	10	17
44	.32116	28	.11367	266	.33913	32	.94872	281	.05594	10	.94702	9	16
45	0.32144	27	3.11101	267	0.33945	33	2.94591	282	1.05604	11	0.94693	9	15
46	.32171	28	.10834	266	.33978	32	.94309	281	.05615	10	.94684	10	14
47	.32199	28	.10568	265	.34010	33	.94028	280	.05625	11	.94674	9	13
48	.32227	27	.10303	265	.34043	32	.93748	280	.05636	10	.94665	9	12
49	.32254	28	.10038	264	.34075	33	.93468	279	.05646	11	.94656	10	11
50	0.32282	27	3.09774	264	0.34108	32	2.93189	279	1.05657	10	0.94646	9	10
51	.32309	28	.09510	264	.34140	33	.92910	278	.05667	11	.94637	10	9
52	.32337	27	.09246	263	.34173	32	.92632	278	.05678	10	.94627	9	8
53	.32364	28	.08983	262	.34205	33	.92354	278	.05688	11	.94618	9	7
54	.32392	27	.08721	262	.34238	32	.92076	277	.05699	10	.94609	10	6
55	0.32419	28	3.08459	262	0.34270	33	2.91799	276	1.05709	11	0.94599	9	5
56	.32447	27	.08197	261	.34303	32	.91523	277	.05720	10	.94590	10	4
57	.32474	28	.07936	261	.34335	33	.91246	275	.05730	11	.94580	9	3
58	.32502	27	.07675	260	.34368	32	.90971	275	.05741	10	.94571	10	2
59	.32529	28	.07415	260	.34400	33	.90696	275	.05751	11	.94561	9	1
60	0.32557		3.07155		0.34433		2.90421		1.05762		0.94552		0
↑ 108°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←71° ↑

TABLE 31

Natural Trigonometric Functions

19°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←160° ↓ ′
0	0. 32557	27	3. 07155	259	0. 34433	32	2. 90421	274	1. 05762	11	0. 94552	10	60
1	. 32584	28	. 06896	259	. 34465	33	. 90147	274	. 05773	10	. 94542	9	59
2	. 32612	27	. 06637	258	. 34498	32	. 89873	273	. 05783	11	. 94533	10	58
3	. 32639	28	. 06379	258	. 34530	33	. 89600	273	. 05794	11	. 94523	9	57
4	. 32667	27	. 06121	257	. 34563	33	. 89327	272	. 05805	10	. 94514	10	56
5	0. 32694	28	3. 05864	257	0. 34596	32	2. 89055	272	1. 05815	11	0. 94504	9	55
6	. 32722	27	. 05607	257	. 34628	33	. 88783	272	. 05826	10	. 94495	10	54
7	. 32749	28	. 05350	256	. 34661	32	. 88511	271	. 05836	11	. 94485	9	53
8	. 32777	27	. 05094	255	. 34693	33	. 88240	270	. 05847	11	. 94476	10	52
9	. 32804	28	. 04839	255	. 34726	32	. 87970	270	. 05858	11	. 94466	9	51
10	0. 32832	27	3. 04584	255	0. 34758	33	2. 87700	270	1. 05869	10	0. 94457	10	50
11	. 32859	28	. 04329	254	. 34791	33	. 87430	269	. 05879	11	. 94447	9	49
12	. 32887	27	. 04075	254	. 34824	32	. 87161	269	. 05890	11	. 94438	10	48
13	. 32914	28	. 03821	253	. 34856	33	. 86892	268	. 05901	10	. 94428	10	47
14	. 32942	27	. 03568	253	. 34889	33	. 86624	268	. 05911	11	. 94418	9	46
15	0. 32969	28	3. 03315	253	0. 34922	32	2. 86356	267	1. 05922	11	0. 94409	10	45
16	. 32997	27	. 03062	252	. 34954	33	. 86089	267	. 05933	11	. 94399	9	44
17	. 33024	27	. 02810	251	. 34987	33	. 85822	267	. 05944	11	. 94390	10	43
18	. 33051	28	. 02559	251	. 35020	32	. 85555	266	. 05955	10	. 94380	10	42
19	. 33079	27	. 02308	251	. 35052	33	. 85289	266	. 05965	11	. 94370	9	41
20	0. 33106	28	3. 02057	250	0. 35085	33	2. 85023	265	1. 05976	11	0. 94361	10	40
21	. 33134	27	. 01807	250	. 35118	32	. 84758	264	. 05987	11	. 94351	9	39
22	. 33161	28	. 01557	249	. 35150	33	. 84494	265	. 05998	11	. 94342	10	38
23	. 33189	27	. 01308	249	. 35183	33	. 84229	264	. 06009	11	. 94332	10	37
24	. 33216	28	. 01059	249	. 35216	32	. 83965	263	. 06020	10	. 94322	9	36
25	0. 33244	27	3. 00810	248	0. 35248	33	2. 83702	263	1. 06030	11	0. 94313	10	35
26	. 33271	27	. 00562	247	. 35281	33	. 83439	263	. 06041	11	. 94303	10	34
27	. 33298	28	. 00315	248	. 35314	32	. 83176	262	. 06052	11	. 94293	9	33
28	. 33326	27	3. 00067	246	. 35346	33	. 82914	261	. 06063	11	. 94284	10	32
29	. 33353	28	2. 99821	247	. 35379	33	. 82653	262	. 06074	11	. 94274	10	31
30	0. 33381	27	2. 99574	245	0. 35412	33	2. 82391	261	1. 06085	11	0. 94264	10	30
31	. 33408	28	. 99329	246	. 35445	32	. 82130	260	. 06096	11	. 94254	9	29
32	. 33436	27	. 99083	245	. 35477	33	. 81870	260	. 06107	11	. 94245	10	28
33	. 33463	27	. 98838	244	. 35510	33	. 81610	260	. 06118	11	. 94235	10	27
34	. 33490	28	. 98594	245	. 35543	33	. 81350	259	. 06129	11	. 94225	10	26
35	0. 33518	27	2. 98349	243	0. 35576	32	2. 81091	258	1. 06140	11	0. 94215	9	25
36	. 33545	28	. 98106	244	. 35608	33	. 80833	259	. 06151	11	. 94206	10	24
37	. 33573	27	. 97862	243	. 35641	33	. 80574	258	. 06162	11	. 94196	10	23
38	. 33600	27	. 97619	242	. 35674	33	. 80316	257	. 06173	11	. 94186	10	22
39	. 33627	28	. 97377	242	. 35707	33	. 80059	257	. 06184	11	. 94176	9	21
40	0. 33655	27	2. 97135	242	0. 35740	32	2. 79802	257	1. 06195	11	0. 94167	10	20
41	. 33682	28	. 96893	241	. 35772	33	. 79545	256	. 06206	11	. 94157	10	19
42	. 33710	27	. 96652	241	. 35805	33	. 79289	256	. 06217	11	. 94147	10	18
43	. 33737	27	. 96411	240	. 35838	33	. 79033	255	. 06228	11	. 94137	10	17
44	. 33764	28	. 96171	240	. 35871	33	. 78778	255	. 06239	11	. 94127	9	16
45	0. 33792	27	2. 95931	240	0. 35904	33	2. 78523	254	1. 06250	11	0. 94118	10	15
46	. 33819	27	. 95691	239	. 35937	32	. 78269	255	. 06261	11	. 94108	10	14
47	. 33846	28	. 95452	239	. 35969	33	. 78014	253	. 06272	11	. 94098	10	13
48	. 33874	27	. 95213	238	. 36002	33	. 77761	254	. 06283	12	. 94088	10	12
49	. 33901	28	. 94975	238	. 36035	33	. 77507	253	. 06295	11	. 94078	10	11
50	0. 33929	27	2. 94737	237	0. 36068	33	2. 77254	252	1. 06306	11	0. 94068	10	10
51	. 33956	27	. 94500	237	. 36101	33	. 77002	252	. 06317	11	. 94058	9	9
52	. 33983	28	. 94263	237	. 36134	33	. 76750	252	. 06328	11	. 94049	10	8
53	. 34011	27	. 94026	236	. 36167	32	. 76498	251	. 06339	11	. 94039	10	7
54	. 34038	27	. 93790	236	. 36199	33	. 76247	251	. 06350	12	. 94029	10	6
55	0. 34065	28	2. 93554	236	0. 36232	33	2. 75996	250	1. 06362	11	0. 94019	10	5
56	. 34093	27	. 93318	235	. 36265	33	. 75746	250	. 06373	11	. 94009	10	4
57	. 34120	27	. 93083	234	. 36298	33	. 75496	250	. 06384	11	. 93999	10	3
58	. 34147	28	. 92849	235	. 36331	33	. 75246	249	. 06395	12	. 93989	10	2
59	. 34175	27	. 92614	234	. 36364	33	. 74997	249	. 06407	11	. 93979	10	1
60	0. 34202		2. 92380		0. 36397		2. 74748		1. 06418		0. 93969		0
↑ 109°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′←70° ↑	

TABLE 31

Natural Trigonometric Functions

20°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←159° ↓ ′
0	0. 34202	27	2. 92380	233	0. 36397	33	2. 74748	249	1. 06418	11	0. 93969	10	60
1	. 34229	28	. 92147	233	. 36430	33	. 74499	248	. 06429	11	. 93959	10	59
2	. 34257	27	. 91914	233	. 36463	33	. 74251	247	. 06440	12	. 93949	10	58
3	. 34284	27	. 91681	232	. 36496	33	. 74004	248	. 06452	11	. 93939	10	57
4	. 34311	28	. 91449	232	. 36529	33	. 73756	247	. 06463	11	. 93929	10	56
5	0. 34339	27	2. 91217	231	0. 36562	33	2. 73509	246	1. 06474	12	0. 93919	10	55
6	. 34366	27	. 90986	232	. 36595	33	. 73263	246	. 06486	11	. 93909	10	54
7	. 34393	28	. 90754	230	. 36628	33	. 73017	246	. 06497	11	. 93899	10	53
8	. 34421	27	. 90524	231	. 36661	33	. 72771	245	. 06508	12	. 93889	10	52
9	. 34448	27	. 90293	230	. 36694	33	. 72526	245	. 06520	11	. 93879	10	51
10	0. 34475	28	2. 90063	229	0. 36727	33	2. 72281	245	1. 06531	11	0. 93869	10	50
11	. 34503	27	. 89834	229	. 36760	33	. 72036	244	. 06542	12	. 93859	10	49
12	. 34530	27	. 89605	229	. 36793	33	. 71792	244	. 06554	11	. 93849	10	48
13	. 34557	27	. 89376	228	. 36826	33	. 71548	243	. 06565	12	. 93839	10	47
14	. 34584	28	. 89148	228	. 36859	33	. 71305	243	. 06577	11	. 93829	10	46
15	0. 34612	27	2. 88920	228	0. 36892	33	2. 71062	243	1. 06588	12	0. 93819	10	45
16	. 34639	27	. 88692	227	. 36925	33	. 70819	242	. 06600	11	. 93809	10	44
17	. 34666	28	. 88465	227	. 36958	33	. 70577	242	. 06611	11	. 93799	10	43
18	. 34694	27	. 88238	227	. 36991	33	. 70335	241	. 06622	12	. 93789	10	42
19	. 34721	27	. 88011	226	. 37024	33	. 70094	241	. 06634	11	. 93779	10	41
20	0. 34748	27	2. 87785	225	0. 37057	33	2. 69853	241	1. 06645	12	0. 93769	10	40
21	. 34775	28	. 87560	226	. 37090	33	. 69612	241	. 06657	11	. 93759	11	39
22	. 34803	27	. 87334	225	. 37123	34	. 69371	240	. 06668	12	. 93748	10	38
23	. 34830	27	. 87109	224	. 37157	33	. 69131	239	. 06680	11	. 93738	10	37
24	. 34857	27	. 86885	224	. 37190	33	. 68892	239	. 06691	12	. 93728	10	36
25	0. 34884	28	2. 86661	224	0. 37223	33	2. 68653	239	1. 06703	12	0. 93718	10	35
26	. 34912	27	. 86437	224	. 37256	33	. 68414	239	. 06715	11	. 93708	10	34
27	. 34939	27	. 86213	223	. 37289	33	. 68175	238	. 06726	12	. 93698	10	33
28	. 34966	27	. 85990	223	. 37322	33	. 67937	237	. 06738	11	. 93688	11	32
29	. 34993	28	. 85767	222	. 37355	33	. 67700	238	. 06749	12	. 93677	10	31
30	0. 35021	27	2. 85545	222	0. 37388	34	2. 67462	237	1. 06761	12	0. 93667	10	30
31	. 35048	27	. 85323	221	. 37422	33	. 67225	236	. 06773	11	. 93657	10	29
32	. 35075	27	. 85102	222	. 37455	33	. 66989	237	. 06784	12	. 93647	10	28
33	. 35102	28	. 84880	221	. 37488	33	. 66752	236	. 06796	11	. 93637	11	27
34	. 35130	27	. 84659	220	. 37521	33	. 66516	235	. 06807	12	. 93626	10	26
35	0. 35157	27	2. 84439	220	0. 37554	34	2. 66281	235	1. 06819	12	0. 93616	10	25
36	. 35184	27	. 84219	220	. 37588	33	. 66046	235	. 06831	11	. 93606	10	24
37	. 35211	28	. 83999	219	. 37621	33	. 65811	235	. 06842	12	. 93596	11	23
38	. 35239	27	. 83780	219	. 37654	33	. 65576	234	. 06854	12	. 93585	10	22
39	. 35266	27	. 83561	219	. 37687	33	. 65342	233	. 06866	12	. 93575	10	21
40	0. 35293	27	2. 83342	218	0. 37720	34	2. 65109	234	1. 06878	11	0. 93565	10	20
41	. 35320	27	. 83124	218	. 37754	33	. 64875	233	. 06889	12	. 93555	11	19
42	. 35347	28	. 82906	218	. 37787	33	. 64642	232	. 06901	12	. 93544	10	18
43	. 35375	27	. 82688	217	. 37820	33	. 64410	233	. 06913	12	. 93534	10	17
44	. 35402	27	. 82471	217	. 37853	34	. 64177	232	. 06925	11	. 93524	10	16
45	0. 35429	27	2. 82254	217	0. 37887	33	2. 63945	231	1. 06936	12	0. 93514	11	15
46	. 35456	28	. 82037	216	. 37920	33	. 63714	231	. 06948	12	. 93503	10	14
47	. 35484	27	. 81821	216	. 37953	33	. 63483	231	. 06960	12	. 93493	10	13
48	. 35511	27	. 81605	215	. 37986	34	. 63252	231	. 06972	12	. 93483	11	12
49	. 35538	27	. 81390	215	. 38020	33	. 63021	230	. 06984	11	. 93472	10	11
50	0. 35565	27	2. 81175	215	0. 38053	33	2. 62791	230	1. 06995	12	0. 93462	10	10
51	. 35592	27	. 80960	214	. 38086	34	. 62561	229	. 07007	12	. 93452	11	9
52	. 35619	28	. 80746	215	. 38120	33	. 62332	229	. 07019	12	. 93441	10	8
53	. 35647	27	. 80531	213	. 38153	33	. 62103	229	. 07031	12	. 93431	11	7
54	. 35674	27	. 80318	214	. 38186	34	. 61874	228	. 07043	12	. 93420	10	6
55	0. 35701	27	2. 80104	213	0. 38220	33	2. 61646	228	1. 07055	12	0. 93410	10	5
56	. 35728	27	. 79891	212	. 38253	33	. 61418	228	. 07067	12	. 93400	11	4
57	. 35755	27	. 79679	213	. 38286	34	. 61190	227	. 07079	12	. 93389	10	3
58	. 35782	28	. 79466	212	. 38320	33	. 60963	227	. 07091	12	. 93379	11	2
59	. 35810	27	. 79254	211	. 38353	33	. 60736	227	. 07103	11	. 93368	10	1
60	0. 35837		2. 79043		0. 38386		2. 60509		1. 07114		0. 93358		0
↑ 110°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←69° ↑

TABLE 31

Natural Trigonometric Functions

21°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←158° ↓ ′
0	0. 35837	27	2. 79043	211	0. 38386	34	2. 60509	226	1. 07114	12	0. 93358	10	60
1	. 35864	27	. 78832	211	. 38420	33	. 60283	226	. 07126	12	. 93348	11	59
2	. 35891	27	. 78621	211	. 38453	34	. 60057	226	. 07138	12	. 93337	10	58
3	. 35918	27	. 78410	210	. 38487	33	. 59831	225	. 07150	12	. 93327	11	57
4	. 35945	28	. 78200	210	. 38520	33	. 59606	225	. 07162	12	. 93316	10	56
5	0. 35973	27	2. 77990	210	0. 38553	34	2. 59381	225	1. 07174	12	0. 93306	11	55
6	. 36000	27	. 77780	209	. 38587	33	. 59156	224	. 07186	13	. 93295	10	54
7	. 36027	27	. 77571	209	. 38620	34	. 58932	224	. 07199	12	. 93285	11	53
8	. 36054	27	. 77362	208	. 38654	33	. 58708	224	. 07211	12	. 93274	10	52
9	. 36081	27	. 77154	209	. 38687	34	. 58484	223	. 07223	12	. 93264	11	51
10	0. 36108	27	2. 76945	208	0. 38721	33	2. 58261	223	1. 07235	12	0. 93253	10	50
11	. 36135	27	. 76737	207	. 38754	33	. 58038	223	. 07247	12	. 93243	11	49
12	. 36162	28	. 76530	207	. 38787	34	. 57815	222	. 07259	12	. 93232	10	48
13	. 36190	27	. 76323	207	. 38821	33	. 57593	222	. 07271	12	. 93222	11	47
14	. 36217	27	. 76116	207	. 38854	34	. 57371	221	. 07283	12	. 93211	10	46
15	0. 36244	27	2. 75909	206	0. 38888	33	2. 57150	222	1. 07295	12	0. 93201	11	45
16	. 36271	27	. 75703	206	. 38921	34	. 56928	221	. 07307	13	. 93190	10	44
17	. 36298	27	. 75497	205	. 38955	33	. 56707	220	. 07320	12	. 93180	11	43
18	. 36325	27	. 75292	206	. 38988	34	. 56487	221	. 07332	12	. 93169	10	42
19	. 36352	27	. 75086	205	. 39022	33	. 56266	220	. 07344	12	. 93159	11	41
20	0. 36379	27	2. 74881	204	0. 39055	34	2. 56046	219	1. 07356	12	0. 93148	11	40
21	. 36406	28	. 74677	204	. 39089	33	. 55827	219	. 07368	12	. 93137	10	39
22	. 36434	27	. 74473	204	. 39122	34	. 55608	219	. 07380	13	. 93127	11	38
23	. 36461	27	. 74269	204	. 39156	34	. 55389	219	. 07393	12	. 93116	10	37
24	. 36488	27	. 74065	203	. 39190	33	. 55170	218	. 07405	12	. 93106	11	36
25	0. 36515	27	2. 73862	203	0. 39223	34	2. 54952	218	1. 07417	12	0. 93095	11	35
26	. 36542	27	. 73659	203	. 39257	33	. 54734	218	. 07429	13	. 93084	10	34
27	. 36569	27	. 73456	202	. 39290	34	. 54516	217	. 07442	12	. 93074	11	33
28	. 36596	27	. 73254	202	. 39324	33	. 54299	217	. 07454	12	. 93063	11	32
29	. 36623	27	. 73052	202	. 39357	34	. 54082	217	. 07466	13	. 93052	10	31
30	0. 36650	27	2. 72850	201	0. 39391	34	2. 53865	217	1. 07479	12	0. 93042	11	30
31	. 36677	27	. 72649	201	. 39425	33	. 53648	216	. 07491	12	. 93031	11	29
32	. 36704	27	. 72448	201	. 39458	34	. 53432	215	. 07503	13	. 93020	10	28
33	. 36731	27	. 72247	200	. 39492	34	. 53217	216	. 07516	12	. 93010	11	27
34	. 36758	27	. 72047	200	. 39526	33	. 53001	215	. 07528	12	. 92999	11	26
35	0. 36785	27	2. 71847	200	0. 39559	34	2. 52786	215	1. 07540	13	0. 92988	10	25
36	. 36812	27	. 71647	199	. 39593	33	. 52571	214	. 07553	12	. 92978	11	24
37	. 36839	28	. 71448	199	. 39626	34	. 52357	215	. 07565	13	. 92967	11	23
38	. 36867	27	. 71249	199	. 39660	34	. 52142	213	. 07578	12	. 92956	11	22
39	. 36894	27	. 71050	199	. 39694	33	. 51929	214	. 07590	12	. 92945	10	21
40	0. 36921	27	2. 70851	198	0. 39727	34	2. 51715	213	1. 07602	13	0. 92935	11	20
41	. 36948	27	. 70653	198	. 39761	34	. 51502	213	. 07615	12	. 92924	11	19
42	. 36975	27	. 70455	197	. 39795	34	. 51289	213	. 07627	13	. 92913	11	18
43	. 37002	27	. 70258	197	. 39829	33	. 51076	212	. 07640	12	. 92902	10	17
44	. 37029	27	. 70061	197	. 39862	34	. 50864	212	. 07652	13	. 92892	11	16
45	0. 37056	27	2. 69864	197	0. 39896	34	2. 50652	212	1. 07665	12	0. 92881	11	15
46	. 37083	27	. 69667	196	. 39930	33	. 50440	211	. 07677	13	. 92870	11	14
47	. 37110	27	. 69471	196	. 39963	34	. 50229	211	. 07690	12	. 92859	10	13
48	. 37137	27	. 69275	196	. 39997	34	. 50018	211	. 07702	13	. 92849	11	12
49	. 37164	27	. 69079	195	. 40031	34	. 49807	210	. 07715	12	. 92838	11	11
50	0. 37191	27	2. 68884	195	0. 40065	33	2. 49597	211	1. 07727	13	0. 92827	11	10
51	. 37218	27	. 68689	195	. 40098	34	. 49386	209	. 07740	12	. 92816	11	9
52	. 37245	27	. 68494	195	. 40132	34	. 49177	210	. 07752	13	. 92805	11	8
53	. 37272	27	. 68299	194	. 40166	34	. 48967	209	. 07765	13	. 92794	10	7
54	. 37299	27	. 68105	194	. 40200	34	. 48758	209	. 07778	12	. 92784	11	6
55	0. 37326	27	2. 67911	193	0. 40234	33	2. 48549	209	1. 07790	13	0. 92773	11	5
56	. 37353	27	. 67718	193	. 40267	34	. 48340	208	. 07803	13	. 92762	11	4
57	. 37380	27	. 67525	193	. 40301	34	. 48132	208	. 07816	12	. 92751	11	3
58	. 37407	27	. 67332	193	. 40335	34	. 47924	208	. 07828	13	. 92740	11	2
59	. 37434	27	. 67139	192	. 40369	34	. 47716	207	. 07841	12	. 92729	11	1
60	0. 37461		2. 66947		0. 40403		2. 47509		1. 07853		0. 92718		0
↑ 111°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←68° ↑

TABLE 31
Natural Trigonometric Functions

22°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←157° ↓ ′
0	0. 37461	27	2. 66947	192	0. 40403	33	2. 47509	207	1. 07853	13	0. 92718	11	60
1	. 37488	27	. 66755	192	. 40436	34	. 47302	207	. 07866	13	. 92707	10	59
2	. 37515	27	. 66563	192	. 40470	34	. 47095	207	. 07879	13	. 92697	11	58
3	. 37542	27	. 66371	191	. 40504	34	. 46888	206	. 07892	12	. 92686	11	57
4	. 37569	26	. 66180	191	. 40538	34	. 46682	206	. 07904	13	. 92675	11	56
5	0. 37595	27	2. 65989	190	0. 40572	34	2. 46476	206	1. 07917	13	0. 92664	11	55
6	. 37622	27	. 65799	190	. 40606	34	. 46270	205	. 07930	13	. 92653	11	54
7	. 37649	27	. 65609	190	. 40640	34	. 46065	205	. 07943	12	. 92642	11	53
8	. 37676	27	. 65419	190	. 40674	33	. 45860	205	. 07955	13	. 92631	11	52
9	. 37703	27	. 65229	189	. 40707	34	. 45655	204	. 07968	13	. 92620	11	51
10	0. 37730	27	2. 65040	189	0. 40741	34	2. 45451	205	1. 07981	13	0. 92609	11	50
11	. 37757	27	. 64851	189	. 40775	34	. 45246	203	. 07994	12	. 92598	11	49
12	. 37784	27	. 64662	189	. 40809	34	. 45043	204	. 08006	13	. 92587	11	48
13	. 37811	27	. 64473	188	. 40843	34	. 44839	203	. 08019	13	. 92576	11	47
14	. 37838	27	. 64285	188	. 40877	34	. 44636	203	. 08032	13	. 92565	11	46
15	0. 37865	27	2. 64097	188	0. 40911	34	2. 44433	203	1. 08045	13	0. 92554	11	45
16	. 37892	27	. 63909	187	. 40945	34	. 44230	203	. 08058	13	. 92543	11	44
17	. 37919	27	. 63722	187	. 40979	34	. 44027	202	. 08071	13	. 92532	11	43
18	. 37946	27	. 63535	187	. 41013	34	. 43825	202	. 08084	13	. 92521	11	42
19	. 37973	26	. 63348	186	. 41047	34	. 43623	201	. 08097	12	. 92510	11	41
20	0. 37999	27	2. 63162	186	0. 41081	34	2. 43422	202	1. 08109	13	0. 92499	11	40
21	. 38026	27	. 62976	186	. 41115	34	. 43220	201	. 08122	13	. 92488	11	39
22	. 38053	27	. 62790	186	. 41149	34	. 43019	200	. 08135	13	. 92477	11	38
23	. 38080	27	. 62604	185	. 41183	34	. 42819	201	. 08148	13	. 92466	11	37
24	. 38107	27	. 62419	185	. 41217	34	. 42618	200	. 08161	13	. 92455	11	36
25	0. 38134	27	2. 62234	185	0. 41251	34	2. 42418	200	1. 08174	13	0. 92444	12	35
26	. 38161	27	. 62049	185	. 41285	34	. 42218	199	. 08187	13	. 92432	11	34
27	. 38188	27	. 61864	184	. 41319	34	. 42019	200	. 08200	13	. 92421	11	33
28	. 38215	26	. 61680	184	. 41353	34	. 41819	199	. 08213	13	. 92410	11	32
29	. 38241	27	. 61496	183	. 41387	34	. 41620	199	. 08226	13	. 92399	11	31
30	0. 38268	27	2. 61313	184	0. 41421	34	2. 41421	198	1. 08239	13	0. 92388	11	30
31	. 38295	27	. 61129	183	. 41455	35	. 41223	198	. 08252	13	. 92377	11	29
32	. 38322	27	. 60946	183	. 41490	34	. 41025	198	. 08265	13	. 92366	11	28
33	. 38349	27	. 60763	182	. 41524	34	. 40827	198	. 08278	13	. 92355	12	27
34	. 38376	27	. 60581	182	. 41558	34	. 40629	197	. 08291	14	. 92343	11	26
35	0. 38403	27	2. 60399	182	0. 41592	34	2. 40432	197	1. 08305	13	0. 92332	11	25
36	. 38430	26	. 60217	182	. 41626	34	. 40235	197	. 08318	13	. 92321	11	24
37	. 38456	27	. 60035	182	. 41660	34	. 40038	197	. 08331	13	. 92310	11	23
38	. 38483	27	. 59853	181	. 41694	34	. 39841	196	. 08344	13	. 92299	12	22
39	. 38510	27	. 59672	181	. 41728	35	. 39645	196	. 08357	13	. 92287	11	21
40	0. 38537	27	2. 59491	180	0. 41763	34	2. 39449	196	1. 08370	13	0. 92276	11	20
41	. 38564	27	. 59311	181	. 41797	34	. 39253	195	. 08383	14	. 92265	11	19
42	. 38591	26	. 59130	180	. 41831	34	. 39058	195	. 08397	13	. 92254	11	18
43	. 38617	27	. 58950	179	. 41865	34	. 38863	195	. 08410	13	. 92243	12	17
44	. 38644	27	. 58771	180	. 41899	34	. 38668	195	. 08423	13	. 92231	11	16
45	0. 38671	27	2. 58591	179	0. 41933	35	2. 38473	194	1. 08436	13	0. 92220	11	15
46	. 38698	27	. 58412	179	. 41968	34	. 38279	195	. 08449	14	. 92209	11	14
47	. 38725	27	. 58233	179	. 42002	34	. 38084	193	. 08463	13	. 92198	12	13
48	. 38752	26	. 58054	178	. 42036	34	. 37891	194	. 08476	13	. 92186	11	12
49	. 38778	27	. 57876	178	. 42070	35	. 37697	193	. 08489	14	. 92175	11	11
50	0. 38805	27	2. 57698	178	0. 42105	34	2. 37504	193	1. 08503	13	0. 92164	12	10
51	. 38832	27	. 57520	178	. 42139	34	. 37311	193	. 08516	13	. 92152	11	9
52	. 38859	27	. 57342	177	. 42173	34	. 37118	193	. 08529	13	. 92141	11	8
53	. 38886	26	. 57165	177	. 42207	35	. 36925	192	. 08542	14	. 92130	11	7
54	. 38912	27	. 56988	177	. 42242	34	. 36733	192	. 08556	13	. 92119	12	6
55	0. 38939	27	2. 56811	177	0. 42276	34	2. 36541	192	1. 08569	13	0. 92107	11	5
56	. 38966	27	. 56634	176	. 42310	35	. 36349	191	. 08582	14	. 92096	11	4
57	. 38993	27	. 56458	176	. 42345	34	. 36158	191	. 08596	13	. 92085	12	3
58	. 39020	26	. 56282	176	. 42379	34	. 35967	191	. 08609	14	. 92073	11	2
59	. 39046	27	. 56106	176	. 42413	34	. 35776	191	. 08623	13	. 92062	12	1
60	0. 39073		2. 55930		0. 42447		2. 35585		1. 08636		0. 92050		0
↑ 112°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ←67°

TABLE 31

Natural Trigonometric Functions

23° → ′ ↓	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	← 156° ′ ↓
0	0. 39073	27	2. 55930	175	0. 42447	35	2. 35585	190	1. 08636	13	0. 92050	11	60
1	. 39100	27	. 55755	175	. 42482	34	. 35395	190	. 08649	14	. 92039	11	59
2	. 39127	26	. 55580	175	. 42516	35	. 35205	190	. 08663	13	. 92028	12	58
3	. 39153	27	. 55405	174	. 42551	34	. 35015	190	. 08676	14	. 92016	11	57
4	. 39180	27	. 55231	174	. 42585	34	. 34825	189	. 08690	13	. 92005	11	56
5	0. 39207	27	2. 55057	174	0. 42619	35	2. 34636	189	1. 08703	14	0. 91994	12	55
6	. 39234	26	. 54883	174	. 42654	34	. 34447	189	. 08717	13	. 91982	11	54
7	. 39260	27	. 54709	173	. 42689	34	. 34258	189	. 08730	14	. 91971	12	53
8	. 39287	27	. 54536	173	. 42722	35	. 34069	188	. 08744	13	. 91959	11	52
9	. 39314	27	. 54363	173	. 42757	34	. 33881	188	. 08757	14	. 91948	12	51
10	0. 39341	26	2. 54190	173	0. 42791	35	2. 33693	188	1. 08771	13	0. 91936	11	50
11	. 39367	27	. 54017	172	. 42826	34	. 33505	188	. 08784	14	. 91925	11	49
12	. 39394	27	. 53845	173	. 42860	34	. 33317	187	. 08798	13	. 91914	12	48
13	. 39421	27	. 53672	172	. 42894	35	. 33130	187	. 08811	14	. 91902	11	47
14	. 39448	26	. 53500	171	. 42929	34	. 32943	187	. 08825	14	. 91891	12	46
15	0. 39474	27	2. 53329	172	0. 42963	35	2. 32756	186	1. 08839	13	0. 91879	11	45
16	. 39501	27	. 53157	171	. 42998	34	. 32570	187	. 08852	14	. 91868	12	44
17	. 39528	27	. 52986	171	. 43032	35	. 32383	186	. 08866	14	. 91856	11	43
18	. 39555	26	. 52815	170	. 43067	34	. 32197	185	. 08880	13	. 91845	12	42
19	. 39581	27	. 52645	171	. 43101	35	. 32012	186	. 08893	14	. 91833	11	41
20	0. 39608	27	2. 52474	170	0. 43136	34	2. 31826	185	1. 08907	13	0. 91822	12	40
21	. 39635	26	. 52304	170	. 43170	35	. 31641	185	. 08920	14	. 91810	11	39
22	. 39661	27	. 52134	169	. 43205	34	. 31456	185	. 08934	14	. 91799	12	38
23	. 39688	27	. 51965	170	. 43239	35	. 31271	185	. 08948	14	. 91787	12	37
24	. 39715	26	. 51795	169	. 43274	34	. 31086	184	. 08962	13	. 91775	11	36
25	0. 39741	27	2. 51626	169	0. 43308	35	2. 30902	184	1. 08975	14	0. 91764	12	35
26	. 39768	27	. 51457	168	. 43343	35	. 30718	184	. 08989	14	. 91752	11	34
27	. 39795	27	. 51289	169	. 43378	34	. 30534	183	. 09003	14	. 91741	12	33
28	. 39822	26	. 51120	168	. 43412	35	. 30351	184	. 09017	13	. 91729	11	32
29	. 39848	27	. 50952	168	. 43447	34	. 30167	183	. 09030	14	. 91718	12	31
30	0. 39875	27	2. 50784	167	0. 43481	35	2. 29984	183	1. 09044	14	0. 91706	12	30
31	. 39902	26	. 50617	168	. 43516	34	. 29801	182	. 09058	14	. 91694	11	29
32	. 39928	27	. 50449	167	. 43550	35	. 29619	182	. 09072	14	. 91683	12	28
33	. 39955	27	. 50282	167	. 43585	35	. 29437	183	. 09086	13	. 91671	11	27
34	. 39982	26	. 50115	167	. 43620	34	. 29254	181	. 09099	14	. 91660	12	26
35	0. 40008	27	2. 49948	166	0. 43654	35	2. 29073	182	1. 09113	14	0. 91648	12	25
36	. 40035	27	. 49782	166	. 43689	35	. 28891	181	. 09127	14	. 91636	11	24
37	. 40062	26	. 49616	166	. 43724	34	. 28710	182	. 09141	14	. 91625	12	23
38	. 40088	27	. 49450	166	. 43758	35	. 28528	180	. 09155	14	. 91613	12	22
39	. 40115	26	. 49284	165	. 43793	35	. 28348	181	. 09169	14	. 91601	11	21
40	0. 40141	27	2. 49119	165	0. 43828	34	2. 28167	180	1. 09183	14	0. 91590	12	20
41	. 40168	27	. 48954	165	. 43862	35	. 27987	181	. 09197	14	. 91578	12	19
42	. 40195	26	. 48789	165	. 43897	35	. 27806	180	. 09211	13	. 91566	11	18
43	. 40221	27	. 48624	165	. 43932	34	. 27626	179	. 09224	14	. 91555	12	17
44	. 40248	27	. 48459	164	. 43966	35	. 27447	180	. 09238	14	. 91543	12	16
45	0. 40275	26	2. 48295	164	0. 44001	35	2. 27267	179	1. 09252	14	0. 91531	12	15
46	. 40301	27	. 48131	164	. 44036	35	. 27088	179	. 09266	14	. 91519	11	14
47	. 40328	27	. 47967	163	. 44071	34	. 26909	179	. 09280	14	. 91508	12	13
48	. 40355	26	. 47804	164	. 44105	35	. 26730	178	. 09294	14	. 91496	12	12
49	. 40381	27	. 47640	163	. 44140	35	. 26552	178	. 09308	15	. 91484	12	11
50	0. 40408	26	2. 47477	163	0. 44175	35	2. 26374	178	1. 09323	14	0. 91472	11	10
51	. 40434	27	. 47314	162	. 44210	34	. 26196	178	. 09337	14	. 91461	12	9
52	. 40461	27	. 47152	163	. 44244	35	. 26018	178	. 09351	14	. 91449	12	8
53	. 40488	26	. 46989	162	. 44279	35	. 25840	177	. 09365	14	. 91437	12	7
54	. 40514	27	. 46827	162	. 44314	35	. 25663	177	. 09379	14	. 91425	11	6
55	0. 40541	26	2. 46665	161	0. 44349	35	2. 25486	177	1. 09393	14	0. 91414	12	5
56	. 40567	27	. 46504	162	. 44384	34	. 25309	177	. 09407	14	. 91402	12	4
57	. 40594	27	. 46342	161	. 44418	35	. 25132	176	. 09421	14	. 91390	12	3
58	. 40621	26	. 46181	161	. 44453	35	. 24956	176	. 09435	14	. 91378	12	2
59	. 40647	27	. 46020	161	. 44488	35	. 24780	176	. 09449	15	. 91366	11	1
60	0. 40674		2. 45859		0. 44523		2. 24604		1. 09464		0. 91355		0
↑ 113° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	← 66° ↑

TABLE 31

Natural Trigonometric Functions

24°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←155° ↓ ′
0	0. 40674	26	2. 45859	160	0. 44523	35	2. 24604	176	1. 09464	14	0. 91355	12	60
1	. 40700	27	. 45699	160	. 44558	35	. 24428	176	. 09478	14	. 91343	12	59
2	. 40727	26	. 45539	161	. 44593	34	. 24252	175	. 09492	14	. 91331	12	58
3	. 40753	27	. 45378	159	. 44627	35	. 24077	175	. 09506	14	. 91319	12	57
4	. 40780	26	. 45219	160	. 44662	35	. 23902	175	. 09520	15	. 91307	12	56
5	0. 40806	27	2. 45059	159	0. 44697	35	2. 23727	174	1. 09535	14	0. 91295	12	55
6	. 40833	27	. 44900	159	. 44732	35	. 23553	175	. 09549	14	. 91283	11	54
7	. 40860	26	. 44741	159	. 44767	35	. 23378	174	. 09563	14	. 91272	12	53
8	. 40886	27	. 44582	159	. 44802	35	. 23204	174	. 09577	15	. 91260	12	52
9	. 40913	26	. 44423	159	. 44837	35	. 23030	173	. 09592	14	. 91248	12	51
10	0. 40939	27	2. 44264	158	0. 44872	35	2. 22857	174	1. 09606	14	0. 91236	12	50
11	. 40966	26	. 44106	158	. 44907	35	. 22683	173	. 09620	15	. 91224	12	49
12	. 40992	27	. 43948	158	. 44942	35	. 22510	173	. 09635	14	. 91212	12	48
13	. 41019	26	. 43790	157	. 44977	35	. 22337	173	. 09649	14	. 91200	12	47
14	. 41045	27	. 43633	157	. 45012	35	. 22164	172	. 09663	15	. 91188	12	46
15	0. 41072	26	2. 43476	158	0. 45047	35	2. 21992	173	1. 09678	14	0. 91176	12	45
16	. 41098	27	. 43318	156	. 45082	35	. 21819	172	. 09692	15	. 91164	12	44
17	. 41125	26	. 43162	157	. 45117	35	. 21647	172	. 09707	14	. 91152	12	43
18	. 41151	27	. 43005	157	. 45152	35	. 21475	171	. 09721	14	. 91140	12	42
19	. 41178	26	. 42848	156	. 45187	35	. 21304	172	. 09735	15	. 91128	12	41
20	0. 41204	27	2. 42692	156	0. 45222	35	2. 21132	171	1. 09750	14	0. 91116	12	40
21	. 41231	26	. 42536	156	. 45257	35	. 20961	171	. 09764	15	. 91104	12	39
22	. 41257	27	. 42380	155	. 45292	35	. 20790	171	. 09779	14	. 91092	12	38
23	. 41284	26	. 42225	155	. 45327	35	. 20619	170	. 09793	15	. 91080	12	37
24	. 41310	27	. 42070	156	. 45362	35	. 20449	171	. 09808	14	. 91068	12	36
25	0. 41337	26	2. 41914	154	0. 45397	35	2. 20278	170	1. 09822	15	0. 91056	12	35
26	. 41363	27	. 41760	155	. 45432	35	. 20108	170	. 09837	14	. 91044	12	34
27	. 41390	26	. 41605	155	. 45467	35	. 19938	169	. 09851	15	. 91032	12	33
28	. 41416	27	. 41450	154	. 45502	36	. 19769	170	. 09866	14	. 91020	12	32
29	. 41443	26	. 41296	154	. 45538	35	. 19599	169	. 09880	15	. 91008	12	31
30	0. 41469	27	2. 41142	154	0. 45573	35	2. 19430	169	1. 09895	14	0. 90996	12	30
31	. 41496	26	. 40988	153	. 45608	35	. 19261	169	. 09909	15	. 90984	12	29
32	. 41522	27	. 40835	154	. 45643	35	. 19092	169	. 09924	15	. 90972	12	28
33	. 41549	26	. 40681	153	. 45678	35	. 18923	168	. 09939	14	. 90960	12	27
34	. 41575	27	. 40528	153	. 45713	35	. 18755	168	. 09953	15	. 90948	12	26
35	0. 41602	26	2. 40375	153	0. 45748	36	2. 18587	168	1. 09968	14	0. 90936	12	25
36	. 41628	27	. 40222	152	. 45784	35	. 18419	168	. 09982	15	. 90924	13	24
37	. 41655	26	. 40070	152	. 45819	35	. 18251	167	. 09997	15	. 90911	12	23
38	. 41681	26	. 39918	152	. 45854	35	. 18084	168	. 10012	14	. 90899	12	22
39	. 41707	27	. 39766	152	. 45889	35	. 17916	167	. 10026	15	. 90887	12	21
40	0. 41734	26	2. 39614	152	0. 45924	36	2. 17749	167	1. 10041	15	0. 90875	12	20
41	. 41760	27	. 39462	151	. 45960	35	. 17582	166	. 10056	15	. 90863	12	19
42	. 41787	26	. 39311	152	. 45995	35	. 17416	167	. 10071	14	. 90851	12	18
43	. 41813	27	. 39159	151	. 46030	35	. 17249	166	. 10085	15	. 90839	13	17
44	. 41840	26	. 39008	151	. 46065	36	. 17083	166	. 10100	15	. 90826	12	16
45	0. 41866	26	2. 38857	150	0. 46101	35	2. 16917	166	1. 10115	15	0. 90814	12	15
46	. 41892	27	. 38707	151	. 46136	35	. 16751	166	. 10130	14	. 90802	12	14
47	. 41919	26	. 38556	150	. 46171	35	. 16585	165	. 10144	15	. 90790	12	13
48	. 41945	27	. 38406	150	. 46206	36	. 16420	165	. 10159	15	. 90778	12	12
49	. 41972	26	. 38256	150	. 46242	35	. 16255	165	. 10174	15	. 90766	13	11
50	0. 41998	26	2. 38106	149	0. 46277	35	2. 16090	165	1. 10189	15	0. 90753	12	10
51	. 42024	27	. 37957	149	. 46312	36	. 15925	165	. 10204	14	. 90741	12	9
52	. 42051	26	. 37808	150	. 46348	35	. 15760	164	. 10218	15	. 90729	12	8
53	. 42077	27	. 37658	149	. 46383	35	. 15596	164	. 10233	15	. 90717	13	7
54	. 42104	26	. 37509	148	. 46418	36	. 15432	164	. 10248	15	. 90704	12	6
55	0. 42130	26	2. 37361	149	0. 46454	35	2. 15268	164	1. 10263	15	0. 90692	12	5
56	. 42156	27	. 37212	148	. 46489	36	. 15104	164	. 10278	15	. 90680	12	4
57	. 42183	26	. 37064	148	. 46525	35	. 14940	163	. 10293	15	. 90668	13	3
58	. 42209	26	. 36916	148	. 46560	35	. 14777	163	. 10308	15	. 90655	12	2
59	. 42235	27	. 36768	148	. 46595	36	. 14614	163	. 10323	15	. 90643	12	1
60	0. 42262		2. 36620		0. 46631		2. 14451		1. 10338		0. 90631		0
↑ 114°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ←65°

TABLE 31

Natural Trigonometric Functions

25° → ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←154° ↓ ′
0	0. 42262	26	2. 36620	147	0. 46631	35	2. 14451	163	1. 10338	15	0. 90631	13	60
1	. 42288	27	. 36473	148	. 46666	36	. 14288	163	. 10353	15	. 90618	12	59
2	. 42315	26	. 36325	147	. 46702	35	. 14125	162	. 10368	15	. 90606	12	58
3	. 42341	26	. 36178	147	. 46737	35	. 13963	162	. 10383	15	. 90594	12	57
4	. 42367	27	. 36031	146	. 46772	36	. 13801	162	. 10398	15	. 90582	13	56
5	0. 42394	26	2. 35885	147	0. 46808	35	2. 13639	162	1. 10413	15	0. 90569	12	55
6	. 42420	26	. 35738	146	. 46843	36	. 13477	161	. 10428	15	. 90557	12	54
7	. 42446	27	. 35592	146	. 46879	35	. 13316	162	. 10443	15	. 90545	13	53
8	. 42473	26	. 35446	146	. 46914	36	. 13154	161	. 10458	15	. 90532	12	52
9	. 42499	26	. 35300	146	. 46950	35	. 12993	161	. 10473	15	. 90520	13	51
10	0. 42525	27	2. 35154	145	0. 46985	36	2. 12832	161	1. 10488	15	0. 90507	12	50
11	. 42552	26	. 35009	146	. 47021	35	. 12671	160	. 10503	15	. 90495	12	49
12	. 42578	26	. 34863	145	. 47056	36	. 12511	161	. 10518	15	. 90483	13	48
13	. 42604	27	. 34718	145	. 47092	36	. 12350	160	. 10533	16	. 90470	12	47
14	. 42631	26	. 34573	144	. 47128	35	. 12190	160	. 10549	15	. 90458	12	46
15	0. 42657	26	2. 34429	145	0. 47163	36	2. 12030	159	1. 10564	15	0. 90446	13	45
16	. 42683	26	. 34284	144	. 47199	35	. 11871	160	. 10579	15	. 90433	12	44
17	. 42709	27	. 34140	144	. 47234	36	. 11711	159	. 10594	15	. 90421	13	43
18	. 42736	26	. 33996	144	. 47270	35	. 11552	160	. 10609	16	. 90408	12	42
19	. 42762	26	. 33852	144	. 47305	36	. 11392	159	. 10625	15	. 90396	13	41
20	0. 42788	27	2. 33708	143	0. 47341	36	2. 11233	158	1. 10640	15	0. 90383	12	40
21	. 42815	26	. 33565	143	. 47377	35	. 11075	159	. 10655	15	. 90371	13	39
22	. 42841	26	. 33422	144	. 47412	36	. 10916	158	. 10670	16	. 90358	12	38
23	. 42867	27	. 33278	143	. 47448	35	. 10758	158	. 10686	15	. 90346	12	37
24	. 42894	26	. 33135	142	. 47483	36	. 10600	158	. 10701	15	. 90334	13	36
25	0. 42920	26	2. 32993	143	0. 47519	36	2. 10442	158	1. 10716	15	0. 90321	12	35
26	. 42946	26	. 32850	142	. 47555	35	. 10284	158	. 10731	16	. 90309	13	34
27	. 42972	27	. 32708	142	. 47590	36	. 10126	157	. 10747	15	. 90296	12	33
28	. 42999	26	. 32566	142	. 47626	36	. 09969	158	. 10762	15	. 90284	13	32
29	. 43025	26	. 32424	142	. 47662	36	. 09811	157	. 10777	16	. 90271	12	31
30	0. 43051	26	2. 32282	142	0. 47698	35	2. 09654	156	1. 10793	15	0. 90259	13	30
31	. 43077	27	. 32140	141	. 47733	36	. 09498	157	. 10808	16	. 90246	13	29
32	. 43104	26	. 31999	141	. 47769	36	. 09341	157	. 10824	15	. 90233	12	28
33	. 43130	26	. 31858	141	. 47805	35	. 09184	156	. 10839	15	. 90221	13	27
34	. 43156	26	. 31717	141	. 47840	36	. 09028	156	. 10854	16	. 90208	12	26
35	0. 43182	27	2. 31576	140	0. 47876	36	2. 08872	156	1. 10870	15	0. 90196	13	25
36	. 43209	26	. 31436	141	. 47912	36	. 08716	156	. 10885	16	. 90183	12	24
37	. 43235	26	. 31295	140	. 47948	36	. 08560	155	. 10901	15	. 90171	13	23
38	. 43261	26	. 31155	140	. 47984	35	. 08405	155	. 10916	16	. 90158	12	22
39	. 43287	26	. 31015	140	. 48019	36	. 08250	156	. 10932	15	. 90146	13	21
40	0. 43313	27	2. 30875	140	0. 48055	36	2. 08094	155	1. 10947	16	0. 90133	13	20
41	. 43340	26	. 30735	139	. 48091	36	. 07939	154	. 10963	15	. 90120	12	19
42	. 43366	26	. 30596	139	. 48127	36	. 07785	155	. 10978	16	. 90108	13	18
43	. 43392	26	. 30457	139	. 48163	35	. 07630	154	. 10994	15	. 90095	13	17
44	. 43418	27	. 30318	139	. 48198	36	. 07476	155	. 11009	16	. 90082	12	16
45	0. 43445	26	2. 30179	139	0. 48234	36	2. 07321	154	1. 11025	16	0. 90070	13	15
46	. 43471	26	. 30040	139	. 48270	36	. 07167	153	. 11041	15	. 90057	12	14
47	. 43497	26	. 29901	138	. 48306	36	. 07014	154	. 11056	16	. 90045	13	13
48	. 43523	26	. 29763	138	. 48342	36	. 06860	154	. 11072	15	. 90032	13	12
49	. 43549	26	. 29625	138	. 48378	36	. 06706	153	. 11087	16	. 90019	12	11
50	0. 43575	27	2. 29487	138	0. 48414	36	2. 06553	153	1. 11103	16	0. 90007	13	10
51	. 43602	26	. 29349	138	. 48450	36	. 06400	153	. 11119	15	. 89994	13	9
52	. 43628	26	. 29211	137	. 48486	35	. 06247	153	. 11134	16	. 89981	13	8
53	. 43654	26	. 29074	137	. 48521	36	. 06094	152	. 11150	16	. 89968	12	7
54	. 43680	26	. 28937	137	. 48557	36	. 05942	152	. 11166	15	. 89956	13	6
55	0. 43706	27	2. 28800	137	0. 48593	36	2. 05790	153	1. 11181	16	0. 89943	13	5
56	. 43733	26	. 28663	137	. 48629	36	. 05637	152	. 11197	16	. 89930	12	4
57	. 43759	26	. 28526	136	. 48665	36	. 05485	152	. 11213	16	. 89918	13	3
58	. 43785	26	. 28390	137	. 48701	36	. 05333	151	. 11229	15	. 89905	13	2
59	. 43811	26	. 28253	136	. 48737	36	. 05182	152	. 11244	16	. 89892	13	1
60	0. 43837		2. 28117		0. 48773		2. 05030		1. 11260		0. 89879		0
↑ 115° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←64° ↑

TABLE 31

Natural Trigonometric Functions

26°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←153° ↓ ′
0	0.43837	26	2.28117	136	0.48773	36	2.05030	151	1.11260	16	0.89879	12	60
1	.43863	26	.27981	136	.48809	36	.04879	151	.11276	16	.89867	13	59
2	.43889	27	.27845	135	.48845	36	.04728	151	.11292	16	.89854	13	58
3	.43916	26	.27710	136	.48881	36	.04577	151	.11308	15	.89841	13	57
4	.43942	26	.27574	135	.48917	36	.04426	150	.11323	16	.89828	12	56
5	0.43968	26	2.27439	135	0.48953	36	2.04276	151	1.11339	16	0.89816	13	55
6	.43994	26	.27304	135	.48989	37	.04125	150	.11355	16	.89803	13	54
7	.44020	26	.27169	134	.49026	36	.03975	150	.11371	16	.89790	13	53
8	.44046	26	.27035	135	.49062	36	.03825	150	.11387	16	.89777	13	52
9	.44072	26	.26900	134	.49098	36	.03675	149	.11403	16	.89764	12	51
10	0.44098	26	2.26766	134	0.49134	36	2.03526	150	1.11419	16	0.89752	13	50
11	.44124	27	.26632	134	.49170	36	.03376	149	.11435	16	.89739	13	49
12	.44151	26	.26498	134	.49206	36	.03227	149	.11451	16	.89726	13	48
13	.44177	26	.26364	134	.49242	36	.03078	149	.11467	16	.89713	13	47
14	.44203	26	.26230	133	.49278	37	.02929	149	.11483	16	.89700	13	46
15	0.44229	26	2.26097	134	0.49315	36	2.02780	149	1.11499	16	0.89687	13	45
16	.44255	26	.25963	133	.49351	36	.02631	148	.11515	16	.89674	12	44
17	.44281	26	.25830	133	.49387	36	.02483	148	.11531	16	.89662	13	43
18	.44307	26	.25697	132	.49423	36	.02335	148	.11547	16	.89649	13	42
19	.44333	26	.25565	133	.49459	36	.02187	148	.11563	16	.89636	13	41
20	0.44359	26	2.25432	132	0.49495	37	2.02039	148	1.11579	16	0.89623	13	40
21	.44385	26	.25300	133	.49532	36	.01891	148	.11595	16	.89610	13	39
22	.44411	26	.25167	132	.49568	36	.01743	147	.11611	16	.89597	13	38
23	.44437	27	.25035	132	.49604	36	.01596	147	.11627	16	.89584	13	37
24	.44464	26	.24903	131	.49640	37	.01449	147	.11643	16	.89571	13	36
25	0.44490	26	2.24772	132	0.49677	36	2.01302	147	1.11659	16	0.89558	13	35
26	.44516	26	.24640	131	.49713	36	.01155	147	.11675	16	.89545	13	34
27	.44542	26	.24509	131	.49749	37	.01008	146	.11691	17	.89532	13	33
28	.44568	26	.24378	131	.49786	36	.00862	147	.11708	16	.89519	13	32
29	.44594	26	.24247	131	.49822	36	.00715	146	.11724	16	.89506	13	31
30	0.44620	26	2.24116	131	0.49858	36	2.00569	146	1.11740	16	0.89493	13	30
31	.44646	26	.23985	130	.49894	37	.00423	146	.11756	16	.89480	13	29
32	.44672	26	.23855	131	.49931	36	.00277	146	.11772	17	.89467	13	28
33	.44698	26	.23724	130	.49967	37	2.00131	145	.11789	16	.89454	13	27
34	.44724	26	.23594	130	.50004	36	1.99986	145	.11805	16	.89441	13	26
35	0.44750	26	2.23464	130	0.50040	36	1.99841	146	1.11821	17	0.89428	13	25
36	.44776	26	.23334	129	.50076	37	.99695	145	.11838	16	.89415	13	24
37	.44802	26	.23205	130	.50113	36	.99550	144	.11854	16	.89402	13	23
38	.44828	26	.23075	129	.50149	36	.99406	145	.11870	16	.89389	13	22
39	.44854	26	.22946	129	.50185	37	.99261	145	.11886	17	.89376	13	21
40	0.44880	26	2.22817	129	0.50222	36	1.99116	144	1.11903	16	0.89363	13	20
41	.44906	26	.22688	129	.50258	37	.98972	144	.11919	17	.89350	13	19
42	.44932	26	.22559	129	.50295	36	.98828	144	.11936	16	.89337	13	18
43	.44958	26	.22430	128	.50331	37	.98684	144	.11952	16	.89324	13	17
44	.44984	26	.22302	128	.50368	36	.98540	144	.11968	17	.89311	13	16
45	0.45010	26	2.22174	129	0.50404	37	1.98396	143	1.11985	16	0.89298	13	15
46	.45036	26	.22045	127	.50441	36	.98253	143	.12001	17	.89285	13	14
47	.45062	26	.21918	128	.50477	37	.98110	144	.12018	16	.89272	13	13
48	.45088	26	.21790	128	.50514	36	.97966	143	.12034	17	.89259	14	12
49	.45114	26	.21662	127	.50550	37	.97823	142	.12051	16	.89245	13	11
50	0.45140	26	2.21535	128	0.50587	36	1.97681	143	1.12067	16	0.89232	13	10
51	.45166	26	.21407	127	.50623	37	.97538	143	.12083	17	.89219	13	9
52	.45192	26	.21280	127	.50660	36	.97395	142	.12100	17	.89206	13	8
53	.45218	25	.21153	127	.50696	37	.97253	142	.12117	16	.89193	13	7
54	.45243	26	.21026	126	.50733	36	.97111	142	.12133	17	.89180	13	6
55	0.45269	26	2.20900	127	0.50769	37	1.96969	142	1.12150	16	0.89167	14	5
56	.45295	26	.20773	126	.50806	37	.96827	142	.12166	17	.89153	13	4
57	.45321	26	.20647	126	.50843	36	.96685	141	.12183	16	.89140	13	3
58	.45347	26	.20521	126	.50879	37	.96544	142	.12199	17	.89127	13	2
59	.45373	26	.20395	126	.50916	37	.96402	141	.12216	17	.89114	13	1
60	0.45399		2.20269		0.50953		1.96261		1.12233		0.89101		0
↑ 116°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←63° ↑

TABLE 31
Natural Trigonometric Functions

27°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←152° ↓ ′
0	0. 45399	26	2. 20269	126	0. 50953	36	1. 96261	141	1. 12233	16	0. 89101	14	60
1	. 45425	26	. 20143	125	. 50989	37	. 96120	141	. 12249	17	. 89087	13	59
2	. 45451	26	. 20018	126	. 51026	37	. 95979	141	. 12266	17	. 89074	13	58
3	. 45477	26	. 19892	125	. 51063	36	. 95838	140	. 12283	16	. 89061	13	57
4	. 45503	26	. 19767	125	. 51099	37	. 95698	141	. 12299	17	. 89048	13	56
5	0. 45529	25	2. 19642	125	0. 51136	37	1. 95557	140	1. 12316	17	0. 89035	14	55
6	. 45554	26	. 19517	124	. 51173	36	. 95417	140	. 12333	16	. 89021	13	54
7	. 45580	26	. 19393	125	. 51209	37	. 95277	140	. 12349	17	. 89008	13	53
8	. 45606	26	. 19268	124	. 51246	37	. 95137	140	. 12366	17	. 88995	14	52
9	. 45632	26	. 19144	125	. 51283	36	. 94997	139	. 12383	17	. 88981	13	51
10	0. 45658	26	2. 19019	124	0. 51319	37	1. 94858	140	1. 12400	16	0. 88968	13	50
11	. 45684	26	. 18895	123	. 51356	37	. 94718	139	. 12416	17	. 88955	13	49
12	. 45710	26	. 18772	124	. 51393	37	. 94579	139	. 12433	17	. 88942	14	48
13	. 45736	26	. 18648	124	. 51430	37	. 94440	139	. 12450	17	. 88928	13	47
14	. 45762	25	. 18524	123	. 51467	36	. 94301	139	. 12467	17	. 88915	13	46
15	0. 45787	26	2. 18401	124	0. 51503	37	1. 94162	139	1. 12484	17	0. 88902	14	45
16	. 45813	26	. 18277	123	. 51540	37	. 94023	138	. 12501	17	. 88888	13	44
17	. 45839	26	. 18154	123	. 51577	37	. 93885	139	. 12518	16	. 88875	13	43
18	. 45865	26	. 18031	122	. 51614	37	. 93746	138	. 12534	17	. 88862	14	42
19	. 45891	26	. 17909	123	. 51651	37	. 93608	138	. 12551	17	. 88848	13	41
20	0. 45917	25	2. 17786	123	0. 51688	36	1. 93470	138	1. 12568	17	0. 88835	13	40
21	. 45942	26	. 17663	122	. 51724	37	. 93332	137	. 12585	17	. 88822	14	39
22	. 45968	26	. 17541	122	. 51761	37	. 93195	138	. 12602	17	. 88808	13	38
23	. 45994	26	. 17419	122	. 51798	37	. 93057	137	. 12619	17	. 88795	13	37
24	. 46020	26	. 17297	122	. 51835	37	. 92920	138	. 12636	17	. 88782	14	36
25	0. 46046	26	2. 17175	122	0. 51872	37	1. 92782	137	1. 12653	17	0. 88768	13	35
26	. 46072	25	. 17053	121	. 51909	37	. 92645	137	. 12670	17	. 88755	14	34
27	. 46097	26	. 16932	122	. 51946	37	. 92508	137	. 12687	17	. 88741	13	33
28	. 46123	26	. 16810	121	. 51983	37	. 92371	136	. 12704	17	. 88728	13	32
29	. 46149	26	. 16689	121	. 52020	37	. 92235	137	. 12721	17	. 88715	14	31
30	0. 46175	26	2. 16568	121	0. 52057	37	1. 92098	136	1. 12738	17	0. 88701	13	30
31	. 46201	25	. 16447	121	. 52094	37	. 91962	136	. 12755	17	. 88688	14	29
32	. 46226	26	. 16326	120	. 52131	37	. 91826	136	. 12772	17	. 88674	13	28
33	. 46252	26	. 16206	121	. 52168	37	. 91690	136	. 12789	18	. 88661	14	27
34	. 46278	26	. 16085	120	. 52205	37	. 91554	136	. 12807	17	. 88647	13	26
35	0. 46304	26	2. 15965	120	0. 52242	37	1. 91418	136	1. 12824	17	0. 88634	14	25
36	. 46330	25	. 15845	120	. 52279	37	. 91282	135	. 12841	17	. 88620	13	24
37	. 46355	26	. 15725	120	. 52316	37	. 91147	135	. 12858	17	. 88607	14	23
38	. 46381	26	. 15605	120	. 52353	37	. 91012	136	. 12875	17	. 88593	13	22
39	. 46407	26	. 15485	119	. 52390	37	. 90876	135	. 12892	18	. 88580	14	21
40	0. 46433	25	2. 15366	120	0. 52427	37	1. 90741	134	1. 12910	17	0. 88566	13	20
41	. 46458	26	. 15246	119	. 52464	37	. 90607	135	. 12927	17	. 88553	14	19
42	. 46484	26	. 15127	119	. 52501	37	. 90472	135	. 12944	17	. 88539	13	18
43	. 46510	26	. 15008	119	. 52538	37	. 90337	134	. 12961	18	. 88526	14	17
44	. 46536	25	. 14889	119	. 52575	38	. 90203	134	. 12979	17	. 88512	13	16
45	0. 46561	26	2. 14770	119	0. 52613	37	1. 90069	134	1. 12996	17	0. 88499	14	15
46	. 46587	26	. 14651	118	. 52650	37	. 89935	134	. 13013	18	. 88485	13	14
47	. 46613	26	. 14533	119	. 52687	37	. 89801	134	. 13031	17	. 88472	14	13
48	. 46639	25	. 14414	118	. 52724	37	. 89667	134	. 13048	17	. 88458	13	12
49	. 46664	26	. 14296	118	. 52761	37	. 89533	133	. 13065	18	. 88445	14	11
50	0. 46690	26	2. 14178	118	0. 52798	38	1. 89400	134	1. 13083	17	0. 88431	14	10
51	. 46716	26	. 14060	118	. 52836	37	. 89266	133	. 13100	17	. 88417	13	9
52	. 46742	25	. 13942	117	. 52873	37	. 89133	133	. 13117	18	. 88404	14	8
53	. 46767	26	. 13825	118	. 52910	37	. 89000	133	. 13135	17	. 88390	13	7
54	. 46793	26	. 13707	117	. 52947	38	. 88867	133	. 13152	18	. 88377	14	6
55	0. 46819	25	2. 13590	117	0. 52985	37	1. 88734	132	1. 13170	17	0. 88363	14	5
56	. 46844	26	. 13473	117	. 53022	37	. 88602	133	. 13187	18	. 88349	13	4
57	. 46870	26	. 13356	117	. 53059	37	. 88469	132	. 13205	17	. 88336	14	3
58	. 46896	25	. 13239	117	. 53096	38	. 88337	132	. 13222	17	. 88322	14	2
59	. 46921	26	. 13122	117	. 53134	37	. 88205	132	. 13239	18	. 88308	13	1
60	0. 46947		2. 13005		0. 53171		1. 88073		1. 13257		0. 88295		0
↑ 117°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←62° ↑

TABLE 31

Natural Trigonometric Functions

28° ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←151° ↓ ′
0	0.46947	26	2.13005	116	0.53171	37	1.88073	132	1.13257	18	0.88295	14	60
1	.46973	26	.12889	116	.53208	38	.87941	132	.13275	17	.88281	14	59
2	.46999	25	.12773	116	.53246	37	.87809	132	.13292	18	.88267	13	58
3	.47024	26	.12657	117	.53283	37	.87677	131	.13310	17	.88254	14	57
4	.47050	26	.12540	115	.53320	38	.87546	131	.13327	18	.88240	14	56
5	0.47076	25	2.12425	116	0.53358	37	1.87415	132	1.13345	17	0.88226	13	55
6	.47101	26	.12309	116	.53395	37	.87283	131	.13362	18	.88213	14	54
7	.47127	26	.12193	115	.53432	38	.87152	131	.13380	18	.88199	14	53
8	.47153	25	.12078	115	.53470	37	.87021	130	.13398	17	.88185	13	52
9	.47178	26	.11963	116	.53507	38	.86891	131	.13415	18	.88172	14	51
10	0.47204	25	2.11847	115	0.53545	37	1.86760	130	1.13433	18	0.88158	14	50
11	.47229	26	.11732	115	.53582	38	.86630	131	.13451	17	.88144	14	49
12	.47255	26	.11617	114	.53620	37	.86499	130	.13468	18	.88130	13	48
13	.47281	25	.11503	115	.53657	37	.86369	130	.13486	18	.88117	14	47
14	.47306	26	.11388	114	.53694	38	.86239	130	.13504	17	.88103	14	46
15	0.47332	26	2.11274	115	0.53732	37	1.86109	130	1.13521	18	0.88089	14	45
16	.47358	25	.11159	114	.53769	38	.85979	129	.13539	18	.88075	13	44
17	.47383	26	.11045	114	.53807	37	.85850	130	.13557	18	.88062	14	43
18	.47409	25	.10931	114	.53844	38	.85720	129	.13575	18	.88048	14	42
19	.47434	26	.10817	113	.53882	38	.85591	129	.13593	17	.88034	14	41
20	0.47460	26	2.10704	114	0.53920	37	1.85462	129	1.13610	18	0.88020	14	40
21	.47486	25	.10590	113	.53957	38	.85333	129	.13628	18	.88006	13	39
22	.47511	26	.10477	114	.53995	37	.85204	129	.13646	18	.87993	14	38
23	.47537	25	.10363	113	.54032	38	.85075	129	.13664	18	.87979	14	37
24	.47562	26	.10250	113	.54070	37	.84946	128	.13682	18	.87965	14	36
25	0.47588	26	2.10137	113	0.54107	38	1.84818	129	1.13700	18	0.87951	14	35
26	.47614	25	.10024	113	.54145	38	.84689	128	.13718	17	.87937	14	34
27	.47639	26	.09911	112	.54183	37	.84561	128	.13735	18	.87923	14	33
28	.47665	25	.09799	113	.54220	38	.84433	128	.13753	18	.87909	13	32
29	.47690	26	.09686	112	.54258	38	.84305	128	.13771	18	.87896	14	31
30	0.47716	25	2.09574	112	0.54296	37	1.84177	128	1.13789	18	0.87882	14	30
31	.47741	26	.09462	112	.54333	38	.84049	127	.13807	18	.87868	14	29
32	.47767	26	.09350	112	.54371	38	.83922	128	.13825	18	.87854	14	28
33	.47793	25	.09238	112	.54409	37	.83794	127	.13843	18	.87840	14	27
34	.47818	26	.09126	112	.54446	38	.83667	127	.13861	18	.87826	14	26
35	0.47844	25	2.09014	111	0.54484	38	1.83540	127	1.13879	18	0.87812	14	25
36	.47869	26	.08903	112	.54522	38	.83413	127	.13897	19	.87798	14	24
37	.47895	25	.08791	111	.54560	37	.83286	127	.13916	18	.87784	14	23
38	.47920	26	.08680	111	.54597	38	.83159	126	.13934	18	.87770	14	22
39	.47946	25	.08569	111	.54635	38	.83033	127	.13952	18	.87756	13	21
40	0.47971	26	2.08458	111	0.54673	38	1.82906	126	1.13970	18	0.87743	14	20
41	.47997	25	.08347	111	.54711	37	.82780	126	.13988	18	.87729	14	19
42	.48022	26	.08236	110	.54748	38	.82654	126	.14006	18	.87715	14	18
43	.48048	25	.08126	111	.54786	38	.82528	126	.14024	18	.87701	14	17
44	.48073	26	.08015	110	.54824	38	.82402	126	.14042	19	.87687	14	16
45	0.48099	25	2.07905	110	0.54862	38	1.82276	126	1.14061	18	0.87673	14	15
46	.48124	26	.07795	110	.54900	38	.82150	125	.14079	18	.87659	14	14
47	.48150	25	.07685	110	.54938	37	.82025	126	.14097	18	.87645	14	13
48	.48175	26	.07575	110	.54975	38	.81899	125	.14115	19	.87631	14	12
49	.48201	25	.07465	109	.55013	38	.81774	125	.14134	18	.87617	14	11
50	0.48226	26	2.07356	110	0.55051	38	1.81649	125	1.14152	18	0.87603	14	10
51	.48252	25	.07246	109	.55089	38	.81524	125	.14170	18	.87589	14	9
52	.48277	26	.07137	110	.55127	38	.81399	125	.14188	19	.87575	14	8
53	.48303	25	.07027	109	.55165	38	.81274	124	.14207	18	.87561	15	7
54	.48328	26	.06918	109	.55203	38	.81150	125	.14225	18	.87546	14	6
55	0.48354	25	2.06809	108	0.55241	38	1.81025	124	1.14243	19	0.87532	14	5
56	.48379	26	.06701	109	.55279	38	.80901	124	.14262	18	.87518	14	4
57	.48405	25	.06592	109	.55317	38	.80777	124	.14280	19	.87504	14	3
58	.48430	26	.06483	108	.55355	38	.80653	124	.14299	18	.87490	14	2
59	.48456	25	.06375	108	.55393	38	.80529	124	.14317	18	.87476	14	1
60	0.48481		2.06267		0.55431		1.80405		1.14335		0.87462		0
↑ 118°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′←61° ↑	

TABLE 31
Natural Trigonometric Functions

29° → ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	← 150° ↓ ′
0	0. 48481	25	2. 06267	109	0. 55431	38	1. 80405	124	1. 14335	19	0. 87462	14	60
1	. 48506	26	. 06158	108	. 55469	38	. 80281	123	. 14354	18	. 87448	14	59
2	. 48532	25	. 06050	108	. 55507	38	. 80158	124	. 14372	19	. 87434	14	58
3	. 48557	26	. 05942	107	. 55545	38	. 80034	123	. 14391	18	. 87420	14	57
4	. 48583	25	. 05835	108	. 55583	38	. 79911	123	. 14409	19	. 87406	15	56
5	0. 48608	26	2. 05727	108	0. 55621	38	1. 79788	123	1. 14428	18	0. 87391	14	55
6	. 48634	25	. 05619	107	. 55659	38	. 79665	123	. 14446	19	. 87377	14	54
7	. 48659	25	. 05512	107	. 55697	39	. 79542	123	. 14465	18	. 87363	14	53
8	. 48684	26	. 05405	107	. 55736	38	. 79419	123	. 14483	19	. 87349	14	52
9	. 48710	25	. 05298	107	. 55774	38	. 79296	122	. 14502	19	. 87335	14	51
10	0. 48735	26	2. 05191	107	0. 55812	38	1. 79174	123	1. 14521	18	0. 87321	15	50
11	. 48761	25	. 05084	107	. 55850	38	. 79051	122	. 14539	19	. 87306	14	49
12	. 48786	25	. 04977	107	. 55888	38	. 78929	122	. 14558	18	. 87292	14	48
13	. 48811	26	. 04870	106	. 55926	38	. 78807	122	. 14576	19	. 87278	14	47
14	. 48837	25	. 04764	107	. 55964	39	. 78685	122	. 14595	19	. 87264	14	46
15	0. 48862	26	2. 04657	106	0. 56003	38	1. 78563	122	1. 14614	18	0. 87250	15	45
16	. 48888	25	. 04551	106	. 56041	38	. 78441	122	. 14632	19	. 87235	14	44
17	. 48913	25	. 04445	106	. 56079	38	. 78319	121	. 14651	19	. 87221	14	43
18	. 48938	26	. 04339	106	. 56117	39	. 78198	121	. 14670	19	. 87207	14	42
19	. 48964	25	. 04233	105	. 56156	38	. 78077	122	. 14689	18	. 87193	15	41
20	0. 48989	25	2. 04128	106	0. 56194	38	1. 77955	121	1. 14707	19	0. 87178	14	40
21	. 49014	26	. 04022	106	. 56232	38	. 77834	121	. 14726	19	. 87164	14	39
22	. 49040	25	. 03916	105	. 56270	39	. 77713	121	. 14745	19	. 87150	14	38
23	. 49065	25	. 03811	105	. 56309	38	. 77592	121	. 14764	18	. 87136	15	37
24	. 49090	26	. 03706	105	. 56347	38	. 77471	120	. 14782	19	. 87121	14	36
25	0. 49116	25	2. 03601	105	0. 56385	39	1. 77351	121	1. 14801	19	0. 87107	14	35
26	. 49141	25	. 03496	105	. 56424	38	. 77230	120	. 14820	19	. 87093	14	34
27	. 49166	26	. 03391	105	. 56462	39	. 77110	120	. 14839	19	. 87079	15	33
28	. 49192	25	. 03286	104	. 56501	38	. 76990	121	. 14858	19	. 87064	14	32
29	. 49217	25	. 03182	105	. 56539	38	. 76869	120	. 14877	19	. 87050	14	31
30	0. 49242	26	2. 03077	104	0. 56577	39	1. 76749	120	1. 14896	18	0. 87036	15	30
31	. 49268	25	. 02973	104	. 56616	38	. 76629	119	. 14914	19	. 87021	14	29
32	. 49293	25	. 02869	104	. 56654	39	. 76510	120	. 14933	19	. 87007	14	28
33	. 49318	26	. 02765	104	. 56693	38	. 76390	119	. 14952	19	. 86993	15	27
34	. 49344	25	. 02661	104	. 56731	38	. 76271	120	. 14971	19	. 86978	14	26
35	0. 49369	25	2. 02557	104	0. 56769	39	1. 76151	119	1. 14990	19	0. 86964	15	25
36	. 49394	25	. 02453	104	. 56808	38	. 76032	119	. 15009	19	. 86949	14	24
37	. 49419	26	. 02349	103	. 56846	39	. 75913	119	. 15028	19	. 86935	14	23
38	. 49445	25	. 02246	103	. 56885	38	. 75794	119	. 15047	19	. 86921	15	22
39	. 49470	25	. 02143	104	. 56923	39	. 75675	119	. 15066	19	. 86906	14	21
40	0. 49495	26	2. 02039	103	0. 56962	38	1. 75556	119	1. 15085	20	0. 86892	14	20
41	. 49521	25	. 01936	103	. 57000	39	. 75437	118	. 15105	19	. 86878	15	19
42	. 49546	25	. 01833	103	. 57039	39	. 75319	119	. 15124	19	. 86863	14	18
43	. 49571	25	. 01730	102	. 57078	38	. 75200	118	. 15143	19	. 86849	15	17
44	. 49596	26	. 01628	103	. 57116	39	. 75082	118	. 15162	19	. 86834	14	16
45	0. 49622	25	2. 01525	103	0. 57155	38	1. 74964	118	1. 15181	19	0. 86820	15	15
46	. 49647	25	. 01422	102	. 57193	39	. 74846	118	. 15200	19	. 86805	14	14
47	. 49672	25	. 01320	102	. 57232	39	. 74728	118	. 15219	20	. 86791	14	13
48	. 49697	26	. 01218	102	. 57271	38	. 74610	118	. 15239	19	. 86777	15	12
49	. 49723	25	. 01116	102	. 57309	39	. 74492	117	. 15258	19	. 86762	14	11
50	0. 49748	25	2. 01014	102	0. 57348	38	1. 74375	118	1. 15277	19	0. 86748	15	10
51	. 49773	25	. 00912	102	. 57386	39	. 74257	117	. 15296	19	. 86733	14	9
52	. 49798	26	. 00810	102	. 57425	39	. 74140	118	. 15315	20	. 86719	15	8
53	. 49824	25	. 00708	101	. 57464	39	. 74022	117	. 15335	19	. 86704	14	7
54	. 49849	25	. 00607	102	. 57503	38	. 73905	117	. 15354	19	. 86690	15	6
55	0. 49874	25	2. 00505	101	0. 57541	39	1. 73788	117	1. 15373	20	0. 86675	14	5
56	. 49899	25	. 00404	101	. 57580	39	. 73671	116	. 15393	19	. 86661	15	4
57	. 49924	26	. 00303	101	. 57619	38	. 73555	117	. 15412	19	. 86646	14	3
58	. 49950	25	. 00202	101	. 57657	39	. 73438	117	. 15431	20	. 86632	15	2
59	. 49975	25	. 00101	101	. 57696	39	. 73321	116	. 15451	19	. 86617	14	1
60	0. 50000		2. 00000		0. 57735		1. 73205		1. 15470		0. 86603		0
↑ 119° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	← 60° ↑

TABLE 31
Natural Trigonometric Functions

30°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←149° ↓ ′
0	0.50000	25	2.00000	101	0.57735	39	1.73205	116	1.15470	19	0.86603	15	60
1	.50025	25	1.99899	100	.57774	39	.73089	116	.15489	20	.86588	15	59
2	.50050	26	.99799	101	.57813	38	.72973	116	.15509	19	.86573	14	58
3	.50076	25	.99698	100	.57851	39	.72857	116	.15528	20	.86559	15	57
4	.50101	25	.99598	100	.57890	39	.72741	116	.15548	19	.86544	14	56
5	0.50126	25	1.99498	100	0.57929	39	1.72625	116	1.15567	20	0.86530	15	55
6	.50151	25	.99398	100	.57968	39	.72509	116	.15587	19	.86515	14	54
7	.50176	25	.99298	100	.58007	39	.72393	115	.15606	20	.86501	15	53
8	.50201	26	.99198	100	.58046	39	.72278	115	.15626	19	.86486	15	52
9	.50227	25	.99098	100	.58085	39	.72163	116	.15645	20	.86471	14	51
10	0.50252	25	1.98998	99	0.58124	38	1.72047	115	1.15665	19	0.86457	15	50
11	.50277	25	.98899	100	.58162	39	.71932	115	.15684	20	.86442	15	49
12	.50302	25	.98799	99	.58201	39	.71817	115	.15704	20	.86427	14	48
13	.50327	25	.98700	99	.58240	39	.71702	114	.15724	19	.86413	15	47
14	.50352	25	.98601	99	.58279	39	.71588	115	.15743	20	.86398	14	46
15	0.50377	26	1.98502	99	0.58318	39	1.71473	115	1.15763	19	0.86384	15	45
16	.50403	25	.98403	99	.58357	39	.71358	114	.15782	20	.86369	15	44
17	.50428	25	.98304	99	.58396	39	.71244	115	.15802	20	.86354	14	43
18	.50453	25	.98205	98	.58435	39	.71129	114	.15822	19	.86340	15	42
19	.50478	25	.98107	99	.58474	39	.71015	114	.15841	20	.86325	15	41
20	0.50503	25	1.98008	98	0.58513	39	1.70901	114	1.15861	20	0.86310	15	40
21	.50528	25	.97910	99	.58552	39	.70787	114	.15881	20	.86295	14	39
22	.50553	25	.97811	98	.58591	40	.70673	113	.15901	19	.86281	15	38
23	.50578	25	.97713	98	.58631	39	.70560	114	.15920	20	.86266	15	37
24	.50603	25	.97615	98	.58670	39	.70446	114	.15940	20	.86251	14	36
25	0.50628	26	1.97517	97	0.58709	39	1.70332	113	1.15960	20	0.86237	15	35
26	.50654	25	.97420	98	.58748	39	.70219	113	.15980	20	.86222	15	34
27	.50679	25	.97322	98	.58787	39	.70106	114	.16000	19	.86207	15	33
28	.50704	25	.97224	97	.58826	39	.69992	113	.16019	20	.86192	14	32
29	.50729	25	.97127	98	.58865	40	.69879	113	.16039	20	.86178	15	31
30	0.50754	25	1.97029	97	0.58905	39	1.69766	113	1.16059	20	0.86163	15	30
31	.50779	25	.96932	97	.58944	39	.69653	112	.16079	20	.86148	15	29
32	.50804	25	.96835	97	.58983	39	.69541	113	.16099	20	.86133	14	28
33	.50829	25	.96738	97	.59022	39	.69428	112	.16119	20	.86119	15	27
34	.50854	25	.96641	97	.59061	40	.69316	113	.16139	20	.86104	15	26
35	0.50879	25	1.96544	96	0.59101	39	1.69203	112	1.16159	20	0.86089	15	25
36	.50904	25	.96448	97	.59140	39	.69091	112	.16179	20	.86074	15	24
37	.50929	25	.96351	96	.59179	39	.68979	113	.16199	20	.86059	14	23
38	.50954	25	.96255	97	.59218	40	.68866	112	.16219	20	.86045	15	22
39	.50979	25	.96158	96	.59258	39	.68754	111	.16239	20	.86030	15	21
40	0.51004	25	1.96062	96	0.59297	39	1.68643	112	1.16259	20	0.86015	15	20
41	.51029	25	.95966	96	.59336	40	.68531	112	.16279	20	.86000	15	19
42	.51054	25	.95870	96	.59376	39	.68419	111	.16299	20	.85985	15	18
43	.51079	25	.95774	96	.59415	39	.68308	112	.16319	20	.85970	14	17
44	.51104	25	.95678	95	.59454	40	.68196	111	.16339	20	.85956	15	16
45	0.51129	25	1.95583	96	0.59494	39	1.68085	111	1.16359	21	0.85941	15	15
46	.51154	25	.95487	95	.59533	40	.67974	111	.16380	20	.85926	15	14
47	.51179	25	.95392	96	.59573	39	.67863	111	.16400	20	.85911	15	13
48	.51204	25	.95296	95	.59612	39	.67752	111	.16420	20	.85896	15	12
49	.51229	25	.95201	95	.59651	40	.67641	111	.16440	20	.85881	15	11
50	0.51254	25	1.95106	95	0.59691	39	1.67530	111	1.16460	21	0.85866	15	10
51	.51279	25	.95011	95	.59730	40	.67419	110	.16481	20	.85851	15	9
52	.51304	25	.94916	95	.59770	39	.67309	111	.16501	20	.85836	15	8
53	.51329	25	.94821	95	.59809	40	.67198	110	.16521	20	.85821	15	7
54	.51354	25	.94726	94	.59849	39	.67088	110	.16541	21	.85806	14	6
55	0.51379	25	1.94632	95	0.59888	40	1.66978	111	1.16562	20	0.85792	15	5
56	.51404	25	.94537	94	.59928	39	.66867	110	.16582	20	.85777	15	4
57	.51429	25	.94443	94	.59967	40	.66757	110	.16602	21	.85762	15	3
58	.51454	25	.94349	95	.60007	39	.66647	109	.16623	20	.85747	15	2
59	.51479	25	.94254	94	.60046	40	.66538	110	.16643	20	.85732	15	1
60	0.51504		1.94160		0.60086		1.66428		1.16663		0.85717		0
↑ 120°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ←59°

TABLE 31
Natural Trigonometric Functions

31° ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←148° ↓ ′
0	0. 51504	25	1. 94160	94	0. 60086	40	1. 66428	110	1. 16663	21	0. 85717	15	60
1	. 51529	25	. 94066	93	. 60126	39	. 66318	109	. 16684	20	. 85702	15	59
2	. 51554	25	. 93973	94	. 60165	40	. 66209	110	. 16704	21	. 85687	15	58
3	. 51579	25	. 93879	94	. 60205	40	. 66099	109	. 16725	20	. 85672	15	57
4	. 51604	24	. 93785	93	. 60245	39	. 65990	109	. 16745	21	. 85657	15	56
5	0. 51628	25	1. 93692	94	0. 60284	40	1. 65881	109	1. 16766	20	0. 85642	15	55
6	. 51653	25	. 93598	93	. 60324	40	. 65772	109	. 16786	20	. 85627	15	54
7	. 51678	25	. 93505	93	. 60364	39	. 65663	109	. 16806	21	. 85612	15	53
8	. 51703	25	. 93412	93	. 60403	40	. 65554	109	. 16827	21	. 85597	15	52
9	. 51728	25	. 93319	93	. 60443	40	. 65445	108	. 16848	20	. 85582	15	51
10	0. 51753	25	1. 93226	93	0. 60483	39	1. 65337	109	1. 16868	21	0. 85567	16	50
11	. 51778	25	. 93133	93	. 60522	40	. 65228	108	. 16889	20	. 85551	15	49
12	. 51803	25	. 93040	93	. 60562	40	. 65120	109	. 16909	21	. 85536	15	48
13	. 51828	24	. 92947	92	. 60602	40	. 65011	108	. 16930	20	. 85521	15	47
14	. 51852	25	. 92855	93	. 60642	39	. 64903	108	. 16950	21	. 85506	15	46
15	0. 51877	25	1. 92762	92	0. 60681	40	1. 64795	108	1. 16971	21	0. 85491	15	45
16	. 51902	25	. 92670	92	. 60721	40	. 64687	108	. 16992	20	. 85476	15	44
17	. 51927	25	. 92578	92	. 60761	40	. 64579	108	. 17012	21	. 85461	15	43
18	. 51952	25	. 92486	92	. 60801	40	. 64471	108	. 17033	21	. 85446	15	42
19	. 51977	25	. 92394	92	. 60841	40	. 64363	107	. 17054	21	. 85431	15	41
20	0. 52002	24	1. 92302	92	0. 60881	40	1. 64256	108	1. 17075	20	0. 85416	15	40
21	. 52026	25	. 92210	92	. 60921	39	. 64148	107	. 17095	21	. 85401	16	39
22	. 52051	25	. 92118	91	. 60960	40	. 64041	107	. 17116	21	. 85385	15	38
23	. 52076	25	. 92027	92	. 61000	40	. 63934	108	. 17137	21	. 85370	15	37
24	. 52101	25	. 91935	91	. 61040	40	. 63826	107	. 17158	20	. 85355	15	36
25	0. 52126	25	1. 91844	92	0. 61080	40	1. 63719	107	1. 17178	21	0. 85340	15	35
26	. 52151	24	. 91752	91	. 61120	40	. 63612	107	. 17199	21	. 85325	15	34
27	. 52175	25	. 91661	91	. 61160	40	. 63505	107	. 17220	21	. 85310	16	33
28	. 52200	25	. 91570	91	. 61200	40	. 63398	106	. 17241	21	. 85294	15	32
29	. 52225	25	. 91479	91	. 61240	40	. 63292	107	. 17262	21	. 85279	15	31
30	0. 52250	25	1. 91388	91	0. 61280	40	1. 63185	106	1. 17283	21	0. 85264	15	30
31	. 52275	24	. 91297	90	. 61320	40	. 63079	107	. 17304	21	. 85249	15	29
32	. 52299	25	. 91207	91	. 61360	40	. 62972	106	. 17325	21	. 85234	16	28
33	. 52324	25	. 91116	90	. 61400	40	. 62866	106	. 17346	21	. 85218	15	27
34	. 52349	25	. 91026	91	. 61440	40	. 62760	106	. 17367	21	. 85203	15	26
35	0. 52374	25	1. 90935	90	0. 61480	40	1. 62654	106	1. 17388	21	0. 85188	15	25
36	. 52399	24	. 90845	90	. 61520	41	. 62548	106	. 17409	21	. 85173	16	24
37	. 52423	25	. 90755	90	. 61561	40	. 62442	106	. 17430	21	. 85157	15	23
38	. 52448	25	. 90665	90	. 61601	40	. 62336	106	. 17451	21	. 85142	15	22
39	. 52473	25	. 90575	90	. 61641	40	. 62230	105	. 17472	21	. 85127	15	21
40	0. 52498	24	1. 90485	90	0. 61681	40	1. 62125	106	1. 17493	21	0. 85112	16	20
41	. 52522	25	. 90395	90	. 61721	40	. 62019	105	. 17514	21	. 85096	15	19
42	. 52547	25	. 90305	89	. 61761	40	. 61914	105	. 17535	21	. 85081	15	18
43	. 52572	25	. 90216	90	. 61801	41	. 61809	106	. 17556	21	. 85066	15	17
44	. 52597	24	. 90126	89	. 61842	40	. 61703	105	. 17577	21	. 85051	16	16
45	0. 52621	25	1. 90037	89	0. 61882	40	1. 61598	105	1. 17598	22	0. 85035	15	15
46	. 52646	25	. 89948	90	. 61922	40	. 61493	105	. 17620	21	. 85020	15	14
47	. 52671	25	. 89858	89	. 61962	41	. 61388	105	. 17641	21	. 85005	16	13
48	. 52696	24	. 89769	89	. 62003	40	. 61283	104	. 17662	21	. 84989	15	12
49	. 52720	25	. 89680	89	. 62043	40	. 61179	105	. 17683	21	. 84974	15	11
50	0. 52745	25	1. 89591	88	0. 62083	41	1. 61074	104	1. 17704	22	0. 84959	16	10
51	. 52770	24	. 89503	89	. 62124	40	. 60970	105	. 17726	21	. 84943	15	9
52	. 52794	25	. 89414	89	. 62164	40	. 60865	104	. 17747	21	. 84928	15	8
53	. 52819	25	. 89325	88	. 62204	41	. 60761	104	. 17768	22	. 84913	16	7
54	. 52844	25	. 89237	89	. 62245	40	. 60657	104	. 17790	21	. 84897	15	6
55	0. 52869	24	1. 89148	88	0. 62285	40	1. 60553	104	1. 17811	21	0. 84882	16	5
56	. 52893	25	. 89060	88	. 62325	41	. 60449	104	. 17832	22	. 84866	15	4
57	. 52918	25	. 88972	88	. 62366	40	. 60345	104	. 17854	21	. 84851	15	3
58	. 52943	24	. 88884	88	. 62406	40	. 60241	104	. 17875	21	. 84836	16	2
59	. 52967	25	. 88796	88	. 62446	41	. 60137	104	. 17896	22	. 84820	15	1
60	0. 52992		1. 88708		0. 62487		1. 60033		1. 17918		0. 84805		0
↑ 121°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←58° ↑

TABLE 31

Natural Trigonometric Functions

32° → ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	← 147° ↓ ′
0	0. 52992	25	1. 88708	88	0. 62487	40	1. 60033	103	1. 17918	21	0. 84805	16	60
1	. 53017	24	. 88620	88	. 62527	41	. 59930	104	. 17939	22	. 84789	15	59
2	. 53041	25	. 88532	87	. 62568	40	. 59826	103	. 17961	21	. 84774	15	58
3	. 53066	25	. 88445	88	. 62608	41	. 59723	103	. 17982	22	. 84759	16	57
4	. 53091	24	. 88357	87	. 62649	40	. 59620	103	. 18004	21	. 84743	15	56
5	0. 53115	25	1. 88270	87	0. 62689	41	1. 59517	103	1. 18025	22	0. 84728	16	55
6	. 53140	24	. 88183	88	. 62730	40	. 59414	103	. 18047	21	. 84712	15	54
7	. 53164	25	. 88095	87	. 62770	41	. 59311	103	. 18068	22	. 84697	16	53
8	. 53189	25	. 88008	87	. 62811	41	. 59208	103	. 18090	21	. 84681	15	52
9	. 53214	24	. 87921	87	. 62852	40	. 59105	103	. 18111	22	. 84666	16	51
10	0. 53238	25	1. 87834	86	0. 62892	41	1. 59002	102	1. 18133	22	0. 84650	15	50
11	. 53263	25	. 87748	87	. 62933	40	. 58900	103	. 18155	21	. 84635	16	49
12	. 53288	24	. 87661	87	. 62973	41	. 58797	102	. 18176	22	. 84619	15	48
13	. 53312	25	. 87574	86	. 63014	41	. 58695	102	. 18198	22	. 84604	16	47
14	. 53337	24	. 87488	87	. 63055	40	. 58593	103	. 18220	21	. 84588	15	46
15	0. 53361	25	1. 87401	86	0. 63095	41	1. 58490	102	1. 18241	22	0. 84573	16	45
16	. 53386	25	. 87315	86	. 63136	41	. 58388	102	. 18263	22	. 84557	15	44
17	. 53411	24	. 87229	87	. 63177	40	. 58286	102	. 18285	22	. 84542	16	43
18	. 53435	25	. 87142	86	. 63217	41	. 58184	101	. 18307	21	. 84526	15	42
19	. 53460	24	. 87056	86	. 63258	41	. 58083	102	. 18328	22	. 84511	16	41
20	0. 53484	25	1. 86970	85	0. 63299	41	1. 57981	102	1. 18350	22	0. 84495	15	40
21	. 53509	25	. 86885	86	. 63340	40	. 57879	101	. 18372	22	. 84480	16	39
22	. 53534	24	. 86799	86	. 63380	41	. 57778	102	. 18394	22	. 84464	16	38
23	. 53558	25	. 86713	86	. 63421	41	. 57676	101	. 18416	21	. 84448	15	37
24	. 53583	24	. 86627	85	. 63462	41	. 57575	101	. 18437	22	. 84433	16	36
25	0. 53607	25	1. 86542	85	0. 63503	41	1. 57474	102	1. 18459	22	0. 84417	15	35
26	. 53632	24	. 86457	86	. 63544	40	. 57372	101	. 18481	22	. 84402	16	34
27	. 53656	25	. 86371	85	. 63584	41	. 57271	101	. 18503	22	. 84386	16	33
28	. 53681	24	. 86286	85	. 63625	41	. 57170	101	. 18525	22	. 84370	15	32
29	. 53705	25	. 86201	85	. 63666	41	. 57069	100	. 18547	22	. 84355	16	31
30	0. 53730	24	1. 86116	85	0. 63707	41	1. 56969	101	1. 18569	22	0. 84339	15	30
31	. 53754	25	. 86031	85	. 63748	41	. 56868	101	. 18591	22	. 84324	16	29
32	. 53779	25	. 85946	85	. 63789	41	. 56767	100	. 18613	22	. 84308	16	28
33	. 53804	24	. 85861	84	. 63830	41	. 56667	101	. 18635	22	. 84292	15	27
34	. 53828	25	. 85777	85	. 63871	41	. 56566	100	. 18657	22	. 84277	16	26
35	0. 53853	24	1. 85692	84	0. 63912	41	1. 56466	100	1. 18679	22	0. 84261	16	25
36	. 53877	25	. 85608	85	. 63953	41	. 56366	101	. 18701	22	. 84245	15	24
37	. 53902	24	. 85523	84	. 63994	41	. 56265	100	. 18723	22	. 84230	16	23
38	. 53926	25	. 85439	84	. 64035	41	. 56165	100	. 18745	22	. 84214	16	22
39	. 53951	24	. 85355	84	. 64076	41	. 56065	99	. 18767	23	. 84198	16	21
40	0. 53975	25	1. 85271	84	0. 64117	41	1. 55966	100	1. 18790	22	0. 84182	15	20
41	. 54000	24	. 85187	84	. 64158	41	. 55866	100	. 18812	22	. 84167	16	19
42	. 54024	25	. 85103	84	. 64199	41	. 55766	100	. 18834	22	. 84151	16	18
43	. 54049	24	. 85019	84	. 64240	41	. 55666	99	. 18856	22	. 84135	15	17
44	. 54073	24	. 84935	83	. 64281	41	. 55567	100	. 18878	23	. 84120	16	16
45	0. 54097	25	1. 84852	84	0. 64322	41	1. 55467	99	1. 18901	22	0. 84104	16	15
46	. 54122	24	. 84768	83	. 64363	41	. 55368	99	. 18923	22	. 84088	16	14
47	. 54146	25	. 84685	84	. 64404	42	. 55269	99	. 18945	22	. 84072	15	13
48	. 54171	24	. 84601	83	. 64446	41	. 55170	99	. 18967	23	. 84057	16	12
49	. 54195	25	. 84518	83	. 64487	41	. 55071	99	. 18990	22	. 84041	16	11
50	0. 54220	24	1. 84435	83	0. 64528	41	1. 54972	99	1. 19012	22	0. 84025	16	10
51	. 54244	25	. 84352	83	. 64569	41	. 54873	99	. 19034	23	. 84009	15	9
52	. 54269	24	. 84269	83	. 64610	42	. 54774	99	. 19057	22	. 83994	16	8
53	. 54293	24	. 84186	83	. 64652	41	. 54675	99	. 19079	23	. 83978	16	7
54	. 54317	25	. 84103	83	. 64693	41	. 54576	98	. 19102	22	. 83962	16	6
55	0. 54342	24	1. 84020	82	0. 64734	41	1. 54478	99	1. 19124	22	0. 83946	16	5
56	. 54366	25	. 83938	83	. 64775	42	. 54379	98	. 19146	23	. 83930	15	4
57	. 54391	24	. 83855	82	. 64817	41	. 54281	98	. 19169	22	. 83915	16	3
58	. 54415	25	. 83773	83	. 64858	41	. 54183	98	. 19191	23	. 83899	16	2
59	. 54440	24	. 83690	82	. 64899	42	. 54085	99	. 19214	22	. 83883	16	1
60	0. 54464		1. 83608		0. 64941		1. 53986		1. 19236		0. 83867		0
↑ 122° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	← 57° ↑

TABLE 31

Natural Trigonometric Functions

33° → ′ ↓	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	← 146° ↓ ′
0	0. 54464	24	1. 83608	82	0. 64941	41	1. 53986	98	1. 19236	23	0. 83867	16	60
1	. 54488	25	. 83526	82	. 64982	42	. 53888	97	. 19259	22	. 83851	16	59
2	. 54513	24	. 83444	82	. 65024	41	. 53791	98	. 19281	23	. 83835	16	58
3	. 54537	24	. 83362	82	. 65065	41	. 53693	98	. 19304	23	. 83819	15	57
4	. 54561	25	. 83280	82	. 65106	42	. 53595	98	. 19327	22	. 83804	16	56
5	0. 54586	24	1. 83198	82	0. 65148	41	1. 53497	97	1. 19349	23	0. 83788	16	55
6	. 54610	25	. 83116	82	. 65189	42	. 53400	98	. 19372	22	. 83772	16	54
7	. 54635	24	. 83034	81	. 65231	41	. 53302	97	. 19394	23	. 83756	16	53
8	. 54659	24	. 82953	82	. 65272	42	. 53205	98	. 19417	23	. 83740	16	52
9	. 54683	25	. 82871	81	. 65314	41	. 53107	97	. 19440	23	. 83724	16	51
10	0. 54708	24	1. 82790	81	0. 65355	42	1. 53010	97	1. 19463	22	0. 83708	16	50
11	. 54732	24	. 82709	82	. 65397	41	. 52913	97	. 19485	23	. 83692	16	49
12	. 54756	25	. 82627	81	. 65438	42	. 52816	97	. 19508	23	. 83676	16	48
13	. 54781	24	. 82546	81	. 65480	41	. 52719	97	. 19531	22	. 83660	15	47
14	. 54805	24	. 82465	81	. 65521	42	. 52622	97	. 19553	23	. 83645	16	46
15	0. 54829	25	1. 82384	81	0. 65563	41	1. 52525	96	1. 19576	23	0. 83629	16	45
16	. 54854	24	. 82303	81	. 65604	42	. 52429	97	. 19599	23	. 83613	16	44
17	. 54878	24	. 82222	80	. 65646	42	. 52332	97	. 19622	23	. 83597	16	43
18	. 54902	25	. 82142	81	. 65688	41	. 52235	96	. 19645	23	. 83581	16	42
19	. 54927	24	. 82061	80	. 65729	42	. 52139	96	. 19668	23	. 83565	16	41
20	0. 54951	24	1. 81981	81	0. 65771	42	1. 52043	97	1. 19691	22	0. 83549	16	40
21	. 54975	24	. 81900	80	. 65813	41	. 51946	96	. 19713	23	. 83533	16	39
22	. 54999	25	. 81820	80	. 65854	42	. 51850	96	. 19736	23	. 83517	16	38
23	. 55024	24	. 81740	81	. 65896	42	. 51754	96	. 19759	23	. 83501	16	37
24	. 55048	24	. 81659	80	. 65938	42	. 51658	96	. 19782	23	. 83485	16	36
25	0. 55072	25	1. 81579	80	0. 65980	41	1. 51562	96	1. 19805	23	0. 83469	16	35
26	. 55097	24	. 81499	80	. 66021	42	. 51466	96	. 19828	23	. 83453	16	34
27	. 55121	24	. 81419	79	. 66063	42	. 51370	95	. 19851	23	. 83437	16	33
28	. 55145	24	. 81340	80	. 66105	42	. 51275	96	. 19874	23	. 83421	16	32
29	. 55169	25	. 81260	80	. 66147	42	. 51179	95	. 19897	23	. 83405	16	31
30	0. 55194	24	1. 81180	79	0. 66189	41	1. 51084	96	1. 19920	24	0. 83389	16	30
31	. 55218	24	. 81101	80	. 66230	42	. 50988	95	. 19944	23	. 83373	17	29
32	. 55242	24	. 81021	79	. 66272	42	. 50893	96	. 19967	23	. 83356	16	28
33	. 55266	25	. 80942	80	. 66314	42	. 50797	95	. 19990	23	. 83340	16	27
34	. 55291	24	. 80862	79	. 66356	42	. 50702	95	. 20013	23	. 83324	16	26
35	0. 55315	24	1. 80783	79	0. 66398	42	1. 50607	95	1. 20036	23	0. 83308	16	25
36	. 55339	24	. 80704	79	. 66440	42	. 50512	95	. 20059	24	. 83292	16	24
37	. 55363	25	. 80625	79	. 66482	42	. 50417	95	. 20083	23	. 83276	16	23
38	. 55388	24	. 80546	79	. 66524	42	. 50322	94	. 20106	23	. 83260	16	22
39	. 55412	24	. 80467	79	. 66566	42	. 50228	95	. 20129	23	. 83244	16	21
40	0. 55436	24	1. 80388	79	0. 66608	42	1. 50133	95	1. 20152	24	0. 83228	16	20
41	. 55460	24	. 80309	78	. 66650	42	. 50038	94	. 20176	23	. 83212	17	19
42	. 55484	25	. 80231	79	. 66692	42	. 49944	95	. 20199	23	. 83195	16	18
43	. 55509	24	. 80152	78	. 66734	42	. 49849	94	. 20222	24	. 83179	16	17
44	. 55533	24	. 80074	79	. 66776	42	. 49755	94	. 20246	23	. 83163	16	16
45	0. 55557	24	1. 79995	78	0. 66818	42	1. 49661	95	1. 20269	23	0. 83147	16	15
46	. 55581	24	. 79917	78	. 66860	42	. 49566	94	. 20292	24	. 83131	16	14
47	. 55605	25	. 79839	78	. 66902	42	. 49472	94	. 20316	23	. 83115	17	13
48	. 55630	24	. 79761	79	. 66944	42	. 49378	94	. 20339	24	. 83098	16	12
49	. 55654	24	. 79682	78	. 66986	42	. 49284	94	. 20363	23	. 83082	16	11
50	0. 55678	24	1. 79604	77	0. 67028	43	1. 49190	93	1. 20386	24	0. 83066	16	10
51	. 55702	24	. 79527	78	. 67071	42	. 49097	94	. 20410	23	. 83050	16	9
52	. 55726	24	. 79449	78	. 67113	42	. 49003	94	. 20433	24	. 83034	17	8
53	. 55750	25	. 79371	78	. 67155	42	. 48909	93	. 20457	23	. 83017	16	7
54	. 55775	24	. 79293	77	. 67197	42	. 48816	94	. 20480	24	. 83001	16	6
55	0. 55799	24	1. 79216	78	0. 67239	43	1. 48722	93	1. 20504	23	0. 82985	16	5
56	. 55823	24	. 79138	77	. 67282	42	. 48629	93	. 20527	24	. 82969	16	4
57	. 55847	24	. 79061	77	. 67324	42	. 48536	94	. 20551	24	. 82953	17	3
58	. 55871	24	. 78984	78	. 67366	43	. 48442	93	. 20575	23	. 82936	16	2
59	. 55895	24	. 78906	77	. 67409	42	. 48349	93	. 20598	24	. 82920	16	1
60	0. 55919		1. 78829		0. 67451		1. 48256		1. 20622		0. 82904		0
↑ 123° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	← 56° ↑

TABLE 31
Natural Trigonometric Functions

34°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←145° ↓ ′
0	0. 55919	24	1. 78829	77	0. 67451	42	1. 48256	93	1. 20622	23	0. 82904	17	60
1	. 55943	25	. 78752	77	. 67493	43	. 48163	93	20645	24	. 82887	16	59
2	. 55968	24	. 78675	77	. 67536	42	. 48070	93	. 20669	24	. 82871	16	58
3	. 55992	24	. 78598	77	. 67578	42	. 47977	92	. 20693	24	. 82855	16	57
4	. 56016	24	. 78521	76	. 67620	43	. 47885	93	. 20717	23	. 82839	17	56
5	0. 56040	24	1. 78445	77	0. 67663	42	1. 47792	93	1. 20740	24	0. 82822	16	55
6	. 56064	24	. 78368	77	. 67705	43	. 47699	92	. 20764	24	. 82806	16	54
7	. 56088	24	. 78291	76	. 67748	42	. 47607	93	. 20788	24	. 82790	17	53
8	. 56112	24	. 78215	77	. 67790	42	. 47514	92	. 20812	24	. 82773	16	52
9	. 56136	24	. 78138	76	. 67832	43	. 47422	92	. 20836	23	. 82757	16	51
10	0. 56160	24	1. 78062	76	0. 67875	42	1. 47330	92	1. 20859	24	0. 82741	17	50
11	. 56184	24	. 77986	76	. 67917	43	. 47238	92	. 20883	24	. 82724	16	49
12	. 56208	24	. 77910	77	. 67960	42	. 47146	93	. 20907	24	. 82708	16	48
13	. 56232	24	. 77833	76	. 68002	43	. 47053	91	. 20931	24	. 82692	17	47
14	. 56256	24	. 77757	76	. 68045	43	. 46962	92	. 20955	24	. 82675	16	46
15	0. 56280	25	1. 77681	75	0. 68088	42	1. 46870	92	1. 20979	24	0. 82659	16	45
16	. 56305	24	. 77606	76	. 68130	43	. 46778	92	. 21003	24	. 82643	17	44
17	. 56329	24	. 77530	76	. 68173	42	. 46686	91	. 21027	24	. 82626	16	43
18	. 56353	24	. 77454	76	. 68215	43	. 46595	92	. 21051	24	. 82610	17	42
19	. 56377	24	. 77378	75	. 68258	43	. 46503	92	. 21075	24	. 82593	16	41
20	0. 56401	24	1. 77303	76	0. 68301	42	1. 46411	91	1. 21099	24	0. 82577	16	40
21	. 56425	24	. 77227	75	. 68343	43	. 46320	91	. 21123	24	. 82561	17	39
22	. 56449	24	. 77152	75	. 68386	43	. 46229	92	. 21147	24	. 82544	16	38
23	. 56473	24	. 77077	76	. 68429	42	. 46137	91	. 21171	24	. 82528	17	37
24	. 56497	24	. 77001	75	. 68471	43	. 46046	91	. 21195	25	. 82511	16	36
25	0. 56521	24	1. 76926	75	0. 68514	43	1. 45955	91	1. 21220	24	0. 82495	17	35
26	. 56545	24	. 76851	75	. 68557	43	. 45864	91	. 21244	24	. 82478	16	34
27	. 56569	24	. 76776	75	. 68600	42	. 45773	91	. 21268	24	. 82462	16	33
28	. 56593	24	. 76701	75	. 68642	43	. 45682	90	. 21292	24	. 82446	17	32
29	. 56617	24	. 76626	74	. 68685	43	. 45592	91	. 21316	25	. 82429	16	31
30	0. 56641	24	1. 76552	75	0. 68728	43	1. 45501	91	1. 21341	24	0. 82413	17	30
31	. 56665	24	. 76477	75	. 68771	43	. 45410	90	. 21365	24	. 82396	16	29
32	. 56689	24	. 76402	74	. 68814	43	. 45320	91	. 21389	25	. 82380	17	28
33	. 56713	23	. 76328	75	. 68857	43	. 45229	90	. 21414	24	. 82363	16	27
34	. 56736	24	. 76253	74	. 68900	42	. 45139	90	. 21438	24	. 82347	17	26
35	0. 56760	24	1. 76179	74	0. 68942	43	1. 45049	91	1. 21462	25	0. 82330	16	25
36	. 56784	24	. 76105	74	. 68985	43	. 44958	90	. 21487	24	. 82314	17	24
37	. 56808	24	. 76031	75	. 69028	43	. 44868	90	. 21511	24	. 82297	16	23
38	. 56832	24	. 75956	74	. 69071	43	. 44778	90	. 21535	25	. 82281	17	22
39	. 56856	24	. 75882	74	. 69114	43	. 44688	90	. 21560	24	. 82264	16	21
40	0. 56880	24	1. 75808	74	0. 69157	43	1. 44598	90	1. 21584	25	0. 82248	17	20
41	. 56904	24	. 75734	73	. 69200	43	. 44508	90	. 21609	24	. 82231	17	19
42	. 56928	24	. 75661	74	. 69243	43	. 44418	89	. 21633	25	. 82214	16	18
43	. 56952	24	. 75587	74	. 69286	43	. 44329	90	. 21658	24	. 82198	17	17
44	. 56976	24	. 75513	73	. 69329	43	. 44239	90	. 21682	25	. 82181	16	16
45	0. 57000	24	1. 75440	74	0. 69372	44	1. 44149	89	1. 21707	24	0. 82165	17	15
46	. 57024	23	. 75366	73	. 69416	43	. 44060	90	. 21731	25	. 82148	16	14
47	. 57047	24	. 75293	74	. 69459	43	. 43970	89	. 21756	25	. 82132	17	13
48	. 57071	24	. 75219	73	. 69502	43	. 43881	89	. 21781	24	. 82115	17	12
49	. 57095	24	. 75146	73	. 69545	43	. 43792	89	. 21805	25	. 82098	16	11
50	0. 57119	24	1. 75073	73	0. 69588	43	1. 43703	89	1. 21830	25	0. 82082	17	10
51	. 57143	24	. 75000	73	. 69631	44	. 43614	89	. 21855	24	. 82065	17	9
52	. 57167	24	. 74927	73	. 69675	43	. 43525	89	. 21879	25	. 82048	16	8
53	. 57191	24	. 74854	73	. 69718	43	. 43436	89	. 21904	25	. 82032	17	7
54	. 57215	23	. 74781	73	. 69761	43	. 43347	89	. 21929	24	. 82015	16	6
55	0. 57238	24	1. 74708	73	0. 69804	43	1. 43258	89	1. 21953	25	0. 81999	17	5
56	. 57262	24	. 74635	73	. 69847	44	. 43169	89	. 21978	25	. 81982	17	4
57	. 57286	24	. 74562	72	. 69891	43	. 43080	88	. 22003	25	. 81965	16	3
58	. 57310	24	. 74490	73	. 69934	43	. 42992	89	. 22028	25	. 81949	17	2
59	. 57334	24	. 74417	72	. 69977	44	. 42903	88	. 22053	24	. 81932	17	1
60	0. 57358		1. 74345		0. 70021		1. 42815		1. 22077		0. 81915		0
↑ 124°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←55° ↑

TABLE 31

Natural Trigonometric Functions

35°→ ↓	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←144° ↓
′													′
0	0. 57358	23	1. 74345	73	0. 70021	43	1. 42815	89	1. 22077	25	0. 81915	16	60
1	. 57381	24	. 74272	72	. 70064	43	. 42726	88	. 22102	25	. 81899	17	59
2	. 57405	24	. 74200	72	. 70107	44	. 42638	88	. 22127	25	. 81882	17	58
3	. 57429	24	. 74128	72	. 70151	43	. 42550	88	. 22152	25	. 81865	17	57
4	. 57453	24	. 74056	73	. 70194	44	. 42462	88	. 22177	25	. 81848	16	56
5	0. 57477	24	1. 73983	72	0. 70238	43	1. 42374	88	1. 22202	25	0. 81832	17	55
6	. 57501	23	. 73911	71	. 70281	44	. 42286	88	. 22227	25	. 81815	17	54
7	. 57524	24	. 73840	72	. 70325	43	. 42198	88	. 22252	25	. 81798	16	53
8	. 57548	24	. 73768	72	. 70368	44	. 42110	88	. 22277	25	. 81782	17	52
9	. 57572	24	. 73696	72	. 70412	43	. 42022	88	. 22302	25	. 81765	17	51
10	0. 57596	23	1. 73624	72	0. 70455	44	1. 41934	87	1. 22327	25	0. 81748	17	50
11	. 57619	24	. 73552	71	. 70499	43	. 41847	88	. 22352	25	. 81731	17	49
12	. 57643	24	. 73481	72	. 70542	44	. 41759	87	. 22377	25	. 81714	16	48
13	. 57667	24	. 73409	71	. 70586	43	. 41672	88	. 22402	26	. 81698	17	47
14	. 57691	24	. 73338	71	. 70629	44	. 41584	87	. 22428	25	. 81681	17	46
15	0. 57715	23	1. 73267	72	0. 70673	44	1. 41497	88	1. 22453	25	0. 81664	17	45
16	. 57738	24	. 73195	71	. 70717	43	. 41409	87	. 22478	25	. 81647	16	44
17	. 57762	24	. 73124	71	. 70760	44	. 41322	87	. 22503	25	. 81631	17	43
18	. 57786	24	. 73053	71	. 70804	44	. 41235	87	. 22528	26	. 81614	17	42
19	. 57810	23	. 72982	71	. 70848	43	. 41148	87	. 22554	25	. 81597	17	41
20	0. 57833	24	1. 72911	71	0. 70891	44	1. 41061	87	1. 22579	25	0. 81580	17	40
21	. 57857	24	. 72840	71	. 70935	44	. 40974	87	. 22604	25	. 81563	17	39
22	. 57881	23	. 72769	71	. 70979	44	. 40887	87	. 22629	26	. 81546	16	38
23	. 57904	24	. 72698	70	. 71023	43	. 40800	86	. 22655	25	. 81530	17	37
24	. 57928	24	. 72628	71	. 71066	44	. 40714	87	. 22680	26	. 81513	17	36
25	0. 57952	24	1. 72557	70	0. 71110	44	1. 40627	87	1. 22706	25	0. 81496	17	35
26	. 57976	23	. 72487	71	. 71154	44	. 40540	86	. 22731	25	. 81479	17	34
27	. 57999	24	. 72416	70	. 71198	44	. 40454	87	. 22756	26	. 81462	17	33
28	. 58023	24	. 72346	71	. 71242	43	. 40367	86	. 22782	25	. 81445	17	32
29	. 58047	23	. 72275	70	. 71285	44	. 40281	86	. 22807	26	. 81428	16	31
30	0. 58070	24	1. 72205	70	0. 71329	44	1. 40195	86	1. 22833	25	0. 81412	17	30
31	. 58094	24	. 72135	70	. 71373	44	. 40109	87	. 22858	26	. 81395	17	29
32	. 58118	23	. 72065	70	. 71417	44	. 40022	86	. 22884	25	. 81378	17	28
33	. 58141	24	. 71995	70	. 71461	44	. 39936	86	. 22909	26	. 81361	17	27
34	. 58165	24	. 71925	70	. 71505	44	. 39850	86	. 22935	25	. 81344	17	26
35	0. 58189	23	1. 71855	70	0. 71549	44	1. 39764	85	1. 22960	26	0. 81327	17	25
36	. 58212	24	. 71785	70	. 71593	44	. 39679	86	. 22986	26	. 81310	17	24
37	. 58236	24	. 71715	69	. 71637	44	. 39593	86	. 23012	25	. 81293	17	23
38	. 58260	23	. 71646	70	. 71681	44	. 39507	86	. 23037	26	. 81276	17	22
39	. 58283	24	. 71576	70	. 71725	44	. 39421	85	. 23063	26	. 81259	17	21
40	0. 58307	23	1. 71506	69	0. 71769	44	1. 39336	86	1. 23089	25	0. 81242	17	20
41	. 58330	24	. 71437	69	. 71813	44	. 39250	85	. 23114	26	. 81225	17	19
42	. 58354	24	. 71368	70	. 71857	44	. 39165	86	. 23140	26	. 81208	17	18
43	. 58378	23	. 71298	69	. 71901	45	. 39079	85	. 23166	26	. 81191	17	17
44	. 58401	24	. 71229	69	. 71946	44	. 38994	85	. 23192	25	. 81174	17	16
45	0. 58425	24	1. 71160	69	0. 71990	44	1. 38909	85	1. 23217	26	0. 81157	17	15
46	. 58449	23	. 71091	69	. 72034	44	. 38824	86	. 23243	26	. 81140	17	14
47	. 58472	24	. 71022	69	. 72078	44	. 38738	85	. 23269	26	. 81123	17	13
48	. 58496	23	. 70953	69	. 72122	45	. 38653	85	. 23295	26	. 81106	17	12
49	. 58519	24	. 70884	69	. 72167	44	. 38568	84	. 23321	26	. 81089	17	11
50	0. 58543	24	1. 70815	69	0. 72211	44	1. 38484	85	1. 23347	26	0. 81072	17	10
51	. 58567	23	. 70746	69	. 72255	44	. 38399	85	. 23373	25	. 81055	17	9
52	. 58590	24	. 70677	68	. 72299	45	. 38314	85	. 23398	26	. 81038	17	8
53	. 58614	23	. 70609	69	. 72344	44	. 38229	84	. 23424	26	. 81021	17	7
54	. 58637	24	. 70540	68	. 72388	44	. 38145	85	. 23450	26	. 81004	17	6
55	0. 58661	23	1. 70472	69	0. 72432	45	1. 38060	84	1. 23476	26	0. 80987	17	5
56	. 58684	24	. 70403	68	. 72477	44	. 37976	85	. 23502	27	. 80970	17	4
57	. 58708	23	. 70335	68	. 72521	44	. 37891	84	. 23529	26	. 80953	17	3
58	. 58731	24	. 70267	69	. 72565	45	. 37807	85	. 23555	26	. 80936	17	2
59	. 58755	24	. 70198	68	. 72610	44	. 37722	84	. 23581	26	. 80919	17	1
60	0. 58779		1. 70130		0. 72654		1. 37638		1. 23607		0. 80902		0
↑ 125°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′←	↑ 54°

TABLE 31
Natural Trigonometric Functions

36° → ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	← 143° ↓ ′
0	0. 58779	23	1. 70130	68	0. 72654	45	1. 37638	84	1. 23607	26	0. 80902	17	60
1	. 58802	24	. 70062	68	. 72699	44	. 37554	84	. 23633	26	. 80885	18	59
2	. 58826	23	. 69994	68	. 72743	45	. 37470	84	. 23659	26	. 80867	17	58
3	. 58849	24	. 69926	68	. 72788	44	. 37386	84	. 23685	26	. 80850	17	57
4	. 58873	23	. 69858	68	. 72832	45	. 37302	84	. 23711	27	. 80833	17	56
5	0. 58896	24	1. 69790	67	0. 72877	44	1. 37218	84	1. 23738	26	0. 80816	17	55
6	. 58920	23	. 69723	68	. 72921	45	. 37134	84	. 23764	26	. 80799	17	54
7	. 58943	24	. 69655	68	. 72966	44	. 37050	83	. 23790	26	. 80782	17	53
8	. 58967	23	. 69587	67	. 73010	45	. 36967	84	. 23816	27	. 80765	17	52
9	. 58990	24	. 69520	68	. 73055	45	. 36883	83	. 23843	26	. 80748	18	51
10	0. 59014	23	1. 69452	67	0. 73100	44	1. 36800	84	1. 23869	26	0. 80730	17	50
11	. 59037	24	. 69385	67	. 73144	45	. 36716	83	. 23895	27	. 80713	17	49
12	. 59061	23	. 69318	68	. 73189	45	. 36633	84	. 23922	26	. 80696	17	48
13	. 59084	24	. 69250	67	. 73234	44	. 36549	83	. 23948	27	. 80679	17	47
14	. 59108	23	. 69183	67	. 73278	45	. 36466	83	. 23975	26	. 80662	18	46
15	0. 59131	23	1. 69116	67	0. 73323	45	1. 36383	83	1. 24001	27	0. 80644	17	45
16	. 59154	24	. 69049	67	. 73368	45	. 36300	83	. 24028	26	. 80627	17	44
17	. 59178	23	. 68982	67	. 73413	44	. 36217	83	. 24054	27	. 80610	17	43
18	. 59201	24	. 68915	67	. 73457	45	. 36134	83	. 24081	26	. 80593	17	42
19	. 59225	23	. 68848	66	. 73502	45	. 36051	83	. 24107	27	. 80576	18	41
20	0. 59248	24	1. 68782	67	0. 73547	45	1. 35968	83	1. 24134	26	0. 80558	17	40
21	. 59272	23	. 68715	67	. 73592	45	. 35885	83	. 24160	27	. 80541	17	39
22	. 59295	23	. 68648	66	. 73637	44	. 35802	83	. 24187	26	. 80524	17	38
23	. 59318	24	. 68582	67	. 73681	45	. 35719	82	. 24213	27	. 80507	18	37
24	. 59342	23	. 68515	66	. 73726	45	. 35637	83	. 24240	27	. 80489	17	36
25	0. 59365	24	1. 68449	67	0. 73771	45	1. 35554	82	1. 24267	26	0. 80472	17	35
26	. 59389	23	. 68382	66	. 73816	45	. 35472	83	. 24293	27	. 80455	17	34
27	. 59412	24	. 68316	66	. 73861	45	. 35389	82	. 24320	27	. 80438	18	33
28	. 59436	23	. 68250	67	. 73906	45	. 35307	83	. 24347	26	. 80420	17	32
29	. 59459	23	. 68183	66	. 73951	45	. 35224	82	. 24373	27	. 80403	17	31
30	0. 59482	24	1. 68117	66	0. 73996	45	1. 35142	82	1. 24400	27	0. 80386	18	30
31	. 59506	23	. 68051	66	. 74041	45	. 35060	82	. 24427	27	. 80368	17	29
32	. 59529	23	. 67985	66	. 74086	45	. 34978	82	. 24454	27	. 80351	17	28
33	. 59552	24	. 67919	66	. 74131	45	. 34896	82	. 24481	27	. 80334	18	27
34	. 59576	23	. 67853	65	. 74176	45	. 34814	82	. 24508	26	. 80316	17	26
35	0. 59599	23	1. 67788	66	0. 74221	46	1. 34732	82	1. 24534	27	0. 80299	17	25
36	. 59622	24	. 67722	66	. 74267	45	. 34650	82	. 24561	27	. 80282	18	24
37	. 59646	23	. 67656	65	. 74312	45	. 34568	81	. 24588	27	. 80264	17	23
38	. 59669	24	. 67591	66	. 74357	45	. 34487	82	. 24615	27	. 80247	17	22
39	. 59693	23	. 67525	65	. 74402	45	. 34405	82	. 24642	27	. 80230	18	21
40	0. 59716	23	1. 67460	66	0. 74447	45	1. 34323	81	1. 24669	27	0. 80212	17	20
41	. 59739	24	. 67394	65	. 74492	46	. 34242	82	. 24696	27	. 80195	17	19
42	. 59763	23	. 67329	65	. 74538	45	. 34160	81	. 24723	27	. 80178	18	18
43	. 59786	23	. 67264	66	. 74583	45	. 34079	81	. 24750	27	. 80160	17	17
44	. 59809	23	. 67198	65	. 74628	46	. 33998	82	. 24777	27	. 80143	18	16
45	0. 59832	24	1. 67133	65	0. 74674	45	1. 33916	81	1. 24804	28	0. 80125	17	15
46	. 59856	23	. 67068	65	. 74719	45	. 33835	81	. 24832	27	. 80108	17	14
47	. 59879	23	. 67003	65	. 74764	46	. 33754	81	. 24859	27	. 80091	18	13
48	. 59902	24	. 66938	65	. 74810	45	. 33673	81	. 24886	27	. 80073	17	12
49	. 59926	23	. 66873	64	. 74855	45	. 33592	81	. 24913	27	. 80056	18	11
50	0. 59949	23	1. 66809	65	0. 74900	46	1. 33511	81	1. 24940	27	0. 80038	17	10
51	. 59972	23	. 66744	65	. 74946	45	. 33430	81	. 24967	28	. 80021	18	9
52	. 59995	24	. 66679	64	. 74991	46	. 33349	81	. 24995	27	. 80003	17	8
53	. 60019	23	. 66615	65	. 75037	45	. 33268	81	. 25022	27	. 79986	18	7
54	. 60042	23	. 66550	64	. 75082	46	. 33187	80	. 25049	28	. 79968	17	6
55	0. 60065	24	1. 66486	65	0. 75128	45	1. 33107	81	1. 25077	27	0. 79951	17	5
56	. 60089	23	. 66421	64	. 75173	46	. 33026	80	. 25104	27	. 79934	18	4
57	. 60112	23	. 66357	65	. 75219	45	. 32946	81	. 25131	28	. 79916	17	3
58	. 60135	23	. 66292	64	. 75264	46	. 32865	80	. 25159	27	. 79899	18	2
59	. 60158	24	. 66228	64	. 75310	45	. 32785	81	. 25186	28	. 79881	17	1
60	0. 60182		1. 66164		0. 75355		1. 32704		1. 25214		0. 79864		0
↑ 126° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	← 53° ↑

TABLE 31
Natural Trigonometric Functions

37°→ ′ ↓	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←142° ′ ↓
0	0. 60182	23	1. 66164	64	0. 75355	46	1. 32704	80	1. 25214	27	0. 79864	18	60
1	. 60205	23	. 66100	64	. 75401	46	. 32624	80	. 25241	28	. 79846	17	59
2	. 60228	23	. 66036	64	. 75447	45	. 32544	80	. 25269	27	. 79829	18	58
3	. 60251	23	. 65972	64	. 75492	46	. 32464	80	. 25296	28	. 79811	18	57
4	. 60274	24	. 65908	64	. 75538	46	. 32384	80	. 25324	27	. 79793	17	56
5	0. 60298	23	1. 65844	64	0. 75584	45	1. 32304	80	1. 25351	28	0. 79776	18	55
6	. 60321	23	. 65780	63	. 75629	46	. 32224	80	. 25379	27	. 79758	17	54
7	. 60344	23	. 65717	64	. 75675	46	. 32144	80	. 25406	28	. 79741	18	53
8	. 60367	23	. 65653	64	. 75721	46	. 32064	80	. 25434	28	. 79723	17	52
9	. 60390	24	. 65589	63	. 75767	45	. 31984	80	. 25462	27	. 79706	18	51
10	0. 60414	23	1. 65526	64	0. 75812	46	1. 31904	79	1. 25489	28	0. 79688	17	50
11	. 60437	23	. 65462	63	. 75858	46	. 31825	80	. 25517	28	. 79671	18	49
12	. 60460	23	. 65399	64	. 75904	46	. 31745	79	. 25545	27	. 79653	18	48
13	. 60483	23	. 65335	63	. 75950	46	. 31666	80	. 25572	28	. 79635	17	47
14	. 60506	23	. 65272	63	. 75996	46	. 31586	79	. 25600	28	. 79618	18	46
15	0. 60529	24	1. 65209	63	0. 76042	46	1. 31507	80	1. 25628	28	0. 79600	17	45
16	. 60553	23	. 65146	63	. 76088	46	. 31427	79	. 25656	27	. 79583	18	44
17	. 60576	23	. 65083	63	. 76134	46	. 31348	79	. 25683	28	. 79565	18	43
18	. 60599	23	. 65020	63	. 76180	46	. 31269	79	. 25711	28	. 79547	17	42
19	. 60622	23	. 64957	63	. 76226	46	. 31190	80	. 25739	28	. 79530	18	41
20	0. 60645	23	1. 64894	63	0. 76272	46	1. 31110	79	1. 25767	28	0. 79512	18	40
21	. 60668	23	. 64831	63	. 76318	46	. 31031	79	. 25795	28	. 79494	17	39
22	. 60691	23	. 64768	63	. 76364	46	. 30952	79	. 25823	28	. 79477	18	38
23	. 60714	24	. 64705	62	. 76410	46	. 30873	78	. 25851	28	. 79459	18	37
24	. 60738	23	. 64643	63	. 76456	46	. 30795	79	. 25879	28	. 79441	17	36
25	0. 60761	23	1. 64580	62	0. 76502	46	1. 30716	79	1. 25907	28	0. 79424	18	35
26	. 60784	23	. 64518	63	. 76548	46	. 30637	79	. 25935	28	. 79406	18	34
27	. 60807	23	. 64455	62	. 76594	46	. 30558	78	. 25963	28	. 79388	17	33
28	. 60830	23	. 64393	63	. 76640	46	. 30480	79	. 25991	28	. 79371	18	32
29	. 60853	23	. 64330	62	. 76686	47	. 30401	78	. 26019	28	. 79353	18	31
30	0. 60876	23	1. 64268	62	0. 76733	46	1. 30323	79	1. 26047	28	0. 79335	17	30
31	. 60899	23	. 64206	62	. 76779	46	. 30244	78	. 26075	29	. 79318	18	29
32	. 60922	23	. 64144	63	. 76825	46	. 30166	79	. 26104	28	. 79300	18	28
33	. 60945	23	. 64081	62	. 76871	47	. 30087	78	. 26132	28	. 79282	18	27
34	. 60968	23	. 64019	62	. 76918	46	. 30009	78	. 26160	28	. 79264	17	26
35	0. 60991	24	1. 63957	62	0. 76964	46	1. 29931	78	1. 26188	28	0. 79247	18	25
36	. 61015	23	. 63895	61	. 77010	47	. 29853	78	. 26216	29	. 79229	18	24
37	. 61038	23	. 63834	62	. 77057	46	. 29775	79	. 26245	28	. 79211	18	23
38	. 61061	23	. 63772	62	. 77103	46	. 29696	78	. 26273	28	. 79193	17	22
39	. 61084	23	. 63710	62	. 77149	47	. 29618	77	. 26301	29	. 79176	18	21
40	0. 61107	23	1. 63648	61	0. 77196	46	1. 29541	78	1. 26330	28	0. 79158	18	20
41	. 61130	23	. 63587	62	. 77242	47	. 29463	78	. 26358	29	. 79140	18	19
42	. 61153	23	. 63525	61	. 77289	46	. 29385	78	. 26387	28	. 79122	17	18
43	. 61176	23	. 63464	62	. 77335	47	. 29307	78	. 26415	28	. 79105	18	17
44	. 61199	23	. 63402	61	. 77382	46	. 29229	77	. 26443	29	. 79087	18	16
45	0. 61222	23	1. 63341	62	0. 77428	47	1. 29152	78	1. 26472	28	0. 79069	18	15
46	. 61245	23	. 63279	61	. 77475	46	. 29074	77	. 26500	29	. 79051	18	14
47	. 61268	23	. 63218	61	. 77521	47	. 28997	78	. 26529	28	. 79033	17	13
48	. 61291	23	. 63157	61	. 77568	47	. 28919	77	. 26557	29	. 79016	18	12
49	. 61314	23	. 63096	61	. 77615	46	. 28842	78	. 26586	29	. 78998	18	11
50	0. 61337	23	1. 63035	61	0. 77661	47	1. 28764	77	1. 26615	28	0. 78980	18	10
51	. 61360	23	. 62974	61	. 77708	46	. 28687	77	. 26643	29	. 78962	18	9
52	. 61383	23	. 62913	61	. 77754	47	. 28610	77	. 26672	29	. 78944	18	8
53	. 61406	23	. 62852	61	. 77801	47	. 28533	77	. 26701	28	. 78926	18	7
54	. 61429	22	. 62791	61	. 77848	47	. 28456	77	. 26729	29	. 78908	17	6
55	0. 61451	23	1. 62730	61	0. 77895	46	1. 28379	77	1. 26758	29	0. 78891	18	5
56	. 61474	23	. 62669	60	. 77941	47	. 28302	77	. 26787	28	. 78873	18	4
57	. 61497	23	. 62609	61	. 77988	47	. 28225	77	. 26815	29	. 78855	18	3
58	. 61520	23	. 62548	61	. 78035	47	. 28148	77	. 26844	29	. 78837	18	2
59	. 61543	23	. 62487	60	. 78082	47	. 28071	77	. 26873	29	. 78819	18	1
60	0. 61566		1. 62427		0. 78129		1. 27994		1. 26902		0. 78801		0
↑ 127°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ←52°

TABLE 31

Natural Trigonometric Functions

38°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←141° ↓ ′
0	0. 61566	23	1. 62427	61	0. 78129	46	1. 27994	77	1. 26902	29	0. 78801	18	60
1	. 61589	23	. 62366	60	. 78175	47	. 27917	76	. 26931	29	. 78783	18	59
2	. 61612	23	. 62306	60	. 78222	47	. 27841	77	. 26960	28	. 78765	18	58
3	. 61635	23	. 62246	61	. 78269	47	. 27764	76	. 26988	29	. 78747	18	57
4	. 61658	23	. 62185	60	. 78316	47	. 27688	77	. 27017	29	. 78729	18	56
5	0. 61681	23	1. 62125	60	0. 78363	47	1. 27611	76	1. 27046	29	0. 78711	17	55
6	. 61704	22	. 62065	60	. 78410	47	. 27535	77	. 27075	29	. 78694	18	54
7	. 61726	23	. 62005	60	. 78457	47	. 27458	76	. 27104	29	. 78676	18	53
8	. 61749	23	. 61945	60	. 78504	47	. 27382	76	. 27133	29	. 78658	18	52
9	. 61772	23	. 61885	60	. 78551	47	. 27306	76	. 27162	29	. 78640	18	51
10	0. 61795	23	1. 61825	60	0. 78598	47	1. 27230	77	1. 27191	30	0. 78622	18	50
11	. 61818	23	. 61765	60	. 78645	47	. 27153	76	. 27221	29	. 78604	18	49
12	. 61841	23	. 61705	59	. 78692	47	. 27077	76	. 27250	29	. 78586	18	48
13	. 61864	23	. 61646	60	. 78739	47	. 27001	76	. 27279	29	. 78568	18	47
14	. 61887	22	. 61586	60	. 78786	48	. 26925	76	. 27308	29	. 78550	18	46
15	0. 61909	23	1. 61526	59	0. 78834	47	1. 26849	75	1. 27337	29	0. 78532	18	45
16	. 61932	23	. 61467	60	. 78881	47	. 26774	76	. 27366	30	. 78514	18	44
17	. 61955	23	. 61407	59	. 78928	47	. 26698	76	. 27396	29	. 78496	18	43
18	. 61978	23	. 61348	60	. 78975	47	. 26622	76	. 27425	29	. 78478	18	42
19	. 62001	23	. 61288	59	. 79022	48	. 26546	75	. 27454	29	. 78460	18	41
20	0. 62024	22	1. 61229	59	0. 79070	47	1. 26471	76	1. 27483	30	0. 78442	18	40
21	. 62046	23	. 61170	59	. 79117	47	. 26395	76	. 27513	29	. 78424	19	39
22	. 62069	23	. 61111	60	. 79164	48	. 26319	75	. 27542	30	. 78405	18	38
23	. 62092	23	. 61051	59	. 79212	47	. 26244	75	. 27572	29	. 78387	18	37
24	. 62115	23	. 60992	59	. 79259	47	. 26169	76	. 27601	29	. 78369	18	36
25	0. 62138	22	1. 60933	59	0. 79306	48	1. 26093	75	1. 27630	30	0. 78351	18	35
26	. 62160	23	. 60874	59	. 79354	47	. 26018	75	. 27660	29	. 78333	18	34
27	. 62183	23	. 60815	59	. 79401	48	. 25943	76	. 27689	30	. 78315	18	33
28	. 62206	23	. 60756	58	. 79449	47	. 25867	75	. 27719	29	. 78297	18	32
29	. 62229	22	. 60698	59	. 79496	48	. 25792	75	. 27748	30	. 78279	18	31
30	0. 62251	23	1. 60639	59	0. 79544	47	1. 25717	75	1. 27778	29	0. 78261	18	30
31	. 62274	23	. 60580	59	. 79591	48	. 25642	75	. 27807	30	. 78243	18	29
32	. 62297	23	. 60521	58	. 79639	47	. 25567	75	. 27837	30	. 78225	19	28
33	. 62320	22	. 60463	59	. 79686	48	. 25492	75	. 27867	29	. 78206	18	27
34	. 62342	23	. 60404	58	. 79734	47	. 25417	74	. 27896	30	. 78188	18	26
35	0. 62365	23	1. 60346	59	0. 79781	48	1. 25343	75	1. 27926	30	0. 78170	18	25
36	. 62388	23	. 60287	58	. 79829	48	. 25268	75	. 27956	29	. 78152	18	24
37	. 62411	22	. 60229	58	. 79877	47	. 25193	75	. 27985	30	. 78134	18	23
38	. 62433	23	. 60171	59	. 79924	48	. 25118	74	. 28015	30	. 78116	18	22
39	. 62456	23	. 60112	58	. 79972	48	. 25044	75	. 28045	30	. 78098	19	21
40	0. 62479	23	1. 60054	58	0. 80020	47	1. 24969	74	1. 28075	30	0. 78079	18	20
41	. 62502	22	. 59996	58	. 80067	48	. 24895	75	. 28105	29	. 78061	18	19
42	. 62524	23	. 59938	58	. 80115	48	. 24820	74	. 28134	30	. 78043	18	18
43	. 62547	23	. 59880	58	. 80163	48	. 24746	74	. 28164	30	. 78025	18	17
44	. 62570	22	. 59822	58	. 80211	47	. 24672	75	. 28194	30	. 78007	19	16
45	0. 62592	23	1. 59764	58	0. 80258	48	1. 24597	74	1. 28224	30	0. 77988	18	15
46	. 62615	23	. 59706	58	. 80306	48	. 24523	74	. 28254	30	. 77970	18	14
47	. 62638	22	. 59648	58	. 80354	48	. 24449	74	. 28284	30	. 77952	18	13
48	. 62660	23	. 59590	57	. 80402	48	. 24375	74	. 28314	30	. 77934	18	12
49	. 62683	23	. 59533	58	. 80450	48	. 24301	74	. 28344	30	. 77916	19	11
50	0. 62706	22	1. 59475	57	0. 80498	48	1. 24227	74	1. 28374	30	0. 77897	18	10
51	. 62728	23	. 59418	58	. 80546	48	. 24153	74	. 28404	30	. 77879	18	9
52	. 62751	23	. 59360	58	. 80594	48	. 24079	74	. 28434	30	. 77861	18	8
53	. 62774	22	. 59302	57	. 80642	48	. 24005	74	. 28464	31	. 77843	19	7
54	. 62796	23	. 59245	57	. 80690	48	. 23931	73	. 28495	30	. 77824	18	6
55	0. 62819	23	1. 59188	58	0. 80738	48	1. 23858	74	1. 28525	30	0. 77806	18	5
56	. 62842	22	. 59130	57	. 80786	48	. 23784	74	. 28555	30	. 77788	19	4
57	. 62864	23	. 59073	57	. 80834	48	. 23710	73	. 28585	30	. 77769	18	3
58	. 62887	22	. 59016	57	. 80882	48	. 23637	74	. 28615	31	. 77751	18	2
59	. 62909	23	. 58959	57	. 80930	48	. 23563	73	. 28646	30	. 77733	18	1
60	0. 62932		1. 58902		0. 80978		1. 23490		1. 28676		0. 77715		0
↑ 128°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ←51°

TABLE 31

Natural Trigonometric Functions

39° → ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←140° ↓ ′
0	0. 62932	23	1. 58902	57	0. 80978	49	1. 23490	74	1. 28676	30	0. 77715	19	60
1	. 62955	22	. 58845	57	. 81027	48	. 23416	73	. 28706	31	. 77696	18	59
2	. 62977	23	. 58788	57	. 81075	48	. 23343	73	. 28737	30	. 77678	18	58
3	. 63000	22	. 58731	57	. 81123	48	. 23270	74	. 28767	30	. 77660	19	57
4	. 63022	23	. 58674	57	. 81171	49	. 23196	73	. 28797	31	. 77641	18	56
5	0. 63045	23	1. 58617	57	0. 81220	48	1. 23123	73	1. 28828	30	0. 77623	18	55
6	. 63068	22	. 58560	57	. 81268	48	. 23050	73	. 28858	31	. 77605	19	54
7	. 63090	23	. 58503	56	. 81316	48	. 22977	73	. 28889	30	. 77586	18	53
8	. 63113	22	. 58447	57	. 81364	49	. 22904	73	. 28919	31	. 77568	18	52
9	. 63135	23	. 58390	57	. 81413	48	. 22831	73	. 28950	30	. 77550	19	51
10	0. 63158	22	1. 58333	56	0. 81461	49	1. 22758	73	1. 28980	31	0. 77531	18	50
11	. 63180	23	. 58277	56	. 81510	48	. 22685	73	. 29011	31	. 77513	19	49
12	. 63203	22	. 58221	57	. 81558	48	. 22612	73	. 29042	30	. 77494	18	48
13	. 63225	23	. 58164	56	. 81606	49	. 22539	72	. 29072	31	. 77476	18	47
14	. 63248	23	. 58108	57	. 81655	48	. 22467	73	. 29103	30	. 77458	19	46
15	0. 63271	22	1. 58051	56	0. 81703	49	1. 22394	73	1. 29133	31	0. 77439	18	45
16	. 63293	23	. 57995	56	. 81752	48	. 22321	72	. 29164	31	. 77421	19	44
17	. 63316	22	. 57939	56	. 81800	49	. 22249	73	. 29195	31	. 77402	18	43
18	. 63338	23	. 57883	56	. 81849	49	. 22176	72	. 29226	30	. 77384	18	42
19	. 63361	22	. 57827	56	. 81898	48	. 22104	73	. 29256	31	. 77366	19	41
20	0. 63383	23	1. 57771	56	0. 81946	49	1. 22031	72	1. 29287	31	0. 77347	18	40
21	. 63406	22	. 57715	56	. 81995	49	. 21959	73	. 29318	31	. 77329	19	39
22	. 63428	23	. 57659	56	. 82044	48	. 21886	72	. 29349	31	. 77310	18	38
23	. 63451	22	. 57603	56	. 82092	49	. 21814	72	. 29380	31	. 77292	19	37
24	. 63473	23	. 57547	56	. 82141	49	. 21742	72	. 29411	31	. 77273	18	36
25	0. 63496	22	1. 57491	55	0. 82190	48	1. 21670	72	1. 29442	31	0. 77255	19	35
26	. 63518	22	. 57436	56	. 82238	49	. 21598	72	. 29473	31	. 77236	18	34
27	. 63540	23	. 57380	56	. 82287	49	. 21526	72	. 29504	31	. 77218	19	33
28	. 63563	22	. 57324	55	. 82336	49	. 21454	72	. 29535	31	. 77199	18	32
29	. 63585	23	. 57269	56	. 82385	49	. 21382	72	. 29566	31	. 77181	19	31
30	0. 63608	22	1. 57213	55	0. 82434	49	1. 21310	72	1. 29597	31	0. 77162	18	30
31	. 63630	23	. 57158	55	. 82483	48	. 21238	72	. 29628	31	. 77144	19	29
32	. 63653	22	. 57103	56	. 82531	49	. 21166	72	. 29659	31	. 77125	18	28
33	. 63675	23	. 57047	55	. 82580	49	. 21094	71	. 29690	31	. 77107	19	27
34	. 63698	22	. 56992	55	. 82629	49	. 21023	72	. 29721	31	. 77088	18	26
35	0. 63720	22	1. 56937	56	0. 82678	49	1. 20951	72	1. 29752	32	0. 77070	19	25
36	. 63742	23	. 56881	55	. 82727	49	. 20879	71	. 29784	31	. 77051	18	24
37	. 63765	22	. 56826	55	. 82776	49	. 20808	72	. 29815	31	. 77033	19	23
38	. 63787	23	. 56771	55	. 82825	49	. 20736	71	. 29846	31	. 77014	18	22
39	. 63810	22	. 56716	55	. 82874	49	. 20665	72	. 29877	32	. 76996	19	21
40	0. 63832	22	1. 56661	55	0. 82923	49	1. 20593	71	1. 29909	31	0. 76977	18	20
41	. 63854	23	. 56606	55	. 82972	50	. 20522	71	. 29940	31	. 76959	19	19
42	. 63877	22	. 56551	54	. 83022	49	. 20451	72	. 29971	32	. 76940	19	18
43	. 63899	23	. 56497	55	. 83071	49	. 20379	71	. 30003	31	. 76921	18	17
44	. 63922	22	. 56442	55	. 83120	49	. 20308	71	. 30034	32	. 76903	19	16
45	0. 63944	22	1. 56387	55	0. 83169	49	1. 20237	71	1. 30066	31	0. 76884	18	15
46	. 63966	23	. 56332	54	. 83218	50	. 20166	71	. 30097	32	. 76866	19	14
47	. 63989	22	. 56278	55	. 83268	49	. 20095	71	. 30129	31	. 76847	19	13
48	. 64011	22	. 56223	54	. 83317	49	. 20024	71	. 30160	32	. 76828	18	12
49	. 64033	23	. 56169	55	. 83366	49	. 19953	71	. 30192	31	. 76810	19	11
50	0. 64056	22	1. 56114	54	0. 83415	50	1. 19882	71	1. 30223	32	0. 76791	19	10
51	. 64078	22	. 56060	55	. 83465	49	. 19811	71	. 30255	32	. 76772	18	9
52	. 64100	23	. 56005	54	. 83514	50	. 19740	71	. 30287	31	. 76754	19	8
53	. 64123	22	. 55951	54	. 83564	49	. 19669	70	. 30318	32	. 76735	18	7
54	. 64145	22	. 55897	54	. 83613	49	. 19599	71	. 30350	32	. 76717	19	6
55	0. 64167	23	1. 55843	54	0. 83662	50	1. 19528	71	1. 30382	31	0. 76698	19	5
56	. 64190	22	. 55789	55	. 83712	49	. 19457	70	. 30413	32	. 76679	18	4
57	. 64212	22	. 55734	54	. 83761	50	. 19387	71	. 30445	32	. 76661	19	3
58	. 64234	22	. 55680	54	. 83811	49	. 19316	70	. 30477	32	. 76642	19	2
59	. 64256	23	. 55626	54	. 83860	50	. 19246	71	. 30509	32	. 76623	19	1
60	0. 64279		1. 55572		0. 83910		1. 19175		1. 30541		0 76604		0
↑ 129° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	← 50° ↑

TABLE 31

Natural Trigonometric Functions

40° → ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	← 139° ↓ ′
0	0. 64279	22	1. 55572	54	0. 83910	50	1. 19175	70	1. 30541	32	0. 76604	18	60
1	. 64301	22	. 55518	53	. 83960	49	. 19105	70	. 30573	32	. 76586	19	59
2	. 64323	23	. 55465	54	. 84009	50	. 19035	71	. 30605	31	. 76567	19	58
3	. 64346	22	. 55411	54	. 84059	49	. 18964	70	. 30636	32	. 76548	18	57
4	. 64368	22	. 55357	54	. 84108	50	. 18894	70	. 30668	32	. 76530	19	56
5	0. 64390	22	1. 55303	53	0. 84158	50	1. 18824	70	1. 30700	32	0. 76511	19	55
6	. 64412	23	. 55250	54	. 84208	50	. 18754	70	. 30732	32	. 76492	19	54
7	. 64435	22	. 55196	53	. 84258	49	. 18684	70	. 30764	32	. 76473	18	53
8	. 64457	22	. 55143	54	. 84307	50	. 18614	70	. 30796	33	. 76455	19	52
9	. 64479	22	. 55089	53	. 84357	50	. 18544	70	. 30829	32	. 76436	19	51
10	0. 64501	23	1. 55036	54	0. 84407	50	1. 18474	70	1. 30861	32	0. 76417	19	50
11	. 64524	22	. 54982	53	. 84457	50	. 18404	70	. 30893	32	. 76398	18	49
12	. 64546	22	. 54929	53	. 84507	49	. 18334	70	. 30925	32	. 76380	19	48
13	. 64568	22	. 54876	54	. 84556	50	. 18264	70	. 30957	32	. 76361	19	47
14	. 64590	22	. 54822	53	. 84606	50	. 18194	69	. 30989	33	. 76342	19	46
15	0. 64612	23	1. 54769	53	0. 84656	50	1. 18125	70	1. 31022	32	0. 76323	19	45
16	. 64635	22	. 54716	53	. 84706	50	. 18055	69	. 31054	32	. 76304	18	44
17	. 64657	22	. 54663	53	. 84756	50	. 17986	70	. 31086	33	. 76286	19	43
18	. 64679	22	. 54610	53	. 84806	50	. 17916	70	. 31119	32	. 76267	19	42
19	. 64701	22	. 54557	53	. 84856	50	. 17846	69	. 31151	32	. 76248	19	41
20	0. 64723	23	1. 54504	53	0. 84906	50	1. 17777	69	1. 31183	33	0. 76229	19	40
21	. 64746	22	. 54451	53	. 84956	50	. 17708	70	. 31216	32	. 76210	18	39
22	. 64768	22	. 54398	53	. 85006	51	. 17638	69	. 31248	33	. 76192	19	38
23	. 64790	22	. 54345	53	. 85057	50	. 17569	69	. 31281	32	. 76173	19	37
24	. 64812	22	. 54292	52	. 85107	50	. 17500	70	. 31313	33	. 76154	19	36
25	0. 64834	22	1. 54240	53	0. 85157	50	1. 17430	69	1. 31346	32	0. 76135	19	35
26	. 64856	22	. 54187	53	. 85207	50	. 17361	69	. 31378	33	. 76116	19	34
27	. 64878	23	. 54134	52	. 85257	51	. 17292	69	. 31411	32	. 76097	19	33
28	. 64901	22	. 54082	53	. 85308	50	. 17223	69	. 31443	33	. 76078	19	32
29	. 64923	22	. 54029	52	. 85358	50	. 17154	69	. 31476	33	. 76059	18	31
30	0. 64945	22	1. 53977	53	0. 85408	50	1. 17085	69	1. 31509	32	0. 76041	19	30
31	. 64967	22	. 53924	52	. 85458	51	. 17016	69	. 31541	33	. 76022	19	29
32	. 64989	22	. 53872	52	. 85509	50	. 16947	69	. 31574	33	. 76003	19	28
33	. 65011	22	. 53820	52	. 85559	50	. 16878	69	. 31607	33	. 75984	19	27
34	. 65033	22	. 53768	53	. 85609	51	. 16809	68	. 31640	32	. 75965	19	26
35	0. 65055	22	1. 53715	52	0. 85660	50	1. 16741	69	1. 31672	33	0. 75946	19	25
36	. 65077	23	. 53663	52	. 85710	51	. 16672	69	. 31705	33	. 75927	19	24
37	. 65100	22	. 53611	52	. 85761	50	. 16603	68	. 31738	33	. 75908	19	23
38	. 65122	22	. 53559	52	. 85811	51	. 16535	69	. 31771	33	. 75889	19	22
39	. 65144	22	. 53507	52	. 85862	50	. 16466	68	. 31804	33	. 75870	19	21
40	0. 65166	22	1. 53455	52	0. 85912	51	1. 16398	69	1. 31837	33	0. 75851	19	20
41	. 65188	22	. 53403	52	. 85963	51	. 16329	68	. 31870	33	. 75832	19	19
42	. 65210	22	. 53351	52	. 86014	50	. 16261	69	. 31903	33	. 75813	19	18
43	. 65232	22	. 53299	52	. 86064	51	. 16192	68	. 31936	33	. 75794	19	17
44	. 65254	22	. 53247	51	. 86115	51	. 16124	68	. 31969	33	. 75775	19	16
45	0. 65276	22	1. 53196	52	0. 86166	50	1. 16056	69	1. 32002	33	0. 75756	18	15
46	. 65298	22	. 53144	52	. 86216	51	. 15987	68	. 32035	33	. 75738	19	14
47	. 65320	22	. 53092	51	. 86267	51	. 15919	68	. 32068	33	. 75719	19	13
48	. 65342	22	. 53041	52	. 86318	50	. 15851	68	. 32101	33	. 75700	20	12
49	. 65364	22	. 52989	51	. 86368	51	. 15783	68	. 32134	34	. 75680	19	11
50	0. 65386	22	1. 52938	52	0. 86419	51	1. 15715	68	1. 32168	33	0. 75661	19	10
51	. 65408	22	. 52886	51	. 86470	51	. 15647	68	. 32201	33	. 75642	19	9
52	. 65430	22	. 52835	51	. 86521	51	. 15579	68	. 32234	33	. 75623	19	8
53	. 65452	22	. 52784	52	. 86572	51	. 15511	68	. 32267	34	. 75604	19	7
54	. 65474	22	. 52732	51	. 86623	51	. 15443	68	. 32301	33	. 75585	19	6
55	0. 65496	22	1. 52681	51	0. 86674	51	1. 15375	67	1. 32334	34	0. 75566	19	5
56	. 65518	22	. 52630	51	. 86725	51	. 15308	68	. 32368	33	. 75547	19	4
57	. 65540	22	. 52579	52	. 86776	51	. 15240	68	. 32401	33	. 75528	19	3
58	. 65562	22	. 52527	51	. 86827	51	. 15172	68	. 32434	34	. 75509	19	2
59	. 65584	22	. 52476	51	. 86878	51	. 15104	67	. 32468	33	. 75490	19	1
60	0. 65606		1. 52425		0. 86929		1. 15037		1. 32501		0. 75471		0
↑ 130° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	← 49° ↑

TABLE 31

Natural Trigonometric Functions

41°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←138° ↓ ′
0	0. 65606	22	1. 52425	51	0. 86929	51	1. 15037	68	1. 32501	34	0. 75471	19	60
1	. 65628	22	. 52374	51	. 86980	51	. 14969	67	. 32535	33	. 75452	19	59
2	. 65650	22	. 52323	50	. 87031	51	. 14902	68	. 32568	34	. 75433	19	58
3	. 65672	22	. 52273	51	. 87082	51	. 14834	67	. 32602	34	. 75414	19	57
4	. 65694	22	. 52222	51	. 87133	51	. 14767	68	. 32636	33	. 75395	20	56
5	0. 65716	22	1. 52171	51	0. 87184	52	1. 14699	67	1. 32669	34	0. 75375	19	55
6	. 65738	21	. 52120	51	. 87236	51	. 14632	67	. 32703	34	. 75356	19	54
7	. 65759	22	. 52069	50	. 87287	51	. 14565	67	. 32737	33	. 75337	19	53
8	. 65781	22	. 52019	51	. 87338	51	. 14498	68	. 32770	34	. 75318	19	52
9	. 65803	22	. 51968	50	. 87389	52	. 14430	67	. 32804	34	. 75299	19	51
10	0. 65825	22	1. 51918	51	0. 87441	51	1. 14363	67	1. 32838	34	0. 75280	19	50
11	. 65847	22	. 51867	50	. 87492	51	. 14296	67	. 32872	33	. 75261	20	49
12	. 65869	22	. 51817	51	. 87543	52	. 14229	67	. 32905	34	. 75241	19	48
13	. 65891	22	. 51766	50	. 87595	51	. 14162	67	. 32939	34	. 75222	19	47
14	. 65913	22	. 51716	51	. 87646	52	. 14095	67	. 32973	34	. 75203	19	46
15	0. 65935	21	1. 51665	50	0. 87698	51	1. 14028	67	1. 33007	34	0. 75184	19	45
16	. 65956	22	. 51615	50	. 87749	52	. 13961	67	. 33041	34	. 75165	19	44
17	. 65978	22	. 51565	50	. 87801	51	. 13894	66	. 33075	34	. 75146	20	43
18	. 66000	22	. 51515	50	. 87852	52	. 13828	67	. 33109	34	. 75126	19	42
19	. 66022	22	. 51465	50	. 87904	51	. 13761	67	. 33143	34	. 75107	19	41
20	0. 66044	22	1. 51415	51	0. 87955	52	1. 13694	67	1. 33177	34	0. 75088	19	40
21	. 66066	22	. 51364	50	. 88007	52	. 13627	66	. 33211	34	. 75069	19	39
22	. 66088	21	. 51314	49	. 88059	51	. 13561	67	. 33245	34	. 75050	20	38
23	. 66109	22	. 51265	50	. 88110	52	. 13494	66	. 33279	35	. 75030	19	37
24	. 66131	22	. 51215	50	. 88162	52	. 13428	67	. 33314	34	. 75011	19	36
25	0. 66153	22	1. 51165	50	0. 88214	51	1. 13361	66	1. 33348	34	0. 74992	19	35
26	. 66175	22	. 51115	50	. 88265	52	. 13295	67	. 33382	34	. 74973	20	34
27	. 66197	21	. 51065	50	. 88317	52	. 13228	66	. 33416	35	. 74953	19	33
28	. 66218	22	. 51015	49	. 88369	52	. 13162	66	. 33451	34	. 74934	19	32
29	. 66240	22	. 50966	50	. 88421	52	. 13096	67	. 33485	34	. 74915	19	31
30	0. 66262	22	1. 50916	50	0. 88473	51	1. 13029	66	1. 33519	35	0. 74896	20	30
31	. 66284	22	. 50866	49	. 88524	52	. 12963	66	. 33554	34	. 74876	19	29
32	. 66306	21	. 50817	50	. 88576	52	. 12897	66	. 33588	34	. 74857	19	28
33	. 66327	22	. 50767	49	. 88628	52	. 12831	66	. 33622	35	. 74838	20	27
34	. 66349	22	. 50718	49	. 88680	52	. 12765	66	. 33657	34	. 74818	19	26
35	0. 66371	22	1. 50669	50	0. 88732	52	1. 12699	66	1. 33691	35	0. 74799	19	25
36	. 66393	21	. 50619	49	. 88784	52	. 12633	66	. 33726	34	. 74780	20	24
37	. 66414	22	. 50570	49	. 88836	52	. 12567	66	. 33760	35	. 74760	19	23
38	. 66436	22	. 50521	50	. 88888	52	. 12501	66	. 33795	35	. 74741	19	22
39	. 66458	22	. 50471	49	. 88940	52	. 12435	66	. 33830	34	. 74722	19	21
40	0. 66480	21	1. 50422	49	0. 88992	53	1. 12369	66	1. 33864	35	0. 74703	20	20
41	. 66501	22	. 50373	49	. 89045	52	. 12303	65	. 33899	35	. 74683	19	19
42	. 66523	22	. 50324	49	. 89097	52	. 12238	66	. 33934	34	. 74664	20	18
43	. 66545	21	. 50275	49	. 89149	52	. 12172	66	. 33968	35	. 74644	19	17
44	. 66566	22	. 50226	49	. 89201	52	. 12106	65	. 34003	35	. 74625	19	16
45	0. 66588	22	1. 50177	49	0. 89253	53	1. 12041	66	1. 34038	35	0. 74606	20	15
46	. 66610	22	. 50128	49	. 89306	52	. 11975	66	. 34073	35	. 74586	19	14
47	. 66632	21	. 50079	49	. 89358	52	. 11909	65	. 34108	34	. 74567	19	13
48	. 66653	22	. 50030	49	. 89410	53	. 11844	66	. 34142	35	. 74548	20	12
49	. 66675	22	. 49981	48	. 89463	52	. 11778	65	. 34177	35	. 74528	19	11
50	0. 66697	21	1. 49933	49	0. 89515	52	1. 11713	65	1. 34212	35	0. 74509	20	10
51	. 66718	22	. 49884	49	. 89567	53	. 11648	66	. 34247	35	. 74489	19	9
52	. 66740	22	. 49835	48	. 89620	52	. 11582	65	. 34282	35	. 74470	19	8
53	. 66762	21	. 49787	49	. 89672	53	. 11517	65	. 34317	35	. 74451	20	7
54	. 66783	22	. 49738	48	. 89725	52	. 11452	65	. 34352	35	. 74431	19	6
55	0. 66805	22	1. 49690	49	0. 89777	53	1. 11387	66	1. 34387	36	0. 74412	20	5
56	. 66827	21	. 49641	48	. 89830	53	. 11321	65	. 34423	35	. 74392	19	4
57	. 66848	22	. 49593	49	. 89883	52	. 11256	65	. 34458	35	. 74373	20	3
58	. 66870	21	. 49544	48	. 89935	53	. 11191	65	. 34493	35	. 74353	19	2
59	. 66891	22	. 49496	48	. 89988	52	. 11126	65	. 34528	35	. 74334	20	1
60	0. 66913		1. 49448		0. 90040		1. 11061		1. 34563		0. 74314		0
↑ 131°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←48° ↑

TABLE 31

Natural Trigonometric Functions

42° → ′ ↓	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	← 137° ′ ↓
0	0.66913	22	1.49448	49	0.90040	53	1.11061	65	1.34563	36	0.74314	19	60
1	.66935	21	.49399	48	.90093	53	.10996	65	.34599	35	.74295	19	59
2	.66956	22	.49351	48	.90146	53	.10931	64	.34634	35	.74276	20	58
3	.66978	21	.49303	48	.90199	52	.10867	65	.34669	35	.74256	19	57
4	.66999	22	.49255	48	.90251	53	.10802	65	.34704	36	.74237	20	56
5	0.67021	22	1.49207	48	0.90304	53	1.10737	65	1.34740	35	0.74217	19	55
6	.67043	21	.49159	48	.90357	53	.10672	65	.34775	36	.74198	20	54
7	.67064	22	.49111	48	.90410	53	.10607	64	.34811	35	.74178	19	53
8	.67086	21	.49063	48	.90463	53	.10543	65	.34846	36	.74159	20	52
9	.67107	22	.49015	48	.90516	53	.10478	64	.34882	35	.74139	19	51
10	0.67129	22	1.48967	48	0.90569	52	1.10414	65	1.34917	36	0.74120	20	50
11	.67151	21	.48919	48	.90621	53	.10349	64	.34953	35	.74100	20	49
12	.67172	22	.48871	47	.90674	53	.10285	65	.34988	36	.74080	19	48
13	.67194	21	.48824	48	.90727	54	.10220	64	.35024	36	.74061	20	47
14	.67215	22	.48776	48	.90781	53	.10156	65	.35060	35	.74041	19	46
15	0.67237	21	1.48728	47	0.90834	53	1.10091	64	1.35095	36	0.74022	20	45
16	.67258	22	.48681	48	.90887	53	.10027	64	.35131	36	.74002	19	44
17	.67280	21	.48633	47	.90940	53	.09963	64	.35167	36	.73983	20	43
18	.67301	22	.48586	48	.90993	53	.09899	65	.35203	35	.73963	19	42
19	.67323	21	.48538	47	.91046	53	.09834	64	.35238	36	.73944	20	41
20	0.67344	22	1.48491	48	0.91099	54	1.09770	64	1.35274	36	0.73924	20	40
21	.67366	21	.48443	47	.91153	53	.09706	64	.35310	36	.73904	19	39
22	.67387	22	.48396	47	.91206	53	.09642	64	.35346	36	.73885	20	38
23	.67409	21	.48349	48	.91259	54	.09578	64	.35382	36	.73865	19	37
24	.67430	22	.48301	47	.91313	53	.09514	64	.35418	36	.73846	20	36
25	0.67452	21	1.48254	47	0.91366	53	1.09450	64	1.35454	36	0.73826	20	35
26	.67473	22	.48207	47	.91419	54	.09386	64	.35490	36	.73806	19	34
27	.67495	21	.48160	47	.91473	53	.09322	64	.35526	36	.73787	20	33
28	.67516	22	.48113	47	.91526	54	.09258	63	.35562	36	.73767	20	32
29	.67538	21	.48066	47	.91580	53	.09195	64	.35598	36	.73747	19	31
30	0.67559	21	1.48019	47	0.91633	54	1.09131	64	1.35634	36	0.73728	20	30
31	.67580	22	.47972	47	.91687	53	.09067	64	.35670	37	.73708	20	29
32	.67602	21	.47925	47	.91740	54	.09003	63	.35707	36	.73688	19	28
33	.67623	22	.47878	47	.91794	53	.08940	64	.35743	36	.73669	20	27
34	.67645	21	.47831	47	.91847	54	.08876	63	.35779	36	.73649	20	26
35	0.67666	22	1.47784	46	0.91901	54	1.08813	64	1.35815	37	0.73629	19	25
36	.67688	21	.47738	47	.91955	53	.08749	63	.35852	36	.73610	20	24
37	.67709	21	.47691	47	.92008	54	.08686	64	.35888	36	.73590	20	23
38	.67730	22	.47644	46	.92062	54	.08622	63	.35924	37	.73570	19	22
39	.67752	21	.47598	47	.92116	54	.08559	63	.35961	36	.73551	20	21
40	0.67773	22	1.47551	47	0.92170	54	1.08496	64	1.35997	37	0.73531	20	20
41	.67795	21	.47504	46	.92224	53	.08432	63	.36034	36	.73511	20	19
42	.67816	21	.47458	47	.92277	54	.08369	63	.36070	37	.73491	19	18
43	.67837	22	.47411	46	.92331	54	.08306	63	.36107	36	.73472	20	17
44	.67859	21	.47365	46	.92385	54	.08243	64	.36143	37	.73452	20	16
45	0.67880	21	1.47319	47	0.92439	54	1.08179	63	1.36180	37	0.73432	19	15
46	.67901	22	.47272	46	.92493	54	.08116	63	.36217	36	.73413	20	14
47	.67923	21	.47226	46	.92547	54	.08053	63	.36253	37	.73393	20	13
48	.67944	21	.47180	46	.92601	54	.07990	63	.36290	37	.73373	20	12
49	.67965	22	.47134	47	.92655	54	.07927	63	.36327	36	.73353	20	11
50	0.67987	21	1.47087	46	0.92709	54	1.07864	63	1.36363	37	0.73333	19	10
51	.68008	21	.47041	46	.92763	54	.07801	63	.36400	37	.73314	20	9
52	.68029	22	.46995	46	.92817	55	.07738	62	.36437	37	.73294	20	8
53	.68051	21	.46949	46	.92872	54	.07676	63	.36474	37	.73274	20	7
54	.68072	21	.46903	46	.92926	54	.07613	63	.36511	37	.73254	20	6
55	0.68093	22	1.46857	46	0.92980	54	1.07550	63	1.36548	37	0.73234	19	5
56	.68115	21	.46811	46	.93034	54	.07487	62	.36585	37	.73215	20	4
57	.68136	21	.46765	46	.93088	55	.07425	63	.36622	37	.73195	20	3
58	.68157	22	.46719	45	.93143	54	.07362	63	.36659	37	.73175	20	2
59	.68179	21	.46674	46	.93197	55	.07299	62	.36696	37	.73155	20	1
60	0.68200		1.46628		0.93252		1.07237		1.36733		0.73135		0
↑ 132° →	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ← 47°

TABLE 31
Natural Trigonometric Functions

43°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←136° ↓ ′
0	0. 68200	21	1. 46628	46	0. 93252	54	1. 07237	63	1. 36733	37	0. 73135	19	60
1	. 68221	21	. 46582	45	. 93306	54	. 07174	62	. 36770	37	. 73116	20	59
2	. 68242	22	. 46537	46	. 93360	55	. 07112	63	. 36807	37	. 73096	20	58
3	. 68264	21	. 46491	46	. 93415	54	. 07049	62	. 36844	37	. 73076	20	57
4	. 68285	21	. 46445	45	. 93469	55	. 06987	62	. 36881	38	. 73056	20	56
5	0. 68306	21	1. 46400	46	0. 93524	54	1. 06925	63	1. 36919	37	0. 73036	20	55
6	. 68327	22	. 46354	45	. 93578	55	. 06862	62	. 36956	37	. 73016	20	54
7	. 68349	21	. 46309	46	. 93633	55	. 06800	62	. 36993	37	. 72996	20	53
8	. 68370	21	. 46263	45	. 93688	54	. 06738	62	. 37030	38	. 72976	19	52
9	. 68391	21	. 46218	45	. 93742	55	. 06676	63	. 37068	37	. 72957	20	51
10	0. 68412	22	1. 46173	46	0. 93797	55	1. 06613	62	1. 37105	38	0. 72937	20	50
11	. 68434	21	. 46127	45	. 93852	54	. 06551	62	. 37143	37	. 72917	20	49
12	. 68455	21	. 46082	45	. 93906	55	. 06489	62	. 37180	38	. 72897	20	48
13	. 68476	21	. 46037	45	. 93961	55	. 06427	62	. 37218	37	. 72877	20	47
14	. 68497	21	. 45992	46	. 94016	55	. 06365	62	. 37255	38	. 72857	20	46
15	0. 68518	21	1. 45946	45	0. 94071	54	1. 06303	62	1. 37293	37	0. 72837	20	45
16	. 68539	22	. 45901	45	. 94125	55	. 06241	62	. 37330	38	. 72817	20	44
17	. 68561	21	. 45856	45	. 94180	55	. 06179	62	. 37368	38	. 72797	20	43
18	. 68582	21	. 45811	45	. 94235	55	. 06117	61	. 37406	37	. 72777	20	42
19	. 68603	21	. 45766	45	. 94290	55	. 06056	62	. 37443	38	. 72757	20	41
20	0. 68624	21	1. 45721	45	0. 94345	55	1. 05994	62	1. 37481	38	0. 72737	20	40
21	. 68645	21	. 45676	45	. 94400	55	. 05932	62	. 37519	37	. 72717	20	39
22	. 68666	22	. 45631	44	. 94455	55	. 05870	61	. 37556	38	. 72697	20	38
23	. 68688	21	. 45587	45	. 94510	55	. 05809	62	. 37594	38	. 72677	20	37
24	. 68709	21	. 45542	45	. 94565	55	. 05747	62	. 37632	38	. 72657	20	36
25	0. 68730	21	1. 45497	45	0. 94620	56	1. 05685	61	1. 37670	38	0. 72637	20	35
26	. 68751	21	. 45452	44	. 94676	55	. 05624	62	. 37708	38	. 72617	20	34
27	. 68772	21	. 45408	45	. 94731	55	. 05562	61	. 37746	38	. 72597	20	33
28	. 68793	21	. 45363	44	. 94786	55	. 05501	62	. 37784	38	. 72577	20	32
29	. 68814	21	. 45319	45	. 94841	55	. 05439	61	. 37822	38	. 72557	20	31
30	0. 68835	22	1. 45274	45	0. 94896	56	1. 05378	61	1. 37860	38	0. 72537	20	30
31	. 68857	21	. 45229	44	. 94952	55	. 05317	62	. 37898	38	. 72517	20	29
32	. 68878	21	. 45185	44	. 95007	55	. 05255	61	. 37936	38	. 72497	20	28
33	. 68899	21	. 45141	45	. 95062	56	. 05194	61	. 37974	38	. 72477	20	27
34	. 68920	21	. 45096	44	. 95118	55	. 05133	61	. 38012	39	. 72457	20	26
35	0. 68941	21	1. 45052	45	0. 95173	56	1. 05072	62	1. 38051	38	0. 72437	20	25
36	. 68962	21	. 45007	44	. 95229	55	. 05010	61	. 38089	38	. 72417	20	24
37	. 68983	21	. 44963	44	. 95284	56	. 04949	61	. 38127	38	. 72397	20	23
38	. 69004	21	. 44919	44	. 95340	55	. 04888	61	. 38165	39	. 72377	20	22
39	. 69025	21	. 44875	44	. 95395	56	. 04827	61	. 38204	38	. 72357	20	21
40	0. 69046	21	1. 44831	44	0. 95451	55	1. 04766	61	1. 38242	38	0. 72337	20	20
41	. 69067	21	. 44787	45	. 95506	56	. 04705	61	. 38280	39	. 72317	20	19
42	. 69088	21	. 44742	44	. 95562	56	. 04644	61	. 38319	38	. 72297	20	18
43	. 69109	21	. 44698	44	. 95618	55	. 04583	61	. 38357	39	. 72277	20	17
44	. 69130	21	. 44654	44	. 95673	56	. 04522	61	. 38396	38	. 72257	21	16
45	0. 69151	21	1. 44610	43	0. 95729	56	1. 04461	60	1. 38434	39	0. 72236	20	15
46	. 69172	21	. 44567	44	. 95785	56	. 04401	61	. 38473	39	. 72216	20	14
47	. 69193	21	. 44523	44	. 95841	56	. 04340	61	. 38512	38	. 72196	20	13
48	. 69214	21	. 44479	44	. 95897	55	. 04279	61	. 38550	39	. 72176	20	12
49	. 69235	21	. 44435	44	. 95952	56	. 04218	60	. 38589	39	. 72156	20	11
50	0. 69256	21	1. 44391	44	0. 96008	56	1. 04158	61	1. 38628	38	0. 72136	20	10
51	. 69277	21	. 44347	43	. 96064	56	. 04097	61	. 38666	39	. 72116	21	9
52	. 69298	21	. 44304	44	. 96120	56	. 04036	60	. 38705	39	. 72095	20	8
53	. 69319	21	. 44260	43	. 96176	56	. 03976	61	. 38744	39	. 72075	20	7
54	. 69340	21	. 44217	44	. 96232	56	. 03915	60	. 38783	39	. 72055	20	6
55	0. 69361	21	1. 44173	44	0. 96288	56	1. 03855	61	1. 38822	38	0. 72035	20	5
56	. 69382	21	. 44129	43	. 96344	56	. 03794	60	. 38860	39	. 72015	20	4
57	. 69403	21	. 44086	44	. 96400	57	. 03734	60	. 38899	39	. 71995	21	3
58	. 69424	21	. 44042	43	. 96457	56	. 03674	61	. 38938	39	. 71974	20	2
59	. 69445	21	. 43999	43	. 96513	56	. 03613	60	. 38977	39	. 71954	20	1
60	0. 69466		1. 43956		0. 96569		1. 03553		1. 39016		0. 71934		0
↑ 133°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	←46° ↑

TABLE 31

Natural Trigonometric Functions

44°→ ↓ ′	sin	Diff. 1′	csc	Diff. 1′	tan	Diff. 1′	cot	Diff. 1′	sec	Diff. 1′	cos	Diff. 1′	←135° ↓ ′
0	0. 69466	21	1. 43956	44	0. 96569	56	1. 03553	60	1. 39016	39	0. 71934	20	60
1	. 69487	21	. 43912	43	. 96625	56	. 03493	60	. 39055	40	. 71914	20	59
2	. 69508	21	. 43869	43	. 96681	57	. 03433	61	. 39095	39	. 71894	21	58
3	. 69529	20	. 43826	43	. 96738	56	. 03372	60	. 39134	39	. 71873	20	57
4	. 69549	21	. 43783	44	. 96794	56	. 03312	60	. 39173	39	. 71853	20	56
5	0. 69570	21	1. 43739	43	0. 96850	57	1. 03252	60	1. 39212	39	0. 71833	20	55
6	. 69591	21	. 43696	43	. 96907	56	. 03192	60	. 39251	40	. 71813	21	54
7	. 69612	21	. 43653	43	. 96963	57	. 03132	60	. 39291	39	. 71792	20	53
8	. 69633	21	. 43610	43	. 97020	56	. 03072	60	. 39330	39	. 71772	20	52
9	. 69654	21	. 43567	43	. 97076	57	. 03012	60	. 39369	40	. 71752	20	51
10	0. 69675	21	1. 43524	43	0. 97133	56	1. 02952	60	1. 39409	39	0. 71732	21	50
11	. 69696	21	. 43481	43	. 97189	57	. 02892	60	. 39448	39	. 71711	20	49
12	. 69717	20	. 43438	43	. 97246	56	. 02832	60	. 39487	40	. 71691	20	48
13	. 69737	21	. 43395	43	. 97302	57	. 02772	59	. 39527	39	. 71671	21	47
14	. 69758	21	. 43352	42	. 97359	57	. 02713	60	. 39566	40	. 71650	20	46
15	0. 69779	21	1. 43310	43	0. 97416	56	1. 02653	60	1. 39606	40	0. 71630	20	45
16	. 69800	21	. 43267	43	. 97472	57	. 02593	60	. 39646	39	. 71610	20	44
17	. 69821	21	. 43224	43	. 97529	57	. 02533	59	. 39685	40	. 71590	21	43
18	. 69842	20	. 43181	42	. 97586	57	. 02474	60	. 39725	39	. 71569	20	42
19	. 69862	21	. 43139	43	. 97643	57	. 02414	59	. 39764	40	. 71549	20	41
20	0. 69883	21	1. 43096	43	0. 97700	56	1. 02355	60	1. 39804	40	0. 71529	21	40
21	. 69904	21	. 43053	42	. 97756	57	. 02295	59	. 39844	40	. 71508	20	39
22	. 69925	21	. 43011	43	. 97813	57	. 02236	60	. 39884	40	. 71488	20	38
23	. 69946	20	. 42968	42	. 97870	57	. 02176	59	. 39924	39	. 71468	21	37
24	. 69966	21	. 42926	43	. 97927	57	. 02117	60	. 39963	40	. 71447	20	36
25	0. 69987	21	1. 42883	42	0. 97984	57	1. 02057	59	1. 40003	40	0. 71427	20	35
26	. 70008	21	. 42841	42	. 98041	57	. 01998	59	. 40043	40	. 71407	21	34
27	. 70029	20	. 42799	43	. 98098	57	. 01939	60	. 40083	40	. 71386	20	33
28	. 70049	21	. 42756	42	. 98155	58	. 01879	59	. 40123	40	. 71366	21	32
29	. 70070	21	. 42714	42	. 98213	57	. 01820	59	. 40163	40	. 71345	20	31
30	0. 70091	21	1. 42672	42	0. 98270	57	1. 01761	59	1. 40203	40	0. 71325	20	30
31	. 70112	20	. 42630	43	. 98327	57	. 01702	60	. 40243	40	. 71305	21	29
32	. 70132	21	. 42587	42	. 98384	57	. 01642	59	. 40283	41	. 71284	20	28
33	. 70153	21	. 42545	42	. 98441	58	. 01583	59	. 40324	40	. 71264	21	27
34	. 70174	21	. 42503	42	. 98499	57	. 01524	59	. 40364	40	. 71243	20	26
35	0. 70195	20	1. 42461	42	0. 98556	57	1. 01465	59	1. 40404	40	0. 71223	20	25
36	. 70215	21	. 42419	42	. 98613	58	. 01406	59	. 40444	41	. 71203	21	24
37	. 70236	21	. 42377	42	. 98671	57	. 01347	59	. 40485	40	. 71182	20	23
38	. 70257	20	. 42335	42	. 98728	58	. 01288	59	. 40525	40	. 71162	21	22
39	. 70277	21	. 42293	42	. 98786	57	. 01229	59	. 40565	41	. 71141	20	21
40	0. 70298	21	1. 42251	42	0. 98843	58	1. 01170	58	1. 40606	40	0. 71121	21	20
41	. 70319	20	. 42209	41	. 98901	57	. 01112	59	. 40646	41	. 71100	20	19
42	. 70339	21	. 42168	42	. 98958	58	. 01053	59	. 40687	40	. 71080	21	18
43	. 70360	21	. 42126	42	. 99016	57	. 00994	59	. 40727	41	. 71059	20	17
44	. 70381	20	. 42084	42	. 99073	58	. 00935	59	. 40768	40	. 71039	20	16
45	0. 70401	21	1. 42042	41	0. 99131	58	1. 00876	58	1. 40808	41	0. 71019	21	15
46	. 70422	21	. 42001	42	. 99189	58	. 00818	59	. 40849	41	. 70998	20	14
47	. 70443	20	. 41959	41	. 99247	57	. 00759	58	. 40890	40	. 70978	21	13
48	. 70463	21	. 41918	42	. 99304	58	. 00701	59	. 40930	41	. 70957	20	12
49	. 70484	21	. 41876	41	. 99362	58	. 00642	59	. 40971	41	. 70937	21	11
50	0. 70505	20	1. 41835	42	0. 99420	58	1. 00583	58	1. 41012	41	0. 70916	20	10
51	. 70525	21	. 41793	41	. 99478	58	. 00525	58	. 41053	40	. 70896	21	9
52	. 70546	21	. 41752	42	. 99536	58	. 00467	59	. 41093	41	. 70875	20	8
53	. 70567	20	. 41710	41	. 99594	58	. 00408	58	. 41134	41	. 70855	21	7
54	. 70587	21	. 41669	42	. 99652	58	. 00350	59	. 41175	41	. 70834	21	6
55	0. 70608	20	1. 41627	41	0. 99710	58	1. 00291	58	1. 41216	41	0. 70813	20	5
56	. 70628	21	. 41586	41	. 99768	58	. 00233	58	. 41257	41	. 70793	21	4
57	. 70649	21	. 41545	41	. 99826	58	. 00175	59	. 41298	41	. 70772	20	3
58	. 70670	20	. 41504	41	. 99884	58	. 00116	58	. 41339	41	. 70752	21	2
59	. 70690	21	. 41463	42	0. 99942	58	. 00058	58	. 41380	41	. 70731	20	1
60	0. 70711		1. 41421		1. 00000		1. 00000		1. 41421		0. 70711		0
↑ 134°→	cos	Diff. 1′	sec	Diff. 1′	cot	Diff. 1′	tan	Diff. 1′	csc	Diff. 1′	sin	Diff. 1′	↑ ←45°

TABLE 32

Logarithms of Numbers

1–250

No.	Log	No.	Log	No.	Log	No.	Log	No.	Log
1	0. 00000	51	1. 70757	101	2. 00432	151	2. 17898	201	2. 30320
2	0. 30103	52	1. 71600	102	2. 00860	152	2. 18184	202	2. 30535
3	0. 47712	53	1. 72428	103	2. 01284	153	2. 18469	203	2. 30750
4	0. 60206	54	1. 73239	104	2. 01703	154	2. 18752	204	2. 30963
5	0. 69897	55	1. 74036	105	2. 02119	155	2. 19033	205	2. 31175
6	0. 77815	56	1. 74819	106	2. 02531	156	2. 19312	206	2. 31387
7	0. 84510	57	1. 75587	107	2. 02938	157	2. 19590	207	2. 31597
8	0. 90309	58	1. 76343	108	2. 03342	158	2. 19866	208	2. 31806
9	0. 95424	59	1. 77085	109	2. 03743	159	2. 20140	209	2. 32015
10	1. 00000	60	1. 77815	110	2. 04139	160	2. 20412	210	2. 32222
11	1. 04139	61	1. 78533	111	2. 04532	161	2. 20683	211	2. 32428
12	1. 07918	62	1. 79239	112	2. 04922	162	2. 20952	212	2. 32634
13	1. 11394	63	1. 79934	113	2. 05308	163	2. 21219	213	2. 32838
14	1. 14613	64	1. 80618	114	2. 05690	164	2 21484	214	2. 33041
15	1. 17609	65	1. 81291	115	2. 06070	165	2. 21748	215	2. 33244
16	1. 20412	66	1. 81954	116	2. 06446	166	2. 22011	216	2. 33445
17	1. 23045	67	1. 82607	117	2. 06819	167	2. 22272	217	2. 33646
18	1. 25527	68	1. 83251	118	2. 07188	168	2. 22531	218	2. 33846
19	1. 27875	69	1. 83885	119	2. 07555	169	2. 22789	219	2. 34044
20	1. 30103	70	1. 84510	120	2. 07918	170	2. 23045	220	2. 34242
21	1. 32222	71	1. 85126	121	2. 08279	171	2. 23300	221	2. 34439
22	1. 34242	72	1. 85733	122	2. 08636	172	2. 23553	222	2. 34635
23	1. 36173	73	1. 86332	123	2. 08991	173	2. 23805	223	2. 34830
24	1. 38021	74	1. 86923	124	2. 09342	174	2. 24055	224	2. 35025
25	1. 39794	75	1. 87506	125	2. 09691	175	2. 24304	225	2. 35218
26	1. 41497	76	1. 88081	126	2. 10037	176	2. 24551	226	2. 35411
27	1. 43136	77	1. 88649	127	2. 10380	177	2. 24797	227	2. 35603
28	1. 44716	78	1. 89209	128	2. 10721	178	2. 25042	228	2. 35793
29	1. 46240	79	1. 89763	129	2. 11059	179	2. 25285	229	2. 35984
30	1. 47712	80	1. 90309	130	2. 11394	180	2. 25527	230	2. 36173
31	1. 49136	81	1. 90849	131	2. 11727	181	2. 25768	231	2. 36361
32	1. 50515	82	1. 91381	132	2. 12057	182	2. 26007	232	2. 36549
33	1. 51851	83	1. 91908	133	2. 12385	183	2. 26245	233	2. 36736
34	1. 53148	84	1. 92428	134	2. 12710	184	2. 26482	234	2. 36922
35	1. 54407	85	1. 92942	135	2. 13033	185	2. 26717	235	2. 37107
36	1. 55630	86	1. 93450	136	2. 13354	186	2. 26951	236	2. 37291
37	1. 56820	87	1. 93952	137	2. 13672	187	2. 27184	237	2. 37475
38	1. 57978	88	1. 94448	138	2. 13988	188	2. 27416	238	2. 37658
39	1. 59106	89	1. 94939	139	2. 14301	189	2. 27646	239	2. 37840
40	1. 60206	90	1. 95424	140	2. 14613	190	2. 27875	240	2. 38021
41	1. 61278	91	1. 95904	141	2. 14922	191	2. 28103	241	2. 38202
42	1. 62325	92	1. 96379	142	2. 15229	192	2. 28330	242	2. 38382
43	1. 63347	93	1. 96848	143	2. 15534	193	2. 28556	243	2. 38561
44	1. 64345	94	1. 97313	144	2. 15836	194	2. 28780	244	2. 38739
45	1. 65321	95	1. 97772	145	2. 16137	195	2. 29003	245	2. 38917
46	1. 66276	96	1. 98227	146	2. 16435	196	2. 29226	246	2. 39094
47	1. 67210	97	1. 98677	147	2. 16732	197	2. 29447	247	2. 39270
48	1. 68124	98	1. 99123	148	2. 17026	198	2. 29667	248	2. 39445
49	1. 69020	99	1. 99564	149	2. 17319	199	2. 29885	249	2. 39620
50	1. 69897	100	2. 00000	150	2. 17609	200	2. 30103	250	2. 39794

TABLE 32

Logarithms of Numbers

1000–1500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
100	00000	43	00043	44	00087	43	00130	43	00173	44	00217	43	00260	43	00303	43	00346	43	00389	43
101	00432	43	00475	43	00518	43	00561	43	00604	43	00647	42	00689	43	00732	43	00775	42	00817	43
102	00860	43	00903	42	00945	43	00988	42	01030	42	01072	43	01115	42	01157	42	01199	43	01242	42
103	01284	42	01326	42	01368	42	01410	42	01452	42	01494	42	01536	42	01578	42	01620	42	01662	41
104	01703	42	01745	42	01787	41	01828	42	01870	42	01912	41	01953	42	01995	41	02036	42	02078	41
105	02119	41	02160	42	02202	41	02243	41	02284	41	02325	41	02366	41	02407	42	02449	41	02490	41
106	02531	41	02572	40	02612	41	02653	41	02694	41	02735	41	02776	40	02816	41	02857	41	02898	40
107	02938	41	02979	40	03019	41	03060	40	03100	41	03141	40	03181	41	03222	40	03262	40	03302	40
108	03342	41	03383	40	03423	40	03463	40	03503	40	03543	40	03583	40	03623	40	03663	40	03703	40
109	03743	39	03782	40	03822	40	03862	40	03902	39	03941	40	03981	40	04021	39	04060	40	04100	39
110	04139	40	04179	39	04218	40	04258	39	04297	39	04336	40	04376	39	04415	39	04454	39	04493	39
111	04532	39	04571	39	04610	40	04650	39	04689	38	04727	39	04766	39	04805	39	04844	39	04883	39
112	04922	39	04961	38	04999	39	05038	39	05077	38	05115	39	05154	38	05192	39	05231	38	05269	39
113	05308	38	05346	39	05385	38	05423	38	05461	39	05500	38	05538	38	05576	38	05614	38	05652	38
114	05690	39	05729	38	05767	38	05805	38	05843	38	05881	37	05918	38	05956	38	05994	38	06032	38
115	06070	38	06108	37	06145	38	06183	38	06221	37	06258	38	06296	37	06333	38	06371	37	06408	38
116	06446	37	06483	38	06521	37	06558	37	06595	38	06633	37	06670	37	06707	37	06744	37	06781	38
117	06819	37	06856	37	06893	37	06930	37	06967	37	07004	37	07041	37	07078	37	07115	36	07151	37
118	07188	37	07225	37	07262	36	07298	37	07335	37	07372	36	07408	37	07445	37	07482	36	07518	37
119	07555	36	07591	37	07628	36	07664	36	07700	37	07737	36	07773	36	07809	37	07846	36	07882	36
120	07918	36	07954	36	07990	37	08027	36	08063	36	08099	36	08135	36	08171	36	08207	36	08243	36
121	08279	35	08314	36	08350	36	08386	36	08422	36	08458	35	08493	36	08529	36	08565	35	08600	36
122	08636	36	08672	35	08707	36	08743	35	08778	36	08814	35	08849	35	08884	36	08920	35	08955	36
123	08991	35	09026	35	09061	35	09096	36	09132	35	09167	35	09202	35	09237	35	09272	35	09307	35
124	09342	35	09377	35	09412	35	09447	35	09482	35	09517	35	09552	35	09587	34	09621	35	09656	35
125	09691	35	09726	34	09760	35	09795	35	09830	34	09864	35	09899	35	09934	34	09968	35	10003	34
126	10037	35	10072	34	10106	34	10140	35	10175	34	10209	34	10243	35	10278	34	10312	34	10346	34
127	10380	35	10415	34	10449	34	10483	34	10517	34	10551	34	10585	34	10619	34	10653	34	10687	34
128	10721	34	10755	34	10789	34	10823	34	10857	33	10890	34	10924	34	10958	34	10992	33	11025	34
129	11059	34	11093	33	11126	34	11160	33	11193	34	11227	34	11261	33	11294	33	11327	34	11361	33
130	11394	34	11428	33	11461	33	11494	34	11528	33	11561	33	11594	34	11628	33	11661	33	11694	33
131	11727	33	11760	33	11793	33	11826	34	11860	33	11893	33	11926	33	11959	33	11992	32	12024	33
132	12057	33	12090	33	12123	33	12156	33	12189	33	12222	32	12254	33	12287	33	12320	32	12352	33
133	12385	33	12418	32	12450	33	12483	33	12516	32	12548	33	12581	32	12613	33	12646	32	12678	32
134	12710	33	12743	32	12775	33	12808	32	12840	32	12872	33	12905	32	12937	32	12969	32	13001	32
135	13033	33	13066	32	13098	32	13130	32	13162	32	13194	32	13226	32	13258	32	13290	32	13322	32
136	13354	32	13386	32	13418	32	13450	31	13481	32	13513	32	13545	32	13577	32	13609	31	13640	32
137	13672	32	13704	31	13735	32	13767	32	13799	31	13830	32	13862	31	13893	32	13925	31	13956	32
138	13988	31	14019	32	14051	31	14082	32	14114	31	14145	31	14176	32	14208	31	14239	31	14270	31
139	14301	32	14333	31	14364	31	14395	31	14426	31	14457	32	14489	31	14520	31	14551	31	14582	31
140	14613	31	14644	31	14675	31	14706	31	14737	31	14768	31	14799	30	14829	31	14860	31	14891	31
141	14922	31	14953	30	14983	31	15014	31	15045	31	15076	30	15106	31	15137	31	15168	30	15198	31
142	15229	30	15259	31	15290	30	15320	31	15351	30	15381	31	15412	30	15442	31	15473	30	15503	31
143	15534	30	15564	30	15594	31	15625	30	15655	30	15685	30	15715	31	15746	30	15776	30	15806	30
144	15836	30	15866	31	15897	30	15927	30	15957	30	15987	30	16017	30	16047	30	16077	30	16107	30
145	16137	30	16167	30	16197	30	16227	29	16256	30	16286	30	16316	30	16346	30	16376	30	16406	29
146	16435	30	16465	30	16495	29	16524	30	16554	30	16584	29	16613	30	16643	30	16673	29	16702	30
147	16732	29	16761	30	16791	29	16820	30	16850	29	16879	30	16909	29	16938	29	16967	30	16997	29
148	17026	30	17056	29	17085	29	17114	29	17143	30	17173	29	17202	29	17231	29	17260	29	17289	30
149	17319	29	17348	29	17377	29	17406	29	17435	29	17464	29	17493	29	17522	29	17551	29	17580	29
150	17609	29	17638	29	17667	29	17696	29	17725	29	17754	28	17782	29	17811	29	17840	29	17869	29
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	44	43		42	41		40	39		38	37		36	35		34	33
1	4	4	1	4	4	1	4	4	1	4	4	1	4	4	1	3	3
2	9	9	2	8	8	2	8	8	2	8	7	2	7	7	2	7	7
3	13	13	3	13	12	3	12	12	3	11	11	3	11	10	3	10	10
4	18	17	4	17	16	4	16	16	4	15	15	4	14	14	4	14	13
5	22	22	5	21	20	5	20	20	5	19	18	5	18	18	5	17	16
6	26	26	6	25	25	6	24	23	6	23	22	6	22	21	6	20	20
7	31	30	7	29	29	7	28	27	7	27	26	7	25	24	7	24	23
8	35	34	8	34	33	8	32	31	8	30	30	8	29	28	8	27	26
9	40	39	9	38	37	9	36	35	9	34	33	9	32	32	9	31	30

TABLE 32

Logarithms of Numbers

1500–2000

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
150	17609	29	17638	29	17667	29	17696	29	17725	29	17754	28	17782	29	17811	29	17840	29	17869	29
151	17898	28	17926	29	17955	29	17984	29	18013	28	18041	29	18070	29	18099	28	18127	29	18156	28
152	18184	29	18213	28	18241	29	18270	28	18298	29	18327	28	18355	29	18384	28	18412	29	18441	28
153	18469	29	18498	28	18526	28	18554	29	18583	28	18611	28	18639	28	18667	29	18696	28	18724	28
154	18752	28	18780	28	18808	29	18837	28	18865	28	18893	28	18921	28	18949	28	18977	28	19005	28
155	19033	28	19061	28	19089	28	19117	28	19145	28	19173	28	19201	28	19229	28	19257	28	19285	27
156	19312	28	19340	28	19368	28	19396	28	19424	27	19451	28	19479	28	19507	28	19535	27	19562	28
157	19590	28	19618	27	19645	28	19673	27	19700	28	19728	28	19756	27	19783	28	19811	27	19838	28
158	19866	27	19893	28	19921	27	19948	28	19976	27	20003	27	20030	28	20058	27	20085	27	20112	28
159	20140	27	20167	27	20194	28	20222	27	20249	27	20276	27	20303	27	20330	28	20358	27	20385	27
160	20412	27	20439	27	20466	27	20493	27	20520	28	20548	27	20575	27	20602	27	20629	27	20656	27
161	20683	27	20710	27	20737	26	20763	27	20790	27	20817	27	20844	27	20871	27	20898	27	20925	27
162	20952	26	20978	27	21005	27	21032	27	21059	26	21085	27	21112	27	21139	26	21165	27	21192	27
163	21219	26	21245	27	21272	27	21299	26	21325	27	21352	26	21378	27	21405	26	21431	27	21458	26
164	21484	27	21511	26	21537	27	21564	26	21590	27	21617	26	21643	26	21669	27	21696	26	21722	26
165	21748	27	21775	26	21801	26	21827	27	21854	26	21880	26	21906	26	21932	26	21958	27	21985	26
166	22011	26	22037	26	22063	26	22089	26	22115	26	22141	26	22167	27	22194	26	22220	26	22246	26
167	22272	26	22298	26	22324	26	22350	26	22376	25	22401	26	22427	26	22453	26	22479	26	22505	26
168	22531	26	22557	26	22583	25	22608	26	22634	26	22660	26	22686	26	22712	25	22737	26	22763	26
169	22789	25	22814	26	22840	26	22866	25	22891	26	22917	26	22943	25	22968	26	22994	25	23019	26
170	23045	25	23070	26	23096	25	23121	26	23147	25	23172	26	23198	25	23223	26	23249	25	23274	26
171	23300	25	23325	25	23350	26	23376	25	23401	25	23426	26	23452	25	23477	25	23502	26	23528	25
172	23553	25	23578	25	23603	26	23629	25	23654	25	23679	25	23704	25	23729	25	23754	25	23779	26
173	23805	25	23830	25	23855	25	23880	25	23905	25	23930	25	23955	25	23980	25	24005	25	24030	25
174	24055	25	24080	25	24105	25	24130	25	24155	25	24180	24	24204	25	24229	25	24254	25	24279	25
175	24304	25	24329	24	24353	25	24378	25	24403	25	24428	24	24452	25	24477	25	24502	25	24527	24
176	24551	25	24576	25	24601	24	24625	25	24650	24	24674	25	24699	25	24724	24	24748	25	24773	24
177	24797	25	24822	24	24846	25	24871	24	24895	25	24920	24	24944	25	24969	24	24993	25	25018	24
178	25042	24	25066	25	25091	24	25115	24	25139	25	25164	24	25188	24	25212	25	25237	24	25261	24
179	25285	25	25310	24	25334	24	25358	24	25382	24	25406	25	25431	24	25455	24	25479	24	25503	24
180	25527	24	25551	24	25575	25	25600	24	25624	24	25648	24	25672	24	25696	24	25720	24	25744	24
181	25768	24	25792	24	25816	24	25840	24	25864	24	25888	24	25912	23	25935	24	25959	24	25983	24
182	26007	24	26031	24	26055	24	26079	23	26102	24	26126	24	26150	24	26174	24	26198	23	26221	24
183	26245	24	26269	24	26293	23	26316	24	26340	24	26364	23	26387	24	26411	24	26435	23	26458	24
184	26482	23	26505	24	26529	24	26553	23	26576	24	26600	23	26623	24	26647	23	26670	24	26694	23
185	26717	24	26741	23	26764	24	26788	23	26811	23	26834	24	26858	23	26881	24	26905	23	26928	23
186	26951	24	26975	23	26998	23	27021	24	27045	23	27068	23	27091	23	27114	24	27138	23	27161	23
187	27184	23	27207	24	27231	23	27254	23	27277	23	27300	23	27323	23	27346	24	27370	23	27393	23
188	27416	23	27439	23	27462	23	27485	23	27508	23	27531	23	27554	23	27577	23	27600	23	27623	23
189	27646	23	27669	23	27692	23	27715	23	27738	23	27761	23	27784	23	27807	23	27830	22	27852	23
190	27875	23	27898	23	27921	23	27944	23	27967	22	27989	23	28012	23	28035	23	28058	23	28081	22
191	28103	23	28126	23	28149	22	28171	23	28194	23	28217	23	28240	22	28262	23	28285	22	28307	23
192	28330	23	28353	22	28375	23	28398	23	28421	22	28443	23	28466	22	28488	23	28511	22	28533	23
193	28556	22	28578	23	28601	22	28623	23	28646	22	28668	23	28691	22	28713	22	28735	23	28758	22
194	28780	23	28803	22	28825	22	28847	23	28870	22	28892	22	28914	23	28937	22	28959	22	28981	22
195	29003	23	29026	22	29048	22	29070	22	29092	23	29115	22	29137	22	29159	22	29181	22	29203	23
196	29226	22	29248	22	29270	22	29292	22	29314	22	29336	22	29358	22	29380	23	29403	22	29425	22
197	29447	22	29469	22	29491	22	29513	22	29535	22	29557	22	29579	22	29601	22	29623	22	29645	22
198	29667	21	29688	22	29710	22	29732	22	29754	22	29776	22	29798	22	29820	22	29842	21	29863	22
199	29885	22	29907	22	29929	22	29951	22	29973	21	29994	22	30016	22	30038	22	30060	21	30081	22
200	30103	22	30125	21	30146	22	30168	22	30190	21	30211	22	30233	22	30255	21	30276	22	30298	22
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	32	31
1	3	3
2	6	6
3	10	9
4	13	12
5	16	16
6	19	19
7	22	22
8	26	25
9	29	28
	30	**29**
1	3	3
2	6	6
3	9	9
4	12	12
5	15	14
6	18	17
7	21	20
8	24	23
9	27	26
	28	**27**
1	3	3
2	6	5
3	8	8
4	11	11
5	14	14
6	17	16
7	20	19
8	22	22
9	25	24
	26	**25**
1	3	2
2	5	5
3	8	8
4	10	10
5	13	12
6	16	15
7	18	18
8	21	20
9	23	22
	24	**23**
1	2	2
2	5	5
3	7	7
4	10	9
5	12	12
6	14	14
7	17	16
8	19	18
9	22	21
	22	**21**
1	2	2
2	4	4
3	7	6
4	9	8
5	11	10
6	13	13
7	15	15
8	18	17
9	20	19

TABLE 32

Logarithms of Numbers

2000–2500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
200	30103	22	30125	21	30146	22	30168	22	30190	21	30211	22	30233	22	30255	21	30276	22	30298	22
201	30320	21	30341	22	30363	21	30384	22	30406	22	30428	21	30449	22	30471	21	30492	22	30514	21
202	30535	22	30557	21	30578	22	30600	21	30621	22	30643	21	30664	21	30685	22	30707	21	30728	22
203	30750	21	30771	21	30792	22	30814	21	30835	21	30856	22	30878	21	30899	21	30920	22	30942	21
204	30963	21	30984	22	31006	21	31027	21	31048	21	31069	22	31091	21	31112	21	31133	21	31154	21
205	31175	22	31197	21	31218	21	31239	21	31260	21	31281	21	31302	21	31323	22	31345	21	31366	21
206	31387	21	31408	21	31429	21	31450	21	31471	21	31492	21	31513	21	31534	21	31555	21	31576	21
207	31597	21	31618	21	31639	21	31660	21	31681	21	31702	21	31723	21	31744	21	31765	20	31785	21
208	31806	21	31827	21	31848	21	31869	21	31890	21	31911	20	31931	21	31952	21	31973	21	31994	21
209	32015	20	32035	21	32056	21	32077	21	32098	20	32118	21	32139	21	32160	21	32181	20	32201	21
210	32222	21	32243	20	32263	21	32284	21	32305	20	32325	21	32346	20	32366	21	32387	21	32408	20
211	32428	21	32449	20	32469	21	32490	20	32510	21	32531	21	32552	20	32572	21	32593	20	32613	21
212	32634	20	32654	21	32675	20	32695	20	32715	21	32736	20	32756	21	32777	20	32797	21	32818	20
213	32838	20	32858	21	32879	20	32899	20	32919	21	32940	20	32960	20	32980	21	33001	20	33021	20
214	33041	21	33062	20	33082	20	33102	20	33122	21	33143	20	33163	20	33183	20	33203	21	33224	20
215	33244	20	33264	20	33284	20	33304	21	33325	20	33345	20	33365	20	33385	20	33405	20	33425	20
216	33445	20	33465	21	33486	20	33506	20	33526	20	33546	20	33566	20	33586	20	33606	20	33626	20
217	33646	20	33666	20	33686	20	33706	20	33726	20	33746	20	33766	20	33786	20	33806	20	33826	20
218	33846	20	33866	19	33885	20	33905	20	33925	20	33945	20	33965	20	33985	20	34005	20	34025	19
219	34044	20	34064	20	34084	20	34104	20	34124	19	34143	20	34163	20	34183	20	34203	20	34223	19
220	34242	20	34262	20	34282	19	34301	20	34321	20	34341	20	34361	19	34380	20	34400	20	34420	19
221	34439	20	34459	20	34479	19	34498	20	34518	19	34537	20	34557	20	34577	19	34596	20	34616	19
222	34635	20	34655	19	34674	20	34694	19	34713	20	34733	20	34753	19	34772	20	34792	19	34811	19
223	34830	20	34850	19	34869	20	34889	19	34908	20	34928	19	34947	20	34967	19	34986	19	35005	20
224	35025	19	35044	20	35064	19	35083	19	35102	20	35122	19	35141	19	35160	20	35180	19	35199	19
225	35218	20	35238	19	35257	19	35276	19	35295	20	35315	19	35334	19	35353	19	35372	20	35392	19
226	35411	19	35430	19	35449	19	35468	20	35488	19	35507	19	35526	19	35545	19	35564	19	35583	20
227	35603	19	35622	19	35641	19	35660	19	35679	19	35698	19	35717	19	35736	19	35755	19	35774	19
228	35793	20	35813	19	35832	19	35851	19	35870	19	35889	19	35908	19	35927	19	35946	19	35965	19
229	35984	19	36003	18	36021	19	36040	19	36059	19	36078	19	36097	19	36116	19	36135	19	36154	19
230	36173	19	36192	19	36211	18	36229	19	36248	19	36267	19	36286	19	36305	19	36324	18	36342	19
231	36361	19	36380	19	36399	19	36418	18	36436	19	36455	19	36474	19	36493	18	36511	19	36530	19
232	36549	19	36568	18	36586	19	36605	19	36624	18	36642	19	36661	19	36680	18	36698	19	36717	19
233	36736	18	36754	19	36773	18	36791	19	36810	19	36829	18	36847	19	36866	18	36884	19	36903	19
234	36922	18	36940	19	36959	18	36977	19	36996	18	37014	19	37033	18	37051	19	37070	18	37088	19
235	37107	18	37125	19	37144	18	37162	19	37181	18	37199	19	37218	18	37236	18	37254	19	37273	18
236	37291	19	37310	18	37328	18	37346	19	37365	18	37383	18	37401	19	37420	18	37438	19	37457	18
237	37475	18	37493	18	37511	19	37530	18	37548	18	37566	19	37585	18	37603	18	37621	18	37639	19
238	37658	18	37676	18	37694	18	37712	19	37731	18	37749	18	37767	18	37785	18	37803	19	37822	18
239	37840	18	37858	18	37876	18	37894	18	37912	19	37931	18	37949	18	37967	18	37985	18	38003	18
240	38021	18	38039	18	38057	18	38075	18	38093	19	38112	18	38130	18	38148	18	38166	18	38184	18
241	38202	18	38220	18	38238	18	38256	18	38274	18	38292	18	38310	18	38328	18	38346	18	38364	18
242	38382	17	38399	18	38417	18	38435	18	38453	18	38471	18	38489	18	38507	18	38525	18	38543	18
243	38561	17	38578	18	38596	18	38614	18	38632	18	38650	18	38668	18	38686	17	38703	18	38721	18
244	38739	18	38757	18	38775	17	38792	18	38810	18	38828	18	38846	17	38863	18	38881	18	38899	18
245	38917	17	38934	18	38952	18	38970	17	38987	18	39005	18	39023	18	39041	17	39058	18	39076	18
246	39094	17	39111	18	39129	17	39146	18	39164	18	39182	17	39199	18	39217	18	39235	17	39252	18
247	39270	17	39287	18	39305	17	39322	18	39340	18	39358	17	39375	18	39393	17	39410	18	39428	17
248	39445	18	39463	17	39480	18	39498	17	39515	18	39533	17	39550	18	39568	17	39585	17	39602	18
249	39620	17	39637	18	39655	17	39672	18	39690	17	39707	17	39724	18	39742	17	39759	18	39777	17
250	39794	17	39811	18	39829	17	39846	17	39863	18	39881	17	39898	17	39915	18	39933	17	39950	17
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	22	21	20	19	18	17
1	2	2	2	2	2	2
2	4	4	4	4	4	3
3	7	6	6	6	5	5
4	9	8	8	8	7	7
5	11	10	10	10	9	8
6	13	13	12	11	11	10
7	15	15	14	13	13	12
8	18	17	16	15	14	14
9	20	19	18	17	16	15

TABLE 32

Logarithms of Numbers

2500–3000

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
250	39794	17	39811	18	39829	17	39846	17	39863	18	39881	17	39898	17	39915	18	39933	17	39950	17
251	39967	18	39985	17	40002	17	40019	18	40037	17	40054	17	40071	17	40088	18	40106	17	40123	17
252	40140	17	40157	18	40175	17	40192	17	40209	17	40226	17	40243	18	40261	17	40278	17	40295	17
253	40312	17	40329	17	40346	18	40364	17	40381	17	40398	17	40415	17	40432	17	40449	17	40466	17
254	40483	17	40500	18	40518	17	40535	17	40552	17	40569	17	40586	17	40603	17	40620	17	40637	17
255	40654	17	40671	17	40688	17	40705	17	40722	17	40739	17	40756	17	40773	17	40790	17	40807	17
256	40824	17	40841	17	40858	17	40875	17	40892	17	40909	17	40926	17	40943	17	40960	16	40976	17
257	40993	17	41010	17	41027	17	41044	17	41061	17	41078	17	41095	16	41111	17	41128	17	41145	17
258	41162	17	41179	17	41196	16	41212	17	41229	17	41246	17	41263	17	41280	16	41296	17	41313	17
259	41330	17	41347	16	41363	17	41380	17	41397	17	41414	16	41430	17	41447	17	41464	17	41481	16
260	41497	17	41514	17	41531	16	41547	17	41564	17	41581	16	41597	17	41614	17	41631	16	41647	17
261	41664	17	41681	16	41697	17	41714	17	41731	16	41747	17	41764	16	41780	17	41797	17	41814	16
262	41830	17	41847	16	41863	17	41880	16	41896	17	41913	16	41929	17	41946	17	41963	16	41979	17
263	41996	16	42012	17	42029	16	42045	17	42062	16	42078	17	42095	16	42111	16	42127	17	42144	16
264	42160	17	42177	16	42193	17	42210	16	42226	17	42243	16	42259	16	42275	17	42292	16	42308	17
265	42325	16	42341	16	42357	17	42374	16	42390	16	42406	17	42423	16	42439	16	42455	17	42472	16
266	42488	16	42504	17	42521	16	42537	16	42553	17	42570	16	42586	16	42602	17	42619	16	42635	16
267	42651	16	42667	17	42684	16	42700	16	42716	16	42732	17	42749	16	42765	16	42781	16	42797	16
268	42813	17	42830	16	42846	16	42862	16	42878	16	42894	17	42911	16	42927	16	42943	16	42959	16
269	42975	16	42991	17	43008	16	43024	16	43040	16	43056	16	43072	16	43088	16	43104	16	43120	16
270	43136	16	43152	17	43169	16	43185	16	43201	16	43217	16	43233	16	43249	16	43265	16	43281	16
271	43297	16	43313	16	43329	16	43345	16	43361	16	43377	16	43393	16	43409	16	43425	16	43441	16
272	43457	16	43473	16	43489	16	43505	16	43521	16	43537	16	43553	16	43569	15	43584	16	43600	16
273	43616	16	43632	16	43648	16	43664	16	43680	16	43696	16	43712	15	43727	16	43743	16	43759	16
274	43775	16	43791	16	43807	16	43823	15	43838	16	43854	16	43870	16	43886	16	43902	15	43917	16
275	43933	16	43949	16	43965	16	43981	15	43996	16	44012	16	44028	16	44044	15	44059	16	44075	16
276	44091	16	44107	15	44122	16	44138	16	44154	16	44170	15	44185	16	44201	16	44217	15	44232	16
277	44248	16	44264	15	44279	16	44295	16	44311	15	44326	16	44342	16	44358	15	44373	16	44389	15
278	44404	16	44420	16	44436	15	44451	16	44467	16	44483	15	44498	16	44514	15	44529	16	44545	15
279	44560	16	44576	16	44592	15	44607	16	44623	15	44638	16	44654	15	44669	16	44685	15	44700	16
280	44716	15	44731	16	44747	15	44762	16	44778	15	44793	16	44809	15	44824	16	44840	15	44855	16
281	44871	15	44886	16	44902	15	44917	15	44932	16	44948	15	44963	16	44979	15	44994	16	45010	15
282	45025	15	45040	16	45056	15	45071	15	45086	16	45102	15	45117	16	45133	15	45148	15	45163	16
283	45179	15	45194	15	45209	16	45225	15	45240	15	45255	16	45271	15	45286	15	45301	16	45317	15
284	45332	15	45347	15	45362	16	45378	15	45393	15	45408	15	45423	16	45439	15	45454	15	45469	15
285	45484	16	45500	15	45515	15	45530	15	45545	16	45561	15	45576	15	45591	15	45606	15	45621	16
286	45637	15	45652	15	45667	15	45682	15	45697	15	45712	16	45728	15	45743	15	45758	15	45773	15
287	45788	15	45803	15	45818	16	45834	15	45849	15	45864	15	45879	15	45894	15	45909	15	45924	15
288	45939	15	45954	15	45969	15	45984	16	46000	15	46015	15	46030	15	46045	15	46060	15	46075	15
289	46090	15	46105	15	46120	15	46135	15	46150	15	46165	15	46180	15	46195	15	46210	15	46225	15
290	46240	15	46255	15	46270	15	46285	15	46300	15	46315	15	46330	15	46345	14	46359	15	46374	15
291	46389	15	46404	15	46419	15	46434	15	46449	15	46464	15	46479	15	46494	15	46509	14	46523	15
292	46538	15	46553	15	46568	15	46583	15	46598	15	46613	14	46627	15	46642	15	46657	15	46672	15
293	46687	15	46702	14	46716	15	46731	15	46746	15	46761	15	46776	14	46790	15	46805	15	46820	15
294	46835	15	46850	14	46864	15	46879	15	46894	15	46909	14	46923	15	46938	15	46953	14	46967	15
295	46982	15	46997	15	47012	14	47026	15	47041	15	47056	14	47070	15	47085	15	47100	14	47114	15
296	47129	15	47144	15	47159	14	47173	15	47188	14	47202	15	47217	15	47232	14	47246	15	47261	15
297	47276	14	47290	15	47305	14	47319	15	47334	15	47349	14	47363	15	47378	14	47392	15	47407	15
298	47422	14	47436	15	47451	14	47465	15	47480	14	47494	15	47509	15	47524	14	47538	15	47553	14
299	47567	15	47582	14	47596	15	47611	14	47625	15	47640	14	47654	15	47669	14	47683	15	47698	14
300	47712	15	47727	14	47741	15	47756	14	47770	14	47784	15	47799	14	47813	15	47828	14	47842	15
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	18	17	16	15	14
1	2	2	2	2	1
2	4	3	3	3	3
3	5	5	5	4	4
4	7	7	6	6	6
5	9	8	8	8	7
6	11	10	10	9	8
7	13	12	11	10	10
8	14	14	13	12	11
9	16	15	14	14	13

TABLE 32

Logarithms of Numbers

3000–3500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
300	47712	15	47727	14	47741	15	47756	14	47770	14	47784	15	47799	14	47813	15	47828	14	47842	15
301	47857	14	47871	14	47885	15	47900	14	47914	15	47929	14	47943	15	47958	14	47972	14	47986	15
302	48001	14	48015	14	48029	15	48044	14	48058	15	48073	14	48087	14	48101	15	48116	14	48130	14
303	48144	15	48159	14	48173	14	48187	15	48202	14	48216	14	48230	14	48244	15	48259	14	48273	14
304	48287	15	48302	14	48316	14	48330	14	48344	15	48359	14	48373	14	48387	14	48401	15	48416	14
305	48430	14	48444	14	48458	15	48473	14	48487	14	48501	14	48515	15	48530	14	48544	14	48558	14
306	48572	14	48586	15	48601	14	48615	14	48629	14	48643	14	48657	14	48671	15	48686	14	48700	14
307	48714	14	48728	14	48742	14	48756	14	48770	15	48785	14	48799	14	48813	14	48827	14	48841	14
308	48855	14	48869	14	48883	14	48897	14	48911	15	48926	14	48940	14	48954	14	48968	14	48982	14
309	48996	14	49010	14	49024	14	49038	14	49052	14	49066	14	49080	14	49094	14	49108	14	49122	14
310	49136	14	49150	14	49164	14	49178	14	49192	14	49206	14	49220	14	49234	14	49248	14	49262	14
311	49276	14	49290	14	49304	14	49318	14	49332	14	49346	14	49360	14	49374	14	49388	14	49402	13
312	49415	14	49429	14	49443	14	49457	14	49471	14	49485	14	49499	14	49513	14	49527	14	49541	13
313	49554	14	49568	14	49582	14	49596	14	49610	14	49624	14	49638	13	49651	14	49665	14	49679	14
314	49693	14	49707	14	49721	13	49734	14	49748	14	49762	14	49776	14	49790	13	49803	14	49817	14
315	49831	14	49845	14	49859	13	49872	14	49886	14	49900	14	49914	13	49927	14	49941	14	49955	14
316	49969	13	49982	14	49996	14	50010	14	50024	13	50037	14	50051	14	50065	14	50079	13	50092	14
317	50106	14	50120	13	50133	14	50147	14	50161	13	50174	14	50188	14	50202	13	50215	14	50229	14
318	50243	13	50256	14	50270	14	50284	13	50297	14	50311	14	50325	13	50338	14	50352	13	50365	14
319	50379	14	50393	13	50406	14	50420	13	50433	14	50447	14	50461	13	50474	14	50488	13	50501	14
320	50515	14	50529	13	50542	14	50556	13	50569	14	50583	13	50596	14	50610	13	50623	14	50637	14
321	50651	13	50664	14	50678	13	50691	14	50705	13	50718	14	50732	13	50745	14	50759	13	50772	14
322	50786	13	50799	14	50813	13	50826	14	50840	13	50853	13	50866	14	50880	13	50893	14	50907	13
323	50920	14	50934	13	50947	14	50961	13	50974	13	50987	14	51001	13	51014	14	51028	13	51041	14
324	51055	13	51068	13	51081	14	51095	13	51108	13	51121	14	51135	13	51148	14	51162	13	51175	13
325	51188	14	51202	13	51215	13	51228	14	51242	13	51255	13	51268	14	51282	13	51295	13	51308	14
326	51322	13	51335	13	51348	14	51362	13	51375	13	51388	14	51402	13	51415	13	51428	13	51441	14
327	51455	13	51468	13	51481	14	51495	13	51508	13	51521	13	51534	14	51548	13	51561	13	51574	13
328	51587	14	51601	13	51614	13	51627	13	51640	14	51654	13	51667	13	51680	13	51693	13	51706	14
329	51720	13	51733	13	51746	13	51759	13	51772	14	51786	13	51799	13	51812	13	51825	13	51838	13
330	51851	14	51865	13	51878	13	51891	13	51904	13	51917	13	51930	13	51943	14	51957	13	51970	13
331	51983	13	51996	13	52009	13	52022	13	52035	13	52048	13	52061	14	52075	13	52088	13	52101	13
332	52114	13	52127	13	52140	13	52153	13	52166	13	52179	13	52192	13	52205	13	52218	13	52231	13
333	52244	13	52257	13	52270	14	52284	13	52297	13	52310	13	52323	13	52336	13	52349	13	52362	13
334	52375	13	52388	13	52401	13	52414	13	52427	13	52440	13	52453	13	52466	13	52479	13	52492	12
335	52504	13	52517	13	52530	13	52543	13	52556	13	52569	13	52582	13	52595	13	52608	13	52621	13
336	52634	13	52647	13	52660	13	52673	13	52686	13	52699	12	52711	13	52724	13	52737	13	52750	13
337	52763	13	52776	13	52789	13	52802	13	52815	12	52827	13	52840	13	52853	13	52866	13	52879	13
338	52892	13	52905	12	52917	13	52930	13	52943	13	52956	13	52969	13	52982	12	52994	13	53007	13
339	53020	13	53033	13	53046	12	53058	13	53071	13	53084	13	53097	13	53110	12	53122	13	53135	13
340	53148	13	53161	12	53173	13	53186	13	53199	13	53212	12	53224	13	53237	13	53250	13	53263	12
341	53275	13	53288	13	53301	13	53314	12	53326	13	53339	13	53352	12	53364	13	53377	13	53390	13
342	53403	12	53415	13	53428	13	53441	12	53453	13	53466	13	53479	12	53491	13	53504	13	53517	12
343	53529	13	53542	13	53555	12	53567	13	53580	13	53593	12	53605	13	53618	13	53631	12	53643	13
344	53656	12	53668	13	53681	13	53694	12	53706	13	53719	13	53732	12	53744	13	53757	12	53769	13
345	53782	12	53794	13	53807	13	53820	12	53832	13	53845	12	53857	13	53870	12	53882	13	53895	13
346	53908	12	53920	13	53933	12	53945	13	53958	12	53970	13	53983	12	53995	13	54008	12	54020	13
347	54033	12	54045	13	54058	12	54070	13	54083	12	54095	13	54108	12	54120	13	54133	12	54145	13
348	54158	12	54170	13	54183	12	54195	13	54208	12	54220	13	54233	12	54245	13	54258	12	54270	13
349	54283	12	54295	12	54307	13	54320	12	54332	13	54345	12	54357	13	54370	12	54382	12	54394	13
350	54407	12	54419	13	54432	12	54444	12	54456	13	54469	12	54481	13	54494	12	54506	12	54518	13
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	15		14		13		12
1	2	1	1	1	1	1	1
2	3	2	3	2	3	2	2
3	4	3	4	3	4	3	4
4	6	4	6	4	5	4	5
5	8	5	7	5	6	5	6
6	9	6	8	6	8	6	7
7	10	7	10	7	9	7	8
8	12	8	11	8	10	8	10
9	14	9	13	9	12	9	11

TABLE 32

Logarithms of Numbers

3500–4000

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
350	54407	12	54419	13	54432	12	54444	12	54456	13	54469	12	54481	13	54494	12	54506	12	54518	13
351	54531	12	54543	12	54555	13	54568	12	54580	13	54593	12	54605	12	54617	13	54630	12	54642	12
352	54654	13	54667	12	54679	12	54691	13	54704	12	54716	12	54728	13	54741	12	54753	12	54765	12
353	54777	13	54790	12	54802	12	54814	13	54827	12	54839	12	54851	13	54864	12	54876	12	54888	12
354	54900	13	54913	12	54925	12	54937	12	54949	13	54962	12	54974	12	54986	12	54998	13	55011	12
355	55023	12	55035	12	55047	13	55060	12	55072	12	55084	12	55096	12	55108	13	55121	12	55133	12
356	55145	12	55157	12	55169	13	55182	12	55194	12	55206	12	55218	12	55230	12	55242	13	55255	12
357	55267	12	55279	12	55291	12	55303	12	55315	13	55328	12	55340	12	55352	12	55364	12	55376	12
358	55388	12	55400	13	55413	12	55425	12	55437	12	55449	12	55461	12	55473	12	55485	12	55497	12
359	55509	13	55522	12	55534	12	55546	12	55558	12	55570	12	55582	12	55594	12	55606	12	55618	12
360	55630	12	55642	12	55654	12	55666	12	55678	13	55691	12	55703	12	55715	12	55727	12	55739	12
361	55751	12	55763	12	55775	12	55787	12	55799	12	55811	12	55823	12	55835	12	55847	12	55859	12
362	55871	12	55883	12	55895	12	55907	12	55919	12	55931	12	55943	12	55955	12	55967	12	55979	12
363	55991	12	56003	12	56015	12	56027	11	56038	12	56050	12	56062	12	56074	12	56086	12	56098	12
364	56110	12	56122	12	56134	12	56146	12	56158	12	56170	12	56182	12	56194	11	56205	12	56217	12
365	56229	12	56241	12	56253	12	56265	12	56277	12	56289	12	56301	11	56312	12	56324	12	56336	12
366	56348	12	56360	12	56372	12	56384	12	56396	11	56407	12	56419	12	56431	12	56443	12	56455	12
367	56467	11	56478	12	56490	12	56502	12	56514	12	56526	12	56538	11	56549	12	56561	12	56573	12
368	56585	12	56597	11	56608	12	56620	12	56632	12	56644	12	56656	11	56667	12	56679	12	56691	12
369	56703	11	56714	12	56726	12	56738	12	56750	11	56761	12	56773	12	56785	12	56797	11	56808	12
370	56820	12	56832	12	56844	11	56855	12	56867	12	56879	12	56891	11	56902	12	56914	12	56926	11
371	56937	12	56949	12	56961	11	56972	12	56984	12	56996	12	57008	11	57019	12	57031	12	57043	11
372	57054	12	57066	12	57078	11	57089	12	57101	12	57113	11	57124	12	57136	12	57148	11	57159	12
373	57171	12	57183	11	57194	12	57206	11	57217	12	57229	12	57241	11	57252	12	57264	12	57276	11
374	57287	12	57299	11	57310	12	57322	12	57334	11	57345	12	57357	11	57368	12	57380	12	57392	11
375	57403	12	57415	11	57426	12	57438	11	57449	12	57461	12	57473	11	57484	12	57496	11	57507	12
376	57519	11	57530	12	57542	11	57553	12	57565	11	57576	12	57588	12	57600	11	57611	12	57623	11
377	57634	12	57646	11	57657	12	57669	11	57680	12	57692	11	57703	12	57715	11	57726	12	57738	11
378	57749	12	57761	11	57772	12	57784	11	57795	12	57807	11	57818	12	57830	11	57841	11	57852	12
379	57864	11	57875	12	57887	11	57898	12	57910	11	57921	12	57933	11	57944	11	57955	12	57967	11
380	57978	12	57990	11	58001	12	58013	11	58024	11	58035	12	58047	11	58058	12	58070	11	58081	11
381	58092	12	58104	11	58115	12	58127	11	58138	11	58149	12	58161	11	58172	12	58184	11	58195	11
382	58206	12	58218	11	58229	11	58240	12	58252	11	58263	11	58274	12	58286	11	58297	12	58309	11
383	58320	11	58331	12	58343	11	58354	11	58365	12	58377	11	58388	11	58399	11	58410	12	58422	11
384	58433	11	58444	12	58456	11	58467	11	58478	12	58490	11	58501	11	58512	12	58524	11	58535	11
385	58546	11	58557	12	58569	11	58580	11	58591	11	58602	12	58614	11	58625	11	58636	11	58647	12
386	58659	11	58670	11	58681	11	58692	12	58704	11	58715	11	58726	11	58737	12	58749	11	58760	11
387	58771	11	58782	12	58794	11	58805	11	58816	11	58827	11	58838	12	58850	11	58861	11	58872	11
388	58883	11	58894	12	58906	11	58917	11	58928	11	58939	11	58950	11	58961	12	58973	11	58984	11
389	58995	11	59006	11	59017	11	59028	12	59040	11	59051	11	59062	11	59073	11	59084	11	59095	11
390	59106	12	59118	11	59129	11	59140	11	59151	11	59162	11	59173	11	59184	11	59195	12	59207	11
391	59218	11	59229	11	59240	11	59251	11	59262	11	59273	11	59284	11	59295	11	59306	12	59318	11
392	59329	11	59340	11	59351	11	59362	11	59373	11	59384	11	59395	11	59406	11	59417	11	59428	11
393	59439	11	59450	11	59461	11	59472	11	59483	11	59494	12	59506	11	59517	11	59528	11	59539	11
394	59550	11	59561	11	59572	11	59583	11	59594	11	59605	11	59616	11	59627	11	59638	11	59649	11
395	59660	11	59671	11	59682	11	59693	11	59704	11	59715	11	59726	11	59737	11	59748	11	59759	11
396	59770	10	59780	11	59791	11	59802	11	59813	11	59824	11	59835	11	59846	11	59857	11	59868	11
397	59879	11	59890	11	59901	11	59912	11	59923	11	59934	11	59945	11	59956	10	59966	11	59977	11
398	59988	11	59999	11	60010	11	60021	11	60032	11	60043	11	60054	11	60065	11	60076	10	60086	11
399	60097	11	60108	11	60119	11	60130	11	60141	11	60152	11	60163	10	60173	11	60184	11	60195	11
400	60206	11	60217	11	60228	11	60239	10	60249	11	60260	11	60271	11	60282	11	60293	11	60304	10
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	13	12	11	10
1	1	1	1	1
2	3	2	2	2
3	4	4	3	3
4	5	5	4	4
5	6	6	6	5
6	8	7	7	6
7	9	8	8	7
8	10	10	9	8
9	12	11	10	9

TABLE 32

Logarithms of Numbers

4000–4500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
400	60206	11	60217	11	60228	11	60239	10	60249	11	60260	11	60271	11	60282	11	60293	11	60304	10
401	60314	11	60325	11	60336	11	60347	11	60358	11	60369	10	60379	11	60390	11	60401	11	60412	11
402	60423	10	60433	11	60444	11	60455	11	60466	11	60477	10	60487	11	60498	11	60509	11	60520	11
403	60531	10	60541	11	60552	11	60563	11	60574	10	60584	11	60595	11	60606	11	60617	10	60627	11
404	60638	11	60649	11	60660	10	60670	11	60681	11	60692	11	60703	10	60713	11	60724	11	60735	11
405	60746	10	60756	11	60767	11	60778	10	60788	11	60799	11	60810	11	60821	10	60831	11	60842	11
406	60853	10	60863	11	60874	11	60885	10	60895	11	60906	11	60917	10	60927	11	60938	11	60949	10
407	60959	11	60970	11	60981	10	60991	11	61002	11	61013	10	61023	11	61034	11	61045	10	61055	11
408	61066	11	61077	10	61087	11	61098	11	61109	10	61119	11	61130	10	61140	11	61151	11	61162	10
409	61172	11	61183	11	61194	10	61204	11	61215	10	61225	11	61236	11	61247	10	61257	11	61268	10
410	61278	11	61289	11	61300	10	61310	11	61321	10	61331	11	61342	10	61352	11	61363	11	61374	10
411	61384	11	61395	10	61405	11	61416	10	61426	11	61437	11	61448	10	61458	11	61469	10	61479	11
412	61490	10	61500	11	61511	10	61521	11	61532	10	61542	11	61553	10	61563	11	61574	10	61584	11
413	61595	11	61606	10	61616	11	61627	10	61637	11	61648	10	61658	11	61669	10	61679	11	61690	10
414	61700	11	61711	10	61721	10	61731	11	61742	10	61752	11	61763	10	61773	11	61784	10	61794	11
415	61805	10	61815	11	61826	10	61836	11	61847	10	61857	11	61868	10	61878	10	61888	11	61899	10
416	61909	11	61920	10	61930	11	61941	10	61951	11	61962	10	61972	10	61982	11	61993	10	62003	11
417	62014	10	62024	10	62034	11	62045	10	62055	11	62066	10	62076	10	62086	11	62097	10	62107	11
418	62118	10	62128	10	62138	11	62149	10	62159	11	62170	10	62180	10	62190	11	62201	10	62211	10
419	62221	11	62232	10	62242	10	62252	11	62263	10	62273	11	62284	10	62294	10	62304	11	62315	10
420	62325	10	62335	11	62346	10	62356	10	62366	11	62377	10	62387	10	62397	11	62408	10	62418	10
421	62428	11	62439	10	62449	10	62459	10	62469	11	62480	10	62490	10	62500	11	62511	10	62521	10
422	62531	11	62542	10	62552	10	62562	10	62572	11	62583	10	62593	10	62603	10	62613	11	62624	10
423	62634	10	62644	11	62655	10	62665	10	62675	10	62685	11	62696	10	62706	10	62716	10	62726	11
424	62737	10	62747	10	62757	10	62767	11	62778	10	62788	10	62798	10	62808	10	62818	11	62829	10
425	62839	10	62849	10	62859	11	62870	10	62880	10	62890	10	62900	10	62910	11	62921	10	62931	10
426	62941	10	62951	10	62961	11	62972	10	62982	10	62992	10	63002	10	63012	10	63022	11	63033	10
427	63043	10	63053	10	63063	10	63073	10	63083	11	63094	10	63104	10	63114	10	63124	10	63134	10
428	63144	11	63155	10	63165	10	63175	10	63185	10	63195	10	63205	10	63215	10	63225	11	63236	10
429	63246	10	63256	10	63266	10	63276	10	63286	10	63296	10	63306	11	63317	10	63327	10	63337	10
430	63347	10	63357	10	63367	10	63377	10	63387	10	63397	10	63407	10	63417	11	63428	10	63438	10
431	63448	10	63458	10	63468	10	63478	10	63488	10	63498	10	63508	10	63518	10	63528	10	63538	10
432	63548	10	63558	10	63568	11	63579	10	63589	10	63599	10	63609	10	63619	10	63629	10	63639	10
433	63649	10	63659	10	63669	10	63679	10	63689	10	63699	10	63709	10	63719	10	63729	10	63739	10
434	63749	10	63759	10	63769	10	63779	10	63789	10	63799	10	63809	10	63819	10	63829	10	63839	10
435	63849	10	63859	10	63869	10	63879	10	63889	10	63899	10	63909	10	63919	10	63929	10	63939	10
436	63949	10	63959	10	63969	10	63979	9	63988	10	63998	10	64008	10	64018	10	64028	10	64038	10
437	64048	10	64058	10	64068	10	64078	10	64088	10	64098	10	64108	10	64118	10	64128	9	64137	10
438	64147	10	64157	10	64167	10	64177	10	64187	10	64197	10	64207	10	64217	10	64227	10	64237	9
439	64246	10	64256	10	64266	10	64276	10	64286	10	64296	10	64306	10	64316	10	64326	9	64335	10
440	64345	10	64355	10	64365	10	64375	10	64385	10	64395	9	64404	10	64414	10	64424	10	64434	10
441	64444	10	64454	10	64464	9	64473	10	64483	10	64493	10	64503	10	64513	10	64523	9	64532	10
442	64542	10	64552	10	64562	10	64572	10	64582	9	64591	10	64601	10	64611	10	64621	10	64631	9
443	64640	10	64650	10	64660	10	64670	10	64680	9	64689	10	64699	10	64709	10	64719	10	64729	9
444	64738	10	64748	10	64758	10	64768	9	64777	10	64787	10	64797	10	64807	9	64816	10	64826	10
445	64836	10	64846	10	64856	9	64865	10	64875	10	64885	10	64895	9	64904	10	64914	10	64924	9
446	64933	10	64943	10	64953	10	64963	9	64972	10	64982	10	64992	10	65002	9	65011	10	65021	10
447	65031	9	65040	10	65050	10	65060	10	65070	9	65079	10	65089	10	65099	9	65108	10	65118	10
448	65128	9	65137	10	65147	10	65157	10	65167	9	65176	10	65186	10	65196	9	65205	10	65215	10
449	65225	9	65234	10	65244	10	65254	9	65263	10	65273	10	65283	9	65292	10	65302	10	65312	9
450	65321	10	65331	10	65341	9	65350	10	65360	9	65369	10	65379	10	65389	9	65398	10	65408	10
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	11
1	1
2	2
3	3
4	4
5	6
6	7
7	8
8	9
9	10

	10
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9

	9
1	1
2	2
3	3
4	4
5	4
6	5
7	6
8	7
9	8

TABLE 32

Logarithms of Numbers

4500–5000

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
450	65321	10	65331	10	65341	9	65350	10	65360	9	65369	10	65379	10	65389	9	65398	10	65408	10
451	65418	9	65427	10	65437	10	65447	9	65456	10	65466	9	65475	10	65485	10	65495	9	65504	10
452	65514	9	65523	10	65533	10	65543	9	65552	10	65562	9	65571	10	65581	10	65591	9	65600	10
453	65610	9	65619	10	65629	10	65639	9	65648	10	65658	9	65667	10	65677	9	65686	10	65696	10
454	65706	9	65715	10	65725	9	65734	10	65744	9	65753	10	65763	9	65772	10	65782	10	65792	9
455	65801	10	65811	9	65820	10	65830	9	65839	10	65849	9	65858	10	65868	9	65877	10	65887	9
456	65896	10	65906	10	65916	9	65925	10	65935	9	65944	10	65954	9	65963	10	65973	9	65982	10
457	65992	9	66001	10	66011	9	66020	10	66030	9	66039	10	66049	9	66058	10	66068	9	66077	10
458	66087	9	66096	10	66106	9	66115	9	66124	10	66134	9	66143	10	66153	9	66162	10	66172	9
459	66181	10	66191	9	66200	10	66210	9	66219	10	66229	9	66238	9	66247	10	66257	9	66266	10
460	66276	9	66285	10	66295	9	66304	10	66314	9	66323	9	66332	10	66342	9	66351	10	66361	9
461	66370	10	66380	9	66389	9	66398	10	66408	9	66417	10	66427	9	66436	9	66445	10	66455	9
462	66464	10	66474	9	66483	9	66492	10	66502	9	66511	10	66521	9	66530	9	66539	10	66549	9
463	66558	9	66567	10	66577	9	66586	10	66596	9	66605	9	66614	10	66624	9	66633	9	66642	10
464	66652	9	66661	10	66671	9	66680	9	66689	10	66699	9	66708	9	66717	10	66727	9	66736	9
465	66745	10	66755	9	66764	9	66773	10	66783	9	66792	9	66801	10	66811	9	66820	9	66829	10
466	66839	9	66848	9	66857	10	66867	9	66876	9	66885	9	66894	10	66904	9	66913	9	66922	10
467	66932	9	66941	9	66950	10	66960	9	66969	9	66978	9	66987	10	66997	9	67006	9	67015	10
468	67025	9	67034	9	67043	9	67052	10	67062	9	67071	9	67080	9	67089	10	67099	9	67108	9
469	67117	10	67127	9	67136	9	67145	9	67154	10	67164	9	67173	9	67182	9	67191	10	67201	9
470	67210	9	67219	9	67228	9	67237	10	67247	9	67256	9	67265	9	67274	10	67284	9	67293	9
471	67302	9	67311	10	67321	9	67330	9	67339	9	67348	9	67357	10	67367	9	67376	9	67385	9
472	67394	9	67403	10	67413	9	67422	9	67431	9	67440	9	67449	10	67459	9	67468	9	67477	9
473	67486	9	67495	9	67504	10	67514	9	67523	9	67532	9	67541	9	67550	10	67560	9	67569	9
474	67578	9	67587	9	67596	9	67605	9	67614	10	67624	9	67633	9	67642	9	67651	9	67660	9
475	67669	10	67679	9	67688	9	67697	9	67706	9	67715	9	67724	9	67733	9	67742	10	67752	9
476	67761	9	67770	9	67779	9	67788	9	67797	9	67806	9	67815	10	67825	9	67834	9	67843	9
477	67852	9	67861	9	67870	9	67879	9	67888	9	67897	9	67906	10	67916	9	67925	9	67934	9
478	67943	9	67952	9	67961	9	67970	9	67979	9	67988	9	67997	9	68006	9	68015	9	68024	10
479	68034	9	68043	9	68052	9	68061	9	68070	9	68079	9	68088	9	68097	9	68106	9	68115	9
480	68124	9	68133	9	68142	9	68151	9	68160	9	68169	9	68178	9	68187	9	68196	9	68205	10
481	68215	9	68224	9	68233	9	68242	9	68251	9	68260	9	68269	9	68278	9	68287	9	68296	9
482	68305	9	68314	9	68323	9	68332	9	68341	9	68350	9	68359	9	68368	9	68377	9	68386	9
483	68395	9	68404	9	68413	9	68422	9	68431	9	68440	9	68449	9	68458	9	68467	9	68476	9
484	68485	9	68494	8	68502	9	68511	9	68520	9	68529	9	68538	9	68547	9	68556	9	68565	9
485	68574	9	68583	9	68592	9	68601	9	68610	9	68619	9	68628	9	68637	9	68646	9	68655	9
486	68664	9	68673	8	68681	9	68690	9	68699	9	68708	9	68717	9	68726	9	68735	9	68744	9
487	68753	9	68762	9	68771	9	68780	9	68789	8	68797	9	68806	9	68815	9	68824	9	68833	9
488	68842	9	68851	9	68860	9	68869	9	68878	8	68886	9	68895	9	68904	9	68913	9	68922	9
489	68931	9	68940	9	68949	9	68958	8	68966	9	68975	9	68984	9	68993	9	69002	9	69011	9
490	69020	8	69028	9	69037	9	69046	9	69055	9	69064	9	69073	9	69082	8	69090	9	69099	9
491	69108	9	69117	9	69126	9	69135	9	69144	8	69152	9	69161	9	69170	9	69179	9	69188	9
492	69197	8	69205	9	69214	9	69223	9	69232	9	69241	8	69249	9	69258	9	69267	9	69276	9
493	69285	9	69294	8	69302	9	69311	9	69320	9	69329	9	69338	8	69346	9	69355	9	69364	9
494	69373	8	69381	9	69390	9	69399	9	69408	9	69417	8	69425	9	69434	9	69443	9	69452	9
495	69461	8	69469	9	69478	9	69487	9	69496	8	69504	9	69513	9	69522	9	69531	8	69539	9
496	69548	9	69557	9	69566	8	69574	9	69583	9	69592	9	69601	8	69609	9	69618	9	69627	9
497	69636	8	69644	9	69653	9	69662	9	69671	8	69679	9	69688	9	69697	8	69705	9	69714	9
498	69723	9	69732	8	69740	9	69749	9	69758	9	69767	8	69775	9	69784	9	69793	8	69801	9
499	69810	9	69819	8	69827	9	69836	9	69845	9	69854	8	69862	9	69871	9	69880	8	69888	9
500	69897	9	69906	8	69914	9	69923	9	69932	8	69940	9	69949	9	69958	8	69966	9	69975	9
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	10	9	8
1	1	1	1
2	2	2	2
3	3	3	2
4	4	4	3
5	5	4	4
6	6	5	5
7	7	6	6
8	8	7	6
9	9	8	7

TABLE 32

Logarithms of Numbers

5000–5500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
500	69897	9	69906	8	69914	9	69923	9	69932	8	69940	9	69949	9	69958	8	69966	9	69975	9
501	69984	8	69992	9	70001	9	70010	8	70018	9	70027	9	70036	8	70044	9	70053	9	70062	8
502	70070	9	70079	9	70088	8	70096	9	70105	9	70114	8	70122	9	70131	9	70140	8	70148	9
503	70157	8	70165	9	70174	9	70183	8	70191	9	70200	9	70209	8	70217	9	70226	8	70234	9
504	70243	9	70252	8	70260	9	70269	9	70278	8	70286	9	70295	8	70303	9	70312	9	70321	8
505	70329	9	70338	8	70346	9	70355	9	70364	8	70372	9	70381	8	70389	9	70398	8	70406	9
506	70415	9	70424	8	70432	9	70441	8	70449	9	70458	9	70467	8	70475	9	70484	8	70492	9
507	70501	8	70509	9	70518	8	70526	9	70535	9	70544	8	70552	9	70561	8	70569	9	70578	8
508	70586	9	70595	8	70603	9	70612	9	70621	8	70629	9	70638	8	70646	9	70655	9	70663	9
509	70672	8	70680	9	70689	8	70697	9	70706	8	70714	9	70723	8	70731	9	70740	9	70749	8
510	70757	9	70766	8	70774	9	70783	8	70791	9	70800	8	70808	9	70817	8	70825	9	70834	8
511	70842	9	70851	8	70859	9	70868	8	70876	9	70885	8	70893	9	70902	8	70910	9	70919	8
512	70927	8	70935	9	70944	8	70952	9	70961	8	70969	9	70978	8	70986	9	70995	8	71003	9
513	71012	8	71020	9	71029	8	71037	9	71046	8	71054	9	71063	8	71071	8	71079	9	71088	8
514	71096	9	71105	8	71113	9	71122	8	71130	9	71139	8	71147	8	71155	9	71164	8	71172	9
515	71181	8	71189	9	71198	8	71206	8	71214	9	71223	8	71231	9	71240	8	71248	9	71257	8
516	71265	8	71273	9	71282	8	71290	9	71299	8	71307	8	71315	9	71324	8	71332	9	71341	8
517	71349	8	71357	9	71366	8	71374	9	71383	8	71391	8	71399	9	71408	8	71416	9	71425	8
518	71433	8	71441	9	71450	8	71458	8	71466	9	71475	8	71483	9	71492	8	71500	8	71508	9
519	71517	8	71525	8	71533	9	71542	8	71550	9	71559	8	71567	8	71575	9	71584	8	71592	8
520	71600	9	71609	8	71617	8	71625	9	71634	8	71642	8	71650	9	71659	8	71667	8	71675	9
521	71684	8	71692	8	71700	9	71709	8	71717	8	71725	9	71734	8	71742	8	71750	9	71759	8
522	71767	8	71775	9	71784	8	71792	8	71800	9	71809	8	71817	8	71825	9	71834	8	71842	8
523	71850	8	71858	9	71867	8	71875	8	71883	9	71892	8	71900	8	71908	9	71917	8	71925	8
524	71933	8	71941	9	71950	8	71958	8	71966	9	71975	8	71983	8	71991	8	71999	9	72008	8
525	72016	8	72024	8	72032	9	72041	8	72049	8	72057	9	72066	8	72074	8	72082	8	72090	9
526	72099	8	72107	8	72115	8	72123	9	72132	8	72140	8	72148	8	72156	9	72165	8	72173	8
527	72181	8	72189	9	72198	8	72206	8	72214	8	72222	8	72230	9	72239	8	72247	8	72255	8
528	72263	9	72272	8	72280	8	72288	8	72296	8	72304	9	72313	8	72321	8	72329	8	72337	9
529	72346	8	72354	8	72362	8	72370	8	72378	9	72387	8	72395	8	72403	8	72411	8	72419	9
530	72428	8	72436	8	72444	8	72452	8	72460	9	72469	8	72477	8	72485	8	72493	8	72501	8
531	72509	9	72518	8	72526	8	72534	8	72542	8	72550	8	72558	9	72567	8	72575	8	72583	8
532	72591	8	72599	8	72607	9	72616	8	72624	8	72632	8	72640	8	72648	8	72656	9	72665	8
533	72673	8	72681	8	72689	8	72697	8	72705	8	72713	9	72722	8	72730	8	72738	8	72746	8
534	72754	8	72762	8	72770	9	72779	8	72787	8	72795	8	72803	8	72811	8	72819	8	72827	8
535	72835	8	72843	9	72852	8	72860	8	72868	8	72876	8	72884	8	72892	8	72900	8	72908	8
536	72916	9	72925	8	72933	8	72941	8	72949	8	72957	8	72965	8	72973	8	72981	8	72989	8
537	72997	9	73006	8	73014	8	73022	8	73030	8	73038	8	73046	8	73054	8	73062	8	73070	8
538	73078	8	73086	8	73094	8	73102	9	73111	8	73119	8	73127	8	73135	8	73143	8	73151	8
539	73159	8	73167	8	73175	8	73183	8	73191	8	73199	8	73207	8	73215	8	73223	8	73231	8
540	73239	8	73247	8	73255	8	73263	9	73272	8	73280	8	73288	8	73296	8	73304	8	73312	8
541	73320	8	73328	8	73336	8	73344	8	73352	8	73360	8	73368	8	73376	8	73384	8	73392	8
542	73400	8	73408	8	73416	8	73424	8	73432	8	73440	8	73448	8	73456	8	73464	8	73472	8
543	73480	8	73488	8	73496	8	73504	8	73512	8	73520	8	73528	8	73536	8	73544	8	73552	8
544	73560	8	73568	8	73576	8	73584	8	73592	8	73600	8	73608	8	73616	8	73624	8	73632	8
545	73640	8	73648	8	73656	8	73664	8	73672	7	73679	8	73687	8	73695	8	73703	8	73711	8
546	73719	8	73727	8	73735	8	73743	8	73751	8	73759	8	73767	8	73775	8	73783	8	73791	8
547	73799	8	73807	8	73815	8	73823	7	73830	8	73838	8	73846	8	73854	8	73862	8	73870	8
548	73878	8	73886	8	73894	8	73902	8	73910	8	73918	8	73926	7	73933	8	73941	8	73949	8
549	73957	8	73965	8	73973	8	73981	8	73989	8	73997	8	74005	8	74013	7	74020	8	74028	8
550	74036	8	74044	8	74052	8	74060	8	74068	8	74076	8	74084	8	74092	7	74099	8	74107	8
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	9	8	7
1	1	1	1
2	2	2	1
3	3	2	2
4	4	3	3
5	4	4	4
6	5	5	4
7	6	6	5
8	7	6	6
9	8	7	6

TABLE 32

Logarithms of Numbers

5500–6000

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
550	74036	8	74044	8	74052	8	74060	8	74068	8	74076	8	74084	8	74092	7	74099	8	74107	8
551	74115	8	74123	8	74131	8	74139	8	74147	8	74155	7	74162	8	74170	8	74178	8	74186	8
552	74194	8	74202	8	74210	8	74218	7	74225	8	74233	8	74241	8	74249	8	74257	8	74265	8
553	74273	7	74280	8	74288	8	74296	8	74304	8	74312	8	74320	7	74327	8	74335	8	74343	8
554	74351	8	74359	8	74367	7	74374	8	74382	8	74390	8	74398	8	74406	8	74414	7	74421	8
555	74429	8	74437	8	74445	8	74453	8	74461	7	74468	8	74476	8	74484	8	74492	8	74500	7
556	74507	8	74515	8	74523	8	74531	8	74539	8	74547	7	74554	8	74562	8	74570	8	74578	8
557	74586	7	74593	8	74601	8	74609	8	74617	7	74624	8	74632	8	74640	8	74648	8	74656	7
558	74663	8	74671	8	74679	8	74687	8	74695	7	74702	8	74710	8	74718	8	74726	7	74733	8
559	74741	8	74749	8	74757	7	74764	8	74772	8	74780	8	74788	8	74796	7	74803	8	74811	8
560	74819	8	74827	7	74834	8	74842	8	74850	8	74858	7	74865	8	74873	8	74881	8	74889	7
561	74896	8	74904	8	74912	8	74920	7	74927	8	74935	8	74943	7	74950	8	74958	8	74966	8
562	74974	7	74981	8	74989	8	74997	8	75005	7	75012	8	75020	8	75028	7	75035	8	75043	8
563	75051	8	75059	7	75066	8	75074	8	75082	7	75089	8	75097	8	75105	8	75113	7	75120	8
564	75128	8	75136	7	75143	8	75151	8	75159	7	75166	8	75174	8	75182	7	75189	8	75197	8
565	75205	8	75213	7	75220	8	75228	8	75236	7	75243	8	75251	8	75259	7	75266	8	75274	8
566	75282	7	75289	8	75297	8	75305	7	75312	8	75320	8	75328	7	75335	8	75343	8	75351	7
567	75358	8	75366	8	75374	7	75381	8	75389	8	75397	7	75404	8	75412	8	75420	7	75427	8
568	75435	7	75442	8	75450	8	75458	7	75465	8	75473	8	75481	7	75488	8	75496	8	75504	7
569	75511	8	75519	7	75526	8	75534	8	75542	7	75549	8	75557	8	75565	7	75572	8	75580	7
570	75587	8	75595	8	75603	7	75610	8	75618	8	75626	7	75633	8	75641	7	75648	8	75656	8
571	75664	7	75671	8	75679	7	75686	8	75694	8	75702	7	75709	8	75717	7	75724	8	75732	8
572	75740	7	75747	8	75755	7	75762	8	75770	8	75778	7	75785	8	75793	7	75800	8	75808	7
573	75815	8	75823	8	75831	7	75838	8	75846	7	75853	8	75861	7	75868	8	75876	8	75884	7
574	75891	8	75899	7	75906	8	75914	7	75921	8	75929	8	75937	7	75944	8	75952	7	75959	8
575	75967	7	75974	8	75982	7	75989	8	75997	8	76005	7	76012	8	76020	7	76027	8	76035	7
576	76042	8	76050	7	76057	8	76065	7	76072	8	76080	7	76087	8	76095	8	76103	7	76110	8
577	76118	7	76125	8	76133	7	76140	8	76148	7	76155	8	76163	7	76170	8	76178	7	76185	8
578	76193	7	76200	8	76208	7	76215	8	76223	7	76230	8	76238	7	76245	8	76253	7	76260	8
579	76268	7	76275	8	76283	7	76290	8	76298	7	76305	8	76313	7	76320	8	76328	7	76335	8
580	76343	7	76350	8	76358	7	76365	8	76373	7	76380	8	76388	7	76395	8	76403	7	76410	8
581	76418	7	76425	8	76433	7	76440	8	76448	7	76455	7	76462	8	76470	7	76477	8	76485	7
582	76492	8	76500	7	76507	8	76515	7	76522	8	76530	7	76537	8	76545	7	76552	7	76559	8
583	76567	7	76574	8	76582	7	76589	8	76597	7	76604	8	76612	7	76619	7	76626	8	76634	7
584	76641	8	76649	7	76656	8	76664	7	76671	7	76678	8	76686	7	76693	8	76701	7	76708	8
585	76716	7	76723	7	76730	8	76738	7	76745	8	76753	7	76760	8	76768	7	76775	7	76782	8
586	76790	7	76797	8	76805	7	76812	7	76819	8	76827	7	76834	8	76842	7	76849	7	76856	8
587	76864	7	76871	8	76879	7	76886	7	76893	8	76901	7	76908	8	76916	7	76923	7	76930	8
588	76938	7	76945	8	76953	7	76960	7	76967	8	76975	7	76982	7	76989	8	76997	7	77004	8
589	77012	7	77019	7	77026	8	77034	7	77041	7	77048	8	77056	7	77063	7	77070	8	77078	7
590	77085	8	77093	7	77100	7	77107	8	77115	7	77122	7	77129	8	77137	7	77144	7	77151	8
591	77159	7	77166	7	77173	8	77181	7	77188	7	77195	8	77203	7	77210	7	77217	8	77225	7
592	77232	8	77240	7	77247	7	77254	8	77262	7	77269	7	77276	7	77283	8	77291	7	77298	7
593	77305	8	77313	7	77320	7	77327	8	77335	7	77342	7	77349	8	77357	7	77364	7	77371	8
594	77379	7	77386	7	77393	8	77401	7	77408	7	77415	7	77422	8	77430	7	77437	7	77444	8
595	77452	7	77459	7	77466	8	77474	7	77481	7	77488	7	77495	8	77503	7	77510	7	77517	8
596	77525	7	77532	7	77539	7	77546	8	77554	7	77561	7	77568	8	77576	7	77583	7	77590	7
597	77597	8	77605	7	77612	7	77619	8	77627	7	77634	7	77641	7	77648	8	77656	7	77663	7
598	77670	7	77677	8	77685	7	77692	7	77699	7	77706	8	77714	7	77721	7	77728	7	77735	8
599	77743	7	77750	7	77757	7	77764	8	77772	7	77779	7	77786	7	77793	8	77801	7	77808	7
600	77815	7	77822	8	77830	7	77837	7	77844	7	77851	8	77859	7	77866	7	77873	7	77880	7
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	8
1	1
2	2
3	2
4	3
5	4
6	5
7	6
8	6
9	7

	7
1	1
2	1
3	2
4	3
5	4
6	4
7	5
8	6
9	6

TABLE 32

Logarithms of Numbers

6000–6500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
600	77815	7	77822	8	77830	7	77837	7	77844	7	77851	8	77859	7	77866	7	77873	7	77880	7
601	77887	8	77895	7	77902	7	77909	7	77916	8	77924	7	77931	7	77938	7	77945	7	77952	8
602	77960	7	77967	7	77974	7	77981	7	77988	8	77996	7	78003	7	78010	7	78017	8	78025	7
603	78032	7	78039	7	78046	7	78053	8	78061	7	78068	7	78075	7	78082	7	78089	8	78097	7
604	78104	7	78111	7	78118	7	78125	7	78132	8	78140	7	78147	7	78154	7	78161	7	78168	8
605	78176	7	78183	7	78190	7	78197	7	78204	7	78211	8	78219	7	78226	7	78233	7	78240	7
606	78247	7	78254	8	78262	7	78269	7	78276	7	78283	7	78290	7	78297	8	78305	7	78312	7
607	78319	7	78326	7	78333	7	78340	7	78347	8	78355	7	78362	7	78369	7	78376	7	78383	7
608	78390	8	78398	7	78405	7	78412	7	78419	7	78426	7	78433	7	78440	7	78447	8	78455	7
609	78462	7	78469	7	78476	7	78483	7	78490	7	78497	7	78504	8	78512	7	78519	7	78526	7
610	78533	7	78540	7	78547	7	78554	7	78561	8	78569	7	78576	7	78583	7	78590	7	78597	7
611	78604	7	78611	7	78618	7	78625	8	78633	7	78640	7	78647	7	78654	7	78661	7	78668	7
612	78675	7	78682	7	78689	7	78696	8	78704	7	78711	7	78718	7	78725	7	78732	7	78739	7
613	78746	7	78753	7	78760	7	78767	7	78774	7	78781	8	78789	7	78796	7	78803	7	78810	7
614	78817	7	78824	7	78831	7	78838	7	78845	7	78852	7	78859	7	78866	7	78873	7	78880	8
615	78888	7	78895	7	78902	7	78909	7	78916	7	78923	7	78930	7	78937	7	78944	7	78951	7
616	78958	7	78965	7	78972	7	78979	7	78986	7	78993	7	79000	7	79007	7	79014	7	79021	8
617	79029	7	79036	7	79043	7	79050	7	79057	7	79064	7	79071	7	79078	7	79085	7	79092	7
618	79099	7	79106	7	79113	7	79120	7	79127	7	79134	7	79141	7	79148	7	79155	7	79162	7
619	79169	7	79176	7	79183	7	79190	7	79197	7	79204	7	79211	7	79218	7	79225	7	79232	7
620	79239	7	79246	7	79253	7	79260	7	79267	7	79274	7	79281	7	79288	7	79295	7	79302	7
621	79309	7	79316	7	79323	7	79330	7	79337	7	79344	7	79351	7	79358	7	79365	7	79372	7
622	79379	7	79386	7	79393	7	79400	7	79407	7	79414	7	79421	7	79428	7	79435	7	79442	7
623	79449	7	79456	7	79463	7	79470	7	79477	7	79484	7	79491	7	79498	7	79505	6	79511	7
624	79518	7	79525	7	79532	7	79539	7	79546	7	79553	7	79560	7	79567	7	79574	7	79581	7
625	79588	7	79595	7	79602	7	79609	7	79616	7	79623	7	79630	7	79637	7	79644	6	79650	7
626	79657	7	79664	7	79671	7	79678	7	79685	7	79692	7	79699	7	79706	7	79713	7	79720	7
627	79727	7	79734	7	79741	7	79748	6	79754	7	79761	7	79768	7	79775	7	79782	7	79789	7
628	79796	7	79803	7	79810	7	79817	7	79824	7	79831	6	79837	7	79844	7	79851	7	79858	7
629	79865	7	79872	7	79879	7	79886	7	79893	7	79900	6	79906	7	79913	7	79920	7	79927	7
630	79934	7	79941	7	79948	7	79955	7	79962	7	79969	6	79975	7	79982	7	79989	7	79996	7
631	80003	7	80010	7	80017	7	80024	6	80030	7	80037	7	80044	7	80051	7	80058	7	80065	7
632	80072	7	80079	6	80085	7	80092	7	80099	7	80106	7	80113	7	80120	7	80127	7	80134	6
633	80140	7	80147	7	80154	7	80161	7	80168	7	80175	7	80182	6	80188	7	80195	7	80202	7
634	80209	7	80216	7	80223	6	80229	7	80236	7	80243	7	80250	7	80257	7	80264	7	80271	6
635	80277	7	80284	7	80291	7	80298	7	80305	7	80312	6	80318	7	80325	7	80332	7	80339	7
636	80346	7	80353	6	80359	7	80366	7	80373	7	80380	7	80387	6	80393	7	80400	7	80407	7
637	80414	7	80421	7	80428	6	80434	7	80441	7	80448	7	80455	7	80462	6	80468	7	80475	7
638	80482	7	80489	7	80496	6	80502	7	80509	7	80516	7	80523	7	80530	6	80536	7	80543	7
639	80550	7	80557	7	80564	6	80570	7	80577	7	80584	7	80591	7	80598	6	80604	7	80611	7
640	80618	7	80625	7	80632	6	80638	7	80645	7	80652	7	80659	6	80665	7	80672	7	80679	7
641	80686	7	80693	6	80699	7	80706	7	80713	7	80720	6	80726	7	80733	7	80740	7	80747	7
642	80754	6	80760	7	80767	7	80774	7	80781	6	80787	7	80794	7	80801	7	80808	6	80814	7
643	80821	7	80828	7	80835	6	80841	7	80848	7	80855	7	80862	6	80868	7	80875	7	80882	7
644	80889	6	80895	7	80902	7	80909	7	80916	6	80922	7	80929	7	80936	7	80943	6	80949	7
645	80956	7	80963	6	80969	7	80976	7	80983	7	80990	6	80996	7	81003	7	81010	7	81017	6
646	81023	7	81030	7	81037	6	81043	7	81050	7	81057	7	81064	6	81070	7	81077	7	81084	6
647	81090	7	81097	7	81104	7	81111	6	81117	7	81124	7	81131	6	81137	7	81144	7	81151	7
648	81158	6	81164	7	81171	7	81178	6	81184	7	81191	7	81198	6	81204	7	81211	7	81218	6
649	81224	7	81231	7	81238	7	81245	6	81251	7	81258	7	81265	6	81271	7	81278	7	81285	6
650	81291	7	81298	7	81305	6	81311	7	81318	7	81325	6	81331	7	81338	7	81345	6	81351	7
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	8
1	1
2	2
3	2
4	3
5	4
6	5
7	6
8	6
9	7

	7
1	1
2	1
3	2
4	3
5	4
6	4
7	5
8	6
9	6

	6
1	1
2	1
3	2
4	2
5	3
6	4
7	4
8	5
9	5

TABLE 32
Logarithms of Numbers

6500–7000

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
650	81291	7	81298	7	81305	6	81311	7	81318	7	81325	6	81331	7	81338	7	81345	6	81351	7
651	81358	7	81365	6	81371	7	81378	7	81385	6	81391	7	81398	7	81405	6	81411	7	81418	7
652	81425	6	81431	7	81438	7	81445	6	81451	7	81458	7	81465	6	81471	7	81478	7	81485	6
653	81491	7	81498	7	81505	6	81511	7	81518	7	81525	6	81531	7	81538	6	81544	7	81551	7
654	81558	6	81564	7	81571	7	81578	6	81584	7	81591	7	81598	6	81604	7	81611	6	81617	7
655	81624	7	81631	6	81637	7	81644	7	81651	6	81657	7	81664	7	81671	6	81677	7	81684	6
656	81690	7	81697	7	81704	6	81710	7	81717	6	81723	7	81730	7	81737	6	81743	7	81750	7
657	81757	6	81763	7	81770	6	81776	7	81783	7	81790	6	81796	7	81803	6	81809	7	81816	7
658	81823	6	81829	7	81836	6	81842	7	81849	7	81856	6	81862	7	81869	6	81875	7	81882	7
659	81889	6	81895	7	81902	6	81908	7	81915	6	81921	7	81928	7	81935	6	81941	7	81948	6
660	81954	7	81961	7	81968	6	81974	7	81981	6	81987	7	81994	6	82000	7	82007	7	82014	6
661	82020	7	82027	6	82033	7	82040	6	82046	7	82053	7	82060	6	82066	7	82073	6	82079	7
662	82086	6	82092	7	82099	6	82105	7	82112	7	82119	6	82125	7	82132	6	82138	7	82145	6
663	82151	7	82158	6	82164	7	82171	7	82178	6	82184	7	82191	6	82197	7	82204	6	82210	7
664	82217	6	82223	7	82230	6	82236	7	82243	6	82249	7	82256	7	82263	6	82269	7	82276	6
665	82282	7	82289	6	82295	7	82302	6	82308	7	82315	6	82321	7	82328	6	82334	7	82341	6
666	82347	7	82354	6	82360	7	82367	6	82373	7	82380	7	82387	6	82393	7	82400	6	82406	7
667	82413	6	82419	7	82426	6	82432	7	82439	6	82445	7	82452	6	82458	7	82465	6	82471	7
668	82478	6	82484	7	82491	6	82497	7	82504	6	82510	7	82517	6	82523	7	82530	6	82536	7
669	82543	6	82549	7	82556	6	82562	7	82569	6	82575	7	82582	6	82588	7	82595	6	82601	6
670	82607	7	82614	6	82620	7	82627	6	82633	7	82640	6	82646	7	82653	6	82659	7	82666	6
671	82672	7	82679	6	82685	7	82692	6	82698	7	82705	6	82711	7	82718	6	82724	6	82730	7
672	82737	6	82743	7	82750	6	82756	7	82763	6	82769	7	82776	6	82782	7	82789	6	82795	7
673	82802	6	82808	6	82814	7	82821	6	82827	7	82834	6	82840	7	82847	6	82853	7	82860	6
674	82866	6	82872	7	82879	6	82885	7	82892	6	82898	7	82905	6	82911	7	82918	6	82924	6
675	82930	7	82937	6	82943	7	82950	6	82956	7	82963	6	82969	6	82975	7	82982	6	82988	7
676	82995	6	83001	7	83008	6	83014	6	83020	7	83027	6	83033	7	83040	6	83046	6	83052	7
677	83059	6	83065	7	83072	6	83078	7	83085	6	83091	6	83097	7	83104	6	83110	7	83117	6
678	83123	6	83129	7	83136	6	83142	7	83149	6	83155	6	83161	7	83168	6	83174	7	83181	6
679	83187	6	83193	7	83200	6	83206	7	83213	6	83219	6	83225	7	83232	6	83238	7	83245	6
680	83251	6	83257	7	83264	6	83270	6	83276	7	83283	6	83289	7	83296	6	83302	6	83308	7
681	83315	6	83321	6	83327	7	83334	6	83340	7	83347	6	83353	6	83359	7	83366	6	83372	6
682	83378	7	83385	6	83391	7	83398	6	83404	6	83410	7	83417	6	83423	6	83429	7	83436	6
683	83442	6	83448	7	83455	6	83461	6	83467	7	83474	6	83480	7	83487	6	83493	6	83499	7
684	83506	6	83512	6	83518	7	83525	6	83531	6	83537	7	83544	6	83550	6	83556	7	83563	6
685	83569	6	83575	7	83582	6	83588	6	83594	7	83601	6	83607	6	83613	7	83620	6	83626	6
686	83632	7	83639	6	83645	6	83651	7	83658	6	83664	6	83670	7	83677	6	83683	6	83689	7
687	83696	6	83702	6	83708	7	83715	6	83721	6	83727	7	83734	6	83740	6	83746	7	83753	6
688	83759	6	83765	6	83771	7	83778	6	83784	6	83790	7	83797	6	83803	6	83809	7	83816	6
689	83822	6	83828	7	83835	6	83841	6	83847	6	83853	7	83860	6	83866	6	83872	7	83879	6
690	83885	6	83891	6	83897	7	83904	6	83910	6	83916	7	83923	6	83929	6	83935	7	83942	6
691	83948	6	83954	6	83960	7	83967	6	83973	6	83979	6	83985	7	83992	6	83998	6	84004	7
692	84011	6	84017	6	84023	6	84029	7	84036	6	84042	6	84048	7	84055	6	84061	6	84067	6
693	84073	7	84080	6	84086	6	84092	6	84098	7	84105	6	84111	6	84117	6	84123	7	84130	6
694	84136	6	84142	6	84148	7	84155	6	84161	6	84167	6	84173	7	84180	6	84186	6	84192	6
695	84198	7	84205	6	84211	6	84217	6	84223	7	84230	6	84236	6	84242	6	84248	7	84255	6
696	84261	6	84267	6	84273	7	84280	6	84286	6	84292	6	84298	7	84305	6	84311	6	84317	6
697	84323	7	84330	6	84336	6	84342	6	84348	6	84354	7	84361	6	84367	6	84373	6	84379	7
698	84386	6	84392	6	84398	6	84404	6	84410	7	84417	6	84423	6	84429	6	84435	7	84442	6
699	84448	6	84454	6	84460	6	84466	7	84473	6	84479	6	84485	6	84491	6	84497	7	84504	6
700	84510	6	84516	6	84522	6	84528	7	84535	6	84541	6	84547	6	84553	6	84559	7	84566	6
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	7
1	1
2	1
3	2
4	3
5	4
6	4
7	5
8	6
9	6

	6
1	1
2	1
3	2
4	2
5	3
6	4
7	4
8	5
9	5

TABLE 32

Logarithms of Numbers

7000–7500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
700	84510	6	84516	6	84522	6	84528	7	84535	6	84541	6	84547	6	84553	6	84559	7	84566	6
701	84572	6	84578	6	84584	6	84590	7	84597	6	84603	6	84609	6	84615	6	84621	7	84628	6
702	84634	6	84640	6	84646	6	84652	6	84658	7	84665	6	84671	6	84677	6	84683	6	84689	7
703	84696	6	84702	6	84708	6	84714	6	84720	6	84726	7	84733	6	84739	6	84745	6	84751	6
704	84757	6	84763	7	84770	6	84776	6	84782	6	84788	6	84794	6	84800	7	84807	6	84813	6
705	84819	6	84825	6	84831	6	84837	7	84844	6	84850	6	84856	6	84862	6	84868	6	84874	6
706	84880	7	84887	6	84893	6	84899	6	84905	6	84911	6	84917	7	84924	6	84930	6	84936	6
707	84942	6	84948	6	84954	6	84960	7	84967	6	84973	6	84979	6	84985	6	84991	6	84997	6
708	85003	6	85009	7	85016	6	85022	6	85028	6	85034	6	85040	6	85046	6	85052	6	85058	7
709	85065	6	85071	6	85077	6	85083	6	85089	6	85095	6	85101	6	85107	7	85114	6	85120	6
710	85126	6	85132	6	85138	6	85144	6	85150	6	85156	7	85163	6	85169	6	85175	6	85181	6
711	85187	6	85193	6	85199	6	85205	6	85211	6	85217	7	85224	6	85230	6	85236	6	85242	6
712	85248	6	85254	6	85260	6	85266	6	85272	6	85278	7	85285	6	85291	6	85297	6	85303	6
713	85309	6	85315	6	85321	6	85327	6	85333	6	85339	6	85345	7	85352	6	85358	6	85364	6
714	85370	6	85376	6	85382	6	85388	6	85394	6	85400	6	85406	6	85412	6	85418	7	85425	6
715	85431	6	85437	6	85443	6	85449	6	85455	6	85461	6	85467	6	85473	6	85479	6	85485	6
716	85491	6	85497	6	85503	6	85509	7	85516	6	85522	6	85528	6	85534	6	85540	6	85546	6
717	85552	6	85558	6	85564	6	85570	6	85576	6	85582	6	85588	6	85594	6	85600	6	85606	6
718	85612	6	85618	7	85625	6	85631	6	85637	6	85643	6	85649	6	85655	6	85661	6	85667	6
719	85673	6	85679	6	85685	6	85691	6	85697	6	85703	6	85709	6	85715	6	85721	6	85727	6
720	85733	6	85739	6	85745	6	85751	6	85757	6	85763	6	85769	6	85775	6	85781	7	85788	6
721	85794	6	85800	6	85806	6	85812	6	85818	6	85824	6	85830	6	85836	6	85842	6	85848	6
722	85854	6	85860	6	85866	6	85872	6	85878	6	85884	6	85890	6	85896	6	85902	6	85908	6
723	85914	6	85920	6	85926	6	85932	6	85938	6	85944	6	85950	6	85956	6	85962	6	85968	6
724	85974	6	85980	6	85986	6	85992	6	85998	6	86004	6	86010	6	86016	6	86022	6	86028	6
725	86034	6	86040	6	86046	6	86052	6	86058	6	86064	6	86070	6	86076	6	86082	6	86088	6
726	86094	6	86100	6	86106	6	86112	6	86118	6	86124	6	86130	6	86136	5	86141	6	86147	6
727	86153	6	86159	6	86165	6	86171	6	86177	6	86183	6	86189	6	86195	6	86201	6	86207	6
728	86213	6	86219	6	86225	6	86231	6	86237	6	86243	6	86249	6	86255	6	86261	6	86267	6
729	86273	6	86279	6	86285	6	86291	6	86297	6	86303	5	86308	6	86314	6	86320	6	86326	6
730	86332	6	86338	6	86344	6	86350	6	86356	6	86362	6	86368	6	86374	6	86380	6	86386	6
731	86392	6	86398	6	86404	6	86410	5	86415	6	86421	6	86427	6	86433	6	86439	6	86445	6
732	86451	6	86457	6	86463	6	86469	6	86475	6	86481	6	86487	6	86493	6	86499	5	86504	6
733	86510	6	86516	6	86522	6	86528	6	86534	6	86540	6	86546	6	86552	6	86558	6	86564	6
734	86570	6	86576	5	86581	6	86587	6	86593	6	86599	6	86605	6	86611	6	86617	6	86623	6
735	86629	6	86635	6	86641	5	86646	6	86652	6	86658	6	86664	6	86670	6	86676	6	86682	6
736	86688	6	86694	6	86700	5	86705	6	86711	6	86717	6	86723	6	86729	6	86735	6	86741	6
737	86747	6	86753	6	86759	5	86764	6	86770	6	86776	6	86782	6	86788	6	86794	6	86800	6
738	86806	6	86812	5	86817	6	86823	6	86829	6	86835	6	86841	6	86847	6	86853	6	86859	5
739	86864	6	86870	6	86876	6	86882	6	86888	6	86894	6	86900	6	86906	5	86911	6	86917	6
740	86923	6	86929	6	86935	6	86941	6	86947	6	86953	5	86958	6	86964	6	86970	6	86976	6
741	86982	6	86988	6	86994	5	86999	6	87005	6	87011	6	87017	6	87023	6	87029	6	87035	5
742	87040	6	87046	6	87052	6	87058	6	87064	6	87070	5	87075	6	87081	6	87087	6	87093	6
743	87099	6	87105	6	87111	5	87116	6	87122	6	87128	6	87134	6	87140	6	87146	5	87151	6
744	87157	6	87163	6	87169	6	87175	6	87181	5	87186	6	87192	6	87198	6	87204	6	87210	6
745	87216	5	87221	6	87227	6	87233	6	87239	6	87245	6	87251	5	87256	6	87262	6	87268	6
746	87274	6	87280	6	87286	5	87291	6	87297	6	87303	6	87309	6	87315	5	87320	6	87326	6
747	87332	6	87338	6	87344	5	87349	6	87355	6	87361	6	87367	6	87373	6	87379	5	87384	6
748	87390	6	87396	6	87402	6	87408	5	87413	6	87419	6	87425	6	87431	6	87437	5	87442	6
749	87448	6	87454	6	87460	6	87466	5	87471	6	87477	6	87483	6	87489	6	87495	5	87500	6
750	87506	6	87512	6	87518	5	87523	6	87529	6	87535	6	87541	6	87547	5	87552	6	87558	6
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	7
1	1
2	1
3	2
4	3
5	4
6	4
7	5
8	6
9	6

	6
1	1
2	1
3	2
4	2
5	3
6	4
7	4
8	5
9	5

	5
1	0
2	1
3	2
4	2
5	2
6	3
7	4
8	4
9	4

TABLE 32

Logarithms of Numbers

7500–8000

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
750	87506	6	87512	6	87518	5	87523	6	87529	6	87535	6	87541	6	87547	5	87552	6	87558	6
751	87564	6	87570	6	87576	5	87581	6	87587	6	87593	6	87599	5	87604	6	87610	6	87616	6
752	87622	6	87628	5	87633	6	87639	6	87645	6	87651	5	87656	6	87662	6	87668	6	87674	5
753	87679	6	87685	6	87691	6	87697	6	87703	5	87708	6	87714	6	87720	6	87726	5	87731	6
754	87737	6	87743	6	87749	5	87754	6	87760	6	87766	6	87772	5	87777	6	87783	6	87789	6
755	87795	5	87800	6	87806	6	87812	6	87818	5	87823	6	87829	6	87835	6	87841	5	87846	6
756	87852	6	87858	6	87864	5	87869	6	87875	6	87881	6	87887	5	87892	6	87898	6	87904	6
757	87910	5	87915	6	87921	6	87927	6	87933	5	87938	6	87944	6	87950	5	87955	6	87961	6
758	87967	6	87973	5	87978	6	87984	6	87990	6	87996	5	88001	6	88007	6	88013	5	88018	6
759	88024	6	88030	6	88036	5	88041	6	88047	6	88053	5	88058	6	88064	6	88070	6	88076	5
760	88081	6	88087	6	88093	5	88098	6	88104	6	88110	6	88116	5	88121	6	88127	6	88133	5
761	88138	6	88144	6	88150	6	88156	5	88161	6	88167	6	88173	5	88178	6	88184	6	88190	5
762	88195	6	88201	6	88207	6	88213	5	88218	6	88224	6	88230	5	88235	6	88241	6	88247	5
763	88252	6	88258	6	88264	6	88270	5	88275	6	88281	6	88287	5	88292	6	88298	6	88304	5
764	88309	6	88315	6	88321	5	88326	6	88332	6	88338	5	88343	6	88349	6	88355	5	88360	6
765	88366	6	88372	5	88377	6	88383	6	88389	6	88395	5	88400	6	88406	6	88412	5	88417	6
766	88423	6	88429	5	88434	6	88440	6	88446	5	88451	6	88457	6	88463	5	88468	6	88474	6
767	88480	5	88485	6	88491	6	88497	5	88502	6	88508	5	88513	6	88519	6	88525	5	88530	6
768	88536	6	88542	5	88547	6	88553	6	88559	5	88564	6	88570	6	88576	5	88581	6	88587	6
769	88593	5	88598	6	88604	6	88610	5	88615	6	88621	6	88627	5	88632	6	88638	5	88643	6
770	88649	6	88655	5	88660	6	88666	6	88672	5	88677	6	88683	6	88689	5	88694	6	88700	5
771	88705	6	88711	6	88717	5	88722	6	88728	6	88734	5	88739	6	88745	5	88750	6	88756	6
772	88762	5	88767	6	88773	6	88779	5	88784	6	88790	5	88795	6	88801	6	88807	5	88812	6
773	88818	6	88824	5	88829	6	88835	5	88840	6	88846	6	88852	5	88857	6	88863	5	88868	6
774	88874	6	88880	5	88885	6	88891	6	88897	5	88902	6	88908	5	88913	6	88919	6	88925	5
775	88930	6	88936	5	88941	6	88947	6	88953	5	88958	6	88964	5	88969	6	88975	6	88981	5
776	88986	6	88992	5	88997	6	89003	6	89009	5	89014	6	89020	5	89025	6	89031	6	89037	5
777	89042	6	89048	5	89053	6	89059	5	89064	6	89070	6	89076	5	89081	6	89087	5	89092	6
778	89098	6	89104	5	89109	6	89115	5	89120	6	89126	5	89131	6	89137	6	89143	5	89148	6
779	89154	5	89159	6	89165	5	89170	6	89176	6	89182	5	89187	6	89193	5	89198	6	89204	5
780	89209	6	89215	6	89221	5	89226	6	89232	5	89237	6	89243	5	89248	6	89254	6	89260	5
781	89265	6	89271	5	89276	6	89282	5	89287	6	89293	5	89298	6	89304	6	89310	5	89315	6
782	89321	5	89326	6	89332	5	89337	6	89343	5	89348	6	89354	6	89360	5	89365	6	89371	5
783	89376	6	89382	5	89387	6	89393	5	89398	6	89404	5	89409	6	89415	6	89421	5	89426	6
784	89432	5	89437	6	89443	5	89448	6	89454	5	89459	6	89465	5	89470	6	89476	5	89481	6
785	89487	5	89492	6	89498	6	89504	5	89509	6	89515	5	89520	6	89526	5	89531	6	89537	5
786	89542	6	89548	5	89553	6	89559	5	89564	6	89570	5	89575	6	89581	5	89586	6	89592	5
787	89597	6	89603	6	89609	5	89614	6	89620	5	89625	6	89631	5	89636	6	89642	5	89647	6
788	89653	5	89658	6	89664	5	89669	6	89675	5	89680	6	89686	5	89691	6	89697	5	89702	6
789	89708	5	89713	6	89719	5	89724	6	89730	5	89735	6	89741	5	89746	6	89752	5	89757	6
790	89763	5	89768	6	89774	5	89779	6	89785	5	89790	6	89796	5	89801	6	89807	5	89812	6
791	89818	5	89823	6	89829	5	89834	6	89840	5	89845	6	89851	5	89856	6	89862	5	89867	6
792	89873	5	89878	5	89883	6	89889	5	89894	6	89900	5	89905	6	89911	5	89916	6	89922	5
793	89927	6	89933	5	89938	6	89944	5	89949	6	89955	5	89960	6	89966	5	89971	6	89977	5
794	89982	6	89988	5	89993	5	89998	6	90004	5	90009	6	90015	5	90020	6	90026	5	90031	6
795	90037	5	90042	6	90048	5	90053	6	90059	5	90064	5	90069	6	90075	5	90080	6	90086	5
796	90091	6	90097	5	90102	6	90108	5	90113	6	90119	5	90124	5	90129	6	90135	5	90140	6
797	90146	5	90151	6	90157	5	90162	6	90168	5	90173	6	90179	5	90184	5	90189	6	90195	5
798	90200	6	90206	5	90211	6	90217	5	90222	5	90227	6	90233	5	90238	6	90244	5	90249	6
799	90255	5	90260	6	90266	5	90271	5	90276	6	90282	5	90287	6	90293	5	90298	6	90304	5
800	90309	5	90314	6	90320	5	90325	6	90331	5	90336	6	90342	5	90347	5	90352	6	90358	5
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	6
1	1
2	1
3	2
4	2
5	3
6	4
7	4
8	5
9	5

	5
1	0
2	1
3	2
4	2
5	2
6	3
7	4
8	4
9	4

TABLE 32

Logarithms of Numbers

8000–8500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
800	90309	5	90314	6	90320	5	90325	6	90331	5	90336	6	90342	5	90347	5	90352	6	90358	5
801	90363	6	90369	5	90374	6	90380	5	90385	5	90390	6	90396	5	90401	6	90407	5	90412	5
802	90417	6	90423	5	90428	6	90434	5	90439	6	90445	5	90450	5	90455	6	90461	5	90466	6
803	90472	5	90477	5	90482	6	90488	5	90493	6	90499	5	90504	5	90509	6	90515	5	90520	6
804	90526	5	90531	5	90536	6	90542	5	90547	6	90553	5	90558	5	90563	6	90569	5	90574	6
805	90580	5	90585	5	90590	6	90596	5	90601	6	90607	5	90612	5	90617	6	90623	5	90628	6
806	90634	5	90639	5	90644	6	90650	5	90655	5	90660	6	90666	5	90671	6	90677	5	90682	5
807	90687	6	90693	5	90698	5	90703	6	90709	5	90714	6	90720	5	90725	5	90730	6	90736	5
808	90741	6	90747	5	90752	5	90757	6	90763	5	90768	5	90773	6	90779	5	90784	5	90789	6
809	90795	5	90800	6	90806	5	90811	5	90816	6	90822	5	90827	5	90832	6	90838	5	90843	6
810	90849	5	90854	5	90859	6	90865	5	90870	5	90875	6	90881	5	90886	5	90891	6	90897	5
811	90902	5	90907	6	90913	5	90918	6	90924	5	90929	5	90934	6	90940	5	90945	5	90950	6
812	90956	5	90961	5	90966	6	90972	5	90977	5	90982	6	90988	5	90993	5	90998	6	91004	5
813	91009	5	91014	6	91020	5	91025	5	91030	6	91036	5	91041	5	91046	6	91052	5	91057	5
814	91062	6	91068	5	91073	5	91078	6	91084	5	91089	5	91094	6	91100	5	91105	5	91110	6
815	91116	5	91121	5	91126	6	91132	5	91137	5	91142	6	91148	5	91153	5	91158	6	91164	5
816	91169	5	91174	6	91180	5	91185	5	91190	6	91196	5	91201	5	91206	6	91212	5	91217	5
817	91222	6	91228	5	91233	5	91238	5	91243	6	91249	5	91254	5	91259	6	91265	5	91270	5
818	91275	6	91281	5	91286	5	91291	6	91297	5	91302	5	91307	5	91312	6	91318	5	91323	5
819	91328	6	91334	5	91339	5	91344	6	91350	5	91355	5	91360	5	91365	6	91371	5	91376	5
820	91381	6	91387	5	91392	5	91397	6	91403	5	91408	5	91413	5	91418	6	91424	5	91429	5
821	91434	6	91440	5	91445	5	91450	5	91455	6	91461	5	91466	5	91471	6	91477	5	91482	5
822	91487	5	91492	6	91498	5	91503	5	91508	6	91514	5	91519	5	91524	5	91529	6	91535	5
823	91540	5	91545	6	91551	5	91556	5	91561	5	91566	6	91572	5	91577	5	91582	5	91587	6
824	91593	5	91598	5	91603	6	91609	5	91614	5	91619	5	91624	6	91630	5	91635	5	91640	5
825	91645	6	91651	5	91656	5	91661	5	91666	6	91672	5	91677	5	91682	5	91687	6	91693	5
826	91698	5	91703	6	91709	5	91714	5	91719	5	91724	6	91730	5	91735	5	91740	5	91745	6
827	91751	5	91756	5	91761	5	91766	6	91772	5	91777	5	91782	5	91787	6	91793	5	91798	5
828	91803	5	91808	6	91814	5	91819	5	91824	5	91829	5	91834	6	91840	5	91845	5	91850	5
829	91855	6	91861	5	91866	5	91871	5	91876	6	91882	5	91887	5	91892	5	91897	6	91903	5
830	91908	5	91913	5	91918	6	91924	5	91929	5	91934	5	91939	5	91944	6	91950	5	91955	5
831	91960	5	91965	6	91971	5	91976	5	91981	5	91986	5	91991	6	91997	5	92002	5	92007	5
832	92012	6	92018	5	92023	5	92028	5	92033	5	92038	6	92044	5	92049	5	92054	5	92059	6
833	92065	5	92070	5	92075	5	92080	5	92085	6	92091	5	92096	5	92101	5	92106	5	92111	6
834	92117	5	92122	5	92127	5	92132	5	92137	6	92143	5	92148	5	92153	5	92158	5	92163	6
835	92169	5	92174	5	92179	5	92184	5	92189	6	92195	5	92200	5	92205	5	92210	5	92215	6
836	92221	5	92226	5	92231	5	92236	5	92241	6	92247	5	92252	5	92257	5	92262	5	92267	6
837	92273	5	92278	5	92283	5	92288	5	92293	5	92298	6	92304	5	92309	5	92314	5	92319	5
838	92324	6	92330	5	92335	5	92340	5	92345	5	92350	5	92355	6	92361	5	92366	5	92371	5
839	92376	5	92381	6	92387	5	92392	5	92397	5	92402	5	92407	5	92412	6	92418	5	92423	5
840	92428	5	92433	5	92438	5	92443	6	92449	5	92454	5	92459	5	92464	5	92469	5	92474	6
841	92480	5	92485	5	92490	5	92495	5	92500	5	92505	6	92511	5	92516	5	92521	5	92526	5
842	92531	5	92536	6	92542	5	92547	5	92552	5	92557	5	92562	5	92567	5	92572	6	92578	5
843	92583	5	92588	5	92593	5	92598	5	92603	6	92609	5	92614	5	92619	5	92624	5	92629	5
844	92634	5	92639	6	92645	5	92650	5	92655	5	92660	5	92665	5	92670	5	92675	6	92681	5
845	92686	5	92691	5	92696	5	92701	5	92706	5	92711	5	92716	6	92722	5	92727	5	92732	5
846	92737	5	92742	5	92747	5	92752	6	92758	5	92763	5	92768	5	92773	5	92778	5	92783	5
847	92788	5	92793	6	92799	5	92804	5	92809	5	92814	5	92819	5	92824	5	92829	5	92834	6
848	92840	5	92845	5	92850	5	92855	5	92860	5	92865	5	92870	5	92875	6	92881	5	92886	5
849	92891	5	92896	5	92901	5	92906	5	92911	5	92916	5	92921	6	92927	5	92932	5	92937	5
850	92942	5	92947	5	92952	5	92957	5	92962	5	92967	6	92973	5	92978	5	92983	5	92988	5
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	6
1	1
2	1
3	2
4	2
5	3
6	4
7	4
8	5
9	5

	5
1	0
2	1
3	2
4	2
5	2
6	3
7	4
8	4
9	4

TABLE 32

Logarithms of Numbers

8500–9000

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
850	92942	5	92947	5	92952	5	92957	5	92962	5	92967	6	92973	5	92978	5	92983	5	92988	5
851	92993	5	92998	5	93003	5	93008	5	93013	5	93018	6	93024	5	93029	5	93034	5	93039	5
852	93044	5	93049	5	93054	5	93059	5	93064	5	93069	6	93075	5	93080	5	93085	5	93090	5
853	93095	5	93100	5	93105	5	93110	5	93115	5	93120	5	93125	6	93131	5	93136	5	93141	5
854	93146	5	93151	5	93156	5	93161	5	93166	5	93171	5	93176	5	93181	5	93186	6	93192	5
855	93197	5	93202	5	93207	5	93212	5	93217	5	93222	5	93227	5	93232	5	93237	5	93242	5
856	93247	5	93252	6	93258	5	93263	5	93268	5	93273	5	93278	5	93283	5	93288	5	93293	5
857	93298	5	93303	5	93308	5	93313	5	93318	5	93323	5	93328	6	93334	5	93339	5	93344	5
858	93349	5	93354	5	93359	5	93364	5	93369	5	93374	5	93379	5	93384	5	93389	5	93394	5
859	93399	5	93404	5	93409	5	93414	6	93420	5	93425	5	93430	5	93435	5	93440	5	93445	5
860	93450	5	93455	5	93460	5	93465	5	93470	5	93475	5	93480	5	93485	5	93490	5	93495	5
861	93500	5	93505	5	93510	5	93515	5	93520	6	93526	5	93531	5	93536	5	93541	5	93546	5
862	93551	5	93556	5	93561	5	93566	5	93571	5	93576	5	93581	5	93586	5	93591	5	93596	5
863	93601	5	93606	5	93611	5	93616	5	93621	5	93626	5	93631	5	93636	5	93641	5	93646	5
864	93651	5	93656	5	93661	5	93666	5	93671	5	93676	6	93682	5	93687	5	93692	5	93697	5
865	93702	5	93707	5	93712	5	93717	5	93722	5	93727	5	93732	5	93737	5	93742	5	93747	5
866	93752	5	93757	5	93762	5	93767	5	93772	5	93777	5	93782	5	93787	5	93792	5	93797	5
867	93802	5	93807	5	93812	5	93817	5	93822	5	93827	5	93832	5	93837	5	93842	5	93847	5
868	93852	5	93857	5	93862	5	93867	5	93872	5	93877	5	93882	5	93887	5	93892	5	93897	5
869	93902	5	93907	5	93912	5	93917	5	93922	5	93927	5	93932	5	93937	5	93942	5	93947	5
870	93952	5	93957	5	93962	5	93967	5	93972	5	93977	5	93982	5	93987	5	93992	5	93997	5
871	94002	5	94007	5	94012	5	94017	5	94022	5	94027	5	94032	5	94037	5	94042	5	94047	5
872	94052	5	94057	5	94062	5	94067	5	94072	5	94077	5	94082	4	94086	5	94091	5	94096	5
873	94101	5	94106	5	94111	5	94116	5	94121	5	94126	5	94131	5	94136	5	94141	5	94146	5
874	94151	5	94156	5	94161	5	94166	5	94171	5	94176	5	94181	5	94186	5	94191	5	94196	5
875	94201	5	94206	5	94211	5	94216	5	94221	5	94226	5	94231	5	94236	4	94240	5	94245	5
876	94250	5	94255	5	94260	5	94265	5	94270	5	94275	5	94280	5	94285	5	94290	5	94295	5
877	94300	5	94305	5	94310	5	94315	5	94320	5	94325	5	94330	5	94335	5	94340	5	94345	4
878	94349	5	94354	5	94359	5	94364	5	94369	5	94374	5	94379	5	94384	5	94389	5	94394	5
879	94399	5	94404	5	94409	5	94414	5	94419	5	94424	5	94429	4	94433	5	94438	5	94443	5
880	94448	5	94453	5	94458	5	94463	5	94468	5	94473	5	94478	5	94483	5	94488	5	94493	5
881	94498	5	94503	4	94507	5	94512	5	94517	5	94522	5	94527	5	94532	5	94537	5	94542	5
882	94547	5	94552	5	94557	5	94562	5	94567	4	94571	5	94576	5	94581	5	94586	5	94591	5
883	94596	5	94601	5	94606	5	94611	5	94616	5	94621	5	94626	4	94630	5	94635	5	94640	5
884	94645	5	94650	5	94655	5	94660	5	94665	5	94670	5	94675	5	94680	5	94685	4	94689	5
885	94694	5	94699	5	94704	5	94709	5	94714	5	94719	5	94724	5	94729	5	94734	4	94738	5
886	94743	5	94748	5	94753	5	94758	5	94763	5	94768	5	94773	5	94778	5	94783	4	94787	5
887	94792	5	94797	5	94802	5	94807	5	94812	5	94817	5	94822	5	94827	5	94832	4	94836	5
888	94841	5	94846	5	94851	5	94856	5	94861	5	94866	5	94871	5	94876	4	94880	5	94885	5
889	94890	5	94895	5	94900	5	94905	5	94910	5	94915	4	94919	5	94924	5	94929	5	94934	5
890	94939	5	94944	5	94949	5	94954	5	94959	4	94963	5	94968	5	94973	5	94978	5	94983	5
891	94988	5	94993	5	94998	4	95002	5	95007	5	95012	5	95017	5	95022	5	95027	5	95032	4
892	95036	5	95041	5	95046	5	95051	5	95056	5	95061	5	95066	5	95071	4	95075	5	95080	5
893	95085	5	95090	5	95095	5	95100	4	95105	4	95109	5	95114	5	95119	5	95124	5	95129	5
894	95134	5	95139	4	95143	5	95148	5	95153	5	95158	5	95163	5	95168	5	95173	4	95177	5
895	95182	5	95187	5	95192	5	95197	5	95202	5	95207	4	95211	5	95216	5	95221	5	95226	5
896	95231	5	95236	4	95240	5	95245	5	95250	5	95255	5	95260	5	95265	5	95270	4	95274	5
897	95279	5	95284	5	95289	5	95294	5	95299	4	95303	5	95308	5	95313	5	95318	5	95323	5
898	95328	4	95332	5	95337	5	95342	5	95347	5	95352	5	95357	4	95361	5	95366	5	95371	5
899	95376	5	95381	5	95386	4	95390	5	95395	5	95400	5	95405	5	95410	5	95415	4	95419	5
900	95424	5	95429	5	95434	5	95439	5	95444	4	95448	5	95453	5	95458	5	95463	5	95468	4
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	6
1	1
2	1
3	2
4	2
5	3
6	4
7	4
8	5
9	5

	5
1	0
2	1
3	2
4	2
5	2
6	3
7	4
8	4
9	4

	4
1	0
2	1
3	1
4	2
5	2
6	2
7	3
8	3
9	4

TABLE 32

Logarithms of Numbers

9000–9500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
900	95424	5	95429	5	95434	5	95439	5	95444	4	95448	5	95453	5	95458	5	95463	5	95468	4
901	95472	5	95477	5	95482	5	95487	5	95492	5	95497	4	95501	5	95506	5	95511	5	95516	5
902	95521	4	95525	5	95530	5	95535	5	95540	5	95545	5	95550	4	95554	5	95559	5	95564	5
903	95569	5	95574	4	95578	5	95583	5	95588	5	95593	5	95598	4	95602	5	95607	5	95612	5
904	95617	5	95622	4	95626	5	95631	5	95636	5	95641	5	95646	4	95650	5	95655	5	95660	5
905	95665	5	95670	4	95674	5	95679	5	95684	5	95689	5	95694	4	95698	5	95703	5	95708	5
906	95713	5	95718	4	95722	5	95727	5	95732	5	95737	5	95742	4	95746	5	95751	5	95756	5
907	95761	5	95766	4	95770	5	95775	5	95780	5	95785	4	95789	5	95794	5	95799	5	95804	5
908	95809	4	95813	5	95818	5	95823	5	95828	4	95832	5	95837	5	95842	5	95847	5	95852	4
909	95856	5	95861	5	95866	5	95871	4	95875	5	95880	5	95885	5	95890	5	95895	4	95899	5
910	95904	5	95909	5	95914	4	95918	5	95923	5	95928	5	95933	5	95938	4	95942	5	95947	5
911	95952	5	95957	4	95961	5	95966	5	95971	5	95976	4	95980	5	95985	5	95990	5	95995	4
912	95999	5	96004	5	96009	5	96014	5	96019	4	96023	5	96028	5	96033	5	96038	4	96042	5
913	96047	5	96052	5	96057	4	96061	5	96066	5	96071	5	96076	4	96080	5	96085	5	96090	5
914	96095	4	96099	5	96104	5	96109	5	96114	4	96118	5	96123	5	96128	5	96133	4	96137	5
915	96142	5	96147	5	96152	4	96156	5	96161	5	96166	5	96171	4	96175	5	96180	5	96185	5
916	96190	4	96194	5	96199	5	96204	5	96209	4	96213	5	96218	5	96223	4	96227	5	96232	5
917	96237	5	96242	4	96246	5	96251	5	96256	5	96261	4	96265	5	96270	5	96275	5	96280	4
918	96284	5	96289	5	96294	4	96298	5	96303	5	96308	5	96313	4	96317	5	96322	5	96327	5
919	96332	4	96336	5	96341	5	96346	4	96350	5	96355	5	96360	5	96365	4	96369	5	96374	5
920	96379	5	96384	4	96388	5	96393	5	96398	4	96402	5	96407	5	96412	5	96417	4	96421	5
921	96426	5	96431	4	96435	5	96440	5	96445	5	96450	4	96454	5	96459	5	96464	4	96468	5
922	96473	5	96478	5	96483	4	96487	5	96492	5	96497	4	96501	5	96506	5	96511	4	96515	5
923	96520	5	96525	5	96530	4	96534	5	96539	5	96544	4	96548	5	96553	5	96558	4	96562	5
924	96567	5	96572	5	96577	4	96581	5	96586	5	96591	4	96595	5	96600	5	96605	4	96609	5
925	96614	5	96619	5	96624	4	96628	5	96633	5	96638	4	96642	5	96647	5	96652	4	96656	5
926	96661	5	96666	4	96670	5	96675	5	96680	5	96685	4	96689	5	96694	5	96699	4	96703	5
927	96708	5	96713	4	96717	5	96722	5	96727	4	96731	5	96736	5	96741	4	96745	5	96750	5
928	96755	4	96759	5	96764	5	96769	5	96774	4	96778	5	96783	5	96788	4	96792	5	96797	5
929	96802	4	96806	5	96811	5	96816	4	96820	5	96825	5	96830	4	96834	5	96839	5	96844	4
930	96848	5	96853	5	96858	4	96862	5	96867	5	96872	4	96876	5	96881	5	96886	4	96890	5
931	96895	5	96900	4	96904	5	96909	5	96914	4	96918	5	96923	5	96928	4	96932	5	96937	5
932	96942	4	96946	5	96951	5	96956	4	96960	5	96965	5	96970	4	96974	5	96979	5	96984	4
933	96988	5	96993	4	96997	5	97002	5	97007	4	97011	5	97016	5	97021	4	97025	5	97030	5
934	97035	4	97039	5	97044	5	97049	4	97053	5	97058	5	97063	4	97067	5	97072	5	97077	4
935	97081	5	97086	4	97090	5	97095	5	97100	4	97104	5	97109	5	97114	4	97118	5	97123	5
936	97128	4	97132	5	97137	5	97142	4	97146	5	97151	4	97155	5	97160	5	97165	4	97169	5
937	97174	5	97179	4	97183	5	97188	4	97192	5	97197	5	97202	4	97206	5	97211	5	97216	4
938	97220	5	97225	5	97230	4	97234	5	97239	4	97243	5	97248	5	97253	4	97257	5	97262	5
939	97267	4	97271	5	97276	4	97280	5	97285	5	97290	4	97294	5	97299	5	97304	4	97308	5
940	97313	4	97317	5	97322	5	97327	4	97331	5	97336	4	97340	5	97345	5	97350	4	97354	5
941	97359	5	97364	4	97368	5	97373	4	97377	5	97382	5	97387	4	97391	5	97396	4	97400	5
942	97405	5	97410	4	97414	5	97419	5	97424	4	97428	5	97433	4	97437	5	97442	5	97447	4
943	97451	5	97456	4	97460	5	97465	5	97470	4	97474	5	97479	4	97483	5	97488	5	97493	4
944	97497	5	97502	4	97506	5	97511	5	97516	4	97520	5	97525	4	97529	5	97534	5	97539	4
945	97543	5	97548	4	97552	5	97557	5	97562	4	97566	5	97571	4	97575	5	97580	5	97585	4
946	97589	5	97594	4	97598	5	97603	4	97607	5	97612	5	97617	4	97621	5	97626	4	97630	5
947	97635	5	97640	4	97644	5	97649	4	97653	5	97658	5	97663	4	97667	5	97672	4	97676	5
948	97681	4	97685	5	97690	5	97695	4	97699	5	97704	4	97708	5	97713	4	97717	5	97722	5
949	97727	4	97731	5	97736	4	97740	5	97745	4	97749	5	97754	5	97759	4	97763	5	97768	4
950	97772	5	97777	5	97782	4	97786	5	97791	4	97795	5	97800	4	97804	5	97809	4	97813	5
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	5
1	0
2	1
3	2
4	2
5	2
6	3
7	4
8	4
9	4

	4
1	0
2	1
3	1
4	2
5	2
6	2
7	3
8	3
9	4

TABLE 32

Logarithms of Numbers

9500–10000

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
950	97772	5	97777	5	97782	4	97786	5	97791	4	97795	5	97800	4	97804	5	97809	4	97813	5
951	97818	5	97823	4	97827	5	97832	4	97836	5	97841	4	97845	5	97850	5	97855	4	97859	5
952	97864	4	97868	5	97873	4	97877	5	97882	4	97886	5	97891	5	97896	4	97900	5	97905	4
953	97909	5	97914	4	97918	5	97923	5	97928	4	97932	5	97937	4	97941	5	97946	4	97950	5
954	97955	4	97959	5	97964	4	97968	5	97973	5	97978	4	97982	5	97987	4	97991	5	97996	4
955	98000	5	98005	4	98009	5	98014	5	98019	4	98023	5	98028	4	98032	5	98037	4	98041	5
956	98046	4	98050	5	98055	4	98059	5	98064	4	98068	5	98073	5	98078	4	98082	5	98087	4
957	98091	5	98096	4	98100	5	98105	4	98109	5	98114	4	98118	5	98123	4	98127	5	98132	5
958	98137	4	98141	5	98146	4	98150	5	98155	4	98159	5	98164	4	98168	5	98173	4	98177	5
959	98182	4	98186	5	98191	4	98195	5	98200	4	98204	5	98209	5	98214	4	98218	5	98223	4
960	98227	5	98232	4	98236	5	98241	4	98245	5	98250	4	98254	5	98259	4	98263	5	98268	4
961	98272	5	98277	4	98281	5	98286	4	98290	5	98295	4	98299	5	98304	4	98308	5	98313	5
962	98318	4	98322	5	98327	4	98331	5	98336	4	98340	5	98345	4	98349	5	98354	4	98358	5
963	98363	4	98367	5	98372	4	98376	5	98381	4	98385	5	98390	4	98394	5	98399	4	98403	5
964	98408	4	98412	5	98417	4	98421	5	98426	4	98430	5	98435	4	98439	5	98444	4	98448	5
965	98453	4	98457	5	98462	4	98466	5	98471	4	98475	5	98480	4	98484	5	98489	4	98493	5
966	98498	4	98502	5	98507	4	98511	5	98516	4	98520	5	98525	4	98529	5	98534	4	98538	5
967	98543	4	98547	5	98552	4	98556	5	98561	4	98565	5	98570	4	98574	5	98579	4	98583	5
968	98588	4	98592	5	98597	4	98601	4	98605	5	98610	4	98614	5	98619	4	98623	5	98628	4
969	98632	5	98637	4	98641	5	98646	4	98650	5	98655	4	98659	5	98664	4	98668	5	98673	4
970	98677	5	98682	4	98686	5	98691	4	98695	5	98700	4	98704	5	98709	4	98713	4	98717	5
971	98722	4	98726	5	98731	4	98735	5	98740	4	98744	5	98749	4	98753	5	98758	4	98762	5
972	98767	4	98771	5	98776	4	98780	4	98784	5	98789	4	98793	5	98798	4	98802	5	98807	4
973	98811	5	98816	4	98820	5	98825	4	98829	5	98834	4	98838	5	98843	4	98847	4	98851	5
974	98856	4	98860	5	98865	4	98869	5	98874	4	98878	5	98883	4	98887	5	98892	4	98896	4
975	98900	5	98905	4	98909	5	98914	4	98918	5	98923	4	98927	5	98932	4	98936	5	98941	4
976	98945	4	98949	5	98954	4	98958	5	98963	4	98967	5	98972	4	98976	5	98981	4	98985	4
977	98989	5	98994	4	98998	5	99003	4	99007	5	99012	4	99016	5	99021	4	99025	4	99029	5
978	99034	4	99038	5	99043	4	99047	5	99052	4	99056	5	99061	4	99065	4	99069	5	99074	4
979	99078	5	99083	4	99087	5	99092	4	99096	4	99100	5	99105	4	99109	5	99114	4	99118	5
980	99123	4	99127	4	99131	5	99136	4	99140	5	99145	4	99149	5	99154	4	99158	4	99162	5
981	99167	4	99171	5	99176	4	99180	5	99185	4	99189	4	99193	5	99198	4	99202	5	99207	4
982	99211	5	99216	4	99220	4	99224	5	99229	4	99233	5	99238	4	99242	5	99247	4	99251	4
983	99255	5	99260	4	99264	5	99269	4	99273	4	99277	5	99282	4	99286	5	99291	4	99295	5
984	99300	4	99304	4	99308	5	99313	4	99317	5	99322	4	99326	4	99330	5	99335	4	99339	5
985	99344	4	99348	4	99352	5	99357	4	99361	5	99366	4	99370	4	99374	5	99379	4	99383	5
986	99388	4	99392	4	99396	5	99401	4	99405	5	99410	4	99414	5	99419	4	99423	4	99427	5
987	99432	4	99436	5	99441	4	99445	4	99449	5	99454	4	99458	5	99463	4	99467	4	99471	5
988	99476	4	99480	4	99484	5	99489	4	99493	5	99498	4	99502	4	99506	5	99511	4	99515	5
989	99520	4	99524	4	99528	5	99533	4	99537	5	99542	4	99546	4	99550	5	99555	4	99559	5
990	99564	4	99568	4	99572	5	99577	4	99581	4	99585	5	99590	4	99594	5	99599	4	99603	4
991	99607	5	99612	4	99616	5	99621	4	99625	4	99629	5	99634	4	99638	4	99642	5	99647	4
992	99651	5	99656	4	99660	4	99664	5	99669	4	99673	4	99677	5	99682	4	99686	5	99691	4
993	99695	4	99699	5	99704	4	99708	4	99712	5	99717	4	99721	5	99726	4	99730	4	99734	5
994	99739	4	99743	4	99747	5	99752	4	99756	4	99760	5	99765	4	99769	5	99774	4	99778	4
995	99782	5	99787	4	99791	4	99795	5	99800	4	99804	4	99808	5	99813	4	99817	5	99822	4
996	99826	4	99830	5	99835	4	99839	4	99843	5	99848	4	99852	4	99856	5	99861	4	99865	5
997	99870	4	99874	4	99878	5	99883	4	99887	4	99891	5	99896	4	99900	4	99904	5	99909	4
998	99913	4	99917	5	99922	4	99926	4	99930	5	99935	4	99939	5	99944	4	99948	4	99952	5
999	99957	4	99961	4	99965	5	99970	4	99974	4	99978	5	99983	4	99987	4	99991	5	99996	4
1000	00000	4	00004	5	00009	4	00013	4	00017	5	00022	4	00026	4	00030	5	00035	4	00039	4
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts

	5
1	0
2	1
3	2
4	2
5	2
6	3
7	4
8	4
9	4

	4
1	0
2	1
3	1
4	2
5	2
6	2
7	3
8	3
9	4

TABLE 33
Logarithms of Trigonometric Functions

0°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←179° ↓ ′
0	∞	—	∞	∞	—	∞	10. 00000	0	10. 00000	60
1	6. 46373	30103	13. 53627	6. 46373	30103	13. 53627	. 00000	0	. 00000	59
2	. 76476	17609	. 23524	. 76476	17609	. 23524	. 00000	0	. 00000	58
3	6. 94085	12494	13. 05915	6. 94085	12494	13. 05915	. 00000	0	. 00000	57
4	7. 06579	9691	12. 93421	7. 06579	9691	12. 93421	. 00000	0	. 00000	56
5	7. 16270	7918	12. 83730	7. 16270	7918	12. 83730	10. 00000	0	10. 00000	55
6	. 24188	6694	. 75812	. 24188	6694	. 75812	. 00000	0	. 00000	54
7	. 30882	5800	. 69118	. 30882	5800	. 69118	. 00000	0	. 00000	53
8	. 36682	5115	. 63318	. 36682	5115	. 63318	. 00000	0	. 00000	52
9	. 41797	4576	. 58203	. 41797	4576	. 58203	. 00000	0	. 00000	51
10	7. 46373	4139	12. 53627	7. 46373	4139	12. 53627	10. 00000	0	10. 00000	50
11	. 50512	3779	. 49488	. 50512	3779	. 49488	. 00000	0	. 00000	49
12	. 54291	3476	. 45709	. 54291	3476	. 45709	. 00000	0	. 00000	48
13	. 57767	3218	. 42233	. 57767	3219	. 42233	. 00000	0	. 00000	47
14	. 60985	2997	. 39015	. 60986	2996	. 39014	. 00000	0	. 00000	46
15	7. 63982	2802	12. 36018	7. 63982	2803	12. 36018	10. 00000	0	10. 00000	45
16	. 66784	2633	. 33216	. 66785	2633	. 33215	. 00000	1	10. 00000	44
17	. 69417	2483	. 30583	. 69418	2482	. 30582	. 00001	0	9. 99999	43
18	. 71900	2348	. 28100	. 71900	2348	. 28100	. 00001	0	. 99999	42
19	. 74248	2227	. 25752	. 74248	2228	. 25752	. 00001	0	. 99999	41
20	7. 76475	2119	12. 23525	7. 76476	2119	12. 23524	10. 00001	0	9. 99999	40
21	. 78594	2021	. 21406	. 78595	2020	. 21405	. 00001	0	. 99999	39
22	. 80615	1930	. 19385	. 80615	1931	. 19385	. 00001	0	. 99999	38
23	. 82545	1848	. 17455	. 82546	1848	. 17454	. 00001	0	. 99999	37
24	. 84393	1773	. 15607	. 84394	1773	. 15606	. 00001	0	. 99999	36
25	7. 86166	1704	12. 13834	7. 86167	1704	12. 13833	10. 00001	0	9. 99999	35
26	. 87870	1639	. 12130	. 87871	1639	. 12129	. 00001	0	. 99999	34
27	. 89509	1579	. 10491	. 89510	1579	. 10490	. 00001	0	. 99999	33
28	. 91088	1524	. 08912	. 91089	1524	. 08911	. 00001	1	. 99999	32
29	. 92612	1472	. 07388	. 92613	1473	. 07387	. 00002	0	. 99998	31
30	7. 94084	1424	12. 05916	7. 94086	1424	12. 05914	10. 00002	0	9. 99998	30
31	. 95508	1379	. 04492	. 95510	1379	. 04490	. 00002	0	. 99998	29
32	. 96887	1336	. 03113	. 96889	1336	. 03111	. 00002	0	. 99998	28
33	. 98223	1297	. 01777	. 98225	1297	. 01775	. 00002	0	. 99998	27
34	7. 99520	1259	12. 00480	7. 99522	1259	12. 00478	. 00002	0	. 99998	26
35	8. 00779	1223	11. 99221	8. 00781	1223	11. 99219	10. 00002	0	9. 99998	25
36	. 02002	1190	. 97998	. 02004	1190	. 97996	. 00002	1	. 99998	24
37	. 03192	1158	. 96808	. 03194	1159	. 96806	. 00003	0	. 99997	23
38	. 04350	1128	. 95650	. 04353	1128	. 95647	. 00003	0	. 99997	22
39	. 05478	1100	. 94522	. 05481	1100	. 94519	. 00003	0	. 99997	21
40	8. 06578	1072	11. 93422	8. 06581	1072	11. 93419	10. 00003	0	9. 99997	20
41	. 07650	1046	. 92350	. 07653	1047	. 92347	. 00003	0	. 99997	19
42	. 08696	1022	. 91304	. 08700	1022	. 91300	. 00003	0	. 99997	18
43	. 09718	999	. 90282	. 09722	998	. 90278	. 00003	1	. 99997	17
44	. 10717	976	. 89283	. 10720	976	. 89280	. 00004	0	. 99996	16
45	8. 11693	954	11. 88307	8. 11696	955	11. 88304	10. 00004	0	9. 99996	15
46	. 12647	934	. 87353	. 12651	934	. 87349	. 00004	0	. 99996	14
47	. 13581	914	. 86419	. 13585	915	. 86415	. 00004	0	. 99996	13
48	. 14495	896	. 85505	. 14500	895	. 85500	. 00004	0	. 99996	12
49	. 15391	877	. 84609	. 15395	878	. 84605	. 00004	1	. 99996	11
50	8. 16268	860	11. 83732	8. 16273	860	11. 83727	10. 00005	0	9. 99995	10
51	. 17128	843	. 82872	. 17133	843	. 82867	. 00005	0	. 99995	9
52	. 17971	827	. 82029	. 17976	828	. 82024	. 00005	0	. 99995	8
53	. 18798	812	. 81202	. 18804	812	. 81196	. 00005	0	. 99995	7
54	. 19610	797	. 80390	. 19616	797	. 80384	. 00005	1	. 99995	6
55	8. 20407	782	11. 79593	8. 20413	782	11. 79587	10. 00006	0	9. 99994	5
56	. 21189	769	. 78811	. 21195	769	. 78805	. 00006	0	. 99994	4
57	. 21958	755	. 78042	. 21964	756	. 78036	. 00006	0	. 99994	3
58	. 22713	743	. 77287	. 22720	742	. 77280	. 00006	0	. 99994	2
59	. 23456	730	. 76544	. 23462	730	. 76538	. 00006	1	. 99994	1
60	8. 24186		11. 75814	8. 24192		11. 75808	10. 00007		9. 99993	0
↑ 90°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←89°

TABLE 33
Logarithms of Trigonometric Functions

1°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←178° ↓ ′
0	8. 24186	717	11. 75814	8. 24192	718	11. 75808	10. 00007	0	9. 99993	60
1	. 24903	706	. 75097	. 24910	706	. 75090	. 00007	0	. 99993	59
2	. 25609	695	. 74391	. 25616	696	. 74384	. 00007	0	. 99993	58
3	. 26304	684	. 73696	. 26312	684	. 73688	. 00007	1	. 99993	57
4	. 26988	673	. 73012	. 26996	673	. 73004	. 00008	0	. 99992	56
5	8. 27661	663	11. 72339	8. 27669	663	11. 72331	10. 00008	0	9. 99992	55
6	. 28324	653	. 71676	. 28332	654	. 71668	. 00008	0	. 99992	54
7	. 28977	644	. 71023	. 28986	643	. 71014	. 00008	0	. 99992	53
8	. 29621	634	. 70379	. 29629	634	. 70371	. 00008	1	. 99992	52
9	. 30255	624	. 69745	. 30263	625	. 69737	. 00009	0	. 99991	51
10	8. 30879	616	11. 69121	8. 30888	617	11. 69112	10. 00009	0	9. 99991	50
11	. 31495	608	. 68505	. 31505	607	. 68495	. 00009	1	. 99991	49
12	. 32103	599	. 67897	. 32112	599	. 67888	. 00010	0	. 99990	48
13	. 32702	590	. 67298	. 32711	591	. 67289	. 00010	0	. 99990	47
14	. 33292	583	. 66708	. 33302	584	. 66698	. 00010	0	. 99990	46
15	8. 33875	575	11. 66125	8. 33886	575	11. 66114	10. 00010	1	9. 99990	45
16	. 34450	568	. 65550	. 34461	568	. 65539	. 00011	0	. 99989	44
17	. 35018	560	. 64982	. 35029	561	. 64971	. 00011	0	. 99989	43
18	. 35578	553	. 64422	. 35590	553	. 64410	. 00011	0	. 99989	42
19	. 36131	547	. 63869	. 36143	546	. 63857	. 00011	1	. 99989	41
20	8. 36678	539	11. 63322	8. 36689	540	11. 63311	10. 00012	0	9. 99988	40
21	. 37217	533	. 62783	. 37229	533	. 62771	. 00012	0	. 99988	39
22	. 37750	526	. 62250	. 37762	527	. 62238	. 00012	1	. 99988	38
23	. 38276	520	. 61724	. 38289	520	. 61711	. 00013	0	. 99987	37
24	. 38796	514	. 61204	. 38809	514	. 61191	. 00013	0	. 99987	36
25	8. 39310	508	11. 60690	8. 39323	509	11. 60677	10. 00013	1	9. 99987	35
26	. 39818	502	. 60182	. 39832	502	. 60168	. 00014	0	. 99986	34
27	. 40320	496	. 59680	. 40334	496	. 59666	. 00014	0	. 99986	33
28	. 40816	491	. 59184	. 40830	491	. 59170	. 00014	1	. 99986	32
29	. 41307	485	. 58693	. 41321	486	. 58679	. 00015	0	. 99985	31
30	8. 41792	480	11. 58208	8. 41807	480	11. 58193	10. 00015	0	9. 99985	30
31	. 42272	474	. 57728	. 42287	475	. 57713	. 00015	1	. 99985	29
32	. 42746	470	. 57254	. 42762	470	. 57238	. 00016	0	. 99984	28
33	. 43216	464	. 56784	. 43232	464	. 56768	. 00016	0	. 99984	27
34	. 43680	459	. 56320	. 43696	460	. 56304	. 00016	1	. 99984	26
35	8. 44139	455	11. 55861	8. 44156	455	11. 55844	10. 00017	0	9. 99983	25
36	. 44594	450	. 55406	. 44611	450	. 55389	. 00017	0	. 99983	24
37	. 45044	445	. 54956	. 45061	446	. 54939	. 00017	1	. 99983	23
38	. 45489	441	. 54511	. 45507	441	. 54493	. 00018	0	. 99982	22
39	. 45930	436	. 54070	. 45948	437	. 54052	. 00018	0	. 99982	21
40	8. 46366	433	11. 53634	8. 46385	432	11. 53615	10. 00018	1	9. 99982	20
41	. 46799	427	. 53201	. 46817	428	. 53183	. 00019	0	. 99981	19
42	. 47226	424	. 52774	. 47245	424	. 52755	. 00019	0	. 99981	18
43	. 47650	419	. 52350	. 47669	420	. 52331	. 00019	1	. 99981	17
44	. 48069	416	. 51931	. 48089	416	. 51911	. 00020	0	. 99980	16
45	8. 48485	411	11. 51515	8. 48505	412	11. 51495	10. 00020	1	9. 99980	15
46	. 48896	408	. 51104	. 48917	408	. 51083	. 00021	0	. 99979	14
47	. 49304	404	. 50696	. 49325	404	. 50675	. 00021	0	. 99979	13
48	. 49708	400	. 50292	. 49729	401	. 50271	. 00021	1	. 99979	12
49	. 50108	396	. 49892	. 50130	397	. 49870	. 00022	0	. 99978	11
50	8. 50504	393	11. 49496	8. 50527	393	11. 49473	10. 00022	1	9. 99978	10
51	. 50897	390	. 49103	. 50920	390	. 49080	. 00023	0	. 99977	9
52	. 51287	386	. 48713	. 51310	386	. 48690	. 00023	0	. 99977	8
53	. 51673	382	. 48327	. 51696	383	. 48304	. 00023	1	. 99977	7
54	. 52055	379	. 47945	. 52079	380	. 47921	. 00024	0	. 99976	6
55	8. 52434	376	11. 47566	8. 52459	376	11. 47541	10. 00024	1	9. 99976	5
56	. 52810	373	. 47190	. 52835	373	. 47165	. 00025	0	. 99975	4
57	. 53183	369	. 46817	. 53208	370	. 46792	. 00025	1	. 99975	3
58	. 53552	367	. 46448	. 53578	367	. 46422	. 00026	0	. 99974	2
59	. 53919	363	. 46081	. 53945	363	. 46055	. 00026	0	. 99974	1
60	8. 54282		11. 45718	8. 54308		11. 45692	10. 00026		9. 99974	0
↑ 91°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←88°

TABLE 33

Logarithms of Trigonometric Functions

2°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←177° ↓ ′
0	8. 54282	360	11. 45718	8. 54308	361	11. 45692	10. 00026	1	9. 99974	60
1	. 54642	357	. 45358	. 54669	358	. 45331	. 00027	0	. 99973	59
2	. 54999	355	. 45001	. 55027	355	. 44973	. 00027	1	. 99973	58
3	. 55354	351	. 44646	. 55382	352	. 44618	. 00028	0	. 99972	57
4	. 55705	349	. 44295	. 55734	349	. 44266	. 00028	1	. 99972	56
5	8. 56054	346	11. 43946	8. 56083	346	11. 43917	10. 00029	0	9. 99971	55
6	. 56400	343	. 43600	. 56429	344	. 43571	. 00029	1	. 99971	54
7	. 56743	341	. 43257	. 56773	341	. 43227	. 00030	0	. 99970	53
8	. 57084	337	. 42916	. 57114	338	. 42886	. 00030	1	. 99970	52
9	. 57421	336	. 42579	. 57452	336	. 42548	. 00031	0	. 99969	51
10	8. 57757	332	11. 42243	8. 57788	333	11. 42212	10. 00031	1	9. 99969	50
11	. 58089	330	. 41911	. 58121	330	. 41879	. 00032	0	. 99968	49
12	. 58419	328	. 41581	. 58451	328	. 41549	. 00032	1	. 99968	48
13	. 58747	325	. 41253	. 58779	326	. 41221	. 00033	0	. 99967	47
14	. 59072	323	. 40928	. 59105	323	. 40895	. 00033	0	. 99967	46
15	8. 59395	320	11. 40605	8. 59428	321	11. 40572	10. 00033	1	9. 99967	45
16	. 59715	318	. 40285	. 59749	319	. 40251	. 00034	0	. 99966	44
17	. 60033	316	. 39967	. 60068	316	. 39932	. 00034	1	. 99966	43
18	. 60349	313	. 39651	. 60384	314	. 39616	. 00035	1	. 99965	42
19	. 60662	311	. 39338	. 60698	311	. 39302	. 00036	0	. 99964	41
20	8. 60973	309	11. 39027	8. 61009	310	11. 38991	10. 00036	1	9. 99964	40
21	. 61282	307	. 38718	. 61319	307	. 38681	. 00037	0	. 99963	39
22	. 61589	305	. 38411	. 61626	305	. 38374	. 00037	1	. 99963	38
23	. 61894	302	. 38106	. 61931	303	. 38069	. 00038	0	. 99962	37
24	. 62196	301	. 37804	. 62234	301	. 37766	. 00038	1	. 99962	36
25	8. 62497	298	11. 37503	8. 62535	299	11. 37465	10. 00039	0	9. 99961	35
26	. 62795	296	. 37205	. 62834	297	. 37166	. 00039	1	. 99961	34
27	. 63091	294	. 36909	. 63131	295	. 36869	. 00040	0	. 99960	33
28	. 63385	293	. 36615	. 63426	292	. 36574	. 00040	1	. 99960	32
29	. 63678	290	. 36322	. 63718	291	. 36282	. 00041	0	. 99959	31
30	8. 63968	288	11. 36032	8. 64009	289	11. 35991	10. 00041	1	9. 99959	30
31	. 64256	287	. 35744	. 64298	287	. 35702	. 00042	0	. 99958	29
32	. 64543	284	. 35457	. 64585	285	. 35415	. 00042	1	. 99958	28
33	. 64827	283	. 35173	. 64870	284	. 35130	. 00043	1	. 99957	27
34	. 65110	281	. 34890	. 65154	281	. 34846	. 00044	0	. 99956	26
35	8. 65391	279	11. 34609	8. 65435	280	11. 34565	10. 00044	1	9. 99956	25
36	. 65670	277	. 34330	. 65715	278	. 34285	. 00045	0	. 99955	24
37	. 65947	276	. 34053	. 65993	276	. 34007	. 00045	1	. 99955	23
38	. 66223	274	. 33777	. 66269	274	. 33731	. 00046	0	. 99954	22
39	. 66497	272	. 33503	. 66543	273	. 33457	. 00046	1	. 99954	21
40	8. 66769	270	11. 33231	8. 66816	271	11. 33184	10. 00047	1	9. 99953	20
41	. 67039	269	. 32961	. 67087	269	. 32913	. 00048	0	. 99952	19
42	. 67308	267	. 32692	. 67356	268	. 32644	. 00048	1	. 99952	18
43	. 67575	266	. 32425	. 67624	266	. 32376	. 00049	0	. 99951	17
44	. 67841	263	. 32159	. 67890	264	. 32110	. 00049	1	. 99951	16
45	8. 68104	263	11. 31896	8. 68154	263	11. 31846	10. 00050	1	9. 99950	15
46	. 68367	260	. 31633	. 68417	261	. 31583	. 00051	0	. 99949	14
47	. 68627	259	. 31373	. 68678	260	. 31322	. 00051	1	. 99949	13
48	. 68886	258	. 31114	. 68938	258	. 31062	. 00052	0	. 99948	12
49	. 69144	256	. 30856	. 69196	257	. 30804	. 00052	1	. 99948	11
50	8. 69400	254	11. 30600	8. 69453	255	11. 30547	10. 00053	1	9. 99947	10
51	. 69654	253	. 30346	. 69708	254	. 30292	. 00054	0	. 99946	9
52	. 69907	252	. 30093	. 69962	252	. 30038	. 00054	1	. 99946	8
53	. 70159	250	. 29841	. 70214	251	. 29786	. 00055	1	. 99945	7
54	. 70409	249	. 29591	. 70465	249	. 29535	. 00056	0	. 99944	6
55	8. 70658	247	11. 29342	8. 70714	248	11. 29286	10. 00056	1	9. 99944	5
56	. 70905	246	. 29095	. 70962	246	. 29038	. 00057	1	. 99943	4
57	. 71151	244	. 28849	. 71208	245	. 28792	. 00058	0	. 99942	3
58	. 71395	243	. 28605	. 71453	244	. 28547	. 00058	1	. 99942	2
59	. 71638	242	. 28362	. 71697	243	. 28303	. 00059	1	. 99941	1
60	8. 71880		11. 28120	8. 71940		11. 28060	10. 00060		9. 99940	0
↑ 92°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←87°

TABLE 33

Logarithms of Trigonometric Functions

3° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 176° ↓ ′
0	8. 71880	240	11. 28120	8. 71940	241	11. 28060	10. 00060	0	9. 99940	60
1	. 72120	239	. 27880	. 72181	239	. 27819	. 00060	1	. 99940	59
2	. 72359	238	. 27641	. 72420	239	. 27580	. 00061	1	. 99939	58
3	. 72597	237	. 27403	. 72659	237	. 27341	. 00062	0	. 99938	57
4	. 72834	235	. 27166	. 72896	236	. 27104	. 00062	1	. 99938	56
5	8. 73069	234	11. 26931	8. 73132	234	11. 26868	10. 00063	1	9. 99937	55
6	. 73303	232	. 26697	. 73366	234	. 26634	. 00064	0	. 99936	54
7	. 73535	232	. 26465	. 73600	232	. 26400	. 00064	1	. 99936	53
8	. 73767	230	. 26233	. 73832	231	. 26168	. 00065	1	. 99935	52
9	. 73997	229	. 26003	. 74063	229	. 25937	. 00066	0	. 99934	51
10	8. 74226	228	11. 25774	8. 74292	229	11. 25708	10. 00066	1	9. 99934	50
11	. 74454	226	. 25546	. 74521	227	. 25479	. 00067	1	. 99933	49
12	. 74680	226	. 25320	. 74748	226	. 25252	. 00068	0	. 99932	48
13	. 74906	224	. 25094	. 74974	225	. 25026	. 00068	1	. 99932	47
14	. 75130	223	. 24870	. 75199	224	. 24801	. 00069	1	. 99931	46
15	8. 75353	222	11. 24647	8. 75423	222	11. 24577	10. 00070	1	9. 99930	45
16	. 75575	220	. 24425	. 75645	222	. 24355	. 00071	0	. 99929	44
17	. 75795	220	. 24205	. 75867	220	. 24133	. 00071	1	. 99929	43
18	. 76015	219	. 23985	. 76087	219	. 23913	. 00072	1	. 99928	42
19	. 76234	217	. 23766	. 76306	219	. 23694	. 00073	1	. 99927	41
20	8. 76451	216	11. 23549	8. 76525	217	11. 23475	10. 00074	0	9. 99926	40
21	. 76667	216	. 23333	. 76742	216	. 23258	. 00074	1	. 99926	39
22	. 76883	214	. 23117	. 76958	215	. 23042	. 00075	1	. 99925	38
23	. 77097	213	. 22903	. 77173	214	. 22827	. 00076	1	. 99924	37
24	. 77310	212	. 22690	. 77387	213	. 22613	. 00077	0	. 99923	36
25	8. 77522	211	11. 22478	8. 77600	211	11. 22400	10. 00077	1	9. 99923	35
26	. 77733	210	. 22267	. 77811	211	. 22189	. 00078	1	. 99922	34
27	. 77943	209	. 22057	. 78022	210	. 21978	. 00079	1	. 99921	33
28	. 78152	208	. 21848	. 78232	209	. 21768	. 00080	0	. 99920	32
29	. 78360	208	. 21640	. 78441	208	. 21559	. 00080	1	. 99920	31
30	8. 78568	206	11. 21432	8. 78649	206	11. 21351	10. 00081	1	9. 99919	30
31	. 78774	205	. 21226	. 78855	206	. 21145	. 00082	1	. 99918	29
32	. 78979	204	. 21021	. 79061	205	. 20939	. 00083	0	. 99917	28
33	. 79183	203	. 20817	. 79266	204	. 20734	. 00083	1	. 99917	27
34	. 79386	202	. 20614	. 79470	203	. 20530	. 00084	1	. 99916	26
35	8. 79588	201	11. 20412	8. 79673	202	11. 20327	10. 00085	1	9. 99915	25
36	. 79789	201	. 20211	. 79875	201	. 20125	. 00086	1	. 99914	24
37	. 79990	199	. 20010	. 80076	201	. 19924	. 00087	0	. 99913	23
38	. 80189	199	. 19811	. 80277	199	. 19723	. 00087	1	. 99913	22
39	. 80388	197	. 19612	. 80476	198	. 19524	. 00088	1	. 99912	21
40	8. 80585	197	11. 19415	8. 80674	198	11. 19326	10. 00089	1	9. 99911	20
41	. 80782	196	. 19218	. 80872	196	. 19128	. 00090	1	. 99910	19
42	. 80978	195	. 19022	. 81068	196	. 18932	. 00091	0	. 99909	18
43	. 81173	194	. 18827	. 81264	195	. 18736	. 00091	1	. 99909	17
44	. 81367	193	. 18633	. 81459	194	. 18541	. 00092	1	. 99908	16
45	8. 81560	192	11. 18440	8. 81653	193	11. 18347	10. 00093	1	9. 99907	15
46	. 81752	192	. 18248	. 81846	192	. 18154	. 00094	1	. 99906	14
47	. 81944	190	. 18056	. 82038	192	. 17962	. 00095	1	. 99905	13
48	. 82134	190	. 17866	. 82230	190	. 17770	. 00096	0	. 99904	12
49	. 82324	189	. 17676	. 82420	190	. 17580	. 00096	1	. 99904	11
50	8. 82513	188	11. 17487	8. 82610	189	11. 17390	10. 00097	1	9. 99903	10
51	. 82701	187	. 17299	. 82799	188	. 17201	. 00098	1	. 99902	9
52	. 82888	187	. 17112	. 82987	188	. 17013	. 00099	1	. 99901	8
53	. 83075	186	. 16925	. 83175	186	. 16825	. 00100	1	. 99900	7
54	. 83261	185	. 16739	. 83361	186	. 16639	. 00101	1	. 99899	6
55	8. 83446	184	11. 16554	8. 83547	185	11. 16453	10. 00102	0	9. 99898	5
56	. 83630	183	. 16370	. 83732	184	. 16268	. 00102	1	. 99898	4
57	. 83813	183	. 16187	. 83916	184	. 16084	. 00103	1	. 99897	3
58	. 83996	181	. 16004	. 84100	182	. 15900	. 00104	1	. 99896	2
59	. 84177	181	. 15823	. 84282	182	. 15718	. 00105	1	. 99895	1
60	8. 84358		11. 15642	8. 84464		11. 15536	10. 00106		9. 99894	0
↑ 93° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 86° ↑

TABLE 33

Logarithms of Trigonometric Functions

4° → ↓	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 175° ↓
′										′
0	8. 84358	181	11. 15642	8. 84464	182	11. 15536	10. 00106	1	9. 99894	60
1	. 84539	179	. 15461	. 84646	180	. 15354	. 00107	1	. 99893	59
2	. 84718	179	. 15282	. 84826	180	. 15174	. 00108	1	. 99892	58
3	. 84897	178	. 15103	. 85006	179	. 14994	. 00109	0	. 99891	57
4	. 85075	177	. 14925	. 85185	178	. 14815	. 00109	1	. 99891	56
5	8. 85252	177	11. 14748	8. 85363	177	11. 14637	10. 00110	1	9. 99890	55
6	. 85429	176	. 14571	. 85540	177	. 14460	. 00111	1	. 99889	54
7	. 85605	175	. 14395	. 85717	176	. 14283	. 00112	1	. 99888	53
8	. 85780	175	. 14220	. 85893	176	. 14107	. 00113	1	. 99887	52
9	. 85955	173	. 14045	. 86069	174	. 13931	. 00114	1	. 99886	51
10	8. 86128	173	11. 13872	8. 86243	174	11. 13757	10. 00115	1	9. 99885	50
11	. 86301	173	. 13699	. 86417	174	. 13583	. 00116	1	. 99884	49
12	. 86474	171	. 13526	. 86591	172	. 13409	. 00117	1	. 99883	48
13	. 86645	171	. 13355	. 86763	172	. 13237	. 00118	1	. 99882	47
14	. 86816	171	. 13184	. 86935	171	. 13065	. 00119	1	. 99881	46
15	8. 86987	169	11. 13013	8. 87106	171	11. 12894	10. 00120	1	9. 99880	45
16	. 87156	169	. 12844	. 87277	170	. 12723	. 00121	0	. 99879	44
17	. 87325	169	. 12675	. 87447	169	. 12553	. 00121	1	. 99879	43
18	. 87494	167	. 12506	. 87616	169	. 12384	. 00122	1	. 99878	42
19	. 87661	168	. 12339	. 87785	168	. 12215	. 00123	1	. 99877	41
20	8. 87829	166	11. 12171	8. 87953	167	11. 12047	10. 00124	1	9. 99876	40
21	. 87995	166	. 12005	. 88120	167	. 11880	. 00125	1	. 99875	39
22	. 88161	165	. 11839	. 88287	166	. 11713	. 00126	1	. 99874	38
23	. 88326	164	. 11674	. 88453	165	. 11547	. 00127	1	. 99873	37
24	. 88490	164	. 11510	. 88618	165	. 11382	. 00128	1	. 99872	36
25	8. 88654	163	11. 11346	8. 88783	165	11. 11217	10. 00129	1	9. 99871	35
26	. 88817	163	. 11183	. 88948	163	. 11052	. 00130	1	. 99870	34
27	. 88980	162	. 11020	. 89111	163	. 10889	. 00131	1	. 99869	33
28	. 89142	162	. 10858	. 89274	163	. 10726	. 00132	1	. 99868	32
29	. 89304	160	. 10696	. 89437	161	. 10563	. 00133	1	. 99867	31
30	8. 89464	161	11. 10536	8. 89598	162	11. 10402	10. 00134	1	9. 99866	30
31	. 89625	159	. 10375	. 89760	160	. 10240	. 00135	1	. 99865	29
32	. 89784	159	. 10216	. 89920	160	. 10080	. 00136	1	. 99864	28
33	. 89943	159	. 10057	. 90080	160	. 09920	. 00137	1	. 99863	27
34	. 90102	158	. 09898	. 90240	159	. 09760	. 00138	1	. 99862	26
35	8. 90260	157	11. 09740	8. 90399	158	11. 09601	10. 00139	1	9. 99861	25
36	. 90417	157	. 09583	. 90557	158	. 09443	. 00140	1	. 99860	24
37	. 90574	156	. 09426	. 90715	157	. 09285	. 00141	1	. 99859	23
38	. 90730	155	. 09270	. 90872	157	. 09128	. 00142	1	. 99858	22
39	. 90885	155	. 09115	. 91029	156	. 08971	. 00143	1	. 99857	21
40	8. 91040	155	11. 08960	8. 91185	155	11. 08815	10. 00144	1	9. 99856	20
41	. 91195	154	. 08805	. 91340	155	. 08660	. 00145	1	. 99855	19
42	. 91349	153	. 08651	. 91495	155	. 08505	. 00146	1	. 99854	18
43	. 91502	153	. 08498	. 91650	153	. 08350	. 00147	1	. 99853	17
44	. 91655	152	. 08345	. 91803	154	. 08197	. 00148	1	. 99852	16
45	8. 91807	152	11. 08193	8. 91957	153	11. 08043	10. 00149	1	9. 99851	15
46	. 91959	151	. 08041	. 92110	152	. 07890	. 00150	2	. 99850	14
47	. 92110	151	. 07890	. 92262	152	. 07738	. 00152	1	. 99848	13
48	. 92261	150	. 07739	. 92414	151	. 07586	. 00153	1	. 99847	12
49	. 92411	150	. 07589	. 92565	151	. 07435	. 00154	1	. 99846	11
50	8. 92561	149	11. 07439	8. 92716	150	11. 07284	10. 00155	1	9. 99845	10
51	. 92710	149	. 07290	. 92866	150	. 07134	. 00156	1	. 99844	9
52	. 92859	148	. 07141	. 93016	149	. 06984	. 00157	1	. 99843	8
53	. 93007	147	. 06993	. 93165	148	. 06835	. 00158	1	. 99842	7
54	. 93154	147	. 06846	. 93313	149	. 06687	. 00159	1	. 99841	6
55	8. 93301	147	11. 06699	8. 93462	147	11. 06538	10. 00160	1	9. 99840	5
56	. 93448	146	. 06552	. 93609	147	. 06391	. 00161	1	. 99839	4
57	. 93594	146	. 06406	. 93756	147	. 06244	. 00162	1	. 99838	3
58	. 93740	145	. 06260	. 93903	146	. 06097	. 00163	1	. 99837	2
59	. 93885	145	. 06115	. 94049	146	. 05951	. 00164	2	. 99836	1
60	8. 94030		11. 05970	8. 94195		11. 05805	10. 00166		9. 99834	0
↑ 94° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ← 85°

TABLE 33
Logarithms of Trigonometric Functions

5°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←174° ↓ ′
0	8. 94030	144	11. 05970	8. 94195	145	11. 05805	10. 00166	1	9. 99834	60
1	. 94174	143	. 05826	. 94340	145	. 05660	. 00167	1	. 99833	59
2	. 94317	144	. 05683	. 94485	145	. 05515	. 00168	1	. 99832	58
3	. 94461	142	. 05539	. 94630	143	. 05370	. 00169	1	. 99831	57
4	. 94603	143	. 05397	. 94773	144	. 05227	. 00170	1	. 99830	56
5	8. 94746	141	11. 05254	8. 94917	143	11. 05083	10. 00171	1	9. 99829	55
6	. 94887	142	. 05113	. 95060	142	. 04940	. 00172	1	. 99828	54
7	. 95029	141	. 04971	. 95202	142	. 04798	. 00173	2	. 99827	53
8	. 95170	140	. 04830	. 95344	142	. 04656	. 00175	1	. 99825	52
9	. 95310	140	. 04690	. 95486	141	. 04514	. 00176	1	. 99824	51
10	8 95450	139	11. 04550	8. 95627	140	11. 04373	10. 00177	1	9. 99823	50
11	.. 95589	139	. 04411	. 95767	141	. 04233	. 00178	1	. 99822	49
12	. 95728	139	. 04272	. 95908	139	. 04092	. 00179	1	. 99821	48
13	. 95867	138	. 04133	. 96047	140	. 03953	. 00180	1	. 99820	47
14	. 96005	138	. 03995	. 96187	138	. 03813	. 00181	2	. 99819	46
15	8. 96143	137	11. 03857	8. 96325	139	11. 03675	10. 00183	1	9. 99817	45
16	. 96280	137	. 03720	. 96464	138	. 03536	. 00184	1	. 99816	44
17	. 96417	136	. 03583	. 96602	137	. 03398	. 00185	1	. 99815	43
18	. 96553	136	. 03447	. 96739	138	. 03261	. 00186	1	. 99814	42
19	. 96689	136	. 03311	. 96877	136	. 03123	. 00187	1	. 99813	41
20	8. 96825	135	11. 03175	8. 97013	137	11. 02987	10. 00188	2	9. 99812	40
21	. 96960	135	. 03040	. 97150	135	. 02850	. 00190	1	. 99810	39
22	. 97095	134	. 02905	. 97285	136	. 02715	. 00191	1	. 99809	38
23	. 97229	134	. 02771	. 97421	135	. 02579	. 00192	1	. 99808	37
24	. 97363	133	. 02637	. 97556	135	. 02444	. 00193	1	. 99807	36
25	8. 97496	133	11. 02504	8. 97691	134	11. 02309	10. 00194	2	9. 99806	35
26	. 97629	133	. 02371	. 97825	134	. 02175	. 00196	1	. 99804	34
27	. 97762	132	. 02238	. 97959	133	. 02041	. 00197	1	. 99803	33
28	. 97894	132	. 02106	. 98092	133	. 01908	. 00198	1	. 99802	32
29	. 98026	131	. 01974	. 98225	133	. 01775	. 00199	1	. 99801	31
30	8. 98157	131	11. 01843	8. 98358	132	11. 01642	10. 00200	2	9. 99800	30
31	. 98288	131	. 01712	. 98490	132	. 01510	. 00202	1	. 99798	29
32	. 98419	130	. 01581	. 98622	131	. 01378	. 00203	1	. 99797	28
33	. 98549	130	. 01451	. 98753	131	. 01247	. 00204	1	. 99796	27
34	. 98679	129	. 01321	. 98884	131	. 01116	. 00205	2	. 99795	26
35	8. 98808	129	11. 01192	8. 99015	130	11. 00985	10. 00207	1	9. 99793	25
36	. 98937	129	. 01063	. 99145	130	. 00855	. 00208	1	. 99792	24
37	. 99066	128	. 00934	. 99275	130	. 00725	. 00209	1	. 99791	23
38	. 99194	128	. 00806	. 99405	129	. 00595	. 00210	2	. 99790	22
39	. 99322	128	. 00678	. 99534	128	. 00466	. 00212	1	. 99788	21
40	8. 99450	127	11. 00550	8. 99662	129	11. 00338	10. 00213	1	9. 99787	20
41	. 99577	127	. 00423	. 99791	128	. 00209	. 00214	1	. 99786	19
42	. 99704	126	. 00296	8. 99919	127	11. 00081	. 00215	2	. 99785	18
43	. 99830	126	. 00170	9. 00046	128	10. 99954	. 00217	1	. 99783	17
44	8. 99956	126	11. 00044	. 00174	127	. 99826	. 00218	1	. 99782	16
45	9. 00082	125	10. 99918	9. 00301	126	10. 99699	10. 00219	1	9. 99781	15
46	. 00207	125	. 99793	. 00427	126	. 99573	. 00220	2	. 99780	14
47	. 00332	124	. 99668	. 00553	126	. 99447	. 00222	1	. 99778	13
48	. 00456	125	. 99544	. 00679	126	. 99321	. 00223	1	. 99777	12
49	. 00581	123	. 99419	. 00805	125	. 99195	. 00224	1	. 99776	11
50	9. 00704	124	10. 99296	9. 00930	125	10. 99070	10. 00225	2	9. 99775	10
51	. 00828	123	. 99172	. 01055	124	. 98945	. 00227	1	. 99773	9
52	. 00951	123	. 99049	. 01179	124	. 98821	. 00228	1	. 99772	8
53	. 01074	122	. 98926	. 01303	124	. 98697	. 00229	2	. 99771	7
54	. 01196	122	. 98804	. 01427	123	. 98573	. 00231	1	. 99769	6
55	9. 01318	122	10. 98682	9. 01550	123	10. 98450	10. 00232	1	9. 99768	5
56	. 01440	121	. 98560	. 01673	123	. 98327	. 00233	2	. 99767	4
57	. 01561	121	. 98439	. 01796	122	. 98204	. 00235	1	. 99765	3
58	. 01682	121	. 98318	. 01918	122	. 98082	. 00236	1	. 99764	2
59	. 01803	120	. 98197	. 02040	122	. 97960	. 00237	2	. 99763	1
60	9. 01923		10. 98077	9. 02162		10. 97838	10. 00239		9. 99761	0
↑ 95°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←84°

TABLE 33
Logarithms of Trigonometric Functions

6°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←173° ↓ ′
0	9. 01923	120	10. 98077	9. 02162	121	10. 97838	10. 00239	1	9. 99761	60
1	. 02043	120	. 97957	. 02283	121	. 97717	. 00240	1	. 99760	59
2	. 02163	120	. 97837	. 02404	121	. 97596	. 00241	2	. 99759	58
3	. 02283	119	. 97717	. 02525	120	. 97475	. 00243	1	. 99757	57
4	. 02402	118	. 97598	. 02645	121	. 97355	. 00244	1	. 99756	56
5	9. 02520	119	10. 97480	9. 02766	119	10. 97234	10. 00245	2	9. 99755	55
6	. 02639	118	. 97361	. 02885	120	. 97115	. 00247	1	. 99753	54
7	. 02757	117	. 97243	. 03005	119	. 96995	. 00248	1	. 99752	53
8	. 02874	118	. 97126	. 03124	118	. 96876	. 00249	2	. 99751	52
9	. 02992	117	. 97008	. 03242	119	. 96758	. 00251	1	. 99749	51
10	9. 03109	117	10. 96891	9. 03361	118	10. 96639	10. 00252	1	9. 99748	50
11	. 03226	116	. 96774	. 03479	118	. 96521	. 00253	2	. 99747	49
12	. 03342	116	. 96658	. 03597	117	. 96403	. 00255	1	. 99745	48
13	. 03458	116	. 96542	. 03714	118	. 96286	. 00256	2	. 99744	47
14	. 03574	116	. 96426	. 03832	116	. 96168	. 00258	1	. 99742	46
15	9. 03690	115	10. 96310	9. 03948	117	10. 96052	10. 00259	1	9. 99741	45
16	. 03805	115	. 96195	. 04065	116	. 95935	. 00260	2	. 99740	44
17	. 03920	114	. 96080	. 04181	116	. 95819	. 00262	1	. 99738	43
18	. 04034	115	. 95966	. 04297	116	. 95703	. 00263	1	. 99737	42
19	. 04149	113	. 95851	. 04413	115	. 95587	. 00264	2	. 99736	41
20	9. 04262	114	10. 95738	9. 04528	115	10. 95472	10. 00266	1	9. 99734	40
21	. 04376	114	. 95624	. 04643	115	. 95357	. 00267	2	. 99733	39
22	. 04490	113	. 95510	. 04758	115	. 95242	. 00269	1	. 99731	38
23	. 04603	112	. 95397	. 04873	114	. 95127	. 00270	2	. 99730	37
24	. 04715	113	. 95285	. 04987	114	. 95013	. 00272	1	. 99728	36
25	9. 04828	112	10. 95172	9. 05101	113	10. 94899	10. 00273	1	9. 99727	35
26	. 04940	112	. 95060	. 05214	114	. 94786	. 00274	2	. 99726	34
27	. 05052	112	. 94948	. 05328	113	. 94672	. 00276	1	. 99724	33
28	. 05164	111	. 94836	. 05441	112	. 94559	. 00277	2	. 99723	32
29	. 05275	111	. 94725	. 05553	113	. 94447	. 00279	1	. 99721	31
30	9. 05386	111	10. 94614	9. 05666	112	10. 94334	10. 00280	2	9. 99720	30
31	. 05497	110	. 94503	. 05778	112	. 94222	. 00282	1	. 99718	29
32	. 05607	110	. 94393	. 05890	112	. 94110	. 00283	1	. 99717	28
33	. 05717	110	. 94283	. 06002	111	. 93998	. 00284	2	. 99716	27
34	. 05827	110	. 94173	. 06113	111	. 93887	. 00286	1	. 99714	26
35	9. 05937	109	10. 94063	9. 06224	111	10. 93776	10. 00287	2	9. 99713	25
36	. 06046	109	. 93954	. 06335	110	. 93665	. 00289	1	. 99711	24
37	. 06155	109	. 93845	. 06445	111	. 93555	. 00290	2	. 99710	23
38	. 06264	108	. 93736	. 06556	110	. 93444	. 00292	1	. 99708	22
39	. 06372	109	. 93628	. 06666	109	. 93334	. 00293	2	. 99707	21
40	9. 06481	108	10. 93519	9. 06775	110	10. 93225	10. 00295	1	9. 99705	20
41	. 06589	107	. 93411	. 06885	109	. 93115	. 00296	2	. 99704	19
42	. 06696	108	. 93304	. 06994	109	. 93006	. 00298	1	. 99702	18
43	. 06804	107	. 93196	. 07103	108	. 92897	. 00299	2	. 99701	17
44	. 06911	107	. 93089	. 07211	109	. 92789	. 00301	1	. 99699	16
45	9. 07018	106	10. 92982	9. 07320	108	10. 92680	10. 00302	2	9. 99698	15
46	. 07124	107	. 92876	. 07428	108	. 92572	. 00304	1	. 99696	14
47	. 07231	106	. 92769	. 07536	107	. 92464	. 00305	2	. 99695	13
48	. 07337	105	. 92663	. 07643	108	. 92357	. 00307	1	. 99693	12
49	. 07442	106	. 92558	. 07751	107	. 92249	. 00308	2	. 99692	11
50	9. 07548	105	10. 92452	9. 07858	106	10. 92142	10. 00310	1	9. 99690	10
51	. 07653	105	. 92347	. 07964	107	. 92036	. 00311	2	. 99689	9
52	. 07758	105	. 92242	. 08071	106	. 91929	. 00313	1	. 99687	8
53	. 07863	105	. 92137	. 08177	106	. 91823	. 00314	2	. 99686	7
54	. 07968	104	. 92032	. 08283	106	. 91717	. 00316	1	. 99684	6
55	9. 08072	104	10. 91928	9. 08389	106	10. 91611	10. 00317	2	9. 99683	5
56	. 08176	104	. 91824	. 08495	105	. 91505	. 00319	1	. 99681	4
57	. 08280	103	. 91720	. 08600	105	. 91400	. 00320	2	. 99680	3
58	. 08383	103	. 91617	. 08705	105	. 91295	. 00322	1	. 99678	2
59	. 08486	103	. 91514	. 08810	104	. 91190	. 00323	2	. 99677	1
60	9. 08589		10. 91411	9. 08914		10. 91086	10. 00325		9. 99675	0
↑ 96°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←83°

TABLE 33

Logarithms of Trigonometric Functions

7° → ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 172° ′
0	9. 08589	103	10. 91411	9. 08914	105	10. 91086	10. 00325	1	9. 99675	60
1	. 08692	103	. 91308	. 09019	104	. 90981	. 00326	2	. 99674	59
2	. 08795	102	. 91205	. 09123	104	. 90877	. 00328	2	. 99672	58
3	. 08897	102	. 91103	. 09227	103	. 90773	. 00330	1	. 99670	57
4	. 08999	102	. 91001	. 09330	104	. 90670	. 00331	2	. 99669	56
5	9. 09101	101	10. 90899	9. 09434	103	10. 90566	10. 00333	1	9. 99667	55
6	. 09202	102	. 90798	. 09537	103	. 90463	. 00334	2	. 99666	54
7	. 09304	101	. 90696	. 09640	102	. 90360	. 00336	1	. 99664	53
8	. 09405	101	. 90595	. 09742	103	. 90258	. 00337	2	. 99663	52
9	. 09506	100	. 90494	. 09845	102	. 90155	. 00339	2	. 99661	51
10	9. 09606	101	10. 90394	9. 09947	102	10. 90053	10. 00341	1	9. 99659	50
11	. 09707	100	. 90293	. 10049	101	. 89951	. 00342	2	. 99658	49
12	. 09807	100	. 90193	. 10150	102	. 89850	. 00344	1	. 99656	48
13	. 09907	99	. 90093	. 10252	101	. 89748	. 00345	2	. 99655	47
14	. 10006	100	. 89994	. 10353	101	. 89647	. 00347	2	. 99653	46
15	9. 10106	99	10. 89894	9. 10454	101	10. 89546	10. 00349	1	9. 99651	45
16	. 10205	99	. 89795	. 10555	101	. 89445	. 00350	2	. 99650	44
17	. 10304	98	. 89696	. 10656	100	. 89344	. 00352	1	. 99648	43
18	. 10402	99	. 89598	. 10756	100	. 89244	. 00353	2	. 99647	42
19	. 10501	98	. 89499	. 10856	100	. 89144	. 00355	2	. 99645	41
20	9. 10599	98	10. 89401	9. 10956	100	10. 89044	10. 00357	1	9. 99643	40
21	. 10697	98	. 89303	. 11056	99	. 88944	. 00358	2	. 99642	39
22	. 10795	98	. 89205	. 11155	99	. 88845	. 00360	2	. 99640	38
23	. 10893	97	. 89107	. 11254	99	. 88746	. 00362	1	. 99638	37
24	. 10990	97	. 89010	. 11353	99	. 88647	. 00363	2	. 99637	36
25	9. 11087	97	10. 88913	9. 11452	99	10. 88548	10. 00365	2	9. 99635	35
26	. 11184	97	. 88816	. 11551	98	. 88449	. 00367	1	. 99633	34
27	. 11281	96	. 88719	. 11649	98	. 88351	. 00368	2	. 99632	33
28	. 11377	97	. 88623	. 11747	98	. 88253	. 00370	1	. 99630	32
29	. 11474	96	. 88526	. 11845	98	. 88155	. 00371	2	. 99629	31
30	9. 11570	96	10. 88430	9. 11943	97	10. 88057	10. 00373	2	9. 99627	30
31	. 11666	95	. 88334	. 12040	98	. 87960	. 00375	1	. 99625	29
32	. 11761	96	. 88239	. 12138	97	. 87862	. 00376	2	. 99624	28
33	. 11857	95	. 88143	. 12235	97	. 87765	. 00378	2	. 99622	27
34	. 11952	95	. 88048	. 12332	96	. 87668	. 00380	2	. 99620	26
35	9. 12047	95	10. 87953	9. 12428	97	10. 87572	10. 00382	1	9. 99618	25
36	. 12142	94	. 87858	. 12525	96	. 87475	. 00383	2	. 99617	24
37	. 12236	95	. 87764	. 12621	96	. 87379	. 00385	2	. 99615	23
38	. 12331	94	. 87669	. 12717	96	. 87283	. 00387	1	. 99613	22
39	. 12425	94	. 87575	. 12813	96	. 87187	. 00388	2	. 99612	21
40	9. 12519	93	10. 87481	9. 12909	95	10. 87091	10. 00390	2	9. 99610	20
41	. 12612	94	. 87388	. 13004	95	. 86996	. 00392	1	. 99608	19
42	. 12706	93	. 87294	. 13099	95	. 86901	. 00393	2	. 99607	18
43	. 12799	93	. 87201	. 13194	95	. 86806	. 00395	2	. 99605	17
44	. 12892	93	. 87108	. 13289	95	. 86711	. 00397	2	. 99603	16
45	9. 12985	93	10. 87015	9. 13384	94	10. 86616	10. 00399	1	9. 99601	15
46	. 13078	93	. 86922	. 13478	95	. 86522	. 00400	2	. 99600	14
47	. 13171	92	. 86829	. 13573	94	. 86427	. 00402	2	. 99598	13
48	. 13263	92	. 86737	. 13667	94	. 86333	. 00404	1	. 99596	12
49	. 13355	92	. 86645	. 13761	93	. 86239	. 00405	2	. 99595	11
50	9. 13447	92	10. 86553	9. 13854	94	10. 86146	10. 00407	2	9. 99593	10
51	. 13539	91	. 86461	. 13948	93	. 86052	. 00409	2	. 99591	9
52	. 13630	92	. 86370	. 14041	93	. 85959	. 00411	1	. 99589	8
53	. 13722	91	. 86278	. 14134	93	. 85866	. 00412	2	. 99588	7
54	. 13813	91	. 86187	. 14227	93	. 85773	. 00414	2	. 99586	6
55	9. 13904	90	10. 86096	9. 14320	92	10. 85680	10. 00416	2	9. 99584	5
56	. 13994	91	. 86006	. 14412	92	. 85588	. 00418	1	. 99582	4
57	. 14085	90	. 85915	. 14504	93	. 85496	. 00419	2	. 99581	3
58	. 14175	91	. 85825	. 14597	91	. 85403	. 00421	2	. 99579	2
59	. 14266	90	. 85734	. 14688	92	. 85312	. 00423	2	. 99577	1
60	9. 14356		10. 85644	9. 14780		10. 85220	10. 00425		9. 99575	0
97° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 82°

TABLE 33

Logarithms of Trigonometric Functions

8°→ ↓	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←171° ↓
′										′
0	9. 14356	89	10. 85644	9. 14780	92	10. 85220	10. 00425	1	9. 99575	60
1	. 14445	90	. 85555	. 14872	91	. 85128	. 00426	2	. 99574	59
2	. 14535	89	. 85465	. 14963	91	. 85037	. 00428	2	. 99572	58
3	. 14624	90	. 85376	. 15054	91	. 84946	. 00430	2	. 99570	57
4	. 14714	89	. 85286	. 15145	91	. 84855	. 00432	2	. 99568	56
5	9. 14803	88	10. 85197	9. 15236	91	10. 84764	10. 00434	1	9. 99566	55
6	. 14891	89	. 85109	. 15327	90	. 84673	. 00435	2	. 99565	54
7	. 14980	89	. 85020	. 15417	91	. 84583	. 00437	2	. 99563	53
8	. 15069	88	. 84931	. 15508	90	. 84492	. 00439	2	. 99561	52
9	. 15157	88	. 84843	. 15598	90	. 84402	. 00441	2	. 99559	51
10	9. 15245	88	10. 84755	9. 15688	89	10. 84312	10. 00443	1	9. 99557	50
11	. 15333	88	. 84667	. 15777	90	. 84223	. 00444	2	. 99556	49
12	. 15421	87	. 84579	. 15867	89	. 84133	. 00446	2	. 99554	48
13	. 15508	88	. 84492	. 15956	90	. 84044	. 00448	2	. 99552	47
14	. 15596	87	. 84404	. 16046	89	. 83954	. 00450	2	. 99550	46
15	9. 15683	87	10. 84317	9. 16135	89	10. 83865	10. 00452	2	9. 99548	45
16	. 15770	87	. 84230	. 16224	88	. 83776	. 00454	1	. 99546	44
17	. 15857	87	. 84143	. 16312	89	. 83688	. 00455	2	. 99545	43
18	. 15944	86	. 84056	. 16401	88	. 83599	. 00457	2	. 99543	42
19	. 16030	86	. 83970	. 16489	88	. 83511	. 00459	2	. 99541	41
20	9. 16116	87	10. 83884	9. 16577	88	10. 83423	10. 00461	2	9. 99539	40
21	. 16203	86	. 83797	. 16665	88	. 83335	. 00463	2	. 99537	39
22	. 16289	85	. 83711	. 16753	88	. 83247	. 00465	2	. 99535	38
23	. 16374	86	. 83626	. 16841	87	. 83159	. 00467	1	. 99533	37
24	. 16460	85	. 83540	. 16928	88	. 83072	. 00468	2	. 99532	36
25	9. 16545	86	10. 83455	9. 17016	87	10. 82984	10. 00470	2	9. 99530	35
26	. 16631	85	. 83369	. 17103	87	. 82897	. 00472	2	. 99528	34
27	. 16716	85	. 83284	. 17190	87	. 82810	. 00474	2	. 99526	33
28	. 16801	85	. 83199	. 17277	86	. 82723	. 00476	2	. 99524	32
29	. 16886	84	. 83114	. 17363	87	. 82637	. 00478	2	. 99522	31
30	9. 16970	85	10. 83030	9. 17450	86	10. 82550	10. 00480	2	9. 99520	30
31	. 17055	84	. 82945	. 17536	86	. 82464	. 00482	1	. 99518	29
32	. 17139	84	. 82861	. 17622	86	. 82378	. 00483	2	. 99517	28
33	. 17223	84	. 82777	. 17708	86	. 82292	. 00485	2	. 99515	27
34	. 17307	84	. 82693	. 17794	86	. 82206	. 00487	2	. 99513	26
35	9. 17391	83	10. 82609	9. 17880	85	10. 82120	10. 00489	2	9. 99511	25
36	. 17474	84	. 82526	. 17965	86	. 82035	. 00491	2	. 99509	24
37	. 17558	83	. 82442	. 18051	85	. 81949	. 00493	2	. 99507	23
38	. 17641	83	. 82359	. 18136	85	. 81864	. 00495	2	. 99505	22
39	. 17724	83	. 82276	. 18221	85	. 81779	. 00497	2	. 99503	21
40	9. 17807	83	10. 82193	9. 18306	85	10. 81694	10. 00499	2	9. 99501	20
41	. 17890	83	. 82110	. 18391	84	. 81609	. 00501	2	. 99499	19
42	. 17973	82	. 82027	. 18475	85	. 81525	. 00503	2	. 99497	18
43	. 18055	82	. 81945	. 18560	84	. 81440	. 00505	1	. 99495	17
44	. 18137	83	. 81863	. 18644	84	. 81356	. 00506	2	. 99494	16
45	9. 18220	82	10. 81780	9. 18728	84	10. 81272	10. 00508	2	9. 99492	15
46	. 18302	81	. 81698	. 18812	84	. 81188	. 00510	2	. 99490	14
47	. 18383	82	. 81617	. 18896	83	. 81104	. 00512	2	. 99488	13
48	. 18465	82	. 81535	. 18979	84	. 81021	. 00514	2	. 99486	12
49	. 18547	81	. 81453	. 19063	83	. 80937	. 00516	2	. 99484	11
50	9. 18628	81	10. 81372	9. 19146	83	10. 80854	10. 00518	2	9. 99482	10
51	. 18709	81	. 81291	. 19229	83	. 80771	. 00520	2	. 99480	9
52	. 18790	81	. 81210	. 19312	83	. 80688	. 00522	2	. 99478	8
53	. 18871	81	. 81129	. 19395	83	. 80605	. 00524	2	. 99476	7
54	. 18952	81	. 81048	. 19478	83	. 80522	. 00526	2	. 99474	6
55	9. 19033	80	10. 80967	9. 19561	82	10. 80439	10. 00528	2	9. 99472	5
56	. 19113	80	. 80887	. 19643	82	. 80357	. 00530	2	. 99470	4
57	. 19193	80	. 80807	. 19725	82	. 80275	. 00532	2	. 99468	3
58	. 19273	80	. 80727	. 19807	82	. 80193	. 00534	2	. 99466	2
59	. 19353	80	. 80647	. 19889	82	. 80111	. 00536	2	. 99464	1
60	9. 19433		10. 80567	9. 19971		10. 80029	10. 00538		9. 99462	0
↑ 98°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←81°

TABLE 33

Logarithms of Trigonometric Functions

9°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←170° ↓ ′
0	9. 19433	80	10. 80567	9. 19971	82	10. 80029	10. 00538	2	9. 99462	60
1	. 19513	79	. 80487	. 20053	81	. 79947	. 00540	2	. 99460	59
2	. 19592	80	. 80408	. 20134	82	. 79866	. 00542	2	. 99458	58
3	. 19672	79	. 80328	. 20216	81	. 79784	. 00544	2	. 99456	57
4	. 19751	79	. 80249	. 20297	81	. 79703	. 00546	2	. 99454	56
5	9. 19830	79	10. 80170	9. 20378	81	10. 79622	10. 00548	2	9. 99452	55
6	. 19909	79	. 80091	. 20459	81	. 79541	. 00550	2	. 99450	54
7	. 19988	79	. 80012	. 20540	81	. 79460	. 00552	2	. 99448	53
8	. 20067	78	. 79933	. 20621	80	. 79379	. 00554	2	. 99446	52
9	. 20145	78	. 79855	. 20701	81	. 79299	. 00556	2	. 99444	51
10	9. 20223	79	10. 79777	9. 20782	80	10. 79218	10. 00558	2	9. 99442	50
11	. 20302	78	. 79698	. 20862	80	. 79138	. 00560	2	. 99440	49
12	. 20380	78	. 79620	. 20942	80	. 79058	. 00562	2	. 99438	48
13	. 20458	77	. 79542	. 21022	80	. 78978	. 00564	2	. 99436	47
14	. 20535	78	. 79465	. 21102	80	. 78898	. 00566	2	. 99434	46
15	9. 20613	78	10. 79387	9. 21182	79	10. 78818	10. 00568	3	9. 99432	45
16	. 20691	77	. 79309	. 21261	80	. 78739	. 00571	2	. 99429	44
17	. 20768	77	. 79232	. 21341	79	. 78659	. 00573	2	. 99427	43
18	. 20845	77	. 79155	. 21420	79	. 78580	. 00575	2	. 99425	42
19	. 20922	77	. 79078	. 21499	79	. 78501	. 00577	2	. 99423	41
20	9. 20999	77	10. 79001	9. 21578	79	10. 78422	10. 00579	2	9. 99421	40
21	. 21076	77	. 78924	. 21657	79	. 78343	. 00581	2	. 99419	39
22	. 21153	76	. 78847	. 21736	78	. 78264	. 00583	2	. 99417	38
23	. 21229	77	. 78771	. 21814	79	. 78186	. 00585	2	. 99415	37
24	. 21306	76	. 78694	. 21893	78	. 78107	. 00587	2	. 99413	36
25	9. 21382	76	10. 78618	9. 21971	78	10. 78029	10. 00589	2	9. 99411	35
26	. 21458	76	. 78542	. 22049	78	. 77951	. 00591	2	. 99409	34
27	. 21534	76	. 78466	. 22127	78	. 77873	. 00593	3	. 99407	33
28	. 21610	75	. 78390	. 22205	78	. 77795	. 00596	2	. 99404	32
29	. 21685	76	. 78315	. 22283	78	. 77717	. 00598	2	. 99402	31
30	9. 21761	75	10. 78239	9. 22361	77	10. 77639	10. 00600	2	9. 99400	30
31	. 21836	76	. 78164	. 22438	78	. 77562	. 00602	2	. 99398	29
32	. 21912	75	. 78088	. 22516	77	. 77484	. 00604	2	. 99396	28
33	. 21987	75	. 78013	. 22593	77	. 77407	. 00606	2	. 99394	27
34	. 22062	75	. 77938	. 22670	77	. 77330	. 00608	2	. 99392	26
35	9. 22137	74	10. 77863	9. 22747	77	10. 77253	10. 00610	2	9. 99390	25
36	. 22211	75	. 77789	. 22824	77	. 77176	. 00612	3	. 99388	24
37	. 22286	75	. 77714	. 22901	76	. 77099	. 00615	2	. 99385	23
38	. 22361	74	. 77639	. 22977	77	. 77023	. 00617	2	. 99383	22
39	. 22435	74	. 77565	. 23054	76	. 76946	. 00619	2	. 99381	21
40	9. 22509	74	10. 77491	9. 23130	76	10. 76870	10. 00621	2	9. 99379	20
41	. 22583	74	. 77417	. 23206	77	. 76794	. 00623	2	. 99377	19
42	. 22657	74	. 77343	. 23283	76	. 76717	. 00625	3	. 99375	18
43	. 22731	74	. 77269	. 23359	76	. 76641	. 00628	2	. 99372	17
44	. 22805	73	. 77195	. 23435	75	. 76565	. 00630	2	. 99370	16
45	9. 22878	74	10. 77122	9. 23510	76	10. 76490	10. 00632	2	9. 99368	15
46	. 22952	73	. 77048	. 23586	75	. 76414	. 00634	2	. 99366	14
47	. 23025	73	. 76975	. 23661	76	. 76339	. 00636	2	. 99364	13
48	. 23098	73	. 76902	. 23737	75	. 76263	. 00638	3	. 99362	12
49	. 23171	73	. 76829	. 23812	75	. 76188	. 00641	2	. 99359	11
50	9. 23244	73	10. 76756	9. 23887	75	10. 76113	10. 00643	2	9. 99357	10
51	. 23317	73	. 76683	. 23962	75	. 76038	. 00645	2	. 99355	9
52	. 23390	72	. 76610	. 24037	75	. 75963	. 00647	2	. 99353	8
53	. 23462	73	. 76538	. 24112	74	. 75888	. 00649	3	. 99351	7
54	. 23535	72	. 76465	. 24186	75	. 75814	. 00652	2	. 99348	6
55	9. 23607	72	10. 76393	9. 24261	74	10. 75739	10. 00654	2	9. 99346	5
56	. 23679	73	. 76321	. 24335	75	. 75665	. 00656	2	. 99344	4
57	. 23752	71	. 76248	. 24410	74	. 75590	. 00658	2	. 99342	3
58	. 23823	72	. 76177	. 24484	74	. 75516	. 00660	3	. 99340	2
59	. 23895	72	. 76105	. 24558	74	. 75442	. 00663	2	. 99337	1
60	9. 23967		10. 76033	9. 24632		10. 75368	10. 00665		9. 99335	0
↑ 99°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	←80° ↑

TABLE 33

Logarithms of Trigonometric Functions

10° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 169° ↓ ′
0	9. 23967	72	10. 76033	9. 24632	74	10. 75368	10. 00665	2	9. 99335	60
1	. 24039	71	. 75961	. 24706	73	. 75294	. 00667	2	. 99333	59
2	. 24110	71	. 75890	. 24779	74	. 75221	. 00669	3	. 99331	58
3	. 24181	72	. 75819	. 24853	73	. 75147	. 00672	2	. 99328	57
4	. 24253	71	. 75747	. 24926	74	. 75074	. 00674	2	. 99326	56
5	9. 24324	71	10. 75676	9. 25000	73	10. 75000	10. 00676	2	9. 99324	55
6	. 24395	71	. 75605	. 25073	73	. 74927	. 00678	3	. 99322	54
7	. 24466	70	. 75534	. 25146	73	. 74854	. 00681	2	. 99319	53
8	. 24536	71	. 75464	. 25219	73	. 74781	. 00683	2	. 99317	52
9	. 24607	70	. 75393	. 25292	73	. 74708	. 00685	2	. 99315	51
10	9. 24677	71	10. 75323	9. 25365	72	10. 74635	10. 00687	3	9. 99313	50
11	. 24748	70	. 75252	. 25437	73	. 74563	. 00690	2	. 99310	49
12	. 24818	70	. 75182	. 25510	72	. 74490	. 00692	2	. 99308	48
13	. 24888	70	. 75112	. 25582	73	. 74418	. 00694	2	. 99306	47
14	. 24958	70	. 75042	. 25655	72	. 74345	. 00696	3	. 99304	46
15	9. 25028	70	10. 74972	9. 25727	72	10. 74273	10. 00699	2	9. 99301	45
16	. 25098	70	. 74902	. 25799	72	. 74201	. 00701	2	. 99299	44
17	. 25168	69	. 74832	. 25871	72	. 74129	. 00703	3	. 99297	43
18	. 25237	70	. 74763	. 25943	72	. 74057	. 00706	2	. 99294	42
19	. 25307	69	. 74693	. 26015	71	. 73985	. 00708	2	. 99292	41
20	9. 25376	69	10. 74624	9. 26086	72	10. 73914	10. 00710	2	9. 99290	40
21	. 25445	69	. 74555	. 26158	71	. 73842	. 00712	3	. 99288	39
22	. 25514	69	. 74486	. 26229	72	. 73771	. 00715	2	. 99285	38
23	. 25583	69	. 74417	. 26301	71	. 73699	. 00717	2	. 99283	37
24	. 25652	69	. 74348	. 26372	71	. 73628	. 00719	3	. 99281	36
25	9. 25721	69	10. 74279	9. 26443	71	10. 73557	10. 00722	2	9. 99278	35
26	. 25790	68	. 74210	. 26514	71	. 73486	. 00724	2	. 99276	34
27	. 25858	69	. 74142	. 26585	70	. 73415	. 00726	3	. 99274	33
28	. 25927	68	. 74073	. 26655	71	. 73345	. 00729	2	. 99271	32
29	. 25995	68	. 74005	. 26726	71	. 73274	. 00731	2	. 99269	31
30	9. 26063	68	10. 73937	9. 26797	70	10. 73203	10. 00733	3	9. 99267	30
31	. 26131	68	. 73869	. 26867	70	. 73133	. 00736	2	. 99264	29
32	. 26199	68	. 73801	. 26937	71	. 73063	. 00738	2	. 99262	28
33	. 26267	68	. 73733	. 27008	70	. 72992	. 00740	3	. 99260	27
34	. 26335	68	. 73665	. 27078	70	. 72922	. 00743	2	. 99257	26
35	9. 26403	67	10. 73597	9. 27148	70	10. 72852	10. 00745	3	9. 99255	25
36	. 26470	68	. 73530	. 27218	70	. 72782	. 00748	2	. 99252	24
37	. 26538	67	. 73462	. 27288	69	. 72712	. 00750	2	. 99250	23
38	. 26605	67	. 73395	. 27357	70	. 72643	. 00752	3	. 99248	22
39	. 26672	67	. 73328	. 27427	69	. 72573	. 00755	2	. 99245	21
40	9. 26739	67	10. 73261	9. 27496	70	10. 72504	10. 00757	2	9. 99243	20
41	. 26806	67	. 73194	. 27566	69	. 72434	. 00759	3	. 99241	19
42	. 26873	67	. 73127	. 27635	69	. 72365	. 00762	2	. 99238	18
43	. 26940	67	. 73060	. 27704	69	. 72296	. 00764	3	. 99236	17
44	. 27007	66	. 72993	. 27773	69	. 72227	. 00767	2	. 99233	16
45	9. 27073	67	10. 72927	9. 27842	69	10. 72158	10. 00769	2	9. 99231	15
46	. 27140	66	. 72860	. 27911	69	. 72089	. 00771	3	. 99229	14
47	. 27206	67	. 72794	. 27980	69	. 72020	. 00774	2	. 99226	13
48	. 27273	66	. 72727	. 28049	68	. 71951	. 00776	3	. 99224	12
49	. 27339	66	. 72661	. 28117	69	. 71883	. 00779	2	. 99221	11
50	9. 27405	66	10. 72595	9. 28186	68	10. 71814	10. 00781	2	9. 99219	10
51	. 27471	66	. 72529	. 28254	69	. 71746	. 00783	3	. 99217	9
52	. 27537	65	. 72463	. 28323	68	. 71677	. 00786	2	. 99214	8
53	. 27602	66	. 72398	. 28391	68	. 71609	. 00788	3	. 99212	7
54	. 27668	66	. 72332	. 28459	68	. 71541	. 00791	2	. 99209	6
55	9. 27734	65	10. 72266	9. 28527	68	10. 71473	10. 00793	3	9. 99207	5
56	. 27799	65	. 72201	. 28595	67	. 71405	. 00796	2	. 99204	4
57	. 27864	66	. 72136	. 28662	68	. 71338	. 00798	2	. 99202	3
58	. 27930	65	. 72070	. 28730	68	. 71270	. 00800	3	. 99200	2
59	. 27995	65	. 72005	. 28798	67	. 71202	. 00803	2	. 99197	1
60	9. 28060		10. 71940	9. 28865		10. 71135	10. 00805		9. 99195	0
↑ 100° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 79° ↑

TABLE 33

Logarithms of Trigonometric Functions

11° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 168° ↓ ′
0	9. 28060	65	10. 71940	9. 28865	68	10. 71135	10. 00805	3	9. 99195	60
1	. 28125	65	. 71875	. 28933	67	. 71067	. 00808	2	. 99192	59
2	. 28190	64	. 71810	. 29000	67	. 71000	. 00810	3	. 99190	58
3	. 28254	65	. 71746	. 29067	67	. 70933	. 00813	2	. 99187	57
4	. 28319	65	. 71681	. 29134	67	. 70866	. 00815	3	. 99185	56
5	9. 28384	64	10. 71616	9. 29201	67	10. 70799	10. 00818	2	9. 99182	55
6	. 28448	64	. 71552	. 29268	67	. 70732	. 00820	3	. 99180	54
7	. 28512	65	. 71488	. 29335	67	. 70665	. 00823	2	. 99177	53
8	. 28577	64	. 71423	. 29402	66	. 70598	. 00825	3	. 99175	52
9	. 28641	64	. 71359	. 29468	67	. 70532	. 00828	2	. 99172	51
10	9. 28705	64	10. 71295	9. 29535	66	10. 70465	10. 00830	3	9. 99170	50
11	. 28769	64	. 71231	. 29601	67	. 70399	. 00833	2	. 99167	49
12	. 28833	63	. 71167	. 29668	66	. 70332	. 00835	3	. 99165	48
13	. 28896	64	. 71104	. 29734	66	. 70266	. 00838	2	. 99162	47
14	. 28960	64	. 71040	. 29800	66	. 70200	. 00840	3	. 99160	46
15	9. 29024	63	10. 70976	9. 29866	66	10. 70134	10. 00843	2	9. 99157	45
16	. 29087	63	. 70913	. 29932	66	. 70068	. 00845	3	. 99155	44
17	. 29150	64	. 70850	. 29998	66	. 70002	. 00848	2	. 99152	43
18	. 29214	63	. 70786	. 30064	66	. 69936	. 00850	3	. 99150	42
19	. 29277	63	. 70723	. 30130	65	. 69870	. 00853	2	. 99147	41
20	9. 29340	63	10. 70660	9. 30195	66	10. 69805	10. 00855	3	9. 99145	40
21	. 29403	63	. 70597	. 30261	65	. 69739	. 00858	2	. 99142	39
22	. 29466	63	. 70534	. 30326	65	. 69674	. 00860	3	. 99140	38
23	. 29529	62	. 70471	. 30391	66	. 69609	. 00863	2	. 99137	37
24	. 29591	63	. 70409	. 30457	65	. 69543	. 00865	3	. 99135	36
25	9. 29654	62	10. 70346	9. 30522	65	10. 69478	10. 00868	2	9. 99132	35
26	. 29716	63	. 70284	. 30587	65	. 69413	. 00870	3	. 99130	34
27	. 29779	62	. 70221	. 30652	65	. 69348	. 00873	3	. 99127	33
28	. 29841	62	. 70159	. 30717	65	. 69283	. 00876	2	. 99124	32
29	. 29903	63	. 70097	. 30782	64	. 69218	. 00878	3	. 99122	31
30	9. 29966	62	10. 70034	9. 30846	65	10. 69154	10. 00881	2	9. 99119	30
31	. 30028	62	. 69972	. 30911	64	. 69089	. 00883	3	. 99117	29
32	. 30090	61	. 69910	. 30975	65	. 69025	. 00886	2	. 99114	28
33	. 30151	62	. 69849	. 31040	64	. 68960	. 00888	3	. 99112	27
34	. 30213	62	. 69787	. 31104	64	. 68896	. 00891	3	. 99109	26
35	9. 30275	61	10. 69725	9. 31168	65	10. 68832	10. 00894	2	9. 99106	25
36	. 30336	62	. 69664	. 31233	64	. 68767	. 00896	3	. 99104	24
37	. 30398	61	. 69602	. 31297	64	. 68703	. 00899	2	. 99101	23
38	. 30459	62	. 69541	. 31361	64	. 68639	. 00901	3	. 99099	22
39	. 30521	61	. 69479	. 31425	64	. 68575	. 00904	3	. 99096	21
40	9. 30582	61	10. 69418	9. 31489	63	10. 68511	10. 00907	2	9. 99093	20
41	. 30643	61	. 69357	. 31552	64	. 68448	. 00909	3	. 99091	19
42	. 30704	61	. 69296	. 31616	63	. 68384	. 00912	2	. 99088	18
43	. 30765	61	. 69235	. 31679	64	. 68321	. 00914	3	. 99086	17
44	. 30826	61	. 69174	. 31743	63	. 68257	. 00917	3	. 99083	16
45	9. 30887	60	10. 69113	9. 31806	64	10. 68194	10. 00920	2	9. 99080	15
46	. 30947	61	. 69053	. 31870	63	. 68130	. 00922	3	. 99078	14
47	. 31008	60	. 68992	. 31933	63	. 68067	. 00925	3	. 99075	13
48	. 31068	61	. 68932	. 31996	63	. 68004	. 00928	2	. 99072	12
49	. 31129	60	. 68871	. 32059	63	. 67941	. 00930	3	. 99070	11
50	9. 31189	61	10. 68811	9. 32122	63	10. 67878	10. 00933	3	9. 99067	10
51	. 31250	60	. 68750	. 32185	63	. 67815	. 00936	2	. 99064	9
52	. 31310	60	. 68690	. 32248	63	. 67752	. 00938	3	. 99062	8
53	. 31370	60	. 68630	. 32311	62	. 67689	. 00941	3	. 99059	7
54	. 31430	60	. 68570	. 32373	63	. 67627	. 00944	2	. 99056	6
55	9. 31490	59	10. 68510	9. 32436	62	10. 67564	10. 00946	3	9. 99054	5
56	. 31549	60	. 68451	. 32498	63	. 67502	. 00949	3	. 99051	4
57	. 31609	60	. 68391	. 32561	62	. 67439	. 00952	2	. 99048	3
58	. 31669	59	. 68331	. 32623	62	. 67377	. 00954	3	. 99046	2
59	. 31728	60	. 68272	. 32685	62	. 67315	. 00957	3	. 99043	1
60	9. 31788		10. 68212	9. 32747		10. 67253	10. 00960		9. 99040	0
↑ 101° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 78° ↑

TABLE 33
Logarithms of Trigonometric Functions

12° → ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 167° ′
0	9.31788	59	10.68212	9.32747	63	10.67253	10.00960	2	9.99040	60
1	.31847	60	.68153	.32810	62	.67190	.00962	3	.99038	59
2	.31907	59	.68093	.32872	61	.67128	.00965	3	.99035	58
3	.31966	59	.68034	.32933	62	.67067	.00968	2	.99032	57
4	.32025	59	.67975	.32995	62	.67005	.00970	3	.99030	56
5	9.32084	59	10.67916	9.33057	62	10.66943	10.00973	3	9.99027	55
6	.32143	59	.67857	.33119	61	.66881	.00976	2	.99024	54
7	.32202	59	.67798	.33180	62	.66820	.00978	3	.99022	53
8	.32261	58	.67739	.33242	61	.66758	.00981	3	.99019	52
9	.32319	59	.67681	.33303	62	.66697	.00984	3	.99016	51
10	9.32378	59	10.67622	9.33365	61	10.66635	10.00987	2	9.99013	50
11	.32437	58	.67563	.33426	61	.66574	.00989	3	.99011	49
12	.32495	58	.67505	.33487	61	.66513	.00992	3	.99008	48
13	.32553	59	.67447	.33548	61	.66452	.00995	3	.99005	47
14	.32612	58	.67388	.33609	61	.66391	.00998	2	.99002	46
15	9.32670	58	10.67330	9.33670	61	10.66330	10.01000	3	9.99000	45
16	.32728	58	.67272	.33731	61	.66269	.01003	3	.98997	44
17	.32786	58	.67214	.33792	61	.66208	.01006	3	.98994	43
18	.32844	58	.67156	.33853	60	.66147	.01009	2	.98991	42
19	.32902	58	.67098	.33913	61	.66087	.01011	3	.98989	41
20	9.32960	58	10.67040	9.33974	60	10.66026	10.01014	3	9.98986	40
21	.33018	57	.66982	.34034	61	.65966	.01017	3	.98983	39
22	.33075	58	.66925	.34095	60	.65905	.01020	2	.98980	38
23	.33133	57	.66867	.34155	60	.65845	.01022	3	.98978	37
24	.33190	58	.66810	.34215	61	.65785	.01025	3	.98975	36
25	9.33248	57	10.66752	9.34276	60	10.65724	10.01028	3	9.98972	35
26	.33305	57	.66695	.34336	60	.65664	.01031	2	.98969	34
27	.33362	58	.66638	.34396	60	.65604	.01033	3	.98967	33
28	.33420	57	.66580	.34456	60	.65544	.01036	3	.98964	32
29	.33477	57	.66523	.34516	60	.65484	.01039	3	.98961	31
30	9.33534	57	10.66466	9.34576	59	10.65424	10.01042	3	9.98958	30
31	.33591	56	.66409	.34635	60	.65365	.01045	2	.98955	29
32	.33647	57	.66353	.34695	60	.65305	.01047	3	.98953	28
33	.33704	57	.66296	.34755	59	.65245	.01050	3	.98950	27
34	.33761	57	.66239	.34814	60	.65186	.01053	3	.98947	26
35	9.33818	56	10.66182	9.34874	59	10.65126	10.01056	3	9.98944	25
36	.33874	57	.66126	.34933	59	.65067	.01059	3	.98941	24
37	.33931	56	.66069	.34992	59	.65008	.01062	2	.98938	23
38	.33987	56	.66013	.35051	60	.64949	.01064	3	.98936	22
39	.34043	57	.65957	.35111	59	.64889	.01067	3	.98933	21
40	9.34100	56	10.65900	9.35170	59	10.64830	10.01070	3	9.98930	20
41	.34156	56	.65844	.35229	59	.64771	.01073	3	.98927	19
42	.34212	56	.65788	.35288	59	.64712	.01076	3	.98924	18
43	.34268	56	.65732	.35347	58	.64653	.01079	2	.98921	17
44	.34324	56	.65676	.35405	59	.64595	.01081	3	.98919	16
45	9.34380	56	10.65620	9.35464	59	10.64536	10.01084	3	9.98916	15
46	.34436	55	.65564	.35523	58	.64477	.01087	3	.98913	14
47	.34491	56	.65509	.35581	59	.64419	.01090	3	.98910	13
48	.34547	55	.65453	.35640	58	.64360	.01093	3	.98907	12
49	.34602	56	.65398	.35698	59	.64302	.01096	3	.98904	11
50	9.34658	55	10.65342	9.35757	58	10.64243	10.01099	3	9.98901	10
51	.34713	56	.65287	.35815	58	.64185	.01102	2	.98898	9
52	.34769	55	.65231	.35873	58	.64127	.01104	3	.98896	8
53	.34824	55	.65176	.35931	58	.64069	.01107	3	.98893	7
54	.34879	55	.65121	.35989	58	.64011	.01110	3	.98890	6
55	9.34934	55	10.65066	9.36047	58	10.63953	10.01113	3	9.98887	5
56	.34989	55	.65011	.36105	58	.63895	.01116	3	.98884	4
57	.35044	55	.64956	.36163	58	.63837	.01119	3	.98881	3
58	.35099	55	.64901	.36221	58	.63779	.01122	3	.98878	2
59	.35154	55	.64846	.36279	57	.63721	.01125	3	.98875	1
60	9.35209		10.64791	9.36336		10.63664	10.01128		9.98872	0
102° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 77°

TABLE 33
Logarithms of Trigonometric Functions

13° → ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 166° ′
0	9. 35209	54	10. 64791	9. 36336	58	10. 63664	10. 01128	3	9. 98872	60
1	. 35263	55	. 64737	. 36394	58	. 63606	. 01131	2	. 98869	59
2	. 35318	55	. 64682	. 36452	57	. 63548	. 01133	3	. 98867	58
3	. 35373	54	. 64627	. 36509	57	. 63491	. 01136	3	. 98864	57
4	. 35427	54	. 64573	. 36566	58	. 63434	. 01139	3	. 98861	56
5	9. 35481	55	10. 64519	9. 36624	57	10. 63376	10. 01142	3	9. 98858	55
6	. 35536	54	. 64464	. 36681	57	. 63319	. 01145	3	. 98855	54
7	. 35590	54	. 64410	. 36738	57	. 63262	. 01148	3	. 98852	53
8	. 35644	54	. 64356	. 36795	57	. 63205	. 01151	3	. 98849	52
9	. 35698	54	. 64302	. 36852	57	. 63148	. 01154	3	. 98846	51
10	9. 35752	54	10. 64248	9. 36909	57	10. 63091	10. 01157	3	9. 98843	50
11	. 35806	54	. 64194	. 36966	57	. 63034	. 01160	3	. 98840	49
12	. 35860	54	. 64140	. 37023	57	. 62977	. 01163	3	. 98837	48
13	. 35914	54	. 64086	. 37080	57	. 62920	. 01166	3	. 98834	47
14	. 35968	54	. 64032	. 37137	56	. 62863	. 01169	3	. 98831	46
15	9. 36022	53	10. 63978	9. 37193	57	10. 62807	10. 01172	3	9. 98828	45
16	. 36075	54	. 63925	. 37250	56	. 62750	. 01175	3	. 98825	44
17	. 36129	53	. 63871	. 37306	57	. 62694	. 01178	3	. 98822	43
18	. 36182	54	. 63818	. 37363	56	. 62637	. 01181	3	. 98819	42
19	. 36236	53	. 63764	. 37419	57	. 62581	. 01184	3	. 98816	41
20	9. 36289	53	10. 63711	9. 37476	56	10. 62524	10. 01187	3	9. 98813	40
21	. 36342	53	. 63658	. 37532	56	. 62468	. 01190	3	. 98810	39
22	. 36395	54	. 63605	. 37588	56	. 62412	. 01193	3	. 98807	38
23	. 36449	53	. 63551	. 37644	56	. 62356	. 01196	3	. 98804	37
24	. 36502	53	. 63498	. 37700	56	. 62300	. 01199	3	. 98801	36
25	9. 36555	53	10. 63445	9. 37756	56	10. 62244	10. 01202	3	9. 98798	35
26	. 36608	52	. 63392	. 37812	56	. 62188	. 01205	3	. 98795	34
27	. 36660	53	. 63340	. 37868	56	. 62132	. 01208	3	. 98792	33
28	. 36713	53	. 63287	. 37924	56	. 62076	. 01211	3	. 98789	32
29	. 36766	53	. 63234	. 37980	55	. 62020	. 01214	3	. 98786	31
30	9. 36819	52	10. 63181	9. 38035	56	10. 61965	10. 01217	3	9. 98783	30
31	. 36871	53	. 63129	. 38091	56	. 61909	. 01220	3	. 98780	29
32	. 36924	52	. 63076	. 38147	55	. 61853	. 01223	3	. 98777	28
33	. 36976	52	. 63024	. 38202	55	. 61798	. 01226	3	. 98774	27
34	. 37028	53	. 62972	. 38257	56	. 61743	. 01229	3	. 98771	26
35	9. 37081	52	10. 62919	9. 38313	55	10. 61687	10. 01232	3	9. 98768	25
36	. 37133	52	. 62867	. 38368	55	. 61632	. 01235	3	. 98765	24
37	. 37185	52	. 62815	. 38423	56	. 61577	. 01238	3	. 98762	23
38	. 37237	52	. 62763	. 38479	55	. 61521	. 01241	3	. 98759	22
39	. 37289	52	. 62711	. 38534	55	. 61466	. 01244	3	. 98756	21
40	9. 37341	52	10. 62659	9. 38589	55	10. 61411	10. 01247	3	9. 98753	20
41	. 37393	52	. 62607	. 38644	55	. 61356	. 01250	4	. 98750	19
42	. 37445	52	. 62555	. 38699	55	. 61301	. 01254	3	. 98746	18
43	. 37497	52	. 62503	. 38754	54	. 61246	. 01257	3	. 98743	17
44	. 37549	51	. 62451	. 38808	55	. 61192	. 01260	3	. 98740	16
45	9. 37600	52	10. 62400	9. 38863	55	10. 61137	10. 01263	3	9. 98737	15
46	. 37652	51	. 62348	. 38918	54	. 61082	. 01266	3	. 98734	14
47	. 37703	52	. 62297	. 38972	55	. 61028	. 01269	3	. 98731	13
48	. 37755	51	. 62245	. 39027	55	. 60973	. 01272	3	. 98728	12
49	. 37806	52	. 62194	. 39082	54	. 60918	. 01275	3	. 98725	11
50	9. 37858	51	10. 62142	9. 39136	54	10. 60864	10. 01278	3	9. 98722	10
51	. 37909	51	. 62091	. 39190	55	. 60810	. 01281	4	. 98719	9
52	. 37960	51	. 62040	. 39245	54	. 60755	. 01285	3	. 98715	8
53	. 38011	51	. 61989	. 39299	54	. 60701	. 01288	3	. 98712	7
54	. 38062	51	. 61938	. 39353	54	. 60647	. 01291	3	. 98709	6
55	9. 38113	51	10. 61887	9. 39407	54	10. 60593	10. 01294	3	9. 98706	5
56	. 38164	51	. 61836	. 39461	54	. 60539	. 01297	3	. 98703	4
57	. 38215	51	. 61785	. 39515	54	. 60485	. 01300	3	. 98700	3
58	. 38266	51	. 61734	. 39569	54	. 60431	. 01303	3	. 98697	2
59	. 38317	51	. 61683	. 39623	54	. 60377	. 01306	4	. 98694	1
60	9. 38368		10. 61632	9. 39677		10. 60323	10. 01310		9. 98690	0
103° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 76°

TABLE 33

Logarithms of Trigonometric Functions

14° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 165° ↓ ′
0	9. 38368	50	10. 61632	9. 39677	54	10. 60323	10. 01310	3	9. 98690	60
1	. 38418	51	. 61582	. 39731	54	. 60269	. 01313	3	. 98687	59
2	. 38469	50	. 61531	. 39785	53	. 60215	. 01316	3	. 98684	58
3	. 38519	51	. 61481	. 39838	54	. 60162	. 01319	3	. 98681	57
4	. 38570	50	. 61430	. 39892	53	. 60108	. 01322	3	. 98678	56
5	9. 38620	50	10. 61380	9. 39945	54	10. 60055	10. 01325	4	9. 98675	55
6	. 38670	51	. 61330	. 39999	53	. 60001	. 01329	3	. 98671	54
7	. 38721	50	. 61279	. 40052	54	. 59948	. 01332	3	. 98668	53
8	. 38771	50	. 61229	. 40106	53	. 59894	. 01335	3	. 98665	52
9	. 38821	50	. 61179	. 40159	53	. 59841	. 01338	3	. 98662	51
10	9. 38871	50	10. 61129	9. 40212	54	10. 59788	10. 01341	3	9. 98659	50
11	. 38921	50	. 61079	. 40266	53	. 59734	. 01344	4	. 98656	49
12	. 38971	50	. 61029	. 40319	53	. 59681	. 01348	3	. 98652	48
13	. 39021	50	. 60979	. 40372	53	. 59628	. 01351	3	. 98649	47
14	. 39071	50	. 60929	. 40425	53	. 59575	. 01354	3	. 98646	46
15	9. 39121	49	10. 60879	9. 40478	53	10. 59522	10. 01357	3	9. 98643	45
16	. 39170	50	. 60830	. 40531	53	. 59469	. 01360	4	. 98640	44
17	. 39220	50	. 60780	. 40584	52	. 59416	. 01364	3	. 98636	43
18	. 39270	49	. 60730	. 40636	53	. 59364	. 01367	3	. 98633	42
19	. 39319	50	. 60681	. 40689	53	. 59311	. 01370	3	. 98630	41
20	9. 39369	49	10. 60631	9. 40742	53	10. 59258	10. 01373	4	9. 98627	40
21	. 39418	49	. 60582	. 40795	52	. 59205	. 01377	3	. 98623	39
22	. 39467	50	. 60533	. 40847	53	. 59153	. 01380	3	. 98620	38
23	. 39517	49	. 60483	. 40900	52	. 59100	. 01383	3	. 98617	37
24	. 39566	49	. 60434	. 40952	53	. 59048	. 01386	4	. 98614	36
25	9. 39615	49	10. 60385	9. 41005	52	10. 58995	10. 01390	3	9. 98610	35
26	. 39664	49	. 60336	. 41057	52	. 58943	. 01393	3	. 98607	34
27	. 39713	49	. 60287	. 41109	52	. 58891	. 01396	3	. 98604	33
28	. 39762	49	. 60238	. 41161	53	. 58839	. 01399	4	. 98601	32
29	. 39811	49	. 60189	. 41214	52	. 58786	. 01403	3	. 98597	31
30	9. 39860	49	10. 60140	9. 41266	52	10. 58734	10. 01406	3	9. 98594	30
31	. 39909	49	. 60091	. 41318	52	. 58682	. 01409	3	. 98591	29
32	. 39958	48	. 60042	. 41370	52	. 58630	. 01412	4	. 98588	28
33	. 40006	49	. 59994	. 41422	52	. 58578	. 01416	3	. 98584	27
34	. 40055	48	. 59945	. 41474	52	. 58526	. 01419	3	. 98581	26
35	9. 40103	49	10. 59897	9. 41526	52	10. 58474	10. 01422	4	9. 98578	25
36	. 40152	48	. 59848	. 41578	51	. 58422	. 01426	3	. 98574	24
37	. 40200	49	. 59800	. 41629	52	. 58371	. 01429	3	. 98571	23
38	. 40249	48	. 59751	. 41681	52	. 58319	. 01432	3	. 98568	22
39	. 40297	49	. 59703	. 41733	51	. 58267	. 01435	4	. 98565	21
40	9. 40346	48	10. 59654	9. 41784	52	10. 58216	10. 01439	3	9. 98561	20
41	. 40394	48	. 59606	. 41836	51	. 58164	. 01442	3	. 98558	19
42	. 40442	48	. 59558	. 41887	52	. 58113	. 01445	4	. 98555	18
43	. 40490	48	. 59510	. 41939	51	. 58061	. 01449	3	. 98551	17
44	. 40538	48	. 59462	. 41990	51	. 58010	. 01452	3	. 98548	16
45	9. 40586	48	10. 59414	9. 42041	52	10. 57959	10. 01455	4	9. 98545	15
46	. 40634	48	. 59366	. 42093	51	. 57907	. 01459	3	. 98541	14
47	. 40682	48	. 59318	. 42144	51	. 57856	. 01462	3	. 98538	13
48	. 40730	48	. 59270	. 42195	51	. 57805	. 01465	4	. 98535	12
49	. 40778	47	. 59222	. 42246	51	. 57754	. 01469	3	. 98531	11
50	9. 40825	48	10. 59175	9. 42297	51	10. 57703	10. 01472	3	9. 98528	10
51	. 40873	48	. 59127	. 42348	51	. 57652	. 01475	4	. 98525	9
52	. 40921	47	. 59079	. 42399	51	. 57601	. 01479	3	. 98521	8
53	. 40968	48	. 59032	. 42450	51	. 57550	. 01482	3	. 98518	7
54	. 41016	47	. 58984	. 42501	51	. 57499	. 01485	4	. 98515	6
55	9. 41063	48	10. 58937	9. 42552	51	10. 57448	10. 01489	3	9. 98511	5
56	. 41111	47	. 58889	. 42603	50	. 57397	. 01492	3	. 98508	4
57	. 41158	47	. 58842	. 42653	51	. 57347	. 01495	4	. 98505	3
58	. 41205	47	. 58795	. 42704	51	. 57296	. 01499	3	. 98501	2
59	. 41252	48	. 58748	. 42755	50	. 57245	. 01502	4	. 98498	1
60	9. 41300		10. 58700	9. 42805		10. 57195	10. 01506		9. 98494	0
↑ 104° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 75° ↑

TABLE 33
Logarithms of Trigonometric Functions

15° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 164° ↓ ′
0	9. 41300	47	10. 58700	9. 42805	51	10. 57195	10. 01506	3	9. 98494	60
1	. 41347	47	. 58653	. 42856	50	. 57144	. 01509	3	. 98491	59
2	. 41394	47	. 58606	. 42906	51	. 57094	. 01512	4	. 98488	58
3	. 41441	47	. 58559	. 42957	50	. 57043	. 01516	3	. 98484	57
4	. 41488	47	. 58512	. 43007	50	. 56993	. 01519	4	. 98481	56
5	9. 41535	47	10. 58465	9. 43057	51	10. 56943	10. 01523	3	9. 98477	55
6	. 41582	46	. 58418	. 43108	50	. 56892	. 01526	3	. 98474	54
7	. 41628	47	. 58372	. 43158	50	. 56842	. 01529	4	. 98471	53
8	. 41675	47	. 58325	. 43208	50	. 56792	. 01533	3	. 98467	52
9	. 41722	46	. 58278	. 43258	50	. 56742	. 01536	4	. 98464	51
10	9. 41768	47	10. 58232	9. 43308	50	10. 56692	10. 01540	3	9. 98460	50
11	. 41815	46	. 58185	. 43358	50	. 56642	. 01543	4	. 98457	49
12	. 41861	47	. 58139	. 43408	50	. 56592	. 01547	3	. 98453	48
13	. 41908	46	. 58092	. 43458	50	. 56542	. 01550	3	. 98450	47
14	. 41954	47	. 58046	. 43508	50	. 56492	. 01553	4	. 98447	46
15	9. 42001	46	10. 57999	9. 43558	49	10. 56442	10. 01557	3	9. 98443	45
16	. 42047	46	. 57953	. 43607	50	. 56393	. 01560	4	. 98440	44
17	. 42093	47	. 57907	. 43657	50	. 56343	. 01564	3	. 98436	43
18	. 42140	46	. 57860	. 43707	49	. 56293	. 01567	4	. 98433	42
19	. 42186	46	. 57814	. 43756	50	. 56244	. 01571	3	. 98429	41
20	9. 42232	46	10. 57768	9. 43806	49	10. 56194	10. 01574	4	9. 98426	40
21	. 42278	46	. 57722	. 43855	50	. 56145	. 01578	3	. 98422	39
22	. 42324	46	. 57676	. 43905	49	. 56095	. 01581	4	. 98419	38
23	. 42370	46	. 57630	. 43954	50	. 56046	. 01585	3	. 98415	37
24	. 42416	45	. 57584	. 44004	49	. 55996	. 01588	3	. 98412	36
25	9. 42461	46	10. 57539	9. 44053	49	10. 55947	10. 01591	4	9. 98409	35
26	. 42507	46	. 57493	. 44102	49	. 55898	. 01595	3	. 98405	34
27	. 42553	46	. 57447	. 44151	50	. 55849	. 01598	4	. 98402	33
28	. 42599	45	. 57401	. 44201	49	. 55799	. 01602	3	. 98398	32
29	. 42644	46	. 57356	. 44250	49	. 55750	. 01605	4	. 98395	31
30	9. 42690	45	10. 57310	9. 44299	49	10. 55701	10. 01609	3	9. 98391	30
31	. 42735	46	. 57265	. 44348	49	. 55652	. 01612	4	. 98388	29
32	. 42781	45	. 57219	. 44397	49	. 55603	. 01616	3	. 98384	28
33	. 42826	46	. 57174	. 44446	49	. 55554	. 01619	4	. 98381	27
34	. 42872	45	. 57128	. 44495	49	. 55505	. 01623	4	. 98377	26
35	9. 42917	45	10. 57083	9. 44544	48	10. 55456	10. 01627	3	9. 98373	25
36	. 42962	46	. 57038	. 44592	49	. 55408	. 01630	4	. 98370	24
37	. 43008	45	. 56992	. 44641	49	. 55359	. 01634	3	. 98366	23
38	. 43053	45	. 56947	. 44690	48	. 55310	. 01637	4	. 98363	22
39	. 43098	45	. 56902	. 44738	49	. 55262	. 01641	3	. 98359	21
40	9. 43143	45	10. 56857	9. 44787	49	10. 55213	10. 01644	4	9. 98356	20
41	. 43188	45	. 56812	. 44836	48	. 55164	. 01648	3	. 98352	19
42	. 43233	45	. 56767	. 44884	49	. 55116	. 01651	4	. 98349	18
43	. 43278	45	. 56722	. 44933	48	. 55067	. 01655	3	. 98345	17
44	. 43323	44	. 56677	. 44981	48	. 55019	. 01658	4	. 98342	16
45	9. 43367	45	10. 56633	9. 45029	49	10. 54971	10. 01662	4	9. 98338	15
46	. 43412	45	. 56588	. 45078	48	. 54922	. 01666	3	. 98334	14
47	. 43457	45	. 56543	. 45126	48	. 54874	. 01669	4	. 98331	13
48	. 43502	44	. 56498	. 45174	48	. 54826	. 01673	3	. 98327	12
49	. 43546	45	. 56454	. 45222	49	. 54778	. 01676	4	. 98324	11
50	9. 43591	44	10. 56409	9. 45271	48	10. 54729	10. 01680	3	9. 98320	10
51	. 43635	45	. 56365	. 45319	48	. 54681	. 01683	4	. 98317	9
52	. 43680	44	. 56320	. 45367	48	. 54633	. 01687	4	. 98313	8
53	. 43724	45	. 56276	. 45415	48	. 54585	. 01691	3	. 98309	7
54	. 43769	44	. 56231	. 45463	48	. 54537	. 01694	4	. 98306	6
55	9. 43813	44	10. 56187	9. 45511	48	10. 54489	10. 01698	3	9. 98302	5
56	. 43857	44	. 56143	. 45559	47	. 54441	. 01701	4	. 98299	4
57	. 43901	45	. 56099	. 45606	48	. 54394	. 01705	4	. 98295	3
58	. 43946	44	. 56054	. 45654	48	. 54346	. 01709	3	. 98291	2
59	. 43990	44	. 56010	. 45702	48	. 54298	. 01712	4	. 98288	1
60	9. 44034		10. 55966	9. 45750		10. 54250	10. 01716		9. 98284	0
↑ 105° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 74° ↑

TABLE 33
Logarithms of Trigonometric Functions

16° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 163° ↓ ′
0	9. 44034	44	10. 55966	9. 45750	47	10. 54250	10. 01716	3	9. 98284	60
1	. 44078	44	. 55922	. 45797	48	. 54203	. 01719	4	. 98281	59
2	. 44122	44	. 55878	. 45845	47	. 54155	. 01723	4	. 98277	58
3	. 44166	44	. 55834	. 45892	48	. 54108	. 01727	3	. 98273	57
4	. 44210	43	. 55790	. 45940	47	. 54060	. 01730	4	. 98270	56
5	9. 44253	44	10. 55747	9. 45987	48	10. 54013	10. 01734	4	9. 98266	55
6	. 44297	44	. 55703	. 46035	47	. 53965	. 01738	3	. 98262	54
7	. 44341	44	. 55659	. 46082	48	. 53918	. 01741	4	. 98259	53
8	. 44385	43	. 55615	. 46130	47	. 53870	. 01745	4	. 98255	52
9	. 44428	44	. 55572	. 46177	47	. 53823	. 01749	3	. 98251	51
10	9. 44472	44	10. 55528	9. 46224	47	10. 53776	10. 01752	4	9. 98248	50
11	. 44516	43	. 55484	. 46271	48	. 53729	. 01756	4	. 98244	49
12	. 44559	43	. 55441	. 46319	47	. 53681	. 01760	3	. 98240	48
13	. 44602	44	. 55398	. 46366	47	. 53634	. 01763	4	. 98237	47
14	. 44646	43	. 55354	. 46413	47	. 53587	. 01767	4	. 98233	46
15	9. 44689	44	10. 55311	9. 46460	47	10. 53540	10. 01771	3	9. 98229	45
16	. 44733	43	. 55267	. 46507	47	. 53493	. 01774	4	. 98226	44
17	. 44776	43	. 55224	. 46554	47	. 53446	. 01778	4	. 98222	43
18	. 44819	43	. 55181	. 46601	47	. 53399	. 01782	3	. 98218	42
19	. 44862	43	. 55138	. 46648	46	. 53352	. 01785	4	. 98215	41
20	9. 44905	43	10. 55095	9. 46694	47	10. 53306	10. 01789	4	9. 98211	40
21	. 44948	44	. 55052	. 46741	47	. 53259	. 01793	3	. 98207	39
22	. 44992	43	. 55008	. 46788	47	. 53212	. 01796	4	. 98204	38
23	. 45035	42	. 54965	. 46835	46	. 53165	. 01800	4	. 98200	37
24	. 45077	43	. 54923	. 46881	47	. 53119	. 01804	4	. 98196	36
25	9. 45120	43	10. 54880	9. 46928	47	10. 53072	10. 01808	3	9. 98192	35
26	. 45163	43	. 54837	. 46975	46	. 53025	. 01811	4	. 98189	34
27	. 45206	43	. 54794	. 47021	47	. 52979	. 01815	4	. 98185	33
28	. 45249	43	. 54751	. 47068	46	. 52932	. 01819	4	. 98181	32
29	. 45292	42	. 54708	. 47114	46	. 52886	. 01823	3	. 98177	31
30	9. 45334	43	10. 54666	9. 47160	47	10. 52840	10. 01826	4	9. 98174	30
31	. 45377	42	. 54623	. 47207	46	. 52793	. 01830	4	. 98170	29
32	. 45419	43	. 54581	. 47253	46	. 52747	. 01834	4	. 98166	28
33	. 45462	42	. 54538	. 47299	47	. 52701	. 01838	3	. 98162	27
34	. 45504	43	. 54496	. 47346	46	. 52654	. 01841	4	. 98159	26
35	9. 45547	42	10. 54453	9. 47392	46	10. 52608	10. 01845	4	9. 98155	25
36	. 45589	43	. 54411	. 47438	46	. 52562	. 01849	4	. 98151	24
37	. 45632	42	. 54368	. 47484	46	. 52516	. 01853	3	. 98147	23
38	. 45674	42	. 54326	. 47530	46	. 52470	. 01856	4	. 98144	22
39	. 45716	42	. 54284	. 47576	46	. 52424	. 01860	4	. 98140	21
40	9. 45758	43	10. 54242	9. 47622	46	10. 52378	10. 01864	4	9. 98136	20
41	. 45801	42	. 54199	. 47668	46	. 52332	. 01868	3	. 98132	19
42	. 45843	42	. 54157	. 47714	46	. 52286	. 01871	4	. 98129	18
43	. 45885	42	. 54115	. 47760	46	. 52240	. 01875	4	. 98125	17
44	. 45927	42	. 54073	. 47806	46	. 52194	. 01879	4	. 98121	16
45	9. 45969	42	10. 54031	9. 47852	45	10. 52148	10. 01883	4	9. 98117	15
46	. 46011	42	. 53989	. 47897	46	. 52103	. 01887	3	. 98113	14
47	. 46053	42	. 53947	. 47943	46	. 52057	. 01890	4	. 98110	13
48	. 46095	41	. 53905	. 47989	46	. 52011	. 01894	4	. 98106	12
49	. 46136	42	. 53864	. 48035	45	. 51965	. 01898	4	. 98102	11
50	9. 46178	42	10. 53822	9. 48080	46	10. 51920	10. 01902	4	9. 98098	10
51	. 46220	42	. 53780	. 48126	45	. 51874	. 01906	4	. 98094	9
52	. 46262	41	. 53738	. 48171	46	. 51829	. 01910	3	. 98090	8
53	. 46303	42	. 53697	. 48217	45	. 51783	. 01913	4	. 98087	7
54	. 46345	41	. 53655	. 48262	45	. 51738	. 01917	4	. 98083	6
55	9. 46386	42	10. 53614	9. 48307	46	10. 51693	10. 01921	4	9. 98079	5
56	. 46428	41	. 53572	. 48353	45	. 51647	. 01925	4	. 98075	4
57	. 46469	42	. 53531	. 48398	45	. 51602	. 01929	4	. 98071	3
58	. 46511	41	. 53489	. 48443	46	. 51557	. 01933	4	. 98067	2
59	. 46552	42	. 53448	. 48489	45	. 51511	. 01937	3	. 98063	1
60	9. 46594		10. 53406	9. 48534		10. 51466	10. 01940		9. 98060	0
↑ 106° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ← 73°

TABLE 33

Logarithms of Trigonometric Functions

17°→ ′ ↓	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos ←162°	′ ↓
0	9. 46594	41	10. 53406	9. 48534	45	10. 51466	10. 01940	4	9. 98060	60
1	. 46635	41	. 53365	. 48579	45	. 51421	. 01944	4	. 98056	59
2	. 46676	41	. 53324	. 48624	45	. 51376	. 01948	4	. 98052	58
3	. 46717	41	. 53283	. 48669	45	. 51331	. 01952	4	. 98048	57
4	. 46758	42	. 53242	. 48714	45	. 51286	. 01956	4	. 98044	56
5	9. 46800	41	10. 53200	9. 48759	45	10. 51241	10. 01960	4	9. 98040	55
6	. 46841	41	. 53159	. 48804	45	. 51196	. 01964	4	. 98036	54
7	. 46882	41	. 53118	. 48849	45	. 51151	. 01968	3	. 98032	53
8	. 46923	41	. 53077	. 48894	45	. 51106	. 01971	4	. 98029	52
9	. 46964	41	. 53036	. 48939	45	. 51061	. 01975	4	. 98025	51
10	9. 47005	40	10. 52995	9. 48984	45	10. 51016	10. 01979	4	9. 98021	50
11	. 47045	41	. 52955	. 49029	44	. 50971	. 01983	4	. 98017	49
12	. 47086	41	. 52914	. 49073	45	. 50927	. 01987	4	. 98013	48
13	. 47127	41	. 52873	. 49118	45	. 50882	. 01991	4	. 98009	47
14	. 47168	41	. 52832	. 49163	44	. 50837	. 01995	4	. 98005	46
15	9. 47209	40	10. 52791	9. 49207	45	10. 50793	10. 01999	4	9. 98001	45
16	. 47249	41	. 52751	. 49252	44	. 50748	. 02003	4	. 97997	44
17	. 47290	40	. 52710	. 49296	45	. 50704	. 02007	4	. 97993	43
18	. 47330	41	. 52670	. 49341	44	. 50659	. 02011	3	. 97989	42
19	. 47371	40	. 52629	. 49385	45	. 50615	. 02014	4	. 97986	41
20	9. 47411	41	10. 52589	9. 49430	44	10. 50570	10. 02018	4	9. 97982	40
21	. 47452	40	. 52548	. 49474	45	. 50526	. 02022	4	. 97978	39
22	. 47492	41	. 52508	. 49519	44	. 50481	. 02026	4	. 97974	38
23	. 47533	40	. 52467	. 49563	44	. 50437	. 02030	4	. 97970	37
24	. 47573	40	. 52427	. 49607	45	. 50393	. 02034	4	. 97966	36
25	9. 47613	41	10. 52387	9. 49652	44	10. 50348	10. 02038	4	9. 97962	35
26	. 47654	40	. 52346	. 49696	44	. 50304	. 02042	4	. 97958	34
27	. 47694	40	. 52306	. 49740	44	. 50260	. 02046	4	. 97954	33
28	. 47734	40	. 52266	. 49784	44	. 50216	. 02050	4	. 97950	32
29	. 47774	40	. 52226	. 49828	44	. 50172	. 02054	4	. 97946	31
30	9. 47814	40	10. 52186	9. 49872	44	10. 50128	10. 02058	4	9. 97942	30
31	. 47854	40	. 52146	. 49916	44	. 50084	. 02062	4	. 97938	29
32	. 47894	40	. 52106	. 49960	44	. 50040	. 02066	4	. 97934	28
33	. 47934	40	. 52066	. 50004	44	. 49996	. 02070	4	. 97930	27
34	. 47974	40	. 52026	. 50048	44	. 49952	. 02074	4	. 97926	26
35	9. 48014	40	10. 51986	9. 50092	44	10. 49908	10. 02078	4	9. 97922	25
36	. 48054	40	. 51946	. 50136	44	. 49864	. 02082	4	. 97918	24
37	. 48094	39	. 51906	. 50180	43	. 49820	. 02086	4	. 97914	23
38	. 48133	40	. 51867	. 50223	44	. 49777	. 02090	4	. 97910	22
39	. 48173	40	. 51827	. 50267	44	. 49733	. 02094	4	. 97906	21
40	9. 48213	39	10. 51787	9. 50311	44	10. 49689	10. 02098	4	9. 97902	20
41	. 48252	40	. 51748	. 50355	43	. 49645	. 02102	4	. 97898	19
42	. 48292	40	. 51708	. 50398	44	. 49602	. 02106	4	. 97894	18
43	. 48332	39	. 51668	. 50442	43	. 49558	. 02110	4	. 97890	17
44	. 48371	40	. 51629	. 50485	44	. 49515	. 02114	4	. 97886	16
45	9. 48411	39	10. 51589	9. 50529	43	10. 49471	10. 02118	4	9. 97882	15
46	. 48450	40	. 51550	. 50572	44	. 49428	. 02122	4	. 97878	14
47	. 48490	39	. 51510	. 50616	43	. 49384	. 02126	4	. 97874	13
48	. 48529	39	. 51471	. 50659	44	. 49341	. 02130	4	. 97870	12
49	. 48568	39	. 51432	. 50703	43	. 49297	. 02134	5	. 97866	11
50	9. 48607	40	10. 51393	9. 50746	43	10. 49254	10. 02139	4	9. 97861	10
51	. 48647	39	. 51353	. 50789	44	. 49211	. 02143	4	. 97857	9
52	. 48686	39	. 51314	. 50833	43	. 49167	. 02147	4	. 97853	8
53	. 48725	39	. 51275	. 50876	43	. 49124	. 02151	4	. 97849	7
54	. 48764	39	. 51236	. 50919	43	. 49081	. 02155	4	. 97845	6
55	9. 48803	39	10. 51197	9. 50962	43	10. 49038	10. 02159	4	9. 97841	5
56	. 48842	39	. 51158	. 51005	43	. 48995	. 02163	4	. 97837	4
57	. 48881	39	. 51119	. 51048	44	. 48952	. 02167	4	. 97833	3
58	. 48920	39	. 51080	. 51092	43	. 48908	. 02171	4	. 97829	2
59	. 48959	39	. 51041	. 51135	43	. 48865	. 02175	4	. 97825	1
60	9. 48998		10. 51002	9. 51178		10. 48822	10. 02179		9. 97821	0
↑ 107°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin ←72°	↑

TABLE 33

Logarithms of Trigonometric Functions

18° → ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 161° ′
0	9.48998	39	10.51002	9.51178	43	10.48822	10.02179	4	9.97821	60
1	.49037	39	.50963	.51221	43	.48779	.02183	5	.97817	59
2	.49076	39	.50924	.51264	42	.48736	.02188	4	.97812	58
3	.49115	38	.50885	.51306	43	.48694	.02192	4	.97808	57
4	.49153	39	.50847	.51349	43	.48651	.02196	4	.97804	56
5	9.49192	39	10.50808	9.51392	43	10.48608	10.02200	4	9.97800	55
6	.49231	38	.50769	.51435	43	.48565	.02204	4	.97796	54
7	.49269	39	.50731	.51478	42	.48522	.02208	4	.97792	53
8	.49308	39	.50692	.51520	43	.48480	.02212	4	.97788	52
9	.49347	38	.50653	.51563	43	.48437	.02216	5	.97784	51
10	9.49385	39	10.50615	9.51606	42	10.48394	10.02221	4	9.97779	50
11	.49424	38	.50576	.51648	43	.48352	.02225	4	.97775	49
12	.49462	38	.50538	.51691	43	.48309	.02229	4	.97771	48
13	.49500	39	.50500	.51734	42	.48266	.02233	4	.97767	47
14	.49539	38	.50461	.51776	43	.48224	.02237	4	.97763	46
15	9.49577	38	10.50423	9.51819	42	10.48181	10.02241	5	9.97759	45
16	.49615	39	.50385	.51861	42	.48139	.02246	4	.97754	44
17	.49654	38	.50346	.51903	43	.48097	.02250	4	.97750	43
18	.49692	38	.50308	.51946	42	.48054	.02254	4	.97746	42
19	.49730	38	.50270	.51988	43	.48012	.02258	4	.97742	41
20	9.49768	38	10.50232	9.52031	42	10.47969	10.02262	4	9.97738	40
21	.49806	38	.50194	.52073	42	.47927	.02266	5	.97734	39
22	.49844	38	.50156	.52115	42	.47885	.02271	4	.97729	38
23	.49882	38	.50118	.52157	43	.47843	.02275	4	.97725	37
24	.49920	38	.50080	.52200	42	.47800	.02279	4	.97721	36
25	9.49958	38	10.50042	9.52242	42	10.47758	10.02283	4	9.97717	35
26	.49996	38	.50004	.52284	42	.47716	.02287	5	.97713	34
27	.50034	38	.49966	.52326	42	.47674	.02292	4	.97708	33
28	.50072	38	.49928	.52368	42	.47632	.02296	4	.97704	32
29	.50110	38	.49890	.52410	42	.47590	.02300	4	.97700	31
30	9.50148	37	10.49852	9.52452	42	10.47548	10.02304	5	9.97696	30
31	.50185	38	.49815	.52494	42	.47506	.02309	4	.97691	29
32	.50223	38	.49777	.52536	42	.47464	.02313	4	.97687	28
33	.50261	37	.49739	.52578	42	.47422	.02317	4	.97683	27
34	.50298	38	.49702	.52620	41	.47380	.02321	5	.97679	26
35	9.50336	38	10.49664	9.52661	42	10.47339	10.02326	4	9.97674	25
36	.50374	37	.49626	.52703	42	.47297	.02330	4	.97670	24
37	.50411	38	.49589	.52745	42	.47255	.02334	4	.97666	23
38	.50449	37	.49551	.52787	42	.47213	.02338	5	.97662	22
39	.50486	37	.49514	.52829	41	.47171	.02343	4	.97657	21
40	9.50523	38	10.49477	9.52870	42	10.47130	10.02347	4	9.97653	20
41	.50561	37	.49439	.52912	41	.47088	.02351	4	.97649	19
42	.50598	37	.49402	.52953	42	.47047	.02355	5	.97645	18
43	.50635	38	.49365	.52995	42	.47005	.02360	4	.97640	17
44	.50673	37	.49327	.53037	41	.46963	.02364	4	.97636	16
45	9.50710	37	10.49290	9.53078	42	10.46922	10.02368	4	9.97632	15
46	.50747	37	.49253	.53120	41	.46880	.02372	5	.97628	14
47	.50784	37	.49216	.53161	41	.46839	.02377	4	.97623	13
48	.50821	37	.49179	.53202	42	.46798	.02381	4	.97619	12
49	.50858	38	.49142	.53244	41	.46756	.02385	5	.97615	11
50	9.50896	37	10.49104	9.53285	42	10.46715	10.02390	4	9.97610	10
51	.50933	37	.49067	.53327	41	.46673	.02394	4	.97606	9
52	.50970	37	.49030	.53368	41	.46632	.02398	5	.97602	8
53	.51007	36	.48993	.53409	41	.46591	.02403	4	.97597	7
54	.51043	37	.48957	.53450	42	.46550	.02407	4	.97593	6
55	9.51080	37	10.48920	9.53492	41	10.46508	10.02411	5	9.97589	5
56	.51117	37	.48883	.53533	41	.46467	.02416	4	.97584	4
57	.51154	37	.48846	.53574	41	.46426	.02420	4	.97580	3
58	.51191	36	.48809	.53615	41	.46385	.02424	5	.97576	2
59	.51227	37	.48773	.53656	41	.46344	.02429	4	.97571	1
60	9.51264		10.48736	9.53697		10.46303	10.02433		9.97567	0
108° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 71°

TABLE 33

Logarithms of Trigonometric Functions

19° → ↓ '	sin	Diff. 1'	csc	tan	Diff. 1'	cot	sec	Diff. 1'	cos	← 160° ↓ '
0	9. 51264	37	10. 48736	9. 53697	41	10. 46303	10. 02433	4	9. 97567	60
1	. 51301	37	. 48699	. 53738	41	. 46262	. 02437	5	. 97563	59
2	. 51338	36	. 48662	. 53779	41	. 46221	. 02442	4	. 97558	58
3	. 51374	37	. 48626	. 53820	41	. 46180	. 02446	4	. 97554	57
4	. 51411	36	. 48589	. 53861	41	. 46139	. 02450	5	. 97550	56
5	9. 51447	37	10. 48553	9. 53902	41	10. 46098	10. 02455	4	9. 97545	55
6	. 51484	36	. 48516	. 53943	41	. 46057	. 02459	5	. 97541	54
7	. 51520	37	. 48480	. 53984	41	. 46016	. 02464	4	. 97536	53
8	. 51557	36	. 48443	. 54025	40	. 45975	. 02468	4	. 97532	52
9	. 51593	36	. 48407	. 54065	41	. 45935	. 02472	5	. 97528	51
10	9. 51629	37	10. 48371	9. 54106	41	10. 45894	10. 02477	4	9. 97523	50
11	. 51666	36	. 48334	. 54147	40	. 45853	. 02481	4	. 97519	49
12	. 51702	36	. 48298	. 54187	41	. 45813	. 02485	5	. 97515	48
13	. 51738	36	. 48262	. 54228	41	. 45772	. 02490	4	. 97510	47
14	. 51774	37	. 48226	. 54269	40	. 45731	. 02494	5	. 97506	46
15	9. 51811	36	10. 48189	9. 54309	41	10. 45691	10. 02499	4	9. 97501	45
16	. 51847	36	. 48153	. 54350	40	. 45650	. 02503	5	. 97497	44
17	. 51883	36	. 48117	. 54390	41	. 45610	. 02508	4	. 97492	43
18	. 51919	36	. 48081	. 54431	40	. 45569	. 02512	4	. 97488	42
19	. 51955	36	. 48045	. 54471	41	. 45529	. 02516	5	. 97484	41
20	9. 51991	36	10. 48009	9. 54512	40	10. 45488	10. 02521	4	9. 97479	40
21	. 52027	36	. 47973	. 54552	41	. 45448	. 02525	5	. 97475	39
22	. 52063	36	. 47937	. 54593	40	. 45407	. 02530	4	. 97470	38
23	. 52099	36	. 47901	. 54633	40	. 45367	. 02534	5	. 97466	37
24	. 52135	36	. 47865	. 54673	41	. 45327	. 02539	4	. 97461	36
25	9. 52171	36	10. 47829	9. 54714	40	10. 45286	10. 02543	4	9. 97457	35
26	. 52207	35	. 47793	. 54754	40	. 45246	. 02547	5	. 97453	34
27	. 52242	36	. 47758	. 54794	41	. 45206	. 02552	4	. 97448	33
28	. 52278	36	. 47722	. 54835	40	. 45165	. 02556	5	. 97444	32
29	. 52314	36	. 47686	. 54875	40	. 45125	. 02561	4	. 97439	31
30	9. 52350	35	10. 47650	9. 54915	40	10. 45085	10. 02565	5	9. 97435	30
31	. 52385	36	. 47615	. 54955	40	. 45045	. 02570	4	. 97430	29
32	. 52421	35	. 47579	. 54995	40	. 45005	. 02574	5	. 97426	28
33	. 52456	36	. 47544	. 55035	40	. 44965	. 02579	4	. 97421	27
34	. 52492	35	. 47508	. 55075	40	. 44925	. 02583	5	. 97417	26
35	9. 52527	36	10. 47473	9. 55115	40	10. 44885	10. 02588	4	9. 97412	25
36	. 52563	35	. 47437	. 55155	40	. 44845	. 02592	5	. 97408	24
37	. 52598	36	. 47402	. 55195	40	. 44805	. 02597	4	. 97403	23
38	. 52634	35	. 47366	. 55235	40	. 44765	. 02601	5	. 97399	22
39	. 52669	36	. 47331	. 55275	40	. 44725	. 02606	4	. 97394	21
40	9. 52705	35	10. 47295	9. 55315	40	10. 44685	10. 02610	5	9. 97390	20
41	. 52740	35	. 47260	. 55355	40	. 44645	. 02615	4	. 97385	19
42	. 52775	36	. 47225	. 55395	39	. 44605	. 02619	5	. 97381	18
43	. 52811	35	. 47189	. 55434	40	. 44566	. 02624	4	. 97376	17
44	. 52846	35	. 47154	. 55474	40	. 44526	. 02628	5	. 97372	16
45	9. 52881	35	10. 47119	9. 55514	40	10. 44486	10. 02633	4	9. 97367	15
46	. 52916	35	. 47084	. 55554	39	. 44446	. 02637	5	. 97363	14
47	. 52951	35	. 47049	. 55593	40	. 44407	. 02642	5	. 97358	13
48	. 52986	35	. 47014	. 55633	40	. 44367	. 02647	4	. 97353	12
49	. 53021	35	. 46979	. 55673	39	. 44327	. 02651	5	. 97349	11
50	9. 53056	36	10. 46944	9. 55712	40	10. 44288	10. 02656	4	9. 97344	10
51	. 53092	34	. 46908	. 55752	39	. 44248	. 02660	5	. 97340	9
52	. 53126	35	. 46874	. 55791	40	. 44209	. 02665	4	. 97335	8
53	. 53161	35	. 46839	. 55831	39	. 44169	. 02669	5	. 97331	7
54	. 53196	35	. 46804	. 55870	40	. 44130	. 02674	4	. 97326	6
55	9. 53231	35	10. 46769	9. 55910	39	10. 44090	10. 02678	5	9. 97322	5
56	. 53266	35	. 46734	. 55949	40	. 44051	. 02683	5	. 97317	4
57	. 53301	35	. 46699	. 55989	39	. 44011	. 02688	4	. 97312	3
58	. 53336	34	. 46664	. 56028	39	. 43972	. 02692	5	. 97308	2
59	. 53370	35	. 46630	. 56067	40	. 43933	. 02697	4	. 97303	1
60	9. 53405		10. 46595	9. 56107		10. 43893	10. 02701		9. 97299	0
↑ 109° →	cos	Diff. 1'	sec	cot	Diff. 1'	tan	csc	Diff. 1'	sin	← 70° ↑

TABLE 33
Logarithms of Trigonometric Functions

20° → ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 159° ′
0	9. 53405	35	10. 46595	9. 56107	39	10. 43893	10. 02701	5	9. 97299	60
1	. 53440	35	. 46560	. 56146	39	. 43854	. 02706	5	. 97294	59
2	. 53475	34	. 46525	. 56185	39	. 43815	. 02711	4	. 97289	58
3	. 53509	35	. 46491	. 56224	40	. 43776	. 02715	5	. 97285	57
4	. 53544	34	. 46456	. 56264	39	. 43736	. 02720	4	. 97280	56
5	9. 53578	35	10. 46422	9. 56303	39	10. 43697	10. 02724	5	9. 97276	55
6	. 53613	34	. 46387	. 56342	39	. 43658	. 02729	5	. 97271	54
7	. 53647	35	. 46353	. 56381	39	. 43619	. 02734	4	. 97266	53
8	. 53682	34	. 46318	. 56420	39	. 43580	. 02738	5	. 97262	52
9	. 53716	35	. 46284	. 56459	39	. 43541	. 02743	5	. 97257	51
10	9. 53751	34	10. 46249	9. 56498	39	10. 43502	10. 02748	4	9. 97252	50
11	. 53785	34	. 46215	. 56537	39	. 43463	. 02752	5	. 97248	49
12	. 53819	35	. 46181	. 56576	39	. 43424	. 02757	5	. 97243	48
13	. 53854	34	. 46146	. 56615	39	. 43385	. 02762	4	. 97238	47
14	. 53888	34	. 46112	. 56654	39	. 43346	. 02766	5	. 97234	46
15	9. 53922	35	10. 46078	9. 56693	39	10. 43307	10. 02771	5	9. 97229	45
16	. 53957	34	. 46043	. 56732	39	. 43268	. 02776	4	. 97224	44
17	. 53991	34	. 46009	. 56771	39	. 43229	. 02780	5	. 97220	43
18	. 54025	34	. 45975	. 56810	39	. 43190	. 02785	5	. 97215	42
19	. 54059	34	. 45941	. 56849	38	. 43151	. 02790	4	. 97210	41
20	9. 54093	34	10. 45907	9. 56887	39	10. 43113	10. 02794	5	9. 97206	40
21	. 54127	34	. 45873	. 56926	39	. 43074	. 02799	5	. 97201	39
22	. 54161	34	. 45839	. 56965	39	. 43035	. 02804	4	. 97196	38
23	. 54195	34	. 45805	. 57004	38	. 42996	. 02808	5	. 97192	37
24	. 54229	34	. 45771	. 57042	39	. 42958	. 02813	5	. 97187	36
25	9. 54263	34	10. 45737	9. 57081	39	10. 42919	10. 02818	4	9. 97182	35
26	. 54297	34	. 45703	. 57120	38	. 42880	. 02822	5	. 97178	34
27	. 54331	34	. 45669	. 57158	39	. 42842	. 02827	5	. 97173	33
28	. 54365	34	. 45635	. 57197	38	. 42803	. 02832	5	. 97168	32
29	. 54399	34	. 45601	. 57235	39	. 42765	. 02837	4	. 97163	31
30	9. 54433	33	10. 45567	9. 57274	38	10. 42726	10. 02841	5	9. 97159	30
31	. 54466	34	. 45534	. 57312	39	. 42688	. 02846	5	. 97154	29
32	. 54500	34	. 45500	. 57351	38	. 42649	. 02851	4	. 97149	28
33	. 54534	33	. 45466	. 57389	39	. 42611	. 02855	5	. 97145	27
34	. 54567	34	. 45433	. 57428	38	. 42572	. 02860	5	. 97140	26
35	9. 54601	34	10. 45399	9. 57466	38	10. 42534	10. 02865	5	9. 97135	25
36	. 54635	33	. 45365	. 57504	39	. 42496	. 02870	4	. 97130	24
37	. 54668	34	. 45332	. 57543	38	. 42457	. 02874	5	. 97126	23
38	. 54702	33	. 45298	. 57581	38	. 42419	. 02879	5	. 97121	22
39	. 54735	34	. 45265	. 57619	39	. 42381	. 02884	5	. 97116	21
40	9. 54769	33	10. 45231	9. 57658	38	10. 42342	10. 02889	4	9. 97111	20
41	. 54802	34	. 45198	. 57696	38	. 42304	. 02893	5	. 97107	19
42	. 54836	33	. 45164	. 57734	38	. 42266	. 02898	5	. 97102	18
43	. 54869	34	. 45131	. 57772	38	. 42228	. 02903	5	. 97097	17
44	. 54903	33	. 45097	. 57810	39	. 42190	. 02908	5	. 97092	16
45	9. 54936	33	10. 45064	9. 57849	38	10. 42151	10. 02913	4	9. 97087	15
46	. 54969	34	. 45031	. 57887	38	. 42113	. 02917	5	. 97083	14
47	. 55003	33	. 44997	. 57925	38	. 42075	. 02922	5	. 97078	13
48	. 55036	33	. 44964	. 57963	38	. 42037	. 02927	5	. 97073	12
49	. 55069	33	. 44931	. 58001	38	. 41999	. 02932	5	. 97068	11
50	9. 55102	34	10. 44898	9. 58039	38	10. 41961	10. 02937	4	9. 97063	10
51	. 55136	33	. 44864	. 58077	38	. 41923	. 02941	5	. 97059	9
52	. 55169	33	. 44831	. 58115	38	. 41885	. 02946	5	. 97054	8
53	. 55202	33	. 44798	. 58153	38	. 41847	. 02951	5	. 97049	7
54	. 55235	33	. 44765	. 58191	38	. 41809	. 02956	5	. 97044	6
55	9. 55268	33	10. 44732	9. 58229	38	10. 41771	10. 02961	4	9. 97039	5
56	. 55301	33	. 44699	. 58267	37	. 41733	. 02965	5	. 97035	4
57	. 55334	33	. 44666	. 58304	38	. 41696	. 02970	5	. 97030	3
58	. 55367	33	. 44633	. 58342	38	. 41658	. 02975	5	. 97025	2
59	. 55400	33	. 44600	. 58380	38	. 41620	. 02980	5	. 97020	1
60	9. 55433		10. 44567	9. 58418		10. 41582	10. 02985		9. 97015	0
↑ 110° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ← 69°

TABLE 33

Logarithms of Trigonometric Functions

21° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 158° ↓ ′
0	9. 55433	33	10. 44567	9. 58418	37	10. 41582	10. 02985	5	9. 97015	60
1	. 55466	33	. 44534	. 58455	38	. 41545	. 02990	5	. 97010	59
2	. 55499	33	. 44501	. 58493	38	. 41507	. 02995	4	. 97005	58
3	. 55532	32	. 44468	. 58531	38	. 41469	. 02999	5	. 97001	57
4	. 55564	33	. 44436	. 58569	37	. 41431	. 03004	5	. 96996	56
5	9. 55597	33	10. 44403	9. 58606	38	10. 41394	10. 03009	5	9. 96991	55
6	. 55630	33	. 44370	. 58644	37	. 41356	. 03014	5	. 96986	54
7	. 55663	32	. 44337	. 58681	38	. 41319	. 03019	5	. 96981	53
8	. 55695	33	. 44305	. 58719	38	. 41281	. 03024	5	. 96976	52
9	. 55728	33	. 44272	. 58757	37	. 41243	. 03029	5	. 96971	51
10	9. 55761	32	10. 44239	9. 58794	38	10. 41206	10. 03034	4	9. 96966	50
11	. 55793	33	. 44207	. 58832	37	. 41168	. 03038	5	. 96962	49
12	. 55826	32	. 44174	. 58869	38	. 41131	. 03043	5	. 96957	48
13	. 55858	33	. 44142	. 58907	37	. 41093	. 03048	5	. 96952	47
14	. 55891	32	. 44109	. 58944	37	. 41056	. 03053	5	. 96947	46
15	9. 55923	33	10. 44077	9. 58981	38	10. 41019	10. 03058	5	9. 96942	45
16	. 55956	32	. 44044	. 59019	37	. 40981	. 03063	5	. 96937	44
17	. 55988	33	. 44012	. 59056	38	. 40944	. 03068	5	. 96932	43
18	. 56021	32	. 43979	. 59094	37	. 40906	. 03073	5	. 96927	42
19	. 56053	32	. 43947	. 59131	37	. 40869	. 03078	5	. 96922	41
20	9. 56085	33	10. 43915	9. 59168	37	10. 40832	10. 03083	5	9. 96917	40
21	. 56118	32	. 43882	. 59205	38	. 40795	. 03088	5	. 96912	39
22	. 56150	32	. 43850	. 59243	37	. 40757	. 03093	4	. 96907	38
23	. 56182	33	. 43818	. 59280	37	. 40720	. 03097	5	. 96903	37
24	. 56215	32	. 43785	. 59317	37	. 40683	. 03102	5	. 96898	36
25	9. 56247	32	10. 43753	9. 59354	37	10. 40646	10. 03107	5	9. 96893	35
26	. 56279	32	. 43721	. 59391	38	. 40609	. 03112	5	. 96888	34
27	. 56311	32	. 43689	. 59429	37	. 40571	. 03117	5	. 96883	33
28	. 56343	32	. 43657	. 59466	37	. 40534	. 03122	5	. 96878	32
29	. 56375	33	. 43625	. 59503	37	. 40497	. 03127	5	. 96873	31
30	9. 56408	32	10. 43592	9. 59540	37	10. 40460	10. 03132	5	9. 96868	30
31	. 56440	32	. 43560	. 59577	37	. 40423	. 03137	5	. 96863	29
32	. 56472	32	. 43528	. 59614	37	. 40386	. 03142	5	. 96858	28
33	. 56504	32	. 43496	. 59651	37	. 40349	. 03147	5	. 96853	27
34	. 56536	32	. 43464	. 59688	37	. 40312	. 03152	5	. 96848	26
35	9. 56568	31	10. 43432	9. 59725	37	10. 40275	10. 03157	5	9. 96843	25
36	. 56599	32	. 43401	. 59762	37	. 40238	. 03162	5	. 96838	24
37	. 56631	32	. 43369	. 59799	36	. 40201	. 03167	5	. 96833	23
38	. 56663	32	. 43337	. 59835	37	. 40165	. 03172	5	. 96828	22
39	. 56695	32	. 43305	. 59872	37	. 40128	. 03177	5	. 96823	21
40	9. 56727	32	10. 43273	9. 59909	37	10. 40091	10. 03182	5	9. 96818	20
41	. 56759	31	. 43241	. 59946	37	. 40054	. 03187	5	. 96813	19
42	. 56790	32	. 43210	. 59983	36	. 40017	. 03192	5	. 96808	18
43	. 56822	32	. 43178	. 60019	37	. 39981	. 03197	5	. 96803	17
44	. 56854	32	. 43146	. 60056	37	. 39944	. 03202	5	. 96798	16
45	9. 56886	31	10. 43114	9. 60093	37	10. 39907	10. 03207	5	9. 96793	15
46	. 56917	32	. 43083	. 60130	36	. 39870	. 03212	5	. 96788	14
47	. 56949	31	. 43051	. 60166	37	. 39834	. 03217	5	. 96783	13
48	. 56980	32	. 43020	. 60203	37	. 39797	. 03222	6	. 96778	12
49	. 57012	32	. 42988	. 60240	36	. 39760	. 03228	5	. 96772	11
50	9. 57044	31	10. 42956	9. 60276	37	10. 39724	10. 03233	5	9. 96767	10
51	. 57075	32	. 42925	. 60313	36	. 39687	. 03238	5	. 96762	9
52	. 57107	31	. 42893	. 60349	37	. 39651	. 03243	5	. 96757	8
53	. 57138	31	. 42862	. 60386	36	. 39614	. 03248	5	. 96752	7
54	. 57169	32	. 42831	. 60422	37	. 39578	. 03253	5	. 96747	6
55	9. 57201	31	10. 42799	9. 60459	36	10. 39541	10. 03258	5	9. 96742	5
56	. 57232	32	. 42768	. 60495	37	. 39505	. 03263	5	. 96737	4
57	. 57264	31	. 42736	. 60532	36	. 39468	. 03268	5	. 96732	3
58	. 57295	31	. 42705	. 60568	37	. 39432	. 03273	5	. 96727	2
59	. 57326	32	. 42674	. 60605	36	. 39395	. 03278	5	. 96722	1
60	9. 57358		10. 42642	9. 60641		10. 39359	10. 03283		9. 96717	0
↑ 111° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ← 68°

TABLE 33

Logarithms of Trigonometric Functions

22°→ ↓ '	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←157° ↓ '
0	9. 57358	31	10. 42642	9. 60641	36	10. 39359	10. 03283	6	9. 96717	60
1	. 57389	31	. 42611	. 60677	37	. 39323	. 03289	5	. 96711	59
2	. 57420	31	. 42580	. 60714	36	. 39286	. 03294	5	. 96706	58
3	. 57451	31	. 42549	. 60750	36	. 39250	. 03299	5	. 96701	57
4	. 57482	32	. 42518	. 60786	37	. 39214	. 03304	5	. 96696	56
5	9. 57514	31	10. 42486	9. 60823	36	10. 39177	10. 03309	5	9. 96691	55
6	. 57545	31	. 42455	. 60859	36	. 39141	. 03314	5	. 96686	54
7	. 57576	31	. 42424	. 60895	36	. 39105	. 03319	5	. 96681	53
8	. 57607	31	. 42393	. 60931	36	. 39069	. 03324	6	. 96676	52
9	. 57638	31	. 42362	. 60967	37	. 39033	. 03330	5	. 96670	51
10	9. 57669	31	10. 42331	9. 61004	36	10. 38996	10. 03335	5	9. 96665	50
11	. 57700	31	. 42300	. 61040	36	. 38960	. 03340	5	. 96660	49
12	. 57731	31	. 42269	. 61076	36	. 38924	. 03345	5	. 96655	48
13	. 57762	31	. 42238	. 61112	36	. 38888	. 03350	5	. 96650	47
14	. 57793	31	. 42207	. 61148	36	. 38852	. 03355	5	. 96645	46
15	9. 57824	31	10. 42176	9. 61184	36	10. 38816	10. 03360	6	9. 96640	45
16	. 57855	30	. 42145	. 61220	36	. 38780	. 03366	5	. 96634	44
17	. 57885	31	. 42115	. 61256	36	. 38744	. 03371	5	. 96629	43
18	. 57916	31	. 42084	. 61292	36	. 38708	. 03376	5	. 96624	42
19	. 57947	31	. 42053	. 61328	36	. 38672	. 03381	5	. 96619	41
20	9. 57978	30	10. 42022	9. 61364	36	10. 38636	10. 03386	6	9. 96614	40
21	. 58008	31	. 41992	. 61400	36	. 38600	. 03392	5	. 96608	39
22	. 58039	31	. 41961	. 61436	36	. 38564	. 03397	5	. 96603	38
23	. 58070	31	. 41930	. 61472	36	. 38528	. 03402	5	. 96598	37
24	. 58101	30	. 41899	. 61508	36	. 38492	. 03407	5	. 96593	36
25	9. 58131	31	10. 41869	9. 61544	35	10. 38456	10. 03412	6	9. 96588	35
26	. 58162	30	. 41838	. 61579	36	. 38421	. 03418	5	. 96582	34
27	. 58192	31	. 41808	. 61615	36	. 38385	. 03423	5	. 96577	33
28	. 58223	30	. 41777	. 61651	36	. 38349	. 03428	5	. 96572	32
29	. 58253	31	41747	. 61687	35	. 38313	. 03433	5	. 96567	31
30	9. 58284	30	10. 41716	9. 61722	36	10. 38278	10. 03438	6	9. 96562	30
31	. 58314	31	. 41686	. 61758	36	. 38242	. 03444	5	. 96556	29
32	. 58345	30	. 41655	. 61794	36	. 38206	. 03449	5	. 96551	28
33	. 58375	31	. 41625	. 61830	35	. 38170	. 03454	5	. 96546	27
34	. 58406	30	. 41594	. 61865	36	. 38135	. 03459	6	. 96541	26
35	9. 58436	31	10. 41564	9. 61901	35	10. 38099	10. 03465	5	9. 96535	25
36	. 58467	30	. 41533	. 61936	36	. 38064	. 03470	5	. 96530	24
37	. 58497	30	. 41503	. 61972	36	. 38028	. 03475	5	. 96525	23
38	. 58527	30	. 41473	. 62008	35	. 37992	. 03480	6	. 96520	22
39	. 58557	31	. 41443	. 62043	36	. 37957	. 03486	5	. 96514	21
40	9. 58588	30	10. 41412	9. 62079	35	10. 37921	10. 03491	5	9. 96509	20
41	. 58618	30	. 41382	. 62114	36	. 37886	. 03496	6	. 96504	19
42	. 58648	30	. 41352	. 62150	35	. 37850	. 03502	5	. 96498	18
43	. 58678	31	. 41322	. 62185	36	. 37815	. 03507	5	. 96493	17
44	. 58709	30	. 41291	. 62221	35	. 37779	. 03512	5	. 96488	16
45	9. 58739	30	10. 41261	9. 62256	36	10. 37744	10. 03517	6	9. 96483	15
46	. 58769	30	. 41231	. 62292	35	. 37708	. 03523	5	. 96477	14
47	. 58799	30	. 41201	. 62327	35	. 37673	. 03528	5	. 96472	13
48	. 58829	30	. 41171	. 62362	36	. 37638	. 03533	6	. 96467	12
49	. 58859	30	. 41141	. 62398	35	. 37602	. 03539	5	. 96461	11
50	9. 58889	30	10. 41111	9. 62433	35	10. 37567	10. 03544	5	9. 96456	10
51	. 58919	30	. 41081	. 62468	36	. 37532	. 03549	6	. 96451	9
52	. 58949	30	. 41051	. 62504	35	. 37496	. 03555	5	. 96445	8
53	. 58979	30	. 41021	. 62539	35	. 37461	. 03560	5	. 96440	7
54	. 59009	30	. 40991	. 62574	35	. 37426	. 03565	6	. 96435	6
55	9. 59039	30	10. 40961	9. 62609	36	10. 37391	10. 03571	5	9. 96429	5
56	. 59069	29	. 40931	. 62645	35	. 37355	. 03576	5	. 96424	4
57	. 59098	30	. 40902	. 62680	35	. 37320	. 03581	6	. 96419	3
58	. 59128	30	. 40872	. 62715	35	. 37285	. 03587	5	. 96413	2
59	. 59158	30	. 40842	. 62750	35	. 37250	. 03592	5	. 96408	1
60	9. 59188		10. 40812	9. 62785		10. 37215	10. 03597		9. 96403	0
↑ 112°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←67°

TABLE 33
Logarithms of Trigonometric Functions

23° ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	156° ′
0	9.59188	30	10.40812	9.62785	35	10.37215	10.03597	6	9.96403	60
1	.59218	29	.40782	.62820	35	.37180	.03603	5	.96397	59
2	.59247	30	.40753	.62855	35	.37145	.03608	5	.96392	58
3	.59277	30	.40723	.62890	36	.37110	.03613	6	.96387	57
4	.59307	29	.40693	.62926	35	.37074	.03619	5	.96381	56
5	9.59336	30	10.40664	9.62961	35	10.37039	10.03624	6	9.96376	55
6	.59366	30	.40634	.62996	35	.37004	.03630	5	.96370	54
7	.59396	29	.40604	.63031	35	.36969	.03635	5	.96365	53
8	.59425	30	.40575	.63066	35	.36934	.03640	6	.96360	52
9	.59455	29	.40545	.63101	34	.36899	.03646	5	.96354	51
10	9.59484	30	10.40516	9.63135	35	10.36865	10.03651	6	9.96349	50
11	.59514	29	.40486	.63170	35	.36830	.03657	5	.96343	49
12	.59543	30	.40457	.63205	35	.36795	.03662	5	.96338	48
13	.59573	29	.40427	.63240	35	.36760	.03667	6	.96333	47
14	.59602	30	.40398	.63275	35	.36725	.03673	5	.96327	46
15	9.59632	29	10.40368	9.63310	35	10.36690	10.03678	6	9.96322	45
16	.59661	29	.40339	.63345	34	.36655	.03684	5	.96316	44
17	.59690	30	.40310	.63379	35	.36621	.03689	6	.96311	43
18	.59720	29	.40280	.63414	35	.36586	.03695	5	.96305	42
19	.59749	29	.40251	.63449	35	.36551	.03700	6	.96300	41
20	9.59778	30	10.40222	9.63484	35	10.36516	10.03706	5	9.96294	40
21	.59808	29	.40192	.63519	34	.36481	.03711	5	.96289	39
22	.59837	29	.40163	.63553	35	.36447	.03716	6	.96284	38
23	.59866	29	.40134	.63588	35	.36412	.03722	5	.96278	37
24	.59895	29	.40105	.63623	34	.36377	.03727	6	.96273	36
25	9.59924	30	10.40076	9.63657	35	10.36343	10.03733	5	9.96267	35
26	.59954	29	.40046	.63692	34	.36308	.03738	6	.96262	34
27	.59983	29	.40017	.63726	35	.36274	.03744	5	.96256	33
28	.60012	29	.39988	.63761	35	.36239	.03749	6	.96251	32
29	.60041	29	.39959	.63796	34	.36204	.03755	5	.96245	31
30	9.60070	29	10.39930	9.63830	35	10.36170	10.03760	6	9.96240	30
31	.60099	29	.39901	.63865	34	.36135	.03766	5	.96234	29
32	.60128	29	.39872	.63899	35	.36101	.03771	6	.96229	28
33	.60157	29	.39843	.63934	34	.36066	.03777	5	.96223	27
34	.60186	29	.39814	.63968	35	.36032	.03782	6	.96218	26
35	9.60215	29	10.39785	9.64003	34	10.35997	10.03788	5	9.96212	25
36	.60244	29	.39756	.64037	35	.35963	.03793	6	.96207	24
37	.60273	29	.39727	.64072	34	.35928	.03799	5	.96201	23
38	.60302	29	.39698	.64106	34	.35894	.03804	6	.96196	22
39	.60331	28	.39669	.64140	35	.35860	.03810	5	.96190	21
40	9.60359	29	10.39641	9.64175	34	10.35825	10.03815	6	9.96185	20
41	.60388	29	.39612	.64209	34	.35791	.03821	5	.96179	19
42	.60417	29	.39583	.64243	35	.35757	.03826	6	.96174	18
43	.60446	28	.39554	.64278	34	.35722	.03832	6	.96168	17
44	.60474	29	.39526	.64312	34	.35688	.03838	5	.96162	16
45	9.60503	29	10.39497	9.64346	35	10.35654	10.03843	6	9.96157	15
46	.60532	29	.39468	.64381	34	.35619	.03849	5	.96151	14
47	.60561	28	.39439	.64415	34	.35585	.03854	6	.96146	13
48	.60589	29	.39411	.64449	34	.35551	.03860	5	.96140	12
49	.60618	28	.39382	.64483	34	.35517	.03865	6	.96135	11
50	9.60646	29	10.39354	9.64517	35	10.35483	10.03871	6	9.96129	10
51	.60675	29	.39325	.64552	34	.35448	.03877	5	.96123	9
52	.60704	28	.39296	.64586	34	.35414	.03882	6	.96118	8
53	.60732	29	.39268	.64620	34	.35380	.03888	5	.96112	7
54	.60761	28	.39239	.64654	34	.35346	.03893	6	.96107	6
55	9.60789	29	10.39211	9.64688	34	10.35312	10.03899	6	9.96101	5
56	.60818	28	.39182	.64722	34	.35278	.03905	5	.96095	4
57	.60846	29	.39154	.64756	34	.35244	.03910	6	.96090	3
58	.60875	28	.39125	.64790	34	.35210	.03916	5	.96084	2
59	.60903	28	.39097	.64824	34	.35176	.03921	6	.96079	1
60	9.60931		10.39069	9.64858		10.35142	10.03927		9.96073	0
113°	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	66°

TABLE 33

Logarithms of Trigonometric Functions

24°→ ′ ↓	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←155° ′ ↓
0	9.60931	29	10.39069	9.64858	34	10.35142	10.03927	6	9.96073	60
1	.60960	28	.39040	.64892	34	.35108	.03933	5	.96067	59
2	.60988	28	.39012	.64926	34	.35074	.03938	6	.96062	58
3	.61016	29	.38984	.64960	34	.35040	.03944	6	.96056	57
4	.61045	28	.38955	.64994	34	.35006	.03950	5	.96050	56
5	9.61073	28	10.38927	9.65028	34	10.34972	10.03955	6	9.96045	55
6	.61101	28	.38899	.65062	34	.34938	.03961	5	.96039	54
7	.61129	29	.38871	.65096	34	.34904	.03966	6	.96034	53
8	.61158	28	.38842	.65130	34	.34870	.03972	6	.96028	52
9	.61186	28	.38814	.65164	33	.34836	.03978	5	.96022	51
10	9.61214	28	10.38786	9.65197	34	10.34803	10.03983	6	9.96017	50
11	.61242	28	.38758	.65231	34	.34769	.03989	6	.96011	49
12	.61270	28	.38730	.65265	34	.34735	.03995	5	.96005	48
13	.61298	28	.38702	.65299	34	.34701	.04000	6	.96000	47
14	.61326	28	.38674	.65333	33	.34667	.04006	6	.95994	46
15	9.61354	28	10.38646	9.65366	34	10.34634	10.04012	6	9.95988	45
16	.61382	29	.38618	.65400	34	.34600	.04018	5	.95982	44
17	.61411	27	.38589	.65434	33	.34566	.04023	6	.95977	43
18	.61438	28	.38562	.65467	34	.34533	.04029	6	.95971	42
19	.61466	28	.38534	.65501	34	.34499	.04035	5	.95965	41
20	9.61494	28	10.38506	9.65535	33	10.34465	10.04040	6	9.95960	40
21	.61522	28	.38478	.65568	34	.34432	.04046	6	.95954	39
22	.61550	28	.38450	.65602	34	.34398	.04052	6	.95948	38
23	.61578	28	.38422	.65636	33	.34364	.04058	5	.95942	37
24	.61606	28	.38394	.65669	34	.34331	.04063	6	.95937	36
25	9.61634	28	10.38366	9.65703	33	10.34297	10.04069	6	9.95931	35
26	.61662	27	.38338	.65736	34	.34264	.04075	5	.95925	34
27	.61689	28	.38311	.65770	33	.34230	.04080	6	.95920	33
28	.61717	28	.38283	.65803	34	.34197	.04086	6	.95914	32
29	.61745	28	.38255	.65837	33	.34163	.04092	6	.95908	31
30	9.61773	27	10.38227	9.65870	34	10.34130	10.04098	5	9.95902	30
31	.61800	28	.38200	.65904	33	.34096	.04103	6	.95897	29
32	.61828	28	.38172	.65937	34	.34063	.04109	6	.95891	28
33	.61856	27	.38144	.65971	33	.34029	.04115	6	.95885	27
34	.61883	28	.38117	.66004	34	.33996	.04121	6	.95879	26
35	9.61911	28	10.38089	9.66038	33	10.33962	10.04127	5	9.95873	25
36	.61939	27	.38061	.66071	33	.33929	.04132	6	.95868	24
37	.61966	28	.38034	.66104	34	.33896	.04138	6	.95862	23
38	.61994	27	.38006	.66138	33	.33862	.04144	6	.95856	22
39	.62021	28	.37979	.66171	33	.33829	.04150	6	.95850	21
40	9.62049	27	10.37951	9.66204	34	10.33796	10.04156	5	9.95844	20
41	.62076	28	.37924	.66238	33	.33762	.04161	6	.95839	19
42	.62104	27	.37896	.66271	33	.33729	.04167	6	.95833	18
43	.62131	28	.37869	.66304	33	.33696	.04173	6	.95827	17
44	.62159	27	.37841	.66337	34	.33663	.04179	6	.95821	16
45	9.62186	28	10.37814	9.66371	33	10.33629	10.04185	5	9.95815	15
46	.62214	27	.37786	.66404	33	.33596	.04190	6	.95810	14
47	.62241	27	.37759	.66437	33	.33563	.04196	6	.95804	13
48	.62268	28	.37732	.66470	33	.33530	.04202	6	.95798	12
49	.62296	27	.37704	.66503	34	.33497	.04208	6	.95792	11
50	9.62323	27	10.37677	9.66537	33	10.33463	10.04214	6	9.95786	10
51	.62350	27	.37650	.66570	33	.33430	.04220	5	.95780	9
52	.62377	28	.37623	.66603	33	.33397	.04225	6	.95775	8
53	.62405	27	.37595	.66636	33	.33364	.04231	6	.95769	7
54	.62432	27	.37568	.66669	33	.33331	.04237	6	.95763	6
55	9.62459	27	10.37541	9.66702	33	10.33298	10.04243	6	9.95757	5
56	.62486	27	.37514	.66735	33	.33265	.04249	6	.95751	4
57	.62513	28	.37487	.66768	33	.33232	.04255	6	.95745	3
58	.62541	27	.37459	.66801	33	.33199	.04261	6	.95739	2
59	.62568	27	.37432	.66834	33	.33166	.04267	5	.95733	1
60	9.62595		10.37405	9.66867		10.33133	10.04272		9.95728	0
↑ 114°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←65°

TABLE 33
Logarithms of Trigonometric Functions

25° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 154° ↓ ′
0	9. 62595	27	10. 37405	9. 66867	33	10. 33133	10. 04272	6	9. 95728	60
1	. 62622	27	. 37378	. 66900	33	. 33100	. 04278	6	. 95722	59
2	. 62649	27	. 37351	. 66933	33	. 33067	. 04284	6	. 95716	58
3	. 62676	27	. 37324	. 66966	33	. 33034	. 04290	6	. 95710	57
4	. 62703	27	. 37297	. 66999	33	. 33001	. 04296	6	. 95704	56
5	9. 62730	27	10. 37270	9. 67032	33	10. 32968	10. 04302	6	9. 95698	55
6	. 62757	27	. 37243	. 67065	33	. 32935	. 04308	6	. 95692	54
7	. 62784	27	. 37216	. 67098	33	. 32902	. 04314	6	. 95686	53
8	. 62811	27	. 37189	. 67131	32	. 32869	. 04320	6	. 95680	52
9	. 62838	27	. 37162	. 67163	33	. 32837	. 04326	6	. 95674	51
10	9. 62865	27	10. 37135	9. 67196	33	10. 32804	10. 04332	5	9. 95668	50
11	. 62892	26	. 37108	. 67229	33	. 32771	. 04337	6	. 95663	49
12	. 62918	27	. 37082	. 67262	33	. 32738	. 04343	6	. 95657	48
13	. 62945	27	. 37055	. 67295	32	. 32705	. 04349	6	. 95651	47
14	. 62972	27	. 37028	. 67327	33	. 32673	. 04355	6	. 95645	46
15	9. 62999	27	10. 37001	9. 67360	33	10. 32640	10. 04361	6	9. 95639	45
16	. 63026	26	. 36974	. 67393	33	. 32607	. 04367	6	. 95633	44
17	. 63052	27	. 36948	. 67426	32	. 32574	. 04373	6	. 95627	43
18	. 63079	27	. 36921	. 67458	33	. 32542	. 04379	6	. 95621	42
19	. 63106	27	. 36894	. 67491	33	. 32509	. 04385	6	. 95615	41
20	9. 63133	26	10. 36867	9. 67524	32	10. 32476	10. 04391	6	9. 95609	40
21	. 63159	27	. 36841	. 67556	33	. 32444	. 04397	6	. 95603	39
22	. 63186	27	. 36814	. 67589	33	. 32411	. 04403	6	. 95597	38
23	. 63213	26	. 36787	. 67622	32	. 32378	. 04409	6	. 95591	37
24	. 63239	27	. 36761	. 67654	33	. 32346	. 04415	6	. 95585	36
25	9. 63266	26	10. 36734	9. 67687	32	10. 32313	10. 04421	6	9. 95579	35
26	. 63292	27	. 36708	. 67719	33	. 32281	. 04427	6	. 95573	34
27	. 63319	26	. 36681	. 67752	33	. 32248	. 04433	6	. 95567	33
28	. 63345	27	. 36655	. 67785	32	. 32215	. 04439	6	. 95561	32
29	. 63372	26	. 36628	. 67817	33	. 32183	. 04445	6	. 95555	31
30	9. 63398	27	10. 36602	9. 67850	32	10. 32150	10. 04451	6	9. 95549	30
31	. 63425	26	. 36575	. 67882	33	. 32118	. 04457	6	. 95543	29
32	. 63451	27	. 36549	. 67915	32	. 32085	. 04463	6	. 95537	28
33	. 63478	26	. 36522	. 67947	33	. 32053	. 04469	6	. 95531	27
34	. 63504	27	. 36496	. 67980	32	. 32020	. 04475	6	. 95525	26
35	9. 63531	26	10. 36469	9. 68012	32	10. 31988	10. 04481	6	9. 95519	25
36	. 63557	26	. 36443	. 68044	33	. 31956	. 04487	6	. 95513	24
37	. 63583	27	. 36417	. 68077	32	. 31923	. 04493	7	. 95507	23
38	. 63610	26	. 36390	. 68109	33	. 31891	. 04500	6	. 95500	22
39	. 63636	26	. 36364	. 68142	32	. 31858	. 04506	6	. 95494	21
40	9. 63662	27	10. 36338	9. 68174	32	10. 31826	10. 04512	6	9. 95488	20
41	. 63689	26	. 36311	. 68206	33	. 31794	. 04518	6	. 95482	19
42	. 63715	26	. 36285	. 68239	32	. 31761	. 04524	6	. 95476	18
43	. 63741	26	. 36259	. 68271	32	. 31729	. 04530	6	. 95470	17
44	. 63767	27	. 36233	. 68303	33	. 31697	. 04536	6	. 95464	16
45	9. 63794	26	10. 36206	9. 68336	32	10. 31664	10. 04542	6	9. 95458	15
46	. 63820	26	. 36180	. 68368	32	. 31632	. 04548	6	. 95452	14
47	. 63846	26	. 36154	. 68400	32	. 31600	. 04554	6	. 95446	13
48	. 63872	26	. 36128	. 68432	33	. 31568	. 04560	6	. 95440	12
49	. 63898	26	. 36102	. 68465	32	. 31535	. 04566	7	. 95434	11
50	9. 63924	26	10. 36076	9. 68497	32	10. 31503	10. 04573	6	9. 95427	10
51	. 63950	26	. 36050	. 68529	32	. 31471	. 04579	6	. 95421	9
52	. 63976	26	. 36024	. 68561	32	. 31439	. 04585	6	. 95415	8
53	. 64002	26	. 35998	. 68593	33	. 31407	. 04591	6	. 95409	7
54	. 64028	26	. 35972	. 68626	32	. 31374	. 04597	6	. 95403	6
55	9. 64054	26	10. 35946	9. 68658	32	10. 31342	10. 04603	6	9. 95397	5
56	. 64080	26	. 35920	. 68690	32	. 31310	. 04609	7	. 95391	4
57	. 64106	26	. 35894	. 68722	32	. 31278	. 04616	6	. 95384	3
58	. 64132	26	. 35868	. 68754	32	. 31246	. 04622	6	. 95378	2
59	. 64158	26	. 35842	. 68786	32	. 31214	. 04628	6	. 95372	1
60	9. 64184		10. 35816	9. 68818		10. 31182	10. 04634		9. 95366	0
↑ 115° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 64° ↑

TABLE 33
Logarithms of Trigonometric Functions

26° ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	153° ↓ ′
0	9. 64184	26	10. 35816	9. 68818	32	10. 31182	10. 04634	6	9. 95366	60
1	. 64210	26	. 35790	. 68850	32	. 31150	. 04640	6	. 95360	59
2	. 64236	26	. 35764	. 68882	32	. 31118	. 04646	6	. 95354	58
3	. 64262	26	. 35738	. 68914	32	. 31086	. 04652	7	. 95348	57
4	. 64288	25	. 35712	. 68946	32	. 31054	. 04659	6	. 95341	56
5	9. 64313	26	10. 35687	9. 68978	32	10. 31022	10. 04665	6	9. 95335	55
6	. 64339	26	. 35661	. 69010	32	. 30990	. 04671	6	. 95329	54
7	. 64365	26	. 35635	. 69042	32	. 30958	. 04677	6	. 95323	53
8	. 64391	26	. 35609	. 69074	32	. 30926	. 04683	7	. 95317	52
9	. 64417	25	. 35583	. 69106	32	. 30894	. 04690	6	. 95310	51
10	9. 64442	26	10. 35558	9. 69138	32	10. 30862	10. 04696	6	9. 95304	50
11	. 64468	26	. 35532	. 69170	32	. 30830	. 04702	6	. 95298	49
12	. 64494	25	. 35506	. 69202	32	. 30798	. 04708	6	. 95292	48
13	. 64519	26	. 35481	. 69234	32	. 30766	. 04714	7	. 95286	47
14	. 64545	26	. 35455	. 69266	32	. 30734	. 04721	6	. 95279	46
15	9. 64571	25	10. 35429	9. 69298	31	10. 30702	10. 04727	6	9. 95273	45
16	. 64596	26	. 35404	. 69329	32	. 30671	. 04733	6	. 95267	44
17	. 64622	25	. 35378	. 69361	32	. 30639	. 04739	7	. 95261	43
18	. 64647	26	. 35353	. 69393	32	. 30607	. 04746	6	. 95254	42
19	. 64673	25	. 35327	. 69425	32	. 30575	. 04752	6	. 95248	41
20	9. 64698	26	10. 35302	9. 69457	31	10. 30543	10. 04758	6	9. 95242	40
21	. 64724	25	. 35276	. 69488	32	. 30512	. 04764	7	. 95236	39
22	. 64749	26	. 35251	. 69520	32	. 30480	. 04771	6	. 95229	38
23	. 64775	25	. 35225	. 69552	32	. 30448	. 04777	6	. 95223	37
24	. 64800	26	. 35200	. 69584	31	. 30416	. 04783	6	. 95217	36
25	9. 64826	25	10. 35174	9. 69615	32	10. 30385	10. 04789	7	9. 95211	35
26	. 64851	26	. 35149	. 69647	32	. 30353	. 04796	6	. 95204	34
27	. 64877	25	. 35123	. 69679	31	. 30321	. 04802	6	. 95198	33
28	. 64902	25	. 35098	. 69710	32	. 30290	. 04808	7	. 95192	32
29	. 64927	26	. 35073	. 69742	32	. 30258	. 04815	6	. 95185	31
30	9. 64953	25	10. 35047	9. 69774	31	10. 30226	10. 04821	6	9. 95179	30
31	. 64978	25	. 35022	. 69805	32	. 30195	. 04827	6	. 95173	29
32	. 65003	26	. 34997	. 69837	31	. 30163	. 04833	7	. 95167	28
33	. 65029	25	. 34971	. 69868	32	. 30132	. 04840	6	. 95160	27
34	. 65054	25	. 34946	. 69900	32	. 30100	. 04846	6	. 95154	26
35	9. 65079	25	10. 34921	9. 69932	31	10. 30068	10. 04852	7	9. 95148	25
36	. 65104	26	. 34896	. 69963	32	. 30037	. 04859	6	. 95141	24
37	. 65130	25	. 34870	. 69995	31	. 30005	. 04865	6	. 95135	23
38	. 65155	25	. 34845	. 70026	32	. 29974	. 04871	7	. 95129	22
39	. 65180	25	. 34820	. 70058	31	. 29942	. 04878	6	. 95122	21
40	9. 65205	25	10. 34795	9. 70089	32	10. 29911	10. 04884	6	9. 95116	20
41	. 65230	25	. 34770	. 70121	31	. 29879	. 04890	7	. 95110	19
42	. 65255	26	. 34745	. 70152	32	. 29848	. 04897	6	. 95103	18
43	. 65281	25	. 34719	. 70184	31	. 29816	. 04903	7	. 95097	17
44	. 65306	25	. 34694	. 70215	32	. 29785	. 04910	6	. 95090	16
45	9. 65331	25	10. 34669	9. 70247	31	10. 29753	10. 04916	6	9. 95084	15
46	. 65356	25	. 34644	. 70278	31	. 29722	. 04922	7	. 95078	14
47	. 65381	25	. 34619	. 70309	32	. 29691	. 04929	6	. 95071	13
48	. 65406	25	. 34594	. 70341	31	. 29659	. 04935	6	. 95065	12
49	. 65431	25	. 34569	. 70372	32	. 29628	. 04941	7	. 95059	11
50	9. 65456	25	10. 34544	9. 70404	31	10. 29596	10. 04948	6	9. 95052	10
51	. 65481	25	. 34519	. 70435	31	. 29565	. 04954	7	. 95046	9
52	. 65506	25	. 34494	. 70466	32	. 29534	. 04961	6	. 95039	8
53	. 65531	25	. 34469	. 70498	31	. 29502	. 04967	6	. 95033	7
54	. 65556	24	. 34444	. 70529	31	. 29471	. 04973	7	. 95027	6
55	9. 65580	25	10. 34420	9. 70560	32	10. 29440	10. 04980	6	9. 95020	5
56	. 65605	25	. 34395	. 70592	31	. 29408	. 04986	7	. 95014	4
57	. 65630	25	. 34370	. 70623	31	. 29377	. 04993	6	. 95007	3
58	. 65655	25	. 34345	. 70654	31	. 29346	. 04999	6	. 95001	2
59	. 65680	25	. 34320	. 70685	32	. 29315	. 05005	7	. 94995	1
60	9. 65705		10. 34295	9. 70717		10. 29283	10. 05012		9. 94988	0
↑ 116°	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ 63°

TABLE 33
Logarithms of Trigonometric Functions

27° → ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 152° ′
0	9. 65705	24	10. 34295	9. 70717	31	10. 29283	10. 05012	6	9. 94988	60
1	. 65729	25	. 34271	. 70748	31	. 29252	. 05018	7	. 94982	59
2	. 65754	25	. 34246	. 70779	31	. 29221	. 05025	6	. 94975	58
3	. 65779	25	. 34221	. 70810	31	. 29190	. 05031	7	. 94969	57
4	. 65804	24	. 34196	. 70841	32	. 29159	. 05038	6	. 94962	56
5	9. 65828	25	10. 34172	9. 70873	31	10. 29127	10. 05044	7	9. 94956	55
6	. 65853	25	. 34147	. 70904	31	. 29096	. 05051	6	. 94949	54
7	. 65878	24	. 34122	. 70935	31	. 29065	. 05057	7	. 94943	53
8	. 65902	25	. 34098	. 70966	31	. 29034	. 05064	6	. 94936	52
9	. 65927	25	. 34073	. 70997	31	. 29003	. 05070	7	. 94930	51
10	9. 65952	24	10. 34048	9. 71028	31	10. 28972	10. 05077	6	9. 94923	50
11	. 65976	25	. 34024	. 71059	31	. 28941	. 05083	6	. 94917	49
12	. 66001	24	. 33999	. 71090	31	. 28910	. 05089	7	. 94911	48
13	. 66025	25	. 33975	. 71121	32	. 28879	. 05096	6	. 94904	47
14	. 66050	25	. 33950	. 71153	31	. 28847	. 05102	7	. 94898	46
15	9. 66075	24	10. 33925	9. 71184	31	10. 28816	10. 05109	6	9. 94891	45
16	. 66099	25	. 33901	. 71215	31	. 28785	. 05115	7	. 94885	44
17	. 66124	24	. 33876	. 71246	31	. 28754	. 05122	7	. 94878	43
18	. 66148	25	. 33852	. 71277	31	. 28723	. 05129	6	. 94871	42
19	. 66173	24	. 33827	. 71308	31	. 28692	. 05135	7	. 94865	41
20	9. 66197	24	10. 33803	9. 71339	31	10. 28661	10. 05142	6	9. 94858	40
21	. 66221	25	. 33779	. 71370	31	. 28630	. 05148	7	. 94852	39
22	. 66246	24	. 33754	. 71401	30	. 28599	. 05155	6	. 94845	38
23	. 66270	25	. 33730	. 71431	31	. 28569	. 05161	7	. 94839	37
24	. 66295	24	. 33705	. 71462	31	. 28538	. 05168	6	. 94832	36
25	9. 66319	24	10. 33681	9. 71493	31	10. 28507	10. 05174	7	9. 94826	35
26	. 66343	25	. 33657	. 71524	31	. 28476	. 05181	6	. 94819	34
27	. 66368	24	. 33632	. 71555	31	. 28445	. 05187	7	. 94813	33
28	. 66392	24	. 33608	. 71586	31	. 28414	. 05194	7	. 94806	32
29	. 66416	25	. 33584	. 71617	31	. 28383	. 05201	6	. 94799	31
30	9. 66441	24	10. 33559	9. 71648	31	10. 28352	10. 05207	7	9. 94793	30
31	. 66465	24	. 33535	. 71679	30	. 28321	. 05214	6	. 94786	29
32	. 66489	24	. 33511	. 71709	31	. 28291	. 05220	7	. 94780	28
33	. 66513	24	. 33487	. 71740	31	. 28260	. 05227	6	. 94773	27
34	. 66537	25	. 33463	. 71771	31	. 28229	. 05233	7	. 94767	26
35	9. 66562	24	10. 33438	9. 71802	31	10. 28198	10. 05240	7	9. 94760	25
36	. 66586	24	. 33414	. 71833	30	. 28167	. 05247	6	. 94753	24
37	. 66610	24	. 33390	. 71863	31	. 28137	. 05253	7	. 94747	23
38	. 66634	24	. 33366	. 71894	31	. 28106	. 05260	6	. 94740	22
39	. 66658	24	. 33342	. 71925	30	. 28075	. 05266	7	. 94734	21
40	9. 66682	24	10. 33318	9. 71955	31	10. 28045	10. 05273	7	9. 94727	20
41	. 66706	25	. 33294	. 71986	31	. 28014	. 05280	6	. 94720	19
42	. 66731	24	. 33269	. 72017	31	. 27983	. 05286	7	. 94714	18
43	. 66755	24	. 33245	. 72048	30	. 27952	. 05293	7	. 94707	17
44	. 66779	24	. 33221	. 72078	31	. 27922	. 05300	6	. 94700	16
45	9. 66803	24	10. 33197	9. 72109	31	10. 27891	10. 05306	7	9. 94694	15
46	. 66827	24	. 33173	. 72140	30	. 27860	. 05313	7	. 94687	14
47	. 66851	24	. 33149	. 72170	31	. 27830	. 05320	6	. 94680	13
48	. 66875	24	. 33125	. 72201	30	. 27799	. 05326	7	. 94674	12
49	. 66899	23	. 33101	. 72231	31	. 27769	. 05333	7	. 94667	11
50	9. 66922	24	10. 33078	9. 72262	31	10. 27738	10. 05340	6	9. 94660	10
51	. 66946	24	. 33054	. 72293	30	. 27707	. 05346	7	. 94654	9
52	. 66970	24	. 33030	. 72323	31	. 27677	. 05353	7	. 94647	8
53	. 66994	24	. 33006	. 72354	30	. 27646	. 05360	6	. 94640	7
54	. 67018	24	. 32982	. 72384	31	. 27616	. 05366	7	. 94634	6
55	9. 67042	24	10. 32958	9. 72415	30	10. 27585	10. 05373	7	9. 94627	5
56	. 67066	24	. 32934	. 72445	31	. 27555	. 05380	6	. 94620	4
57	. 67090	23	. 32910	. 72476	30	. 27524	. 05386	7	. 94614	3
58	. 67113	24	. 32887	. 72506	31	. 27494	. 05393	7	. 94607	2
59	. 67137	24	. 32863	. 72537	30	. 27463	. 05400	7	. 94600	1
60	9. 67161		10. 32839	9. 72567		10. 27433	10. 05407		9. 94593	0
↑ 117° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 62° ↑

TABLE 33

Logarithms of Trigonometric Functions

28°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←151° ↓ ′
0	9. 67161	24	10. 32839	9. 72567	31	10. 27433	10. 05407	6	9. 94593	60
1	. 67185	23	. 32815	. 72598	30	. 27402	. 05413	7	. 94587	59
2	. 67208	24	. 32792	. 72628	31	. 27372	. 05420	7	. 94580	58
3	. 67232	24	. 32768	. 72659	30	. 27341	. 05427	6	. 94573	57
4	. 67256	24	. 32744	. 72689	31	. 27311	. 05433	7	. 94567	56
5	9. 67280	23	10. 32720	9. 72720	30	10. 27280	10. 05440	7	9. 94560	55
6	. 67303	24	. 32697	. 72750	30	. 27250	. 05447	7	. 94553	54
7	. 67327	23	. 32673	. 72780	31	. 27220	. 05454	6	. 94546	53
8	. 67350	24	. 32650	. 72811	30	. 27189	. 05460	7	. 94540	52
9	. 67374	24	. 32626	. 72841	31	. 27159	. 05467	7	. 94533	51
10	9. 67398	23	10. 32602	9. 72872	30	10. 27128	10. 05474	7	9. 94526	50
11	. 67421	24	. 32579	. 72902	30	. 27098	. 05481	6	. 94519	49
12	. 67445	23	. 32555	. 72932	31	. 27068	. 05487	7	. 94513	48
13	. 67468	24	. 32532	. 72963	30	. 27037	. 05494	7	. 94506	47
14	. 67492	23	. 32508	. 72993	30	. 27007	. 05501	7	. 94499	46
15	9. 67515	24	10. 32485	9. 73023	31	10. 26977	10. 05508	7	9. 94492	45
16	. 67539	23	. 32461	. 73054	30	. 26946	. 05515	6	. 94485	44
17	. 67562	24	. 32438	. 73084	30	. 26916	. 05521	7	. 94479	43
18	. 67586	23	. 32414	. 73114	30	. 26886	. 05528	7	. 94472	42
19	. 67609	24	. 32391	. 73144	31	. 26856	. 05535	7	. 94465	41
20	9. 67633	23	10. 32367	9. 73175	30	10. 26825	10. 05542	7	9. 94458	40
21	. 67656	24	. 32344	. 73205	30	. 26795	. 05549	6	. 94451	39
22	. 67680	23	. 32320	. 73235	30	. 26765	. 05555	7	. 94445	38
23	. 67703	23	. 32297	. 73265	30	. 26735	. 05562	7	. 94438	37
24	. 67726	24	. 32274	. 73295	31	. 26705	. 05569	7	. 94431	36
25	9. 67750	23	10. 32250	9. 73326	30	10. 26674	10. 05576	7	9. 94424	35
26	. 67773	23	. 32227	. 73356	30	. 26644	. 05583	7	. 94417	34
27	. 67796	24	. 32204	. 73386	30	. 26614	. 05590	6	. 94410	33
28	. 67820	23	. 32180	. 73416	30	. 26584	. 05596	7	. 94404	32
29	. 67843	23	32157	. 73446	30	. 26554	. 05603	7	. 94397	31
30	9. 67866	24	10. 32134	9. 73476	31	10. 26524	10. 05610	7	9. 94390	30
31	. 67890	23	. 32110	. 73507	30	. 26493	. 05617	7	. 94383	29
32	. 67913	23	. 32087	. 73537	30	. 26463	. 05624	7	. 94376	28
33	. 67936	23	. 32064	. 73567	30	. 26433	. 05631	7	. 94369	27
34	. 67959	23	. 32041	. 73597	30	. 26403	. 05638	7	. 94362	26
35	9. 67982	24	10. 32018	9. 73627	30	10. 26373	10. 05645	6	9. 94355	25
36	. 68006	23	. 31994	. 73657	30	. 26343	. 05651	7	. 94349	24
37	. 68029	23	. 31971	. 73687	30	. 26313	. 05658	7	. 94342	23
38	. 68052	23	. 31948	. 73717	30	. 26283	. 05665	7	. 94335	22
39	. 68075	23	. 31925	. 73747	30	. 26253	. 05672	7	. 94328	21
40	9. 68098	23	10. 31902	9. 73777	30	10. 26223	10. 05679	7	9. 94321	20
41	. 68121	23	. 31879	. 73807	30	. 26193	. 05686	7	. 94314	19
42	. 68144	23	. 31856	. 73837	30	. 26163	. 05693	7	. 94307	18
43	. 68167	23	. 31833	. 73867	30	. 26133	. 05700	7	. 94300	17
44	. 68190	23	. 31810	. 73897	30	. 26103	. 05707	7	. 94293	16
45	9. 68213	24	10. 31787	9. 73927	30	10. 26073	10. 05714	7	9. 94286	15
46	. 68237	23	. 31763	. 73957	30	. 26043	. 05721	6	. 94279	14
47	. 68260	23	. 31740	. 73987	30	. 26013	. 05727	7	. 94273	13
48	. 68283	22	. 31717	. 74017	30	. 25983	. 05734	7	. 94266	12
49	. 68305	23	. 31695	. 74047	30	. 25953	. 05741	7	. 94259	11
50	9. 68328	23	10. 31672	9. 74077	30	10. 25923	10. 05748	7	9. 94252	10
51	. 68351	23	. 31649	. 74107	30	. 25893	. 05755	7	. 94245	9
52	. 68374	23	. 31626	. 74137	29	. 25863	. 05762	7	. 94238	8
53	. 68397	23	. 31603	. 74166	30	. 25834	. 05769	7	. 94231	7
54	. 68420	23	. 31580	. 74196	30	. 25804	. 05776	7	. 94224	6
55	9. 68443	23	10. 31557	9. 74226	30	10. 25774	10. 05783	7	9. 94217	5
56	. 68466	23	. 31534	. 74256	30	. 25744	. 05790	7	. 94210	4
57	. 68489	23	. 31511	. 74286	30	. 25714	. 05797	7	. 94203	3
58	. 68512	22	. 31488	. 74316	29	. 25684	. 05804	7	. 94196	2
59	. 68534	23	. 31466	. 74345	30	. 25655	. 05811	7	. 94189	1
60	9. 68557		10. 31443	9. 74375		10. 25625	10. 05818		9. 94182	0
↑ 118°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←61°

TABLE 33

Logarithms of Trigonometric Functions

29° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 150° ↓ ′
0	9. 68557	23	10. 31443	9. 74375	30	10. 25625	10. 05818	7	9. 94182	60
1	. 68580	23	. 31420	. 74405	30	. 25595	. 05825	7	. 94175	59
2	. 68603	22	. 31397	. 74435	30	. 25565	. 05832	7	. 94168	58
3	. 68625	23	. 31375	. 74465	29	. 25535	. 05839	7	. 94161	57
4	. 68648	23	. 31352	. 74494	30	. 25506	. 05846	7	. 94154	56
5	9. 68671	23	10. 31329	9. 74524	30	10. 25476	10. 05853	7	9. 94147	55
6	. 68694	22	. 31306	. 74554	29	. 25446	. 05860	7	. 94140	54
7	. 68716	23	. 31284	. 74583	30	. 25417	. 05867	7	. 94133	53
8	. 68739	23	. 31261	. 74613	30	. 25387	. 05874	7	. 94126	52
9	. 68762	22	. 31238	. 74643	30	. 25357	. 05881	7	. 94119	51
10	9. 68784	23	10. 31216	9. 74673	29	10. 25327	10. 05888	7	9. 94112	50
11	. 68807	22	. 31193	. 74702	30	. 25298	. 05895	7	. 94105	49
12	. 68829	23	. 31171	. 74732	30	. 25268	. 05902	8	. 94098	48
13	. 68852	23	. 31148	. 74762	29	. 25238	. 05910	7	. 94090	47
14	. 68875	22	. 31125	. 74791	30	. 25209	. 05917	7	. 94083	46
15	9. 68897	23	10. 31103	9. 74821	30	10. 25179	10. 05924	7	9. 94076	45
16	. 68920	22	. 31080	. 74851	29	. 25149	. 05931	7	. 94069	44
17	. 68942	23	. 31058	. 74880	30	. 25120	. 05938	7	. 94062	43
18	. 68965	22	. 31035	. 74910	29	. 25090	. 05945	7	. 94055	42
19	. 68987	23	. 31013	. 74939	30	. 25061	. 05952	7	. 94048	41
20	9. 69010	22	10. 30990	9. 74969	29	10. 25031	10. 05959	7	9. 94041	40
21	. 69032	23	. 30968	. 74998	30	. 25002	. 05966	7	. 94034	39
22	. 69055	22	. 30945	. 75028	30	. 24972	. 05973	7	. 94027	38
23	. 69077	23	. 30923	. 75058	29	. 24942	. 05980	8	. 94020	37
24	. 69100	22	. 30900	. 75087	30	. 24913	. 05988	7	. 94012	36
25	9. 69122	22	10. 30878	9. 75117	29	10. 24883	10. 05995	7	9. 94005	35
26	. 69144	23	. 30856	. 75146	30	. 24854	. 06002	7	. 93998	34
27	. 69167	22	. 30833	. 75176	29	. 24824	. 06009	7	. 93991	33
28	. 69189	23	. 30811	. 75205	30	. 24795	. 06016	7	. 93984	32
29	. 69212	22	. 30788	. 75235	29	. 24765	. 06023	7	. 93977	31
30	9. 69234	22	10. 30766	9. 75264	30	10. 24736	10. 06030	7	9. 93970	30
31	. 69256	23	. 30744	. 75294	29	. 24706	. 06037	8	. 93963	29
32	. 69279	22	. 30721	. 75323	30	. 24677	. 06045	7	. 93955	28
33	. 69301	22	. 30699	. 75353	29	. 24647	. 06052	7	. 93948	27
34	. 69323	22	. 30677	. 75382	29	. 24618	. 06059	7	. 93941	26
35	9. 69345	23	10. 30655	9. 75411	30	10. 24589	10. 06066	7	9. 93934	25
36	. 69368	22	. 30632	. 75441	29	. 24559	. 06073	7	. 93927	24
37	. 69390	22	. 30610	. 75470	30	. 24530	. 06080	8	. 93920	23
38	. 69412	22	. 30588	. 75500	29	. 24500	. 06088	7	. 93912	22
39	. 69434	22	. 30566	. 75529	29	. 24471	. 06095	7	. 93905	21
40	9. 69456	23	10. 30544	9. 75558	30	10. 24442	10. 06102	7	9. 93898	20
41	. 69479	22	. 30521	. 75588	29	. 24412	. 06109	7	. 93891	19
42	. 69501	22	. 30499	. 75617	30	. 24383	. 06116	8	. 93884	18
43	. 69523	22	. 30477	. 75647	29	. 24353	. 06124	7	. 93876	17
44	. 69545	22	. 30455	. 75676	29	. 24324	. 06131	7	. 93869	16
45	9. 69567	22	10. 30433	9. 75705	30	10. 24295	10. 06138	7	9. 93862	15
46	. 69589	22	. 30411	. 75735	29	. 24265	. 06145	8	. 93855	14
47	. 69611	22	. 30389	. 75764	29	. 24236	. 06153	7	. 93847	13
48	. 69633	22	. 30367	. 75793	29	. 24207	. 06160	7	. 93840	12
49	. 69655	22	. 30345	. 75822	30	. 24178	. 06167	7	. 93833	11
50	9. 69677	22	10. 30323	9. 75852	29	10. 24148	10. 06174	7	9. 93826	10
51	. 69699	22	. 30301	. 75881	29	. 24119	. 06181	8	. 93819	9
52	. 69721	22	. 30279	. 75910	29	. 24090	. 06189	7	. 93811	8
53	. 69743	22	. 30257	. 75939	30	. 24061	. 06196	7	. 93804	7
54	. 69765	22	. 30235	. 75969	29	. 24031	. 06203	8	. 93797	6
55	9. 69787	22	10. 30213	9. 75998	29	10. 24002	10. 06211	7	9. 93789	5
56	. 69809	22	. 30191	. 76027	29	. 23973	. 06218	7	. 93782	4
57	. 69831	22	. 30169	. 76056	30	. 23944	. 06225	7	. 93775	3
58	. 69853	22	. 30147	. 76086	29	. 23914	. 06232	8	. 93768	2
59	. 69875	22	. 30125	. 76115	29	. 23885	. 06240	7	. 93760	1
60	9. 69897		10. 30103	9. 76144		10. 23856	10. 06247		9. 93753	0
↑ 119° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ← 60°

TABLE 33
Logarithms of Trigonometric Functions

30°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←149° ↓ ′
0	9. 69897	22	10. 30103	9. 76144	29	10. 23856	10. 06247	7	9. 93753	60
1	. 69919	22	. 30081	. 76173	29	. 23827	. 06254	8	. 93746	59
2	. 69941	22	. 30059	. 76202	29	. 23798	. 06262	7	. 93738	58
3	. 69963	21	. 30037	. 76231	30	. 23769	. 06269	7	. 93731	57
4	. 69984	22	. 30016	. 76261	29	. 23739	. 06276	7	. 93724	56
5	9. 70006	22	10. 29994	9. 76290	29	10. 23710	10. 06283	8	9. 93717	55
6	. 70028	22	. 29972	. 76319	29	. 23681	. 06291	7	. 93709	54
7	. 70050	22	. 29950	. 76348	29	. 23652	. 06298	7	. 93702	53
8	. 70072	21	. 29928	. 76377	29	. 23623	. 06305	8	. 93695	52
9	. 70093	22	. 29907	. 76406	29	. 23594	. 06313	7	. 93687	51
10	9. 70115	22	10. 29885	9. 76435	29	10. 23565	10. 06320	7	9. 93680	50
11	. 70137	22	. 29863	. 76464	29	. 23536	. 06327	8	. 93673	49
12	. 70159	21	. 29841	. 76493	29	. 23507	. 06335	7	. 93665	48
13	. 70180	22	. 29820	. 76522	29	. 23478	. 06342	8	. 93658	47
14	. 70202	22	. 29798	. 76551	29	. 23449	. 06350	7	. 93650	46
15	9. 70224	21	10. 29776	9. 76580	29	10. 23420	10. 06357	7	9. 93643	45
16	. 70245	22	. 29755	. 76609	30	. 23391	. 06364	8	. 93636	44
17	. 70267	21	. 29733	. 76639	29	. 23361	. 06372	7	. 93628	43
18	. 70288	22	. 29712	. 76668	29	. 23332	. 06379	7	. 93621	42
19	. 70310	22	. 29690	. 76697	28	. 23303	. 06386	8	. 93614	41
20	9. 70332	21	10. 29668	9. 76725	29	10. 23275	10. 06394	7	9. 93606	40
21	. 70353	22	. 29647	. 76754	29	. 23246	. 06401	8	. 93599	39
22	. 70375	21	. 29625	. 76783	29	. 23217	. 06409	7	. 93591	38
23	. 70396	22	. 29604	. 76812	29	. 23188	. 06416	7	. 93584	37
24	. 70418	21	. 29582	. 76841	29	. 23159	. 06423	8	. 93577	36
25	9. 70439	22	10. 29561	9. 76870	29	10. 23130	10. 06431	7	9. 93569	35
26	. 70461	21	. 29539	. 76899	29	. 23101	. 06438	8	. 93562	34
27	. 70482	22	. 29518	. 76928	29	. 23072	. 06446	7	. 93554	33
28	. 70504	21	. 29496	. 76957	29	. 23043	. 06453	8	. 93547	32
29	. 70525	22	. 29475	. 76986	29	. 23014	. 06461	7	. 93539	31
30	9. 70547	21	10. 29453	9. 77015	29	10. 22985	10. 06468	7	9. 93532	30
31	. 70568	22	. 29432	. 77044	29	. 22956	. 06475	8	. 93525	29
32	. 70590	21	. 29410	. 77073	28	. 22927	. 06483	7	. 93517	28
33	. 70611	22	. 29389	. 77101	29	. 22899	. 06490	8	. 93510	27
34	. 70633	21	. 29367	. 77130	29	. 22870	. 06498	7	. 93502	26
35	9. 70654	21	10. 29346	9. 77159	29	10. 22841	10. 06505	8	9. 93495	25
36	. 70675	22	. 29325	. 77188	29	. 22812	. 06513	7	. 93487	24
37	. 70697	21	. 29303	. 77217	29	. 22783	. 06520	8	. 93480	23
38	. 70718	21	. 29282	. 77246	28	. 22754	. 06528	7	. 93472	22
39	. 70739	22	. 29261	. 77274	29	. 22726	. 06535	8	. 93465	21
40	9. 70761	21	10. 29239	9. 77303	29	10. 22697	10. 06543	7	9. 93457	20
41	. 70782	21	. 29218	. 77332	29	. 22668	. 06550	8	. 93450	19
42	. 70803	21	. 29197	. 77361	29	. 22639	. 06558	7	. 93442	18
43	. 70824	22	. 29176	. 77390	28	. 22610	. 06565	8	. 93435	17
44	. 70846	21	. 29154	. 77418	29	. 22582	. 06573	7	. 93427	16
45	9. 70867	21	10. 29133	9. 77447	29	10. 22553	10. 06580	8	9. 93420	15
46	. 70888	21	. 29112	. 77476	29	. 22524	. 06588	7	. 93412	14
47	. 70909	22	. 29091	. 77505	28	. 22495	. 06595	8	. 93405	13
48	. 70931	21	. 29069	. 77533	29	. 22467	. 06603	7	. 93397	12
49	. 70952	21	. 29048	. 77562	29	. 22438	. 06610	8	. 93390	11
50	9. 70973	21	10. 29027	9. 77591	28	10. 22409	10. 06618	7	9. 93382	10
51	. 70994	21	. 29006	. 77619	29	. 22381	. 06625	8	. 93375	9
52	. 71015	21	. 28985	. 77648	29	. 22352	. 06633	7	. 93367	8
53	. 71036	22	. 28964	. 77677	29	. 22323	. 06640	8	. 93360	7
54	. 71058	21	. 28942	. 77706	28	. 22294	. 06648	8	. 93352	6
55	9. 71079	21	10. 28921	9. 77734	29	10. 22266	10. 06656	7	9. 93344	5
56	. 71100	21	. 28900	. 77763	28	. 22237	. 06663	8	. 93337	4
57	. 71121	21	. 28879	. 77791	29	. 22209	. 06671	7	. 93329	3
58	. 71142	21	. 28858	. 77820	29	. 22180	. 06678	8	. 93322	2
59	. 71163	21	. 28837	. 77849	28	. 22151	. 06686	7	. 93314	1
60	9. 71184		10. 28816	9. 77877		10. 22123	10. 06693		9. 93307	0
↑ 120°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←59°

TABLE 33

Logarithms of Trigonometric Functions

31° ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos 148°	′
0	9. 71184	21	10. 28816	9. 77877	29	10. 22123	10. 06693	8	9. 93307	60
1	. 71205	21	. 28795	. 77906	29	. 22094	. 06701	8	. 93299	59
2	. 71226	21	. 28774	. 77935	28	. 22065	. 06709	7	. 93291	58
3	. 71247	21	. 28753	. 77963	29	. 22037	. 06716	8	. 93284	57
4	. 71268	21	. 28732	. 77992	28	. 22008	. 06724	7	. 93276	56
5	9. 71289	21	10. 28711	9. 78020	29	10. 21980	10. 06731	8	9. 93269	55
6	. 71310	21	. 28690	. 78049	28	. 21951	. 06739	8	. 93261	54
7	. 71331	21	. 28669	. 78077	29	. 21923	. 06747	7	. 93253	53
8	. 71352	21	. 28648	. 78106	29	. 21894	. 06754	8	. 93246	52
9	. 71373	20	. 28627	. 78135	28	. 21865	. 06762	8	. 93238	51
10	9. 71393	21	10. 28607	9. 78163	29	10. 21837	10. 06770	7	9. 93230	50
11	. 71414	21	. 28586	. 78192	28	. 21808	. 06777	8	. 93223	49
12	. 71435	21	. 28565	. 78220	29	. 21780	. 06785	8	. 93215	48
13	. 71456	21	. 28544	. 78249	28	. 21751	. 06793	7	. 93207	47
14	. 71477	21	. 28523	. 78277	29	. 21723	. 06800	8	. 93200	46
15	9. 71498	21	10. 28502	9. 78306	28	10. 21694	10. 06808	8	9. 93192	45
16	. 71519	20	. 28481	. 78334	29	. 21666	. 06816	7	. 93184	44
17	. 71539	21	. 28461	. 78363	28	. 21637	. 06823	8	. 93177	43
18	. 71560	21	. 28440	. 78391	28	. 21609	. 06831	8	. 93169	42
19	. 71581	21	. 28419	. 78419	29	. 21581	. 06839	7	. 93161	41
20	9. 71602	20	10. 28398	9. 78448	28	10. 21552	10. 06846	8	9. 93154	40
21	. 71622	21	. 28378	. 78476	29	. 21524	. 06854	8	. 93146	39
22	. 71643	21	. 28357	. 78505	28	. 21495	. 06862	7	. 93138	38
23	. 71664	21	. 28336	. 78533	29	. 21467	. 06869	8	. 93131	37
24	. 71685	20	. 28315	. 78562	28	. 21438	. 06877	8	. 93123	36
25	9. 71705	21	10. 28295	9. 78590	28	10. 21410	10. 06885	7	9. 93115	35
26	. 71726	21	. 28274	. 78618	29	. 21382	. 06892	8	. 93108	34
27	. 71747	20	. 28253	. 78647	28	. 21353	. 06900	8	. 93100	33
28	. 71767	21	. 28233	. 78675	29	. 21325	. 06908	8	. 93092	32
29	. 71788	21	. 28212	. 78704	28	. 21296	. 06916	7	. 93084	31
30	9. 71809	20	10. 28191	9. 78732	28	10. 21268	10. 06923	8	9. 93077	30
31	. 71829	21	. 28171	. 78760	29	. 21240	. 06931	8	. 93069	29
32	. 71850	20	. 28150	. 78789	28	. 21211	. 06939	8	. 93061	28
33	. 71870	21	. 28130	. 78817	28	. 21183	. 06947	7	. 93053	27
34	. 71891	20	. 28109	. 78845	29	. 21155	. 06954	8	. 93046	26
35	9. 71911	21	10. 28089	9. 78874	28	10. 21126	10. 06962	8	9. 93038	25
36	. 71932	20	. 28068	. 78902	28	. 21098	. 06970	8	. 93030	24
37	. 71952	21	. 28048	. 78930	29	. 21070	. 06978	8	. 93022	23
38	. 71973	21	. 28027	. 78959	28	. 21041	. 06986	7	. 93014	22
39	. 71994	20	. 28006	. 78987	28	. 21013	. 06993	8	. 93007	21
40	9. 72014	20	10. 27986	9. 79015	28	10. 20985	10. 07001	8	9. 92999	20
41	. 72034	21	. 27966	. 79043	29	. 20957	. 07009	8	. 92991	19
42	. 72055	20	. 27945	. 79072	28	. 20928	. 07017	7	. 92983	18
43	. 72075	21	. 27925	. 79100	28	. 20900	. 07024	8	. 92976	17
44	. 72096	20	. 27904	. 79128	28	. 20872	. 07032	8	. 92968	16
45	9. 72116	21	10. 27884	9. 79156	29	10. 20844	10. 07040	8	9. 92960	15
46	. 72137	20	. 27863	. 79185	28	. 20815	. 07048	8	. 92952	14
47	. 72157	20	. 27843	. 79213	28	. 20787	. 07056	8	. 92944	13
48	. 72177	21	. 27823	. 79241	28	. 20759	. 07064	7	. 92936	12
49	. 72198	20	. 27802	. 79269	28	. 20731	. 07071	8	. 92929	11
50	9. 72218	20	10. 27782	9. 79297	29	10. 20703	10. 07079	8	9. 92921	10
51	. 72238	21	. 27762	. 79326	28	. 20674	. 07087	8	. 92913	9
52	. 72259	20	. 27741	. 79354	28	. 20646	. 07095	8	. 92905	8
53	. 72279	20	. 27721	. 79382	28	. 20618	. 07103	8	. 92897	7
54	. 72299	21	. 27701	. 79410	28	. 20590	. 07111	8	. 92889	6
55	9. 72320	20	10. 27680	9. 79438	28	10. 20562	10. 07119	7	9. 92881	5
56	. 72340	20	. 27660	. 79466	29	. 20534	. 07126	8	. 92874	4
57	. 72360	21	. 27640	. 79495	28	. 20505	. 07134	8	. 92866	3
58	. 72381	20	. 27619	. 79523	28	. 20477	. 07142	8	. 92858	2
59	. 72401	20	. 27599	. 79551	28	. 20449	. 07150	8	. 92850	1
60	9. 72421		10. 27579	9. 79579		10. 20421	10. 07158		9. 92842	0
121°	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin 58°	

TABLE 33
Logarithms of Trigonometric Functions

32° ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←147° ↓ ′
0	9. 72421	20	10. 27579	9. 79579	28	10. 20421	10. 07158	8	9. 92842	60
1	. 72441	20	. 27559	. 79607	28	. 20393	. 07166	8	. 92834	59
2	. 72461	21	. 27539	. 79635	28	. 20365	. 07174	8	. 92826	58
3	. 72482	20	. 27518	. 79663	28	. 20337	. 07182	8	. 92818	57
4	. 72502	20	. 27498	. 79691	28	. 20309	. 07190	7	. 92810	56
5	9. 72522	20	10. 27478	9. 79719	28	10. 20281	10. 07197	8	9. 92803	55
6	. 72542	20	. 27458	. 79747	29	. 20253	. 07205	8	. 92795	54
7	. 72562	20	. 27438	. 79776	28	. 20224	. 07213	8	. 92787	53
8	. 72582	20	. 27418	. 79804	28	. 20196	. 07221	8	. 92779	52
9	. 72602	20	. 27398	. 79832	28	. 20168	. 07229	8	. 92771	51
10	9. 72622	21	10. 27378	9. 79860	28	10. 20140	10. 07237	8	9. 92763	50
11	. 72643	20	. 27357	. 79888	28	. 20112	. 07245	8	. 92755	49
12	. 72663	20	. 27337	. 79916	28	. 20084	. 07253	8	. 92747	48
13	. 72683	20	. 27317	. 79944	28	. 20056	. 07261	8	. 92739	47
14	. 72703	20	. 27297	. 79972	28	. 20028	. 07269	8	. 92731	46
15	9. 72723	20	10. 27277	9. 80000	28	10. 20000	10. 07277	8	9. 92723	45
16	. 72743	20	. 27257	. 80028	28	. 19972	. 07285	8	. 92715	44
17	. 72763	20	. 27237	. 80056	28	. 19944	. 07293	8	. 92707	43
18	. 72783	20	. 27217	. 80084	28	. 19916	. 07301	8	. 92699	42
19	. 72803	20	. 27197	. 80112	28	. 19888	. 07309	8	. 92691	41
20	9. 72823	20	10. 27177	9. 80140	28	10. 19860	10. 07317	8	9. 92683	40
21	. 72843	20	. 27157	. 80168	27	. 19832	. 07325	8	. 92675	39
22	. 72863	20	. 27137	. 80195	28	. 19805	. 07333	8	. 92667	38
23	. 72883	19	. 27117	. 80223	28	. 19777	. 07341	8	. 92659	37
24	. 72902	20	. 27098	. 80251	28	. 19749	. 07349	8	. 92651	36
25	9. 72922	20	10. 27078	9. 80279	28	10. 19721	10. 07357	8	9. 92643	35
26	. 72942	20	. 27058	. 80307	28	. 19693	. 07365	8	. 92635	34
27	. 72962	20	. 27038	. 80335	28	. 19665	. 07373	8	. 92627	33
28	. 72982	20	27018	. 80363	28	. 19637	. 07381	8	. 92619	32
29	. 73002	20	. 26998	. 80391	28	. 19609	. 07389	8	. 92611	31
30	9. 73022	19	10. 26978	9. 80419	28	10. 19581	10. 07397	8	9. 92603	30
31	. 73041	20	. 26959	. 80447	27	. 19553	. 07405	8	. 92595	29
32	. 73061	20	. 26939	. 80474	28	. 19526	. 07413	8	. 92587	28
33	. 73081	20	. 26919	. 80502	28	. 19498	. 07421	8	. 92579	27
34	. 73101	20	. 26899	. 80530	28	. 19470	. 07429	8	. 92571	26
35	9. 73121	19	10. 26879	9. 80558	28	10. 19442	10. 07437	8	9. 92563	25
36	. 73140	20	. 26860	. 80586	28	. 19414	. 07445	9	. 92555	24
37	. 73160	20	. 26840	. 80614	28	. 19386	. 07454	8	. 92546	23
38	. 73180	20	. 26820	. 80642	27	. 19358	. 07462	8	. 92538	22
39	. 73200	19	. 26800	. 80669	28	. 19331	. 07470	8	. 92530	21
40	9. 73219	20	10. 26781	9. 80697	28	10. 19303	10. 07478	8	9. 92522	20
41	. 73239	20	. 26761	. 80725	28	. 19275	. 07486	8	. 92514	19
42	. 73259	19	. 26741	. 80753	28	. 19247	. 07494	8	. 92506	18
43	. 73278	20	. 26722	. 80781	27	. 19219	. 07502	8	. 92498	17
44	. 73298	20	. 26702	. 80808	28	. 19192	. 07510	8	. 92490	16
45	9. 73318	19	10. 26682	9. 80836	28	10. 19164	10. 07518	9	9. 92482	15
46	. 73337	20	. 26663	. 80864	28	. 19136	. 07527	8	. 92473	14
47	. 73357	20	. 26643	. 80892	27	. 19108	. 07535	8	. 92465	13
48	. 73377	19	. 26623	. 80919	28	. 19081	. 07543	8	. 92457	12
49	. 73396	20	. 26604	. 80947	28	. 19053	. 07551	8	. 92449	11
50	9. 73416	19	10. 26584	9. 80975	28	10. 19025	10. 07559	8	9. 92441	10
51	. 73435	20	. 26565	. 81003	27	. 18997	. 07567	8	. 92433	9
52	. 73455	19	. 26545	. 81030	28	. 18970	. 07575	9	. 92425	8
53	. 73474	20	. 26526	. 81058	28	. 18942	. 07584	8	. 92416	7
54	. 73494	19	. 26506	. 81086	27	. 18914	. 07592	8	. 92408	6
55	9. 73513	20	10. 26487	9. 81113	28	10. 18887	10. 07600	8	9. 92400	5
56	. 73533	19	. 26467	. 81141	28	. 18859	. 07608	8	. 92392	4
57	. 73552	20	. 26448	. 81169	27	. 18831	. 07616	8	. 92384	3
58	. 73572	19	. 26428	. 81196	28	. 18804	. 07624	9	. 92376	2
59	. 73591	20	. 26409	. 81224	28	. 18776	. 07633	8	. 92367	1
60	9. 73611		10. 26389	9. 81252		10. 18748	10. 07641		9. 92359	0
↑ 122° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←57°

TABLE 33
Logarithms of Trigonometric Functions

33°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←146° ↓ ′
0	9. 73611	19	10. 26389	9. 81252	27	10. 18748	10. 07641	8	9. 92359	60
1	. 73630	20	. 26370	. 81279	28	. 18721	. 07649	8	. 92351	59
2	. 73650	19	. 26350	. 81307	28	. 18693	. 07657	8	. 92343	58
3	. 73669	20	. 26331	. 81335	27	. 18665	. 07665	9	. 92335	57
4	. 73689	19	. 26311	. 81362	28	. 18638	. 07674	8	. 92326	56
5	9. 73708	19	10. 26292	9. 81390	28	10. 18610	10. 07682	8	9. 92318	55
6	. 73727	20	. 26273	. 81418	27	. 18582	. 07690	8	. 92310	54
7	. 73747	19	. 26253	. 81445	28	. 18555	. 07698	9	. 92302	53
8	. 73766	19	. 26234	. 81473	27	. 18527	. 07707	8	. 92293	52
9	. 73785	20	. 26215	. 81500	28	. 18500	. 07715	8	. 92285	51
10	9. 73805	19	10. 26195	9. 81528	28	10. 18472	10. 07723	8	9. 92277	50
11	. 73824	19	. 26176	. 81556	27	. 18444	. 07731	9	. 92269	49
12	. 73843	20	. 26157	. 81583	28	. 18417	. 07740	8	. 92260	48
13	. 73863	19	. 26137	. 81611	27	. 18389	. 07748	8	. 92252	47
14	. 73882	19	. 26118	. 81638	28	. 18362	. 07756	9	. 92244	46
15	9. 73901	20	10. 26099	9. 81666	27	10. 18334	10. 07765	8	9. 92235	45
16	. 73921	19	. 26079	. 81693	28	. 18307	. 07773	8	. 92227	44
17	. 73940	19	. 26060	. 81721	27	. 18279	. 07781	8	. 92219	43
18	. 73959	19	. 26041	. 81748	28	. 18252	. 07789	9	. 92211	42
19	. 73978	19	. 26022	. 81776	27	. 18224	. 07798	8	. 92202	41
20	9. 73997	20	10. 26003	9. 81803	28	10. 18197	10. 07806	8	9. 92194	40
21	. 74017	19	. 25983	. 81831	27	. 18169	. 07814	9	. 92186	39
22	. 74036	19	. 25964	. 81858	28	. 18142	. 07823	8	. 92177	38
23	. 74055	19	. 25945	. 81886	27	. 18114	. 07831	8	. 92169	37
24	. 74074	19	. 25926	. 81913	28	. 18087	. 07839	9	. 92161	36
25	9. 74093	20	10. 25907	9. 81941	27	10. 18059	10. 07848	8	9. 92152	35
26	. 74113	19	. 25887	. 81968	28	. 18032	. 07856	8	. 92144	34
27	. 74132	19	. 25868	. 81996	27	. 18004	. 07864	9	. 92136	33
28	. 74151	19	. 25849	. 82023	28	. 17977	. 07873	8	. 92127	32
29	. 74170	19	. 25830	. 82051	27	. 17949	. 07881	8	. 92119	31
30	9. 74189	19	10. 25811	9. 82078	28	10. 17922	10. 07889	9	9. 92111	30
31	. 74208	19	. 25792	. 82106	27	. 17894	. 07898	8	. 92102	29
32	. 74227	19	. 25773	. 82133	28	. 17867	. 07906	8	. 92094	28
33	. 74246	19	. 25754	. 82161	27	. 17839	. 07914	9	. 92086	27
34	. 74265	19	. 25735	. 82188	27	. 17812	. 07923	8	. 92077	26
35	9. 74284	19	10. 25716	9. 82215	28	10. 17785	10. 07931	9	9. 92069	25
36	. 74303	19	. 25697	. 82243	27	. 17757	. 07940	8	. 92060	24
37	. 74322	19	. 25678	. 82270	28	. 17730	. 07948	8	. 92052	23
38	. 74341	19	. 25659	. 82298	27	. 17702	. 07956	9	. 92044	22
39	. 74360	19	. 25640	. 82325	27	. 17675	. 07965	8	. 92035	21
40	9. 74379	19	10. 25621	9. 82352	28	10. 17648	10. 07973	9	9. 92027	20
41	. 74398	19	. 25602	. 82380	27	. 17620	. 07982	8	. 92018	19
42	. 74417	19	. 25583	. 82407	28	. 17593	. 07990	8	. 92010	18
43	. 74436	19	. 25564	. 82435	27	. 17565	. 07998	9	. 92002	17
44	. 74455	19	. 25545	. 82462	27	. 17538	. 08007	8	. 91993	16
45	9. 74474	19	10. 25526	9. 82489	28	10. 17511	10. 08015	9	9. 91985	15
46	. 74493	19	. 25507	. 82517	27	. 17483	. 08024	8	. 91976	14
47	. 74512	19	. 25488	. 82544	27	. 17456	. 08032	9	. 91968	13
48	. 74531	18	. 25469	. 82571	28	. 17429	. 08041	8	. 91959	12
49	. 74549	19	. 25451	. 82599	27	. 17401	. 08049	9	. 91951	11
50	9. 74568	19	10. 25432	9. 82626	27	10. 17374	10. 08058	8	9. 91942	10
51	. 74587	19	. 25413	. 82653	28	. 17347	. 08066	9	. 91934	9
52	. 74606	19	. 25394	. 82681	27	. 17319	. 08075	8	. 91925	8
53	. 74625	19	. 25375	. 82708	27	. 17292	. 08083	9	. 91917	7
54	. 74644	18	. 25356	. 82735	27	. 17265	. 08092	8	. 91908	6
55	9. 74662	19	10. 25338	9. 82762	28	10. 17238	10. 08100	9	9. 91900	5
56	. 74681	19	. 25319	. 82790	27	. 17210	. 08109	8	. 91891	4
57	. 74700	19	. 25300	. 82817	27	. 17183	. 08117	9	. 91883	3
58	. 74719	18	. 25281	. 82844	27	. 17156	. 08126	8	. 91874	2
59	. 74737	19	. 25263	. 82871	28	. 17129	. 08134	9	. 91866	1
60	9. 74756		10. 25244	9. 82899		10. 17101	10. 08143		9. 91857	0
↑ 123°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←56°

TABLE 33

Logarithms of Trigonometric Functions

34°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←145° ↓ ′
0	9. 74756	19	10. 25244	9. 82899	27	10. 17101	10. 08143	8	9. 91857	60
1	. 74775	19	. 25225	. 82926	27	. 17074	. 08151	9	. 91849	59
2	. 74794	18	. 25206	. 82953	27	. 17047	. 08160	8	. 91840	58
3	. 74812	19	. 25188	. 82980	28	. 17020	. 08168	9	. 91832	57
4	. 74831	19	. 25169	. 83008	27	. 16992	. 08177	8	. 91823	56
5	9. 74850	18	10. 25150	9. 83035	27	10. 16965	10. 08185	9	9. 91815	55
6	. 74868	19	. 25132	. 83062	27	. 16938	. 08194	8	. 91806	54
7	. 74887	19	. 25113	. 83089	28	. 16911	. 08202	9	. 91798	53
8	. 74906	18	. 25094	. 83117	27	. 16883	. 08211	8	. 91789	52
9	. 74924	19	. 25076	. 83144	27	. 16856	. 08219	9	. 91781	51
10	9. 74943	18	10. 25057	9. 83171	27	10. 16829	10. 08228	9	9. 91772	50
11	. 74961	19	. 25039	. 83198	27	. 16802	. 08237	8	. 91763	49
12	. 74980	19	. 25020	. 83225	27	. 16775	. 08245	9	. 91755	48
13	. 74999	18	. 25001	. 83252	28	. 16748	. 08254	8	. 91746	47
14	. 75017	19	. 24983	. 83280	27	. 16720	. 08262	9	. 91738	46
15	9. 75036	18	10. 24964	9. 83307	27	10. 16693	10. 08271	9	9. 91729	45
16	. 75054	19	. 24946	. 83334	27	. 16666	. 08280	8	. 91720	44
17	. 75073	18	. 24927	. 83361	27	. 16639	. 08288	9	. 91712	43
18	. 75091	19	. 24909	. 83388	27	. 16612	. 08297	8	. 91703	42
19	. 75110	18	. 24890	. 83415	27	. 16585	. 08305	9	. 91695	41
20	9. 75128	19	10. 24872	9. 83442	28	10. 16558	10. 08314	9	9. 91686	40
21	. 75147	18	. 24853	. 83470	27	. 16530	. 08323	8	. 91677	39
22	. 75165	19	. 24835	. 83497	27	. 16503	. 08331	9	. 91669	38
23	. 75184	18	. 24816	. 83524	27	. 16476	. 08340	9	. 91660	37
24	. 75202	19	. 24798	. 83551	27	. 16449	. 08349	8	. 91651	36
25	9. 75221	18	10. 24779	9. 83578	27	10. 16422	10. 08357	9	9. 91643	35
26	. 75239	19	. 24761	. 83605	27	. 16395	. 08366	9	. 91634	34
27	. 75258	18	. 24742	. 83632	27	. 16368	. 08375	8	. 91625	33
28	. 75276	18	24724	. 83659	27	. 16341	. 08383	9	. 91617	32
29	. 75294	19	. 24706	. 83686	27	. 16314	. 08392	9	. 91608	31
30	9. 75313	18	10. 24687	9. 83713	27	10. 16287	10. 08401	8	9. 91599	30
31	. 75331	19	. 24669	. 83740	28	. 16260	. 08409	9	. 91591	29
32	. 75350	18	. 24650	. 83768	27	. 16232	. 08418	9	. 91582	28
33	. 75368	18	. 24632	. 83795	27	. 16205	. 08427	8	. 91573	27
34	. 75386	19	. 24614	. 83822	27	. 16178	. 08435	9	. 91565	26
35	9. 75405	18	10. 24595	9. 83849	27	10. 16151	10. 08444	9	9. 91556	25
36	. 75423	18	. 24577	. 83876	27	. 16124	. 08453	9	. 91547	24
37	. 75441	18	. 24559	. 83903	27	. 16097	. 08462	8	. 91538	23
38	. 75459	19	. 24541	. 83930	27	. 16070	. 08470	9	. 91530	22
39	. 75478	18	. 24522	. 83957	27	. 16043	. 08479	9	. 91521	21
40	9. 75496	18	10. 24504	9. 83984	27	10. 16016	10. 08488	8	9. 91512	20
41	. 75514	19	. 24486	. 84011	27	. 15989	. 08496	9	. 91504	19
42	. 75533	18	. 24467	. 84038	27	. 15962	. 08505	9	. 91495	18
43	. 75551	18	. 24449	. 84065	27	. 15935	. 08514	9	. 91486	17
44	. 75569	18	. 24431	. 84092	27	. 15908	. 08523	8	. 91477	16
45	9. 75587	18	10. 24413	9. 84119	27	10. 15881	10. 08531	9	9. 91469	15
46	. 75605	19	. 24395	. 84146	27	. 15854	. 08540	9	. 91460	14
47	. 75624	18	. 24376	. 84173	27	. 15827	. 08549	9	. 91451	13
48	. 75642	18	. 24358	. 84200	27	. 15800	. 08558	9	. 91442	12
49	. 75660	18	. 24340	. 84227	27	. 15773	. 08567	8	. 91433	11
50	9. 75678	18	10. 24322	9. 84254	26	10. 15746	10. 08575	9	9. 91425	10
51	. 75696	18	. 24304	. 84280	27	. 15720	. 08584	9	. 91416	9
52	. 75714	19	. 24286	. 84307	27	. 15693	. 08593	9	. 91407	8
53	. 75733	18	. 24267	. 84334	27	. 15666	. 08602	9	. 91398	7
54	. 75751	18	. 24249	. 84361	27	. 15639	. 08611	8	. 91389	6
55	9. 75769	18	10. 24231	9. 84388	27	10. 15612	10. 08619	9	9. 91381	5
56	. 75787	18	. 24213	. 84415	27	. 15585	. 08628	9	. 91372	4
57	. 75805	18	. 24195	. 84442	27	. 15558	. 08637	9	. 91363	3
58	. 75823	18	. 24177	. 84469	27	. 15531	. 08646	9	. 91354	2
59	. 75841	18	. 24159	. 84496	27	. 15504	. 08655	9	. 91345	1
60	9. 75859		10. 24141	9. 84523		10. 15477	10. 08664		9. 91336	0
↑ 124°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←55°

TABLE 33
Logarithms of Trigonometric Functions

35°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←144° ↓ ′
0	9. 75859	18	10. 24141	9. 84523	27	10. 15477	10. 08664	8	9. 91336	60
1	. 75877	18	. 24123	. 84550	26	. 15450	. 08672	9	. 91328	59
2	. 75895	18	. 24105	. 84576	27	. 15424	. 08681	9	. 91319	58
3	. 75913	18	. 24087	. 84603	27	. 15397	. 08690	9	. 91310	57
4	. 75931	18	. 24069	. 84630	27	. 15370	. 08699	9	. 91301	56
5	9. 75949	18	10. 24051	9. 84657	27	10. 15343	10. 08708	9	9. 91292	55
6	. 75967	18	. 24033	. 84684	27	. 15316	. 08717	9	. 91283	54
7	. 75985	18	. 24015	. 84711	27	. 15289	. 08726	8	. 91274	53
8	. 76003	18	. 23997	. 84738	26	. 15262	. 08734	9	. 91266	52
9	. 76021	18	. 23979	. 84764	27	. 15236	. 08743	9	. 91257	51
10	9. 76039	18	10. 23961	9. 84791	27	10. 15209	10. 08752	9	9. 91248	50
11	. 76057	18	. 23943	. 84818	27	. 15182	. 08761	9	. 91239	49
12	. 76075	18	. 23925	. 84845	27	. 15155	. 08770	9	. 91230	48
13	. 76093	18	. 23907	. 84872	27	. 15128	. 08779	9	. 91221	47
14	. 76111	18	. 23889	. 84899	26	. 15101	. 08788	9	. 91212	46
15	9. 76129	17	10. 23871	9. 84925	27	10. 15075	10. 08797	9	9. 91203	45
16	. 76146	18	. 23854	. 84952	27	. 15048	. 08806	9	. 91194	44
17	. 76164	18	. 23836	. 84979	27	. 15021	. 08815	9	. 91185	43
18	. 76182	18	. 23818	. 85006	27	. 14994	. 08824	9	. 91176	42
19	. 76200	18	. 23800	. 85033	26	. 14967	. 08833	9	. 91167	41
20	9. 76218	18	10. 23782	9. 85059	27	10. 14941	10. 08842	9	9. 91158	40
21	. 76236	17	. 23764	. 85086	27	. 14914	. 08851	8	. 91149	39
22	. 76253	18	. 23747	. 85113	27	. 14887	. 08859	9	. 91141	38
23	. 76271	18	. 23729	. 85140	26	. 14860	. 08868	9	. 91132	37
24	. 76289	18	. 23711	. 85166	27	. 14834	. 08877	9	. 91123	36
25	9. 76307	17	10. 23693	9. 85193	27	10. 14807	10. 08886	9	9. 91114	35
26	. 76324	18	. 23676	. 85220	27	. 14780	. 08895	9	. 91105	34
27	. 76342	18	. 23658	. 85247	26	. 14753	. 08904	9	. 91096	33
28	. 76360	18	. 23640	. 85273	27	. 14727	. 08913	9	. 91087	32
29	. 76378	17	. 23622	. 85300	27	. 14700	. 08922	9	. 91078	31
30	9. 76395	18	10. 23605	9. 85327	27	10. 14673	10. 08931	9	9. 91069	30
31	. 76413	18	. 23587	. 85354	26	. 14646	. 08940	9	. 91060	29
32	. 76431	17	. 23569	. 85380	27	. 14620	. 08949	9	. 91051	28
33	. 76448	18	. 23552	. 85407	27	. 14593	. 08958	9	. 91042	27
34	. 76466	18	. 23534	. 85434	26	. 14566	. 08967	10	. 91033	26
35	9. 76484	17	10. 23516	9. 85460	27	10. 14540	10. 08977	9	9. 91023	25
36	. 76501	18	. 23499	. 85487	27	. 14513	. 08986	9	. 91014	24
37	. 76519	18	. 23481	. 85514	26	. 14486	. 08995	9	. 91005	23
38	. 76537	17	. 23463	. 85540	27	. 14460	. 09004	9	. 90996	22
39	. 76554	18	. 23446	. 85567	27	. 14433	. 09013	9	. 90987	21
40	9. 76572	18	10. 23428	9. 85594	26	10. 14406	10. 09022	9	9. 90978	20
41	. 76590	17	. 23410	. 85620	27	. 14380	. 09031	9	. 90969	19
42	. 76607	18	. 23393	. 85647	27	. 14353	. 09040	9	. 90960	18
43	. 76625	17	. 23375	. 85674	26	. 14326	. 09049	9	. 90951	17
44	. 76642	18	. 23358	. 85700	27	. 14300	. 09058	9	. 90942	16
45	9. 76660	17	10. 23340	9. 85727	27	10. 14273	10. 09067	9	9. 90933	15
46	. 76677	18	. 23323	. 85754	26	. 14246	. 09076	9	. 90924	14
47	. 76695	17	. 23305	. 85780	27	. 14220	. 09085	9	. 90915	13
48	. 76712	18	. 23288	. 85807	27	. 14193	. 09094	10	. 90906	12
49	. 76730	17	. 23270	. 85834	26	. 14166	. 09104	9	. 90896	11
50	9. 76747	18	10. 23253	9. 85860	27	10. 14140	10. 09113	9	9. 90887	10
51	. 76765	17	. 23235	. 85887	26	. 14113	. 09122	9	. 90878	9
52	. 76782	18	. 23218	. 85913	27	. 14087	. 09131	9	. 90869	8
53	. 76800	17	. 23200	. 85940	27	. 14060	. 09140	9	. 90860	7
54	. 76817	18	. 23183	. 85967	26	. 14033	. 09149	9	. 90851	6
55	9. 76835	17	10. 23165	9. 85993	27	10. 14007	10. 09158	10	9. 90842	5
56	. 76852	18	. 23148	. 86020	26	. 13980	. 09168	9	. 90832	4
57	. 76870	17	. 23130	. 86046	27	. 13954	. 09177	9	. 90823	3
58	. 76887	17	. 23113	. 86073	27	. 13927	. 09186	9	. 90814	2
59	. 76904	18	. 23096	. 86100	26	. 13900	. 09195	9	. 90805	1
60	9. 76922		10. 23078	9. 86126		10. 13874	10. 09204		9. 90796	0
↑ 125°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←54°

TABLE 33

Logarithms of Trigonometric Functions

36° ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	′ ↓ 143°
0	9.76922	17	10.23078	9.86126	27	10.13874	10.09204	9	9.90796	60
1	.76939	18	.23061	.86153	26	.13847	.09213	10	.90787	59
2	.76957	17	.23043	.86179	27	.13821	.09223	9	.90777	58
3	.76974	17	.23026	.86206	26	.13794	.09232	9	.90768	57
4	.76991	18	.23009	.86232	27	.13768	.09241	9	.90759	56
5	9.77009	17	10.22991	9.86259	26	10.13741	10.09250	9	9.90750	55
6	.77026	17	.22974	.86285	27	.13715	.09259	10	.90741	54
7	.77043	18	.22957	.86312	26	.13688	.09269	9	.90731	53
8	.77061	17	.22939	.86338	27	.13662	.09278	9	.90722	52
9	.77078	17	.22922	.86365	27	.13635	.09287	9	.90713	51
10	9.77095	17	10.22905	9.86392	26	10.13608	10.09296	10	9.90704	50
11	.77112	18	.22888	.86418	27	.13582	.09306	9	.90694	49
12	.77130	17	.22870	.86445	26	.13555	.09315	9	.90685	48
13	.77147	17	.22853	.86471	27	.13529	.09324	9	.90676	47
14	.77164	17	.22836	.86498	26	.13502	.09333	10	.90667	46
15	9.77181	18	10.22819	9.86524	27	10.13476	10.09343	9	9.90657	45
16	.77199	17	.22801	.86551	26	.13449	.09352	9	.90648	44
17	.77216	17	.22784	.86577	26	.13423	.09361	9	.90639	43
18	.77233	17	.22767	.86603	27	.13397	.09370	10	.90630	42
19	.77250	18	.22750	.86630	26	.13370	.09380	9	.90620	41
20	9.77268	17	10.22732	9.86656	27	10.13344	10.09389	9	9.90611	40
21	.77285	17	.22715	.86683	26	.13317	.09398	10	.90602	39
22	.77302	17	.22698	.86709	27	.13291	.09408	9	.90592	38
23	.77319	17	.22681	.86736	26	.13264	.09417	9	.90583	37
24	.77336	17	.22664	.86762	27	.13238	.09426	9	.90574	36
25	9.77353	17	10.22647	9.86789	26	10.13211	10.09435	10	9.90565	35
26	.77370	17	.22630	.86815	27	.13185	.09445	9	.90555	34
27	.77387	18	.22613	.86842	26	.13158	.09454	9	.90546	33
28	.77405	17	.22595	.86868	26	.13132	.09463	10	.90537	32
29	.77422	17	.22578	.86894	27	.13106	.09473	9	.90527	31
30	9.77439	17	10.22561	9.86921	26	10.13079	10.09482	9	9.90518	30
31	.77456	17	.22544	.86947	27	.13053	.09491	10	.90509	29
32	.77473	17	.22527	.86974	26	.13026	.09501	9	.90499	28
33	.77490	17	.22510	.87000	27	.13000	.09510	10	.90490	27
34	.77507	17	.22493	.87027	26	.12973	.09520	9	.90480	26
35	9.77524	17	10.22476	9.87053	26	10.12947	10.09529	9	9.90471	25
36	.77541	17	.22459	.87079	27	.12921	.09538	10	.90462	24
37	.77558	17	.22442	.87106	26	.12894	.09548	9	.90452	23
38	.77575	17	.22425	.87132	26	.12868	.09557	9	.90443	22
39	.77592	17	.22408	.87158	27	.12842	.09566	10	.90434	21
40	9.77609	17	10.22391	9.87185	26	10.12815	10.09576	9	9.90424	20
41	.77626	17	.22374	.87211	27	.12789	.09585	10	.90415	19
42	.77643	17	.22357	.87238	26	.12762	.09595	9	.90405	18
43	.77660	17	.22340	.87264	26	.12736	.09604	10	.90396	17
44	.77677	17	.22323	.87290	27	.12710	.09614	9	.90386	16
45	9.77694	17	10.22306	9.87317	26	10.12683	10.09623	9	9.90377	15
46	.77711	17	.22289	.87343	26	.12657	.09632	10	.90368	14
47	.77728	16	.22272	.87369	27	.12631	.09642	9	.90358	13
48	.77744	17	.22256	.87396	26	.12604	.09651	10	.90349	12
49	.77761	17	.22239	.87422	26	.12578	.09661	9	.90339	11
50	9.77778	17	10.22222	9.87448	27	10.12552	10.09670	10	9.90330	10
51	.77795	17	.22205	.87475	26	.12525	.09680	9	.90320	9
52	.77812	17	.22188	.87501	26	.12499	.09689	10	.90311	8
53	.77829	17	.22171	.87527	27	.12473	.09699	9	.90301	7
54	.77846	16	.22154	.87554	26	.12446	.09708	10	.90292	6
55	9.77862	17	10.22138	9.87580	26	10.12420	10.09718	9	9.90282	5
56	.77879	17	.22121	.87606	27	.12394	.09727	10	.90273	4
57	.77896	17	.22104	.87633	26	.12367	.09737	9	.90263	3
58	.77913	17	.22087	.87659	26	.12341	.09746	10	.90254	2
59	.77930	16	.22070	.87685	26	.12315	.09756	9	.90244	1
60	9.77946		10.22054	9.87711		10.12289	10.09765		9.90235	0
↑ 126° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ← 53°

TABLE 33

Logarithms of Trigonometric Functions

37° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos ← 142°	↓ ′
0	9.77946	17	10.22054	9.87711	27	10.12289	10.09765	10	9.90235	60
1	.77963	17	.22037	.87738	26	.12262	.09775	9	.90225	59
2	.77980	17	.22020	.87764	26	.12236	.09784	10	.90216	58
3	.77997	16	.22003	.87790	27	.12210	.09794	9	.90206	57
4	.78013	17	.21987	.87817	26	.12183	.09803	10	.90197	56
5	9.78030	17	10.21970	9.87843	26	10.12157	10.09813	9	9.90187	55
6	.78047	16	.21953	.87869	26	.12131	.09822	10	.90178	54
7	.78063	17	.21937	.87895	27	.12105	.09832	9	.90168	53
8	.78080	17	.21920	.87922	26	.12078	.09841	10	.90159	52
9	.78097	16	.21903	.87948	26	.12052	.09851	10	.90149	51
10	9.78113	17	10.21887	9.87974	26	10.12026	10.09861	9	9.90139	50
11	.78130	17	.21870	.88000	27	.12000	.09870	10	.90130	49
12	.78147	16	.21853	.88027	26	.11973	.09880	9	.90120	48
13	.78163	17	.21837	.88053	26	.11947	.09889	10	.90111	47
14	.78180	17	.21820	.88079	26	.11921	.09899	10	.90101	46
15	9.78197	16	10.21803	9.88105	26	10.11895	10.09909	9	9.90091	45
16	.78213	17	.21787	.88131	27	.11869	.09918	10	.90082	44
17	.78230	16	.21770	.88158	26	.11842	.09928	9	.90072	43
18	.78246	17	.21754	.88184	26	.11816	.09937	10	.90063	42
19	.78263	17	.21737	.88210	26	.11790	.09947	10	.90053	41
20	9.78280	16	10.21720	9.88236	26	10.11764	10.09957	9	9.90043	40
21	.78296	17	.21704	.88262	27	.11738	.09966	10	.90034	39
22	.78313	16	.21687	.88289	26	.11711	.09976	10	.90024	38
23	.78329	17	.21671	.88315	26	.11685	.09986	9	.90014	37
24	.78346	16	.21654	.88341	26	.11659	.09995	10	.90005	36
25	9.78362	17	10.21638	9.88367	26	10.11633	10.10005	10	9.89995	35
26	.78379	16	.21621	.88393	27	.11607	.10015	9	.89985	34
27	.78395	17	.21605	.88420	26	.11580	.10024	10	.89976	33
28	.78412	16	.21588	.88446	26	.11554	.10034	10	.89966	32
29	.78428	17	.21572	.88472	26	.11528	.10044	9	.89956	31
30	9.78445	16	10.21555	9.88498	26	10.11502	10.10053	10	9.89947	30
31	.78461	17	.21539	.88524	26	.11476	.10063	10	.89937	29
32	.78478	16	.21522	.88550	27	.11450	.10073	9	.89927	28
33	.78494	16	.21506	.88577	26	.11423	.10082	10	.89918	27
34	.78510	17	.21490	.88603	26	.11397	.10092	10	.89908	26
35	9.78527	16	10.21473	9.88629	26	10.11371	10.10102	10	9.89898	25
36	.78543	17	.21457	.88655	26	.11345	.10112	9	.89888	24
37	.78560	16	.21440	.88681	26	.11319	.10121	10	.89879	23
38	.78576	16	.21424	.88707	26	.11293	.10131	10	.89869	22
39	.78592	17	.21408	.88733	26	.11267	.10141	10	.89859	21
40	9.78609	16	10.21391	9.88759	27	10.11241	10.10151	9	9.89849	20
41	.78625	17	.21375	.88786	26	.11214	.10160	10	.89840	19
42	.78642	16	.21358	.88812	26	.11188	.10170	10	.89830	18
43	.78658	16	.21342	.88838	26	.11162	.10180	10	.89820	17
44	.78674	17	.21326	.88864	26	.11136	.10190	9	.89810	16
45	9.78691	16	10.21309	9.88890	26	10.11110	10.10199	10	9.89801	15
46	.78707	16	.21293	.88916	26	.11084	.10209	10	.89791	14
47	.78723	16	.21277	.88942	26	.11058	.10219	10	.89781	13
48	.78739	17	.21261	.88968	26	.11032	.10229	10	.89771	12
49	.78756	16	.21244	.88994	26	.11006	.10239	9	.89761	11
50	9.78772	16	10.21228	9.89020	26	10.10980	10.10248	10	9.89752	10
51	.78788	17	.21212	.89046	27	.10954	.10258	10	.89742	9
52	.78805	16	.21195	.89073	26	.10927	.10268	10	.89732	8
53	.78821	16	.21179	.89099	26	.10901	.10278	10	.89722	7
54	.78837	16	.21163	.89125	26	.10875	.10288	10	.89712	6
55	9.78853	16	10.21147	9.89151	26	10.10849	10.10298	9	9.89702	5
56	.78869	17	.21131	.89177	26	.10823	.10307	10	.89693	4
57	.78886	16	.21114	.89203	26	.10797	.10317	10	.89683	3
58	.78902	16	.21098	.89229	26	.10771	.10327	10	.89673	2
59	.78918	16	.21082	.89255	26	.10745	.10337	10	.89663	1
60	9.78934		10.21066	9.89281		10.10719	10.10347		9.89653	0
↑ 127° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin ← 52°	↑

TABLE 33

Logarithms of Trigonometric Functions

38° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 141° ↓ ′
0	9.78934	16	10.21066	9.89281	26	10.10719	10.10347	10	9.89653	60
1	.78950	17	.21050	.89307	26	.10693	.10357	10	.89643	59
2	.78967	16	.21033	.89333	26	.10667	.10367	9	.89633	58
3	.78983	16	.21017	.89359	26	.10641	.10376	10	.89624	57
4	.78999	16	.21001	.89385	26	.10615	.10386	10	.89614	56
5	9.79015	16	10.20985	9.89411	26	10.10589	10.10396	10	9.89604	55
6	.79031	16	.20969	.89437	26	.10563	.10406	10	.89594	54
7	.79047	16	.20953	.89463	26	.10537	.10416	10	.89584	53
8	.79063	16	.20937	.89489	26	.10511	.10426	10	.89574	52
9	.79079	16	.20921	.89515	26	.10485	.10436	10	.89564	51
10	9.79095	16	10.20905	9.89541	26	10.10459	10.10446	10	9.89554	50
11	.79111	17	.20889	.89567	26	.10433	.10456	10	.89544	49
12	.79128	16	.20872	.89593	26	.10407	.10466	10	.89534	48
13	.79144	16	.20856	.89619	26	.10381	.10476	10	.89524	47
14	.79160	16	.20840	.89645	26	.10355	.10486	10	.89514	46
15	9.79176	16	10.20824	9.89671	26	10.10329	10.10496	9	9.89504	45
16	.79192	16	.20808	.89697	26	.10303	.10505	10	.89495	44
17	.79208	16	.20792	.89723	26	.10277	.10515	10	.89485	43
18	.79224	16	.20776	.89749	26	.10251	.10525	10	.89475	42
19	.79240	16	.20760	.89775	26	.10225	.10535	10	.89465	41
20	9.79256	16	10.20744	9.89801	26	10.10199	10.10545	10	9.89455	40
21	.79272	16	.20728	.89827	26	.10173	.10555	10	.89445	39
22	.79288	16	.20712	.89853	26	.10147	.10565	10	.89435	38
23	.79304	15	.20696	.89879	26	.10121	.10575	10	.89425	37
24	.79319	16	.20681	.89905	26	.10095	.10585	10	.89415	36
25	9.79335	16	10.20665	9.89931	26	10.10069	10.10595	10	9.89405	35
26	.79351	16	.20649	.89957	26	.10043	.10605	10	.89395	34
27	.79367	16	.20633	.89983	26	.10017	.10615	10	.89385	33
28	.79383	16	.20617	.90009	26	.09991	.10625	11	.89375	32
29	.79399	16	.20601	.90035	26	.09965	.10636	10	.89364	31
30	9.79415	16	10.20585	9.90061	25	10.09939	10.10646	10	9.89354	30
31	.79431	16	.20569	.90086	26	.09914	.10656	10	.89344	29
32	.79447	16	.20553	.90112	26	.09888	.10666	10	.89334	28
33	.79463	15	.20537	.90138	26	.09862	.10676	10	.89324	27
34	.79478	16	.20522	.90164	26	.09836	.10686	10	.89314	26
35	9.79494	16	10.20506	9.90190	26	10.09810	10.10696	10	9.89304	25
36	.79510	16	.20490	.90216	26	.09784	.10706	10	.89294	24
37	.79526	16	.20474	.90242	26	.09758	.10716	10	.89284	23
38	.79542	16	.20458	.90268	26	.09732	.10726	10	.89274	22
39	.79558	15	.20442	.90294	26	.09706	.10736	10	.89264	21
40	9.79573	16	10.20427	9.90320	26	10.09680	10.10746	10	9.89254	20
41	.79589	16	.20411	.90346	25	.09654	.10756	11	.89244	19
42	.79605	16	.20395	.90371	26	.09629	.10767	10	.89233	18
43	.79621	15	.20379	.90397	26	.09603	.10777	10	.89223	17
44	.79636	16	.20364	.90423	26	.09577	.10787	10	.89213	16
45	9.79652	16	10.20348	9.90449	26	10.09551	10.10797	10	9.89203	15
46	.79668	16	.20332	.90475	26	.09525	.10807	10	.89193	14
47	.79684	15	.20316	.90501	26	.09499	.10817	10	.89183	13
48	.79699	16	.20301	.90527	26	.09473	.10827	11	.89173	12
49	.79715	16	.20285	.90553	25	.09447	.10838	10	.89162	11
50	9.79731	15	10.20269	9.90578	26	10.09422	10.10848	10	9.89152	10
51	.79746	16	.20254	.90604	26	.09396	.10858	10	.89142	9
52	.79762	16	.20238	.90630	26	.09370	.10868	10	.89132	8
53	.79778	15	.20222	.90656	26	.09344	.10878	10	.89122	7
54	.79793	16	.20207	.90682	26	.09318	.10888	11	.89112	6
55	9.79809	16	10.20191	9.90708	26	10.09292	10.10899	10	9.89101	5
56	.79825	15	.20175	.90734	25	.09266	.10909	10	.89091	4
57	.79840	16	.20160	.90759	26	.09241	.10919	10	.89081	3
58	.79856	16	.20144	.90785	26	.09215	.10929	11	.89071	2
59	.79872	15	.20128	.90811	26	.09189	.10940	10	.89060	1
60	9.79887		10.20113	9.90837		10.09163	10.10950		9.89050	0
↑ 128° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 51° ↑

TABLE 33
Logarithms of Trigonometric Functions

39° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 140° ↓ ′
0	9.79887	16	10.20113	9.90837	26	10.09163	10.10950	10	9.89050	60
1	.79903	15	.20097	.90863	26	.09137	.10960	10	.89040	59
2	.79918	16	.20082	.90889	25	.09111	.10970	10	.89030	58
3	.79934	16	.20066	.90914	26	.09086	.10980	11	.89020	57
4	.79950	15	.20050	.90940	26	.09060	.10991	10	.89009	56
5	9.79965	16	10.20035	9.90966	26	10.09034	10.11001	10	9.88999	55
6	.79981	15	.20019	.90992	26	.09008	.11011	11	.88989	54
7	.79996	16	.20004	.91018	25	.08982	.11022	10	.88978	53
8	.80012	15	.19988	.91043	26	.08957	.11032	10	.88968	52
9	.80027	16	.19973	.91069	26	.08931	.11042	10	.88958	51
10	9.80043	15	10.19957	9.91095	26	10.08905	10.11052	11	9.88948	50
11	.80058	16	.19942	.91121	26	.08879	.11063	10	.88937	49
12	.80074	15	.19926	.91147	25	.08853	.11073	10	.88927	48
13	.80089	16	.19911	.91172	26	.08828	.11083	11	.88917	47
14	.80105	15	.19895	.91198	26	.08802	.11094	10	.88906	46
15	9.80120	16	10.19880	9.91224	26	10.08776	10.11104	10	9.88896	45
16	.80136	15	.19864	.91250	26	.08750	.11114	11	.88886	44
17	.80151	15	.19849	.91276	25	.08724	.11125	10	.88875	43
18	.80166	16	.19834	.91301	26	.08699	.11135	10	.88865	42
19	.80182	15	.19818	.91327	26	.08673	.11145	11	.88855	41
20	9.80197	16	10.19803	9.91353	26	10.08647	10.11156	10	9.88844	40
21	.80213	15	.19787	.91379	25	.08621	.11166	10	.88834	39
22	.80228	16	.19772	.91404	26	.08596	.11176	11	.88824	38
23	.80244	15	.19756	.91430	26	.08570	.11187	10	.88813	37
24	.80259	15	.19741	.91456	26	.08544	.11197	10	.88803	36
25	9.80274	16	10.19726	9.91482	25	10.08518	10.11207	11	9.88793	35
26	.80290	15	.19710	.91507	26	.08493	.11218	10	.88782	34
27	.80305	15	.19695	.91533	26	.08467	.11228	11	.88772	33
28	.80320	16	.19680	.91559	26	.08441	.11239	10	.88761	32
29	.80336	15	.19664	.91585	25	.08415	.11249	10	.88751	31
30	9.80351	15	10.19649	9.91610	26	10.08390	10.11259	11	9.88741	30
31	.80366	16	.19634	.91636	26	.08364	.11270	10	.88730	29
32	.80382	15	.19618	.91662	26	.08338	.11280	11	.88720	28
33	.80397	15	.19603	.91688	25	.08312	.11291	10	.88709	27
34	.80412	16	.19588	.91713	26	.08287	.11301	11	.88699	26
35	9.80428	15	10.19572	9.91739	26	10.08261	10.11312	10	9.88688	25
36	.80443	15	.19557	.91765	26	.08235	.11322	10	.88678	24
37	.80458	15	.19542	.91791	25	.08209	.11332	11	.88668	23
38	.80473	16	.19527	.91816	26	.08184	.11343	10	.88657	22
39	.80489	15	.19511	.91842	26	.08158	.11353	11	.88647	21
40	9.80504	15	10.19496	9.91868	25	10.08132	10.11364	10	9.88636	20
41	.80519	15	.19481	.91893	26	.08107	.11374	11	.88626	19
42	.80534	16	.19466	.91919	26	.08081	.11385	10	.88615	18
43	.80550	15	.19450	.91945	26	.08055	.11395	11	.88605	17
44	.80565	15	.19435	.91971	25	.08029	.11406	10	.88594	16
45	9.80580	15	10.19420	9.91996	26	10.08004	10.11416	11	9.88584	15
46	.80595	15	.19405	.92022	26	.07978	.11427	10	.88573	14
47	.80610	15	.19390	.92048	25	.07952	.11437	11	.88563	13
48	.80625	16	.19375	.92073	26	.07927	.11448	10	.88552	12
49	.80641	15	.19359	.92099	26	.07901	.11458	11	.88542	11
50	9.80656	15	10.19344	9.92125	25	10.07875	10.11469	10	9.88531	10
51	.80671	15	.19329	.92150	26	.07850	.11479	11	.88521	9
52	.80686	15	.19314	.92176	26	.07824	.11490	11	.88510	8
53	.80701	15	.19299	.92202	25	.07798	.11501	10	.88499	7
54	.80716	15	.19284	.92227	26	.07773	.11511	11	.88489	6
55	9.80731	15	10.19269	9.92253	26	10.07747	10.11522	10	9.88478	5
56	.80746	16	.19254	.92279	25	.07721	.11532	11	.88468	4
57	.80762	15	.19238	.92304	26	.07696	.11543	10	.88457	3
58	.80777	15	.19223	.92330	26	.07670	.11553	11	.88447	2
59	.80792	15	.19208	.92356	25	.07644	.11564	11	.88436	1
60	9.80807		10.19193	9.92381		10.07619	10.11575		9.88425	0
↑ 129° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 50° ↑

TABLE 33

Logarithms of Trigonometric Functions

40°→ ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←139° ↓ ′
0	9. 80807	15	10. 19193	9. 92381	26	10. 07619	10. 11575	10	9. 88425	60
1	. 80822	15	. 19178	. 92407	26	. 07593	. 11585	11	. 88415	59
2	. 80837	15	. 19163	. 92433	25	. 07567	. 11596	10	. 88404	58
3	. 80852	15	. 19148	. 92458	26	. 07542	. 11606	11	. 88394	57
4	. 80867	15	. 19133	. 92484	26	. 07516	. 11617	11	. 88383	56
5	9. 80882	15	10. 19118	9. 92510	25	10. 07490	10. 11628	10	9. 88372	55
6	. 80897	15	. 19103	. 92535	26	. 07465	. 11638	11	. 88362	54
7	. 80912	15	. 19088	. 92561	26	. 07439	. 11649	11	. 88351	53
8	. 80927	15	. 19073	. 92587	25	. 07413	. 11660	10	. 88340	52
9	. 80942	15	. 19058	. 92612	26	. 07388	. 11670	11	. 88330	51
10	9. 80957	15	10. 19043	9. 92638	25	10. 07362	10. 11681	11	9. 88319	50
11	. 80972	15	. 19028	. 92663	26	. 07337	. 11692	10	. 88308	49
12	. 80987	15	. 19013	. 92689	26	. 07311	. 11702	11	. 88298	48
13	. 81002	15	. 18998	. 92715	25	. 07285	. 11713	11	. 88287	47
14	. 81017	15	. 18983	. 92740	26	. 07260	. 11724	10	. 88276	46
15	9. 81032	15	10. 18968	9. 92766	26	10. 07234	10. 11734	11	9. 88266	45
16	. 81047	14	. 18953	. 92792	25	. 07208	. 11745	11	. 88255	44
17	. 81061	15	. 18939	. 92817	26	. 07183	. 11756	10	. 88244	43
18	. 81076	15	. 18924	. 92843	25	. 07157	. 11766	11	. 88234	42
19	. 81091	15	. 18909	. 92868	26	. 07132	. 11777	11	. 88223	41
20	9. 81106	15	10. 18894	9. 92894	26	10. 07106	10. 11788	11	9. 88212	40
21	. 81121	15	. 18879	. 92920	25	. 07080	. 11799	10	. 88201	39
22	. 81136	15	. 18864	. 92945	26	. 07055	. 11809	11	. 88191	38
23	. 81151	15	. 18849	. 92971	25	. 07029	. 11820	11	. 88180	37
24	. 81166	14	. 18834	. 92996	26	. 07004	. 11831	11	. 88169	36
25	9. 81180	15	10. 18820	9. 93022	26	10. 06978	10. 11842	10	9. 88158	35
26	. 81195	15	. 18805	. 93048	25	. 06952	. 11852	11	. 88148	34
27	. 81210	15	. 18790	. 93073	26	. 06927	. 11863	11	. 88137	33
28	. 81225	15	. 18775	. 93099	25	. 06901	. 11874	11	. 88126	32
29	. 81240	14	. 18760	. 93124	26	. 06876	. 11885	10	. 88115	31
30	9. 81254	15	10. 18746	9. 93150	25	10. 06850	10. 11895	11	9. 88105	30
31	. 81269	15	. 18731	. 93175	26	. 06825	. 11906	11	. 88094	29
32	. 81284	15	. 18716	. 93201	26	. 06799	. 11917	11	. 88083	28
33	. 81299	15	. 18701	. 93227	25	. 06773	. 11928	11	. 88072	27
34	. 81314	14	. 18686	. 93252	26	. 06748	. 11939	10	. 88061	26
35	9. 81328	15	10. 18672	9. 93278	25	10. 06722	10. 11949	11	9. 88051	25
36	. 81343	15	. 18657	. 93303	26	. 06697	. 11960	11	. 88040	24
37	. 81358	14	. 18642	. 93329	25	. 06671	. 11971	11	. 88029	23
38	. 81372	15	. 18628	. 93354	26	. 06646	. 11982	11	. 88018	22
39	. 81387	15	. 18613	. 93380	26	. 06620	. 11993	11	. 88007	21
40	9. 81402	15	10. 18598	9. 93406	25	10. 06594	10. 12004	11	9. 87996	20
41	. 81417	14	. 18583	. 93431	26	. 06569	. 12015	10	. 87985	19
42	. 81431	15	. 18569	. 93457	25	. 06543	. 12025	11	. 87975	18
43	. 81446	15	. 18554	. 93482	26	. 06518	. 12036	11	. 87964	17
44	. 81461	14	. 18539	. 93508	25	. 06492	. 12047	11	. 87953	16
45	9. 81475	15	10. 18525	9. 93533	26	10. 06467	10. 12058	11	9. 87942	15
46	. 81490	15	. 18510	. 93559	25	. 06441	. 12069	11	. 87931	14
47	. 81505	14	. 18495	. 93584	26	. 06416	. 12080	11	. 87920	13
48	. 81519	15	. 18481	. 93610	26	. 06390	. 12091	11	. 87909	12
49	. 81534	15	. 18466	. 93636	25	. 06364	. 12102	11	. 87898	11
50	9. 81549	14	10. 18451	9. 93661	26	10. 06339	10. 12113	10	9. 87887	10
51	. 81563	15	. 18437	. 93687	25	. 06313	. 12123	11	. 87877	9
52	. 81578	14	. 18422	. 93712	26	. 06288	. 12134	11	. 87866	8
53	. 81592	15	. 18408	. 93738	25	. 06262	. 12145	11	. 87855	7
54	. 81607	15	. 18393	. 93763	26	. 06237	. 12156	11	. 87844	6
55	9. 81622	14	10. 18378	9. 93789	25	10. 06211	10. 12167	11	9. 87833	5
56	. 81636	15	. 18364	. 93814	26	. 06186	. 12178	11	. 87822	4
57	. 81651	14	. 18349	. 93840	25	. 06160	. 12189	11	. 87811	3
58	. 81665	15	. 18335	. 93865	26	. 06135	. 12200	11	. 87800	2
59	. 81680	14	. 18320	. 93891	25	. 06109	. 12211	11	. 87789	1
60	9. 81694		10. 18306	9. 93916		10. 06084	10. 12222		9. 87778	0
↑ 130°→	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	↑ ←49°

TABLE 33
Logarithms of Trigonometric Functions

41° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	←138° ↓ ′
0	9. 81694	15	10. 18306	9. 93916	26	10. 06084	10. 12222	11	9. 87778	60
1	. 81709	14	. 18291	. 93942	25	. 06058	. 12233	11	. 87767	59
2	. 81723	15	. 18277	. 93967	26	. 06033	. 12244	11	. 87756	58
3	. 81738	14	. 18262	. 93993	25	. 06007	. 12255	11	. 87745	57
4	. 81752	15	. 18248	. 94018	26	. 05982	. 12266	11	. 87734	56
5	9. 81767	14	10. 18233	9. 94044	25	10. 05956	10. 12277	11	9. 87723	55
6	. 81781	15	. 18219	. 94069	26	. 05931	. 12288	11	. 87712	54
7	. 81796	14	. 18204	. 94095	25	. 05905	. 12299	11	. 87701	53
8	. 81810	15	. 18190	. 94120	26	. 05880	. 12310	11	. 87690	52
9	. 81825	14	. 18175	. 94146	25	. 05854	. 12321	11	. 87679	51
10	9. 81839	15	10. 18161	9. 94171	26	10. 05829	10. 12332	11	9. 87668	50
11	. 81854	14	. 18146	. 94197	25	. 05803	. 12343	11	. 87657	49
12	. 81868	14	. 18132	. 94222	26	. 05778	. 12354	11	. 87646	48
13	. 81882	15	. 18118	. 94248	25	. 05752	. 12365	11	. 87635	47
14	. 81897	14	. 18103	. 94273	26	. 05727	. 12376	11	. 87624	46
15	9. 81911	15	10. 18089	9. 94299	25	10. 05701	10. 12387	12	9. 87613	45
16	. 81926	14	. 18074	. 94324	26	. 05676	. 12399	11	. 87601	44
17	. 81940	15	. 18060	. 94350	25	. 05650	. 12410	11	. 87590	43
18	. 81955	14	. 18045	. 94375	26	. 05625	. 12421	11	. 87579	42
19	. 81969	14	. 18031	. 94401	25	. 05599	. 12432	11	. 87568	41
20	9. 81983	15	10. 18017	9. 94426	26	10. 05574	10. 12443	11	9. 87557	40
21	. 81998	14	. 18002	. 94452	25	. 05548	. 12454	11	. 87546	39
22	. 82012	14	. 17988	. 94477	26	. 05523	. 12465	11	. 87535	38
23	. 82026	15	. 17974	. 94503	25	. 05497	. 12476	11	. 87524	37
24	. 82041	14	. 17959	. 94528	26	. 05472	. 12487	12	. 87513	36
25	9. 82055	14	10. 17945	9. 94554	25	10. 05446	10. 12499	11	9. 87501	35
26	. 82069	15	. 17931	. 94579	25	. 05421	. 12510	11	. 87490	34
27	. 82084	14	. 17916	. 94604	26	. 05396	. 12521	11	. 87479	33
28	. 82098	14	. 17902	. 94630	25	. 05370	. 12532	11	. 87468	32
29	. 82112	14	. 17888	. 94655	26	. 05345	. 12543	11	. 87457	31
30	9. 82126	15	10. 17874	9. 94681	25	10. 05319	10. 12554	12	9. 87446	30
31	. 82141	14	. 17859	. 94706	26	. 05294	. 12566	11	. 87434	29
32	. 82155	14	. 17845	. 94732	25	. 05268	. 12577	11	. 87423	28
33	. 82169	15	. 17831	. 94757	26	. 05243	. 12588	11	. 87412	27
34	. 82184	14	. 17816	. 94783	25	. 05217	. 12599	11	. 87401	26
35	9. 82198	14	10. 17802	9. 94808	26	10. 05192	10. 12610	12	9. 87390	25
36	. 82212	14	. 17788	. 94834	25	. 05166	. 12622	11	. 87378	24
37	. 82226	14	. 17774	. 94859	25	. 05141	. 12633	11	. 87367	23
38	. 82240	15	. 17760	. 94884	26	. 05116	. 12644	11	. 87356	22
39	. 82255	14	. 17745	. 94910	25	. 05090	. 12655	11	. 87345	21
40	9. 82269	14	10. 17731	9. 94935	26	10. 05065	10. 12666	12	9. 87334	20
41	. 82283	14	. 17717	. 94961	25	. 05039	. 12678	11	. 87322	19
42	. 82297	14	. 17703	. 94986	26	. 05014	. 12689	11	. 87311	18
43	. 82311	15	. 17689	. 95012	25	. 04988	. 12700	12	. 87300	17
44	. 82326	14	. 17674	. 95037	25	. 04963	. 12712	11	. 87288	16
45	9. 82340	14	10. 17660	9. 95062	26	10. 04938	10. 12723	11	9. 87277	15
46	. 82354	14	. 17646	. 95088	25	. 04912	. 12734	11	. 87266	14
47	. 82368	14	. 17632	. 95113	26	. 04887	. 12745	12	. 87255	13
48	. 82382	14	. 17618	. 95139	25	. 04861	. 12757	11	. 87243	12
49	. 82396	14	. 17604	. 95164	26	. 04836	. 12768	11	. 87232	11
50	9. 82410	14	10. 17590	9. 95190	25	10. 04810	10. 12779	12	9. 87221	10
51	. 82424	15	. 17576	. 95215	25	. 04785	. 12791	11	. 87209	9
52	. 82439	14	. 17561	. 95240	26	. 04760	. 12802	11	. 87198	8
53	. 82453	14	. 17547	. 95266	25	. 04734	. 12813	12	. 87187	7
54	. 82467	14	. 17533	. 95291	26	. 04709	. 12825	11	. 87175	6
55	9. 82481	14	10. 17519	9. 95317	25	10. 04683	10. 12836	11	9. 87164	5
56	. 82495	14	. 17505	. 95342	26	. 04658	. 12847	12	. 87153	4
57	. 82509	14	. 17491	. 95368	25	. 04632	. 12859	11	. 87141	3
58	. 82523	14	. 17477	. 95393	25	. 04607	. 12870	11	. 87130	2
59	. 82537	14	. 17463	. 95418	26	. 04582	. 12881	12	. 87119	1
60	9. 82551		10. 17449	9. 95444		10. 04556	10. 12893		9. 87107	0
↑ 131° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	←48° ↑

TABLE 33

Logarithms of Trigonometric Functions

42° → ↓ ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 137° ↓ ′
0	9. 82551	14	10. 17449	9. 95444	25	10. 04556	10. 12893	11	9. 87107	60
1	. 82565	14	. 17435	. 95469	26	. 04531	. 12904	11	. 87096	59
2	. 82579	14	. 17421	. 95495	25	. 04505	. 12915	12	. 87085	58
3	. 82593	14	. 17407	. 95520	25	. 04480	. 12927	11	. 87073	57
4	. 82607	14	. 17393	. 95545	26	. 04455	. 12938	12	. 87062	56
5	9. 82621	14	10. 17379	9. 95571	25	10. 04429	10. 12950	11	9. 87050	55
6	. 82635	14	. 17365	. 95596	26	. 04404	. 12961	11	. 87039	54
7	. 82649	14	. 17351	. 95622	25	. 04378	. 12972	12	. 87028	53
8	. 82663	14	. 17337	. 95647	25	. 04353	. 12984	11	. 87016	52
9	. 82677	14	. 17323	. 95672	26	. 04328	. 12995	12	. 87005	51
10	9. 82691	14	10. 17309	9. 95698	25	10. 04302	10. 13007	11	9. 86993	50
11	. 82705	14	. 17295	. 95723	25	. 04277	. 13018	12	. 86982	49
12	. 82719	14	. 17281	. 95748	26	. 04252	. 13030	11	. 86970	48
13	. 82733	14	. 17267	. 95774	25	. 04226	. 13041	12	. 86959	47
14	. 82747	14	. 17253	. 95799	26	. 04201	. 13053	11	. 86947	46
15	9. 82761	14	10. 17239	9. 95825	25	10. 04175	10. 13064	12	9. 86936	45
16	. 82775	13	. 17225	. 95850	25	. 04150	. 13076	11	. 86924	44
17	. 82788	14	. 17212	. 95875	26	. 04125	. 13087	11	. 86913	43
18	. 82802	14	. 17198	. 95901	25	. 04099	. 13098	12	. 86902	42
19	. 82816	14	. 17184	. 95926	26	. 04074	. 13110	11	. 86890	41
20	9. 82830	14	10. 17170	9. 95952	25	10. 04048	10. 13121	12	9. 86879	40
21	. 82844	14	. 17156	. 95977	25	. 04023	. 13133	12	. 86867	39
22	. 82858	14	. 17142	. 96002	26	. 03998	. 13145	11	. 86855	38
23	. 82872	13	. 17128	. 96028	25	. 03972	. 13156	12	. 86844	37
24	. 82885	14	. 17115	. 96053	25	. 03947	. 13168	11	. 86832	36
25	9. 82899	14	10. 17101	9. 96078	26	10. 03922	10. 13179	12	9. 86821	35
26	. 82913	14	. 17087	. 96104	25	. 03896	. 13191	11	. 86809	34
27	. 82927	14	. 17073	. 96129	26	. 03871	. 13202	12	. 86798	33
28	. 82941	14	. 17059	. 96155	25	. 03845	. 13214	11	. 86786	32
29	. 82955	13	. 17045	. 96180	25	. 03820	. 13225	12	. 86775	31
30	9. 82968	14	10. 17032	9. 96205	26	10. 03795	10. 13237	11	9. 86763	30
31	. 82982	14	. 17018	. 96231	25	. 03769	. 13248	12	. 86752	29
32	. 82996	14	. 17004	. 96256	25	. 03744	. 13260	12	. 86740	28
33	. 83010	13	. 16990	. 96281	26	. 03719	. 13272	11	. 86728	27
34	. 83023	14	. 16977	. 96307	25	. 03693	. 13283	12	. 86717	26
35	9. 83037	14	10. 16963	9. 96332	25	10. 03668	10. 13295	11	9. 86705	25
36	. 83051	14	. 16949	. 96357	26	. 03643	. 13306	12	. 86694	24
37	. 83065	13	. 16935	. 96383	25	. 03617	. 13318	12	. 86682	23
38	. 83078	14	. 16922	. 96408	25	. 03592	. 13330	11	. 86670	22
39	. 83092	14	. 16908	. 96433	26	. 03567	. 13341	12	. 86659	21
40	9. 83106	14	10. 16894	9. 96459	25	10. 03541	10. 13353	12	9. 86647	20
41	. 83120	13	. 16880	. 96484	26	. 03516	. 13365	11	. 86635	19
42	. 83133	14	. 16867	. 96510	25	. 03490	. 13376	12	. 86624	18
43	. 83147	14	. 16853	. 96535	25	. 03465	. 13388	12	. 86612	17
44	. 83161	13	. 16839	. 96560	26	. 03440	. 13400	11	. 86600	16
45	9. 83174	14	10. 16826	9. 96586	25	10. 03414	10. 13411	12	9. 86589	15
46	. 83188	14	. 16812	. 96611	25	. 03389	. 13423	12	. 86577	14
47	. 83202	13	. 16798	. 96636	26	. 03364	. 13435	11	. 86565	13
48	. 83215	14	. 16785	. 96662	25	. 03338	. 13446	12	. 86554	12
49	. 83229	13	. 16771	. 96687	25	. 03313	. 13458	12	. 86542	11
50	9. 83242	14	10. 16758	9. 96712	26	10. 03288	10. 13470	12	9. 86530	10
51	. 83256	14	. 16744	. 96738	25	. 03262	. 13482	11	. 86518	9
52	. 83270	13	. 16730	. 96763	25	. 03237	. 13493	12	. 86507	8
53	. 83283	14	. 16717	. 96788	26	. 03212	. 13505	12	. 86495	7
54	. 83297	13	. 16703	. 96814	25	. 03186	. 13517	11	. 86483	6
55	9. 83310	14	10. 16690	9. 96839	25	10. 03161	10. 13528	12	9. 86472	5
56	. 83324	14	. 16676	. 96864	26	. 03136	. 13540	12	. 86460	4
57	. 83338	13	. 16662	. 96890	25	. 03110	. 13552	12	. 86448	3
58	. 83351	14	. 16649	. 96915	25	. 03085	. 13564	11	. 86436	2
59	. 83365	13	. 16635	. 96940	26	. 03060	. 13575	12	. 86425	1
60	9. 83378		10. 16622	9. 96966		10. 03034	10. 13587		9. 86413	0
↑ 132° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 47° ↑

TABLE 33
Logarithms of Trigonometric Functions

43° → ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 136° ′
0	9. 83378	14	10. 16622	9. 96966	25	10. 03034	10. 13587	12	9. 86413	60
1	. 83392	13	. 16608	. 96991	25	. 03009	. 13599	12	. 86401	59
2	. 83405	14	. 16595	. 97016	26	. 02984	. 13611	12	. 86389	58
3	. 83419	13	. 16581	. 97042	25	. 02958	. 13623	11	. 86377	57
4	. 83432	14	. 16568	. 97067	25	. 02933	. 13634	12	. 86366	56
5	9. 83446	13	10. 16554	9. 97092	26	10. 02908	10. 13646	12	9. 86354	55
6	. 83459	14	. 16541	. 97118	25	. 02882	. 13658	12	. 86342	54
7	. 83473	13	. 16527	. 97143	25	. 02857	. 13670	12	. 86330	53
8	. 83486	14	. 16514	. 97168	25	. 02832	. 13682	12	. 86318	52
9	. 83500	13	. 16500	. 97193	26	. 02807	. 13694	11	. 86306	51
10	9. 83513	14	10. 16487	9. 97219	25	10. 02781	10. 13705	12	9. 86295	50
11	. 83527	13	. 16473	. 97244	25	. 02756	. 13717	12	. 86283	49
12	. 83540	14	. 16460	. 97269	26	. 02731	. 13729	12	. 86271	48
13	. 83554	13	. 16446	. 97295	25	. 02705	. 13741	12	. 86259	47
14	. 83567	14	. 16433	. 97320	25	. 02680	. 13753	12	. 86247	46
15	9. 83581	13	10. 16419	9. 97345	26	10. 02655	10. 13765	12	9. 86235	45
16	. 83594	14	. 16406	. 97371	25	. 02629	. 13777	12	. 86223	44
17	. 83608	13	. 16392	. 97396	25	. 02604	. 13789	11	. 86211	43
18	. 83621	13	. 16379	. 97421	26	. 02579	. 13800	12	. 86200	42
19	. 83634	14	. 16366	. 97447	25	. 02553	. 13812	12	. 86188	41
20	9. 83648	13	10. 16352	9. 97472	25	10. 02528	10. 13824	12	9. 86176	40
21	. 83661	13	. 16339	. 97497	26	. 02503	. 13836	12	. 86164	39
22	. 83674	14	. 16326	. 97523	25	. 02477	. 13848	12	. 86152	38
23	. 83688	13	. 16312	. 97548	25	. 02452	. 13860	12	. 86140	37
24	. 83701	14	. 16299	. 97573	25	. 02427	. 13872	12	. 86128	36
25	9. 83715	13	10. 16285	9. 97598	26	10. 02402	10. 13884	12	9. 86116	35
26	. 83728	13	. 16272	. 97624	25	. 02376	. 13896	12	. 86104	34
27	. 83741	14	. 16259	. 97649	25	. 02351	. 13908	12	. 86092	33
28	. 83755	13	. 16245	. 97674	26	. 02326	. 13920	12	. 86080	32
29	. 83768	13	. 16232	. 97700	25	. 02300	. 13932	12	. 86068	31
30	9. 83781	14	10. 16219	9. 97725	25	10. 02275	10. 13944	12	9. 86056	30
31	. 83795	13	. 16205	. 97750	26	. 02250	. 13956	12	. 86044	29
32	. 83808	13	. 16192	. 97776	25	. 02224	. 13968	12	. 86032	28
33	. 83821	13	. 16179	. 97801	25	. 02199	. 13980	12	. 86020	27
34	. 83834	14	. 16166	. 97826	25	. 02174	. 13992	12	. 86008	26
35	9. 83848	13	10. 16152	9. 97851	26	10. 02149	10. 14004	12	9. 85996	25
36	. 83861	13	. 16139	. 97877	25	. 02123	. 14016	12	. 85984	24
37	. 83874	13	. 16126	. 97902	25	. 02098	. 14028	12	. 85972	23
38	. 83887	14	. 16113	. 97927	26	. 02073	. 14040	12	. 85960	22
39	. 83901	13	. 16099	. 97953	25	. 02047	. 14052	12	. 85948	21
40	9. 83914	13	10. 16086	9. 97978	25	10. 02022	10. 14064	12	9. 85936	20
41	. 83927	13	. 16073	. 98003	26	. 01997	. 14076	12	. 85924	19
42	. 83940	14	. 16060	. 98029	25	. 01971	. 14088	12	. 85912	18
43	. 83954	13	. 16046	. 98054	25	. 01946	. 14100	12	. 85900	17
44	. 83967	13	. 16033	. 98079	25	. 01921	. 14112	12	. 85888	16
45	9. 83980	13	10. 16020	9. 98104	26	10. 01896	10. 14124	12	9. 85876	15
46	. 83993	13	. 16007	. 98130	25	. 01870	. 14136	13	. 85864	14
47	. 84006	14	. 15994	. 98155	25	. 01845	. 14149	12	. 85851	13
48	. 84020	13	. 15980	. 98180	26	. 01820	. 14161	12	. 85839	12
49	. 84033	13	. 15967	. 98206	25	. 01794	. 14173	12	. 85827	11
50	9. 84046	13	10. 15954	9. 98231	25	10. 01769	10. 14185	12	9. 85815	10
51	. 84059	13	. 15941	. 98256	25	. 01744	. 14197	12	. 85803	9
52	. 84072	13	. 15928	. 98281	26	. 01719	. 14209	12	. 85791	8
53	. 84085	13	. 15915	. 98307	25	. 01693	. 14221	13	. 85779	7
54	. 84098	14	. 15902	. 98332	25	. 01668	. 14234	12	. 85766	6
55	9. 84112	13	10. 15888	9. 98357	26	10. 01643	10. 14246	12	9. 85754	5
56	. 84125	13	. 15875	. 98383	25	. 01617	. 14258	12	. 85742	4
57	. 84138	13	. 15862	. 98408	25	. 01592	. 14270	12	. 85730	3
58	. 84151	13	. 15849	. 98433	25	. 01567	. 14282	12	. 85718	2
59	. 84164	13	. 15836	. 98458	26	. 01542	. 14294	13	. 85706	1
60	9. 84177		10. 15823	9. 98484		10. 01516	10. 14307		9. 85693	0
133° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 46°

TABLE 33
Logarithms of Trigonometric Functions

44° → ′	sin	Diff. 1′	csc	tan	Diff. 1′	cot	sec	Diff. 1′	cos	← 135° ′
0	9.84177	13	10.15823	9.98484	25	10.01516	10.14307	12	9.85693	60
1	.84190	13	.15810	.98509	25	.01491	.14319	12	.85681	59
2	.84203	13	.15797	.98534	26	.01466	.14331	12	.85669	58
3	.84216	13	.15784	.98560	25	.01440	.14343	12	.85657	57
4	.84229	13	.15771	.98585	25	.01415	.14355	13	.85645	56
5	9.84242	13	10.15758	9.98610	25	10.01390	10.14368	12	9.85632	55
6	.84255	14	.15745	.98635	26	.01365	.14380	12	.85620	54
7	.84269	13	.15731	.98661	25	.01339	.14392	12	.85608	53
8	.84282	13	.15718	.98686	25	.01314	.14404	13	.85596	52
9	.84295	13	.15705	.98711	26	.01289	.14417	12	.85583	51
10	9.84308	13	10.15692	9.98737	25	10.01263	10.14429	12	9.85571	50
11	.84321	13	.15679	.98762	25	.01238	.14441	12	.85559	49
12	.84334	13	.15666	.98787	25	.01213	.14453	13	.85547	48
13	.84347	13	.15653	.98812	26	.01188	.14466	12	.85534	47
14	.84360	13	.15640	.98838	25	.01162	.14478	12	.85522	46
15	9.84373	12	10.15627	9.98863	25	10.01137	10.14490	13	9.85510	45
16	.84385	13	.15615	.98888	25	.01112	.14503	12	.85497	44
17	.84398	13	.15602	.98913	26	.01087	.14515	12	.85485	43
18	.84411	13	.15589	.98939	25	.01061	.14527	13	.85473	42
19	.84424	13	.15576	.98964	25	.01036	.14540	12	.85460	41
20	9.84437	13	10.15563	9.98989	26	10.01011	10.14552	12	9.85448	40
21	.84450	13	.15550	.99015	25	.00985	.14564	13	.85436	39
22	.84463	13	.15537	.99040	25	.00960	.14577	12	.85423	38
23	.84476	13	.15524	.99065	25	.00935	.14589	12	.85411	37
24	.84489	13	.15511	.99090	26	.00910	.14601	13	.85399	36
25	9.84502	13	10.15498	9.99116	25	10.00884	10.14614	12	9.85386	35
26	.84515	13	.15485	.99141	25	.00859	.14626	13	.85374	34
27	.84528	12	.15472	.99166	25	.00834	.14639	12	.85361	33
28	.84540	13	.15460	.99191	26	.00809	.14651	12	.85349	32
29	.84553	13	.15447	.99217	25	.00783	.14663	13	.85337	31
30	9.84566	13	10.15434	9.99242	25	10.00758	10.14676	12	9.85324	30
31	.84579	13	.15421	.99267	26	.00733	.14688	13	.85312	29
32	.84592	13	.15408	.99293	25	.00707	.14701	12	.85299	28
33	.84605	13	.15395	.99318	25	.00682	.14713	13	.85287	27
34	.84618	12	.15382	.99343	25	.00657	.14726	12	.85274	26
35	9.84630	13	10.15370	9.99368	26	10.00632	10.14738	12	9.85262	25
36	.84643	13	.15357	.99394	25	.00606	.14750	13	.85250	24
37	.84656	13	.15344	.99419	25	.00581	.14763	12	.85237	23
38	.84669	13	.15331	.99444	25	.00556	.14775	13	.85225	22
39	.84682	12	.15318	.99469	26	.00531	.14788	12	.85212	21
40	9.84694	13	10.15306	9.99495	25	10.00505	10.14800	13	9.85200	20
41	.84707	13	.15293	.99520	25	.00480	.14813	12	.85187	19
42	.84720	13	.15280	.99545	25	.00455	.14825	13	.85175	18
43	.84733	12	.15267	.99570	26	.00430	.14838	12	.85162	17
44	.84745	13	.15255	.99596	25	.00404	.14850	13	.85150	16
45	9.84758	13	10.15242	9.99621	25	10.00379	10.14863	12	9.85137	15
46	.84771	13	.15229	.99646	26	.00354	.14875	13	.85125	14
47	.84784	12	.15216	.99672	25	.00328	.14888	12	.85112	13
48	.84796	13	.15204	.99697	25	.00303	.14900	13	.85100	12
49	.84809	13	.15191	.99722	25	.00278	.14913	13	.85087	11
50	9.84822	13	10.15178	9.99747	26	10.00253	10.14926	12	9.85074	10
51	.84835	12	.15165	.99773	25	.00227	.14938	13	.85062	9
52	.84847	13	.15153	.99798	25	.00202	.14951	12	.85049	8
53	.84860	13	.15140	.99823	25	.00177	.14963	13	.85037	7
54	.84873	12	.15127	.99848	26	.00152	.14976	12	.85024	6
55	9.84885	13	10.15115	9.99874	25	10.00126	10.14988	13	9.85012	5
56	.84898	13	.15102	.99899	25	.00101	.15001	13	.84999	4
57	.84911	12	.15089	.99924	25	.00076	.15014	12	.84986	3
58	.84923	13	.15077	.99949	26	.00051	.15026	13	.84974	2
59	.84936	13	.15064	9.99975	25	.00025	.15039	12	.84961	1
60	9.84949		10.15051	10.00000		10.00000	10.15051		9.84949	0
134° →	cos	Diff. 1′	sec	cot	Diff. 1′	tan	csc	Diff. 1′	sin	← 45°

TABLE 34

Haversines

	0°		1°		2°		3°		4°		
′	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	′
0	Inf. Neg.	0. 00000	5. 88168	0. 00008	6. 48371	0. 00030	6. 83584	0. 00069	7. 08564	0. 00122	60
1	2. 32539	. 00000	. 89604	. 00008	. 49092	. 00031	. 84065	. 00069	. 08925	. 00123	59
2	2. 92745	. 00000	. 91016	. 00008	. 49807	. 00031	. 84543	. 00070	. 09284	. 00124	58
3	3. 27963	. 00000	. 92406	. 00008	. 50516	. 00032	. 85019	. 00071	. 09642	. 00125	57
4	. 52951	. 00000	. 93774	. 00009	. 51219	. 00033	. 85492	. 00072	. 09999	. 00126	56
5	3. 72333	0. 00000	5. 95121	0. 00009	6. 51916	0. 00033	6. 85963	0. 00072	7. 10354	0. 00127	55
6	3. 88169	. 00000	. 96447	. 00009	. 52608	. 00034	. 86431	. 00073	. 10708	. 00128	54
7	4. 01559	. 00000	. 97753	. 00009	. 53295	. 00034	. 86897	. 00074	. 11060	. 00129	53
8	. 13157	. 00000	5. 99040	. 00010	. 53976	. 00035	. 87360	. 00075	. 11411	. 00130	52
9	. 23388	. 00000	6. 00308	. 00010	. 54652	. 00035	. 87821	. 00076	. 11760	. 00131	51
10	4. 32539	0. 00000	6. 01557	0. 00010	6. 55323	0. 00036	6. 88279	0. 00076	7. 12108	0. 00132	50
11	. 40818	. 00000	. 02789	. 00011	. 55988	. 00036	. 88735	. 00077	. 12455	. 00133	49
12	. 48375	. 00000	. 04004	. 00011	. 56649	. 00037	. 89188	. 00078	. 12800	. 00134	48
13	. 55328	. 00000	. 05202	. 00011	. 57304	. 00037	. 89639	. 00079	. 13144	. 00135	47
14	. 61765	. 00000	. 06384	. 00012	. 57955	. 00038	. 90088	. 00080	. 13486	. 00136	46
15	4. 67757	0. 00000	6. 07550	0. 00012	6. 58600	0. 00039	6. 90535	0. 00080	7. 13827	0. 00137	45
16	. 73363	. 00001	. 08700	. 00012	. 59241	. 00039	. 90979	. 00081	. 14167	. 00139	44
17	. 78629	. 00001	. 09836	. 00013	. 59878	. 00040	. 91421	. 00082	. 14506	. 00140	43
18	. 83594	. 00001	. 10956	. 00013	. 60509	. 00040	. 91860	. 00083	. 14843	. 00141	42
19	. 88290	. 00001	. 12063	. 00013	. 61136	. 00041	. 92298	. 00084	. 15179	. 00142	41
20	4. 92745	0. 00001	6. 13155	0. 00014	6. 61759	0. 00041	6. 92733	0. 00085	7. 15513	0. 00143	40
21	4. 96983	. 00001	. 14234	. 00014	. 62377	. 00042	. 93166	. 00085	. 15846	. 00144	39
22	5. 01024	. 00001	. 15300	. 00014	. 62991	. 00043	. 93597	. 00086	. 16178	. 00145	38
23	. 04885	. 00001	. 16353	. 00015	. 63600	. 00043	. 94026	. 00087	. 16509	. 00146	37
24	. 08581	. 00001	. 17393	. 00015	. 64205	. 00044	. 94453	. 00088	. 16839	. 00147	36
25	5. 12127	0. 00001	6. 18421	0. 00015	6. 64806	0. 00044	6. 94877	0. 00089	7. 17167	0. 00148	35
26	. 15534	. 00001	. 19437	. 00016	. 65403	. 00045	. 95300	. 00090	. 17494	. 00150	34
27	. 18812	. 00002	. 20441	. 00016	. 65996	. 00046	. 95720	. 00091	. 17820	. 00151	33
28	. 21971	. 00002	. 21433	. 00016	. 66585	. 00046	. 96139	. 00091	. 18144	. 00152	32
29	. 25019	. 00002	. 22415	. 00017	. 67170	. 00047	. 96555	. 00092	. 18468	. 00153	31
30	5. 27963	0. 00002	6. 23385	0. 00017	6. 67751	0. 00048	6. 96970	0. 00093	7. 18790	0. 00154	30
31	. 30811	. 00002	. 24345	. 00018	. 68328	. 00048	. 97382	. 00094	. 19111	. 00155	29
32	. 33569	. 00002	. 25294	. 00018	. 68901	. 00049	. 97793	. 00095	. 19430	. 00156	28
33	. 36242	. 00002	. 26233	. 00018	. 69470	. 00050	. 98201	. 00096	. 19749	. 00158	27
34	. 38835	. 00002	. 27162	. 00019	. 70036	. 00050	. 98608	. 00097	. 20066	. 00159	26
35	5. 41352	0. 00003	6. 28081	0. 00019	6. 70598	0. 00051	6. 99013	0. 00098	7. 20383	0. 00160	25
36	. 43799	. 00003	. 28991	. 00019	. 71157	. 00051	. 99416	. 00099	. 20698	. 00161	24
37	. 46179	. 00003	. 29891	. 00020	. 71712	. 00052	6. 99817	. 00100	. 21012	. 00162	23
38	. 48496	. 00003	. 30781	. 00020	. 72263	. 00053	7. 00216	. 00100	. 21325	. 00163	22
39	. 50752	. 00003	. 31663	. 00021	. 72811	. 00053	. 00613	. 00101	. 21636	. 00165	21
40	5. 52951	0. 00003	6. 32536	0. 00021	6. 73355	0. 00054	7. 01009	0. 00102	7. 21947	0. 00166	20
41	. 55095	. 00004	. 33400	. 00022	. 73896	. 00055	. 01403	. 00103	. 22256	. 00167	19
42	. 57189	. 00004	. 34256	. 00022	. 74434	. 00056	. 01795	. 00104	. 22565	. 00168	18
43	. 59232	. 00004	. 35103	. 00022	. 74969	. 00056	. 02185	. 00105	. 22872	. 00169	17
44	. 61229	. 00004	. 35943	. 00023	. 75500	. 00057	. 02573	. 00106	. 23178	. 00171	16
45	5. 63181	0. 00004	6. 36774	0. 00023	6. 76028	0. 00058	7. 02960	0. 00107	7. 23483	0. 00172	15
46	. 65090	. 00004	. 37597	. 00024	. 76552	. 00058	. 03345	. 00108	. 23787	. 00173	14
47	. 66958	. 00005	. 38412	. 00024	. 77074	. 00059	. 03729	. 00109	. 24090	. 00174	13
48	. 68787	. 00005	. 39220	. 00025	. 77592	. 00060	. 04110	. 00110	. 24392	. 00175	12
49	. 70578	. 00005	. 40021	. 00025	. 78108	. 00060	. 04490	. 00111	. 24693	. 00177	11
50	5. 72332	0. 00005	6. 40814	0. 00026	6. 78620	0. 00061	7. 04869	0. 00112	7. 24993	0. 00178	10
51	. 74052	. 00006	. 41600	. 00026	. 79129	. 00062	. 05245	. 00113	. 25292	. 00179	9
52	. 75739	. 00006	. 42379	. 00027	. 79636	. 00063	. 05620	. 00114	. 25590	. 00180	8
53	. 77394	. 00006	. 43151	. 00027	. 80139	. 00063	. 05994	. 00115	. 25886	. 00181	7
54	. 79017	. 00006	. 43916	. 00027	. 80640	. 00064	. 06366	. 00116	. 26182	. 00183	6
55	5. 80611	0. 00006	6. 44675	0. 00028	6. 81137	0. 00065	7. 06736	0. 00117	7. 26477	0. 00184	5
56	. 82176	. 00007	. 45427	. 00028	. 81632	. 00066	. 07105	. 00118	. 26771	. 00185	4
57	. 83713	. 00007	. 46172	. 00029	. 82124	. 00066	. 07472	. 00119	. 27064	. 00186	3
58	. 85224	. 00007	. 46911	. 00029	. 82614	. 00067	. 07837	. 00120	. 27355	. 00188	2
59	. 86709	. 00007	. 47644	. 00030	. 83100	. 00068	. 08201	. 00121	. 27646	. 00189	1
60	5. 88168	0. 00008	6. 48371	0. 00030	6. 83584	0. 00069	7. 08564	0. 00122	7. 27936	0. 00190	0
	359°		358°		357°		356°		355°		

TABLE 34

Haversines

′	5° Log Hav	5° Nat. Hav	6° Log Hav	6° Nat. Hav	7° Log Hav	7° Nat. Hav	8° Log Hav	8° Nat. Hav	9° Log Hav	9° Nat. Hav	′
0	7. 27936	0. 00190	7. 43760	0. 00274	7. 57135	0. 00373	7. 68717	0. 00487	7. 78929	0. 00616	60
1	. 28225	. 00192	. 44001	. 00275	. 57341	. 00374	. 68897	. 00489	. 79089	. 00618	59
2	. 28513	. 00193	. 44241	. 00277	. 57547	. 00376	. 69077	. 00491	. 79249	. 00620	58
3	. 28800	. 00194	. 44480	. 00278	. 57752	. 00378	. 69257	. 00493	. 79409	. 00622	57
4	. 29086	. 00195	. 44719	. 00280	. 57957	. 00380	. 69437	. 00495	. 79568	. 00625	56
5	7. 29371	0. 00197	7. 44957	0. 00282	7. 58162	0. 00382	7. 69616	0. 00497	7. 79728	0. 00627	55
6	. 29655	. 00198	. 45194	. 00283	. 58366	. 00383	. 69794	. 00499	. 79886	. 00629	54
7	. 29938	. 00199	. 45431	. 00285	. 58569	. 00385	. 69972	. 00501	. 80045	. 00632	53
8	. 30220	. 00201	. 45667	. 00286	. 58772	. 00387	. 70150	. 00503	. 80203	. 00634	52
9	. 30502	. 00202	. 45903	. 00288	. 58974	. 00389	. 70328	. 00505	. 80361	. 00636	51
10	7. 30782	0. 00203	7. 46138	0. 00289	7. 59176	0. 00391	7. 70505	0. 00507	7. 80519	0. 00639	50
11	. 31062	. 00204	. 46372	. 00291	. 59378	. 00392	. 70682	. 00509	. 80677	. 00641	49
12	. 31340	. 00206	. 46605	. 00292	. 59579	. 00394	. 70858	. 00511	. 80834	. 00643	48
13	. 31618	. 00207	. 46838	. 00294	. 59779	. 00396	. 71034	. 00513	. 80991	. 00646	47
14	. 31895	. 00208	. 47071	. 00296	. 59979	. 00398	. 71210	. 00515	. 81147	. 00648	46
15	7. 32171	0. 00210	7. 47302	0. 00297	7. 60179	0. 00400	7. 71385	0. 00517	7. 81303	0. 00650	45
16	. 32446	. 00211	. 47533	. 00299	. 60378	. 00402	. 71560	. 00520	. 81459	. 00653	44
17	. 32720	. 00212	. 47764	. 00300	. 60577	. 00403	. 71735	. 00522	. 81615	. 00655	43
18	. 32994	. 00214	. 47994	. 00302	. 60775	. 00405	. 71909	. 00524	. 81771	. 00657	42
19	. 33266	. 00215	. 48223	. 00304	. 60973	. 00407	. 72083	. 00526	. 81926	. 00660	41
20	7. 33538	0. 00216	7. 48452	0. 00305	7. 61170	0. 00409	7. 72257	0. 00528	7. 82081	0. 00662	40
21	. 33809	. 00218	. 48680	. 00307	. 61367	. 00411	. 72430	. 00530	. 82235	. 00664	39
22	. 34079	. 00219	. 48907	. 00308	. 61564	. 00413	. 72603	. 00532	. 82390	. 00667	38
23	. 34348	. 00221	. 49134	. 00310	. 61760	. 00415	. 72775	. 00534	. 82544	. 00669	37
24	. 34616	. 00222	. 49360	. 00312	. 61955	. 00416	. 72948	. 00536	. 82698	. 00671	36
25	7. 34884	0. 00223	7. 49586	0. 00313	7. 62151	0. 00418	7. 73119	0. 00539	7. 82851	0. 00674	35
26	. 35150	. 00225	. 49811	. 00315	. 62345	. 00420	. 73291	. 00541	. 83004	. 00676	34
27	. 35416	. 00226	. 50036	. 00316	. 62540	. 00422	. 73462	. 00543	. 83157	. 00679	33
28	. 35681	. 00227	. 50259	. 00318	. 62733	. 00424	. 73633	. 00545	. 83310	. 00681	32
29	. 35945	. 00229	. 50483	. 00320	. 62927	. 00426	. 73803	. 00547	. 83463	. 00683	31
30	7. 36209	0. 00230	7. 50706	0. 00321	7. 63120	0. 00428	7. 73974	0. 00549	7. 83615	0. 00686	30
31	. 36471	. 00232	. 50928	. 00323	. 63312	. 00430	. 74143	. 00551	. 83767	. 00688	29
32	. 36733	. 00233	. 51149	. 00325	. 63504	. 00432	. 74313	. 00554	. 83918	. 00691	28
33	. 36994	. 00234	. 51370	. 00326	. 63696	. 00433	. 74482	. 00556	. 84070	. 00693	27
34	. 37254	. 00236	. 51591	. 00328	. 63887	. 00435	. 74651	. 00558	. 84221	. 00695	26
35	7. 37514	0. 00237	7. 51811	0. 00330	7. 64078	0. 00437	7. 74819	0. 00560	7. 84372	0. 00698	25
36	. 37773	. 00239	. 52030	. 00331	. 64269	. 00439	. 74988	. 00562	. 84522	. 00700	24
37	. 38030	. 00240	. 52249	. 00333	. 64458	. 00441	. 75155	. 00564	. 84672	. 00703	23
38	. 38288	. 00241	. 52467	. 00335	. 64648	. 00443	. 75323	. 00567	. 84822	. 00705	22
39	. 38544	. 00243	. 52685	. 00336	. 64837	. 00445	. 75490	. 00569	. 84972	. 00707	21
40	7. 38800	0. 00244	7. 52902	0. 00338	7. 65026	0. 00447	7. 75657	0. 00571	7. 85122	0. 00710	20
41	. 39054	. 00246	. 53119	. 00340	. 65214	. 00449	. 75824	. 00573	. 85271	. 00712	19
42	. 39309	. 00247	. 53335	. 00341	. 65402	. 00451	. 75990	. 00575	. 85420	. 00715	18
43	. 39562	. 00249	. 53550	. 00343	. 65590	. 00453	. 76156	. 00578	. 85569	. 00717	17
44	. 39815	. 00250	. 53766	. 00345	. 65777	. 00455	. 76321	. 00580	. 85717	. 00720	16
45	7. 40067	0. 00252	7. 53980	0. 00347	7. 65964	0. 00457	7. 76487	0. 00582	7. 85866	0. 00722	15
46	. 40318	. 00253	. 54194	. 00348	. 66150	. 00459	. 76652	. 00584	. 86014	. 00725	14
47	. 40568	. 00255	. 54407	. 00350	. 66336	. 00461	. 76816	. 00586	. 86161	. 00727	13
48	. 40818	. 00256	. 54620	. 00352	. 66521	. 00463	. 76981	. 00589	. 86309	. 00730	12
49	. 41067	. 00257	. 54833	. 00353	. 66706	. 00465	. 77145	. 00591	. 86456	. 00732	11
50	7. 41315	0. 00259	7. 55045	0. 00355	7. 66891	0. 00467	7. 77308	0. 00593	7. 86603	0. 00735	10
51	. 41563	. 00260	. 55256	. 00357	. 67075	. 00469	. 77472	. 00595	. 86750	. 00737	9
52	. 41810	. 00262	. 55467	. 00359	. 67259	. 00471	. 77635	. 00598	. 86896	. 00740	8
53	. 42056	. 00263	. 55677	. 00360	. 67443	. 00473	. 77798	. 00600	. 87042	. 00742	7
54	. 42301	. 00265	. 55887	. 00362	. 67626	. 00475	. 77960	. 00602	. 87188	. 00745	6
55	7. 42546	0. 00266	7. 56096	0. 00364	7. 67809	0. 00477	7. 78122	0. 00604	7. 87334	0. 00747	5
56	. 42790	. 00268	. 56305	. 00366	. 67991	. 00479	. 78284	. 00607	. 87480	. 00750	4
57	. 43034	. 00269	. 56513	. 00367	. 68173	. 00481	. 78446	. 00609	. 87625	. 00752	3
58	. 43277	. 00271	. 56721	. 00369	. 68355	. 00483	. 78607	. 00611	. 87770	. 00755	2
59	. 43519	. 00272	. 56928	. 00371	. 68536	. 00485	. 78768	. 00613	. 87915	. 00757	1
60	7. 43760	0. 00274	7. 57135	0. 00373	7. 68717	0. 00487	7. 78929	0. 00616	7. 88059	0. 00760	0
	354°		353°		352°		351°		350°		

TABLE 34

Haversines

'	10° Log Hav	10° Nat. Hav	11° Log Hav	11° Nat. Hav	12° Log Hav	12° Nat. Hav	13° Log Hav	13° Nat. Hav	14° Log Hav	14° Nat. Hav	'
0	7. 88059	0. 00760	7. 96315	0. 00919	8. 03847	0. 01093	8. 10772	0. 01281	8. 17179	0. 01485	60
1	. 88203	. 00762	. 96446	. 00921	. 03967	. 01096	. 10883	. 01285	. 17282	. 01489	59
2	. 88348	. 00765	. 96577	. 00924	. 04087	. 01099	. 10993	. 01288	. 17384	. 01492	58
3	. 88491	. 00767	. 96707	. 00927	. 04207	. 01102	. 11104	. 01291	. 17487	. 01496	57
4	. 88635	. 00770	. 96838	. 00930	. 04326	. 01105	. 11214	. 01295	. 17590	. 01499	56
5	7. 88778	0. 00772	7. 96968	0. 00933	8. 04446	0. 01108	8. 11324	0. 01298	8. 17692	0. 01503	55
6	. 88921	. 00775	. 97098	. 00935	. 04565	. 01111	. 11435	. 01301	. 17794	. 01506	54
7	. 89064	. 00777	. 97228	. 00938	. 04684	. 01114	. 11544	. 01305	. 17896	. 01510	53
8	. 89207	. 00780	. 97358	. 00941	. 04803	. 01117	. 11654	. 01308	. 17998	. 01513	52
9	. 89349	. 00783	. 97487	. 00944	. 04922	. 01120	. 11764	. 01311	. 18100	. 01517	51
10	7. 89491	0. 00785	7. 97617	0. 00947	8. 05041	0. 01123	8. 11873	0. 01314	8. 18202	0. 01521	50
11	. 89633	. 00788	. 97746	. 00949	. 05159	. 01126	. 11983	. 01318	. 18303	. 01524	49
12	. 89775	. 00790	. 97875	. 00952	. 05277	. 01129	. 12092	. 01321	. 18405	. 01528	48
13	. 89916	. 00793	. 98003	. 00955	. 05395	. 01132	. 12201	. 01324	. 18506	. 01531	47
14	. 90057	. 00795	. 98132	. 00958	. 05513	. 01135	. 12310	. 01328	. 18607	. 01535	46
15	7. 90198	0. 00798	7. 98260	0. 00961	8. 05631	0. 01138	8. 12419	0. 01331	8. 18709	0. 01538	45
16	. 90339	. 00801	. 98389	. 00964	. 05749	. 01142	. 12528	. 01334	. 18810	. 01542	44
17	. 90480	. 00803	. 98517	. 00966	. 05866	. 01145	. 12636	. 01338	. 18910	. 01546	43
18	. 90620	. 00806	. 98644	. 00969	. 05984	. 01148	. 12745	. 01341	. 19011	. 01549	42
19	. 90760	. 00808	. 98772	. 00972	. 06101	. 01151	. 12853	. 01344	. 19112	. 01553	41
20	7. 90900	0. 00811	7. 98899	0. 00975	8. 06218	0. 01154	8. 12961	0. 01348	8. 19212	0. 01556	40
21	. 91039	. 00814	. 99027	. 00978	. 06335	. 01157	. 13069	. 01351	. 19313	. 01560	39
22	. 91179	. 00816	. 99154	. 00981	. 06451	. 01160	. 13177	. 01354	. 19413	. 01564	38
23	. 91318	. 00819	. 99281	. 00984	. 06568	. 01163	. 13285	. 01358	. 19513	. 01567	37
24	. 91457	. 00821	. 99407	. 00986	. 06684	. 01166	. 13392	. 01361	. 19613	. 01571	36
25	7. 91596	0. 00824	7. 99534	0. 00989	8. 06800	0. 01170	8. 13500	0. 01365	8. 19713	0. 01574	35
26	. 91734	. 00827	. 99660	. 00992	. 06917	. 01173	. 13607	. 01368	. 19813	. 01578	34
27	. 91872	. 00829	. 99786	. 00995	. 07032	. 01176	. 13714	. 01371	. 19913	. 01582	33
28	. 92010	. 00832	7. 99912	. 00998	. 07148	. 01179	. 13822	. 01375	. 20012	. 01585	32
29	. 92148	. 00835	8. 00038	. 01001	. 07264	. 01182	. 13928	. 01378	. 20112	. 01589	31
30	7. 92286	0. 00837	8. 00163	0. 01004	8. 07379	0. 01185	8. 14035	0. 01382	8. 20211	0. 01593	30
31	. 92423	. 00840	. 00289	. 01007	. 07494	. 01188	. 14142	. 01385	. 20310	. 01596	29
32	. 92560	. 00843	. 00414	. 01010	. 07610	. 01192	. 14248	. 01388	. 20410	. 01600	28
33	. 92697	. 00845	. 00539	. 01012	. 07725	. 01195	. 14355	. 01392	. 20509	. 01604	27
34	. 92834	. 00848	. 00664	. 01015	. 07839	. 01198	. 14461	. 01395	. 20608	. 01607	26
35	7. 92970	0. 00851	8. 00788	0. 01018	8. 07954	0. 01201	8. 14567	0. 01399	8. 20706	0. 01611	25
36	. 93107	. 00853	. 00913	. 01021	. 08069	. 01204	. 14673	. 01402	. 20805	. 01615	24
37	. 93243	. 00856	. 01037	. 01024	. 08183	. 01207	. 14779	. 01405	. 20904	. 01618	23
38	. 93379	. 00859	. 01161	. 01027	. 08297	. 01211	. 14885	. 01409	. 21002	. 01622	22
39	. 93514	. 00861	. 01285	. 01030	. 08411	. 01214	. 14991	. 01412	. 21100	. 01626	21
40	7. 93650	0. 00864	8. 01409	0. 01033	8. 08525	0. 01217	8. 15096	0. 01416	8. 21199	0. 01629	20
41	. 93785	. 00867	. 01532	. 01036	. 08639	. 01220	. 15201	. 01419	. 21297	. 01633	19
42	. 93920	. 00869	. 01656	. 01039	. 08752	. 01223	. 15307	. 01423	. 21395	. 01637	18
43	. 94055	. 00872	. 01779	. 01042	. 08866	. 01226	. 15412	. 01426	. 21493	. 01640	17
44	. 94189	. 00875	. 01902	. 01045	. 08979	. 01230	. 15517	. 01429	. 21590	. 01644	16
45	7. 94324	0. 00877	8. 02025	0. 01048	8. 09092	0. 01233	8. 15622	0. 01433	8. 21688	0. 01648	15
46	. 94458	. 00880	. 02148	. 01051	. 09205	. 01236	. 15726	. 01436	. 21785	. 01651	14
47	. 94592	. 00883	. 02270	. 01054	. 09318	. 01239	. 15831	. 01440	. 21883	. 01655	13
48	. 94726	. 00886	. 02392	. 01057	. 09431	. 01243	. 15935	. 01443	. 21980	. 01659	12
49	. 94859	. 00888	. 02515	. 01060	. 09543	. 01246	. 16040	. 01447	. 22077	. 01663	11
50	7. 94992	0. 00891	8. 02637	0. 01063	8. 09656	0. 01249	8. 16144	0. 01450	8. 22175	0. 01666	10
51	. 95126	. 00894	. 02758	. 01066	. 09768	. 01252	. 16248	. 01454	. 22272	. 01670	9
52	. 95259	. 00897	. 02880	. 01069	. 09880	. 01255	. 16352	. 01457	. 22368	. 01674	8
53	. 95391	. 00899	. 03001	. 01072	. 09992	. 01259	. 16456	. 01461	. 22465	. 01677	7
54	. 95524	. 00902	. 03123	. 01075	. 10104	. 01262	. 16559	. 01464	. 22562	. 01681	6
55	7. 95656	0. 00905	8. 03244	0. 01078	8. 10216	0. 01265	8. 16663	0. 01468	8. 22658	0. 01685	5
56	. 95788	. 00908	. 03365	. 01081	. 10327	. 01268	. 16766	. 01471	. 22755	. 01689	4
57	. 95920	. 00910	. 03486	. 01084	. 10439	. 01272	. 16870	. 01475	. 22851	. 01692	3
58	. 96052	. 00913	. 03606	. 01087	. 10550	. 01275	. 16973	. 01478	. 22947	. 01696	2
59	. 96183	. 00916	. 03727	. 01090	. 10661	. 01278	. 17076	. 01482	. 23044	. 01700	1
60	7. 96315	0. 00919	8. 03847	0. 01093	8. 10772	0. 01281	8. 17179	0. 01485	8. 23140	0. 01704	0
	349°		348°		347°		346°		345°		

TABLE 34

Haversines

′	15° Log Hav	15° Nat. Hav	16° Log Hav	16° Nat. Hav	17° Log Hav	17° Nat. Hav	18° Log Hav	18° Nat. Hav	19° Log Hav	19° Nat. Hav	′
0	8. 23140	0. 01704	8. 28711	0. 01937	8. 33940	0. 02185	8. 38867	0. 02447	8. 43522	0. 02724	60
1	. 23235	. 01707	. 28801	. 01941	. 34025	. 02189	. 38946	. 02452	. 43597	. 02729	59
2	. 23331	. 01711	. 28891	. 01945	. 34109	. 02193	. 39026	. 02456	. 43673	. 02734	58
3	. 23427	. 01715	. 28980	. 01949	. 34194	. 02198	. 39105	. 02461	. 43748	. 02738	57
4	. 23523	. 01719	. 29070	. 01953	. 34278	. 02202	. 39185	. 02465	. 43823	. 02743	56
5	8. 23618	0. 01723	8. 29159	0. 01957	8. 34362	0. 02206	8. 39264	0. 02470	8. 43899	0. 02748	55
6	. 23713	. 01726	. 29249	. 01961	. 34446	. 02210	. 39344	. 02474	. 43974	. 02753	54
7	. 23809	. 01730	. 29338	. 01965	. 34530	. 02215	. 39423	. 02479	. 44049	. 02757	53
8	. 23904	. 01734	. 29427	. 01969	. 34614	. 02219	. 39502	. 02483	. 44124	. 02762	52
9	. 23999	. 01738	. 29516	. 01973	. 34698	. 02223	. 39581	. 02488	. 44199	. 02767	51
10	8. 24094	0. 01742	8. 29605	0. 01977	8. 34782	0. 02227	8. 39660	0. 02492	8. 44273	0. 02772	50
11	. 24189	. 01745	. 29694	. 01981	. 34865	. 02232	. 39739	. 02497	. 44348	. 02776	49
12	. 24283	. 01749	. 29783	. 01985	. 34949	. 02236	. 39818	. 02501	. 44423	. 02781	48
13	. 24378	. 01753	. 29872	. 01989	. 35032	. 02240	. 39897	. 02506	. 44498	. 02786	47
14	. 24473	. 01757	. 29960	. 01993	. 35116	. 02245	. 39976	. 02510	. 44572	. 02791	46
15	8. 24567	0. 01761	8. 30049	0. 01998	8. 35199	0. 02249	8. 40055	0. 02515	8. 44647	0. 02796	45
16	. 24661	. 01764	. 30137	. 02002	. 35282	. 02253	. 40133	. 02520	. 44721	. 02800	44
17	. 24755	. 01768	. 30226	. 02006	. 35365	. 02258	. 40212	. 02524	. 44796	. 02805	43
18	. 24850	. 01772	. 30314	. 02010	. 35449	. 02262	. 40290	. 02529	. 44870	. 02810	42
19	. 24944	. 01776	. 30402	. 02014	. 35532	. 02266	. 40369	. 02533	. 44944	. 02815	41
20	8. 25037	0. 01780	8. 30490	0. 02018	8. 35614	0. 02271	8. 40447	0. 02538	8. 45018	0. 02820	40
21	. 25131	. 01784	. 30578	. 02022	. 35697	. 02275	. 40525	. 02542	. 45093	. 02824	39
22	. 25225	. 01788	. 30666	. 02026	. 35780	. 02279	. 40603	. 02547	. 45167	. 02829	38
23	. 25319	. 01791	. 30754	. 02030	. 35863	. 02284	. 40681	. 02552	. 45241	. 02834	37
24	. 25412	. 01795	. 30842	. 02034	. 35945	. 02288	. 40760	. 02556	. 45315	. 02839	36
25	8. 25505	0. 01799	8. 30929	0. 02038	8. 36028	0. 02292	8. 40837	0. 02561	8. 45388	0. 02844	35
26	. 25599	. 01803	. 31017	. 02043	. 36110	. 02297	. 40915	. 02565	. 45462	. 02849	34
27	. 25692	. 01807	. 31104	. 02047	. 36193	. 02301	. 40993	. 02570	. 45536	. 02853	33
28	. 25785	. 01811	. 31192	. 02051	. 36275	. 02305	. 41071	. 02575	. 45610	. 02858	32
29	. 25878	. 01815	. 31279	. 02055	. 36357	. 02310	. 41149	. 02579	. 45683	. 02863	31
30	8. 25971	0. 01818	8. 31366	0. 02059	8. 36439	0. 02314	8. 41226	0. 02584	8. 45757	0. 02868	30
31	. 26064	. 01822	. 31453	. 02063	. 36521	. 02319	. 41304	. 02588	. 45830	. 02873	29
32	. 26156	. 01826	. 31540	. 02067	. 36603	. 02323	. 41381	. 02593	. 45904	. 02878	28
33	. 26249	. 01830	. 31627	. 02071	. 36685	. 02327	. 41459	. 02598	. 45977	. 02883	27
34	. 26341	. 01834	. 31714	. 02076	. 36767	. 02332	. 41536	. 02602	. 46050	. 02887	26
35	8. 26434	0. 01838	8. 31800	0. 02080	8. 36849	0. 02336	8. 41613	0. 02607	8. 46124	0. 02892	25
36	. 26526	. 01842	. 31887	. 02084	. 36930	. 02340	. 41690	. 02612	. 46197	. 02897	24
37	. 26618	. 01846	. 31974	. 02088	. 37012	. 02345	. 41767	. 02616	. 46270	. 02902	23
38	. 26710	. 01850	. 32060	. 02092	. 37093	. 02349	. 41845	. 02621	. 46343	. 02907	22
39	. 26802	. 01854	. 32147	. 02096	. 37175	. 02354	. 41921	. 02626	. 46416	. 02912	21
40	8. 26894	0. 01858	8. 32233	0. 02101	8. 37256	0. 02358	8. 41998	0. 02630	8. 46489	0. 02917	20
41	. 26986	. 01861	. 32319	. 02105	. 37337	. 02363	. 42075	. 02635	. 46562	. 02922	19
42	. 27078	. 01865	. 32405	. 02109	. 37419	. 02367	. 42152	. 02639	. 46634	. 02926	18
43	. 27169	. 01869	. 32491	. 02113	. 37500	. 02371	. 42229	. 02644	. 46707	. 02931	17
44	. 27261	. 01873	. 32577	. 02117	. 37581	. 02376	. 42305	. 02649	. 46780	. 02936	16
45	8. 27352	0. 01877	8. 32663	0. 02121	8. 37662	0. 02380	8. 42382	0. 02653	8. 46852	0. 02941	15
46	. 27443	. 01881	. 32749	. 02126	. 37742	. 02385	. 42458	. 02658	. 46925	. 02946	14
47	. 27534	. 01885	. 32834	. 02130	. 37823	. 02389	. 42535	. 02663	. 46998	. 02951	13
48	. 27626	. 01889	. 32920	. 02134	. 37904	. 02394	. 42611	. 02668	. 47070	. 02956	12
49	. 27717	. 01893	. 33006	. 02138	. 37985	. 02398	. 42687	. 02672	. 47142	. 02961	11
50	8. 27807	0. 01897	8. 33091	0. 02142	8. 38065	0. 02402	8. 42764	0. 02677	8. 47215	0. 02966	10
51	. 27898	. 01901	. 33176	. 02147	. 38146	. 02407	. 42840	. 02682	. 47287	. 02971	9
52	. 27989	. 01905	. 33262	. 02151	. 38226	. 02411	. 42916	. 02686	. 47359	. 02976	8
53	. 28080	. 01909	. 33347	. 02155	. 38306	. 02416	. 42992	. 02691	. 47431	. 02981	7
54	. 28170	. 01913	. 33432	. 02159	. 38387	. 02420	. 43068	. 02696	. 47503	. 02986	6
55	8. 28260	0. 01917	8. 33517	0. 02164	8. 38467	0. 02425	8. 43144	0. 02700	8. 47575	0. 02991	5
56	. 28351	. 01921	. 33602	. 02168	. 38547	. 02429	. 43219	. 02705	. 47647	. 02996	4
57	. 28441	. 01925	. 33686	. 02172	. 38627	. 02434	. 43295	. 02710	. 47719	. 03000	3
58	. 28531	. 01929	. 33771	. 02176	. 38707	. 02438	. 43371	. 02715	. 47791	. 03005	2
59	. 28621	. 01933	. 33856	. 02181	. 38787	. 02443	. 43446	. 02719	. 47862	. 03010	1
60	8. 28711	0. 01937	8. 33940	0. 02185	8. 38867	0. 02447	8. 43522	0. 02724	8. 47934	0. 03015	0
	344°		343°		342°		341°		340°		

TABLE 34

Haversines

′	20° Log Hav	20° Nat. Hav	21° Log Hav	21° Nat. Hav	22° Log Hav	22° Nat. Hav	23° Log Hav	23° Nat. Hav	24° Log Hav	24° Nat. Hav	′
0	8. 47934	0. 03015	8. 52127	0. 03321	8. 56120	0. 03641	8. 59931	0. 03975	8. 63576	0. 04323	60
1	. 48006	. 03020	. 52195	. 03326	. 56185	. 03646	. 59993	. 03980	. 63635	. 04329	59
2	. 48077	. 03025	. 52263	. 03331	. 56250	. 03652	. 60055	. 03986	. 63695	. 04335	58
3	. 48149	. 03030	. 52331	. 03337	. 56315	. 03657	. 60117	. 03992	. 63754	. 04340	57
4	. 48220	. 03035	. 52399	. 03342	. 56379	. 03663	. 60179	. 03998	. 63813	. 04346	56
5	8. 48292	0. 03040	8. 52467	0. 03347	8. 56444	0. 03668	8. 60241	0. 04003	8. 63872	0. 04352	55
6	. 48363	. 03045	. 52535	. 03352	. 56509	. 03674	. 60303	. 04009	. 63932	. 04358	54
7	. 48434	. 03050	. 52602	. 03358	. 56574	. 03679	. 60365	. 04015	. 63991	. 04364	53
8	. 48505	. 03055	. 52670	. 03363	. 56638	. 03685	. 60426	. 04020	. 64050	. 04370	52
9	. 48576	. 03060	. 52738	. 03368	. 56703	. 03690	. 60488	. 04026	. 64109	. 04376	51
10	8. 48648	0. 03065	8. 52806	0. 03373	8. 56767	0. 03695	8. 60550	0. 04032	8. 64168	0. 04382	50
11	. 48719	. 03070	. 52873	. 03379	. 56832	. 03701	. 60611	. 04038	. 64227	. 04388	49
12	. 48789	. 03075	. 52941	. 03384	. 56896	. 03706	. 60673	. 04043	. 64286	. 04394	48
13	. 48860	. 03080	. 53008	. 03389	. 56960	. 03712	. 60734	. 04049	. 64345	. 04400	47
14	. 48931	. 03085	. 53076	. 03394	. 57025	. 03717	. 60796	. 04055	. 64404	. 04406	46
15	8. 49002	0. 03090	8. 53143	0. 03400	8. 57089	0. 03723	8. 60857	0. 04060	8. 64463	0. 04412	45
16	. 49073	. 03095	. 53210	. 03405	. 57153	. 03728	. 60919	. 04066	. 64521	. 04418	44
17	. 49143	. 03101	. 53277	. 03410	. 57217	. 03734	. 60980	. 04072	. 64580	. 04424	43
18	. 49214	. 03106	. 53345	. 03415	. 57282	. 03740	. 61041	. 04078	. 64639	. 04430	42
19	. 49284	. 03111	. 53412	. 03421	. 57346	. 03745	. 61103	. 04083	. 64697	. 04436	41
20	8. 49355	0. 03116	8. 53479	0. 03426	8. 57410	0. 03751	8. 61164	0. 04089	8. 64756	0. 04442	40
21	. 49425	. 03121	. 53546	. 03431	. 57474	. 03756	. 61225	. 04095	. 64815	. 04448	39
22	. 49496	. 03126	. 53613	. 03437	. 57538	. 03762	. 61286	. 04101	. 64873	. 04454	38
23	. 49566	. 03131	. 53680	. 03442	. 57601	. 03767	. 61347	. 04106	. 64932	. 04460	37
24	. 49636	. 03136	. 53747	. 03447	. 57665	. 03773	. 61408	. 04112	. 64990	. 04466	36
25	8. 49706	0. 03141	8. 53814	0. 03453	8. 57729	0. 03778	8. 61469	0. 04118	8. 65049	0. 04472	35
26	. 49777	. 03146	. 53880	. 03458	. 57793	. 03784	. 61530	. 04124	. 65107	. 04478	34
27	. 49847	. 03151	. 53947	. 03463	. 57856	. 03789	. 61591	. 04130	. 65165	. 04484	33
28	. 49917	. 03156	. 54014	. 03468	. 57920	. 03795	. 61652	. 04135	. 65224	. 04490	32
29	. 49987	. 03161	. 54080	. 03474	. 57984	. 03800	. 61713	. 04141	. 65282	. 04496	31
30	8. 50056	0. 03166	8. 54147	0. 03479	8. 58047	0. 03806	8. 61773	0. 04147	8. 65340	0. 04502	30
31	. 50126	. 03171	. 54214	. 03484	. 58111	. 03812	. 61834	. 04153	. 65398	. 04508	29
32	. 50196	. 03177	. 54280	. 03490	. 58174	. 03817	. 61895	. 04159	. 65456	. 04514	28
33	. 50266	. 03182	. 54346	. 03495	. 58238	. 03823	. 61955	. 04164	. 65514	. 04520	27
34	. 50335	. 03187	. 54413	. 03500	. 58301	. 03828	. 62016	. 04170	. 65572	. 04526	26
35	8. 50405	0. 03192	8. 54479	0. 03506	8. 58364	0. 03834	8. 62077	0. 04176	8. 65630	0. 04532	25
36	. 50475	. 03197	. 54545	. 03511	. 58427	. 03839	. 62137	. 04182	. 65688	. 04538	24
37	. 50544	. 03202	. 54612	. 03517	. 58491	. 03845	. 62197	. 04188	. 65746	. 04544	23
38	. 50614	. 03207	. 54678	. 03522	. 58554	. 03851	. 62258	. 04194	. 65804	. 04550	22
39	. 50683	. 03212	. 54744	. 03527	. 58617	. 03856	. 62318	. 04199	. 65862	. 04556	21
40	8. 50752	0. 03218	8. 54810	0. 03533	8. 58680	0. 03862	8. 62379	0. 04205	8. 65920	0. 04562	20
41	. 50821	. 03223	. 54876	. 03538	. 58743	. 03867	. 62439	. 04211	. 65978	. 04569	19
42	. 50891	. 03228	. 54942	. 03543	. 58806	. 03873	. 62499	. 04217	. 66035	. 04575	18
43	. 50960	. 03233	. 55008	. 03549	. 58869	. 03879	. 62559	. 04223	. 66093	. 04581	17
44	. 51029	. 03238	. 55073	. 03554	. 58932	. 03884	. 62619	. 04229	. 66151	. 04587	16
45	8. 51098	0. 03243	8. 55139	0. 03560	8. 58994	0. 03890	8. 62680	0. 04234	8. 66208	0. 04593	15
46	. 51167	. 03248	. 55205	. 03565	. 59057	. 03896	. 62740	. 04240	. 66266	. 04599	14
47	. 51236	. 03254	. 55271	. 03570	. 59120	. 03901	. 62800	. 04246	. 66323	. 04605	13
48	. 51305	. 03259	. 55336	. 03576	. 59183	. 03907	. 62860	. 04252	. 66381	. 04611	12
49	. 51374	. 03264	. 55402	. 03581	. 59245	. 03912	. 62919	. 04258	. 66438	. 04617	11
50	8. 51442	0. 03269	8. 55467	0. 03587	8. 59308	0. 03918	8. 62979	0. 04264	8. 66496	0. 04623	10
51	. 51511	. 03274	. 55533	. 03592	. 59370	. 03924	. 63039	. 04270	. 66553	. 04629	9
52	. 51580	. 03279	. 55598	. 03597	. 59433	. 03929	. 63099	. 04276	. 66610	. 04636	8
53	. 51648	. 03285	. 55664	. 03603	. 59495	. 03935	. 63159	. 04281	. 66668	. 04642	7
54	. 51717	. 03290	. 55729	. 03608	. 59558	. 03941	. 63218	. 04287	. 66725	. 04648	6
55	8. 51785	0. 03295	8. 55794	0. 03614	8. 59620	0. 03946	8. 63278	0. 04293	8. 66782	0. 04654	5
56	. 51854	. 03300	. 55859	. 03619	. 59682	. 03952	. 63338	. 04299	. 66839	. 04660	4
57	. 51922	. 03305	. 55925	. 03624	. 59745	. 03958	. 63397	. 04305	. 66896	. 04666	3
58	. 51990	. 03311	. 55990	. 03630	. 59807	. 03963	. 63457	. 04311	. 66953	. 04672	2
59	. 52058	. 03316	. 56055	. 03635	. 59869	. 03969	. 63516	. 04317	. 67010	. 04678	1
60	8. 52127	0. 03321	8. 56120	0. 03641	8. 59931	0. 03975	8. 63576	0. 04323	8. 67067	0. 04685	0
	339°		338°		337°		336°		335°		

TABLE 34

Haversines

	25°		26°		27°		28°		29°		
	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	′
0	8. 67067	0. 04685	8. 70418	0. 05060	8. 73637	0. 05450	8. 76735	0. 05853	8. 79720	0. 06269	60
1	. 67124	. 04691	. 70472	. 05067	. 73690	. 05456	. 76786	. 05859	. 79769	. 06276	59
2	. 67181	. 04697	. 70527	. 05073	. 73742	. 05463	. 76836	. 05866	. 79818	. 06283	58
3	. 67238	. 04703	. 70582	. 05079	. 73795	. 05469	. 76887	. 05873	. 79866	. 06290	57
4	. 67295	. 04709	. 70636	. 05086	. 73847	. 05476	. 76938	. 05880	. 79915	. 06297	56
5	8. 67352	0. 04715	8. 70691	0. 05092	8. 73900	0. 05483	8. 76988	0. 05887	8. 79964	0. 06304	55
6	. 67409	. 04722	. 70745	. 05099	. 73952	. 05489	. 77039	. 05894	. 80013	. 06311	54
7	. 67465	. 04728	. 70800	. 05105	. 74005	. 05496	. 77089	. 05901	. 80061	. 06318	53
8	. 67522	. 04734	. 70854	. 05111	. 74057	. 05503	. 77139	. 05907	. 80110	. 06326	52
9	. 67579	. 04740	. 70909	. 05118	. 74109	. 05509	. 77190	. 05914	. 80158	. 06333	51
10	8. 67635	0. 04746	8. 70963	0. 05124	8. 74162	0. 05516	8. 77240	0. 05921	8. 80207	0. 06340	50
11	. 67692	. 04752	. 71017	. 05131	. 74214	. 05523	. 77291	. 05928	. 80256	. 06347	49
12	. 67748	. 04759	. 71072	. 05137	. 74266	. 05529	. 77341	. 05935	. 80304	. 06354	48
13	. 67805	. 04765	. 71126	. 05144	. 74318	. 05536	. 77391	. 05942	. 80353	. 06361	47
14	. 67861	. 04771	. 71180	. 05150	. 74371	. 05542	. 77441	. 05949	. 80401	. 06368	46
15	8. 67918	0. 04777	8. 71234	0. 05156	8. 74423	0. 05549	8. 77492	0. 05955	8. 80449	0. 06375	45
16	. 67974	. 04783	. 71289	. 05163	. 74475	. 05556	. 77542	. 05962	. 80498	. 06382	44
17	. 68030	. 04790	. 71343	. 05169	. 74527	. 05562	. 77592	. 05969	. 80546	. 06389	43
18	. 68087	. 04796	. 71397	. 05176	. 74579	. 05569	. 77642	. 05976	. 80595	. 06397	42
19	. 68143	. 04802	. 71451	. 05182	. 74631	. 05576	. 77692	. 05983	. 80643	. 06404	41
20	8. 68199	0. 04808	8. 71505	0. 05189	8. 74683	0. 05582	8. 77742	0. 05990	8. 80691	0. 06411	40
21	. 68256	. 04815	. 71559	. 05195	. 74735	. 05589	. 77792	. 05997	. 80739	. 06418	39
22	. 68312	. 04821	. 71613	. 05201	. 74787	. 05596	. 77842	. 06004	. 80788	. 06425	38
23	. 68368	. 04827	. 71667	. 05208	. 74839	. 05603	. 77892	. 06011	. 80836	. 06432	37
24	. 68424	. 04833	. 71721	. 05214	. 74890	. 05609	. 77942	. 06018	. 80884	. 06439	36
25	8. 68480	0. 04839	8. 71774	0. 05221	8. 74942	0. 05616	8. 77992	0. 06024	8. 80932	0. 06446	35
26	. 68536	. 04846	. 71828	. 05227	. 74994	. 05623	. 78042	. 06031	. 80980	. 06454	34
27	. 68592	. 04852	. 71882	. 05234	. 75046	. 05629	. 78092	. 06038	. 81028	. 06461	33
28	. 68648	. 04858	. 71936	. 05240	. 75097	. 05636	. 78142	. 06045	. 81076	. 06468	32
29	. 68704	. 04864	. 71989	. 05247	. 75149	. 05643	. 78191	. 06052	. 81124	. 06475	31
30	8. 68760	0. 04871	8. 72043	0. 05253	8. 75201	0. 05649	8. 78241	0. 06059	8. 81172	0. 06482	30
31	. 68815	. 04877	. 72097	. 05260	. 75252	. 05656	. 78291	. 06066	. 81220	. 06489	29
32	. 68871	. 04883	. 72150	. 05266	. 75304	. 05663	. 78341	. 06073	. 81268	. 06497	28
33	. 68927	. 04890	. 72204	. 05273	. 75355	. 05670	. 78390	. 06080	. 81316	. 06504	27
34	. 68983	. 04896	. 72257	. 05279	. 75407	. 05676	. 78440	. 06087	. 81364	. 06511	26
35	8. 69038	0. 04902	8. 72311	0. 05286	8. 75458	0. 05683	8. 78490	0. 06094	8. 81412	0. 06518	25
36	. 69094	. 04908	. 72364	. 05292	. 75510	. 05690	. 78539	. 06101	. 81460	. 06525	24
37	. 69149	. 04915	. 72418	. 05299	. 75561	. 05697	. 78589	. 06108	. 81508	. 06532	23
38	. 69205	. 04921	. 72471	. 05305	. 75613	. 05703	. 78638	. 06115	. 81555	. 06540	22
39	. 69260	. 04927	. 72525	. 05312	. 75664	. 05710	. 78688	. 06122	. 81603	. 06547	21
40	8. 69316	0. 04934	8. 72578	0. 05318	8. 75715	0. 05717	8. 78737	0. 06129	8. 81651	0. 06554	20
41	. 69371	. 04940	. 72631	. 05325	. 75767	. 05724	. 78787	. 06136	. 81699	. 06561	19
42	. 69427	. 04946	. 72684	. 05331	. 75818	. 05730	. 78836	. 06143	. 81746	. 06568	18
43	. 69482	. 04952	. 72738	. 05338	. 75869	. 05737	. 78885	. 06150	. 81794	. 06576	17
44	. 69537	. 04959	. 72791	. 05345	. 75920	. 05744	. 78935	. 06157	. 81841	. 06583	16
45	8. 69593	0. 04965	8. 72844	0. 05351	8. 75972	0. 05751	8. 78984	0. 06164	8. 81889	0. 06590	15
46	. 69648	. 04971	. 72897	. 05358	. 76023	. 05757	. 79033	. 06171	. 81937	. 06597	14
47	. 69703	. 04978	. 72950	. 05364	. 76074	. 05764	. 79082	. 06178	. 81984	. 06605	13
48	. 69758	. 04984	. 73003	. 05371	. 76125	. 05771	. 79132	. 06185	. 82032	. 06612	12
49	. 69814	. 04990	. 73056	. 05377	. 76176	. 05778	. 79181	. 06192	. 82079	. 06619	11
50	8. 69869	0. 04997	8. 73109	0. 05384	8. 76227	0. 05785	8. 79230	0. 06199	8. 82126	0. 06626	10
51	. 69924	. 05003	. 73162	. 05390	. 76278	. 05791	. 79279	. 06206	. 82174	. 06633	9
52	. 69979	. 05009	. 73215	. 05397	. 76329	. 05798	. 79328	. 06213	. 82221	. 06641	8
53	. 70034	. 05016	. 73268	. 05404	. 76380	. 05805	. 79377	. 06220	. 82269	. 06648	7
54	. 70089	. 05022	. 73321	. 05410	. 76431	. 05812	. 79426	. 06227	. 82316	. 06655	6
55	8. 70144	0. 05028	8. 73374	0. 05417	8. 76481	0. 05819	8. 79475	0. 06234	8. 82363	0. 06662	5
56	. 70198	. 05035	. 73426	. 05423	. 76532	. 05825	. 79524	. 06241	. 82410	. 06670	4
57	. 70253	. 05041	. 73479	. 05430	. 76583	. 05832	. 79573	. 06248	. 82458	. 06677	3
58	. 70308	. 05048	. 73532	. 05436	. 76634	. 05839	. 79622	. 06255	. 82505	. 06684	2
59	. 70363	. 05054	. 73584	. 05443	. 76684	. 05846	. 79671	. 06262	. 82552	. 06691	1
60	8. 70418	0. 05060	8. 73637	0. 05450	8. 76735	0. 05853	8. 79720	0. 06269	8. 82599	0. 06699	0
	334°		333°		332°		331°		330°		

TABLE 34

Haversines

′	30° Log Hav	30° Nat. Hav	31° Log Hav	31° Nat. Hav	32° Log Hav	32° Nat. Hav	33° Log Hav	33° Nat. Hav	34° Log Hav	34° Nat. Hav	
0	8. 82599	0. 06699	8. 85380	0. 07142	8. 88068	0. 07598	8. 90668	0. 08066	8. 93187	0. 08548	60
1	. 82646	. 06706	. 85425	. 07149	. 88112	. 07605	. 90711	. 08074	. 93228	. 08556	59
2	. 82694	. 06713	. 85471	. 07157	. 88156	. 07613	. 90754	. 08082	. 93270	. 08564	58
3	. 82741	. 06721	. 85516	. 07164	. 88200	. 07621	. 90796	. 08090	. 93311	. 08573	57
4	. 82788	. 06728	. 85562	. 07172	. 88244	. 07628	. 90839	. 08098	. 93352	. 08581	56
5	8. 82835	0. 06735	8. 85607	0. 07179	8. 88288	0. 07636	8. 90881	0. 08106	8. 93393	0. 08589	55
6	. 82882	. 06742	. 85653	. 07187	. 88332	. 07644	. 90924	. 08114	. 93435	. 08597	54
7	. 82929	. 06750	. 85698	. 07194	. 88375	. 07652	. 90966	. 08122	. 93476	. 08605	53
8	. 82976	. 06757	. 85743	. 07202	. 88419	. 07659	. 91009	. 08130	. 93517	. 08613	52
9	. 83023	. 06764	. 85789	. 07209	. 88463	. 07667	. 91051	. 08138	. 93558	. 08621	51
10	8. 83069	0. 06772	8. 85834	0. 07217	8. 88507	0. 07675	8. 91094	0. 08146	8. 93599	0. 08630	50
11	. 83116	. 06779	. 85879	. 07224	. 88551	. 07683	. 91136	. 08154	. 93640	. 08638	49
12	. 83163	. 06786	. 85925	. 07232	. 88595	. 07690	. 91179	. 08162	. 93681	. 08646	48
13	. 83210	. 06794	. 85970	. 07239	. 88638	. 07698	. 91221	. 08170	. 93722	. 08654	47
14	. 83257	. 06801	. 86015	. 07247	. 88682	. 07706	. 91263	. 08178	. 93764	. 08662	46
15	8. 83303	0. 06808	8. 86060	0. 07254	8. 88726	0. 07714	8. 91306	0. 08186	8. 93805	0. 08671	45
16	. 83350	. 06816	. 86105	. 07262	. 88769	. 07721	. 91348	. 08194	. 93846	. 08679	44
17	. 83397	. 06823	. 86151	. 07270	. 88813	. 07729	. 91390	. 08202	. 93886	. 08687	43
18	. 83444	. 06830	. 86196	. 07277	. 88857	. 07737	. 91432	. 08210	. 93927	. 08695	42
19	. 83490	. 06838	. 86241	. 07285	. 88900	. 07745	. 91475	. 08218	. 93968	. 08703	41
20	8. 83537	0. 06845	8. 86286	0. 07292	8. 88944	0. 07752	8. 91517	0. 08226	8. 94009	0. 08711	40
21	. 83583	. 06852	. 86331	. 07300	. 88988	. 07760	. 91559	. 08234	. 94050	. 08720	39
22	. 83630	. 06860	. 86376	. 07307	. 89031	. 07768	. 91601	. 08242	. 94091	. 08728	38
23	. 83676	. 06867	. 86421	. 07315	. 89075	. 07776	. 91643	. 08250	. 94132	. 08736	37
24	. 83723	. 06874	. 86466	. 07322	. 89118	. 07784	. 91685	. 08258	. 94173	. 08744	36
25	8. 83769	0. 06882	8. 86511	0. 07330	8. 89162	0. 07791	8. 91728	0. 08266	8. 94213	0. 08753	35
26	. 83816	. 06889	. 86556	. 07338	. 89205	. 07799	. 91770	. 08274	. 94254	. 08761	34
27	. 83862	. 06896	. 86600	. 07345	. 89248	. 07807	. 91812	. 08282	. 94295	. 08769	33
28	. 83909	. 06904	. 86645	. 07353	. 89292	. 07815	. 91854	. 08290	. 94336	. 08777	32
29	. 83955	. 06911	. 86690	. 07360	. 89335	. 07823	. 91896	. 08298	. 94376	. 08785	31
30	8. 84002	0. 06919	8. 86735	0. 07368	8. 89379	0. 07830	8. 91938	0. 08306	8. 94417	0. 08794	30
31	. 84048	. 06926	. 86780	. 07376	. 89422	. 07838	. 91980	. 08314	. 94458	. 08802	29
32	. 84094	. 06933	. 86825	. 07383	. 89465	. 07846	. 92022	. 08322	. 94498	. 08810	28
33	. 84140	. 06941	. 86869	. 07391	. 89509	. 07854	. 92064	. 08330	. 94539	. 08818	27
34	. 84187	. 06948	. 86914	. 07398	. 89552	. 07862	. 92105	. 08338	. 94580	. 08827	26
35	8. 84233	0. 06955	8. 86959	0. 07406	8. 89595	0. 07870	8. 92147	0. 08346	8. 94620	0. 08835	25
36	. 84279	. 06963	. 87003	. 07414	. 89638	. 07877	. 92189	. 08354	. 94661	. 08843	24
37	. 84325	. 06970	. 87048	. 07421	. 89681	. 07885	. 92231	. 08362	. 94701	. 08851	23
38	. 84371	. 06978	. 87093	. 07429	. 89725	. 07893	. 92273	. 08370	. 94742	. 08860	22
39	. 84417	. 06985	. 87137	. 07437	. 89768	. 07901	. 92315	. 08378	. 94782	. 08868	21
40	8. 84464	0. 06993	8. 87182	0. 07444	8. 89811	0. 07909	8. 92356	0. 08386	8. 94823	0. 08876	20
41	. 84510	. 07000	. 87226	. 07452	. 89854	. 07917	. 92398	. 08394	. 94863	. 08885	19
42	. 84556	. 07007	. 87271	. 07459	. 89897	. 07924	. 92440	. 08402	. 94904	. 08893	18
43	. 84602	. 07015	. 87315	. 07467	. 89940	. 07932	. 92482	. 08410	. 94944	. 08901	17
44	. 84648	. 07022	. 87360	. 07475	. 89983	. 07940	. 92523	. 08418	. 94985	. 08909	16
45	8. 84694	0. 07030	8. 87404	0. 07482	8. 90026	0. 07948	8. 92565	0. 08427	8. 95025	0. 08918	15
46	. 84740	. 07037	. 87448	. 07490	. 90069	. 07956	. 92607	. 08435	. 95065	. 08926	14
47	. 84785	. 07045	. 87493	. 07498	. 90112	. 07964	. 92648	. 08443	. 95106	. 08934	13
48	. 84831	. 07052	. 87537	. 07505	. 90155	. 07972	. 92690	. 08451	. 95146	. 08943	12
49	. 84877	. 07059	. 87582	. 07513	. 90198	. 07980	. 92731	. 08459	. 95186	. 08951	11
50	8. 84923	0. 07067	8. 87626	0. 07521	8. 90241	0. 07987	8. 92773	0. 08467	8. 95227	0. 08959	10
51	. 84969	. 07074	. 87670	. 07528	. 90284	. 07995	. 92814	. 08475	. 95267	. 08967	9
52	. 85015	. 07082	. 87714	. 07536	. 90326	. 08003	. 92856	. 08483	. 95307	. 08976	8
53	. 85060	. 07089	. 87759	. 07544	. 90369	. 08011	. 92897	. 08491	. 95347	. 08984	7
54	. 85106	. 07097	. 87803	. 07551	. 90412	. 08019	. 92939	. 08499	. 95388	. 08992	6
55	8. 85152	0. 07104	8. 87847	0. 07559	8. 90455	0. 08027	8. 92980	0. 08507	8. 95428	0. 09001	5
56	. 85197	. 07112	. 87891	. 07567	. 90498	. 08035	. 93022	. 08516	. 95468	. 09009	4
57	. 85243	. 07119	. 87935	. 07574	. 90540	. 08043	. 93063	. 08524	. 95508	. 09017	3
58	. 85289	. 07127	. 87980	. 07582	. 90583	. 08051	. 93104	. 08532	. 95548	. 09026	2
59	. 85334	. 07134	. 88024	. 07590	. 90626	. 08059	. 93146	. 08540	. 95588	. 09034	1
60	8. 85380	0. 07142	8. 88068	0. 07598	8. 90668	0. 08066	8. 93187	0. 08548	8. 95628	0. 09042	0
	329°		328°		327°		326°		325°		

TABLE 34

Haversines

′	35° Log Hav	35° Nat. Hav	36° Log Hav	36° Nat. Hav	37° Log Hav	37° Nat. Hav	38° Log Hav	38° Nat. Hav	39° Log Hav	39° Nat. Hav	′
0	8.95628	0.09042	8.97997	0.09549	9.00295	0.10068	9.02528	0.10599	9.04699	0.11143	60
1	.95668	.09051	.98035	.09558	.00333	.10077	.02565	.10608	.04735	.11152	59
2	.95709	.09059	.98074	.09566	.00371	.10086	.02602	.10617	.04770	.11161	58
3	.95749	.09067	.98113	.09575	.00408	.10094	.02638	.10626	.04806	.11170	57
4	.95789	.09076	.98152	.09583	.00446	.10103	.02675	.10635	.04842	.11179	56
5	8.95828	0.09084	8.98191	0.09592	9.00484	0.10112	9.02712	0.10644	9.04877	0.11189	55
6	.95868	.09093	.98229	.09601	.00522	.10121	.02748	.10653	.04913	.11198	54
7	.95908	.09101	.98268	.09609	.00559	.10130	.02785	.10662	.04948	.11207	53
8	.95948	.09109	.98307	.09618	.00597	.10138	.02821	.10671	.04984	.11216	52
9	.95988	.09118	.98346	.09626	.00634	.10147	.02858	.10680	.05019	.11225	51
10	8.96028	0.09126	8.98384	0.09635	9.00672	0.10156	9.02894	0.10689	9.05055	0.11234	50
11	.96068	.09134	.98423	.09643	.00710	.10165	.02931	.10698	.05090	.11244	49
12	.96108	.09143	.98462	.09652	.00747	.10174	.02967	.10707	.05126	.11253	48
13	.96148	.09151	.98500	.09661	.00785	.10182	.03004	.10716	.05161	.11262	47
14	.96187	.09160	.98539	.09669	.00822	.10191	.03040	.10725	.05197	.11271	46
15	8.96227	0.09168	8.98578	0.09678	9.00860	0.10200	9.03077	0.10734	9.05232	0.11280	45
16	.96267	.09176	.98616	.09686	.00897	.10209	.03113	.10743	.05268	.11290	44
17	.96307	.09185	.98655	.09695	.00935	.10218	.03150	.10752	.05303	.11299	43
18	.96346	.09193	.98693	.09704	.00972	.10226	.03186	.10761	.05339	.11308	42
19	.96386	.09202	.98732	.09712	.01009	.10235	.03222	.10770	.05374	.11317	41
20	8.96426	0.09210	8.98770	0.09721	9.01047	0.10244	9.03259	0.10779	9.05409	0.11326	40
21	.96465	.09218	.98809	.09729	.01084	.10253	.03295	.10788	.05445	.11336	39
22	.96505	.09227	.98847	.09738	.01122	.10262	.03331	.10797	.05480	.11345	38
23	.96545	.09235	.98886	.09747	.01159	.10270	.03368	.10806	.05515	.11354	37
24	.96584	.09244	.98924	.09755	.01196	.10279	.03404	.10815	.05551	.11363	36
25	8.96624	0.09252	8.98963	0.09764	9.01234	0.10288	9.03440	0.10824	9.05586	0.11373	35
26	.96663	.09260	.99001	.09773	.01271	.10297	.03476	.10833	.05621	.11382	34
27	.96703	.09269	.99039	.09781	.01308	.10306	.03513	.10842	.05656	.11391	33
28	.96742	.09277	.99078	.09790	.01345	.10315	.03549	.10851	.05692	.11400	32
29	.96782	.09286	.99116	.09799	.01383	.10323	.03585	.10861	.05727	.11410	31
30	8.96821	0.09294	8.99154	0.09807	9.01420	0.10332	9.03621	0.10870	9.05762	0.11419	30
31	.96861	.09303	.99193	.09816	.01457	.10341	.03657	.10879	.05797	.11428	29
32	.96900	.09311	.99231	.09824	.01494	.10350	.03694	.10888	.05832	.11437	28
33	.96940	.09320	.99269	.09833	.01531	.10359	.03730	.10897	.05867	.11447	27
34	.96979	.09328	.99307	.09842	.01569	.10368	.03766	.10906	.05903	.11456	26
35	8.97018	0.09336	8.99346	0.09850	9.01606	0.10377	9.03802	0.10915	9.05938	0.11465	25
36	.97058	.09345	.99384	.09859	.01643	.10386	.03838	.10924	.05973	.11474	24
37	.97097	.09353	.99422	.09868	.01680	.10394	.03874	.10933	.06008	.11484	23
38	.97136	.09362	.99460	.09876	.01717	.10403	.03910	.10942	.06043	.11493	22
39	.97176	.09370	.99498	.09885	.01754	.10412	.03946	.10951	.06078	.11502	21
40	8.97215	0.09379	8.99536	0.09894	9.01791	0.10421	9.03982	0.10960	9.06113	0.11511	20
41	.97254	.09387	.99575	.09903	.01828	.10430	.04018	.10969	.06148	.11521	19
42	.97294	.09396	.99613	.09911	.01865	.10439	.04054	.10978	.06183	.11530	18
43	.97333	.09404	.99651	.09920	.01902	.10448	.04090	.10988	.06218	.11539	17
44	.97372	.09413	.99689	.09929	.01939	.10457	.04126	.10997	.06253	.11549	16
45	8.97411	0.09421	8.99727	0.09937	9.01976	0.10466	9.04162	0.11006	9.06288	0.11558	15
46	.97450	.09430	.99765	.09946	.02013	.10474	.04198	.11015	.06323	.11567	14
47	.97489	.09438	.99803	.09955	.02050	.10483	.04234	.11024	.06358	.11577	13
48	.97529	.09447	.99841	.09963	.02087	.10492	.04270	.11033	.06393	.11586	12
49	.97568	.09455	.99879	.09972	.02124	.10501	.04306	.11042	.06428	.11595	11
50	8.97607	0.09464	8.99917	0.09981	9.02161	0.10510	9.04341	0.11051	9.06462	0.11604	10
51	.97646	.09472	.99955	.09990	.02197	.10519	.04377	.11060	.06497	.11614	9
52	.97685	.09481	8.99993	.09998	.02234	.10528	.04413	.11070	.06532	.11623	8
53	.97724	.09489	9.00031	.10007	.02271	.10537	.04449	.11079	.06567	.11632	7
54	.97763	.09498	.00068	.10016	.02308	.10546	.04485	.11088	.06602	.11642	6
55	8.97802	0.09506	9.00106	0.10025	9.02345	0.10555	9.04520	0.11097	9.06637	0.11651	5
56	.97841	.09515	.00144	.10033	.02381	.10564	.04556	.11106	.06671	.11660	4
57	.97880	.09524	.00182	.10042	.02418	.10573	.04592	.11115	.06706	.11670	3
58	.97919	.09532	.00220	.10051	.02455	.10582	.04628	.11124	.06741	.11679	2
59	.97958	.09541	.00258	.10059	.02492	.10591	.04663	.11134	.06776	.11688	1
60	8.97997	0.09549	9.00295	0.10068	9.02528	0.10599	9.04699	0.11143	9.06810	0.11698	0
	324°		323°		322°		321°		320°		

TABLE 34

Haversines

′	40° Log Hav	40° Nat. Hav	41° Log Hav	41° Nat. Hav	42° Log Hav	42° Nat. Hav	43° Log Hav	43° Nat. Hav	44° Log Hav	44° Nat. Hav	′
0	9. 06810	0. 11698	9. 08865	0. 12265	9. 10866	0. 12843	9. 12815	0. 13432	9. 14715	0. 14033	60
1	. 06845	. 11707	. 08899	. 12274	. 10899	. 12852	. 12847	. 13442	. 14746	. 14043	59
2	. 06880	. 11716	. 08933	. 12284	. 10932	. 12862	. 12879	. 13452	. 14778	. 14053	58
3	. 06914	. 11726	. 08966	. 12293	. 10965	. 12872	. 12911	. 13462	. 14809	. 14063	57
4	. 06949	. 11735	. 09000	. 12303	. 10997	. 12882	. 12943	. 13472	. 14840	. 14073	56
5	9. 06984	0. 11745	9. 09034	0. 12312	9. 11030	0. 12891	9. 12975	0. 13482	9. 14871	0. 14084	55
6	. 07018	. 11754	. 09068	. 12322	. 11063	. 12901	. 13007	. 13492	. 14902	. 14094	54
7	. 07053	. 11763	. 09101	. 12331	. 11096	. 12911	. 13039	. 13502	. 14934	. 14104	53
8	. 07088	. 11773	. 09135	. 12341	. 11129	. 12921	. 13071	. 13512	. 14965	. 14114	52
9	. 07122	. 11782	. 09169	. 12351	. 11161	. 12930	. 13103	. 13522	. 14996	. 14124	51
10	9. 07157	0. 11791	9. 09202	0. 12360	9. 11194	0. 12940	9. 13135	0. 13532	9. 15027	0. 14134	50
11	. 07191	. 11801	. 09236	. 12370	. 11227	. 12950	. 13167	. 13542	. 15058	. 14144	49
12	. 07226	. 11810	. 09269	. 12379	. 11260	. 12960	. 13199	. 13552	. 15089	. 14154	48
13	. 07260	. 11820	. 09303	. 12389	. 11292	. 12970	. 13231	. 13562	. 15120	. 14165	47
14	. 07295	. 11829	. 09337	. 12398	. 11325	. 12979	. 13263	. 13571	. 15152	. 14175	46
15	9. 07329	0. 11838	9. 09370	0. 12408	9. 11358	0. 12989	9. 13295	0. 13581	9. 15183	0. 14185	45
16	. 07364	. 11848	. 09404	. 12418	. 11391	. 12999	. 13326	. 13591	. 15214	. 14195	44
17	. 07398	. 11857	. 09437	. 12427	. 11423	. 13009	. 13358	. 13601	. 15245	. 14205	43
18	. 07433	. 11867	. 09471	. 12437	. 11456	. 13018	. 13390	. 13611	. 15276	. 14215	42
19	. 07467	. 11876	. 09504	. 12446	. 11489	. 13028	. 13422	. 13621	. 15307	. 14226	41
20	9. 07501	0. 11885	9. 09538	0. 12456	9. 11521	0. 13038	9. 13454	0. 13631	9. 15338	0. 14236	40
21	. 07536	. 11895	. 09571	. 12466	. 11554	. 13048	. 13486	. 13641	. 15369	. 14246	39
22	. 07570	. 11904	. 09605	. 12475	. 11586	. 13058	. 13517	. 13651	. 15400	. 14256	38
23	. 07605	. 11914	. 09638	. 12485	. 11619	. 13067	. 13549	. 13661	. 15431	. 14266	37
24	. 07639	. 11923	. 09672	. 12494	. 11652	. 13077	. 13581	. 13671	. 15462	. 14276	36
25	9. 07673	0. 11933	9. 09705	0. 12504	9. 11684	0. 13087	9. 13613	0. 13681	9. 15493	0. 14287	35
26	. 07708	. 11942	. 09739	. 12514	. 11717	. 13097	. 13644	. 13691	. 15524	. 14297	34
27	. 07742	. 11951	. 09772	. 12523	. 11749	. 13107	. 13676	. 13701	. 15555	. 14307	33
28	. 07776	. 11961	. 09805	. 12533	. 11782	. 13116	. 13708	. 13711	. 15585	. 14317	32
29	. 07810	. 11970	. 09839	. 12543	. 11814	. 13126	. 13739	. 13721	. 15616	. 14327	31
30	9. 07845	0. 11980	9. 09872	0. 12552	9. 11847	0. 13136	9. 13771	0. 13731	9. 15647	0. 14337	30
31	. 07879	. 11989	. 09905	. 12562	. 11879	. 13146	. 13803	. 13741	. 15678	. 14348	29
32	. 07913	. 11999	. 09939	. 12571	. 11912	. 13156	. 13834	. 13751	. 15709	. 14358	28
33	. 07947	. 12008	. 09972	. 12581	. 11944	. 13166	. 13866	. 13761	. 15740	. 14368	27
34	. 07981	. 12018	. 10005	. 12591	. 11977	. 13175	. 13898	. 13771	. 15771	. 14378	26
35	9. 08016	0. 12027	9. 10039	0. 12600	9. 12009	0. 13185	9. 13929	0. 13781	9. 15802	0. 14388	25
36	. 08050	. 12036	. 10072	. 12610	. 12041	. 13195	. 13961	. 13791	. 15832	. 14399	24
37	. 08084	. 12046	. 10105	. 12620	. 12074	. 13205	. 13992	. 13801	. 15863	. 14409	23
38	. 08118	. 12055	. 10138	. 12629	. 12106	. 13215	. 14024	. 13811	. 15894	. 14419	22
39	. 08152	. 12065	. 10172	. 12639	. 12139	. 13225	. 14056	. 13822	. 15925	. 14429	21
40	9. 08186	0. 12074	9. 10205	0. 12649	9. 12171	0. 13235	9. 14087	0. 13832	9. 15955	0. 14440	20
41	. 08220	. 12084	. 10238	. 12658	. 12203	. 13244	. 14119	. 13842	. 15986	. 14450	19
42	. 08254	. 12093	. 10271	. 12668	. 12236	. 13254	. 14150	. 13852	. 16017	. 14460	18
43	. 08288	. 12103	. 10304	. 12678	. 12268	. 13264	. 14182	. 13862	. 16048	. 14470	17
44	. 08323	. 12112	. 10337	. 12687	. 12300	. 13274	. 14213	. 13872	. 16078	. 14480	16
45	9. 08357	0. 12122	9. 10371	0. 12697	9. 12332	0. 13284	9. 14245	0. 13882	9. 16109	0. 14491	15
46	. 08391	. 12131	. 10404	. 12707	. 12365	. 13294	. 14276	. 13892	. 16140	. 14501	14
47	. 08425	. 12141	. 10437	. 12717	. 12397	. 13304	. 14307	. 13902	. 16170	. 14511	13
48	. 08459	. 12150	. 10470	. 12726	. 12429	. 13314	. 14339	. 13912	. 16201	. 14521	12
49	. 08492	. 12160	. 10503	. 12736	. 12461	. 13323	. 14370	. 13922	. 16232	. 14532	11
50	9. 08526	0. 12169	9. 10536	0. 12746	9. 12494	0. 13333	9. 14402	0. 13932	9. 16262	0. 14542	10
51	. 08560	. 12179	. 10569	. 12755	. 12526	. 13343	. 14433	. 13942	. 16293	. 14552	9
52	. 08594	. 12188	. 10602	. 12765	. 12558	. 13353	. 14465	. 13952	. 16324	. 14562	8
53	. 08628	. 12198	. 10635	. 12775	. 12590	. 13363	. 14496	. 13962	. 16354	. 14573	7
54	. 08662	. 12207	. 10668	. 12784	. 12622	. 13373	. 14527	. 13972	. 16385	. 14583	6
55	9. 08696	0. 12217	9. 10701	0. 12794	9. 12655	0. 13383	9. 14559	0. 13983	9. 16415	0. 14593	5
56	. 08730	. 12226	. 10734	. 12804	. 12687	. 13393	. 14590	. 13993	. 16446	. 14604	4
57	. 08764	. 12236	. 10767	. 12814	. 12719	. 13403	. 14621	. 14003	. 16476	. 14614	3
58	. 08797	. 12245	. 10800	. 12823	. 12751	. 13412	. 14653	. 14013	. 16507	. 14624	2
59	. 08831	. 12255	. 10833	. 12833	. 12783	. 13422	. 14684	. 14023	. 16537	. 14634	1
60	9. 08865	0. 12265	9. 10866	0. 12843	9. 12815	0. 13432	9. 14715	0. 14033	9. 16568	0. 14645	0
	319°		318°		317°		316°		315°		

TABLE 34
Haversines

′	45°		46°		47°		48°		49°		′
	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	
0	9. 16568	0. 14645	9. 18376	0. 15267	9. 20140	0. 15900	9. 21863	0. 16543	9. 23545	0. 17197	60
1	. 16598	. 14655	. 18405	. 15278	. 20169	. 15911	. 21891	. 16554	. 23573	. 17208	59
2	. 16629	. 14665	. 18435	. 15288	. 20198	. 15921	. 21919	. 16565	. 23601	. 17219	58
3	. 16659	. 14676	. 18465	. 15298	. 20227	. 15932	. 21948	. 16576	. 23629	. 17230	57
4	. 16690	. 14686	. 18495	. 15309	. 20256	. 15943	. 21976	. 16587	. 23656	. 17241	56
5	9. 16720	0. 14696	9. 18524	0. 15319	9. 20285	0. 15953	9. 22004	0. 16598	9. 23684	0. 17252	55
6	. 16751	. 14706	. 18554	. 15330	. 20314	. 15964	. 22033	. 16608	. 23712	. 17263	54
7	. 16781	. 14717	. 18584	. 15340	. 20343	. 15975	. 22061	. 16619	. 23739	. 17274	53
8	. 16812	. 14727	. 18613	. 15351	. 20372	. 15985	. 22089	. 16630	. 23767	. 17285	52
9	. 16842	. 14737	. 18643	. 15361	. 20401	. 15996	. 22118	. 16641	. 23794	. 17296	51
10	9. 16872	0. 14748	9. 18673	0. 15372	9. 20430	0. 16007	9. 22146	0. 16652	9. 23822	0. 17307	50
11	. 16903	. 14758	. 18702	. 15382	. 20459	. 16017	. 22174	. 16663	. 23850	. 17318	49
12	. 16933	. 14768	. 18732	. 15393	. 20488	. 16028	. 22202	. 16673	. 23877	. 17329	48
13	. 16963	. 14779	. 18762	. 15403	. 20517	. 16039	. 22231	. 16684	. 23905	. 17340	47
14	. 16994	. 14789	. 18791	. 15414	. 20546	. 16049	. 22259	. 16695	. 23932	. 17351	46
15	9. 17024	0. 14799	9. 18821	0. 15424	9. 20574	0. 16060	9. 22287	0. 16706	9. 23960	0. 17362	45
16	. 17054	. 14810	. 18850	. 15435	. 20603	. 16071	. 22315	. 16717	. 23988	. 17373	44
17	. 17085	. 14820	. 18880	. 15445	. 20632	. 16081	. 22343	. 16728	. 24015	. 17384	43
18	. 17115	. 14830	. 18909	. 15456	. 20661	. 16092	. 22372	. 16738	. 24043	. 17395	42
19	. 17145	. 14841	. 18939	. 15466	. 20690	. 16103	. 22400	. 16749	. 24070	. 17406	41
20	9. 17175	0. 14851	9. 18968	0. 15477	9. 20719	0. 16113	9. 22428	0. 16760	9. 24098	0. 17417	40
21	. 17206	. 14861	. 18998	. 15487	. 20748	. 16124	. 22456	. 16771	. 24125	. 17428	39
22	. 17236	. 14872	. 19027	. 15498	. 20776	. 16135	. 22484	. 16782	. 24153	. 17439	38
23	. 17266	. 14882	. 19057	. 15508	. 20805	. 16145	. 22512	. 16793	. 24180	. 17450	37
24	. 17296	. 14892	. 19086	. 15519	. 20834	. 16156	. 22540	. 16804	. 24208	. 17461	36
25	9. 17327	0. 14903	9. 19116	0. 15530	9. 20863	0. 16167	9. 22569	0. 16815	9. 24235	0. 17472	35
26	. 17357	. 14913	. 19145	. 15540	. 20891	. 16178	. 22597	. 16825	. 24263	. 17483	34
27	. 17387	. 14923	. 19175	. 15551	. 20920	. 16188	. 22625	. 16836	. 24290	. 17494	33
28	. 17417	. 14934	. 19204	. 15561	. 20949	. 16199	. 22653	. 16847	. 24317	. 17505	32
29	. 17447	. 14944	. 19234	. 15572	. 20978	. 16210	. 22681	. 16858	. 24345	. 17517	31
30	9. 17477	0. 14955	9. 19263	0. 15582	9. 21006	0. 16220	9. 22709	0. 16869	9. 24372	0. 17528	30
31	. 17507	. 14965	. 19292	. 15593	. 21035	. 16231	. 22737	. 16880	. 24400	. 17539	29
32	. 17538	. 14975	. 19322	. 15603	. 21064	. 16242	. 22765	. 16891	. 24427	. 17550	28
33	. 17568	. 14986	. 19351	. 15614	. 21092	. 16253	. 22793	. 16902	. 24454	. 17561	27
34	. 17598	. 14996	. 19381	. 15624	. 21121	. 16263	. 22821	. 16913	. 24482	. 17572	26
35	9. 17628	0. 15006	9. 19410	0. 15635	9. 21150	0. 16274	9. 22849	0. 16923	9. 24509	0. 17583	25
36	. 17658	. 15017	. 19439	. 15646	. 21178	. 16285	. 22877	. 16934	. 24536	. 17594	24
37	. 17688	. 15027	. 19469	. 15656	. 21207	. 16296	. 22905	. 16945	. 24564	. 17605	23
38	. 17718	. 15038	. 19498	. 15667	. 21236	. 16306	. 22933	. 16956	. 24591	. 17616	22
39	. 17748	. 15048	. 19527	. 15677	. 21264	. 16317	. 22961	. 16967	. 24618	. 17627	21
40	9. 17778	0. 15058	9. 19557	0. 15688	9. 21293	0. 16328	9. 22989	0. 16978	9. 24646	0. 17638	20
41	. 17808	. 15069	. 19586	. 15698	. 21322	. 16339	. 23017	. 16989	. 24673	. 17649	19
42	. 17838	. 15079	. 19615	. 15709	. 21350	. 16349	. 23045	. 17000	. 24700	. 17661	18
43	. 17868	. 15090	. 19644	. 15720	. 21379	. 16360	. 23073	. 17011	. 24728	. 17672	17
44	. 17898	. 15100	. 19674	. 15730	. 21407	. 16371	. 23100	. 17022	. 24755	. 17683	16
45	9. 17928	0. 15110	9. 19703	0. 15741	9. 21436	0. 16382	9. 23128	0. 17033	9. 24782	0. 17694	15
46	. 17958	. 15121	. 19732	. 15751	. 21464	. 16392	. 23156	. 17044	. 24809	. 17705	14
47	. 17988	. 15131	. 19761	. 15762	. 21493	. 16403	. 23184	. 17055	. 24837	. 17716	13
48	. 18018	. 15142	. 19790	. 15773	. 21521	. 16414	. 23212	. 17066	. 24864	. 17727	12
49	. 18048	. 15152	. 19820	. 15783	. 21550	. 16425	. 23240	. 17076	. 24891	. 17738	11
50	9. 18077	0. 15163	9. 19849	0. 15794	9. 21578	0. 16436	9. 23268	0. 17087	9. 24918	0. 17749	10
51	. 18107	. 15173	. 19878	. 15804	. 21607	. 16446	. 23295	. 17098	. 24945	. 17760	9
52	. 18137	. 15183	. 19907	. 15815	. 21635	. 16457	. 23323	. 17109	. 24973	. 17772	8
53	. 18167	. 15194	. 19936	. 15826	. 21664	. 16468	. 23351	. 17120	. 25000	. 17783	7
54	. 18197	. 15204	. 19965	. 15836	. 21692	. 16479	. 23379	. 17131	. 25027	. 17794	6
55	9. 18227	0. 15215	9. 19995	0. 15847	9. 21721	0. 16489	9. 23407	0. 17142	9. 25054	0. 17805	5
56	. 18256	. 15225	. 20024	. 15858	. 21749	. 16500	. 23434	. 17153	. 25081	. 17816	4
57	. 18286	. 15236	. 20053	. 15868	. 21778	. 16511	. 23462	. 17164	. 25108	. 17827	3
58	. 18316	. 15246	. 20082	. 15879	. 21806	. 16522	. 23490	. 17175	. 25135	. 17838	2
59	. 18346	. 15257	. 20111	. 15889	. 21834	. 16533	. 23518	. 17186	. 25163	. 17849	1
60	9. 18376	0. 15267	9. 20140	0. 15900	9. 21863	0. 16543	9. 23545	0. 17197	9. 25190	0. 17861	0
	314°		313°		312°		311°		310°		

TABLE 34

Haversines

′	50°		51°		52°		53°		54°		′
	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	
0	9. 25190	0. 17861	9. 26797	0. 18534	9. 28368	0. 19217	9. 29906	0. 19909	9. 31409	0. 20611	60
1	. 25217	. 17872	. 26823	. 18545	. 28394	. 19228	. 29931	. 19921	. 31434	. 20623	59
2	. 25244	. 17883	. 26850	. 18557	. 28420	. 19240	. 29956	. 19932	. 31459	. 20634	58
3	. 25271	. 17894	. 26876	. 18568	. 28446	. 19251	. 29981	. 19944	. 31484	. 20646	57
4	. 25298	. 17905	. 26903	. 18579	. 28472	. 19263	. 30007	. 19956	. 31508	. 20658	56
5	9. 25325	0. 17916	9. 26929	0. 18591	9. 28498	0. 19274	9. 30032	0. 19967	9. 31533	0. 20670	55
6	. 25352	. 17928	. 26956	. 18602	. 28524	. 19286	. 30057	. 19979	. 31558	. 20681	54
7	. 25379	. 17939	. 26982	. 18613	. 28549	. 19297	. 30083	. 19991	. 31583	. 20693	53
8	. 25406	. 17950	. 27008	. 18624	. 28575	. 19309	. 30108	. 20002	. 31607	. 20705	52
9	. 25433	. 17961	. 27035	. 18636	. 28601	. 19320	. 30133	. 20014	. 31632	. 20717	51
10	9. 25460	0. 17972	9. 27061	0. 18647	9. 28627	0. 19332	9. 30158	0. 20026	9. 31657	0. 20729	50
11	. 25487	. 17983	. 27088	. 18658	. 28653	. 19343	. 30184	. 20037	. 31682	. 20740	49
12	. 25514	. 17995	. 27114	. 18670	. 28679	. 19355	. 30209	. 20049	. 31706	. 20752	48
13	. 25541	. 18006	. 27140	. 18681	. 28704	. 19366	. 30234	. 20060	. 31731	. 20764	47
14	. 25568	. 18017	. 27167	. 18692	. 28730	. 19378	. 30259	. 20072	. 31756	. 20776	46
15	9. 25595	0. 18028	9. 27193	0. 18704	9. 28756	0. 19389	9. 30285	0. 20084	9. 31780	0. 20788	45
16	. 25622	. 18039	. 27219	. 18715	. 28782	. 19401	. 30310	. 20095	. 31805	. 20799	44
17	. 25649	. 18050	. 27246	. 18727	. 28807	. 19412	. 30335	. 20107	. 31830	. 20811	43
18	. 25676	. 18062	. 27272	. 18738	. 28833	. 19424	. 30360	. 20119	. 31854	. 20823	42
19	. 25703	. 18073	. 27298	. 18749	. 28859	. 19435	. 30385	. 20130	. 31879	. 20835	41
20	9. 25729	0. 18084	9. 27325	0. 18761	9. 28885	0. 19447	9. 30410	0. 20142	9. 31903	0. 20847	40
21	. 25756	. 18095	. 27351	. 18772	. 28910	. 19458	. 30436	. 20154	. 31928	. 20858	39
22	. 25783	. 18106	. 27377	. 18783	. 28936	. 19470	. 30461	. 20165	. 31953	. 20870	38
23	. 25810	. 18118	. 27403	. 18795	. 28962	. 19481	. 30486	. 20177	. 31977	. 20882	37
24	. 25837	. 18129	. 27430	. 18806	. 28987	. 19493	. 30511	. 20189	. 32002	. 20894	36
25	9. 25864	0. 18140	9. 27456	0. 18817	9. 29013	0. 19504	9. 30536	0. 20200	9. 32026	0. 20906	35
26	. 25891	. 18151	. 27482	. 18829	. 29039	. 19516	. 30561	. 20212	. 32051	. 20918	34
27	. 25917	. 18162	. 27508	. 18840	. 29064	. 19527	. 30586	. 20224	. 32076	. 20929	33
28	. 25944	. 18174	. 27535	. 18852	. 29090	. 19539	. 30611	. 20235	. 32100	. 20941	32
29	. 25971	. 18185	. 27561	. 18863	. 29116	. 19550	. 30636	. 20247	. 32125	. 20953	31
30	9. 25998	0. 18196	9. 27587	0. 18874	9. 29141	0. 19562	9. 30662	0. 20259	9. 32149	0. 20965	30
31	. 26025	. 18207	. 27613	. 18886	. 29167	. 19573	. 30687	. 20271	. 32174	. 20977	29
32	. 26051	. 18219	. 27639	. 18897	. 29192	. 19585	. 30712	. 20282	. 32198	. 20989	28
33	. 26078	. 18230	. 27666	. 18908	. 29218	. 19597	. 30737	. 20294	. 32223	. 21000	27
34	. 26105	. 18241	. 27692	. 18920	. 29244	. 19608	. 30762	. 20306	. 32247	. 21012	26
35	9. 26132	0. 18252	9. 27718	0. 18931	9. 29269	0. 19620	9. 30787	0. 20317	9. 32272	0. 21024	25
36	. 26158	. 18263	. 27744	. 18943	. 29295	. 19631	. 30812	. 20329	. 32296	. 21036	24
37	. 26185	. 18275	. 27770	. 18954	. 29320	. 19643	. 30837	. 20341	. 32321	. 21048	23
38	. 26212	. 18286	. 27796	. 18965	. 29346	. 19654	. 30862	. 20352	. 32345	. 21060	22
39	. 26238	. 18297	. 27822	. 18977	. 29371	. 19666	. 30887	. 20364	. 32370	. 21072	21
40	9. 26265	0. 18308	9. 27848	0. 18988	9. 29397	0. 19677	9. 30912	0. 20376	9. 32394	0. 21083	20
41	. 26292	. 18320	. 27875	. 19000	. 29422	. 19689	. 30937	. 20388	. 32418	. 21095	19
42	. 26319	. 18331	. 27901	. 19011	. 29448	. 19701	. 30962	. 20399	. 32443	. 21107	18
43	. 26345	. 18342	. 27927	. 19022	. 29473	. 19712	. 30987	. 20411	. 32467	. 21119	17
44	. 26372	. 18353	. 27953	. 19034	. 29499	. 19724	. 31012	. 20423	. 32492	. 21131	16
45	9. 26398	0. 18365	9. 27979	0. 19045	9. 29524	0. 19735	9. 31036	0. 20435	9. 32516	0. 21143	15
46	. 26425	. 18376	. 28005	. 19057	. 29550	. 19747	. 31061	. 20446	. 32541	. 21155	14
47	. 26452	. 18387	. 28031	. 19068	. 29575	. 19758	. 31086	. 20458	. 32565	. 21167	13
48	. 26478	. 18399	. 28057	. 19080	. 29601	. 19770	. 31111	. 20470	. 32589	. 21178	12
49	. 26505	. 18410	. 28083	. 19091	. 29626	. 19782	. 31136	. 20481	. 32614	. 21190	11
50	9. 26532	0. 18421	9. 28109	0. 19102	9. 29652	0. 19793	9. 31161	0. 20493	9. 32638	0. 21202	10
51	. 26558	. 18432	. 28135	. 19114	. 29677	. 19805	. 31186	. 20505	. 32662	. 21214	9
52	. 26585	. 18444	. 28161	. 19125	. 29703	. 19816	. 31211	. 20517	. 32687	. 21226	8
53	. 26611	. 18455	. 28187	. 19137	. 29728	. 19828	. 31236	. 20528	. 32711	. 21238	7
54	. 26638	. 18466	. 28213	. 19148	. 29753	. 19840	. 31260	. 20540	. 32735	. 21250	6
55	9. 26664	0. 18477	9. 28239	0. 19160	9. 29779	0. 19851	9. 31285	0. 20552	9. 32760	0. 21262	5
56	. 26691	. 18489	. 28265	. 19171	. 29804	. 19863	. 31310	. 20564	. 32784	. 21274	4
57	. 26717	. 18500	. 28291	. 19183	. 29829	. 19874	. 31335	. 20575	. 32808	. 21285	3
58	. 26744	. 18511	. 28317	. 19194	. 29855	. 19886	. 31360	. 20587	. 32833	. 21297	2
59	. 26770	. 18523	. 28342	. 19205	. 29880	. 19898	. 31385	. 20599	. 32857	. 21309	1
60	9. 26797	0. 18534	9. 28368	0. 19217	9. 29906	0. 19909	9. 31409	0. 20611	9. 32881	0. 21321	0
	309°		308°		307°		306°		305°		

TABLE 34

Haversines

′	55° Log Hav	55° Nat. Hav	56° Log Hav	56° Nat. Hav	57° Log Hav	57° Nat. Hav	58° Log Hav	58° Nat. Hav	59° Log Hav	59° Nat. Hav	′
0	9. 32881	0. 21321	9. 34322	0. 22040	9. 35733	0. 22768	9. 37114	0. 23504	9. 38468	0. 24248	60
1	. 32905	. 21333	. 34346	. 22052	. 35756	. 22780	. 37137	. 23516	. 38490	. 24261	59
2	. 32930	. 21345	. 34369	. 22064	. 35779	. 22792	. 37160	. 23529	. 38512	. 24273	58
3	. 32954	. 21357	. 34393	. 22077	. 35802	. 22805	. 37183	. 23541	. 38535	. 24286	57
4	. 32978	. 21369	. 34417	. 22089	. 35826	. 22817	. 37205	. 23553	. 38557	. 24298	56
5	9. 33002	0. 21381	9. 34441	0. 22101	9. 35849	0. 22829	9. 37228	0. 23566	9. 38579	0. 24310	55
6	. 33027	. 21393	. 34464	. 22113	. 35872	. 22841	. 37251	. 23578	. 38602	. 24323	54
7	. 33051	. 21405	. 34488	. 22125	. 35895	. 22853	. 37274	. 23590	. 38624	. 24335	53
8	. 33075	. 21417	. 34512	. 22137	. 35918	. 22866	. 37296	. 23603	. 38646	. 24348	52
9	. 33099	. 21429	. 34535	. 22149	. 35942	. 22878	. 37319	. 23615	. 38668	. 24360	51
10	9. 33123	0. 21440	9. 34559	0. 22161	9. 35965	0. 22890	9. 37342	0. 23627	9. 38691	0. 24373	50
11	. 33148	. 21452	. 34583	. 22173	. 35988	. 22902	. 37364	. 23640	. 38713	. 24385	49
12	. 33172	. 21464	. 34606	. 22185	. 36011	. 22915	. 37387	. 23652	. 38735	. 24398	48
13	. 33196	. 21476	. 34630	. 22197	. 36034	. 22927	. 37410	. 23665	. 38757	. 24410	47
14	. 33220	. 21488	. 34654	. 22209	. 36058	. 22939	. 37433	. 23677	. 38780	. 24423	46
15	9. 33244	0. 21500	9. 34677	0. 22221	9. 36081	0. 22951	9. 37455	0. 23689	9. 38802	0. 24435	45
16	. 33268	. 21512	. 34701	. 22234	. 36104	. 22964	. 37478	. 23702	. 38824	. 24448	44
17	. 33292	. 21524	. 34725	. 22246	. 36127	. 22976	. 37501	. 23714	. 38846	. 24460	43
18	. 33317	. 21536	. 34748	. 22258	. 36150	. 22988	. 37523	. 23726	. 38868	. 24473	42
19	. 33341	. 21548	. 34772	. 22270	. 36173	. 23000	. 37546	. 23739	. 38891	. 24485	41
20	9. 33365	0. 21560	9. 34795	0. 22282	9. 36196	0. 23012	9. 37569	0. 23751	9. 38913	0. 24498	40
21	. 33389	. 21572	. 34819	. 22294	. 36219	. 23025	. 37591	. 23764	. 38935	. 24510	39
22	. 33413	. 21584	. 34843	. 22306	. 36243	. 23037	. 37614	. 23776	. 38957	. 24523	38
23	. 33437	. 21596	. 34866	. 22318	. 36266	. 23049	. 37636	. 23788	. 38979	. 24535	37
24	. 33461	. 21608	. 34890	. 22330	. 36289	. 23061	. 37659	. 23801	. 39002	. 24548	36
25	9. 33485	0. 21620	9. 34913	0. 22343	9. 36312	0. 23074	9. 37682	0. 23813	9. 39024	0. 24560	35
26	. 33509	. 21632	. 34937	. 22355	. 36335	. 23086	. 37704	. 23825	. 39046	. 24573	34
27	. 33533	. 21644	. 34960	. 22367	. 36358	. 23098	. 37727	. 23838	. 39068	. 24585	33
28	. 33557	. 21656	. 34984	. 22379	. 36381	. 23110	. 37749	. 23850	. 39090	. 24598	32
29	. 33581	. 21668	. 35007	. 22391	. 36404	. 23123	. 37772	. 23863	. 39112	. 24611	31
30	9. 33605	0. 21680	9. 35031	0. 22403	9. 36427	0. 23135	9. 37794	0. 23875	9. 39134	0. 24623	30
31	. 33629	. 21692	. 35054	. 22415	. 36450	. 23147	. 37817	. 23887	. 39156	. 24636	29
32	. 33653	. 21704	. 35078	. 22427	. 36473	. 23160	. 37840	. 23900	. 39178	. 24648	28
33	. 33677	. 21716	. 35101	. 22440	. 36496	. 23172	. 37862	. 23912	. 39201	. 24661	27
34	. 33701	. 21728	. 35125	. 22452	. 36519	. 23184	. 37885	. 23925	. 39223	. 24673	26
35	9. 33725	0. 21740	9. 35148	0. 22464	9. 36542	0. 23196	9. 37907	0. 23937	9. 39245	0. 24686	25
36	. 33749	. 21752	. 35172	. 22476	. 36565	. 23209	. 37930	. 23950	. 39267	. 24698	24
37	. 33773	. 21764	. 35195	. 22488	. 36588	. 23221	. 37952	. 23962	. 39289	. 24711	23
38	. 33797	. 21776	. 35219	. 22500	. 36611	. 23233	. 37975	. 23974	. 39311	. 24723	22
39	. 33821	. 21788	. 35242	. 22512	. 36634	. 23246	. 37997	. 23987	. 39333	. 24736	21
40	9. 33845	0. 21800	9. 35266	0. 22525	9. 36657	0. 23258	9. 38020	0. 23999	9. 39355	0. 24749	20
41	. 33869	. 21812	. 35289	. 22537	. 36680	. 23270	. 38042	. 24012	. 39377	. 24761	19
42	. 33893	. 21824	. 35312	. 22549	. 36703	. 23282	. 38065	. 24024	. 39399	. 24774	18
43	. 33917	. 21836	. 35336	. 22561	. 36726	. 23295	. 38087	. 24036	. 39421	. 24786	17
44	. 33941	. 21848	. 35359	. 22573	. 36749	. 23307	. 38110	. 24049	. 39443	. 24799	16
45	9. 33965	0. 21860	9. 35383	0. 22585	9. 36772	0. 23319	9. 38132	0. 24061	9. 39465	0. 24811	15
46	. 33988	. 21872	. 35406	. 22598	. 36794	. 23332	. 38154	. 24074	. 39487	. 24824	14
47	. 34012	. 21884	. 35429	. 22610	. 36817	. 23344	. 38177	. 24086	. 39509	. 24836	13
48	. 34036	. 21896	. 35453	. 22622	. 36840	. 23356	. 38199	. 24099	. 39531	. 24849	12
49	. 34060	. 21908	. 35476	. 22634	. 36863	. 23368	. 38222	. 24111	. 39553	. 24862	11
50	9. 34084	0. 21920	9. 35500	0. 22646	9. 36886	0. 23381	9. 38244	0. 24124	9. 39575	0. 24874	10
51	. 34108	. 21932	. 35523	. 22658	. 36909	. 23393	. 38267	. 24136	. 39597	. 24887	9
52	. 34132	. 21944	. 35546	. 22671	. 36932	. 23405	. 38289	. 24148	. 39619	. 24899	8
53	. 34155	. 21956	. 35570	. 22683	. 36955	. 23418	. 38311	. 24161	. 39641	. 24912	7
54	. 34179	. 21968	. 35593	. 22695	. 36977	. 23430	. 38334	. 24173	. 39663	. 24924	6
55	9. 34203	0. 21980	9. 35616	0. 22707	9. 37000	0. 23442	9. 38356	0. 24186	9. 39685	0. 24937	5
56	. 34227	. 21992	. 35639	. 22719	. 37023	. 23455	. 38378	. 24198	. 39706	. 24950	4
57	. 34251	. 22004	. 35663	. 22731	. 37046	. 23467	. 38401	. 24211	. 39728	. 24962	3
58	. 34274	. 22016	. 35686	. 22744	. 37069	. 23479	. 38423	. 24223	. 39750	. 24975	2
59	. 34298	. 22028	. 35709	. 22756	. 37091	. 23492	. 38445	. 24236	. 39772	. 24987	1
60	9. 34322	0. 22040	9. 35733	0. 22768	9. 37114	0. 23504	9. 38468	0. 24248	9. 39794	0. 25000	0
	304°		303°		302°		301°		300°		

TABLE 34

Haversines

′	60°		61°		62°		63°		64°		′
	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	
0	9.39794	0.25000	9.41094	0.25760	9.42368	0.26526	9.43617	0.27300	9.44842	0.28081	60
1	.39816	.25013	.41115	.25772	.42389	.26539	.43638	.27313	.44862	.28095	59
2	.39838	.25025	.41137	.25785	.42410	.26552	.43658	.27326	.44882	.28108	58
3	.39860	.25038	.41158	.25798	.42431	.26565	.43679	.27339	.44903	.28121	57
4	.39881	.25050	.41180	.25810	.42452	.26578	.43699	.27352	.44923	.28134	56
5	9.39903	0.25063	9.41201	0.25823	9.42473	0.26591	9.43720	0.27365	9.44943	0.28147	55
6	.39925	.25076	.41222	.25836	.42494	.26604	.43741	.27378	.44963	.28160	54
7	.39947	.25088	.41244	.25849	.42515	.26616	.43761	.27391	.44983	.28173	53
8	.39969	.25101	.41265	.25861	.42536	.26629	.43782	.27404	.45003	.28186	52
9	.39991	.25113	.41287	.25874	.42557	.26642	.43802	.27417	.45024	.28199	51
10	9.40012	0.25126	9.41308	0.25887	9.42578	0.26655	9.43823	0.27430	9.45044	0.28212	50
11	.40034	.25139	.41329	.25900	.42599	.26668	.43843	.27443	.45064	.28225	49
12	.40056	.25151	.41351	.25912	.42620	.26681	.43864	.27456	.45084	.28238	48
13	.40078	.25164	.41372	.25925	.42641	.26694	.43884	.27469	.45104	.28252	47
14	.40100	.25177	.41393	.25938	.42662	.26706	.43905	.27482	.45124	.28265	46
15	9.40121	0.25189	9.41415	0.25951	9.42682	0.26719	9.43926	0.27495	9.45144	0.28278	45
16	.40143	.25202	.41436	.25963	.42703	.26732	.43946	.27508	.45165	.28291	44
17	.40165	.25214	.41457	.25976	.42724	.26745	.43967	.27521	.45185	.28304	43
18	.40187	.25227	.41479	.25989	.42745	.26758	.43987	.27534	.45205	.28317	42
19	.40208	.25240	.41500	.26002	.42766	.26771	.44008	.27547	.45225	.28330	41
20	9.40230	0.25252	9.41521	0.26014	9.42787	0.26784	9.44028	0.27560	9.45245	0.28343	40
21	.40252	.25265	.41543	.26027	.42808	.26797	.44048	.27573	.45265	.28356	39
22	.40274	.25278	.41564	.26040	.42829	.26809	.44069	.27586	.45285	.28369	38
23	.40295	.25290	.41585	.26053	.42850	.26822	.44089	.27599	.45305	.28383	37
24	.40317	.25303	.41606	.26065	.42870	.26835	.44110	.27612	.45325	.28396	36
25	9.40339	0.25316	9.41628	0.26078	9.42891	0.26848	9.44130	0.27625	9.45345	0.28409	35
26	.40360	.25328	.41649	.26091	.42912	.26861	.44151	.27638	.45365	.28422	34
27	.40382	.25341	.41670	.26104	.42933	.26874	.44171	.27651	.45385	.28435	33
28	.40404	.25354	.41692	.26117	.42954	.26887	.44192	.27664	.45405	.28448	32
29	.40425	.25366	.41713	.26129	.42975	.26900	.44212	.27677	.45426	.28461	31
30	9.40447	0.25379	9.41734	0.26142	9.42996	0.26913	9.44232	0.27690	9.45446	0.28474	30
31	.40469	.25391	.41755	.26155	.43016	.26925	.44253	.27703	.45466	.28488	29
32	.40490	.25404	.41776	.26168	.43037	.26938	.44273	.27716	.45486	.28501	28
33	.40512	.25417	.41798	.26180	.43058	.26951	.44294	.27729	.45506	.28514	27
34	.40534	.25429	.41819	.26193	.43079	.26964	.44314	.27742	.45526	.28527	26
35	9.40555	0.25442	9.41840	0.26206	9.43100	0.26977	9.44334	0.27755	9.45546	0.28540	25
36	.40577	.25455	.41861	.26219	.43120	.26990	.44355	.27768	.45566	.28553	24
37	.40599	.25467	.41882	.26232	.43141	.27003	.44375	.27781	.45586	.28566	23
38	.40620	.25480	.41904	.26244	.43162	.27016	.44396	.27794	.45606	.28580	22
39	.40642	.25493	.41925	.26257	.43183	.27029	.44416	.27807	.45625	.28593	21
40	9.40663	0.25506	9.41946	0.26270	9.43203	0.27042	9.44436	0.27820	9.45645	0.28606	20
41	.40685	.25518	.41967	.26283	.43224	.27055	.44457	.27833	.45665	.28619	19
42	.40707	.25531	.41988	.26296	.43245	.27068	.44477	.27846	.45685	.28632	18
43	.40728	.25544	.42009	.26308	.43266	.27080	.44497	.27859	.45705	.28645	17
44	.40750	.25556	.42031	.26321	.43286	.27093	.44518	.27873	.45725	.28658	16
45	9.40771	0.25569	9.42052	0.26334	9.43307	0.27106	9.44538	0.27886	9.45745	0.28672	15
46	.40793	.25582	.42073	.26347	.43328	.27119	.44558	.27899	.45765	.28685	14
47	.40814	.25594	.42094	.26360	.43348	.27132	.44579	.27912	.45785	.28698	13
48	.40836	.25607	.42115	.26372	.43369	.27145	.44599	.27925	.45805	.28711	12
49	.40858	.25620	.42136	.26385	.43390	.27158	.44619	.27938	.45825	.28724	11
50	9.40879	0.25632	9.42157	0.26398	9.43411	0.27171	9.44639	0.27951	9.45845	0.28737	10
51	.40900	.25645	.42178	.26411	.43431	.27184	.44660	.27964	.45865	.28751	9
52	.40922	.25658	.42199	.26424	.43452	.27197	.44680	.27977	.45884	.28764	8
53	.40943	.25671	.42221	.26437	.43473	.27210	.44700	.27990	.45904	.28777	7
54	.40965	.25683	.42242	.26449	.43493	.27223	.44721	.28003	.45924	.28790	6
55	9.40986	0.25696	9.42263	0.26462	9.43514	0.27236	9.44741	0.28016	9.45944	0.28803	5
56	.41008	.25709	.42284	.26475	.43535	.27249	.44761	.28029	.45964	.28816	4
57	.41029	.25721	.42305	.26488	.43555	.27262	.44781	.28042	.45984	.28830	3
58	.41051	.25734	.42326	.26501	.43576	.27275	.44801	.28055	.46004	.28843	2
59	.41072	.25747	.42347	.26514	.43596	.27288	.44822	.28068	.46023	.28856	1
60	9.41094	0.25760	9.42368	0.26526	9.43617	0.27300	9.44842	0.28081	9.46043	0.28869	0
	299°		298°		297°		296°		295°		

TABLE 34

Haversines

′	65° Log Hav	65° Nat. Hav	66° Log Hav	66° Nat. Hav	67° Log Hav	67° Nat. Hav	68° Log Hav	68° Nat. Hav	69° Log Hav	69° Nat. Hav	′
0	9. 46043	0. 28869	9. 47222	0. 29663	9. 48378	0. 30463	9. 49512	0. 31270	9. 50626	0. 32082	60
1	. 46063	. 28882	. 47241	. 29676	. 48397	. 30477	. 49531	. 31283	. 50644	. 32095	59
2	. 46083	. 28895	. 47261	. 29690	. 48416	. 30490	. 49550	. 31297	. 50662	. 32109	58
3	. 46103	. 28909	. 47280	. 29703	. 48435	. 30504	. 49568	. 31310	. 50681	. 32122	57
4	. 46123	. 28922	. 47300	. 29716	. 48454	. 30517	. 49587	. 31324	. 50699	. 32136	56
5	9. 46142	0. 28935	9. 47319	0. 29730	9. 48473	0. 30530	9. 49606	0. 31337	9. 50717	0. 32150	55
6	. 46162	. 28948	. 47338	. 29743	. 48492	. 30544	. 49625	. 31351	. 50736	. 32163	54
7	. 46182	. 28961	. 47358	. 29756	. 48511	. 30557	. 49643	. 31364	. 50754	. 32177	53
8	. 46202	. 28975	. 47377	. 29770	. 48530	. 30571	. 49662	. 31378	. 50772	. 32190	52
9	. 46222	. 28988	. 47397	. 29783	. 48549	. 30584	. 49681	. 31391	. 50791	. 32204	51
10	9. 46241	0. 29001	9. 47416	0. 29796	9. 48568	0. 30597	9. 49699	0. 31405	9. 50809	0. 32217	50
11	. 46261	. 29014	. 47435	. 29809	. 48587	. 30611	. 49718	. 31418	. 50827	. 32231	49
12	. 46281	. 29027	. 47455	. 29823	. 48607	. 30624	. 49737	. 31432	. 50846	. 32245	48
13	. 46301	. 29041	. 47474	. 29836	. 48626	. 30638	. 49755	. 31445	. 50864	. 32258	47
14	. 46320	. 29054	. 47493	. 29849	. 48645	. 30651	. 49774	. 31459	. 50882	. 32272	46
15	9. 46340	0. 29067	9. 47513	0. 29863	9. 48664	0. 30664	9. 49793	0. 31472	9. 50901	0. 32285	45
16	. 46360	. 29080	. 47532	. 29876	. 48683	. 30678	. 49811	. 31486	. 50919	. 32299	44
17	. 46380	. 29093	. 47552	. 29889	. 48702	. 30691	. 49830	. 31499	. 50937	. 32313	43
18	. 46399	. 29107	. 47571	. 29903	. 48720	. 30705	. 49849	. 31513	. 50956	. 32326	42
19	. 46419	. 29120	. 47590	. 29916	. 48739	. 30718	. 49867	. 31526	. 50974	. 32340	41
20	9. 46439	0. 29133	9. 47610	0. 29929	9. 48758	0. 30732	9. 49886	0. 31540	9. 50992	0. 32353	40
21	. 46458	. 29146	. 47629	. 29943	. 48777	. 30745	. 49904	. 31553	. 51010	. 32367	39
22	. 46478	. 29160	. 47648	. 29956	. 48796	. 30758	. 49923	. 31567	. 51029	. 32381	38
23	. 46498	. 29173	. 47668	. 29969	. 48815	. 30772	. 49942	. 31580	. 51047	. 32394	37
24	. 46517	. 29186	. 47687	. 29983	. 48834	. 30785	. 49960	. 31594	. 51065	. 32408	36
25	9. 46537	0. 29199	9. 47706	0. 29996	9. 48853	0. 30799	9. 49979	0. 31607	9. 51083	0. 32422	35
26	. 46557	. 29212	. 47725	. 30009	. 48872	. 30812	. 49997	. 31621	. 51102	. 32435	34
27	. 46576	. 29226	. 47745	. 30023	. 48891	. 30826	. 50016	. 31634	. 51120	. 32449	33
28	. 46596	. 29239	. 47764	. 30036	. 48910	. 30839	. 50034	. 31648	. 51138	. 32462	32
29	. 46616	. 29252	. 47783	. 30049	. 48929	. 30852	. 50053	. 31661	. 51156	. 32476	31
30	9. 46635	0. 29265	9. 47803	0. 30063	9. 48948	0. 30866	9. 50072	0. 31675	9. 51174	0. 32490	30
31	. 46655	. 29279	. 47822	. 30076	. 48967	. 30879	. 50090	. 31688	. 51193	. 32503	29
32	. 46675	. 29292	. 47841	. 30089	. 48986	. 30893	. 50109	. 31702	. 51211	. 32517	28
33	. 46694	. 29305	. 47860	. 30103	. 49004	. 30906	. 50127	. 31716	. 51229	. 32531	27
34	. 46714	. 29318	. 47880	. 30116	. 49023	. 30920	. 50146	. 31729	. 51247	. 32544	26
35	9. 46733	0. 29332	9. 47899	0. 30129	9. 49042	0. 30933	9. 50164	0. 31743	9. 51265	0. 32558	25
36	. 46753	. 29345	. 47918	. 30143	. 49061	. 30946	. 50183	. 31756	. 51284	. 32571	24
37	. 46773	. 29358	. 47937	. 30156	. 49080	. 30960	. 50201	. 31770	. 51302	. 32585	23
38	. 46792	. 29371	. 47957	. 30169	. 49099	. 30973	. 50220	. 31783	. 51320	. 32599	22
39	. 46812	. 29385	. 47976	. 30183	. 49118	. 30987	. 50238	. 31797	. 51338	. 32612	21
40	9. 46831	0. 29398	9. 47995	0. 30196	9. 49137	0. 31000	9. 50257	0. 31810	9. 51356	0. 32626	20
41	. 46851	. 29411	. 48014	. 30209	. 49155	. 31014	. 50275	. 31824	. 51374	. 32640	19
42	. 46871	. 29424	. 48033	. 30223	. 49174	. 31027	. 50294	. 31837	. 51393	. 32653	18
43	. 46890	. 29438	. 48053	. 30236	. 49193	. 31041	. 50312	. 31851	. 51411	. 32667	17
44	. 46910	. 29451	. 48072	. 30249	. 49212	. 31054	. 50331	. 31865	. 51429	. 32681	16
45	9. 46929	0. 29464	9. 48091	0. 30263	9. 49231	0. 31068	9. 50349	0. 31878	9. 51447	0. 32694	15
46	. 46949	. 29477	. 48110	. 30276	. 49250	. 31081	. 50368	. 31892	. 51465	. 32708	14
47	. 46968	. 29491	. 48129	. 30290	. 49268	. 31094	. 50386	. 31905	. 51483	. 32721	13
48	. 46988	. 29504	. 48148	. 30303	. 49287	. 31108	. 50405	. 31919	. 51501	. 32735	12
49	. 47007	. 29517	. 48168	. 30316	. 49306	. 31121	. 50423	. 31932	. 51519	. 32749	11
50	9. 47027	0. 29530	9. 48187	0. 30330	9. 49325	0. 31135	9. 50442	0. 31946	9. 51538	0. 32762	10
51	. 47046	. 29544	. 48206	. 30343	. 49344	. 31148	. 50460	. 31959	. 51556	. 32776	9
52	. 47066	. 29557	. 48225	. 30356	. 49362	. 31162	. 50478	. 31973	. 51574	. 32790	8
53	. 47085	. 29570	. 48244	. 30370	. 49381	. 31175	. 50497	. 31987	. 51592	. 32803	7
54	. 47105	. 29583	. 48263	. 30383	. 49400	. 31189	. 50515	. 32000	. 51610	. 32817	6
55	9. 47124	0. 29597	9. 48282	0. 30397	9. 49419	0. 31202	9. 50534	0. 32014	9. 51628	0. 32831	5
56	. 47144	. 29610	. 48302	. 30410	. 49437	. 31216	. 50552	. 32027	. 51646	. 32844	4
57	. 47163	. 29623	. 48321	. 30423	. 49456	. 31229	. 50570	. 32041	. 51664	. 32858	3
58	. 47183	. 29637	. 48340	. 30437	. 49475	. 31243	. 50589	. 32054	. 51682	. 32872	2
59	. 47202	. 29650	. 48359	. 30450	. 49494	. 31256	. 50607	. 32068	. 51700	. 32885	1
60	9. 47222	0. 29663	9. 48378	0. 30463	9. 49512	0. 31270	9. 50626	0. 32082	9. 51718	0. 32899	0
	294°		293°		292°		291°		290°		

TABLE 34

Haversines

	70°		71°		72°		73°		74°		
′	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	′
0	9. 51718	0. 32899	9. 52791	0. 33722	9. 53844	0. 34549	9. 54878	0. 35381	9. 55893	0. 36218	60
1	. 51736	. 32913	. 52809	. 33735	. 53861	. 34563	. 54895	. 35395	. 55909	. 36232	59
2	. 51754	. 32926	. 52826	. 33749	. 53879	. 34577	. 54912	. 35409	. 55926	. 36246	58
3	. 51772	. 32940	. 52844	. 33763	. 53896	. 34591	. 54929	. 35423	. 55943	. 36260	57
4	. 51790	. 32954	. 52862	. 33777	. 53913	. 34604	. 54946	. 35437	. 55960	. 36274	56
5	9. 51808	0. 32967	9. 52879	0. 33790	9. 53931	0. 34618	9. 54963	0. 35451	9. 55976	0. 36288	55
6	. 51826	. 32981	. 52897	. 33804	. 53948	. 34632	. 54980	. 35465	. 55993	. 36302	54
7	. 51844	. 32995	. 52915	. 33818	. 53966	. 34646	. 54997	. 35479	. 56010	. 36316	53
8	. 51862	. 33008	. 52932	. 33832	. 53983	. 34660	. 55014	. 35493	. 56027	. 36330	52
9	. 51880	. 33022	. 52950	. 33845	. 54000	. 34674	. 55031	. 35507	. 56043	. 36344	51
10	9. 51898	0. 33036	9. 52968	0. 33859	9. 54017	0. 34688	9. 55048	0. 35521	9. 56060	0. 36358	50
11	. 51916	. 33049	. 52985	. 33873	. 54035	. 34701	. 55065	. 35534	. 56077	. 36372	49
12	. 51934	. 33063	. 53003	. 33887	. 54052	. 34715	. 55082	. 35548	. 56093	. 36386	48
13	. 51952	. 33077	. 53021	. 33900	. 54069	. 34729	. 55099	. 35562	. 56110	. 36400	47
14	. 51970	. 33090	. 53038	. 33914	. 54087	. 34743	. 55116	. 35576	. 56127	. 36414	46
15	9. 51988	0. 33104	9. 53056	0. 33928	9. 54104	0. 34757	9. 55133	0. 35590	9. 56144	0. 36428	45
16	. 52006	. 33118	. 53073	. 33942	. 54121	. 34771	. 55150	. 35604	. 56160	. 36442	44
17	. 52024	. 33132	. 53091	. 33956	. 54139	. 34784	. 55167	. 35618	. 56177	. 36456	43
18	. 52042	. 33145	. 53109	. 33969	. 54156	. 34798	. 55184	. 35632	. 56194	. 36470	42
19	. 52060	. 33159	. 53126	. 33983	. 54173	. 34812	. 55201	. 35646	. 56210	. 36484	41
20	9. 52078	0. 33173	9. 53144	0. 33997	9. 54190	0. 34826	9. 55218	0. 35660	9. 56227	0. 36498	40
21	. 52096	. 33186	. 53162	. 34011	. 54208	. 34840	. 55235	. 35674	. 56244	. 36512	39
22	. 52114	. 33200	. 53179	. 34024	. 54225	. 34854	. 55252	. 35688	. 56260	. 36526	38
23	. 52132	. 33214	. 53197	. 34038	. 54242	. 34868	. 55269	. 35702	. 56277	. 36540	37
24	. 52150	. 33227	. 53214	. 34052	. 54260	. 34882	. 55286	. 35716	. 56294	. 36554	36
25	9. 52168	0. 33241	9. 53232	0. 34066	9. 54277	0. 34895	9. 55303	0. 35730	9. 56310	0. 36568	35
26	. 52185	. 33255	. 53249	. 34080	. 54294	. 34909	. 55320	. 35743	. 56327	. 36582	34
27	. 52203	. 33269	. 53267	. 34093	. 54311	. 34923	. 55337	. 35757	. 56343	. 36596	33
28	. 52221	. 33282	. 53285	. 34107	. 54329	. 34937	. 55354	. 35771	. 56360	. 36610	32
29	. 52239	. 33296	. 53302	. 34121	. 54346	. 34951	. 55370	. 35785	. 56377	. 36624	31
30	9. 52257	0. 33310	9. 53320	0. 34135	9. 54363	0. 34965	9. 55387	0. 35799	9. 56393	0. 36638	30
31	. 52275	. 33323	. 53337	. 34149	. 54380	. 34979	. 55404	. 35813	. 56410	. 36652	29
32	. 52293	. 33337	. 53355	. 34162	. 54397	. 34992	. 55421	. 35827	. 56426	. 36666	28
33	. 52311	. 33351	. 53372	. 34176	. 54415	. 35006	. 55438	. 35841	. 56443	. 36680	27
34	. 52328	. 33365	. 53390	. 34190	. 54432	. 35020	. 55455	. 35855	. 56460	. 36694	26
35	9. 52346	0. 33378	9. 53407	0. 34204	9. 54449	0. 35034	9. 55472	0. 35869	9. 56476	0. 36708	25
36	. 52364	. 33392	. 53425	. 34218	. 54466	. 35048	. 55489	. 35883	. 56493	. 36722	24
37	. 52382	. 33406	. 53442	. 34231	. 54483	. 35062	. 55506	. 35897	. 56509	. 36736	23
38	. 52400	. 33419	. 53460	. 34245	. 54501	. 35076	. 55523	. 35911	. 56526	. 36750	22
39	. 52418	. 33433	. 53477	. 34259	. 54518	. 35090	. 55539	. 35925	. 56543	. 36764	21
40	9. 52436	0. 33447	9. 53495	0. 34273	9. 54535	0. 35103	9. 55556	0. 35939	9. 56559	0. 36778	20
41	. 52453	. 33461	. 53512	. 34287	. 54552	. 35117	. 55573	. 35953	. 56576	. 36792	19
42	. 52471	. 33474	. 53530	. 34300	. 54569	. 35131	. 55590	. 35967	. 56592	. 36806	18
43	. 52489	. 33488	. 53547	. 34314	. 54587	. 35145	. 55607	. 35981	. 56609	. 36820	17
44	. 52507	. 33502	. 53565	. 34328	. 54604	. 35159	. 55624	. 35995	. 56625	. 36834	16
45	9. 52525	0. 33515	9. 53582	0. 34342	9. 54621	0. 35173	9. 55641	0. 36009	9. 56642	0. 36848	15
46	. 52542	. 33529	. 53600	. 34356	. 54638	. 35187	. 55657	. 36023	. 56658	. 36862	14
47	. 52560	. 33543	. 53617	. 34369	. 54655	. 35201	. 55674	. 36036	. 56675	. 36877	13
48	. 52578	. 33557	. 53635	. 34383	. 54672	. 35215	. 55691	. 36050	. 56692	. 36891	12
49	. 52596	. 33570	. 53652	. 34397	. 54689	. 35228	. 55708	. 36064	. 56708	. 36905	11
50	9. 52613	0. 33584	9. 53670	0. 34411	9. 54707	0. 35242	9. 55725	0. 36078	9. 56725	0. 36919	10
51	. 52631	. 33598	. 53687	. 34425	. 54724	. 35256	. 55742	. 36092	. 56741	. 36933	9
52	. 52649	. 33612	. 53704	. 34439	. 54741	. 35270	. 55758	. 36106	. 56758	. 36947	8
53	. 52667	. 33625	. 53722	. 34452	. 54758	. 35284	. 55775	. 36120	. 56774	. 36961	7
54	. 52684	. 33639	. 53739	. 34466	. 54775	. 35298	. 55792	. 36134	. 56791	. 36975	6
55	9. 52702	0. 33653	9. 53757	0. 34480	9. 54792	0. 35312	9. 55809	0. 36148	9. 56807	0. 36989	5
56	. 52720	. 33667	. 53774	. 34494	. 54809	. 35326	. 55826	. 36162	. 56824	. 37003	4
57	. 52738	. 33680	. 53792	. 34508	. 54826	. 35340	. 55842	. 36176	. 56840	. 37017	3
58	. 52755	. 33694	. 53809	. 34521	. 54843	. 35354	. 55859	. 36190	. 56856	. 37031	2
59	. 52773	. 33708	. 53826	. 34535	. 54860	. 35368	. 55876	. 36204	. 56873	. 37045	1
60	9. 52791	0. 33722	9. 53844	0. 34549	9. 54878	0. 35381	9. 55893	0. 36218	9. 56889	0. 37059	0
	289°		288°		287°		286°		285°		

TABLE 34

Haversines

′	75° Log Hav	75° Nat. Hav	76° Log Hav	76° Nat. Hav	77° Log Hav	77° Nat. Hav	78° Log Hav	78° Nat. Hav	79° Log Hav	79° Nat. Hav	′
0	9.56889	0.37059	9.57868	0.37904	9.58830	0.38752	9.59774	0.39604	9.60702	0.40460	60
1	.56906	.37073	.57885	.37918	.58846	.38767	.59790	.39619	.60717	.40474	59
2	.56922	.37087	.57901	.37932	.58862	.38781	.59806	.39633	.60733	.40488	58
3	.56939	.37101	.57917	.37946	.58878	.38795	.59821	.39647	.60748	.40502	57
4	.56955	.37115	.57933	.37960	.58893	.38809	.59837	.39661	.60763	.40517	56
5	9.56972	0.37129	9.57949	0.37974	9.58909	0.38823	9.59852	0.39676	9.60779	0.40531	55
6	.56988	.37143	.57965	.37989	.58925	.38837	.59868	.39690	.60794	.40545	54
7	.57005	.37157	.57981	.38003	.58941	.38852	.59883	.39704	.60809	.40560	53
8	.57021	.37171	.57998	.38017	.58957	.38866	.59899	.39718	.60825	.40574	52
9	.57037	.37186	.58014	.38031	.58973	.38880	.59915	.39732	.60840	.40588	51
10	9.57054	0.37200	9.58030	0.38045	9.58989	0.38894	9.59930	0.39747	9.60855	0.40602	50
11	.57070	.37214	.58046	.38059	.59004	.38908	.59946	.39761	.60870	.40617	49
12	.57087	.37228	.58062	.38073	.59020	.38923	.59961	.39775	.60886	.40631	48
13	.57103	.37242	.58078	.38087	.59036	.38937	.59977	.39789	.60901	.40645	47
14	.57119	.37256	.58094	.38102	.59052	.38951	.59992	.39804	.60916	.40660	46
15	9.57136	0.37270	9.58110	0.38116	9.59068	0.38965	9.60008	0.39818	9.60931	0.40674	45
16	.57152	.37284	.58126	.38130	.59083	.38979	.60023	.39832	.60947	.40688	44
17	.57169	.37298	.58143	.38144	.59099	.38994	.60039	.39846	.60962	.40702	43
18	.57185	.37312	.58159	.38158	.59115	.39008	.60054	.39861	.60977	.40717	42
19	.57201	.37326	.58175	.38172	.59131	.39022	.60070	.39875	.60992	.40731	41
20	9.57218	0.37340	9.58191	0.38186	9.59147	0.39036	9.60085	0.39889	9.61008	0.40745	40
21	.57234	.37354	.58207	.38200	.59162	.39050	.60101	.39903	.61023	.40760	39
22	.57250	.37368	.58223	.38215	.59178	.39064	.60116	.39918	.61038	.40774	38
23	.57267	.37382	.58239	.38229	.59194	.39079	.60132	.39932	.61053	.40788	37
24	.57283	.37397	.58255	.38243	.59210	.39093	.60147	.39946	.61069	.40802	36
25	9.57299	0.37411	9.58271	0.38257	9.59225	0.39107	9.60163	0.39960	9.61084	0.40817	35
26	.57316	.37425	.58287	.38271	.59241	.39121	.60178	.39975	.61099	.40831	34
27	.57332	.37439	.58303	.38285	.59257	.39135	.60194	.39989	.61114	.40845	33
28	.57348	.37453	.58319	.38299	.59273	.39150	.60209	.40003	.61129	.40860	32
29	.57365	.37467	.58335	.38314	.59289	.39164	.60225	.40017	.61145	.40874	31
30	9.57381	0.37481	9.58351	0.38328	9.59304	0.39178	9.60240	0.40032	9.61160	0.40888	30
31	.57397	.37495	.58367	.38342	.59320	.39192	.60256	.40046	.61175	.40903	29
32	.57414	.37509	.58383	.38356	.59336	.39206	.60271	.40060	.61190	.40917	28
33	.57430	.37523	.58399	.38370	.59351	.39221	.60287	.40074	.61205	.40931	27
34	.57446	.37537	.58415	.38384	.59367	.39235	.60302	.40089	.61221	.40945	26
35	9.57463	0.37551	9.58431	0.38398	9.59383	0.39249	9.60318	0.40103	9.61236	0.40960	25
36	.57479	.37566	.58447	.38413	.59399	.39263	.60333	.40117	.61251	.40974	24
37	.57495	.37580	.58463	.38427	.59414	.39277	.60348	.40131	.61266	.40988	23
38	.57511	.37594	.58479	.38441	.59430	.39292	.60364	.40146	.61281	.41003	22
39	.57528	.37608	.58495	.38455	.59446	.39306	.60379	.40160	.61296	.41017	21
40	9.57544	0.37622	9.58511	0.38469	9.59461	0.39320	9.60395	0.40174	9.61312	0.41031	20
41	.57560	.37636	.58527	.38483	.59477	.39334	.60410	.40188	.61327	.41046	19
42	.57577	.37650	.58543	.38498	.59493	.39348	.60426	.40203	.61342	.41060	18
43	.57593	.37664	.58559	.38512	.59508	.39363	.60441	.40217	.61357	.41074	17
44	.57609	.37678	.58575	.38526	.59524	.39377	.60456	.40231	.61372	.41089	16
45	9.57625	0.37692	9.58591	0.38540	9.59540	0.39391	9.60472	0.40245	9.61387	0.41103	15
46	.57642	.37706	.58607	.38554	.59556	.39405	.60487	.40260	.61402	.41117	14
47	.57658	.37721	.58623	.38568	.59571	.39420	.60502	.40274	.61417	.41131	13
48	.57674	.37735	.58639	.38582	.59587	.39434	.60518	.40288	.61433	.41146	12
49	.57690	.37749	.58655	.38597	.59602	.39448	.60533	.40303	.61448	.41160	11
50	9.57706	0.37763	9.58671	0.38611	9.59618	0.39462	9.60549	0.40317	9.61463	0.41174	10
51	.57723	.37777	.58687	.38625	.59634	.39476	.60564	.40331	.61478	.41189	9
52	.57739	.37791	.58703	.38639	.59649	.39491	.60579	.40345	.61493	.41203	8
53	.57755	.37805	.58719	.38653	.59665	.39505	.60595	.40360	.61508	.41217	7
54	.57771	.37819	.58735	.38667	.59681	.39519	.60610	.40374	.61523	.41232	6
55	9.57787	0.37833	9.58750	0.38682	9.59696	0.39533	9.60625	0.40388	9.61538	0.41246	5
56	.57804	.37847	.58766	.38696	.59712	.39548	.60641	.40402	.61553	.41260	4
57	.57820	.37862	.58782	.38710	.59728	.39562	.60656	.40417	.61568	.41275	3
58	.57836	.37876	.58798	.38724	.59743	.39576	.60671	.40431	.61583	.41289	2
59	.57852	.37890	.58814	.38738	.59759	.39590	.60687	.40445	.61598	.41303	1
60	9.57868	0.37904	9.58830	0.38752	9.59774	0.39604	9.60702	0.40460	9.61614	0.41318	0
	284°		283°		282°		281°		280°		

TABLE 34

Haversines

′	80°		81°		82°		83°		84°		′
	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	
0	9. 61614	0. 41318	9. 62509	0. 42178	9. 63389	0. 43041	9. 64253	0. 43907	9. 65102	0. 44774	60
1	. 61629	. 41332	. 62524	. 42193	. 63403	. 43056	. 64267	. 43921	. 65116	. 44788	59
2	. 61644	. 41346	. 62538	. 42207	. 63418	. 43070	. 64281	. 43935	. 65130	. 44803	58
3	. 61659	. 41361	. 62553	. 42221	. 63432	. 43085	. 64296	. 43950	. 65144	. 44817	57
4	. 61674	. 41375	. 62568	. 42236	. 63447	. 43099	. 64310	. 43964	. 65158	. 44831	56
5	9. 61689	0. 41389	9. 62583	0. 42250	9. 63461	0. 43113	9. 64324	0. 43979	9. 65172	0. 44846	55
6	. 61704	. 41404	. 62598	. 42264	. 63476	. 43128	. 64339	. 43993	. 65186	. 44860	54
7	. 61719	. 41418	. 62612	. 42279	. 63490	. 43142	. 64353	. 44008	. 65200	. 44875	53
8	. 61734	. 41432	. 62627	. 42293	. 63505	. 43157	. 64367	. 44022	. 65214	. 44889	52
9	. 61749	. 41447	. 62642	. 42308	. 63519	. 43171	. 64381	. 44036	. 65228	. 44904	51
10	9. 61764	0. 41461	9. 62657	0. 42322	9. 63534	0. 43185	9. 64396	0. 44051	9. 65242	0. 44918	50
11	. 61779	. 41475	. 62671	. 42336	. 63548	. 43200	. 64410	. 44065	. 65256	. 44933	49
12	. 61794	. 41490	. 62686	. 42351	. 63563	. 43214	. 64424	. 44080	. 65270	. 44947	48
13	. 61809	. 41504	. 62701	. 42365	. 63577	. 43229	. 64438	. 44094	. 65284	. 44962	47
14	. 61824	. 41518	. 62716	. 42379	. 63592	. 43243	. 64452	. 44109	. 65298	. 44976	46
15	9. 61839	0. 41533	9. 62730	0. 42394	9. 63606	0. 43257	9. 64467	0. 44123	9. 65312	0. 44991	45
16	. 61854	. 41547	. 62745	. 42408	. 63621	. 43272	. 64481	. 44138	. 65326	. 45005	44
17	. 61869	. 41561	. 62760	. 42423	. 63635	. 43286	. 64495	. 44152	. 65340	. 45020	43
18	. 61884	. 41576	. 62774	. 42437	. 63649	. 43301	. 64509	. 44166	. 65354	. 45034	42
19	. 61899	. 41590	. 62789	. 42451	. 63664	. 43315	. 64523	. 44181	. 65368	. 45048	41
20	9. 61914	0. 41604	9. 62804	0. 42466	9. 63678	0. 43330	9. 64538	0. 44195	9. 65382	0. 45063	40
21	. 61929	. 41619	. 62819	. 42480	. 63693	. 43344	. 64552	. 44210	. 65396	. 45077	39
22	. 61944	. 41633	. 62833	. 42494	. 63707	. 43358	. 64566	. 44224	. 65410	. 45092	38
23	. 61959	. 41647	. 62848	. 42509	. 63722	. 43373	. 64580	. 44239	. 65424	. 45106	37
24	. 61974	. 41662	. 62863	. 42523	. 63736	. 43387	. 64594	. 44253	. 65438	. 45121	36
25	9. 61989	0. 41676	9. 62877	0. 42538	9. 63751	0. 43402	9. 64609	0. 44268	9. 65452	0. 45135	35
26	. 62003	. 41690	. 62892	. 42552	. 63765	. 43416	. 64623	. 44282	. 65466	. 45150	34
27	. 62018	. 41705	. 62907	. 42566	. 63779	. 43430	. 64637	. 44296	. 65480	. 45164	33
28	. 62033	. 41719	. 62921	. 42581	. 63794	. 43445	. 64651	. 44311	. 65493	. 45179	32
29	. 62048	. 41733	. 62936	. 42595	. 63808	. 43459	. 64665	. 44325	. 65507	. 45193	31
30	9. 62063	0. 41748	9. 62951	0. 42610	9. 63823	0. 43474	9. 64679	0. 44340	9. 65521	0. 45208	30
31	. 62078	. 41762	. 62965	. 42624	. 63837	. 43488	. 64694	. 44354	. 65535	. 45222	29
32	. 62093	. 41776	. 62980	. 42638	. 63851	. 43503	. 64708	. 44369	. 65549	. 45237	28
33	. 62108	. 41791	. 62995	. 42653	. 63866	. 43517	. 64722	. 44383	. 65563	. 45251	27
34	. 62123	. 41805	. 63009	. 42667	. 63880	. 43531	. 64736	. 44398	. 65577	. 45266	26
35	9. 62138	0. 41819	9. 63024	0. 42681	9. 63895	0. 43546	9. 64750	0. 44412	9. 65591	0. 45280	25
36	. 62153	. 41834	. 63039	. 42696	. 63909	. 43560	. 64764	. 44427	. 65605	. 45295	24
37	. 62168	. 41848	. 63053	. 42710	. 63923	. 43575	. 64778	. 44441	. 65619	. 45309	23
38	. 62182	. 41862	. 63068	. 42725	. 63938	. 43589	. 64793	. 44455	. 65632	. 45324	22
39	. 62197	. 41877	. 63082	. 42739	. 63952	. 43603	. 64807	. 44470	. 65646	. 45338	21
40	9. 62212	0. 41891	9. 63097	0. 42753	9. 63966	0. 43618	9. 64821	0. 44484	9. 65660	0. 45353	20
41	. 62227	. 41905	. 63112	. 42768	. 63981	. 43632	. 64835	. 44499	. 65674	. 45367	19
42	. 62242	. 41920	. 63126	. 42782	. 63995	. 43647	. 64849	. 44513	. 65688	. 45381	18
43	. 62257	. 41934	. 63141	. 42797	. 64010	. 43661	. 64863	. 44528	. 65702	. 45396	17
44	. 62272	. 41949	. 63156	. 42811	. 64024	. 43676	. 64877	. 44542	. 65716	. 45410	16
45	9. 62287	0. 41963	9. 63170	0. 42825	9. 64038	0. 43690	9. 64891	0. 44557	9. 65729	0. 45425	15
46	. 62301	. 41977	. 63185	. 42840	. 64053	. 43704	. 64905	. 44571	. 65743	. 45439	14
47	. 62316	. 41992	. 63199	. 42854	. 64067	. 43719	. 64919	. 44586	. 65757	. 45454	13
48	. 62331	. 42006	. 63214	. 42869	. 64081	. 43733	. 64934	. 44600	. 65771	. 45468	12
49	. 62346	. 42020	. 63228	. 42883	. 64096	. 43748	. 64948	. 44614	. 65785	. 45483	11
50	9. 62361	0. 42035	9. 63243	0. 42897	9. 64110	0. 43762	9. 64962	0. 44629	9. 65799	0. 45497	10
51	. 62376	. 42049	. 63258	. 42912	. 64124	. 43777	. 64976	. 44643	. 65812	. 45512	9
52	. 62390	. 42063	. 63272	. 42926	. 64139	. 43791	. 64990	. 44658	. 65826	. 45526	8
53	. 62405	. 42078	. 63287	. 42941	. 64153	. 43805	. 65004	. 44672	. 65840	. 45541	7
54	. 62420	. 42092	. 63301	. 42955	. 64167	. 43820	. 65018	. 44687	. 65854	. 45555	6
55	9. 62435	0. 42106	9. 63316	0. 42969	9. 64181	0. 43834	9. 65032	0. 44701	9. 65868	0. 45570	5
56	. 62450	. 42121	. 63330	. 42984	. 64196	. 43849	. 65046	. 44716	. 65881	. 45584	4
57	. 62464	. 42135	. 63345	. 42998	. 64210	. 43863	. 65060	. 44730	. 65895	. 45599	3
58	. 62479	. 42150	. 63360	. 43013	. 64224	. 43878	. 65074	. 44745	. 65909	. 45613	2
59	. 62494	. 42164	. 63374	. 43027	. 64239	. 43892	. 65088	. 44759	. 65923	. 45628	1
60	9. 62509	0. 42178	9. 63389	0. 43041	9. 64253	0. 43907	9. 65102	0. 44774	9. 65937	0. 45642	0
	279°		278°		277°		276°		275°		

TABLE 34

Haversines

	85°		86°		87°		88°		89°		
′	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	′
0	9. 65937	0. 45642	9. 66757	0. 46512	9. 67562	0. 47383	9. 68354	0. 48255	9. 69132	0. 49127	60
1	. 65950	. 45657	. 66770	. 46527	. 67576	. 47398	. 68367	. 48270	. 69145	. 49142	59
2	. 65964	. 45671	. 66784	. 46541	. 67589	. 47412	. 68380	. 48284	. 69158	. 49156	58
3	. 65978	. 45686	. 66797	. 46556	. 67602	. 47427	. 68393	. 48299	. 69171	. 49171	57
4	. 65992	. 45700	. 66811	. 46570	. 67616	. 47441	. 68407	. 48313	. 69184	. 49186	56
5	9. 66006	0. 45715	9. 66824	0. 46585	9. 67629	0. 47456	9. 68420	0. 48328	9. 69197	0. 49200	55
6	. 66019	. 45729	. 66838	. 46599	. 67642	. 47470	. 68433	. 48342	. 69209	. 49215	54
7	. 66033	. 45744	. 66851	. 46614	. 67656	. 47485	. 68446	. 48357	. 69222	. 49229	53
8	. 66047	. 45758	. 66865	. 46628	. 67669	. 47499	. 68459	. 48371	. 69235	. 49244	52
9	. 66061	. 45773	. 66878	. 46643	. 67682	. 47514	. 68472	. 48386	. 69248	. 49258	51
10	9. 66074	0. 45787	9. 66892	0. 46657	9. 67695	0. 47528	9. 68485	0. 48400	9. 69261	0. 49273	50
11	. 66088	. 45802	. 66905	. 46672	. 67709	. 47543	. 68498	. 48415	. 69274	. 49287	49
12	. 66102	. 45816	. 66919	. 46686	. 67722	. 47558	. 68511	. 48429	. 69286	. 49302	48
13	. 66116	. 45831	. 66932	. 46701	. 67735	. 47572	. 68524	. 48444	. 69299	. 49316	47
14	. 66129	. 45845	. 66946	. 46715	. 67748	. 47587	. 68537	. 48459	. 69312	. 49331	46
15	9. 66143	0. 45860	9. 66959	0. 46730	9. 67762	0. 47601	9. 68550	0. 48473	9. 69325	0. 49346	45
16	. 66157	. 45874	. 66973	. 46744	. 67775	. 47616	. 68563	. 48488	. 69338	. 49360	44
17	. 66170	. 45889	. 66986	. 46759	. 67788	. 47630	. 68576	. 48502	. 69350	. 49375	43
18	. 66184	. 45903	. 67000	. 46773	. 67801	. 47645	. 68589	. 48517	. 69363	. 49389	42
19	. 66198	. 45918	. 67013	. 46788	. 67815	. 47659	. 68602	. 48531	. 69376	. 49404	41
20	9. 66212	0. 45932	9. 67027	0. 46802	9. 67828	0. 47674	9. 68615	0. 48546	9. 69389	0. 49418	40
21	. 66225	. 45947	. 67040	. 46817	. 67841	. 47688	. 68628	. 48560	. 69402	. 49433	39
22	. 66239	. 45961	. 67054	. 46831	. 67854	. 47703	. 68641	. 48575	. 69414	. 49447	38
23	. 66253	. 45976	. 67067	. 46846	. 67868	. 47717	. 68654	. 48589	. 69427	. 49462	37
24	. 66266	. 45990	. 67081	. 46860	. 67881	. 47732	. 68667	. 48604	. 69440	. 49476	36
25	9. 66280	0. 46005	9. 67094	0. 46875	9. 67894	0. 47746	9. 68680	0. 48618	9. 69453	0. 49491	35
26	. 66294	. 46019	. 67108	. 46890	. 67907	. 47761	. 68693	. 48633	. 69465	. 49505	34
27	. 66307	. 46034	. 67121	. 46904	. 67920	. 47775	. 68706	. 48648	. 69478	. 49520	33
28	. 66321	. 46048	. 67134	. 46919	. 67934	. 47790	. 68719	. 48662	. 69491	. 49535	32
29	. 66335	. 46063	. 67148	. 46933	. 67947	. 47804	. 68732	. 48677	. 69504	. 49549	31
30	9. 66348	0. 46077	9. 67161	0. 46948	9. 67960	0. 47819	9. 68745	0. 48691	9. 69516	0. 49564	30
31	. 66362	. 46092	. 67175	. 46962	. 67973	. 47834	. 68758	. 48706	. 69529	. 49578	29
32	. 66376	. 46106	. 67188	. 46977	. 67986	. 47848	. 68771	. 48720	. 69542	. 49593	28
33	. 66389	. 46121	. 67202	. 46991	. 68000	. 47863	. 68784	. 48735	. 69555	. 49607	27
34	. 66403	. 46135	. 67215	. 47006	. 68013	. 47877	. 68797	. 48749	. 69567	. 49622	26
35	9. 66417	0. 46150	9. 67228	0. 47020	9. 68026	0. 47892	9. 68810	0. 48764	9. 69580	0. 49636	25
36	. 66430	. 46164	. 67242	. 47035	. 68039	. 47906	. 68823	. 48778	. 69593	. 49651	24
37	. 66444	. 46179	. 67255	. 47049	. 68052	. 47921	. 68836	. 48793	. 69605	. 49665	23
38	. 66458	. 46193	. 67269	. 47064	. 68066	. 47935	. 68849	. 48807	. 69618	. 49680	22
39	. 66471	. 46208	. 67282	. 47078	. 68079	. 47950	. 68862	. 48822	. 69631	. 49695	21
40	9. 66485	0. 46222	9. 67295	0. 47093	9. 68092	0. 47964	9. 68875	0. 48837	9. 69644	0. 49709	20
41	. 66499	. 46237	. 67309	. 47107	. 68105	. 47979	. 68887	. 48851	. 69656	. 49724	19
42	. 66512	. 46251	. 67322	. 47122	. 68118	. 47993	. 68900	. 48866	. 69669	. 49738	18
43	. 66526	. 46266	. 67336	. 47136	. 68131	. 48008	. 68913	. 48880	. 69682	. 49753	17
44	. 66539	. 46280	. 67349	. 47151	. 68144	. 48022	. 68926	. 48895	. 69694	. 49767	16
45	9. 66553	0. 46295	9. 67362	0. 47165	9. 68158	0. 48037	9. 68939	0. 48909	9. 69707	0. 49782	15
46	. 66567	. 46309	. 67376	. 47180	. 68171	. 48052	. 68952	. 48924	. 69720	. 49796	14
47	. 66580	. 46324	. 67389	. 47194	. 68184	. 48066	. 68965	. 48938	. 69732	. 49811	13
48	. 66594	. 46338	. 67402	. 47209	. 68197	. 48081	. 68978	. 48953	. 69745	. 49825	12
49	. 66607	. 46353	. 67416	. 47223	. 68210	. 48095	. 68991	. 48967	. 69758	. 49840	11
50	9. 66621	0. 46367	9. 67429	0. 47238	9. 68223	0. 48110	9. 69004	0. 48982	9. 69770	0. 49855	10
51	. 66635	. 46382	. 67443	. 47252	. 68236	. 48124	. 69017	. 48997	. 69783	. 49869	9
52	. 66648	. 46396	. 67456	. 47267	. 68249	. 48139	. 69029	. 49011	. 69796	. 49884	8
53	. 66662	. 46411	. 67469	. 47282	. 68263	. 48153	. 69042	. 49026	. 69808	. 49898	7
54	. 66675	. 46425	. 67483	. 47296	. 68276	. 48168	. 69055	. 49040	. 69821	. 49913	6
55	9. 66689	0. 46440	9. 67496	0. 47311	9. 68289	0. 48182	9. 69068	0. 49055	9. 69834	0. 49927	5
56	. 66702	. 46454	. 67509	. 47325	. 68302	. 48197	. 69081	. 49069	. 69846	. 49942	4
57	. 66716	. 46469	. 67522	. 47340	. 68315	. 48211	. 69094	. 49084	. 69859	. 49956	3
58	. 66730	. 46483	. 67536	. 47354	. 68328	. 48226	. 69107	. 49098	. 69872	. 49971	2
59	. 66743	. 46498	. 67549	. 47369	. 68341	. 48240	. 69120	. 49113	. 69884	. 49985	1
60	9. 66757	0. 46512	9. 67562	0. 47383	9. 68354	0. 48255	9. 69132	0. 49127	9. 69897	0. 50000	0
	274°		273°		272°		271°		270°		

TABLE 34

Haversines

′	90°		91°		92°		93°		94°		′
	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	
0	9. 69897	0. 50000	9. 70648	0. 50873	9. 71387	0. 51745	9. 72112	0. 52617	9. 72825	0. 53488	60
1	. 69910	. 50015	. 70661	. 50887	. 71399	. 51760	. 72124	. 52631	. 72837	. 53502	59
2	. 69922	. 50029	. 70673	. 50902	. 71411	. 51774	. 72136	. 52646	. 72849	. 53517	58
3	. 69935	. 50044	. 70686	. 50916	. 71423	. 51789	. 72148	. 52660	. 72861	. 53531	57
4	. 69948	. 50058	. 70698	. 50931	. 71436	. 51803	. 72160	. 52675	. 72873	. 53546	56
5	9. 69960	0. 50073	9. 70710	0. 50945	9. 71448	0. 51818	9. 72172	0. 52689	9. 72884	0. 53560	55
6	. 69973	. 50087	. 70723	. 50960	. 71460	. 51832	. 72184	. 52704	. 72896	. 53575	54
7	. 69985	. 50102	. 70735	. 50974	. 71472	. 51847	. 72196	. 52718	. 72908	. 53589	53
8	. 69998	. 50116	. 70748	. 50989	. 71484	. 51861	. 72208	. 52733	. 72920	. 53604	52
9	. 70011	. 50131	. 70760	. 51003	. 71496	. 51876	. 72220	. 52748	. 72931	. 53618	51
10	9. 70023	0. 50145	9. 70772	0. 51018	9. 71509	0. 51890	9. 72232	0. 52762	9. 72943	0. 53633	50
11	. 70036	. 50160	. 70785	. 51033	. 71521	. 51905	. 72244	. 52777	. 72955	. 53647	49
12	. 70048	. 50175	. 70797	. 51047	. 71533	. 51919	. 72256	. 52791	. 72967	. 53662	48
13	. 70061	. 50189	. 70809	. 51062	. 71545	. 51934	. 72268	. 52806	. 72978	. 53676	47
14	. 70074	. 50204	. 70822	. 51076	. 71557	. 51948	. 72280	. 52820	. 72990	. 53691	46
15	9. 70086	0. 50218	9. 70834	0. 51091	9. 71569	0. 51963	9. 72292	0. 52835	9. 73002	0. 53705	45
16	. 70099	. 50233	. 70847	. 51105	. 71582	. 51978	. 72304	. 52849	. 73014	. 53720	44
17	. 70111	. 50247	. 70859	. 51120	. 71594	. 51992	. 72316	. 52864	. 73025	. 53734	43
18	. 70124	. 50262	. 70871	. 51134	. 71606	. 52007	. 72328	. 52878	. 73037	. 53749	42
19	. 70136	. 50276	. 70884	. 51149	. 71618	. 52021	. 72340	. 52893	. 73049	. 53763	41
20	9. 70149	0. 50291	9. 70896	0. 51163	9. 71630	0. 52036	9. 72352	0. 52907	9. 73060	0. 53778	40
21	. 70161	. 50305	. 70908	. 51178	. 71642	. 52050	. 72363	. 52922	. 73072	. 53792	39
22	. 70174	. 50320	. 70921	. 51193	. 71654	. 52065	. 72375	. 52936	. 73084	. 53807	38
23	. 70187	. 50335	. 70933	. 51207	. 71666	. 52079	. 72387	. 52951	. 73096	. 53821	37
24	. 70199	. 50349	. 70945	. 51222	. 71679	. 52094	. 72399	. 52965	. 73107	. 53836	36
25	9. 70212	0. 50364	9. 70958	0. 51236	9. 71691	0. 52108	9. 72411	0. 52980	9. 73119	0. 53850	35
26	. 70224	. 50378	. 70970	. 51251	. 71703	. 52123	. 72423	. 52994	. 73131	. 53865	34
27	. 70237	. 50393	. 70982	. 51265	. 71715	. 52137	. 72435	. 53009	. 73142	. 53879	33
28	. 70249	. 50407	. 70995	. 51280	. 71727	. 52152	. 72447	. 53023	. 73154	. 53894	32
29	. 70262	. 50422	. 71007	. 51294	. 71739	. 52166	. 72459	. 53038	. 73166	. 53908	31
30	9. 70274	0. 50436	9. 71019	0. 51309	9. 71751	0. 52181	9. 72471	0. 53052	9. 73177	0. 53923	30
31	. 70287	. 50451	. 71032	. 51323	. 71763	. 52195	. 72482	. 53067	. 73189	. 53937	29
32	. 70299	. 50465	. 71044	. 51338	. 71775	. 52210	. 72494	. 53081	. 73201	. 53952	28
33	. 70312	. 50480	. 71056	. 51352	. 71787	. 52225	. 72506	. 53096	. 73212	. 53966	27
34	. 70324	. 50495	. 71068	. 51367	. 71800	. 52239	. 72518	. 53110	. 73224	. 53981	26
35	9. 70337	0. 50509	9. 71081	0. 51382	9. 71812	0. 52254	9. 72530	0. 53125	9. 73236	0. 53995	25
36	. 70349	. 50524	. 71093	. 51396	. 71824	. 52268	. 72542	. 53140	. 73247	. 54010	24
37	. 70362	. 50538	. 71105	. 51411	. 71836	. 52283	. 72554	. 53154	. 73259	. 54024	23
38	. 70374	. 50553	. 71118	. 51425	. 71848	. 52297	. 72565	. 53169	. 73271	. 54039	22
39	. 70387	. 50567	. 71130	. 51440	. 71860	. 52312	. 72577	. 53183	. 73282	. 54053	21
40	9. 70399	0. 50582	9. 71142	0. 51454	9. 71872	0. 52326	9. 72589	0. 53198	9. 73294	0. 54068	20
41	. 70412	. 50596	. 71154	. 51469	. 71884	. 52341	. 72601	. 53212	. 73306	. 54082	19
42	. 70424	. 50611	. 71167	. 51483	. 71896	. 52355	. 72613	. 53227	. 73317	. 54097	18
43	. 70437	. 50625	. 71179	. 51498	. 71908	. 52370	. 72625	. 53241	. 73329	. 54111	17
44	. 70449	. 50640	. 71191	. 51512	. 71920	. 52384	. 72637	. 53256	. 73341	. 54126	16
45	9. 70462	0. 50654	9. 71203	0. 51527	9. 71932	0. 52399	9. 72648	0. 53270	9. 73352	0. 54140	15
46	. 70474	. 50669	. 71216	. 51541	. 71944	. 52413	. 72660	. 53285	. 73364	. 54155	14
47	. 70487	. 50684	. 71228	. 51556	. 71956	. 52428	. 72672	. 53299	. 73375	. 54169	13
48	. 70499	. 50698	. 71240	. 51571	. 71968	. 52442	. 72684	. 53314	. 73387	. 54184	12
49	. 70512	. 50713	. 71252	. 51585	. 71980	. 52457	. 72696	. 53328	. 73399	. 54198	11
50	9. 70524	0. 50727	9. 71265	0. 51600	9. 71992	0. 52472	9. 72708	0. 53343	9. 73410	0. 54213	10
51	. 70537	. 50742	. 71277	. 51614	. 72004	. 52486	. 72719	. 53357	. 73422	. 54227	9
52	. 70549	. 50756	. 71289	. 51629	. 72016	. 52501	. 72731	. 53372	. 73433	. 54242	8
53	. 70561	. 50771	. 71301	. 51643	. 72028	. 52515	. 72743	. 53386	. 73445	. 54256	7
54	. 70574	. 50785	. 71314	. 51658	. 72040	. 52530	. 72755	. 53401	. 73457	. 54271	6
55	9. 70586	0. 50800	9. 71326	0. 51672	9. 72052	0. 52544	9. 72767	0. 53415	9. 73468	0. 54285	5
56	. 70599	. 50814	. 71338	. 51687	. 72064	. 52559	. 72778	. 53430	. 73480	. 54300	4
57	. 70611	. 50829	. 71350	. 51701	. 72076	. 52573	. 72790	. 53444	. 73491	. 54314	3
58	. 70624	. 50844	. 71362	. 51716	. 72088	. 52588	. 72802	. 53459	. 73503	. 54329	2
59	. 70636	. 50858	. 71375	. 51730	. 72100	. 52602	. 72814	. 53473	. 73515	. 54343	1
60	9. 70648	0. 50873	9. 71387	0. 51745	9. 72112	0. 52617	9. 72825	0. 53488	9. 73526	0. 54358	0
	269°		268°		267°		266°		265°		

TABLE 34

Haversines

	95°		96°		97°		98°		99°		
	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	′
0	9. 73526	0. 54358	9. 74215	0. 55226	9. 74891	0. 56093	9. 75556	0. 56959	9. 76209	0. 57822	60
1	. 73538	. 54372	. 74226	. 55241	. 74902	. 56108	. 75567	. 56973	. 76220	. 57836	59
2	. 73549	. 54387	. 74237	. 55255	. 74914	. 56122	. 75578	. 56987	. 76231	. 57850	58
3	. 73561	. 54401	. 74249	. 55270	. 74925	. 56137	. 75589	. 57002	. 76241	. 57865	57
4	. 73572	. 54416	. 74260	. 55284	. 74936	. 56151	. 75600	. 57016	. 76252	. 57879	56
5	9. 73584	0. 54430	9. 74272	0. 55299	9. 74947	0. 56166	9. 75611	0. 57031	9. 76263	0. 57894	55
6	. 73596	. 54445	. 74283	. 55313	. 74958	. 56180	. 75622	. 57045	. 76274	. 57908	54
7	. 73607	. 54459	. 74294	. 55328	. 74969	. 56195	. 75633	. 57059	. 76285	. 57922	53
8	. 73619	. 54474	. 74306	. 55342	. 74981	. 56209	. 75644	. 57074	. 76296	. 57937	52
9	. 73630	. 54488	. 74317	. 55357	. 74992	. 56223	. 75655	. 57088	. 76306	. 57951	51
10	9. 73642	0. 54503	9. 74328	0. 55371	9. 75003	0. 56238	9. 75666	0. 57103	9. 76317	0. 57965	50
11	. 73653	. 54517	. 74340	. 55386	. 75014	. 56252	. 75677	. 57117	. 76328	. 57980	49
12	. 73665	. 54532	. 74351	. 55400	. 75025	. 56267	. 75688	. 57131	. 76338	. 57994	48
13	. 73676	. 54546	. 74362	. 55414	. 75036	. 56281	. 75698	. 57146	. 76349	. 58008	47
14	. 73688	. 54561	. 74374	. 55429	. 75047	. 56296	. 75709	. 57160	. 76360	. 58023	46
15	9. 73699	0. 54575	9. 74385	0. 55443	9. 75059	0. 56310	9. 75720	0. 57175	9. 76371	0. 58037	45
16	. 73711	. 54590	. 74396	. 55458	. 75070	. 56324	. 75731	. 57189	. 76381	. 58051	44
17	. 73722	. 54604	. 74408	. 55472	. 75081	. 56339	. 75742	. 57203	. 76392	. 58066	43
18	. 73734	. 54619	. 74419	. 55487	. 75092	. 56353	. 75753	. 57218	. 76403	. 58080	42
19	. 73746	. 54633	. 74430	. 55501	. 75103	. 56368	. 75764	. 57232	. 76414	. 58095	41
20	9. 73757	0. 54647	9. 74442	0. 55516	9. 75114	0. 56382	9. 75775	0. 57247	9. 76424	0. 58109	40
21	. 73769	. 54662	. 74453	. 55530	. 75125	. 56397	. 75786	. 57261	. 76435	. 58123	39
22	. 73780	. 54676	. 74464	. 55545	. 75136	. 56411	. 75797	. 57275	. 76446	. 58138	38
23	. 73792	. 54691	. 74475	. 55559	. 75147	. 56425	. 75808	. 57290	. 76456	. 58152	37
24	. 73803	. 54705	. 74487	. 55573	. 75159	. 56440	. 75819	. 57304	. 76467	. 58166	36
25	9. 73815	0. 54720	9. 74498	0. 55588	9. 75170	0. 56454	9. 75830	0. 57319	9. 76478	0. 58181	35
26	. 73826	. 54734	. 74509	. 55602	. 75181	. 56469	. 75840	. 57333	. 76489	. 58195	34
27	. 73838	. 54749	. 74521	. 55617	. 75192	. 56483	. 75851	. 57347	. 76499	. 58209	33
28	. 73849	. 54763	. 74532	. 55631	. 75203	. 56497	. 75862	. 57362	. 76510	. 58224	32
29	. 73860	. 54778	. 74543	. 55646	. 75214	. 56512	. 75873	. 57376	. 76521	. 58238	31
30	9. 73872	0. 54792	9. 74554	0. 55660	9. 75225	0. 56526	9. 75884	0. 57390	9. 76531	0. 58252	30
31	. 73883	. 54807	. 74566	. 55675	. 75236	. 56541	. 75895	. 57405	. 76542	. 58267	29
32	. 73895	. 54821	. 74577	. 55689	. 75247	. 56555	. 75906	. 57419	. 76553	. 58281	28
33	. 73906	. 54836	. 74588	. 55704	. 75258	. 56570	. 75917	. 57434	. 76563	. 58295	27
34	. 73918	. 54850	. 74600	. 55718	. 75269	. 56584	. 75927	. 57448	. 76574	. 58310	26
35	9. 73929	0. 54865	9. 74611	0. 55732	9. 75280	0. 56598	9. 75938	0. 57462	9. 76585	0. 58324	25
36	. 73941	. 54879	. 74622	. 55747	. 75291	. 56613	. 75949	. 57477	. 76595	. 58338	24
37	. 73952	. 54894	. 74633	. 55761	. 75303	. 56627	. 75960	. 57491	. 76606	. 58353	23
38	. 73964	. 54908	. 74645	. 55776	. 75314	. 56642	. 75971	. 57506	. 76617	. 58367	22
39	. 73975	. 54923	. 74656	. 55790	. 75325	. 56656	. 75982	. 57520	. 76627	. 58381	21
40	9. 73987	0. 54937	9. 74667	0. 55805	9. 75336	0. 56670	9. 75993	0. 57534	9. 76638	0. 58396	20
41	. 73998	. 54952	. 74678	. 55819	. 75347	. 56685	. 76004	. 57549	. 76649	. 58410	19
42	. 74009	. 54966	. 74690	. 55834	. 75358	. 56699	. 76014	. 57563	. 76659	. 58424	18
43	. 74021	. 54980	. 74701	. 55848	. 75369	. 56714	. 76025	. 57577	. 76670	. 58439	17
44	. 74032	. 54995	. 74712	. 55862	. 75380	. 56728	. 76036	. 57592	. 76681	. 58453	16
45	9. 74044	0. 55009	9. 74723	0. 55877	9. 75391	0. 56743	9. 76047	0. 57606	9. 76691	0. 58467	15
46	. 74055	. 55024	. 74734	. 55891	. 75402	. 56757	. 76058	. 57621	. 76702	. 58482	14
47	. 74067	. 55038	. 74746	. 55906	. 75413	. 56771	. 76069	. 57635	. 76713	. 58496	13
48	. 74078	. 55053	. 74757	. 55920	. 75424	. 56786	. 76079	. 57649	. 76723	. 58510	12
49	. 74089	. 55067	. 74768	. 55935	. 75435	. 56800	. 76090	. 57664	. 76734	. 58525	11
50	9. 74101	0. 55082	9. 74779	0. 55949	9. 75446	0. 56815	9. 76101	0. 57678	9. 76745	0. 58539	10
51	. 74112	. 55096	. 74791	. 55964	. 75457	. 56829	. 76112	. 57692	. 76755	. 58553	9
52	. 74124	. 55111	. 74802	. 55978	. 75468	. 56843	. 76123	. 57707	. 76766	. 58568	8
53	. 74135	. 55125	. 74813	. 55992	. 75479	. 56858	. 76134	. 57721	. 76777	. 58582	7
54	. 74146	. 55140	. 74824	. 56007	. 75490	. 56872	. 76144	. 57736	. 76787	. 58596	6
55	9. 74158	0. 55154	9. 74835	0. 56021	9. 75501	0. 56887	9. 76155	0. 57750	9. 76798	0. 58611	5
56	. 74169	. 55169	. 74846	. 56036	. 75512	. 56901	. 76166	. 57764	. 76808	. 58625	4
57	. 74181	. 55183	. 74858	. 56050	. 75523	. 56915	. 76177	. 57779	. 76819	. 58639	3
58	. 74192	. 55197	. 74869	. 56065	. 75534	. 56930	. 76188	. 57793	. 76830	. 58654	2
59	. 74203	. 55212	. 74880	. 56079	. 75545	. 56944	. 76198	. 57807	. 76840	. 58668	1
60	9. 74215	0. 55226	9. 74891	0. 56093	9. 75556	0. 56959	9. 76209	0. 57822	9. 76851	0. 58682	0
	264°		263°		262°		261°		260°		

TABLE 34

Haversines

′	100°		101°		102°		103°		104°		′
	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	
0	9. 76851	0. 58682	9. 77481	0. 59540	9. 78101	0. 60396	9. 78709	0. 61248	9. 79306	0. 62096	60
1	. 76861	. 58697	. 77492	. 59555	. 78111	. 60410	. 78719	. 61262	. 79316	. 62110	59
2	. 76872	. 58711	. 77502	. 59569	. 78121	. 60424	. 78729	. 61276	. 79326	. 62124	58
3	. 76883	. 58725	. 77512	. 59583	. 78131	. 60438	. 78739	. 61290	. 79336	. 62138	57
4	. 76893	. 58740	. 77523	. 59598	. 78141	. 60452	. 78749	. 61304	. 79346	. 62153	56
5	9. 76904	0. 58754	9. 77533	0. 59612	9. 78152	0. 60467	9. 78759	0. 61318	9. 79356	0. 62167	55
6	. 76914	. 58768	. 77544	. 59626	. 78162	. 60481	. 78769	. 61333	. 79366	. 62181	54
7	. 76925	. 58783	. 77554	. 59640	. 78172	. 60495	. 78779	. 61347	. 79376	. 62195	53
8	. 76936	. 58797	. 77564	. 59655	. 78182	. 60509	. 78789	. 61361	. 79385	. 62209	52
9	. 76946	. 58811	. 77575	. 59669	. 78192	. 60524	. 78799	. 61375	. 79395	. 62223	51
10	9. 76957	0. 58826	9. 77585	0. 59683	9. 78203	0. 60538	9. 78809	0. 61389	9. 79405	0. 62237	50
11	. 76967	. 58840	. 77596	. 59697	. 78213	. 60552	. 78819	. 61403	. 79415	. 62251	49
12	. 76978	. 58854	. 77606	. 59712	. 78223	. 60566	. 78829	. 61418	. 79425	. 62265	48
13	. 76988	. 58869	. 77616	. 59726	. 78233	. 60580	. 78839	. 61432	. 79434	. 62279	47
14	. 76999	. 58883	. 77627	. 59740	. 78243	. 60595	. 78849	. 61446	. 79444	. 62294	46
15	9. 77009	0. 58897	9. 77637	0. 59755	9. 78254	0. 60609	9. 78859	0. 61460	9. 79454	0. 62308	45
16	. 77020	. 58911	. 77647	. 59769	. 78264	. 60623	. 78869	. 61474	. 79464	. 62322	44
17	. 77031	. 58926	. 77658	. 59783	. 78274	. 60637	. 78879	. 61488	. 79474	. 62336	43
18	. 77041	. 58940	. 77668	. 59797	. 78284	. 60652	. 78889	. 61502	. 79484	. 62350	42
19	. 77052	. 58954	. 77679	. 59812	. 78294	. 60666	. 78899	. 61517	. 79493	. 62364	41
20	9. 77062	0. 58969	9. 77689	0. 59826	9. 78305	0. 60680	9. 78909	0. 61531	9. 79503	0. 62378	40
21	. 77073	. 58983	. 77699	. 59840	. 78315	. 60694	. 78919	. 61545	. 79513	. 62392	39
22	. 77083	. 58997	. 77710	. 59854	. 78325	. 60708	. 78929	. 61559	. 79523	. 62406	38
23	. 77094	. 59012	. 77720	. 59869	. 78335	. 60723	. 78939	. 61573	. 79533	. 62420	37
24	. 77104	. 59026	. 77730	. 59883	. 78345	. 60737	. 78949	. 61587	. 79542	. 62434	36
25	9. 77115	0. 59040	9. 77741	0. 59897	9. 78355	0. 60751	9. 78959	0. 61602	9. 79552	0. 62449	35
26	. 77125	. 59055	. 77751	. 59911	. 78365	. 60765	. 78969	. 61616	. 79562	. 62463	34
27	. 77136	. 59069	. 77761	. 59926	. 78376	. 60779	. 78979	. 61630	. 79572	. 62477	33
28	. 77146	. 59083	. 77772	. 59940	. 78386	. 60794	. 78989	. 61644	. 79582	. 62491	32
29	. 77157	. 59097	. 77782	. 59954	. 78396	. 60808	. 78999	. 61658	. 79591	. 62505	31
30	9. 77167	0. 59112	9. 77792	0. 59968	9. 78406	0. 60822	9. 79009	0. 61672	9. 79601	0. 62519	30
31	. 77178	. 59126	. 77803	. 59983	. 78416	. 60836	. 79019	. 61686	. 79611	. 62533	29
32	. 77188	. 59140	. 77813	. 59997	. 78426	. 60850	. 79029	. 61701	. 79621	. 62547	28
33	. 77199	. 59155	. 77823	. 60011	. 78436	. 60865	. 79039	. 61715	. 79631	. 62561	27
34	. 77209	. 59169	. 77834	. 60025	. 78447	. 60879	. 79049	. 61729	. 79640	. 62575	26
35	9. 77220	0. 59183	9. 77844	0. 60040	9. 78457	0. 60893	9. 79059	0. 61743	9. 79650	0. 62589	25
36	. 77230	. 59198	. 77854	. 60054	. 78467	. 60907	. 79069	. 61757	. 79660	. 62603	24
37	. 77241	. 59212	. 77864	. 60068	. 78477	. 60921	. 79079	. 61771	. 79670	. 62618	23
38	. 77251	. 59226	. 77875	. 60082	. 78487	. 60936	. 79089	. 61785	. 79679	. 62632	22
39	. 77262	. 59240	. 77885	. 60097	. 78497	. 60950	. 79099	. 61800	. 79689	. 62646	21
40	9. 77272	0. 59255	9. 77895	0. 60111	9. 78507	0. 60964	9. 79108	0. 61814	9. 79699	0. 62660	20
41	. 77283	. 59269	. 77906	. 60125	. 78517	. 60978	. 79118	. 61828	. 79709	. 62674	19
42	. 77293	. 59283	. 77916	. 60139	. 78528	. 60992	. 79128	. 61842	. 79718	. 62688	18
43	. 77304	. 59298	. 77926	. 60154	. 78538	. 61006	. 79138	. 61856	. 79728	. 62702	17
44	. 77314	. 59312	. 77936	. 60168	. 78548	. 61021	. 79148	. 61870	. 79738	. 62716	16
45	9. 77325	0. 59326	9. 77947	0. 60182	9. 78558	0. 61035	9. 79158	0. 61884	9. 79748	0. 62730	15
46	. 77335	. 59340	. 77957	. 60196	. 78568	. 61049	. 79168	. 61898	. 79757	. 62744	14
47	. 77346	. 59355	. 77967	. 60211	. 78578	. 61063	. 79178	. 61913	. 79767	. 62758	13
48	. 77356	. 59369	. 77978	. 60225	. 78588	. 61077	. 79188	. 61927	. 79777	. 62772	12
49	. 77366	. 59383	. 77988	. 60239	. 78598	. 61092	. 79198	. 61941	. 79787	. 62786	11
50	9. 77377	0. 59398	9. 77998	0. 60253	9. 78608	0. 61106	9. 79208	0. 61955	9. 79796	0. 62800	10
51	. 77387	. 59412	. 78008	. 60268	. 78618	. 61120	. 79217	. 61969	. 79806	. 62814	9
52	. 77398	. 59426	. 78019	. 60282	. 78628	. 61134	. 79227	. 61983	. 79816	. 62829	8
53	. 77408	. 59440	. 78029	. 60296	. 78638	. 61148	. 79237	. 61997	. 79825	. 62843	7
54	. 77419	. 59455	. 78039	. 60310	. 78649	. 61163	. 79247	. 62011	. 79835	. 62857	6
55	9. 77429	0. 59469	9. 78049	0. 60324	9. 78659	0. 61177	9. 79257	0. 62026	9. 79845	0. 62871	5
56	. 77440	. 59483	. 78060	. 60339	. 78669	. 61191	. 79267	. 62040	. 79855	. 62885	4
57	. 77450	. 59498	. 78070	. 60353	. 78679	. 61205	. 79277	. 62054	. 79864	. 62899	3
58	. 77460	. 59512	. 78080	. 60367	. 78689	. 61219	. 79287	. 62068	. 79874	. 62913	2
59	. 77471	. 59526	. 78090	. 60381	. 78699	. 61233	. 79297	. 62082	. 79884	. 62927	1
60	9. 77481	0. 59540	9. 78101	0. 60396	9. 78709	0. 61248	9. 79306	0. 62096	9. 79893	0. 62941	0
	259°		258°		257°		256°		255°		

TABLE 34

Haversines

′	105° Log Hav	105° Nat. Hav	106° Log Hav	106° Nat. Hav	107° Log Hav	107° Nat. Hav	108° Log Hav	108° Nat. Hav	109° Log Hav	109° Nat. Hav	′
0	9.79893	0.62941	9.80470	0.63782	9.81036	0.64619	9.81592	0.65451	9.82137	0.66278	60
1	.79903	.62955	.80479	.63796	.81045	.64632	.81601	.65465	.82146	.66292	59
2	.79913	.62969	.80489	.63810	.81054	.64646	.81610	.65479	.82155	.66306	58
3	.79922	.62983	.80498	.63824	.81064	.64660	.81619	.65492	.82164	.66320	57
4	.79932	.62997	.80508	.63838	.81073	.64674	.81628	.65506	.82173	.66333	56
5	9.79942	0.63011	9.80517	0.63852	9.81082	0.64688	9.81637	0.65520	9.82182	0.66347	55
6	.79951	.63025	.80527	.63866	.81092	.64702	.81647	.65534	.82191	.66361	54
7	.79961	.63039	.80536	.63880	.81101	.64716	.81656	.65548	.82200	.66375	53
8	.79971	.63053	.80546	.63894	.81110	.64730	.81665	.65561	.82209	.66388	52
9	.79980	.63067	.80555	.63908	.81120	.64744	.81674	.65575	.82218	.66402	51
10	9.79990	0.63081	9.80565	0.63922	9.81129	0.64758	9.81683	0.65589	9.82227	0.66416	50
11	.80000	.63095	.80574	.63936	.81138	.64772	.81692	.65603	.82236	.66430	49
12	.80009	.63109	.80584	.63950	.81148	.64785	.81701	.65617	.82245	.66443	48
13	.80019	.63123	.80593	.63964	.81157	.64799	.81711	.65631	.82254	.66457	47
14	.80029	.63138	.80603	.63977	.81166	.64813	.81720	.65644	.82263	.66471	46
15	9.80038	0.63152	9.80612	0.63991	9.81176	0.64827	9.81729	0.65658	9.82272	0.66485	45
16	.80048	.63166	.80622	.64005	.81185	.64841	.81738	.65672	.82281	.66498	44
17	.80058	.63180	.80631	.64019	.81194	.64855	.81747	.65686	.82290	.66512	43
18	.80067	.63194	.80641	.64033	.81204	.64869	.81756	.65700	.82299	.66526	42
19	.80077	.63208	.80650	.64047	.81213	.64883	.81765	.65713	.82308	.66539	41
20	9.80087	0.63222	9.80660	0.64061	9.81222	0.64897	9.81775	0.65727	9.82317	0.66553	40
21	.80096	.63236	.80669	.64075	.81231	.64910	.81784	.65741	.82326	.66567	39
22	.80106	.63250	.80678	.64089	.81241	.64924	.81793	.65755	.82335	.66581	38
23	.80116	.63264	.80688	.64103	.81250	.64938	.81802	.65769	.82344	.66594	37
24	.80125	.63278	.80697	.64117	.81259	.64952	.81811	.65782	.82353	.66608	36
25	9.80135	0.63292	9.80707	0.64131	9.81269	0.64966	9.81820	0.65796	9.82362	0.66622	35
26	.80144	.63306	.80716	.64145	.81278	.64980	.81829	.65810	.82371	.66635	34
27	.80154	.63320	.80726	.64159	.81287	.64994	.81838	.65824	.82380	.66649	33
28	.80164	.63334	.80735	.64173	.81296	.65008	.81847	.65838	.82388	.66663	32
29	.80173	.63348	.80745	.64187	.81306	.65021	.81857	.65851	.82397	.66677	31
30	9.80183	0.63362	9.80754	0.64201	9.81315	0.65035	9.81866	0.65865	9.82406	0.66690	30
31	.80192	.63376	.80763	.64215	.81324	.65049	.81875	.65879	.82415	.66704	29
32	.80202	.63390	.80773	.64229	.81333	.65063	.81884	.65893	.82424	.66718	28
33	.80212	.63404	.80782	.64243	.81343	.65077	.81893	.65907	.82433	.66731	27
34	.80221	.63418	.80792	.64257	.81352	.65091	.81902	.65920	.82442	.66745	26
35	9.80231	0.63432	9.80801	0.64270	9.81361	0.65105	9.81911	0.65934	9.82451	0.66759	25
36	.80240	.63446	.80811	.64284	.81370	.65118	.81920	.65948	.82460	.66773	24
37	.80250	.63460	.80820	.64298	.81380	.65132	.81929	.65962	.82469	.66786	23
38	.80260	.63474	.80829	.64312	.81389	.65146	.81938	.65976	.82478	.66800	22
39	.80269	.63488	.80839	.64326	.81398	.65160	.81947	.65989	.82487	.66814	21
40	9.80279	0.63502	9.80848	0.64340	9.81407	0.65174	9.81956	0.66003	9.82495	0.66827	20
41	.80288	.63516	.80858	.64354	.81417	.65188	.81965	.66017	.82504	.66841	19
42	.80298	.63530	.80867	.64368	.81426	.65202	.81975	.66031	.82513	.66855	18
43	.80307	.63544	.80876	.64382	.81435	.65216	.81984	.66044	.82522	.66868	17
44	.80317	.63558	.80886	.64396	.81444	.65229	.81993	.66058	.82531	.66882	16
45	9.80327	0.63572	9.80895	0.64410	9.81454	0.65243	9.82002	0.66072	9.82540	0.66896	15
46	.80336	.63586	.80905	.64424	.81463	.65257	.82011	.66086	.82549	.66910	14
47	.80346	.63600	.80914	.64438	.81472	.65271	.82020	.66100	.82558	.66923	13
48	.80355	.63614	.80923	.64452	.81481	.65285	.82029	.66113	.82567	.66937	12
49	.80365	.63628	.80933	.64466	.81490	.65299	.82038	.66127	.82575	.66951	11
50	9.80374	0.63642	9.80942	0.64479	9.81500	0.65312	9.82047	0.66141	9.82584	0.66964	10
51	.80384	.63656	.80952	.64493	.81509	.65326	.82056	.66155	.82593	.66978	9
52	.80393	.63670	.80961	.64507	.81518	.65340	.82065	.66168	.82602	.66992	8
53	.80403	.63684	.80970	.64521	.81527	.65354	.82074	.66182	.82611	.67005	7
54	.80413	.63698	.80980	.64535	.81536	.65368	.82083	.66196	.82620	.67019	6
55	9.80422	0.63712	9.80989	0.64549	9.81546	0.65382	9.82092	0.66210	9.82629	0.67033	5
56	.80432	.63726	.80998	.64563	.81555	.65396	.82101	.66223	.82638	.67046	4
57	.80441	.63740	.81008	.64577	.81564	.65409	.82110	.66237	.82646	.67060	3
58	.80451	.63754	.81017	.64591	.81573	.65423	.82119	.66251	.82655	.67074	2
59	.80460	.63768	.81026	.64605	.81582	.65437	.82128	.66265	.82664	.67087	1
60	9.80470	0.63782	9.81036	0.64619	9.81592	0.65451	9.82137	0.66278	9.82673	0.67101	0
	254°		253°		252°		251°		250°		

TABLE 34

Haversines

′	110° Log Hav	110° Nat. Hav	111° Log Hav	111° Nat. Hav	112° Log Hav	112° Nat. Hav	113° Log Hav	113° Nat. Hav	114° Log Hav	114° Nat. Hav	′
0	9. 82673	0. 67101	9. 83199	0. 67918	9. 83715	0. 68730	9. 84221	0. 69537	9. 84718	0. 70337	60
1	. 82682	. 67115	. 83207	. 67932	. 83723	. 68744	. 84230	. 69550	. 84726	. 70350	59
2	. 82691	. 67128	. 83216	. 67946	. 83732	. 68757	. 84238	. 69563	. 84735	. 70363	58
3	. 82699	. 67142	. 83225	. 67959	. 83740	. 68771	. 84246	. 69577	. 84743	. 70377	57
4	. 82708	. 67156	. 83233	. 67973	. 83749	. 68784	. 84255	. 69590	. 84751	. 70390	56
5	9. 82717	0. 67169	9. 83242	0. 67986	9. 83757	0. 68798	9. 84263	0. 69603	9. 84759	0. 70403	55
6	. 82726	. 67183	. 83251	. 68000	. 83766	. 68811	. 84271	. 69617	. 84767	. 70417	54
7	. 82735	. 67197	. 83259	. 68013	. 83774	. 68825	. 84280	. 69630	. 84776	. 70430	53
8	. 82744	. 67210	. 83268	. 68027	. 83783	. 68838	. 84288	. 69644	. 84784	. 70443	52
9	. 82752	. 67224	. 83277	. 68041	. 83791	. 68852	. 84296	. 69657	. 84792	. 70456	51
10	9. 82761	0. 67238	9. 83285	0. 68054	9. 83800	0. 68865	9. 84305	0. 69670	9. 84800	0. 70470	50
11	. 82770	. 67251	. 83294	. 68068	. 83808	. 68879	. 84313	. 69684	. 84808	. 70483	49
12	. 82779	. 67265	. 83303	. 68081	. 83817	. 68892	. 84321	. 69697	. 84817	. 70496	48
13	. 82788	. 67279	. 83311	. 68095	. 83825	. 68906	. 84330	. 69710	. 84825	. 70509	47
14	. 82796	. 67292	. 83320	. 68108	. 83834	. 68919	. 84338	. 69724	. 84833	. 70523	46
15	9. 82805	0. 67306	9. 83329	0. 68122	9. 83842	0. 68932	9. 84346	0. 69737	9. 84841	0. 70536	45
16	. 82814	. 67319	. 83337	. 68135	. 83851	. 68946	. 84355	. 69751	. 84849	. 70549	44
17	. 82823	. 67333	. 83346	. 68149	. 83859	. 68959	. 84363	. 69764	. 84857	. 70562	43
18	. 82832	. 67347	. 83355	. 68163	. 83868	. 68973	. 84371	. 69777	. 84866	. 70576	42
19	. 82840	. 67360	. 83363	. 68176	. 83876	. 68986	. 84380	. 69791	. 84874	. 70589	41
20	9. 82849	0. 67374	9. 83372	0. 68190	9. 83885	0. 69000	9. 84388	0. 69804	9. 84882	0. 70602	40
21	. 82858	. 67388	. 83380	. 68203	. 83893	. 69013	. 84396	. 69817	. 84890	. 70615	39
22	. 82867	. 67401	. 83389	. 68217	. 83902	. 69027	. 84405	. 69831	. 84898	. 70629	38
23	. 82876	. 67415	. 83398	. 68230	. 83910	. 69040	. 84413	. 69844	. 84906	. 70642	37
24	. 82884	. 67429	. 83406	. 68244	. 83919	. 69054	. 84421	. 69857	. 84914	. 70655	36
25	9. 82893	0. 67442	9. 83415	0. 68257	9. 83927	0. 69067	9. 84430	0. 69871	9. 84923	0. 70668	35
26	. 82902	. 67456	. 83424	. 68271	. 83935	. 69080	. 84438	. 69884	. 84931	. 70682	34
27	. 82911	. 67469	. 83432	. 68284	. 83944	. 69094	. 84446	. 69897	. 84939	. 70695	33
28	. 82920	. 67483	. 83441	. 68298	. 83952	. 69107	. 84454	. 69911	. 84947	. 70708	32
29	. 82928	. 67497	. 83449	. 68312	. 83961	. 69121	. 84463	. 69924	. 84955	. 70721	31
30	9. 82937	0. 67510	9. 83458	0. 68325	9. 83969	0. 69134	9. 84471	0. 69937	9. 84963	0. 70735	30
31	. 82946	. 67524	. 83467	. 68339	. 83978	. 69148	. 84479	. 69951	. 84971	. 70748	29
32	. 82955	. 67538	. 83475	. 68352	. 83986	. 69161	. 84488	. 69964	. 84979	. 70761	28
33	. 82963	. 67551	. 83484	. 68366	. 83995	. 69174	. 84496	. 69977	. 84988	. 70774	27
34	. 82972	. 67565	. 83492	. 68379	. 84003	. 69188	. 84504	. 69991	. 84996	. 70788	26
35	9. 82981	0. 67578	9. 83501	0. 68393	9. 84011	0. 69201	9. 84512	0. 70004	9. 85004	0. 70801	25
36	. 82990	. 67592	. 83510	. 68406	. 84020	. 69215	. 84521	. 70017	. 85012	. 70814	24
37	. 82998	. 67606	. 83518	. 68420	. 84028	. 69228	. 84529	. 70031	. 85020	. 70827	23
38	. 83007	. 67619	. 83527	. 68433	. 84037	. 69242	. 84537	. 70044	. 85028	. 70840	22
39	. 83016	. 67633	. 83535	. 68447	. 84045	. 69255	. 84545	. 70057	. 85036	. 70854	21
40	9. 83025	0. 67647	9. 83544	0. 68460	9. 84054	0. 69268	9. 84554	0. 70071	9. 85044	0. 70867	20
41	. 83033	. 67660	. 83552	. 68474	. 84062	. 69282	. 84562	. 70084	. 85052	. 70880	19
42	. 83042	. 67674	. 83561	. 68487	. 84070	. 69295	. 84570	. 70097	. 85061	. 70893	18
43	. 83051	. 67687	. 83570	. 68501	. 84079	. 69309	. 84578	. 70111	. 85069	. 70907	17
44	. 83059	. 67701	. 83578	. 68514	. 84087	. 69322	. 84587	. 70124	. 85077	. 70920	16
45	9. 83068	0. 67715	9. 83587	0. 68528	9. 84096	0. 69336	9. 84595	0. 70137	9. 85085	0. 70933	15
46	. 83077	. 67728	. 83595	. 68541	. 84104	. 69349	. 84603	. 70151	. 85093	. 70946	14
47	. 83086	. 67742	. 83604	. 68555	. 84112	. 69362	. 84611	. 70164	. 85101	. 70959	13
48	. 83094	. 67755	. 83612	. 68568	. 84121	. 69376	. 84620	. 70177	. 85109	. 70973	12
49	. 83103	. 67769	. 83621	. 68582	. 84129	. 69389	. 84628	. 70191	. 85117	. 70986	11
50	9. 83112	0. 67783	9. 83630	0. 68595	9. 84138	0. 69403	9. 84636	0. 70204	9. 85125	0. 70999	10
51	. 83120	. 67796	. 83638	. 68609	. 84146	. 69416	. 84644	. 70217	. 85133	. 71012	9
52	. 83129	. 67810	. 83647	. 68622	. 84154	. 69429	. 84653	. 70230	. 85141	. 71025	8
53	. 83138	. 67823	. 83655	. 68636	. 84163	. 69443	. 84661	. 70244	. 85149	. 71039	7
54	. 83147	. 67837	. 83664	. 68649	. 84171	. 69456	. 84669	. 70257	. 85158	. 71052	6
55	9. 83155	0. 67850	9. 83672	0. 68663	9. 84179	0. 69470	9. 84677	0. 70270	9. 85166	0. 71065	5
56	. 83164	. 67864	. 83681	. 68676	. 84188	. 69483	. 84685	. 70284	. 85174	. 71078	4
57	. 83173	. 67878	. 83689	. 68690	. 84196	. 69496	. 84694	. 70297	. 85182	. 71091	3
58	. 83181	. 67891	. 83698	. 68703	. 84205	. 69510	. 84702	. 70310	. 85190	. 71105	2
59	. 83190	. 67905	. 83706	. 68717	. 84213	. 69523	. 84710	. 70324	. 85198	. 71118	1
60	9. 83199	0. 67918	9. 83715	0. 68730	9. 84221	0. 69537	9. 84718	0. 70337	9. 85206	0. 71131	0
	249°		248°		247°		246°		245°		

TABLE 34

Haversines

′	115° Log Hav	115° Nat. Hav	116° Log Hav	116° Nat. Hav	117° Log Hav	117° Nat. Hav	118° Log Hav	118° Nat. Hav	119° Log Hav	119° Nat. Hav	′
0	9. 85206	0. 71131	9. 85684	0. 71919	9. 86153	0. 72700	9. 86613	0. 73474	9. 87064	0. 74240	60
1	. 85214	. 71144	. 85692	. 71932	. 86161	. 72712	. 86621	. 73486	. 87072	. 74253	59
2	. 85222	. 71157	. 85700	. 71945	. 86169	. 72725	. 86628	. 73499	. 87079	. 74266	58
3	. 85230	. 71170	. 85708	. 71958	. 86176	. 72738	. 86636	. 73512	. 87086	. 74279	57
4	. 85238	. 71184	. 85716	. 71971	. 86184	. 72751	. 86643	. 73525	. 87094	. 74291	56
5	9. 85246	0. 71197	9. 85724	0. 71984	9. 86192	0. 72764	9. 86651	0. 73538	9. 87101	0. 74304	55
6	. 85254	. 71210	. 85731	. 71997	. 86200	. 72777	. 86659	. 73551	. 87109	. 74317	54
7	. 85262	. 71223	. 85739	. 72010	. 86207	. 72790	. 86666	. 73563	. 87116	. 74329	53
8	. 85270	. 71236	. 85747	. 72023	. 86215	. 72803	. 86674	. 73576	. 87124	. 74342	52
9	. 85278	. 71249	. 85755	. 72036	. 86223	. 72816	. 86681	. 73589	. 87131	. 74355	51
10	9. 85286	0. 71263	9. 85763	0. 72049	9. 86230	0. 72829	9. 86689	0. 73602	9. 87138	0. 74368	50
11	. 85294	. 71276	. 85771	. 72062	. 86238	. 72842	. 86696	. 73615	. 87146	. 74380	49
12	. 85302	. 71289	. 85779	. 72075	. 86246	. 72855	. 86704	. 73628	. 87153	. 74393	48
13	. 85310	. 71302	. 85787	. 72088	. 86254	. 72868	. 86712	. 73640	. 87161	. 74406	47
14	. 85318	. 71315	. 85794	. 72101	. 86261	. 72881	. 86719	. 73653	. 87168	. 74418	46
15	9. 85326	0. 71328	9. 85802	0. 72114	9. 86269	0. 72894	9. 86727	0. 73666	9. 87175	0. 74431	45
16	. 85334	. 71342	. 85810	. 72127	. 86277	. 72907	. 86734	. 73679	. 87183	. 74444	44
17	. 85342	. 71355	. 85818	. 72141	. 86284	. 72920	. 86742	. 73692	. 87190	. 74456	43
18	. 85350	. 71368	. 85826	. 72154	. 86292	. 72932	. 86749	. 73704	. 87198	. 74469	42
19	. 85358	. 71381	. 85834	. 72167	. 86300	. 72945	. 86757	. 73717	. 87205	. 74482	41
20	9. 85366	0. 71394	9. 85841	0. 72180	9. 86307	0. 72958	9. 86764	0. 73730	9. 87212	0. 74494	40
21	. 85374	. 71407	. 85849	. 72193	. 86315	. 72971	. 86772	. 73743	. 87220	. 74507	39
22	. 85382	. 71420	. 85857	. 72206	. 86323	. 72984	. 86780	. 73756	. 87227	. 74520	38
23	. 85390	. 71434	. 85865	. 72219	. 86331	. 72997	. 86787	. 73768	. 87235	. 74533	37
24	. 85398	. 71447	. 85873	. 72232	. 86338	. 73010	. 86795	. 73781	. 87242	. 74545	36
25	9. 85406	0. 71460	9. 85881	0. 72245	9. 86346	0. 73023	9. 86802	0. 73794	9. 87249	0. 74558	35
26	. 85414	. 71473	. 85888	. 72258	. 86354	. 73036	. 86810	. 73807	. 87257	. 74571	34
27	. 85422	. 71486	. 85896	. 72271	. 86361	. 73049	. 86817	. 73820	. 87264	. 74583	33
28	. 85430	. 71499	. 85904	. 72284	. 86369	. 73062	. 86825	. 73832	. 87271	. 74596	32
29	. 85438	. 71512	. 85912	. 72297	. 86377	. 73075	. 86832	. 73845	. 87279	. 74609	31
30	9. 85446	0. 71526	9. 85920	0. 72310	9. 86384	0. 73087	9. 86840	0. 73858	9. 87286	0. 74621	30
31	. 85454	. 71539	. 85928	. 72323	. 86392	. 73100	. 86847	. 73871	. 87294	. 74634	29
32	. 85462	. 71552	. 85935	. 72336	. 86400	. 73113	. 86855	. 73883	. 87301	. 74646	28
33	. 85470	. 71565	. 85943	. 72349	. 86407	. 73126	. 86862	. 73896	. 87308	. 74659	27
34	. 85478	. 71578	. 85951	. 72362	. 86415	. 73139	. 86870	. 73909	. 87316	. 74672	26
35	9. 85486	0. 71591	9. 85959	0. 72375	9. 86423	0. 73152	9. 86877	0. 73922	9. 87323	0. 74684	25
36	. 85494	. 71604	. 85967	. 72388	. 86430	. 73165	. 86885	. 73935	. 87330	. 74697	24
37	. 85502	. 71617	. 85974	. 72401	. 86438	. 73178	. 86892	. 73947	. 87338	. 74710	23
38	. 85510	. 71631	. 85982	. 72414	. 86446	. 73191	. 86900	. 73960	. 87345	. 74722	22
39	. 85518	. 71644	. 85990	. 72427	. 86453	. 73203	. 86907	. 73973	. 87352	. 74735	21
40	9. 85526	0. 71657	9. 85998	0. 72440	9. 86461	0. 73216	9. 86915	0. 73986	9. 87360	0. 74748	20
41	. 85534	. 71670	. 86006	. 72453	. 86468	. 73229	. 86922	. 73998	. 87367	. 74760	19
42	. 85542	. 71683	. 86013	. 72466	. 86476	. 73242	. 86930	. 74011	. 87374	. 74773	18
43	. 85550	. 71696	. 86021	. 72479	. 86484	. 73255	. 86937	. 74024	. 87382	. 74786	17
44	. 85557	. 71709	. 86029	. 72492	. 86491	. 73268	. 86945	. 74037	. 87389	. 74798	16
45	9. 85565	0. 71722	9. 86037	0. 72505	9. 86499	0. 73281	9. 86952	0. 74049	9. 87396	0. 74811	15
46	. 85573	. 71735	. 86045	. 72518	. 86507	. 73294	. 86960	. 74062	. 87404	. 74823	14
47	. 85581	. 71748	. 86052	. 72531	. 86514	. 73306	. 86967	. 74075	. 87411	. 74836	13
48	. 85589	. 71762	. 86060	. 72544	. 86522	. 73319	. 86975	. 74088	. 87418	. 74849	12
49	. 85597	. 71775	. 86068	. 72557	. 86529	. 73332	. 86982	. 74100	. 87426	. 74861	11
50	9. 85605	0. 71788	9. 86076	0. 72570	9. 86537	0. 73345	9. 86990	0. 74113	9. 87433	0. 74874	10
51	. 85613	. 71801	. 86083	. 72583	. 86545	. 73358	. 86997	. 74126	. 87440	. 74887	9
52	. 85621	. 71814	. 86091	. 72596	. 86552	. 73371	. 87004	. 74139	. 87448	. 74899	8
53	. 85629	. 71827	. 86099	. 72609	. 86560	. 73384	. 87012	. 74151	. 87455	. 74912	7
54	. 85637	. 71840	. 86107	. 72622	. 86568	. 73396	. 87019	. 74164	. 87462	. 74924	6
55	9. 85645	0. 71853	9. 86114	0. 72635	9. 86575	0. 73409	9. 87027	0. 74177	9. 87470	0. 74937	5
56	. 85653	. 71866	. 86122	. 72648	. 86583	. 73422	. 87034	. 74190	. 87477	. 74950	4
57	. 85660	. 71879	. 86130	. 72661	. 86590	. 73435	. 87042	. 74202	. 87484	. 74962	3
58	. 85668	. 71892	. 86138	. 72674	. 86598	. 73448	. 87049	. 74215	. 87492	. 74975	2
59	. 85676	. 71905	. 86145	. 72687	. 86606	. 73461	. 87057	. 74228	. 87499	. 74987	1
60	9. 85684	0. 71919	9. 86153	0. 72700	9. 86613	0. 73474	9. 87064	0. 74240	9. 87506	0. 75000	0
	244°		243°		242°		241°		240°		

TABLE 34

Haversines

′	120°		121°		122°		123°		124°		′
	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	
0	9. 87506	0. 75000	9. 87939	0. 75752	9. 88364	0. 76496	9. 88780	0. 77232	9. 89187	0. 77960	60
1	. 87513	. 75013	. 87947	. 75764	. 88371	. 76508	. 88787	. 77244	. 89194	. 77972	59
2	. 87521	. 75025	. 87954	. 75777	. 88378	. 76521	. 88793	. 77256	. 89200	. 77984	58
3	. 87528	. 75038	. 87961	. 75789	. 88385	. 76533	. 88800	. 77269	. 89207	. 77996	57
4	. 87535	. 75050	. 87968	. 75802	. 88392	. 76545	. 88807	. 77281	. 89214	. 78008	56
5	9. 87543	0. 75063	9. 87975	0. 75814	9. 88399	0. 76558	9. 88814	0. 77293	9. 89221	0. 78020	55
6	. 87550	. 75076	. 87982	. 75827	. 88406	. 76570	. 88821	. 77305	. 89227	. 78032	54
7	. 87557	. 75088	. 87989	. 75839	. 88413	. 76582	. 88828	. 77317	. 89234	. 78044	53
8	. 87564	. 75101	. 87996	. 75852	. 88420	. 76595	. 88835	. 77329	. 89241	. 78056	52
9	. 87572	. 75113	. 88004	. 75864	. 88427	. 76607	. 88841	. 77342	. 89247	. 78068	51
10	9. 87579	0. 75126	9. 88011	0. 75876	9. 88434	0. 76619	9. 88848	0. 77354	9. 89254	0. 78080	50
11	. 87586	. 75138	. 88018	. 75889	. 88441	. 76632	. 88855	. 77366	. 89261	. 78092	49
12	. 87593	. 75151	. 88025	. 75901	. 88448	. 76644	. 88862	. 77378	. 89267	. 78104	48
13	. 87601	. 75164	. 88032	. 75914	. 88455	. 76656	. 88869	. 77390	. 89274	. 78116	47
14	. 87608	. 75176	. 88039	. 75926	. 88462	. 76668	. 88876	. 77402	. 89281	. 78128	46
15	9. 87615	0. 75189	9. 88046	0. 75939	9. 88469	0. 76681	9. 88882	0. 77415	9. 89287	0. 78140	45
16	. 87623	. 75201	. 88053	. 75951	. 88476	. 76693	. 88889	. 77427	. 89294	. 78152	44
17	. 87630	. 75214	. 88061	. 75964	. 88483	. 76705	. 88896	. 77439	. 89301	. 78164	43
18	. 87637	. 75226	. 88068	. 75976	. 88490	. 76718	. 88903	. 77451	. 89308	. 78176	42
19	. 87644	. 75239	. 88075	. 75988	. 88496	. 76730	. 88910	. 77463	. 89314	. 78188	41
20	9. 87652	0. 75251	9. 88082	0. 76001	9. 88503	0. 76742	9. 88916	0. 77475	9. 89321	0. 78200	40
21	. 87659	. 75264	. 88089	. 76013	. 88510	. 76754	. 88923	. 77488	. 89328	. 78212	39
22	. 87666	. 75277	. 88096	. 76026	. 88517	. 76767	. 88930	. 77500	. 89334	. 78224	38
23	. 87673	. 75289	. 88103	. 76038	. 88524	. 76779	. 88937	. 77512	. 89341	. 78236	37
24	. 87680	. 75302	. 88110	. 76050	. 88531	. 76791	. 88944	. 77524	. 89348	. 78248	36
25	9. 87688	0. 75314	9. 88117	0. 76063	9. 88538	0. 76804	9. 88950	0. 77536	9. 89354	0. 78260	35
26	. 87695	. 75327	. 88124	. 76075	. 88545	. 76816	. 88957	. 77548	. 89361	. 78272	34
27	. 87702	. 75339	. 88131	. 76088	. 88552	. 76828	. 88964	. 77560	. 89368	. 78284	33
28	. 87709	. 75352	. 88139	. 76100	. 88559	. 76840	. 88971	. 77573	. 89374	. 78296	32
29	. 87717	. 75364	. 88146	. 76113	. 88566	. 76853	. 88978	. 77585	. 89381	. 78308	31
30	9. 87724	0. 75377	9. 88153	0. 76125	9. 88573	0. 76865	9. 88984	0. 77597	9. 89387	0. 78320	30
31	. 87731	. 75389	. 88160	. 76137	. 88580	. 76877	. 88991	. 77609	. 89394	. 78332	29
32	. 87738	. 75402	. 88167	. 76150	. 88587	. 76890	. 88998	. 77621	. 89400	. 78344	28
33	. 87745	. 75415	. 88174	. 76162	. 88594	. 76902	. 89005	. 77633	. 89407	. 78356	27
34	. 87753	. 75427	. 88181	. 76175	. 88600	. 76914	. 89012	. 77645	. 89414	. 78368	26
35	9. 87760	0. 75440	9. 88188	0. 76187	9. 88607	0. 76926	9. 89018	0. 77657	9. 89421	0. 78380	25
36	. 87767	. 75452	. 88195	. 76199	. 88614	. 76939	. 89025	. 77670	. 89427	. 78392	24
37	. 87774	. 75465	. 88202	. 76212	. 88621	. 76951	. 89032	. 77682	. 89434	. 78404	23
38	. 87782	. 75477	. 88209	. 76224	. 88628	. 76963	. 89039	. 77694	. 89441	. 78416	22
39	. 87789	. 75490	. 88216	. 76236	. 88635	. 76975	. 89045	. 77706	. 89447	. 78428	21
40	9. 87796	0. 75502	9. 88223	0. 76249	9. 88642	0. 76988	9. 89052	0. 77718	9. 89454	0. 78440	20
41	. 87803	. 75515	. 88230	. 76261	. 88649	. 77000	. 89059	. 77730	. 89460	. 78452	19
42	. 87810	. 75527	. 88237	. 76274	. 88656	. 77012	. 89066	. 77742	. 89467	. 78464	18
43	. 87818	. 75540	. 88244	. 76286	. 88663	. 77024	. 89072	. 77754	. 89474	. 78476	17
44	. 87825	. 75552	. 88252	. 76298	. 88670	. 77036	. 89079	. 77766	. 89480	. 78488	16
45	9. 87832	0. 75565	9. 88259	0. 76311	9. 88677	0. 77049	9. 89086	0. 77779	9. 89487	0. 78500	15
46	. 87839	. 75577	. 88266	. 76323	. 88683	. 77061	. 89093	. 77791	. 89493	. 78512	14
47	. 87846	. 75590	. 88273	. 76335	. 88690	. 77073	. 89099	. 77803	. 89500	. 78524	13
48	. 87853	. 75602	. 88280	. 76348	. 88697	. 77085	. 89106	. 77815	. 89507	. 78536	12
49	. 87861	. 75615	. 88287	. 76360	. 88704	. 77098	. 89113	. 77827	. 89513	. 78548	11
50	9. 87868	0. 75627	9. 88294	0. 76373	9. 88711	0. 77110	9. 89120	0. 77839	9. 89520	0. 78560	10
51	. 87875	. 75640	. 88301	. 76385	. 88718	. 77122	. 89126	. 77851	. 89527	. 78571	9
52	. 87882	. 75652	. 88308	. 76397	. 88725	. 77134	. 89133	. 77863	. 89533	. 78583	8
53	. 87889	. 75665	. 88315	. 76410	. 88732	. 77147	. 89140	. 77875	. 89540	. 78595	7
54	. 87896	. 75677	. 88322	. 76422	. 88739	. 77159	. 89147	. 77887	. 89546	. 78607	6
55	9. 87904	0. 75690	9. 88329	0. 76434	9. 88745	0. 77171	9. 89153	0. 77899	9. 89553	0. 78619	5
56	. 87911	. 75702	. 88336	. 76447	. 88752	. 77183	. 89160	. 77911	. 89559	. 78631	4
57	. 87918	. 75714	. 88343	. 76459	. 88759	. 77195	. 89167	. 77923	. 89566	. 78643	3
58	. 87925	. 75727	. 88350	. 76471	. 88766	. 77208	. 89174	. 77936	. 89573	. 78655	2
59	. 87932	. 75739	. 88357	. 76484	. 88773	. 77220	. 89180	. 77948	. 89579	. 78667	1
60	9. 87939	0. 75752	9. 88364	0. 76496	9. 88780	0. 77232	9. 89187	0. 77960	9. 89586	0. 78679	0
	239°		238°		237°		236°		235°		

TABLE 34

Haversines

′	125° Log Hav	125° Nat. Hav	126° Log Hav	126° Nat. Hav	127° Log Hav	127° Nat. Hav	128° Log Hav	128° Nat. Hav	129° Log Hav	129° Nat. Hav	′
0	9.89586	0.78679	9.89976	0.79389	9.90358	0.80091	9.90732	0.80783	9.91098	0.81466	60
1	.89592	.78691	.89983	.79401	.90365	.80102	.90738	.80795	.91104	.81477	59
2	.89599	.78703	.89989	.79413	.90371	.80114	.90744	.80806	.91110	.81489	58
3	.89606	.78715	.89995	.79425	.90377	.80126	.90751	.80817	.91116	.81500	57
4	.89612	.78726	.90002	.79436	.90383	.80137	.90757	.80829	.91122	.81511	56
5	9.89619	0.78738	9.90008	0.79448	9.90390	0.80149	9.90763	0.80840	9.91128	0.81523	55
6	.89625	.78750	.90015	.79460	.90396	.80160	.90769	.80852	.91134	.81534	54
7	.89632	.78762	.90021	.79472	.90402	.80172	.90775	.80863	.91140	.81545	53
8	.89638	.78774	.90028	.79483	.90409	.80184	.90781	.80875	.91146	.81556	52
9	.89645	.78786	.90034	.79495	.90415	.80195	.90787	.80886	.91152	.81568	51
10	9.89651	0.78798	9.90040	0.79507	9.90421	0.80207	9.90794	0.80898	9.91158	0.81579	50
11	.89658	.78810	.90047	.79519	.90428	.80218	.90800	.80909	.91164	.81590	49
12	.89665	.78822	.90053	.79530	.90434	.80230	.90806	.80920	.91170	.81601	48
13	.89671	.78833	.90060	.79542	.90440	.80242	.90812	.80932	.91176	.81613	47
14	.89678	.78845	.90066	.79554	.90446	.80253	.90818	.80943	.91182	.81624	46
15	9.89684	0.78857	9.90072	0.79565	9.90452	0.80265	9.90824	0.80955	9.91188	0.81635	45
16	.89691	.78869	.90079	.79577	.90459	.80276	.90830	.80966	.91194	.81647	44
17	.89697	.78881	.90085	.79589	.90465	.80288	.90836	.80978	.91200	.81658	43
18	.89704	.78893	.90092	.79601	.90471	.80299	.90843	.80989	.91206	.81669	42
19	.89710	.78905	.90098	.79612	.90478	.80311	.90849	.81000	.91212	.81680	41
20	9.89717	0.78917	9.90104	0.79624	9.90484	0.80323	9.90855	0.81012	9.91218	0.81692	40
21	.89723	.78928	.90111	.79636	.90490	.80334	.90861	.81023	.91224	.81703	39
22	.89730	.78940	.90117	.79648	.90496	.80346	.90867	.81035	.91230	.81714	38
23	.89736	.78952	.90124	.79659	.90503	.80357	.90873	.81046	.91236	.81725	37
24	.89743	.78964	.90130	.79671	.90509	.80369	.90879	.81057	.91242	.81737	36
25	9.89749	0.78976	9.90136	0.79683	9.90515	0.80380	9.90885	0.81069	9.91248	0.81748	35
26	.89756	.78988	.90143	.79694	.90521	.80392	.90892	.81080	.91254	.81759	34
27	.89763	.79000	.90149	.79706	.90527	.80403	.90898	.81092	.91260	.81770	33
28	.89769	.79011	.90156	.79718	.90534	.80415	.90904	.81103	.91265	.81781	32
29	.89776	.79023	.90162	.79729	.90540	.80427	.90910	.81114	.91271	.81793	31
30	9.89782	0.79035	9.90168	0.79741	9.90546	0.80438	9.90916	0.81126	9.91277	0.81804	30
31	.89789	.79047	.90175	.79753	.90552	.80450	.90922	.81137	.91283	.81815	29
32	.89795	.79059	.90181	.79765	.90559	.80461	.90928	.81148	.91289	.81826	28
33	.89802	.79071	.90187	.79776	.90565	.80473	.90934	.81160	.91295	.81838	27
34	.89808	.79082	.90194	.79788	.90571	.80484	.90940	.81171	.91301	.81849	26
35	9.89815	0.79094	9.90200	0.79800	9.90577	0.80496	9.90946	0.81183	9.91307	0.81860	25
36	.89821	.79106	.90206	.79811	.90584	.80507	.90952	.81194	.91313	.81871	24
37	.89828	.79118	.90213	.79823	.90590	.80519	.90958	.81205	.91319	.81882	23
38	.89834	.79130	.90219	.79835	.90596	.80530	.90965	.81217	.91325	.81894	22
39	.89840	.79142	.90225	.79846	.90602	.80542	.90971	.81228	.91331	.81905	21
40	9.89847	0.79153	9.90232	0.79858	9.90608	0.80553	9.90977	0.81239	9.91337	0.81916	20
41	.89853	.79165	.90238	.79870	.90615	.80565	.90983	.81251	.91343	.81927	19
42	.89860	.79177	.90244	.79881	.90621	.80576	.90989	.81262	.91349	.81938	18
43	.89866	.79189	.90251	.79893	.90627	.80588	.90995	.81273	.91355	.81950	17
44	.89873	.79201	.90257	.79905	.90633	.80599	.91001	.81285	.91361	.81961	16
45	9.89879	0.79212	9.90264	0.79916	9.90639	0.80611	9.91007	0.81296	9.91367	0.81972	15
46	.89886	.79224	.90270	.79928	.90646	.80622	.91013	.81308	.91372	.81983	14
47	.89892	.79236	.90276	.79940	.90652	.80634	.91019	.81319	.91378	.81994	13
48	.89899	.79248	.90282	.79951	.90658	.80645	.91025	.81330	.91384	.82005	12
49	.89905	.79260	.90289	.79963	.90664	.80657	.91031	.81342	.91390	.82017	11
50	9.89912	0.79271	9.90295	0.79974	9.90670	0.80668	9.91037	0.81353	9.91396	0.82028	10
51	.89918	.79283	.90301	.79986	.90676	.80680	.91043	.81364	.91402	.82039	9
52	.89925	.79295	.90308	.79998	.90683	.80691	.91049	.81376	.91408	.82050	8
53	.89931	.79307	.90314	.80009	.90689	.80703	.91055	.81387	.91414	.82061	7
54	.89938	.79319	.90320	.80021	.90695	.80714	.91061	.81398	.91420	.82072	6
55	9.89944	0.79330	9.90327	0.80033	9.90701	0.80726	9.91067	0.81409	9.91426	0.82084	5
56	.89950	.79342	.90333	.80044	.90707	.80737	.91074	.81421	.91432	.82095	4
57	.89957	.79354	.90339	.80056	.90714	.80749	.91080	.81432	.91437	.82106	3
58	.89963	.79366	.90346	.80068	.90720	.80760	.91086	.81443	.91443	.82117	2
59	.89970	.79377	.90352	.80079	.90726	.80772	.91092	.81455	.91449	.82128	1
60	9.89976	0.79389	9.90358	0.80091	9.90732	0.80783	9.91098	0.81466	9.91455	0.82139	0
	234°		233°		232°		231°		230°		

TABLE 34

Haversines

′	130°		131°		132°		133°		134°		′
	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	
0	9. 91455	0. 82139	9. 91805	0. 82803	9. 92146	0. 83457	9. 92480	0. 84100	9. 92805	0. 84733	60
1	. 91461	. 82151	. 91810	. 82814	. 92152	. 83467	. 92485	. 84111	. 92811	. 84743	59
2	. 91467	. 82162	. 91816	. 82825	. 92157	. 83478	. 92491	. 84121	. 92816	. 84754	58
3	. 91473	. 82173	. 91822	. 82836	. 92163	. 83489	. 92496	. 84132	. 92821	. 84764	57
4	. 91479	. 82184	. 91828	. 82847	. 92169	. 83500	. 92502	. 84142	. 92827	. 84775	56
5	9. 91485	0. 82195	9. 91833	0. 82858	9. 92174	0. 83511	9. 92507	0. 84153	9. 92832	0. 84785	55
6	. 91490	. 82206	. 91839	. 82869	. 92180	. 83521	. 92512	. 84164	. 92837	. 84796	54
7	. 91496	. 82217	. 91845	. 82880	. 92185	. 83532	. 92518	. 84174	. 92843	. 84806	53
8	. 91502	. 82228	. 91851	. 82891	. 92191	. 83543	. 92523	. 84185	. 92848	. 84817	52
9	. 91508	. 82240	. 91856	. 82902	. 92197	. 83554	. 92529	. 84196	. 92853	. 84827	51
10	9. 91514	0. 82251	9. 91862	0. 82913	9. 92202	0. 83564	9. 92534	0. 84206	9. 92859	0. 84837	50
11	. 91520	. 82262	. 91868	. 82924	. 92208	. 83575	. 92540	. 84217	. 92864	. 84848	49
12	. 91526	. 82273	. 91874	. 82934	. 92213	. 83586	. 92545	. 84227	. 92869	. 84858	48
13	. 91532	. 82284	. 91879	. 82945	. 92219	. 83597	. 92551	. 84238	. 92875	. 84869	47
14	. 91537	. 82295	. 91885	. 82956	. 92225	. 83608	. 92556	. 84249	. 92880	. 84879	46
15	9. 91543	0. 82306	9. 91891	0. 82967	9. 92230	0. 83618	9. 92562	0. 84259	9. 92885	0. 84890	45
16	. 91549	. 82317	. 91896	. 82978	. 92236	. 83629	. 92567	. 84270	. 92891	. 84900	44
17	. 91555	. 82328	. 91902	. 82989	. 92241	. 83640	. 92573	. 84280	. 92896	. 84910	43
18	. 91561	. 82339	. 91908	. 83000	. 92247	. 83651	. 92578	. 84291	. 92901	. 84921	42
19	. 91567	. 82351	. 91914	. 83011	. 92253	. 83661	. 92584	. 84302	. 92907	. 84931	41
20	9. 91573	0. 82362	9. 91919	0. 83022	9. 92258	0. 83672	9. 92589	0. 84312	9. 92912	0. 84942	40
21	. 91578	. 82373	. 91925	. 83033	. 92264	. 83683	. 92594	. 84323	. 92917	. 84952	39
22	. 91584	. 82384	. 91931	. 83044	. 92269	. 83694	. 92600	. 84333	. 92923	. 84962	38
23	. 91590	. 82395	. 91936	. 83055	. 92275	. 83704	. 92605	. 84344	. 92928	. 84973	37
24	. 91596	. 82406	. 91942	. 83066	. 92280	. 83715	. 92611	. 84354	. 92933	. 84983	36
25	9. 91602	0. 82417	9. 91948	0. 83077	9. 92286	0. 83726	9. 92616	0. 84365	9. 92939	0. 84994	35
26	. 91608	. 82428	. 91954	. 83087	. 92292	. 83737	. 92622	. 84376	. 92944	. 85004	34
27	. 91613	. 82439	. 91959	. 83098	. 92297	. 83747	. 92627	. 84386	. 92949	. 85014	33
28	. 91619	. 82450	. 91965	. 83109	. 92303	. 83758	. 92633	. 84397	. 92955	. 85025	32
29	. 91625	. 82461	. 91971	. 83120	. 92308	. 83769	. 92638	. 84407	. 92960	. 85035	31
30	9. 91631	0. 82472	9. 91976	0. 83131	9. 92314	0. 83780	9. 92643	0. 84418	9. 92965	0. 85045	30
31	. 91637	. 82483	. 91982	. 83142	. 92319	. 83790	. 92649	. 84428	. 92970	. 85056	29
32	. 91643	. 82495	. 91988	. 83153	. 92325	. 83801	. 92654	. 84439	. 92975	. 85066	28
33	. 91648	. 82506	. 91993	. 83164	. 92330	. 83812	. 92660	. 84449	. 92981	. 85077	27
34	. 91654	. 82517	. 91999	. 83175	. 92336	. 83822	. 92665	. 84460	. 92986	. 85087	26
35	9. 91660	0. 82528	9. 92005	0. 83185	9. 92342	0. 83833	9. 92670	0. 84470	9. 92992	0. 85097	25
36	. 91666	. 82539	. 92010	. 83196	. 92347	. 83844	. 92676	. 84481	. 92997	. 85108	24
37	. 91672	. 82550	. 92016	. 83207	. 92353	. 83855	. 92681	. 84492	. 93002	. 85118	23
38	. 91677	. 82561	. 92022	. 83218	. 92358	. 83865	. 92687	. 84502	. 93007	. 85128	22
39	. 91683	. 82572	. 92027	. 83229	. 92364	. 83876	. 92692	. 84513	. 93013	. 85139	21
40	9. 91689	0. 82583	9. 92033	0. 83240	9. 92369	0. 83887	9. 92698	0. 84523	9. 93018	0. 85149	20
41	. 91695	. 82594	. 92039	. 83251	. 92375	. 83897	. 92703	. 84534	. 93023	. 85159	19
42	. 91701	. 82605	. 92044	. 83262	. 92380	. 83908	. 92708	. 84544	. 93029	. 85170	18
43	. 91706	. 82616	. 92050	. 83272	. 92386	. 83919	. 92714	. 84555	. 93034	. 85180	17
44	. 91712	. 82627	. 92056	. 83283	. 92391	. 83929	. 92719	. 84565	. 93039	. 85190	16
45	9. 91718	0. 82638	9. 92061	0. 83294	9. 92397	0. 83940	9. 92725	0. 84576	9. 93044	0. 85201	15
46	. 91724	. 82649	. 92067	. 83305	. 92402	. 83951	. 92730	. 84586	. 93050	. 85211	14
47	. 91730	. 82660	. 92073	. 83316	. 92408	. 83961	. 92735	. 84597	. 93055	. 85221	13
48	. 91735	. 82671	. 92078	. 83327	. 92413	. 83972	. 92741	. 84607	. 93060	. 85232	12
49	. 91741	. 82682	. 92084	. 83337	. 92419	. 83983	. 92746	. 84618	. 93065	. 85242	11
50	9. 91747	0. 82693	9. 92090	0. 83348	9. 92425	0. 83993	9. 92751	0. 84628	9. 93071	0. 85252	10
51	. 91753	. 82704	. 92095	. 83359	. 92430	. 84004	. 92757	. 84639	. 93076	. 85263	9
52	. 91758	. 82715	. 92101	. 83370	. 92436	. 84015	. 92762	. 84649	. 93081	. 85273	8
53	. 91764	. 82726	. 92107	. 83381	. 92441	. 84025	. 92768	. 84660	. 93086	. 85283	7
54	. 91770	. 82737	. 92112	. 83392	. 92447	. 84036	. 92773	. 84670	. 93092	. 85294	6
55	9. 91776	0. 82748	9. 92118	0. 83402	9. 92452	0. 84047	9. 92778	0. 84681	9. 93097	0. 85304	5
56	. 91782	. 82759	. 92124	. 83413	. 92458	. 84057	. 92784	. 84691	. 93102	. 85314	4
57	. 91787	. 82770	. 92129	. 83424	. 92463	. 84068	. 92789	. 84702	. 93107	. 85324	3
58	. 91793	. 82781	. 92135	. 83435	. 92469	. 84079	. 92794	. 84712	. 93113	. 85335	2
59	. 91799	. 82792	. 92140	. 83446	. 92474	. 84089	. 92800	. 84722	. 93118	. 85345	1
60	9. 91805	0. 82803	9. 92146	0. 83457	9. 92480	0. 84100	9. 92805	0. 84733	9. 93123	0. 85355	0
	229°		228°		227°		226°		225°		

TABLE 34
Haversines

′	135° Log Hav	135° Nat. Hav	136° Log Hav	136° Nat. Hav	137° Log Hav	137° Nat. Hav	138° Log Hav	138° Nat. Hav	139° Log Hav	139° Nat. Hav	′
0	9. 93123	0. 85355	9. 93433	0. 85967	9. 93736	0. 86568	9. 94030	0. 87157	9. 94318	0. 87735	60
1	. 93128	. 85366	. 93438	. 85977	. 93741	. 86578	. 94035	. 87167	. 94322	. 87745	59
2	. 93134	. 85376	. 93443	. 85987	. 93746	. 86588	. 94040	. 87177	. 94327	. 87755	58
3	. 93139	. 85386	. 93448	. 85997	. 93751	. 86597	. 94045	. 87186	. 94332	. 87764	57
4	. 93144	. 85396	. 93454	. 86007	. 93755	. 86607	. 94050	. 87196	. 94336	. 87774	56
5	9. 93149	0. 85407	9. 93459	0. 86017	9. 93760	0. 86617	9. 94055	0. 87206	9. 94341	0. 87783	55
6	. 93154	. 85417	. 93464	. 86028	. 93765	. 86627	. 94059	. 87216	. 94346	. 87793	54
7	. 93160	. 85427	. 93469	. 86038	. 93770	. 86637	. 94064	. 87225	. 94351	. 87802	53
8	. 93165	. 85438	. 93474	. 86048	. 93775	. 86647	. 94069	. 87235	. 94355	. 87812	52
9	. 93170	. 85448	. 93479	. 86058	. 93780	. 86657	. 94074	. 87245	. 94360	. 87821	51
10	9. 93175	0. 85458	9. 93484	0. 86068	9. 93785	0. 86667	9. 94079	0. 87254	9. 94365	0. 87831	50
11	. 93181	. 85468	. 93489	. 86078	. 93790	. 86677	. 94084	. 87264	. 94369	. 87840	49
12	. 93186	. 85479	. 93494	. 86088	. 93795	. 86686	. 94088	. 87274	. 94374	. 87850	48
13	. 93191	. 85489	. 93499	. 86098	. 93800	. 86696	. 94093	. 87283	. 94379	. 87859	47
14	. 93196	. 85499	. 93504	. 86108	. 93805	. 86706	. 94098	. 87293	. 94383	. 87869	46
15	9. 93201	0. 85509	9. 93509	0. 86118	9. 93810	0. 86716	9. 94103	0. 87303	9. 94388	0. 87878	45
16	. 93207	. 85520	. 93515	. 86128	. 93815	. 86726	. 94108	. 87313	. 94393	. 87888	44
17	. 93212	. 85530	. 93520	. 86138	. 93820	. 86736	. 94112	. 87322	. 94398	. 87897	43
18	. 93217	. 85540	. 93525	. 86148	. 93825	. 86746	. 94117	. 87332	. 94402	. 87907	42
19	. 93222	. 85550	. 93530	. 86158	. 93830	. 86756	. 94122	. 87342	. 94407	. 87916	41
20	9. 93227	0. 85560	9. 93535	0. 86168	9. 93835	0. 86765	9. 94127	0. 87351	9. 94412	0. 87926	40
21	. 93232	. 85571	. 93540	. 86178	. 93840	. 86775	. 94132	. 87361	. 94416	. 87935	39
22	. 93238	. 85581	. 93545	. 86189	. 93845	. 86785	. 94137	. 87371	. 94421	. 87945	38
23	. 93243	. 85591	. 93550	. 86199	. 93849	. 86795	. 94141	. 87380	. 94426	. 87954	37
24	. 93248	. 85601	. 93555	. 86209	. 93854	. 86805	. 94146	. 87390	. 94430	. 87964	36
25	9. 93253	0. 85612	9. 93560	0. 86219	9. 93859	0. 86815	9. 94151	0. 87400	9. 94435	0. 87973	35
26	. 93258	. 85622	. 93565	. 86229	. 93864	. 86825	. 94156	. 87409	. 94440	. 87982	34
27	. 93264	. 85632	. 93570	. 86239	. 93869	. 86834	. 94161	. 87419	. 94444	. 87992	33
28	. 93269	. 85642	. 93575	. 86249	. 93874	. 86844	. 94165	. 87429	. 94449	. 88001	32
29	. 93274	. 85652	. 93580	. 86259	. 93879	. 86854	. 94170	. 87438	. 94454	. 88011	31
30	9. 93279	0. 85663	9. 93585	0. 86269	9. 93884	0. 86864	9. 94175	0. 87448	9. 94458	0. 88020	30
31	. 93284	. 85673	. 93590	. 86279	. 93889	. 86874	. 94180	. 87457	. 94463	. 88030	29
32	. 93289	. 85683	. 93595	. 86289	. 93894	. 86884	. 94184	. 87467	. 94468	. 88039	28
33	. 93295	. 85693	. 93600	. 86299	. 93899	. 86893	. 94189	. 87477	. 94472	. 88049	27
34	. 93300	. 85703	. 93605	. 86309	. 93904	. 86903	. 94194	. 87486	. 94477	. 88058	26
35	9. 93305	0. 85713	9. 93611	0. 86319	9. 93908	0. 86913	9. 94199	0. 87496	9. 94482	0. 88067	25
36	. 93310	. 85724	. 93616	. 86329	. 93913	. 86923	. 94204	. 87506	. 94486	. 88077	24
37	. 93315	. 85734	. 93621	. 86339	. 93918	. 86933	. 94208	. 87515	. 94491	. 88086	23
38	. 93320	. 85744	. 93626	. 86349	. 93923	. 86942	. 94213	. 87525	. 94496	. 88096	22
39	. 93326	. 85754	. 93631	. 86359	. 93928	. 86952	. 94218	. 87534	. 94500	. 88105	21
40	9. 93331	0. 85764	9. 93636	0. 86369	9. 93933	0. 86962	9. 94223	0. 87544	9. 94505	0. 88115	20
41	. 93336	. 85774	. 93641	. 86379	. 93938	. 86972	. 94227	. 87554	. 94509	. 88124	19
42	. 93341	. 85785	. 93646	. 86389	. 93943	. 86982	. 94232	. 87563	. 94514	. 88133	18
43	. 93346	. 85795	. 93651	. 86399	. 93948	. 86991	. 94237	. 87573	. 94519	. 88143	17
44	. 93351	. 85805	. 93656	. 86409	. 93952	. 87001	. 94242	. 87582	. 94523	. 88152	16
45	9. 93356	0. 85815	9. 93661	0. 86419	9. 93957	0. 87011	9. 94246	0. 87592	9. 94528	0. 88162	15
46	. 93362	. 85825	. 93666	. 86429	. 93962	. 87021	. 94251	. 87602	. 94533	. 88171	14
47	. 93367	. 85835	. 93671	. 86438	. 93967	. 87030	. 94256	. 87611	. 94537	. 88180	13
48	. 93372	. 85846	. 93676	. 86448	. 93972	. 87040	. 94261	. 87621	. 94542	. 88190	12
49	. 93377	. 85856	. 93681	. 86458	. 93977	. 87050	. 94265	. 87630	. 94546	. 88199	11
50	9. 93382	0. 85866	9. 93686	0. 86468	9. 93982	0. 87060	9. 94270	0. 87640	9. 94551	0. 88209	10
51	. 93387	. 85876	. 93691	. 86478	. 93987	. 87070	. 94275	. 87649	. 94556	. 88218	9
52	. 93392	. 85886	. 93696	. 86488	. 93991	. 87079	. 94280	. 87659	. 94560	. 88227	8
53	. 93397	. 85896	. 93701	. 86498	. 93996	. 87089	. 94284	. 87669	. 94565	. 88237	7
54	. 93403	. 85906	. 93706	. 86508	. 94001	. 87099	. 94289	. 87678	. 94570	. 88246	6
55	9. 93408	0. 85916	9. 93711	0. 86518	9. 94006	0. 87109	9. 94294	0. 87688	9. 94574	0. 88255	5
56	. 93413	. 85927	. 93716	. 86528	. 94011	. 87118	. 94299	. 87697	. 94579	. 88265	4
57	. 93418	. 85937	. 93721	. 86538	. 94016	. 87128	. 94303	. 87707	. 94583	. 88274	3
58	. 93423	. 85947	. 93726	. 86548	. 94021	. 87138	. 94308	. 87716	. 94588	. 88284	2
59	. 93428	. 85957	. 93731	. 86558	. 94026	. 87148	. 94313	. 87726	. 94593	. 88293	1
60	9. 93433	0. 85967	9. 93736	0. 86568	9. 94030	0. 87157	9. 94318	0. 87735	9. 94597	0. 88302	0
	224°		223°		222°		221°		220°		

TABLE 34

Haversines

	140°		141°		142°		143°		144°		
′	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	′
0	9. 94597	0. 88302	9. 94869	0. 88857	9. 95134	0. 89401	9. 95391	0. 89932	9. 95641	0. 90451	60
1	. 94602	. 88312	. 94874	. 88866	. 95138	. 89409	. 95396	. 89941	. 95645	. 90459	59
2	. 94606	. 88321	. 94878	. 88876	. 95143	. 89418	. 95400	. 89949	. 95649	. 90468	58
3	. 94611	. 88330	. 94883	. 88885	. 95147	. 89427	. 95404	. 89958	. 95654	. 90476	57
4	. 94616	. 88340	. 94887	. 88894	. 95151	. 89436	. 95408	. 89967	. 95658	. 90485	56
5	9. 94620	0. 88349	9. 94892	0. 88903	9. 95156	0. 89445	9. 95412	0. 89975	9. 95662	0. 90494	55
6	. 94625	. 88358	. 94896	. 88912	. 95160	. 89454	. 95417	. 89984	. 95666	. 90502	54
7	. 94629	. 88368	. 94901	. 88921	. 95164	. 89463	. 95421	. 89993	. 95670	. 90511	53
8	. 94634	. 88377	. 94905	. 88930	. 95169	. 89472	. 95425	. 90002	. 95674	. 90519	52
9	. 94638	. 88386	. 94909	. 88940	. 95173	. 89481	. 95429	. 90010	. 95678	. 90528	51
10	9. 94643	0. 88396	9. 94914	0. 88949	9. 95177	0. 89490	9. 95433	0. 90019	9. 95682	0. 90536	50
11	. 94648	. 88405	. 94918	. 88958	. 95182	. 89499	. 95438	. 90028	. 95686	. 90545	49
12	. 94652	. 88414	. 94923	. 88967	. 95186	. 89508	. 95442	. 90037	. 95690	. 90553	48
13	. 94657	. 88423	. 94927	. 88976	. 95190	. 89517	. 95446	. 90045	. 95694	. 90562	47
14	. 94661	. 88433	. 94932	. 88985	. 95195	. 89526	. 95450	. 90054	. 95699	. 90570	46
15	9. 94666	0. 88442	9. 94936	0. 88994	9. 95199	0. 89534	9. 95454	0. 90063	9. 95703	0. 90579	45
16	. 94670	. 88451	. 94941	. 89003	. 95203	. 89543	. 95459	. 90071	. 95707	. 90587	44
17	. 94675	. 88461	. 94945	. 89012	. 95208	. 89552	. 95463	. 90080	. 95711	. 90596	43
18	. 94680	. 88470	. 94950	. 89022	. 95212	. 89561	. 95467	. 90089	. 95715	. 90604	42
19	. 94684	. 88479	. 94954	. 89031	. 95216	. 89570	. 95471	. 90097	. 95719	. 90613	41
20	9. 94689	0. 88489	9. 94958	0. 89040	9. 95221	0. 89579	9. 95475	0. 90106	9. 95723	0. 90621	40
21	. 94693	. 88498	. 94963	. 89049	. 95225	. 89588	. 95480	. 90115	. 95727	. 90630	39
22	. 94698	. 88507	. 94967	. 89058	. 95229	. 89597	. 95484	. 90124	. 95731	. 90638	38
23	. 94702	. 88516	. 94972	. 89067	. 95234	. 89606	. 95488	. 90132	. 95735	. 90647	37
24	. 94707	. 88526	. 94976	. 89076	. 95238	. 89614	. 95492	. 90141	. 95739	. 90655	36
25	9. 94711	0. 88535	9. 94981	0. 89085	9. 95242	0. 89623	9. 95496	0. 90150	9. 95743	0. 90664	35
26	. 94716	. 88544	. 94985	. 89094	. 95246	. 89632	. 95501	. 90158	. 95747	. 90672	34
27	. 94721	. 88553	. 94989	. 89103	. 95251	. 89641	. 95505	. 90167	. 95751	. 90680	33
28	. 94725	. 88563	. 94994	. 89112	. 95255	. 89650	. 95509	. 90176	. 95755	. 90689	32
29	. 94730	. 88572	. 94998	. 89121	. 95259	. 89659	. 95513	. 90184	. 95759	. 90697	31
30	9. 94734	0. 88581	9. 95003	0. 89130	9. 95264	0. 89668	9. 95517	0. 90193	9. 95763	0. 90706	30
31	. 94739	. 88590	. 95007	. 89139	. 95268	. 89677	. 95521	. 90201	. 95768	. 90714	29
32	. 94743	. 88600	. 95011	. 89149	. 95272	. 89685	. 95526	. 90210	. 95772	. 90723	28
33	. 94748	. 88609	. 95016	. 89158	. 95276	. 89694	. 95530	. 90219	. 95776	. 90731	27
34	. 94752	. 88618	. 95020	. 89167	. 95281	. 89703	. 95534	. 90227	. 95780	. 90740	26
35	9. 94757	0. 88627	9. 95025	0. 89176	9. 95285	0. 89712	9. 95538	0. 90236	9. 95784	0. 90748	25
36	. 94761	. 88637	. 95029	. 89185	. 95289	. 89721	. 95542	. 90245	. 95788	. 90756	24
37	. 94766	. 88646	. 95033	. 89194	. 95294	. 89730	. 95546	. 90253	. 95792	. 90765	23
38	. 94770	. 88655	. 95038	. 89203	. 95298	. 89738	. 95550	. 90262	. 95796	. 90773	22
39	. 94774	. 88664	. 95042	. 89212	. 95302	. 89747	. 95555	. 90271	. 95800	. 90782	21
40	9. 94779	0. 88674	9. 95047	0. 89221	9. 95306	0. 89756	9. 95559	0. 90279	9. 95804	0. 90790	20
41	. 94784	. 88683	. 95051	. 89230	. 95311	. 89765	. 95563	. 90288	. 95808	. 90798	19
42	. 94788	. 88692	. 95055	. 89239	. 95315	. 89774	. 95567	. 90296	. 95812	. 90807	18
43	. 94793	. 88701	. 95060	. 89248	. 95319	. 89782	. 95571	. 90305	. 95816	. 90815	17
44	. 94797	. 88710	. 95064	. 89257	. 95323	. 89791	. 95575	. 90314	. 95820	. 90824	16
45	9. 94802	0. 88720	9. 95069	0. 89266	9. 95328	0. 89800	9. 95579	0. 90322	9. 95824	0. 90832	15
46	. 94806	. 88729	. 95073	. 89275	. 95332	. 89809	. 95584	. 90331	. 95828	. 90840	14
47	. 94811	. 88738	. 95077	. 89284	. 95336	. 89818	. 95588	. 90339	. 95832	. 90849	13
48	. 94815	. 88747	. 95082	. 89293	. 95340	. 89826	. 95592	. 90348	. 95836	. 90857	12
49	. 94820	. 88756	. 95086	. 89302	. 95345	. 89835	. 95596	. 90357	. 95840	. 90866	11
50	9. 94824	0. 88766	9. 95090	0. 89311	9. 95349	0. 89844	9. 95600	0. 90365	9. 95844	0. 90874	10
51	. 94829	. 88775	. 95095	. 89320	. 95353	. 89853	. 95604	. 90374	. 95848	. 90882	9
52	. 94833	. 88784	. 95099	. 89329	. 95357	. 89862	. 95608	. 90382	. 95852	. 90891	8
53	. 94838	. 88793	. 95104	. 89338	. 95362	. 89870	. 95613	. 90391	. 95856	. 90899	7
54	. 94842	. 88802	. 95108	. 89347	. 95366	. 89879	. 95617	. 90399	. 95860	. 90907	6
55	9. 94847	0. 88811	9. 95112	0. 89356	9. 95370	0. 89888	9. 95621	0. 90408	9. 95864	0. 90916	5
56	. 94851	. 88821	. 95117	. 89365	. 95374	. 89897	. 95625	. 90417	. 95868	. 90924	4
57	. 94856	. 88830	. 95121	. 89374	. 95379	. 89906	. 95629	. 90425	. 95872	. 90933	3
58	. 94860	. 88839	. 95125	. 89383	. 95383	. 89914	. 95633	. 90434	. 95876	. 90941	2
59	. 94865	. 88848	. 95130	. 89392	. 95387	. 89923	. 95637	. 90442	. 95880	. 90949	1
60	9. 94869	0. 88857	9. 95134	0. 89401	9. 95391	0. 89932	9. 95641	0. 90451	9. 95884	0. 90958	0
	219°		218°		217°		216°		215°		

TABLE 34

Haversines

′	145° Log Hav	145° Nat. Hav	146° Log Hav	146° Nat. Hav	147° Log Hav	147° Nat. Hav	148° Log Hav	148° Nat. Hav	149° Log Hav	149° Nat. Hav	′
0	9.95884	0.90958	9.96119	0.91452	9.96347	0.91934	9.96568	0.92402	9.96782	0.92858	60
1	.95888	.90966	.96123	.91460	.96351	.91941	.96572	.92410	.96786	.92866	59
2	.95892	.90974	.96127	.91468	.96355	.91949	.96576	.92418	.96789	.92873	58
3	.95896	.90983	.96131	.91476	.96359	.91957	.96579	.92426	.96793	.92881	57
4	.95900	.90991	.96135	.91484	.96362	.91965	.96583	.92433	.96796	.92888	56
5	9.95904	0.90999	9.96139	0.91493	9.96366	0.91973	9.96586	0.92441	9.96800	0.92896	55
6	.95908	.91008	.96142	.91501	.96370	.91981	.96590	.92449	.96803	.92903	54
7	.95912	.91016	.96146	.91509	.96374	.91989	.96594	.92456	.96807	.92911	53
8	.95916	.91024	.96150	.91517	.96377	.91997	.96597	.92464	.96810	.92918	52
9	.95920	.91033	.96154	.91525	.96381	.92005	.96601	.92472	.96814	.92926	51
10	9.95924	0.91041	9.96158	0.91533	9.96385	0.92013	9.96604	0.92479	9.96817	0.92933	50
11	.95928	.91049	.96162	.91541	.96388	.92020	.96608	.92487	.96821	.92941	49
12	.95932	.91057	.96165	.91549	.96392	.92028	.96612	.92495	.96824	.92948	48
13	.95936	.91066	.96169	.91557	.96396	.92036	.96615	.92502	.96827	.92955	47
14	.95939	.91074	.96173	.91565	.96400	.92044	.96619	.92510	.96831	.92963	46
15	9.95943	0.91082	9.96177	0.91573	9.96403	0.92052	9.96622	0.92518	9.96834	0.92970	45
16	.95947	.91091	.96181	.91582	.96407	.92060	.96626	.92525	.96837	.92978	44
17	.95951	.91099	.96185	.91590	.96411	.92068	.96630	.92533	.96841	.92985	43
18	.95955	.91107	.96188	.91598	.96414	.92076	.96633	.92541	.96845	.92993	42
19	.95959	.91115	.96192	.91606	.96418	.92083	.96637	.92548	.96848	.93000	41
20	9.95963	0.91124	9.96196	0.91614	9.96422	0.92091	9.96640	0.92556	9.96852	0.93007	40
21	.95967	.91132	.96200	.91622	.96426	.92099	.96644	.92563	.96855	.93015	39
22	.95971	.91140	.96204	.91630	.96429	.92107	.96648	.92571	.96859	.93022	38
23	.95975	.91149	.96208	.91638	.96433	.92115	.96651	.92579	.96862	.93030	37
24	.95979	.91157	.96211	.91646	.96437	.92123	.96655	.92586	.96866	.93037	36
25	9.95983	0.91165	9.96215	0.91654	9.96440	0.92130	9.96658	0.92594	9.96869	0.93045	35
26	.95987	.91173	.96219	.91662	.96444	.92138	.96662	.92602	.96873	.93052	34
27	.95991	.91182	.96223	.91670	.96448	.92146	.96665	.92609	.96876	.93059	33
28	.95995	.91190	.96227	.91678	.96451	.92154	.96669	.92617	.96879	.93067	32
29	.95999	.91198	.96230	.91686	.96455	.92162	.96673	.92624	.96883	.93074	31
30	9.96002	0.91206	9.96234	0.91694	9.96459	0.92170	9.96676	0.92632	9.96886	0.93081	30
31	.96006	.91215	.96238	.91702	.96462	.92177	.96680	.92640	.96890	.93089	29
32	.96010	.91223	.96242	.91710	.96466	.92185	.96683	.92647	.96894	.93096	28
33	.96014	.91231	.96246	.91718	.96470	.92193	.96687	.92655	.96897	.93104	27
34	.96018	.91239	.96249	.91726	.96473	.92201	.96690	.92662	.96900	.93111	26
35	9.96022	0.91247	9.96253	0.91734	9.96477	0.92209	9.96694	0.92670	9.96904	0.93118	25
36	.96026	.91256	.96257	.91742	.96481	.92216	.96697	.92678	.96907	.93126	24
37	.96030	.91264	.96261	.91750	.96484	.92224	.96701	.92685	.96910	.93133	23
38	.96034	.91272	.96265	.91758	.96488	.92232	.96705	.92693	.96914	.93140	22
39	.96038	.91280	.96268	.91766	.96492	.92240	.96708	.92700	.96917	.93148	21
40	9.96042	0.91289	9.96272	0.91774	9.96495	0.92248	9.96712	0.92708	9.96921	0.93155	20
41	.96046	.91297	.96276	.91782	.96499	.92255	.96715	.92715	.96924	.93162	19
42	.96049	.91305	.96280	.91790	.96503	.92263	.96719	.92723	.96928	.93170	18
43	.96053	.91313	.96283	.91798	.96506	.92271	.96722	.92730	.96931	.93177	17
44	.96057	.91321	.96287	.91806	.96510	.92279	.96726	.92738	.96934	.93184	16
45	9.96061	0.91329	9.96291	0.91814	9.96514	0.92286	9.96729	0.92746	9.96938	0.93192	15
46	.96065	.91338	.96295	.91822	.96517	.92294	.96733	.92753	.96941	.93199	14
47	.96069	.91346	.96299	.91830	.96521	.92302	.96736	.92761	.96945	.93206	13
48	.96073	.91354	.96302	.91838	.96525	.92310	.96740	.92768	.96948	.93214	12
49	.96077	.91362	.96306	.91846	.96528	.92317	.96743	.92776	.96951	.93221	11
50	9.96081	0.91370	9.96310	0.91854	9.96532	0.92325	9.96747	0.92783	9.96955	0.93228	10
51	.96084	.91379	.96314	.91862	.96536	.92333	.96750	.92791	.96958	.93236	9
52	.96088	.91387	.96317	.91870	.96539	.92341	.96754	.92798	.96962	.93243	8
53	.96092	.91395	.96321	.91878	.96543	.92348	.96758	.92806	.96965	.93250	7
54	.96096	.91403	.96325	.91886	.96547	.92356	.96761	.92813	.96968	.93258	6
55	9.96100	0.91411	9.96329	0.91894	9.96550	0.92364	9.96765	0.92821	9.96972	0.93265	5
56	.96104	.91419	.96332	.91902	.96554	.92372	.96768	.92828	.96975	.93272	4
57	.96108	.91427	.96336	.91910	.96557	.92379	.96772	.92836	.96979	.93279	3
58	.96112	.91436	.96340	.91918	.96561	.92387	.96775	.92843	.96982	.93287	2
59	.96115	.91444	.96344	.91926	.96565	.92395	.96779	.92851	.96985	.93294	1
60	9.96119	0.91452	9.96347	0.91934	9.96568	0.92402	9.96782	0.92858	9.96989	0.93301	0
	214°		213°		212°		211°		210°		

TABLE 34

Haversines

′	150° Log Hav	150° Nat. Hav	151° Log Hav	151° Nat. Hav	152° Log Hav	152° Nat. Hav	153° Log Hav	153° Nat. Hav	154° Log Hav	154° Nat. Hav	′
0	9. 96989	0. 93301	9. 97188	0. 93731	9. 97381	0. 94147	9. 97566	0. 94550	9. 97745	0. 94940	60
1	. 96992	. 93309	. 97192	. 93738	. 97384	. 94154	. 97569	. 94557	. 97748	. 94946	59
2	. 96996	. 93316	. 97195	. 93745	. 97387	. 94161	. 97572	. 94564	. 97751	. 94952	58
3	. 96999	. 93323	. 97198	. 93752	. 97390	. 94168	. 97575	. 94570	. 97754	. 94959	57
4	. 97002	. 93330	. 97201	. 93759	. 97393	. 94175	. 97578	. 94577	. 97756	. 94965	56
5	9. 97006	0. 93338	9. 97205	0. 93766	9. 97397	0. 94181	9. 97581	0. 94583	9. 97759	0. 94972	55
6	. 97009	. 93345	. 97208	. 93773	. 97400	. 94188	. 97584	. 94590	. 97762	. 94978	54
7	. 97012	. 93352	. 97211	. 93780	. 97403	. 94195	. 97587	. 94596	. 97765	. 94984	53
8	. 97016	. 93359	. 97214	. 93787	. 97406	. 94202	. 97591	. 94603	. 97768	. 94991	52
9	. 97019	. 93367	. 97218	. 93794	. 97409	. 94209	. 97594	. 94610	. 97771	. 94997	51
10	9. 97022	0. 93374	9. 97221	0. 93801	9. 97412	0. 94215	9. 97597	0. 94616	9. 97774	0. 95003	50
11	. 97026	. 93381	. 97224	. 93808	. 97415	. 94222	. 97600	. 94623	. 97777	. 95010	49
12	. 97029	. 93388	. 97227	. 93815	. 97418	. 94229	. 97603	. 94629	. 97780	. 95016	48
13	. 97033	. 93395	. 97231	. 93822	. 97422	. 94236	. 97606	. 94636	. 97783	. 95022	47
14	. 97036	. 93403	. 97234	. 93829	. 97425	. 94243	. 97609	. 94642	. 97785	. 95029	46
15	9. 97039	0. 93410	9. 97237	0. 93836	9. 97428	0. 94249	9. 97612	0. 94649	9. 97788	0. 95035	45
16	. 97043	. 93417	. 97240	. 93843	. 97431	. 94256	. 97615	. 94655	. 97791	. 95041	44
17	. 97046	. 93424	. 97244	. 93850	. 97434	. 94263	. 97618	. 94662	. 97794	. 95048	43
18	. 97049	. 93432	. 97247	. 93857	. 97437	. 94270	. 97621	. 94669	. 97797	. 95054	42
19	. 97052	. 93439	. 97250	. 93864	. 97440	. 94276	. 97624	. 94675	. 97800	. 95060	41
20	9. 97056	0. 93446	9. 97253	0. 93871	9. 97443	0. 94283	9. 97627	0. 94682	9. 97803	0. 95066	40
21	. 97059	. 93453	. 97257	. 93878	. 97447	. 94290	. 97630	. 94688	. 97806	. 95073	39
22	. 97063	. 93460	. 97260	. 93885	. 97450	. 94297	. 97633	. 94695	. 97808	. 95079	38
23	. 97066	. 93468	. 97263	. 93892	. 97453	. 94303	. 97636	. 94701	. 97811	. 95085	37
24	. 97069	. 93475	. 97266	. 93899	. 97456	. 94310	. 97639	. 94708	. 97814	. 95092	36
25	9. 97073	0. 93482	9. 97269	0. 93906	9. 97459	0. 94317	9. 97642	0. 94714	9. 97817	0. 95098	35
26	. 97076	. 93489	. 97273	. 93913	. 97462	. 94324	. 97645	. 94721	. 97820	. 95104	34
27	. 97079	. 93496	. 97276	. 93920	. 97465	. 94330	. 97647	. 94727	. 97823	. 95110	33
28	. 97083	. 93503	. 97279	. 93927	. 97468	. 94337	. 97650	. 94734	. 97826	. 95117	32
29	. 97086	. 93511	. 97282	. 93934	. 97471	. 94344	. 97653	. 94740	. 97829	. 95123	31
30	9. 97089	0. 93518	9. 97285	0. 93941	9. 97474	0. 94351	9. 97656	0. 94747	9. 97831	0. 95129	30
31	. 97093	. 93525	. 97289	. 93948	. 97478	. 94357	. 97659	. 94753	. 97834	. 95136	29
32	. 97096	. 93532	. 97292	. 93955	. 97481	. 94364	. 97662	. 94760	. 97837	. 95142	28
33	. 97099	. 93539	. 97295	. 93962	. 97484	. 94371	. 97665	. 94766	. 97840	. 95148	27
34	. 97103	. 93546	. 97298	. 93969	. 97487	. 94377	. 97668	. 94773	. 97843	. 95154	26
35	9. 97106	0. 93554	9. 97301	0. 93976	9. 97490	0. 94384	9. 97671	0. 94779	9. 97846	0. 95161	25
36	. 97109	. 93561	. 97305	. 93982	. 97493	. 94391	. 97674	. 94786	. 97849	. 95167	24
37	. 97113	. 93568	. 97308	. 93989	. 97496	. 94397	. 97677	. 94792	. 97851	. 95173	23
38	. 97116	. 93575	. 97311	. 93996	. 97499	. 94404	. 97680	. 94799	. 97854	. 95179	22
39	. 97119	. 93582	. 97314	. 94003	. 97502	. 94411	. 97683	. 94805	. 97857	. 95185	21
40	9. 97123	0. 93589	9. 97317	0. 94010	9. 97505	0. 94418	9. 97686	0. 94811	9. 97860	0. 95192	20
41	. 97126	. 93596	. 97321	. 94017	. 97508	. 94424	. 97689	. 94818	. 97863	. 95198	19
42	. 97129	. 93603	. 97324	. 94024	. 97511	. 94431	. 97692	. 94824	. 97866	. 95204	18
43	. 97132	. 93611	. 97327	. 94031	. 97514	. 94438	. 97695	. 94831	. 97868	. 95210	17
44	. 97136	. 93618	. 97330	. 94038	. 97518	. 94444	. 97698	. 94837	. 97871	. 95217	16
45	9. 97139	0. 93625	9. 97333	0. 94045	9. 97521	0. 94451	9. 97701	0. 94844	9. 97874	0. 95223	15
46	. 97142	. 93632	. 97337	. 94051	. 97524	. 94458	. 97704	. 94850	. 97877	. 95229	14
47	. 97146	. 93639	. 97340	. 94058	. 97527	. 94464	. 97707	. 94856	. 97880	. 95235	13
48	. 97149	. 93646	. 97343	. 94065	. 97530	. 94471	. 97710	. 94863	. 97883	. 95241	12
49	. 97152	. 93653	. 97346	. 94072	. 97533	. 94477	. 97713	. 94869	. 97885	. 95248	11
50	9. 97156	0. 93660	9. 97349	0. 94079	9. 97536	0. 94484	9. 97716	0. 94876	9. 97888	0. 95254	10
51	. 97159	. 93667	. 97352	. 94086	. 97539	. 94491	. 97718	. 94882	. 97891	. 95260	9
52	. 97162	. 93674	. 97356	. 94093	. 97542	. 94497	. 97721	. 94889	. 97894	. 95266	8
53	. 97165	. 93682	. 97359	. 94099	. 97545	. 94504	. 97724	. 94895	. 97897	. 95272	7
54	. 97169	. 93689	. 97362	. 94106	. 97548	. 94511	. 97727	. 94901	. 97899	. 95278	6
55	9. 97172	0. 93696	9. 97365	0. 94113	9. 97551	0. 94517	9. 97730	0. 94908	9. 97902	0. 95285	5
56	. 97175	. 93703	. 97368	. 94120	. 97554	. 94524	. 97733	. 94914	. 97905	. 95291	4
57	. 97179	. 93710	. 97371	. 94127	. 97557	. 94531	. 97736	. 94921	. 97908	. 95297	3
58	. 97182	. 93717	. 97375	. 94134	. 97560	. 94537	. 97739	. 94927	. 97911	. 95303	2
59	. 97185	. 93724	. 97378	. 94141	. 97563	. 94544	. 97742	. 94933	. 97914	. 95309	1
60	9. 97188	0. 93731	9. 97381	0. 94147	9. 97566	0. 94550	9. 97745	0. 94940	9. 97916	0. 95315	0
	209°		208°		207°		206°		205°		

TABLE 34

Haversines

′	155° Log Hav	155° Nat. Hav	156° Log Hav	156° Nat. Hav	157° Log Hav	157° Nat. Hav	158° Log Hav	158° Nat. Hav	159° Log Hav	159° Nat. Hav	′
0	9. 97916	0. 95315	9. 98081	0. 95677	9. 98239	0. 96025	9. 98389	0. 96359	9. 98533	0. 96679	60
1	. 97919	. 95322	. 98084	. 95683	. 98241	. 96031	. 98392	. 96365	. 98536	. 96684	59
2	. 97922	. 95328	. 98086	. 95689	. 98244	. 96037	. 98394	. 96370	. 98538	. 96689	58
3	. 97925	. 95334	. 98089	. 95695	. 98246	. 96042	. 98397	. 96376	. 98540	. 96695	57
4	. 97927	. 95340	. 98092	. 95701	. 98249	. 96048	. 98399	. 96381	. 98543	. 96700	56
5	9. 97930	0. 95346	9. 98094	0. 95707	9. 98251	0. 96054	9. 98402	0. 96386	9. 98545	0. 96705	55
6	. 97933	. 95352	. 98097	. 95713	. 98254	. 96059	. 98404	. 96392	. 98547	. 96710	54
7	. 97936	. 95358	. 98100	. 95719	. 98256	. 96065	. 98406	. 96397	. 98550	. 96715	53
8	. 97939	. 95364	. 98102	. 95724	. 98259	. 96071	. 98409	. 96403	. 98552	. 96721	52
9	. 97941	. 95371	. 98105	. 95730	. 98262	. 96076	. 98411	. 96408	. 98554	. 96726	51
10	9. 97944	0. 95377	9. 98108	0. 95736	9. 98264	0. 96082	9. 98414	0. 96413	9. 98557	0. 96731	50
11	. 97947	. 95383	. 98110	. 95742	. 98267	. 96088	. 98416	. 96419	. 98559	. 96736	49
12	. 97950	. 95389	. 98113	. 95748	. 98269	. 96093	. 98419	. 96424	. 98561	. 96741	48
13	. 97953	. 95395	. 98116	. 95754	. 98272	. 96099	. 98421	. 96430	. 98564	. 96746	47
14	. 97955	. 95401	. 98118	. 95760	. 98274	. 96104	. 98424	. 96435	. 98566	. 96752	46
15	9. 97958	0. 95407	9. 98121	0. 95766	9. 98277	0. 96110	9. 98426	0. 96440	9. 98568	0. 96757	45
16	. 97961	. 95413	. 98124	. 95771	. 98279	. 96116	. 98428	. 96446	. 98570	. 96762	44
17	. 97964	. 95419	. 98126	. 95777	. 98282	. 96121	. 98431	. 96451	. 98573	. 96767	43
18	. 97966	. 95425	. 98129	. 95783	. 98285	. 96127	. 98433	. 96457	. 98575	. 96772	42
19	. 97969	. 95431	. 98132	. 95789	. 98287	. 96133	. 98436	. 96462	. 98577	. 96777	41
20	9. 97972	0. 95438	9. 98134	0. 95795	9. 98290	0. 96138	9. 98438	0. 96467	9. 98580	0. 96782	40
21	. 97975	. 95444	. 98137	. 95801	. 98292	. 96144	. 98440	. 96473	. 98582	. 96788	39
22	. 97977	. 95450	. 98139	. 95806	. 98295	. 96149	. 98443	. 96478	. 98584	. 96793	38
23	. 97980	. 95456	. 98142	. 95812	. 98297	. 96155	. 98445	. 96483	. 98587	. 96798	37
24	. 97983	. 95462	. 98145	. 95818	. 98300	. 96161	. 98448	. 96489	. 98589	. 96803	36
25	9. 97986	0. 95468	9. 98147	0. 95824	9. 98302	0. 96166	9. 98450	0. 96494	9. 98591	0. 96808	35
26	. 97988	. 95474	. 98150	. 95830	. 98305	. 96172	. 98453	. 96500	. 98593	. 96813	34
27	. 97991	. 95480	. 98153	. 95836	. 98307	. 96177	. 98455	. 96505	. 98596	. 96818	33
28	. 97994	. 95486	. 98155	. 95841	. 98310	. 96183	. 98457	. 96510	. 98598	. 96823	32
29	. 97997	. 95492	. 98158	. 95847	. 98312	. 96188	. 98460	. 96516	. 98600	. 96829	31
30	9. 97999	0. 95498	9. 98161	0. 95853	9. 98315	0. 96194	9. 98462	0. 96521	9. 98603	0. 96834	30
31	. 98002	. 95504	. 98163	. 95859	. 98317	. 96200	. 98465	. 96526	. 98605	. 96839	29
32	. 98005	. 95510	. 98166	. 95865	. 98320	. 96205	. 98467	. 96532	. 98607	. 96844	28
33	. 98008	. 95516	. 98168	. 95870	. 98322	. 96211	. 98469	. 96537	. 98609	. 96849	27
34	. 98010	. 95522	. 98171	. 95876	. 98325	. 96216	. 98472	. 96542	. 98612	. 96854	26
35	9. 98013	0. 95528	9. 98174	0. 95882	9. 98327	0. 96222	9. 98474	0. 96547	9. 98614	0. 96859	25
36	. 98016	. 95534	. 98176	. 95888	. 98330	. 96227	. 98476	. 96553	. 98616	. 96864	24
37	. 98019	. 95540	. 98179	. 95894	. 98332	. 96233	. 98479	. 96558	. 98619	. 96869	23
38	. 98021	. 95546	. 98182	. 95899	. 98335	. 96238	. 98481	. 96563	. 98621	. 96874	22
39	. 98024	. 95552	. 98184	. 95905	. 98337	. 96244	. 98484	. 96569	. 98623	. 96879	21
40	9. 98027	0. 95558	9. 98187	0. 95911	9. 98340	0. 96249	9. 98486	0. 96574	9. 98625	0. 96884	20
41	. 98030	. 95564	. 98189	. 95917	. 98342	. 96255	. 98488	. 96579	. 98628	. 96889	19
42	. 98032	. 95570	. 98192	. 95922	. 98345	. 96260	. 98491	. 96585	. 98630	. 96894	18
43	. 98035	. 95576	. 98195	. 95928	. 98347	. 96266	. 98493	. 96590	. 98632	. 96899	17
44	. 98038	. 95582	. 98197	. 95934	. 98350	. 96272	. 98496	. 96595	. 98634	. 96905	16
45	9. 98040	0. 95588	9. 98200	0. 95940	9. 98352	0. 96277	9. 98498	0. 96600	9. 98637	0. 96910	15
46	. 98043	. 95594	. 98202	. 95945	. 98355	. 96283	. 98500	. 96606	. 98639	. 96915	14
47	. 98046	. 95600	. 98205	. 95951	. 98357	. 96288	. 98503	. 96611	. 98641	. 96920	13
48	. 98049	. 95606	. 98208	. 95957	. 98360	. 96294	. 98505	. 96616	. 98643	. 96925	12
49	. 98051	. 95612	. 98210	. 95962	. 98362	. 96299	. 98507	. 96621	. 98646	. 96930	11
50	9. 98054	0. 95618	9. 98213	0. 95968	9. 98365	0. 96305	9. 98510	0. 96627	9. 98648	0. 96935	10
51	. 98057	. 95624	. 98215	. 95974	. 98367	. 96310	. 98512	. 96632	. 98650	. 96940	9
52	. 98059	. 95630	. 98218	. 95980	. 98370	. 96315	. 98514	. 96637	. 98652	. 96945	8
53	. 98062	. 95636	. 98221	. 95985	. 98372	. 96321	. 98517	. 96642	. 98655	. 96950	7
54	. 98065	. 95642	. 98223	. 95991	. 98375	. 96326	. 98519	. 96648	. 98657	. 96955	6
55	9. 98067	0. 95648	9. 98226	0. 95997	9. 98377	0. 96332	9. 98521	0. 96653	9. 98659	0. 96960	5
56	. 98070	. 95654	. 98228	. 96002	. 98379	. 96337	. 98524	. 96658	. 98661	. 96965	4
57	. 98073	. 95660	. 98231	. 96008	. 98382	. 96343	. 98526	. 96663	. 98664	. 96970	3
58	. 98076	. 95665	. 98233	. 96014	. 98384	. 96348	. 98529	. 96669	. 98666	. 96975	2
59	. 98078	. 95671	. 98236	. 96020	. 98387	. 96354	. 98531	. 96674	. 98668	. 96980	1
60	9. 98081	0. 95677	9. 98239	0. 96025	9. 98389	0. 96359	9. 98533	0. 96679	9. 98670	0. 96985	0
	204°		203°		202°		201°		200°		

TABLE 34

Haversines

′	160° Log Hav	160° Nat. Hav	161° Log Hav	161° Nat. Hav	162° Log Hav	162° Nat. Hav	163° Log Hav	163° Nat. Hav	164° Log Hav	164° Nat. Hav	′
0	9. 98670	0. 96985	9. 98801	0. 97276	9. 98924	0. 97553	9. 99041	0. 97815	9. 99151	0. 98063	60
1	. 98673	. 96990	. 98803	. 97281	. 98926	. 97557	. 99043	. 97819	. 99152	. 98067	59
2	. 98675	. 96995	. 98805	. 97285	. 98928	. 97562	. 99044	. 97824	. 99154	. 98071	58
3	. 98677	. 97000	. 98807	. 97290	. 98930	. 97566	. 99046	. 97828	. 99156	. 98075	57
4	. 98679	. 97004	. 98809	. 97295	. 98932	. 97571	. 99048	. 97832	. 99158	. 98079	56
5	9. 98681	0. 97009	9. 98811	0. 97300	9. 98934	0. 97575	9. 99050	0. 97836	9. 99159	0. 98083	55
6	. 98684	. 97014	. 98813	. 97304	. 98936	. 97580	. 99052	. 97841	. 99161	. 98087	54
7	. 98686	. 97019	. 98815	. 97309	. 98938	. 97584	. 99054	. 97845	. 99163	. 98091	53
8	. 98688	. 97024	. 98817	. 97314	. 98940	. 97589	. 99056	. 97849	. 99165	. 98095	52
9	. 98690	. 97029	. 98819	. 97318	. 98942	. 97593	. 99058	. 97853	. 99166	. 98099	51
10	9. 98692	0. 97034	9. 98822	0. 97323	9. 98944	0. 97598	9. 99059	0. 97858	9. 99168	0. 98103	50
11	. 98695	. 97039	. 98824	. 97328	. 98946	. 97602	. 99061	. 97862	. 99170	. 98107	49
12	. 98697	. 97044	. 98826	. 97332	. 98948	. 97606	. 99063	. 97866	. 99172	. 98111	48
13	. 98699	. 97049	. 98828	. 97337	. 98950	. 97611	. 99065	. 97870	. 99173	. 98115	47
14	. 98701	. 97054	. 98830	. 97342	. 98952	. 97615	. 99067	. 97874	. 99175	. 98119	46
15	9. 98703	0. 97059	9. 98832	0. 97347	9. 98954	0. 97620	9. 99069	0. 97879	9. 99177	0. 98123	45
16	. 98706	. 97064	. 98834	. 97351	. 98956	. 97624	. 99071	. 97883	. 99179	. 98127	44
17	. 98708	. 97069	. 98836	. 97356	. 98958	. 97629	. 99072	. 97887	. 99180	. 98131	43
18	. 98710	. 97074	. 98838	. 97361	. 98960	. 97633	. 99074	. 97891	. 99182	. 98135	42
19	. 98712	. 97078	. 98840	. 97365	. 98962	. 97637	. 99076	. 97895	. 99184	. 98139	41
20	9. 98714	0. 97083	9. 98842	0. 97370	9. 98964	0. 97642	9. 99078	0. 97899	9. 99186	0. 98142	40
21	. 98717	. 97088	. 98845	. 97374	. 98966	. 97646	. 99080	. 97904	. 99187	. 98146	39
22	. 98719	. 97093	. 98847	. 97379	. 98968	. 97651	. 99082	. 97908	. 99189	. 98150	38
23	. 98721	. 97098	. 98849	. 97384	. 98970	. 97655	. 99084	. 97912	. 99191	. 98154	37
24	. 98723	. 97103	. 98851	. 97388	. 98971	. 97660	. 99085	. 97916	. 99193	. 98158	36
25	9. 98725	0. 97108	9. 98853	0. 97393	9. 98973	0. 97664	9. 99087	0. 97920	9. 99194	0. 98162	35
26	. 98728	. 97113	. 98855	. 97398	. 98975	. 97668	. 99089	. 97924	. 99196	. 98166	34
27	. 98730	. 97117	. 98857	. 97402	. 98977	. 97673	. 99091	. 97929	. 99198	. 98170	33
28	. 98732	. 97122	. 98859	. 97407	. 98979	. 97677	. 99093	. 97933	. 99200	. 98174	32
29	. 98734	. 97127	. 98861	. 97412	. 98981	. 97681	. 99095	. 97937	. 99201	. 98178	31
30	9. 98736	0. 97132	9. 98863	0. 97416	9. 98983	0. 97686	9. 99096	0. 97941	9. 99203	0. 98182	30
31	. 98738	. 97137	. 98865	. 97421	. 98985	. 97690	. 99098	. 97945	. 99205	. 98185	29
32	. 98741	. 97142	. 98867	. 97425	. 98987	. 97695	. 99100	. 97949	. 99206	. 98189	28
33	. 98743	. 97147	. 98869	. 97430	. 98989	. 97699	. 99102	. 97953	. 99208	. 98193	27
34	. 98745	. 97151	. 98871	. 97435	. 98991	. 97703	. 99104	. 97957	. 99210	. 98197	26
35	9. 98747	0. 97156	9. 98873	0. 97439	9. 98993	0. 97708	9. 99106	0. 97962	9. 99212	0. 98201	25
36	. 98749	. 97161	. 98875	. 97444	. 98995	. 97712	. 99107	. 97966	. 99213	. 98205	24
37	. 98751	. 97166	. 98877	. 97448	. 98997	. 97716	. 99109	. 97970	. 99215	. 98209	23
38	. 98754	. 97171	. 98880	. 97453	. 98999	. 97721	. 99111	. 97974	. 99217	. 98212	22
39	. 98756	. 97176	. 98882	. 97458	. 99001	. 97725	. 99113	. 97978	. 99218	. 98216	21
40	9. 98758	0. 97180	9. 98884	0. 97462	9. 99003	0. 97729	9. 99115	0. 97982	9. 99220	0. 98220	20
41	. 98760	. 97185	. 98886	. 97467	. 99004	. 97734	. 99116	. 97986	. 99222	. 98224	19
42	. 98762	. 97190	. 98888	. 97471	. 99006	. 97738	. 99118	. 97990	. 99223	. 98228	18
43	. 98764	. 97195	. 98890	. 97476	. 99008	. 97742	. 99120	. 97994	. 99225	. 98232	17
44	. 98766	. 97200	. 98892	. 97480	. 99010	. 97747	. 99122	. 97998	. 99227	. 98236	16
45	9. 98769	0. 97204	9. 98894	0. 97485	9. 99012	0. 97751	9. 99124	0. 98002	9. 99229	0. 98239	15
46	. 98771	. 97209	. 98896	. 97490	. 99014	. 97755	. 99126	. 98007	. 99230	. 98243	14
47	. 98773	. 97214	. 98898	. 97494	. 99016	. 97760	. 99127	. 98011	. 99232	. 98247	13
48	. 98775	. 97219	. 98900	. 97499	. 99018	. 97764	. 99129	. 98015	. 99234	. 98251	12
49	. 98777	. 97224	. 98902	. 97503	. 99020	. 97768	. 99131	. 98019	. 99235	. 98255	11
50	9. 98779	0. 97228	9. 98904	0. 97508	9. 99022	0. 97773	9. 99133	0. 98023	9. 99237	0. 98258	10
51	. 98781	. 97233	. 98906	. 97512	. 99024	. 97777	. 99135	. 98027	. 99239	. 98262	9
52	. 98784	. 97238	. 98908	. 97517	. 99026	. 97781	. 99136	. 98031	. 99240	. 98266	8
53	. 98786	. 97243	. 98910	. 97521	. 99027	. 97785	. 99138	. 98035	. 99242	. 98270	7
54	. 98788	. 97247	. 98912	. 97526	. 99029	. 97790	. 99140	. 98039	. 99244	. 98274	6
55	9. 98790	0. 97252	9. 98914	0. 97530	9. 99031	0. 97794	9. 99142	0. 98043	9. 99245	0. 98277	5
56	. 98792	. 97257	. 98916	. 97535	. 99033	. 97798	. 99143	. 98047	. 99247	. 98281	4
57	. 98794	. 97262	. 98918	. 97539	. 99035	. 97802	. 99145	. 98051	. 99249	. 98285	3
58	. 98796	. 97266	. 98920	. 97544	. 99037	. 97807	. 99147	. 98055	. 99250	. 98289	2
59	. 98798	. 97271	. 98922	. 97548	. 99039	. 97811	. 99149	. 98059	. 99252	. 98293	1
60	9. 98801	0. 97276	9. 98924	0. 97553	9. 99041	0. 97815	9. 99151	0. 98063	9. 99254	0. 98296	0
	199°		198°		197°		196°		195°		

TABLE 34

Haversines

′	165° Log Hav	165° Nat. Hav	166° Log Hav	166° Nat. Hav	167° Log Hav	167° Nat. Hav	168° Log Hav	168° Nat. Hav	169° Log Hav	169° Nat. Hav	′
0	9. 99254	0. 98296	9. 99350	0. 98515	9. 99440	0. 98719	9. 99523	0. 98907	9. 99599	0. 99081	60
1	. 99255	. 98300	. 99352	. 98518	. 99441	. 98722	. 99524	. 98910	. 99600	. 99084	59
2	. 99257	. 98304	. 99353	. 98522	. 99443	. 98725	. 99526	. 98913	. 99602	. 99087	58
3	. 99259	. 98308	. 99355	. 98525	. 99444	. 98728	. 99527	. 98916	. 99603	. 99090	57
4	. 99260	. 98311	. 99356	. 98529	. 99446	. 98732	. 99528	. 98919	. 99604	. 99092	56
5	9. 99262	0. 98315	9. 99358	0. 98532	9. 99447	0. 98735	9. 99529	0. 98922	9. 99605	0. 99095	55
6	. 99264	. 98319	. 99359	. 98536	. 99448	. 98738	. 99531	. 98925	. 99606	. 99098	54
7	. 99265	. 98323	. 99361	. 98539	. 99450	. 98741	. 99532	. 98928	. 99608	. 99101	53
8	. 99267	. 98326	. 99362	. 98543	. 99451	. 98745	. 99533	. 98931	. 99609	. 99103	52
9	. 99269	. 98330	. 99364	. 98546	. 99453	. 98748	. 99535	. 98934	. 99610	. 99106	51
10	9. 99270	0. 98334	9. 99366	0. 98550	9. 99454	0. 98751	9. 99536	0. 98937	9. 99611	0. 99109	50
11	. 99272	. 98337	. 99367	. 98553	. 99456	. 98754	. 99537	. 98940	. 99612	. 99112	49
12	. 99274	. 98341	. 99369	. 98557	. 99457	. 98757	. 99539	. 98943	. 99614	. 99114	48
13	. 99275	. 98345	. 99370	. 98560	. 99458	. 98761	. 99540	. 98946	. 99615	. 99117	47
14	. 99277	. 98349	. 99372	. 98564	. 99460	. 98764	. 99541	. 98949	. 99616	. 99120	46
15	9. 99278	0. 98352	9. 99373	0. 98567	9. 99461	0. 98767	9. 99543	0. 98952	9. 99617	0. 99123	45
16	. 99280	. 98356	. 99375	. 98571	. 99463	. 98770	. 99544	. 98955	. 99618	. 99125	44
17	. 99282	. 98360	. 99376	. 98574	. 99464	. 98774	. 99545	. 98958	. 99620	. 99128	43
18	. 99283	. 98363	. 99378	. 98577	. 99465	. 98777	. 99546	. 98961	. 99621	. 99131	42
19	. 99285	. 98367	. 99379	. 98581	. 99467	. 98780	. 99548	. 98964	. 99622	. 99133	41
20	9. 99287	0. 98371	9. 99381	0. 98584	9. 99468	0. 98783	9. 99549	0. 98967	9. 99623	0. 99136	40
21	. 99288	. 98374	. 99382	. 98588	. 99470	. 98786	. 99550	. 98970	. 99624	. 99139	39
22	. 99290	. 98378	. 99384	. 98591	. 99471	. 98789	. 99552	. 98973	. 99626	. 99141	38
23	. 99291	. 98382	. 99385	. 98595	. 99472	. 98793	. 99553	. 98976	. 99627	. 99144	37
24	. 99293	. 98385	. 99387	. 98598	. 99474	. 98796	. 99554	. 98979	. 99628	. 99147	36
25	9. 99295	0. 98389	9. 99388	0. 98601	9. 99475	0. 98799	9. 99555	0. 98982	9. 99629	0. 99149	35
26	. 99296	. 98393	. 99390	. 98605	. 99477	. 98802	. 99557	. 98985	. 99630	. 99152	34
27	. 99298	. 98396	. 99391	. 98608	. 99478	. 98805	. 99558	. 98988	. 99631	. 99155	33
28	. 99300	. 98400	. 99393	. 98612	. 99479	. 98808	. 99559	. 98990	. 99633	. 99157	32
29	. 99301	. 98404	. 99394	. 98615	. 99481	. 98812	. 99561	. 98993	. 99634	. 99160	31
30	9. 99303	0. 98407	9. 99396	0. 98618	9. 99482	0. 98815	9. 99562	0. 98996	9. 99635	0. 99163	30
31	. 99304	. 98411	. 99397	. 98622	. 99484	. 98818	. 99563	. 98999	. 99636	. 99165	29
32	. 99306	. 98415	. 99399	. 98625	. 99485	. 98821	. 99564	. 99002	. 99637	. 99168	28
33	. 99308	. 98418	. 99400	. 98629	. 99486	. 98824	. 99566	. 99005	. 99638	. 99171	27
34	. 99309	. 98422	. 99402	. 98632	. 99488	. 98827	. 99567	. 99008	. 99639	. 99173	26
35	9. 99311	0. 98426	9. 99403	0. 98635	9. 99489	0. 98830	9. 99568	0. 99011	9. 99641	0. 99176	25
36	. 99312	. 98429	. 99405	. 98639	. 99490	. 98834	. 99569	. 99014	. 99642	. 99179	24
37	. 99314	. 98433	. 99406	. 98642	. 99492	. 98837	. 99571	. 99016	. 99643	. 99181	23
38	. 99316	. 98436	. 99408	. 98646	. 99493	. 98840	. 99572	. 99019	. 99644	. 99184	22
39	. 99317	. 98440	. 99409	. 98649	. 99495	. 98843	. 99573	. 99022	. 99645	. 99186	21
40	9. 99319	0. 98444	9. 99411	0. 98652	9. 99496	0. 98846	9. 99575	0. 99025	9. 99646	0. 99189	20
41	. 99320	. 98447	. 99412	. 98656	. 99497	. 98849	. 99576	. 99028	. 99648	. 99192	19
42	. 99322	. 98451	. 99414	. 98659	. 99499	. 98852	. 99577	. 99031	. 99649	. 99194	18
43	. 99324	. 98454	. 99415	. 98662	. 99500	. 98855	. 99578	. 99034	. 99650	. 99197	17
44	. 99325	. 98458	. 99417	. 98666	. 99501	. 98858	. 99580	. 99036	. 99651	. 99199	16
45	9. 99327	0. 98462	9. 99418	0. 98669	9. 99503	0. 98862	9. 99581	0. 99039	9. 99652	0. 99202	15
46	. 99328	. 98465	. 99420	. 98672	. 99504	. 98865	. 99582	. 99042	. 99653	. 99205	14
47	. 99330	. 98469	. 99421	. 98676	. 99505	. 98868	. 99583	. 99045	. 99654	. 99207	13
48	. 99331	. 98472	. 99422	. 98679	. 99507	. 98871	. 99584	. 99048	. 99655	. 99210	12
49	. 99333	. 98476	. 99424	. 98682	. 99508	. 98874	. 99586	. 99051	. 99657	. 99212	11
50	9. 99335	0. 98479	9. 99425	0. 98686	9. 99510	0. 98877	9. 99587	0. 99053	9. 99658	0. 99215	10
51	. 99336	. 98483	. 99427	. 98689	. 99511	. 98880	. 99588	. 99056	. 99659	. 99217	9
52	. 99338	. 98487	. 99429	. 98692	. 99512	. 98883	. 99589	. 99059	. 99660	. 99220	8
53	. 99339	. 98490	. 99430	. 98695	. 99514	. 98886	. 99591	. 99062	. 99661	. 99223	7
54	. 99341	. 98494	. 99431	. 98699	. 99515	. 98889	. 99592	. 99065	. 99662	. 99225	6
55	9. 99342	0. 98497	9. 99433	0. 98702	9. 99516	0. 98892	9. 99593	0. 99067	9. 99663	0. 99228	5
56	. 99344	. 98501	. 99434	. 98705	. 99518	. 98895	. 99594	. 99070	. 99664	. 99230	4
57	. 99345	. 98504	. 99436	. 98709	. 99519	. 98898	. 99596	. 99073	. 99666	. 99233	3
58	. 99347	. 98508	. 99437	. 98712	. 99520	. 98901	. 99597	. 99076	. 99667	. 99235	2
59	. 99349	. 98511	. 99438	. 98715	. 99522	. 98904	. 99598	. 99079	. 99668	. 99238	1
60	9. 99350	0. 98515	9. 99440	0. 98719	9. 99523	0. 98907	9. 99599	0. 99081	9. 99669	0. 99240	0
	194°		193°		192°		191°		190°		

TABLE 34

Haversines

′	170° Log Hav	170° Nat. Hav	171° Log Hav	171° Nat. Hav	172° Log Hav	172° Nat. Hav	173° Log Hav	173° Nat. Hav	174° Log Hav	174° Nat. Hav	′
0	9. 99669	0. 99240	9. 99732	0. 99384	9. 99788	0. 99513	9. 99838	0. 99627	9. 99881	0. 99726	60
1	. 99670	. 99243	. 99733	. 99387	. 99789	. 99515	. 99839	. 99629	. 99882	. 99728	59
2	. 99671	. 99245	. 99734	. 99389	. 99790	. 99517	. 99839	. 99631	. 99882	. 99729	58
3	. 99672	. 99248	. 99735	. 99391	. 99791	. 99519	. 99840	. 99633	. 99883	. 99731	57
4	. 99673	. 99250	. 99736	. 99393	. 99792	. 99521	. 99841	. 99634	. 99884	. 99732	56
5	9. 99674	0. 99253	9. 99737	0. 99396	9. 99793	0. 99523	9. 99842	0. 99636	9. 99884	0. 99734	55
6	. 99675	. 99255	. 99738	. 99398	. 99793	. 99525	. 99842	. 99638	. 99885	. 99735	54
7	. 99677	. 99258	. 99739	. 99400	. 99794	. 99527	. 99843	. 99640	. 99885	. 99737	53
8	. 99678	. 99260	. 99740	. 99402	. 99795	. 99529	. 99844	. 99641	. 99886	. 99738	52
9	. 99679	. 99263	. 99741	. 99405	. 99796	. 99531	. 99845	. 99643	. 99887	. 99740	51
10	9. 99680	0. 99265	9. 99742	0. 99407	9. 99797	0. 99533	9. 99845	0. 99645	9. 99887	0. 99741	50
11	. 99681	. 99268	. 99743	. 99409	. 99798	. 99535	. 99846	. 99647	. 99888	. 99743	49
12	. 99682	. 99270	. 99744	. 99411	. 99799	. 99537	. 99847	. 99648	. 99889	. 99744	48
13	. 99683	. 99273	. 99745	. 99414	. 99800	. 99539	. 99848	. 99650	. 99889	. 99746	47
14	. 99684	. 99275	. 99746	. 99416	. 99800	. 99541	. 99848	. 99652	. 99890	. 99747	46
15	9. 99685	0. 99278	9. 99747	0. 99418	9. 99801	0. 99543	9. 99849	0. 99653	9. 99891	0. 99748	45
16	. 99686	. 99280	. 99748	. 99420	. 99802	. 99545	. 99850	. 99655	. 99891	. 99750	44
17	. 99687	. 99283	. 99748	. 99422	. 99803	. 99547	. 99851	. 99657	. 99892	. 99751	43
18	. 99688	. 99285	. 99749	. 99425	. 99804	. 99549	. 99851	. 99659	. 99893	. 99753	42
19	. 99690	. 99288	. 99750	. 99427	. 99805	. 99551	. 99852	. 99660	. 99893	. 99754	41
20	9. 99691	0. 99290	9. 99751	0. 99429	9. 99805	0. 99553	9. 99853	0. 99662	9. 99894	0. 99756	40
21	. 99692	. 99293	. 99752	. 99431	. 99806	. 99555	. 99854	. 99664	. 99894	. 99757	39
22	. 99693	. 99295	. 99753	. 99433	. 99807	. 99557	. 99854	. 99665	. 99895	. 99759	38
23	. 99694	. 99297	. 99754	. 99436	. 99808	. 99559	. 99855	. 99667	. 99896	. 99760	37
24	. 99695	. 99300	. 99755	. 99438	. 99809	. 99561	. 99856	. 99669	. 99896	. 99761	36
25	9. 99696	0. 99302	9. 99756	0. 99440	9. 99810	0. 99563	9. 99857	0. 99670	9. 99897	0. 99763	35
26	. 99697	. 99305	. 99757	. 99442	. 99811	. 99565	. 99857	. 99672	. 99897	. 99764	34
27	. 99698	. 99307	. 99758	. 99444	. 99811	. 99567	. 99858	. 99674	. 99898	. 99766	33
28	. 99699	. 99309	. 99759	. 99446	. 99812	. 99568	. 99859	. 99675	. 99899	. 99767	32
29	. 99700	. 99312	. 99760	. 99449	. 99813	. 99570	. 99859	. 99677	. 99899	. 99768	31
30	9. 99701	0. 99314	9. 99761	0. 99451	9. 99814	0. 99572	9. 99860	0. 99679	9. 99900	0. 99770	30
31	. 99702	. 99317	. 99762	. 99453	. 99815	. 99574	. 99861	. 99680	. 99901	. 99771	29
32	. 99703	. 99319	. 99763	. 99455	. 99815	. 99576	. 99862	. 99682	. 99901	. 99773	28
33	. 99704	. 99321	. 99764	. 99457	. 99816	. 99578	. 99862	. 99684	. 99902	. 99774	27
34	. 99705	. 99324	. 99765	. 99459	. 99817	. 99580	. 99863	. 99685	. 99902	. 99775	26
35	9. 99706	0. 99326	9. 99766	0. 99461	9. 99818	0. 99582	9. 99864	0. 99687	9. 99903	0. 99777	25
36	. 99707	. 99329	. 99766	. 99464	. 99819	. 99584	. 99864	. 99688	. 99904	. 99778	24
37	. 99708	. 99331	. 99767	. 99466	. 99820	. 99585	. 99865	. 99690	. 99904	. 99779	23
38	. 99710	. 99333	. 99768	. 99468	. 99820	. 99587	. 99866	. 99692	. 99905	. 99781	22
39	. 99711	. 99336	. 99769	. 99470	. 99821	. 99589	. 99867	. 99693	. 99905	. 99782	21
40	9. 99712	0. 99338	9. 99770	0. 99472	9. 99822	0. 99591	9. 99867	0. 99695	9. 99906	0. 99784	20
41	. 99713	. 99340	. 99771	. 99474	. 99823	. 99593	. 99868	. 99696	. 99906	. 99785	19
42	. 99714	. 99343	. 99772	. 99476	. 99824	. 99595	. 99869	. 99698	. 99907	. 99786	18
43	. 99715	. 99345	. 99773	. 99478	. 99824	. 99597	. 99869	. 99700	. 99908	. 99788	17
44	. 99716	. 99347	. 99774	. 99480	. 99825	. 99598	. 99870	. 99701	. 99908	. 99789	16
45	9. 99717	0. 99350	9. 99774	0. 99483	9. 99826	0. 99600	9. 99871	0. 99703	9. 99909	0. 99790	15
46	. 99718	. 99352	. 99775	. 99485	. 99827	. 99602	. 99871	. 99704	. 99909	. 99792	14
47	. 99719	. 99354	. 99776	. 99487	. 99828	. 99604	. 99872	. 99706	. 99910	. 99793	13
48	. 99720	. 99357	. 99777	. 99489	. 99828	. 99606	. 99873	. 99708	. 99911	. 99794	12
49	. 99721	. 99359	. 99778	. 99491	. 99829	. 99608	. 99874	. 99709	. 99911	. 99796	11
50	9. 99722	0. 99361	9. 99779	0. 99493	9. 99830	0. 99609	9. 99874	0. 99711	9. 99912	0. 99797	10
51	. 99723	. 99364	. 99780	. 99495	. 99831	. 99611	. 99875	. 99712	. 99912	. 99798	9
52	. 99724	. 99366	. 99781	. 99497	. 99832	. 99613	. 99876	. 99714	. 99913	. 99799	8
53	. 99725	. 99368	. 99782	. 99499	. 99832	. 99615	. 99876	. 99715	. 99913	. 99801	7
54	. 99726	. 99371	. 99783	. 99501	. 99833	. 99617	. 99877	. 99717	. 99914	. 99802	6
55	9. 99727	0. 99373	9. 99784	0. 99503	9. 99834	0. 99618	9. 99878	0. 99718	9. 99915	0. 99803	5
56	. 99728	. 99375	. 99785	. 99505	. 99835	. 99620	. 99878	. 99720	. 99915	. 99805	4
57	. 99729	. 99378	. 99786	. 99507	. 99836	. 99622	. 99879	. 99722	. 99916	. 99806	3
58	. 99730	. 99380	. 99786	. 99509	. 99836	. 99624	. 99880	. 99723	. 99916	. 99807	2
59	. 99731	. 99382	. 99787	. 99511	. 99837	. 99626	. 99880	. 99725	. 99917	. 99808	1
60	9. 99732	0. 99384	9. 99788	0. 99513	9. 99838	0. 99627	9. 99881	0. 99726	9. 99917	0. 99810	0
	189°		188°		187°		186°		185°		

TABLE 34

Haversines

	175°		176°		177°		178°		179°		
′	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	Log Hav	Nat. Hav	′
0	9.99917	0.99810	9.99947	0.99878	9.99970	0.99931	9.99987	0.99970	9.99997	0.99992	60
1	.99918	.99811	.99948	.99879	.99971	.99932	.99987	.99970	.99997	.99993	59
2	.99918	.99812	.99948	.99880	.99971	.99933	.99987	.99971	.99997	.99993	58
3	.99919	.99814	.99948	.99881	.99971	.99934	.99987	.99971	.99997	.99993	57
4	.99919	.99815	.99949	.99882	.99972	.99934	.99988	.99972	.99997	.99993	56
5	9.99920	0.99816	9.99949	0.99883	9.99972	0.99935	9.99988	0.99972	9.99997	0.99994	55
6	.99921	.99817	.99950	.99884	.99972	.99936	.99988	.99973	.99997	.99994	54
7	.99921	.99819	.99950	.99885	.99973	.99937	.99988	.99973	.99997	.99994	53
8	.99922	.99820	.99951	.99886	.99973	.99937	.99988	.99973	.99998	.99994	52
9	.99922	.99821	.99951	.99887	.99973	.99938	.99989	.99974	.99998	.99994	51
10	9.99923	0.99822	9.99951	0.99888	9.99973	0.99939	9.99989	0.99974	9.99998	0.99995	50
11	.99923	.99823	.99952	.99889	.99974	.99940	.99989	.99975	.99998	.99995	49
12	.99924	.99825	.99952	.99890	.99974	.99940	.99989	.99975	.99998	.99995	48
13	.99924	.99826	.99953	.99891	.99974	.99941	.99989	.99976	.99998	.99995	47
14	.99925	.99827	.99953	.99892	.99975	.99942	.99990	.99976	.99998	.99996	46
15	9.99925	0.99828	9.99953	0.99893	9.99975	0.99942	9.99990	0.99977	9.99998	0.99996	45
16	.99926	.99829	.99954	.99894	.99975	.99943	.99990	.99977	.99998	.99996	44
17	.99926	.99831	.99954	.99895	.99976	.99944	.99990	.99978	.99998	.99996	43
18	.99927	.99832	.99954	.99896	.99976	.99944	.99990	.99978	.99998	.99996	42
19	.99927	.99833	.99955	.99897	.99976	.99945	.99991	.99978	.99998	.99996	41
20	9.99928	0.99834	9.99955	0.99898	9.99976	0.99946	9.99991	0.99979	9.99999	0.99997	40
21	.99928	.99835	.99956	.99899	.99977	.99947	.99991	.99979	.99999	.99997	39
22	.99929	.99837	.99956	.99900	.99977	.99947	.99991	.99980	.99999	.99997	38
23	.99929	.99838	.99957	.99900	.99977	.99948	.99991	.99980	.99999	.99997	37
24	.99930	.99839	.99957	.99901	.99978	.99949	.99992	.99981	.99999	.99997	36
25	9.99931	0.99840	9.99958	0.99902	9.99978	0.99949	9.99992	0.99981	9.99999	0.99997	35
26	.99931	.99841	.99958	.99903	.99978	.99950	.99992	.99981	.99999	.99998	34
27	.99932	.99842	.99958	.99904	.99978	.99950	.99992	.99982	.99999	.99998	33
28	.99932	.99844	.99959	.99905	.99979	.99951	.99992	.99982	.99999	.99998	32
29	.99933	.99845	.99959	.99906	.99979	.99952	.99992	.99982	.99999	.99998	31
30	9.99933	0.99846	9.99959	0.99907	9.99979	0.99952	9.99993	0.99983	9.99999	0.99998	30
31	.99934	.99847	.99960	.99908	.99980	.99953	.99993	.99983	.99999	.99998	29
32	.99934	.99848	.99960	.99909	.99980	.99954	.99993	.99984	.99999	.99998	28
33	.99935	.99849	.99961	.99909	.99980	.99954	.99993	.99984	.99999	.99998	27
34	.99935	.99850	.99961	.99910	.99980	.99955	.99993	.99984	.99999	.99999	26
35	9.99935	0.99852	9.99961	0.99911	9.99981	0.99956	9.99993	0.99985	9.99999	0.99999	25
36	.99936	.99853	.99962	.99912	.99981	.99956	.99994	.99985	9.99999	.99999	24
37	.99936	.99854	.99962	.99913	.99981	.99957	.99994	.99985	0.00000	.99999	23
38	.99937	.99855	.99963	.99914	.99981	.99957	.99994	.99986	.00000	.99999	22
39	.99937	.99856	.99963	.99915	.99982	.99958	.99994	.99986	.00000	.99999	21
40	9.99938	0.99857	9.99963	0.99915	9.99982	0.99959	9.99994	0.99986	0.00000	0.99999	20
41	.99938	.99858	.99964	.99916	.99982	.99959	.99994	.99987	.00000	.99999	19
42	.99939	.99859	.99964	.99917	.99983	.99960	.99994	.99987	.00000	.99999	18
43	.99939	.99860	.99964	.99918	.99983	.99960	.99995	.99987	.00000	.99999	17
44	.99940	.99861	.99965	.99919	.99983	.99961	.99995	.99988	.00000	.99999	16
45	9.99940	0.99863	9.99965	0.99920	9.99983	0.99961	9.99995	0.99988	0.00000	1.00000	15
46	.99941	.99864	.99965	.99920	.99983	.99962	.99995	.99988	.00000	.00000	14
47	.99941	.99865	.99966	.99921	.99984	.99963	.99995	.99988	.00000	.00000	13
48	.99942	.99866	.99966	.99922	.99984	.99963	.99995	.99989	.00000	.00000	12
49	.99942	.99867	.99966	.99923	.99984	.99964	.99995	.99989	.00000	.00000	11
50	9.99943	0.99868	9.99967	0.99924	9.99984	0.99964	9.99996	0.99990	0.00000	1.00000	10
51	.99943	.99869	.99967	.99924	.99985	.99965	.99996	.99990	.00000	.00000	9
52	.99943	.99870	.99968	.99925	.99985	.99965	.99996	.99990	.00000	.00000	8
53	.99944	.99871	.99968	.99926	.99985	.99966	.99996	.99991	.00000	.00000	7
54	.99944	.99872	.99968	.99927	.99985	.99966	.99996	.99991	.00000	.00000	6
55	9.99945	0.99873	9.99969	0.99928	9.99986	0.99967	9.99996	0.99991	0.00000	1.00000	5
56	.99945	.99874	.99969	.99928	.99986	.99967	.99996	.99991	.00000	.00000	4
57	.99946	.99875	.99969	.99929	.99986	.99968	.99996	.99992	.00000	.00000	3
58	.99946	.99876	.99970	.99930	.99986	.99969	.99996	.99992	.00000	.00000	2
59	.99947	.99877	.99970	.99931	.99987	.99969	.99997	.99992	.00000	.00000	1
60	9.99947	0.99878	9.99970	0.99931	9.99987	0.99970	9.99997	0.99992	0.00000	1.00000	0
	184°		183°		182°		181°		180°		

INDEX

U. S. GOVERNMENT PRINTING OFFICE : 1969 O - 353-583